SOIL MECHANICS AND FOUNDATION ENGG.

→ The process of formation of soil is termed as "Pedogenesis".

→ Soil is formed due to the weathering of rocks
↓
erosion/wear & tear

→ Weathering of the soil can occur either physically or chemically.
↓
mechanical disintegration

→ Weathering of rocks can be done by

a. Physical

→ Impact & grinding of water, air, ice (or wind)

→ Splitting action of ice

→ Gravitational force

→ Plants and animals

b. Chemical

→ oxidation

→ reduction

→ carbonation

→ Hydration

→ leaching by organic acid and H_2O.

NOTE: Leaching is the process of extracting a substance from a solid material, i.e. dissolved in a liquid.

* If soil is formed by physical weathering there is no change in mineral content of the soil but if it is formed by chemical weathering mineral content of soil differs from that of rock.

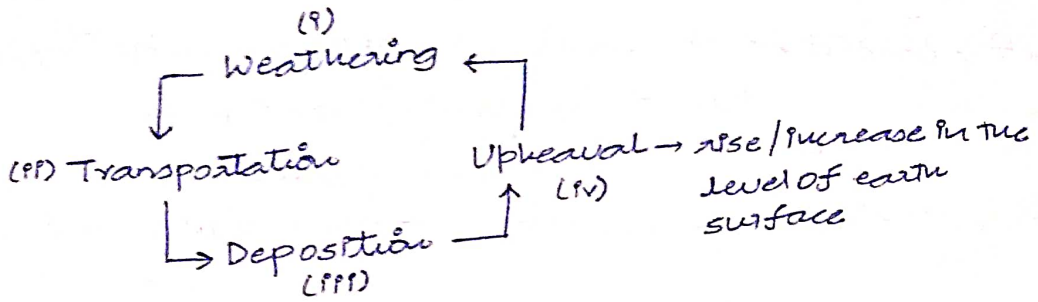
* If the weathered material retains over the parent rock, it is termed as residual soil & if it is transported it is termed as transported soil.

* The characteristics of the soil: size of particles, shape & roundness, surface texture, degree of sortness depends upon mode/agency of transportation.

Soled particle → ⌓, Angular → ⬠

Properties	Water	Air	Ice	Gravity	Organisms
* Size	Reduction	considerable reduction	considerable grinding & impact	considerable impact	Minor abrasion
* Shape & roundness	rounding of sand & gravel	high degree of rounding	Angular, soled particle	Angular, non-spherical particle	—
* Surface texture	smooth, shiny polished	Impact produces frosted surface	striated surface	striated surface	—
* Sorting	considerable sorting		Very little sorting	No sorting	Limited sorting

→ General cycle involved in the formation of soil,



* Types of soil:

→ On the basis of mode of transportation

a. Alluvial soil: It is the soil which is deposited from suspension in running water
→ Transported soil (water)
→ Weathering is physical
→ Found alongs the bank of river

b. Lacustrine soil: It is the soil which is deposited from suspension in fresh, still water of lake.

c. Marine soil: It is the soil which is deposited from sea water.

d. Aeoline soil / Sand dunes: It is the soil which is transported by blowing wind.

e. Glacial soil: It is the soil which is transported by ice.

f. Colluvial / Talus soil: It is formed due to transportation by gravity.

g. Loess soil: It is uniformely graded wind blown silt, slightly cemented by calcium compound or Montmorillonite. It is also termed as collapseable soil (imparts binding property)

h. Marl soil: It is finely grained calcium carbonated soil of marine origin which is formed due to the decomporition of plants & boves of animals. (aquatic)

i. Tuff soil: A fine grained slightly cemented volcanic ash transported by wind or water.

j. Bentonite soil: It is chemically weathered volcanic ash. It consists of high % of montmorillonite. It is a residual soil.
↓ It is highly plastic, poness high shrinkage and swelling prop. & low shear strength
→ It is used as lubricant in drilling operations.

K. Black cotton soil: It is residual soil formed from basalt & have excess of montmorillonite in it. It is dark in colour & is suitable for growing cottom.

L. Laterite soil: It is the type of soil formed due to leaching (washing out of siliceous compound & deposition of Fe_2O_3 & Al_2O_3). It is found in hilly areas.

M. Muck soil: It is a mixture of fine particled inorganic soil & black decomposed organic matter.
→ It is found in marshy / swampy / after the over flow of river.

N. Peat soil: It is highly organic soil which almost entirely consist of black decomposed vegetative matter in different stages. It is highly fibrous, comprenible

NOTE: Muck soil + Peat soil ⟶ Cumulose soil (found near the river)

O. Loam soil: It is a mixture of clay, sand & silt

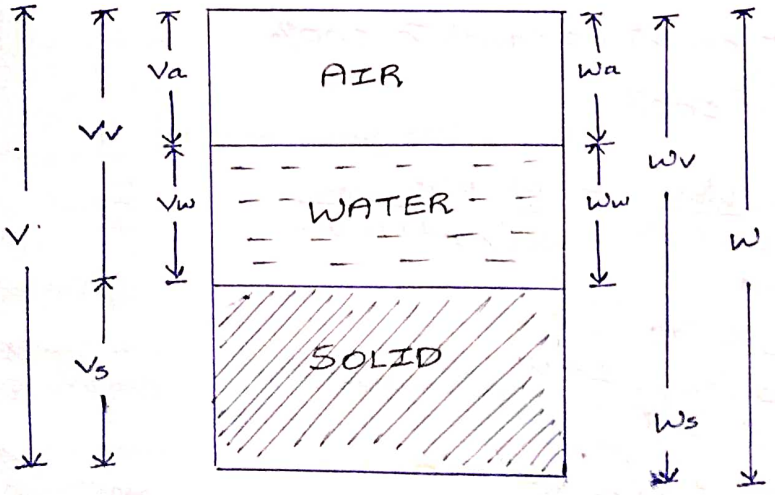
P. Gumbo soil: It is black coloured, sticky, highly plastic soil.
→ It can be formed either by physical weathering or by chemical weathering.

* Properties of the soil:

→ Soil is a 3-phase system which consists of solid, liquid and gaseous matter that do not occupy seperate spaces but are blended with each other in definite proportion, which in turn governs the property of soil.

Solids → Inorganic minerals
Solids → organic matter

Liquid ⟶ water
Gaseous matter ⟶ air



3-phase diagram

$V = V_v + V_s$

$V = (V_a + V_w) + V_s$

$W = W_v + W_s$

$W = (W_a + W_w) + W_s$

$\rho_{air} \approx 1.2\ kg/m^3$

$\rho_{water} \approx 1000\ kg/m^3$

$\rho_{solids} \approx 2600\ kg/m^3$

$W_{air} <<< W_w$ or W_s

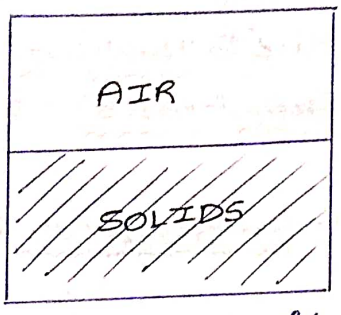
Hence, it can be neglected

V_a & V_w or V_s → but volume of air cannot be neglected

$W_{soil} = W_w + W_s$ but $V_{soil} \neq V_w + V_s$

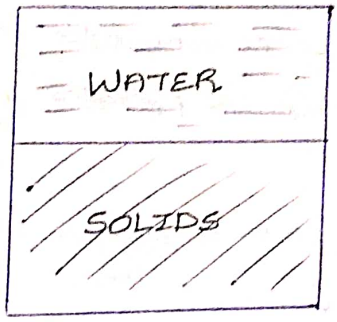
→ In some limiting case, it can be represented as 2-phase system also.

* Case 1:



Dry soil → air, solids

* case 2:



Saturated soil → water, solids

* Properties of soil:

1. Water content (w): It is defined as the weight of water to the weight of solids present in the given soil mass.

** $w = \frac{W_w}{W_s} \times 100$

→ Water content can be equal to zero but it has no upper limit.

$$w = \frac{W_w}{W_s} \Rightarrow w+1 = \frac{W_w}{W_s} + 1$$

$$w+1 = \frac{W_w + W_s}{W_s} = \frac{W'}{W_s}$$

** $W_s = \frac{W'}{1+w}$

$W' \rightarrow$ weight of soil

→ Water content can also be reported in terms of the total weight of the soil.

$$w'' = \frac{\text{wt. of water}}{\text{wt. of soil}} = \frac{W_w}{W'} \times 100$$

→ w'' can be equal to zero but cannot be equal to 100%

$$0\% \leq w'' < 100\%$$

$$w'' = \frac{W_w}{W'} = \frac{W_w}{W_w + W_s} = \frac{W_w/W_w}{W_w/W_w + W_s/W_w}$$

$$w'' = \frac{1}{1+\frac{1}{w}} = \frac{w}{1+w}$$

** $w'' = \frac{w}{1+w}$

** $w' = \frac{w''}{1-w''}$

NOTE: Weight of solid is comparatively stable in comparison to wt of soil as it does not changes with change in water, hence engg. significance of 'w' is more than 'w'.

2. Void ratio (e): It is defined as the ratio of volume of voids to volume of solids in given soil mass.

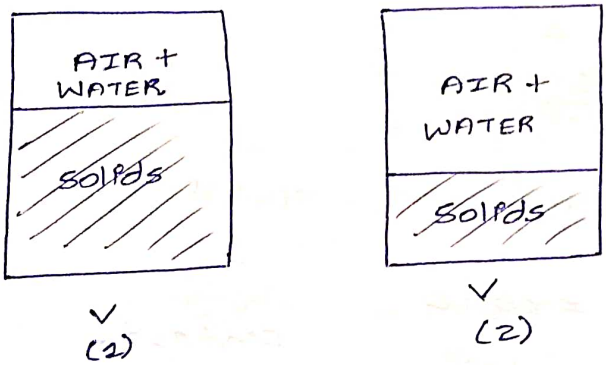
$e = \frac{V_v}{V_s}$ $\quad e > 0$

→ Volume of void cannot be zero for soil as it is atleast 2-phase system, however it can be zero for rocks.

$$e = \frac{V_v}{V_s} \Rightarrow e+1 = \frac{V_v}{V_s} + 1 \Rightarrow e+1 = \frac{V_v + V_s}{V_s} = \frac{V}{V_s}$$

$V_s = \frac{V}{1+e}$ → volume of soil

→ Void ratio can also be used to represent degree of denseness of soil.



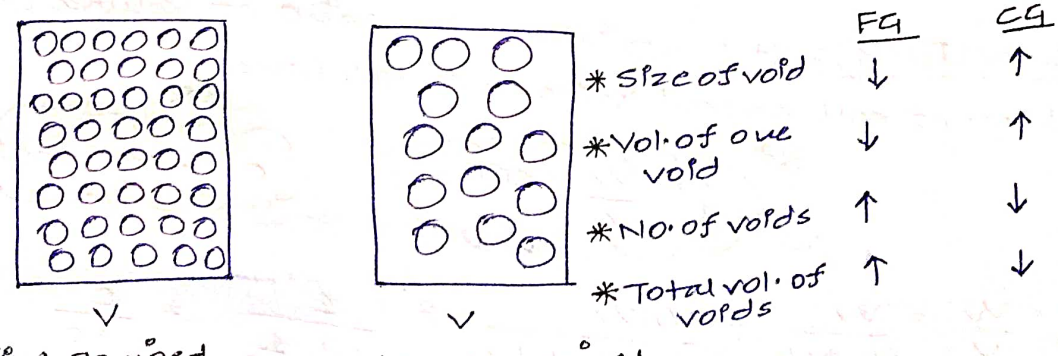
* $e_2 > e_1$
* $V_{v2} > V_{v1}$
* $V_{s2} < V_{s1}$

* (Denseness)$_2$ < (Denseness)$_1$

**

$e \propto \frac{1}{\text{Denseness}}$

→ Though volume of one void is more for coarse grained but total vol. of voids is more for fine grained soil, hence void ratio & water content fine grained soil is more than coarse grained soil.



	FG	CG
* Size of void	↓	↑
* Vol. of one void	↓	↑
* No. of voids	↑	↓
* Total vol. of voids	↑	↓

→ When 2 or more soil samples are added or mixed, quantity (vol & wt) of solids will not change.

$$A + B \longrightarrow C$$

V_{SA} V_{SB} V_{SC}

W_{SA} W_{SB} W_{SC}

→ Quantity (wt & volume) of solids remain same if soil is displaced from one to another location.

Q1.

	Soil A	Soil B
W	1Kg	1Kg
w	100%	50%

Find water content of soil 'C'

$A + B \longrightarrow C$.

$$W_{SA} = \frac{1}{1+1} = 0.5\,Kg \qquad W_{SB} = \frac{1}{1.5} = \frac{2}{3}\,Kg$$

$$W_{SC} = W_{SA} + W_{SB} = 0.5 + 0.67 = 1.17\,Kg$$

$$1.17 = \frac{2}{1+w}$$

$$1 + w = 1.709$$

$$w = 0.709 = 70.9\%$$

Q2. Soil A + Soil B = Soil C

V $1M^3$ $1M^3$ $1M^3$

e 1 2

Find e_C?

$$e_{SA} = \frac{V_V}{V_S} \qquad V_{SA} = \frac{1}{1+1}$$

$$\frac{V_C}{1+e_C} = \frac{V_A}{1+e_A} + \frac{V_B}{1+e_B}$$

$$0.5 + \frac{1}{1+2} = \frac{1+e}{1+e}$$

$$1 + e = \frac{1}{0.833}$$

$$e = 0.2$$

3. **Porosity (η):** It is defined as the ratio of volume of voids to the volume of soil present in the given soil mass.

** $\eta = \frac{V_v}{V} \times 100$

** $0\% < \eta < 100\%$

$$\eta = \frac{V_v}{V} = \frac{V_v}{V_v + V_s} = \frac{V_v/V_v}{V_v/V_v + V_s/V_v} = \frac{1}{1 + \frac{1}{e}} = \frac{e}{1+e}$$

** $\eta = \frac{e}{1+e}$ ** $e = \frac{\eta}{1-\eta}$

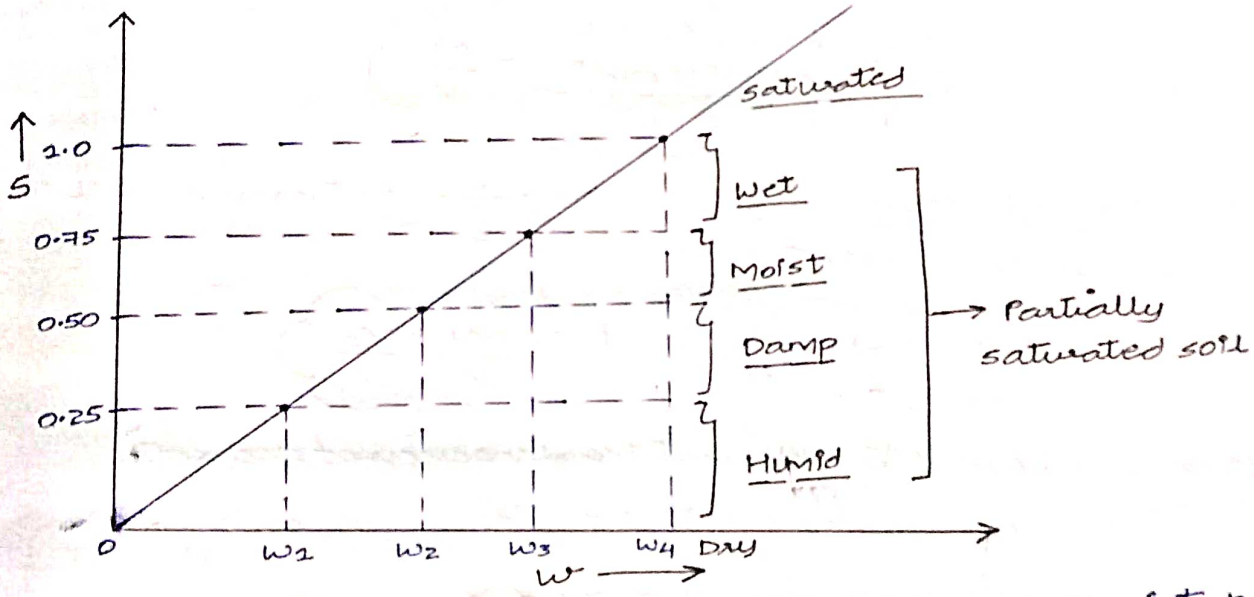
→ Both 'e' & 'η' signifies the water storage capacity of soil as they represent volume of voids in it.

→ Volume of solids is comparitively stable in comparison to the volume of soil as it does not change with the change in volume of voids, hence engg. significance of $e > \eta$.

4. Degree of saturation: (S): It is defined as the ratio of volume of water to the volume of voids present in the given soil mass.

$S = \frac{V_w}{V_v} \times 100$ $\quad 0 \leq S \leq 100\%$

→ Depending upon degree of saturation in given soil mass, it can be classified as dry, humid, damp, moist, wet & saturated.



5. Air content (a_c): It is defined as the ratio of volume of air to the volume of voids present in the given sample of soil mass.

$a_c = \frac{V_a}{V_v} \times 100$ $\quad 0 \leq a_c \leq 100\%$

$V_v = V_a + V_w \Rightarrow \frac{V_v}{V_v} = \frac{V_a}{V_v} + \frac{V_w}{V_v}$

** $a_c + S = 1$

6. Percentage air voids (η_a): It is defined as the volume of air to the volume of soil mass present in the given soil mass.

$$\eta_a = \frac{V_a}{V} \times 100 \qquad 0 \leq \eta_a < 100\%$$

$$\eta_a = \frac{V_a}{V} = \frac{V_a}{V} \cdot \frac{V_v}{V_v} = ac \cdot \eta$$

** $\eta_a = ac \cdot \eta$

7. Unit weight / density:

a. Bulk unit wt / Bulk density: It is ratio of wt/mass of soil in existing condition to the total volume of soil mass.

$$\gamma_b = \frac{\text{wt. of soil in existing condn}}{\text{Vol. of soil}} = \frac{W}{V}$$

$$\rho_b = \frac{\text{mass of soil in existing condn}}{\text{Vol}^m \text{ of soil}} = \frac{M}{V}$$

b. Dry unit wt / dry density: It is the ratio of wt/mass of soil in dry condition to the volume of soil mass.

$$\gamma_d = \frac{W_d}{V} = \frac{W_s}{V} \qquad \rho_d = \frac{M_d}{V} = \frac{M_s}{V}$$

→ It can also be used to represent the denseness of the soil.

Denseness of soil $= f[\gamma_d]$

** Denseness of soil $\propto \gamma_d \propto \frac{1}{e}$

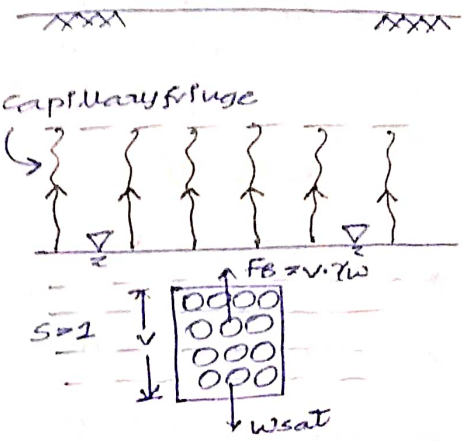
c. Saturated unit wt / saturated density: It is defined as the ratio of wt/mass of soil in saturated condition to the volume of soil.

$$\gamma_{sat} = \frac{W_{sat}}{V} \qquad \rho_{sat} = \frac{M_{sat}}{V}$$

NOTE: If soil is dry $\Rightarrow S = 0$, $\gamma_b = \gamma_d$ or $\rho_b = \rho_d$

→ If soil is saturated $\Rightarrow S = 1$, $\gamma_b = \gamma_{sat}$ or $\rho_b = \rho_{sat}$

d. Submerged unit wt / submerged density: If the soil mass is submerged below the W.T. level, it is subjected to force of buoyancy in vertically upward direction, magnitude of which is equal to the wt of the water displaced by soil, which in turn reduces the / its weight. This reduced wt of soil is termed as reduced / effective / apparent / buoyant / submerged unit wt.



$$\gamma_{sub}/\gamma' = \frac{W_{sub}}{V}$$

$$\gamma_{sub}/\gamma' = \frac{W_{sat} - F_B}{V} = \frac{W_{sat} - V\gamma_w}{V}$$

$$\gamma_{sub}/\gamma' = \frac{W_{sat}}{V} - \gamma_w$$

** $\gamma_{sub}/\gamma' = \gamma_{sat} - \gamma_w$

→ A submerged soil will always be saturated but the vice-versa is not true/necessary.

→ Soil below W.T. is submerged + saturated but above WT in capillary fringe is saturated but not submerged.

e. Unit wt of solids / density of solids: It is defined as the ratio of wt / mass of solids to the volume of solids in the given soil mass.

$$\gamma_s = \frac{\text{wt of solids}}{\text{Vol. of solids}} = \frac{W_s}{V_s} \qquad \rho_s = \frac{M_s}{V_s}$$

NOTE: $\gamma_s > \gamma_{sat} > \gamma_b > \gamma_d > \gamma_{sub}$

$\gamma'/\gamma_{sub} \approx \frac{\gamma_{sat}}{2}$ $\quad \gamma = \rho g$

	γ_s	γ_{sat}	γ_b	γ_d	γ_{sub}
$\frac{KN}{M^3}$	26-29	20-21	18-20	16-18	10

8. Specific gravity:

a. True / absolute specific gravity: (G):

$$G_s/G = \frac{\text{wt of solid of given vol}}{\text{wt of std fluid (water) of same vol}^m} \qquad [V_s = V_w]$$

** $G_s/G = \frac{W_s}{W_w}$

$$G_s/G = \frac{W_s}{W_w} \times \frac{V_s}{V_s} = \frac{\gamma_s}{\gamma_w}$$

** $G_s/G = \frac{\gamma_s}{\gamma_w}$

** $G/G_s = \frac{\rho_s}{\rho_w}$

b. Mass / Bulk / Apparent spec. gravity (G_M):

$$G_M = \frac{\text{wt of soil of given vol}^m}{\text{wt of std. fluid of same vol}^m} \quad [V_s = V_w]$$

(std. fluid → water)

** $G_M = \frac{W}{W_w} = \frac{\gamma}{\gamma_w} = \frac{\rho}{\rho_w}$

** $G > G_M$

Soil Type	Specific gravity (G)
* Sand & gravel	2.65 – 2.68
* Silt & silty sand	2.66 – 2.70
* Inorganic clay	2.70 – 2.80
* Soil high in mica, Iron	2.75 – 2.85
* Organic soil	1 – 2

** → Spec. gravity (G) of fine grained soil is more than coarse grained soil.
(fine grained → silt / clay; coarse grained → sand / gravel)

** → Spec. gravity (G) of inorganic soil is more than organic soil.

** → Spec. gravity (G) increases with increase in mineral content of soil.

→ As per Indian standards 'G' is reported at 27°C.

At any other temperature, γ_s = constant

$\frac{\gamma_s}{\gamma_w} = G$

$\gamma_s(27°C) = \gamma_{s\,T°C}$

** $G_{27°C}\,\gamma_{w27°C} = G_{T°C} \times \gamma_{wT°C}$

If std. fluid is changed, γ_s = const.

$G'_f = \frac{\gamma_s}{\gamma_f}$ $\quad G = \frac{\gamma_s}{\gamma_w}$

$G'_f \gamma_f = \gamma_w G$

$G'_f = G\left(\frac{\gamma_w}{\gamma_f}\right)$

** $G'_f = \frac{G}{G_{fluid}}$

9. Density index / Relative density / Degree of density:

→ Density index is used to represent the relative degree of compactness of natural cohesionless soil in absolute term unlike void ratio and dry unit wt.

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

Obviously

$$0\% \leq I_D \leq 100\%$$

→ It is defined as the ratio of diff. of void ratio of soil in loosest state & natural state to the diff. of void ratio of soil in loosest state & densest state.

e_{max} = void ratio of soil in loosest state

e = " " " " " natural "

e_{min} = " " " " " densest "

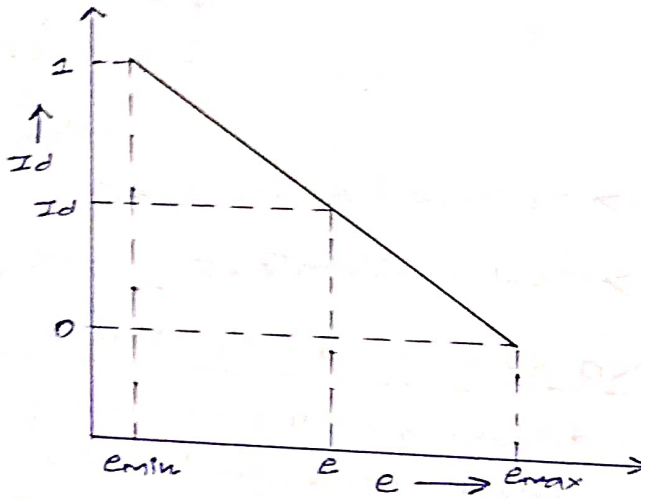
→ If soil is naturally in loosest state,

$e = e_{max}$, $I_D = 0$

→ If soil is naturally in densest state

$e = e_{min}$, $I_D = 100\%$

$I_d = f(e)$



→ $e \propto \frac{1}{\gamma_d}$

$$I_D = \frac{\frac{1}{\gamma_{dmin}} - \frac{1}{\gamma_d}}{\frac{1}{\gamma_{dmin}} - \frac{1}{\gamma_{dmax}}} \times 100$$

I_d (%)	Degree of denseness
0-15	Very loose
15-35	Loose
35-65	Medium dense
65-85	Dense
85-100	Very dense

→ 'Id' is not used for the analysis of cohesive soils as there are large uncertainities involved in finding the void ratio of cohesive soils in its loosest state in laboratory.

→ Two sand deposits having same shape & size can exhibit diff. prop. if they possess different I_D.

→ Sand having more I_D is more compact, dense, stable, less compressible, has higher shear strength.

10. Relative compaction (R_c): It is used to indicate the degree of denseness or compactness in absolute terms as density index (I_D), but it can be applied for both cohesionless & cohesive soil.

$$R_c = \frac{\gamma_d}{\gamma_{dmax}} \times 100$$

γ_d → dry unit wt of soil in existing/natural state

γ_{dmax} → " " " " " " densest state.

$$R_c = \frac{\gamma_d (1+e_{min})}{(1+e)\gamma_d}$$

** $R_c = \frac{1+e_{min}}{1+e} \times 100$

** $R_c = 80 + 0.2 I_D$

→ In loosest state, $I_D = 0\%$, $R_c = 80\%$

→ In densest state, $I_D = 100\%$, $R_c = 100\%$

NOTE: Packing of uniform sphere:

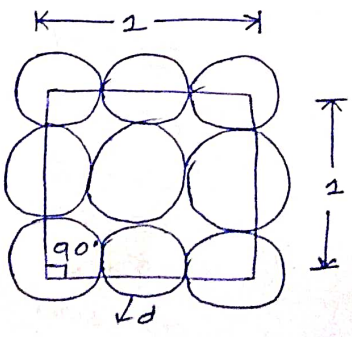
→ Packing of the uniform sphere is the only system void ratio & porosity of which can be found mathematically.

→ Spheres of uniform size attains stable loosest packing in "cubical array"

→ Spheres of uniform size attains stable densest packing in "Rhombohedral array".

→ Loosest state:

→ Here each particle is in contact with 6 more similar particle.



cubical array

Vol. of soil = $1 \times 1 \times 1 = 1 m^3$

no. of solids in soil = $\frac{1}{d} \times \frac{1}{d} \times \frac{1}{d} = \frac{1}{d^3}$

Vol. of one solid = $\frac{\pi d^3}{6}$

Total vol. of solids (V_s) = $\frac{\pi d^3}{6} \times \frac{1}{d^3} = \frac{\pi}{6}$

Volume of voids $(V_v) = V - V_s = 1 - \frac{\pi}{6} = 0.476 m^3$

$$e_{max} = \frac{V_v}{V_s} = \frac{0.476 \times 6}{\pi} = 0.91$$

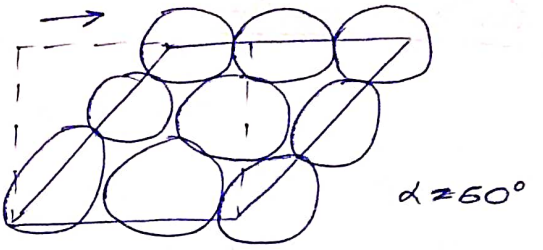
** $e_{max} = 0.91$

$$\gamma_{max} = \frac{e_{max}}{1+e_{max}} = \frac{0.91}{1+0.91} = 0.47$$

** $\gamma_{max} = 0.47$

→ Densest state:

→ Here each particle is in contact with 12 more similar particles.



"Rhombohedral array"

Vol. of unit rhombohedral (V)

$$= \sqrt{1+2\cos\alpha}\,(1-\cos\alpha)$$

$$V = \sqrt{1+\cos 60^\circ}\,(1-\cos 60^\circ) = 0.707$$

Vol. of solids $(V_s) = \frac{\pi}{6}$ Vol. of voids $(V_v) = 0.707 - \frac{\pi}{6} = 0.183 m^3$

$$e_{min} = \frac{V_v}{V_s} = \frac{0.183 \times 6}{\pi} = 0.35$$

$$\gamma_{min} = \frac{e_{min}}{1+e_{min}} = \frac{V_v}{V} = 0.26$$

** $e_{min} = 0.35$ ** $\gamma_{min} = 0.26$

Q. If void ratio of soil is 0.5. Indicate its degree of denseness.

$e = 0.5$

$$I_p = \frac{0.91 - 0.5}{0.91 - 0.35} = \frac{0.41}{0.56} \times 100 = 73.21\% \rightarrow \text{dense soil}$$

* Interelationship b/w the properties of soil:

1. e, w, G, S

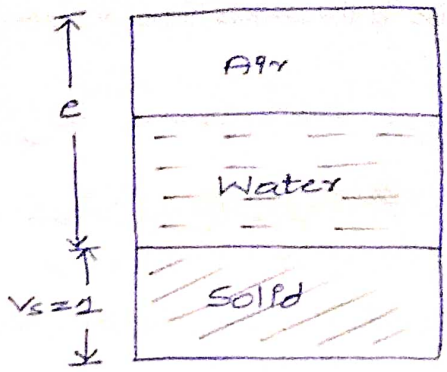
$$\gamma_w = \frac{W_w}{V_w}$$

$$e = \frac{V_v}{V_s} = \frac{V_v \cdot V_w}{V_s \cdot V_w} = \frac{1}{S} \cdot \left(\frac{V_w}{V_s}\right) = \frac{1}{S} \cdot \left(\frac{W_w/\gamma_w}{W_s/\gamma_s}\right)$$

$$e = \frac{1}{S} \cdot (w \cdot G)$$

** $Se = w \cdot G$

→ $e = \frac{V_v}{V_s} = \frac{V_v}{1} = V_v$



2. $\gamma_b, G, e, S, \gamma_w$

$$\gamma_b = \frac{W}{V} = \frac{W_s + W_w}{V_v + V_s} \Rightarrow$$

$$G = \frac{\gamma_s}{\gamma_w} = \frac{W_s}{V_s \cdot \gamma_w}$$

$$\gamma_w = \frac{W_w}{V_w}$$

$$\gamma_b = \frac{G V_s \gamma_w + \gamma_w V_w}{V_s [e+1]}$$

$$\gamma_b = \frac{G\gamma_w + \gamma_w \cdot S \cdot e}{e+1}$$

***

$$\gamma_b = \frac{\gamma_w (G + Se)}{e+1}$$

***

$$\gamma_b = \frac{(G+Se)\gamma_w}{1+e}$$

3. γ_d, G, e, γ_w

→ For dry soil

$S = 0$, $\gamma_b = \gamma_d$

**

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

→ For saturated soil

**

$$\gamma_{sat} = \gamma_d (1+w)$$

$S = 1$, $\gamma_b = \gamma_{sat}$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

**

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

4. $\gamma'/\gamma_{sub}, G, e, \gamma_w$

$$\gamma_{sub}/\gamma' = \gamma_{sat} - \gamma_w = \frac{(G+e)\gamma_w}{1+e} - G\gamma_w$$

$$\gamma' = \frac{(G+e)\gamma_w - \gamma_w(1+e)}{1+e}$$

$$e = \frac{\eta}{1-\eta}$$

$$1+e = \frac{\eta}{1-\eta} + 1$$

$$1+e = \frac{1}{1-\eta}$$

**

$$\gamma' = \frac{(G-1)\gamma_w}{1+e}$$

$$\gamma' = \frac{G\gamma_w}{1+e} - \frac{\gamma_w}{1+e} = \gamma_d - (1-\eta)\gamma_w$$

**

$$\frac{1}{1+e} = 1-\eta$$

**

$$\gamma' = \gamma_d + (\eta-1)\gamma_w$$

**

$$G_M = G(1-\eta) = \frac{G}{1+e}$$

5. γ_d, γ_b, w

$$\gamma_d = \frac{W_d}{V} = \frac{W_s}{V} = \frac{W'}{(1+w)V}$$

**

$$\gamma_d = \frac{\gamma_b}{1+w}$$

6. γ_d, G, γ_w, w

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{G\gamma_w}{1+\frac{wG}{S}}$$

If $S=1 \Rightarrow e = wG$ → water content at which soil is saturated

**

$$\gamma_d = \frac{G\gamma_w}{1+wG}$$

→ The above relationship is significant to understand the compactness of the soil.

7. $\gamma_d, \eta_a, G, w, \gamma_w$

$$V = V_v + V_s = V_a + V_w + V_s$$

$$\frac{V}{V} = \frac{V_a}{V} + \frac{V_w}{V} + \frac{V_s}{V}$$

$$1 = \eta_a + \frac{W_w}{\gamma_w \cdot V} + \frac{W_s}{\gamma_w \cdot G \cdot V}$$

$$1-\eta_a = \frac{w \cdot W_s}{\gamma_w \cdot V} + \frac{W_s}{G \cdot \gamma_w \cdot V} = \frac{w \cdot W_d}{\gamma_w \cdot V} + \frac{W_d}{G \cdot \gamma_w \cdot V}$$

$$1-\eta_a = \frac{w\gamma_d}{\gamma_w} + \frac{\gamma_d}{G \cdot \gamma_w} = \frac{\gamma_d}{\gamma_w}\left[w + \frac{1}{G}\right]$$

**

$$\gamma_d = \frac{(1-\eta_a) G \gamma_w}{1+wG}$$

→ This relationship is also significant to understand the compactness of the soil.

Q. A soil has 28 gms of soil solids, 10 cm^3 of voids, 9 gm of water, spec. gravity of soil grain of 2.7. Consider following statements

a. $w = \frac{9}{28} \times 100\%$

b. $e = \frac{10 \times 2.7}{28}$

c. $\frac{9}{10 \times 2.7} \times 100 = 5$

d. $\eta = \frac{10 \times 2.7}{28 + 10 \times 27}$

$W_s = 28$ gms $\quad W_w = 9$ gm $\quad \gamma_w = \frac{W_w}{V_w}$

$V_v = 10\ cm^3 \quad G = 2.7$

$w = \frac{9}{28} \times 100$

$e = \frac{10 \times 2.7}{28}$.

$S = \frac{W_w}{\gamma_w \times 10} = \frac{9}{10 \times \gamma_w} \times 100$ (since $\gamma_w \downarrow 1$)

$\eta = \frac{10 \times 2.7}{28 + 10 \times 2.7}$

$G = \frac{W_s}{V}$

$2.7 = \frac{28}{V}$

$V = \frac{28}{2.7}$

$\frac{1}{1 + \frac{10 \times 2.7}{28}} = 1 - \eta$

$\frac{28}{28 + 10 \times 2.7} = 1 - \eta$

$\eta = \frac{28 + 10 \times 2.7 - 1}{28}$

Q. A saturated soil has a total density 22 KN/M^3 & a water content of 10%. The bulk density & dry density of soil are

Total density = Bulk density = 22 KN/M^3

$\gamma_d = \frac{\gamma_b}{1+w} = \frac{22}{1+0.10} = \frac{22}{1.1} 20 = 20\ KN/M^3$

* Index properties of the soil:

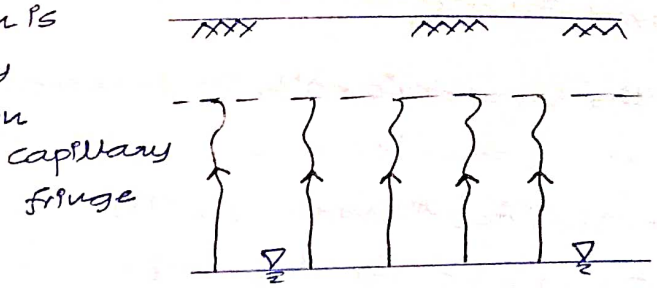
→ Index prop. of soils are those which helps in classification & identification of the soil.

1. Water content (w):

→ Soil water:

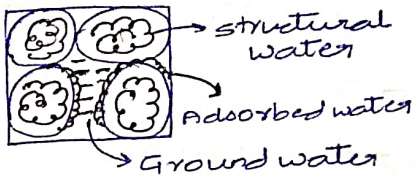
a. Ground / Free / Gravity water: It is subsurface water that fills the voids continuously upto ground water table level. This water is subjected to no force other than its own weight. It obeys all the laws of hydraulics and is capable of moving under hydrodynamic forces.

b. Capillary water: It is the water which is lifted above ground water table level by capillary forces. It remains in suspension in the voids of the soil. It also obeys laws of hydraulics & moves under hydrodynamic forces.



NOTE: Ground water & capillary water is also termed as pore water.

→ If GW is present then no capillary water present.



c. Structural water: It is the water which is chemically combined with the crystal structure of water soil. Under loading condition observed in engg. structural water can not be removed even by boiling at 105°-110°C.

d. Adsorbed water: Pellicular water

e. Hygroscopic water: It is the water which is adsorbed by soil solids from the atmosphere by physical forces of attraction & is held over its surface due to adhesion.

→ Extent of water adsorped by soil solids $\propto \frac{1}{d} \propto SSA$

→ Hygroscopic adsorption capacity of different soils

Sand : Silt : clay → 1-2% : 6-7% : 16-17%

→ Hygroscopic water is not affected by gravity or capillary forces.

→ It has higher density, viscosity, boiling point & lower freezing point than ground water.

ff. Film water: It is the water that is present as a film over the layer of hygroscopic water.

→ It is formed due to the condensation of aqueous vapour.

→ It is also held by soil solids. but has lower adhesion than hygroscopic water.

1. **Water content:** It can be determined by any of following methods.

a. **Oven dry method:**

→ This is most accurate laboratory method

→ In this method, moist sample of soil is placed in dry container of mass (M_1) & its total mass is noted (M_2).

→ Moist sample along with the container is then placed in temp. controlled oven for dry sample.

→ For inorganic soil, temp. is maintained at 105°-110°C range.

→ " organic " , " " " " less than 60°C to avoid oxidation of organic matter.

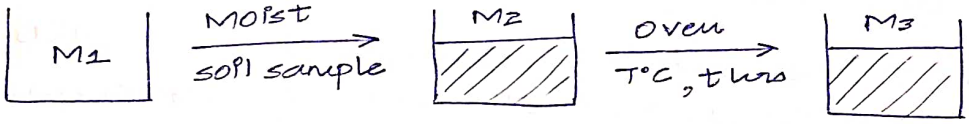
→ For soil having high gypsum content, temp. is kept $\leq$ 80°C (to avoid loss of water of crystallisation)

→ For sand & gravel, drying is required for 4-6 hrs.

→ For clay & silt , " " " " 16-20 hrs, however drying is for 24 hrs to ensure complete loss of water.

→ In no case temp. should exceed 110°C as it would lead to the loss of structural water.

→ In this method all types of soil water is removed, i.e. ground, capillary, adsorbed except structural water.



$$w = \frac{M_w}{M_s} = \frac{M_2 - M_3}{M_3 - M_1} \times 100$$

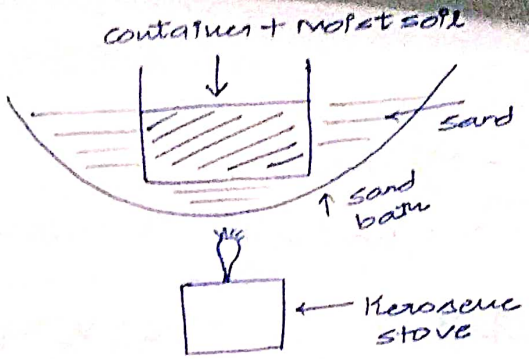
b. **Sand bath method:**

→ It is a quick field method that is used when the facility of oven is not available

→ In this sample, the moist sample of the soil along with the container is placed in sand bath & is heated over kerosene stove.

→ As there is no control over temp. in this case it is not suitable for high organic soil & soil having high gypsum content, moreover it gives inaccurate result due to loss of structural water.

→ Steps of observation are same as in Oven dry method.



c. Alcohol method:

→ It is also a quick field method.

→ In this method, methylated spirit (alcohol) is added in the sample as it increases the rate of drying & is ignited to note the water content.

→ Here also, there is no control over temp., hence it is not suitable for organic soil or soil having high gypsum content, moreover it is also inaccurate method as there is tendency of structural water to be lost.

→ Series of observation is same as oven drying.

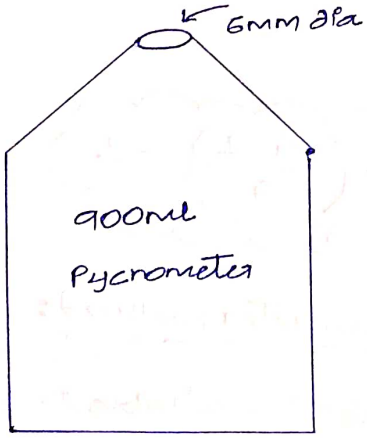
d. Pycnometer method:

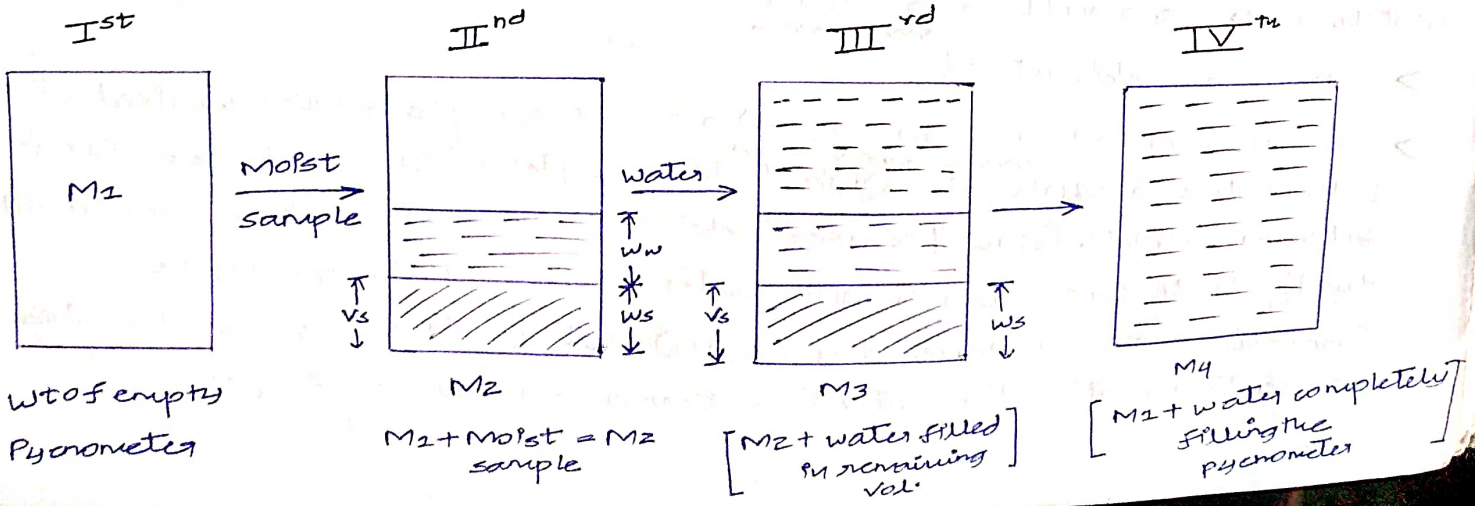
→ It is a quick method which gives the results in 10-20 mins.

→ This is suitable for those soil whose "G" is known.

→ This method is used for cohesionless soil as in case of cohesion soil it is difficult to remove entrapped air from soil sample during the test.

→ Pycnometer is 900ml flask having conical top with 6mm diameter circular hole at top to remove air from the sample.





$$w = \frac{M_w}{M_s}$$

Mass of water in soil $(M_w) = M_2 - M_1 - M_s$

$M_4 = M_3 - M_s + M_w$ (having volume same as that of solids)

$M_4 = M_3 - M_s + V_w \rho_w$

$M_4 = M_3 - M_s + V_s \rho_w$

$M_4 = M_3 - M_s + \frac{M_s}{G \rho_w} \cdot \rho_w$

$M_4 = M_3 - M_s \left[1 - \frac{1}{G}\right]$

$G = \frac{\rho_s}{\rho_w} = \frac{M_s}{V_s \cdot \rho_w}$

$V_s = \frac{M_s}{G \rho_w}$

$$\text{or} \quad M_s = \frac{(M_3 - M_4) \cdot G}{[G - 1]}$$

**

$$M_s = \frac{(M_3 - M_4) G}{(G - 1)}$$

$$w = \frac{M_w}{M_s} = \frac{M_2 - M_1 - M_s}{M_s} = \frac{M_2 - M_1}{M_s} - 1$$

$$w = \frac{(M_2 - M_1)(G - 1)}{(M_3 - M_4) \cdot G} - 1$$

**

$$w = \frac{M_2 - M_1}{M_3 - M_4}\left(\frac{G - 1}{G}\right) - 1$$

e. Calcium carbide or moisture meter method:

→ It is one of the quickest method available to find water content which gives the result in "5-7 min"

→ It is a field method.

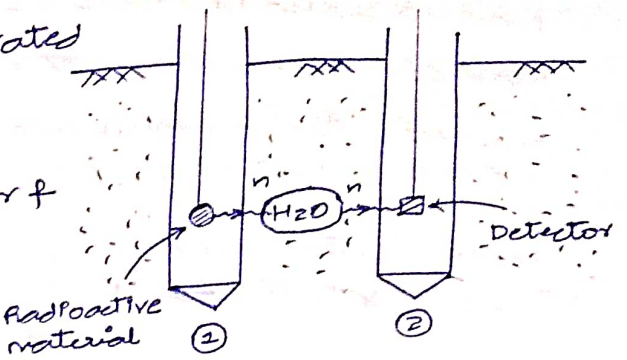
→ In this method 4-6 gm (5 gms) of soil sample is taken (fixed wt is taken due to calibration) which is then placed in moisture meter & calcium carbide [CaC_2] is used/added & sample is shaken vigourously due to which CaC_2 reacts with water present in it & results in formation of acetylene gas (C_2H_2) that exerts the pr. on calibrated gauge to report water content in terms of total wt of soil.

5. Radiation method:

→ It is a field method in which two steel casings are lowered into the soil mass to be tested.

→ In first casing, a radioactive material (:Co-60) is placed & in another casing detector is placed.

→ When radioactive material is activated it emits neutrons which when strike "Hydrogen atoms" of water loses its energy, i.e. further detected by detector & is calibrated with the water content of the soil.



3. Torsional balance method (Infrared method):

→ In this method, Infrared rays are used for drying of the sample.

→ This method is suitable for those soils which quickly absorbs the moisture from the atmosphere, as in this case drying & weighing of this sample is done simultaneously.

→ Infrared rays are generated by 250W, 230V, 50Hz frequency bulb.

→ Here also water content of the soil in terms of total wt of the soil is measured.

2. Specific gravity:

→ Specific gravity of the soil can be measured by

a. 50ml density bottle

b. 500ml flask

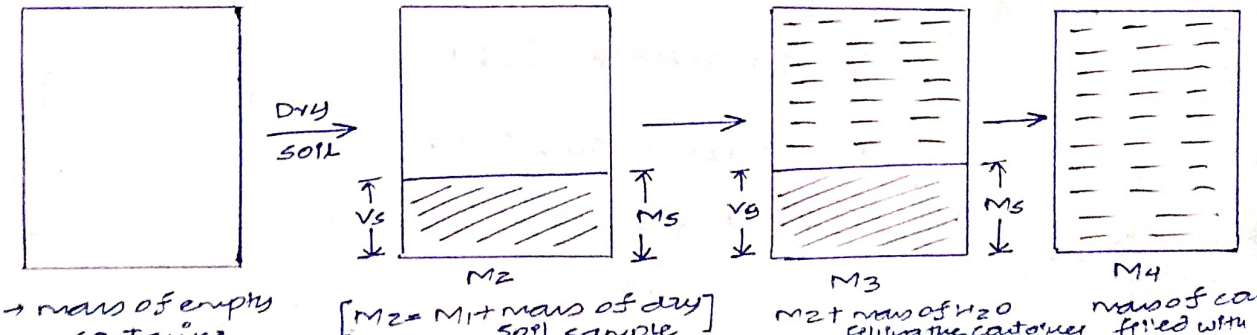
c. Pycnometer

→ Density bottle method is the most accurate amongst all & is suitable for all types of soil.

→ Flask & pycnometer is suitable to be used for coarse grained soil & if it is used for fine grained soil then kerosene is used instead of water as it is better wetting reagent than water.

→ In density bottle only kerosene is used.

→ Steps of obs. is same for all.



M1 → mass of empty container

[M2 = M1 + mass of dry soil sample]

M2 + mass of H2O filling the container

Mass of container filled with H2O

$G = \frac{\text{wt of solids of given vol.}}{\text{wt of std fluid of same vol.}} = \frac{W_s}{W_w/W_K} = \frac{\rho_s}{\rho_w/\rho_K}$

(std fluid: water / kerosene)

Mass of solids $\Rightarrow M_s = M_2 - M_1 =$ Mass of dry soil $= M_d$

Mass of water in stage III $= M_3 - M_2$

Mass of water in stage IV $= M_4 - M_1$

Mass of water having volume same as that of solids = Mass of water in Stage IV − Mass of water in stage III

Mass of water having Vol. same as that of solids $= (M_4 - M_1) - (M_3 - M_2)$

$= (M_4 - M_3) + (M_2 - M_1)$

$= (M_4 - M_3) + M_d$

$$G = \frac{M_2 - M_1}{(M_4 - M_3) + (M_2 - M_1)} = \frac{M_d}{(M_4 - M_3) + M_d}$$

** $G = \frac{M_d}{(M_4 - M_3) + M_d}$

γ_w at 27°C = 996.5

γ_w at 4°C = 1000

$G' = \frac{G}{G_f}$

$G = G' \cdot G_f$

** $G = \left(\frac{M_d}{(M_4 - M_3) + M_d}\right) \cdot G_K$

Q. $M_1 = 62.129$ gm

$M_2 = 83.499$ gm

$M_3 = 264.419$ gm

$M_4 = 249.249$ gm

$G_K = 0.773$

$$G = \frac{(M_2 - M_1)}{(-M_3 + M_4) + (M_2 - M_1)}$$

$$G = \frac{21.37}{-15.17 + 21.37} \times 0.773$$

↓ 36.54 6.2

$G =$ ~~0.445~~ 2.664

$G_{27°C}\,\gamma_{w27°} = G_{4°C}\,\gamma_{w4°C}$

$2.66 \times 996.5 = G_{4°C} \times 1000$

$G_{4°C} = 2.65$

3. Unit weight of soil:

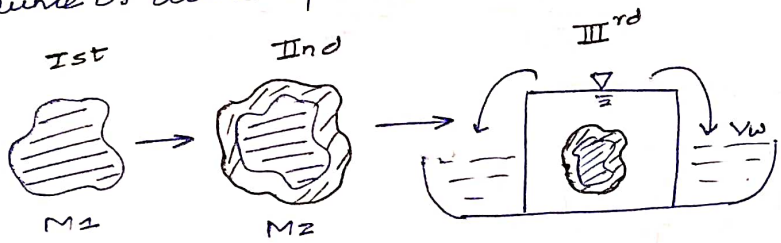
a. Water displacement method:

→ This method is suitable for cohesive soil which is highly sticky in nature.

→ In this method, sample of soil trimmed into more or less uniform shape and weighed (M_1).

→ The sample is then coated with paraffin wax to prevent the entry of water into the soil & is again weighed (M_2).

→ The coated specimen is then immersed in the container full of water which displaces the water from the water, volume of which is equal to the volume of coated specimen (V_w).



Mass of paraffin wax = $M_2 - M_1$

Volume " " " = $V_p = \frac{M_2 - M_1}{\rho_p}$ $\left[\rho_p = 0.908\ g/ml\right]$

Vol. of water displaced V_w = Vol. of soil + Vol. of paraffin wax

$$V_w = V + V_p$$

$$V = V_w - V_p$$

$$\rho = \frac{\text{Mass of soil}}{\text{Vol. of soil}} = \frac{M_1}{V} = \frac{M_1}{V_w - \left(\frac{M_2 - M_1}{\rho_p}\right)}$$

**

$$\rho = \frac{M_1}{V_w - \left(\frac{M_2 - M_1}{\rho_p}\right)}$$

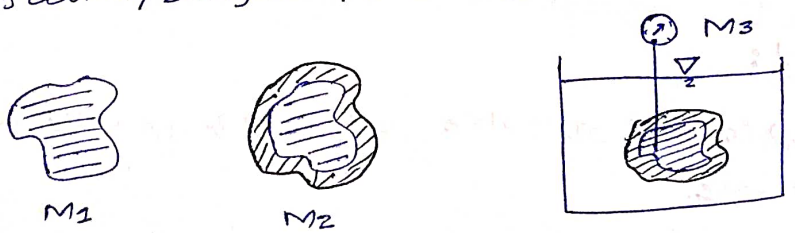
b. Submerged unit weight method: Submerged mass density method

→ It is suitable for cohesive soil which is highly sticky in nature.

→ Same as above test

→ Same as above test

→ In this method, coated specimen is immersed in water & its reduced/effective/buoyant/submerged wt is noted (M_3).



Mass of paraffin wax = $M_2 - M_1$

Volume " " " = $V_p = \frac{(M_2 - M_1)}{\rho_p}$

$M_3 = M_2 - F_B$

$M_3 = M_2 - V_w\gamma_w / V_w\rho_w$ [V_w = Vol. of coated specimen]

$M_3 = M_2 - V_w\gamma_w / V_w\rho_w$

$V_w = \frac{M_2 - M_3}{\rho_w}$

$V_w = V + V_p$

$V = V_w - V_p = \frac{M_2 - M_3}{\rho_w} - \frac{(M_2 - M_1)}{\rho_p}$

***

$$\rho = \frac{M_1}{\left(\frac{M_2 - M_3}{\rho_w}\right) - \left(\frac{M_2 - M_1}{\rho_p}\right)}$$

c. Core cutter method:

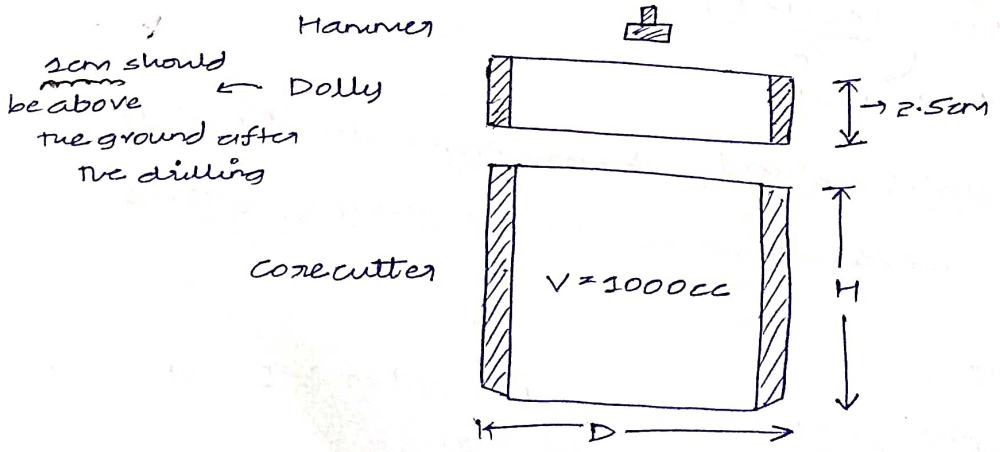
→ It is a field method which is suitable to be used for cohesive soils.

→ This method consists of a core cutter of volume 1000 cc [D = 10cm, H = 13cm], a dolly of height 2.5 cm & a hammer

→ In this test, core cutter is drilled in the soil mass which is to be tested & cutter with the soil is taken out to note the mass of the soil in the cutter (M)

$$\rho = \frac{\text{Mass of soil}}{\text{Volume of soil}} = \frac{M}{V}$$

Vol. of soil in cutter = Vol. of core cutter = V



d. Sand replacement method:

→ Since core cutter method is not suitable for hard & gravelly soil, this method is used in this case.

→ This method consists of pouring cylinder mounted on a pouring cone & a calibrating cylinder of known volume.

→ In this test, first bulk mass density of sand used is found followed by the unit weight of the soil.

→ Sand is completely filled in the pouring cylinder & its mass is noted (M_2).

→ It is allowed to run into pouring cone & mass of sand filling the pouring cone is noted (M_2).

→ Now pouring cylinder along with pouring cone is placed over calibrating cylinder & the mass of sand left in pouring cylinder after allowing it to run in calibrating cylinder is noted (M_3).

→ Mass of sand filling the calibrating cylinder is calculated

$$M' = M_1 - M_2 - M_3$$

$$\rho_{sand} = \left[\frac{\text{Mass of sand}}{\text{Volume of sand}}\right]_{\text{in cylinder}}$$

$$\rho_{sand} = \frac{M'}{V} \qquad (V_{sand} = V_{calibrating\ cylinder})$$

→ Pouring cylinder along with pouring cone is placed over the hole created by excavating the sample of soil of mass "M" to be tested and sand is allowed to run into this hole.

→ Mass of sand left in pouring cylinder is noted (M_4)

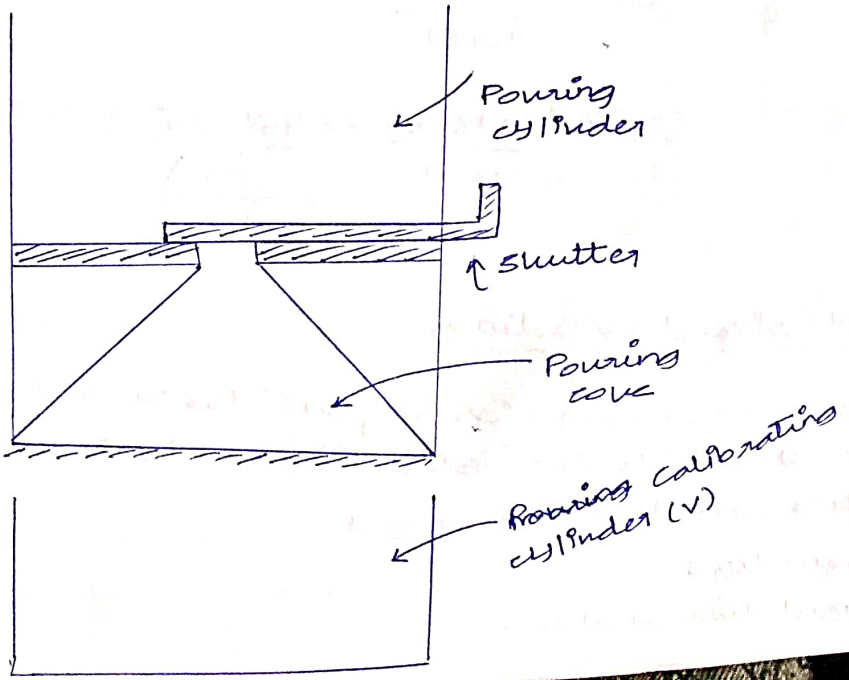
Mass of the sand filling the hole is calculated

$$M'' = M_1 - M_2 - M_4$$

$$\text{Vol. of the sand filling the hole} = \frac{M''}{\rho_{sand}}$$

$$\text{Vol. of sand filling the hole } (V) = \text{vol. of soil excavated}$$

$$\text{Density of soil} = \frac{M}{V}$$



Q. $\gamma = \frac{W}{V} = \frac{W_2 - W_1}{V} = \frac{(2770.6 - 946.0)}{1000} = \frac{1.8238 \times 10^{-3}}{10^{-6}} \times 9.81$

$\rightarrow \gamma = 17.89 \text{ KN/m}^3$

$\rightarrow \gamma_d = \frac{\gamma_b}{1+w} = \frac{17.89}{1 + 0.1045} = 16.198 \text{ KN/m}^3$

$\gamma_d = \frac{G\gamma_w}{1+e}$

$16.198 = \frac{2.65 \times 9.81}{1+e}$

$\rightarrow e = 0.604$

$Se = wG$

$S \times 0.604 = 0.1045 \times 2.65$

$\rightarrow S = 0.45$

NOTE: In both core cutter method & sand replacement method, soil sample is further tested for its water content by oven drying method.

$\rightarrow S = 1$, If there is no change in volume of soil

$\rightarrow V_v = \text{constant}$

$\Rightarrow e = \text{constant}$

$e = \frac{wG}{S} \Rightarrow w = \frac{eS}{G} = \frac{0.604 \times 100}{2.65} = 22.83\%$

Q. $\rho_{sand} = \left.\frac{M_{sand}}{V_{sand}}\right]_{\text{in calibrating can}}$

$\rho_{sand} = \frac{1500}{1000} = 1.5 \text{ gm/cc}$

$\rho_{soil} = \frac{M_{soil}}{V_{soil}} = \frac{761.25}{\left(\frac{445}{1.5}\right)} = 2.56 \text{ gm/cc}$

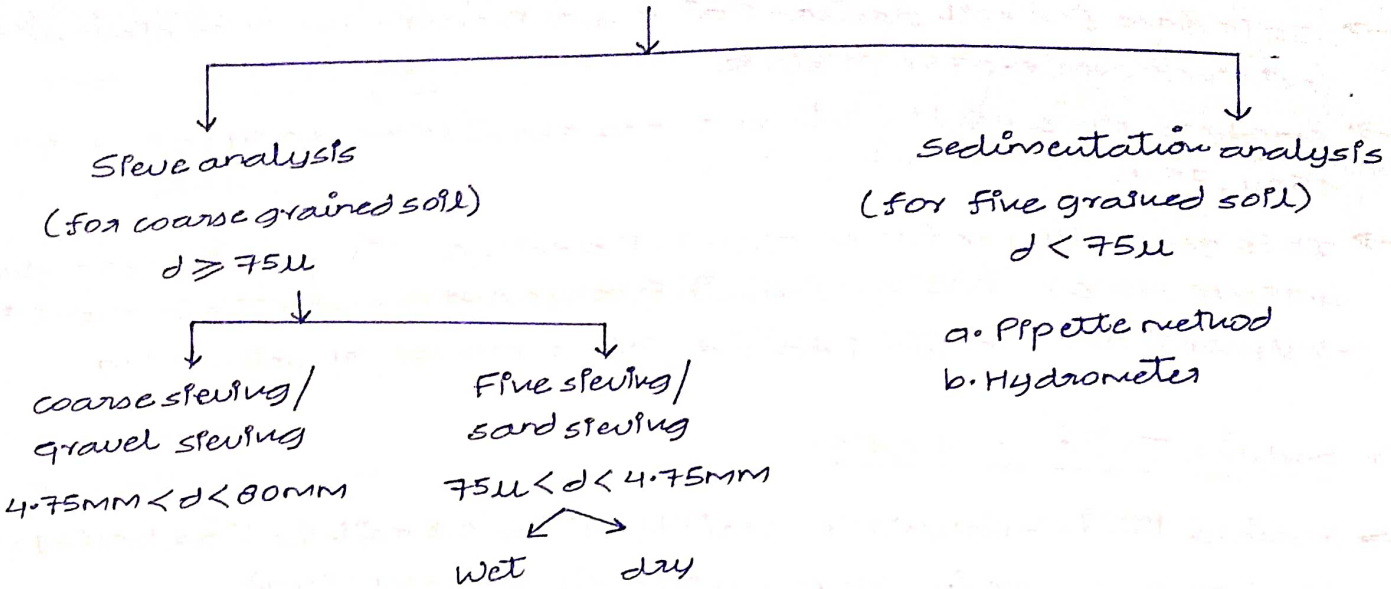
4. Particle size distribution:

→ The % of various size of particles in a given dry soil sample is found by particle size distribution analysis or mechanical analysis, which is carried out in 2 stages.

a. Sieve analysis

b. Sedimentation analysis

Particle size analysis



a. Sieve analysis:

→ It is done for the soil fraction having size greater than 0.075MM or 75μ.

→ As per IS 460 : 1962, sieves are designed by the size of the square opening (aperture) in MM or μ.

→ In this method, different sieves are placed one over each other in descending order of their size & dry sample of the soil to be tested is placed on topmost sieve & sieving is done continously for atleast 10 min either manually or in sieve shaker.

→ Weight of the soil fraction retained over each sieve is noted to find the % finer corresponding to given size of particle.

$$\% \text{ finer} = \%N = 100 - \text{cumulative } \% \text{ wt retained}$$

1. Coarse sieving / Gravel sieving:

→ It is done for soil fraction having size > 4.75MM (soil particles retained over 4.75 MM sieve).

→ Standard sieves used in this cases are 80MM, 40MM, 20MM, 10MM, 4.75MM.

Sieve size	wt retained	% wt retained	cumulative % wt retained	% finer
80MM → D60				
40MM	10gM	10%	10%	90%
→ D30				
20 MM	30gM	30%	40%	60%
→ D10				
10 MM	30gM	30%	70%	30%
4.75MM	20gM	20%	90%	10%
	10gM	10%	100%	0%
	100gM			

a. Fine sieving / sand sieving:

→ It is done for soil fraction that passes through 4.75mm sieve and retained over 0.075mm sieve.

→ Standard sieve used in this case are 2mm, 1mm, 600μ, 425μ, 212μ, 150μ, 75μ.

→ It is generally preferred to wash the soil fraction passing through 4.75mm sieve before carrying out fine sieving in order to remove silt & clay present over the sand particles. This is termed as wet sieving.

b. Sedimentation analysis:

→ Sedimentation analysis is carried out for the soil fraction having size less than 0.075mm (which passes through 0.075mm sieve)

→ Particles having size smaller than 0.2μ cannot be analysed even by sedimentation analysis as they undergo brownian motion.

→ For them special technique x-ray diffraction or electron microscopy is used.

→ Sedimentation analysis is based upon "Stoke's law" according to which settling velocity of spherical particle undergoing settlement in an infinite medium, discretely, keeping all the other factors to be constant depends upon shape, size & mass of the particle & is given by

$$v_s = \frac{(G-1)\gamma_w d^2}{18\mu}$$

→ At 20°C, $\mu = 0.01$ Poise, $G = 2.65$

1 Poise = 0.1 N-s

$$v_s = \frac{(2.65-1)\,9.81\times10^3 d^2}{18\times0.01\times10^{-1}} = 899250\,d^2\times10^{-6}\ M/sec$$

$$* \quad v_s = 0.899\,d^2\ M/sec \approx 0.9\,d^2\ (M/sec)$$

(d in MM)

→ At 27°C, $\mu = 0.00855$ Poise, $G = 2.65$

$$* \quad v_s = 1.05\,d^2\ M/sec$$

(d in MM)

→ Limitations of Stoke's law:

1. Particles undergoing settlement are assumed to be spherical but in actual fine grained soil is flaky.
2. Medium in which settlement takes place is assumed to be infinite, but in actual sedimentation jar is finite.

3. During settlement, discrete settling is considered but in actual agglomeration of particles during settlement takes place.

4. Stoke's law is valid for particles having size in the range of 0.1MM to 0.2µ as above 0.1MM size gravity acceleration sets in & constant velocity is not attained & below 0.2µ Brownian motion takes place.

* General procedure for sedimentation analysis:

→ Sedimentation analysis can be carried out by

a. Pipette method

b. Hydrometer method

→ Methods of preparation of soil suspension is same in both the methods as follows.

a. Pre-treatment: Before preparing soil suspension, organic matter & calcium compounds present in the soil sample is removed to avoid agglomeration of soil particles during settlement.

→ To remove OM, oxidising reagent like H_2O_2 is added which oxidise organic matter & to remove calcium compound, 0.2N HCl is used.

b. Preparing the soil suspension: In pipette method, 12-30gM of soil is added to make 500ml of soil suspension (total volume) & in hydrometer method (24-60gM) of soil is added to make 1000ml of soil suspension.

c. Post treatment: After preparing the soil suspension dispersing/ deflocculating reagent is added to avoid the agglomeration of fine grained soil solids (as they are sticky in nature).

→ Commonly used deflocculating reagent are sodium Hexametaphosp-hate, Sodium oxalate, Sodium carbonate, Sodium silicate.

a. Pipette method:

→ In this method, 10ml of sample is collected from soil suspension at fixed sampling depth (Hc) of 10cm at diff. time intervals.

→ The collected sample is further being tested for the mass of solids ($M_s = M_d$) present in it by oven drying method.

→ The size of the particle that settles in soil suspension by sampling depth "Hc" in time 't' is computed using stoke's law.

$$v_s = \frac{(G-1)\gamma_w d^2}{18\mu} = \frac{H_c}{t}$$

** $d = K\sqrt{\frac{H_c}{t}}$

** $d = \sqrt{\frac{18\mu}{(G-1)\gamma_w} \cdot \frac{H_c}{t}}$

$K = \sqrt{\frac{18\mu}{(G-1)\gamma_w}}$

$t = \frac{1}{2}, 1, 2, 4, 8, 15, 30\text{Min}, 1\text{hr}, 2, 4, 8, 16, 24\text{ hrs}$

→ % finer (%N) corresponding to computed size of the particle which settles by distance 'He' in the suspension in time 't' is given by,

* $\%N = \frac{\text{Mass of solids per unit volume of depth He, \& time 't'}}{\text{Mass of the solids per unit volume initially}}$

** $\%N = \frac{Md/Vp}{MD/V} \times 100$

$Vp = 10\text{ml}$

$V = 500\text{ml}$

$MD = 12-30\text{gm}$

Md = by oven drying method

→ If affect of deflocculating reagent is also to be considered then,

** $\%N = \frac{\frac{Md}{Vp} - \frac{M}{V}}{\frac{MD}{V}}$

Md = mass of solids in sample collected from 'Ho' height after time 't'.

MD = Total mass of solids taken initially

M = Mass of deflocculating reagent added.

V = Vol. of soil suspension

Vp = " " pipette

b. Hydrometer method:

→ Hydrometer is a device which is used for the measurement of density or specific gravity of the suspension (in our case soil suspension)

→ ~~Variation~~ Reading of hydrometer unlike normal case increases d/w.

→ For simplicity in understanding, reduced hydrometer readings are marked on it (RH).

$$\rho_{ss} = 1 + \frac{RH}{1000} \quad \text{or} \quad RH = (\rho_{ss} - 1) \times 1000$$

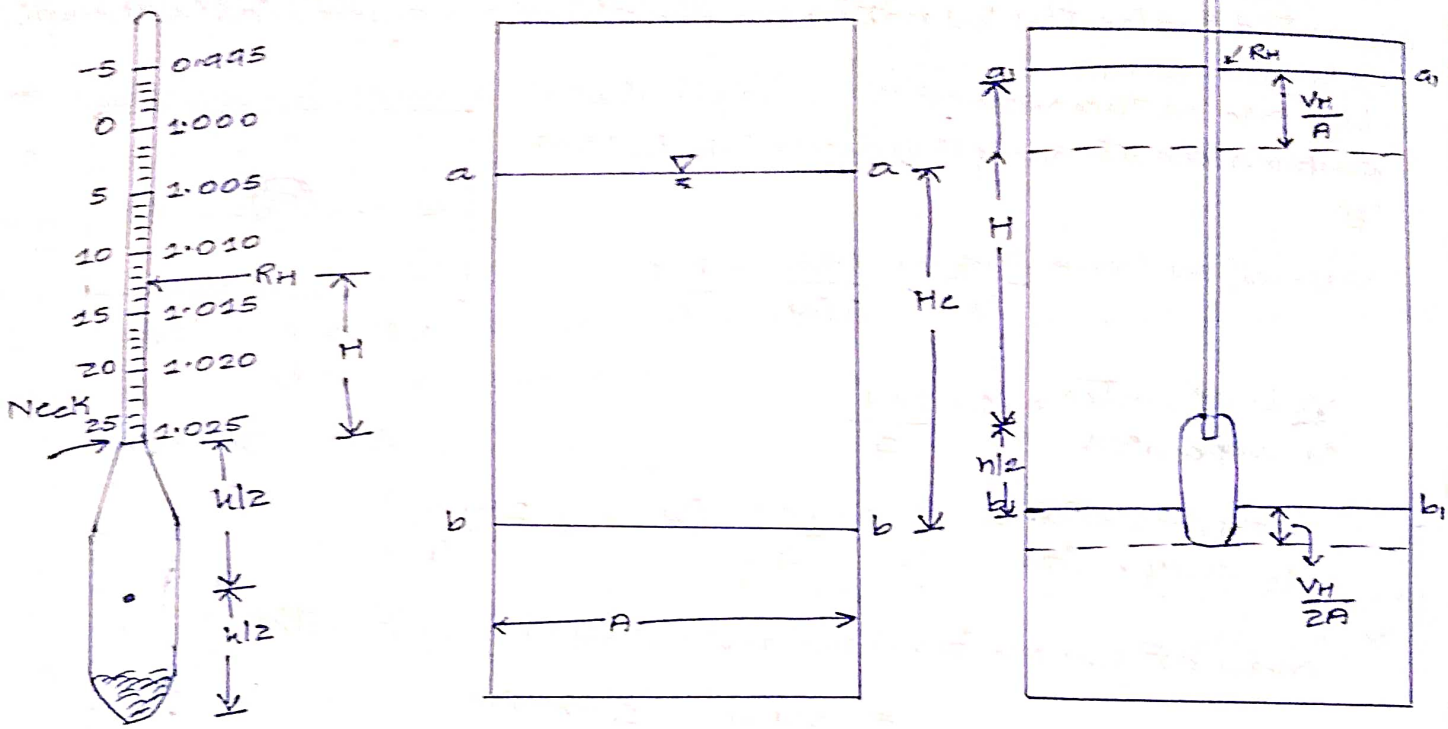
→ Volume of the hydrometer below the centre of the bulb is approx. 50% of its total volume

→ Principle of testing in both Pipette & Hydrometer method is same but they are different in following ways.

a. Mass of solids present at depth 'He' in time 't' is computed directly in Pipette method by ovendrying method, but is found

Particularly in hydrometer method by noting the density of soil suspension.

b. In pipette method, sampling depth is kept constant whereas in hydrometer method it increases with increase in time of settlement of particles, hence hydrometer & sedimentation jar is calibrated prior to every test as follows.

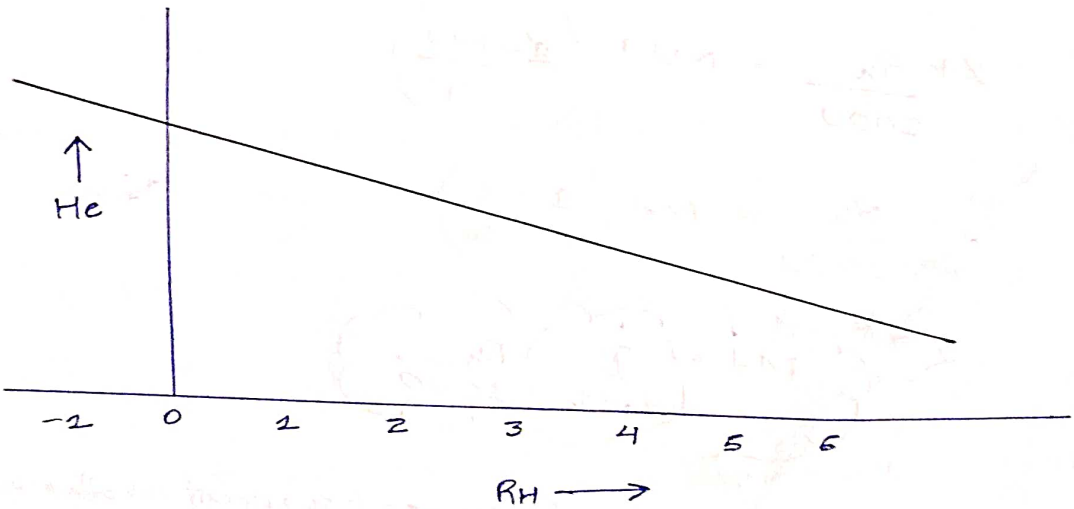


$$Hc = H + \frac{h}{2} + \frac{V_H}{2A} - \frac{V_H}{A}$$

$$Hc = H + \frac{1}{2}\left(h - \frac{V_H}{A}\right)$$

$$H \propto \frac{1}{R_H}$$

$$Hc = f(R_H)$$



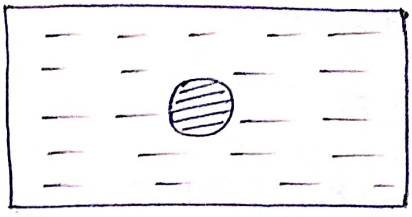
→ The size of the particles that settles in suspension upto sampling depth "Hc" in time 't' is found using stoke's law.

$$v_s = \frac{(G-1)\gamma_w d^2}{18\mu} = \frac{H_e}{t}$$

$$d = K\sqrt{\frac{H_e}{t}} \qquad K = \sqrt{\frac{18\mu}{(G-1)\gamma_w}}$$

→ % finer (%N) corresponding to noted size of particle "d" is computed indirectly by noting the density of the suspension (soil susp.).

Let 'Md' be the mass of the solid part of suspension at sampling depth 'He' in time 't'.



Soil suspension (1ml)

$$\text{Vol. of solids} = \frac{M_d}{\rho_s} = \frac{M_d}{G\rho_w} = \frac{M_d}{G}$$

$$\text{Vol. of water in suspension} = 1 - \frac{M_d}{G}$$

$\rho_w = 1\ gm/cc$

$$\text{Mass of water in suspension} = \left(1 - \frac{M_d}{G}\right)\rho_w \xrightarrow{1} \left(1 - \frac{M_d}{G}\right)$$

Mass of suspension = Mass of solids + Mass of water

$$= \frac{M_d}{G} + \left(1 - \frac{M_d}{G}\right)$$

$$\text{Density of suspension} = \frac{M_{suspension}}{V_{suspension}} = M_d + \left(1 - \frac{M_d}{G}\right)$$

Density of suspension as read from hydrometer

$$\rho_{ss} = 1 + \frac{R_h}{1000}$$

$$1 + \frac{R_h}{1000} = M_d + \left(1 - \frac{M_d}{G}\right)$$

$$\frac{R_h}{1000} = M_d\left(1 - \frac{1}{G}\right)$$

$$M_d = \left(\frac{G}{G-1}\right)\frac{R_h}{1000} \quad ***$$

$$\% \text{ finer (\%N)} = \frac{\text{Mass of solids / Volume of suspension at sampling depth 'He' \& time 't'}}{\text{Mass of solids / Volume of suspension taken initially}}$$

$$\%N = \frac{M_d}{\frac{1}{\frac{M_D}{1000}}} \times 100 = \left(\frac{G}{G-1}\right)\frac{R_h}{1000} \times \frac{1000}{M_D} \times 100$$

** $\%N = \left(\frac{G}{G-1}\right)\frac{R_h}{M_D} \times 100$

→ The observed reduced hydrometer readings are further corrected for the following

a. Meniscus correction:

→ Due to presence of turbidity in soil suspension upper level of meniscus is noted instead of lower level, which leads to reduced value of the obs. being noted. Hence meniscus corrected is applied which is (+)ve.

$$R_h = R_h' + C_M$$

b. Dispersing agent correction:

→ Addition of dispersing agent results in increased density of soil suspension hence higher value of observation is noted thereby dispersing agent correction is applied which is (-)ve.

$$R_h = R_h' - C_d$$

c. Temperature correction:

→ The calibration of hydrometer is done at standard temp. of 27°C, hence if temp during the test differs, correction due to temp is to be applied over observed hydrometer reading

* If $T > 27°C$ $\quad R_h = R_h' + C_t$

* If $T < 27°C$ $\quad R_h = R_h' - C_t$

NOTE: In some cases composite correction can also be applied directly where composite correction C_{M_c}

$$C_{M_c} = C_M \pm C_T - C_d \qquad R_h = R_h' + C_{M_c}$$

Q. * Particle size distribution curve:

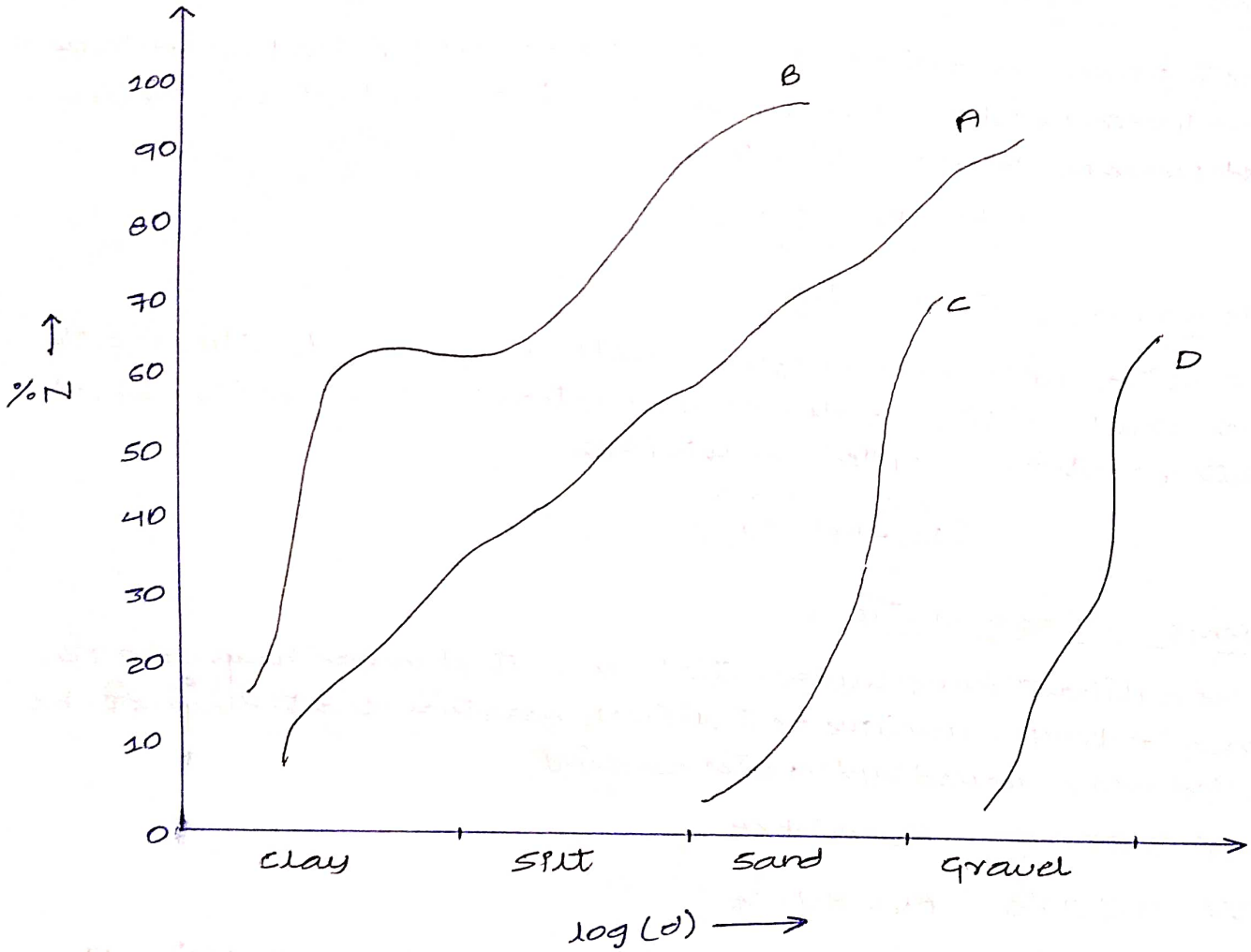
→ The result of the particle size distribution analysis is reported in the form of a curve termed as particle size distribution curve in which % finer is expressed on y-axis & size of the particle is expressed on x-axis on log scale.

→ This curve helps in analysing the type of soil & gradation of soil.

→ The soil may be termed as well graded soil if it has good representation of all sizes of particles in it.

→ If soil consists of excess of one size of particle & deficiency of another size of particle or it consists of all the particles of same size it is termed as poorly graded soil, in latter case it is termed as uniformly graded soil.

→ If certain size of particles are ~~mixed~~ missing from soil, it is termed as gap graded soil. / flat graded soil



Soil type	Gradation	Type of soil
A	Well graded soil	—
B	Gap graded soil	Fine grained soil
C	Well graded soil	Coarse grained soil
D	Poorly graded soil	Coarse grained soil

→ For coarse grained soil, D_{10}, D_{30}, D_{60} size of particles holds higher significance & can be found using particle size distribution curve

→ D_{10} = size in "mm" such that 10% of particles are finer than this size

→ D_{30} = " " " " " 30% " " " " " " " " "

→ D_{60} = " " " " " 60% " " " " " " " " "

* D_{10} is also termed as effective size or effective diameter or designed Allen Hazen.

→ Their significance is more as these size helps in finding the gradation of CG soil in terms of following parameters

a. Coefficient of uniformity (C_u): It signifies the range of particles size distribution curve.

* It is defined as ratio of D_{60} & D_{10}

$$C_u = \frac{D_{60}}{D_{10}}$$

→ For well graded sand, $C_u > 6$

→ " " " gravel, $C_u > 4$

→ " uniformly graded CG soil, $C_u = 1$

b. Coefficient of curvature (C_c): It signifies the shape of particle size distribution curve

$$C_c = \frac{D_{30}^2}{D_{60} \cdot D_{10}}$$

$C_c > 3$ or $C_c < 1$ gap graded soil

→ For well graded CG soil, C_c → 1 to 3 i.e. $1 \leq C_c \leq 3$

e. Consistency of soil:

→ It is the relative ease with which soil can be deformed.

→ This term is generally used for fine grained soil, in which it is related with large water content.

→ It also signifies degree of firmness of soil which may be stated as soft, stiff, hard (It indirectly relates with the strength of soil)

→ Fine grained soil may be mixed with the water so as to form a plastic paste which can be moulded into form by pressure

→ The addition of water reduces the cohesion making the soil still easier to mould.

→ Further addition of water reduces cohesion until it no longer retain its shape under its own wt & starts to flow.

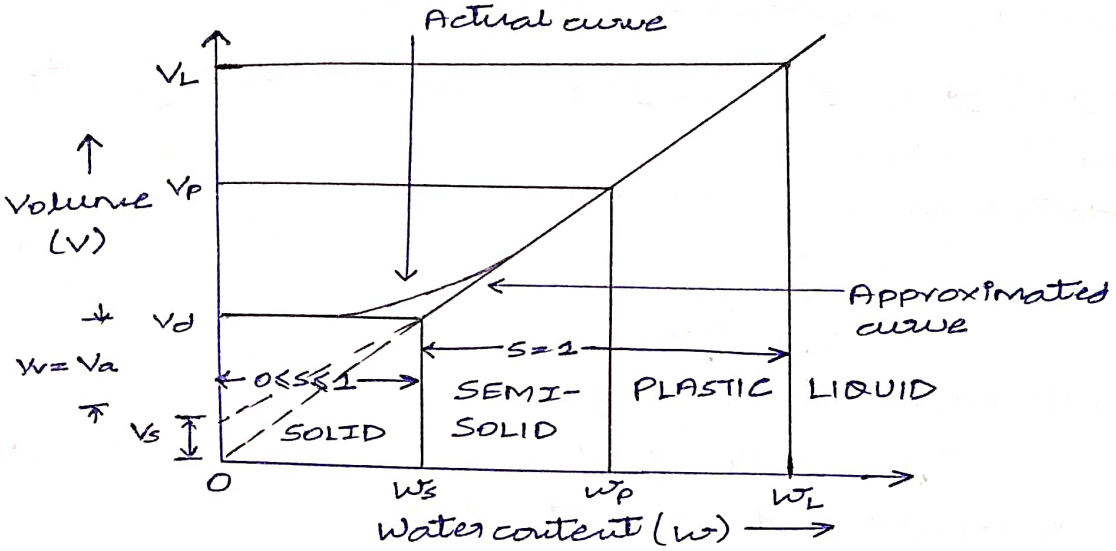
→ Enough water may be added until soil suspension is prepared.

→ Now when this water is removed/dryed from the soil mass it passes through diff. stages of consistency

→ Atterberg analysed this in 4 stages of consistency, i.e. solid state, semi-solid state, plastic state, liquid state.

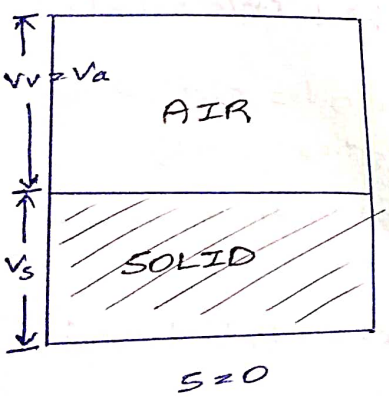
→ The water content at which soil passes from one stage of consistency to another stage is termed as consistency limit (Atterberg's limit)

* Consistency curve (Atterberg's curve):

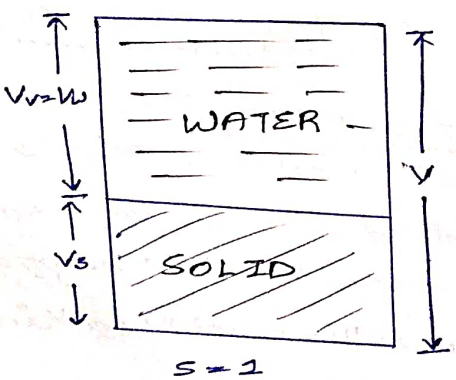


→ If soil is partially saturated or just saturated, any change in water content does not leads to change in its volume of voids, volume of soil, e or n.

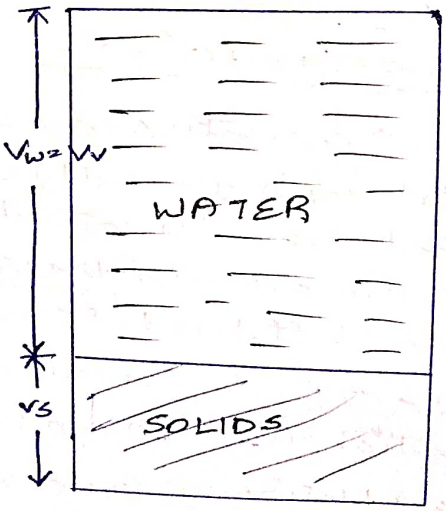
→ But if the soil is in "i.e supersaturated state ($S = 1$)", any change in water content leads to corresponding change in volume of voids, vol. of soil, e & n.



$S = 0$

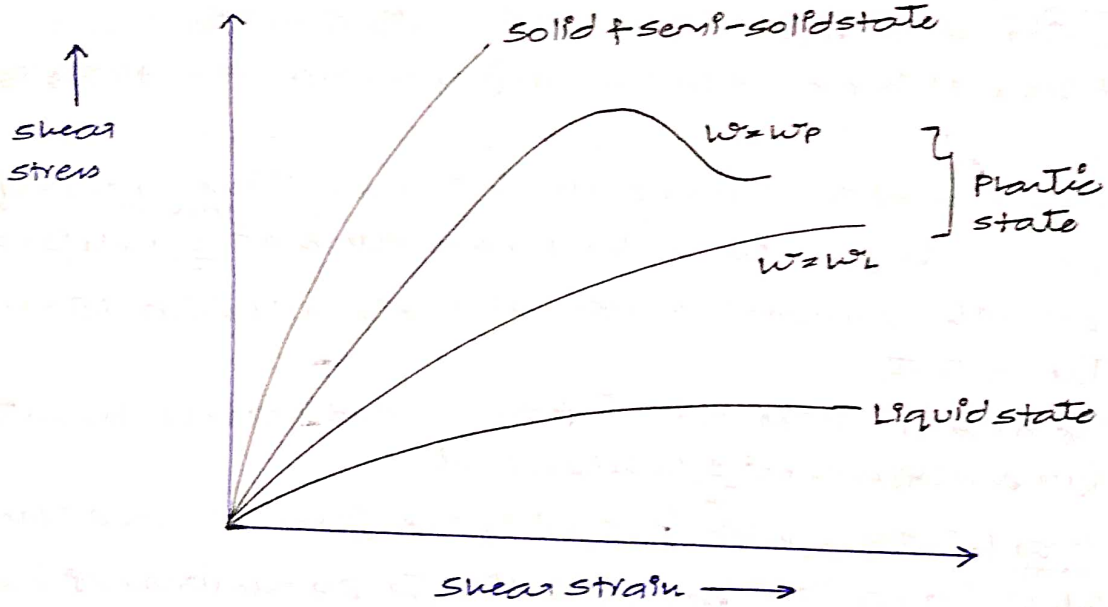


$S = 1$



$S = 1$

State	Liquid	Plastic	Semi-solid	Solid
w (%)	w_L	w_P	w_s	
Consistency	← Liquid →	← Very soft to stiff →	← Very stiff to very hard →	
Saturation	← 100% →		← 100-0% →	
Volume	← Volume decreases →		← Volume is constant →	
Strength	← shown graphically →			



→ slope of stress-strain curve represents strength.

* Consistency (Atterberg's) limit:

a. Liquid limit (w_L):

→ It is defined as min. water content at which soil has tendency to flow.

→ At liquid limit, soil paves from liquid state of consistency to plastic state of consistency & vice-versa.

→ At liquid limit, all the soils paves same negligible shear strength of 2.7 kN/m², which can be just measured.

→

Type of soil		w_L (%)
* Gravel	→	Non-plastic
* Sand	→	Non-plastic
* Silt	→	30-40
* Clay (alluvial soil)	→	40-150
* Clay (black soil)	→	400-500
* Clay (bentonite soil)	→	500-800

→ Higher is the w_L of the soil, higher is its compressibility (volume changes).

→ From determination pt of view, liquid limit (w_L) may also be defined as min. water content at which a part of soil cut by a groove of std. size flow together by the distance of $\frac{1}{2}$" ("=inch) (12mm) under the impact of 25 number blows, where ht of fall is adjusted to 1cm for each blow.

→ The tool used for cutting the soil may be "CASSAGRANDE TOOL or ASTM TOOL".

* Test to find liquid limit (casagrande test)

→ About 120gm of soil passing through 425μ sieve is mixed with sufficient water so as to form a uniform paste from it.

→ A portion of the soil is placed in the cup & a groove of std. size is made in it.

→ The handle is rotated at the rate of about 2 revolutions per second & no. of blows req. to flow the soil by the distance of $\frac{1}{2}$ inch is noted.

→ About 10gm of soil from near the closed groove is taken for determination of water content.

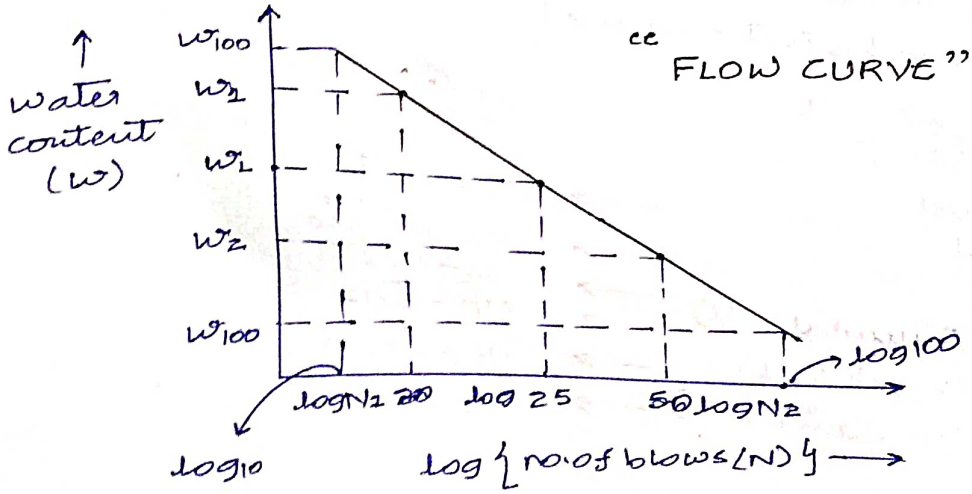
→ Test is repeated at diff. water content & corresponding no. of blows to flow the soil by the distance of $\frac{1}{2}$ inch is noted.

→ The result of test is then noted/plotted in the form of curve termed as "flow curve" & water content corresponding to 25 numbers of blows to flow the soil by distance of $\frac{1}{2}$ inch is interpolated & termed as liquid limit.

NOTE: In some cases soil slips instead of flowing in the test, the result of such test is discarded.

→ In flow curve, water content is marked on y-axis & corresponding no. of blows is marked on x-axis on log scale

→ Test is repeated such that no. of blows range from "20-50"



Groove	Cassagrande tool	ATSM tool
* Bottom width	2MM	2MM
* Top width	11MM	13.5MM
* Height	8MM	10MM

* Remark: ATSM tool is suitable for more sandy tool where cassagrande tool tends to tear the sides of groove.

→ The slope of flow curve is termed as flow index which signifies rate of loss of shear strength of soil with increase in water content.

* → Higher is the value of If, lower is the shear strength of soil.

→ During the analysis, if curve is extrapolated to find water content corresponding to 10 & 100 no. of blows, diff. of water content directly gives If.

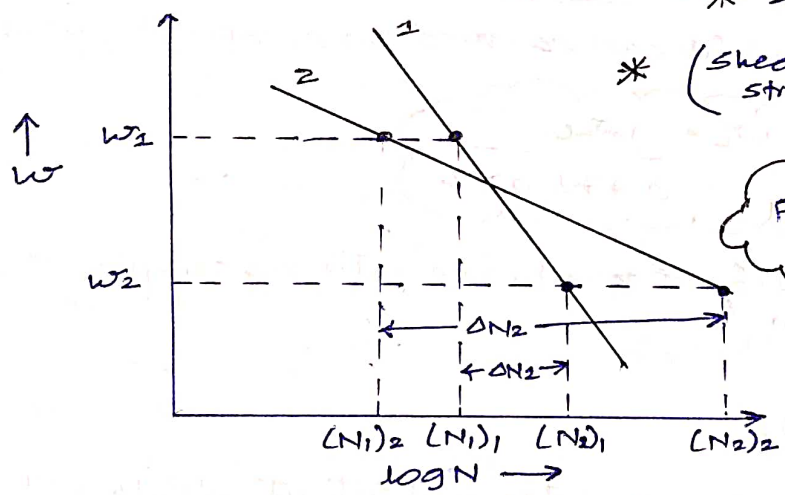
* Flow index = $I_f = \frac{w_1 - w_2}{\log_{10} N_2 - \log_{10} N_1} = \frac{w_1 - w_2}{\log_{10}(N_2/N_1)}$

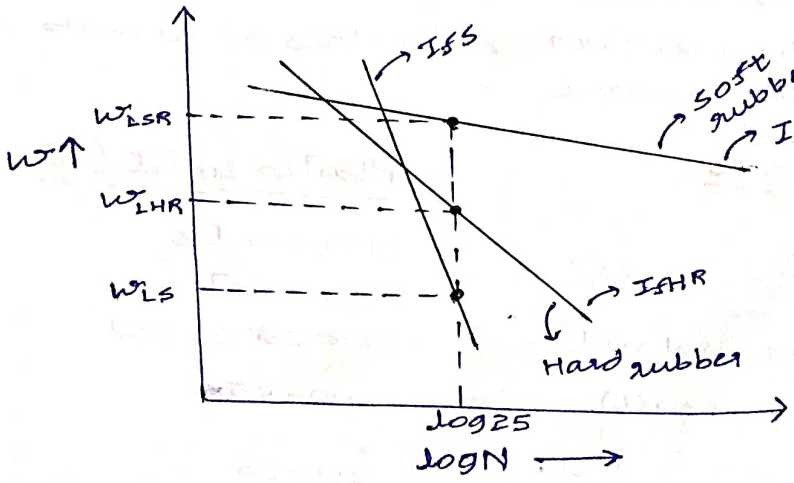
$I_f = w_{10} - w_{100}$

* $I_{f1} > I_{f2}$

* $(\text{Shear strength})_1 < (\text{Shear strength})_2$

Flow index $\propto \frac{1}{\text{Shear Strength}}$





$I_{fHR} > I_{fSR}$

$w_{LSR} > w_{LHR} > w_{LS}$.

** $I_{fS} > I_{fHR} > I_{fSR}$

→ Liquid limit of the soil can also be found by one point method as time req. in the previous method was comparatively more as follows

* $w_L = w_N \left(\frac{N}{25}\right)^{-x}$

$x = 0.058 - 0.121$ (by Mohan)

* If $N = 20$ to 30, $x = 0.1$

w_N = water content corresponding to 'N' no. of blows required to flow the soil by $\frac{1}{2}$ inch distance.

→ Alternatively liquid limit can also be found by IS code method. (IS2720)

$$w_L = \frac{w_N}{1.3213 - 0.23 \log_{10} N}$$

* N = 15 to 35

→ Liquid limit can also be found by "static cone penetration test."

→ It consists of static cone penetrometer having central cone angle of 31° & sliding mass of 80gms.

→ The sample to be tested is filled in cylindrical containers of dia. & ht 5cm each.

→ The cone is lowered upto the top surface of the sample & released.

→ Liquid limit is taken as that water content which allows the penetration of cone into the sample by 20mm.

NOTE: Here also liquid limit can be found by one-point method,

$$w_L = \frac{w_x}{0.77 \log_{10} x}$$

where x = penetration of cone in 'mm' into the sample at water content of 'w'.

b. Plastic limit (w_P):

→ It is defined as minimum water content at which soil is in plastic stage of consistency or behaves like a plastic material.

→ At plastic limit, soil passes from plastic state of consistency to semi-solid state of consistency & vice-versa.

Type of soil	Plastic limit (%)
* Gravel	Non-plastic
* Sand	Non-plastic
* clay (alluvial soil)	25-50
* clay (black soil)	200-250
* Silt	20-25

NOTE: Plasticity is the property of the soil which it to deformed rapidly without elastic rebound, without volume change & without rupture.

→ The plasticity of fine grained soil is due to the presence of charge over the surface of particle & bipolar nature of water.

→ F.G. soil does not become plastic when a non-polarising liquid like Kerosene, naphthalene is added in it.

* Plastic limit determination:

→ From determination pt of view, PL is defined as min. water at which soil just began to crumble or crack when rolled in a thread of 3MM diameter

→ 15 gm airdried sample soil paring through 425μ sieve is mixed with sufficient amount of water so as to make it plastic.

→ A ball is formed with about 8 gm of this plastic soil & is rolled b/w fingers & glass plate into thread of 3MM diameter.

→ The soil is remoulded again into a ball & rolled again into the thread of 3MM ϕ upto an extent it begins to crumble.

→ The crumble thread is tested for its water content & termed as plastic limit (w_p).

→ An avg. of 3 test is considered as final results

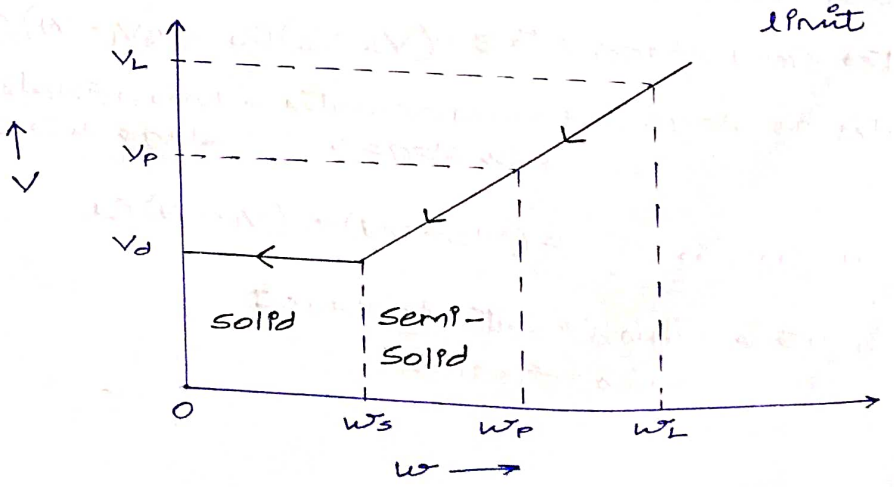
* Shrinkage limit: (w_s):

→ It is defined as maximum water content at which further reduction in the water content of soil does not lead to reduction in the volume of soil, as below shrinkage limit replacement of the water by air in equal volume takes place while reducing the water content.

→ It may also be defined as minimum water content at which soil is completely saturated.

→ At shrinkage limit, soil passes from semi-solid state of consistency to solid state of consistency & vice-versa.

Shrinkage limit $\propto \frac{1}{\text{Volume change}}$



NOTE: Shrinkage limit governs the compressibility of the soil, higher is the shrinkage limit, lower is the compressibility or volume changes

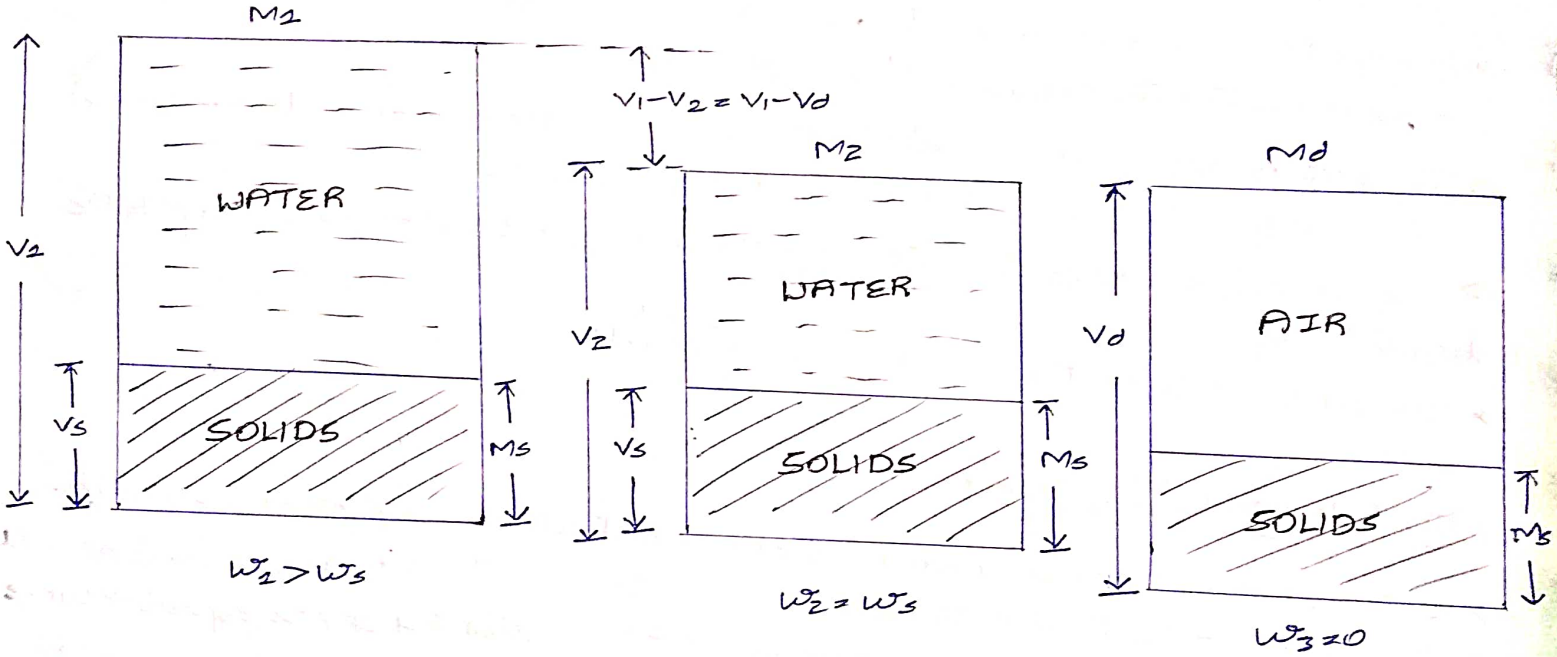
→ For soil to have higher compressibility it should have lower shrinkage limit & higher liquid limit.

* Determination

→ Consider a soil sample having water content more than shrinkage limit let its mass & volume be M_1, V_1.

→ The soil is subjected to drying as a result of which its volume decreases. At a particular stage of drying, when its water content is equal to its shrinkage limit, it mass & volume M_2 & V_2.

→ On complete drying, let its mass & volume be M_d & V_d.



$V_2 = V_d$

$V_2 \neq V_s$

$V_d \neq V_s$

$M_d = M_s$

Mass of water in stage 1 = $M_1 - M_s = M_1 - M_d$

Loss of water from stage 1 to 2 = $(V_1 - V_2)\rho_w = (V_1 - V_d)\rho_w$

Mass of water in stage 2 = Mass of water in stage 1 − Loss of water from stage 1 to 2

" " " " " = $(M_1 - M_d) - (V_1 - V_d)\rho_w$

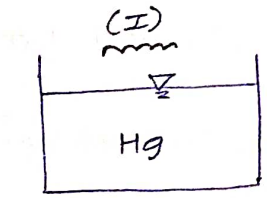
$$w_s = \frac{\text{Mass of water in stage 2}}{\text{Mass of solids}}$$

$$w_s = \frac{(M_1 - M_d) - (V_1 - V_d)\rho_w}{M_d}$$

$$w_s = \left(\frac{M_1 - M_d}{M_d}\right) - \left(\frac{V_1 - V_d}{M_d}\right)\rho_w$$

** $w_s = w_1 - \left(\frac{V_1 - V_d}{M_d}\right)\rho_w$

→ To find shrinkage limit, 30gms of soil passing through 425μ IS sieve is mixed with water content greater than shrinkage limit & following test is performed.



shrinkage dish (SD)

$Hg \longrightarrow M_{Hg}$

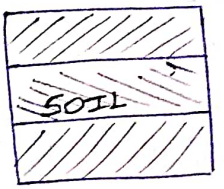
$V_{Hg} \longrightarrow \frac{M_{Hg}}{\rho_{Hg}}$

$V_{Hg} = V_{SD}$

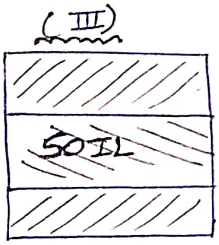
(II)

* M_1 (30gms)

* $V_1 = V_{Hg} = V_{SD}$



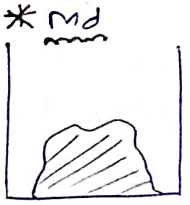
SD

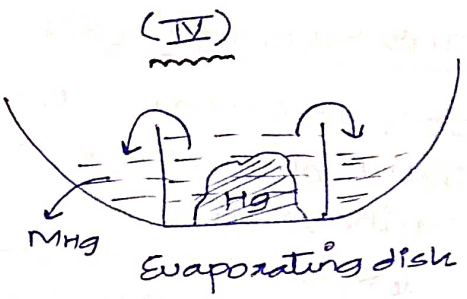


SD

oven dry →

* M_d





Evaporating dish

* Vol. of mercury displaced $= \frac{M_{Hg}}{\rho_{Hg}} = M_d V_d$

* Mass of water in stage 2 $= (V_2 - V_s)\rho_w$

$$w_s = \frac{\text{Mass of water in stage II}}{\text{Mass of solids}} = \frac{(V_2 - V_s)\rho_w}{M_s}$$

$$w_s = \frac{(V_d - V_s)\rho_w}{M_s} = \frac{V_d \cdot \rho_w}{M_s} - \frac{V_s \rho_w}{M_s}$$

$$w_s = \frac{V_d \rho_w}{M_d} - \frac{V_s \rho_w}{M_s}$$

*** $w_s = \frac{\rho_w}{\rho_d} - \frac{\rho_w}{\rho_s}$

$$w_s = \frac{\rho_w}{G\rho_w}(1+e) - \frac{1}{G}$$

$$w_s = \frac{1}{G}(e) = \frac{e}{G}$$

**** $w_s = \frac{e}{G}$

NOTES: $G = \frac{M_s}{V_s \rho_w}$; $M_s = M_d$

Volume of solid (V_s) = $V_1 - \left(\frac{M_1 - M_d}{\rho_w}\right)$

** $G = \frac{M_d \rho_w}{V_1 \rho_w - (M_1 - M_d)\rho_w}$

* spec. gravity can also be found by the data of shrinkage limit test.

** $G = \frac{M_d}{V_1 \rho_w - (M_1 - M_d)}$

→ Shrinkage Index: (I_s):

→ It is the range of consistency in which soil is in semi-solid state of consistency.

→ It is defined as numerical diff. of plastic limit & shrinkage limit.

$$I_s = w_p - w_s$$

→ Shrinkage ratio (R):

→ It is defined as ratio of decrease in volume of soil expressed as a % of its dry volume to the corresponding change in its water content above shrinkage limit.

** S.R. or $R = \frac{\frac{V_1 - V_2}{V_d} \times 100}{w_1 - w_2}$

$w_1 - w_2 = \frac{\text{Change in mass of water}}{\text{Mass of solids}}$

$w_1 - w_2 = \frac{(V_1 - V_2)\rho_w}{M_s} = \frac{(V_1 - V_2)\rho_w}{M_d}$

→ If water content is reduced upto shrinkage limit, $w_2 = w_s$ & $V_2 = V_d$

* SR or $R = \frac{\frac{V_1 - V_d}{V_d} \times 100}{w_1 - w_s}$

$$R = \frac{\frac{V_1 - V_2}{V_d} \times 100}{\frac{(V_1 - V_2)\rho_w}{M_d}} = \frac{M_d}{V_d \rho_w} \times 100$$

$$R = \frac{\rho_d}{\rho_w} \times 100 = G_d \times 100$$

* R = mass specific gravity of soil in its dry state

$$w_s = \frac{\rho_w}{\rho_d} - \frac{\rho_w}{\rho_s} = \frac{1}{R} - \frac{1}{G}$$

***

$$w_s = \frac{1}{R} - \frac{1}{G}$$

**

NOTE: $$w_1 - w_2 = \frac{(V_1 - V_2)\rho_w}{M_s} = \frac{V_1\rho_w}{M_s} - \frac{V_2\rho_w}{M_s} = \frac{V_1\rho_w}{M_d} - \frac{V_2\rho_w}{M_d}$$

→ Volumetric shrinkage (VS):

→ It is defined as decrease in volume of soil expressed as the % of its dry volume when water content is reduced from its given value upto shrinkage limit.

$$VS = \frac{V_1 - V_d}{V_d} \times 100$$

$$R = \frac{\frac{V_1 - V_2}{V_d}}{w_1 - w_2} \times 100$$

If $w_2 = w_s$

$$R = \frac{\frac{V_1 - V_d}{V_d}}{w_1 - w_s} \times 100 \Rightarrow R = \frac{VS}{w_1 - w_s}$$

***

$$V.S. = (w_1 - w_s)R$$

→ Linear shrinkage (LS):

→ It is defined as decrease in one dimension of the soil expressed as % of its original dimension when water content is reduced from its given value upto shrinkage limit.

→ In case of soil, linear dimension is considered as the thickness (as area of soil is infinite).

$$LS = 100\left[1 - \left(\frac{100}{100 + VS}\right)^{1/3}\right]$$

* Plasticity Index (I_P):

→ It is the range of consistency in which soil exhibits plastic properties or behave like plastic material.

→ It is defined as numerical diff. of liquid limit & plastic limit.

$$I_P = w_L - w_P$$

→ LL, PL, and plasticity index are all dependent upon type & amount of clay mineral [In 'I_P', significance of amount is more].

NOTE: Table 1:

IP	Description
0	Non-plastic
<7	Low plastic
7-17	Medium plastic
>17	Highly plastic

* Table 2:

Type of soil	IP
Gravel	0
Sand	0
Silt	10-15
Clay (alluvial)	15-100
Clay (black cotton)	100-250

→ For sand & gravel, plastic limit is not attained hence they are reported as non-plastic soil.

→ If plastic limit of the soil comes equal or greater than liquid limit, I_P is reported as zero.

→ When a non-plastic soil, like sand (I_{P1}) is mixed with plastic soil like clay (I_{P2}), then plasticity index (I_P)$_{mix}$ of mixture is given by

$$I_{P_{mix}} = \frac{I_{P1} \cdot x_1 + I_{P2} x_2}{x_1 + x_2}$$

x_1, x_2 → prop. of soils present

* Consistency index / Relative consistency:

→ In situ behaviour of saturated fine grained soil at its natural water content is represented by consistency index.

→ It is the ratio of difference of liquid limit & natural water content to the plasticity index.

$$I_c = \frac{w_L - w_N}{I_P} = \frac{w_L - w_N}{w_L - w_P}$$

Case 1: If $w_N > w_L$ → Liquid, $I_c < 0$

Case 2: If $w_N < w_L$ → soil is in solid or semi-solid state of consistency,

$I_c > 1$

Case 3: If $w_P < w_N < w_L$ → Plastic state of consistency.

$0 \leq I_c \leq 1$

* Liquidity index (I_L)/water plasticity ratio:

→ Its significance is same as that of consistency index.

→ It is defined as the ratio of diff. of natural water content (w_N) & plastic limit to the plasticity index.

$$I_L = \frac{w_N - w_P}{I_P} = \frac{w_N - w_P}{w_L - w_P}$$

$$I_c + I_L = \frac{w_L - w_N}{w_L - w_P} + \frac{w_N - w_P}{w_L - w_P}$$

$$I_c + I_L = 1$$

→ Case 1: If $w_N > w_L$ → Liquid, $I_L > 1$

→ Case 2: If $w_N < w_P$ → solid or semisolid, $I_L < 0$

→ Case 3: If $w_P < w_N < w_L$ → Plastic, $0 \leq I_L \leq 1$

NOTE:

Consistency stage	Description	I_c	I_L	UCS (kg/cm^2)
Liquid	Liquid	<0	>1	—
Plastic	Very soft	0-0.25	1-0.75	0-0.25
	Soft	0.25-0.5	0.75-0.5	0.25-0.50
	Medium stiff	0.5-0.75	0.5-0.25	0.5-1
	Stiff	0.75-1	0.25-0	1-2
Semi solid	Very stiff to hard	>1	<0	2-4
Solid	Hard to very hard	>1	<0	>4

→ Unconfined comp. strength (UCS): It is defined as the load per unit area at which unconfined specimen of soil of std dimension fail in simple compression test.

* Toughness index (I_T):

→ It represents the shear strength of the soil at its plastic limit.

→ It is defined as ratio of plasticity index & flow index.

$$I_T = \frac{I_P}{I_f}$$

→ Generally, $0 \leq I_T \leq 3$ but if "$I_T < 1$", soil is considered to be easily

friable (crumble) at plastic limit

NOTE: a. Compressibility / Volume change / shrinkage → w_L, w_s, I_s

b. Plasticity → w_p, I_p

c. Rate of loss of shear strength → I_f
or shear strength

d. In situ behaviour of fine grained soil → I_c, I_L

e. Shear strength at plastic limit → I_T

→ General relationship b/w Atterberg's limit & engg. properties:

Characteristics	Comparing soil at equal I_A w_L with I_p inc.	Comparing soil at equal I_p with w_L increases
* Dry strength	Increases	Decreases
* Toughness index near w_p	Increases	Decreases
* Compressibility	almost constant	Increases
* Permeability	decreases	Increases
* Rate of volume change	decreases	Increases

Q. Soil 1

$w_p = 25\%$

$w_L = w_N \left(\frac{N}{25}\right)^x$

$w_L = \frac{40}{100}\left(\frac{20}{25}\right)^{0.1}$

$w_L =$

Soil 2

$w_p = 30\%$

$\frac{30-25}{25-20} = \frac{37-x}{x-39}$

$5x - 195 = 185 - 5x$

$w_{LA}\% = 38\%$

$I_{fA} = 11.35$

$I_{PA} = 13\%$

$w_{LB} = 60\%$

$I_{fB} = 6.64$

$I_{PB} = 30\%$

→ Soil 'B' is more plastic than 'A'

$I_{CA} = \frac{25-40}{13} < 0$

$I_{CB} = \frac{60-50}{30} = 0.33$

→ Soil 'B' is better for remoulding

→ Soil 'B' is more compressible than 'A'

→ $(I_{fA}) > (I_{fB})$ → Rate of loss of shear strength is more in soil 'A'

→ $(I_{fA}) = 1.14$ $(I_{fB}) = 4.51$

* soil 'B' has higher strength at plastic limit

**

Q.

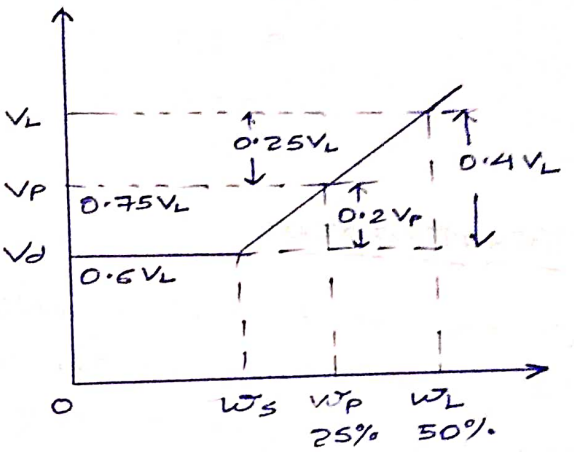
$V_d = V_L - 0.4V_L = 0.6V_L$ —①

$V_d = V_P - 0.2V_P = 0.8V_P$ —②

$0.6V_L = 0.8V_P$

$V_L = \frac{4}{3}V_P$ —③

$V_P = 0.75V_L$ —③



$\frac{w_L - w_s}{0.4V_L} = \frac{50-25}{0.25V_L}$

$50 - w_s = \frac{25 \times 0.40}{0.25}$

$w_s = 10\%$

$R = \frac{\frac{V_P - V_d}{V_d}}{w_P - w_s} \times 100 = \frac{\frac{0.75-0.6}{0.6}}{15} \times 100 = \frac{0.15}{0.6 \times 15} \times 100$

$R = 1.67\%$

f. Sensitivity:

→ consistency of undisturbed sample of clay may change upon remoulding even at the same water content

→ This change in consistency or decrease in degree of firmness or decrease in shear strength takes place due to following reasons.

a. due to permanent destruction of soil solids upon remoulding (due to breaking of intermolecular bonds) {not recoverable}

b. due to reorientation of the water molecule in the adsorbed layer of soil solids {recoverable}

→ This loss in strength is represented in terms of its sensitivity which signifies the degree of disturbance of soil sample upon remoulding.

→ It is defined as ratio of UCS in undisturbed/natural state to UCS of soil in remoulded state. eg: Piling in clay

$$S = \frac{(UCS)_{undisturbed}}{(UCS)_{remoulded}}$$

Sensitivity	Description
<1	In-sensitive (Honeycomb)
1-4	Less sensitive/normal sensitive
4-8	Sensitive (Flocculant/H.C)
8-16	Extra-sensitive (Flocculant)
>16	Unstable/quick

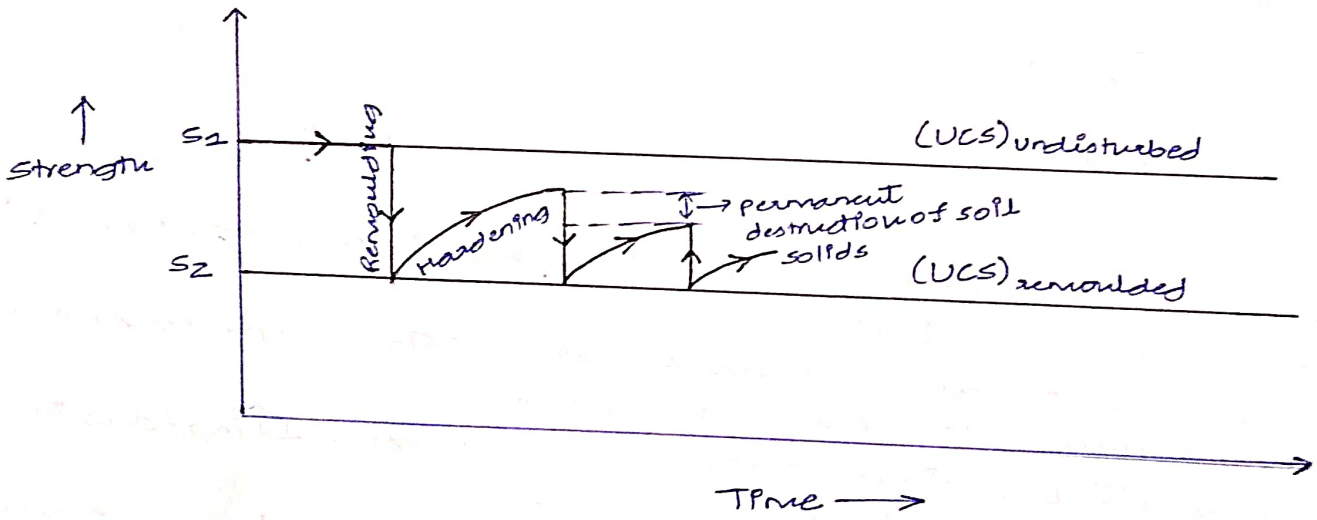
g. Thixotrophy:

→ Over a period of time, soil regains a part of the lost strength on the account of remoulding due to rehabilitation of water molecules in the adsorped layer of soil to attain chemical equilibrium

→ This property of soil (FG) by virtue of which it attains or regains a part of its lost strength is termed as thixotrophy.

→ Higher the sensitivity of soil, higher is its thixotropic hardening.

→ This property of soil plays an important role where construction activities involves remoulding of natural clay deposits eg: piling



h. Activity:

→ The behaviour or plasticity of soil depends upon type of mineral, amount of mineral & amount of water present in the adsorbed layer of the soil.

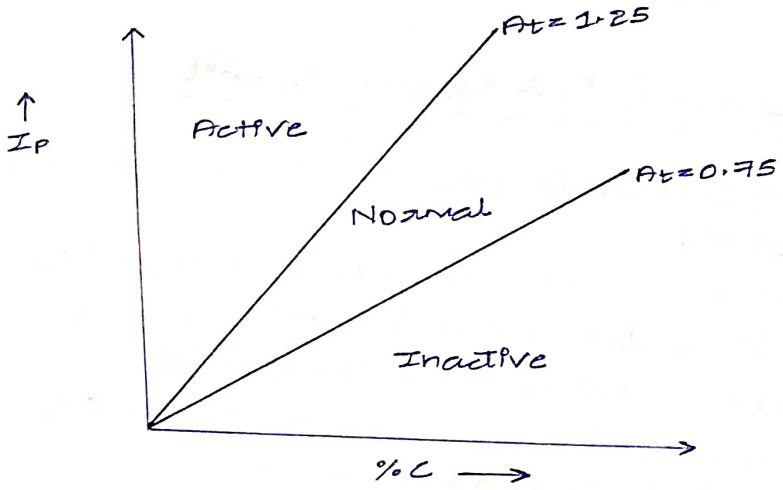
→ Skempton defined a parameter termed as activity which represents the compressibility or volume changes in soil with change in water content.

→ It is defined as ratio of plasticity index & percentage of the particles present in it having sizes less than 2μ (clay size)

$$A_t = \frac{I_p}{C}$$

C = % of particles finer than 2μ

Activity	Description
< 0.75	Inactive
0.75 – 1.25	Normal
> 1.25	Active



Clay Mineral	Activity (A_t)
Kaolinite	0.4 – 0.5
Illite	0.5 – 1
Montmorillonite	1 – 7
Na – Montmorillonite	4 – 7 (7.2)
Ca – Montmorillonite	1.5

NOTE: Highly active soils are not suitable for construction activities eg: Black cotton soil.

9. Collapsibility:

→ The soil which shows large decrease in its volume due to increase in its water content without any increase in external pressure or load are termed as collapsible soil. eg: Loess soil, sand dunes, fine sand & silt slightly cemented due to the presence of calcium.

→ Collapsibility of the soil is measured in terms of the parameter collapse potential (CP), which is defined as ratio of decrease in volume of soil to its original volume with increase in water content.

$$CP = \frac{\Delta V}{V_o} = \frac{\Delta H}{H_o} = \frac{\Delta e}{1+e_o}$$

→ All collapsible soils are characterised with bulky coarse grained solids loosely held by capillary forces.

Collapse potential (CP)	Severity of problems
0-1%	No problem
1-5%	Moderate problem
5-10%	Trouble
10-20%	Severe trouble
>20%	Very severe trouble

→ Difference b/w organic & inorganic soil

→ Liquid limit of inorganic soil not of volcanic origin rarely has value greater 100%.

→ Organic soil have generally high value of liquid limit (>50%) but their plastic limit is also comparatively high, hence their plasticity index value are not as high as its liquid limit

→ For organic clays, liquid limit of oven dried sample is 0.7 times less than the liquid limit of air dried sample.

CLASSIFICATION OF SOIL

→ Classification of soil is done in order to arrange the soils into diff. groups on basis of its engg. properties,

→ It can be done by any of the following method

a. **Particle size classification**: In this method, soil is classified on the basis of its particle size composition & is termed as Clay, Silt, Sand, Gravel, Pebble & Boulder.

a. **Clay**: $d < 2\mu$

b. **Silt**: $2\mu \leq d < 75\mu$

→ Fine silt $2\mu \leq d < 10\mu$

→ Medium silt $10\mu \leq d < 20\mu$

→ Coarse silt $20\mu \leq d < 75\mu$

c. **Sand**: $75\mu \leq d < 4.75\,mm$

→ Fine sand $75\mu \leq d < 0.425\,mm$

→ Medium sand $0.425\,mm \leq d < 2\,mm$

→ Coarse sand $2\,mm \leq d < 4.75\,mm$

d. **Gravel**: $4.75\,mm \leq d < 80\,mm$

→ Fine gravel $4.75\,mm \leq d < 20\,mm$

→ Coarse gravel $20\,mm \leq d < 80\,mm$

e. **Cobble**: $80\,mm \leq d < 300\,mm$

f. **Boulder**: $d \geq 300\,mm$

b. **Textural chart classification**:

→ In this method, classification of soil is done on the basis of relative % of clay, silt & sand present in the soil, which is further read from the textural chart.

→ This method is suitable for describing the coarse grained soil rather than clay soils prop. of which are less dependent upon particle size distribution.

→ To use the chart, for the given % of three constituents forming a soil, lines are drawn parallel to the three sides of an equilateral Δ & their common pt of intersection is noted to fall in a particular region of the textural chart.

C. Highway Research Board classification system:

→ It is also termed as Public Road Administration classification (PRA) system, which is generally used for classification of soils used for pavement construction.

→ In this method, soil is divided into 7 main groups, which is further divided into several subgroups.

→ In this method, a "characteristic group index" is used to describe the performance of the soil when used for pavement construction, however, it is not used to place the soil in particular group.

→ The higher the value of "group index" (GI), poorer is the quality of the material used for the pavement construction.

→ Group index (GI) depends upon

P. Liquid limit (w_L)

PP. Plastic limit (w_P)

PPP. % of the particle paring through 75μ sieve

$$GI = 0.2a + 0.01bd + 0.005ac$$

a = portion of % paring through 75μ sieve greater than 35 but not exceeding 75. expressed as a whole number b/w '0-40'

b = portion of % paring through 75μ sieve greater than 15 but not exceeding 55, expressed as a whole no. b/w '0-40'

c = portion of numerical liquid limit greater than 40 but not exceeding 60 expressed as whole number b/w '0-20'.

d = portion of numerical plasticity index greater than 10 but not exceeding 10 expressed as whole number b/w '0-20'

GI	Quality of material
0-4	Excellent to good
4-20	Average to poor or fair

NOTE: When $a=b=c=d=0$, $GI_{min}=0$

When $a=b=40$, $c=d=20$, $GI_{max}=20$

Q. The GI for a soil whose liquid limit = 40, & I_P = 10%, % paring 75μ IS sieve is 35.

$a=0 = 35-35$ $b=35-15=20$

$c=0 = 40-40$

$d=0 = 10-10$

$GI = 0$

Q. consider a soil sample for which test yield the following results.

% Paning @ 75μ sieve = 62%

$w_L = 35\%$

$I_p = 14\%$

As per the group index classification of soil,

$a = 62 - 35 = 27\%$

$b = 62 - 15 = 47\% = 40\%$

$c = 0$

$d = 14 - 10 = 4\%$

$GI = 7$ (POOR)

d. Indian soil classification system:

→ In this system, classification of soil is done on the basis of

i. Particle size distribution

ii. Plasticity characteristics

iii. Compressibility (pt's)

→ In this method, soil is classified in broadly 2 categories. (step 1)

a. coarse grained soil

b. Fine " "

a. Coarse grained soil:

→ A soil is termed as C.G. soil if 50% or more ($\geq 50\%$) soil fraction are greater than 0.075MM or are retained on 0.075 MM sieve

→ Here C.G. soil is further classified into (step 2)

a. Gravel (G)

b. Sand (S)

→ If more than 50% or more ($\geq 50\%$) of coarse fraction (having size greater than 75μ) have size greater than 4.75MM ϕ is retained on 4.75MM sieve, it is termed as gravel or else termed as sand.

→ If Gravel fraction ($d \geq 4.75MM$) $\geq$ Sand fraction ($4.75MM < d \leq 75\mu$)

soil is termed as Gravel (G)

→ If sand fraction > Gravel fraction

($4.75MM < d \leq 75\mu$) ($d \geq 4.75MM$)

soil is termed as sand.

→ In this method, CG soil is further classified on the basis of % fineness (% of the particles finer than 0.075MM are termed as % fineness) (step 3)

→ Case a: When % fineness < 5% (we indicate gradation of soil)

* G
  - → GW: Well graded gravel ($C_u > 4$, $1 \leq C_c \leq 3$)
  - → GP: Poorly graded gravel ($C_u \neq 4$, $C_c < 1$ & $C_c > 3$)

* S
  - → SW: Well graded sand ($C_u > 6$, $1 \leq C_c \leq 3$)
  - → SP: Poorly graded sand ($C_u \neq 6$, $C_c < 1$ & $C_c > 3$)

→ Case b: When % fineness > 12 (we indicate type of fine grained soil also)

* G
  - → GC: Clayey gravel ($I_P > 7$ or above A-line) — C > M
  - → GM: Silty gravel ($I_P < 4$ or below A-line) — M > C
  - → GM-GC: Silty clayey gravel ($4 \leq I_P \leq 7$) — C = M

* S
  - → SC: Clayey sand ($I_P > 7$ or above A-line) — C > M
  - → SM: Silty sand ($I_P < 4$ or below A-line) — M > C
  - → SM-SC: Silty clayey sand ($4 \leq I_P \leq 7$) — M = C

→ Case c: If % fineness is in between "5 to 12" (we indicate both gradation of fineness of soil also) (Dual notation is adopted)

* Gravel: G > S

GW-GC: Well graded gravel with clay ($C_u > 4$, $1 \leq C_c \leq 3$, $I_P > 7$)

GP-GC: Poorly graded gravel with clay ($C_u \neq 4$, $C_c < 1$ or $C_c > 3$, $I_P > 7$)

GW-GM: Well graded gravel with silt ($C_u > 4$, $1 \leq C_c \leq 3$, $I_P < 4$)

GP-GM: Poorly graded gravel with silt ($C_u \neq 4$, $C_c < 1$ or $C_c > 3$, $I_P < 4$)

* Sand: S > G

SW-SC: Well graded sand with clay ($C_u > 6$, $1 \leq C_c \leq 3$, $I_P > 7$)

SP-SC: Poorly graded sand with clay ($C_u \neq 6$, $C_c < 1$ or $C_c > 3$, $I_P > 7$)

SW-SM: Well graded sand with silt ($C_u > 6$, $1 \leq C_c \leq 3$, $I_P < 4$)

SP-SM: Poorly graded sand with silt ($I_P < 4$, $C_u \neq 6$, $C_c < 1$ or $C_c > 3$)

NOTE: If % fineness is in between 5 to 12% & I_P of the soil is in b/w "4 to 7" in such cases non-plastic classification is favoured.

b. Fine Grained Soil:

→ A soil is termed as fine grained soil, if more than 50% (>50%) of soil have size smaller than 0.075 mm (or passes through 0.075mm sieve) (Step 1)

→ F.G. soil is further classified into (Step 2)

i. Clay (C)
ii. Silt (M)
iii. Organic soil (O)

→ F.G. soil are further classified as on the basis of their compressibility also. (Step 3)

i. Highly compressible soil (H): when $w_L > 50\%$
ii. Intermediate compressible soil (I): $w_L \geq 35\%$ & $w_L \leq 50\%$
iii. Low compressible soil (L): $w_L < 35\%$.

where classification can be

CH: High compressible clay
CI = Intermediate comp. clay
CL: Low compressible clay

MH: High compressible silt
MI: Intermediate comp. silt
ML: Low compressible silt

OH: High compressible organic soil
OI: Intermediate compressible organic soil
OL: Low compressible organic soil

→ In order to separate, inorganic clay from silt & organic soil, A-casagrande defined a line termed as "A-line" which shows relationship b/w plasticity index & liquid limit of the soil

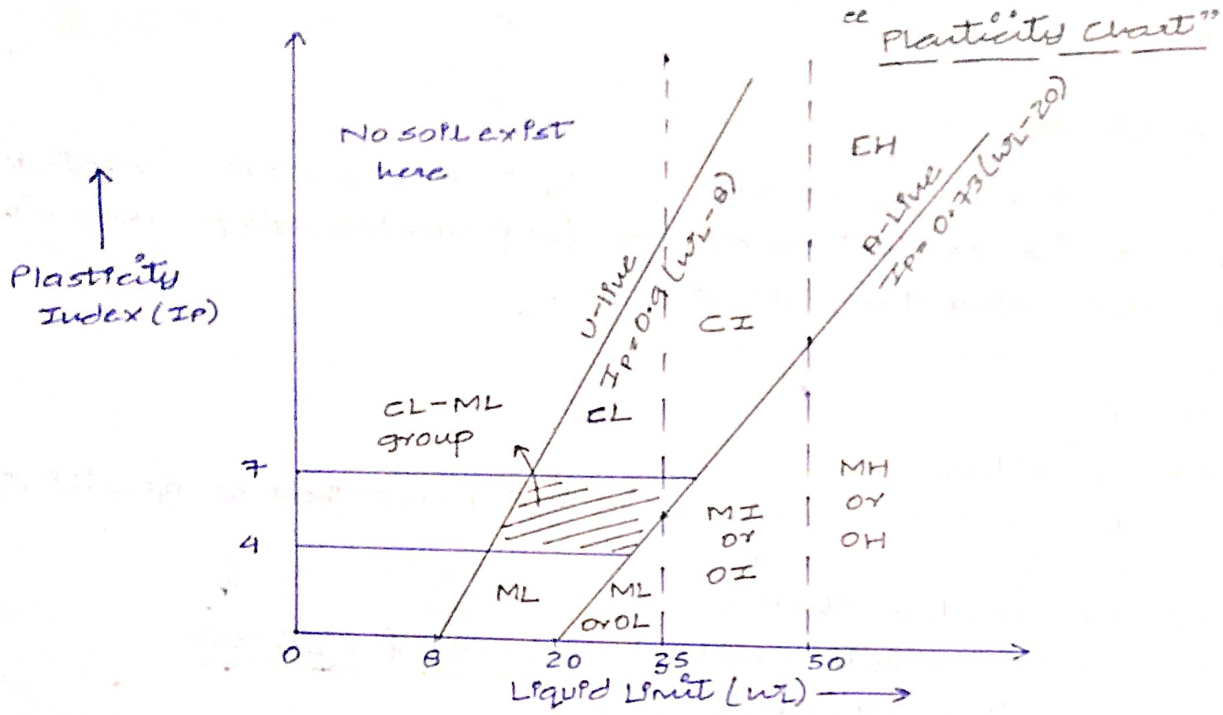
→ Soil above A-line is "clay" & below A-line is "silt" or "Organic soil".

→ Eq^n of A-line, $I_P = 0.73(w_L - 20)$

→ In order to mark the presence of upper limit of soil, a line termed as "U-line" is defined which also shows the relationship b/w 'I_P' & 'w_L'.

→ No soil exists above U-line.

→ Eqn of U-line, $I_p = 0.9(w_L - 8)$



Q. A soil mass contain 40% gravel, 50% sand & 10% silt, these soil can be classified as

a. silty sand gravel having $C_u < 60$

b. silty gravelly sand having $C_u = 10$

✓ c. gravelly silty sand having $C_u > 60$

d. gravelly silt sand, where C_u cannot be determined.

			cumulative % wt retained
Gravel	40%	4.75mm	40%
Sand	50%	0.075mm	90%
Silt	10%	0.002mm	10%

$$C_u = \frac{D_{60}}{D_{10}} = \frac{4.75}{0.075} = 63.33$$

$D_{10} = 0.075mm$

$D_{60} = 4.75mm$

Q. Sieve analysis on a dry soil sample of mass 100gm showed that 980gm & 270gm of soil pass through 4.75mm & 0.075mm sieve resp. The liq. limit & Ip of soil passing through 425μ sieve are 40% & 18% respectively

		cumulative % wt retained	% finer
20gm ← 2000gm	4.75mm	2%	98%
730gm	0.075mm	73%	27%

$I_p = W_L - W_p = 40 - 18 = 22\%$

Clayey sand (SC)

Q. Sieve analysis is done on a sample of soil & following observations were made

Sieve size	% retained	% finer
4.75mm	36	
75μ	90	10% ("5-12")

Size of particle for which 60% particles are finer = 5mm
" " " " " 30% " " " = 3mm
" " " " " 10% " " " = 1mm

On further testing the finer particles, it was found, $W_L = 50\%$, $W_p = 35\%$.

$C_c = \frac{9}{5} = 1.8$ $\quad C_u = \frac{5}{1} = 5$

$I_p = 50 - 35 = 15\%$

$I_p = 0.73(50 - 20) = 21.9 > 15$

Hence below A-line, split

"SP-SM soil"

Q. The results of laboratory test conducted on the 2 soil samples 'A' & 'B' are are as follows.

Characteristics	% passing 75μ	% passing 4.75mm	D_{10}	D_{30}	D_{60}	W_L	W_p
Soil A	14	92	0.14	0.33	1.0	16	8
Soil B	75	75	—	—	—	58	24

Soil A (SC soil) $\quad C_u = \frac{D_{60}}{D_{10}} = \frac{1}{0.14} = 7.14$ $\quad$ Soil B (CH)

$I_p = 8\%$

A line = 27.74
$I_p = 44$

$I_p >$ A-line
Above A-line (clay)

SOIL STRUCTURE

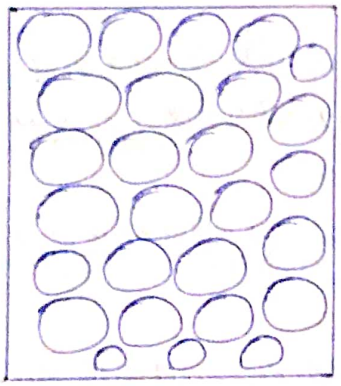
→ The arrangement of soil particles in the soil per mass is termed as structure of soil.

→ A given sample of soil may have any of the following structure

a. Coarse grained structure / Single grained structure

→ This type of str. is formed for the soil having particle size > 0.02 mm i.e. Gravel, sand, coarse silt.

→ In the formation of these structures, gravitational force plays predominant role & surface electrical force are insignificant as the result of which soil particles when settles down in suspension under the effect of their own wt tends to come in contact with other particles leading to particle to particle contact.



Coarse / Single grained structure

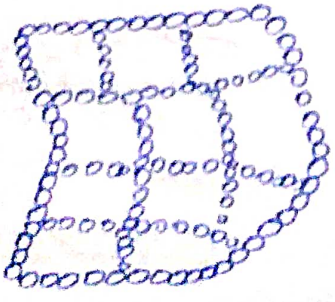
b. Honeycomb structure:

→ This type of structure is found for the soil having particle size in the range (0.2 mm - 0.02 mm) i.e. for silt & clay.

→ In the formation of these str. both gravitational force & surface electrical forces plays equally important role, as a result of which when the particles settles under the effect of their own weight in suspension, surface electrical forces holds them in contact with each other in the form of clusters of honeycomb enclosing large volume of voids b/w them.

→ If the str. is unbroken it possess high strength & permeability but once it is broken both strength & permeability decreases.





Honeycomb structure

c. Flocculant & dispersed structure:

→ This type of structure is formed if size of particle is < 0.2μ fine clays & colloids.

→ In the formation of these structures, surface electrical forces plays the predominant role & gravitational force are insignificant.

→ The particles of fine grained soil in which this type of structure is formed are flaky in nature & carries surface electric charge which is generally positive at the edges & negative at the centre.

* Flocculant structure: If the net force during the deposition b/w these particles are attractive, they are aligned into head to head or head to face orientation. termed as flocculant structures.

→ These structures are found to have high strength & low compressibility & high permeability.

* Dispersed structure: If the net forces b/w these particles is repulsive during deposition, they are aligned into face to face orientation termed as Dispersed structure.

→ These str's are found to have low strength & ~~high~~ low permeability & ~~low~~ high compressibility.

NOTE: Tendency of formation of flocculant structure increases

a. With increase in

i. concentration of electrolyte

ii. valency of ion

iii. temp.

b. Increases with decrease in

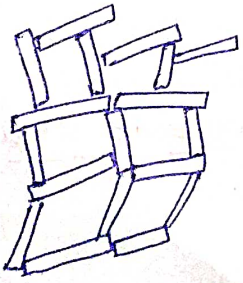
i. dielectric constant of the medium

ii. size of ions

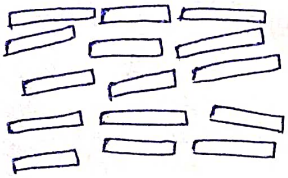
iii. pH

iv. anion adsorption

→ Hence eg. marine clay is found to have flocculant str. & lacustrine clay is found to have dispersed structure.



Flocculant structure



Dispersed structure

* Structure of composite soil:

→ Structure of composite soil depends upon relative proportion of coarse and fine grained particles in it.

→ According to which, these are classified as

a. Coarse grained skeleton structure

b. Cohesive matrix

a. Coarse grained skeleton structure:

→ This structure is formed when relative % of coarse grained particle is more than fine grained particles.

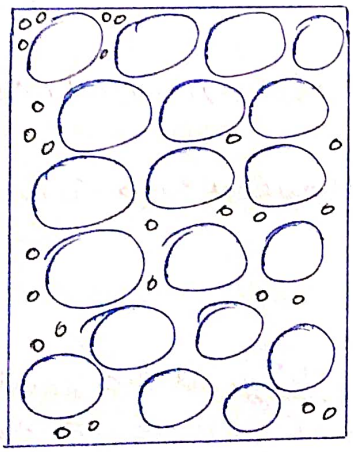
→ Voids of coarse grained str. are filled with fine grained particle.

→ In this case, coarse grained particles acts as load bearing members & results in formation of low compressible soil.

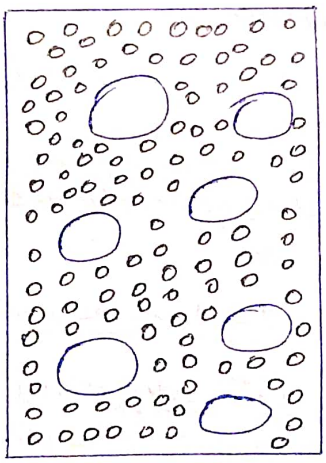
b. Cohesive matrix:

→ This structure is formed when relative % of fine grained particle is more than coarse grained particles, due to which now coarse grained particles are not in contact with each other.

→ In this case, fine grained particles acts as load bearing members & results in formation of high compressible soil.



Coarse grained skeleton structure



Cohesive matrix

CLAY MINEROLOGY

→ The clay material are basically composed of tiny crystalline substances of one or more members of a small group of minerals termed as clay minerals.

→ These clay minerals are evolved mainly from the chemical weathering of certain rocks forming minerals

→ These minerals are hydrous aluminosilicates with other metallic ions.

→ These clay minerals on the basis of their crystalline arrangement can be divided into following groups

Name of Mineral group		Structural formula
1. Kaoline group		
a. Kaolinite	} 1:1 Mineral }	$(OH)_4Al_4Si_4O_{10}$
b. Halloysite		$(OH)_4Al_4Si_4O_{10} \cdot 4H_2O$
2. Montmorillonite group		
a. Montmorillonite	} 2:1 Mineral }	$(OH)_4Al_4Si_xO_{20} \cdot nH_2O$
3. Illite group		
a. Illite	} 2:1 Mineral }	$(OH)_4K_4(Si_{x-4}Al_4)Al_4Mg_5Fe_4$

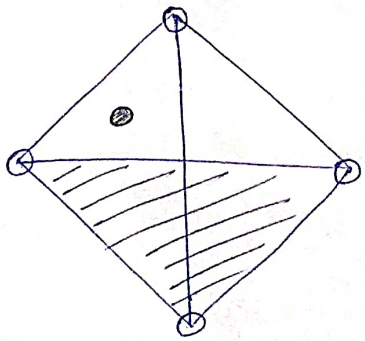
→ All the clay minerals are found to have following into fundamental building blocks

a. Silica tetrahedral unit:

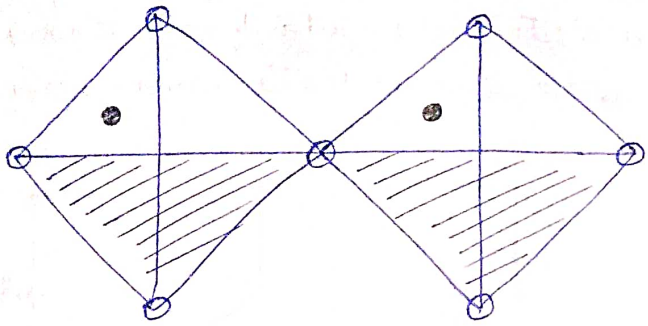
→ In this unit, four oxygen atoms encloses silica at the centre of the tetrahedron geometry with all the oxygen at the base of the tetrahedron lying in same common plane & is shared by 2 tetrahedral units.

→ charge present over tetrahedron unit is '−1'

→ It is symbolically represented as trapezium.



● → Silicon
O → oxygen



b. Octahedron unit (Octahedral):

→ In this unit, 6 hydroxyl atom encloses Al, Mg or Fe at the centre of octahedral geometry.

→ If 'Al' is at the centre, it is termed as "GIBBSITE UNIT."

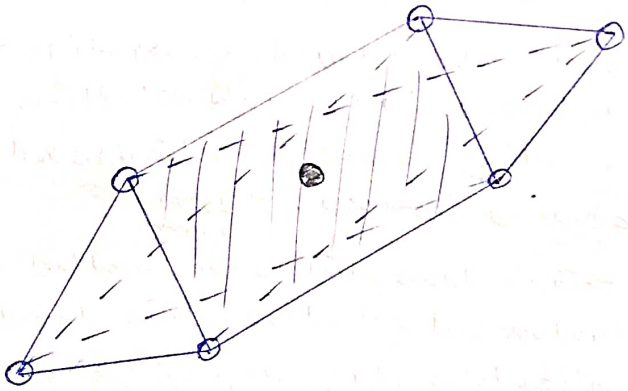
→ If 'Fe' is at centre, it is termed as "FERRITE UNIT"

→ If 'Mg' is at centre, it is termed as "BRUCITE UNIT".

→ Net charge present over Gibbsite unit is "(+)1"

→ It is symbollically represented by rectangle.

→ Here each hydroxyl atom is shared b/w 3 units



→ These 2 fundamental building blocks joins with each other in different geometries resulting in the formation of diff. clay minerals as follows

a. Kaolinite: (1:1 Mineral)

→ The structural unit is formed when one octahedral unit combines with silica tetrahedral unit through the oxygen at the apex of silica tetrahedron unit having thickness of 7Å ($1 Å = 10^{-10} m$).

→ Several such units are joined by each other by strong hydrogen bond which is relatively difficult to displace off, leading the formation of soil having comparatively low swelling + shrinkage properties.

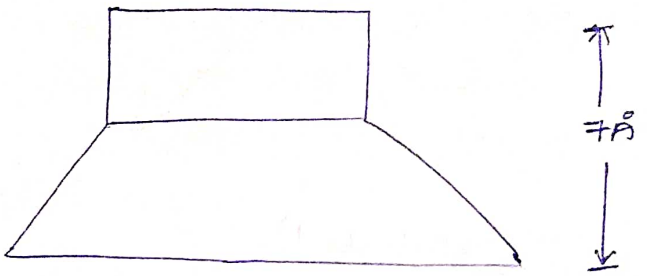
The total thickness of these layers varies b/w 50-2000nm ($1nm = 10^{-9} m$)

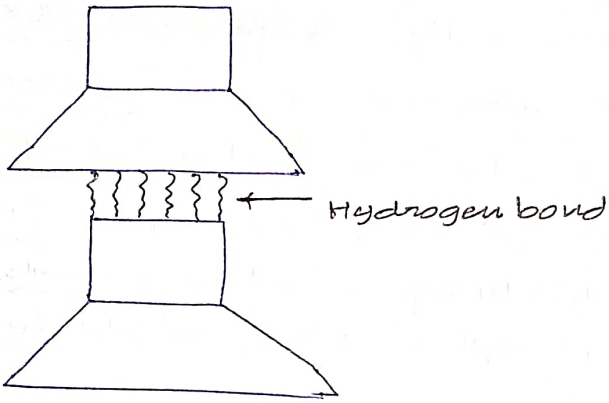
**

→ China clay is the purest form of Kaolinite

→ Its plasticity index is approximately 15.

→ Kaolinite mineral is electrically neutral but it may show negative charge in the presence of water or due to isomorphous substitution.





b. Montmorillonite (smectite) {2:1 mineral}

→ It is the most common mineral of clay whose structure is formed when one of octahedral is sandwiched b/w two units of silica tetrahedron units through oxygen present at the apex of silica tetrahedron unit resulting in the structure having thickness of 10Å

→ In this case, silica in tetrahedron unit is replaced by aluminium in some cases resulting in development of net negative charge, which is balanced by any of the cations like Na^+, Ca^{2+}, Mg^{+2}, K^+ present in the water/mineral.

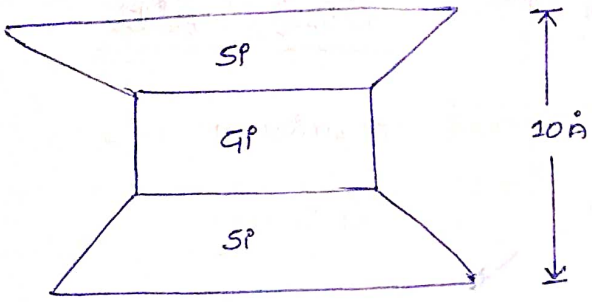
→ These structural changes in which replacement of one atom by another atom takes place is termed as "Isomorphous substitution"

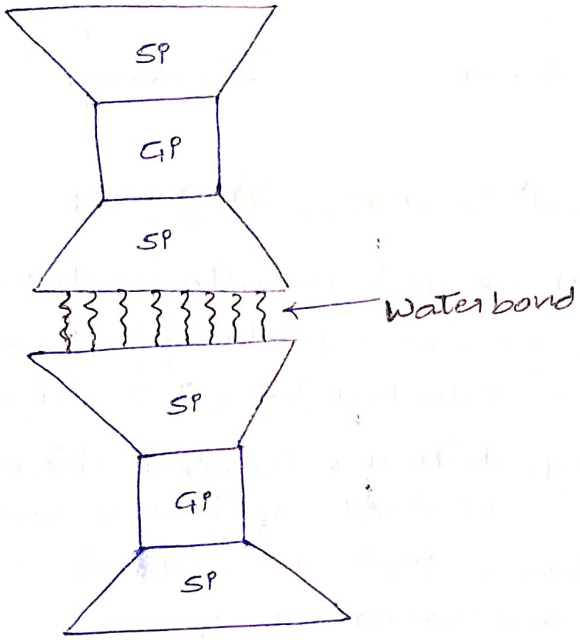
→ several such structural units of montmorillonite combines with each other or are joined with each other with weak water bond that is easily displaced off hence soil having montmorillonite mineral shows excess swelling & shrinkage characteristics with change in water content.

→ Total thickness of these units may extend upto 3nm.

→ Plasticity index of montmorillonite is approx. 300.

→ Black cotton soil is found to have excess of montmorillonite





c. Illite {2:1 minerals}

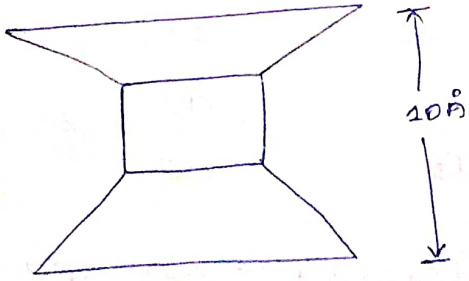
→ The structure of illite is same as that of montmorillonite with the only diff. that substantial (>20%) replacement of silica in tetrahedron unit by alumina takes place & negative charge developed is compensated by K^+ ions.

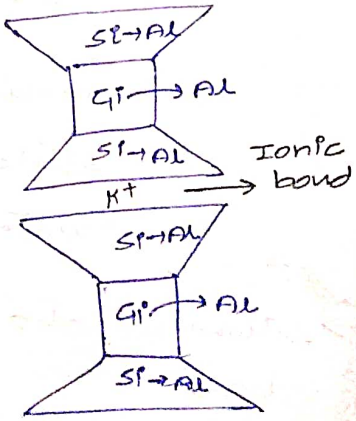
→ Several such structural units are joined with each other by ionic bond of potassium which is stronger than water bond of montmorillonite but weaker than hydrogen bond of kaolinite. Hence soil having this mineral shows intermediate swelling & shrinkage characteristics

→ The total thickness of these structural units is approx. 30nm.

→ It's plasticity index is in range of 30.

→ Laterite soil has excess of illite present in it.





Properties	Kaolinite	Montmorillonite	Illite
* size grain	Maximum	Minimum	All Intermediate
* Swelling & shrinkage behaviour	Minimum	Maximum	Intermediate
* Plasticity Index	Minimum	Maximum	Intermediate
* Dry strength	Minimum	Maximum	Intermediate

NOTE: BASE (CATION) EXCHANGE CAPACITY:

→ The surface atoms are predominantly negative like [oxygen (-2), hydroxyl (-1)] charged, whereas metallic ions (Al^{+3}, Si^{+4}) which occupy central position in tetrahedral & gibbsite unit are positive.

→ Thus, although the particle as a whole is electrically neutral but its surface can be considered to be strongly negatively charged.

→ In case of Kaoline, isomorphous substitution may also lead to development of negative charge on the surface.

→ Positive ions or cations in the water are attracted towards the surface of these clay particles to make them electrically neutral.

→ The ability of clay particles to absorb ion or cation on its surface or edges is called its base or cation exchange capacity.

→ This prop. is function of mineral structure & the size of the particle.

→ This base exchange capacity is measured/expressed in terms of weight of cations which may be held over the surface of 100gms of dry soil.

→ It is expressed as meq/100gm of dry soil

Here meq = 1 mg of hydrogen or any other cation which can displace 1mg of hydrogen

Mineral	Base exchange capacity (meq/100gm)
Kaolinite	3-8
Illite	40
Montmorillonite	80

***

→ Cations can also be placed in order of their replacement ability as follows

$Li^{+} < Na^{+} < H^{+} < K^{+} < NH_4^{+} < Mg^{+2} < Ca^{+2} < Al^{+3}$

Q. Determine the base exchange capacity of clay if 100gm of dry clay soil adsorbs 160mg of calcium. Find the quantity of hydrogen that can be adsorbed by $10M^3$ of this clay if $\rho_d = 1500 kg/M^3$.

BEC → Meq. of $Ca^{+2} = \frac{160}{\frac{40}{2}} = 8 meq/100gm$ of dry soil

100gm of soil can adsorb 8mg of hydrogen over its surface

Mass of $10M^3$ of dry soil $= 1500 \times 10 = 15000kg = 15000 \times 10^3$

$100gm \longrightarrow 8mg$ hydrogen

$1gm \longrightarrow \frac{8 \times 10^{-3}}{100}$

$15000 \times 10^3 \longrightarrow \frac{8 \times 10^{-3}}{100} \times 15 \times 10^6 = \frac{120 \times 10^3}{100} = 1200gm$

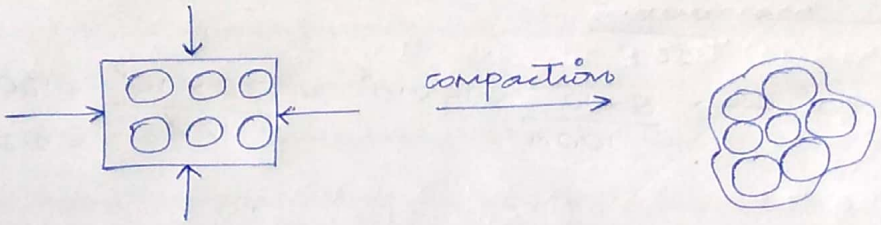
$= 1.2kg$ of H

COMPACTION OF SOIL

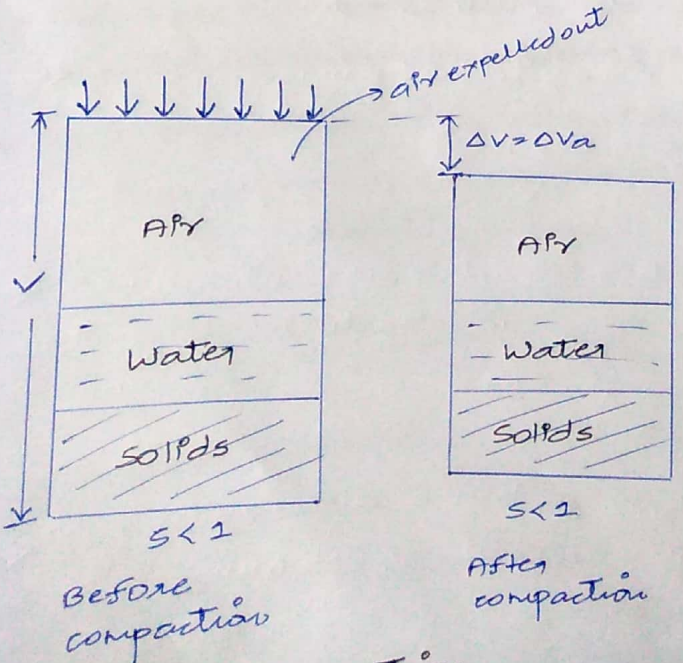
→ Compaction is the process of artificially rearranging of soil particles & packing them together into closer state of contact by mechanical means (rammer, roller & vibrator), so as to modify its engg. properties.

→ Compaction reduces void ratio, permeability & compressibility & increases shear strength, degree of denseness, stability & bearing capacity.

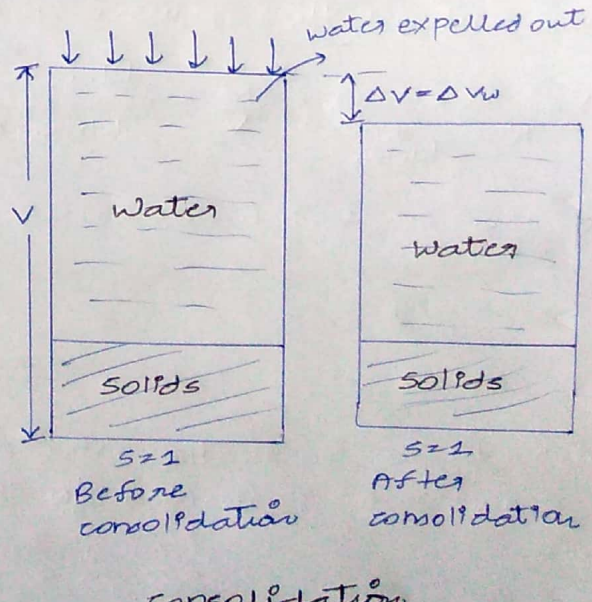
NOTE: Compaction must not be confused with consolidation which is another soil phenomenon in which volume reduction takes place.



Compaction	Consolidation
→ It is almost instantaneous process	→ It is time dependent process.
→ Soil is always unsaturated ($S<1$)	→ Soil is always saturated ($S=1$)
→ Volume reduction in this case is due to expulsion of air.	→ Volume reduction in this case is due to expulsion of water
→ Specified mechanical techniques are used (rollers, rammers)	→ It occurs on the account of load placed on the soil.



Compaction



Consolidation

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→ The compaction of the soil is reported in terms of dry density of soil, i.e. amount of solids which can be packed into unit volume of soil $\left(\rho_d = \frac{M_d}{V_d} = \frac{M_s}{V_d}\right)$

→ Compaction characteristics of the soil are determined in the laboratory by performing following lab test

→ Compaction lab test:

→ These test consists of a standard cylindrical mould in which soil to be tested is filled in number of layers & each layer is subjected to 25 number of blows with the help of standard hammer.

→ These are following 4 lab test being performed over the soil.

* Compaction characteristics:	Standard Proctor test	IS 2720 Light compaction test
→ Volume of mould	$\frac{1}{30}$ ft³ (945cc)	1000cc
→ No. of layers	3	3
→ No. of blows on each layer	25	25
→ Wt of hammer	5.5 lb (2.495kg)	2.6kg
→ Ht of free fall of hammer	12 inches (304.8 mm)	310mm

Compaction characteristics	Modified Proctor test	IS 2720 Heavy compaction test
→ Volume of mould	945cc	1000cc
→ No. of layers	5	5
→ No. of blows on each layer	25	25
→ Wt of hammer	10 lb (4.54kg)	4.9kg
→ Ht of freefall of hammer	18 inch (457.2 mm)	450mm

→ In order to perform IS compaction test, take 3kg of air dried sample paning from 4.75mm sieve.

→ It is mixed thoroughly with water in small quantity.

→ The mixture is covered with wet cloth & left for a maturing time of 5-30 minutes (higher time for FG soil)

→ The quantity of water to be added in the first trial is kept in the range of 4-10% (4% for CG soil & 10% for FG soil)

→ The soil is then filled in mould in 3 eff. layers & each layer is properly compacted by subjecting it to 25 no. of blows by standard hammer.

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→ Mass of the soil filling the mould is noted (M).

→ A representative sample is taken from the centre of the compacted specimen to be tested for water content by oven drying method (w)

→ The compacted specimen is then taken out of the mould & broken with hand & remixed with raised water content (2-4% more). After allowing for maturing time, the test is repeated again at diff. water content

→ The result of the test is reported in terms of a curve termed as "compaction curve" in which water content is reported on x-axis & corresponding dry density is reported on y-axis.

NOTE: Compaction energy provided in standard proctor test is 595 KJ/M^3 whereas in modified proctor test, it is 2674 KJ/M^3.

→ Energy provided in IS light compaction test (E_L) = Mgh

$$= \frac{2.6 \times 9.81 \times \frac{310}{1000} (25 \times 3)}{10^3 \times (10^{-2})^3}$$

$$= 593 \text{ KJ/M}^3$$

$E_L = 593$ KJ/M^3

Energy provided in IS heavy compaction test (E_H),

$$E_H = \frac{4.9 \times 9.81 \times 0.45 \times (25 \times 5)}{10^3 \times (10^{-2})^3}$$

$E_H = 2703$ KJ/M^3

$$\frac{E_H}{E_L} = \frac{2703}{593} = 4.55$$

→ After performing the test on the soil specimen, its bulk density is found

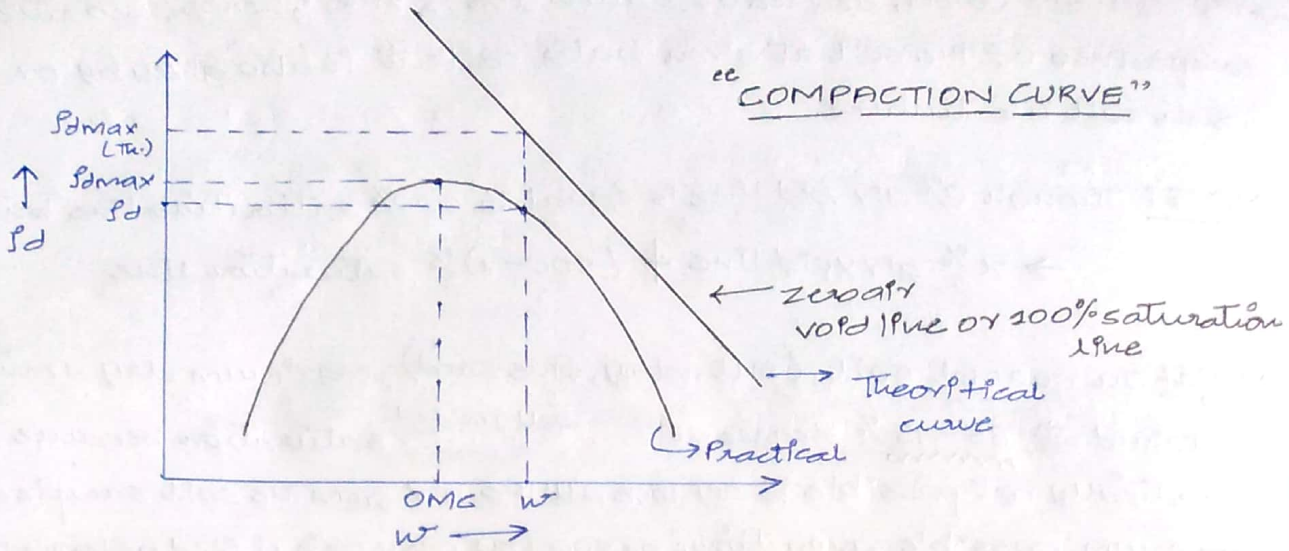
$$\rho_b = \frac{\text{Mass of soil in mould}}{\text{Vol. of soil in mould}} = \frac{M}{V} \quad \left\{ \text{Vol. of soil in mould} = \text{Vol. of mould} = V \right\}$$

→ Dry density of the soil under test is further calculated with the help of water content of soil.

$$\rho_d = \frac{\rho_b}{1+w}$$

→ Compaction curve is used to find the water content of the soil which gives maximum dry density (max. compactness/denseness) of the soil which is also termed as "Optimum moisture content" (OMC).

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→ A line showing the relationship b/w water content & dry density of compacted soil at constant % of air voids is termed as "air void line".

→ Eqn of air void line, $\rho_d = \frac{(1-\eta_a) G \rho_w}{1+wG}$

→ The theoritical maximum compaction of these soil at given water content corresponds to zero air void condition ($\eta_a = 0$). The line

→ The line showing the relationship b/w water content & dry density of compacted soil at 0% of air voids is called "zero air void line".

→ Eqn of zero air void line, $\rho_d = \frac{G \rho_w}{1+wG}$

AIR VOID LINE $\rho_d = \frac{(1-\eta_a) G \rho_w}{1+wG}$

ZERO AIR VOID LINE $\rho_d = \frac{G \rho_w}{1+wG}$

→ A line showing the relationship b/w water content & dry density of compacted soil, at constant degree of saturation (%) is termed as "saturation line."

→ Eqn of saturation line,

$$\rho_d = \frac{G \rho_w}{1+e} = \frac{G \rho_w}{1+\frac{wG}{S}}$$

→ At zero % air voids, degree of saturation is 100%, hence theoritical maximum compactness of the soil at the given water content is also may be stated to be achieved when the soil is saturated.

0% air void line = 100% saturation line

$$\rho_d = \left.\frac{(1-\eta_a) G \rho_w}{1+wG}\right|_{\eta_a=0} = \left.\frac{G \rho_w}{1+\frac{wG}{S}}\right|_{S=1}$$

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→ At 0% air voids, degree of saturation is 100%, hence, theoritical max. compactness of the soil at given water content is also given by or achieved when soil is saturated.

NOTE: Though 0% air void line is equal to 100% saturation line but,

→ x% air void line ≠ (100−x)% saturation line

NOTE: In normal soil (silt, clay, fine sand), maximum dry density is attained at 85-95% degree of saturation because it is practically not possible to remove all the air from the soil sample even under maximum possible compaction (as soon as hammer is lifted after the impact some amount of air always again enters into the soil.

* Field compaction control:

→ Field compaction control consists of determination of

a. Water content at which soil is compacted

b. Dry density of compacted soil hence degree of compaction

→ Water content of the soil in the field can be computed rapidly by

a. Calcium carbide method

b. Proctor needle method

* Proctor needle test

→ This method consists of a proctor needle, having needle point at the base which is connected with needle shank, that is further connected with spring controlled plunger.

→ Proctor needle is inserted in the sample to be tested upto the depth of 7.5 cms at the rate of 1.25 cm/sec & resistance offered by the soil to the needle pt against penetration is noted.

→ The test is repeated at diff. water content & corresponding penetration resistance is noted in the laboratory.

→ These results of the tests are then expressed in the form of curve termed as "Penetration resistance curve"

→ This curve is further used in field to find the water content at which soil is compacted

→ Bulk density of the soil is found by any of field test discussed before to calculate dry density of the soil in the field.

→ The dry density achieved in the field is compared with the max. dry density obtained in laboratory. with the help of relative compaction. (RC)

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→ Relative compaction (RC) = $\frac{(\rho_d)_{\text{in the field}}}{(\rho_{dmax})_{\text{obt. in lab}}} \times 100$

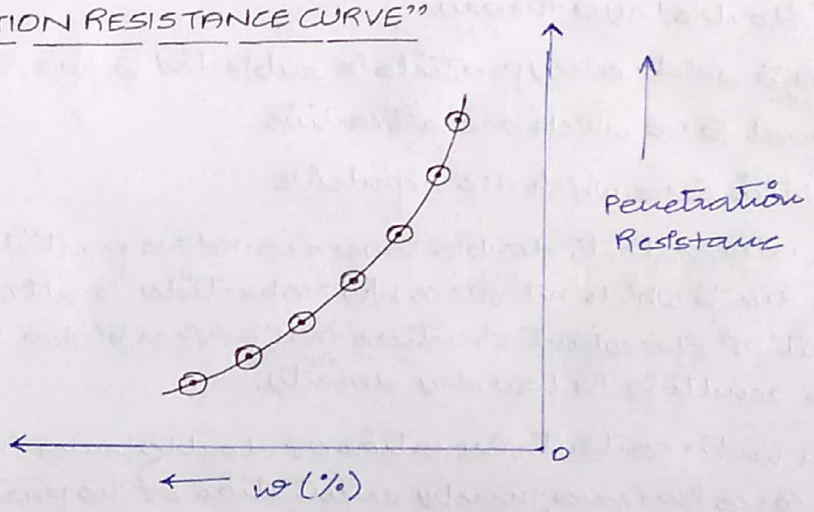
$$RC = \frac{\frac{G\rho_w}{1+e}}{\frac{G\rho_w}{1+e_{min}}} \times 100 = \frac{1+e_{min}}{1+e_{field}} \times 100$$

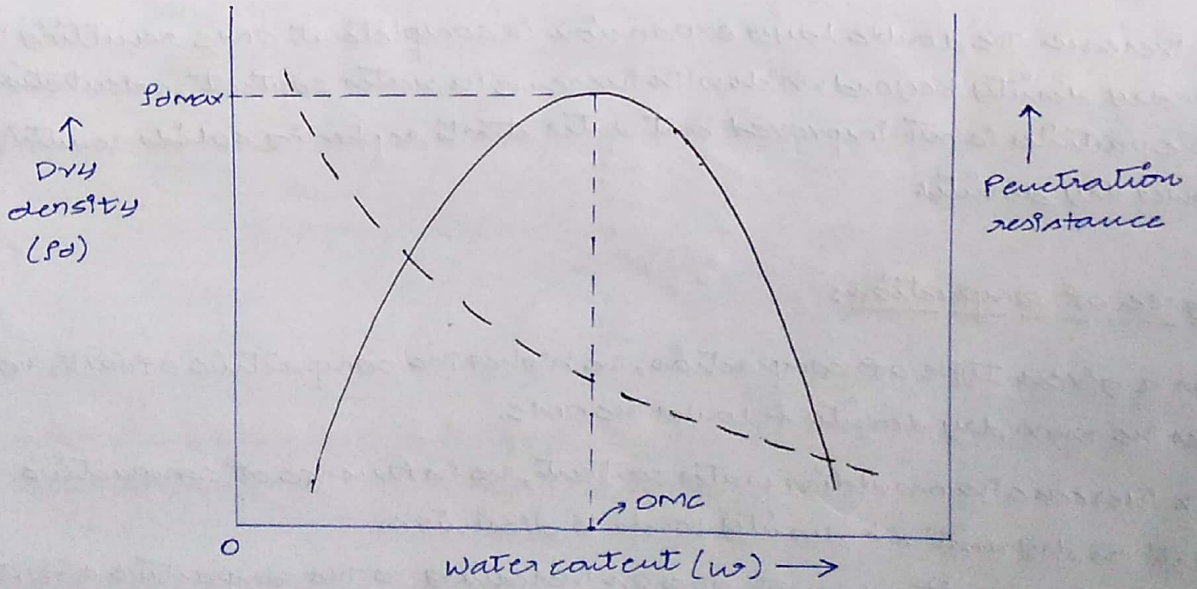
$RC = \frac{1+e_{min}}{1+e_{field}} \times 100$

$RC = 80 + 0.2 I_D$

→ For cohesionless soil, compactness can be reported in terms of density index only.

"PENETRATION RESISTANCE CURVE"





* Factors affecting compaction:

a. Water content:

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→ At low water content, soil is stiff & soil grains offers more resistance to compaction thereby gets less compacted giving lower dry density.

→ As the water content increases, the particle develops larger & larger water films around them which tends to lubricate the particles and make them easier to compact, resulting in higher dry density.

→ This increase in dry density takes place upto OMC at which maximum lubrication is attained.

→ Beyond OMC, if water content is increased there is no increase in lubrication of particles but now this water starts replacing solids resulting in decrease of dry density ($\rho_w < \rho_s$).

→ This entire theory given by Proctor is termed as "Lubrication theory" i.e. not rational enough & the same was further explained by "LAMBE" double layer theory.

→ According to which every particle is subjected to two forces
a. Vander waal force which are attractive
b. Electro static force which are repulsive

→ At lower water content, double layer around the particles is not formed completely due to which net force b/w the particles is attractive leading to development of flocculant structure that offers higher resistance to compaction resulting in low dry density.

→ With inc. in water content formation of double layer increases, hence repulsive force increases, thereby orientation of the particles is improved resulting in higher compaction i.e. higher dry density.

→ This increase the double layer expansion is complete at OMC resulting in max. dry density beyond which with increase in water content, orientation of the particles is not improved but water starts replacing solids resulting in lower dry density.

b. Degree of compaction:

→ For a given type of compaction, the higher the compactive effort, the higher the max. dry density & lower the OMC.

→ With increase in moulding water content, the influence of compactive effort on dry ~~unit~~ wb density tends to diminish.

→ The max. dry density does not goes on increasing as the compactive effort is increased

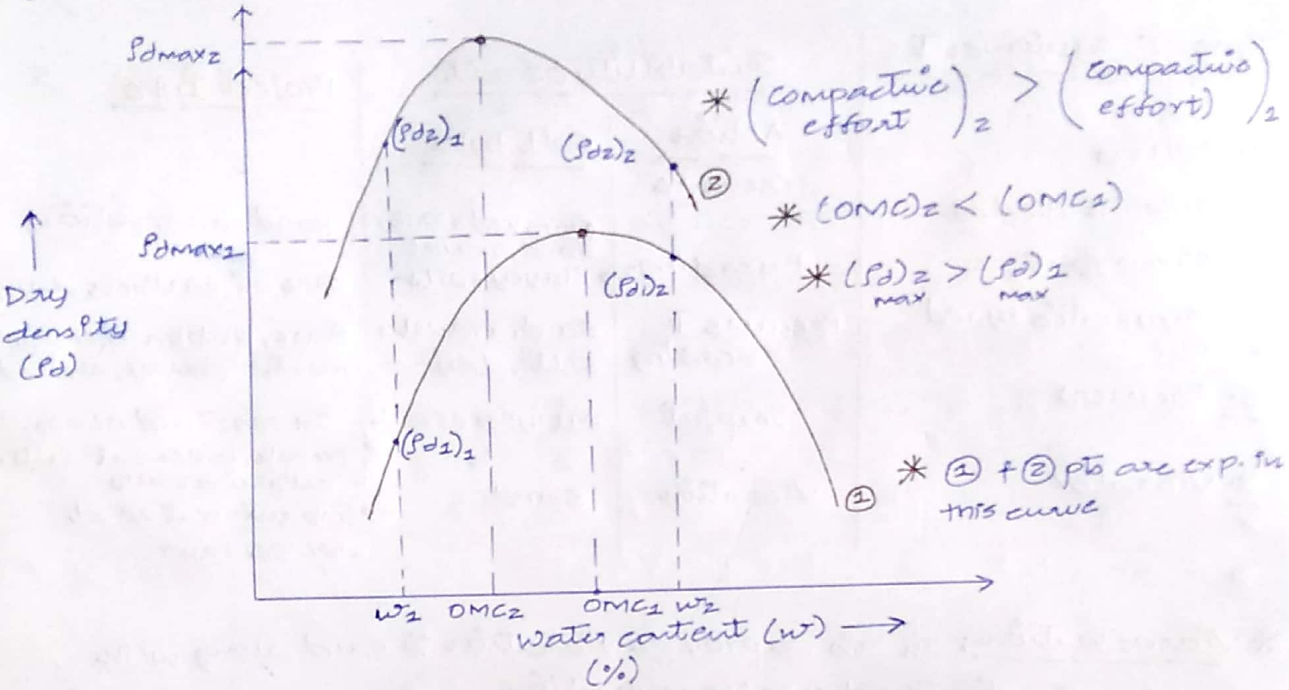
→ The margin of increase becomes smaller & smaller even on dry side of OMC while on wet side of OMC there is hardly any increase.

→ If all the peaks of compaction curve for diff. compactive effort are

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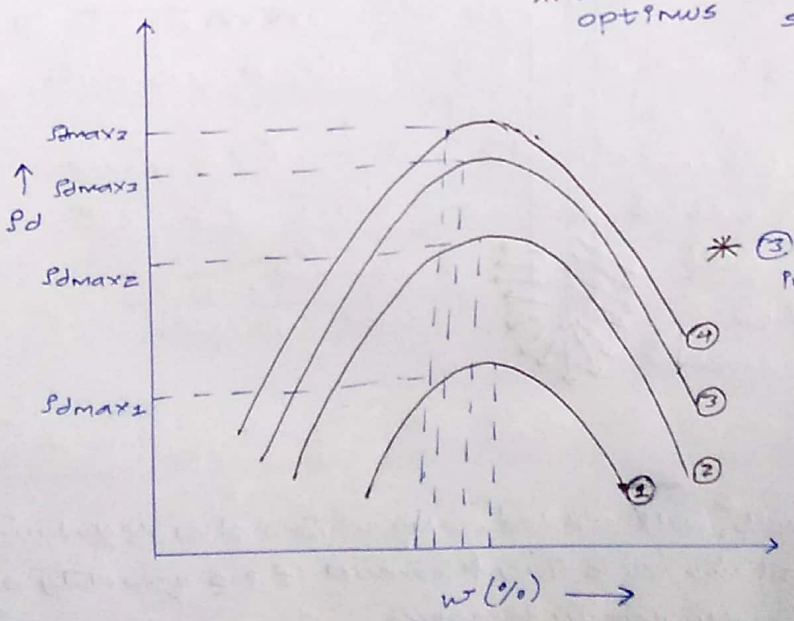
Joined together, a "line of optimums" is obtained which is nearly parallel to zero air void line which shows that even a higher compactive effort does not results in higher efficiency of compaction.

→ The degree of saturation at OMC remains almost same in all cases (85-95%).



$$(\rho_{d_2})_1 - (\rho_{d_1})_1 > (\rho_{d_2})_2 - (\rho_{d_1})_2$$

* Line of optimums → 85-95% degree of saturation line



* ③ + ④ pts are explained in this graph/curve

c. Method of compaction:

→ The density obtained during compaction for a given soil depends upon type of compaction or the manner in which compactive effort is applied.

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→ The various aspects linked with it are

P. Wt of compacting equipment

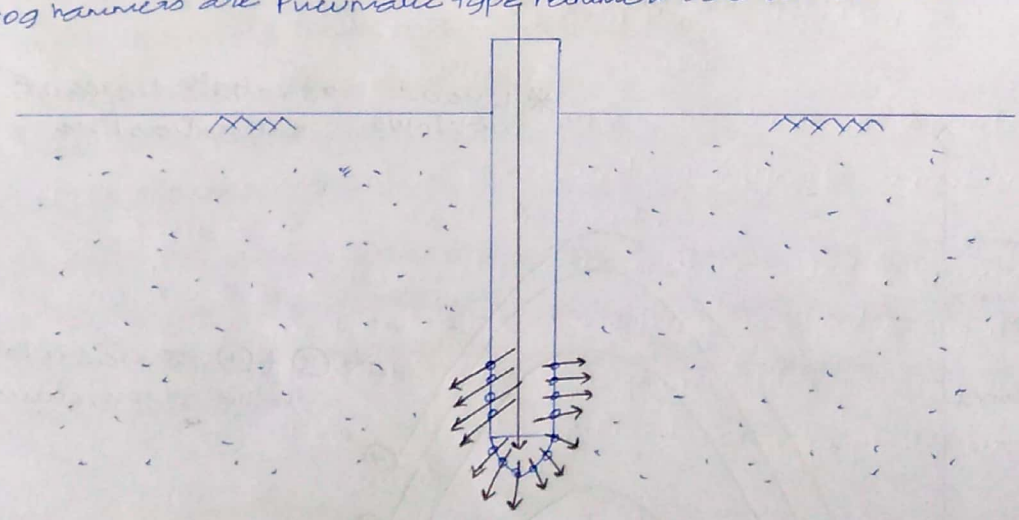
PP. Time & area of contact b/w compacting element & soil.

PPP. The manner in which compaction is done such as dynamic or impact, static, kneading or rolling.

Types of equipment	Suitability of soil		Project Type
	Action	Soil type	
1. Rollers	Pressure		
a. Smooth wheeled		Crushed stone, sand, gravel	Road constructions
b. Sheep foot	Kneading →	clayey soils	Core of earthen dams
c. Pneumatic tyred	Pressure + Kneading	Sand, Gravel, silt, clay	Base, subbase, embankment, highway, airfield
2. Rammers	Impact	All types of soil	In confined areas: Trench, basement walls, retaining walls
3. Vibrator	Vibration	Sand	→ Embankment of oil storage tanks.

* Vibrofloation: In this we provide vibration to sand along with flushing of water to enhance the compaction

→ Frog rammers are pneumatic type hammers used for cohesionless soil.



d. Type of soil:

→ Coarse grained soil, well graded, compact to higher dry density especially if they contain some fines & however if the quantity of fines is excessive max. dry density decreases

→ Poorly graded or uniform sand leads to lower dry density.

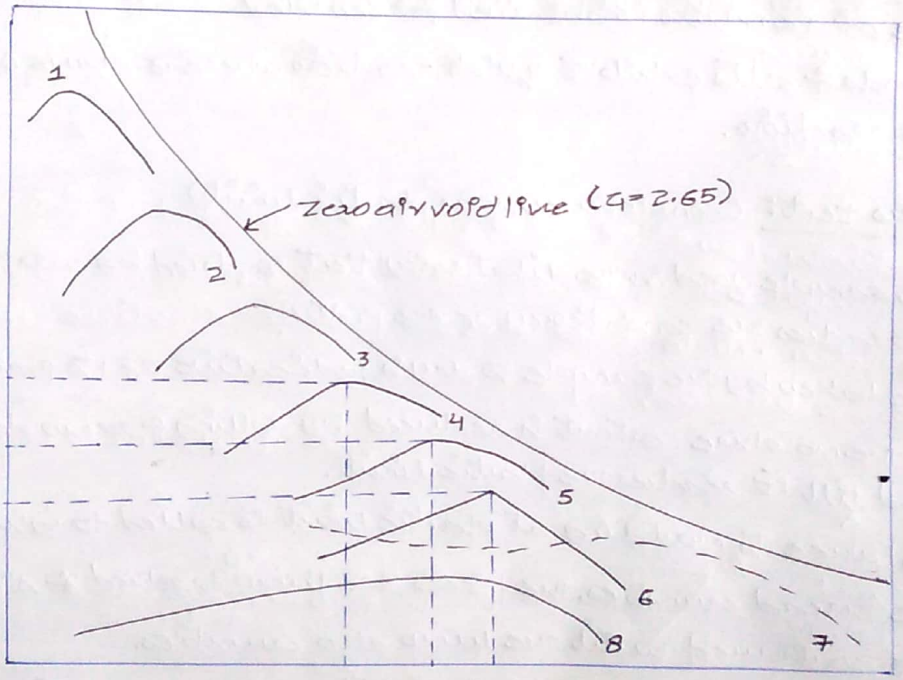
→ In clay soils, the max. dry density tends to decrease as plasticity increases

→ Cohesive soils have generally high value of OMC.

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→ Heavy clays with high plasticity have very low max. dry density & very high OMC.

Curve no.	Type of soil
1	Well graded sand with silt (SW-SM)
2	Silty sand (SM)
3	Silty sand with medium gradation (SM)
4	Low compressible clay (CL)
5	Intermediate compressible clay (CI)
6	Low compressible silt (ML)
7	Heavy / fat clay (CH)
8	Poorly graded sand (SP)



→ Since type of soil also effects the compaction of soil, one should know diff. methods to identify the soil in field.

→ ISSCS recommends field identification procedure through following tests.

a. Visual examination test: only the particles of coarse grained soil can be seen by naked eye.

→ Particles of silt & clay are not visible to naked eye.

→ Visual examination is done by taking a representative sample of soil and spreading it on a flat surface or palm of the hand.

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→ Visual examination is carried out wrt size, angularity, texture & grading.

b. Dilatency test: (reaction to shaking)

→ About 5cc of soil sample is taken & enough water is added in it to nearly saturated.
→ The pat of soil is then placed in the open palm of the hand & shaken horizontally, striking vigorously against the other hand.
→ The pat is done then squeezed b/w the fingers.
→ The appearance & disappearance of the water with shaking & squeezing is referred as positive reaction.
→ This reaction is called quick if water appears & disappears rapidly.
→ This reaction is called normal, if water appears & disappears slowly & no reaction if water does not appear to change.
→ The type of reaction is observed & recorded
→ Fine sands & silts exhibit quick reaction whereas clays exhibit no or normal reaction.

c. Toughness test: (consistency near plastic limit)

→ The soil sample used in the dilatency test is dried by working & moulding until it reaches the consistency of the putty.
→ The time taken by the sample to dry is indicative of its plasticity
→ Further the moisture content is reduced by rolling & remoulding it into 3mm dia thread till it reaches the plastic limit.
→ The resistance to moulding at plastic limit is called toughness
→ After the thread crumbles, the pieces are lumped together & slight kneading action is continued until the lump also crumbles.
→ If the lump can still be moulded slightly drier than plastic limit, soil is termed to have high toughness.
→ If the lump cannot be moulded easily, the soil is termed to have medium toughness.
→ If the lump cannot be moulded, the soil is termed to have low toughness

d. Dry strength test (crushing resistance)

→ The prepared soil sample is completely dried in the sun or by air drying method.
→ Its strength is tested by breaking the lump b/w the fingers.
→ Resistance to breaking is termed as dry strength, is a measure of plasticity of soil.

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→ If the sample breaks easily it has low dry strength whereas if considerable finger pr. is required soil possess high dry strength.
→ A high dry strength is characteristics of clay of high plasticity.
→ Inorganic silts have only a slight dry strength.
→ Silty fine sand & silt have practically same low dry strength.

e. Organic content & colour:

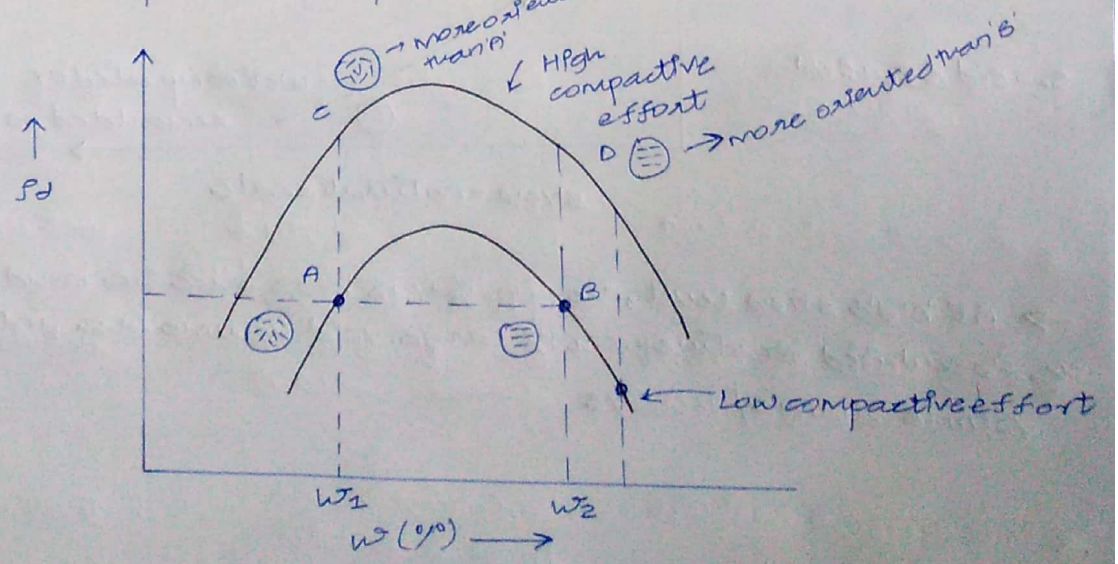
→ Fresh, wet organic soils have the characteristic pungent odour of organic matter.
→ Organic soils have usually dark colour.

* Effect on compaction on engg. prop. of the soil:

→ An increase in density would produce a soil which is more strong, less compressible, less permeable is possible if there is no change in structure of the soil during compaction but as structural changes becomes imp. part of compaction, the above logic fails.
→ Thus soils are said to be showing following effects on its properties during compaction

a. Structure:

→ For a given compaction effort, the soil tends to have a flocculated str. on dry of optimum, whereas on wet of optimum structure tends to be more dispersed.
→ For a given moulding water content if the compactive effort is increased the soil tends to be more oriented i.e. dispersed specially on dry side of optimum.
→ Among the diff. methods of compaction, the kneading compaction as induced by sheepfoot rollers tends to produce a more oriented structure as compared to either static or impact type compaction.



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→ Degree of orientation

B > A
D > C
} w%

C > A
D > B
} compactive effort

b. Permeability:

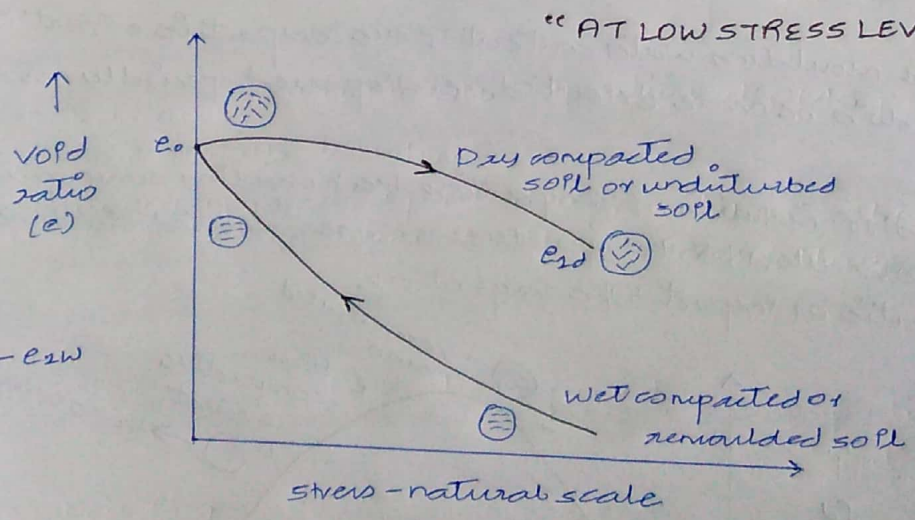
→ For a given compactive effort, the permeability decreases sharply with increase in water content on dry of optimum (it is on the account of improved orientation of the particles which results in a decrease in size of voids)

→ The minimum permeability occurs at or slightly above OMC

→ Beyond OMC, the permeability may show a slight increase (due to effect of decrease in dry density which is more pronounced than the effect of improved orientation), but it always remains much smaller than on dry side of optimum.

c. Compressibility:

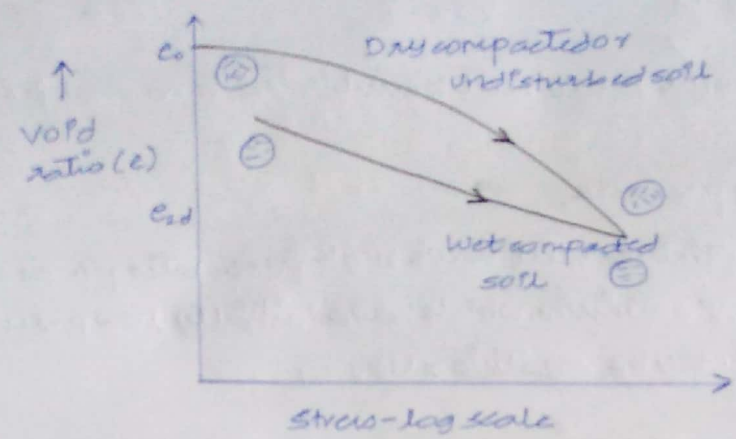
→ At relatively slow stress levels, a soil compacted at wet of optimum is more compressible than the flocculated soil on dry of optimum, because the strong interparticle bond in a flocculated structure do not allow the particles to be displaced.



$e_0 - e_{1d} < e_0 - e_{1w}$

→ At high stress level, the opp. is true bcoz once the bond are broken, the flocculated structure with its large void volume can undergo a larger volume change/decrease

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"AT HIGH STRESS LEVEL"

$e_o - e_{sd} > e_o - e_{sw}$

$e_{sd} < e_{sw}$

→ A steam or gasoline driven roller in which wt is divided b/w the 2 heavy iron rolls one behind each other is termed as tandem roller.

d. Swelling:

→ A soil on dry of optimum has higher water deficiency & a more random arrangement of particles

→ It can therefore imbibe (accept) more water than the soil on wet of optimum and in this process, it swells more.

e. Shrinkage:

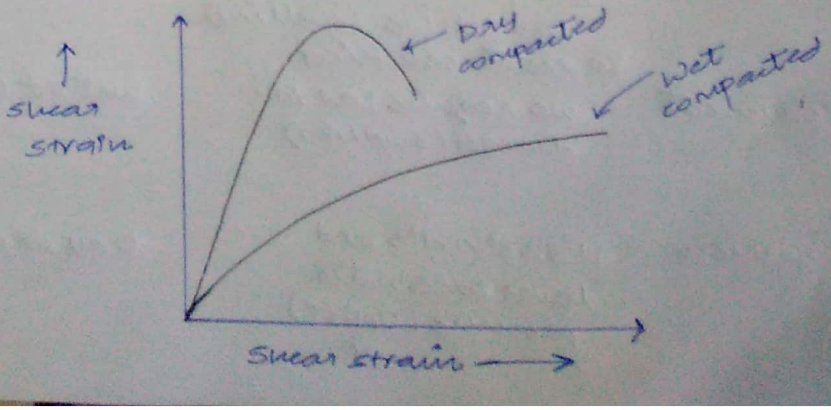
→ Soil compacted on wet of optimum tends to exhibit more shrinkage upon drying than those compacted on dry of optimum.

→ The more orderly, nearly parallel arrangement of particles allows the particle to pack more efficiently as compared to the randomly oriented particles on dry side of optimum.

f. Stress-strain relationship (strength):

→ Soil compacted on dry of optimum with their strong interparticle bond resist deformation upon application of stress, hence, stress strain curve rises steeply to a peak (indicating higher strength), then falls off as the interparticle bond are broken (showing brittle failure).

→ Soil compacted to wet of optimum have a flatter stress-strain curve without any peak showing low strength & progressive failure.



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3. Pore water pressure:

→ During compaction, the soil below will be subjected to normal & shear stress

→ This induces pore water pressure.

→ Soil compacted wet of optimum will have high pore water pr. as compared to soil compacted to dry of optimum. which have initially low pressure. (Pore water pressure) that may be negative also.

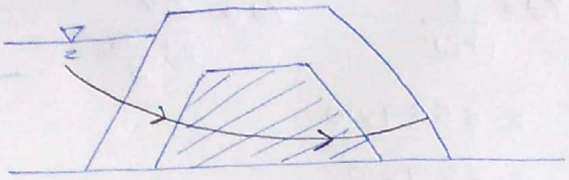
Property of soil	Dry of optimum	Wet of optimum
* Structure	Flocculant	Dispersed
* Water deficiency	more	less
* Permeability	more (isotropic)	less (anisotropic) ($K_H > K_V$)
* compressibility		
a. Low stress	less	more
b. High stress	more	less
* Swellability	more	less
* Shrinkage	less	more
* Strength	more (brittle)	less (ductile)
* Pore water pressure	less	more

→ Conclusion drawn: Even though compaction at OMC produces max. dry density, it is not always desirable to do so.

→ In a certain engg. project, a particular property of soil may be required on the basis of which compaction of soil is done as follows.

Engineering construction	Properties required	Compaction water content
* Core of earthen dam	Low permeability & low vol. inc. (swelling) to resist cracking	wet of OMC
* Subgrade of pavement	we require low vol. changes (swelling)	wet of OMC
* Homogeneous earthen dam	High strength and lower pore water pressure (uplift) pr	Dry of OMC

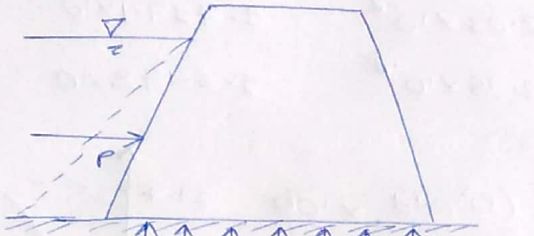
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core of earthen dam



subgrade of pavement



Homogeneous earthen dam

Q. In a standard proctor test, 1.8 kg of moist soil was filling the mould ($V = 944$ cc) after compaction. A soil sample weighing 23gm was taken from the mould & oven dried for 24 hrs at a temp of 110°C. Wt of dry sample was found to be 20gm, $G = 2.7$. The theoritical max. value of dry unit of soil at the water content is equal to?

$$\gamma_b = \frac{W}{V} = \frac{1.8 \times 9.81 \times 10^{-3}}{944 \times (10^{-2})^3} = 18.705\ KN/M^3$$

$$w = \frac{W_w}{W_s} = \frac{23-20}{20} \times 100 = \frac{3}{20} \times 100 = 15\%$$

Theoritical → zero air void line: $\gamma_{dmax} = \frac{G\gamma_w}{1+wG} = \frac{2.7 \times 9.81}{1+0.15 \times 2.7} = 18.85\ KN/M^3$

At field → $\gamma_d = \frac{\gamma_b}{1+w} = \frac{18.705}{1+0.15} = 16.26\ KN/M^3$

Q. The following data refer to a compaction test as per Indian standard light compaction test

w (%)	wt of wet sample (Kg)
8.5	1.8
12.2	1.94
13.75	2.00
15.5	2.05
18.2	2.03
20.2	1.98

If the $G = 2.7$

i. Plot the compaction curve & find γ_{dmax} & OMC

ii. Plot 80% saturation line & 100% saturation line.

iii. If it is proposed to secure a relative compaction of 95% in the field what is the range of water content that can be allowed?

iv. Would the 20% air void line be same as 80% saturation line.

Assume 1 unit = 1 cm

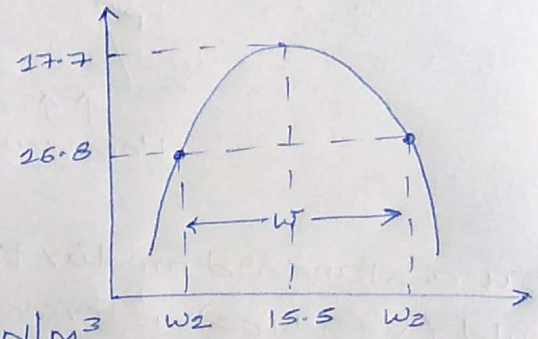
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w (%)	W	$\gamma = \frac{W}{V}$	$\gamma_d = \frac{\gamma}{1+w}$	$\gamma_d = \frac{G\gamma}{1+\frac{wG}{S}}$
8.5	1.8	1.8×10^{-3}	1.6589×10	
12.2	1.94	1.94×10^{-3}	1.729×10	
13.75	2.00	2×10^{-3}	1.7582×10	
15.5	2.05	2.05×10^{-3}	1.7748×10	17.7
18.2	2.03	2.03×10^{-3}	1.7174×10	
20.2	1.98	1.98×10^{-3}	1.6472×10	

$$R.C = \frac{(\rho_d)_{field}}{(\rho_d)_{max}} \times 100$$

$$0.95 = \frac{(\rho_d)_{field}}{17.7}$$

$$(\rho_d)_{field} = 16.81 \text{ KN/M}^3$$



$$\frac{17.7 - 16.81}{15.5 - w_1} = \frac{16.81 - 0}{w_1}$$

$$0.89 w_1 = 260.555 - 16.81 w_1$$

$$w_1 = 10$$

$$w_2 = 17$$

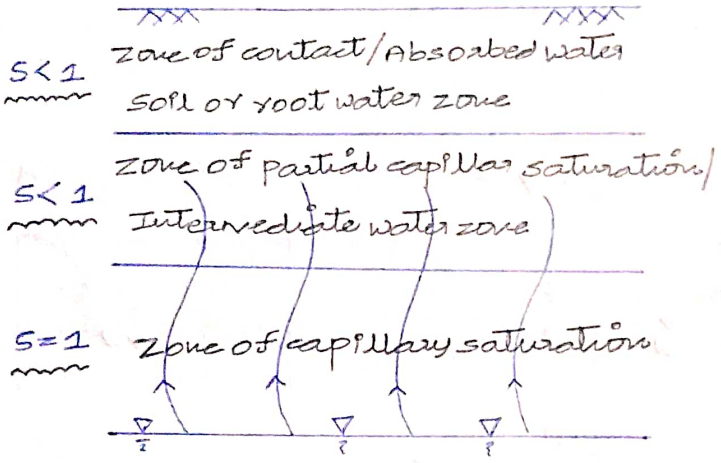
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CAPILLARITY IN SOIL

→ Capillary water:

→ It is the water which is being lifted by capillary forces above the free water surface level.

→ This water remains in suspension in the voids of the soils & fills it upto a certain distance above the ground water table level, i.e. further classified into following zones.



* Zone of capillary saturation:

→ It is the zone in which soil with 100% voids are filled with capillary water

* Zone of partial capillary saturation

→ It is the zone in which only small voids are filled with capillary water & large voids are filled with air.

* Zone of contact/absorbed water:

→ It is the zone in which water surrounding the particles at contact point is present, without having any continuity

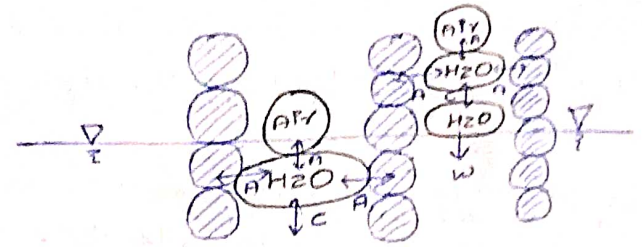
* Capillary forces:

→ capillary forces depends upon surface tension, pressure in water in relation to atmospheric pr. & size & conformation of soil pores

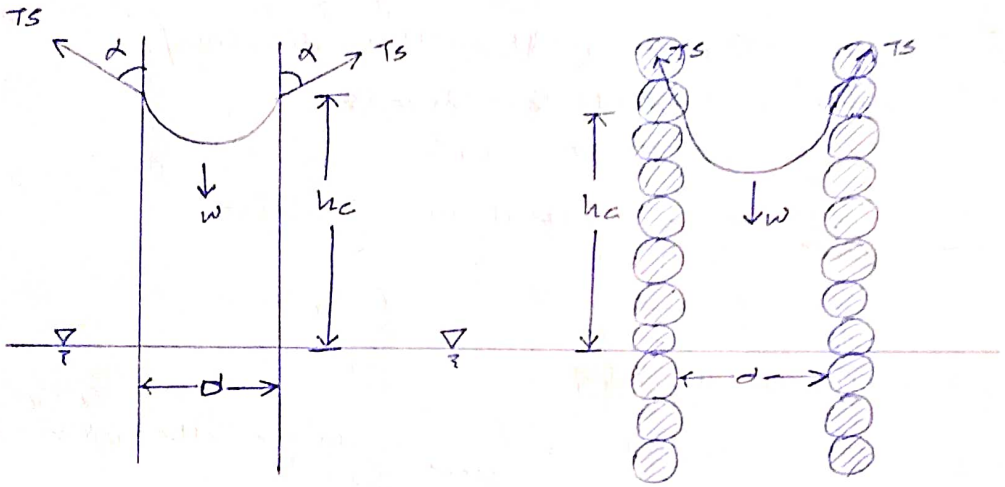
* Surface tension:

→ It is the property which exists in the surface film of water which tends to contract the contained volume into a form having minimum surface area.

→ Water at surface since is in the state of tension, it is termed as surface tension



→ Rise of the water in capillary tube / voids of the soil takes place upto an extent when surface tension force is balanced by wt of the water in capillary tube / voids of the soil.



→ For equilibrium of water in capillary tube / voids of soil,

$$\Sigma F_y = 0$$

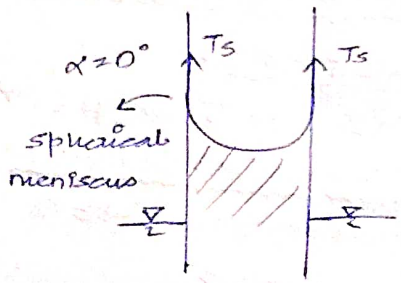
$$T_s (\pi d) \cos\alpha - W = V_w \gamma_w = 0$$

$$T_s (\pi d) \cos\alpha = h_c \frac{\pi d^2}{4} \times \gamma_w$$

** $h_c = \frac{4 T_s \cos\alpha}{\gamma_w d}$ → T_s: surface tension, α: contact angle

→ For height of capillary rise to be maximum ⇒ $\cos\alpha = 1 \Rightarrow \alpha = 0°$

$$h_{c\,max} = \frac{4 T_s}{\gamma_w d}$$

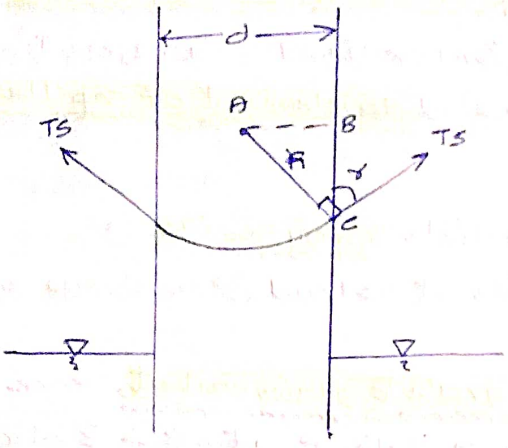


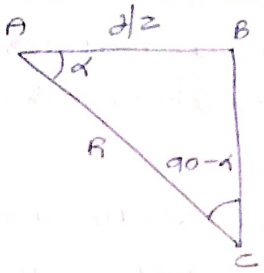
→ At 20°C, $T_s = 72.8 \times 10^{-8}$ KN/cm

** $h_{c\,max} = \frac{0.29\ (cm)}{d}$ → (cm)

→ At 4°C, $h_{c\,max} = \frac{0.3}{d}$ (cm) (d → cm) $T_s = 75.8 \times 10^{-6}$ KN/cm

*
→ With increase in temp., height of capillary rise decreases





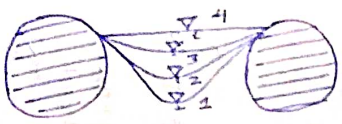
$\cos\alpha = \frac{d/2}{R}$

$d = 2R\cos\alpha$

$$h_c = \frac{4T_s\cos\alpha}{\gamma_w d} = \frac{4T_s\cos\alpha}{\gamma_w \cdot 2R\cos\alpha}$$

**
$h_c = \frac{2T_s}{\gamma_w R}$ → radius of meniscus

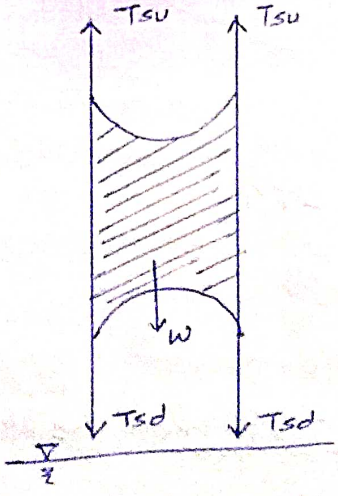
$h_c \propto \frac{1}{R}$



→ With increase in water content of the soil, the radius of an meniscus increases hence height of capillary rise decreases in soil & when water content rises to such an extent soil becomes completely saturated, radius of meniscus becomes infinity (meniscus is destroyed) hence ht of capillary rise reduces to zero.

Capillarity is maximum when soil is dry & water is added.

**
→ capillarity is not found in saturated soil.



$T_{su} = T_{sd}$

→ Entire water will drain out (If d = const)

→ Water will retains (If d = variable)

$T_{su} > T_{sd}$

→ Water in the capillary tube above free water surface level is in the state of tension which is maximum at the level of menisacus & gradually reduces to zero upto free water surface level however water in tube below free WSL will be in hydrostatic compression as usual.

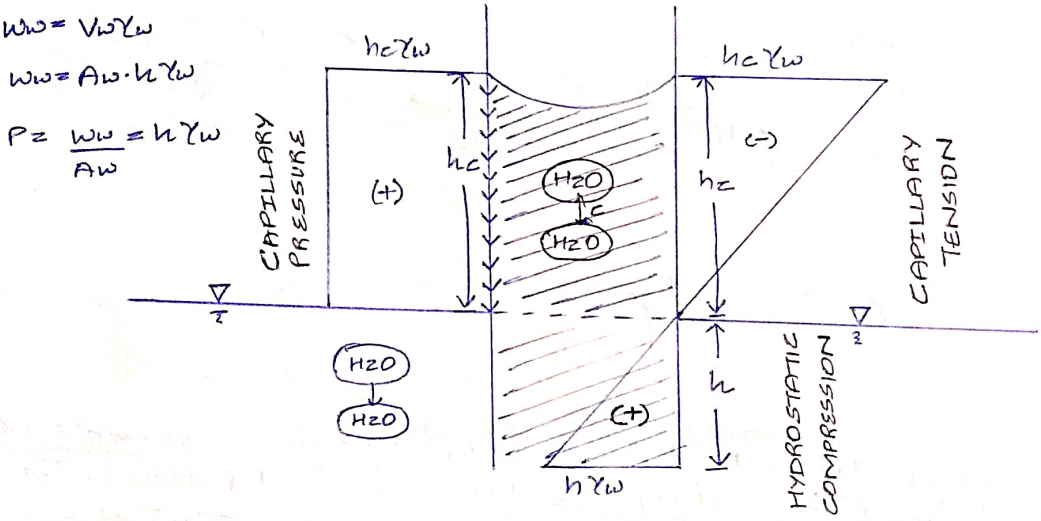
→ This tensile stress in capillary water is termed as capillary tension/ capillary potential/ soil suction / Pressure deficiency/ suction pressure.

→ The menisacus transfers to surface tension from the liquid continuously to the wall of capillary tube/soil solids causing development of capillary pressure in it.

→ The capillary pressure induced on soil particle is compressive.

→ This pressure is equal to capillary tension at the level of menisacus and remains same throughout its height.

→ This capillary pressure is transferred by grain to grain contact, hence tends to force the soil solids to come in closer state of contact & is also termed as effective stress or intergranular pressure.



→ This capillary tension in water depends upon following (CT = $h_c\gamma_w$)

a. Particle size (d) ↑ CT ↓
b. Water content (w) ↑ CT ↓
c. Plasticity Index (I_P) ↑ CT ↑
d. Temperature ↑ CT ↓
e. Denseness of soil (γ_d) ↑ CT ↑
f. Angle of contact (α) ↑ CT ↓
*g. Dissolved ~~salts~~ salts in porewater ↑ CT ↑

* Contact moisture:

→ Water can also be held by surface tension at the pt of contact

b/w the soil solids.

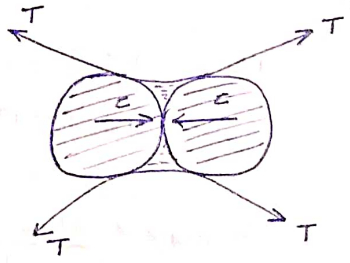
→ capillary water in this form is termed as "contact moisture"

→ Tension is present in this water, which tends to force the soil solids in closer state of contact, leading to the development of pressure b/w them termed as contact pressure.

→ This contact pressure gives rise to phenomenon called "apparent cohesion" due to which soil (sand) may acquire strength.

→ Factors affecting contact pressure are as follows

a. Particle size
b. Density of packing
c. Angle of contact
d. Water content



NOTE: Increase in water content results in increase in radius of meniscus which in turn reduces contact pressure.

→ When soil is completely saturated, meniscus is destroyed, contact pressure reduces to zero, hence there is no apparent cohesion.

→ The theory of capillarity is not entirely valid in case of soils bcoz size of voids, extent of packing, arrangement of particles is not uniform.

→ Moreover interconnection b/w the voids not only extends in vertical dir" but also in hor. direction.

→ Hence following modifications are to be applied in case of soil to analyse capillarity.

a. The size of the void may be assumed to be 20% of effective size of the solids.

$$d = 20\% D_{10} = \frac{D_{10}}{5}$$

$$h_{cmax} = \frac{4T_s}{\gamma_w d} = \frac{4T_s \cdot 5}{\gamma_w D_{10}} = \frac{20T_s}{\gamma_w D_{10}} \text{ (cm)} \quad D_{10} \rightarrow \text{cm}$$

b. Capillarity in soil as per Allen Hazen

** $h_{cmax} = \frac{C}{eD_{10}}$ (cm)

* C → constant (0.1 – 0.5 cm²)
* D_{10} → cm

c. Capillarity in soil can be computed by assuming voids to be spherical

$$V_v \propto d^3$$

$$V_s \propto D_{10}^3$$

$$e = \frac{V_v}{V_s} = \frac{d^3}{D_{10}^3}$$

* $d = e^{1/3} D_{10}$

** $h_{cmax} = \frac{4T_s}{\gamma_w e^{1/3} D_{10}}$

Type of soil	Capillary rise (cm)
Gravel	2-10
Sand	10-100
Silt	100-1000
Clay	1000-3000

Q. The capillary rise in soil 'x' with $D_{10} = 0.05$ mm is 45cm. Estimate the capillary rise in soil 'y' with $D_{10} = 0.25$ mm. Assume e = same

$$45 = \frac{4T_s}{\gamma_w e^{1/3} \times 0.05} \times 10^{-1} \quad \text{—①}$$

$$h_{cmax} = \frac{4T_s}{\gamma_w e^{1/3} \times 0.15} \times 10^{-1} \quad \text{—②}$$

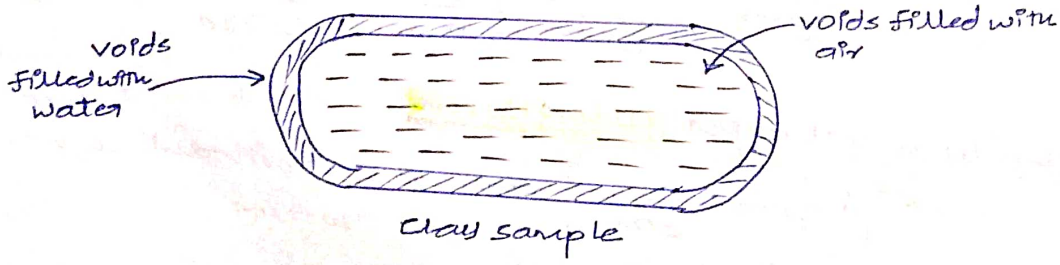
$$\frac{45}{h_{cmax}} = \frac{0.15}{0.05}$$

$$h_{cmax} = 15\,cm$$

* Slaking of clay

→ When a sample of clay is dried below the shrinkage limit to its minimum volume & is suddenly immersed in water, it will cause its slaking (disintegration into a soft wet mass)

→ The slaking is due to entry of air in the voids of clay during drying & when water enters in these voids due to immersion, meniscus is formed, causing high pressure & finally disintegration & explosion of the voids.



* Bulking of sand

→ If a dry mass of sand is moistened slightly & then dumped loosely, its volume increases considerably relative to its dry state.

→ This phenomenon is termed as "bulking of sand."

→ It takes place due to contact moisture which results in apparent

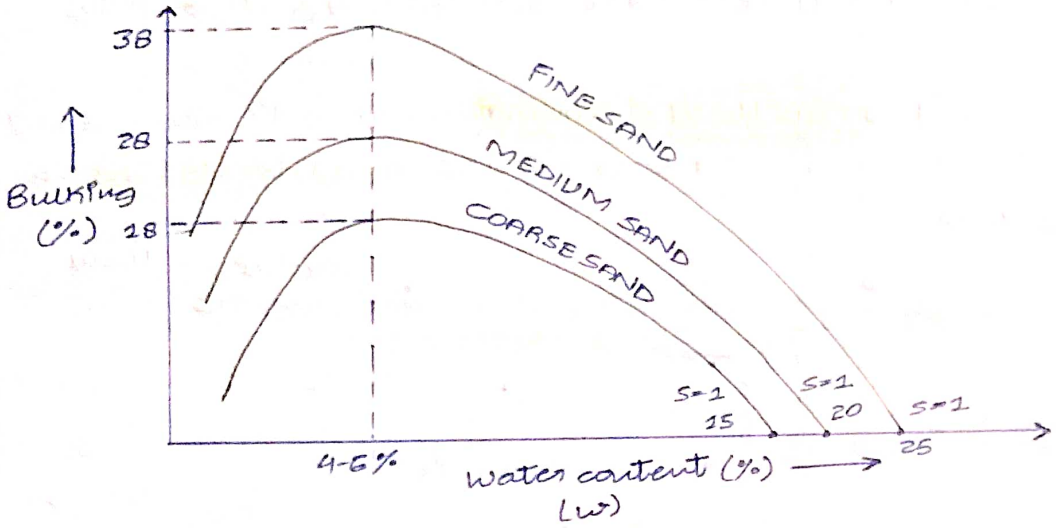
cohesion b/w sand particles that in turn holds the particles together in clusters of honeycomb, enclosing large volume of voids b/w them.

→ The finer the particle, more the apparent cohesion, more is the bulking

→ The volume change (bulking) also depends upon water content

→ Bulking increases initially with increase in water content upto 4-6% at which it becomes maximum, beyond it if water content is increased, meniscus (radius) also increases, reducing apparent cohesion & bulking

→ Finally when soil is saturated, menisci is destroyed, bulking is reduced to zero.



* Frost action

→ The freezing & thawing (melting) of water present in soil and the resultant effect on soil and its structure is termed as frost action.

→ Frost action consists of 2 processes

a. Frost heave

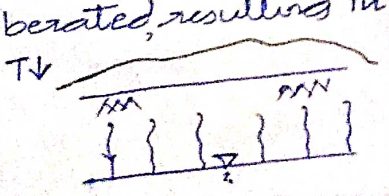
b. Frost boil

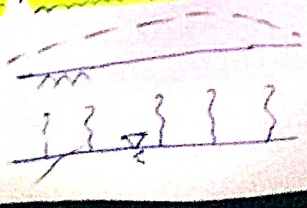
a. **Frost heave**: It is defined as rise of ground water level surface due to frost action.

→ When the temp. falls below freezing pt, the water in the capillary fringe may freeze leading to the formation of ice.

→ Since volume of ice is more (≈9%) than water, volume of voids & subsequently volume of soil increases

b. **Frost boil**: If the temp. rises above the freezing pt in the soil in which frost heave has occured the frozen soil mass thaws and free water is liberated, resulting in softening of soil / settling of soil.





NOTE: Frost action (heave) takes place in the soil that is highly porous and permeable eg. fine sand & coarse silt

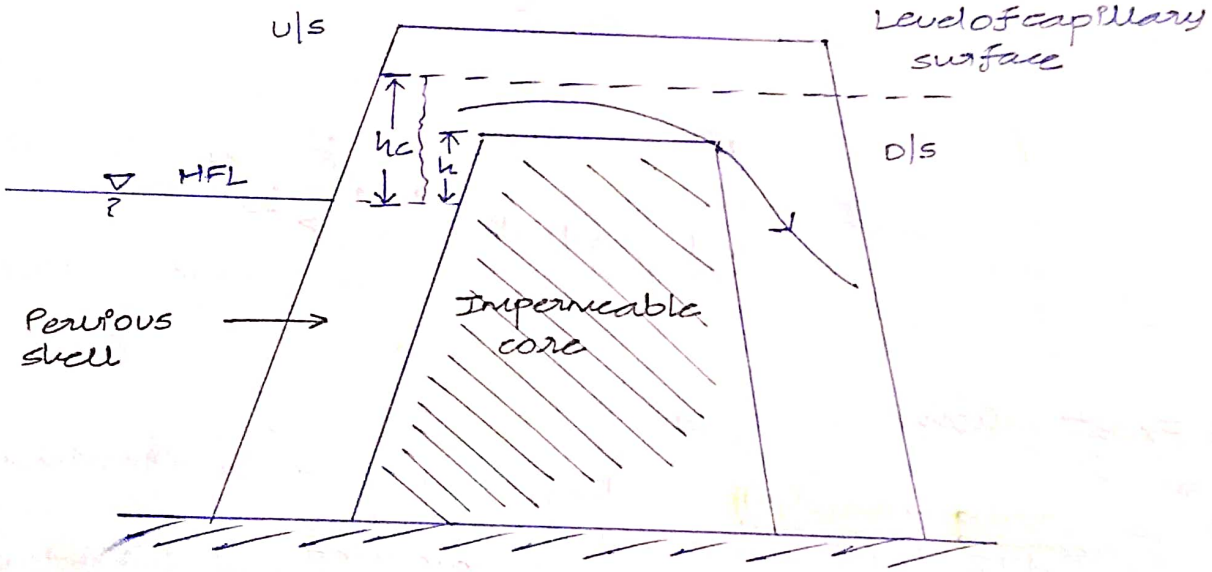
* Capillary siphoning:

→ When the water level in the reservoir is corresponding to highest flood level, the portion on the upstream of the dam will be saturated. The water level in the upstream pervious shell will be same as highest flood level.

→ Due to capillarity, water will rise through a height 'hc'

→ If top of the core of dam is situated at a height "h < hc" above the HFL, the capillary forces will pull the water in the descending portions of the dam.

→ This process is termed as "capillary siphoning" & may continue for a long time thereby not only causes water loss but also causes damage on the d/s slope of the dam.



**

→ capillary moisture in fine sand & silt allows excavation to be made because the stability it provides by virtue of induced capillary pressure.

→ Such excavations would be impossible in dry sand/silt conditions or under water table

→ capillary meniscus are easily destroyed by saturation due to rainfall or by any disturbances like evaporation.

STRESSES IN SOIL

* Total stress (σ):

→ At any given plane section in the soil mass, total stress (load per unit area) is due to self wt of soil (wt of solid + wt of water) or due to overburden pressure (uniform surcharge).

→ The total stress is further constituted of 2 diff. types of stress

a. Effective stress (Intergranular stress/pressure): ($\bar{\sigma}$)

→ It is the stress which is being transferred in the soil mass by grain to grain contact which tends to force the soil solids to come in contact (closer contact) with each other, resulting in increased denseness, stability, strength and reduced void ratio and mobilisation of shear strength

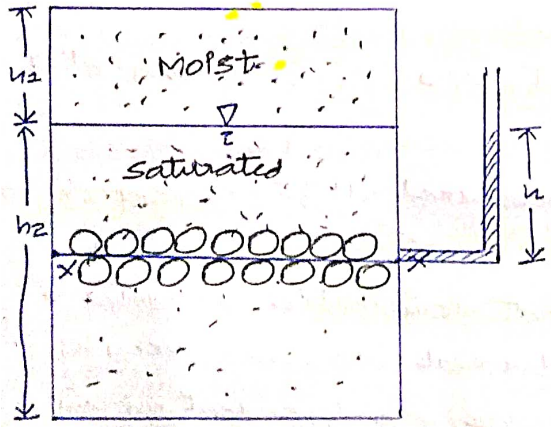
→ Since this pressure is transferred by grain to grain contact it is termed as intergranular pressure

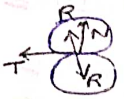
b. Pore water pressure (neutral pressure): (u)

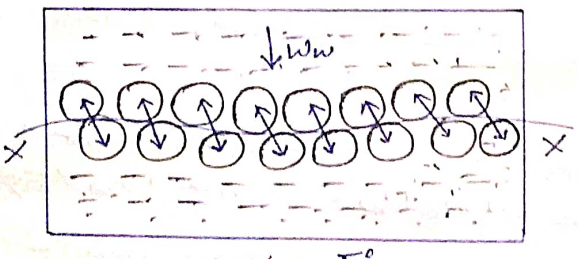
→ It is the pressure which is being transferred by pore fluid and is equal to the height of wt of fluid column (water) above the concerned section in soil mass.

→ This water pressure acts all around the soil solids hence does not force the solids to come in closer contact with each other (doesn't have any shear component).

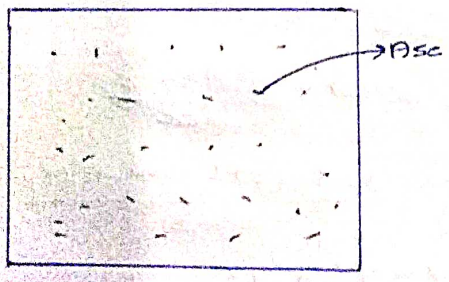
→ It is also termed as "NEUTRAL PRESSURE"



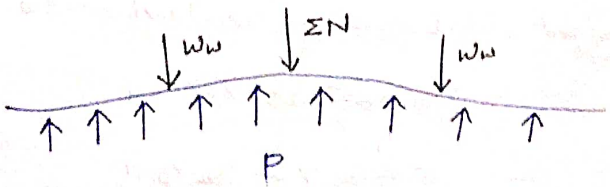




X–X section



Plan



→ For static eqm of soil mass at section x-x in normal direction

$$\Sigma F_y = 0$$

$$P - \Sigma N - W_w = 0$$

$$P = \Sigma N + W_w$$

$$\frac{P}{A} = \frac{\Sigma N}{A} + \frac{W_w}{A}$$

$\frac{P}{A}$ = Total stress (σ)

$\frac{\Sigma N}{A}$ = Effective ($\bar{\sigma}$) stress

$$\sigma = \bar{\sigma} + u\frac{A_w}{A}$$

Total area of soil at x-x ← $A = A_w + A_{sc}$ → Total area of solids at contact

A_w → Total area of water

$$\frac{W_w}{A} = \frac{V_w \gamma_w}{A} = \frac{h A_w \gamma_w}{A}$$

$h\gamma_w = u$

$A_{sc} <<< A$

Assume, $A \cong A_w$

$$\sigma = \bar{\sigma} + u$$

→ Total stress & pore water pressure are measurable.

→ The effective stress ($\bar{\sigma}$) is not a physical parameter and cannot be measured.

→ It can be only calculated by subtracting the neutral stress from total stress.

→ Effective stress is not a physical parameter reason being it is considered to be acting upon total area of soil, but in actual it acts at the pt of contact of the soil solids. Hence

$$\text{Actual contact stress } \bar{\sigma}'_A = \frac{\Sigma N}{A_{sc}}$$

Since $A_{sc} <<< A$, actual contact stress, $\bar{\sigma}'_A$ is much greater than $\bar{\sigma}$.

→ Since area of solids at contact cannot be measured, all the engg. aspects of the soil are related to effective stress ($\bar{\sigma}$)

→ The concept of effective stress even if it is not measurable parameter is still very useful as all prop. of the soil (mechanical like shear strength, consolidation, bearing capacity, lateral pressure) are dependent on this instead of total stress or neutral stress.

At section x-x,

$$\sigma = h_1\gamma + h_2\gamma_{sat}$$

$$u = h\gamma_w$$

$$\text{Stress} = \frac{\text{load/force}}{\text{area}}$$

$$= \frac{\text{wt of soil/water}}{\text{area}}$$

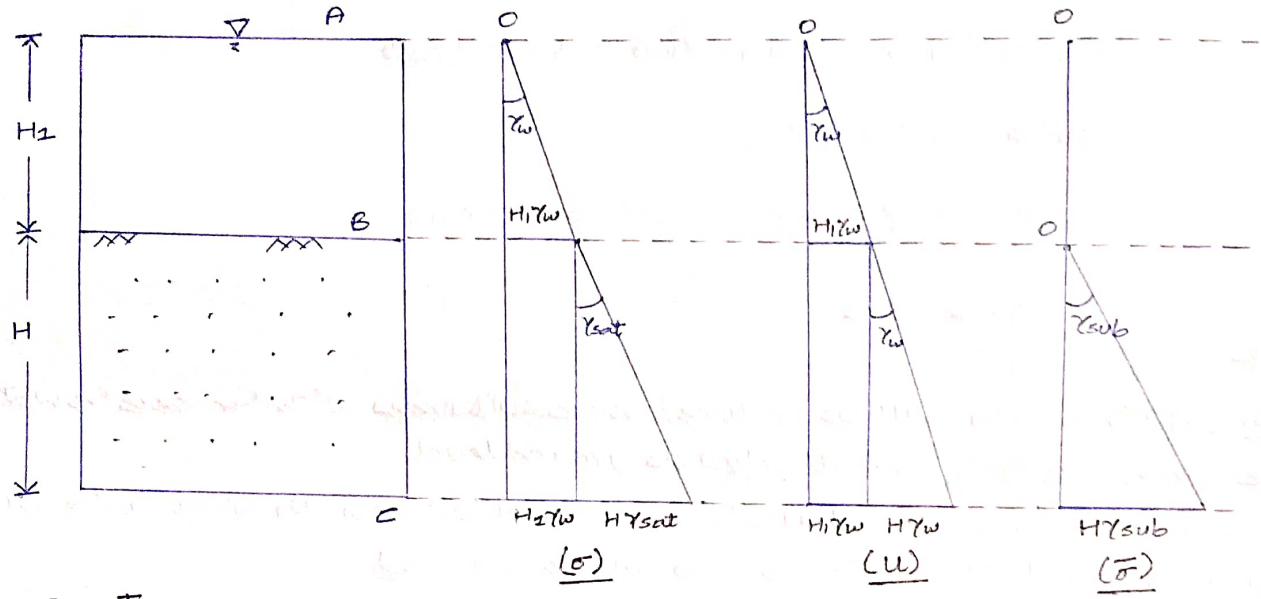
$$\text{Stress} = \frac{V\gamma}{A} = h\gamma$$

$$\bar{\sigma} = \sigma - u = h_1\gamma + h\gamma_{sat} - h\gamma_w$$

$$\bar{\sigma} = h_1\gamma + h(\gamma_{sat} - \gamma_w)$$

$$\bar{\sigma} = h_1\gamma + h\gamma_{sub}$$

CASE 1: Submerged soil mass

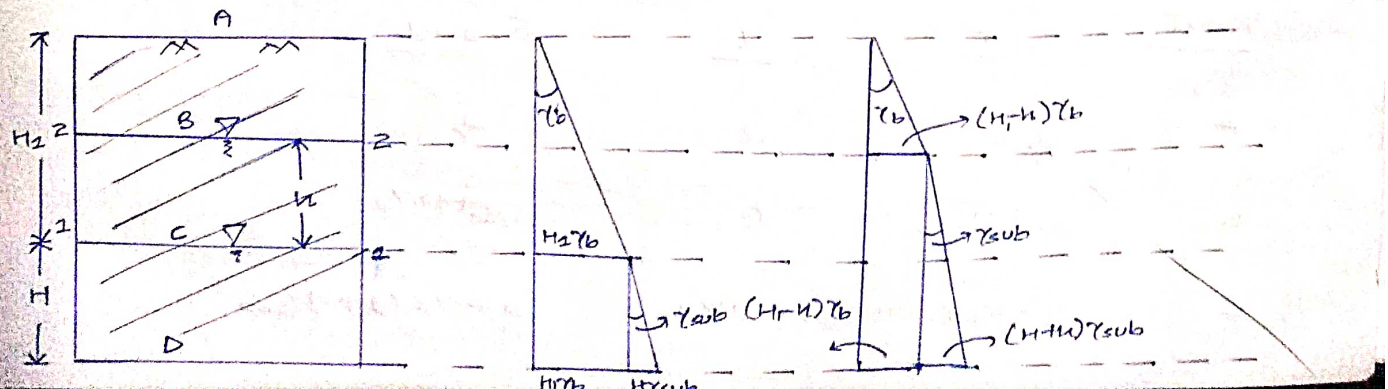


Point	σ	u	$\bar{\sigma} = \sigma - u$
A	0	0	0
B	$H_1\gamma_w$	$H_1\gamma_w$	0
C	$H_1\gamma_w + H\gamma_{sat}$	$(H+H_1)\gamma_w$	$H\gamma_{sub}$

NOTE: If the water table level is above ground level, the variation in ground water table level will have no effect over effective stresses (like in case of lakes or ponds), as total stress and pore water pressure changes by same magnitude.

→ But if water table level is below ground level, change in ground water table level also causes corresponding change in effective stresses.

CASE II: When water table is below the ground level and is rising



→ Effective stress at pt 'C' when water table level is at section 1-1

$$\bar{\sigma} = \sigma - u = (H_1\gamma + H\gamma_{sat}) - H\gamma_w = H_1\gamma + H\gamma_{sat} - H\gamma_{sub}$$

→ Eff. stress at pt 'C' when W.T. level rises by 'h' and is at section 2-2

$$\bar{\sigma} = \sigma - u = (H_1 - h)\gamma + (H+h)\gamma_{sat} - (H+h)\gamma_w$$

$$= (H_1 - h)\gamma + (H+h)\gamma_{sub}$$

→ Change in effective stress at pt 'C'

$$\Delta\bar{\sigma} = (H_1 - h)\gamma + (H+h)\gamma_{sub} - H_1\gamma - H\gamma_{sub}$$

$$\Delta\bar{\sigma} = -h\gamma + h\gamma_{sub}$$

$$\Delta\bar{\sigma} = h(\gamma_{sub} - \gamma) \quad \text{since } \gamma > \gamma_{sub}$$

$$\Delta\bar{\sigma} = (-)ve$$

**

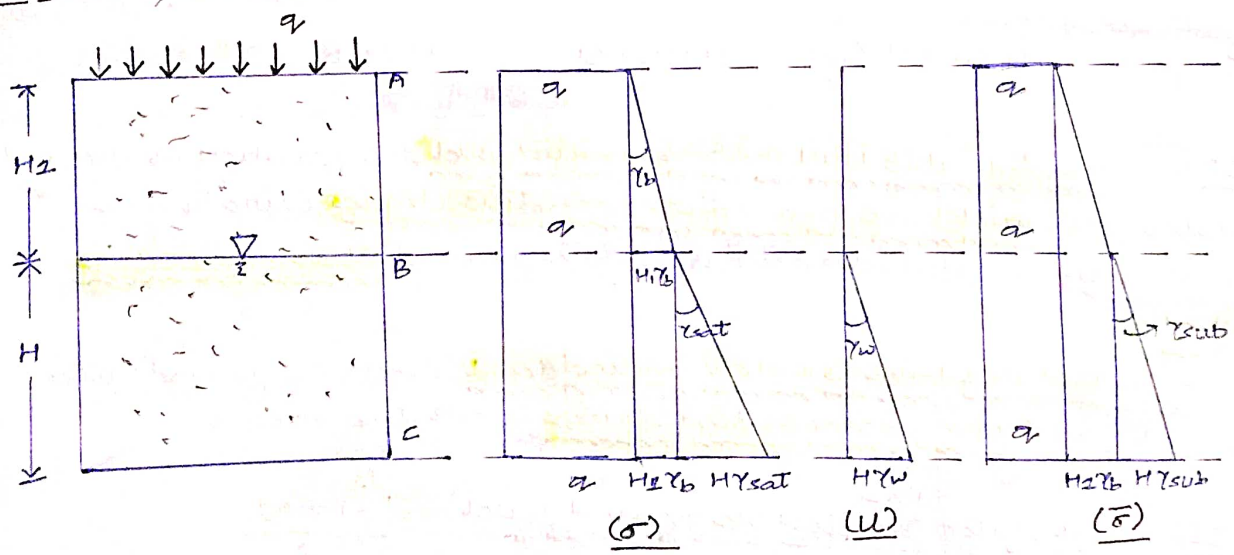
NOTE: Effective stress will be reduced in the soil mass with increase in water table level when it is present below the ground level.

→ If water table is lowered due to pumping out of the water, when water table is below the ground level, effective stress will be increased.

a. In short term: soil over lowered water table remains saturated

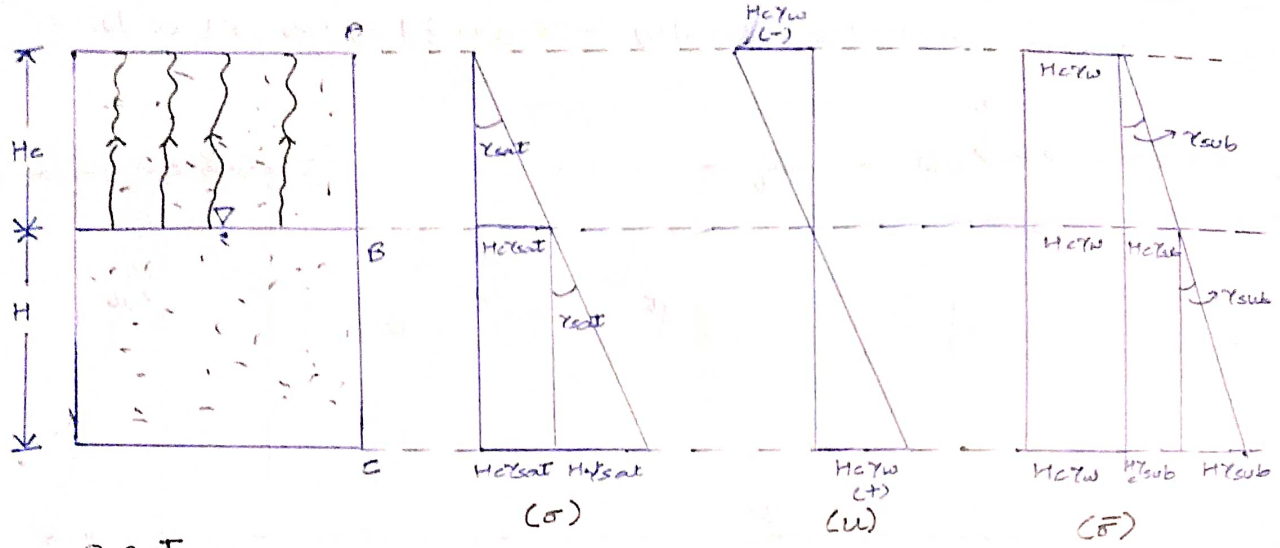
b. In long term: soil over lowered water table will reach its bulk state.

CASE 3: Soil mass with surcharge & water table below the ground level



Point	σ	u	$\bar{\sigma} = \sigma - u$
A	q	0	q
B	$q + H_2\gamma_b$	0	$q + H_2\gamma_b$
C	$q + H_2\gamma_b + H\gamma_{sat}$	$H\gamma_w$	$q + H_2\gamma_b + H\gamma_{sub}$

CASE 4: Soil mass with capillary fringe (soil is saturated with capillary water above GWT)



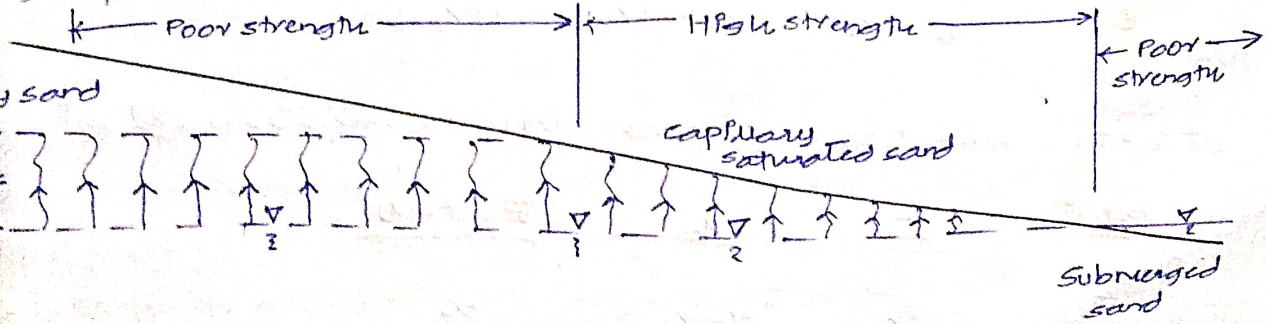
Point	σ	u	$\bar{\sigma} = \sigma - u$
A	0	Capillary tension $-Hc\gamma_w$ ← Meniscous → $Hc\gamma_w$ Capillary pressure	
B	$Hc\gamma_{sat}$	0	$Hc\gamma_{sat}$ or $Hc\gamma_w + Hc\gamma_{sub}$
C	$Hc\gamma_{sat} + H\gamma_{sat}$	$H\gamma_w$	$Hc\gamma_{sat} + H\gamma_{sub}$ or $Hc\gamma_w + (Hc+H)\gamma_{sub}$

NOTE: The effect of capillarity is same as that of surcharge ($q = Hc\gamma_w$) bcoz it helps in increasing the effective stress.

* → If the soil above pt 'B' would have been saturated due to GW instead of capillary water, effective stress in the soil would have decreased due to increase in porewater pressure (by $Hc\gamma_w$)

* The effect of capillarity on eff. stress can be calculated by considering soil to be saturated by ground water + adding capillary pressure ($Hc\gamma_w$).

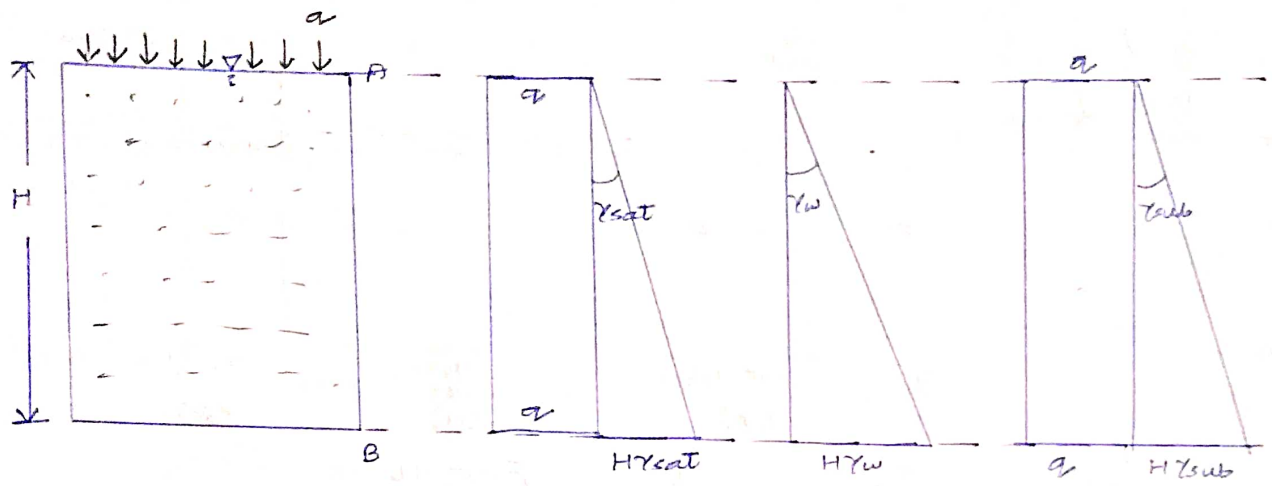
NOTE: This is the reason why we can walk comfortably on a sand beach adjacent to the sea, on a patch falling within capillary zone, but not on dry sand above capillary zone some distance further away from the sea, neither on the submerged soil patch below seawater both of which exhibit less bearing capacity or strength, due to absence of capillary pressure.



CASE V: Soil mass with surcharge and water table is at ground level

a. When surcharge is applied gradually

→ If surcharge is applied gradually, it is carried by the soil solids.



Point	σ	u	$\bar{\sigma} = \sigma - u$
A	q	0	q
B	$q + H\gamma_{sat}$	$H\gamma_w$	$q + H\gamma_{sub}$

b. When surcharge is applied instantly/suddenly

→ When surcharge is applied instantly & WTL is at ground level, it is carried by the pore water which causes inc. in pore water pressure due to which pore water seeps out from the voids of the soil subsequently transferring the surcharge over the soil solids.

→ When the entire excess pore water pr. is dissipated out, total surcharge is carried by soil solids.

at $t = 0$

Point	σ	u	$\bar{\sigma} = \sigma - u$
A	q	q	0
B	$q + H\gamma_{sat}$	$q + H\gamma_w$	$H\gamma_{sub}$

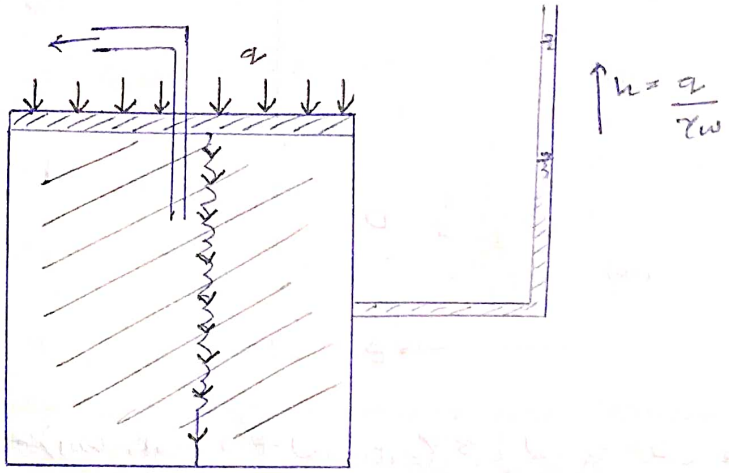
at $t = t$, when entire excess pore water pr. is not dissipated out

Point	σ	u	$\bar{\sigma} = \sigma - u$
A	q	$q - x$	x
B	$q + H\gamma_{sat}$	$q - x + H\gamma_w$	$x + H\gamma_{sub}$

at $t = t_{final}$

A	q	0	q
B	$q + H\gamma_{sat}$	$H\gamma_w$	$q + H\gamma_{sub}$

* Spring analogy:



CASE VI: Partially saturated soil

→ If soil is partially saturated, air is also present along with water in the voids of the soil mass hence in analysis of effective stress pore air pressure is also considered along with pore water pressure

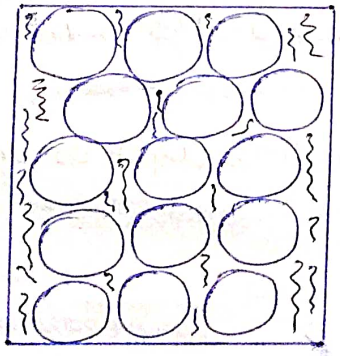
→ In this case, effective stress is given by

$$\bar{\sigma} = \sigma - u_a + \chi(u_a - u_w)$$

u_a = pore air pressure
u_w = pore water pressure
χ = fraction of area of soil occupied by water

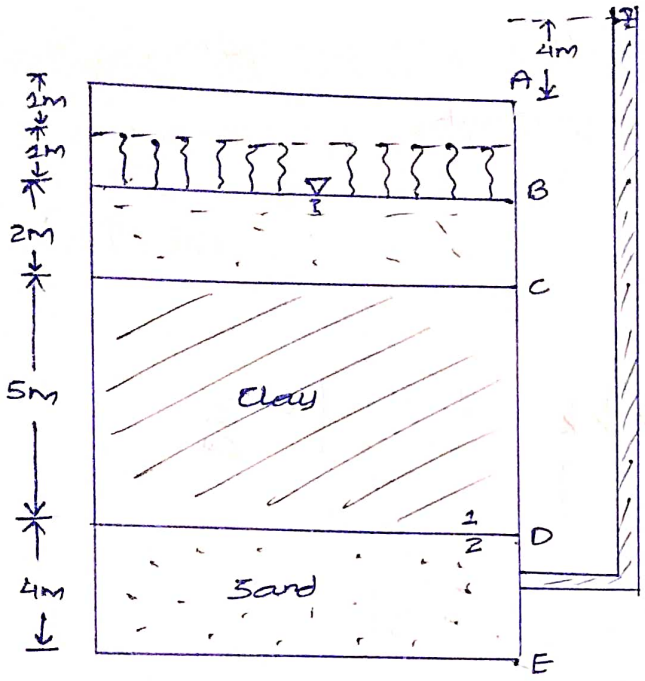
depends upon degree of saturation

$$\chi = \frac{A_w}{A}$$



Q. A 5m thick clay layer lies b/w 2 layers of sand each of 4m thick, the top upper layer being at ground level. The water table is 2m below GL, but the lower layer layer of sand is under artesian pr., the piezometric surface is 4m above the ground level. The saturated unit wt of clay is 20KN/m³ & that of sand is 19 KN/m³. Above the WT unit wt of sand is 16.5 KN/m³. Calculate total stress, pore w.p., eff. stress at the top & bottom of clay

If there is capillary fringe also available over the WTL of 1M.



confined aquifer,
Pressure > Hydrostatic pr.

$$\bar{\sigma}_{D_1} = (1 \times \gamma_{sand} + 3\gamma_{sat,sand} + 5\gamma_{sat,clay}) - (7\gamma_w)$$

$$\bar{\sigma}_{D_1} = 104.83 \text{ kN/m}^2$$

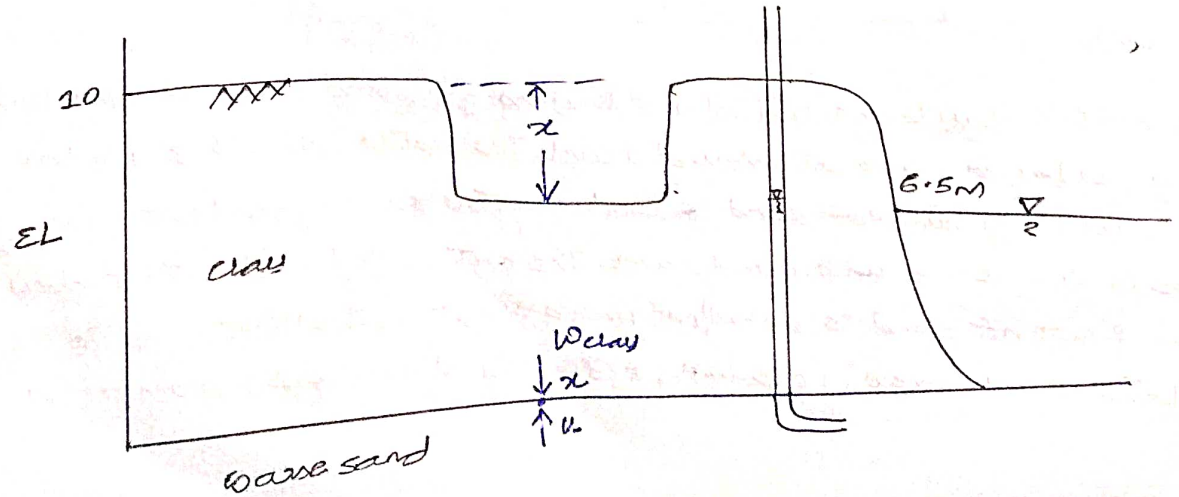
$$\bar{\sigma}_{D_2} = (1 \times \gamma_{sand} + 3\gamma_{sat,sand} + 5\gamma_{sat,clay}) - (3 \cdot 13\gamma_w)$$

$$\bar{\sigma}_{D_2} = 45.97 \text{ kN/m}^2$$

***

Q. A foundation trench is to be excavated in a stratum of stiff clay, 10m thick, underlain by a bed of coarse sand. In a trial bore hole, the GWT level was observed to rise at an elevation of 3.5m below the ground surface. determine the depth upto which excavation can be safely carried out with danger of the bottom becomes unstable under the artesian pr. in sand. $G_{clay} = 2.75$, $e = 0.8$

pp. If the excavation is to be carried upto the depth of 8M, how much water table level should be lowered



P. For eqm of bottom of excavation

$$\Sigma F_{dx} = 0$$

$$W_{day} - W_w = 0$$

or

$$(10 - x)\gamma_{sat,day} - 6.5\gamma_w = 0$$

$$x = 6.7\text{M}$$

PP.

$$(8 - 6.7)\gamma_{sat,day} - x\gamma_w = 0 \quad \text{or} \quad 2\gamma_{sat} - h\gamma_w = 0$$

$x = 2.55\text{M}$ ← decrease in water table level

$h = \checkmark$ → final water level

Final WT level = $6.5 - 2.55 = 3.95\text{M}$

PERMEABILITY

→ It is the ability of the medium of to permit the flow of fluid (water) through its interconnecting voids.

→ It is also termed as "Hydraulic conductivity"

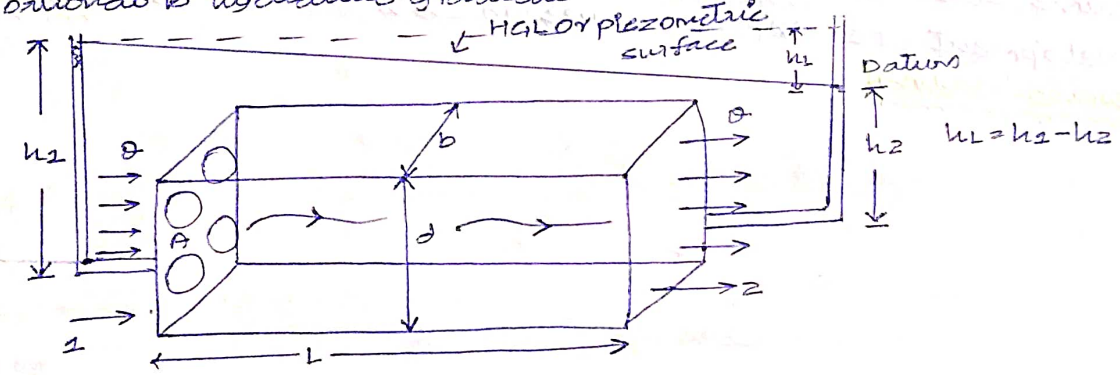
→ Permeability of coarse grained soil is more than fine grained soil

Type of soil	Permeability (cm/sec)
Gravel	>1
Sand	$1-10^{-3}$
Silt	$10^{-3}-10^{-7}$
Clay	$<10^{-7}$

→ Darcy's law: For saturated soil mass, in laminar flow condition, rate of flow through the medium per unit area under unit hydraulic gradient is termed as 'Permeability'

or

→ Velocity of flow, in laminar flow conditions for saturated soil mass is proportional to hydraulic gradient



$v \propto i$ i = hydraulic gradient

$i = \frac{h_L}{L}$

$i = \frac{\text{Hyd. head loss}}{\text{Length of flow}}$

$v = Ki$

$i = \frac{(HL)_1 - (HL)_2}{\text{Length of flow}}$

$Q = Av$

$i = \frac{(-h_2 + h_1)_1 - (-h_2 + h_2)_2}{L'}$ $L' > L$

$Q = KiA$

$i = \frac{(h_1 - h_2)}{L'} = \frac{h_L}{L'}$

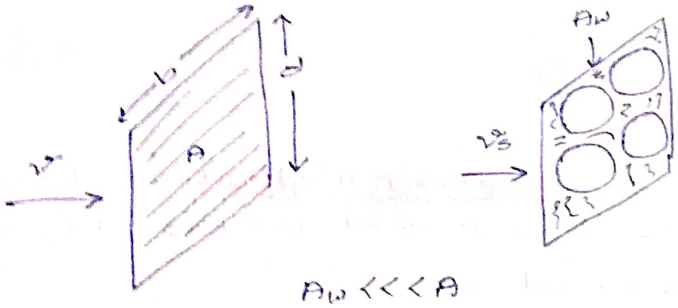
If $i = 1$

Assume $L' = L$

$Q = AK$

** $K = \frac{Q}{A}$ $(L/T$ or $L^3/L^2/T)$

NOTE: The velocity of flow considered by Darcy is average / discharge / superficial / virtual / macroscopic velocity as, flow is assume to be take place through total area of soil but in actual flow of the water takes place through interconnecting voids, area of which is much smaller than total area of soil. Hence actual or seepage velocity of flow comes out to be much more than avg. velocity



$A_w << A$ $\qquad A = A_s + A_w + A_a$

$v_s >> v$

$Q = Av = A_w v_s$

$$\frac{v_s}{v} = \frac{A_w}{A} = \frac{A_w L}{AL} = \frac{A_w L}{AL}$$

$$\frac{v_s}{v} = \frac{V}{V_w}$$

$$\frac{v_s}{v} = \frac{V}{V_w} \cdot \frac{V_v}{V_v} = \frac{1}{\left(\frac{V_v}{V}\right)\left(\frac{V_w}{V_v}\right)} = \frac{1}{\eta \cdot S}$$

$$v_s = \frac{v}{\eta S}$$

$0 \le S \le 1$
$0 < \eta < 1$

$0 < \eta S < 1$

If $S = 1$,

$$v_s = \frac{v}{\eta}$$

NOTE:

$\rightarrow v \propto i \Rightarrow v = ki$

$\rightarrow v_s \propto i \Rightarrow v_s = k_p i$

k = coeff. of permeability

k_p = " " percolation

$$v_s = \frac{v}{\eta} \Rightarrow k_p i = \frac{k i}{\eta}$$

$$k_p = \frac{k}{\eta}$$

→ Percolation is flow in soil itself

NOTE: Darcy's law is not valid in case of gravel, as flow through gravel is not laminar, it is turbulent.

→ For flow to be laminar in case of soil, $Re \leq 1$

* Determination of permeability

→ Permeability of soil can be found by any of the following method.

a. Lab method:

i. Constant head method

→ This method is generally used for coarse grained soil (gravel & sand) for which substantial discharge can be obtained during the test in small time.

→ It is not suitable for fine grained soil.

→ In this method, water is allowed to flow through soil mass/medium under constant head & volume of water passed through it in given time 't' is noted.

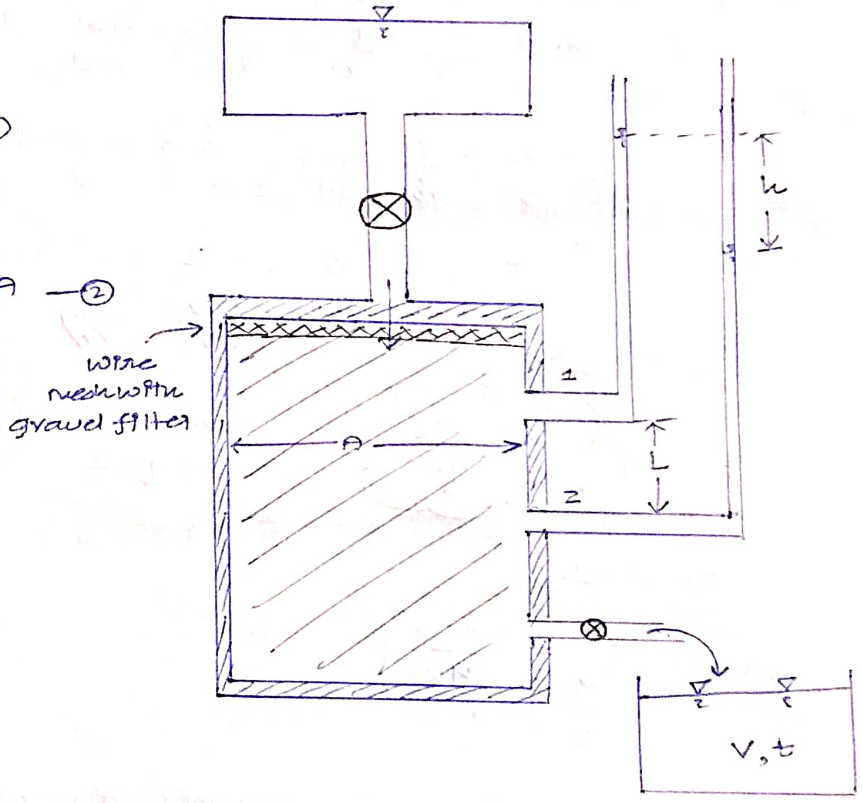
$$Q = \frac{V}{t} \quad \text{—①}$$

$$Q = KiA$$

$$Q = KiA = K\frac{h}{L}A \quad \text{—②}$$

$$K\frac{h}{L}A = \frac{V}{t}$$

** $$K = \frac{V \cdot L}{hAt}$$



ii. Falling head method / Variable head permeability test

→ This method is generally used for fine grained soil for which constant head method is not suitable.

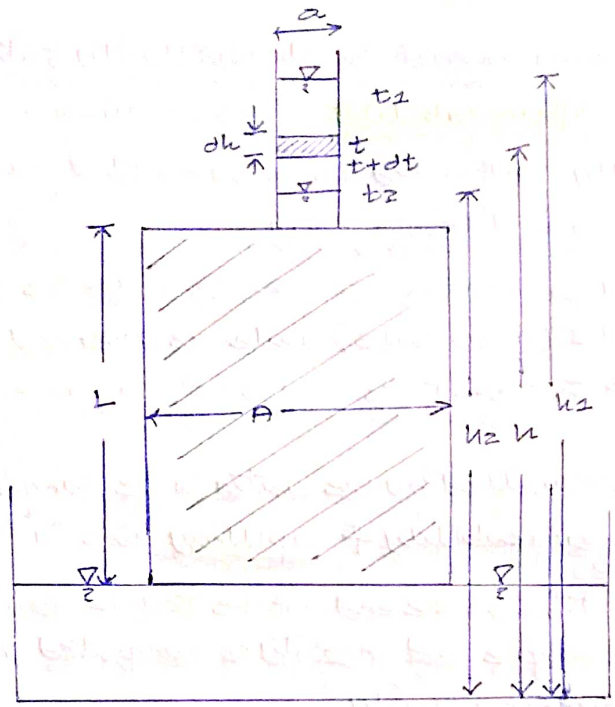
→ In this method, a stand pipe of known area "a" is inserted into the medium & water is allowed to flow through it.

→ As the flow of water takes place through the medium over period of time, lvl of water in stand pipe reduces.

→ In order to compute permeability of medium, rate of water in stand pipe at different time interval is noted.

→ Let at any time interval 't', flow of water through medium takes place under the head of 'h'

→ If water moves by 'dh' distance in stand pipe in time 'dt'.



Volume of water flowing through sand pipe $(dv) = a\,dh$

Discharge through the sand pipe $(dq) = \frac{dv}{dt} = -a\frac{dh}{dt}$ ——①

Discharge through soil $(dq) = KiA$ ——②

$$-a\frac{dh}{dt} = KiA$$

$$-a\frac{dh}{dt} = K\frac{h}{L}A$$

$$\frac{KA}{L}\int_{t_1}^{t_2} dt = \int_{h_1}^{h_2} -a\frac{dh}{h}$$

$$\frac{KA}{L}(t_2 - t_1) = -a\ln h\Big]_{h_1}^{h_2}$$

$$\frac{KA}{L}(t_2 - t_1) = a\ln(h_1/h_2) \qquad \text{let } t_2 - t_1 = t$$

$$\frac{KAt}{L} = a\ln(h_1/h_2)$$

** $K = \frac{La}{At}\ln(h_1/h_2)$

** $K = \frac{2.303La}{At}\log_{10}(h_1/h_2)$

NOTE: If in given time interval 't', wt of water in sand pipe falls from 'h2' to 'h2' & in same time interval it falls from 'h2' to 'h3' then

** $h_2 = \sqrt{h_1 h_3}$

iii. Capillarity permeability test

→ This test is also known as horizontal capillarity test

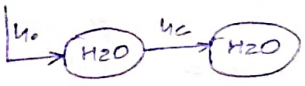
→ It is used to find both permeability of the medium and capillary rise in it.

→ In this test, a partially saturated sample of soil is placed in a cylindrical glass tube of dia 4cm and length 35cm.

→ Water is allowed to flow through the sample under the constant head applied at the inlet of the tube (glass tube) while the other end of the tube is kept open to the atmosphere to permit the escape of air present in the voids of the sample.

→ Horizontal distance travelled by the water in the sample at diff. time is noted to analyse its permeability & capillary rise in it.

→ The degree of saturation at the end of the test is found by taking the wet mass of the soil sample and porosity is computed from the known dry mass, volume and specific gravity.



Head at inlet pt A = h_0

" " point B = $-h_c$

" under which flow takes place from point 'A' to 'B' = $h_0 - (-h_c)$

$= h_0 + h_c$

Hydraulic gradient, $i = \frac{h_0 + h_c}{x}$

Let water move by distance 'dx' in time 'dt'

Velocity (seepage/actual) of water (v_s) = $\frac{dx}{dt}$

$$v_s = \frac{v}{\eta_s}$$

$$v = v_s \eta_s \quad \text{—①}$$

As per Darcy's law, $v = ki$ (But for $S = 1$)

For partially saturated soil sample, Darcy's law is modified,

$v = k_w i$ —② k_w = permeability of partially saturated soil

$$v_s \eta^s = K_u i^o$$

$$\frac{\partial x}{\partial t} \eta^s = K_u \left(\frac{h_o + h_c}{x} \right)$$

$$\int_{x_1}^{x_2} x \partial x = \frac{K_u}{\eta^s} (h_o + h_c) \int_{t_1}^{t_2} \partial t$$

$$\frac{x^2}{2} \Big|_{x_1}^{x_2} = \frac{K_u}{\eta^s} (h_o + h_c)(t_2 - t_1)$$

$$\frac{x_2^2 - x_1^2}{2} = \frac{K_u}{\eta^s} (h_o + h_c)(t_2 - t_1)$$

** $$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2K_u}{\eta^s} (h_o + h_c)$$

→ In the above equation, there are 2 unknown, i.e K_u & h_c, hence two sets of readings of distance (x), travelled by the water in time 't' is noted in this test under the constant head 'h_{o1} + h_{o2}'

$$\left(\frac{x_2^2 - x_1^2}{t_2 - t_1} \right)_1 = \frac{2K_u}{\eta^s} \left(h_{o1} + h_c \right) \quad \text{——} (1)$$

$$\left(\frac{x_2^2 - x_1^2}{t_2 - t_1} \right)_2 = \frac{2K_u}{\eta^s} \left(h_{o2} + h_c \right) \quad \text{——} (2)$$

Q. $$\frac{(10^2) - (3)^2}{8} = \frac{2K_u}{0.32 \times 0.92} (60 + h_c) \quad \text{——} (1)$$

SG = WG

0.92

$$\frac{(29)^2 - (10)^2}{24} = \frac{2K_u}{0.32 \times 0.92} (232 + h_c) \quad \text{——} (2)$$

(2) ÷ (1)

$$429 \frac{\cancel{444}}{\cancel{24}_3 \times 91} \times 8 = \frac{232 + h_c}{60 + h_c}$$

$$104.615 + 1.743 h_c = 232 + h_c$$

h_c = 237.5 cm

K_u = 9.38 × 10^{-5} cm/sec

b. Indirect methods to find permeability

i. Kozney Carman method

** $K = \frac{1}{K_{KS}} \cdot \frac{\gamma_w}{\mu} \cdot \frac{e^3}{1+e} \cdot d^2$

** $K = \frac{1}{K_{K'}} \cdot \frac{\gamma_w}{\mu} \cdot \frac{e^3}{1+e} \cdot \frac{1}{(S_S)^2}$

* Here K_{KS} & $K_{K'}$ are Kozney Carman constant which depends upon shape & tortuosity (curvature) of voids

d = size of particle

S_S = specific surface area = surface area per unit volume

μ = dynamic viscosity

→ For spherical particle,

$$S_S = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r} = \frac{6}{d}$$

→ If concerned particle is not spherical and passes through sieve of size 'A' & retains over sieve of size 'B', then specific surface area

* Specific surface area = $\frac{6}{\sqrt{AB}}$

ii. Allen Hazen method

$K = C \cdot D_{10}^2$ (cm/sec)

$C = 100$

** $K = 100 D_{10}^2$ cm/sec D_{10} = cm

iii. consolidation

** $K = C_v \cdot M_v \cdot \gamma_M$

C_v = coeff. of consolidation

M_v = coeff. of volume compressibility

iv. Terzaghi's method

**

$K = 200 e^2 De^2$ cm/sec

cm

e = void ratio

De = size of sphere for which ratio of volume to surface area is same as that of given particle



$\left(\frac{V}{SA}\right)_{actual\ particle} = \left(\frac{V}{SA}\right)_s$

**

$De = 6\left(\frac{V}{SA}\right)_{AP}$

v. Louden's method:

**

$\log_{10}(K \cdot SS^2) = a + b\eta$ (cm/sec)

SS = spec. surface area = $\frac{6}{d \rightarrow cm}$

$a = 1.3$, $b = 5.1$

η = porosity

vi. Jaky's method

$K = 100\, Dm^2$

→ "Dm" denotes the size of the particle whose frequency of occurence is maximum. (cm)

c. Field method

→ Permeability of the soil found in laboratory is representing properties of sample which differs from the actual permeability of the soil in field.

→ Lab test must be performed as far as possible on undisturbed soil sample but in the type of soil for which permeability is important (sand, silty sand etc), undisturbed sample possess large uncertainities, hence as these samples lab tests are not reliable & should be tested by field methods to note true values of permeability.

→ Permeability in the fields can be found by any of the following methods

p. Pumping out test

→ This method is used for soil having large area of influence eg. Dams, reservoirs.

→ In this method, water is pumped out from the medium with the help of wells in order to find its permeability.

Case a: Well in unconfined aquifer

→ Dupit's theory:

$$Q = \frac{\pi K (H^2 - h_0^2)}{2.303 \log_{10}(R/r_w)}$$

**
$$K = \frac{2.303 Q \log_{10}(R/r_w)}{\pi (H^2 - h_0^2)}$$

→ Theim's theory:

$$Q = \frac{\pi K (h_2^2 - h_1^2)}{2.303 \log_{10}(r_2/r_1)}$$

**
$$K = \frac{2.303 Q \log_{10}(r_2/r_1)}{\pi (h_2^2 - h_1^2)}$$

Case b: Well in confined aquifer

→ Dupit's theory:

$$Q = \frac{2\pi KD (H - h_0)}{2.303 \log_{10}(R/r_w)}$$

**
$$K = \frac{2.303 Q \log_{10}(R/r_w)}{2\pi D (H - h_0)}$$

→ Theim's theory:

$$Q = \frac{2\pi KD (h_2 - h_2)}{2.303 \log_{10}(r_2/r_1)}$$

T = KD = Transmissibility or coeff of transmissibility

$T = \frac{Q}{b}, i = 1$

**
$$K = \frac{2.303 Q \log_{10}(r_2/r_1)}{2\pi D (h_2 - h_2)}$$

ii. Pumping in test

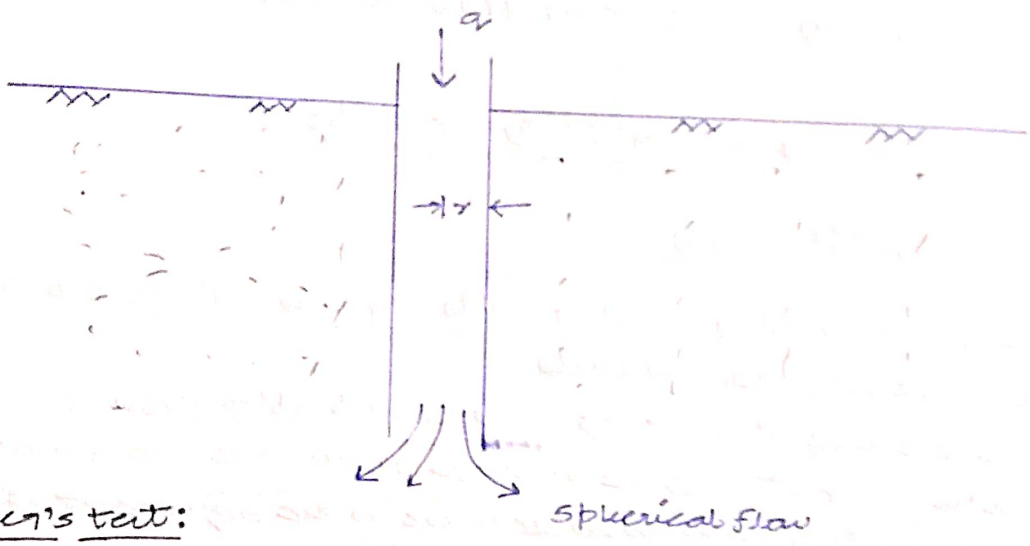
→ This method is used for small area of influence
→ These method includes following test

a. Open end test: In this test, an open end pipe is sunk into the soil medium and soil is taken out of the pipe just to the bottom
→ clean water having temp. slightly greater than the ground water is added through a metering system to maintain gravity flow under the constant head.
→ water may also be allowed to enter into the hole under some pressure head.

→ The permeability of the medium is given by,

$$K = \frac{q}{5.5 r h}$$

h = head under which flow takes place
r = radius of well/casing
q = constant rate of flow.



Spherical flow

b. Packer's test:

→ In this test, pipe plugged at bottom & perforated over it's surface upto distance 'L' from the bottom is sunk into the medium & water is allowed to flow in the medium under constant head.

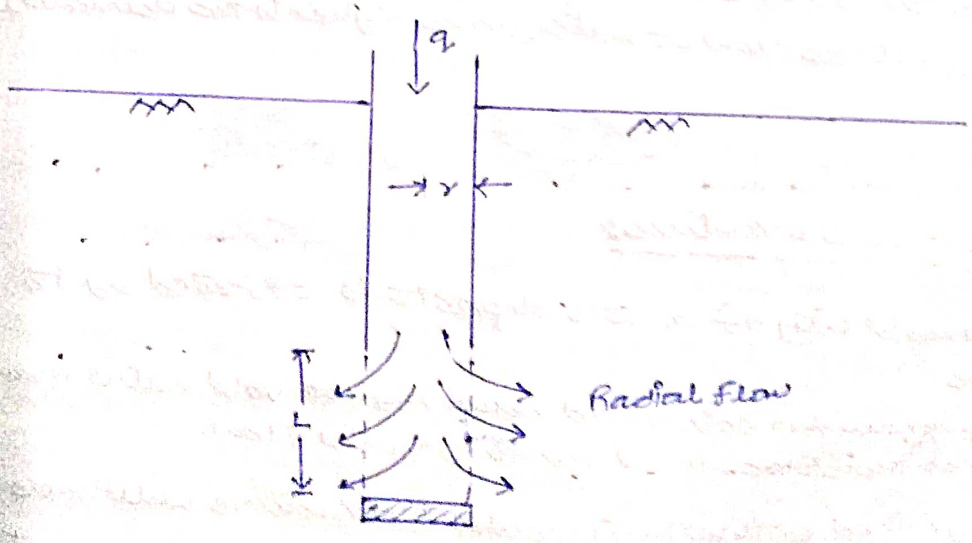
Permeability of the medium is given by

* If $L \geq 10r$, **

$$K = \frac{q}{2\pi L h} \log_{10}\left(\frac{L}{r}\right)$$

* If $r \leq L < 10r$, **

$$K = \frac{q}{2\pi L h} \sinh^{-1}\left(\frac{L}{2r}\right)$$



Radial flow

NOTE: Tests suitable for a particular type of soil is as follows

→ Clean gravel and sand : constant head method

→ Fine sand, sandy silt, silt : Falling head method

→ Homogeneous clay : consolidation test

* Factors affecting permeability of soil:

1. Size of the particle (D_{10})

→ The permeability of granular soils depends mainly on the cross-sectional area of pore channels.

→ Since the average dia. of the pores in soil at a given porosity increase in proportion to the average grain size, the permeability of granular soil is expected to increase as the square of effective size of grain

$K \propto D_{10}^2$ or $K \propto d^2$ d = size of void

2. Specific surface area (SS):

→ Higher is the specific surface area, more is the resistance available to the flow of water, hence lower is the permeability

$$K \propto \frac{1}{(SS)^2}$$

3. Degree of saturation (S):

→ Higher is the degree of saturation, more will be the flow channels available for water to flow & lesser will be the resistance offered by air against the flow of water, hence higher is the permeability

$K \propto S$

4. Effect of soil structure:

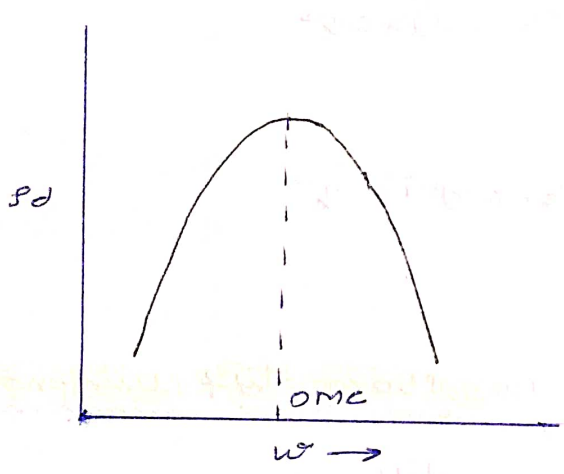
→ The permeability of a soil deposit is effected by its in-place structure

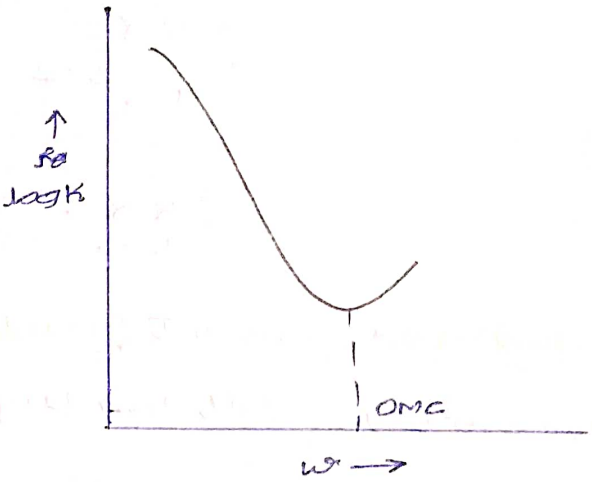
→ A loose granular soil would have higher void ratio than a dense soil, hence therefore would permit greater flow.

→ A fine grained soil with flocculant structure will have higher permeability than soil with dispersed structure even at same

void ratio.

NOTE: Variation of permeability with water content (w) & dry density (γ_d)





5. Effect of soil type:

→ The volume of water that can flow through a soil mass is related more to the size of void opening than the number of openings (voids).

$K_{coarse\ grained\ soil} > K_{fine\ grained\ soil}$

* $K_{coarse\ grained\ soil} = f$ [particle size, gradation, particle shape, roughness & void ratio]

* $K_{fine\ grained\ soil} = f$ [Type of clay mineral & adsorbed ions on the particles]

→ Depending upon type of cation adsorbed on the surface of montmorillonite clay mineral, the permeability is increased in following order,

$K < Na < H < Ca$

→ For Kaolinite,

$Na < K < Ca < H$

→ From the above relationship, Na clays have a relatively low permeability & it can be utilised in construction activities like core of earthen dam, where after construction it is treated with salt water to reduce the seepage.

6. Degree of compaction:

→ With increase in degree of compaction, denseness of soil increases which reduces area of flow channel that in turn reduces permeability

7. Void ratio

→ Keeping all the factors to be constant, higher is the void ratio, higher is the area of flow channel, hence higher is the permeability.

$$K \propto \frac{e^3}{1+e}$$ Kozney's eqn

$$K \propto e^2$$ Terzaghi's eqn

8. Properties of permeant (fluid)

→ Permeability of soil depends upon both soil property & fluid property

$$K = f[\text{fluid property, medium property}]$$

→ Hence, it is difficult to compare the permeability of 2 different soils having 2 diff. fluids flowing through it.

→ In order to avoid this a parameter termed as coefficient of intrinsic or absolute permeability is defined which depends only upon fluid medium property.

$$K = C\frac{\gamma_w}{\mu}\frac{e^3}{1+e}D_{10}^2$$ ** 1 darcy = 0.987×10^{-8} cm²

$$K' = \frac{K\mu}{\gamma_w}$$ $\frac{K\mu}{\gamma_w} = K'$

K' or $K_0 = \frac{L}{T}\frac{FT}{L^2}L^3$ **

$$K' = C\frac{e^3}{1+e}D_{10}^2$$

K' or $K_0 = L^2$

γ_w, μ = fluid properties
e, D_{10}, C = medium property

$$K = f[\text{fluid property, medium property}]$$

m², cm², mm² or darcy ← K' or $K_0 = \frac{K\mu}{\gamma_w}$ = coeff. of absolute or intrinsic permeability

→ Fluid prop. 'γ_w' & 'μ' depends upon temperature

If $T \uparrow \Rightarrow \gamma_w \downarrow$, $\mu \downarrow$

→ With change in temp., both the density & viscosity of water changes but change in density is very less in comparison to viscosity, hence the dependency of permeability maybe considered to be on viscosity for calculation purpose.

$$K = C \frac{\gamma_w}{\mu}$$

If $T \uparrow \Rightarrow K = C \frac{\gamma_w \downarrow}{\mu \downarrow\downarrow}$

**



→ Hence for a given void ratio & structural arrangement of particles 'K' remains constant irrespective of fluid property.

9. Entrapped gases:

→ Higher is the presence of entrapped gases, more is the resistance offered to the flow of water & lesser is the area of flow channel hence smaller is the permeability.

10. Foreign Impurities:

→ More is the presence of foreign impurities, higher is the resistance available to the flow of water, hence smaller is the permeability.

11. Adsorbed water:

→ Higher is the presence of adsorbed water, lower is the permeability as lower is the area of flow channel available for free water to flow.

* → It is assumed that in the presence of adsorbed water, void ratio is reduced by 0.1.

* Effective void ratio (e') = $e - 0.1$

* Permeability of stratified soil deposits

→ In nature, soil mass consists of several layers deposited one above each other. Their bedding planes may be horizontal or vertical or inclined.

→ Each layer is assumed to be homogeneous & isotropic has its own value of coeff. of permeability.

→ The avg. permeability of the whole deposit will depend upon direction of flow with relation to the bedding plane as follows

9. Parallel to the bedding plane:

→ When flow takes place parallel to bedding plane through each layer head loss is same but rate of flow is different.

$$\theta = q_1 + q_2 + q_3 + \ldots + q_n$$

$$\theta = k_{avg\parallel} iA$$

$$Q = K_{avg\,||}\, i\, ((z_1 + z_2 + z_3 + \ldots + z_n) \times 1)$$

$$Q = K_{avg\,||}\, i \sum_{i=1}^{n} z_i$$

$$q_1 = K_1 i_1 A_1 = K_1 i_1 (z_1 \times 1)$$

$$q_2 = K_2 i_2 A_2 = K_2 i_2 (z_2 \times 1)$$

$$\vdots$$

$$q_n = K_n i_n A_n = K_n i_n (z_n \times 1)$$

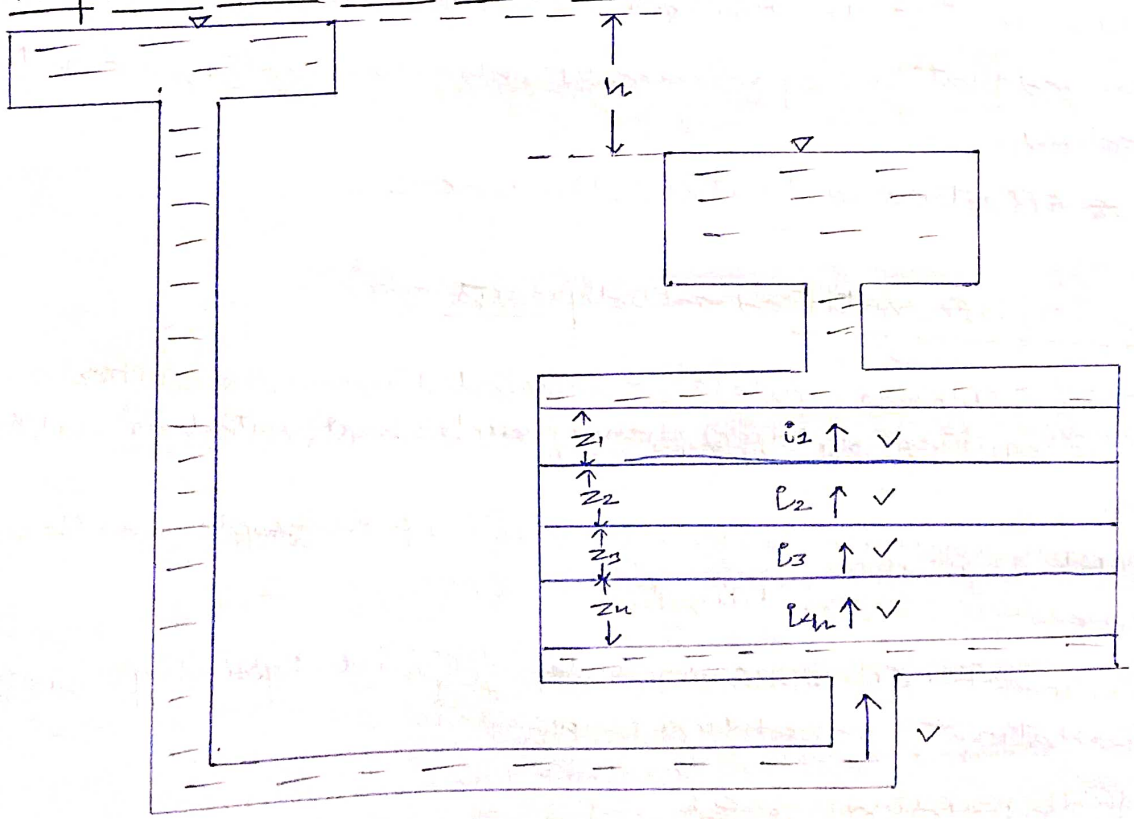
Here $i = \frac{h}{L}$, $i_1 = i_2 = \ldots = i_n = \frac{h}{L}$

$$K_{avg\,||} \cdot \frac{h}{L} \cdot \sum_{i=1}^{n} z_i = (K_1 z_1 + K_2 z_2 + \ldots + K_n z_n)\frac{h}{L}$$

**

$$K_{avg\,||} = \frac{\sum_{i=1}^{n} K_i z_i}{\sum_{i=1}^{n} z_i}$$

b. Perpendicular to the bedding plane



→ In this case, the velocity of flow & discharge remains same, through each layer but loss of head differs for each layer.

$h = h_1 - h_2$

$h = h_1 + h_2 + \dots + h_n$

$Q = K_{avg\perp} \, i \, A$

$Q = K_{avg\perp} \frac{h}{Z} \cdot A$

$A_1 = A_2 = \dots A_n = A = L \times 1$

$Q_1 = Q_2 = \dots Q_n = Q$

$$h = \frac{Q \sum_{i=1}^{n} Z_i}{K_{avg\perp} \cdot A}$$

$Q_1 = K_1 i_1 A_1 = K_1 \cdot \frac{h_1}{Z_1} \cdot A \Rightarrow h_1 = \frac{Q_1 Z_2}{K_1 A} = \frac{Q Z_1}{K_1 A}$

$Q_2 = K_2 i_2 A_2 = K_2 \frac{h_2}{Z_2} A \Rightarrow h_2 = \frac{Q Z_2}{K_2 A}$

$Q_n = K_n i_n A_n = K_n \frac{h_n}{Z_n} A \Rightarrow h_n = \frac{Q Z_n}{K_n A}$

$$\frac{Q \sum_{i=1}^{n} Z_i}{K_{avg\perp} A} = \frac{Q Z_1}{K_1 A} + \frac{Q Z_2}{K_2 A} + \frac{Q Z_3}{K_3 A} + \dots + \frac{Q Z_n}{K_n A}$$

** $$K_{avg\perp} = \frac{\sum_{i=1}^{n} Z_i}{\sum_{i=1}^{n} \frac{Z_i}{K_i}}$$

NOTE: stresses in vertical direction are observed to be comparatively more than in horizontal direction,

$\sigma_V > \sigma_H \Rightarrow K_{avg\parallel} > K_{avg\perp}$

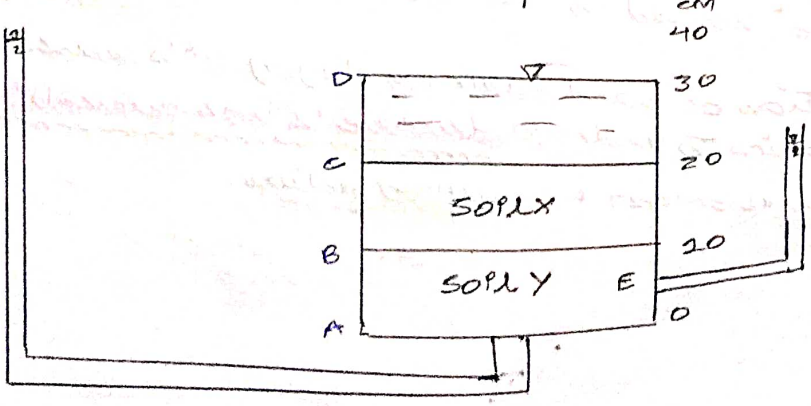
Q. In the given soil 'X' has permeability of 4×10^{-3} cm/sec and headloss in soil 'Y' is 'q' times the headloss in soil 'X', then find $A = 10 cm^2$

i. permeability of soil Y

ii. Quantity of flow per hour

iii. To what elevation would water rise in a piezometer inserted in soil 'Y' at elevation of 5cm. What is pressure head at this point?

Sol.



Points	Datum head	Pr. head	Total head	Head loss
A	0	40	40	0
B	10	21	31	9
C	20	10	30	10
D	30	0	30	10
E	5	30.5	35.5	4.5

Total headloss (h) = $h_A - h_C$ = 40 − 30 = 10cm

$h_L = h_{Lx} + h_{Ly} = 10$ ——①

$h_{Ly} = 9 h_{Lx}$

$10 = h_L = 10 h_{Lx}$

$h_{Lx} = 1cm$

$h_{Ly} = 9cm$

$i_y = \frac{9}{10} = 0.9$ cm/cm

$h_{LE} = 0.9 \times 5 = 4.5cm$

Since, flow is ⊥ʳ to bedding plane,

$Q_x = K_x i_x A$

$Q_y = K_y i_y A$

$4 \times 10^{-3} \times \frac{1}{10} \times 10 = K_x \times \frac{9}{10} \times 10$

∴ $K_x = 4.44 \times 10^{-4}$ cm/sec

∴ $Q_x = K_x i_x A = 4 \times 10^{-3} \times \frac{1}{10} \times 10 \ \frac{cm^3}{sec}$

$= 4 \times 10^{-3} \times (10^{-2})^3 \times 24 \times 3600$

SEEPAGE ANALYSIS

→ When the water flows through saturated soil mass, total head at any point consists of pressure head

a. Pressure head (Piezometric head)

b. Datum head (Elevation head)

c. Velocity head (Kinetic head)

→ Flow through the soil mass is considerably very less (negligible), hence velocity head is neglected.

→ Thereby, total head at any point consists of Pressure head + datum head. also termed as hydraulic head.

Total head = Pressure head + datum head

→ Elevation head or datum head at any pt is the vertical distance of that pt measured from the datum point or reference point which is normally taken at the tail water level for convenience.

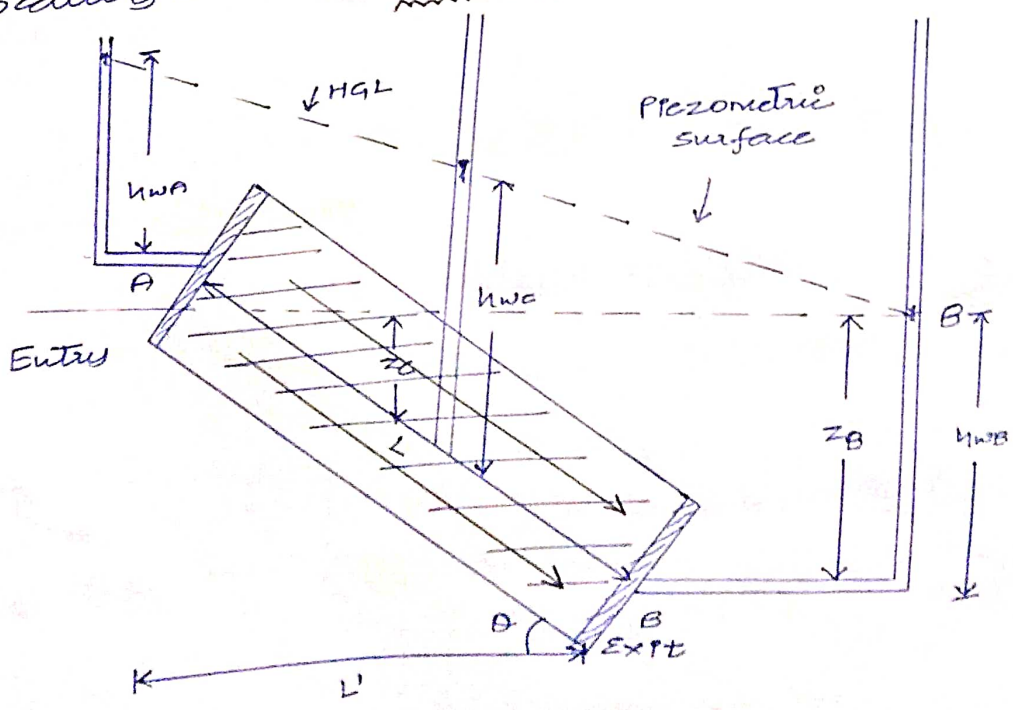
→ If concerned point is above datum, then datum head is positive and if pt is below datum, datum head is negative

→ Pressure head at any any given point is equal to height of the water above the concerned point or ht of water in piezometer or stand pipe inserted at the concerned point.

Total head = Pressure head + datum head

$$H = h_w \pm Z$$

→ Difference of between total head at any 2 pts in soil through which flow is occurring is termed as head loss.



Head	A	B	C
Pressure (h_w)	h_{wA}	h_{wB}	h_{wC}
Datum (z)	$-z_A$	$-z_B$	$-z_C$
Total (H)	$h_{wA}-z_A$	$h_{wB}-z_B=0$ ($h_{wB}=z_B$)	$h_{wC}-z_C$

→ Hydraulic head under which flow is taking place from pt 'A' to 'B'

$$h = H_A - H_B$$

$$h = (h_{wA}-z_A)_B - 0 = h_{wA}-z_A$$

→ Hydraulic gradient, i = $\frac{\text{hyd. head under which flow takes place}}{\text{length of flow}}$

$$i = \frac{h}{L}$$ {(In actual length of flow is more than L) but for simplicity in calculation, it is consd. equal to L}

$$\cos\theta = \frac{L'}{L}$$

$$L = \frac{L'}{\cos\theta}$$

* Seepage pressure:

→ When water flows through saturated soil mass, it exerts the pressure over the soil skeleton by virtue of frictional drag, this pressure exerted by water over the soil solids is termed as **seepage pressure**.

→ If "h" is the hydraulic head or seepage head under which flow takes place, then seepage pressure is given as

$$p_s = h\gamma_w$$

$$i = \frac{h}{L}$$

$$p_s = iL\gamma_w$$

→ This pressure leads to the development of force over the soil mass termed as **seepage force**.

$$P_s = p_s A = iLA\gamma_w$$

A = total x-sectional area of the soil mass

→ This seepage force, when expressed in terms of unit volume of soil is termed as **specific seepage force**.

$$SP_s = \frac{P_s}{V} = \frac{iLA\gamma_w}{LA}$$

$$SP_s = i\gamma_w$$

→ Seepage pressure always act in the direction of flow, hence effective stresses in the soil mass, may either increase or decrease due to seepage pressure depending upon direction of flow.

$$\sigma' = Z\gamma_{sub} \mp P_s$$

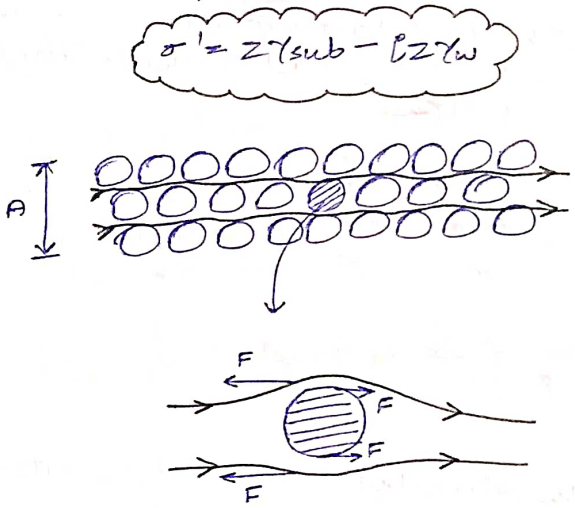
→ If flow takes in downward direction, seepage pr. also acts in the downward direction. resulting in increase of effective stress.

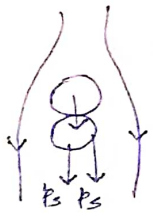
$$\sigma' = Z\gamma_{sub} + P_s \qquad \text{(here } L = Z\text{)}$$

$$\sigma' = Z\gamma_{sub} + iL\gamma_w = Z\gamma_{sub} + iZ\gamma_w$$

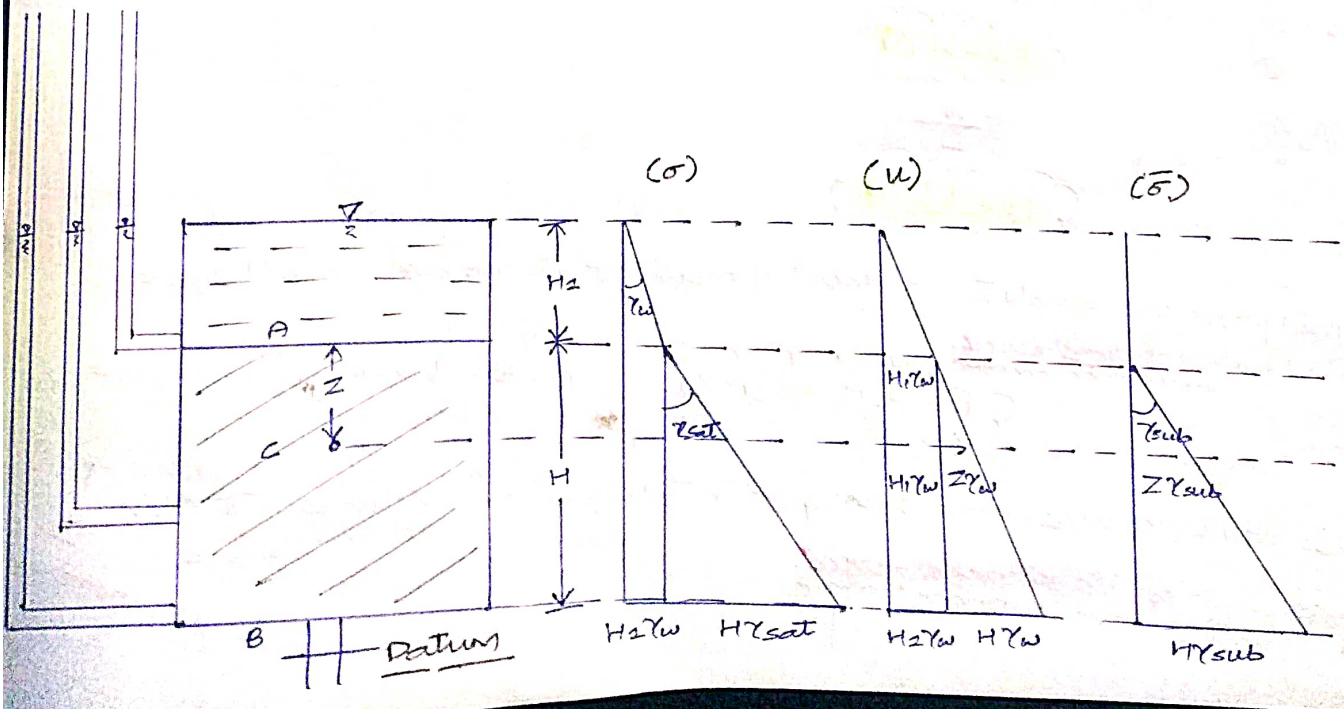
→ If flow takes place in upward direction, seepage pr. also acts in upward direction, resulting in decrease of effective stresses.

$$\sigma' = Z\gamma_{sub} - iZ\gamma_w$$





9. No flow condition:

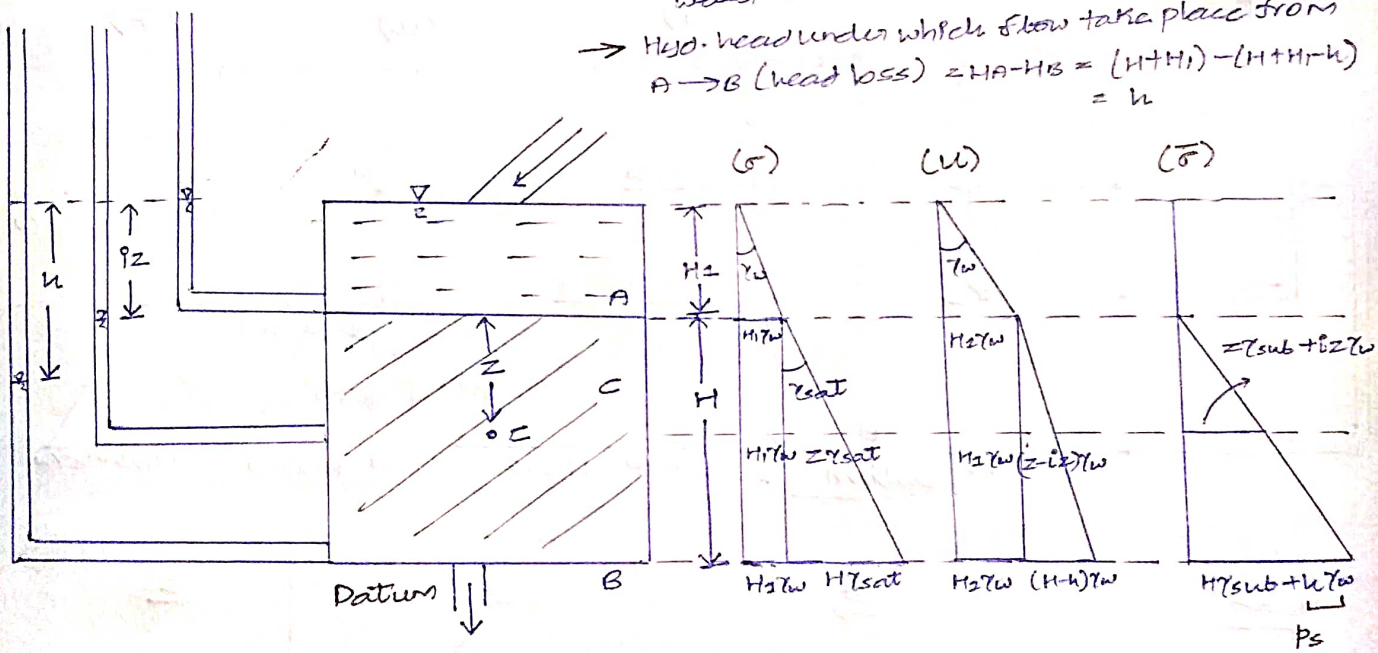


Head	A	B	C
Datum	H	0	H−Z
Pressure	H_1	$(H+H_1)$	H_1+Z
Total stress	$H+H_1$	H_1+H	$H+H_1$
Total (σ)	$H_1\gamma_w$	$H_1\gamma_w + H\gamma_{sat}$	$H_1\gamma_w + \gamma_{sat}\cdot Z$
Pore water pr. (u)	$H_1\gamma_w$	$(H+H_1)\gamma_w$	$(H_1+Z)\gamma_w$
Eff. stress (σ')	0	$H\gamma_{sub}$	$Z\gamma_{sub}$

ii. Downward flow condition:

→ Since (Total head)A > (Total head)B, water flows from A → B

→ Hyd. head under which flow take place from A → B (head loss) = $H_A - H_B = (H+H_1) - (H+H_1-h)$ = h



Head	A	B	C
Datum	H	0	H−Z
Pressure	H_1	$H+H_1-h$	H_1+Z-iz
Total stress	$H+H_1$	$H+H_1-h$	$H+H_1-iz$
Total (σ)	$H_1\gamma_w$	$H_1\gamma_w + H\gamma_{sat}$	$H_1\gamma_w + Z\gamma_{sat}$
Porewater pr. (u)	$H_2\gamma_w$	$(H+H_1)\gamma_w - h$	$(H_1+z)\gamma_w - iz\gamma_w$
Effective stress ($\bar{\sigma}$)	0	$H\gamma_{sub} + h\gamma_w$ = P_s	$z_{sat} - z\gamma_w + iz\gamma_w$ = $z\gamma_{sub} + P_s$

Seepage pr. at pt B = $h\gamma_w$
(p_s)

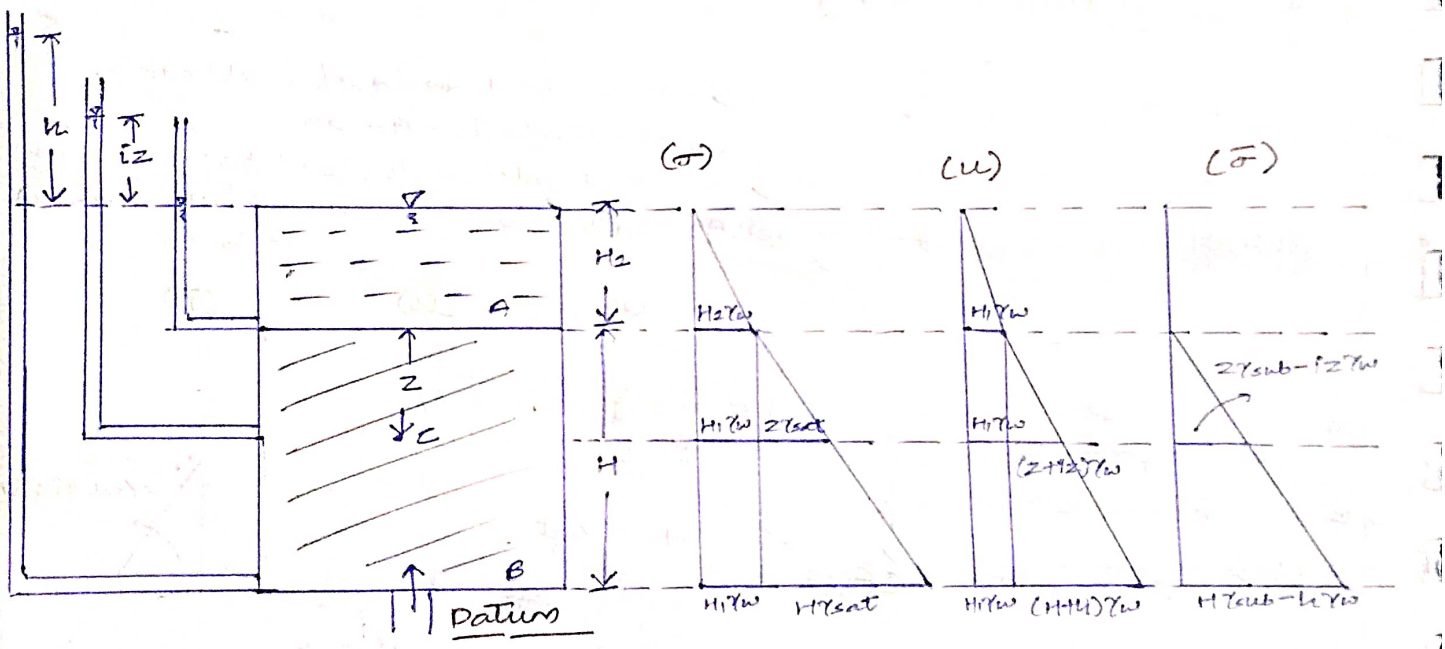
Hydraulic gradient (i) = $\frac{h}{H} = \frac{h}{H}$

Hence head loss from pt A to C, $h_{AC} = \frac{h}{H} z = iz$

→ In d/w flow condition, the valve below the tank is open & a constant level of water in the soil is maintained by adjusting the supply from the top & outflow from the top.

iii. Upward flow condition:

→ In upward flow condition, valve at the bottom of the soil is opened & the water is supplied at the constant rate.



	A	B	C
Head			
Datum	H	0	H − Z
Pressure	H_2	$H + H_2 + h$	$Z + H_2 + iz$
Total	$H + H_2$	$H + H_2 + h$	$(H + H_2 + h) - i(H - Z)$
Stress			$H + H_2 + iz$
Total (σ)	$H_2\gamma_w$	$H_1\gamma_w + H\gamma_{sat}$	$H_2\gamma_w + Z\gamma_{sat}$
Porewater (u)	$H_2\gamma_w$	$(H + H_2 + h)\gamma_w$	$(H_1 + Z + iz)\gamma_w$
Eff. stress (σ')	0	$H\gamma_{sub} - h\gamma_w$ (p_s)	$Z\gamma_{sub} - iz\gamma_w$ (p_s)

→ Head under which flow takes place from pt B → A = $H_B - H_A$

$= (H + H_1 + h) - (H + H_1)$

$= h$

$i = \frac{h}{H}$

→ Head under which flow takes place from pt C → A = $\frac{hz}{H} = iz$

→ Head under which flow takes place from B → C = $\frac{h}{H}(H - z) = h - iz$

$= i(H - z)$

* Special case:

→ When flow takes place in upward direction, seepage pressure also acts in upward direction resulting in decrease of effective stresses

→ If seepage pr. results equals the submerged wt of soil mass, effective stresses in soil reduces to zero.

→ In such case, cohesionless soil mass loses all its shear strength & has the tendency to flow along with flowing water.

→ This phenomenon in which particles of the soil leaves the soil mass & flow along with water is termed as "i.e. quick sand condition / piping / sand boiling / floating condition".

* For quick sand condition,

$\bar{\sigma} = z\gamma_{sub} - p_s = 0$ (coz flow is in u/s dirn)

$z\gamma_{sub} = p_s$

$z\gamma_{sub} = i z \gamma_w$

$i = \frac{\gamma_{sub}}{\gamma_w} = \frac{(G-1)\gamma_w}{(1+e)\gamma_w}$

**

$$i = \frac{(G-1)}{(1+e)}$$

→ The hydraulic gradient under which quick sand condition occurs is termed as critical hydraulic gradient / piping gradient / floating / boiling gradient

**

$$i_c = \frac{(G-1)}{(1+e)}$$

→ To avoid quick sand condition, working or available gradient 'i' must be less than critical hyd. gradient.

**

$$i < i_c$$

NOTE: Factor of safety against quick sand condition,

$$FOS = \frac{i_c}{i} \quad \text{(generally 2)}$$

→ For fine sand and coarse silt, for which, specific gravity & void ratio 2.65 & 0.65 respectively, hence

$$i_c = \frac{(G-1)}{(1+e)} = \frac{(2.65-1)}{(1+0.65)} = \frac{1.65}{1.65} \approx 1$$

$$i_c \approx 1$$

→ Quick sand is not a type of sand, it is a flow cond^n observed in cohesionless soil mass when eff. stresses in it are reduced to zero in u/w flow condition.

→ It is found in fine sand & coarse silt and is not found in gravels, coarse sand & clays.

→ In cohesive soils, like clay, shear strength is not reduced to zero even if eff. stresses are reduced to zero due to inherent cohesion possessed by these soils which holds soil particles together.

NOTE: Shear strength, $\tau = c + \sigma' \tan\phi$

* $\tau = c + \sigma' \tan\phi$

τ = cohesion
σ' = eff. stress
ϕ = friction angle

* For cohesive soil

$\tau = c + \sigma' \tan\phi$

For cohesionless soil

$\tau = c + \sigma \tan\phi$

In quick sand condition ($\sigma' = 0$)

$\tau = c$

$\tau = 0 + 0 = 0$

→ In gravels & coarse sand which are highly permeable soil, high discharge is required to obtain critical hydraulic gradient, that practically does not occur in nature

$$Q = KiA$$

for unit area of soil

$$Q = Ki$$

$$i = \frac{Q}{K}$$

→ For gravel & coarse sand, $k \uparrow\uparrow$, hence 'i' required is also very high to attain the critical condition ($i = i_c$).

→ Quick sand condition can also be analysed by considering equilibrium of critical section.

→ For equilibrium of section at B,

$$\Sigma F_y = 0$$

$$\downarrow (H_1\gamma_w + H\gamma_{sat}) - (H + H_1 + h)\gamma_w \uparrow = 0$$

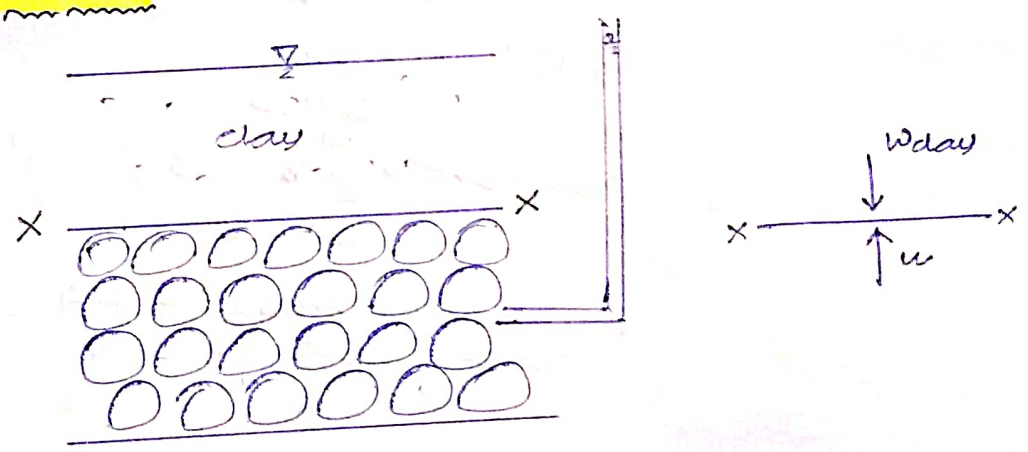
$$H\gamma_{sub} - h\gamma_w = 0$$

$$h\gamma_w = H\gamma_{sub}$$

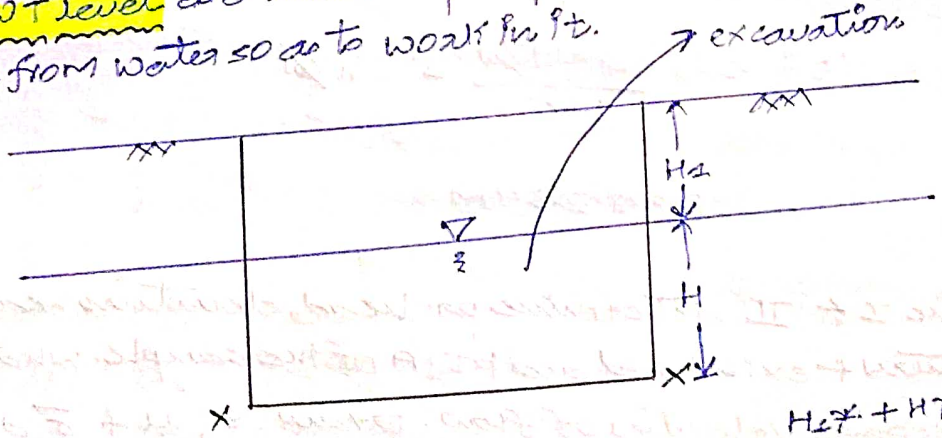
$$\frac{h}{H} = \frac{\gamma_{sub}}{\gamma_w}$$

$$i = \frac{\gamma_{sub}}{\gamma_w} = \frac{(G-1)}{(1+e)}$$

* → Quick sand cond^n occurs when the sand under artesian pr. is overlain by impervious layer of clay.



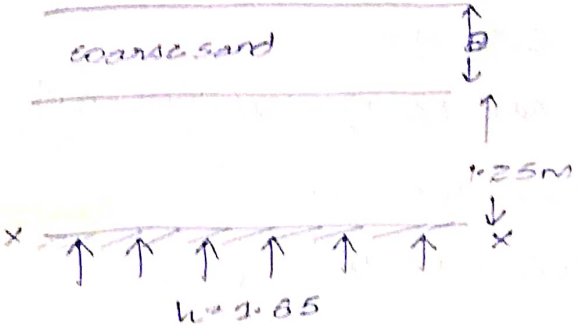
→ In practice, quick sand condition is found when excavation is made below the WT level and water is pumped out from the excavation to keep the pit free from water so as to work in it.



$H_1\gamma_w + H\gamma_{sat} - H\gamma_w$ —— ①

$H\gamma_w - H\gamma_w$ —— ②

Q. A 1.25m layer of soil ($\eta = 0.5$, $G = 2.65$) is subjected to an upward seepage head of 1.95m, what depth of coarse sand would be required above the existing soil to provide a FOS of 2 against piping. Assume that the coarse sand has the same porosity & specific gravity as the soil & there is negligible head loss in sand.



$$FOS = 2 = \frac{i_c}{i}$$

$$i_c = 2i$$

$$\frac{(G-1)}{1+e} = \frac{2h}{(D+1.25)}$$

$$(G-1)(1-\eta) = \frac{2h}{D+1.25}$$

$$1.65 \times 0.5 = \frac{2 \times 1.65 \times 1.95}{D+1.25}$$

$$1.65D + 2.0625 = 7.4$$

$$D = 3.234m$$

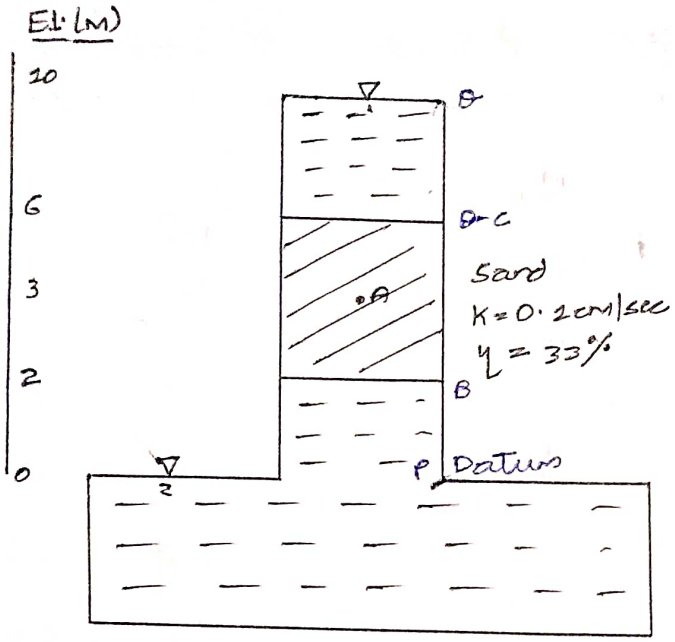
For FOS of 2 against piping,

$$2\gamma_{sub} = 2p_s$$

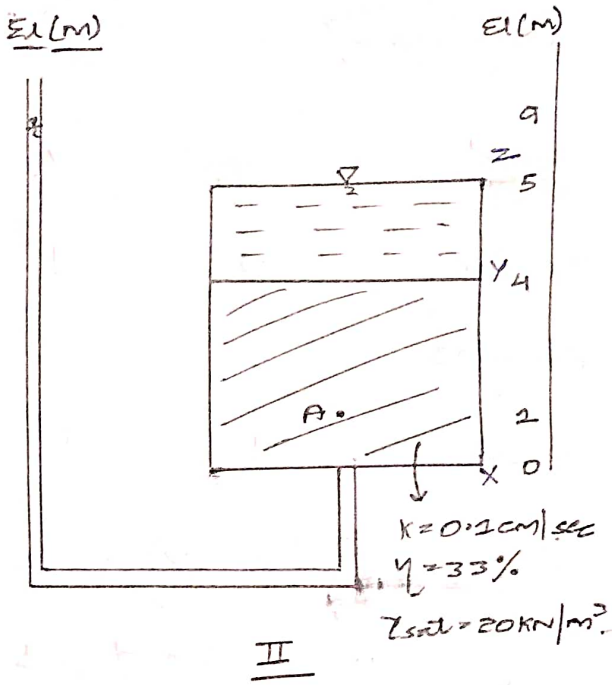
$$(D+1.25)\frac{(G-1)\gamma_w}{(1+e)} = 2h\gamma_w$$

$$D = 3.234m$$

Q. For each case I & II determine pr. head, elevation head & total head at the entry & exit head and pt A of the sample. What are superficial & seepage velocities of flow. Draw σ, u & $\bar{\sigma}$ diagrams from elevation of 0m to 5m for case 2.



I



II

For case I

Head	A	B	C	P	Q
Datum	3	2	6	0	10
Pressure	-0.5	A-2	4	0	0
Total	2.5	10	10	0	10

Head loss from C → B = 10

$$i = \frac{10}{6-2} = 2.5 \text{ M/M}$$

$$H_L \cdot (C \rightarrow A) = 2.5 \times 3$$

Case I: $v = Ki$

$$v = 0.1 \times 2.5 = 0.25 \text{ cm/s}$$

$$v_s = \frac{0.25}{0.33} = 0.75 \text{ cm/s}$$

Case 2:

$$v = Ki$$

$$v = 0.1 \times 1 = 0.1 \text{ cm/s}$$

$$v_s = \frac{v}{\eta} = \frac{0.1}{0.33} = 0.33 \text{ cm/sec}$$

For case 2

Head	X	Y	Z	A
Datum	0	4	5	1
Pressure	9	1	0	7
Total	9	5	5	8

$h_L (X \rightarrow Y) = 9 - 5 = 4$

$i = \frac{4}{4} = 1 M/M$

Stress	X	Y	Z	A
Total	$2\times\gamma_w + 4\gamma_{sat}$	$1\gamma_w$	0	$1\gamma_w + 3\gamma_{sat}$
Pore water	$9\gamma_w$	$1\gamma_w$	0	$7\gamma_w$
Effective	$4\gamma_{sat} - 8\gamma_w$ $4\gamma_{sub} - 4\gamma_w$ γ_s	0	0	$3\gamma_{sub} - 3\gamma_w$ γ_s

Q. In the test setup shown 2 different granular soils are placed in permeameter and flow is allowed to take place under the constant head of 30cm.

a. Determine the total head & pr. head at pt A

b. If 30% of the total head is lost as water flows upward through lower layer of soil what is the total head + pressure head at pt B.

c. If permeability of layer is 3×10^{-2} cm/sec. Calculate the quantity of water flowing per sec through unit area of soil

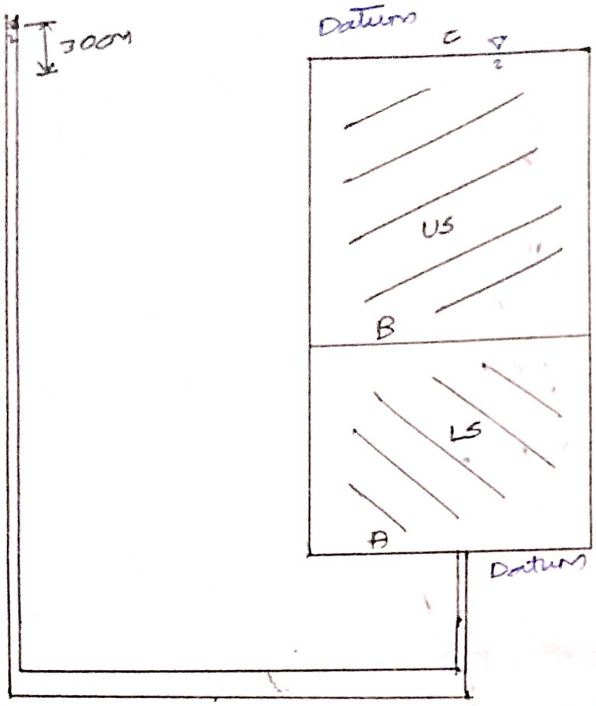
d. What is the permeability of upper soil layer

e.

	e	G
Upper soil	0.68	2.70
Lower soil	0.52	2.66

Determine the discharge velocity & seepage velocity of each soil.

f. If the total head is increased, determine at what value of head will effects of the soil be moved out of the container.



	Head			
	Datum	Pr.	Total	Loss
	0	0	0	0
-25	46	21	21	
-45	75	30	30	

$h_L (A \to C) = 30$

$h_L (A \to B) = 30 \times 0.30 = 9cm$

$h_L (B \to C) = 30 \times 0.70 = 21cm$

a. Datum head = 0

Pressure head = 75cm

Total head = 75cm

b. Datum head = 20cm

Pressure head = 32.5cm

Total head = 75 − 22.5 = 52.5

c. $Q = K_L i A$

$Q = K_L i$

$v_L = \frac{Q}{A} = 3 \times 10^{-2} \times \frac{9}{20} = 1.35 \times 10^{-2} \frac{cm^3}{sec}/cm^2$

d. $\frac{Q_U}{A_L} = \frac{Q_L}{A_L}$

$K_L i = K_U i$

$3 \times 10^{-7} \times \frac{9}{4} = K_U \times \frac{21}{5}$

$K_U = 1.6 \times 10^{-2} cm/sec$

e. $v_{s_U} = \frac{v}{\eta} = 3.35 \times 10^{-2} cm/sec$ $\quad v_{s_L} = \frac{v}{\eta} = \frac{v(1+e)}{e} = 3.9 \times 10^{-2} cm/sec$

f. for quick sand conditions.

U.S

$i_c = \frac{G-1}{1+e} = 1.012$

L.S

$i_c = \frac{G-1}{1+e} = 1.09$

Let h_1 be the head at which upper soil becomes quick

$$i = \frac{0.7 h_1}{25} = i_c = 1.011$$

$$h_1 = 36.144 \text{ cm}$$

* Laplace equation and flow net

→ If seepage takes place in 2-D, it can be analysed using laplace eqn which represents the energy loss in any resistive medium.

→ It can be used to describe flow of heat in thermal medium, flow of current in conductive medium and in our case flow of water through permeable medium (soil).

a. Laplace eqn for isotropic medium:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

(Permeability is same in all dirn)

b. Laplace eqn for non-isotropic medium:

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0$$

(Permeability is diff in all dirn)

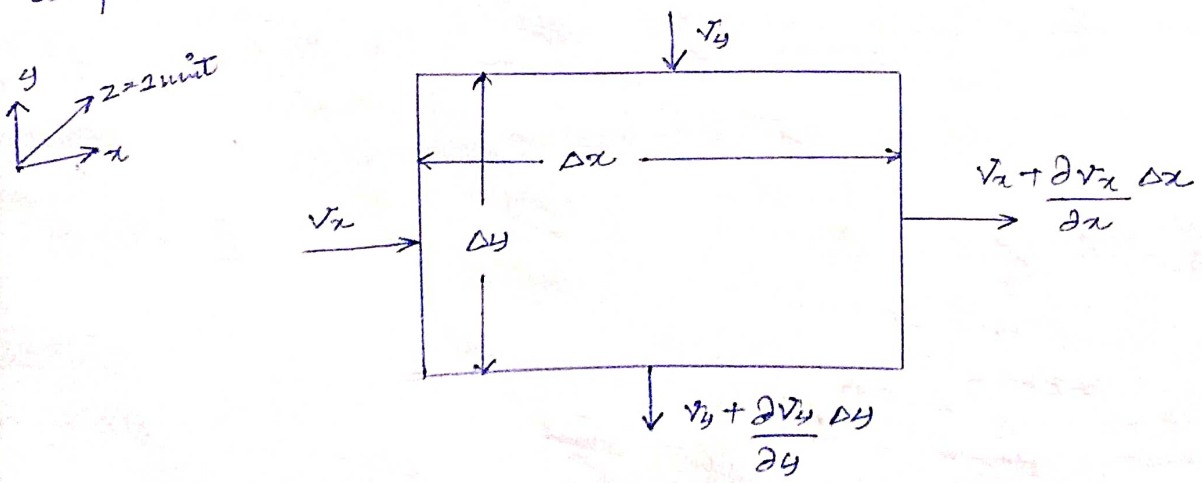
→ Assumptions in Laplace theory

1. Darcy's law is valid, i.e. flow is laminar and medium is saturated.
2. Pore fluid & soil solids are incompressible.
3. Soil is homogeneous and isotropic.

→ consider a soil element of size Δx & Δy & of unit thickness ⊥ to the plane of screen/view.

→ Let v_x & v_y be the entry velocity components in 'x' & 'y' dirn

→ Then $\left(v_x + \frac{\partial v_x}{\partial x}\Delta x\right)$ & $\left(v_y + \frac{\partial v_y}{\partial y}\Delta y\right)$ will be the corresponding velocity component at the exit of the element.



As per conservation of mass,

$$q_{entry} = q_{exit}$$

$$v_x(\Delta y \times 1) + v_y(\Delta x \times 1) = \left(v_x + \frac{\partial v_x}{\partial x}\Delta x\right)(\Delta y \times 1) + \left(v_y + \frac{\partial v_y}{\partial y}\Delta y\right)(\Delta x \times 1)$$

$$0 = \frac{\partial v_x}{\partial x}\Delta x \Delta y + \frac{\partial v_y}{\partial y}\Delta x \Delta y$$

$$\Delta x \Delta y \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = 0$$

$$\Delta x \Delta y \neq 0$$

$$\Rightarrow \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

As per Darcy's law,

$$v_x = K_x i_x = K_x\left(\frac{\partial H}{\partial x}\right)$$

$$v_y = K_y i_y = K_y\left(\frac{\partial H}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial H}{\partial y}\right) = 0$$

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0$$

→ For isotropic medium, $K_x = K_y$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

Q. $\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = 0$ Boundary cond^n $x=0, H=5M, \frac{\partial H}{\partial x} = -1$

$$\frac{\partial^2 H}{\partial y^2} = \frac{\partial^2 H}{\partial z^2} = 0$$

If there is no headloss in 'y' & 'z' dirn.

$$\int \frac{\partial^2 H}{\partial x^2} = 0$$

at $x=0, H=5$

$$c_2 = 5$$

$$\int \frac{\partial H}{\partial x} = c_1$$

$$c_1 = -1.$$

$$H = c_1 x + c_2$$

at $x = 1.2$

$$H = -1 \times 1.2 + 5 = 3.8\,M$$

→ The solution of laplace eq^n by graphical method is termed as "FLOW NET" which represents the description of streamlines and (flow lines) equipotential lines.

→ Stream lines shows the path along which individual particle of the water flows or seeps in the soil medium.

→ Equipotential line represents the contours or points of equal total head or potential.

* Properties of flownet:

→ Equipotential lines & flow lines intercept each other perpendicularly/orthogonally.

→ There can be no flow across flow line and velocity of flow is always $\perp^r$ to equipotential line.

→ Area bounded b/w 2 equipotential line & flow line is known as "flow field" which is approx. square for isotropic medium that may be linear or curvilinear and is approx. rectangle for non-isotropic medium that may also be linear or curvilinear.

→ Loss of head b/w 2 equipotential line is always same & is termed as equipotential drop. (Δh)

→ The area b/w 2 flow line is termed "Flow channel" and discharge through each flow channel is same.

*→ Smaller is the dimension of flow field, greater will be hydraulic gradient & velocity of flow through it.

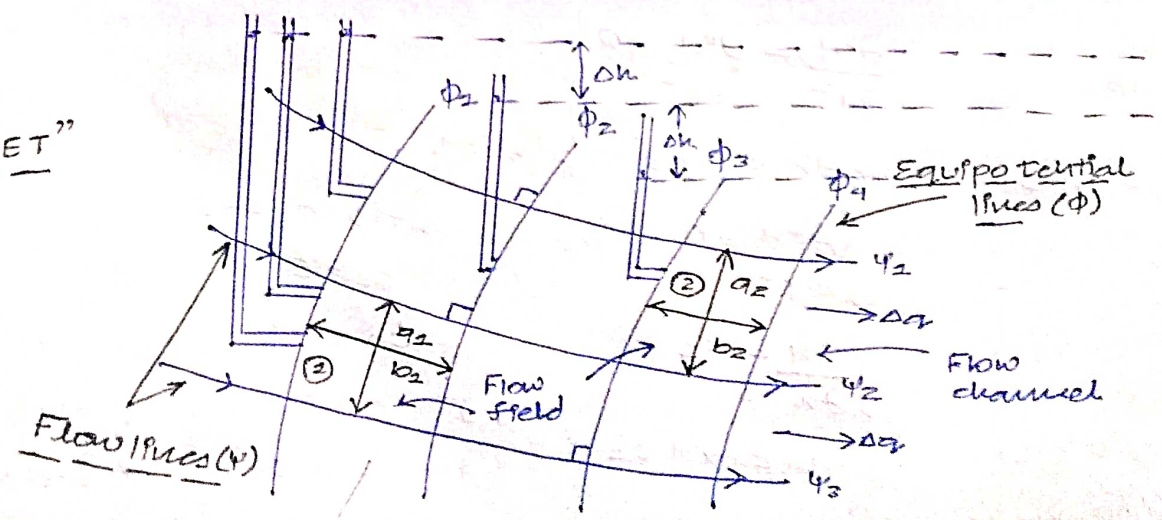
*→ If water level are reversed on u/s & d/s side or permeability of soil is changed or head under which flow takes place is changed, keeping the boundary cond^n to be same (without altering the specific geometry of flow), there is no change in flow net

*→ Flow net is unique for given set of boundary conditions.

"FLOW NET"

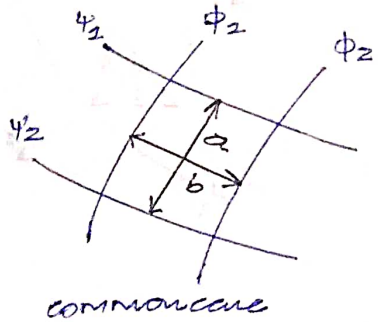
$a_1 = b_1$

$a_2 = b_2$

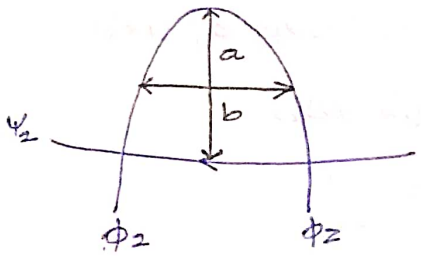


NOTE: An elementary square, which as a rule is adopted in flow net is not a real square, it only means the average distance b/w opp. sides of any field is equal. (a=b)

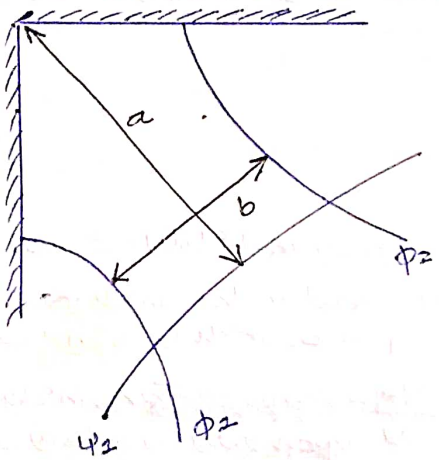
→ Many of fields in flow net are in fact far from resembling real squares but they still confer to the definition of elementary square.



common case



Special case



* In all these cases, a=b

→ Total discharge (q)

$q = \Delta q \times N_f$

Δq = discharge passing through each flow channel

N_f = no. of flow channels = No. of flow lines − 1

→ Equipotential drop (Δh)

$\Delta h = \frac{H}{N_d}$

H = Total head under which flow takes place

N_d = no. of equipotential drop = No. of equipotential lines − 1

* Applications of Flow net

→ Flow net can be used for following purposes

a. To find seepage discharge

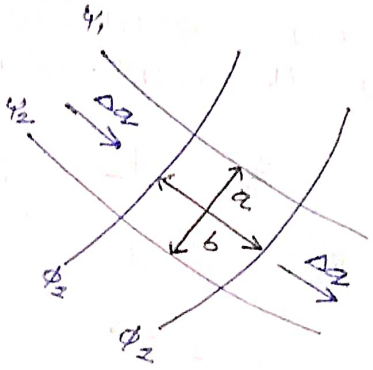
Case 1: For Isotropic medium

Consider a flow field of unit width in the plane of your screen of size (a×b) units

Let Δq be the discharge passing through this flow field.

$$\Delta q = K i A = K \frac{\Delta h}{b} (a \times 1)$$

$$\Delta q = K \Delta h \left(\frac{a}{b}\right)$$



Since medium is isotropic, $a = b$

$$\Delta q = K \Delta h$$

$$\Delta h = \frac{H}{N_d}$$

$$\Delta q = \frac{K H}{N_d}$$

As per conservation of mass, same is the discharge through the flow channel

Total discharge (q) = $\Delta q \cdot N_f$

$$q = \frac{K H}{N_d} \cdot N_f$$

** $q = K H \left(\frac{N_f}{N_d}\right)$

$$q = f\left[K, H, \frac{N_f}{N_d}\right]$$

K = permeability of the medium

H = head under which flow is taking place, which is a hydraulic condⁿ

$\frac{N_f}{N_d}$ = shape factor, which depends upon only boundary condⁿ

NOTE: If soil is replaced (K is changed) with other soil or if the headloss (H) during flow changes or if level of the water on u/s & d/s side are reversed change in only seepage discharge will be observed but shape factor will remain same. (flow net is same for same set of boundary condⁿ)

Case 2: For non-isotropic medium:

→ In non-isotropic medium, flow field is rectangular & not square.

→ A non-isotropic medium can be converted into an isotropic medium in which flow field is square by applying the following transformation.

→ For non-isotropic medium,

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0$$

$$\frac{K_x}{K_y} \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

$\frac{K_y}{K_x}$

Let x_T or x_m is the x-dimension on the transformed scale

$$x_T = x\sqrt{\frac{K_y}{K_x}}$$

$$x_T^2 = x^2\left(\frac{K_y}{K_x}\right)$$

On double diff. on both sides.

$$\partial x_T^2 = \left(\frac{K_y}{K_x}\right)\partial x^2$$

$$\frac{\partial^2 H}{\partial x_T^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

$$\frac{\partial^2 H}{\partial x_T^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

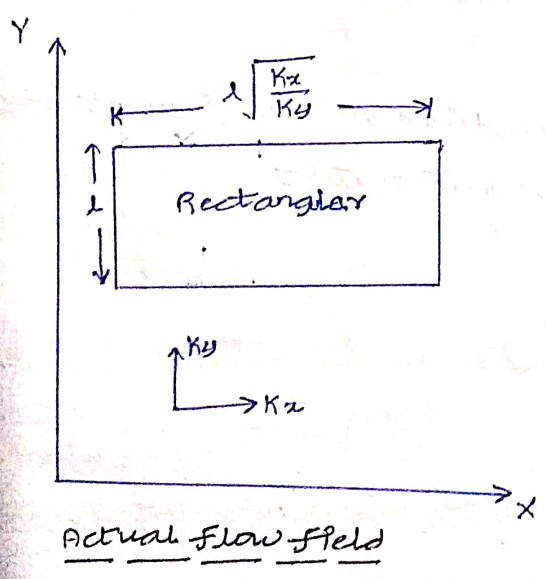
* This above eqn corresponds to isotropic medium

→ To plot the flow net for non-isotropic medium, the cross-section through non-isotropic medium is plotted to a natural scale on y-axis whereas in x-direction all the dimensions parallel to x-axis are being reduced by factor $\sqrt{\frac{K_y}{K_x}}$.

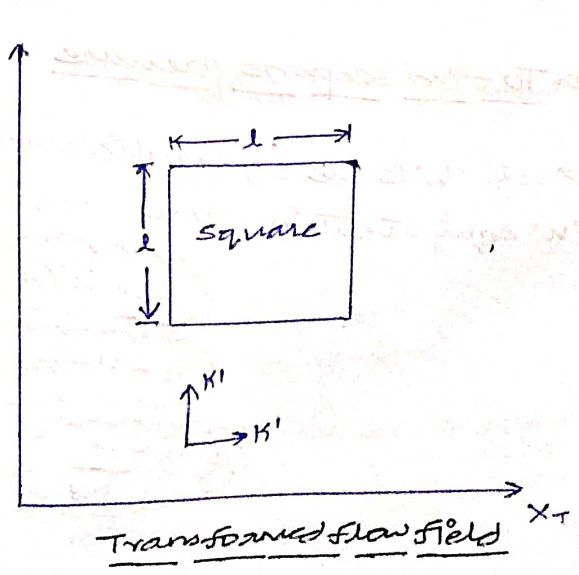
→ The flow net for this transformed section will now be constructed in normal manner as if soil is isotropic

→ The actual flow net is then obtained by retransforming the cross-section including the flow net back to normal scale by multiplying it with factor $\sqrt{\frac{K_x}{K_y}}$.

→ The actual flow net thus will not have orthogonal net of curves



Actual flow field



Transformed flow field

→ Discharge through flow field:

a. For actual flow field:

$$\Delta q = K_x i_x n = K_x \frac{\Delta h}{l\sqrt{\frac{K_x}{K_y}}} \cdot 1 \times 1$$

$$\Delta q = K_x \Delta h \sqrt{\frac{K_y}{K_x}}$$

$$** \quad \Delta q = \sqrt{K_x K_y}\, \Delta h \quad \text{——①}$$

b. For transformed flow field:

$$\Delta q = K' \frac{\Delta h}{1} \times (1 \times 1)$$

$$\Delta q = K' \Delta h \quad \text{——②}$$

① = ②

$$K' \Delta h = \sqrt{K_x K_y}\, \Delta h$$

$$** \quad K' = \sqrt{K_x K_y}$$

We know discharge from isotropic medium is given by

$$q = K H \frac{N_f}{N_d}$$

→ In our case, $K = K'$, hence discharge from equivalent transformed medium

$$** \quad \Delta q = K' H \frac{N_f}{N_d} \qquad \text{where } K' = \sqrt{K_x K_y}$$

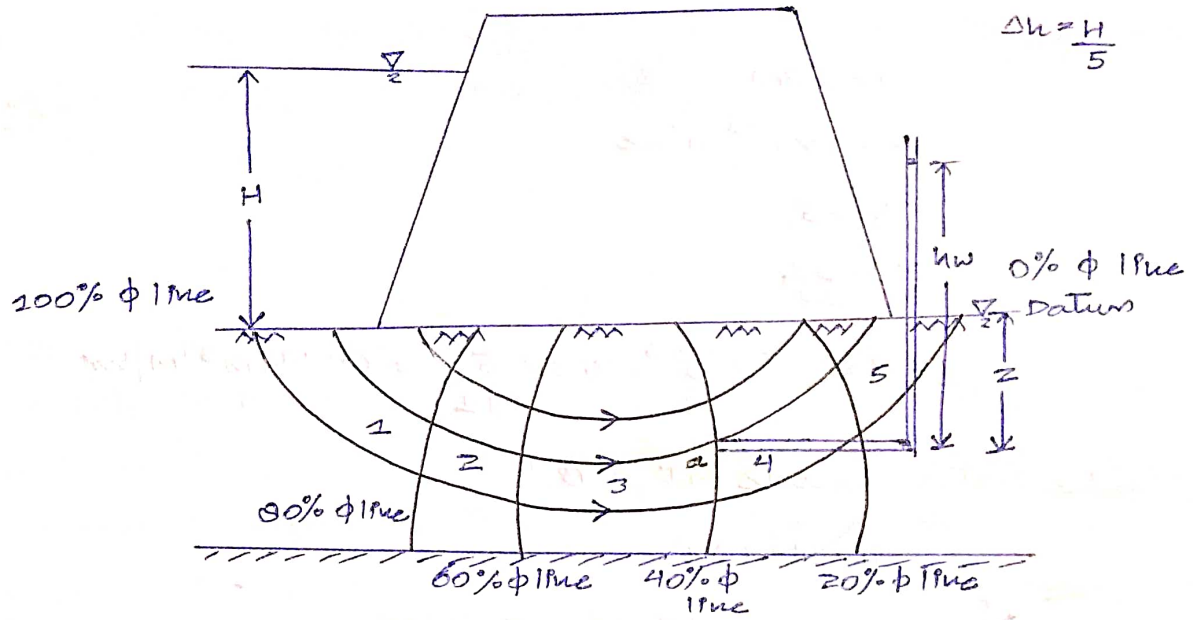
b. To find seepage pressure

→ Let 'n' is the hydraulic head in the medium at any given pt after 'n' equipotential drops, then seepage ps. at that pt

$$p_s = h \gamma_w$$

$$h = H - n\Delta h$$

$$\Delta h = \frac{H}{N_d}$$



c. To find the hydrostatic/pore water pressure

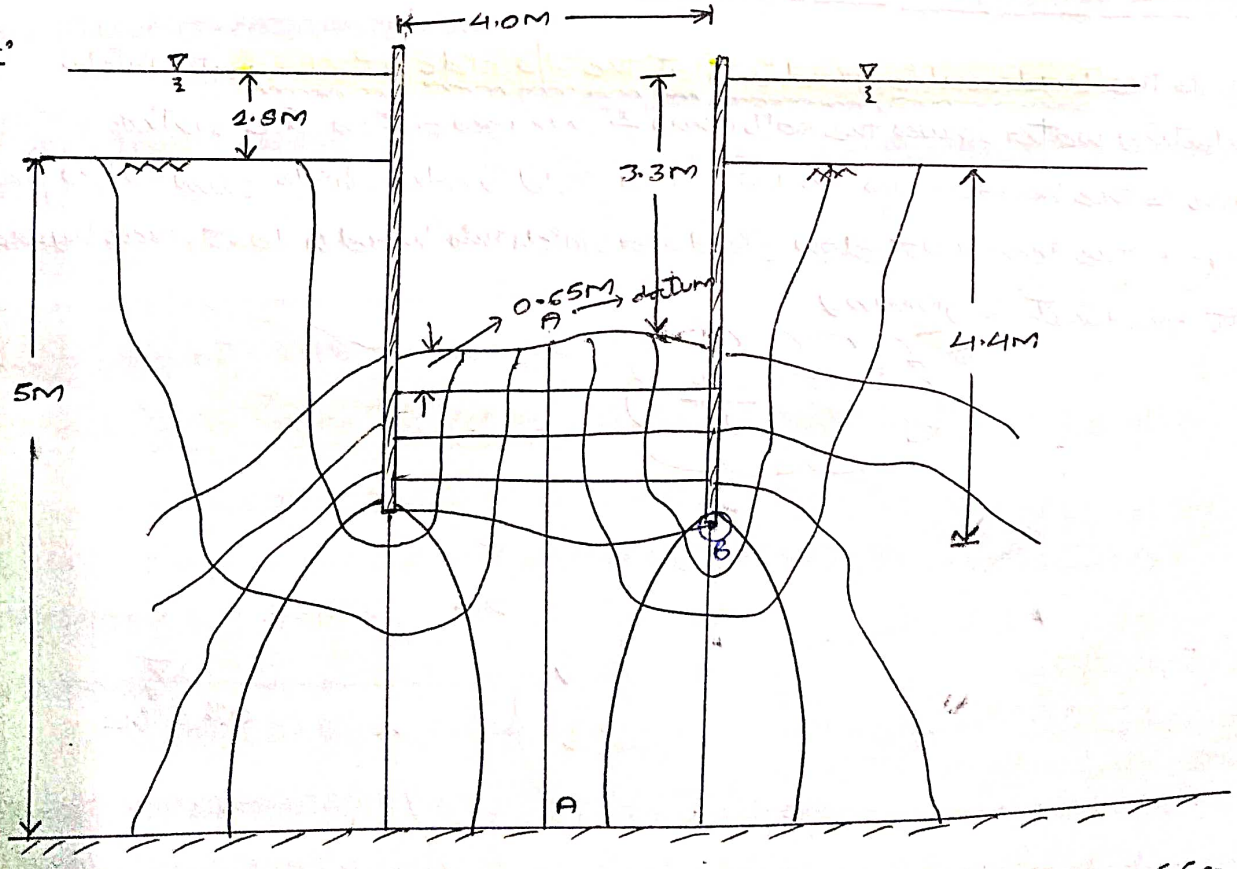
→ If 'hw' is the pressure head at any pt in the medium after 'n' equipotential drop then hydrostatic pr. at this pt is given by

$$u = h_w \gamma_w$$

Total head (h) = $h_w + z$

$$h_w = h \mp z \quad \text{and} \quad h = H - n\Delta h \quad \Delta h = \frac{H}{N_d}$$

Q.



If $K = 3 \times 10^{-3}$ cm/sec. Compute the quantity of flow in the coffer dam per hour per meter length of sheet pile walls.

$$q = \frac{KHN_f}{N_d}$$

$H = 3.3M$

$K = 3 \times 10^{-3}$ cm/sec

$N_f = 6$

$N_d = 11$

$$q = 3 \times 10^{-3} \times 10^{-2} \times 3.3 \times \frac{6}{11} = 0.1944 \, M^3/M/hr$$

Q. Find hydrostatic pressure at pt 'B'

$$h = H - n\Delta h$$

$$h = 3.3 - 4 \times \frac{3.3}{11} = 2.1M$$

$$h = h_w + z$$

$$2.1 = h_w + (-\{4.14 + 1.8 - 3.3\})$$

$$h_w = 5M$$

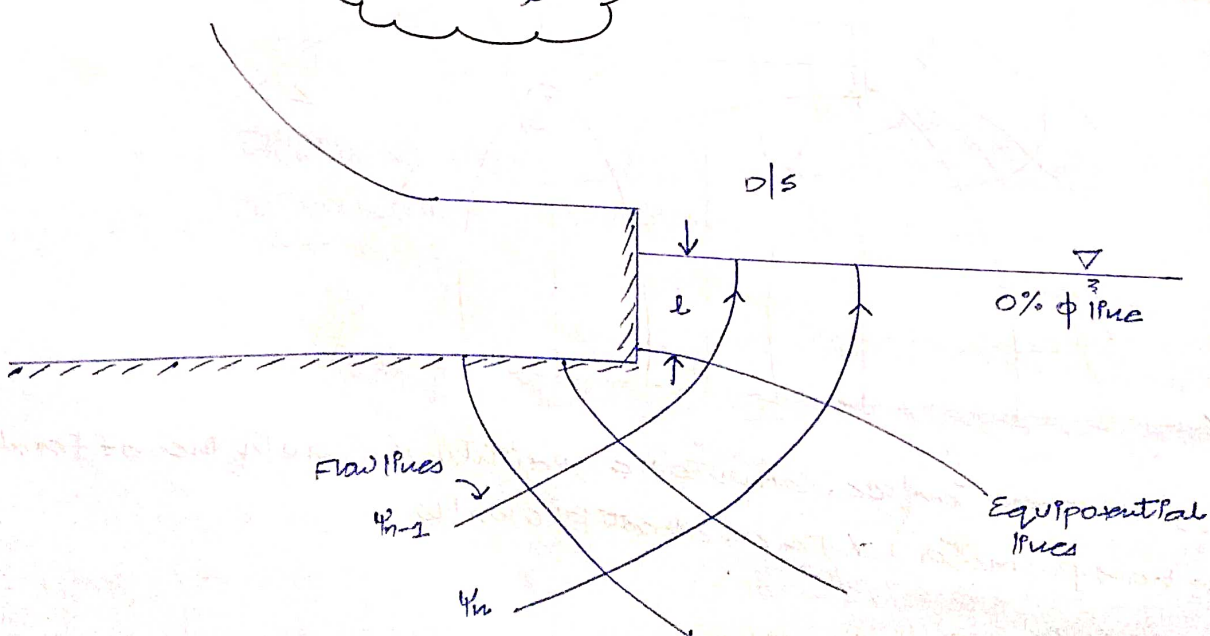
$$u = h_w \gamma_w = 5 \times 10 = 50$$

* Determination of exit gradient:

→ It is the hydraulic gradient at the d/s side of flow at which percolating water leaves the soil mass & emerges out as free water.

→ If Δh is the head across the last flow field under which flow takes place and 'l' is the length of flow field over which this head is lost, then hydraulic or exit gradient is given by

** $$i_{exit} = \frac{\Delta h}{l}$$



→ Exit gradient is the governing criteria for "piping" failure amongst all the hydraulic gradient.

→ To avoid piping, FOS of "6-7" is provided in case of fine sand & FOS of "4-5" is provided in case of coarse sand

$i_{exit} < i_c$

$$FOS = \frac{i_c}{i_{exit}} \quad [i_c \geq 1]$$

→ If the seepage force are large enough to erode the soil particles, piping can initiate and may progressively lead to structural collapse/failure.

→ Soil most susceptible to piping by erosion are fine sand & coarse silt.

→ To avoid the possibility of erosion & piping, two approaches are used.

a. control of seepage and seepage forces by providing a cut off wall/sheet pile wall, trench to stop seepage or lengthening the flow path with the help of impervious blanket is used.

b. By the use of protective filters which prevents erosion & reduces the uplift pressure.

→ A protective file consits of one or more layer of coarse grained, free draining material placed over a less drained pervious soil called base.

→ A filter would prevent migration of finer particles but without inhibiting flow of water so there is hardly any loss of head, which ensures that within the filter itself, seepage forces are reduced.

NOTE: If the voids in the filter layer are much larger than finer grains of the protected material, then these grains are likely to washed into the voids of filter & would lead to obstruction of flow.

→ On the other hand, if the voids of the filter are reduced, seepage forces will increase to unacceptable level.

→ Conditions to be satisfied by filter medium:

a. The filter material should be sufficiently fine & so graded that the voids of the filter are small enough to prevent the base material from penetrating and clogging the voids.

b. The filter material should be sufficiently coarse & pervious compared to the base material so that the flowing material (water) is rapidly removed without any build up of seepage forces.

c. The filter material should be coarse enough not to be carried away through the opening in drainage pipe.

d. The filter layer should be sufficiently thick to provide good distribution of all

particle sizes throughout the filter and to be able to carry seepage discharge

** $\frac{D_{15}\text{ (filter)}}{D_{85}\text{ (protected soil)}} < 5$ → upper limit

* conditions proposed by Terzaghi

** $4 < \frac{D_{15}\text{ (filter)}}{D_{15}\text{ (protected soil)}} < 20$ ← Lower limit

** $\frac{D_{50}\text{ (filter)}}{D_{50}\text{ (protected soil)}} < 25$

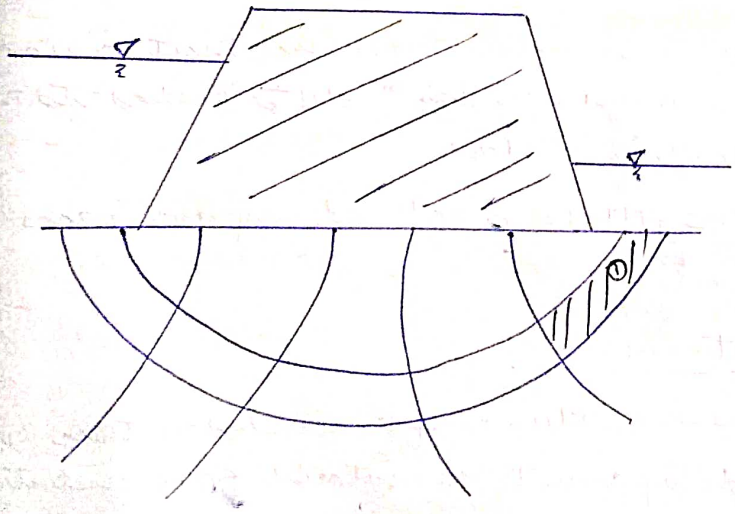
→ The specification ensures no significant intrusion of particles into the filter takes place hence it governs the upper limit of the grain size of filter medium.

→ The first part of 2nd specification ensures that sufficient headloss through filter takes place without build up of excess seepage pressure hence it governs the lower limit of grain size of filter medium.

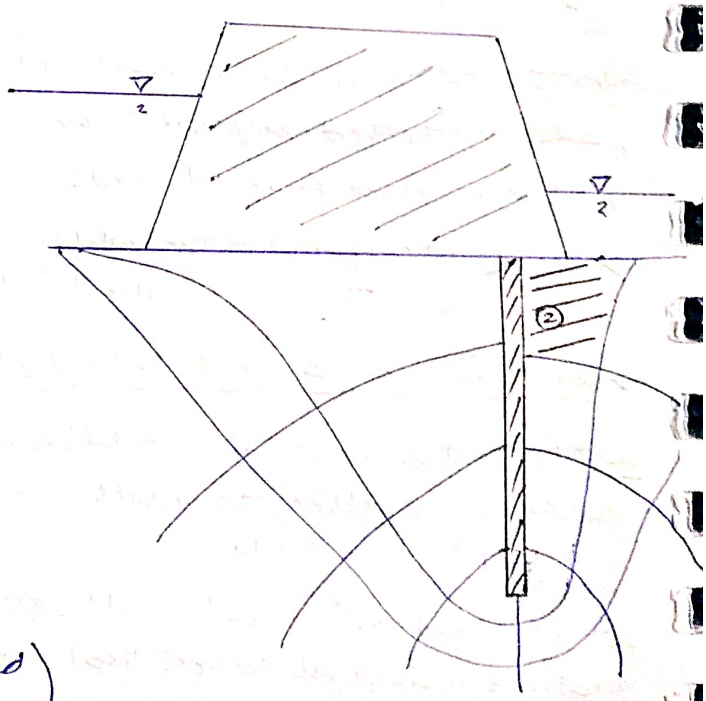
→ Rest are additional conditions.

NOTE:

(1)

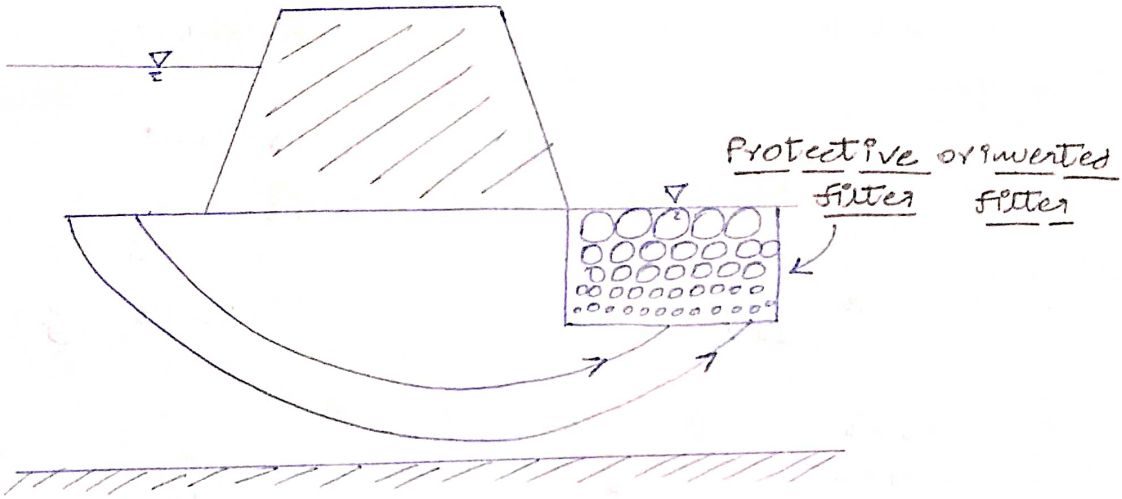


(2)



* $\left(\text{Flow field size}\right)_2 > \left(\text{flow field size}\right)_1$

* $(i_{exit})_2 < (i_{exit})_1$



NOTE: Phreatic line is the top flow line which seperates saturated zone from unsaturated zone and below which positive hydrostatic pressure exists in the dam section.

→ Along the phreatic line, the atmospheric pressure exists. It's shape is approximately of a parabola.

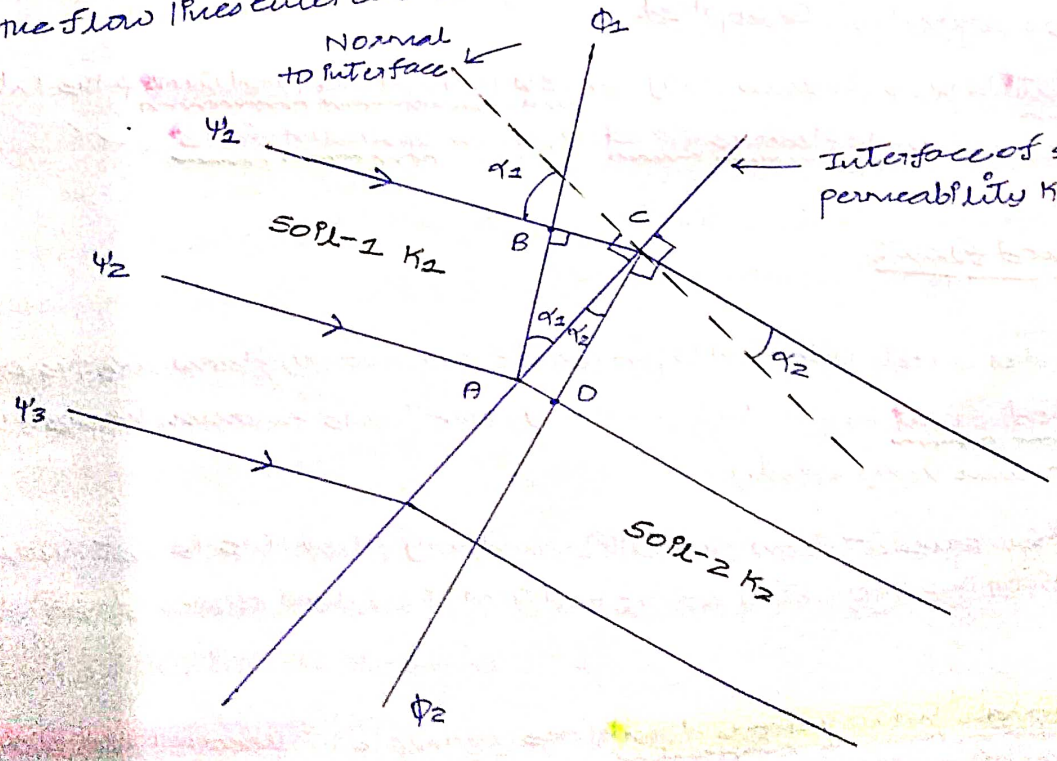
→ Quick condition and liquefaction of saturated sands are based on similar phenomenon.

→ Liquefaction is associated with increase in pore water pressure due to vibrations.

* Flow through non-homogeneous section:

→ If the permeability of the soil changes along the flow path, the flow lines are deflected at the junction of 2 soil masses.

→ Let the flow lines enter and exit the soil masses having permeability 'K₂' & 'K₂'



In flow field ABCD, discharge through both the soil zones will be same as per conservation of mass.

In soil-1

$\Delta q = K_1 i A$

$= K_1 \frac{\Delta h}{BC} (AB \times 1)$

$= K_1 \Delta h \left(\frac{AB}{BC}\right)$

$\Delta q = \frac{K_1 \Delta h}{\tan \alpha_1}$ —— (1)

In soil-2

$\Delta q = K_2 i A$

$\Delta q = K_2 \frac{\Delta h}{AB} (CD \times 1)$

$= K_2 \Delta h \left(\frac{CD}{AD}\right)$

$\Delta q = \frac{K_2 \Delta h}{\tan \alpha_2}$ —— (2)

from (1) & (2)

$\Delta q = \frac{K_1 \Delta h}{\tan \alpha_1} = \frac{K_2 \Delta h}{\tan \alpha_2}$

$\frac{K_1}{K_2} = \frac{\tan \alpha_1}{\tan \alpha_2}$

→ If K_1 is greater than K_2, then angle 'α_1' is greater than 'α_2' and flow line gets deflected towards the normal after the interface.

→ If 'K_2' is smaller than 'K_2', then angle 'α_2' is smaller than 'α_2' & flow line gets deflected away from the normal.

→ When the permeability of one of the soil is 10 times more than the other soil then the resistance offered by more permeable soil is not considered & no deflection correction is applied.

→ The flow net is then drawn only for less permeable mediums & the interface will act either as a discharge face or as an upstream face.

* Unconfined flow:

→ Flow under sheet pile walls, concrete or masonry dams are the examples of confined flow in which boundary cond^n are known, hence flow net can be drawn very easily.

→ In certain cases of flow, the top boundary flow line is not known or evident from the geometry of the medium, i.e. flow space is not fully defined.

→ Flow through homogeneous dam is the example of unconfined flow in which top flow line is drawn by the following method

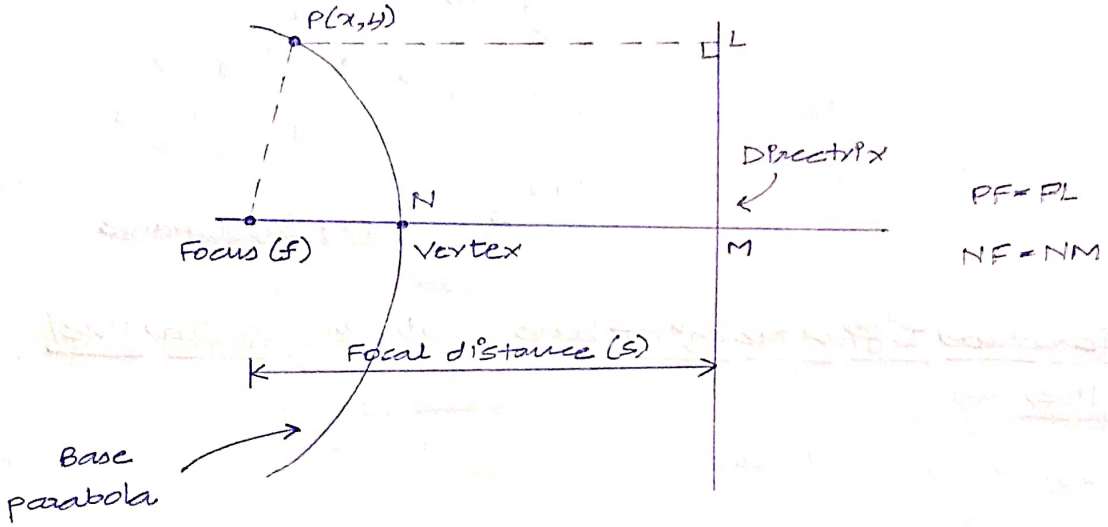
NOTE: This topmost flow line below which seepage takes place is termed as "Phreatic line" separate

→ It is used to saturated soil mass from unsaturated soil.

→ Pressure below phreatic line is hydrostatic whereas pressure above or on phreatic line is atmospheric

→ Phreatic line follows the path of base parabola

→ Base parabola is the parabola for which distance of any pt lying on it is equidistant from its focus & directrix.



Case 1: To draw the Phreatic line in homogeneous earthen dam when draining filter is provided

a. Let AB is the U/S face of the dam

→ Let its horizontal projection on water surface be 'L' = BD

b. Measure a distance of BC = 0.3L from the U/S face. Then the pt C is the starting pt of these base parabola.

c. To locate the directrix of parabola, we utilize the principle that any pt on base parabola is equidistant from its focus & directrix. Hence with pt 'C' as centre & 'F' as focus (Interface of perveable & impervious medium/ starting pt of filter), a circle is drawn to cut the horizontal projection of water surface at pt 'E'

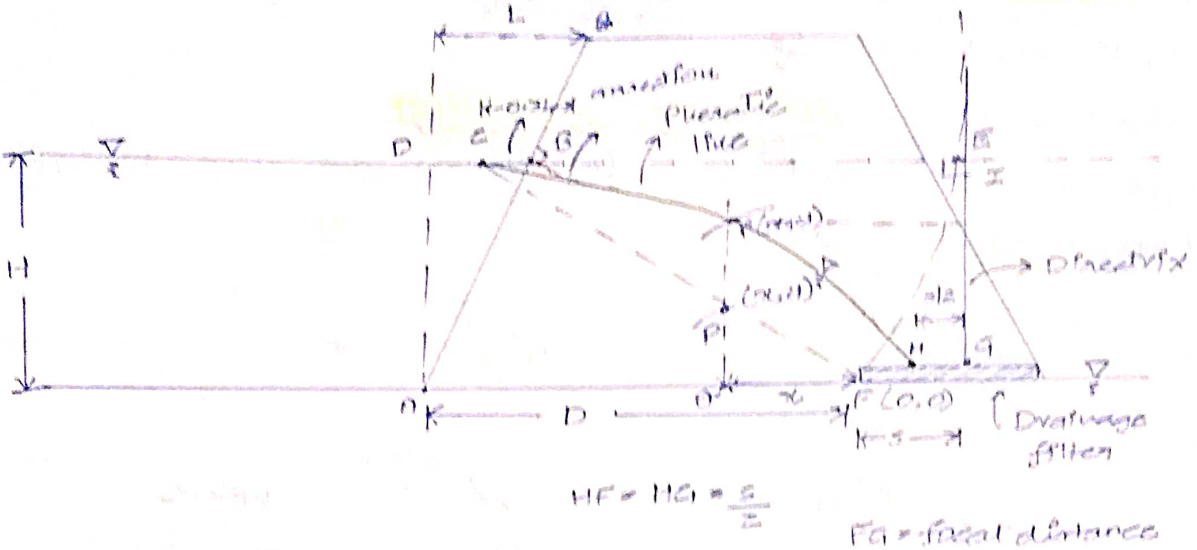
d. Vertical tangent at pt 'E' drawn to the arc circle is directrix of the parabola meeting the filter at pt 'G'

e. The last pt of parabola 'H' is located in b/w directrix & focus of parabola

f. In order to locate intermediate pts of parabola, a circle is drawn with foars 'F' as centre & radius of 'x+S' to meet the vertical line drawn at a distance of 'x' from the focus 'f'.

g. Join all these points to get base parabola.

4. Hence corrections are to be applied at the entry pt of the parabola. As Phreatic line is top flow it must be perpendicular to u/s face of the dam i.e 'AB' i.e. 100% of the must start from pt 'B'.



→ Analytical method to find the eqn of base parabola (Top flow line/ phreatic line)

From property of base parabola,

$PF = PI$

$\sqrt{x^2+y^2} = x+s$ — (1)

$x^2+y^2 = (x+s)^2$

$x^2+y^2 = x^2+s^2+2xs$

$y^2 = 2xs+s^2$ — (2)

** This is the eqn of base parabola

Since pt (C) is also on base parabola

At pt 'C', $x=D$, $y=H$

$H^2 = 2Ds+s^2$

From (1)

$s = \sqrt{x^2+y^2} - x$

$s = \sqrt{D^2+H^2} - D$ — (3)



→ Discharge through body of dam can also be computed directly from section PQ,

$q = KiA = K \times \frac{dy}{dx} \times (y \times 1)$

$$q = K_y \frac{\partial y}{\partial x}$$

$$y^2 = 2xs + s^2$$

$$y = (2xs + s^2)^{1/2}$$

$$\frac{\partial y}{\partial x} = \frac{1}{2}\left(2xs + s^{\frac{2}{2}}\right)^{-\frac{1}{2}} \times \left[2s\right]$$

$$\frac{\partial y}{\partial x} = \frac{s}{\sqrt{2xs + s^2}} = \frac{s}{y}$$

$$q = K_y \frac{\partial y}{\partial x} = K_y \cdot \frac{s}{y} = Ks$$

** $q = K \cdot s$

** $q = K\left[\sqrt{D^2 + H^2} - D\right]$ → valid for isotropic soil

NOTE: If medium is non-isotropic,

$q = K's$ $\qquad K' = \sqrt{K_x K_y}$

We know $x_T = x\sqrt{\frac{K_y}{K_x}}$

$$\rightarrow s = \sqrt{D_T^2 + H^2} - D_T \qquad D_T = D\sqrt{\frac{K_y}{K_x}}$$

Q.

$K_x = 8 \times 10^{-7}$ cm/s

$K_y = 3.6 \times 10^{-7}$ cm/s

$K' = \sqrt{K_x K_y} = 5.36 \times 10^{-7}$ cm/sec

$L = 32.3$ M

$D = 45.46$ M

$D_T = 45.46\sqrt{\frac{K_y}{K_x}} = 30.5$

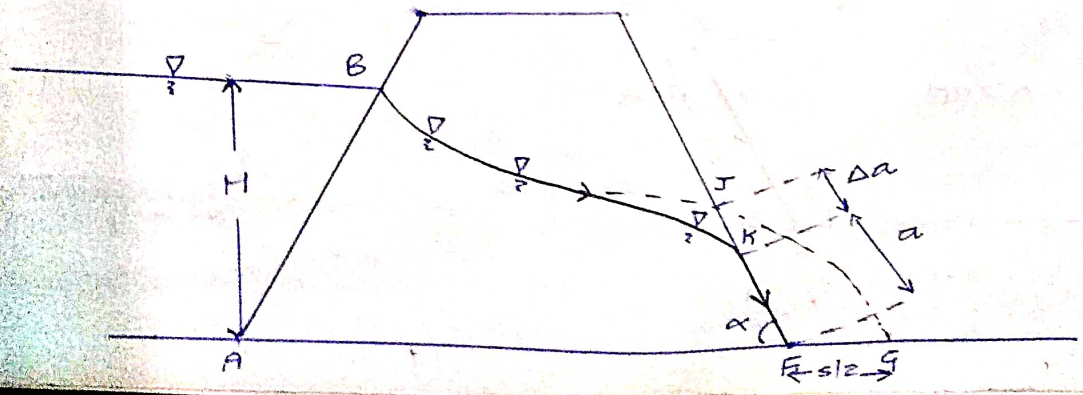
$q = K's$

$s = \sqrt{D_T^2 + H^2} - D_T = 3.055$

$q = 5.36 \times 10^{-9} \times 3.055$

$q = 1.64 \times 10^{-6}$ M³/M/sec

Case 2: To draw the pheratic line in earthen dam with no drainage filter



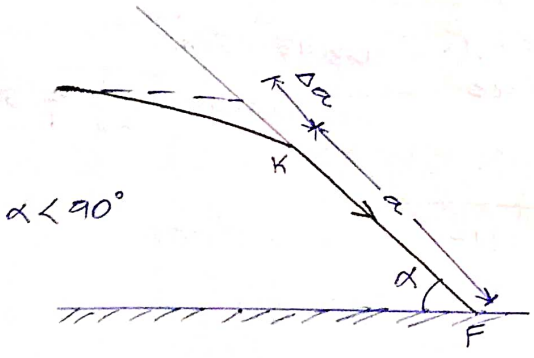
→ In case homogeneous earthern dam has no drainage filter at the d/s side, the focus in this case will be lowest point "Toe" ('F') of d/s side. slope and the base parabola 'BJG' will cut the d/s stream slope at 'J' & extend beyond the limit of the dam.

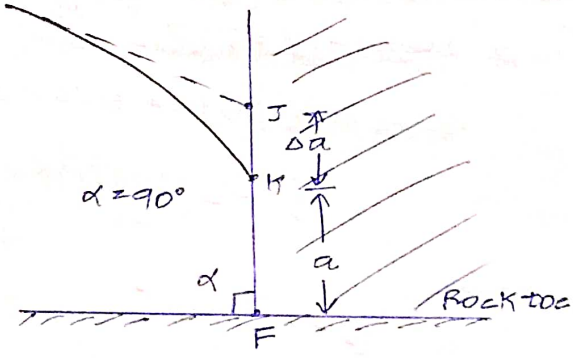
→ The phreatic line must emerge out at some point 'K' meeting the d/s face tangentially.

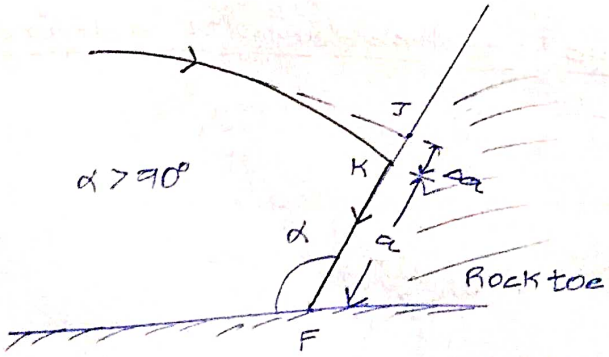
→ The portion 'KF' is then termed as 'discharge face' & always remains wet.

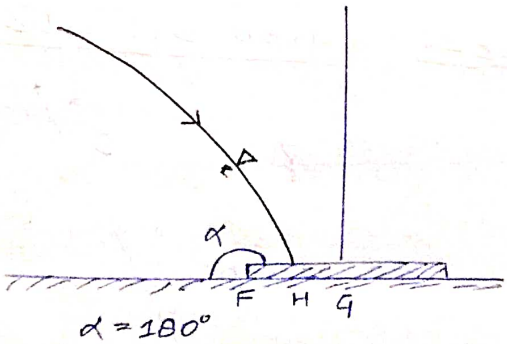
→ The correction Δa by which the parabola is to be shifted downward is found by the values of $\frac{\Delta a}{a+\Delta a}$ given by Casagrande which depends upon d/s slope angle 'α' (which may also be greater than 90°) as follows

$\alpha°$	$\frac{\Delta a}{a+\Delta a}$
30°	0.36
60°	0.32 (0.32)
90°	0.26
120°	0.18
~~150°~~ 135°	0.14
~~180°~~ 150°	0.10
180°	0







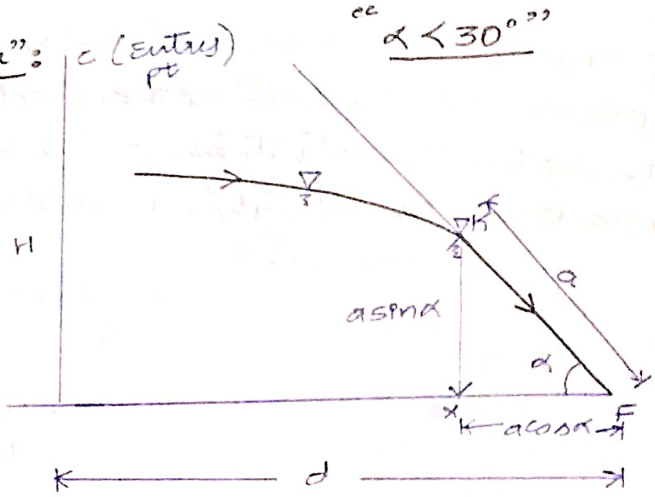


→ From the d/s slope, $\Delta a + a = JF$ —①

→ From the table $\frac{\Delta a}{a+\Delta a} = x$ —②

→ From ① & ②, Δa can be computed & correction is applied

→ **Analytical solution to find "a":**



Through section 'KX'

$$q = KiA = K\frac{\partial y}{\partial x}(y \times 1)$$

$$= K\frac{\partial y}{\partial x} y \quad —①$$

$$q = K\frac{a\sin\alpha}{a\cos\alpha} \times a\cos\alpha \quad (a\sin\alpha \times 1) \quad —①$$

$$q = Kx\frac{a\sin\alpha}{a\cos\alpha}(a\sin\alpha \times 1) \quad —②$$

$$K\frac{\partial y}{\partial x}\cdot y = Kx\tan\alpha \times a\sin\alpha$$

$$\int_{a\sin\alpha}^{H} y\,\partial y = \int_{a\cos\alpha}^{d} a\sin\alpha\tan\alpha\,\partial x$$

$$\left|\frac{y^2}{2}\right|_{a\sin\alpha}^{H} = a\sin\alpha\tan\alpha \cdot x$$

$$\frac{1}{2}\left[H^2 - a^2\sin^2\alpha\right] = \left[\frac{1}{2}a\sin\alpha\cos\alpha\tan\alpha + a\sin\alpha\tan\alpha\, d\right]$$

** $$a = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}}$$

pp. **If $30° < \alpha < 60°$**

Discharge through 'KX'

$$q = KiA = K\frac{\partial y}{\partial x}(y \times 1) = K\frac{\partial y}{ds}\cdot y \quad —①$$

$$q = KiA = K\frac{a\sin\alpha}{a}(a\sin\alpha \times 1) \quad —②$$

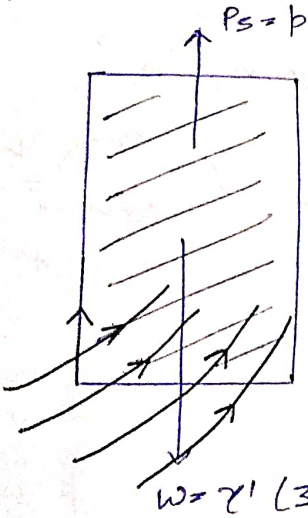
$$K\frac{\partial y}{ds}\cdot y = K\sin^2\alpha \times a$$

$$\int_{a\sin\alpha}^{H} y\,\partial y = a\sin^2\alpha \int_{s=a}^{s} ds$$

Here $S = \sqrt{H^2 + d^2}$

$$a = \sqrt{H^2 + d^2} - \sqrt{d^2 - H^2 \cot^2 \alpha}$$

Q. Compute the FOS against piping in front of sheet pile. Assume the prism of soil in front of the sheet pile of width equal to about half the depth of penetration of pile & depth equal to the depth of sheet pile itself in critical/susceptible to piping.



$$p_s = \left\{ \frac{6\Delta h + 2.75\Delta h}{2} \right\} \gamma_w = 7.29 \text{ KN/M}^2$$

$$\Delta h = \frac{H}{Nd} = \frac{2.25 - 0.25}{12} = \frac{1}{6}$$

$$P_s = 7.28 \times 1.5 \times 1 = 10.93 \text{ KN/M}$$

$$FOS = \frac{W}{P_s} = \frac{45}{10.93} = 4.11$$

$= 45 \text{ KN/M}^2/\text{M}$

$= 45 \text{ KN/M}$

STRESS DISTRIBUTION

→ At a certain point within the soil mass, the stresses are caused due to both surface loadings as well as due to self wt of the soil.

→ The stress components due to both the loadings i.e. self wt & surface load can be found seperately & then added to get the final stresses at the concerned pt.

→ Stresses due to self wt are already discussed and stresses due to surface loading are computed as follows.

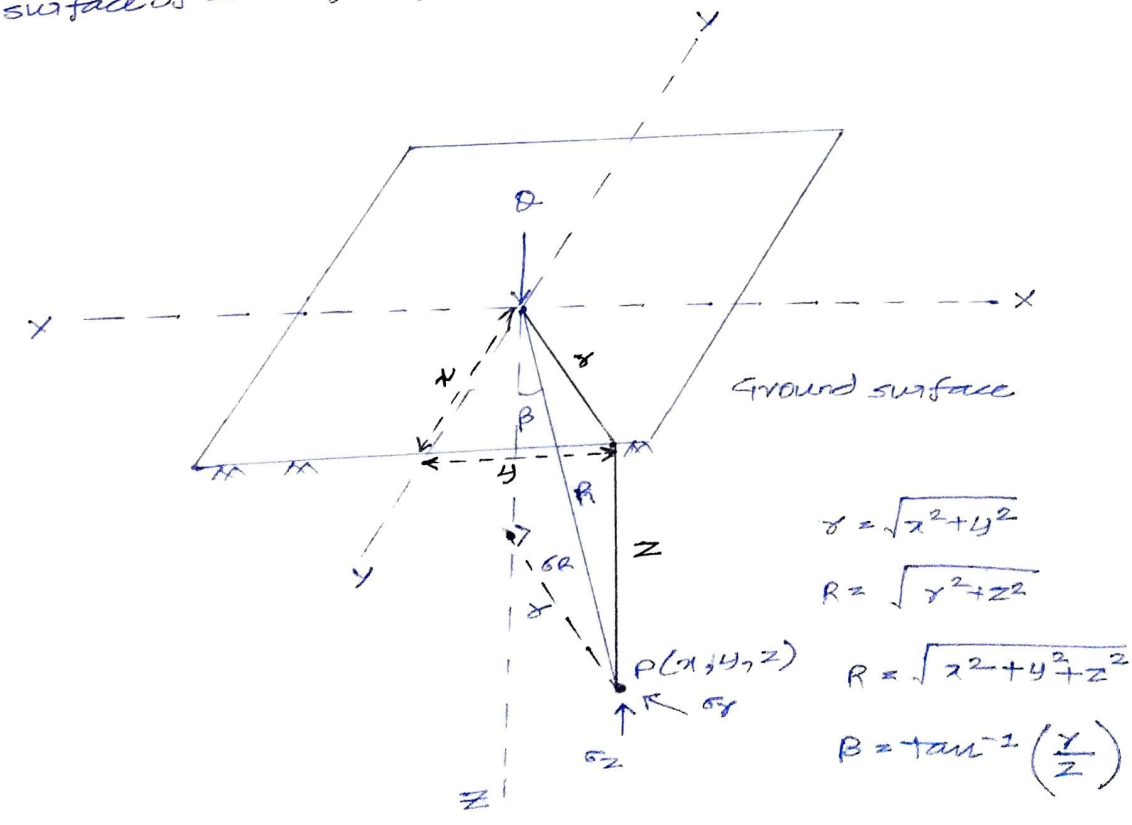
I. Vertical pressure due to pt load

→ Stress at any pt 'P' in the soil mass due to pt load 'Q' applied at the surface at pt 'O' can be analysed either either using "BOUSSINESQ'S THEORY" or "WESTERGAARD'S THEORY".

a. Boussinesq's theory:

→ **Assumptions:**

→ Soil is homogeneous, isotropic, semi-infinite & elastic
→ HOOKE's law is valid
→ Self wt of the soil is neglected.
→ Soil is unstressed initially
→ Change in volume of soil due to application of load is neglected
→ Distribution of stresses about vertical axis is symmetrical
→ The top surface of soil is free from shear stress & is only subjected to pt load.

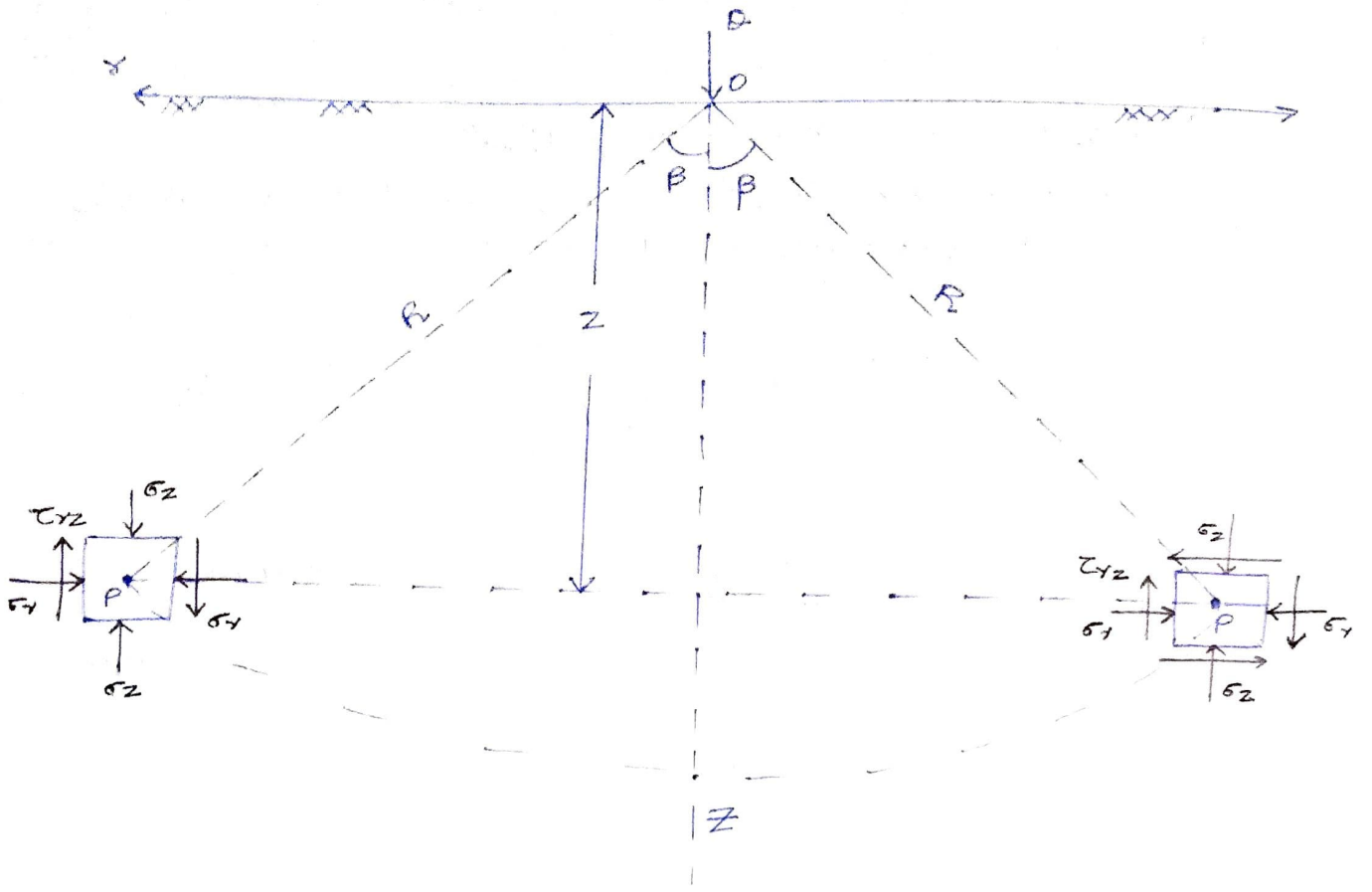


$r = \sqrt{x^2+y^2}$

$R = \sqrt{r^2+z^2}$

$R = \sqrt{x^2+y^2+z^2}$

$\beta = \tan^{-1}\left(\frac{r}{z}\right)$



→ Acc. to Boussinesq's theory,

→ $\sigma_z = K_B \cdot \frac{Q}{z^2}$ Here K_B = Boussinesq's influence factor

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

**

$$\sigma_z = \frac{3Q}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \cdot \frac{1}{z^2}$$

→ $\tau_{rz} = \sigma_z \cdot \frac{r}{z}$

**

$$\tau_{rz} = \frac{3Qr}{2\pi z^3}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

NOTE: It can be referred both vertical normal stress & shear stress are independent of elastic constant (E & μ) but are very much dependent on the assumptions of elastic theory.

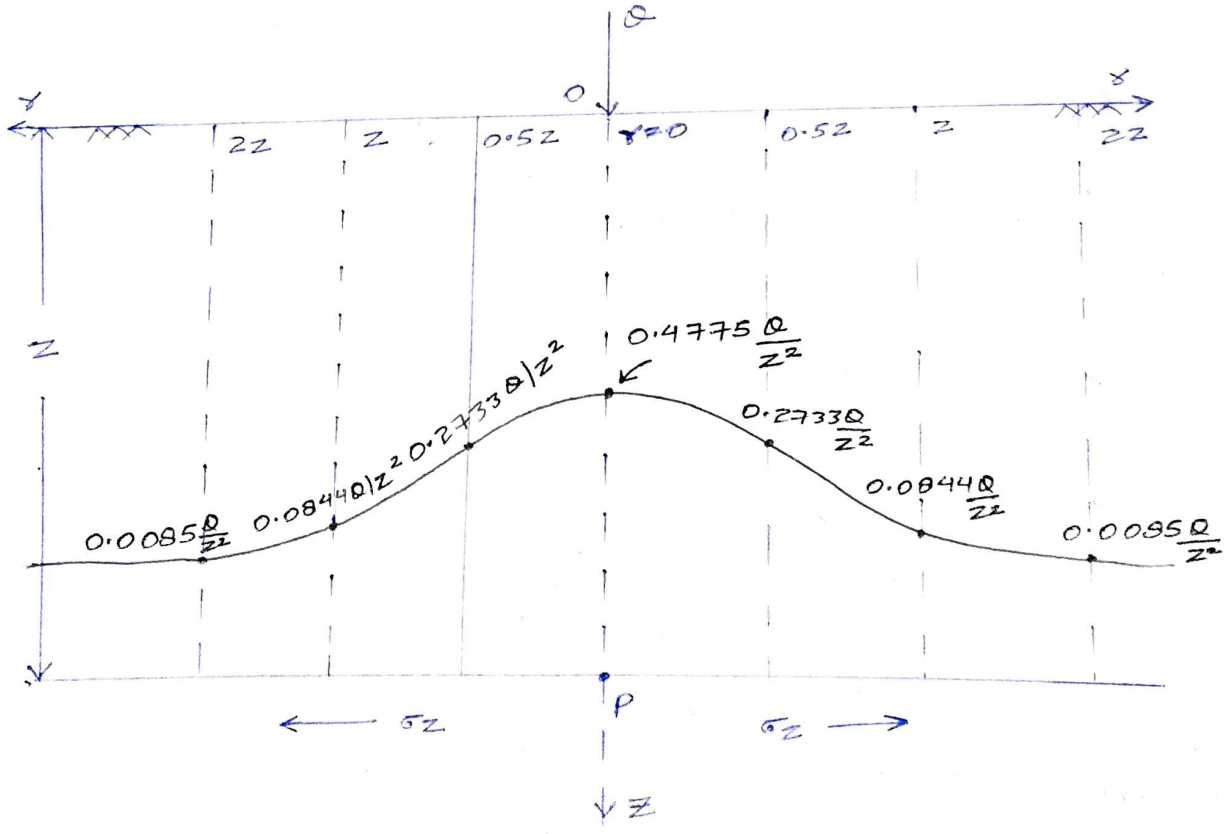
→ Boussinesq's stress distribution theory can further be used to analyse follow pressure distribution diagram.

a. Vertical Pressure distribution diagram on any horizontal plane at constant depth.

→ In this case, depth 'z' is known & constant

→ By selecting diff. values of 'r', K_B can be computed from which σ_z on this horizontal plane can be found.

$\frac{r}{z}$	$K_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$	$\sigma_z = \frac{K_B Q}{z^2}$
0	0.4775	Maximum
0.5	0.2733	57% of maximum
1	0.0844	17.7% of maximum
2	0.0085	1.8% of maximum

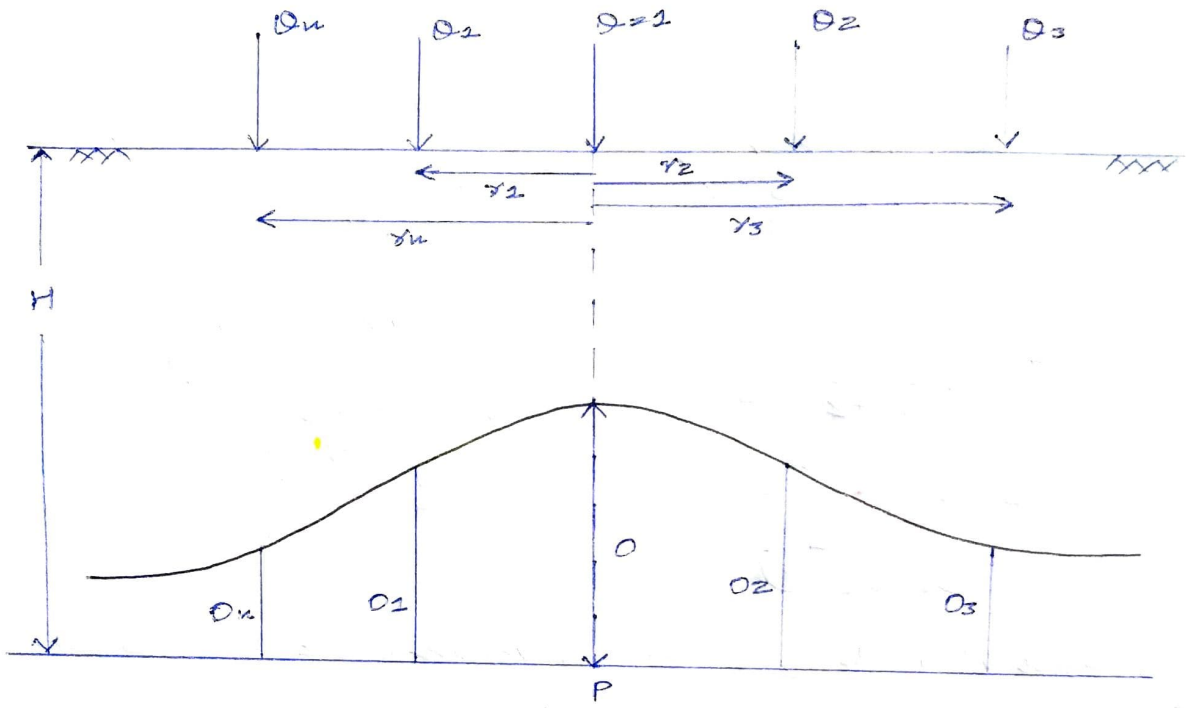


→ If this pressure distribution diagram is plotted for unit load, i.e. ($Q = 1$), it is called "Influence diagram" for pt 'P' below the axis.

→ Such diagrams can be used in computing the vertical stress (σ_z) at pt 'P' due to number of concentrated loads $Q_1, Q_2, \ldots, Q_n$ situated at a radial distance of $r_2, r_2, \ldots, r_n$ from the vertical axis through pt 'P'.

**

NOTE: It can be concluded that at a given depth when horizontal radial distance i.e. more than or equal to three the depth, σ_z due to pt load is neglected.

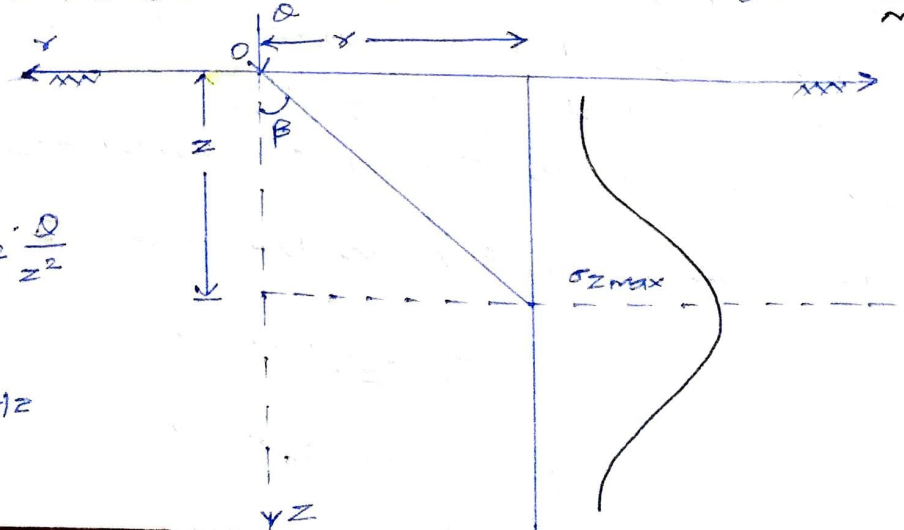


* $\sigma_z = Q_1 \cdot O_1 + Q_2 \cdot O_2 + Q_3 \cdot O_3 + \ldots + Q_n O_n$ → Law of superposition

b. Vertical Pressure distribution diagram on vertical line at a constant radial distance:

→ Vertical pr. on vertical line at a constant radial distance from pt load first increases with increase in depth reaches its maximum value & further starts decreasing.

→ The max. vertical pr. on vertical line is obtained at pt of intersection of vertical line with the radial line drawn at an angle of "39°13'" from pt load.



$$\sigma_z = K_\beta \frac{Q}{z^2}$$

$$= \frac{3}{2\pi} \cdot \frac{1}{\left[1 + \frac{r^2}{z^2}\right]^{5/2}} \cdot \frac{Q}{z^2}$$

$$\sigma_z = \frac{3}{2\pi} \cdot \frac{z^3}{(r^2 + z^2)^{5/2}}$$

For $\sigma_z = \sigma_{z\max}$,

$$\frac{\partial(\sigma_z)}{\partial z} = 0$$

$$\frac{\partial}{\partial z}\left[\frac{3Q}{2\pi}\cdot\frac{z^3}{(r^2+z^2)^{5/2}}\right] = 0$$

$$5r^2 = 2z^2$$

$$\frac{r}{z} = \sqrt{\frac{2}{3}}$$

$$z = \sqrt{\frac{3}{2}}\cdot r$$

$$\tan\beta = \frac{r}{z} = \sqrt{\frac{2}{3}}$$

$$\beta = \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

$$\beta = 39°15'$$

$$\sigma_{z\max} = \frac{3Q}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

$$\sigma_{z\max} = 0.088\frac{Q}{r^2}$$

→ The depth at which σ_z is max. on vertical line, shear stress at this point,

$$\tau_{rz} = \sigma_z\cdot\frac{r}{z} = 0.071\frac{Q}{r^2}$$

c. Stress Isobar / Isobar Diagram:

→ An isobar is a curve or contour joining all pts below the ground surface of equal vertical pr.

→ An isobar is a spatial, curved surface of the shape of the bulb acc the vertical pr. on a given horizontal plane is same in all directions at the pts located at equal radial distance around the axis of loading.

→ The zone in the loaded soil mass bounded by an isobar of given vertical pr. is called "Pressure bulb".

→ The vertical pressure at every pt on the surface of pr. bulb is same.

Ex: Suppose an isobar of value $\sigma_z = 0.5Q$ is to be drawn.

$$\sigma_z = KB\frac{Q}{z^2} = 0.5Q$$

$$KB = 0.5z^2 \quad \text{—①}$$

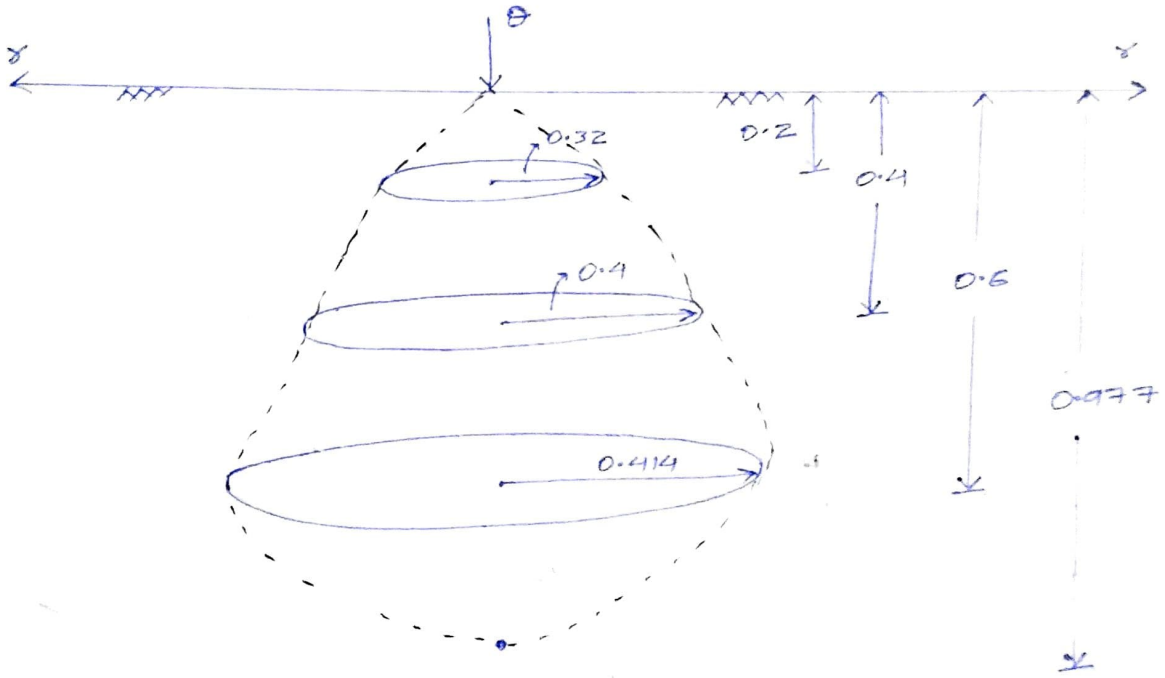
Just below the pt load, at $r = 0$

$$K_B = 0.4775 \quad \text{---} (2)$$

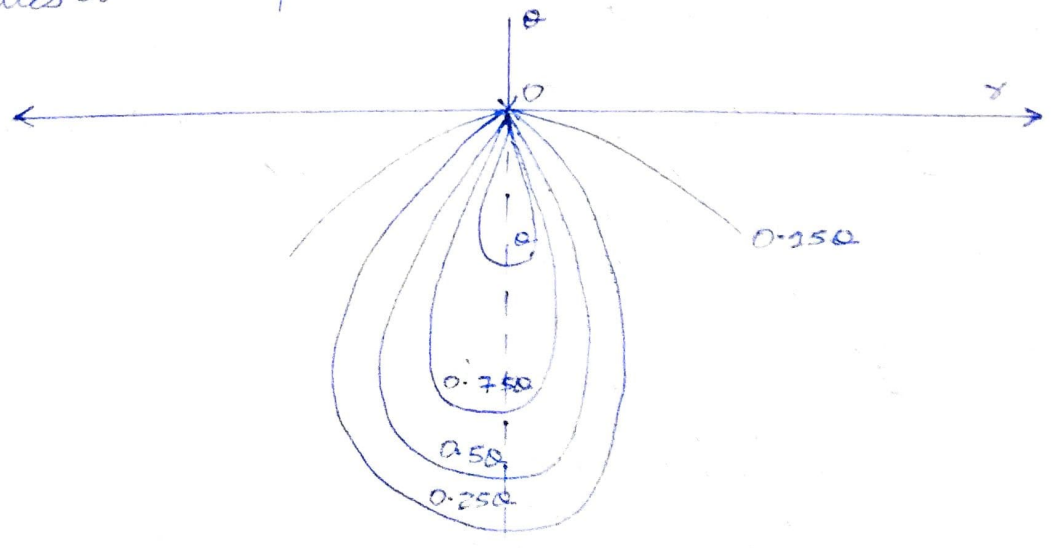
from (1) & (2), $0.5 Z^2 = 0.4775$

$$Z = 0.977$$

Z	K_B	r/z	r
0.2	0.02	1.599	0.32
0.4	0.08	1.022	0.41
0.6	0.18	0.69	0.4114
0.977	0.4775	0	0



→ For a given load, a number of isobars corresponding to various intensities of vertical pr. can be drawn & is termed as "isobar diagram"



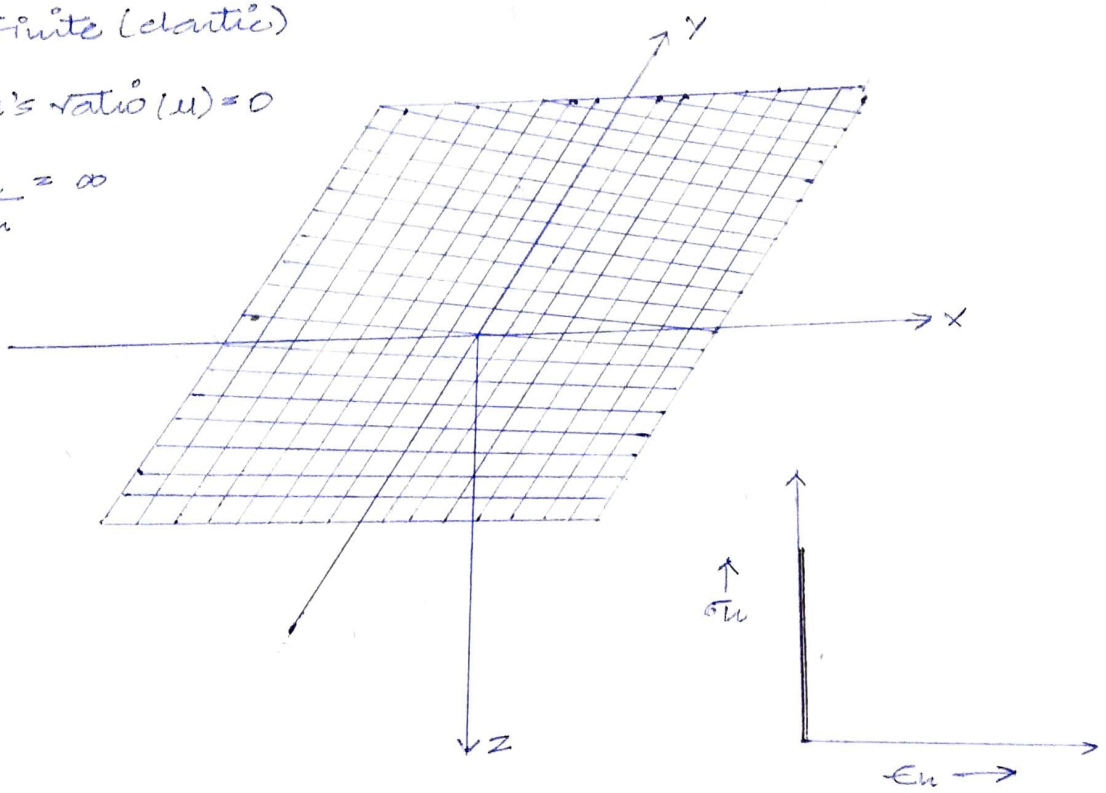
b. Westergaard's theory:

→ Assumptions:

→ Soil is homogeneous, semi-infinite & non-isotropic (hence it is applicable in stratified soils)

→ Medium is laterally reinforced with numerous closely spaced sheets of negligible thickness (medium is rigid horizontally & elastic vertically)

* $\epsilon_H = 0$ (rigid horizontally)
* ϵ_V = finite (elastic)
* Poisson's ratio (μ) = 0
* $E_h = \frac{\sigma_h}{\epsilon_h} = \infty$



→ According to Westergaard,

$$\sigma_z = K_w \frac{Q}{z^2}$$

$$\rightarrow K_w = \frac{1}{\pi}\left[\frac{1}{1+\frac{2r^2}{z^2}}\right]^{3/2}$$

**

$$\sigma_z = \frac{Q}{\pi z^2}\left[\frac{1}{1+\frac{2r^2}{z^2}}\right]^{3/2}$$

K_w = Westergaard's influence factor

→ If $r = 0$, $\frac{r}{z} = 0$

$K_w = 0.318$

$$\sigma_z = 0.318\frac{K}{z^2}$$

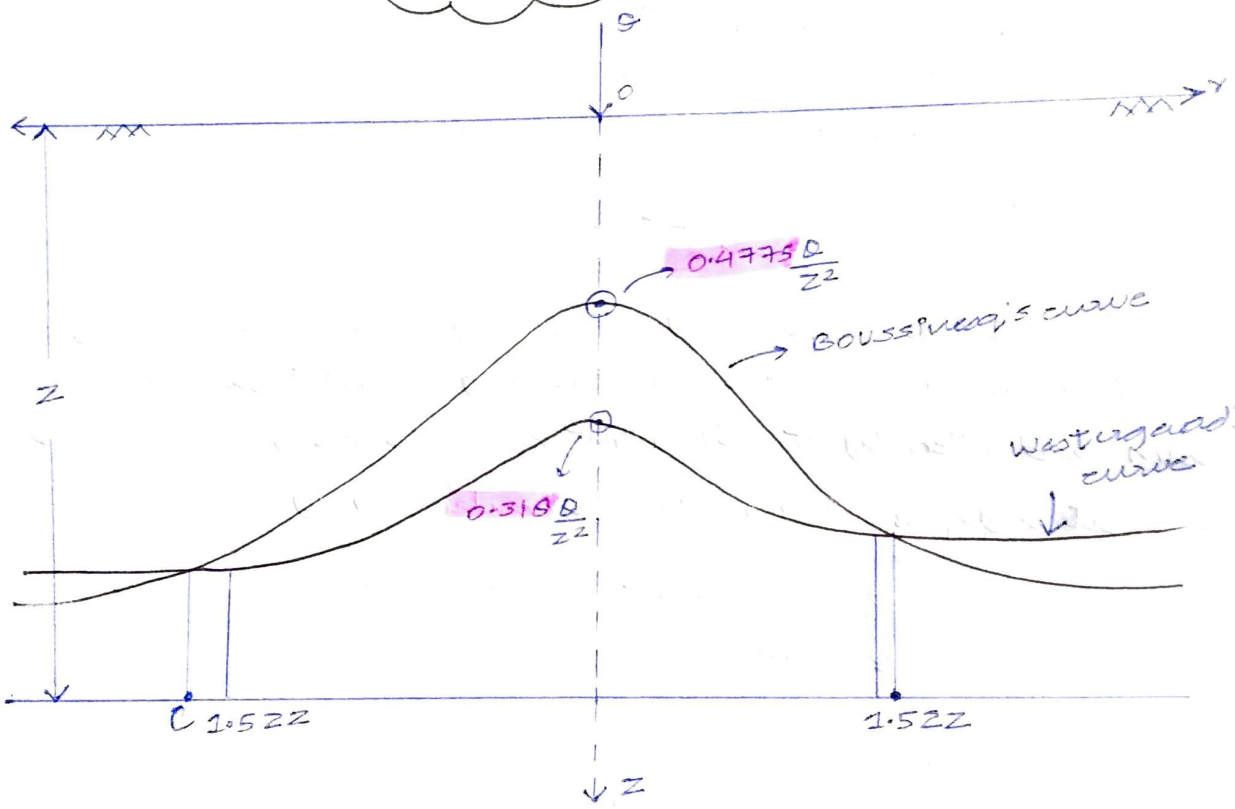
→ For some $\frac{r}{z}$, $\sigma_{zB} = \sigma_{zW}$

$$\frac{K_B Q}{z^2} = \frac{K_W Q}{z^2}$$

$$K_B = K_W$$

$$\frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = \frac{1}{\pi}\left[\frac{1}{1+2\frac{r^2}{z^2}}\right]^{3/2}$$

** $\frac{r}{z} = 1.5$



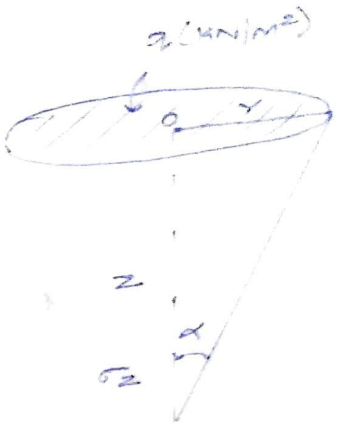
→ If

$\frac{r}{z} = 1.5$	$K_B = K_W$
$\frac{r}{z} < 1.5$	$K_B > K_W$
$\frac{r}{z} > 1.5$	$K_W > K_B$

NOTE: The soil in practical is non-isotropic hence results obtained using Westergaard's theory are more accurate & closer to the actual result but practically Boussinesq theory is used as it gives more conservative results.

→ Beyond '$r = 1.5z$' when results obtained by Westergaard's theory are more, the values are insignificant (<17.7% of max.)

→ Vertical pressure due to uniformly loaded circular area below the centre:



$$\sigma_z = K_\theta q$$

$$K_\theta = 1 - \cos^3\alpha$$

$$\cos\alpha = \frac{z}{\sqrt{r^2+z^2}}$$

$$K_\theta = 1 - \frac{z^3}{(r^2+z^2)^{3/2}}$$

**

$$K_\theta = 1 - \left(\frac{1}{1+\frac{r^2}{z^2}}\right)^{3/2}$$

Q. A water tank is supported by a ring foundation having outer dia. 10m & inner dia. of 7.5m. The ring foundation transmits uniform load intensity of 160 KN/m². Compute the vertical stress induced at a depth of 4m below the centre of ring foundation.

$q = 160 KN/m^2$ $\quad r_o = 5m$

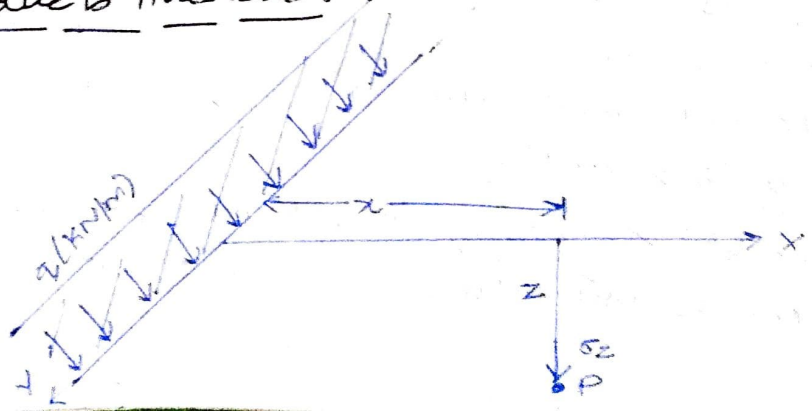
$z = 4m$ $\quad r_i = 3.75m$

$$(K_\theta)_o = 1 - \left(\frac{1}{1+\frac{(5)^2}{(4)^2}}\right)^{3/2}$$

$$\sigma_z = q\left[(K_\theta)_o - (K_\theta)_i\right] = 23.114 KN/m^2$$

NOTE: In this case, $K_\theta = 1 - \left(\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right)^{3/2}$

→ Vertical pressure due to line load:

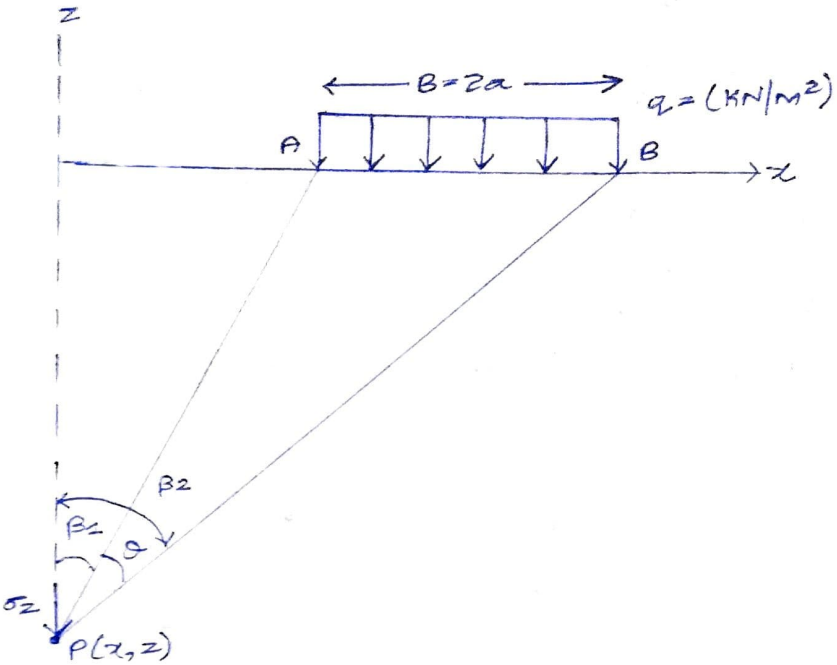


$$\sigma_z = \frac{2q}{\pi z}\left(\frac{1}{1+\frac{x^2}{z^2}}\right)^2$$

ex: railway lines

→ Vertical pressure due to strip load:

→ L >>> B (for strip)



**

$$\sigma_z = \frac{q}{\pi}\left[\theta + \sin\theta \cos(\beta_1+\beta_2)\right]$$

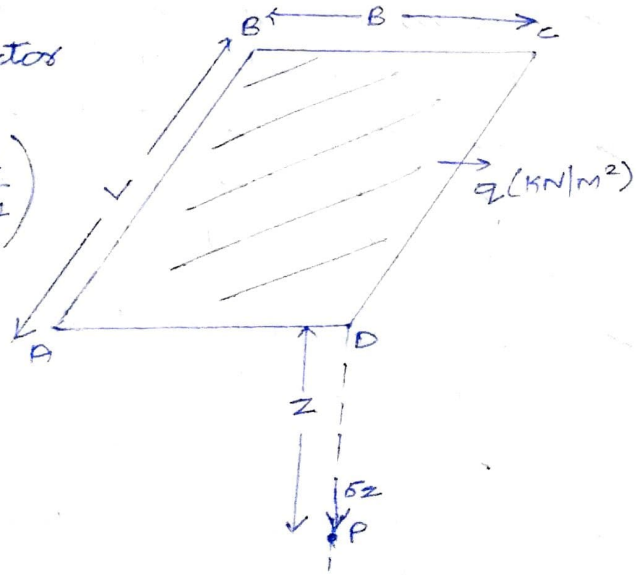
NOTE: If pt 'P' is situated under the centre of strip, $\beta_1 = -\frac{\theta}{2}$ & $\beta_2 = \frac{\theta}{2}$

$$\sigma_z = \frac{q}{\pi}\left[\theta + \sin\theta\right]$$

→ Vertical pressure below the corner of uniformly distributed rectangular area:

$\sigma_z = Kq$ K = influence factor

$$* \; K = \frac{1}{4\pi}\left\{\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\cdot\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)\right] + \tan^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right\}$$



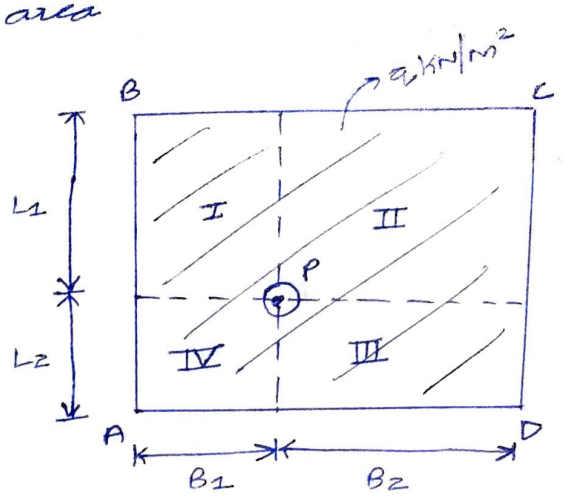
* $m = \frac{L}{z}$ * $n = \frac{B}{z}$

→ 'm' & 'n' are interchangeable

Special case:

→ If pt (P) lies inside the loaded area

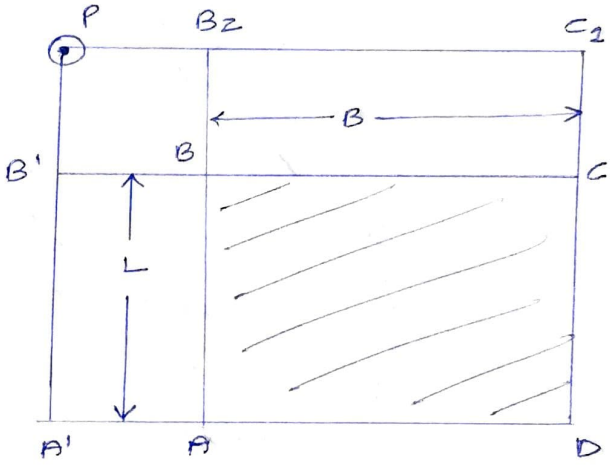
$$\sigma_z = q(K_1 + K_2 + K_3 + K_4)$$



* $m_1 = \frac{L_1}{z}$, $n_1 = \frac{B_1}{z}$
* $m_2 = \frac{L_1}{z}$, $n_2 = \frac{B_2}{z}$
* $m_3 = \frac{L_2}{z}$, $n_3 = \frac{B_2}{z}$
* $m_4 = \frac{L_2}{z}$, $n_4 = \frac{B_1}{z}$

→ If point (P) lies outside the rectangular loaded area

$$\sigma_z = q\left[K_{PA_1O C_1} - K_{PB_1 C C_1} - K_{PB_2 A A_1} + K_{PB_1 B B_2}\right]$$



→ Approximate methods to calculate the vertical stress:

a. Equivalent point load method:

→ It is an approximate method to calculate the vertical pressure at any pt in the soil mass due to any shape of loaded area.

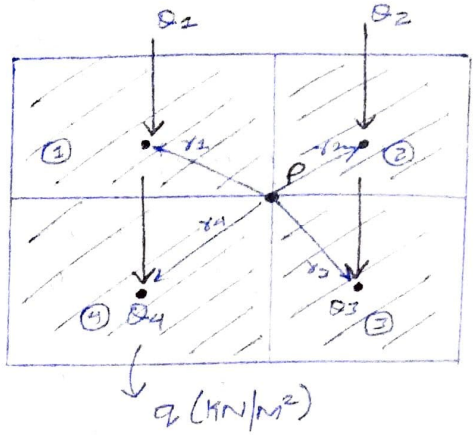
→ In this method, the entire loaded area is divided into number of smaller area units & load acting on these area units is replaced by equivalent pt load acting at the centroids of these area units

→ Hence the entire loaded area is divided & replaced by equivalent number of pt loads influence of which is considered seperately to find the stress at concerned pt.

→ Accuracy of this method depends upon the size of the area units considered, if the length of side of these area units is smaller

near one third the depth at which stress is required, the error involved is less than 3% in this method.

$Q_1 = qA_1$, $Q_2 = qA_2$, $Q_3 = qA_3$, $Q_4 = qA_4$



$$\sigma_z = \frac{K_{B1}Q_1}{z^2} + \frac{K_{B2}Q_2}{z^2} + \frac{K_{B3}Q_3}{z^2} + \frac{K_{B4}Q_4}{z^2} + \dots \frac{K_{Bn}Q_n}{z^2}$$

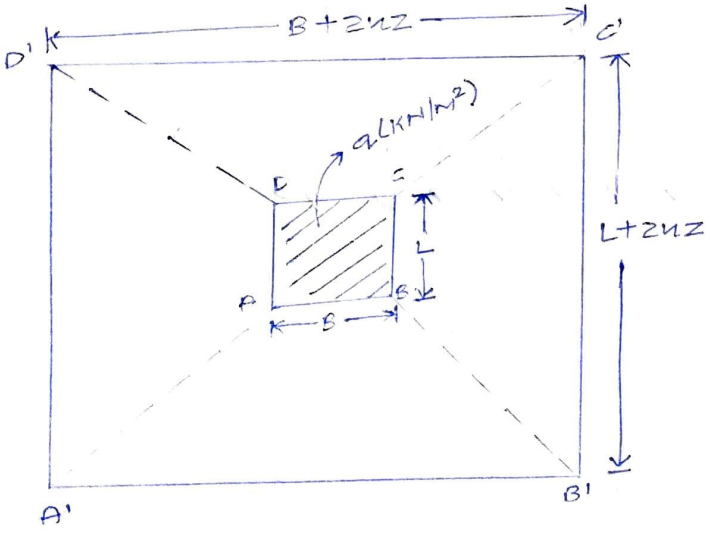
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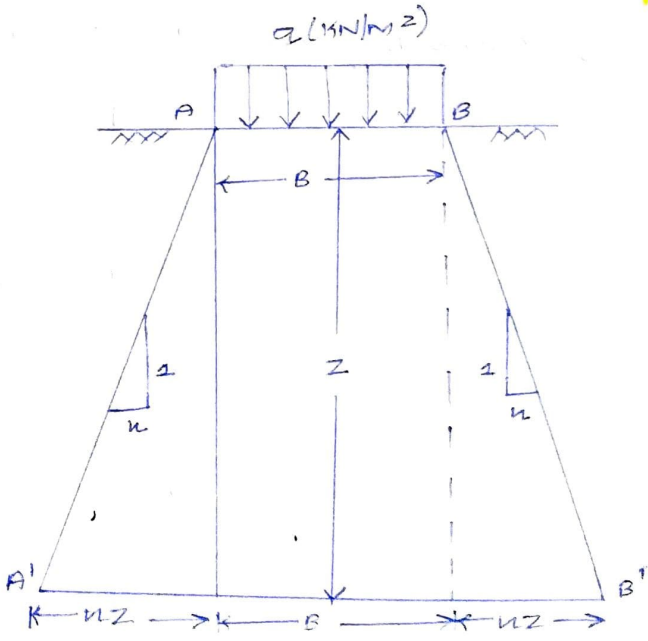
$$\sigma_z = \sum_{i=1}^{n} \frac{K_{Bi}Q_i}{z^2}$$

b. Trapezoidal method / load distribution 1 to 2 method:

→ In this method, load over the surface is assumed to be distributed over an area, shape of which is same as that of loaded area but size of which increases in proportion to its depth.

→ In this method, generally "2V : 1H" is adopted (distribution) below the loaded area.





a. For rectangular loading:

**

$$\sigma_z = \frac{q(BL)}{(B+2nz)(L+2nz)}$$

b. For square loading:

**

$$\sigma_z = \frac{qB^2}{(B+zn z)^2}$$

c. For circular loading:

**

$$\sigma_z = q\left(\frac{\pi D^2}{4}\right) \Big/ \frac{\pi(D+zn z)^2}{4}$$

d. For strip loading:

$$\sigma_z = \frac{q(BL)}{(B+zn z)(L+zn z)}$$

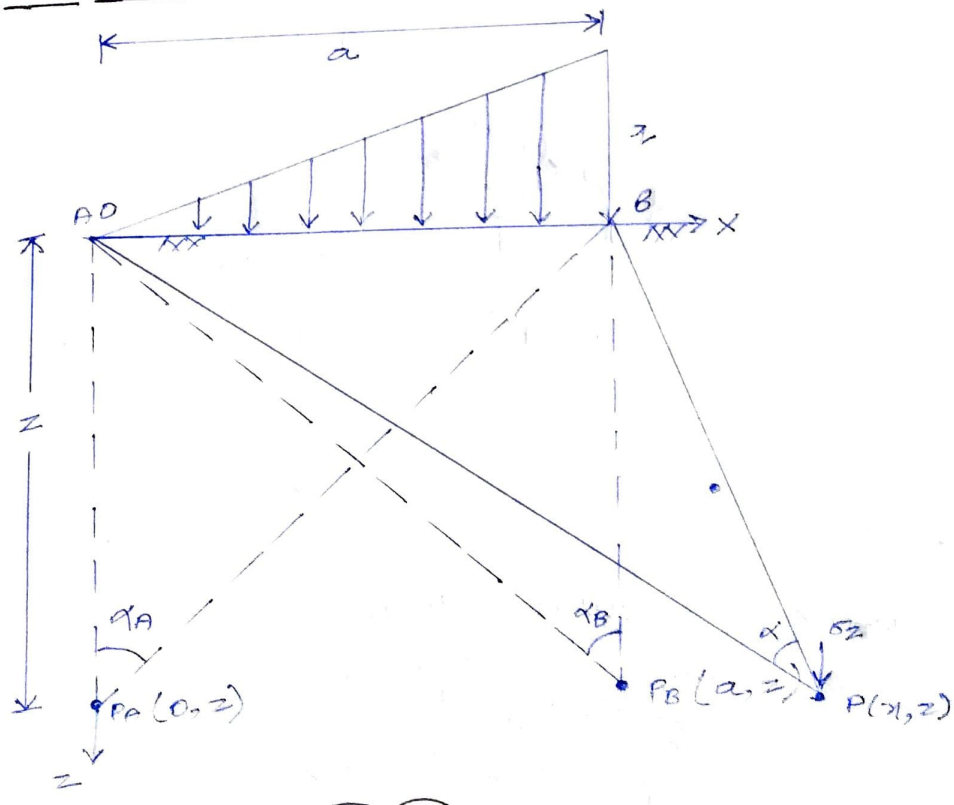
Here $L >> B$

$L + zn z \neq L$

**

$$\sigma_z = \frac{qB}{B+zn z}$$

→ Vertical pressure due to triangular loading:



**

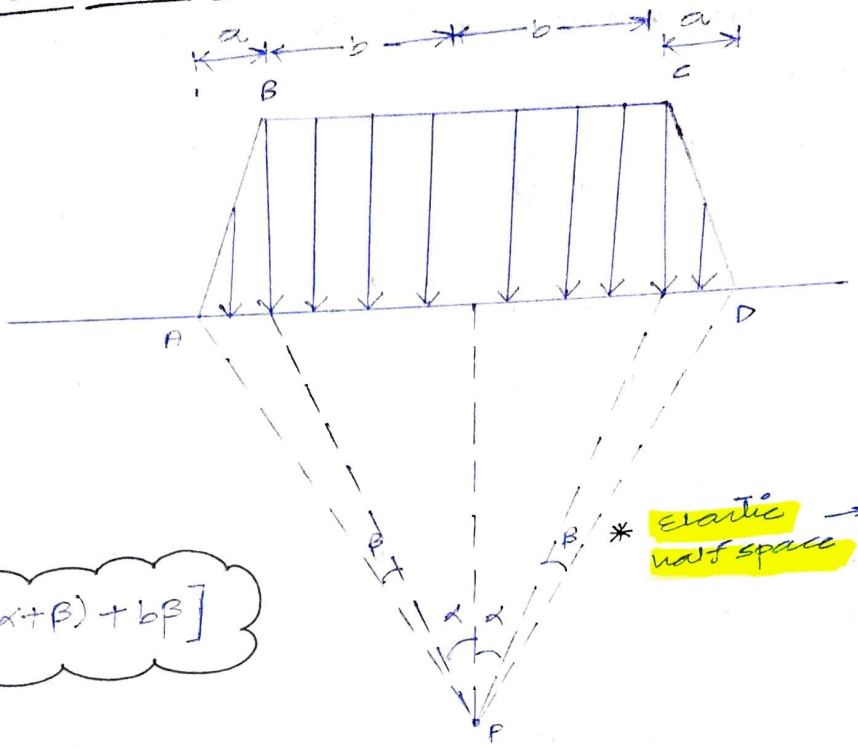
$$\sigma_z = \frac{q}{a\pi}\left[x\alpha - \frac{az}{(x-a)^2+z^2}(x-a)\right]$$

* Special case:

** $\sigma_{zA} = \frac{q}{\pi}\left[\frac{az}{a^2+z^2}\right]$ {Here $x=0$}

** $\sigma_{zB} = \frac{q.a}{\pi}$ {Here $x=a$}

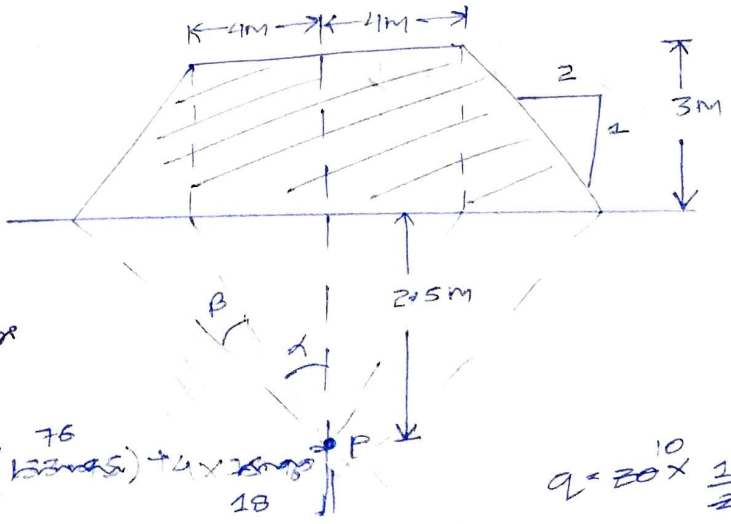
→ Vertical pressure due to trapezoidal load:



* Elastic half space → semi-infinite medium

** $\sigma_z = \frac{2q_1}{\pi a}\left[a(\alpha+\beta)+b\beta\right]$

Q. A railway embankment as shown is constructed at the site. Assume unit wt of the soil to be 20 kN/m³. Compute the increase in vertical stress under the centre line at depth of 2.5m.



$\tan\alpha = \frac{4}{2.5} = 1.6$

$\tan\beta = \frac{1}{2.5} = 0.4$

$\alpha = 58$ $\beta = 18°$

$\alpha+\beta = 75.963$

$\sigma_z = \frac{2\times 20}{\pi\times 6}\left[6\left(\frac{76}{180}\pi\right)+6\times\frac{18}{180}\pi\right]$

$\sigma_z = \frac{2\times 840}{\pi\times 6}\left[528\right]$

$\frac{1}{2} = \frac{3}{x}$

$x = 6$

$q = 20\times\frac{1}{2}(8+20)\times 3$

$q = 10\times 3\times 28 = 840$

→ Newmark's Influence chart method:

→ In this method, soil is assumed to be homogeneous, semi-infinite, elastic & isotropic (it means it is not applicable for stratified soil)

→ The greatest advantage of this method is that it can be used for any given shape of the loaded area

→ It is more accurate method for determining the vertical stress at any pt under the uniform area

→ Influence chart consists of 'n' numbers of concentric circles & 'n' numbers of radial lines which divides the influence chart into numbers of area units ($m \times n$) where the influence of each area unit at the centre of the circle is same

→ Influence factor $= \frac{1}{(m \times n)}$ m = no. of conc. circles (20)
n = " " radial line (20)

$I_f = \frac{1}{20 \times 20} = 0.05$ 0.005

→ In order to use influence chart, plan of the loaded area is drawn on the tracing paper on a scale such that distance 'AB' marked on influence chart is equal to the depth at which vertical stress is req

→ The plan of the loaded area is then placed on influence chart such that pt at which stress is required coincides with the centre of the influence chart.

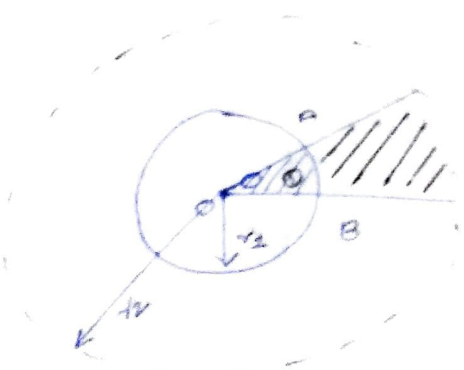
NOTE: The concerned point may be inside or outside the loaded area

→ The no. of area units occupied by the plan on influence chart is noted or counted (including the fractions ones)

→ Vertical stress at req pt 'P'

$\sigma_z = I_f \cdot N_a \cdot q$

I_f = Influence factor
N_a = Total no. of area units occupied by the plan of loaded area



AB
z = 5cm
I_f = 0.005

→ Let the uniformly loaded circular area of radius 'r₂' is divided into 20 sectors (area units)

→ If 'q' is the intensity of loading & σ_z is the vertical stress at depth 'z' below the centre of the area.

→ Then each area unit OAB, exerts a pressure equal to $\sigma_z/20$ at the centre.

$$\frac{\sigma_z}{20} = \frac{q}{20}\left[1 - \left(\frac{1}{1+\left(\frac{r_1}{z}\right)^2}\right)^{3/2}\right] \quad \text{—①}$$

→ Let influence of each area unit be If.q —②

$$\frac{q}{20}\left[1 - \left(\frac{1}{1+\left(\frac{r_1}{z}\right)^2}\right)^{3/2}\right] = 0.005q$$

$$\frac{r_1}{z} = 0.27$$

Let $z = 5cm \Rightarrow r_1 = 5 \times 0.27 = 1.35cm$

→ Let the radius of second concentric circle be 'r_2' cm.

→ By extending the 20 radial lines, the space b/w 2 consecutive of 2 concentric circles is again divided in 20 equal area units.

→ Hence in total 40 area units have same influence at the centre of the circle of radius 'r_2' of magnitude $\frac{\sigma_z}{40}$.

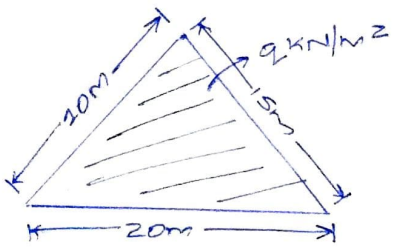
$$\frac{\sigma_z}{40} = \frac{q}{40}\left[1 - \left(\frac{1}{1+\left(\frac{r_2}{z}\right)^2}\right)^{3/2}\right] \quad \text{—①}$$

→ Influence of each area unit be If q —②

$$\frac{q}{40}\left[1 - \left(\frac{1}{1+\left(\frac{r_2}{z}\right)^2}\right)^{3/2}\right] = 0.005q$$

$$\frac{r_2}{z} = 0.4$$

If $z = 5cm \Rightarrow r_2 = 5 \times 0.4 = 2cm$

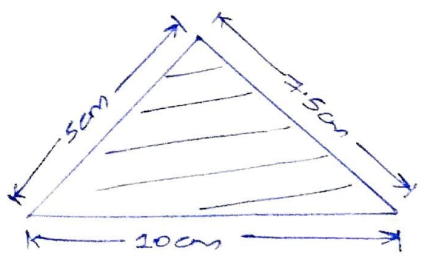


• P (Z=10M)

Scale

10m = AB = 5cm

10M = 5cm



COMPRESSIBILITY OF SOIL AND CONSOLIDATION

→ Whenever a material is stressed, it undergoes strain, so is the case with soil.

→ When a soil is loaded because of any reason (construction of material structure), its volume will decrease, due to rearrangement of soil particles.

→ The vertical downward displacement brought about by this volume change is termed as settlement of the structure standing over the soil.

→ The total vertical deformation of soil layer, under which stress, is directly proportional to

a. Compressibility of the soil
b. Magnitude of stress increase
c. The volume of soil or thickness of layer

→ Here first is the soil property, second is the loading condition & third is subsoil boundary condition

→ The compressibility of soil is defined as gradual decrease in volume of soil on application of stresses / load on soil.

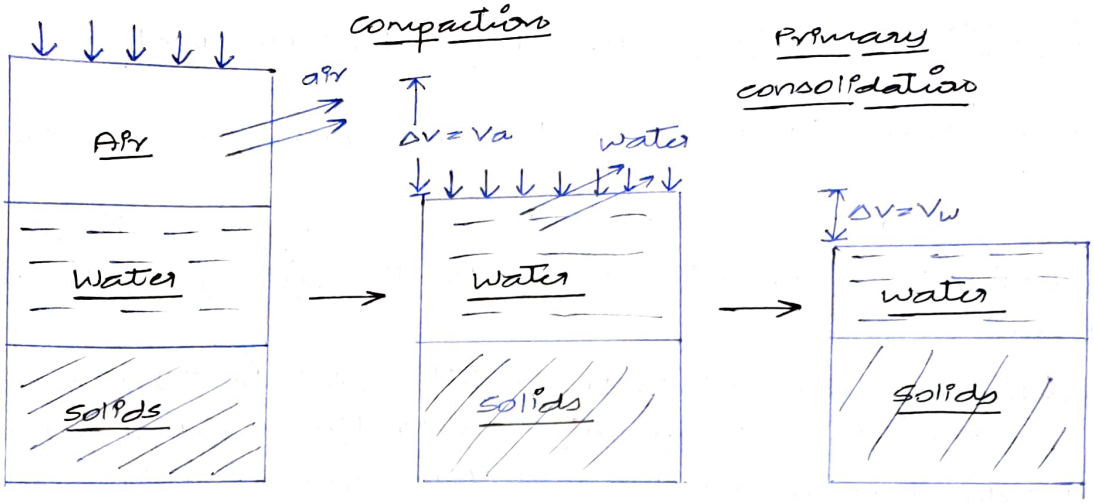
→ The volume decrease can be due to

a. compression & expulsion of pore air
b. " " " " " water
c. " of soil solid
d. Plastic rearrangement of soil particles.

NOTE: water & soil solids are considered to be incompressible, hence compression of water & solid is neglected.

COMPRESSIBILITY

- Initial compression/compaction (due to compression/expulsion of air)
- Consolidation
  - Primary (due to expulsion of water)
  - Secondary (due to plastic rearrangement of particles)



a. Initial compression/compaction

→ It is an instantaneous process which occurs immediately after loading & is due to compression & expulsion of air.

b Primary consolidation:

→ It begins when the soil reaches to full saturation state (after expulsion of pore air) & remains saturated during the entire process of primary consolidation

→ It takes place due to expulsion of the pore water

→ It is completed when expulsion of pore water stops

→ It is a time taking process & depends upon loading rate, permeability & length of the drainage path.

c. Secondary consolidation:

→ It starts after completion of primary consolidation

→ If load is constant & volume change is observed with passage of time without expulsion of pore water then volume change is due to secondary consolidation

→ It occurs due to plastic rearrangement of soil solids & expulsion of highly viscous water.

→ In case of e.g. soils (gravel, sand) stiff clay & pre-consolidated clay (fat clay) it is insignificant but in case of highly plastic clays, soft organic clays eg. peat it may extend upto 20% of total settlement.

→ Hence total settlement 'St' of loaded soil is

$$S_t = S_i + S_c + S_s$$

S_i = initial/immediate settlement
S_c = primary consolidation
S_s = secondary " /creep

→ Consolidation characteristics of the soil:

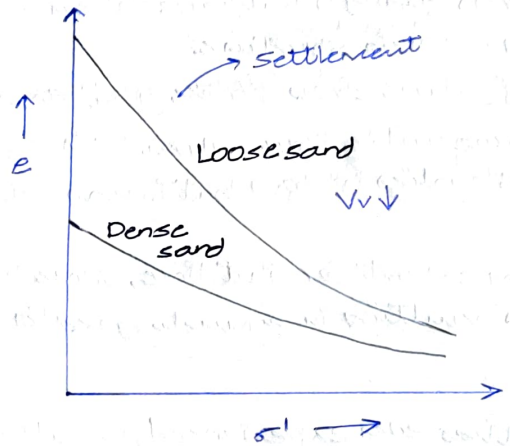
a. Effect of soil:

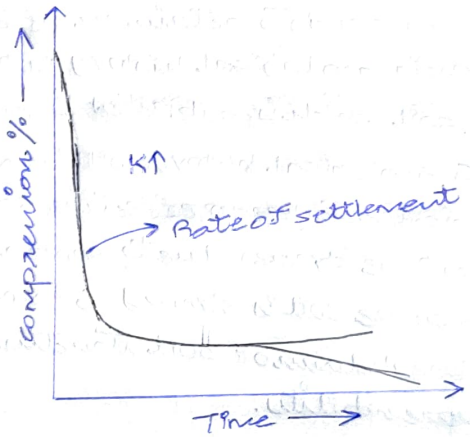
→ Granular soils exhibit a compressibility behaviour different from that of clay.

→ Due to its very high permeability, rate of consolidation is very high & compression occurs almost instantaneously, as it does not take much time for pore water to drain out (this is the reason why a structure on a sand soil experience very little settlement after it has been constructed as most of the settlement will already take place during construction only)

→ But due to its less volume of voids (void ratio, porosity), its total settlement is comparatively/relatively less hence for all practical purposes, compressibility of granular soil is neglected on account of consolidation.

NOTE: Dense sands are much less compressible than loose sand.

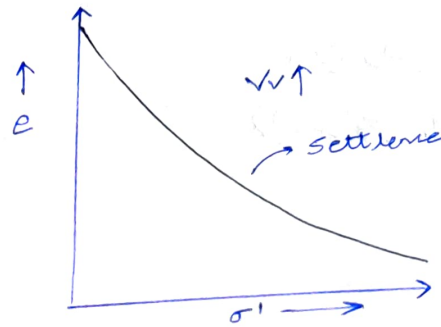


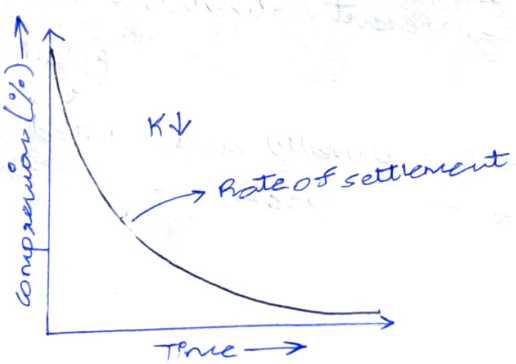


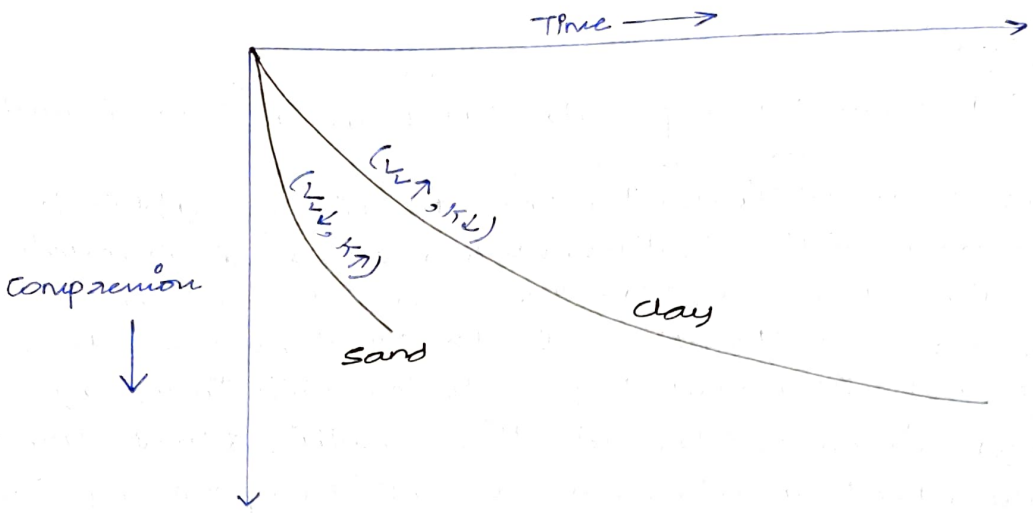
→ For saturated fine grained soil, the major factor contributing in comp. is consolidation.

→ In contrast to granular soils, where expulsion of pore water is instantaneous, here it takes much long time to expel out due to its low permeability, but the magnitude of consolidation in this case is much more due to its higher volume of voids.

→ The magnitude of time in this case may extend upto several months to several years also.







NOTE: Rate of settlement decreases with increase in effective stress.

b. Effect of stress:

→ soil tends to retain the effect of stress changes that have taken place in their geological history in the form of their structure.

→ A soil which is subjected to certain effective stress for the first time in its geological history, will be more compressible than, when it is subjected to larger effective stress in its earlier history but is now relieved from these stresses due to some reason.

→ When the soil is stressed to its maximum extent for first time, some kind of breakdown of soil structure occurs resulting in a much greater compressibility.

→ The maximum value of stress the soil has ever experienced, is called "Preconsolidated stress" (σ_c')

→ A soil is said to be "normally consolidated" when the existing effective stress σ', is the max. that is ever experienced by the soil in its stress history, i.e. $\sigma_c' = \sigma'$

→ A soil is said to be "Preconsolidated/overconsolidated", if the existing stresses is less than preconsolidated stress, i.e. $\sigma' < \sigma_c'$

→ Overconsolidation ratio (OCR) is the ratio of preconsolidation stress to the present vertical effective stress.

* $OCR = \frac{\sigma_c'}{\sigma'}$

* For normally consolidated soil, $OCR = 1$ ($\sigma' = \sigma_c'$)

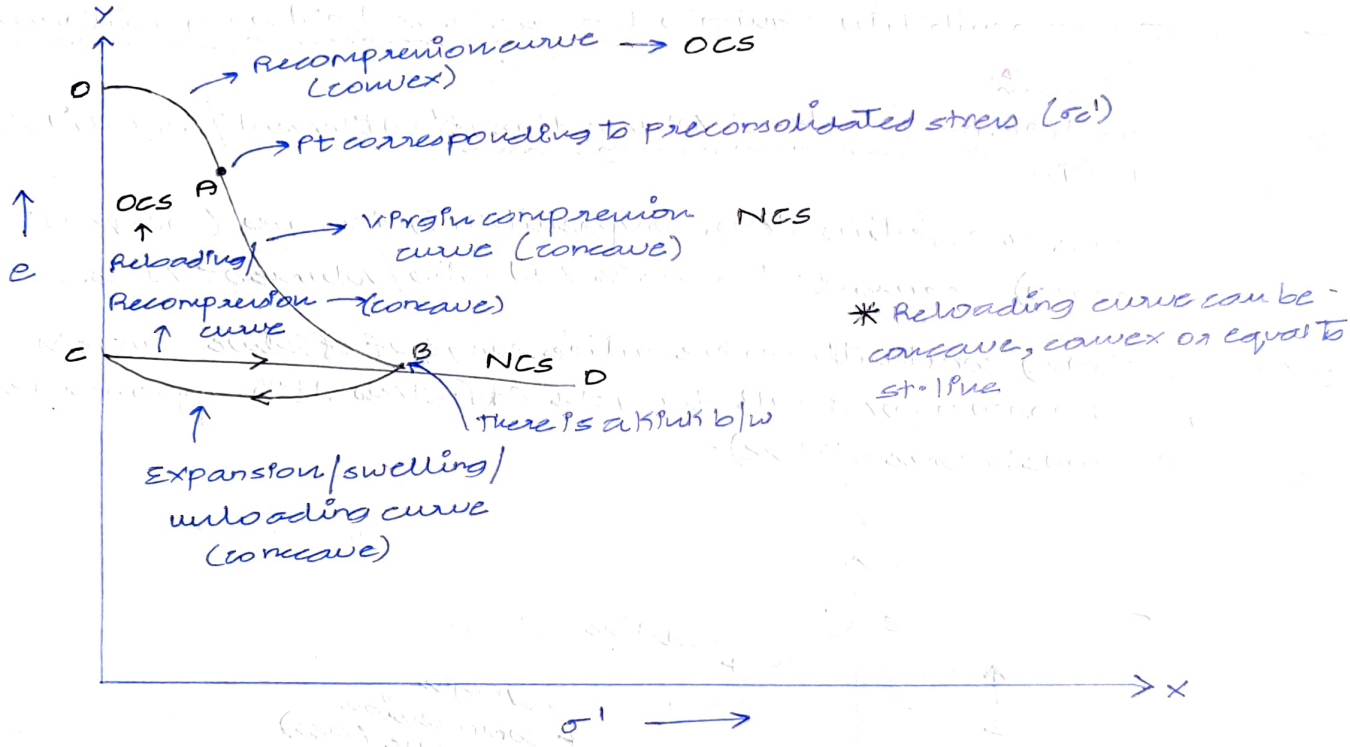
* " pre- " " , $OCR > 1$ ($\sigma_c' > \sigma'$)

*

→ soil which are yet to be consolidated under the existing overburden pressure, eg. in case of recently placed fill 'OCR < 1' & soil is termed as "Underconsolidated soil"

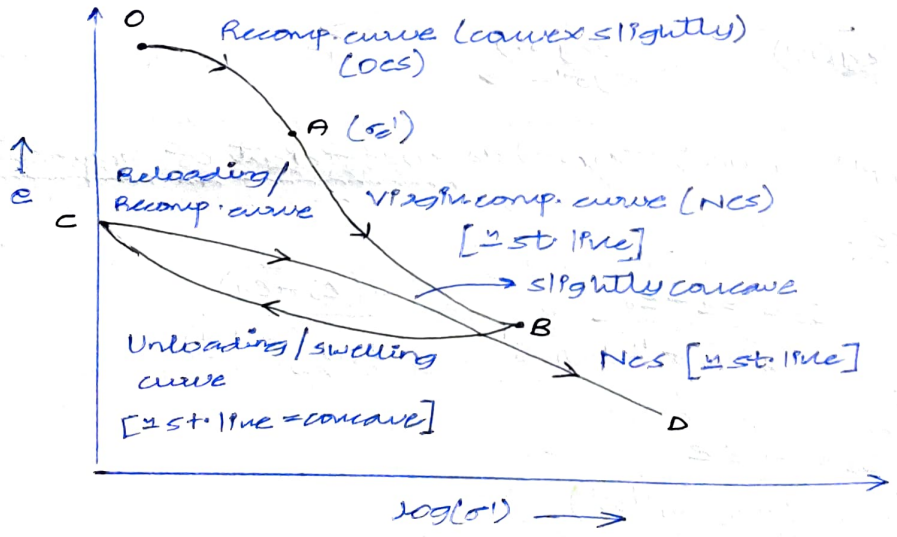
→ Preconsolidation of a soil stratum may occur due to several reasons

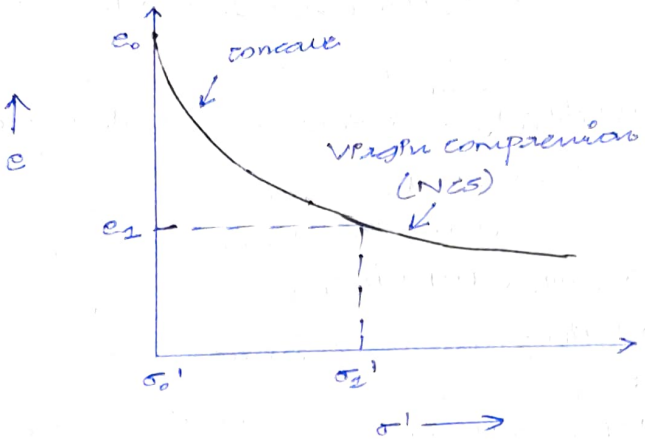
a. Erosion of overburden soil
b. Melting of glacier
c. Desiccation of upper layer of soil (drying)
d. Change in elevation of water table
e. Desiccation due to plant life
f. Removal of construction after sometime



* Reloading curve can be concave, convex or equal to straight

→ NCS has more settlement than OCS.





→ Slope of 'e' v/s 'σ'' curve is termed as **coefficient of compressibility 'av'**

$$av = \frac{e_1 - e_0}{\sigma_1' - \sigma_0'}$$

$$av = -\frac{(e_0 - e_1)}{\sigma_1' - \sigma_0'}$$

**

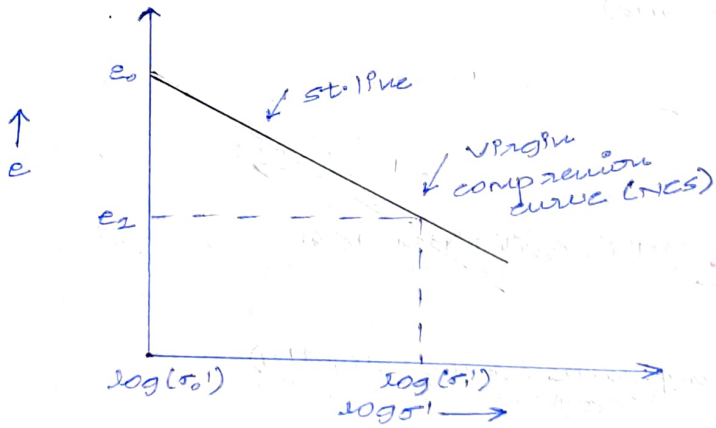
$$av = -\frac{\Delta e}{\Delta \sigma'}$$

→ Coeff. of compressibility signifies decrease in void ratio per unit increase in effective stress.

→ It represents the magnitude of volume change/ settlement/ consolidation/ compressibility

→ With increase in effective stress, compressibility decreases. (coeff. of comp. decreases) because of decrease in void ratio / volume of voids.

→ 'av' not being constant over the entire range of effective range is not widely used, however for small change in eff. stress, it is approximated as constant (curve become straight line)



→ Slope of 'e' v/s 'logσ'' curve is termed as **coefficient of compression/ compression index (Cc).**

$$C_c = \frac{e_1 - e_0}{\log_{10}\sigma_1' - \log_{10}\sigma_0'} = -\frac{(e_0 - e_1)}{\log(\sigma_1'/\sigma_0')}$$

**

$$C_c = -\frac{\Delta e}{\log_{10}(\sigma_1'/\sigma_0')}$$

→ Compression index is constant for NCS hence is the meaning ful parameter for this soil & is not preferred for OCS

→ For most of soils (clays),

* $C_c = 0.1 - 0.8$

→ Compression index also signifies the compressibility / volume change (magnitude) in soil.

→ It can be found empirically by following relations

a. For undisturbed clay of medium low sensitivity

$$C_c = 0.009 (w_L - 10)$$ w_L = liquid limit (%)

b. For remoulded clays,

$$C_c = 0.007 (w_L - 7)$$

c. For all clays,

$$C_c = 1.15 (e_0 - 0.35)$$

d. For inorganic cohesive soil, clay, silts,

$$C_c = 0.30 (e_0 - 0.27)$$

e. For organic soil,

$$C_c = 1.15 \times 10^{-2} \times w_n$$ w_n = natural water content

f. For low plastic clays,

$$C_c = 0.75 (e_0 - 0.5)$$

→ Slope of "e" v/s "log σ'" in recompression stage or overconsolidation stage is termed as "coefficient of recompression / recomp. index" (C_r)

** $$C_r = \frac{-\Delta e}{\log_{10}\left(\frac{\sigma_2'}{\sigma_1'}\right)}$$

→ OCS is less compressible than NCS, since it has already undergone volume change due to permanent destruction of soil structure

→ Hence, $C_r < C_c$ $\left[C_r = \frac{1}{5} - \frac{1}{10} C_c \right]$

→ Highly OC clays behave like dense sand [i.e. are less compressible]

→ The slope of "e" v/s "log (σ')" curve in portion 'BC' is termed as "coeff. of swelling / expansion or swelling / expansion index" (C_s)

** $$C_s = \frac{-\Delta e}{\log_{10}\left(\frac{\sigma_1'}{\sigma_0'}\right)}$$

NOTE: In order to identify the stage in which soil is in NCS or OCS stage, preconsolidation stress is required to be found which is obtained by performing the test on undisturbed overconsolidated soil.

→ A Casagrande suggested following method to find the value of 'σ_o'.

→ By Judgement of eye, point A, the pt of max. curvature (min. radius) is located on consolidation curve

→ At 'A', a horizontal line is drawn

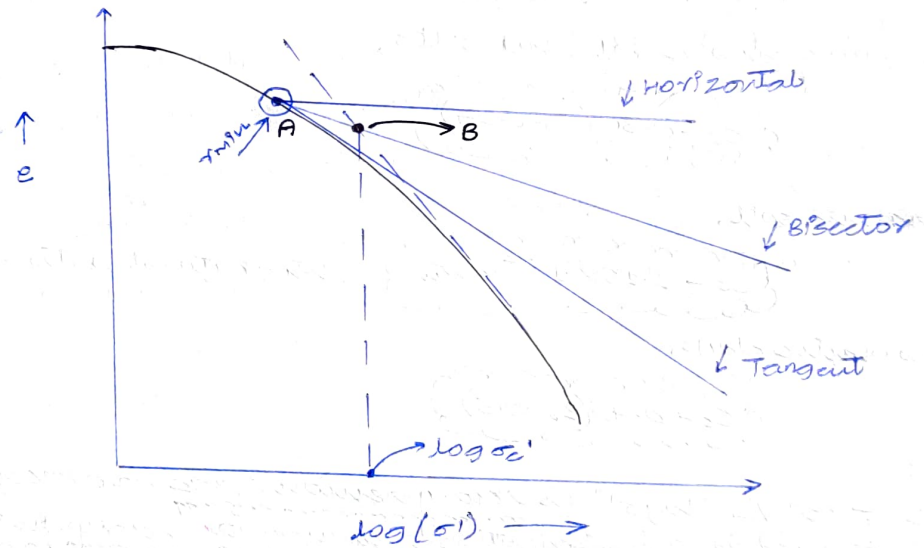
→ A tangent is drawn to the curve at 'A'

→ The angle b/w above 2 lines is bisected

→ The st. line part of the curve is extended back to meet the bisector line at pt 'B'.

→ Stress at 'pt' "B" is preconsolidated stress.

"CONSOLIDATION CURVE"



→ Terzaghi's one-dimensional consolidation theory:

→ Mechanism of consolidation:

→ Terzaghi demonstrated with the help of mechanistic model (spring-piston), the process of consolidation & the factors controlling it.

→ This model consists of cylindrical vessel with series of piston separated by springs.

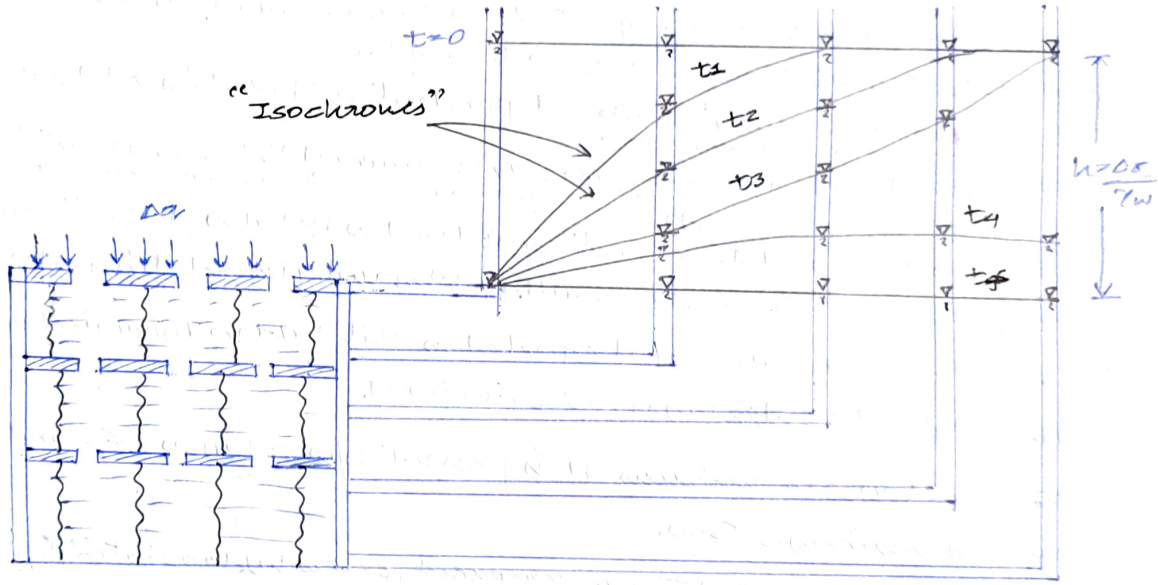
→ The space b/w the springs is filled with water & the pistons are perforated so as to allow through communication to water.

→ Since the area of piston is approx. same as that of vessel, the comp. is 1-D.

→ In this vessel, water can drain out only in vertical direction (upwards)

→ Similarity b/w this model of consolidation is as follows

Spring piston model	Soil
→ Spring	→ Soil solids
→ Perforation in piston	→ Voids in soil
→ Water surrounding the spring	→ Pore water
→ Vessel	→ Soil
→ Stress in spring	→ Effective stress in soil solids



→ At $t=0$, $\bar{u}_0 = \Delta\sigma = \gamma_w h$ → At $t=t_f$, $\bar{u}_f = 0 \Rightarrow h_{w_f} = 0$ → $\Delta\sigma = \Delta\sigma'$

$h_{w_0} = \frac{\Delta\sigma}{\gamma_w}$ → At $t=t$ $\bar{u}$, $\Delta\sigma = \Delta\sigma' + \bar{u}$

→ Hence dissipation / decrease in volume of soil by squeezing out of pore water on account of gradual dissipation of excess hydrostatic pressure induced by imposed total stress is termed as "consolidation".

→ Volume change / consolidation is taking place due to effective stress & not total stress.

→ Volume decrease & shear strength transfer from water over soil solids is simultaneous.

→ Time required by water to escape out from voids of soil depends upon its permeability, rate of loading & length of drainage and is termed as "hydrodynamic lag".

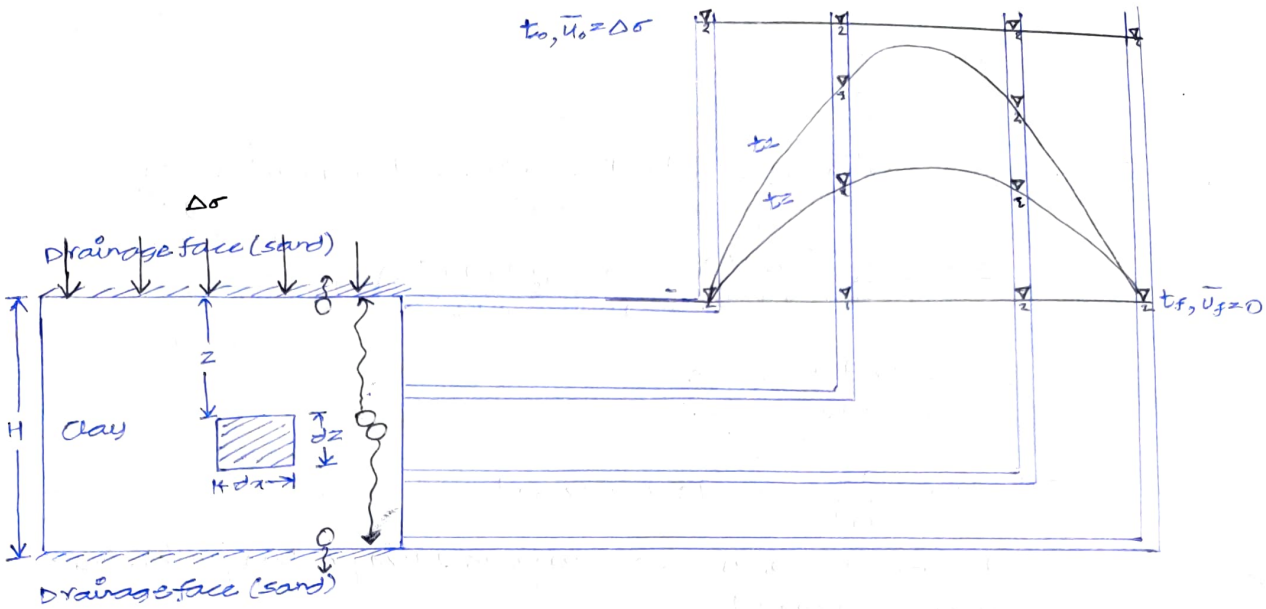
→ Line joining the water level at different / any given time in different piezometers is termed as "Isochrones"

→ **Terzaghi's theory of 1-D consolidation:** for small load increment only

→ The rate of settlement is directly related to the rate of dissipation of excess pore water pr., hence in order to predict time-rate of consolidation of soil, Terzaghi's theory is used, which works on following assumptions.

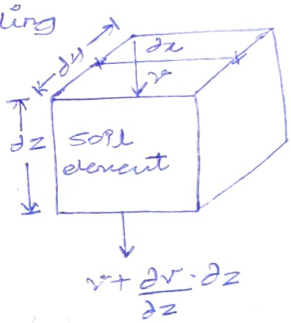
a. Compression & flow are one dimensional (vertical)
b. Darcy's law is valid
c. Soil is homogenous & isotropic.
d. Soil is completely saturated
e. The soil solids & water are both incompressible
→ strains are small i.e. the applied load increment produces virtually no change in the thickness of the soil sample.
f. There is unique relationship independent of time b/w void ratio & eff. stresses [i.e. $a_v = \frac{-de}{d\sigma'}$, here a_v is assumed to be constant] i.e. secondary consolidation is not considered as it takes place at constant stress, hence in such case there will not be unique relationship b/w 'e' & 'σ'' or it can also be interpreted as that hydrodynamic lag is considered but plastic lag is not considered.

→ Let a clay layer of thickness 'H' is present b/w 2 layers of sand which serves as drainage faces.
→ When clay layer is subjected to pr. increment, excess hydrostatic pr. is setup in the clay layer
→ At time t_0, the instant pressure is applied whole of the consolidation pr $\Delta\sigma$ is carried by the pore water so initial excess pore water pr., $\bar{u}_0 = \Delta\sigma$
→ This excess hydrostatic pressure cause water to expel out which leads to transfer of the certain excess hydrostatic pr. over the soil solids, resulting in increase of eff. stress & consolidation.
→ After infinite time t_f, the whole of the excess hydrostatic pr. is dissipated out out, i.e. $\bar{u}_f = 0$
→ Let a soil sample/element at depth 'z' is considered from top having dimension dx, dy & dz & excess hydrostatic pr. at this element of soil at time 't' be $\bar{u}$.



→ At any time 't', the hydraulic head corresponding to excess hydraulic/hydrostatic pr., $\bar{u}$,

$$h = \frac{\bar{u}}{\gamma_w}$$



→ Hydraulic gradient, $i = \frac{\partial h}{\partial z}$

$$i = \frac{1}{\gamma_w} \frac{\partial}{\partial z}(\bar{u})$$

→ As per Darcy's law, $v = ki = \frac{k}{\gamma_w} \frac{\partial}{\partial z}(\bar{u})$

→ The Rate of change of velocity wrt depth in soil element

$$\frac{\partial v}{\partial z} = \frac{\partial}{\partial z}\left[\frac{k}{\gamma_w} \cdot \frac{\partial}{\partial z}(\bar{u})\right]$$

$$\frac{\partial v}{\partial z} = \frac{k}{\gamma_w} \frac{\partial^2(\bar{u})}{\partial z^2} \quad \text{—— (1)}$$

→ The quantity of water entering the soil element = $v\,dx\,dy$

→ " " " " leaving " " " $= \left(v + \frac{\partial v}{\partial z} \cdot dz\right) dx\,dy$

→ Net quantity of the water squeezed out of the soil element (Δq) $= \frac{\partial v}{\partial z} dx\,dy\,dz$

$$\Delta q = \frac{k}{\gamma_w} \frac{\partial^2(\bar{u})}{\partial z^2} dx\,dy\,dz \quad \text{—— (A)}$$

→ Vol. of soil element, $V = dx\,dy\,dz$

As we know, $V_s = \frac{V}{1+e} =$

$[V = V_s + V_v]$

$V_v = V - V_s$

$$V_v = V - V_s = \frac{e_0 V}{1+e_0} \qquad [\text{If } S=1,\ V_v = V_w]$$

$$V_w = \frac{e_0 V}{1+e_0}$$

→ Net quantity of water squeezed out from the soil element (Δq) $= \frac{\partial V_w}{\partial t}$

$$\Delta q = \frac{\partial}{\partial t}\left(\frac{e_0 V}{1+e_0}\right) = \frac{\partial}{\partial t}\left(\frac{e\,\partial x\,\partial y\,\partial z}{1+e_0}\right)$$

$$\Delta q = \left(\frac{\partial x\,\partial y\,\partial z}{1+e_0}\right)\frac{\partial e}{\partial t}$$

As we know, $a_v = -\frac{\partial e}{\partial \sigma'} \Rightarrow \partial e = -a_v \Delta\sigma'$

→ In the consolidation process, as the excess porewater pr. dissipates, the effective stresses are increased

$$\partial\sigma' = -\partial\bar{u}$$

$$\partial e = a_v \partial\bar{u}$$

$$\Delta q = \frac{\partial x\,\partial y\,\partial z}{1+e_0} \cdot a_v \cdot \frac{\partial \bar{u}}{\partial t}$$

$$\Delta q = \frac{a_v}{1+e_0} \cdot \frac{\partial \bar{u}}{\partial t}\,\partial x\,\partial y\,\partial z \quad \text{—— (B)}$$

from (A) & (B)

$$\frac{a_v}{1+e_0}\frac{\partial \bar{u}}{\partial t}\,\partial x\,\partial y\,\partial z = \frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}\,\partial x\,\partial y\,\partial z$$

$$\frac{\partial \bar{u}}{\partial t} = \frac{k(1+e_0)}{a_v \gamma_w}\frac{\partial^2 u}{\partial z^2}$$

Here $\frac{k(1+e_0)}{a_v \gamma_w} = c_v$ = coeff. of consolidation

** $\boxed{\frac{\partial \bar{u}}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}}$

→ It is the basic diff. eqn which relates the rate of change of excess hydrostatic pr. to the rate of change of expulsion of excess pore water pr. from a unit volume of soil mass during the same time interval.

→ Here, $c_v = \frac{k(1+e_0)}{a_v \gamma_w}$ termed as coeff. of consolidation that governs the

time rate of consolidation.

$$C_v = \frac{L|T \cdot F' \cdot L^2}{L^2 \cdot F} = \frac{1 \cdot L^2}{T} = \frac{L^2}{T}$$

NOTE: Magnitude of consolidation is given by 'av' & time rate of consolidation is given by 'Cv'.

→ Here change in volume of soil expressed in terms of original volume of soil to the corresponding change in effective stress is termed as coeff. of volume change or coeff. of volume compressibility (m_v).

→ $m_v = \frac{-\Delta V/V}{\Delta \sigma'} = \frac{-\Delta H/H}{\Delta \sigma'} = \frac{-\Delta e/1+e}{\Delta \sigma'}$

→ $a_v = \frac{-\Delta e}{\Delta \sigma'}$ → $m_v = \frac{a_v}{1+e_0}$ $\left[\frac{L^2}{F}\right]$

→ coeff. of volume change (m_v) represents inverse of compressibility modulus

→ Hence, $C_v = \frac{K}{m_v \gamma_w}$

→ With increase in consolidation pr., coeff. of consolidation 'Cv' increases as water expels out at faster rate but its variation is small hence can be neglected.

→ consolidation pressure ↑ settlement ↓ Rate of consolidation ↑

→ The solution of differential eq^n of consolidation is obtained by means of Fourier series, that must satisfy these boundary conditions

* At $t=0$, at any distance z, $\bar{u} = \bar{u}_0 = \Delta\sigma$
* " $t=\infty$, " " " " , $\bar{u} = \bar{u}_f = 0$
* " $t=t$, " " " $z=0$, $\bar{u} = 0$
* " $t=t$, at $z=H$, $\bar{u} = 0$

→ In Terzaghi solution, three non-dimensional factors are provided

a. Drainage path ratio, $z = \frac{z}{d}$

$Z = \frac{z}{d}$

* d = max. distance that water has to travel to reach the drainage face or it is length of the longest drainage path

→ If there are 2 drainage surface, one at top & other at bottom (double drainage)

$d = \frac{H}{2}$

→ If there is single drainage path, either at top or at bottom (single drainage)

$d = H$

→ Hence drainage path ratio Z, is the ratio of distance travelled by the

water to reach the drainage face to the length of the longest drainage path.

b. Degree of consolidation (U%):

→ It is the fraction of ultimate consolidation which is completed at any stage of time during consolidation

* At $t=0$, $U(\%)=0$
* " $t=t_f$, $U(\%)=100$

→ It can be computed by any of the following method

i. When settlement at any stage is known

→ Let ΔH is ultimate consolidation settlement & Δh is settlement at time 't'.

$$U(\%) = \frac{\Delta h}{\Delta H} \times 100$$

ii. When pore water pressure is known

→ Let $\bar{u}$ = excess pore water pressure at time 't'

$\bar{u}_i$ = " " " " " " $t=0$

$\bar{u}_f$ = " " " " " " $t=t_f$

$$U(\%) = \frac{\bar{u}_i - \bar{u}}{\bar{u}_i - \bar{u}_f} \times 100$$

iii. When void ratio is known

Let e_0 = void ratio at the beginning

e = " " " any time 't'

e_{100} = " " " end of consolidation

$$U(\%) = \frac{e_0 - e}{e_0 - e_{100}} \times 100$$

c. Time Factor (T_v):

→ It is a parameter which relates the degree of consolidation & the time required for that consolidation.

→ It is given by,

$$T_v = \frac{c_v t}{d^2}$$

c_v = coeff. of consolidation
d = length of longest drainage path
t = time after which particular degree of consolidation is achieved

→ Solution of Terzaghi's 1-D diff. eqn is given by

$$U = 1 - \sum_{n=0}^{\infty} f_1(Z) f_2(T_v)$$

→ If approximate relationship b/w U (%) & T_v is represented, it is given by

* For $U \leq 60\%$, $T_v = \frac{\pi}{4} U^2$

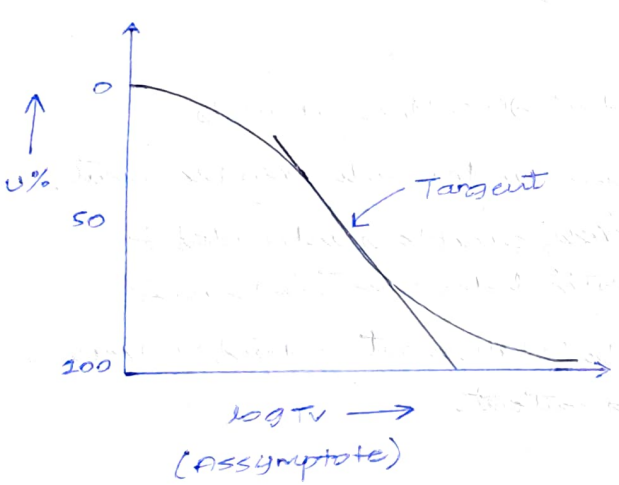
* For $U > 60\%$, $T_v = 1.781 - 0.933 \log_{10}(100 - U)$ 'U' → in %

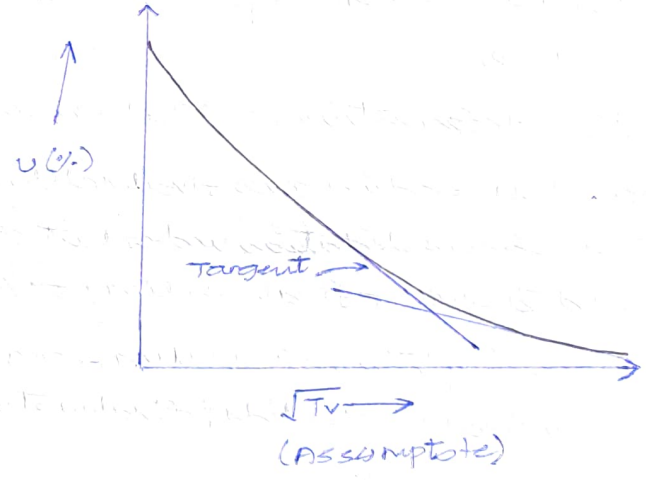
NOTE: If 'U' is in fraction,

$$T_v = -0.0851 - 0.9332 \log_{10}(2 - U)$$

2. Theoritically, for 100% primary consolidation, infinite time is required but it is assumed to be completed if degree of consolidation has reached ≥90%

3. If approx. result of 1-D consolidation eqn is referred graphically, it is observed as follows





→ Laboratory consolidation test:

→ The consolidation parameter that controls the magnitude of consolidation in soil is 'av' & time rate consolidation is controlled by 'Cv.'

→ These two parameters along with coeff. of permeability & swelling behaviour of the soil is found by performing consolidation test.

→ This test simulates 1-D consolidation with double drainage.

→ This test is carried out in the device termed as oedometer or consolidometer.

→ It consists of a loading mechanism & specimen containing known as consolidation cell.

→ An undisturbed soil specimen, representing the in-situ soil layer is placed in the metallic ring (confining) which does not allow any lateral deformation.

of soil.

→ Porous stone ~~bodies~~ disc are provided at the top & bottom of the soil sample which permits the drainage

→ Two types of consolidation cells are used.

a. Floating ring type
b. Fixed " "

→ In floating ring type test, compression occurs from both top & bottom and friction available b/w the soil sample & ring is comparatively less.

→ In fixed ring type, the soil sample moves only in d/w direction, however drainage at the bottom porous ~~face~~ disc can be controlled or measured in this case, hence it can be used for measurement of permeability also of soil sample.

→ The soil sample is loaded in increments of vertical stress & under each stress increment the sample is allowed to consolidate, till there is little or no further compression (usually a stress increment is kept for 24 hrs)

→ Vertical deformation of soil is noted at diff. time intervals

→ The 24 hr reading gives final settlement under each stress increment.

→ After the consolidation under last stress, sample is unloaded & allowed to swell & final reading of vertical deformation is noted

→ After the completion of swelling, sample is taken out & dried in oven to determine wt of solids & final water content.

→ IS 2720 Part XV

* Diameter → 65-110MM
* Thickness → 15-30MM

→ The consolidation test data are further used to determine the following

a. Void ratio & coeff. of volume change

→ It can be computed by any of following methods

i. Height of solid methods
ii. Change in void ratio method

NOTE: The change in void ratio method is only used for fully saturated soil whereas, ht of solids methods can be used for ~~partially~~ partially saturated soil also.

a. Height of solids method:

→ The ht of solids of specimen is calculated as

$$G = \frac{\rho_s}{\rho_w} = \frac{M_s}{V_s \rho_w} \Rightarrow V_s = \frac{M_s}{G\rho_w} = \frac{M_s}{H_s A \rho_w}$$

$$H_s = \frac{M_s}{GA\rho_w} = \frac{W_s}{GA}$$

→ The void ratio of the soil sample is given as

$$* \; e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \frac{HA - H_s A}{H_s} = \frac{H - H_s}{H_s}$$

H = specimen ht in 'cm' at eqm under various applied pr

→ $H = H_0 + \Sigma \Delta H = H_1 + \Delta H$

→ H_0 = initial ht of specimen

→ ΔH = change in specimen thickness under any pr. increment

→ H_1 = ht of specimen at beginning of load increment

Applied pr. (σ') $\left(\frac{KN}{M^2}\right)$	Final dial reading	ΔH (MM)	$H = H_1 + \Delta H$	Height of voids $H - H_s$	$e = \frac{H - H_s}{H_s}$
0	a	ΔH_1	–	✓	e_0
10	b	ΔH_2	–	✓	e_1
20	c		–		
50			–		
100			–		
200			–		
400			–		
800		ΔH_8	–		
1000		ΔH_9	–	✓	e_{10}
0 → swelling					

** b. change in void ratio method:

→ Assuming the specimen to be fully saturated, the void ratio 'e_f' at the end of test is determined as follows,

$$e = \frac{w G}{S}$$

$$e_f = w_f G$$

→ The change in void ratio Δe under each pr. increment is calculated as follows

$$\frac{\Delta V}{V} = \frac{\Delta e}{1+e} = \frac{\Delta H}{H} \Rightarrow \Delta e = (1+e_f)\frac{\Delta H}{\Delta H_f}$$

*

$$\Delta e = (1+e_f)\frac{\Delta H}{H_f}$$

H_f → final ht of specimen at end of the test

→ Knowing Δe & working backward from the known value of e_f, the eqm void ratio can be calculated for each pr. increment.

→ With the help of 'e' & corresponding σ', plot b/w the 2 is drawn & its slope is termed as 'coeff. of compressibility (a_v)'.

→ Coeff. of volume change can also be computed as

$$M_v = \frac{a_v}{1+e_0} = \frac{-\Delta e}{1+e_0}\cdot\frac{1}{\Delta\sigma'} = \frac{-\Delta H}{H}\cdot\frac{1}{\Delta\sigma'}$$

b. Determination of coefficient of consolidation:

→ Coeff. of consolidation increases with increase in magnitude of consolidation pressure, hence should be determined in the laboratory consolidation test for a range of pressure that can be encountered in practice.

→ It also depends upon type of soil (K) and change in effective stress hence must be computed at the centre of compressible layer

→ Its value is observed to decrease with increase in liquid limit of the soil.

→ 'Cv' can be determined by taking the advantage of characteristics of degree of consolidation v/s time factor curve.

→ This curve is in fact similar to that obtained from laboratory test b/w dial reading 'R' & time required to reach a particular degree of consolidation

→ These methods are termed as 'curve-fitting methods'.

i. Casagrande's logarithmic of time fitting method:

→ The shape of theoritical 'U' v/s 'T_v' (log) curve & experimental dial reading v/s log(t) curve are similar.

→ The tangent & the asymptote intersect at U = 100%, since the time req. for U = 100% is infinity, Casagrande suggested that if tangents are drawn to the experimental curve over the straight portion, their intersection will correspond to U = 100% & dial reading at U = 100% is R_{100}.

→ To determine R_0, two values of time t_1 & t_2 are chosen in the initial parabolic (curved) portion of the curve such that $t_2 = 4t_1$.

→ The pts corresponding to these times are marked on the curve & the vertical distance b/w them is measured (z).

→ Now, a horizontal line is drawn above the first pt at same vertical distance (z).

→ This line cuts the ordinate at R_0, called corrected zero reading.

→ The compression that occurs b/w R_0 & R_{100} is primary consolidation & from R_{100} onwards is secondary consolidation

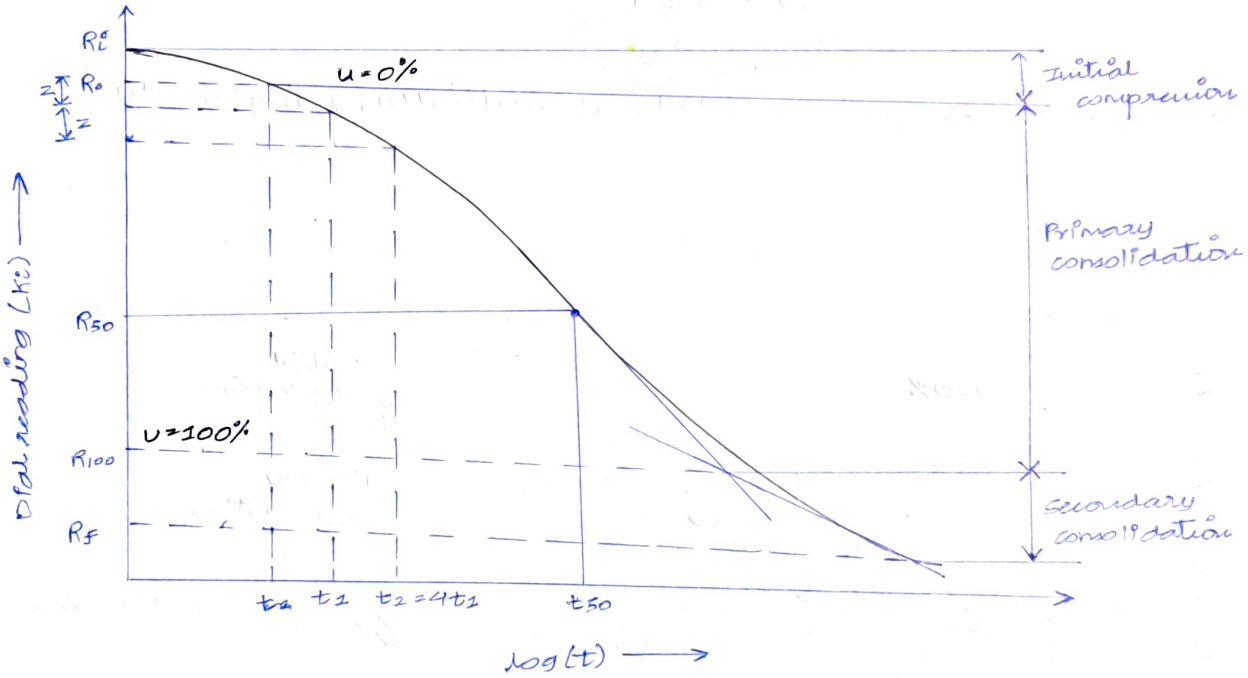
→ After locating R_0 & R_{100}, R_{50} is marked midway b/w these 2 pts & the time corresponding to R_{50} is read t_{50}.

$$T_v = \frac{c_v t}{d^2}$$

→ For $U = 50\%$, T_v can be found mathematically/graphically

** $c_v = \frac{T_{v50} d^2}{t_{50}}$

$$d = \frac{H_{avg}}{2} = \left(\frac{H_0 + H_f}{2}\right) \cdot \frac{1}{2}$$



pp. Taylor square root of time fitting method:

→ This method uses the characteristics of U (%) vs $\sqrt{T_v}$ curve

→ This curve is straight upto U = 60% and the abscissa at U = 90% is equal to 1.15 times of abscissa at U = 60%

→ The use of this characteristic of theoritical curve to determine c_v was suggested by Taylor.

→ This method consists of drawing the curve b/w $\sqrt{T_v}$ on x-axis & dial gauge reading on y-axis representing the comp. of the specimen.

→ The initial dial gauge reading R_0 corresponds to 0% degree of consolid. at $t = 0$.

→ The st. position of the pro curve is produced back to meet the ordinate reading at R_0 which is called corrected zero / initial reading.

→ The consolidation b/w R_0 & R_i signifies initial compression.

→ From R_0, another st. line is drawn such that its abscissa at every pt is 1.15 times that of line drawn previously.

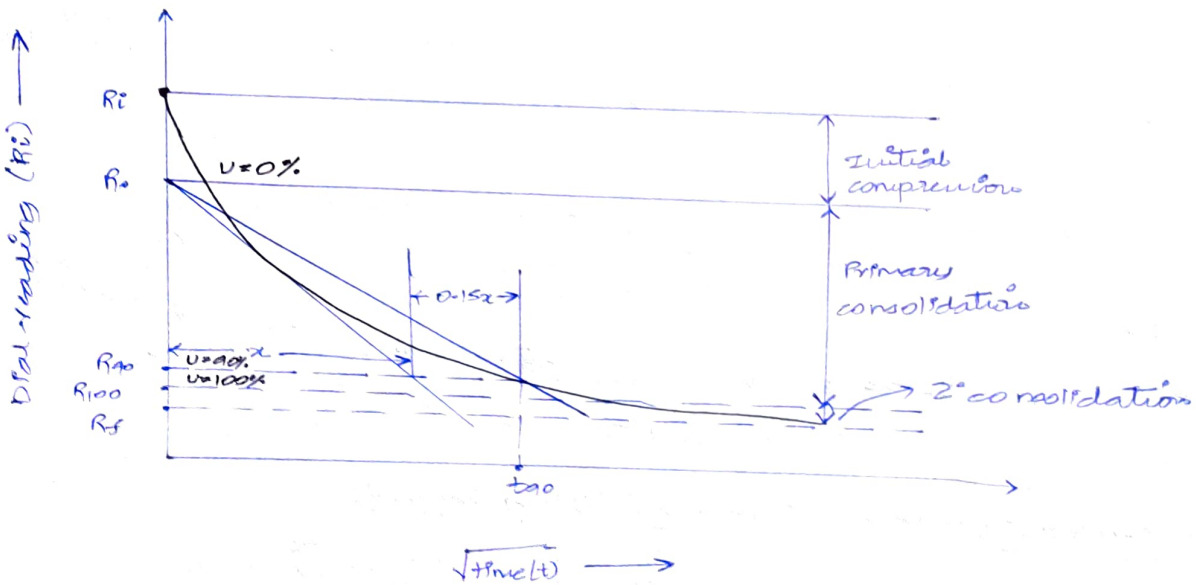
→ Wherever this new st. line meet the curve, it corresponds to U = 90%, whose dial gauge reading R_{90} is read from the curve.

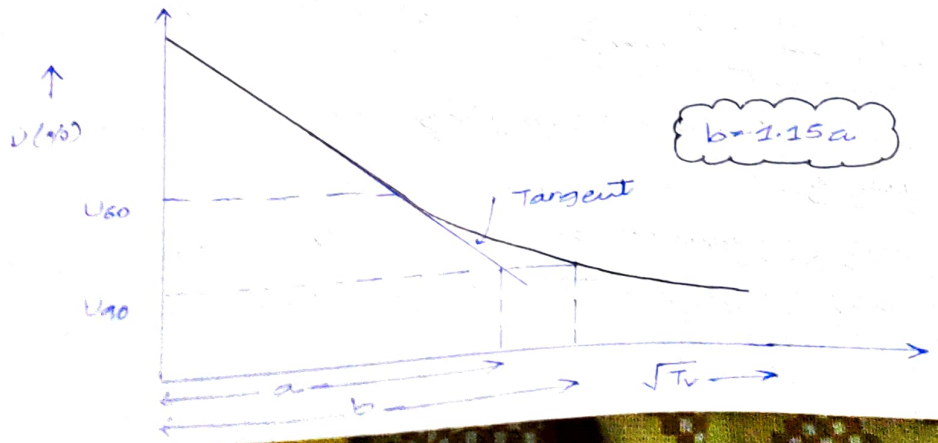
→ Time required to reach U = 90%, i.e. t_{90} is also read from the curve to compute degree of consolidation.

$$T_v = C_v \frac{t}{d^2}$$

$$C_v = \frac{T_{v90} d^2}{t_{90}}$$

→ For U = 90% ⇒ T_{90} is computed mathematically or graphically

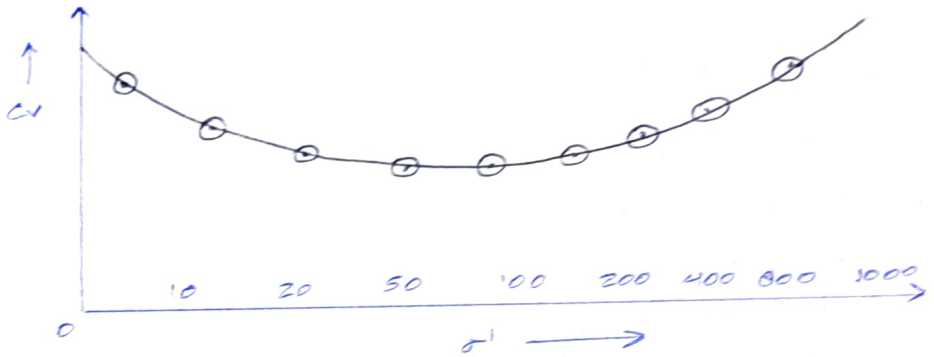




NOTE: square root of time fitting method is better for those soils which have high compressibility.

→ An advantage of this method is that test can be finished by plotting the curve upto u=90% degree of consolidation

→ The cv value of cv will be obtained for each one of the stress increment, hence cv obtained from any one curve fitting method can be plotted as ordinate with the avg. effective stress as abscissa & the req. value of cv can be read from it.



NOTE: The compression $R_0 - R_i$ is due to many factors which include

* compression of small quantities of air in soil
* Imperfect saturation
* Vertical elastical comp. of soil specimen
* Lateral expansion of soil specimen

→ This compression is termed as "Initial compression"

→ The relative magnitude of Initial compression, primary consolidation & secondary consolidation can be computed as

→ Initial comp. ratio, r_{i0}

* $r_0 = \frac{R_i - R_0}{R_i - R_f}$

→ Primary consolidation comp. ratio, $r_p = \frac{R_0 - R_{100}}{R_i - R_f}$ (log time)

→ Primary compression ratio, $r_p = \frac{10}{9}\left(\frac{R_0 - R_{90}}{R_i - R_f}\right)$ (root time)

* $r_p = \frac{10}{9}\left(\frac{R_0 - R_{90}}{R_i - R_f}\right)$

→ Secondary compression ratio, $r_s = 1 - (r_0 + r_p)$

* $r_s = 1 - (r_0 + r_p)$

c. Determination of coeff. of permeability:

→ A falling head permeability test can be performed on the consolidation specimen, attaching a stand pipe to fixed ring consolidometer. When consolidation of the specimen is complete under a particular pressure increment or can be computed as **

$$C_v = \frac{K}{M_v \gamma_w}$$

$$K = C_v \cdot M_v \cdot \gamma_w$$

NOTE: Initial compression ratio,

**

$$\gamma_0 = \frac{R_i - R_0}{R_i - R_f}$$

→ Primary consolidation ratio, (log time)

**

$$\gamma_p = \frac{R_0 - R_{100}}{R_i - R_f}$$

→ Primary consolidation ratio (root time),

**

$$\gamma_p = \frac{10}{9}\left(\frac{R_0 - R_{90}}{R_i - R_f}\right)$$

→ secondary consolidation ratio,

**

$$\gamma_s = 1 - (\gamma_0 + \gamma_p)$$

→ Computation of settlement:

→ Total settlement of loaded soil consist of (S_t)

a. Initial compression/compaction (S_i)

b. Primary consolidation settlement (S_c)

c. Secondary " " (S_s)

$$S_t = S_i + S_c + S_s$$

NOTE: Here primary & secondary consolidation settlement is referred.

f. Determination of primary consolidation: (S_c):

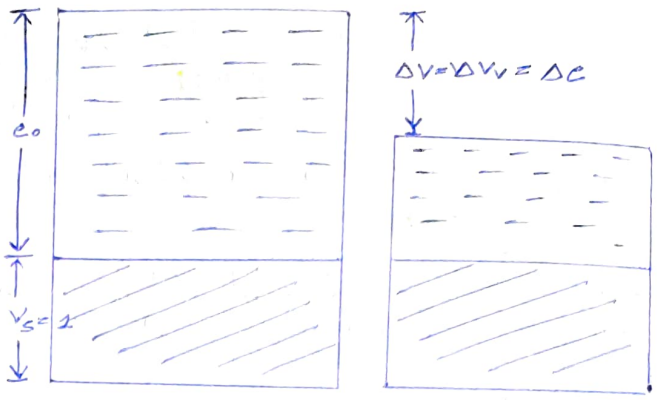
a. When void ratio is known:

$$\frac{\Delta V}{V_0} = \frac{\Delta HA}{HA} = \frac{\Delta H}{H_0}$$

∵ soil is semi-infinite & 1-D consolidation is considered.

$$e = \frac{V_v}{V_s} = \frac{V_v}{1}$$

$$\frac{\Delta V}{V} = \frac{\Delta e}{1+e_0} = \frac{\Delta H}{H}$$



→ Ultimate primary consolidation settlement (ΔH),

** $$\Delta H = \frac{\Delta e}{1+e_0} \cdot H_0$$

H_0 = Initial thickness of soil mass
Δe = change in void ratio
e_0 = original " "

b. When coeff. of compressibility is known:

$$a_v = \frac{\Delta e}{\Delta \sigma'}$$

$$\Delta e = a_v \cdot \Delta \sigma'$$

** $$\Delta H = \frac{a_v \cdot \Delta \sigma'}{1+e_0} \cdot H_0$$

c. When coeff. of volume compressibility is known:

$$M_v = \frac{\Delta e}{1+e_0} \cdot \frac{1}{\Delta \sigma'} = \frac{a_v}{1+e_0}$$

** $$\Delta H = M_v \cdot \Delta \sigma' \cdot H_0$$

NOTE: 'av' & 'Mv' is obtained for a particular stress range as their value is not constant & depends of stress increment.

→ If there are different consolidation layer with different compressibility parameter settlement in each layer (at the centre) is seperately computed & thus total settlement is then sum of settlement in different layers.

d. When coeff. of compression is known:

$$C_c = \frac{\Delta e}{\log\left(\frac{\sigma_1'}{\sigma_0'}\right)}$$

$$\Delta e = C_c \log_{10}\left(\frac{\sigma_1'}{\sigma_0'}\right)$$

**

$$\Delta H = \frac{C_c H_0 \log_{10}\left(\frac{\sigma_1'}{\sigma_0'}\right)}{1+e_0}$$

NOTE: The previous method is applicable only if soil is normally consolidated.

→ If the soil is pre/over consolidated then coeff. of recompression (C_r) is used instead of coeff. of compression as follows

i. If $\sigma_1' < \sigma_c'$

$$\Delta H = \frac{C_r \cdot H_0}{1+e_0} \log_{10}\left(\frac{\sigma_1'}{\sigma_0'}\right)$$

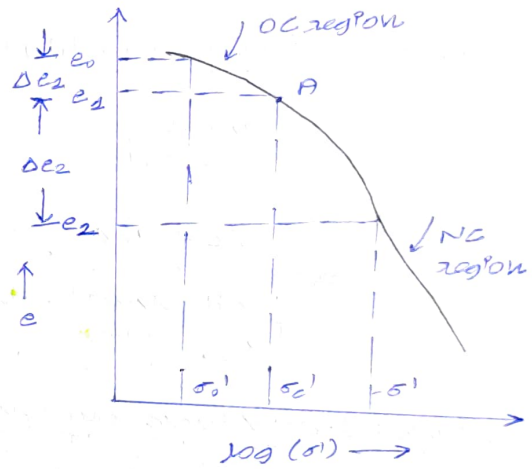
ii. If $\sigma_1' > \sigma_c'$ ($\sigma_0' \xrightarrow[OCS]{} \sigma_c' \xrightarrow[NCS]{} \sigma_1'$)

→ For OC region

$$\Delta H_1 = \frac{C_R H_0}{1+e_0} \log_{10}\left(\frac{\sigma_c'}{\sigma_0'}\right)$$

→ For NC region

$$\Delta H_2 = \frac{C_c H_0}{1+e_0} \log_{10}\left(\frac{\sigma_1'}{\sigma_c'}\right)$$



→ Total settlement ΔH,

**

$$\Delta H = \frac{C_R H_0}{1+e_0} \log_{10}\left(\frac{\sigma_c'}{\sigma_0'}\right) + \frac{C_c H_0}{1+e_0} \log_{10}\left(\frac{\sigma_1'}{\sigma_c'}\right)$$

NOTE: The use of 'e_0' instead of void ratio at 'σ_c'' in second term would not produce much error as the contribution of second term is much more than first. (comp of NC soil $\Delta H_2 / \Delta e_2$ is much greater than OC soil $\Delta H_1 / \Delta e_1$)

→ When the sample is loaded in the laboratory during consolidation testing, it is in fact subjected to reloading

→ Thus, even with every care possible exercised during the sampling to avoid the disturbance, the so called undisturbed sample produces a comp. curve whose slope is flatter than that of field v/s e/p curve

pp. Determination of secondary consolidation settlement (S_s):

→ secondary consolidation settlement / creep continues indefinitely and primary consolidation settlement occurs at a much smaller / slower rate.

→ The time at which secondary consolidation can be said to begin, it is not clear but it takes place at constant effective stress & is not considered associated with dissipation of excess pore water pressure.

→ It takes place due to readjustment of soil skeleton after disturbance during 1-D consolidation & its rate is controlled by highly viscous adsorped water surrounding the clay particles

→ Its magnitude is given "SECONDARY COMPRESSION INDEX" (C_α)

$$C_\alpha = \frac{\Delta e}{\Delta \log t}$$

$$\Delta e = C_\alpha \Delta \log t$$

$$\Delta H = \frac{C_\alpha H_0 \Delta \log(t)}{1+e_0}$$

**
$$\Delta H = \frac{C_\alpha H_{100}/H_f}{1+e_{100}/e_f} \log\left(\frac{t}{t_{100}}\right)$$

* $H_0 = H_{100} = H_f$ → thickness of the soil at the starting of secondary consolidation or at the end of 1° consolidation

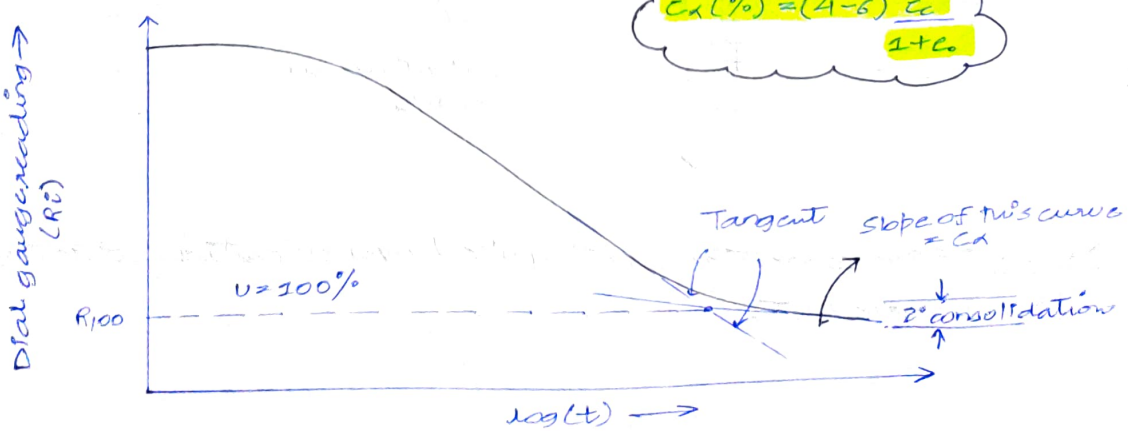
* $e_0 = e_{100} = e_f$ → void ratio at the starting of secondary consolidation

* t_{100} → time required for 100% completion of 1° consolidation ($\pm t_{90}$) [since $t_{100} = \infty$]

* t → time at which secondary consolidation is required.

NOTE: 'C_α' is measured as the slope of the st. line portion of dial gauge v/s log(t) curve.

**
$$C_\alpha(\%) = (4-6)\frac{C_c}{1+e_0}$$



NOTE: correction for construction period

→ It is assumed that the structural load producing consolidation is applied over the soil strata instantaneously.

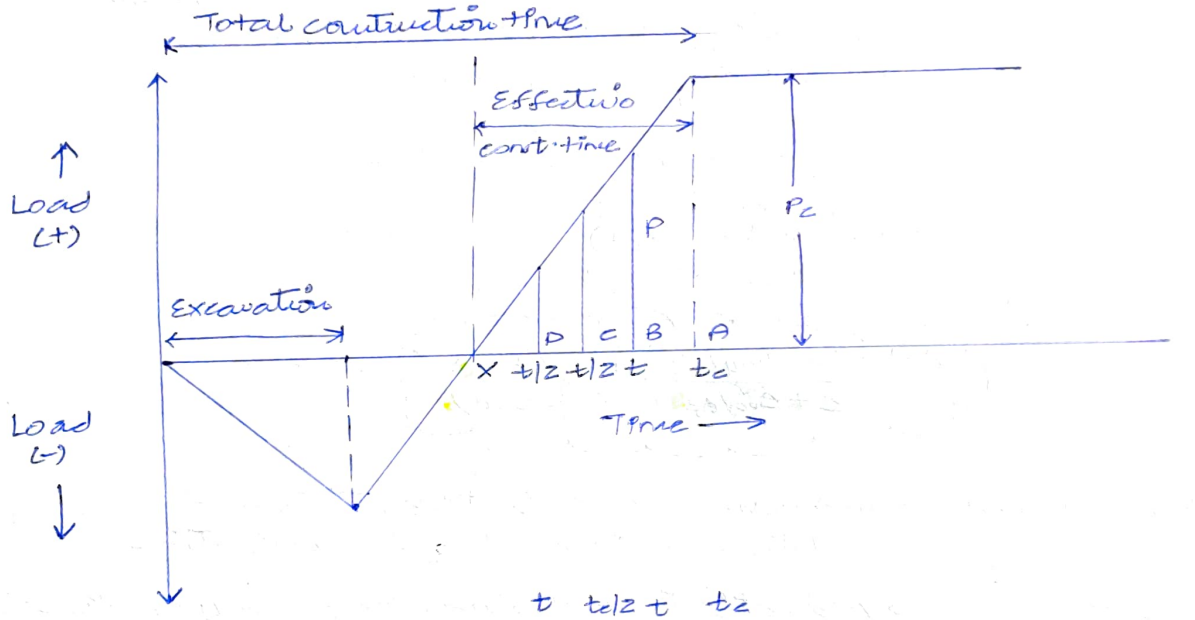
→ But in practice, the construction may extend upto several months/years.

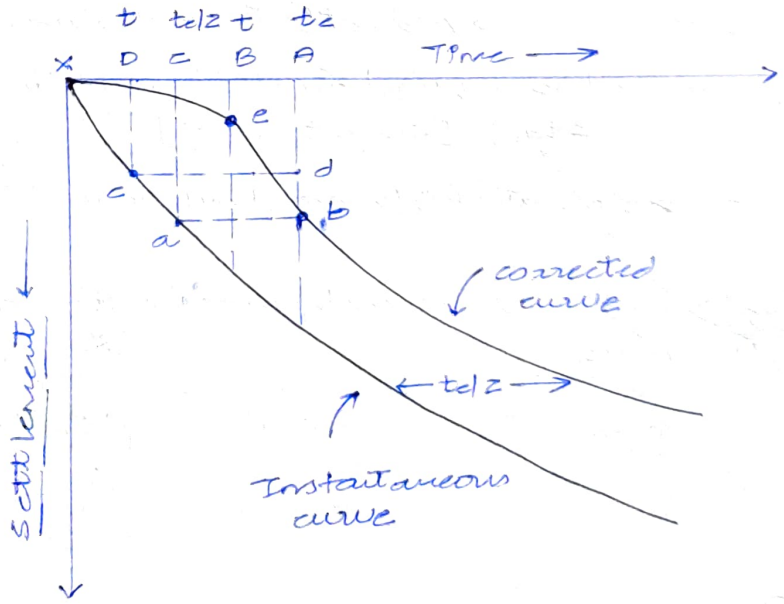
→ The loading hence progress gradually & is not instantaneous.

→ The net load becomes positive only after the load release due to excavation of soil is replaced by the wt of the structure.

→ The consolidation hence begins only at this point (X).

→ Terzaghi proposed a method of correcting the instantaneous curve to allow for the construction period.





NOTE: If this correction is not applied over estimation of settlement will be carried out.

→ Vertical sand drains:

→ The principle of the functioning of a sand drain is based on the consolidation theory of radially & vertically drained soil.

→ The slow rate of consolidation of saturated clay of low permeability may be accelerated by means of vertical sand drains, which provide path for radial drainage, resulting in shortening of drainage path.

→ consolidation is then mainly due to horizontal radial drainage, which results in faster dissipation of excess pore water pressure.

* → The magnitude of consolidation would remain same & only the rate of settlement would increase.

* → Sand drains are provided when embankments to be constructed over a highly compressible clay layer

* → These sand drains also helps increasing the rate of development of shear strength in soil, thereby would permit a faster rate of construction of the embankment.

→ Sand drains are installed by driving a casing or hollow cylinder/ mandrel through soil layer & making vertical boreholes.

→ The casing is withdrawn after the borehole is back filled with a suitable graded, having permeability approx 1000 times than that of clay.

→ The sand drains vary in diameter from 450mm - 600mm.

→ They are either constructed in square or triangular pattern in plan.

* → In these drains, ~~not~~ its diameter is not of much significance as in the spacing b/w these drains which controls the length of drainage path.

→ spacing is the range 1.9 - 4.5m (It is determined on the basis of fact that 90% consolidation takes during construction of fill itself)

* → spacing b/w the drains must be less than thickness of clay.

* → During the installation of sand drains, the clay around the drains may get remolded thus reducing the value of coeff. of consolidation, this is termed as "SMEAR EFFECT"

→ The general eqn of 3D consolidation

$$\frac{\partial u}{\partial t} = c_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + c_v \frac{\partial^2 u}{\partial z^2}$$

r = radial cylindrical coordinate

z = axial " "

a. For radial flow

$$\frac{\partial u}{\partial t} = c_h \frac{\partial^2 u}{\partial r^2}$$

b. For vertical flow

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

$$c_h = \frac{K_h}{m_v \gamma_w} \quad , \quad c_v = \frac{K_v}{m_v \gamma_w} \quad \left[\text{as } K_h > K_v \Rightarrow c_h > c_v\right]$$

→ If U_r & U_v are avg. degree of consolidation due to radial & vertical flow

$$U_r = f(T_r), \quad U_v = f(T_v)$$

$$T_v = \frac{c_v t}{d^2} \quad , \quad T_r = \frac{c_h t}{(2R)^2}$$

d = length of longest drainage path

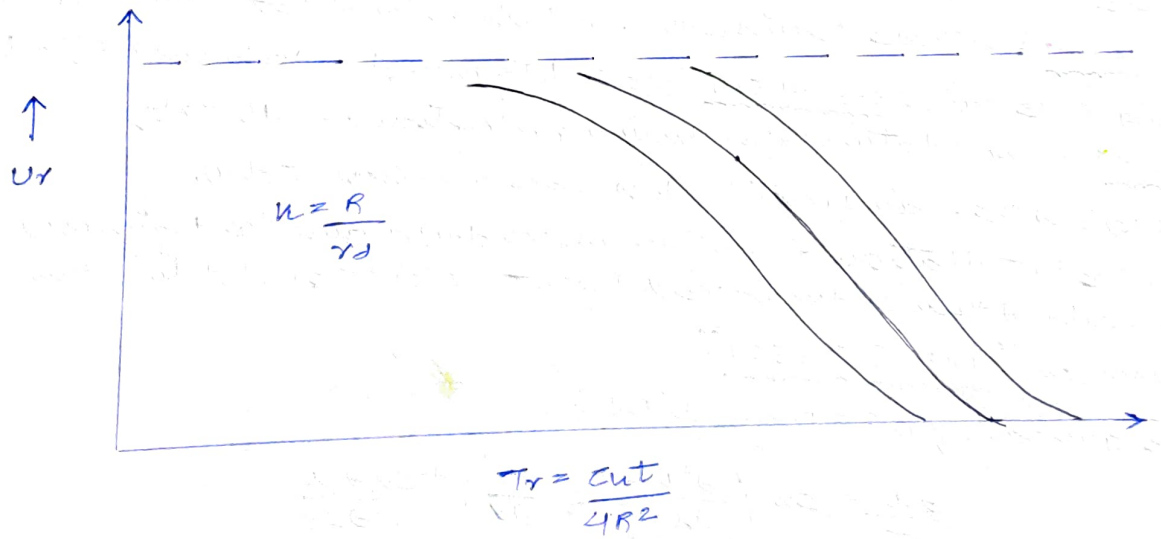
R = It is the radius cylindrical block of soil having same cross-section area as that of vertical prismatic block of soil surrounding the sand drain

R = 0.564 S for square pattern (0.564 S) $\quad S^2 = \pi R^2$

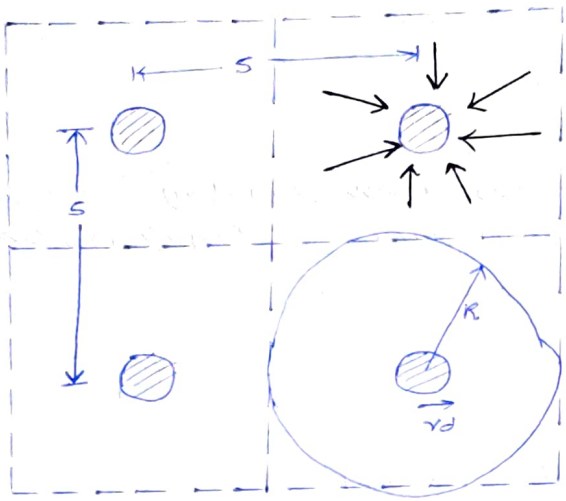
0.525 S " triangular " (0.525 S) $\quad R = \sqrt{\frac{1}{\pi}} S$

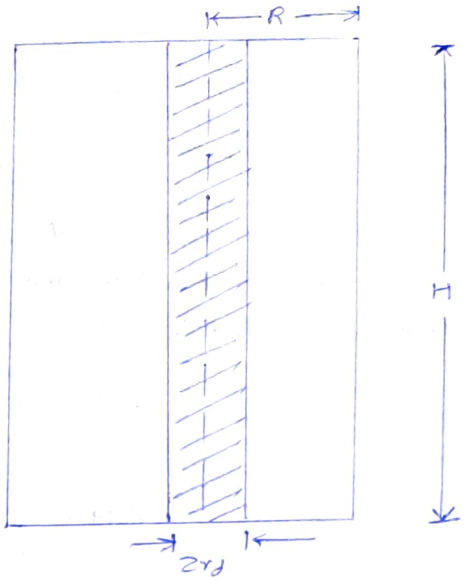
→ Overall degree of consolidation is given by

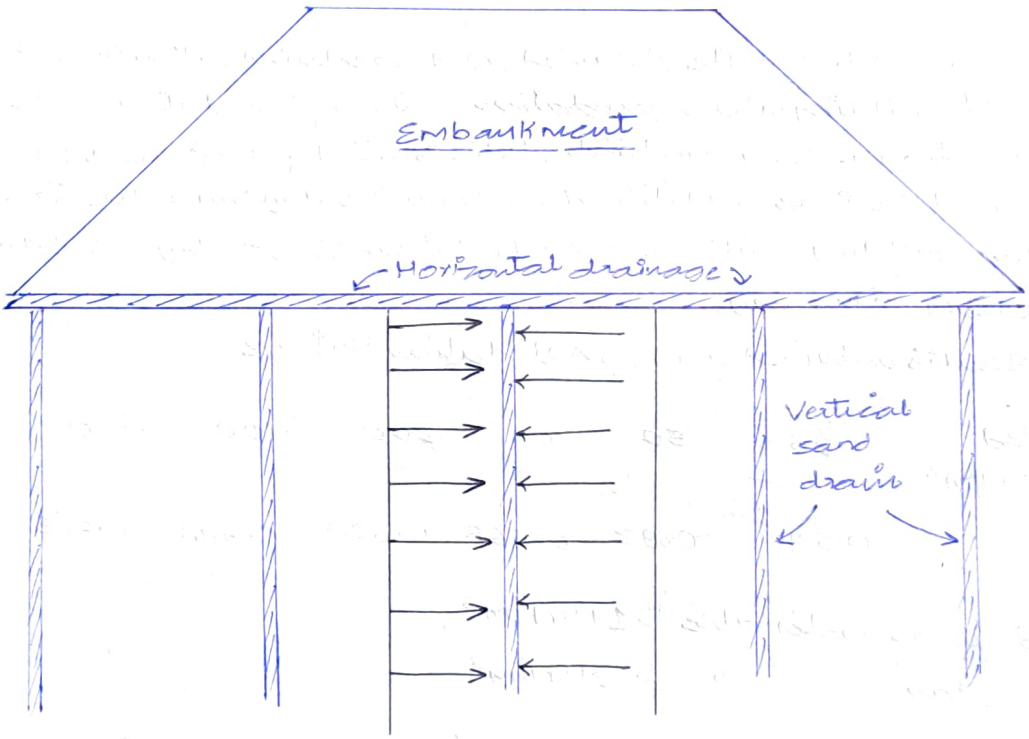
→ $(1-U) = (1-U_r)(1-U_v)$



r_d = radius of sand drains







Q. A clay soil in a consolidometer, showed a decrease in void ratio from 1.20 to 1.10 when the pressure was increased from 0.25 to 0.5 Kgf/cm². If the coeff. of consolidation in the test for the given stress increment was 10 m²/yr. calculate K (cm/sec). Also compute a_v & M_v.

$$a_v = \frac{0.10}{0.25} = 0.4 \text{ cm}^2/\text{kgf}$$

$$M_v = \frac{a_v}{1+e_0} = \frac{0.4}{1+1.2} = \frac{0.4}{2.2} = 0.18 \text{ cm}^2/\text{kgf}$$

$$C_v = \frac{K}{M_v \gamma_w} \rightarrow K = C_v M_v \gamma_w$$

$$K = \frac{10 \times 10^{4}}{365 \times 86400} \times 0.18 \times 10^{-3}$$

$$K = 5.75 \times 10^{-7} \text{ cm/sec}$$

→ If the sample tested at the site was taken from clay layer, 3m thick. Determine the consolidation settlement resulting from given stress increment.

$$\Delta H = \frac{\Delta e}{1+e_0} \cdot H$$

$$\Delta H = \frac{0.10}{1+1.2} \times 3 = 13.36 \text{cm}$$

Or $\Delta H = m_v \Delta \sigma' \cdot H_0 = 0.18 \times 0.25 \times 300 = 13.5 \text{cm}$

Q. Average soil profile obtained from boreholes, at site of a propored building where foundation is to be placed at a depth of 2.5m is shown. The groundwater table is at depth of 3.5M. It is estimated that the additional stress on the clay layer due to wt of the building will be 150kN/m² at the top of clay & 70kN/m² at the bottom of the clay.

Results obtained from consolidation test are

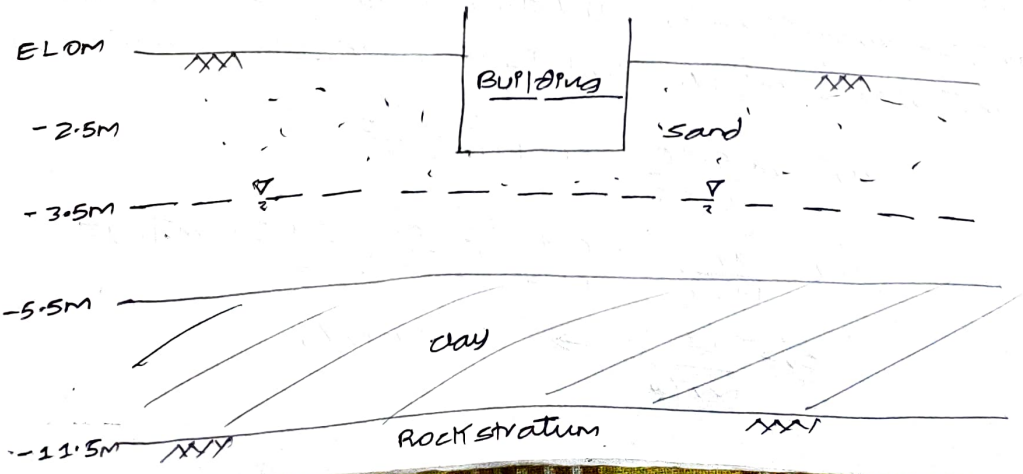
Applied stress kN/m²	25	50	100	200	400	800
e	0.99	0.882	0.865	0.830	0.798	0.765

γ_{sand} above watertable = 19 kN/m³

" below " " = 21 kN/m³

γ_{clay} " " " = 20 kN/m³

Determine the settlement of clay



Initial effective stress at the centre of the clay layer

$$\sigma_0' = 3.5 \times 19 + 2 \times (21-10) + \frac{3}{2} \times (20-10)$$

$$\sigma_0' = 118.5 \text{ KN/M}^2$$

Increase in eff. stress at the centre of clay layer

$$\Delta\sigma' = \frac{150+70}{2} = 110 \text{KN/M}^2$$

$$\sigma_1' = \sigma_0' + \Delta\sigma' = 118.5 + 110 = 228.5 \text{KN/M}^2$$

From given table, for $\sigma_0' = 118.5$

$$\frac{200 - 118.5}{0.83 - e_0} = \frac{118.5 - 100}{e_0 - 0.865}$$

$$e_0 = 0.858$$

for $\sigma_1' = 228.5$, $e_1 = 0.824$

$$\Delta H = \frac{\Delta e}{1+e_0} \cdot H_0 = \frac{(0.858 - 0.824)}{1+0.858} \times 6 = 0.109\text{M} = 10.9\text{cm}$$

Q. $H = 8\text{M}$ $\quad C_v = 6 \times 10^{-3} \text{ cm}^2/\text{sec}$

$\Delta H = 120\text{MM}$

$t = 2 \text{ years}$

$$U(\%) = \frac{\Delta H}{H} \times 100$$

$$T_v = \frac{C_v t}{d^2} = \frac{6 \times 10^{-3} \times 2 \times 365 \times 86400}{(800)^2} = 0.5913$$

$$T_v = 1.781 - 0.933 \log(100 - U\%)$$

$$0.5913 = 1.781 - 0.933 \log(100-u)$$

$$\log(100 - u\%) = 1.275$$

$$100 - U\% = 18.836$$

$$U\% = 81.164 > 60\%$$

$$U = \frac{\Delta h}{\Delta H} \times 100$$

$$81.164 = \frac{120}{\Delta H} \times 100$$

$$\Delta H = 148 \text{ MM}$$

pp. $U = 90\%$, $> 60\%$

$$T_v = 1.781 - 0.933 \log(100 - 90)$$

$$T_v = 0.848$$

$$T_v = \frac{C_v t}{d^2}$$

$$0.848 = \frac{6\times10^{-3} \times t \times 365 \times 86400}{(900)^2}$$

$$t = 2.896 \text{ years}$$

Q. For test: $U = 50\%$, $T_v = 0.196$

For field: $U = 50\%$, $T_v = 0.196$

$$T_v = \frac{C_v t}{d^2}$$

If T_v = constant

$$\frac{C_v t}{d^2} = \text{constant}$$

$$\frac{t}{d^2} = \text{constant}$$

$$\frac{10}{\left(\frac{20}{2}\right)^2_T} = \frac{t}{(5000)^2}$$

$$t_f = \frac{10 \times 25 \times 10^{8} \times 4}{100} = 25 \times 10^5 = 2.5 \times 10^6 \text{ MPH}$$

$$= 4.76 \text{ years}$$

PP. $U = 90\%$ $T_v = 0.848$

$$T_v = \frac{C_v t}{d^2} = Kt$$

$$\frac{T_{v90}}{T_{50}} = \frac{t_{90}}{t_{50}}$$

$$\frac{0.848}{0.196} = \frac{t_{90}}{4.76}$$

$$t_{90} = 20.59 \text{ years}$$

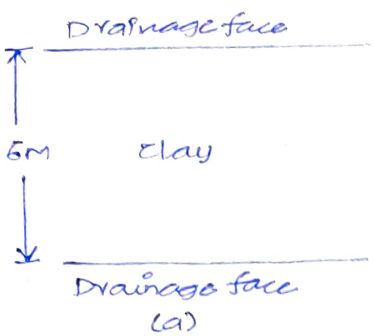
PPP. $\frac{t}{d^2} = \text{constant}$

$$\frac{t}{d^2_T} = \frac{t}{d^2_{F1}} = \frac{t}{d^2_{F2}}$$

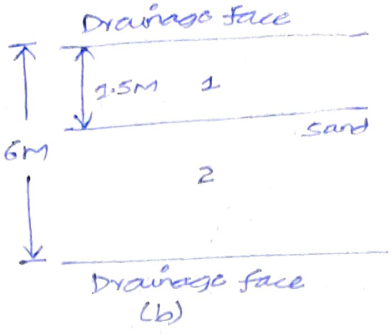
$$\frac{4.76}{\left(\frac{20}{2}\right)^2} = \frac{t}{\left(\frac{20}{2}\right)^2} \qquad t = 1.19 \text{ years}$$

Q. d H = 6M

Double drainage, $d = \frac{H}{2} = 3M$



(a)



(b)

i. The introduction of sand layer in clay has no effect on compressibility, rate of loading & thickness of the clay layer, hence ultimate settlement would remain unchanged

$\Delta H = 120MM$

ii. For case (a), at 3 yrs, $\Delta h = 30MM$, $\Delta H = 120MM$

$$U = \frac{\Delta h}{\Delta H} \times 100 \Rightarrow \frac{30}{120} \times 100$$

$$U = 25\%$$

$$T_v = \frac{\pi}{4} U^2 = 0.049$$

$$T_v = \frac{c_v t}{d^2} \Rightarrow T_v d^2 = \text{constant}$$

For portion 1,

$$0.049 \times \left(\frac{6}{2}\right)^2 = T_{VB_1} \left(\frac{1.5}{2}\right)^2$$

$$T_{VB_1} = 0.784$$

$$T_v = \frac{\pi}{4} (U)^2 \Rightarrow U_{B_1} = 88.31\%$$

$$T_v = 1.781 - 0.933 \log(100 - U)$$

$$100 - U = 11.694$$

$$U = 88.306\%$$

For portion 2,

$$0.049 \left(\frac{6}{2}\right)^2 = T_{VB_2} \left(\frac{4.5}{2}\right)^2$$

$$T_{VB} = 0.087$$

$$T_{VB} = \frac{\pi}{4} U^2 \Rightarrow U = 33.28\%$$

Now, $\Delta H_1 + \Delta H_2 = \Delta h$

$$U_2 \Delta H_1 + U_2 \Delta H_2 = \Delta h \cdot U \cdot \Delta H$$

$$U_2 \cdot K H_{o_1} + U_2 K H_{o_2} = U(K H_o)$$

$$88.31 \times 1.5 + 33.3 \times 4.5 = U \times 6$$

$$\frac{\Delta H}{H_o} = \frac{\Delta e}{1 + e_o}$$

$$\Delta H = K H_o$$

$U = 47.1\%$

$\Delta h = U \times \Delta H = 0.471 \times 120 = 56.4\,mm$

Q.

$U = 50\%$

$T_v = \frac{\pi}{4} U^2 = 0.196$

$T_v = \frac{C_v t}{d^2}$

$0.196 = \frac{C_v \times 1}{(5)^2}$

$C_v = 4.9\,m^2/yr$

$T_v = \frac{4.9 \times 1}{25 \times 25} = 0.00784$

After 1 yr, $U = 50\%$, $\Delta H = 9\,cm$, $H_0 = 5\,cm$

$U = \frac{\Delta h}{\Delta H} \times 100$

$\Delta H = \frac{50}{0.5} \times 100 = 16\,cm$

$T_v = \frac{C_v t}{d^2} = \frac{\pi}{4} U^2 \Rightarrow$ constant

$U \cdot d =$ constant

$(U \cdot d)_1 = (U \cdot d)_2$

$0.5 \times 5 = U \times 25$

$10\% = U$

$U = 10\%$

$U = \frac{\Delta h}{\Delta H}$, $\Delta H = K H_0$

$U = \frac{\Delta h}{K H_0} \Rightarrow \Delta h = K U \cdot H_0$

$\frac{\Delta h_2}{\Delta h_1} = \frac{K U_2 H_{0_2}}{K U_1 H_{0_1}}$

$\frac{\Delta h}{8} = \frac{20 \times 25}{50 \times 5}$

$\Delta h = 8\,cm$

After 4 years,

$$T_v = C_v \frac{t}{d^2} = \frac{\pi}{4} U^2 \Rightarrow \frac{U}{\sqrt{t}} = \text{constant}$$

$$\frac{U_2}{\sqrt{t_2}} = \frac{U_3}{\sqrt{t_3}}$$

$$\frac{10}{\sqrt{1}} = \frac{U_3}{\sqrt{4}}$$

$$U_3 = 20\%$$

$$U = \frac{\Delta h}{\Delta H} \times 100$$

$$\Delta H = \frac{\Delta h}{U} \times 100$$

$$\frac{\Delta h_2}{U_2} = \frac{\Delta h_3}{U_3}$$

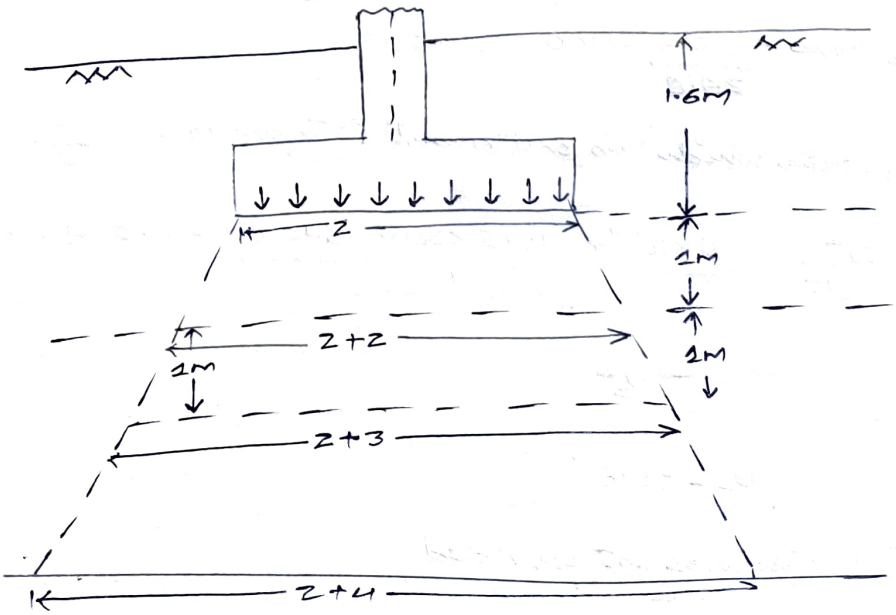
$$\frac{8}{10} = \frac{\Delta h_3}{20}$$

$$\Delta h_3 = 16\text{cm}$$

Q. A building column has a footing area of 2M x 3M, & transmits a pressure increment of 150KN/M^2 & its base embedded 1.6M below GL, Assuming a pr. distribution of 2:1. Determine the consolidation settlement at the middle of clay layer. Consider the pr. variation across the thickness of clay layer also.

P. For sand, $\gamma = 16.5\text{KN/M}^3$ & $\gamma_{sat} = 18.5\text{KN/M}^3$

PP. For clay, $\gamma_{sat} = 16\text{KN/M}^3$, $e_o = 0.95$, $C_c = 0.26$



$\Delta\sigma_0' = 2.6 \times 16.5 + \frac{1}{2}(18.5-10) + 1\,(16-10) = 57.4\ KN/m^2$

$\Delta\sigma' = \frac{150 \times BL}{(B+2nz)(L+2nz)} = \frac{150 \times 2 \times 3}{(2+2\times0.5\times3)(3+2\times0.5\times3)}$

$\Delta\sigma' = 30\ KN/m^2$

$\Delta H = \frac{C_c H_0}{1+e_0} \log_{10}\left(\frac{\sigma_1'}{\sigma_0'}\right) = \frac{C_c H_0}{1+e_0} \log_{10}\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}\right)$

$\Delta H = 48.7\ mm$

Q. An embankment of 10m height is to be constructed on a 8m thick clay layer overlying rock. The embankment will increase the mean effective vertical stress in clay after consolidation from a value of 80 KN/M² to 200 KN/M². The embankment will carry a road & will be laid in 5 months. The surface will be laid after 13 months of commencement of construction. Only 3cm of settlement can be accepted after the surface is being laid. Design a suitable sand drain installation to achieve the above requirements. Take $C_{v_z} = C_{v_r} = 8\times10^{-4}\ cm^2/sec$ & $M_v = 0.3\ \frac{M^2}{MN}$

$\Delta H = M_v \Delta\sigma' H_0 = \frac{0.3}{10^3} \times (200-80) \times 8 = 0.288\ M$

$\Delta H = 28.8\ cm$

Settlement to relable after 13 months = 3cm

" to be achieved at the end of 13 months = 25.8 cm (28.8 - 3)

U% in this time = $\frac{25.8}{28.8} \times 100 = 89.58\%$

Eff. time within which this settlement is to occur = $13 - \frac{5}{2} = 10.5$ months

Now, $T_v = \frac{C_v t}{d_e^2} = \frac{8\times10^{-4} \times (10.5\times30\times86400)}{8\times10^4} = 0.034 < 0.28$

$T_v = \frac{\pi}{4} U_z^2$

$U_z = 20.8\%$

→ If sand drains were not provided

For $U_z = 89.58$, $T_v = 0.831$ $\Rightarrow t = 253$ months

$t' = 253 + 2.5 = 255.5$ months

Let dia. of sand drains be 30cm spaced 2M apart

Assume square pattern arrangement

$$R = 0.5645 s = 0.564 \times 2 = 1.128 M = 112.8 cm$$

$$n = \frac{R}{r_w} = \frac{112.8}{15} = 7.52$$

$$T_r = \frac{C_h t}{(2R)^2} = \frac{8 \times 10^{-4} \times (10.5 \times 30 \times 86400)}{(2 \times 112.8)^2} = 0.4278$$

For $n = 7.52$ & $T_r = 0.4278$, from curve

$U_r = 90\%$

$$(1-U) = (1-U_z)(1-U_r)$$

$$(1-U) = (1-0.208)(1-0.9)$$

$U = 92.08\% > 89.58\%$

Designing is safe

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Complete Course on Geo-Technical & Foundation Engineering

Shallow Foundation - Part III

Lesson 57 • July 25, 2020 • Jaspal Singh

Question

Anmol Agrawal

sir isme 2 3 kya hoga

(c) Over toppling due to insufficient spillway capacity

(d) Burrowing activities of animals from both side

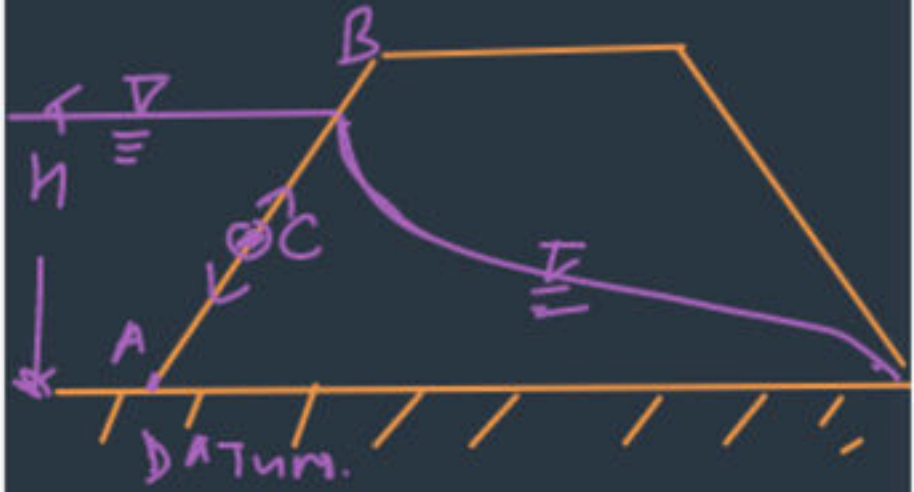
The upstream face of an earth dam is a

(a) Flow line

(b) An equi-potential line

(c) Phreatic line

(d) Atmospheric pressure line



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Question

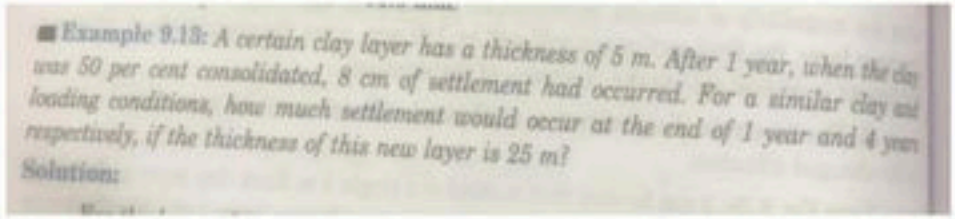
Pooja

Sir bhut doubt ooo rha h cv bi thickness(H) pr dependent hona chaie na ??? Kuki agr H increse hua to time jydha lagega to same consolidation ko to rate dec o jaigi na

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Question

Pooja



■ Example 9.13: A certain clay layer has a thickness of 5 m. After 1 year, when the clay was 50 per cent consolidated, 8 cm of settlement had occurred. For a similar clay and loading conditions, how much settlement would occur at the end of 1 year and 4 years respectively, if the thickness of this new layer is 25 m?

Solution:

Slide 18 of 30

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■ **Example 9.9:** *A 8 m thick clay layer with single drainage settles by 120 mm in 2 years. The coefficient of consolidation for this clay was found to be 6×10^{-3} cm^2/s. Calculate the likely ultimate consolidation settlement and find out how long it will take to undergo 90 per cent of this settlement.*

■ **Example 9.10:** *In a laboratory consolidometer test on a 20 mm thick sample of saturated clay taken from a site, 50 per cent consolidation point was reached in 10 mm. Estimate the time required for the clay layer of 5 m thickness at the site for 50 per cent compression if there is drainage only towards the top. Assume that the laboratory sample and the clay layer at the site are both subject to the same increase in stress. How much time is required for the clay layer to reach 90 per cent consolidation? What is the time required for the clay layer to reach 50 per cent consolidation if the layer has double drainage instead of single drainage?*

$(S_c)_1$.

Example 9.12: *A clay layer 6 m thick, having double drainage, settles by 30 mm in 3 years after it has been subject to a certain load. Its final consolidation settlement is calculated as equal to 120 mm. If a layer of sand of negligible thickness were to be present at a depth of 1.5 m from the top of the clay layer, what will be the final consolidation settlement of the clay layer? What will be its settlement after 3 years?*

Solution:

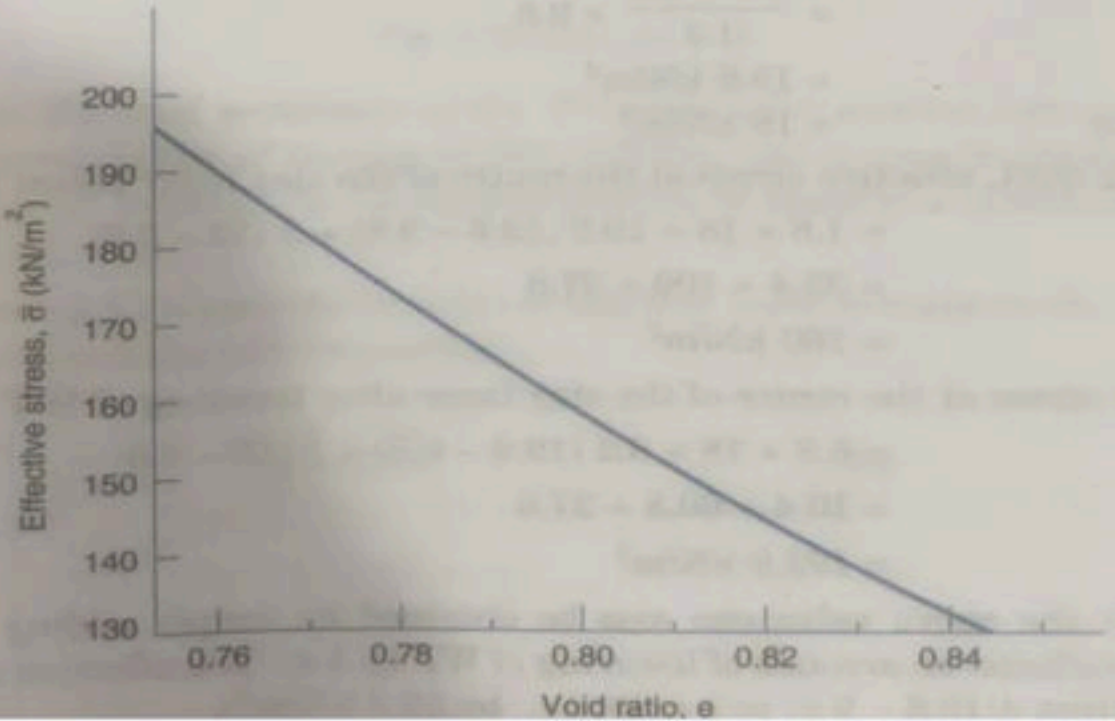
■ **Example 9.13:** *A certain clay layer has a thickness of 5 m. After 1 year, when the clay was 50 per cent consolidated, 8 cm of settlement had occurred. For a similar clay and loading conditions, how much settlement would occur at the end of 1 year and 4 years respectively, if the thickness of this new layer is 25 m?*

Solution:

Example 9.14: *A school building stands over a 12 m thick stratum of sand. Beneath the sand stratum is a layer of clay 6 m thick and this is underlain by an impermeable rock stratum. The sand has a void ratio of 0.7. The water table lies at a depth of 1.8 m below ground surface. Above the water table, the average degree of saturation is 0.6.*

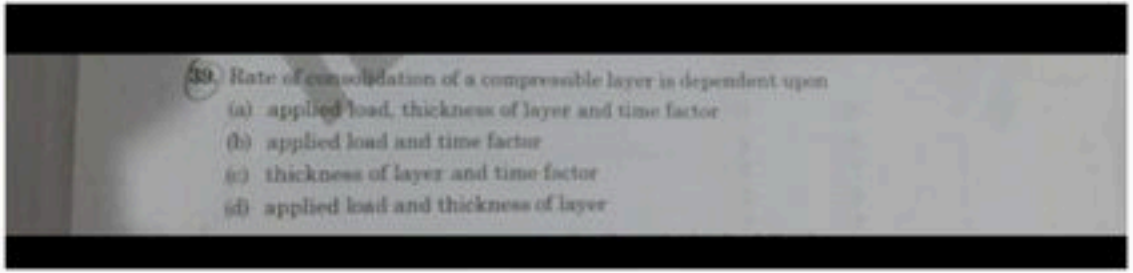
Consolidation test data for the clay soil gave the curve of $e - \bar{\sigma}$ as shown in Fig. 9.40. G_s for both sand and clay can be taken as 2.7. γ_{sat} (clay) = 19 kN/m^3.

It is planned to supply water to the school by pumping from the sand stratum and it is expected that as a result of this scheme, the water table will be drawn down by 4 m. Assuming that the school building is light (insignificant load), calculate the probable settlement of the school as a result of the pumping.



Question

Pooja



39. Rate of consolidation of a compressible layer is dependent upon
(a) applied load, thickness of layer and time factor
(b) applied load and time factor
(c) thickness of layer and time factor
(d) applied load and thickness of layer

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$mv = \frac{cv}{1+e}$

$\frac{\partial u}{\partial t} = cv \frac{\partial^2 u}{\partial z^2}$

$K_3 \Delta\sigma' H_0$

$T \quad cv = \frac{T d^2}{t}$

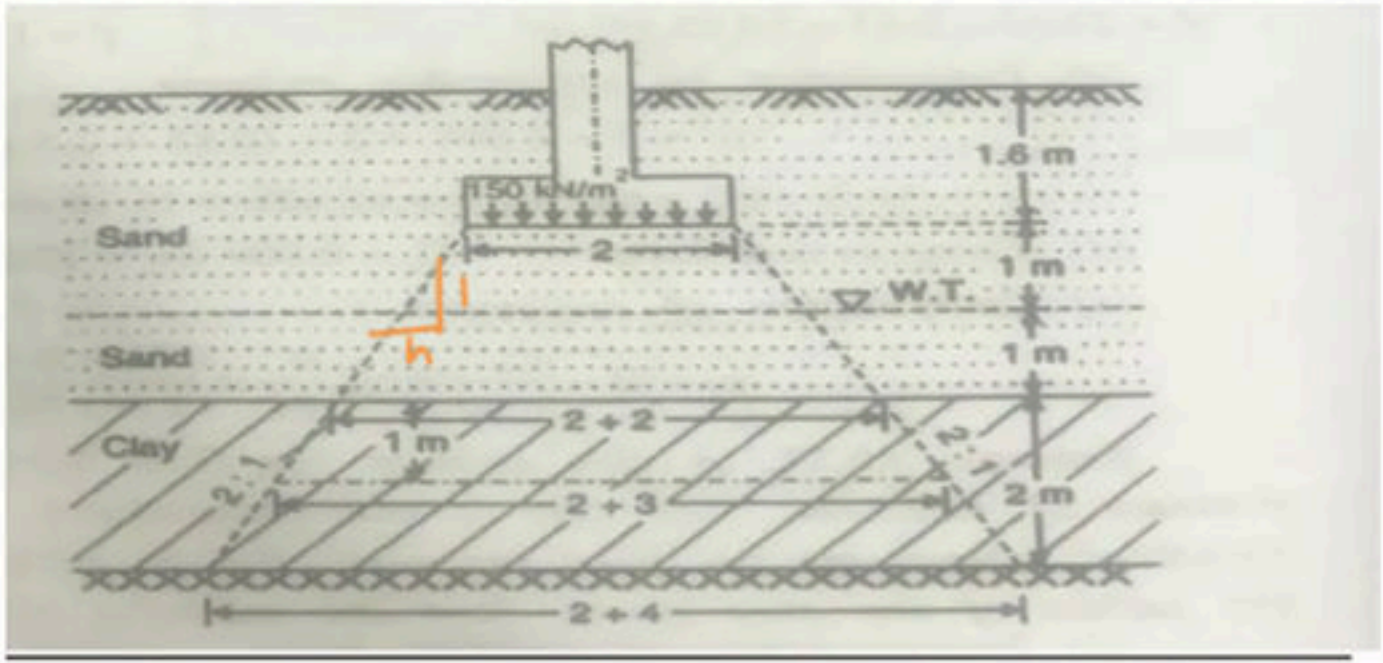
$Tv = \frac{cv\, t}{d^2}$

neglected.

Example 15.13. *A building column has a footing area of 2 m × 3 m, and transmits a pressure increment of 150 kN/m² at its base embedded 1.6 m below ground level, as shown in Fig. 15.16. Assuming a pressure distribution of 2 vertical to 1 horizontal, determine the consolidation settlement at the middle of clay layer. Consider the pressure variation across the thickness of clay layer also. Given the following.*

(i) *For sand* : $\gamma = 16.5\ kN/m^3$ *and* $\gamma_{sat} = 18.5\ kN/m^3$

(ii) *For clay* : $\gamma_{sat} = 16\ kN/m^3$; $e_0 = 0.95$; $C_c = 0.26$



Q1

$$\Delta\sigma_0' = 2.6 \times 14.5 + 1(19.5 - 10) + 1(16 - 10) = 57.4 \ \frac{kN}{m^2}$$

$$\Delta\sigma' = \frac{150 \times BL}{(B + 2nz)(L + 2nz)} = \frac{150 \times 2 \times 3}{(2 + 2 \times 0.5 \times 3)(3 + 2 \times 0.5 \times 3)}$$

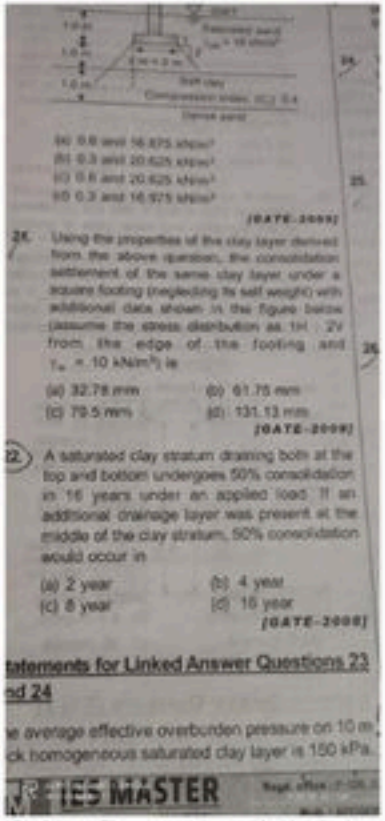
$$\Delta\sigma' = 30 \ kN/m^2$$

$$\Delta H = \frac{C_c \cdot H_0}{1 + e_0} \log_{10}\left(\frac{\sigma_1'}{\sigma_0'}\right) = \frac{C_c H_0}{1 + e_0} \log_{10}\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}\right)$$

$$\Delta H = 48.7 \ mm$$

Question

Rakshit Raj



(a) 0.6 and 16.875 kN/m²
(b) 0.3 and 20.625 kN/m²
(c) 0.6 and 20.625 kN/m²
(d) 0.3 and 16.875 kN/m²

[GATE-2009]

24. Using the properties of the clay layer derived from the above question, the consolidation settlement of the same clay layer under a square footing (neglecting its self weight) with additional data shown in the figure below (assume the stress distribution as 1H : 2V from the edge of the footing and γ_w = 10 kN/m³) is

(a) 32.78 mm (b) 61.75 mm
(c) 79.5 mm (d) 131.13 mm

[GATE-2009]

22. A saturated clay stratum draining both at the top and bottom undergoes 50% consolidation in 16 years under an applied load. If an additional drainage layer was present at the middle of the clay stratum, 50% consolidation would occur in

(a) 2 year (b) 4 year
(c) 8 year (d) 16 year

[GATE-2008]

tatements for Linked Answer Questions 23 nd 24

e average effective overburden pressure on 10 m ck homogeneous saturated clay layer is 150 kPa.

IES MASTER

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Example 15.30. *A building was constructed on a clay stratum. Preliminary analysis indicated settlement of 50 mm in 6 years and an ultimate settlement of 250 mm. The average increase of pressure in clay stratum was 24 kN/m^2.*

The following variations occurred from the assumptions used in the preliminary analysis.

(i) The loading period was 3 years, which was not considered in the preliminary analysis.

(ii) Borings indicated 20% more thickness for the clay stratum than originally assumed.

(iii) During construction, the water table got lowered permanently by 1 m.

Estimate : 1. The ultimate settlement 2. The settlement at the end of the loading period

3. The settlement at 2 years after completion of building. **(Civil Services Exam. 2001)**

Sol (i) By lowering of water table level effective stress would increase $= (\gamma - \gamma') \Delta h_w$

(Assuming $\gamma = \gamma_{sat}$) $= (\gamma_{sat} - \gamma') \Delta h_w$

$= \gamma_w \Delta h_w = 10 \times 1 = 10 \frac{kN}{m^2}$

$\Delta H = m_v \cdot \Delta \sigma' H_0 = 250$ mm — (i)

$\Delta H_c = m_v \Delta \sigma'_c (1.2 H_0)$ — (ii)

From $m_v H_0 = \frac{250}{24}$

$\Delta H_c = \frac{250}{24} \times 34 \times 1.2 = 425$ mm

(ii) After $t = 6$ yrs

$\Delta h = 50\text{mm}$ $\Delta H = 250\text{mm}$

$$U = \frac{\Delta h}{\Delta H} \times 100 = \frac{50}{250} \times 100 = 20\%.$$

$$T_v = \frac{c_v t}{d^2} = \frac{\pi}{4} U^2 \Rightarrow \frac{U}{\sqrt{t}} = \text{const}$$

$$\frac{U_1}{\sqrt{t_1}} = \frac{U_2}{\sqrt{t_2}} \Rightarrow \frac{20}{\sqrt{6}} = \frac{U_2}{\sqrt{1.5}}$$

$$U_2 = 10\%.$$

$$\Delta h_2 = U_2 \Delta H_c = 0.1 \times 425 = 42.5\text{mm}$$

(iii) settlement after 2 yrs of completion

$t_3 = 2 + 1.5 = 3.5 \text{ yrs}$

$$\frac{U}{\sqrt{t}} = \text{const}$$

$$\frac{U_2}{\sqrt{t_2}} = \frac{U_3}{\sqrt{t_3}} \Rightarrow \frac{10}{\sqrt{1.5}} = \frac{U_3}{\sqrt{3.5}}$$

$$U_3 = 15.27\%$$

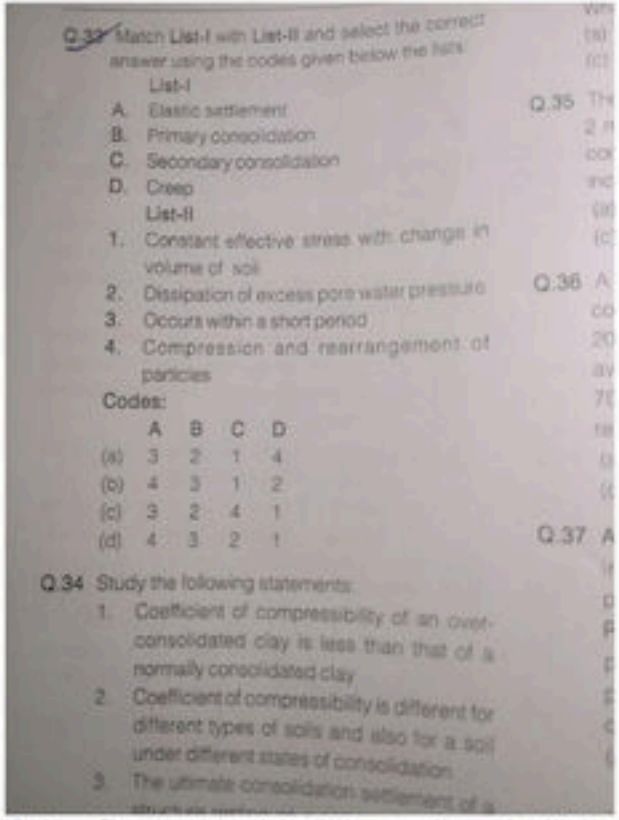
$$\Delta h_3 = U_3 \, \Delta h_c$$

$$= 15.27 \times 425 = 64.91 \text{ mm}$$

Question

Govinda Madhava

Sir creep and secondary seltlement to same hota h na??



Q.33 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I

A. Elastic settlement
B. Primary consolidation
C. Secondary consolidation
D. Creep

List-II

1. Constant effective stress with change in volume of soil
2. Dissipation of excess pore water pressure
3. Occurs within a short period
4. Compression and rearrangement of particles

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	4	3	1	2
(c)	3	2	4	1
(d)	4	3	2	1

Q.34 Study the following statements:

1. Coefficient of compressibility of an over-consolidated clay is less than that of a normally consolidated clay.
2. Coefficient of compressibility is different for different types of soils and also for a soil under different states of consolidation.
3. The ultimate consolidation settlement of a

Q.35 Th

Q.36 A

Q.37 A

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Example 15.32. *A 20 m thick isotropic clay stratum overlies an impervious rock. The coefficient of consolidation of soil is* 5×10^{-4} *cm*^{2}*/sec.*

(i) Find the time required for 50% and 90% consolidation. Time factor for $U = 50\%$ *is 0.2 and for* $U = 90\%$ *is 0.85, where U is the degree of consolidation.*

(ii) In order to accelerate the settlement ratio, vertical sand drains of 1 m diameter were made at 5 m centre to centre in the soil stratum throughout the area.

Calculate how much the settlement is accelerated due to provision of sand drains. For the provided ratio of five between the spacing of sand drains (centre to centre) and diameter of drains, values of time factor for $U = 50\%$ *is 0.078 and for* $U = 90\%$ is 0.28. **(Engg. Services Exam. 1986)**

31

For $U = 50\%$, $T_v = 0.2$

$$T_v = \frac{c_v t}{d^2} \Rightarrow 0.2 = \frac{5\times10^{-4} \times t \times 365 \times 86400}{(20\times10^2)^2}$$

$$t = 50.73 \text{ yrs}$$

For $U = 90\%$, $T_v = 0.85$

$$T_v = \frac{c_v t}{d^2} \Rightarrow t = 215.62 \text{ yrs}$$

(iii) Acceleration of consolidation due to provision of sand drains

$$T_R = \frac{C_h t}{(2R)^2}$$

Assume square pattern of drains

$R = 0.564 S = 0.564 \times 5 = 2.82 m$

Since soil is isotropic $C_v = C_h$

for $\frac{S}{d_w} = 5$, $U = 50\%$, $T_v = 0.078$

$$T_v = \frac{5 \times 10^{-4} \times t_{50}}{(2 \times 2.82)^2} = 0.078$$

$$t_{50} = 1.57 \text{ yrs}$$

$$\frac{t_{50}}{t_{50s}} = \frac{50.73}{1.57} = 32.3 \text{ times}$$

Similarly for $\frac{S}{d_w} = 5$, $U = 70\%$, $T_v = 0.28$

$$T_v = \frac{C_v t_{70}}{(2K)^2} = 0.28$$

$$\frac{5 \times 10^{-4} t_{70}}{(2 \times 2.82)^2} = 0.28 \Rightarrow t_{70} = 5.648 \text{ yrs}$$

$$\frac{t_{70}}{t_{70s}} = \frac{215.6}{5.648} = 39.18 \text{ times}$$

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

$$U(\%) = \left[1 - \frac{8}{\pi^2} \sum \frac{1}{(2N+1)^2} e^{-\frac{(2N+1)^2 \pi^2 c_v t}{d^2}}\right] \times 100$$

$$T_v = \frac{c_v t}{d_3^2}$$

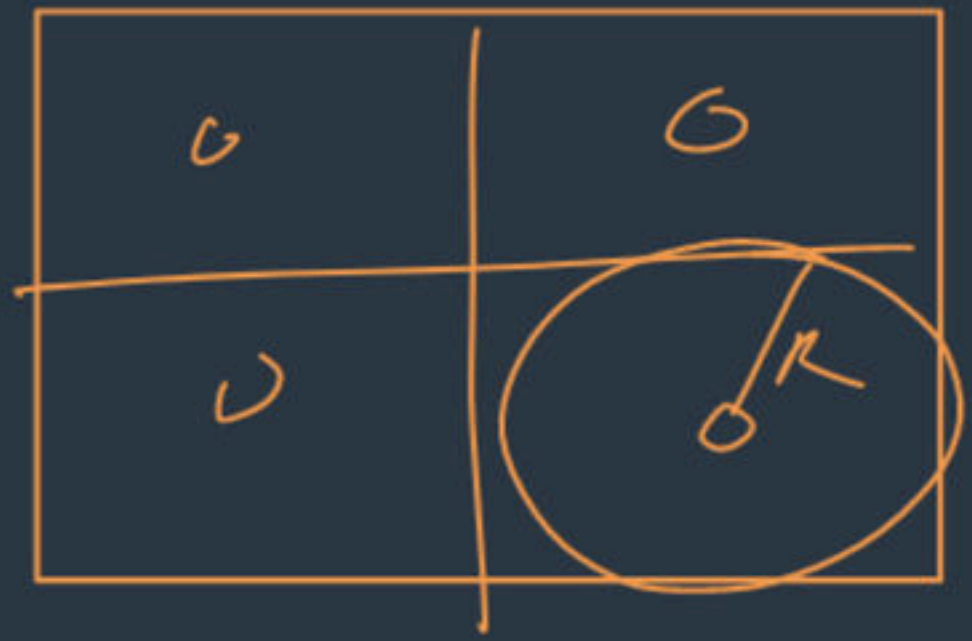
$$\frac{\partial u}{\partial t} = c_v \left(\quad \right)$$

$$U\% = 1 - \sum f(z) f(T_v)$$

$$\frac{c_v t}{u\%^2} = T_{v_2}$$

$$T_v = \frac{\pi}{4} u^2 \qquad u \leq 60\%.$$

$$T_v = \underline{\qquad\qquad}$$


G
G
J
R

Example 15.34. *A 2 m wide strip footing is to be placed in a sand layer 2 m thick at a depth of 1 m below the ground surface. The sand is underlain by a layer of saturated clay which is 1 m thick. The clay overlies a bed of dense sand. The ground water table is at the level of the top of the clay layer. The bulk unit weight of sand above the clay layer is 2 t/m³ and the submerged unit weight of clay is 0.8 t/m³. The footing is designed to carry a load of 22 t/m². Compute the probable ultimate settlement of the footing below its centre. Also, determine the elapsed time in which 10 percent and 90 percent of the ultimate settlement will occur.*

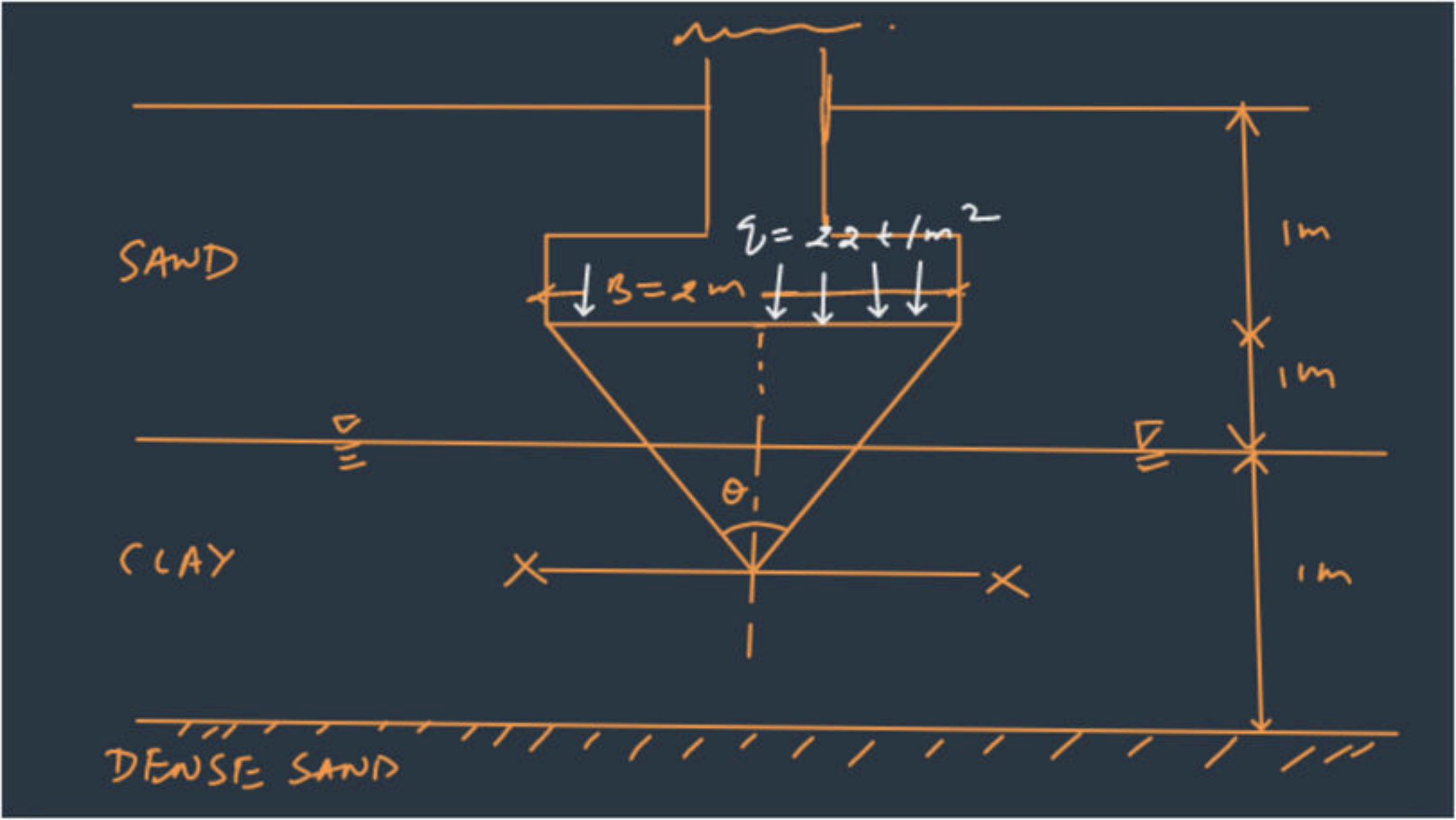
It is known from a graph between pressure and void ratio for clay that the void ratio corresponding to the vertical pressure of 4.4 t/m² and 18.1 t/m² are respectively 1.16 and 1.02. The coefficient of consolidation for the soil is 4×10^{-4} cm²/s.

The vertical pressure (σ_z) below the centre line of uniformly loaded strip footing (intensity q) of width B may be taken as

Depth	σ_z
0.1 B	0.997 q
0.5 B	0.817 q
1.0 B	0.550 q
2.0 B	0.306 q.

The time factor (T_v) corresponding to the degree of consolidation (U %) is as follows.

(U%)	10	90
T_v	0.008	0.848


SAND
$q = 22 t/m^2$
B = 2 m
1m
1m
1m
θ
CLAY
DENSE SAND

Q7 $\sigma_0' = 2 \times 2 + 0.5 \times 6.8 = 4.4\ t/m^2 \Rightarrow e_0 = 1.16$

Net increase in the pressure intensity at footing level $q = 22 - 2 \times 1 = 20\ t/m^2$

At depth of 1.5 m $\Rightarrow$ $z = B = 1.5$ m

$$x = \frac{1.5}{2} = 0.25B$$

From given table $\Delta\sigma' = 0.6835\, q$

$$\Delta\sigma' = 0.6835 \times 20$$

$$= 13.67\ t/m^2$$

$$\sigma_1' = \sigma_0' + \Delta\sigma' = 4.4 + 13.67 = 18.07 \Rightarrow e_1 = 1.02$$

$$\Delta n = \frac{\Delta \varepsilon}{1+\varepsilon_0} n_0 = \frac{1.16-1.02}{1+1.16} \times 1 = 64.8\ mm.$$

Q A 20m thick clay layer is sandwiched b/w a silty sand layer & a gravelly sand layer. The layer experience 30mm settlement in 2 yrs

$$T_v = \begin{cases} \frac{\pi}{4} U^2, & U \leq 60 \\ 1.781 - 0.933 \log_{10}(100 - U) & U > 60 \end{cases}$$

If the $C_v = 0.003\ cm^2/sec$, the deposit will experience a total 150 mm settlement in next how many yrs.

Sol

After $t = 2yrs$, $\Delta h = 30mm$

$$T_v = \frac{cvt}{d^2} = \frac{0.003 \times 2 \times 365 \times 86400}{\left(\frac{20 \times 10^2}{2}\right)^2}$$

$$T_v = 0.189$$

$$T_{v50} = 0.196$$

$$T_v < T_{v50}$$

$$T_v = \frac{\pi}{4} U^2 \Rightarrow 0.189 = \frac{\pi}{4} U \Rightarrow U = 49\%.$$

$$U = \frac{\Delta h}{\Delta H} \times 100 \Rightarrow \Delta H = \frac{30}{0.49} = 61.22\ m$$

at settlement of $\Delta h = 50mm$

$$U = \frac{\Delta h}{\Delta H} \times 100 = \frac{50}{61.22} \times 100 = 81.66 > 60\%.$$

$T_v = 1.781 - 0.933 \log_{10}(100 - U)$

$T_v = 1.781 - 0.933 \log_{10}(100 - 81.61)$

$T_v = 0.602$

$T_v = \frac{cvt}{d^2} \Rightarrow 0.602 = \frac{6.603 \times t \times 365 \times 86400}{\left(\frac{20 \times 10^2}{2}\right)^2}$

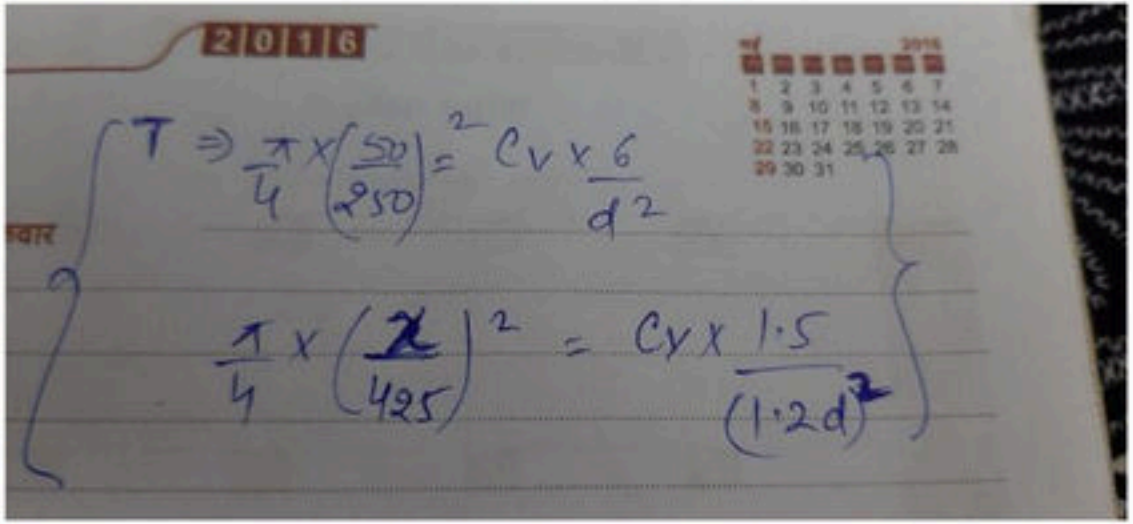
$t = 6.36 \text{ yrs}$

$(6.31 - 2) = 4.36 \text{ yrs}$ after 2yrs total settlement would be 50mm

Question

Manish

Sir tc/2 wale ques k second part m ye ni hga??







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SHEAR STRENGTH OF SOILS

→ Volume changes in soil due to 1-D loading is referred to consolidation, in which deformation was due to normal stress, compressive in nature

→ But in most of problems in soil engg encountered has to withstand shearing stresses like earthwork, slopes, ground foundation etc, nature of which is not same as that of compressive stress.

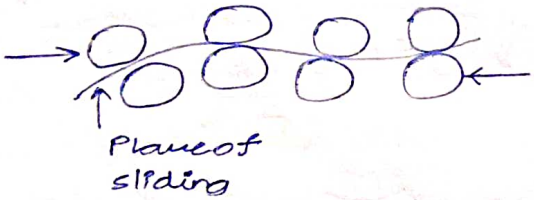
→ 'SHEAR STRENGTH' of soil is the capacity of the soil to resist shearing stress on it can also be designed as maximum value of shear stress that can be controlled within the soil mass

→ If this value is equalled by shear stress on any plane or a surface at a point, failure will occur in soil because of movement of portion of soil mass along that particular plane or surface, thus the soil is then said to fail in shear.

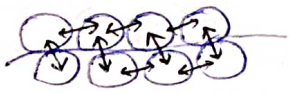
→ The determination of shear strength of soil is not simple & is not merely related to the material itself but is also the function of application of load

→ A soil may derive its shear strength due to following

→ Interlocking b/w particles (gravel & sand)

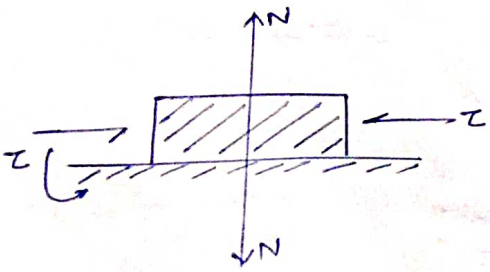


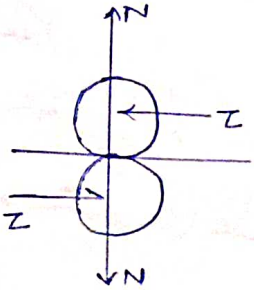
Before shear

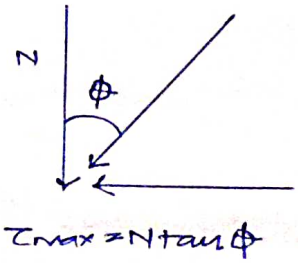


After shear

PP. Friction b/w the particles due to sliding & rolling of particles (sand, gravel, silt)







NOTE: To express the friction resistence of soil, we can consider the analogy of force transmitted by two solid bodies in contact with each other. If 'N' is the normal force perpendicular to the surface of contact & tangential force 'τ' parallel to the surface.

→ When body begins to slide, the shearing force will have reached its max. value τ_{max},

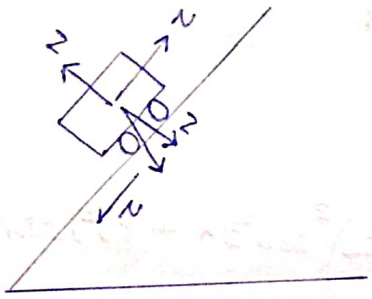
where $\tau_{max} = \mu N$ $\quad \mu$ = Coeff. of friction

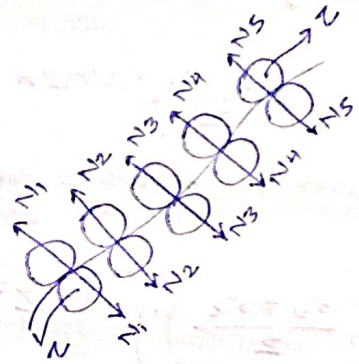
→ In case of soil, an angle of friction ϕ is used such that $\mu = \tan\phi$

* $\tau_{max} = N \tan\phi$

→ In soil like sand, gravel, the resistence to sliding on any plane within the soil mass depends upon the normal force & angle of internal friction.

→ However in case of these soils, there is rolling friction also along with sliding friction.



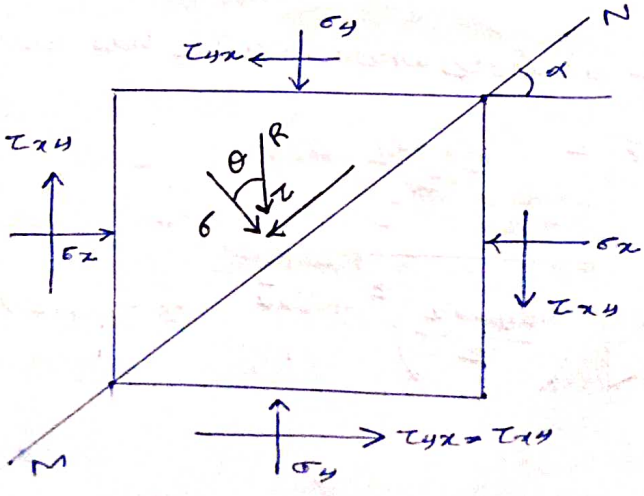


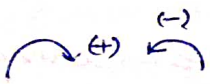
ppp. Intermolecular attraction due to cohesion or adhesion (clay & silt)

* Mohr's stress circle:

→ Through a pt in a loaded soil mass, infinite plane panes & stress components on each plane depends upon the direction of the plane.

→ If problem in soil engg is approximated by considering two dimensional stress conditions, it can be analysed as follows





→ Consider a soil element subjected to 2-D stress system. From consideration of the eq^{m} of the element, we get

$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_y-\sigma_x}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha \quad \text{——(i)}$$

$$\tau = \left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\alpha - \tau_{xy}\cos 2\alpha \quad \text{——(ii)}$$

→ Here 'σ_y' & 'σ_x' are normal stresses on planes ⊥^{r} to 'y' & 'x' axis resp.

$\sigma_y > \sigma_x$

$\tau_{xy} = \tau_{yx}$ = Shear stress on these 2 planes

From (i) & (ii)

$$\sigma - \left(\frac{\sigma_y+\sigma_x}{2}\right) = \left(\frac{\sigma_y-\sigma_x}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$\tau = \left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\alpha - \tau_{xy}\cos 2\alpha$$

Sq. and add above eq^{n}

$$\left(\sigma - \left(\frac{\sigma_y+\sigma_x}{2}\right)\right)^2 + \tau^2 = \left(\frac{\sigma_y-\sigma_x}{2}\right)^2\cos^2 2\alpha + \tau_{xy}^2\sin^2 2\alpha + \ldots\ldots$$

**

$$\left[\sigma - \left(\frac{\sigma_y+\sigma_x}{2}\right)\right]^2 + \tau^2 = \left(\frac{\sigma_y-\sigma_x}{2}\right)^2 + \tau_{xy}^2$$

→ Eq^{n} of circle with centre (a,b) & radius 'r'

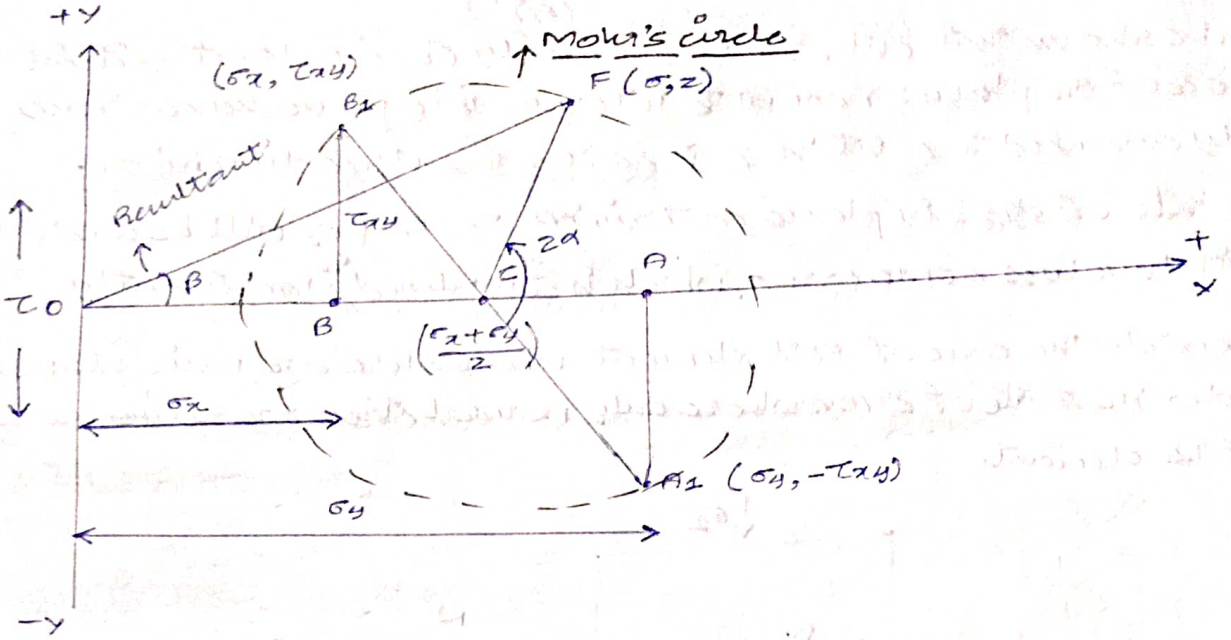
$$(x-a)^2 + (y-b)^2 = r^2$$

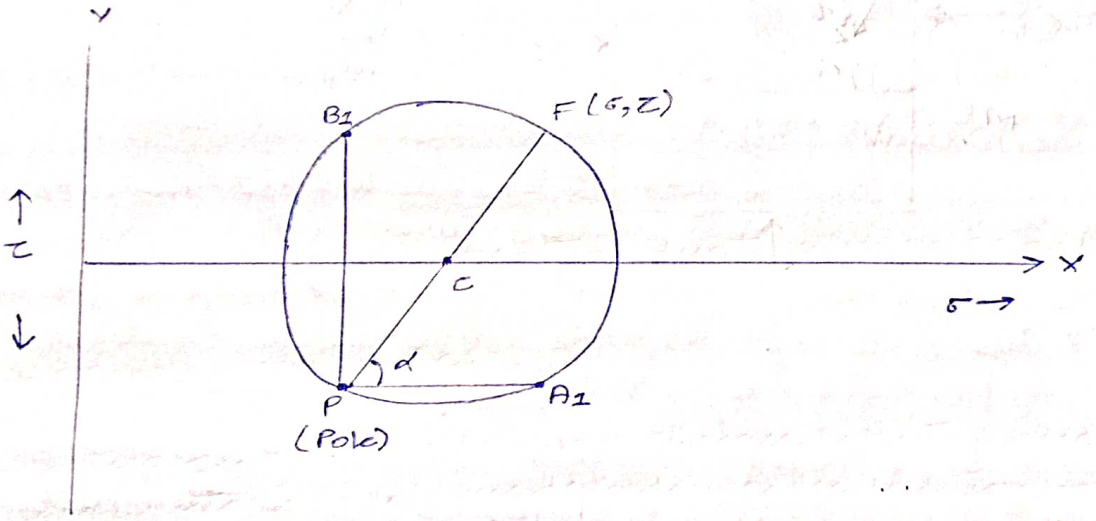
→ The previous eq^{n} of a circle whose centre has coordinate & radius

* coordinates = $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$

* Radius = $\sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2 + \tau_{xy}^2}$ is termed as Mohr's circle.

→ The coordinates of the pt on this circle represents the normal & shearing stresses on inclined plane at given points, hence it can be stated as the locus of normal & shear stress on inclined planes at (σ, τ) given pt.





→ If from point B_1, on a circle representing the state of stress on vertical plane, a line is drawn ∥ to this plane (i.e. vertical), it intersects the circle at a pt 'P'.

→ Also, if from point A_1, on the circle representing state of stress on the horizontal plane, a line is drawn ∥ to this plane (i.e. horizontal), it also intersects the circle at point 'P'

→ In general, if through a pt 'F' representing the state of stress on a given plane (at an inclination of 'α' with horizontal), a line is drawn ∥ to this plane, it will also intersect the circle in the pt 'P'.

→ The point 'P' is therefore a unique point called **origin of plane or the pole.**

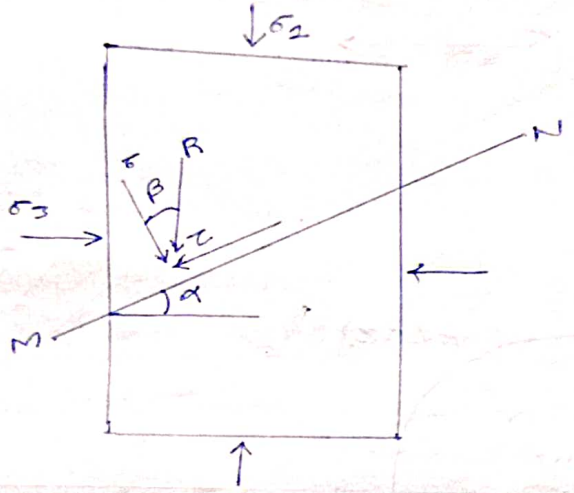
→ It can be shown that out of infinite plane passing through a point, three mutually orthogonal plane exists on which stress is wholly normal & no shear stress act on these plane.

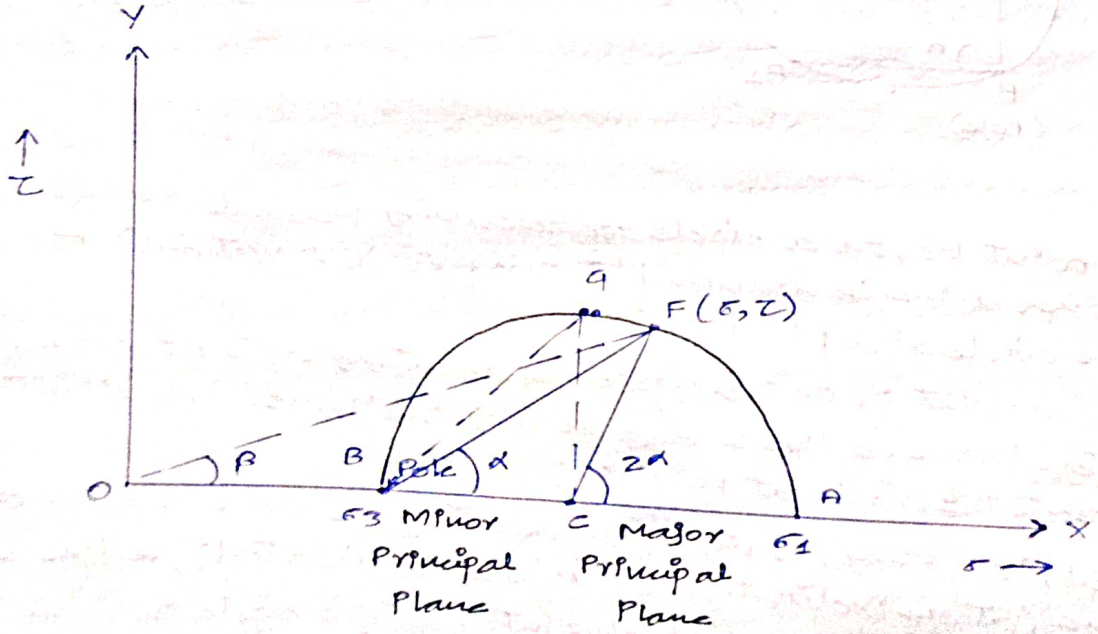
→ These planes are termed as **Principal planes** & the normal stress acting on these planes is termed as **Principal stress**, i.e. Major principal stress, (σ_1) Intermediate Principal stress & Minor principal stress (σ_3).

→ It can be shown that out. It is generally observed that critical stress occur on planes normal to intermediate plane, hence stress commonly assumed to exist in 2-D rather than three dimension.

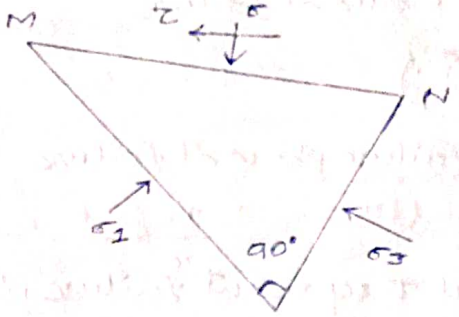
→ Or the state of stress in plane containing σ_1, σ_2 & σ_3 will be considered however it involves some error, which is considered insignificantly

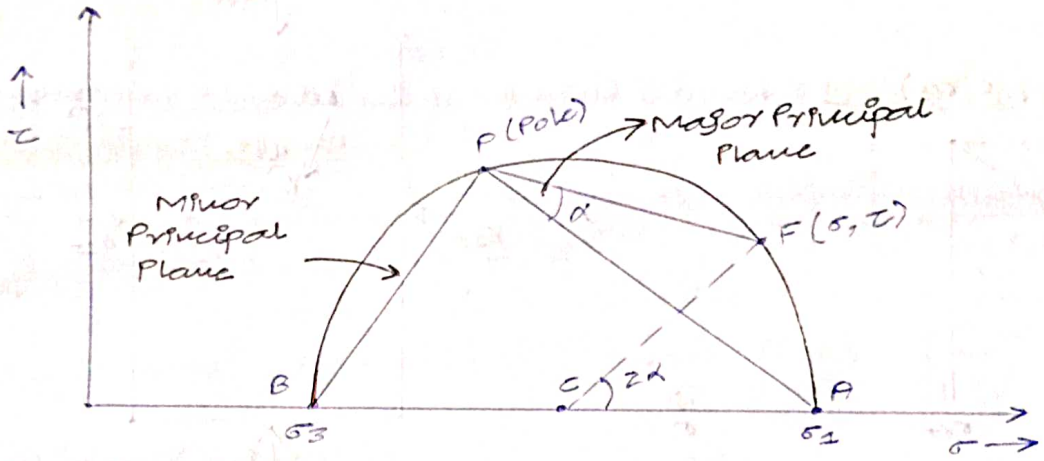
→ Let us consider the case of soil element where sides are principle plane i.e. consider the state of stress where only normal stress are acting on the faces of the element





→ Consider an element in which the principal planes are not horizontal & vertical, but are inclined to 'y' & 'x' directions





→ State of stress at pt 'F' on given plane at an angle 'α' with major principal plane is

* $\sigma = \frac{\sigma_1+\sigma_3}{2} + \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\alpha$

* $\tau = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\alpha$

→ Resultant stress on any plane (R)

$$R = \sqrt{\sigma^2+\tau^2}$$

→ Angle of inclination / obliquity ⇒ * $\beta = \tan^{-1}\left(\frac{\tau}{\sigma}\right)$

→ The max. value of shear stress,

$\tau_{max} = \frac{\sigma_1-\sigma_3}{2}$ and it occurs on planes with $\alpha = 45°$ with major principal plane & normal stress on this plane $\left(\frac{\sigma_1+\sigma_3}{2}\right)$

** $\tau_{max} = \frac{\sigma_1-\sigma_3}{2}$

* Mohr failure criteria & Mohr failure hypothesis:

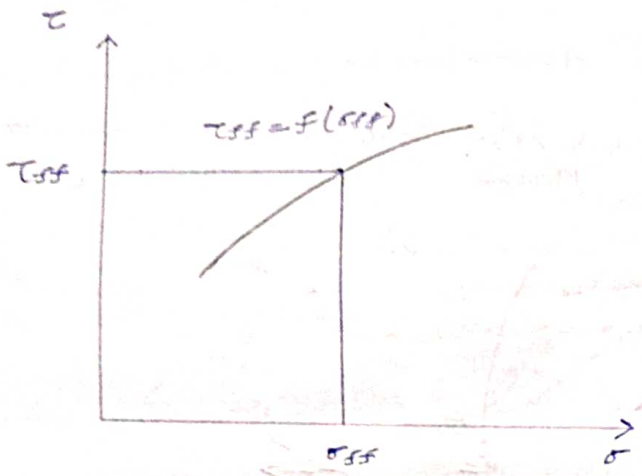
→ According to Mohr hypothesis, the material fail when the shear stress on the failure plane at failure reaches a value which is unique function of normal stress on that plane at failure.

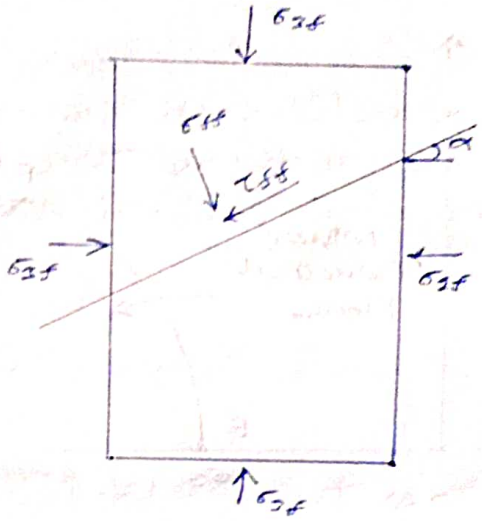
$$\tau_{ff} = f(\sigma_{ff})$$

τ_{ff} = Shear stress on failure plane at failure

σ_{ff} = Normal " " " " " "

NOTE: The first 'f' in the subscript represent "failure plane" & second one represent "at failure".





→ Here σ_{1f} & σ_{3f} are principal stresses at failure.

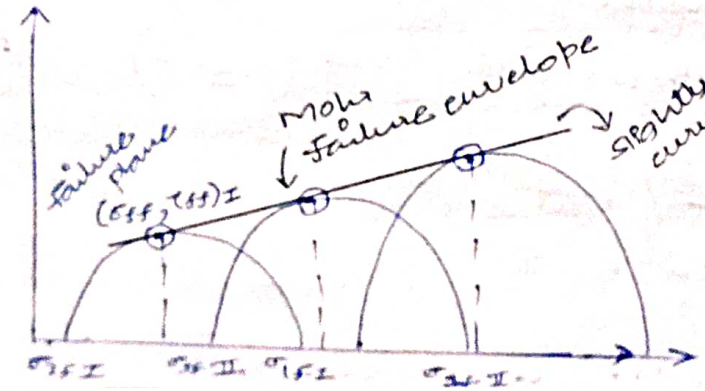
→ If the value of principal stresses σ_{1f} & σ_{3f} are known (determined from test), Mohr circle corresponding to it can be drawn.

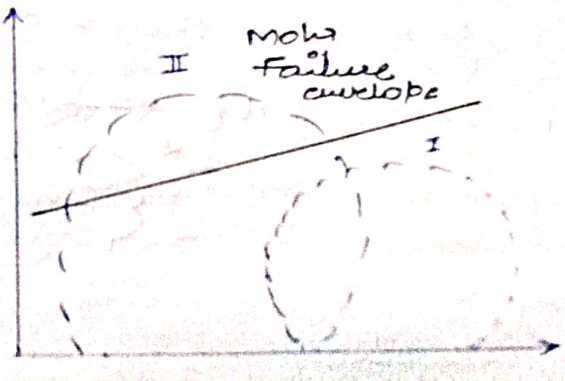
→ If the data from several test, carried out on different sample upto failure is available, series of Mohr circle can be drawn, each circle can be representing the data from one test from it.

→ It is convenient to show these Mohr circle in terms of their upper half.

→ A line tangential to these Mohr circle can now be drawn. This curve is called "Mohr failure envelope"

→ The Mohr failure envelope expresses the functional form of the relationship b/w τ_{ff} & σ_{ff}.





$$\tau_f = c + \sigma \tan\phi$$

$$\tau_f = c + \left[\frac{\sigma_1+\sigma_3}{2} - \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\alpha\right]\tan\phi$$

$$\tau = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\alpha$$

$$\tau_f - \tau = c + \left(\frac{\sigma_1+\sigma_3}{2}\right)\tan\phi + \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\alpha \tan\phi - \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\alpha$$

→ The most dangerous plane, i.e. plane on which failure will take place is the plane on which the diff. of $(\tau_f - \tau)$ is min.

Differentiate wrt 'α'

$$\frac{\partial}{\partial\alpha}(\tau_f - \tau) = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\alpha\,(-2) + \tan\phi - \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\alpha \cdot \cos 2\alpha\,(2)$$

for minimum, $\tau_f - \tau$, $\frac{\partial}{\partial\alpha}(\tau_f - \tau) = 0$

$$-(\sigma_1-\sigma_3)\left[\sin 2\alpha \tan\phi + \cos 2\alpha\right] = 0$$

$$\sigma_1 - \sigma_3 \neq 0$$

$$\sin 2\alpha \tan\phi + \cos 2\alpha = 0$$

$$\cos 2\alpha = -\sin 2\alpha \tan\phi$$

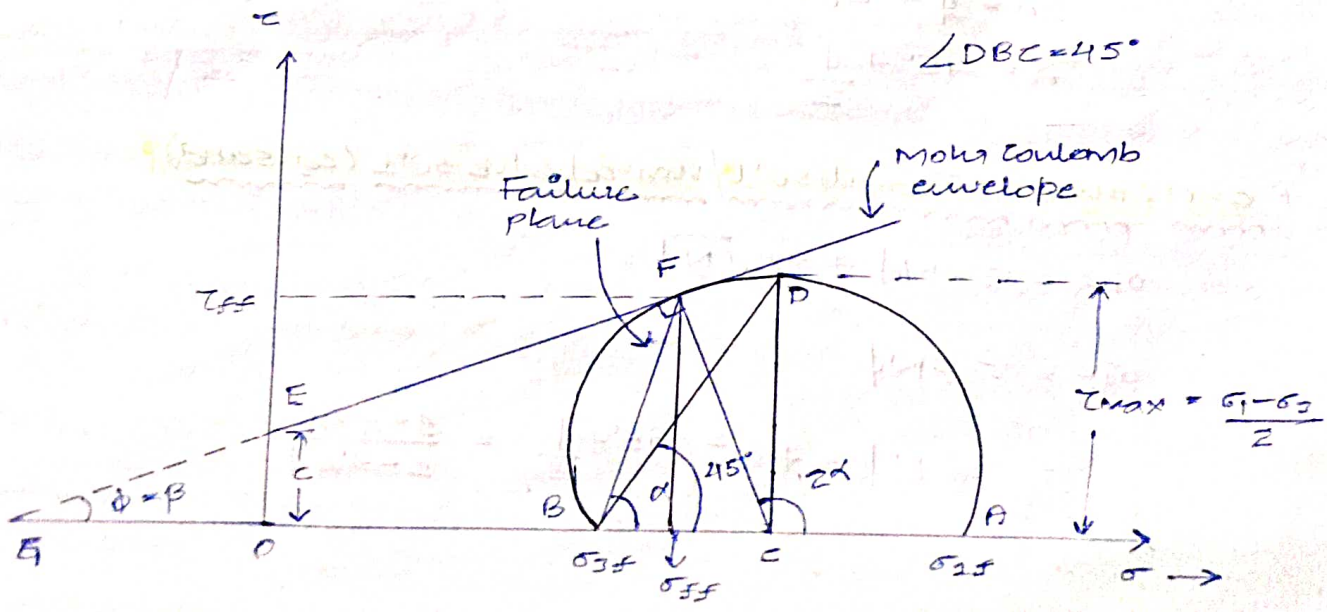
$$\cot 2\alpha = -\tan\phi$$

$$\cot 2\alpha = \cot(90^\circ + \phi)$$

$$2\alpha = 90^\circ + \phi$$

$$\alpha = 45^\circ + \frac{\phi}{2}$$

→ Mohr coulombs failure criterion can be expressed in terms of the relationship b/w the principle stress, σ_{1f} & σ_{3f}.



→ In $\triangle FGC$, $\sin\phi = \frac{CF}{CG} = \frac{CF}{CO+OG}$

$$\sin\phi = \frac{\frac{\sigma_{1f}-\sigma_{3f}}{2}}{\frac{\sigma_{1f}+\sigma_{3f}}{2} + C\cot\phi}$$

$$\left(\frac{\sigma_{1f}-\sigma_{3f}}{2}\right) = \left(\frac{\sigma_{1f}+\sigma_{3f}}{2}\right)\sin\phi + C\cot\phi\sin\phi$$

$$\sigma_{1f} - \sigma_{3f} = (\sigma_{1f}+\sigma_{3f})\sin\phi + 2C\cos\phi$$

$$\sigma_{1f}(1-\sin\phi) = \sigma_{3f}(1+\sin\phi) + 2C\cos\phi$$

$$\sigma_{1f} = \sigma_{3f}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2C\frac{\cos\phi}{1-\sin\phi}$$

$$\sigma_{1f} = \sigma_{3f}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + \frac{2C\sqrt{1-\sin^2\phi}}{(1-\sin\phi)}$$

$$\sigma_{1f} = \sigma_{3f}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2C\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\frac{1+\sin\phi}{1-\sin\phi} = \tan^2(45^\circ+\phi/2) = N\phi$$

**

$$\sigma_{1f} = \sigma_{3f}\tan^2(45^\circ+\phi/2) + 2C\tan\left(45+\frac{\phi}{2}\right)$$

**

$$\sigma_{1f} = \sigma_{3f}N\phi + 2C\sqrt{N\phi}$$

→ By Mohr failure hypothesis

$$2\alpha = \phi + 90^\circ$$

$$\alpha = 45^\circ + \frac{\phi}{2}$$

→ If $C=0$, pure frictional soil / non-cohesive soil (eg. sand)

$$\sigma_{1f} = \sigma_{3f}N\phi + 2C\sqrt{N\phi}$$

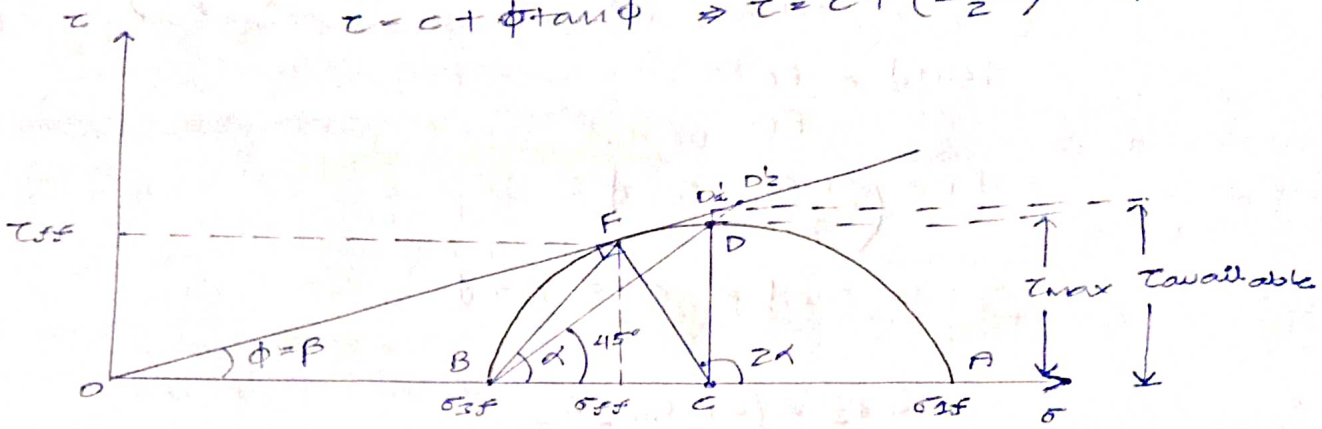
$$\sigma_{1f} = \sigma_{3f}N\phi$$

$$\frac{\sigma_{1f}}{\sigma_{3f}} = N\phi = \tan^2\left(45^\circ+\frac{\phi}{2}\right) = \frac{1+\sin\phi}{1-\sin\phi}$$

** $\frac{\sigma_{1f}}{\sigma_{3f}} = \frac{1+\sin\phi}{1-\sin\phi}$

→ This relation is known as "OBLIQUITY RELATIONSHIP" & is valid for soil having $c=0$, i.e. cohesionless soil only.

$\tau = c + \sigma \tan\phi \Rightarrow \tau = c + \left(\frac{\sigma_1+\sigma_3}{2}\right)\tan\phi$



→ Plane at which sand and silt failure takes place, shear stress is not maximum as shear strength exceeds shear stress.

→ In case of sand, failure takes place on the plane at which resultant stress is most inclined with the normal stress on the same plane at failure.

$\tan\phi$ or $\tan\beta = \frac{\tau_{ff}}{\sigma_{ff}}$ → This is same as obliquity relationship

** $\tau_{ff} = \sigma_{ff}\tan\phi$

→ Hence, here ϕ or β is termed as angle of maximum obliquity also.

In $\triangle FOC$, $\tan\phi = \frac{FC}{FO}$

$FO = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cot\phi$

In $\triangle FOE$, $\sin\phi = \frac{FE}{FO}$

$FE = FO\sin\phi = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cot\phi\sin\phi$

$FE = \tau_{ff} = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos\phi$

$\cos\phi = \frac{OE}{FO} \Rightarrow OE = FO\cos\phi$

$\sigma_{ff} = OE = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cot\phi\cos\phi$

$\tau_{ff} = \sigma_{ff}\tan\phi$

$\left(\frac{\sigma_1-\sigma_3}{2}\right)\cos\phi = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cot\phi\cos\phi\tan\phi$

$$\cos\phi = \frac{\tau_{sf}}{\left(\frac{\sigma_1-\sigma_3}{2}\right)} \tan\phi$$

$$\tau_{sf} = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos\phi \times \left(\frac{\sigma_1-\sigma_3}{2}\right)\frac{\sigma_{sf}}{\left(\frac{\sigma_1-\sigma_3}{2}\right)} \tan\phi$$

$$\tan\phi = \frac{FC}{FO}$$

$$FO = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cot\phi$$

$$\frac{\sigma_1+\sigma_3}{2}\cos\phi = \frac{\sigma_1-\sigma_3}{2}\cot\phi$$

$$\sigma_1+\sigma_3 = (\sigma_1-\sigma_3)\frac{1}{\sin\theta}$$

$$\frac{\sigma_1}{\sigma_3} = \frac{1+\sin\phi}{1-\sin\phi}$$

* → The Mohr failure hypothesis shows an interesting fact. The max. shear stress on the surface failure plane τ_{sf}, is not the max. shear stress in the element (τ_{max})

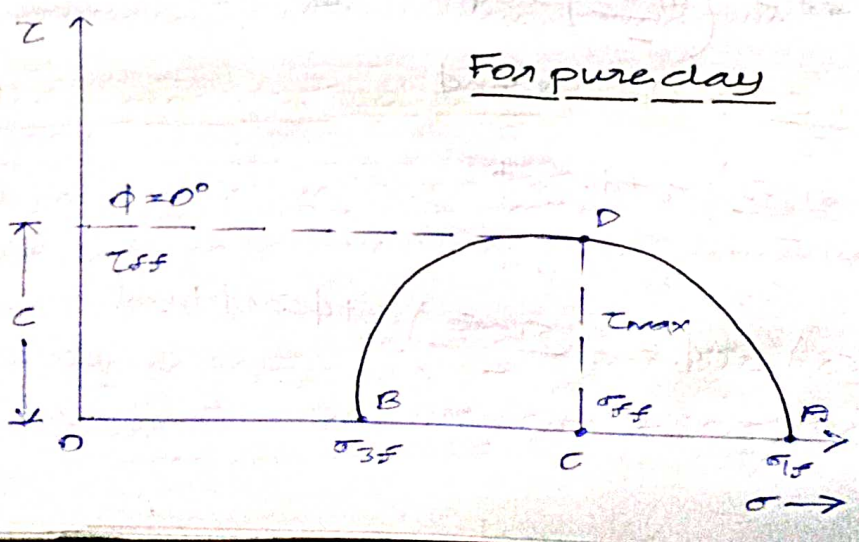
* → The max. shear stress (τ_{max}) acts on plane inclined at 45° to major principal plane & its magnitude is

$$\tau_{max} = \frac{\sigma_{1f}-\sigma_{3f}}{2}$$

→ The failure does not takes place on this plane because the available shear strength on this plane CD_1 is greater than the max. shear stress. ($CD = \tau_{max}$)

→ The failure on plane of max. shear stress can occur only at $\alpha = 45°$, or $\phi = 0°$, that is when Mohr envelope is horizontal line, which will happen in the case of pure cohesive soil ($\phi = 0$) eg. clay.

For pure clay



NOTE: Hence for sand + silt, shear strength is always less than max. shear stress but for clay, it is equal to max. shear stress.

→ The original form of coulomb's eqn is in terms of total normal stress, this however is found to be incorrect.

→ After Terzaghi introduced his effective stress principle, it was established that shear strength is related to effective stress.

→ Hence modified form of coulomb's eqn is

$\sigma' = \sigma - u$

** $\tau_f = c' + \sigma' \tan\phi'$

c' = effective cohesion / cohesion referred to eff. stresses

σ' = eff. normal stress on the plane of failure.

ϕ' = angle of shearing resistance referred to eff. stresses

u = pore water pressure on the plane of failure / rupture at failure

→ In terms of total stress, the same eqn is

$\tau_f = c_u + \sigma \tan\phi_u$

c_u = cohesion referred to total stress

ϕ_u = angle of shearing resistance referred to total stress

NOTE: The strength parameters in terms of effective stresses; $c' + \phi'$ would be taken as constant for given soil provided void ratio, density, pore pr. all remain same for diff. normal stress values.

→ since this never happens in actual, it is represented in terms of test & cond^n under which these tests are performed.

→ Types of shear strength

a. Undrained shear strength:

→ If saturated soil mass is subjected to shear loading & soil is undrained (pore pressure is developed), then the shear strength corresponds to undrained condition.

$\tau_{fu} = c_u + \sigma \tan\phi_u$

or

$\tau_{fu} = C + \sigma \tan\phi$

c_u / C = undrained / total cohesion

ϕ_u / ϕ = undrained / total frictional angle

σ = total normal stress on failure plane.

NOTE: If fast loading is done on saturated clay then, in short term condition, undrained shear strength should be determined.

b. Drained shear strength:

→ If loading rate is slow & expulsion of pore water completes during

loading then pore pressure is dissipated out & the shear strength corresponds to drained condition

$\tau_{fd} = c_d + \sigma' \tan\phi_d$

or

$\tau_{fd} = c' + \sigma' \tan\phi'$

c_d / c' = drained / eff. cohesion

ϕ_d / ϕ' = drained / eff. friction angle

σ' = eff. normal stress on critical plane.

NOTE: If loading is slow in both in short term & long term condition on c and ϕ for long term condition or day, drained shear strength should be determined.

→ Type of shear test on the basis of drainage:

→ The selection of test will depend upon type of soil, purpose of test & drainage condition.

→ There are 2 stages of soil loading

a. confining pressure stage / cell pressure stage

b. Deviator stage / back pressure / shear stage

i. Unconsolidated undrained test (UU test) / Quick test:

→ This test is completed in "5–15 min"

→ In this test, expulsion of pore water is not permitted in both the stage

→ It is suitable for saturated clays, in short term analysis under undrained condition at fast loading rate.

ii. Consolidated undrained test (CU test):

→ In this test, expulsion of pore water may occur / is permitted in first stage which takes approx. 24 hrs, but not in second stage (it is completed in approx. 2 hrs).

** → It is suitable computated for investigation of safety against failure of earthen dam which may occur due to sudden drawdown of the water table.

iii. Consolidated drained test (CD test):

→ It is the maximum time taking test.

→ The rate of loading is very slow, hence it may take few weeks (generally two) for fine clays.

→ In this test, expulsion of pore water is permitted in both stages

→ It is suitable for saturated sand (short + long term) + can also be both

used for long term stability analysis of slope

Drainage condition in 2 stages		Time required for test
Confining stage (Before shear)	Deviator stage (After shear)	
a. Unconsolidated (drainage not allowed)	Undrained	5 - 15 min
b. Consolidated (drainage is allowed)	Undrained	I: 24 hrs + II: 2 hrs
c. Consolidated (drainage is allowed)	Drained	2 weeks

→ Shear stress parameters determined

* C_u, ϕ_u → Unconsolidated Undrained
* C_{cu}, ϕ_{cu} & C'_{cu}, ϕ'_{cu} → Consolidated Undrained
* C_d, ϕ_d → Consolidated, drained

NOTE: Since more time is required for CD test, CU test is the one usually preferred to determine effective stress parameters.

NOTE: Unconsolidated drained test is not performed practically, as confining pressure for very long time is not consolidating the soil, then it can't be drained in small period of shear loading.

→ Measurement of shear strength:

→ Measurement of shear strength can be done by any of the following method.

a. Direct shear test:

→ This test is also called as "shear box test" & it is the oldest test available
→ The soil specimen to be tested is confined in a metal box, square or circular in cross-section that splits into two halves horizontally.
→ If the specimen is fully or partially saturated, perforated metal plates & porous stone are placed above & below the specimen to allow free drainage.
→ Solid metal plates are used if specimen is dry.
→ A pressure pad is placed on the top on the entire specimen is placed in the trolley.

→ One half of box is fixed & the other half is pushed or pulled relative to the fixed one.

→ A vertical load is applied to the specimen through a static weight hanger & the soil is sheared gradually by applying a horizontal force which causes the two halves of the box to move relative to each other.

→ The shear is normally applied at "CONSTANT RATE OF STRAIN"

→ The magnitude of shear load is measured by the mean of proving ring.

→ The shear deformation as well as vertical deformation (ΔH) are measured during the test with the help of dial gauge.

→ The process is repeated on 4-5 specimen, each subjected to a diff. vertical load.

→ Normal & shear stress on the failure plane are obtained by dividing the normal force & shear force by nominal area of the specimen.

→ Value of shear stress at failure are plotted against the normal & stress for each test.

→ The shear strength parameters are obtained from the Mohr-Coulomb method.

→ The direct shear test has the advantage of being quick, inexpensive and simple test in case of sand, moreover the ease of preparation of test is also a plus point.

→ The test however suffers from many disadvantages

a. Drainage condition cannot be controlled & pore water pressure cannot be measured.

b. Hence, it is not of much use for fine saturated soil where drainage cond^n plays an important role in influencing the shear strength.

c. This test is not recommended for conditions other than "full drained"

d. As a result, its use/ failure is limited to only free draining soils, where effective stress parameters & shear strength are to be determined.

e. The failure plane is always horizontal in this test, which may not be direction of weakest plane in the soil.

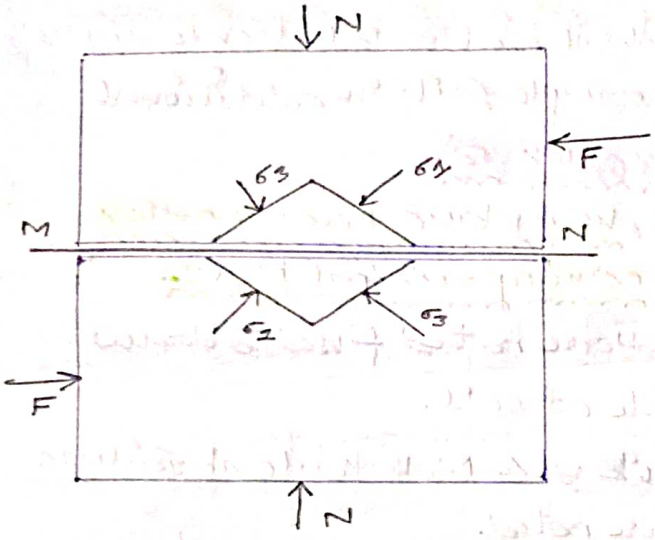
f. There are stress concentration at the sample boundary, leading to non-uniform shear stress distribution on the failure plane.

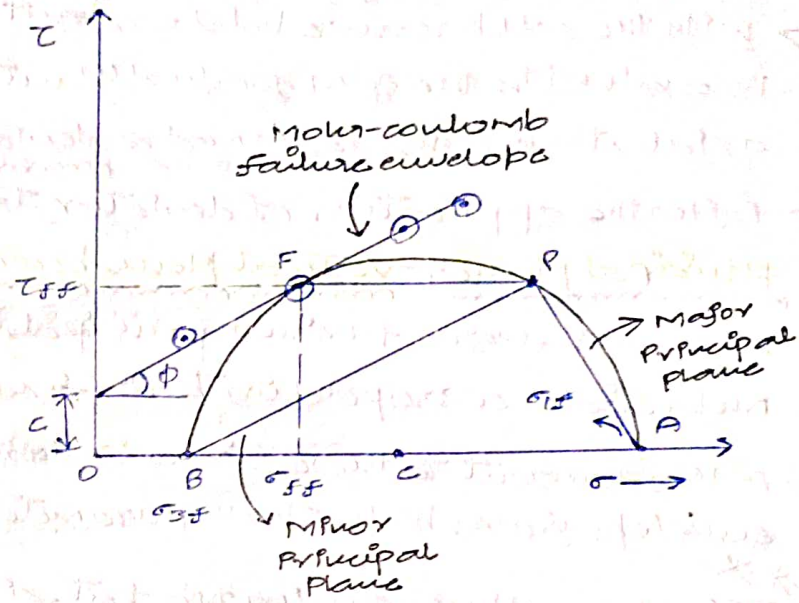
g. Failure of soil in this case takes place progressively from edges towards the centre.

h. The area of specimen under normal & shear loading doesn't remain constant for the duration of test, hence stresses are calculated on the basis of constant nominal area of specimen which are inaccurate.

1. Directions of principal plane are not known at every stage of test, It is only after Mohr failure envelope is known, the magnitude & direction of the principal stresses can be determined.

2. There is rotation of the principal planes throughout the test, due to this, the field loading situation cannot be simulated accurately in lab sample.





b. Triaxial shear test:

→ This test is very accurate & is generally used for research purposes.
→ This is the most versatile of all the test available, however is little complicated.
→ Drainage conditions can be controlled, depending upon type of soil.
→ Pore water pressure measurements can be done accurately.
→ Volume changes can also be measured.
→ There is no rotation of principal plane during test.
→ The failure plane is not forced/predetermined. The specimen can fail on any weak plane or can bulge also.
→ The stress distribution is uniform on failure plane.
→ The test is carried out on a cylindrical specimen having $\frac{L}{D} = 2$ (generally 76×38 mm or 100×50 mm size is used)
→ The cylindrical specimen is placed on saturated porous disc resting on the pedestal of triaxial cell.
→ The specimen is encased in rubber membrane & sealed to the pedestal & at bottom & at top.

→ The membrane is used to prevent the entry of pressurised cell fluid (water) from penetrating into the soil specimen.

→ The triaxial cell is filled with water at the required pressure, thus subjecting the soil specimen at an all round pressure (This is called **cell pressure or confining pressure σ_c**)

→ It acts radially on the vertical surface of the specimen & axially at the top & bottom of specimen.

→ With the cell pressure held constant, additional axial stress is applied through with the ram gradually, until sample fails in additional axial stress (This is termed as **deviator stress σ_d**).

→ After the application of deviator stress, hor. plane becomes major principal plane & vertical plane becomes minor principal plane.

→ At failure, major & minor principal stresses are noted & used to draw Mohr circle corresponding to diff. sample of soil.

→ A line tangent to these circles is drawn to give Mohr Coulomb failure envelope from which shear parameters are noted.

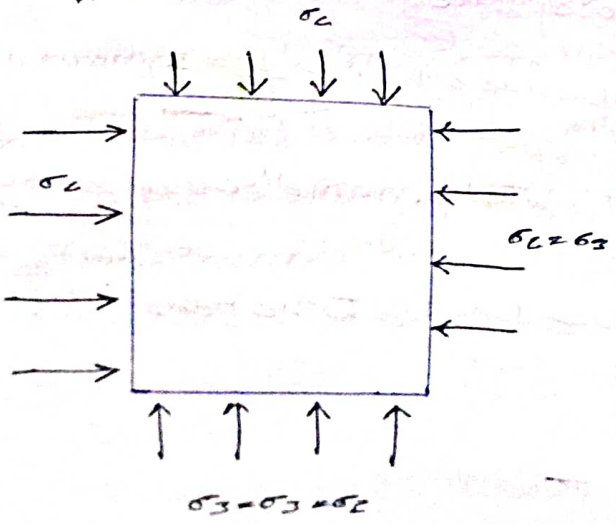
NOTE: At any time, during this test, either pore water line is open & drainage line is closed or vice versa.

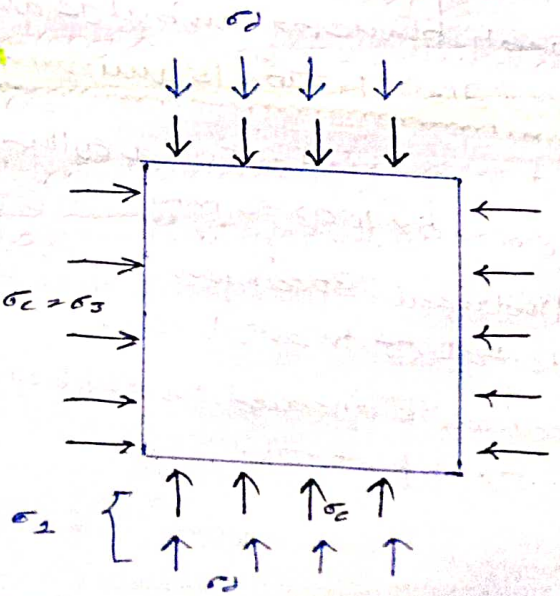
→ That is either the pore water pressure measurement will be made under undrained conditions or the volume changes will be measured during drained condition.

→ This test can be performed for all the drainage conditions.

→ Pore water pressure measurements in this test is done using **Bishop's pore-pressure** measuring apparatus.

→ In this test, the **pore water pressure** developed in the specimen can be **negative** also in some case.





NOTE: Here, $\sigma_c = \sigma_3$ = Minor principal stress

$\sigma_1 = \sigma_d + \sigma_3$ = " " "

$\therefore \sigma_d = \sigma_3 + \sigma_1 = \sigma_1 - \sigma_3$

→ Deviator stress at failure is also termed as confined compressive strength, which depends on the value of "σ_3".

→ Higher is the value of σ_3, higher is the deviator stress

* Confined comp. strength, $\sigma_{df} = (\sigma_1 - \sigma_3)_f = \frac{P}{A_f}$

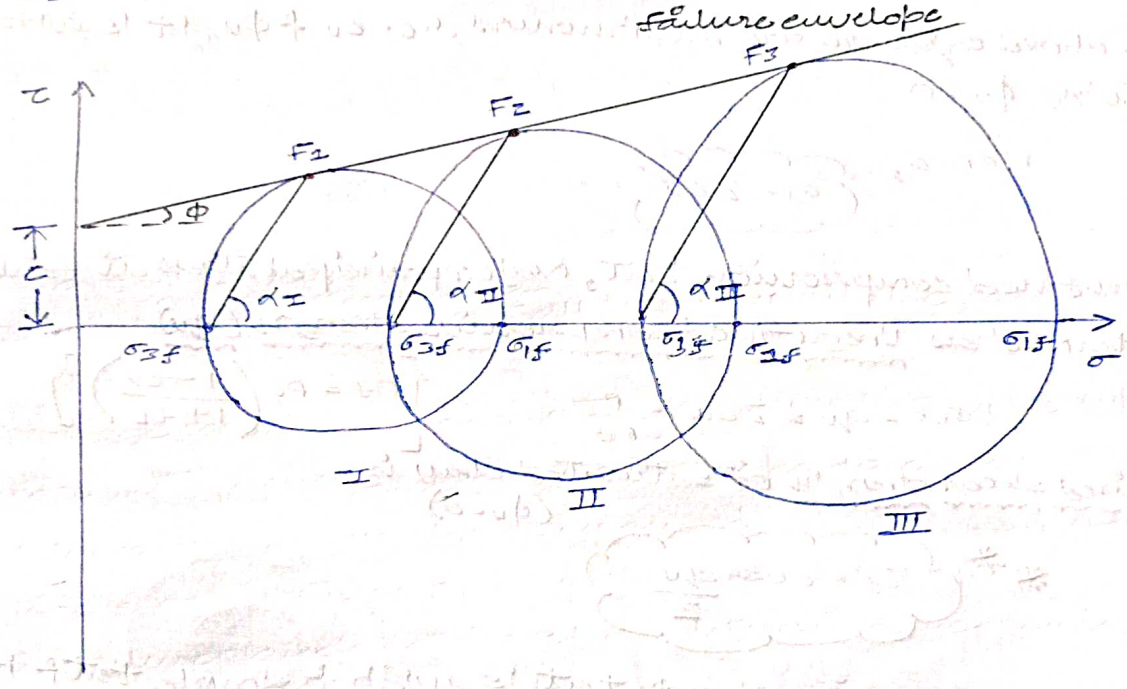
A_f = area of cross-section at the time of failure

Let A_0 = area of cross-section before loading = $\frac{\pi D_0^2}{4}$

$$A_f = \frac{(\text{Volume})_f}{(\text{length})_f} = \frac{V_0 \pm \Delta V}{L - \Delta V} = \frac{V_0\left(1 \mp \frac{\Delta V}{V_0}\right)}{L_0\left(1 - \frac{\Delta L}{L_0}\right)} = \frac{A_0(1 \pm \epsilon_v)}{(1 - \epsilon_L)}$$

ϵ_v = volumetric strain

ϵ_L = axial strain



→ For undrained cond^n,

$\Delta V = 0$

$\epsilon_v = 0$

** $A_f = \frac{A_0}{1 + \epsilon_L}$

c. Unconfined compression test:

→ This is a special case of triaxial test carried out at zero cell pressure i.e. ($\sigma_c = \sigma_3 = \sigma_2 = 0$).

→ The test is conducted in lab using a triaxial setup or modified triaxial apparatus.

→ The soil specimen is subjected to gradually increasing axial stress until it fails in shear

→ Since, it is quick, water is not allowed to drain out of the sample, hence it is an undrained test / quick test.

→ Since the test produces only one mohr CIRCLE (corresponding to $\sigma_3 = 0$), this is only to soils for which $\phi_u = 0$ i.e. fully saturated fine grained clays.

Here, $\sigma_3 = 0$

$$\sigma_1 = \sigma_3 \tan^2\alpha + 2c_u \tan\alpha$$

$$\sigma_1 = 2c_u \tan\alpha$$

$$\sigma_1 = 2c_u \sqrt{\frac{1+\sin\phi_u}{1-\sin\phi_u}}$$

NOTE: Since in the above eqn there are 2 unknowns, i.e. c_u & ϕ_u, it is suitable for soil having $\phi_u = 0$

Hence,

$$\sigma_1 = 2c_u$$

→ In the unconfined compression test, major principal stress at failure σ_{1f} is also termed as Unconfined compressive strength (q_u)

$$\sigma_{1f} = q_u = 2c_u = \frac{P}{A_f} \qquad \left[A_f = A_0\left(\frac{1-\epsilon_v}{1+\epsilon_L}\right)\right]$$

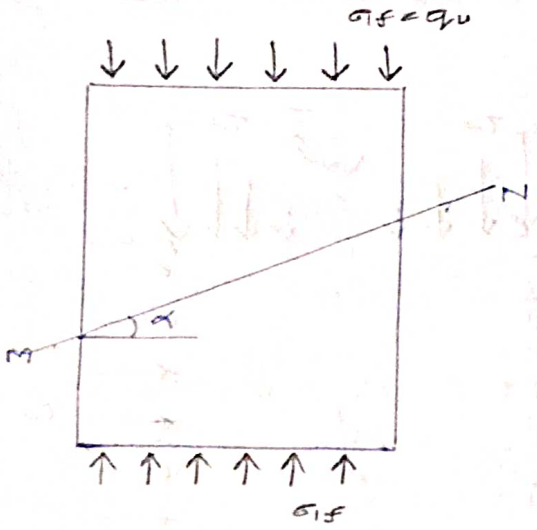
→ And undrained shear strength of saturated clay is ($\phi_u = 0$)

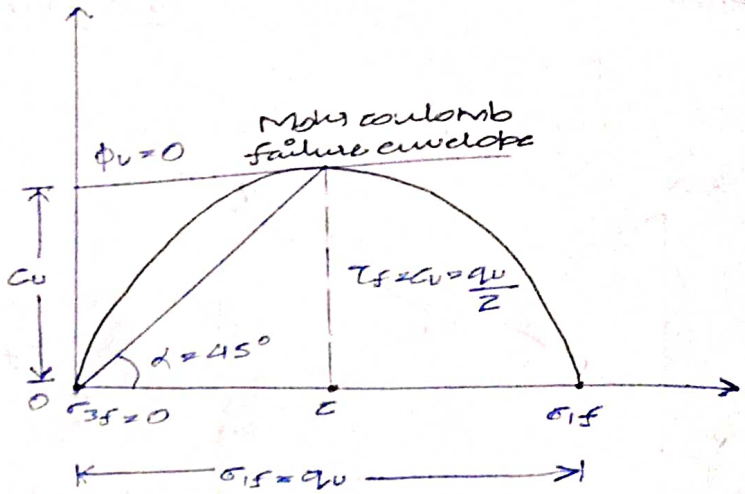
** $$\tau_f = c_u = \frac{q_u}{2}$$

→ The unconfined compressive strength test is quick & simple test & is often used to determine the in situ strength of soft, saturated, fine grained soil deposits for which $\phi_u = 0$

→ This test is preferred to UU triaxial test in view of its simplicity

→ This test is also used to determine the sensitivity of clay soils other than fissured clay





d. Vane shear test:

→ In this test, there is no mechanism to measure pore pressure & no drainage facility is provided, hence it can be conducted only under undrained condition.

→ It is suitable for soft saturated clays eg. marine clays to find undrained shear strength & it can also be used to find sensitivity of clay.

→ It can be conducted in lab as well as in field.

→ The mechanism in both case is same, the only diff. lies is in the size of vane.

→ Size of vane that can be used in this case is of 37.5, 50, 65, 75 or 100mm.

→ The height of the vane shall be twice the overall diameter of vane.

→ The vane is punched into the soil and torque is applied by rotating the vane at 6°/min

→ Vane is calibrated to a spring having torsional stiffness (K), hence the torque at shear failure is determined as

* $T = K\theta$

θ = angular rotation of vane at which complete shear failure occurs

→ While punching the vane in the soil following 2 cond^n may occurs.

i. Two way shearing:

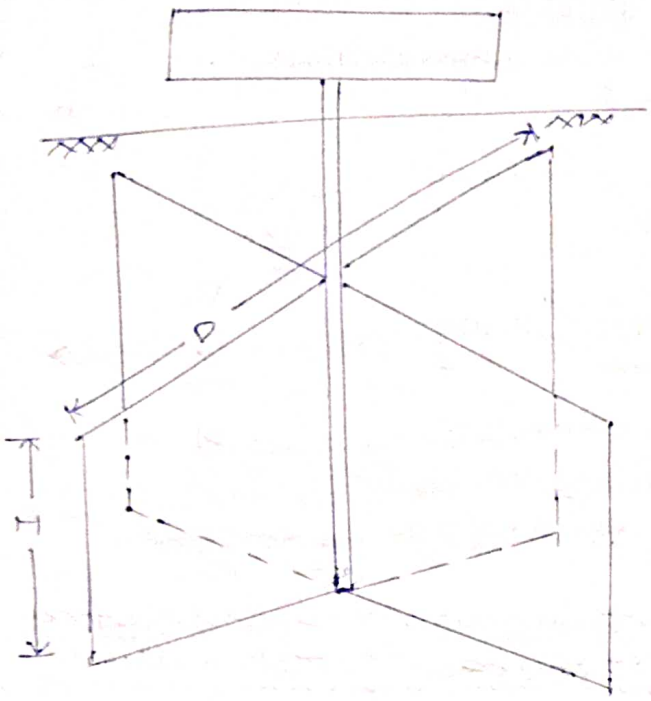
→ If vane is punched such that shearing occur at sides & top & bottom of vane at a certain depth below the ground level.

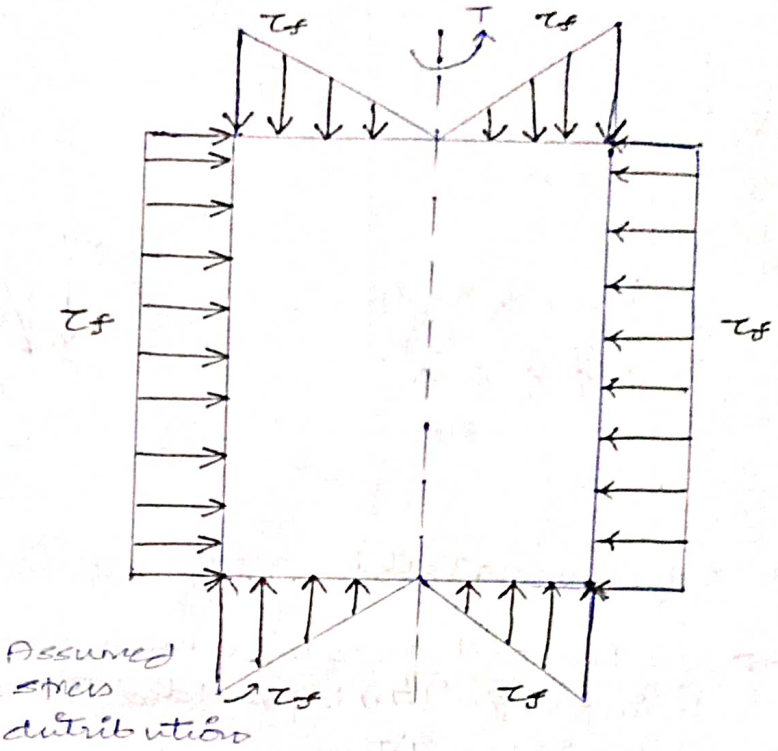
ii. One way shearing:

→ If vane is punched into the soil where exactly top of vane is at

ground level. Then shearing will occur at sides & bottom, but not on top.

Two way shear





Torsion (T_2) = $T_{1\,sides} + T_{2\,top\,\&\,bottom}$

Torque = force × distance

$T_{side} = f_{side} \times \frac{D}{2}$

$= \tau_f (\pi D H) \times \frac{D}{2}$

$T_{side} = \tau_f \frac{\pi D^2 H}{2}$

$T_{2\,top\,\&\,bottom} = 2 \int \partial T$

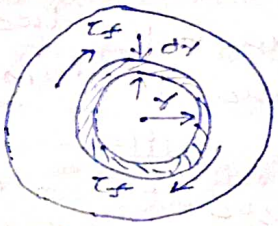
$= 2 [\tau_f (2\pi r\, dr)] \cdot r$

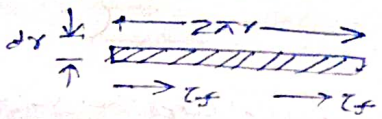
$= 4 \tau_f \pi \int_0^{D/2} r^2 dr$

$= 4 \tau_f \pi \cdot \frac{r^3}{3} \Big|_0^{D/2}$

$= 4 \tau_f \pi \left[\frac{D^3}{2^3 \times 3}\right]$

$T_2 = \frac{\pi \tau_f D^3}{6}$





$$T = T_1 + T_2$$

$$T = \frac{\tau_f \times D^2 H}{2} + \frac{\times \tau_f D^3}{6}$$

$$T_f = \tau_f \frac{\times D^2}{2}\left[H + \frac{D}{3}\right]$$

→ shear strength $\tau_f = \frac{T_f}{\frac{\times D^2}{2}\left(H + \frac{D}{3}\right)}$

for 2 way shearing

NOTE: Determination of sensitivity of soil

$$S_t = \frac{(UCS)\text{ undisturbed soil}}{(UCS)\text{ remoulded soil}}$$

$$S_t = \frac{(q_u)\text{ undisturbed soil}}{(q_u)\text{ remoulded soil}}$$

$$S_t = \frac{(2C_u)\text{ undisturbed soil}}{(2C_u)\text{ remoulded soil}}$$

$$S_t = \frac{(C_u)\text{ undisturbed soil}}{(C_u)\text{ remoulded soil}}$$

or

$$S_t = \frac{(\text{Vane shear strength})\text{ undisturbed soil}}{(\text{Vane shear strength})\text{ remoulded soil}}$$

NOTE: Vane shear strength α T_f

$$S_t = \frac{(T_f)\text{ undisturbed soil}}{(T_f)\text{ remoulded soil}}$$

→ To find the sensitivity of clay, initially vane shear test is performed on undisturbed soil sample hence shear strength of undisturbed clay is known.

→ Vane is further quickly rotated several times, hence soil becomes remoulded After sometime again vane shear test is performed on remoulded soil to find shear strength.

NOTE: There are some other lab test also available to find shear strength of soil like

→ Hollow cylinder test
→ Plane strain test
→ True triaxial or cuboidal test
→ Torsional or ring shear test
→ Simple shear test
→ Among the field shear test, the vane shear test, direct shear test, static cone penetrometer test, standard penetration test are used.

* Shear strength of clay soils:

→ Fine grained soils like fine silts, clays which are referred as cohesive soil powers one property which set them apart from C.G. soil like sand i.e. its permeability.
→ Drainage of pore water under loading takes much longer for a cohesive soil than sand.
→ Under shear load, volume changes occur quickly in granular soil due to their high permeability
→ In actual, loads are applied faster than rate of escape of pore waters which, leads to development of pore water.
→ If it is sufficiently high, it may lead to shear failure as water cannot resist shear.
→ In such cases, effective shear stress approach cannot be used & total stress approach based on undrained shear strength is utilised.
→ There is one more important considerations influencing shear stress of cohesive soil i.e. stress history.
→ The drained & undrained shear strength depends to a great degree of whether soil is NCS or OCS.

NOTE: strength behaviour of cohesive soil is with reference to diff. type of triaxial test is analysed as it relates real field situation.

I. Undrained strength from UU Test:

→ In this UU test, drainage is not permitted during the first stage i.e. cell pressure stage & deviator stage.
→ If soil is completely saturated, no consolidation can occur even during cell pressure application.

→ Atleast 3 samples with identical conditions, i.e. w% and 'e' are to be tested.

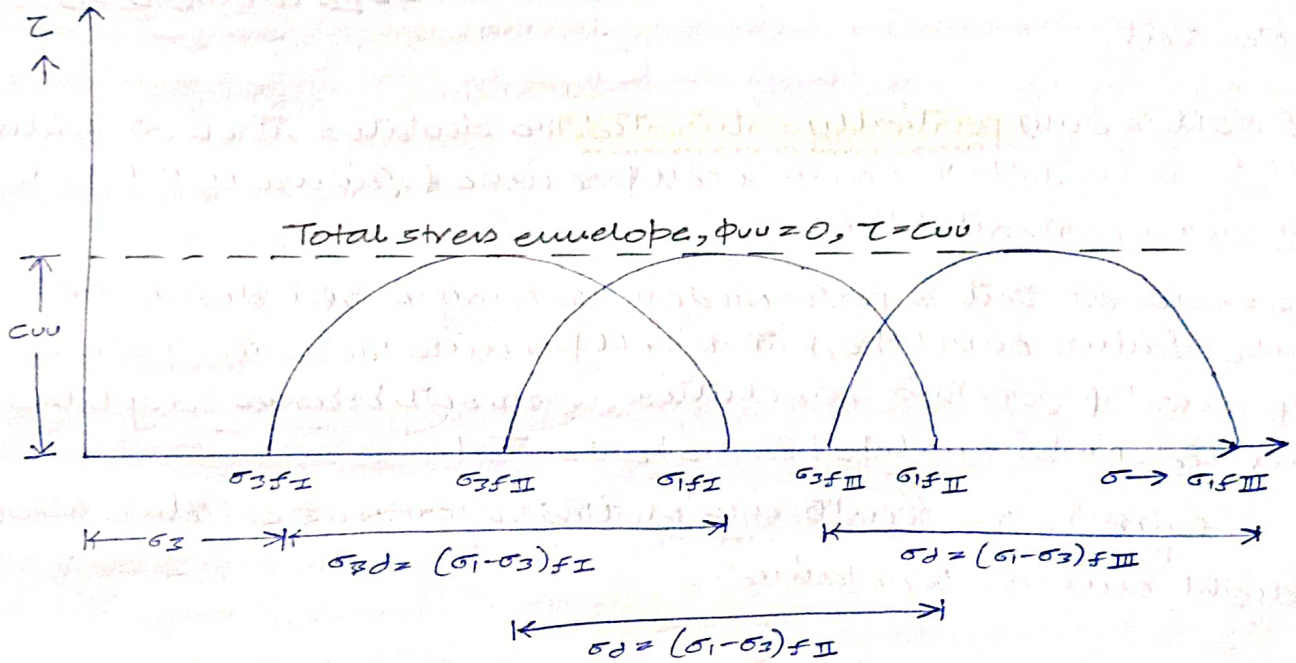
→ Mohr circle at failure & resulting Mohr envelope is then drawn.

* → The diameter of Mohr circle is equal to "$\sigma_{df} = \sigma_{1f} - \sigma_{3f}$" for all samples & is equal.

→ Even though, the test are conducted at diff. cell pressure, different from one used for previous test, but due to same initial condition of soil, shear strength comes out to be same.

→ The UU test gives value $\phi_{uu} = 0$, The intercept of the envelope on y-axis (τ-axis) gives the value of c_{uu}, which are total stress parameter.

** $$\tau_f = c_{uu} = \frac{(\sigma_1 - \sigma_3)_f}{2}$$

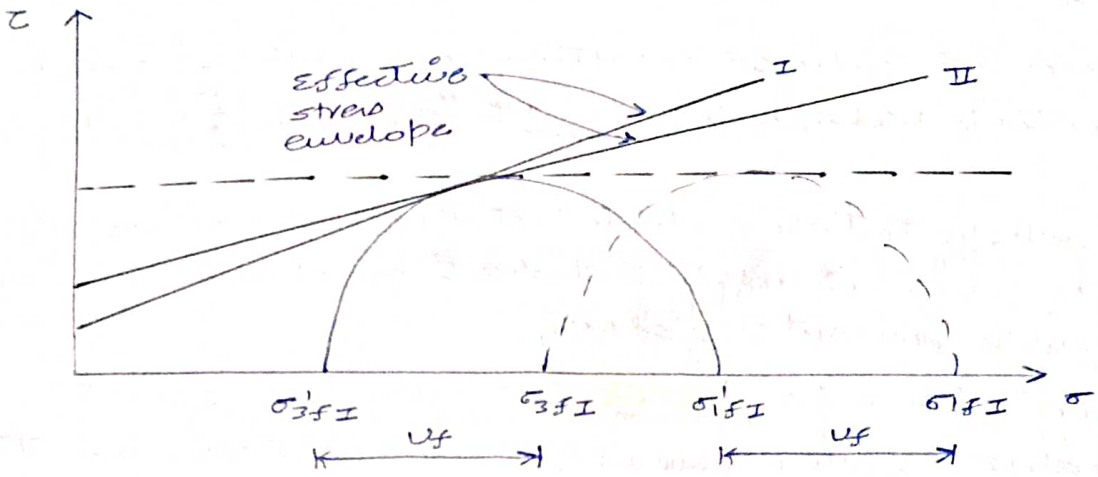


* → If pore water pressure measurement are made in UU test & results are plotted in terms of effective stress, only one Mohr circle is obtained for all the test samples.

→ The increase in cell pressure for test II & III, simply shows as as increase in pore water pressure of same magnitude under no drainage condition.

→ Hence in this case, no failure envelope can be defined.

→ Thereby determination of shear strength parameters is not practically possible, hence pore pressure are not measured in this case.

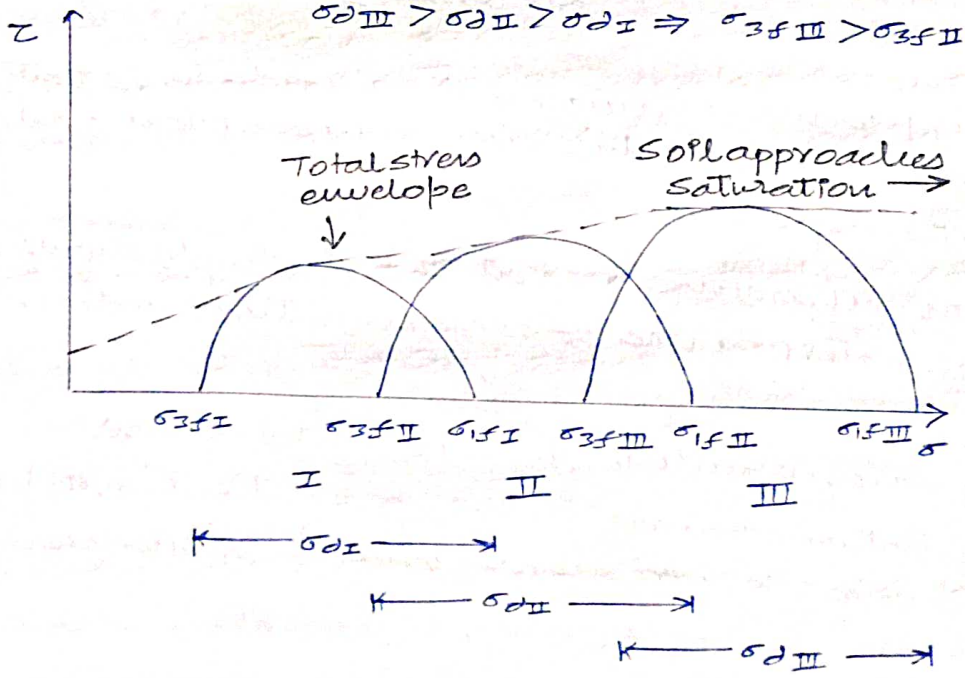


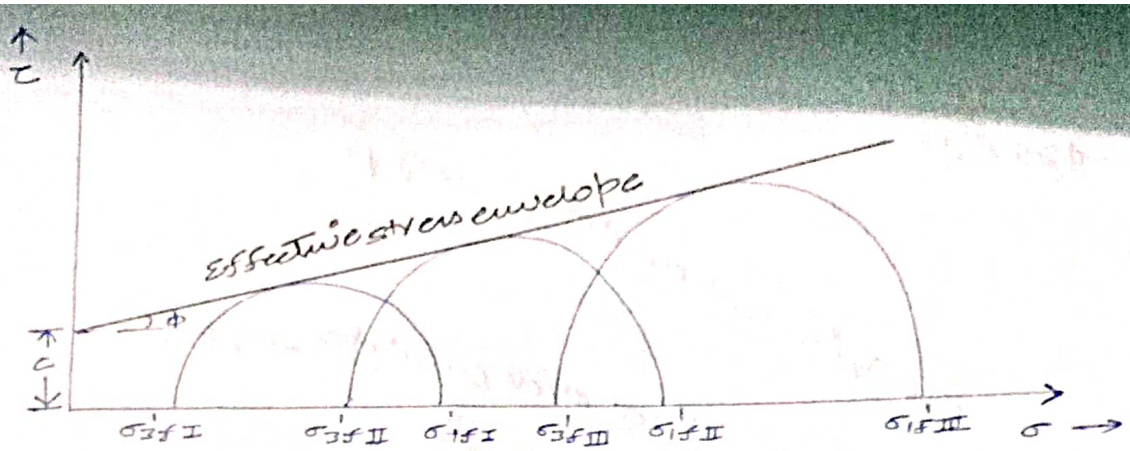
NOTE: Since failure envelope in UU test is horizontal, the unconfined compression test can be used to determine the value C_u, due to the ease of its performance but results will not be as accurate as of triaxial test.

→ If soil is only partially saturated, the deviator stress at failure will increase, with increase in cell pressure & failure envelope is also no longer a horizontal line.

→ The series of test is performed on the samples that define an initially curved failure envelope, but as cell pressure increases, air in voids is compressed & goes into the solution, when soil becomes completely saturated, Mohr envelope become horizontal.

→ It is found if the results are plotted in terms of effective stress, a straight line envelope is obtained.





NOTE: The ratio of undrained shear strength of a normally consolidated clay to the effective overburden pressure at the corresponding depth (from which the sample is taken), i.e. $\frac{C_u}{\sigma_z'}$, was been found to be constant & is related to the I_P & as

** $\frac{C_u}{\sigma_z'} = 0.11 + 0.0037\, I_P\%$ (Relation developed by Skempton)

→ This relation helps in quick estimation of the increase in the undrained shear strength of NCS with depth on the basis of Atterberg limit.

→ It also helps in identifying the state of soil, that whether it is NCS or OCS

II. Consolidated undrained strength from CU test

→ In consolidated undrained test, the saturated sample is first consolidated at the desired cell pressure by keeping drainage line open.

→ After consolidation is over, the drainage line is closed & sample is sheared upto failure, under undrained condition, keeping cell pressure constant.

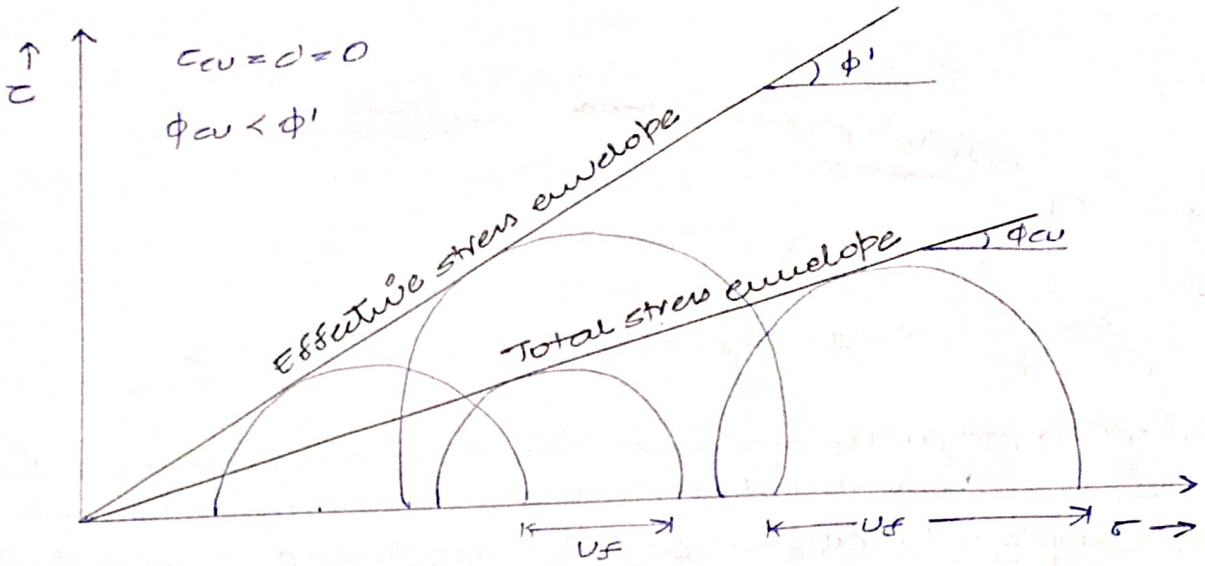
→ During the deviator stress application, the pore pressure is measured.

→ The test results can be plotted in terms of total or effective stresses.

NOTE: One of the important characteristic of clay soil is that at the end of first stage of the test there is no excess pore water pressure in the sample & when specimen under undrained condition is sheared, the pore water pressure developed can be positive (for NCS) or negative (for OCS)

→ Mohr envelope in terms of both total & effective stress for a series of CU test for a NC clays comes out to be straight line passing through origin.

→ The angle of shearing resistance in terms of total stress ϕ_{cu} comes to be less than that observed in terms of effective stress ϕ'_{cu}.



→ If the clay is overconsolidated & pore water pressure is negative during shear, the effective stress becomes greater than the total stress and the effective stress Mohr circle will be shifted to right of total stress Mohr circle.

→ To depict the effect of preconsolidation on the shape of envelopes, large range of consolidation stresses are considered.

→ It is observed that pt of transition in the shape of total stress envelope occurs at a much higher value of cell pressure, than it is in the case of effective stress envelope. ($\sigma_3 > \sigma_3'$)

→ The failure envelope for overconsolidated clay is not a straight line but is curved. Since the diff. b/w deviator stress necessary to cause failure in an overconsolidated clay & the one necessary to cause the failure in NCC reduces as OCR decreases.

→ For a particular effective consolidation stress, before shear, the shear strength of an over consolidated clay is much higher than that of NCS & margin of difference reduces with decrease in overconsolidation ratio.

→ The shear strength for CU test in terms of total stress for OCS clays,

$$\tau_f = C_{cu} + \sigma \tan \phi_{cu}$$

→ For normally consolidated clays, $C_{cu} = 0$

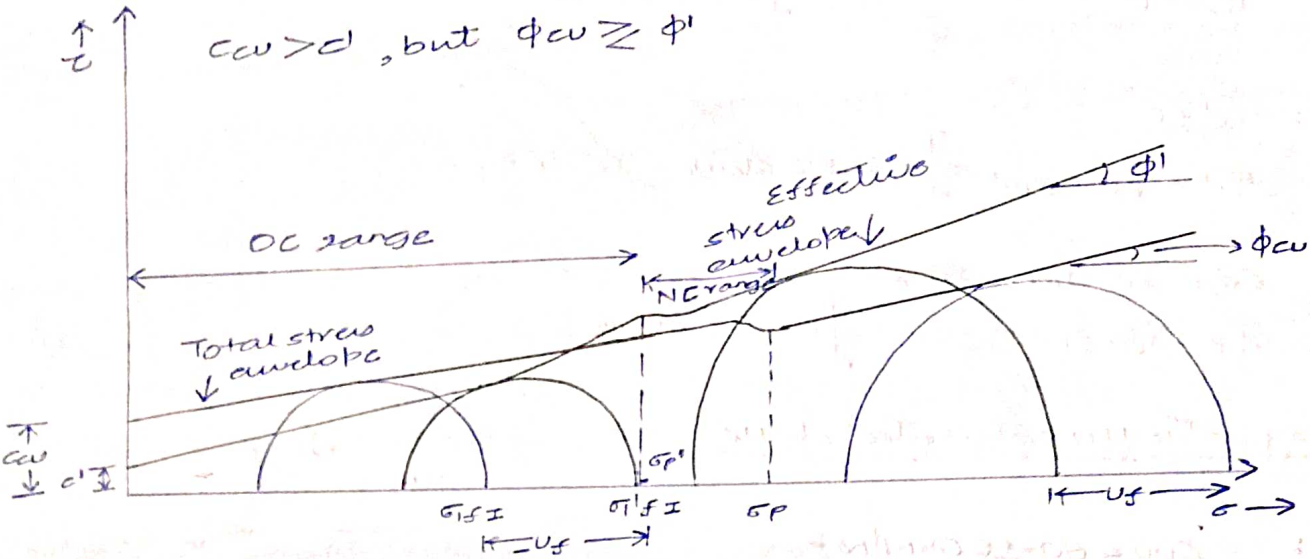
$$\tau_f = \sigma \tan \phi_{cu}$$

→ In terms of effective stress, shear strength for OC clays,

$$\tau_f = C'_{cu} + \sigma' \tan \phi'_{cu}$$

→ for NC clays, $c'_{cu} = 0$

$\tau_f = \sigma' \tan \phi'_{cu}$



III. Consolidated drained strength from CD test:

→ In CD test, the test specimen is first consolidated under an appropriate cell pressure, with cell pressure kept constant, the test specimen is then sheared by applying the deviator stress, so slowly that excess pore water pressure does not develop during the test.

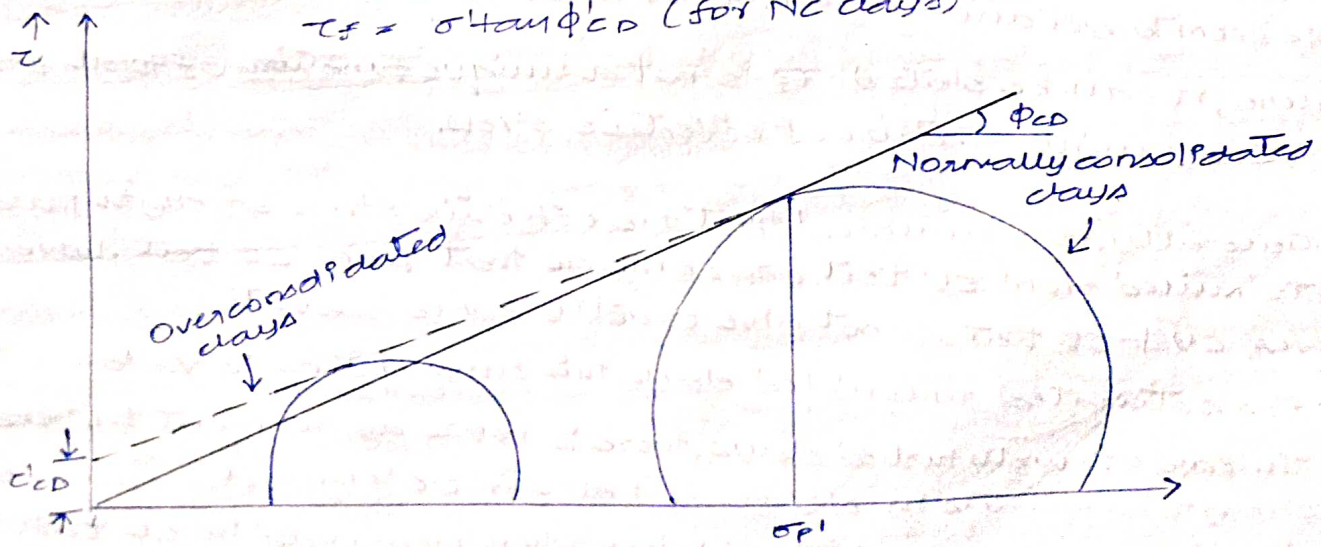
→ Thus at any stage during the test, the total stresses are effective stresses.

→ The Mohr envelope obtained from CD test on normally consolidated clay is always straight line & for OC clays is slightly curved.

→ This envelope for NC clay pass through origin as in CU test but not for OC clays.

$\tau_f = c'_{CD} + \sigma' \tan \phi'_{CD}$ (for OC clays)

$\tau_f = \sigma' \tan \phi'_{CD}$ (for NC clays)



NOTE: $C_{UU} = 25-30\ KN/M^2$, $\phi_{UU} = 0$] → NC clays

Case I: $C_{UU} = 100-200 KN/M^2$, $\phi_{UU} = 0$] → OC clays

Case II: $C_d = 0$, $\phi_d = 20°-30°$] → NC clays

$C_d = 60-100\ KN/M^2$, $\phi_d = 10-50°$] → OC clays

→ For partially saturated clays

Case I: $C_{UU} = 80-150 KN/M^2$, $\phi_{UU} = 0-50°$

Case II: $C_d = 60-70\ KN/M^2$, $\phi_d = 28-30°$

99. The UU test data indicates, that irrespective of magnitude of total stress to which soil sample is subjected, it requires the same additional axial/deviator stress to cause the failure. This means the shear stress at failure is constant but not a function of total stress.

→ The CU test indicates that with increase in total stress, the additional axial stress is required to cause failure & also the increase is linear, i.e. it is linear function of total stress.

→ In CD test, where total stress is the effective stress only shear strength is seem to get another linear function of its total stress.

→ Here, it can be stated 'f' is not a unique function of total stress but is unique function of effective stress.

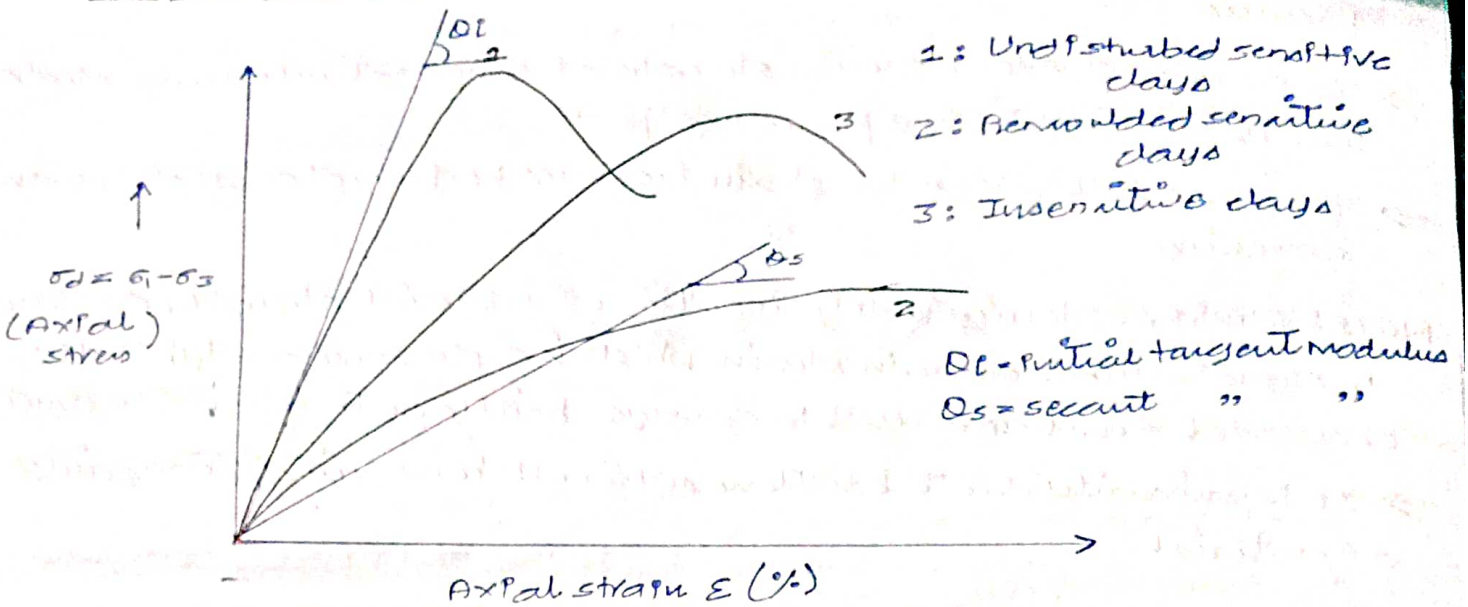
999. Generally, it is assumed that the effective stress strength parameters determined from CU test are same as that from CD test, however most CU & CD test do not give exactly same results.

→ For saturated remoulded clays, this assumption is valid.

→ In case of undisturbed clays, there is more agreement in the strength parameters in NC clays but for OC clays, the values of c & ϕ' are usually a little higher when measured in CD test, than

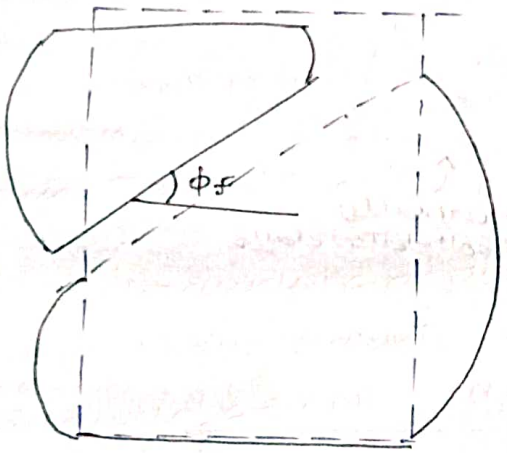
in CU test, because of the work done during the volume change under shear in CD test.

→ Stress strain for clays:

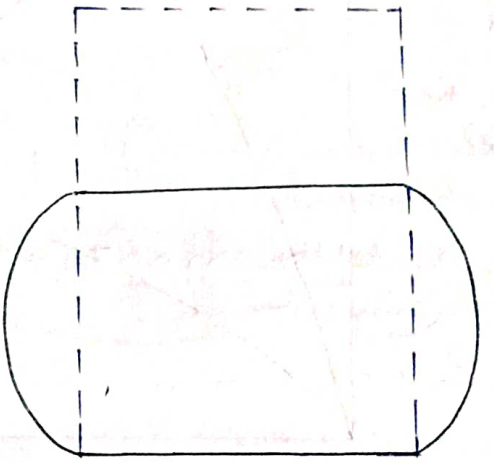


1: Undisturbed sensitive clays
2: Remoulded sensitive clays
3: Insensitive clays

E_i - Initial tangent modulus
E_s = secant " "



Brittle failure



Plastic failure

→ Stress strain curves obtained from UU test are represented

→ Curve 1 for undisturbed clay of high sensitivity, shows a sharp peak at low strain. This specimen shears along a clearly along a well designed/defined shear plane, also termed as brittle failure

→ Curve 2 for remoulded sensitive clay show plastic failure since the specimen shows a continuous yield at a more or less at constant stress. It results in increased cross-section area of the specimen due to bulging effect.

→ curve 3 for an insensitive clay, shows an increase of stress with strain upto a certain stage & thereafter the stress may drop slightly or almost remains constant for increase in strain, failure in this

this, corresponds to horizontal portion of curve

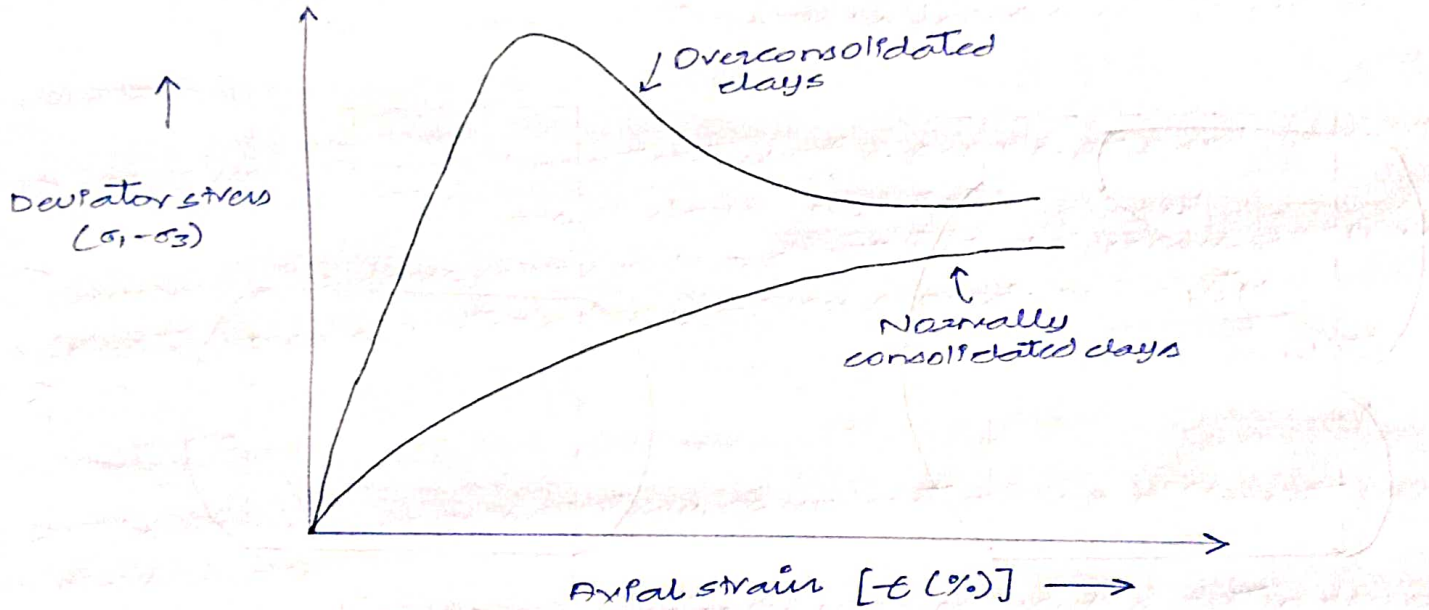
→ If stress strain curve for NCC & OCC clays from CD test are analysed, where both are considered under the same magnitude of cell pressure.

→ The OC clays shows a greater strength at comparatively lower strain than the NC clays & has a pronounced peak.

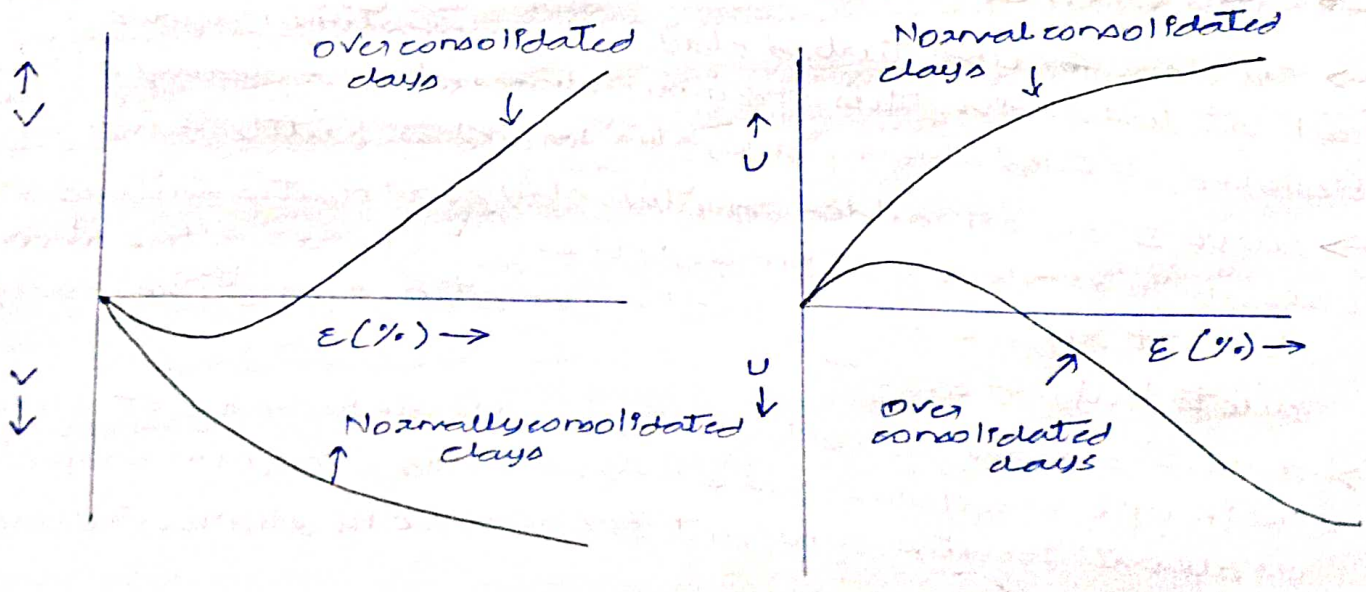
→ The stress falls off as the strain increases i.e. termed as work / strain softening.

NOTE: strain softening is deterioration of material strength / decrease in strength with increase in strain which is a phenomenon typically observed at a continuous level in damaged brittle materials RCC struct.

→ It is primarily due to brittleness and non-homogenity/heterogenity of material.



→ Volume & pore water pressure variation in clays:



→ The OC clays, after a small initial decrease in volume, shows a volume increase upon shear while the NC clays get compressed when sheared, as the pore water pressure increases during the CU test in NC clays increases & in OC clays after the initial increase becomes negative.

→ This process in increase in volume of OC clays under shear is termed as "DILATION".

→ Dilatation is the process in which solid materials has the tendency to show increase in volume when sheared.

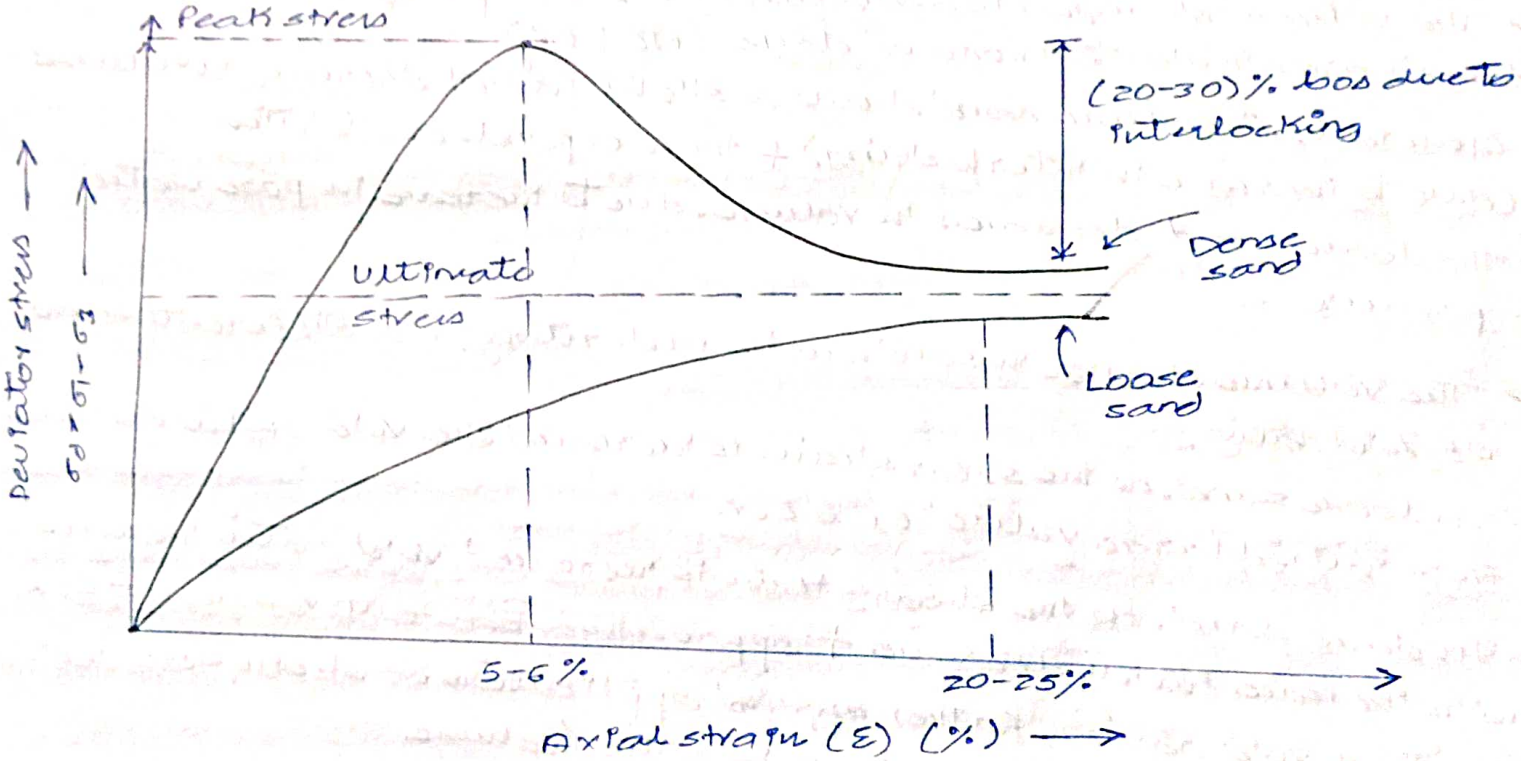
* Shear strength of sand:

→ In dense sand, the shear strength is due to interlocking & friction (sliding & rolling) both.

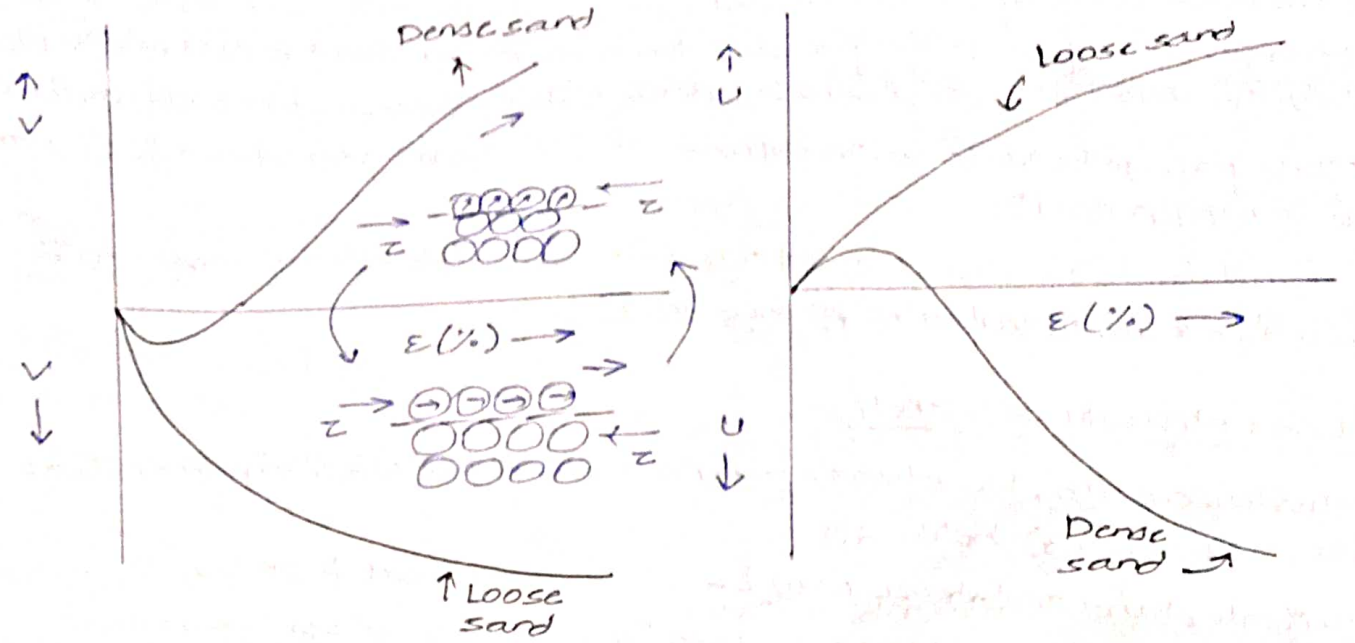
→ Interlocking resistance may be 20-30% of total & at 5-6% of strain interlocking and sudden decrease in shear strength is recorded.

→ In loose sand, the shear strength is due to friction resistance only.

→ In loose sand, volume continue to decrease & clear failure is not observed, hence when strain is reached 20-25% failure is assumed to take place.



→ The deviator str. soil is continued to be sheared after it attain peak stress, the stress reduces & becomes more or less constant termed as Ultimate stress.



NOTE: When loose sand is sheared by horizontal arrows, it tries to achieve the configuration of dense sand & when dense sand is sheared it tries to achieve the configuration as in loose sand (termed as dilation).

→ The volume change characteristics of a sample of sand (loose & dense) are similar to those of clays (NC & OC).

→ Generally, the dense sand shows a slight initial decrease in volume (due to increase in interlocking) & then expands or dilates.

→ The loose sand decreases in volume, due to increase in pore water pressure

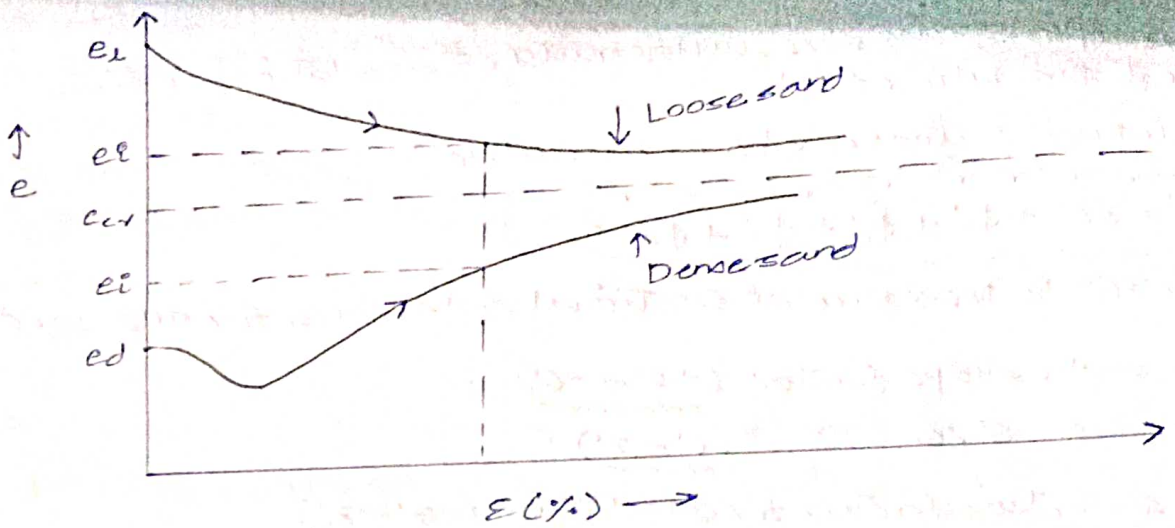
→ The volume change in sand can be understood in a different terms of void ratio.

→ In loose sand, as the shear strain is increased the void ratio decreases from initial large value e_i to e_{cr}.

→ In dense sand, as the shear strain is increased, void ratio increase with increase initial decrease & approaches a constant value of e_{cr} (critical void ratio), further applied application of strain after which do not produce any change in void ratio / volume.

→ Hence, $e_i < e_{cr}$ V↑

$e_i > e_{cr}$ V↓



NOTE: Loose saturated sand deposits may loose a part or all of their shear strength when subjected to sudden shock or other dynamic loading like pile driving, vibrations of machinery, explosive blasting earthquake shock (seismic loading) or rapid drawdown in dams.

→ There is a tendency for a rapid decrease in volume of loose sand under these sudden loading, since it cannot occur within such a short period of time, the soil mass becomes behaves like undrained system.

→ Hence, there is a sudden increase in pore water pressure & consequently an equal decrease in the effective stress.

→ If this decrease is such that the effective stress reduces to zero, the soil in the localised zone will be transferred / transformed into liquid-like mass with zero shear strength.

→ This process is termed as LIQUIFACTION, which is naturally found near river banks & sea shores.

→ Once the complete loss of strength takes place in a limited mass of soil, the stresses are then transferred to the nearby soil & liquifaction process may continue.

→ Factors affecting shearing resistance:

→ Shearing resistance of sands & gravels is due to sliding friction, rolling friction & by interlocking.

→ Interlocking effect is more significant for dense sands & is the chief contributing factor for the increased angle of shearing resistance.

→ However, the angle of shearing resistance corresponding to ultimate strength is not much diff. for the dense sand & loose sand.

→ The angle of shearing resistance decreases rapidly with increase in % of stresses.

→ A fine content of 20-25% may reduce the ϕ' value of sand by 5-7°.

→ The grain shape, grain size, uniformity of gradation, relative density affects the value of ϕ'.

→ Effect of all these factors can be expressed as

$$\phi' = 36° + \phi_1° + \phi_2° + \phi_3° + \phi_4°$$

Here $\phi' = 36°$ is the value of frictional resistance for avg. condn

ϕ_1 = grain shape factor (+1° – -6°)

ϕ_2 = " size " (0° – 2°)

ϕ_3 = correction factor for gradation (-3° – 3°)

ϕ_4 = " " " " relative density

NOTE: The influence of relative density is most important amongst the all

$$\phi' = 20-40°$$

Type of sand	Round grains, uniform grain size distribution	Angular grains, well graded distribution
Loose sand	28.5°	34°
Dense sand	35°	46°

→ Angle of Repose:

→ The maximum slope at which, material dropped down through a natural process, remains stable is called angle of repose.

→ This concept is limited only to dry, granular soils, which have more or less constant value of angle of repose, irrespective of ht of slope, but in case of cohesive soils, this concept has no relevance since its value depends upon height of slope.

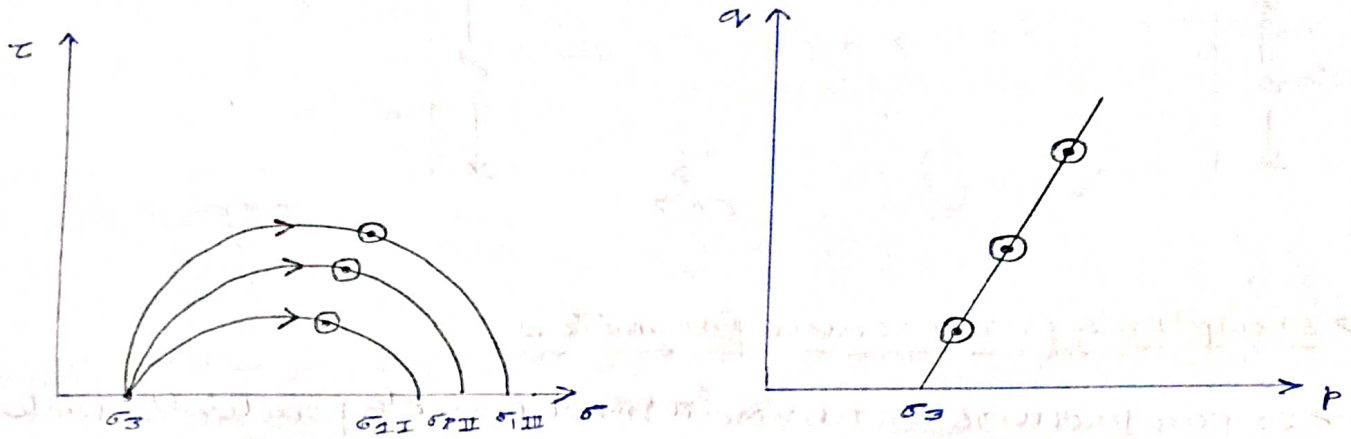
→ When sand or gravel is dumped, the material is in a loose state near its surface, hence the angle of repose is approx equal to angle of internal friction in its loose state.

→ Stress paths:

→ In some cases, the successive state of stress in a test specimen or in situ element of soil during loading or unloading are to be represented.

→ For this series of Mohr circle can be drawn but it is completely confusing & tedious hence to represent these series of stresses point, locus of these points can be shown in the forms of straight line or curve

→ This locus is called stress path.



Here, $p = \frac{\sigma_v + \sigma_h}{2}$ and $q = \frac{\sigma_v - \sigma_h}{2}$

Assuming $\sigma_v > \sigma_h$

→ Here stress path may also be defined as the locus of points of maximum shear stress.

→ Three possible stress paths can be drawn

i. Effective stress path (ESP)

ii. Total " " (TSP)

iii. stress path of total stress minus static pore water pressure. (TSSP)

NOTE: Mohr-Coulomb failure envelope can also be expressed in terms of stress path as follows

$$\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha$$

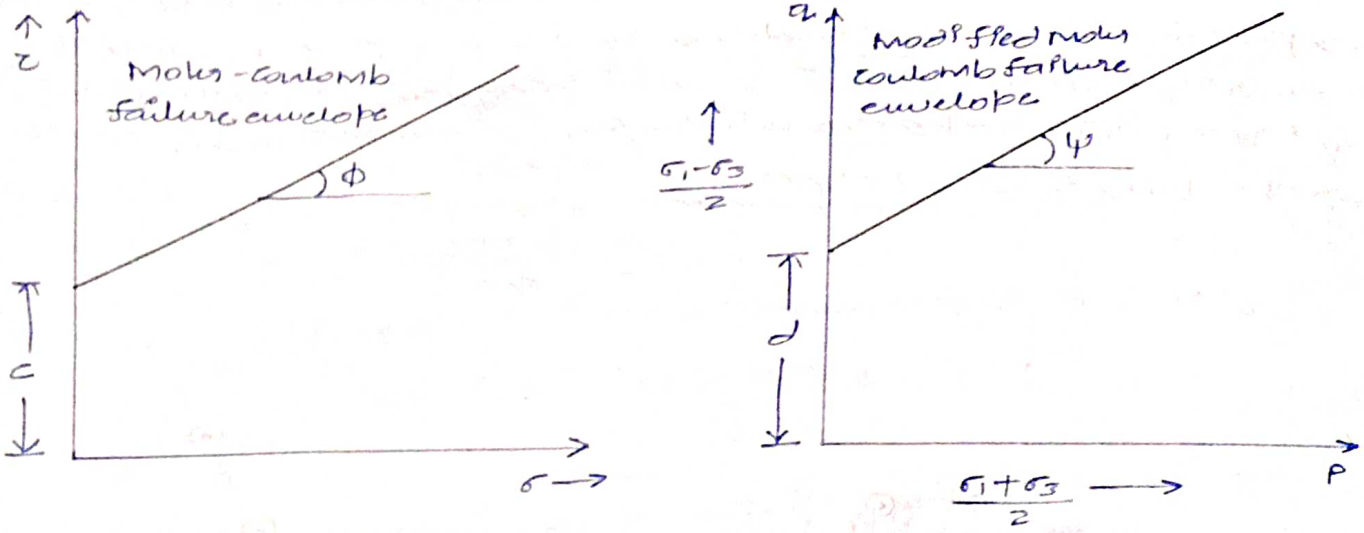
or $\sigma_1 - \sigma_3 = 2c\cos\phi + (\sigma_1 + \sigma_3)\sin\phi$ —— ①

→ The same relationship can also be written in terms of $\frac{\sigma_1 - \sigma_3}{2}$ i.e. q & $\frac{\sigma_1 + \sigma_3}{2}$ i.e. p

$$\frac{\sigma_1 - \sigma_3}{2} = d' + \frac{1}{2}(\sigma_1 + \sigma_3)\tan\psi$$ —— ②

$d = c\cos\phi$ → $c = \frac{d}{\cos\phi}$

$\sin\phi = \tan\psi$



→ Skempton's pore pressure parameters :

→ If pore pressure measurement is not possible practically while performing test, it can be expressed empirically in terms of pore pressure parameters.

→ Let us consider a soil mass or soil skeleton subjected to increase in three principal stress, $\Delta\sigma_1, \Delta\sigma_2, \Delta\sigma_3$ resulting in volume decrease of ΔV & a consequent increase in pore pressure ΔU

$$\Delta\sigma_1' = \Delta\sigma_1 - \Delta U, \quad \Delta\sigma_2' = \Delta\sigma_2 - \Delta U, \quad \Delta\sigma_3' = \Delta\sigma_3 - \Delta U$$

→ If $\epsilon_1, \epsilon_2, \epsilon_3$ denotes triaxial strain in 3 direction

$$\epsilon_1 = \frac{\Delta\sigma_1' - \mu(\Delta\sigma_2' + \Delta\sigma_3')}{E}$$

$$\epsilon_2 = \frac{\Delta\sigma_2' - \mu(\Delta\sigma_1' + \Delta\sigma_3')}{E}$$

$$\epsilon_3 = \frac{\Delta\sigma_3' - \mu(\Delta\sigma_2' + \Delta\sigma_1')}{E}$$

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = \frac{(1-2\mu)}{E}(\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3')$$

Here $\epsilon_1 + \epsilon_2 + \epsilon_3 = \epsilon_V = \frac{\Delta V}{V}$

$$\frac{\Delta V}{V} = \frac{(1-2\mu)}{E}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta U)$$

$$\frac{\Delta V}{V} = \frac{3(1-2\mu)}{E}\left\{\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta U\right\}$$

Here $\frac{3(1-2\mu)}{E} = C_c$ = compressibility of soil skeleton

* $C_c = \frac{3(1-2\mu)}{E}$

$$\frac{\Delta V}{V} = C_c \left\{ \frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta U \right\} \quad \text{——— ①}$$

→ If 'η' is porosity of soil, then volume of voids or volume of pore fluid,

$$V_w = V_v = \eta V$$

→ Let C_v is the coeff. of compressibility of pore fluid + decrease in volume of soil skeleton is almost entirely due to expulsion of water/ decrease in volume of soil.

Here, $C_v = \frac{\Delta V_w}{V_v \cdot \Delta U} = \frac{\Delta V_w}{\eta V \Delta U} = \frac{\Delta V}{V \cdot \eta \Delta U}$

$$\frac{\Delta V}{V} = \eta C_v \Delta U \quad \text{——— ②}$$

from ① + ②

$$\eta C_v \Delta U = C_c \left\{ \frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta U \right\}$$

→ In case of soil in triaxial test, $\Delta\sigma_2 = \Delta\sigma_3$

$$\eta C_v \Delta U = C_c \left\{ \frac{1}{3}(\Delta\sigma_1 + 2\Delta\sigma_3) - \Delta U \right\}$$

*** $$\Delta U = \frac{1}{1 + \eta \frac{C_v}{C_c}} \left[\Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3) \right]$$

$$\Delta U = B\left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\right] \quad \text{——— ①}$$

→ Here 'B' & 'A' are termed as Skempton's pore pressure parameters.

→ In undrained triaxial test, stress changes are usually made in 2 stage

a. cell pressure stage ($\Delta\sigma_3$)

b. Deviator stress stage ($\Delta\sigma_d = \Delta\sigma_1 - \Delta\sigma_3$)

Hence, total change in pore water pressure (ΔU) can be written in terms of change of pore pressure in 2 stages.

$$\Delta U = \Delta U_1 + \Delta U_2 \quad \text{——— ②}$$

from ① & ②

$$B\Delta\sigma_3 = \Delta U_1$$

** $$B = \frac{\Delta U_1}{\Delta\sigma_3}$$

$B A (\Delta\sigma_1 - \Delta\sigma_3) = \Delta u_2$

Let $BA = \bar{A}$

$$BA = \frac{\Delta u_2}{\Delta\sigma_1 - \Delta\sigma_3}$$

**

$$\bar{A} = \frac{\Delta u_2}{\Delta\sigma_1 - \Delta\sigma_3}$$

→ Factors affecting A & B

$$B = \frac{1}{1 + n\frac{C_v}{C_c}}$$

→ If soil is fully saturated, the compressibility of water C_v is much less than C_c i.e. compressibility of soil skeleton

Hence, $\frac{C_v}{C_c} \neq 0 \Rightarrow B = 1$

→ If however, soil is dry, $\frac{C_v}{C_c} \neq \infty \Rightarrow B = 0$

Hence $0 \leq B \leq 1$ for $0 \leq S \leq 1$

→ Here A depends upon strain, anisotropy, sample disturbance, over consolidation ratio.

→ The value of A at failure for different soil is as follows.

Type of soil	A_f
→ Very loose, fine saturated sand or saturated clay	2-3
→ Extra sensitive / quick soil	1.2-2.5
→ Normally consolidated	0.7-1.3
→ Lightly "	0.3-0.7
→ Heavily pre "	-0.5-0

a. Hydrostatic compression:

$\sigma_v = \sigma_h \quad (\sigma_1 = \sigma_3)$

$p = \frac{\sigma_v + \sigma_h}{2} = \frac{\sigma_1 + \sigma_3}{2}$

$q = \frac{\sigma_v - \sigma_h}{2} = \frac{\sigma_1 - \sigma_3}{2}$

$p_0 = \frac{\sigma_v + \sigma_h}{2} = \frac{2\sigma_v}{2} = \sigma_v$

$q_0 = \frac{\sigma_v - \sigma_h}{2} = 0$

} Initial conditions

$p_f = \frac{(\sigma_v + \Delta\sigma_v) + (\sigma_h + \Delta\sigma_h)}{2}$

$q_f = \frac{(\sigma_v + \Delta\sigma_v) - (\sigma_h + \Delta\sigma_h)}{2}$

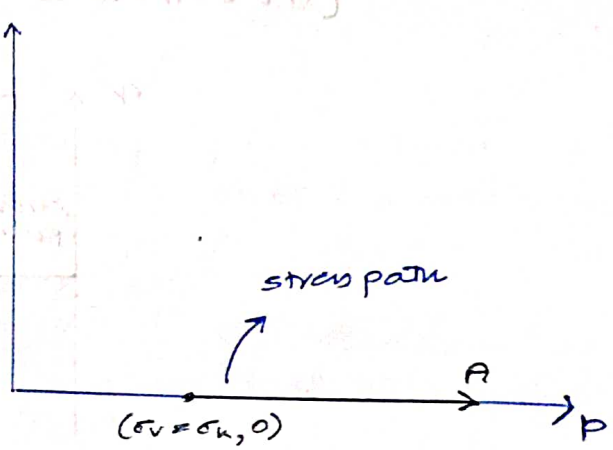
} final cond^n

Here $\Delta\sigma_h = \Delta\sigma_v$

$p_f = \sigma_v + \Delta\sigma_v = p_0 + \Delta\sigma_v$

$\Delta p = p_f - p_0 = \Delta\sigma_v$

$q_f = 0$



b. Non-hydrostatic pressure:

$\sigma_v \neq \sigma_h \neq 0$

$p_0 = \frac{\sigma_v + \sigma_h}{2}$, $q_0 = \frac{\sigma_v - \sigma_h}{2}$

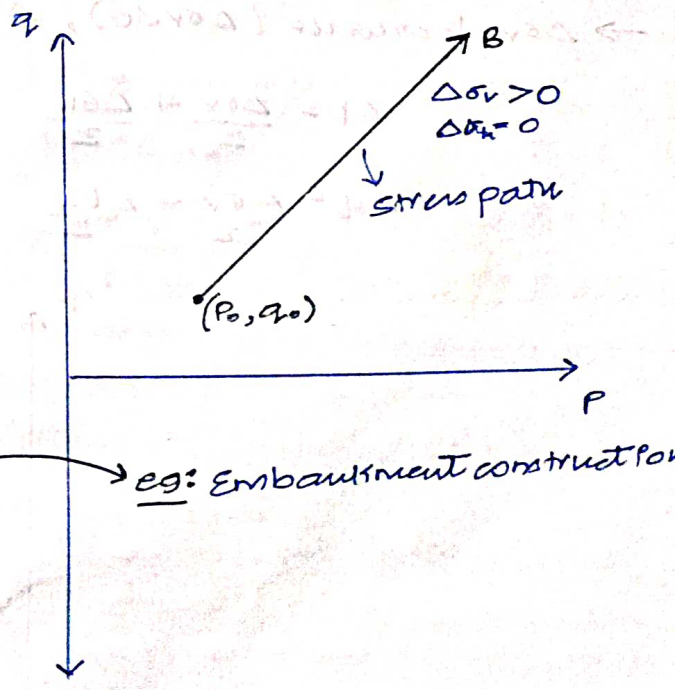
$p_f = \frac{(\sigma_v + \Delta\sigma_v) + (\sigma_h + \Delta\sigma_h)}{2}$

$q_f = \frac{(\sigma_v + \Delta\sigma_v) - (\sigma_h - \Delta\sigma_h)}{2}$

→ $\Delta\sigma_v$ is increasing $(\Delta\sigma_v > 0)$, $\Delta\sigma_h = 0$ → eg: Embankment construction

$p_f = p_0 + \frac{\Delta\sigma_v}{2} \Rightarrow \Delta p = p_f - p_0 = \frac{\Delta\sigma_v}{2}$

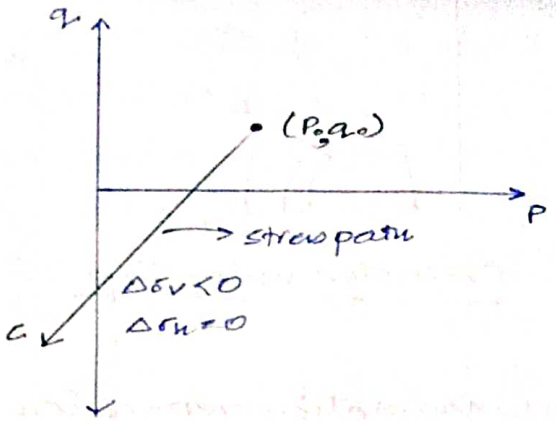
$q_f = q_0 + \frac{\Delta\sigma_v}{2} \Rightarrow \Delta q = q_f - q_0 = \frac{\Delta\sigma_v}{2}$



→ $\Delta\sigma_v$ decreases ($\Delta\sigma_v < 0$), $\Delta\sigma_h = 0$ → eg. Excavation in soil mass

$$\Delta p = \frac{\Delta\sigma_v}{2}$$

$$\Delta q = \frac{\Delta\sigma_v}{2}$$



→ $\Delta\sigma_v$ increases ($\Delta\sigma_v > 0$), $\Delta\sigma_h$ decreases ($\Delta\sigma_h < 0$)

$$p_f = \frac{(\sigma_v + \Delta\sigma_v) + (\sigma_h + \Delta\sigma_h)}{2}$$

$$p_f = p_0 + \frac{\Delta\sigma_v}{2} + \frac{\Delta\sigma_h}{2}$$

$$\Delta p = p_f - p_0 = \frac{\Delta\sigma_v}{2} + \frac{\Delta\sigma_h}{2}$$

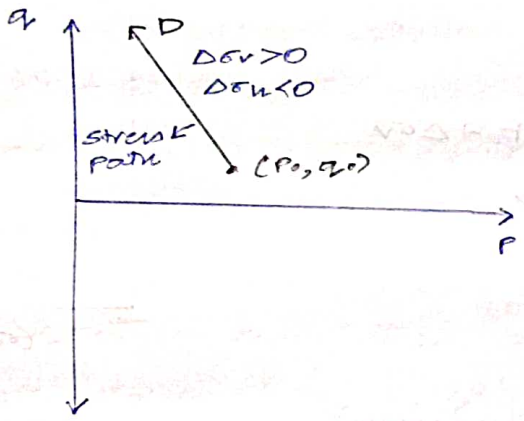
(let $\Delta\sigma_h > \Delta\sigma_v$)

$$q_f = \frac{(\sigma_v + \Delta\sigma_v) - (\sigma_h - \Delta\sigma_h)}{2}$$

$$q_f = q_0 + \frac{\Delta\sigma_v}{2} - \frac{\Delta\sigma_h}{2}$$

$$\Delta q = q_f - q_0 = \frac{\Delta\sigma_v}{2} - \frac{\Delta\sigma_h}{2}$$

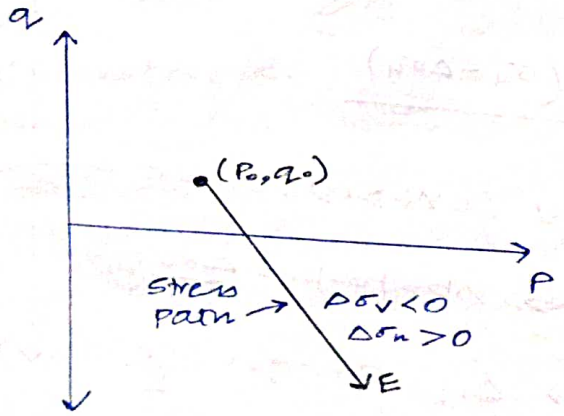
(let $\Delta\sigma_h > \Delta\sigma_v$)



→ $\Delta\sigma_v$ decreases ($\Delta\sigma_v < 0$), $\Delta\sigma_h > 0$ (increases)

$$\Delta p = \frac{\Delta\sigma_v}{2} + \frac{\Delta\sigma_h}{2}$$ (let $\Delta\sigma_h > \Delta\sigma_v$)

$$\Delta q = \frac{\Delta\sigma_v}{2} - \frac{\Delta\sigma_h}{2}$$ (let $\Delta\sigma_h > \Delta\sigma_v$)



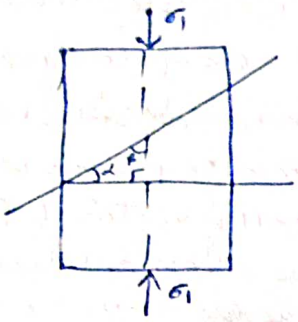
Q. A soil specimen having cohesion $c = 106\ KN/M^2$ + $\phi = 6°$ is tested in a unconfined compression test apparatus. The angle which the failure plane of the sample will make with the axis of the sample is

a. 48° b. 45° ✓ 42° d. 51°

$$\alpha = 45° + \frac{\phi}{2}$$

$$\alpha = 45° + \frac{6}{2} = 48°$$

$$\beta = 90° - \alpha = 90° - 48° = 42°$$



Q. The representative w_L & w_P values of a saturated normally consolidated clay deposits are 60% & 30% respectively. The sat. unit wt of the soil is 19KN/M3. Estimate the undrained shear strength of soil at a depth of 10M from ground surface. Water table level is at 8M below ground level.

a. $41\ KN/M^2$ b. $39\ KN/M^2$ c. $36\ KN/M^2$ d. $52\ KN/M^2$

$$\frac{C_u}{\sigma_3'} = 0.11 + 0.0037 I_P$$

$$I_P = w_L - w_P = 60 - 30 = 30\%$$

$$\sigma_3' = 8 \times 19 + 2(19 - 9.81) = 170.30\ KN/M^2$$

$$C_u = (0.11 + 0.0037 \times 30) \times 170.38 = 37.65\ KN/M^2$$

Q. Stress path eqn for triaxial test upon application of deviator stress is $q = 10\sqrt{3} + 0.5p$. The respective values of cohesion (c) KN/M^2 & angle of internal friction (ϕ) are

a. 20 & 20°
b. 30 & 30°
c. 30 & 20°
✓ d. 20 & 30°

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)_q = d + \left(\frac{\sigma_1 + \sigma_3}{2}\right)_p \tan\phi$$

$$c = \frac{d}{\cos\phi} = \frac{10\sqrt{3}}{\cos\phi} = \frac{10\sqrt{3}}{\cos\phi} = 20$$

$$\sin\phi = \tan\phi = 0.5$$

$$\phi = 30°$$

Q. A saturated clay sample was obtained from the field without allowing the water content & void ratio to change. The sample was subjected to the following stresses in the field $\sigma = 240 KN/M^2$, $u = 140 KN/M^2$, $\bar{\sigma} = 100 KN/M^2$

Compute

i. On sampling, what are the stresses acting on soil sample (σ, u, $\bar{\sigma}$)

a. 240, 140, 100

b. 0, 140, 100

c. 0, -100, +100

d. 100, 0, 100

For sample after being collected, $\sigma = 0$

since sampling is perfect, $\bar{\sigma} = 100 KN/m^2$

$\sigma = \bar{\sigma} + u \Rightarrow u = -100 KN/m^2$

ii. The sample was placed in a triaxial cell & was subjected to a cell pressure of $100 KN/m^2$ under undrained cond^n. What are the stresses acting on soil mass

a. 0, 0, 200

b. 100, 0, 100

c. 100, 100, 0

d. 0, 100, 0

Since cell pressure applied σ_3 is $= 100 KN/m^2$

$\sigma = 100 KN/m^2$, $u = -100 + 100 = 0 KN/m^2$ $\quad \bar{\sigma} = \sigma - u = 100 - 0 = 100 KN/m^2$

iii. The cell pr. was raised to $320 KN/m^2$ with the drainage valve closed. Find the value of stresses in soil

a. 220, 320, 100

b. 320, 220, 100

c. 100, 320, 220

d. 220, 100, 220

$\sigma = 320 KN/m^2$ $\quad u = -100 + 320 = 220 KN/m^2$

$\bar{\sigma} = \sigma - u = 320 - 220 = 100 KN/m^2$

iv. The drainage valve are opened now & soil is allowed to consolidate under the cell pressure. Compute the stresses in soil

a. 320, 0, 320

b. 320, 220, 100

c. 320, 100, 220

d. 320, 0, 0

Since drainage occurs from soil, the excess pore water pressure will dissipate out & stresses will be transferred over soil's

$\sigma = 320 KN/m^2$, $u = 0$, $\bar{\sigma} = 320 KN/m^2$

v. Comment upon the nature of change in w(%) & S(%) of soil sample after consolidation

a. w ↑, S < 1

b. w ↓, S > 1

c. w ↓, S < 1

d. w ↓, S = 1

Since water is expelled out during consolidation w(%) decreases, but soil remains saturated.

vi. The drainage valve is now closed & additional axial stress was applied with the cell pressure held constant whereas additional axial stress was $100 KN/m^2$, pore water pressure was $80 KN/m^2$. Compute the value of A at this stage of test

a. 1 b. 2 c. 0.8 d. −0.25

$\bar{A} = AB = \frac{\Delta U_2}{\Delta\sigma_1 - \Delta\sigma_3}$

$A = \frac{80}{100} = 0.8$

$\Delta U_2 = 80 kN/m^2$

$\Delta\sigma_3 = 0$

$\Delta\sigma_1 = \Delta\sigma_3 + \sigma$

$\Delta\sigma_1 = \Delta\sigma_2 = 100 kN/m^2$

If $S = 1$, $B = 1$

vii. When an additional axial stress of $200 kN/m^2$ had been applied, the sample failed at that stage, the value of 'A' was 1. Compute the value of $\bar{\sigma}_{1f}$ (kN/m^2)

a. 220 b. 120 c. 320 d. 0

Since drainage is not allowed in 2nd stage, the additional axial stress is borne by water only, hence there is no change in major principal eff. stress

$\bar{\sigma}_{1f} = 320 kN/m^2$

$\bar{A}_f = (AB)_f = \frac{\Delta U_{2f}}{(\Delta\sigma_1 - \Delta\sigma_3)_f}$

$A \times 1 = \frac{\Delta U_{2f}}{\Delta\sigma_f}$

$\Delta U_{2f} = 1 \times 1 \times 200 = 200 kN/m^2$

$\sigma_{1f} = \sigma_{3f} + \sigma_f = 320 + 200 = 520 kN/m^2$

$\bar{\sigma}_{1f} = \sigma_{1f} - U_f = 520 - 200 = 320 kN/m^2$

viii. What is value of ϕ' of soil if $c' = 0$

a. 20° b. 25° c. 30° d. 35°

$c' = 0$

$\bar{\sigma}_{1f} = \bar{\sigma}_{3f} \tan^2\alpha + 2c' \tan\alpha$

$320 = (\sigma_{3f} - U_f) \tan^2\alpha$

$320 = (320 - 200) \tan^2\alpha$

$\alpha = 59.51°$

$45° + \frac{\phi}{2} = 59.51° \Rightarrow \phi = 27°$

Q. An unconfined compression test was conducted on an undisturbed sample of clay. The sample had a dia of 37.5mm & was 80mm long. The load at failure was 28N & the axial deformation of the sample at failure was 13mm. Find the undrained shear strength of clay (kN/m^2)

$UCS = \sigma_1 = q_u = 2C = \frac{P}{A_f}$

$A_f = \frac{A_o}{1 - \frac{\Delta L}{L}} = \frac{\pi D_o^2}{4\left(1 - \frac{\Delta L}{L}\right)} = 1318.7 mm^2$

$UCS = \frac{25}{1318.7} = 0.0212\ N/mm^2$

$UCS = 21.23\ kN/m^2$

$\tau_f = c_u + \sigma \tan\phi_u$

$\phi_u = 0$

$\tau_f = c_u = \frac{UCS}{2} = \frac{21.23}{2} = 10.61\ kN/m^2$

Q. In a CD triaxial test, a specimen of clay fails at a cell pressure of 60 kN/m². The effective shear parameters are found $c' = 15\ kN/m^2$, $\phi' = 20°$. Determine the compressive strength of clay

$\sigma_3'_f = 60\ kN/m^2$

$\sigma_1'_f = \sigma_3'_f \tan^2\alpha + 2c' \tan\alpha$

$\sigma_1'_f = 60 \tan^2(45+10) + 2 \times 15 \tan(45+10)$

$\sigma_1'_f = 165.22\ kN/m^2$

$UCS = \sigma_{1f} = \sigma_1'_f - \sigma_3'_f = 165.22 - 60 = 105.22\ kN/m^2$

Q. A CD test was conducted on a saturated clay which gave following data. Cell pressure is 250 kN/m² & deviator stress at failure is 275 kN/m², then determine

i. Effective angle of friction

ii. Angle of failure plane with major principal plane

iii. Normal & shear stress on failure plane

iv. Normal stress on the plane of max. shear stress

i. $\sigma_{3f} = 250\ kN/m^2$ $\quad \sigma_{2f} = 275\ kN/m^2$

$\sigma_{1f} = 275 + 250 = 525\ kN/m^2$

For saturated clay, $c = 0$

$\sigma_{1f} = \sigma_{3f} \tan^2\alpha + 2\cancel{c}^{0}\sqrt{\sigma_{3f}} \tan\alpha$

$525 = 250 \tan^2\alpha$

ii. $\alpha = 45° + \frac{\phi}{2} = 55.39 \Rightarrow \phi = 20.75°$

iii. $\sigma_{ff} = \left(\frac{\sigma_1+\sigma_3}{2}\right)_f + \left(\frac{\sigma_1-\sigma_3}{2}\right)_f \cos 2\alpha$

$\tau_{ff} = \left(\frac{\sigma_1-\sigma_3}{2}\right)_f \sin 2\alpha$

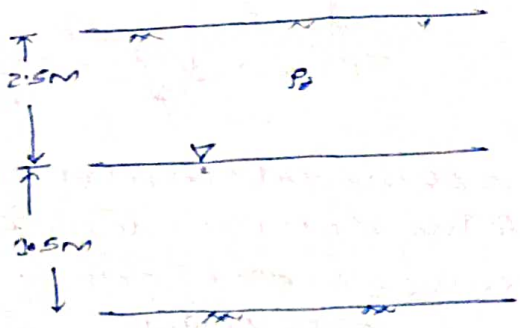
$\sigma_{ff} = 338.71\ kN/m^2$ $\quad \tau_{ff} = 128.55\ kN/m^2$

iv. Normal stress on the plane of max. shear stress,

$\sigma = \left(\frac{\sigma_1+\sigma_3}{2}\right)_f = \frac{525+250}{2} = 387.5\ kN/m^2$

Q. Lab results on a soil show that unconfined compressive strength is 2.5 Kg/cm². In another triaxial test, the same soil under chamber pressure of 0.4 Kg/cm² fails at deviator stress of 2.6 Kg/cm² under drained cond^n. Find the

Shear strength of some soil sample in the field at 4m from the ground level on a horizontal plane. The ground water table is 2.5m from the ground level. Considering the soil to be dry above the ground water table having dry density of 1.7gm/cc & G=2.7



$\tau = c' + \sigma' \tan\phi'$

$UCS = 1.5 \text{ Kg/cm}^2$

$\sigma_3 = 0.4 \text{ Kg/cm}^2$

$\sigma_2 = 1.6 \text{ Kg/cm}^2$

$\sigma_1 = \sigma_2 + \sigma_3 = 0.215 \text{ g/cm}^2$

$UCS = \sigma_1 f = 2c \tan\alpha \qquad \sigma_3 f = 0$

$1.5 = 2c \tan(45° + \phi/2)$ ——— (1)

In triaxial test,

$\sigma_1 f = \sigma_3 \tan^2\alpha + 2c \tan\alpha$

$0.2 = 0.4 \tan^2(45° + \phi/2) + 1.5$

$\phi' = 6.38°$

$1.5 = 2c \tan\left(45° + \frac{6.38}{2}\right) \Rightarrow c = 0.671 \text{ Kg/cm}^2$

$\rho_d = 1.7 \text{ gm/cm}^3$

$\rho_d = \frac{G\rho_w}{1+e} \Rightarrow 1.7 = \frac{2.7 \times 1}{1+e} \Rightarrow e = 0.588$

$\rho' = \frac{(G-1)\rho_w}{1+e} = \frac{(2.7-1)\times 1}{1+0.588} = 1.07 \text{ gm/cc}$

$\sigma' = \rho_d \times 2.5 + \rho' \times 1.5$

$\sigma' = (1.7 \times 2.5 + 1.07 \times 1.5)\, 10^2 = 585.5 \text{ gm/cm}^2$

$\tau = 0.671 + 585.5 \times 10^{-3} \tan(6.38°)$

$\tau = 0.735 \text{ Kg/cm}^2 \Rightarrow 72.241 \text{ KN/m}^2$