
INDIAN RAILWAYS
भारतीय रेल
RRB JE CBT 2
Marathon Class
Complete Strength of Materials
Special Class
wifistudy

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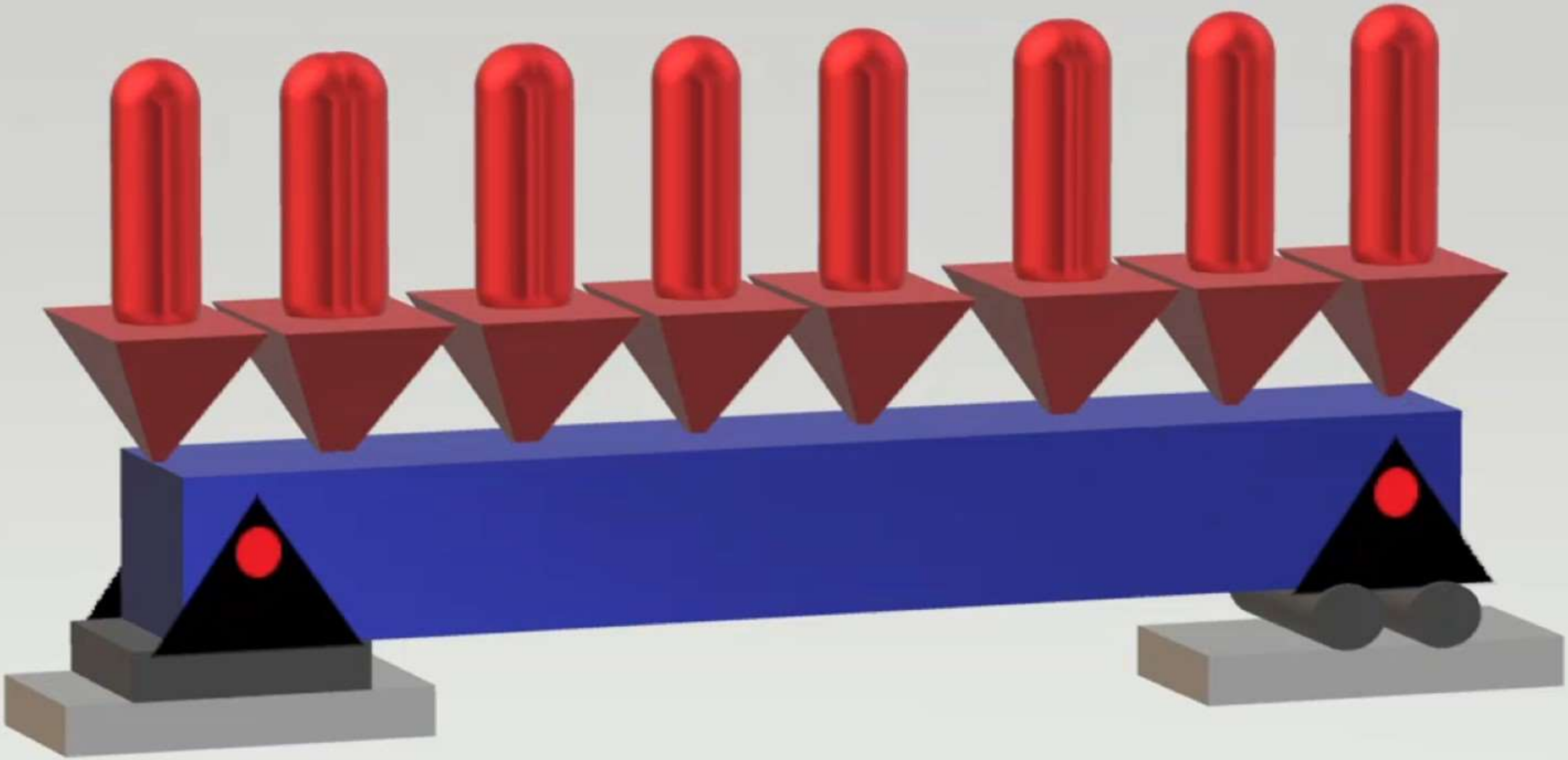
Civil Engineering by Sandeep Jyani

Happy Learning !!



Strength of Materials

or

Mechanics of Solids

Mechanics (Branch of Science which deals with study of Forces and their Effects on bodies)

1. Engineering Mechanics

- Branch of Science that deals with study of forces and their effects on **RIGID BODIES**
- Displacement, Velocity, Acceleration, etc

2. Strength of Materials

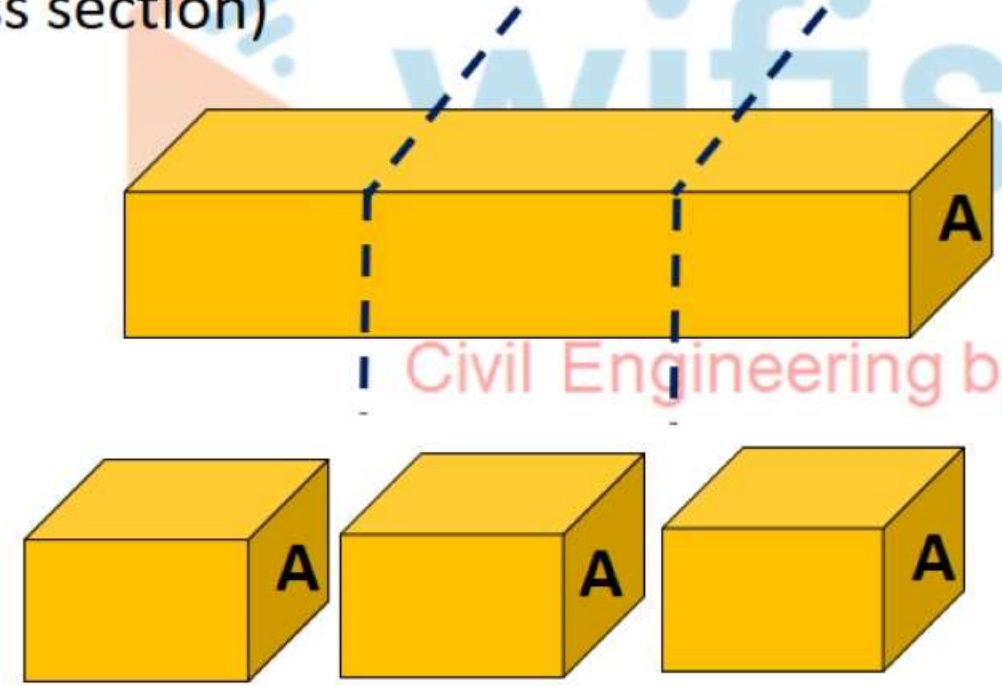
- Branch of Science that deals with study of forces and their effects on **DEFORMABLE BODIES**
- Stress, Strain, etc

3. Fluid Mechanics

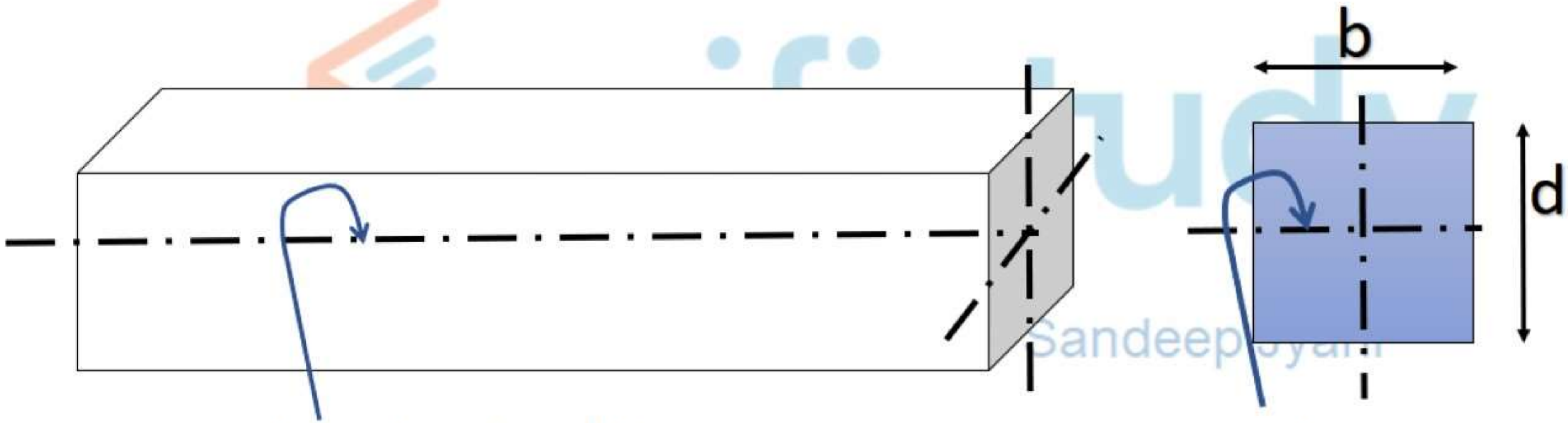
- Branch of science that deals with study of forces and their effects on **Fluids**
- Flow, continuous deformation, viscosity, etc

Some Basic Terms

- **Prismatic Bar** – Area throughout the length should be constant(same cross section)



Some Basic Terms



Longitudinal Line
Or
Centroidal Line or Axial Line

Longitudinal Axis

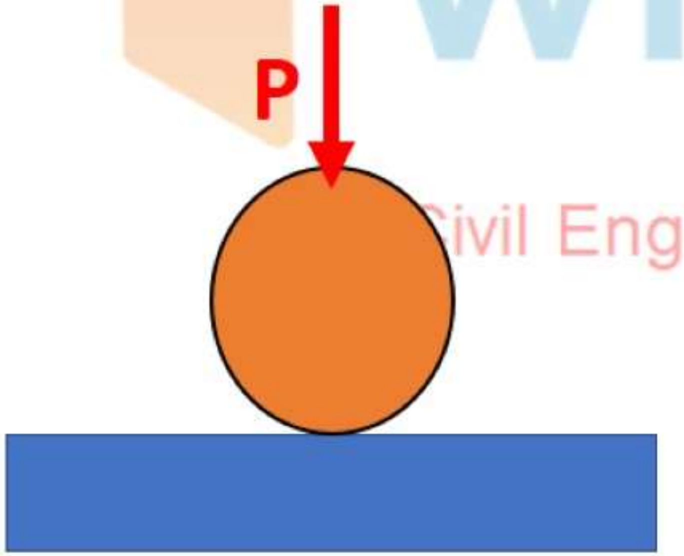
Mechanical Properties

- While we use any material, it is subjected to the action of external forces, which create stresses that cause deformation
- To keep these stresses, and deformation within permissible limits it is necessary to select suitable materials for the Components of various designs and knowing their properties such as strength, ductility, toughness etc.
- For this reason the specification of metals, used in the manufacture of various products and structure, are based on the results of mechanical tests or we say that the mechanical tests conducted on the specially prepared specimens (test pieces) of standard form and size on special machines to obtained the strength, ductility and toughness characteristics of the metal.

Hardness

A material's ability to withstand friction, essentially abrasion resistance, is known as hardness

- Hardness is the resistance of a metal to the penetration of another harder body which does not receive a permanent set.
- **Ball indentation Tests:**



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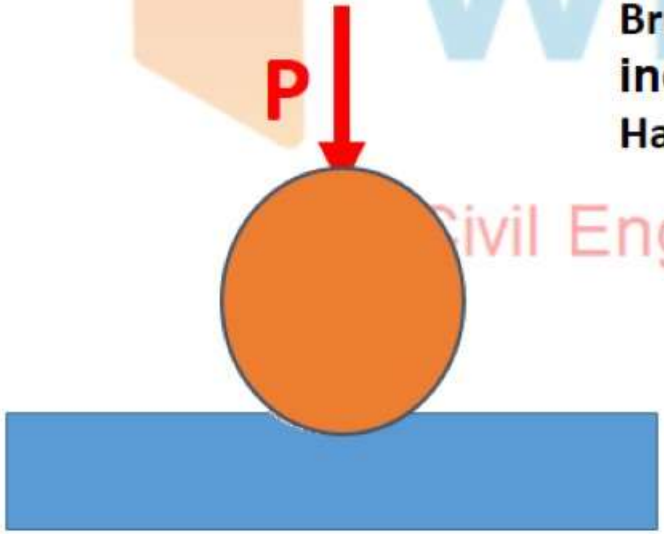


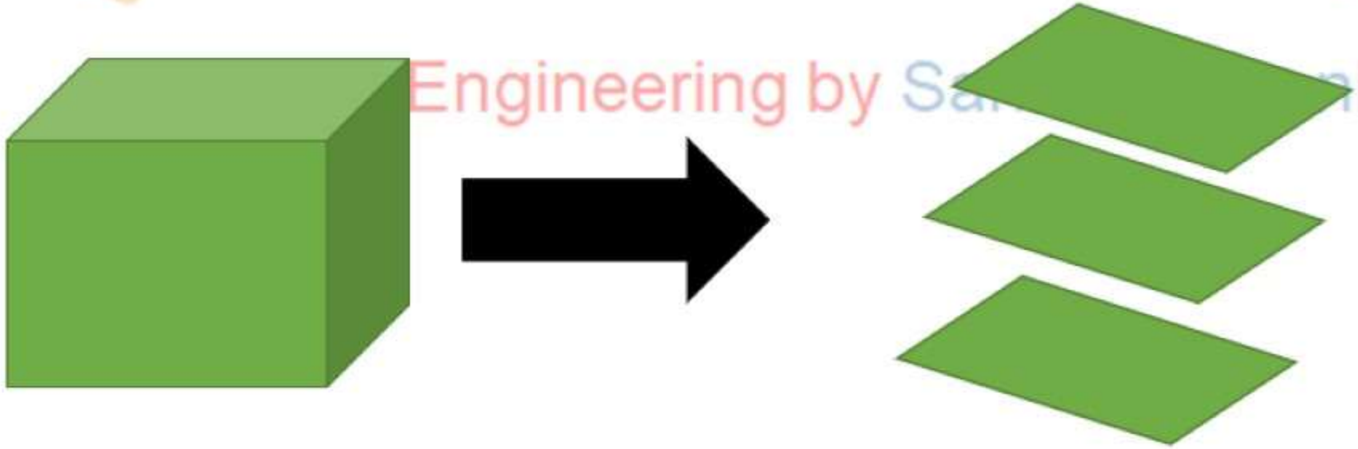
Brunel Hardness test

indentation remains on the surface of the test specimen

Hard steel ball is used

Malleability

- Property of Material due to which it can be Spread or sheets can be formed
- A material can be malleable but not ductile (exp. lead)

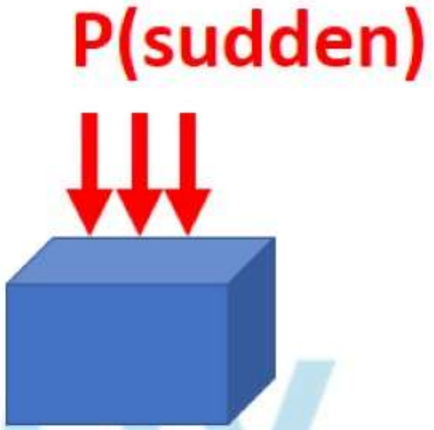


Elasticity and Elastic Limit

- **When an external force acts on a body and the body tends to undergo some deformation. If the external force is removed, and the body comes back to its original shape and size, the body is known as Elastic Body**
- **The maximum value of stress at which the body's deformation disappears on removal of force is called as Elastic Limit**

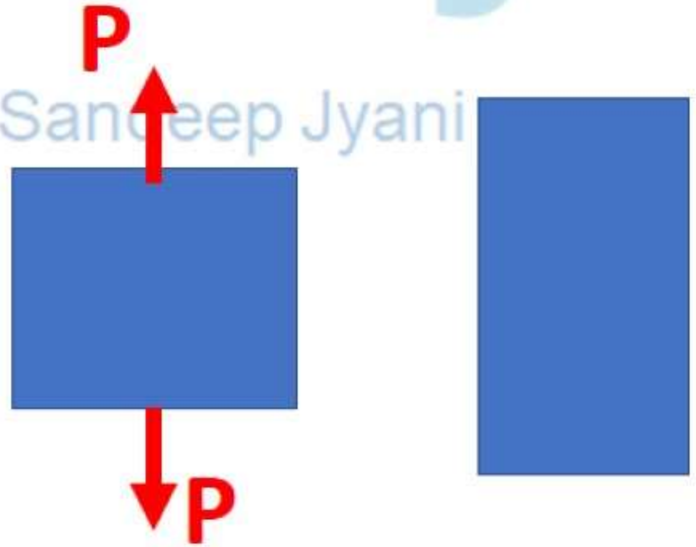
Toughness

- **Toughness is resistance to sudden loading or to absorb mechanical energy upto fracture**

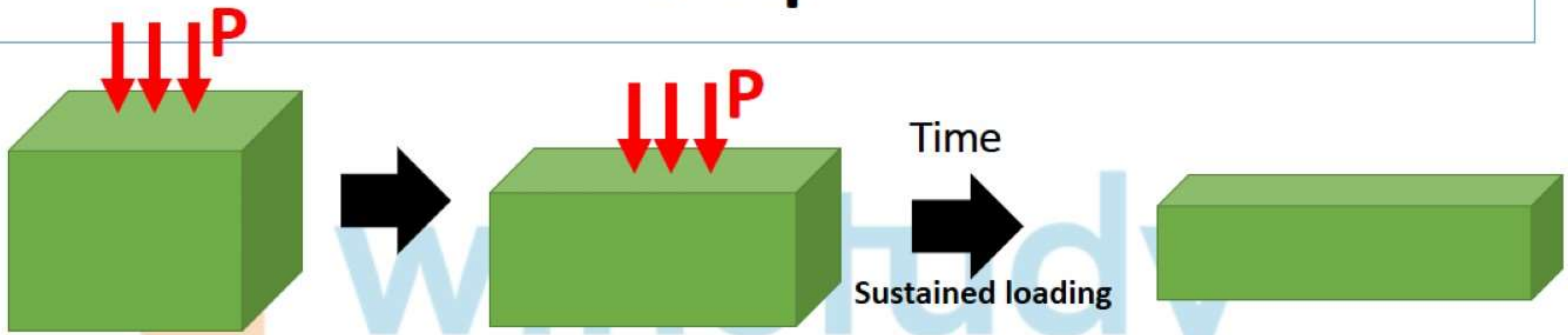


Plasticity

- **The property of material due to which it undergoes inelastic strain beyond elastic limit is called as Plasticity**

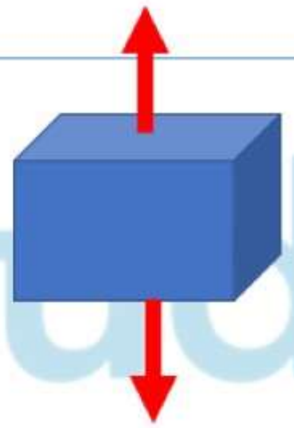


Creep



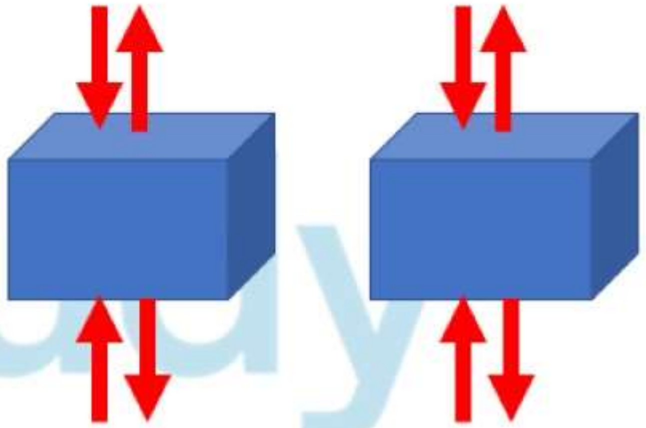
- It is property of material due to which it undergoes additional deformation(elastic strain) with passage of time under sustained loading is called CREEP
- Creep occurs due to dead load and is important when temperature is high or stress is high

Fatigue



Fatigue

- **Deterioration of material under repeated cycles of load resulting in Progressive cracking ultimately leading to Fracture is called Fatigue**

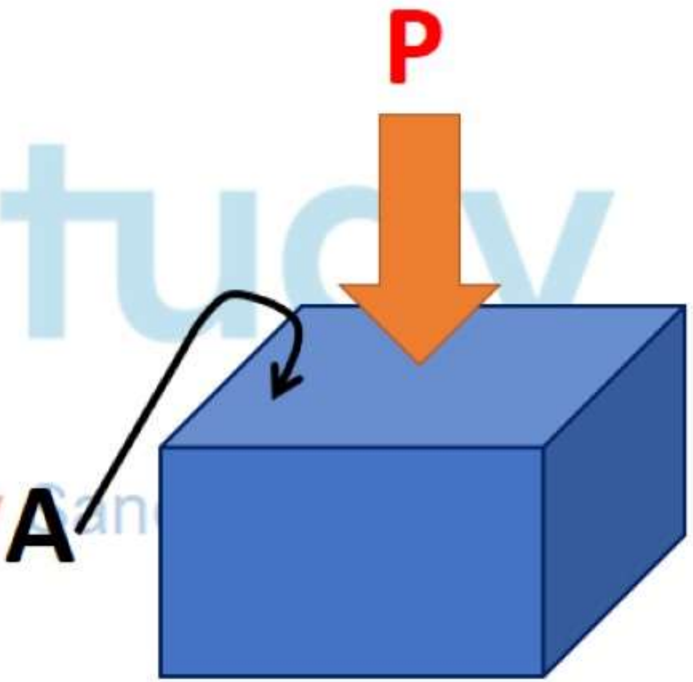


Resilience

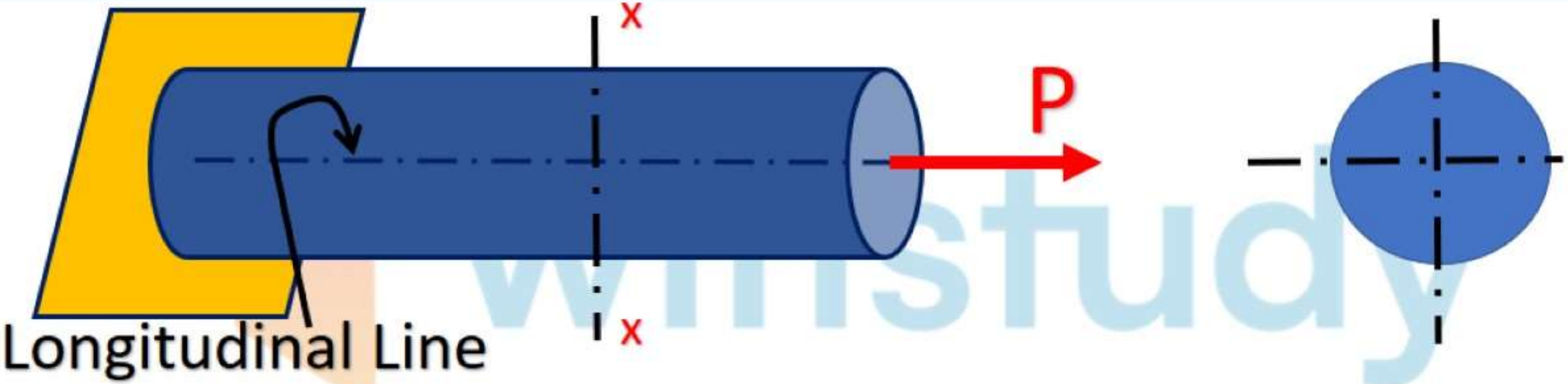
- **The property by virtue of which material can absorb energy when deform Elastically is called "Resilience"**

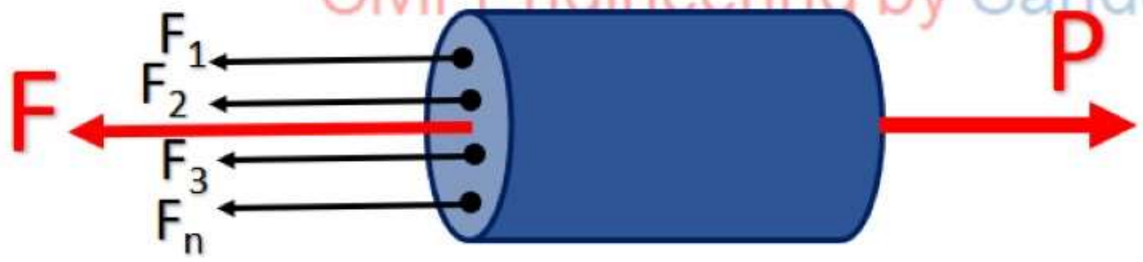
Stress

Stress is defined as **Internal Resisting force** produced at a **point** against the deformation due to External Force



Analysis of Stress





F_1 = IRF at point 1

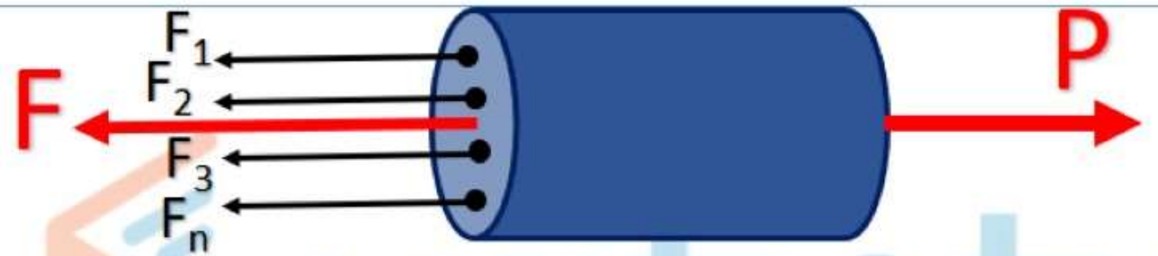
F_2 = IRF at point 2

F_3 = IRF at point 3

F_n = IRF at n^{th} point

F = Total IRF at a section

Analysis of Stress



F_1 = IRF at point 1

F_2 = IRF at point 2

F_3 = IRF at point 3

F_n = IRF at n^{th} point

F = Total IRF at a section

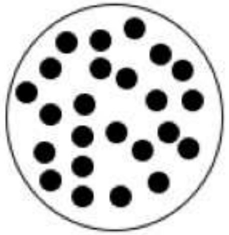
$$F = F_1 + F_2 + F_3 + \ldots + F_n$$

And if $F_1 = F_2 = F_3 = \ldots = F_n$

$$F = F_1 + F_1 + F_1 + \ldots + F_1$$

$$F = nF_1$$

$$F_1 = \frac{F}{n}$$



If all points meet, n=Area=A

$$\sigma_{avg} = F_1 = \frac{F}{A}$$

$$\sigma\ avg = \frac{Total\ IRF}{Area}$$

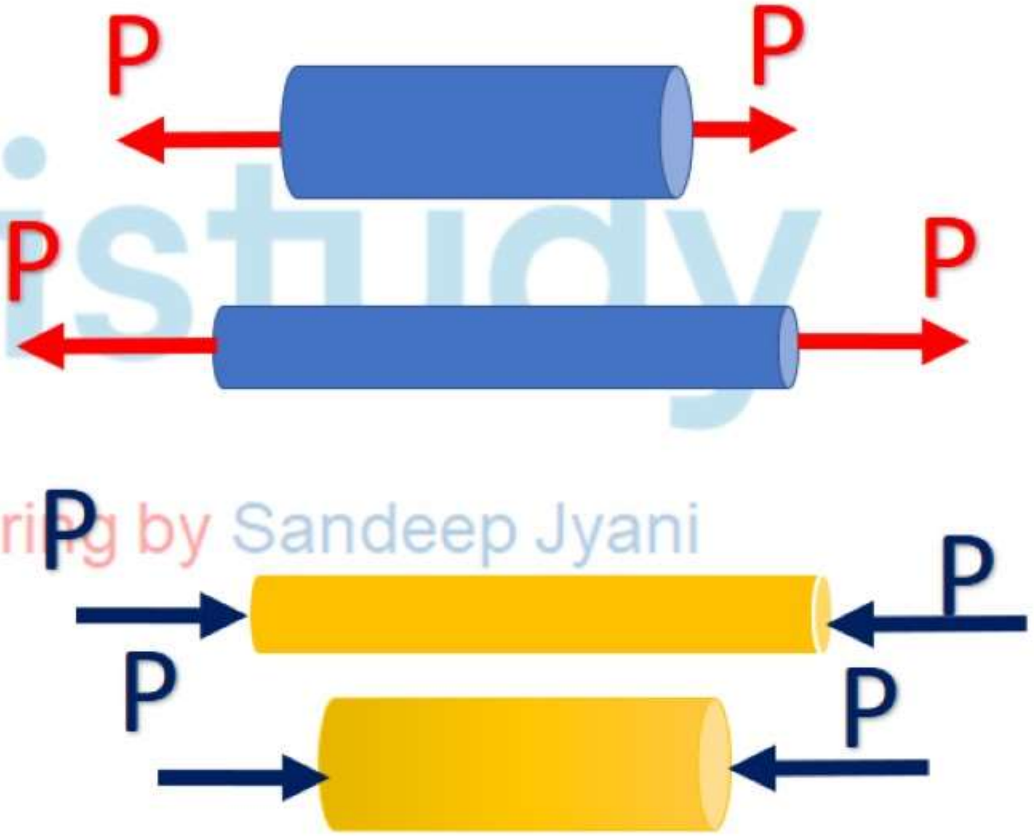
Unit Of Stress

$$\boldsymbol{\sigma}\ avg = \frac{Total\ IRF}{Area}$$

- $Pa = \frac{N}{m^2}$
- $MPa = \frac{10^6 N}{m^2} = \frac{10^6 N}{10^6\ mm^2} = \frac{N}{mm^2}$
- GPa
- $1\ kgF/cm^2$

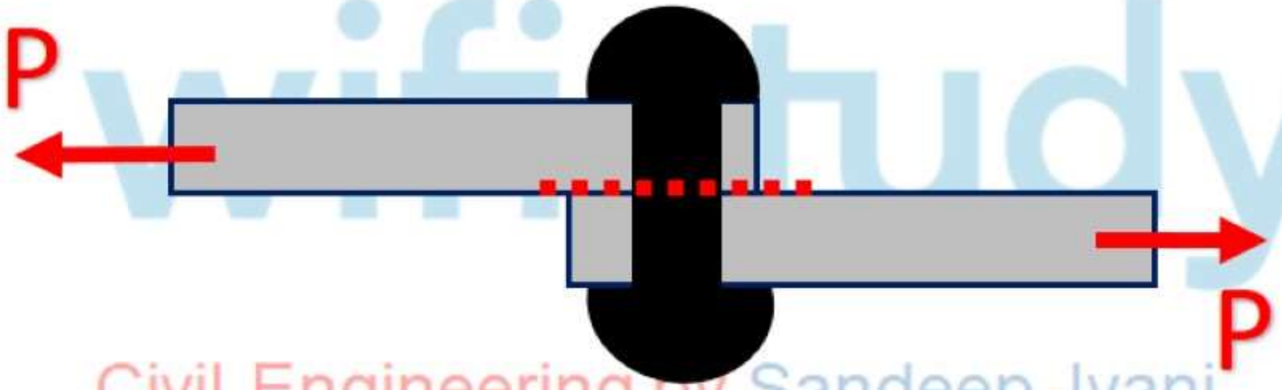
Type of Stress

1. **Tensile Stress**
   - Two equal and opposite pulls are applied as a result of which length is increased
2. **Compressive Stress**
   - Two equal and opposite pushes are applied as a result of which length is decreased



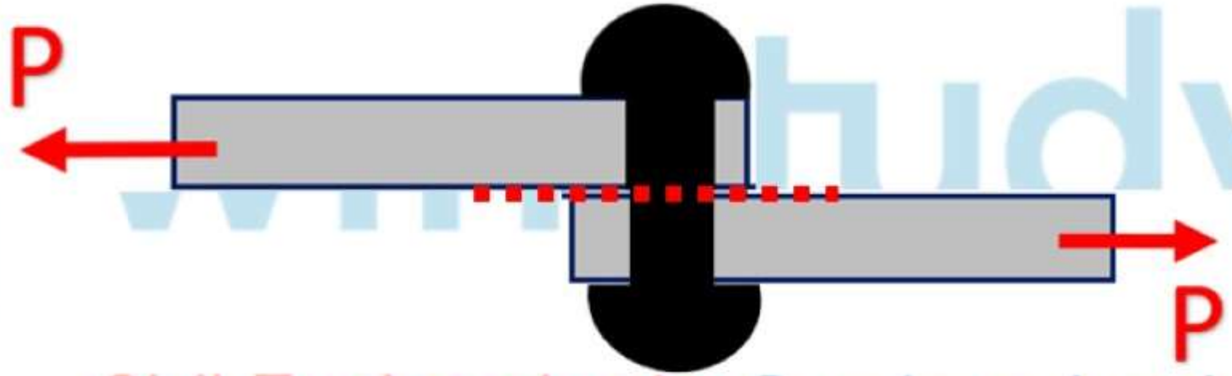
Type of Stress

3. Shear Stress



Type of Stress

3. Shear Stress



- Two forces, equal and opposite in nature, when act tangential to the resisting section, as a result of which the body shear off across the section is known as **Shear Stress.**

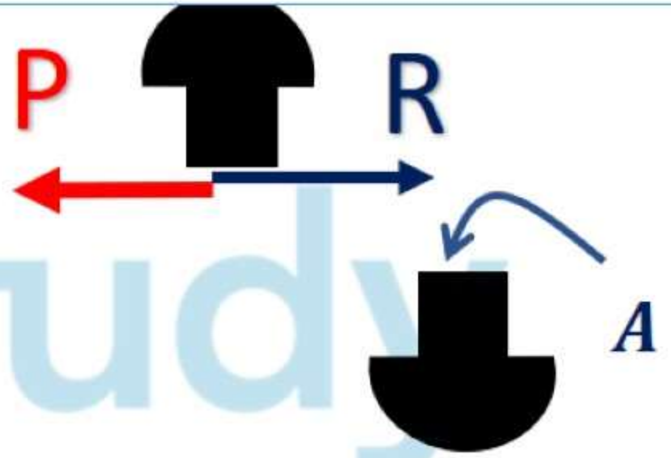
Type of Stress

3. Shear Stress

$$\text{Shear Stress} = \frac{\text{Shear resistance}}{\text{Shear Area}}$$

$$= \frac{R}{A}$$

$$= \frac{P}{A}$$



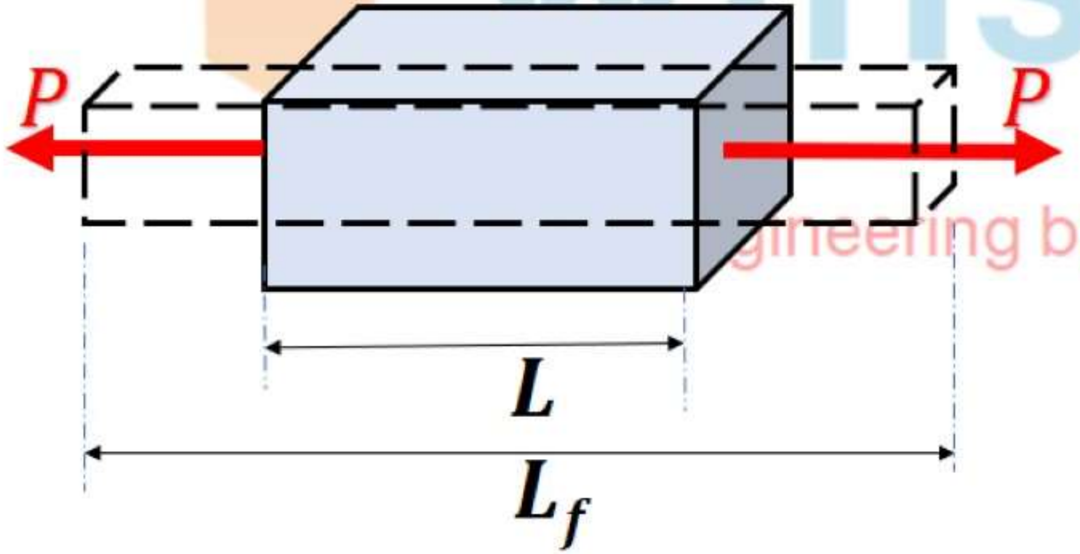
Stress vs Pressure

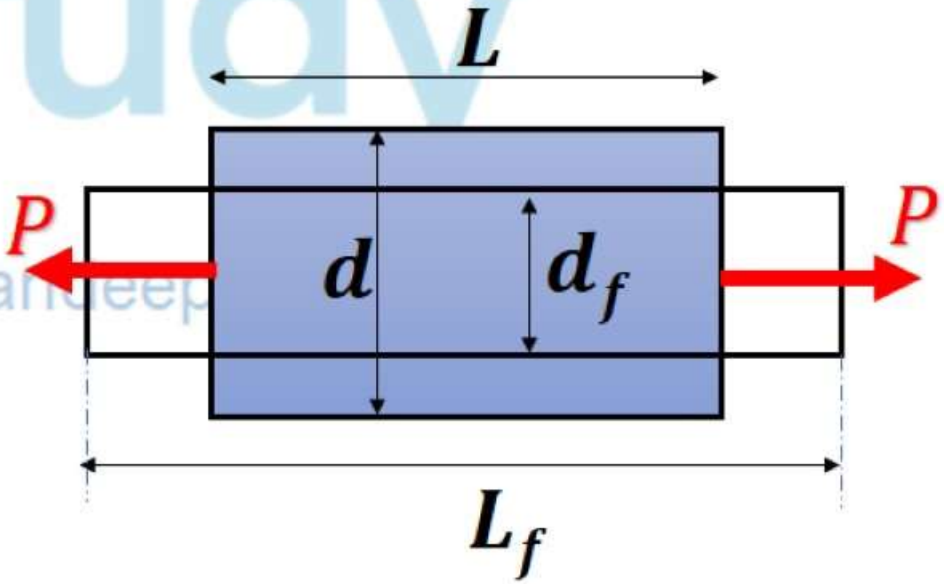
1. Pressure is an external quantity
2. Pressure is scalar quantity
3. Pressure can be measured
4. Due to pressure, stress can be produced in the body
5. Pressure force is always normal to the surface

1. Stress is an internal quantity
2. Stress is a TENSOR quantity
3. Stress is not a measurable quantity
4. Due to stress, pressure can not be created
5. Stress may be parallel or perpendicular to the cross section

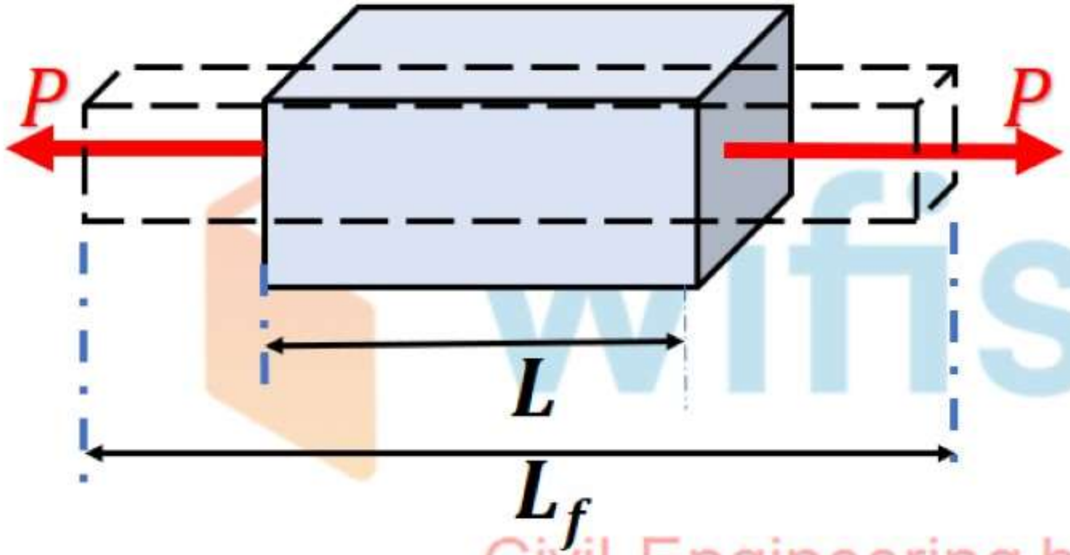
Strain

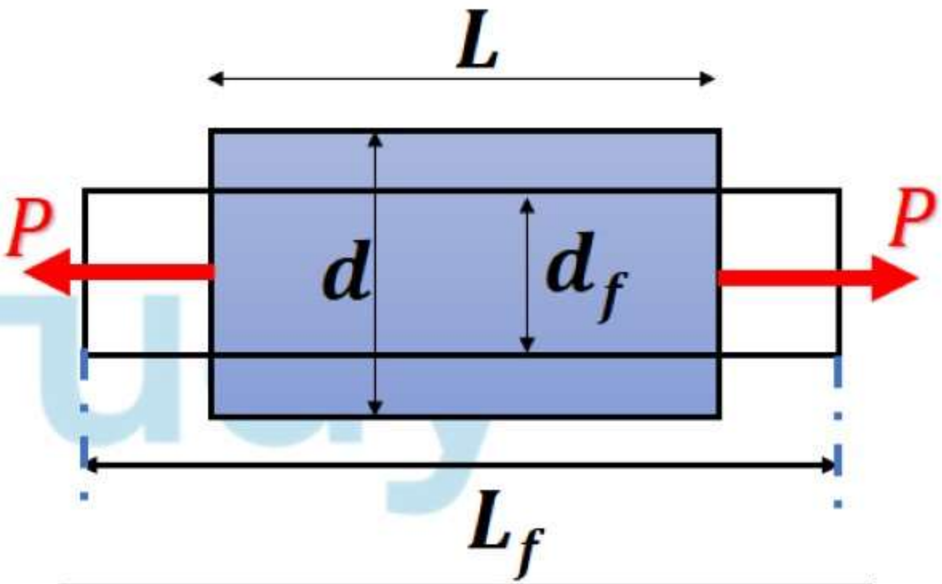
A body is said to be strained when relative position between particles will change under the application of external force.





Strain





- The change in relative position between particles is called Deformation
- Strain is defined as ratio of Change in Dimension to Original Dimension

$$\varepsilon = \frac{\text{change in dimension}}{\text{Original Dimension}}$$

$$\varepsilon = \frac{L_f - L}{L}$$

$$\varepsilon = \frac{\Delta L}{L}$$

Strain

$$\boldsymbol{\varepsilon} = \frac{\boldsymbol{change\ in\ dimension}}{\boldsymbol{Original\ Dimension}}$$

$$\boldsymbol{\varepsilon} = \frac{\boldsymbol{L_f} - \boldsymbol{L}}{\boldsymbol{L}}$$

$$\boldsymbol{\varepsilon} = \frac{\Delta \boldsymbol{L}}{\boldsymbol{L}}$$

- If $\boldsymbol{L_f} > \boldsymbol{L}$,

  $\Delta \boldsymbol{L}$= +ve

  $\boldsymbol{\varepsilon}$=+ve

  Tensile Strain

- If $\boldsymbol{L_f} < \boldsymbol{L}$,

  $\Delta \boldsymbol{L}$ = -ve

  $\boldsymbol{\varepsilon}$ = -ve

  Compressive Strain

Elasticity and Elastic Limit

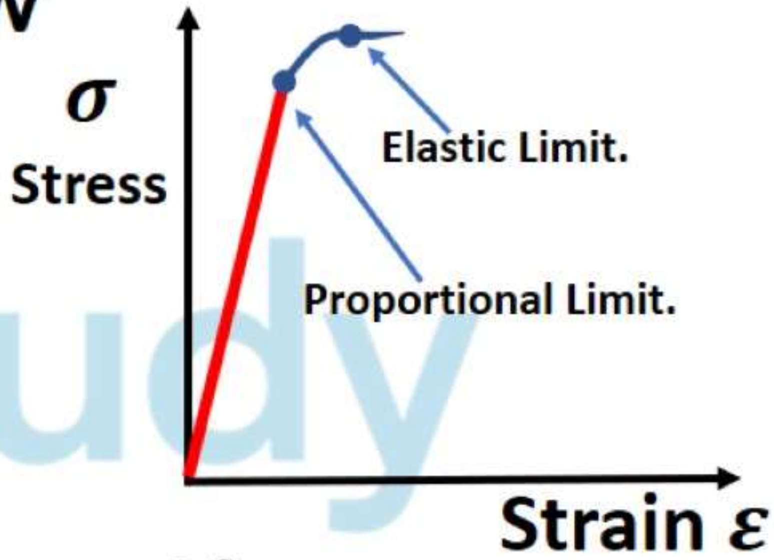
- **When an external force acts on a body and the body tends to undergo some deformation. If the external force is removed, and the body comes back to its original shape and size, the body is known as Elastic Body**
- **The maximum value of stress at which the body's deformation disappears on removal of force is called as Elastic Limit**

Hooke's Law

- When a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress upto **Proportional Limit.**

Or

- Ratio of the Stress to the corresponding strain is constant upto **Proportional Limit.**
- ***E= Young's Modulus of Elasticity or Modulus of Elasticity***



$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

$$\boxed{E = \frac{\sigma}{\epsilon}}$$

Que. 1 Every material obeys Hooke's law within

a) Elastic Limit

b) Plastic limit

c) Limit of Proportionality

d) None of the above

Que. 1 Every material obeys Hooke's law within

a) Elastic Limit

b) Plastic limit

c) <u>Limit of Proportionality</u>

d) None of the above

Que. 2 The ratio between stress and strain is called as

a) Modulus of elasticity

b) Modulus of rigidity

c) Bulk modulus

d) None of the above

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a) Modulus of elasticity

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d) None of the above

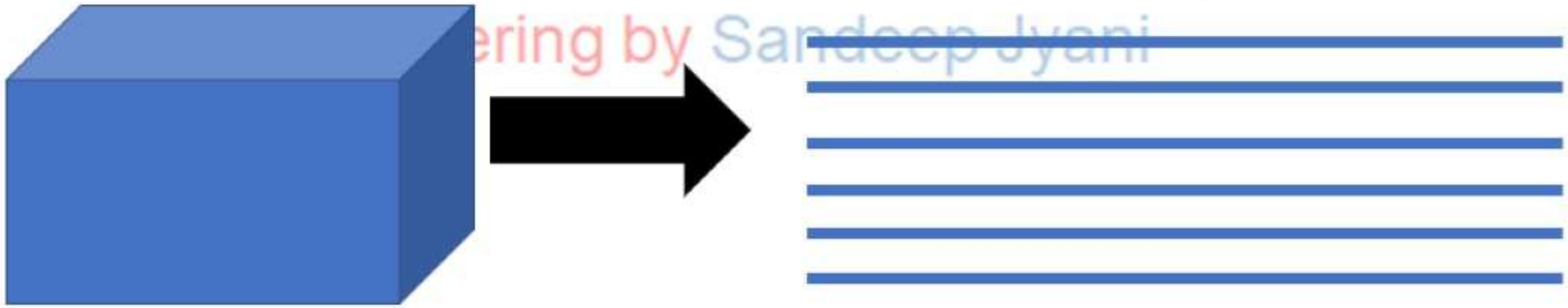
Que. 3 The % elongation of test piece under tension indicates its

a) Brittleness
b) Malleability
c) Stiffness
d) Ductility

Que. 3 The % elongation of test piece under tension indicates its

a) Brittleness
b) Malleability
c) Stiffness
d) **<u>Ductility</u>**

Ductility



Que. 4 A linear force deformation relation is obtained in materials

a) Having elastic stress strain property
b) Having plastic stress-strain properties
c) **Following Hooke's law**
d) Which are rigid elastic materials

Que. 5 When a load of 1960N is raised at the end of steel wire. The minimum dia of the wire so that stress in the wire does not exceed 100 N/mm^2

a) 4mm

b) 4.5mm

c) 5mm

d) 5.5mm

Que. 5 When a load of 1960N is raised at the end of steel wire. The minimum dia of the wire so that stress in the wire does not exceed 100 N/mm^2

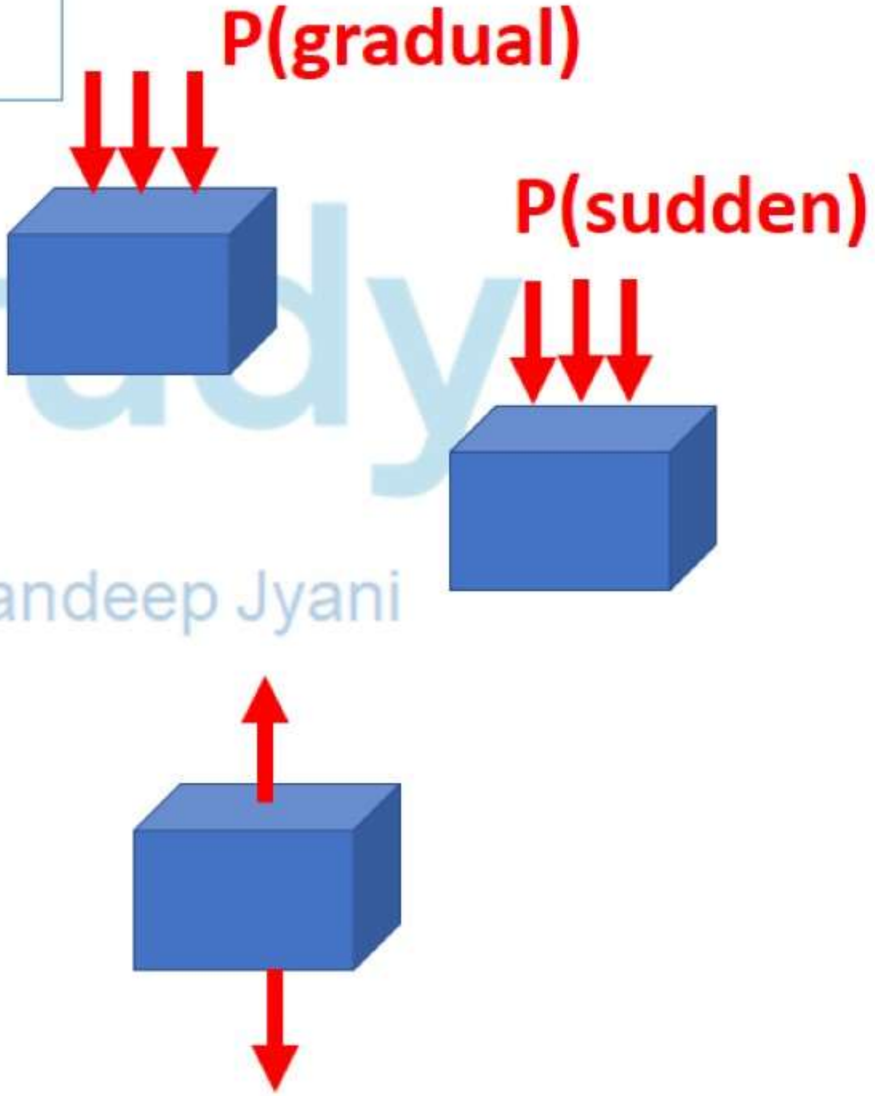
a) 4mm

b) 4.5mm

c) **5mm**

d) 5.5mm

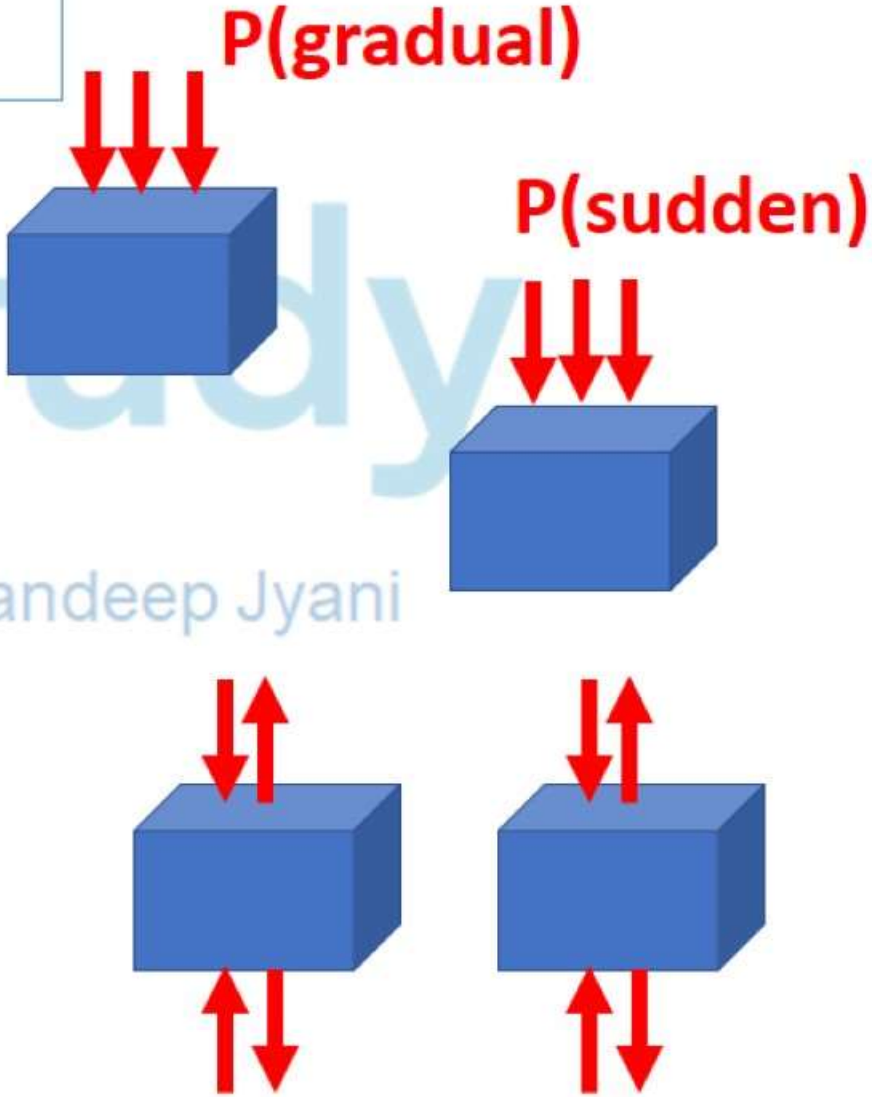
Loading Conditions



Loading Conditions

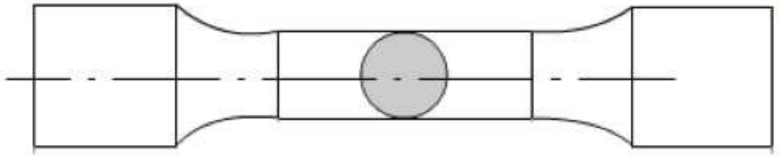
1. **Static**
   - **When the load is increased slowly and gradually and the metal is loaded by tension, compression, torsion or bending.**
2. **Dynamic**
   - **when the load increases rapidly as in impact**

3. **Repeated or Fatigue:** (both static and impact type
   - **when the load repeatedly varies in the course of test either in value or both in value and direction**

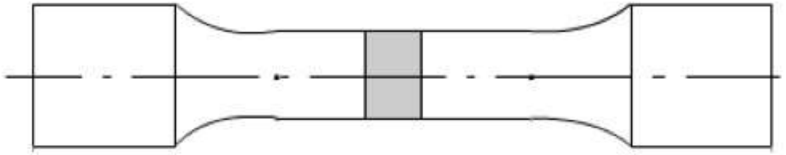


Uniaxial Tension Test

- This test is of static type i.e. the load is increased comparatively slowly from zero to a certain value.
- UTM or Tensile Testing Machine is used



Specimen with CIrcular Cross Section



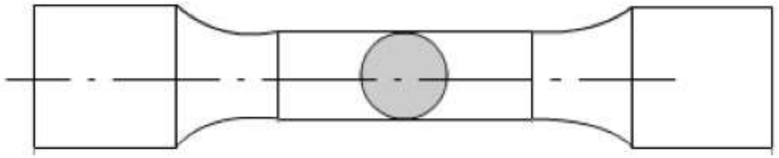
Specimen with Rectangular Cross Section

Uniaxial Tension Test

(i) The ends of the specimen's are secured in the grips of the testing machine.

(ii) There is a unit for applying a load to the specimen with a hydraulic or mechanical drive.

(iii) There must be a some recording device by which you should be able to measure the final output in the form of Load or stress.



Specimen with CIrcular Cross Section

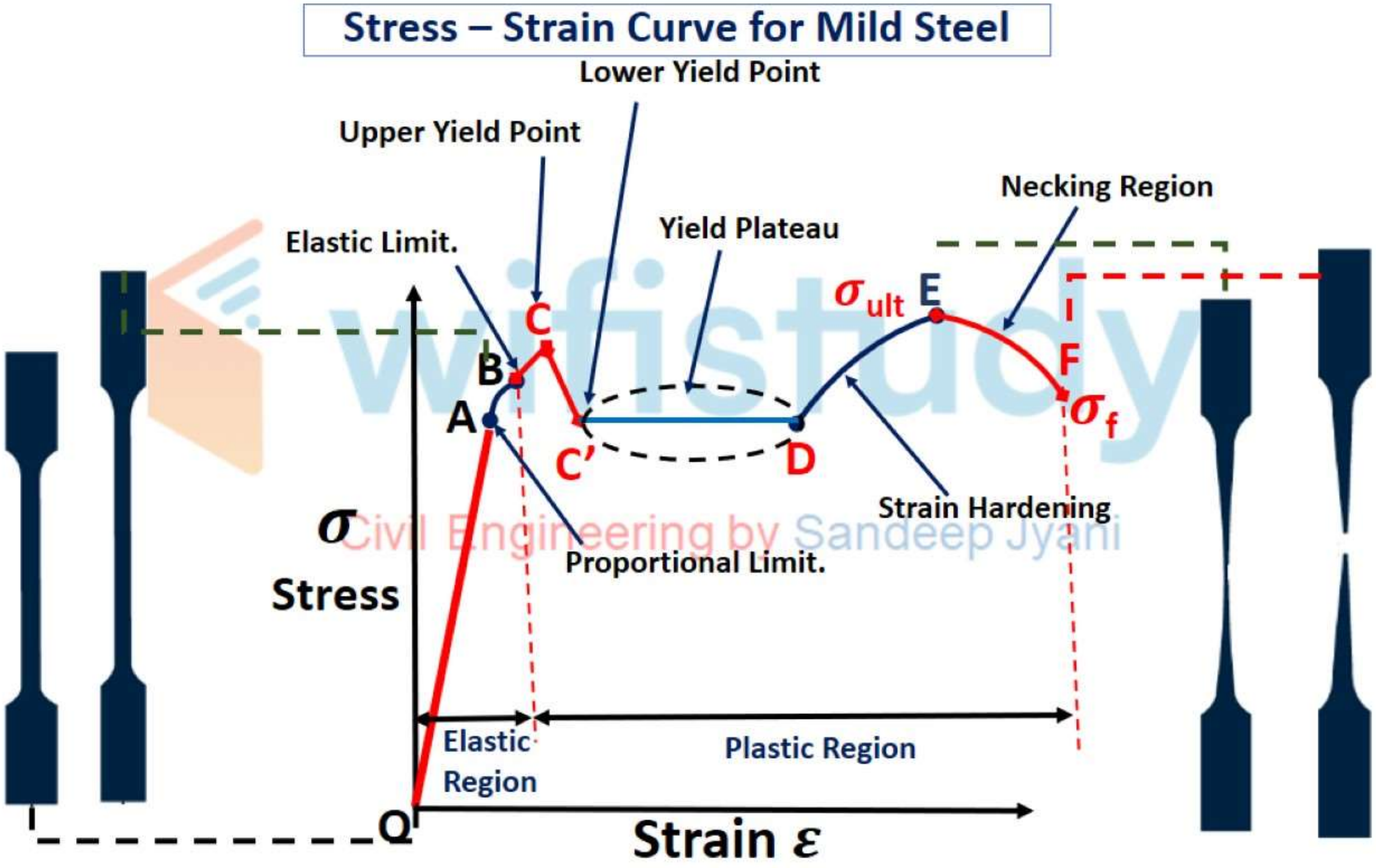
True Stress & Nominal Stress

1. **Nominal stress – Strain OR Conventional Stress – Strain diagrams:**

   Stresses are usually computed on the basis of the original area of the specimen; such stresses are often referred to as conventional or nominal stresses.

2. **True stress – Strain Diagram:**

   Since when a material is subjected to a uniaxial load, some contraction or expansion always takes place. Thus, dividing the applied force by the corresponding actual area of the specimen at the same instant gives the so called true stress.



Stress – Strain Curve for Mild Steel
Lower Yield Point
Upper Yield Point
Necking Region
Elastic Limit.
Yield Plateau
σ ult
E
C
B
F
A
σ f
C'
D
Strain Hardening
σ
Stress
Proportional Limit.
Elastic Region
Plastic Region
Q
Strain ε
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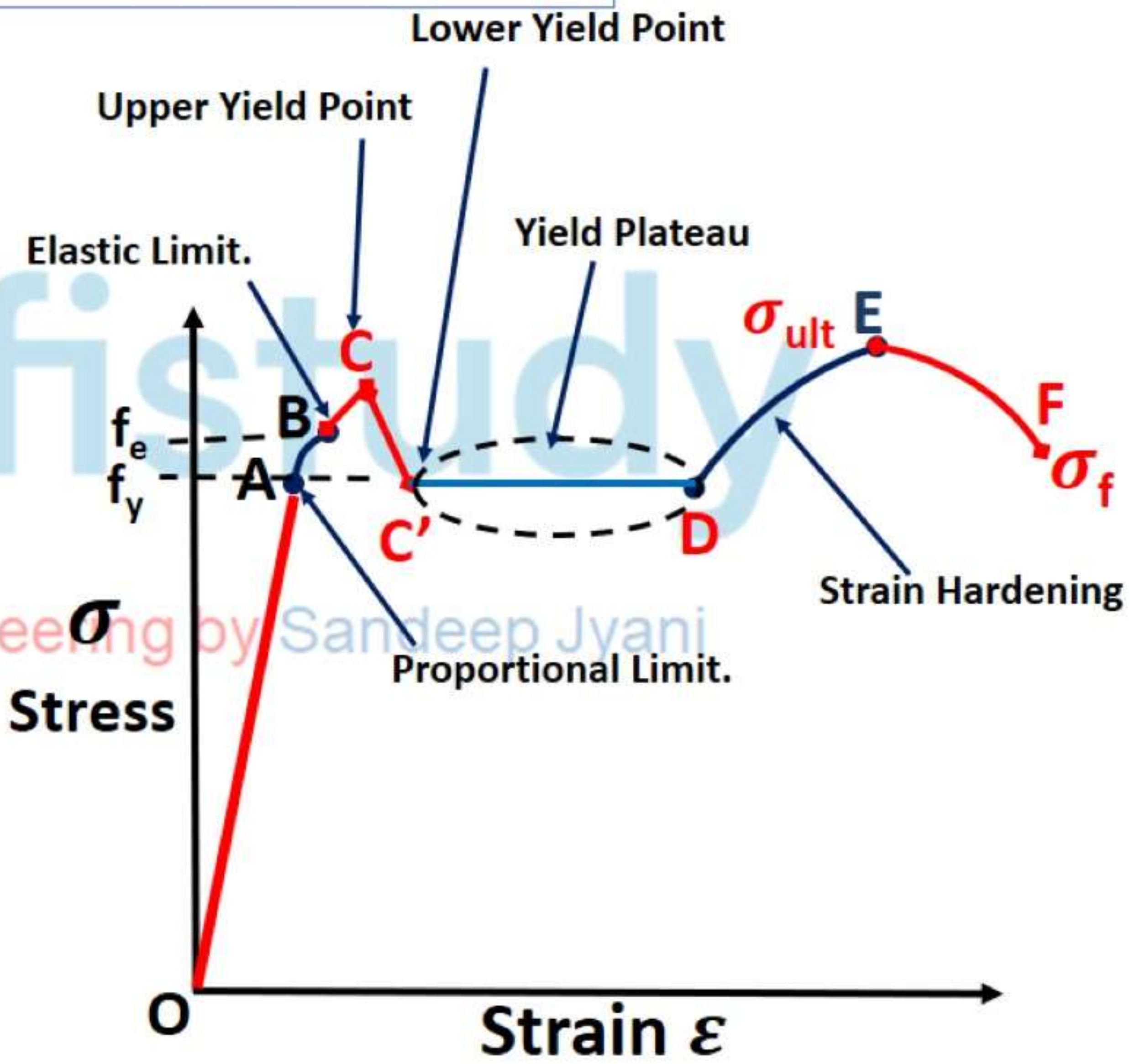
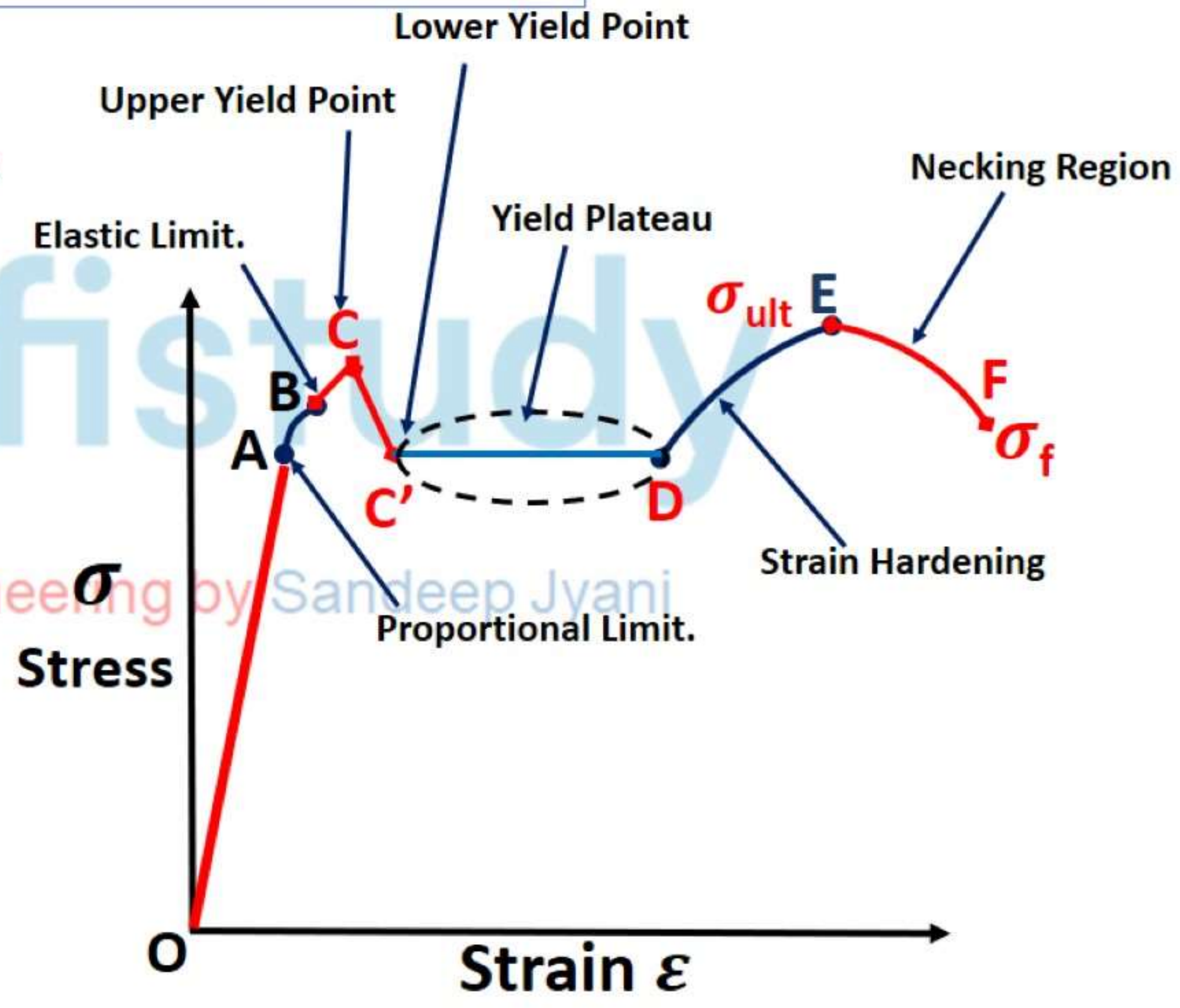
Stress – Strain Curve for Mild Steel

- **OA** is Proportionality limit
- **OB** is Elastic limit but **OB** is Non linear
- *The slippage of the carbon atom within a molecular mass leads to drop down of stress marginally from C to C'*
- C is upper yield point
- C' is lower yield point (also known as Yield Stress f_y)
  - For exp Fe-250 => $f_y = 250 N/mm^2$
- C'D is constant stress region called Yield Plateau



Stress – Strain Curve for Mild Steel

- DE is Strain Hardening region, material starts offering resistance against deformation
- EF is Necking region where drop down of stresses occur upto Failure point
- Necking region exists only in ductile material
- In mild steel, ABC are closer to each other, therefore it is known as Linear Elastic Metal, and Yield stress and elastic stress is taken as 250N/mm²
- The Fracture or Failure in mild steel depends upon Percentage of carbon present in a steel



Ductile and Brittle Materials

1. **<u>Ductile Materials:</u>**
   - **The Capacity of materials to allow these large deformations or large extensions without failure is termed as ductility. The materials with high ductility are termed as ductile materials.**

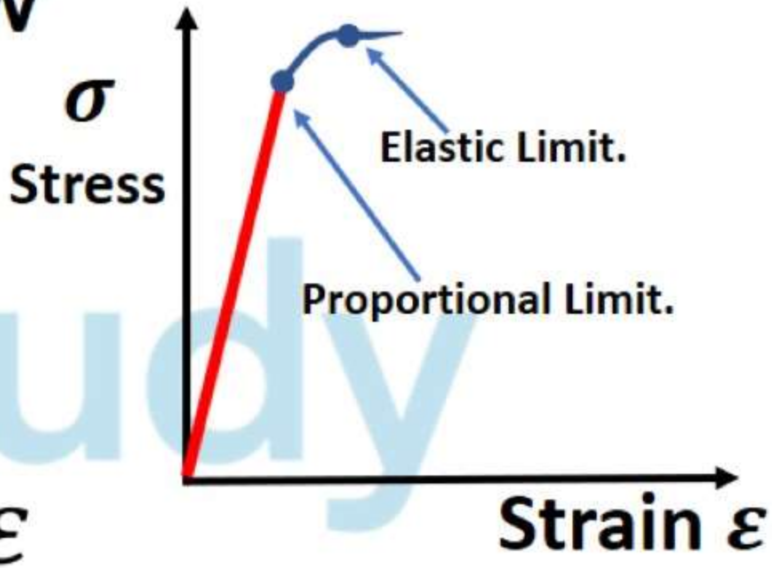
2. **<u>Brittle Materials:</u>**
   - **A brittle material is one which exhibits a relatively small extensions or deformations to fracture, so that the partially plastic region of the tensile test graph is much reduced.**

Hooke's Law

- When a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress upto **Proportional Limit.**

Or

- Ratio of the Stress to the corresponding strain is constant upto **Proportional Limit.**
- ***E= Young's Modulus of Elasticity or Modulus of Elasticity***
- E_{steel} = *200 GPa* E_{rubber}=*50 Gpa*
- E_{rigid} = ***infinte***



$$\sigma \propto \varepsilon$$

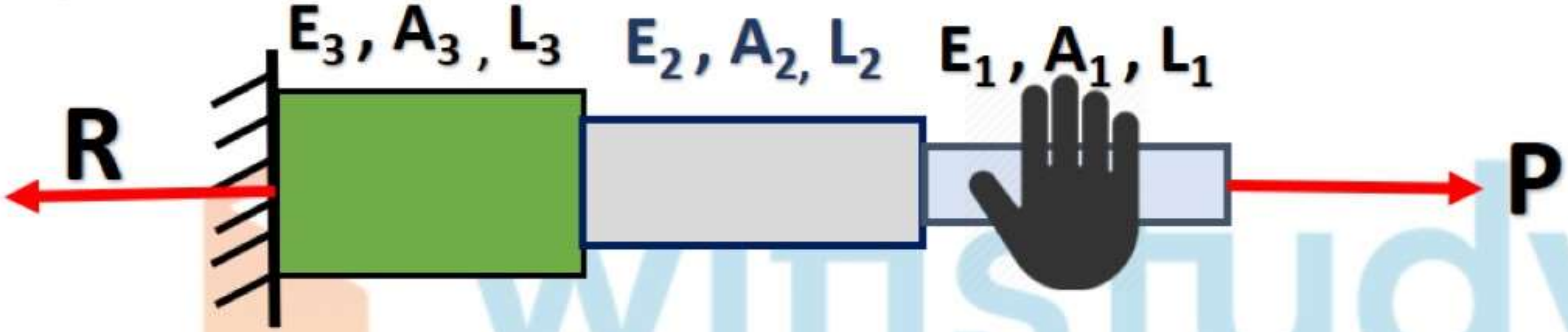
$$\sigma = E\varepsilon$$

$$E = \frac{\sigma}{\epsilon}$$

$$\triangle L = \frac{PL}{AE}$$

Application of Hooke's Law

1) Bar in Series



Step 1: Calculate the reaction

$$R = -\text{ (total net load)}$$

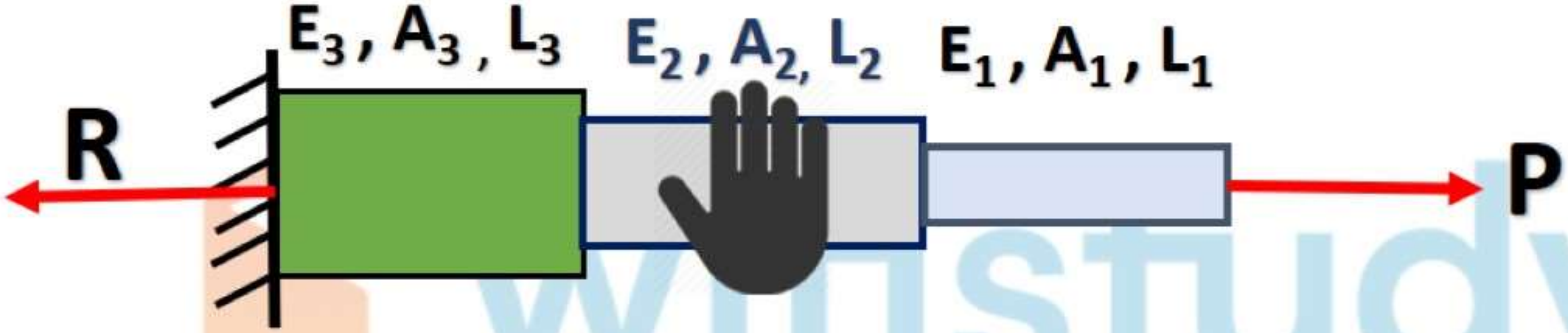
{- sign indicates opposite direction to net load}

Step 2: Calculate the Forces on each and every member

$$P_1 = P$$

Application of Hooke's Law

1. Bar in Series



Step 1: Calculate the reaction

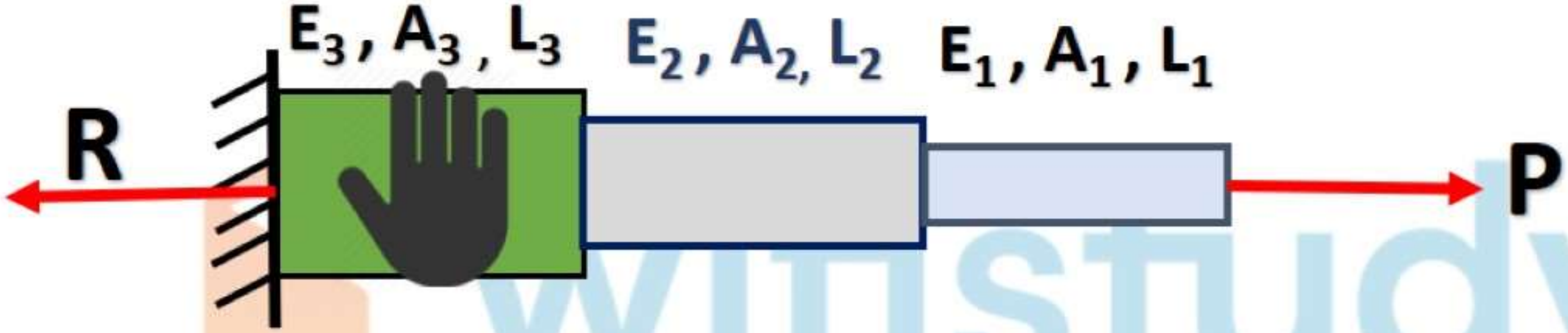
R = - (total net load) {- sign indicates opposite direction to net load}

Step 2: Calculate the Forces on each and every member

$P_1 = P$ $P_2 = P$

Application of Hooke's Law

1. Bar in Series



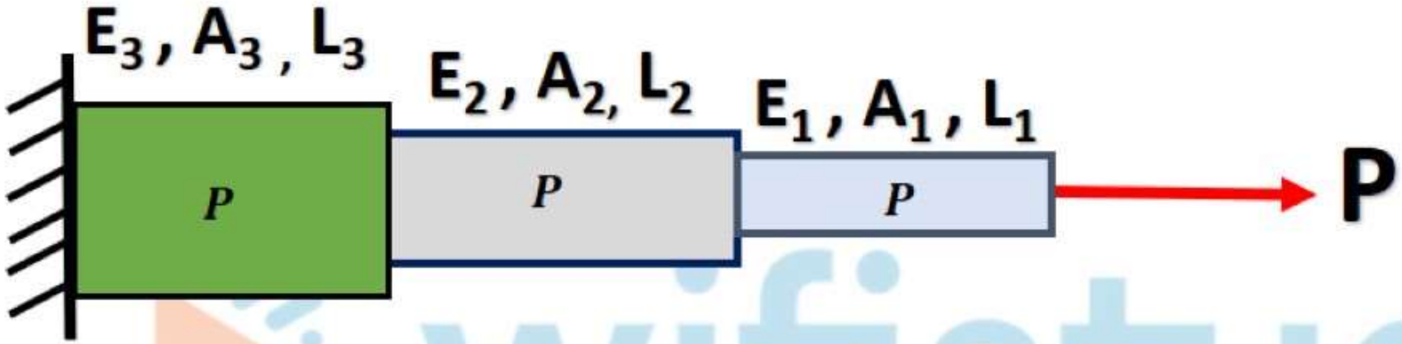
Step 1: Calculate the reaction

R = - (total net load) {- sign indicates opposite direction to net load}

Step 2: Calculate the Forces on each and every member

$P_1 = P \quad P_2 = P \quad P_3 = P$

1. Bar in Series



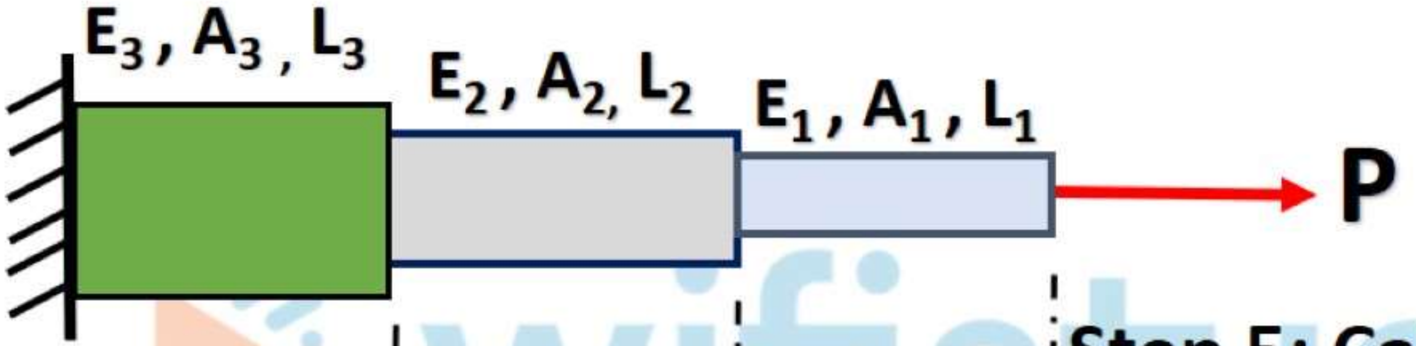
Step 3: Calculate the **Stresses** on each and every member

$$\sigma = \frac{P}{A} \quad \sigma_1 = \frac{P}{A_1} \quad \sigma_2 = \frac{P}{A_2} \quad \sigma_3 = \frac{P}{A_3}$$

Step 4: Calculate the **Elongation** on each and every member

$$\triangle L = \frac{PL}{AE} \quad \triangle L_1 = \frac{P_1 L_1}{A_1 E_1} \quad \triangle L_2 = \frac{P_2 L_2}{A_2 E_2} \quad \triangle L_3 = \frac{P_3 L_3}{A_3 E_3}$$

1. Bar in Series



Step 5: Calculate Total Elongation

$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3$$

Step 6: Draw Axial Load Diagram

$P_1 = P$,

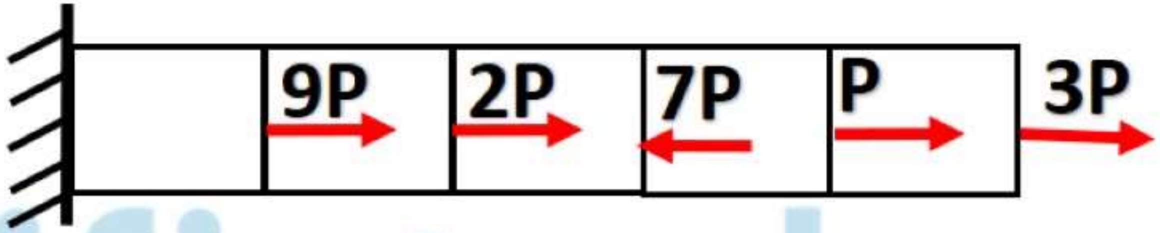
$P_2 = P$,

$P_3 = P$

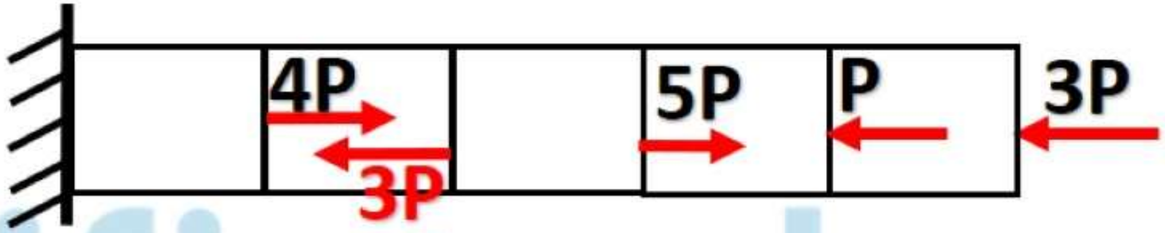
Que. 6 In the given series of bars, Length, Area and Young's modulus of elasticity are same. Calculate

a. $\sigma_{max} / \sigma_{min}$

b. $\triangle L_{total}$ Elongation



Que. 7 In the given series of bars, Length, Area and Young's modulus of elasticity are same. Calculate



a. $\sigma_{max} / \sigma_{min}$

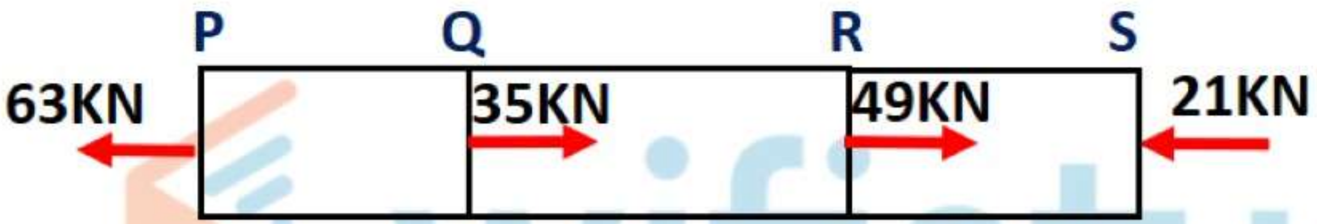
b. $\triangle L_{total}$ Elongation

HOMEWORK

Comment the answer a and b

Que. 8 Find total elongation. E =200GPa, Area = 25mm^2

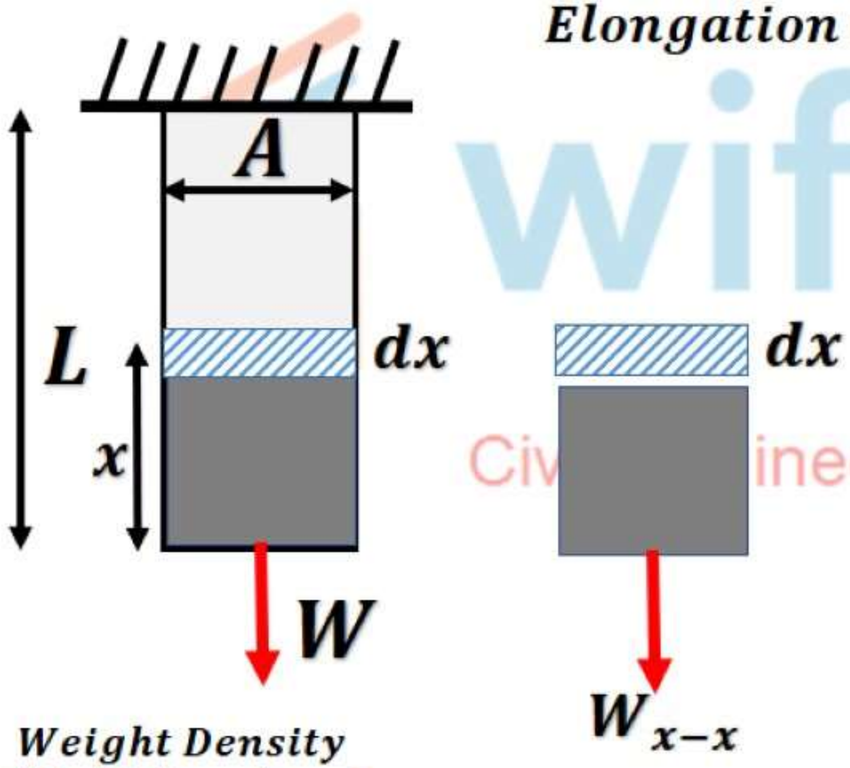
Que. 9 Find stress σ_{QR} if Area = 700mm^2



HOMEWORK !

Application of Hooke's Law

2) Elongation of Prismatic bar due to Self Weight



Weight Density (Wt/Volume) = λ

$$W_{x-x} = P_{x-x}$$

Elongation of strip whose length is dx and Area is A

$$\Delta x = \frac{P_{x-x} \times dx}{AE} \quad \dots(1)$$

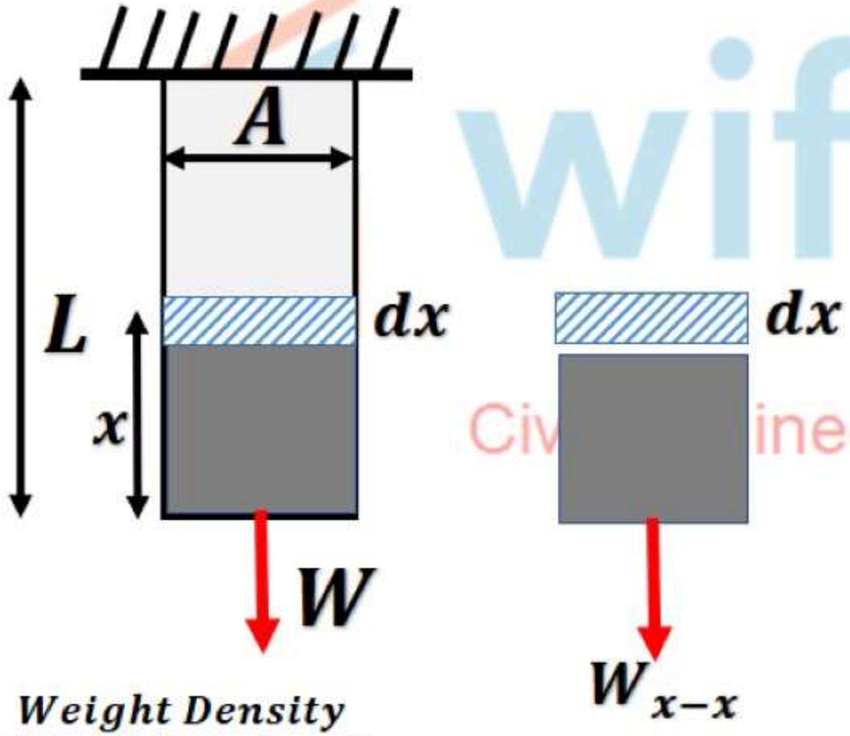
$$\lambda = \frac{W}{V} = \frac{W_{x-x}}{A \times x} = \frac{P_{x-x}}{A \times x}$$

$$\Rightarrow P_{x-x} = \lambda\, Ax$$ *Force at any point*

$$\Rightarrow P_{max} = \lambda\, AL$$

Application of Hooke's Law

2) Elongation of Prismatic bar due to Self Weight



Weight Density (Wt/Volume) $= \lambda$

$$W_{x-x} = P_{x-x}$$

$$\Delta x = \frac{P_{x-x} \times dx}{AE} \quad \ldots (1)$$

$$\Rightarrow P_{x-x} = \lambda\, Ax \quad \ldots (2)$$

Elongation

$$\Delta x = \frac{\lambda\, Ax \times dx}{AE}$$

$$\Rightarrow \Delta x = \frac{\lambda\, x dx}{E}$$

For total Elongation of the bar,

$$\Delta L_{self\ wt} = \int_0^L \Delta x = \int_0^L \frac{\lambda\, x dx}{E}$$

$$\boxed{\Delta L_{self\ wt} = \frac{\lambda L^2}{2E}}$$

Que. 10 If cross section area of a bar is doubled, then elongation due to self weight of the bar will be...

a) Doubled

b) Halved

c) Remains same

d) Four times

Que. 10 If cross section area of a bar is doubled, then elongation due to self weight of the bar will be...

a) Doubled

b) Halved

c) **Remains same**

d) Four times

$$\Delta L_{self\ wt} = \frac{\lambda L^2}{2E}$$

NOTE: Elongation due to self weight depends only on Length and does not depend upon area.

Que. 11 If all the dimensions of a bar are doubled, then elongation due to self weight of the bar will be...

a) Doubled

b) Halved

c) Remains same

d) Four times

Que. 11 If all the dimensions of a bar are doubled, then elongation due to self weight of the bar will be...

a) Doubled

b) Halved

c) Remains same

d) <u>Four times</u>

$$\Delta L_{self\ wt} = \frac{\lambda L^2}{2E}$$

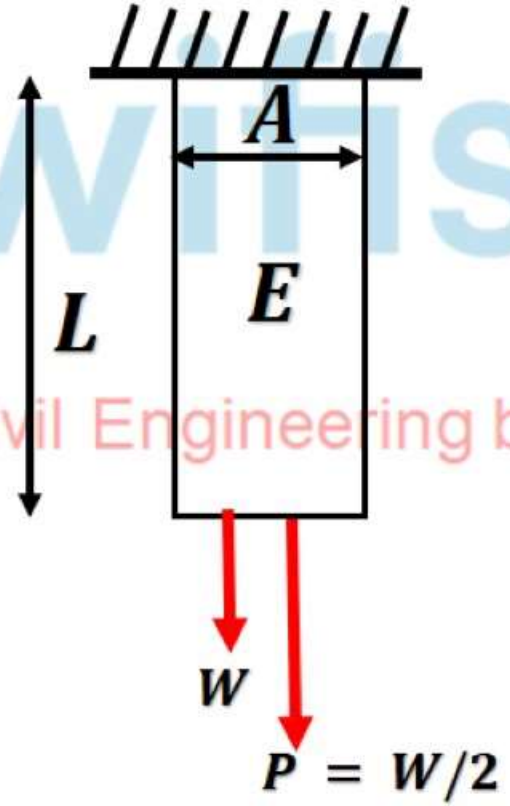
Que. 12 If Load P is applied on a bar having self weight W, the elongation will be

a) WL/2AE

b) 2WL/AE

c) WL/4AE

d) WL/AE



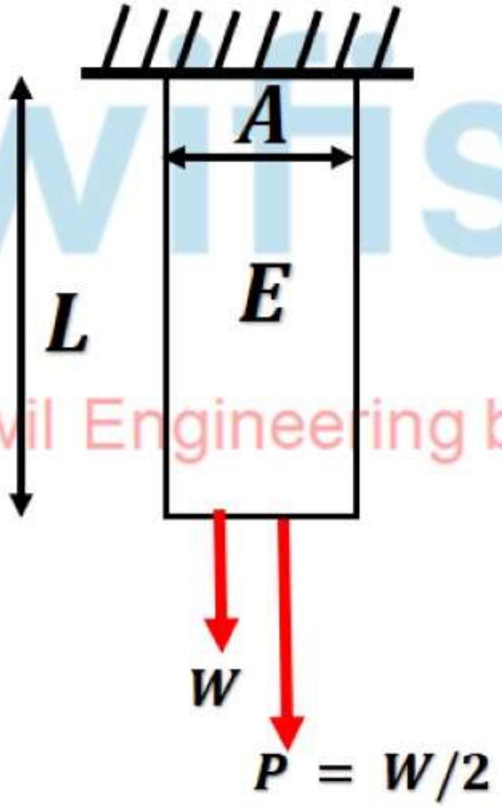
Que. 12 If Load P is applied on a bar having self weight W, the elongation will be

a) WL/2AE

b) 2WL/AE

c) WL/4AE

d) <u>WL/AE</u>



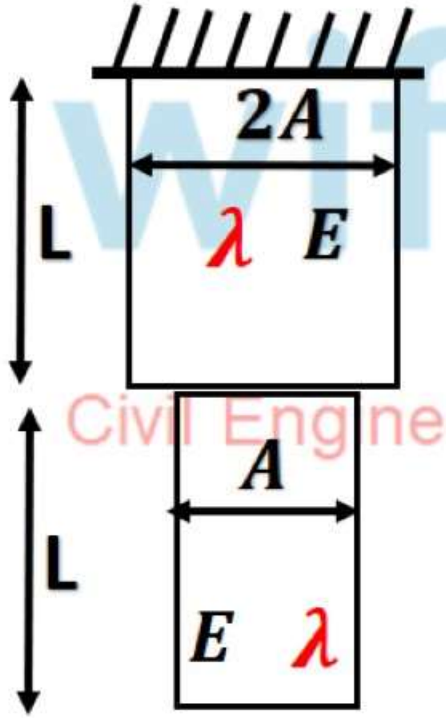
$$\Delta L_{total} = \Delta L_{self\ wt} + \Delta L_{external\ load}$$

$$\Delta L_{total} = \frac{\lambda L^2}{2E} + \frac{PL}{AE}$$

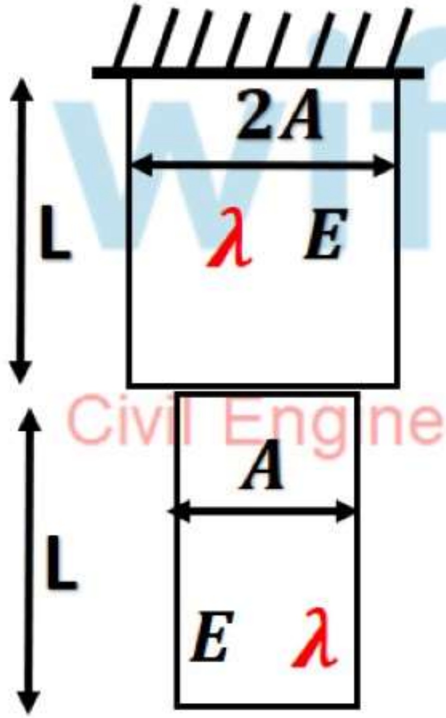
$$\Delta L_{total} = \frac{(\frac{W}{AL})L^2}{2E} + \frac{(\frac{W}{2})L}{AE}$$

$$\Delta L_{total} = \frac{WL}{AE}$$

Que. 13 Total elongation due to self weight of the two bars will be equal to... ?



Que. 13 Total elongation due to self weight of the two bars will be equal to... ?

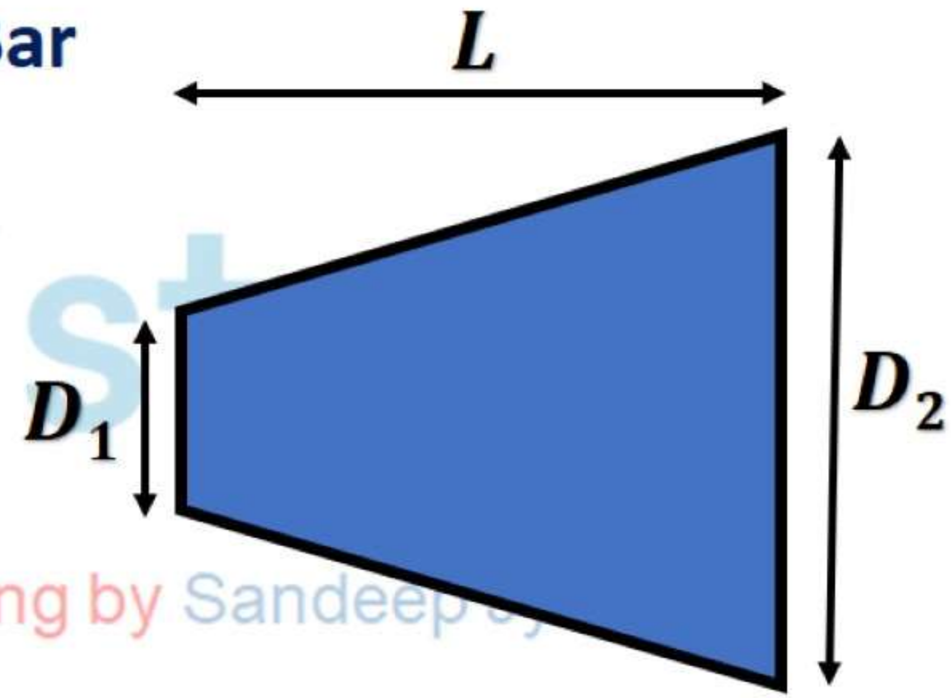


$$\Delta L_{total} = \Delta L_{self\ wt} + \Delta L_{external\ load}$$

Application of Hooke's Law

3. Elongation of Tapered Bar

$$\Delta L = \frac{4PL}{\pi D_1 D_2 E}$$



Application of Hooke's Law

Que. 14 For same elongation, what is the relation between the two (Prismatic Bar and Tapered Bar)?

a) $\mathbf{D} = \boldsymbol{D_1 D_2}$

b) $\mathbf{D} = \sqrt{\boldsymbol{D_1 D_2}}$

c) $\boldsymbol{D_1} = \sqrt{\boldsymbol{D D_2}}$

d) $\boldsymbol{D_2} = \sqrt{\boldsymbol{D D_1}}$

Application of Hooke's Law

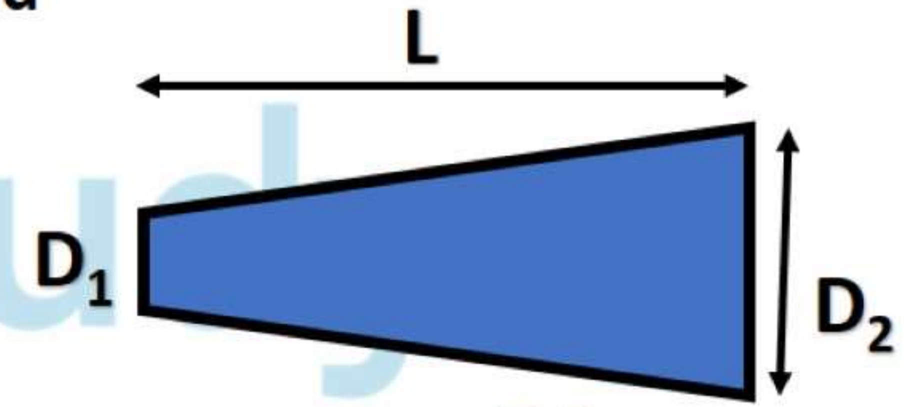
Que. 14 For same elongation, what is the relation between the two (Prismatic Bar and Tapered Bar)?

a) $\mathrm{D} = D_1 D_2$

b) $D = \sqrt{D_1 D_2}$

c) $D_1 = \sqrt{D D_2}$

d) $D_2 = \sqrt{D D_1}$



$$\Delta L = \frac{PL}{AE}$$

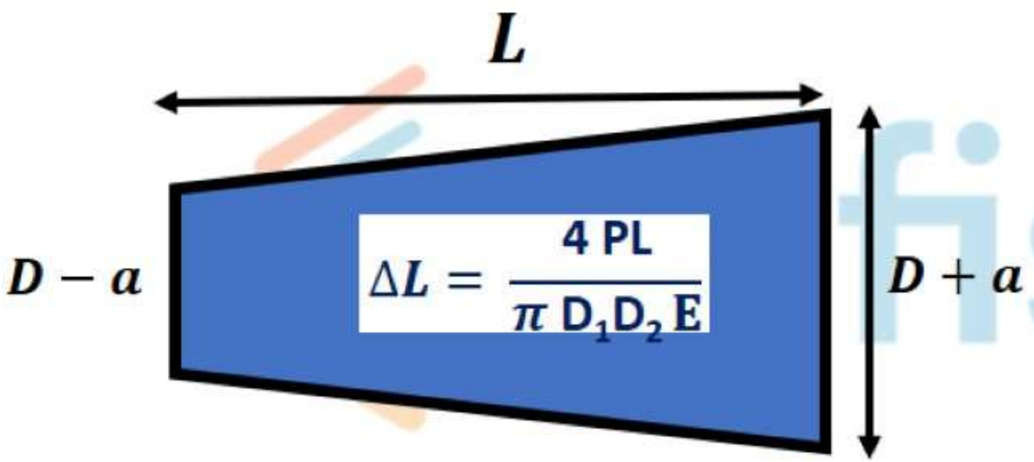
$$\Delta L = \frac{4PL}{\pi \, D_1 D_2 \, E}$$

$$\boxed{D = \sqrt{D_1 D_2}}$$

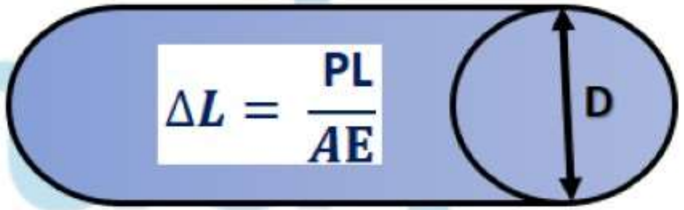
Application of Hooke's Law

Que. 15 What is the value of percentage elongation error in Tapered bar?

By using mean dia of Tapered bar, make a prismatic bar of dia equal to mean dia



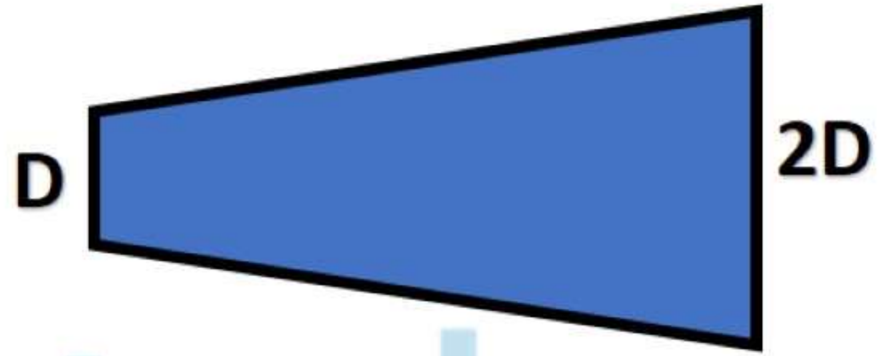
$$D_{mean} = \frac{D + a + D - a}{2} = D$$



$$\% \; error \; in \; elongation = \frac{(\Delta L)_{Tapered} - (\Delta L)_{Prismatic}}{(\Delta L)_{Tapered}} \times 100$$

$$= \frac{\frac{4\,PL}{\pi\,(D+a)(D-a)\,E} - \frac{4PL}{\pi D^2 E}}{\frac{4\,PL}{\pi\,(D+a)(D-a)\,E}} \times 100 \quad = \left\{\frac{10a}{D_{mean}}\right\}^2 \% \text{ of error}$$

Que. 16 Determine the % error of elongation in given Tapered bar.



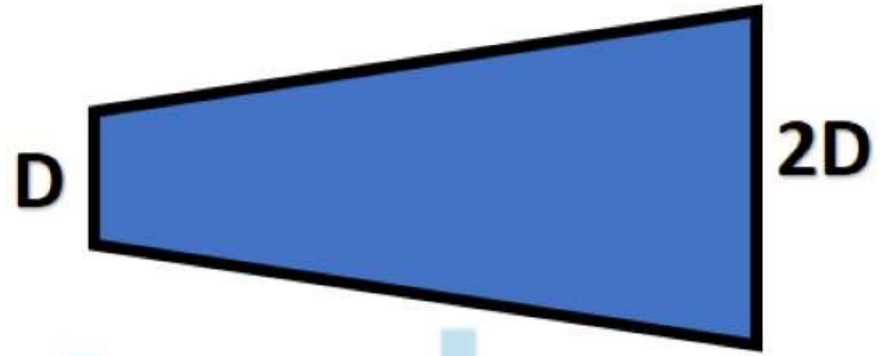
a) 3.33 %

b) 11.11%

c) 33.3%

d) 22.2%

Que. 16 Determine the % error of elongation in given Tapered bar.



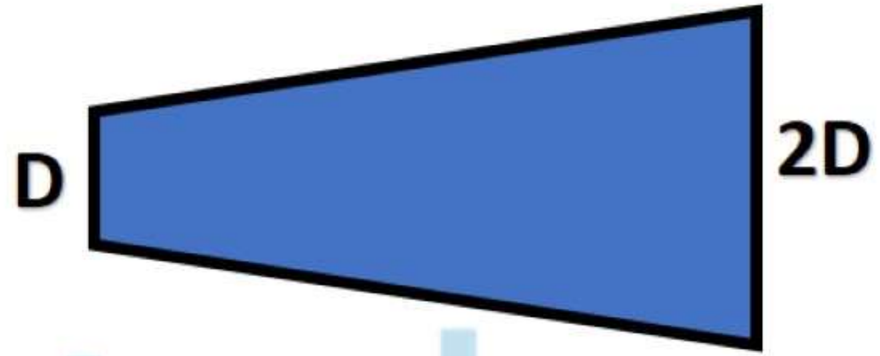
a) 3.33 %

b) **11.11%**

c) 33.3%

d) 22.2%

Que. 16 Determine the % error of elongation in given Tapered bar.



Let mean dia be d_m , so

$D = d_m - a$

$2D = d_m + a$, *adding*

$3D = 2d_m$

$\Rightarrow d_m = \frac{3}{2}D$ and

$\Rightarrow a = D/2$

$$\% \text{ elongation of error} = \left\{\frac{10a}{d_{mean}}\right\}^2 \%$$

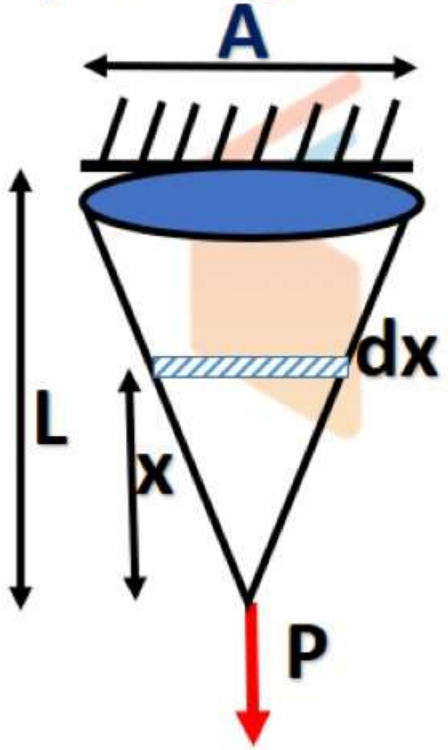
$$= \left\{ \frac{10 \times \frac{D}{2}}{\frac{3}{2}D} \right\}^2 \%$$

= ~~3.33%~~ (you did not square)

=11.11 %

Application of Hooke's Law

4) Elongation of Conical bar due to Self Weight



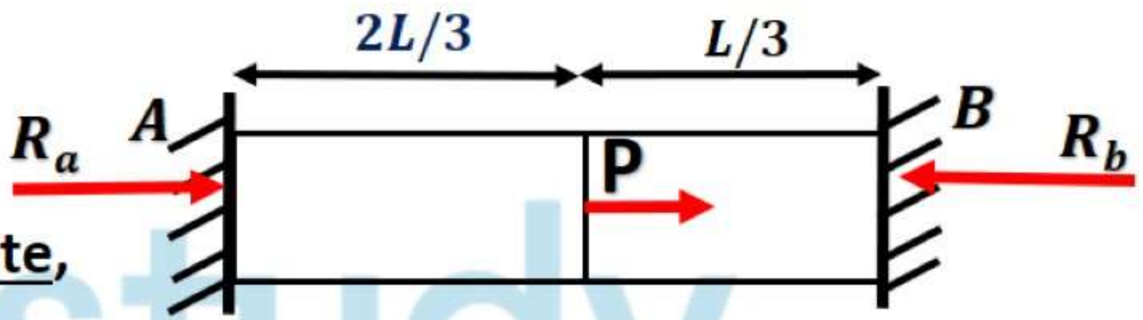
Elongation due to self weight of a conical bar in terms of weight

$$\Delta L = \frac{1}{3} \times \frac{\lambda L^2}{2E}$$

Weight Density (Wt/Volume)= λ

Application of Hooke's Law

5) Bar is fixed at Both Ends



Step 1: Calculate Reactions

$$R_a + Rb = P$$

Step 2: Since it is statically indeterminate, we use total elongation=0

$\Delta L_{total} = 0$

$$\Rightarrow \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} = 0$$

$$\Rightarrow P_1 L_1 + P_2 L_2 = 0$$

Step 3: By Section method

$$P_1 = -R_b \text{ and}$$

$$P_2 = P - R_b$$

$$= R_a + Rb - R_b$$

$$= Ra$$

Step 4: $P_1 L_1 + P_2 L_2 = 0$

Value of ...

$R_a = P/3$

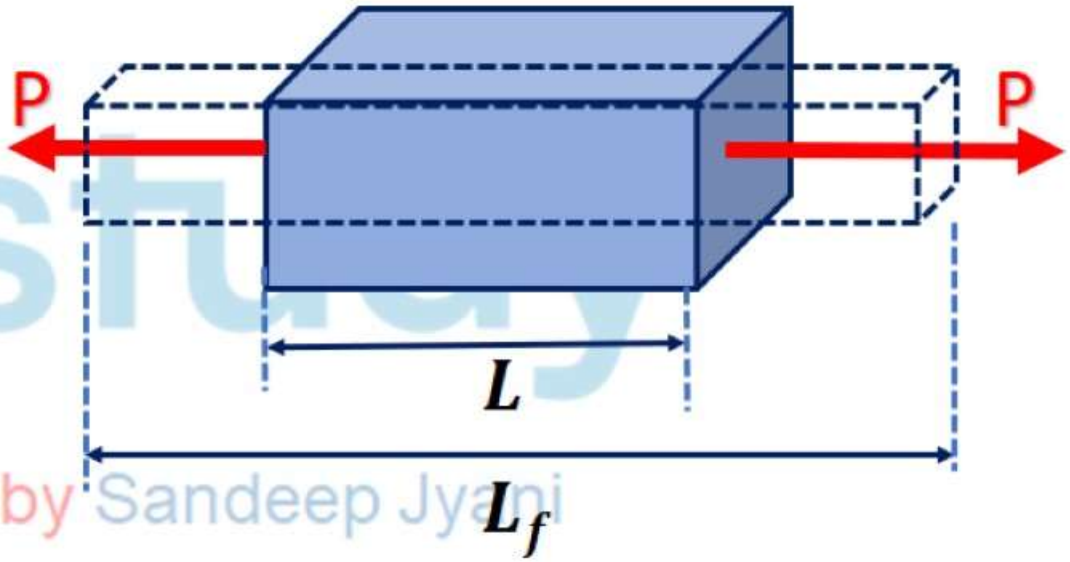
$R_b = 2P/3$

Elastic Constants

- Strain
- Poisson's ratio
- Volumetric Strain
- Bulk Modulus
- Relation between Young' modulus and Bulk Modulus

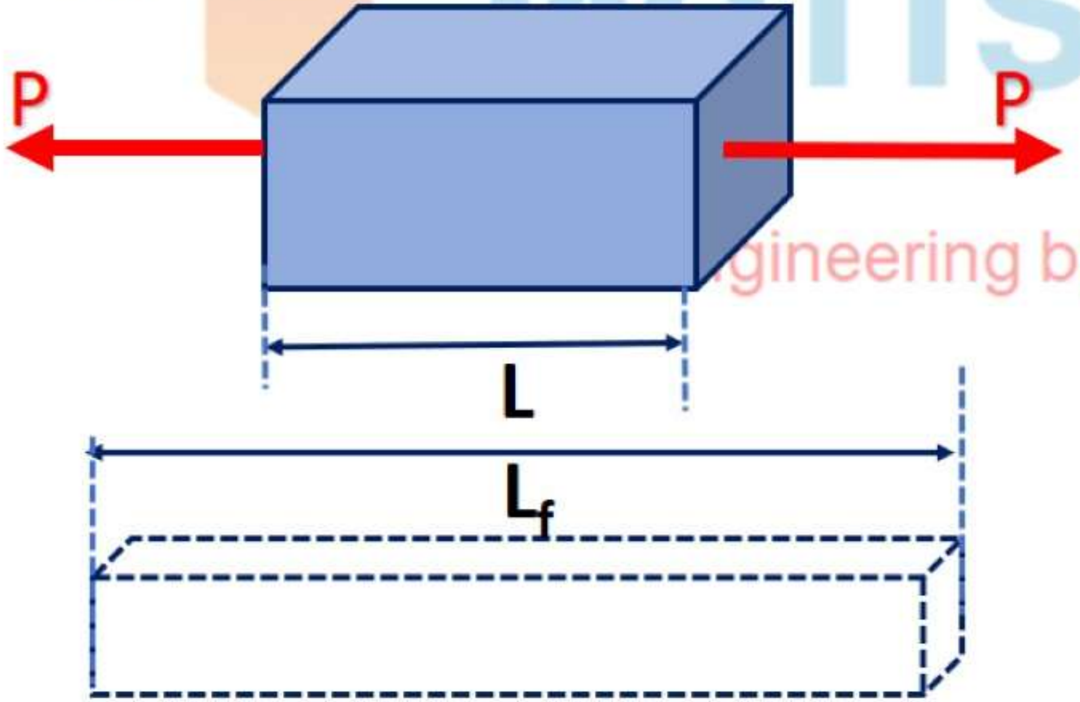
Elastic Constants

- When a body is subjected to tensile load, there is an increase in the length of the body, but at the same time there is a decrease in other dimensions of the body at right angles to the direction of applied load.
- Thus the body is having ***axial deformation*** and also deformation at right angles to the line of action of applied load i.e. ***lateral deformation***



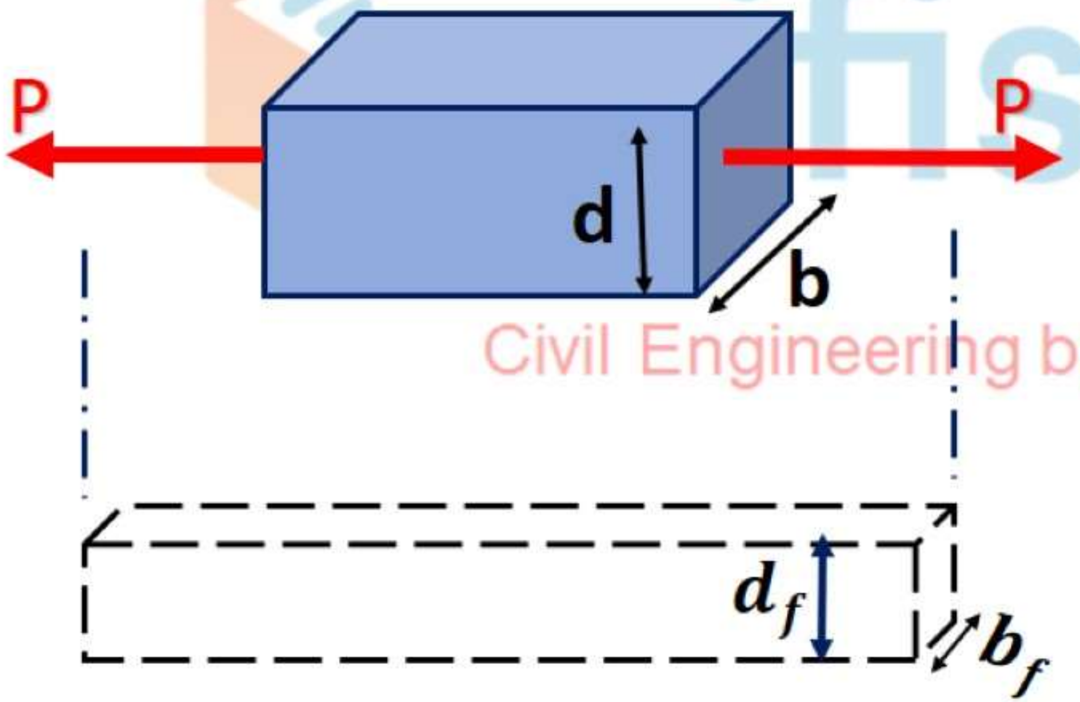
Strain

Longitudinal Strain: When body is subjected to axial load (tensile or compressive), there is an axial deformation in the length of the body or Longitudinal strain is the strain produced in the direction of applied load



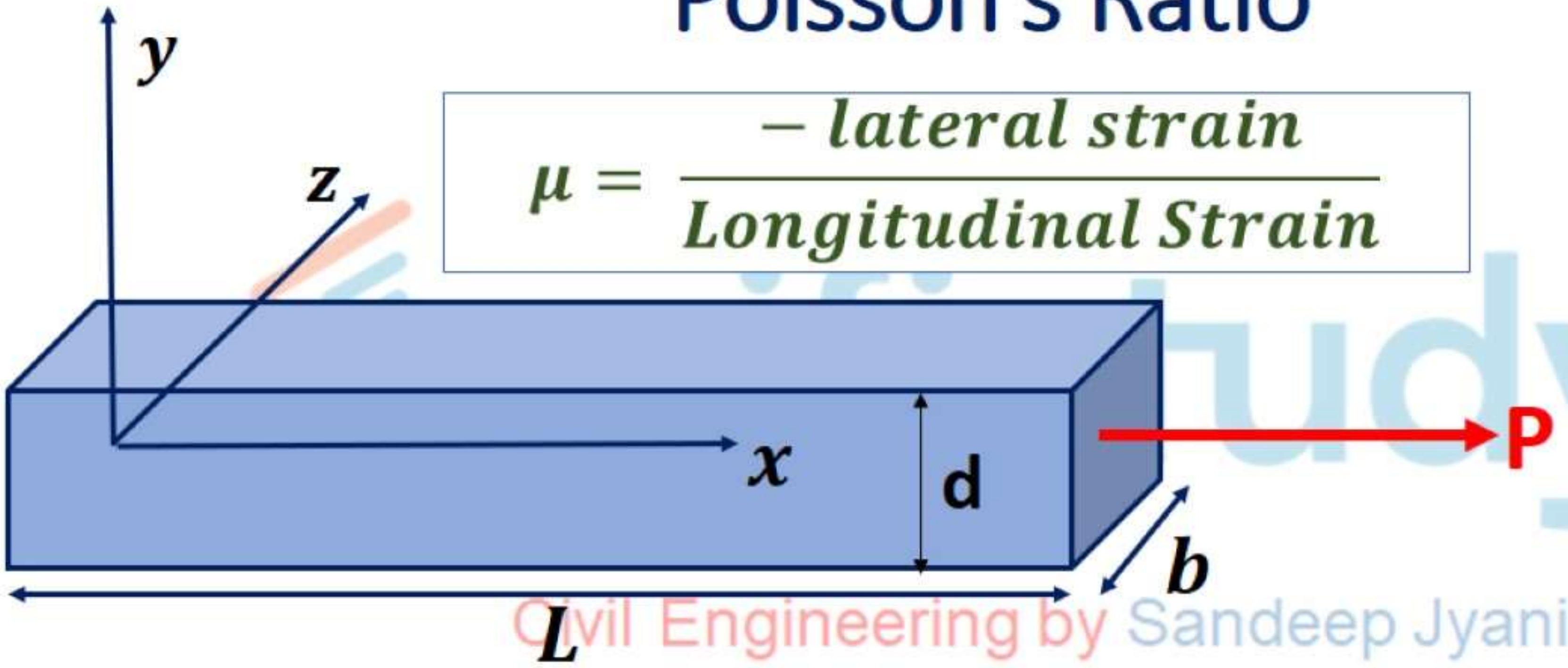
Strain

Lateral Strain: The strain at right angles to direction of applied load is known as lateral strain



Poisson's Ratio

$$\mu = \frac{-\,lateral\ strain}{Longitudinal\ Strain}$$



$$(x\ direction)\ Longitudinal\ Strain = \frac{\Delta L}{L}$$

$$(y\ direction)\ Lateral\ Strain = \frac{\Delta d}{d}$$

$$(z\ direction)\ Lateral\ Strain = \frac{\Delta b}{b}$$

Poisson's Ratio

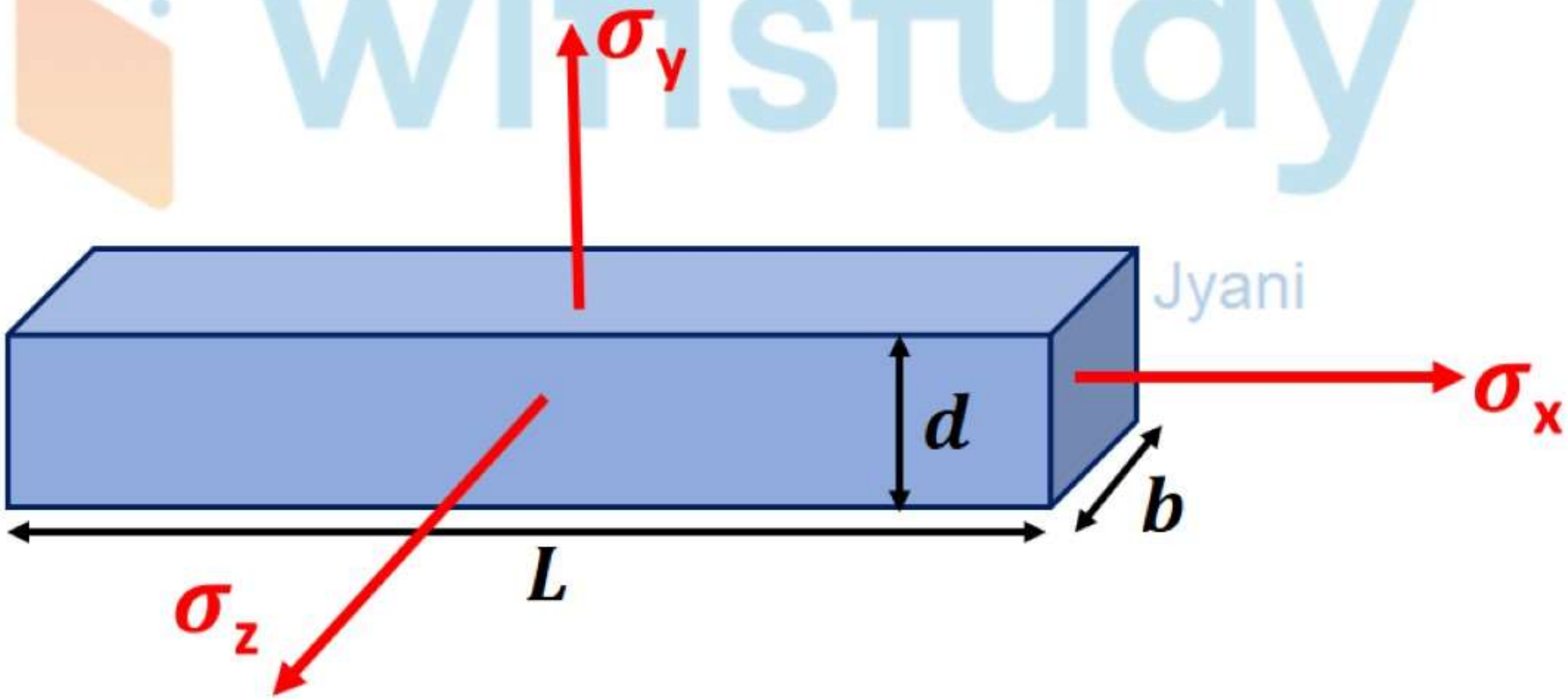
- ✓ μ_{cork} = 0
- ✓ μ_{metal} = 0.25-0.33
- ✓ $\mu_{human\ tissue/foam}$ = negative
- ✓ μ_{rubber} = 0.5
- ✓ μ_{steel} = 0.286
- ✓ $\mu_{concrete}$ = 0.1-0.2

- $\mu_{generally}$ $0 \le \mu \le 0.5$
- μ_{ideal} $-1 \le \mu \le 0.5$

Volumetric Strain under Triaxial Loading

$$Volmetric\ Strain = \frac{Change\ in\ volume}{Original\ Volume}$$

$$Volmetric\ Strain = \frac{\Delta V}{V}$$



Volumetric Strain

Strain in x – direction,

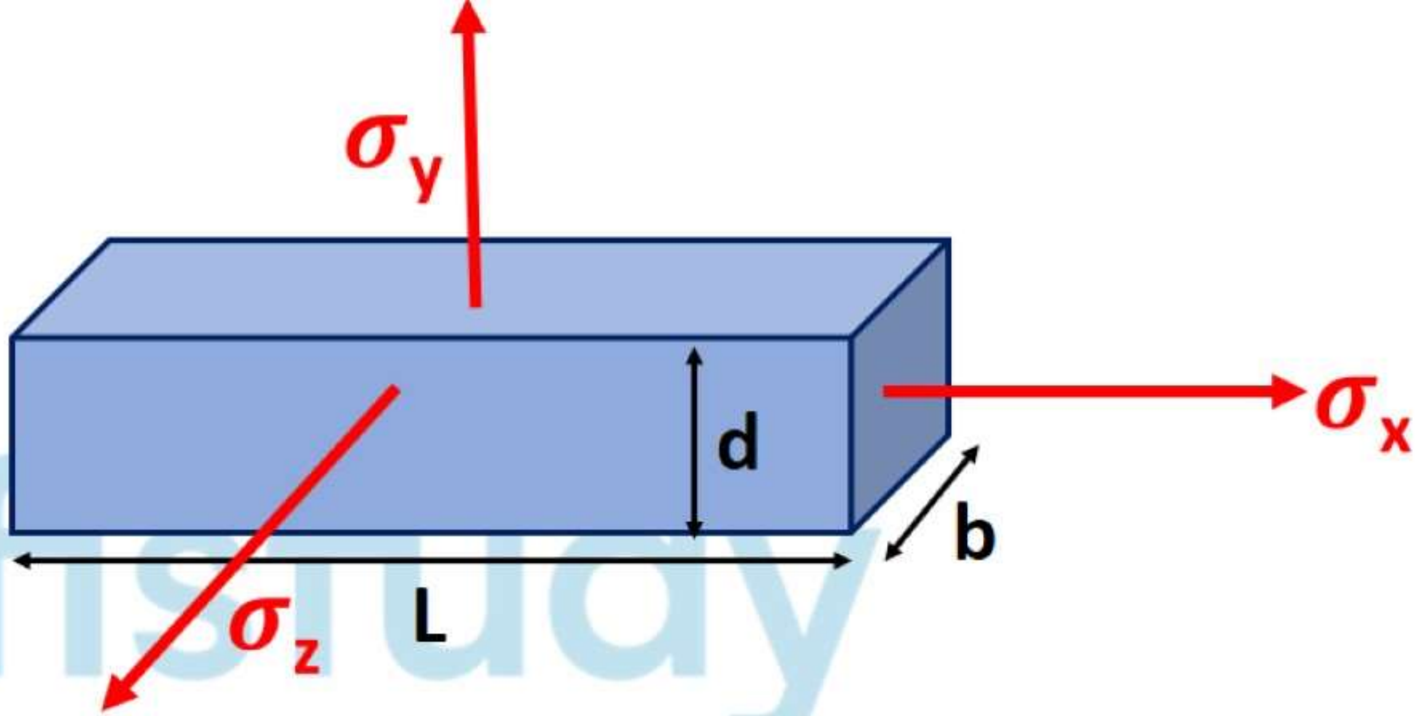
$$\varepsilon_x = \frac{\Delta L}{L} = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

Strain in y – direction,

$$\varepsilon_y = \frac{\Delta d}{d} = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

Strain in z – direction,

$$\varepsilon_z = \frac{\Delta b}{b} = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$



$\therefore$ ***Volumetric Strain***

$$e_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\boxed{e_v = \frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu)}$$

Volumetric Strain

$$e_v = \frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu)$$

1. If there is uniaxial loading, then volumetric strain will be

$$Volmetric\ Strain = \frac{\sigma_x\,(1-2\mu)}{E}$$

2. If the poisson's ratio of material is μ= 0.5, then volumetric strain will be 0 under any state of loading
3. If the poisson's ratio of any material is less than 0.5 i.e. (μ<0.5), the change in volume or volumetric strain will be 0 if sum of all the normal stresses is 0

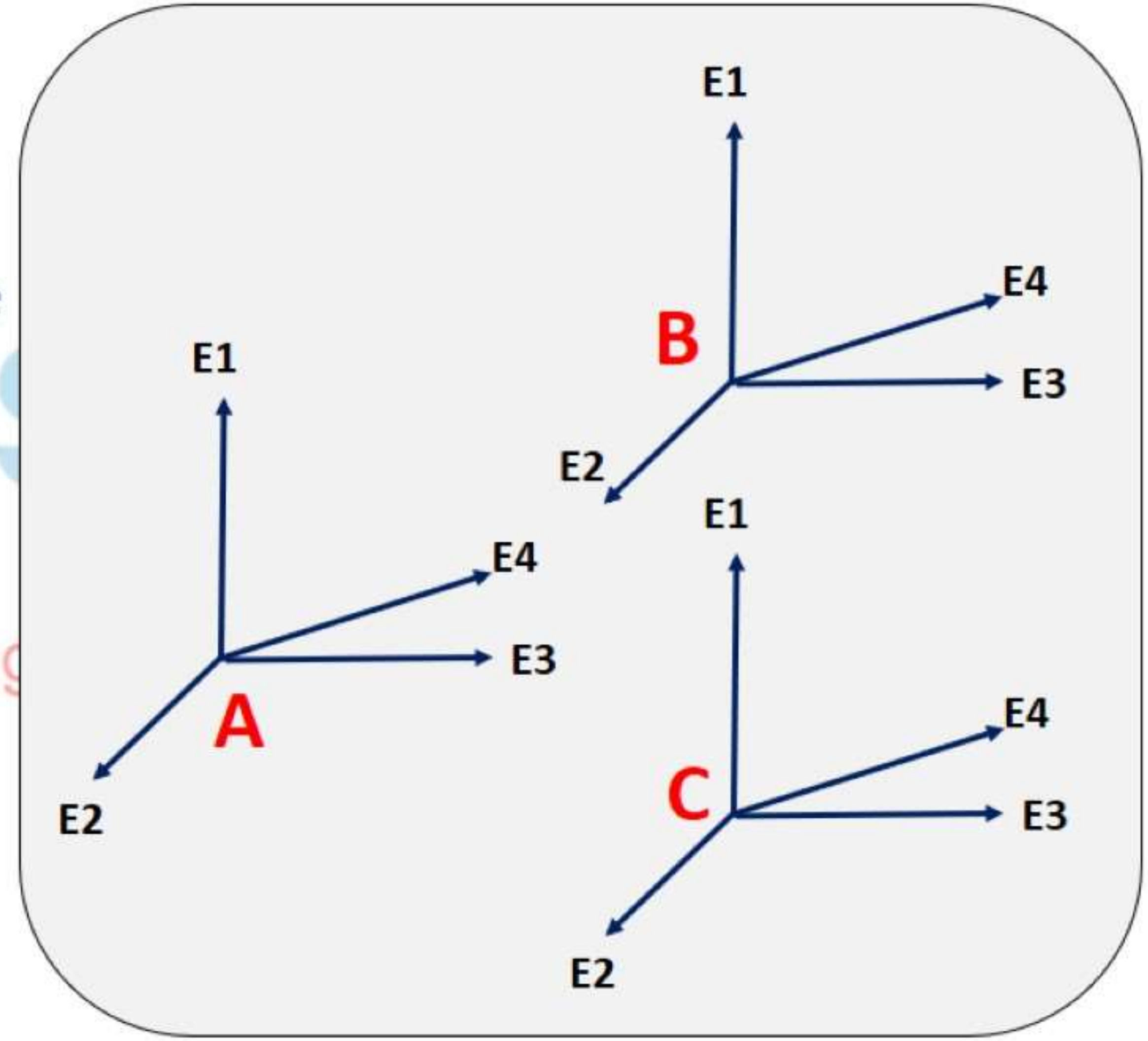
Elastic Constants

- **A constant or coefficient that express the elasticity of the material.**
- **Elastic constants are basically used obtain relationship between Stress and Strain.**
- **For a homogenous and Isotropic material, the number of total Elastic constants are 4 (E, G, μ, K)**

- **E = Young's Modulus of Elasticity**
- **G = Shear Modulus/Modulus of Rigidity**
- $\mu = Poisson's\ Ratio,$
- **K = Bulk Modulus/Modulus**

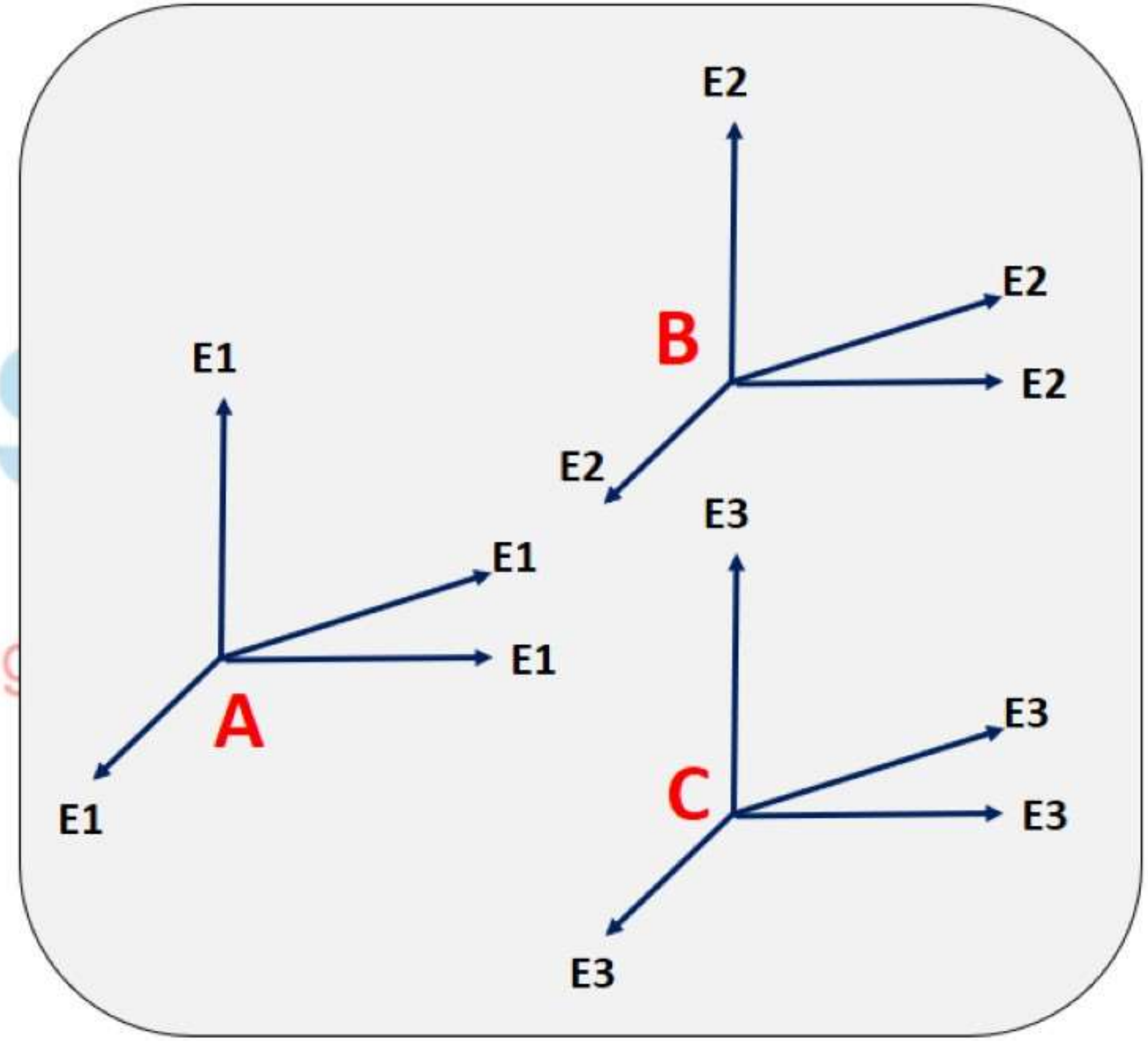
1. Homogenous Material

- A material is said to be homogenous when it shows same elastic properties at ANY POINT of material IN A GIVEN DIRECTION



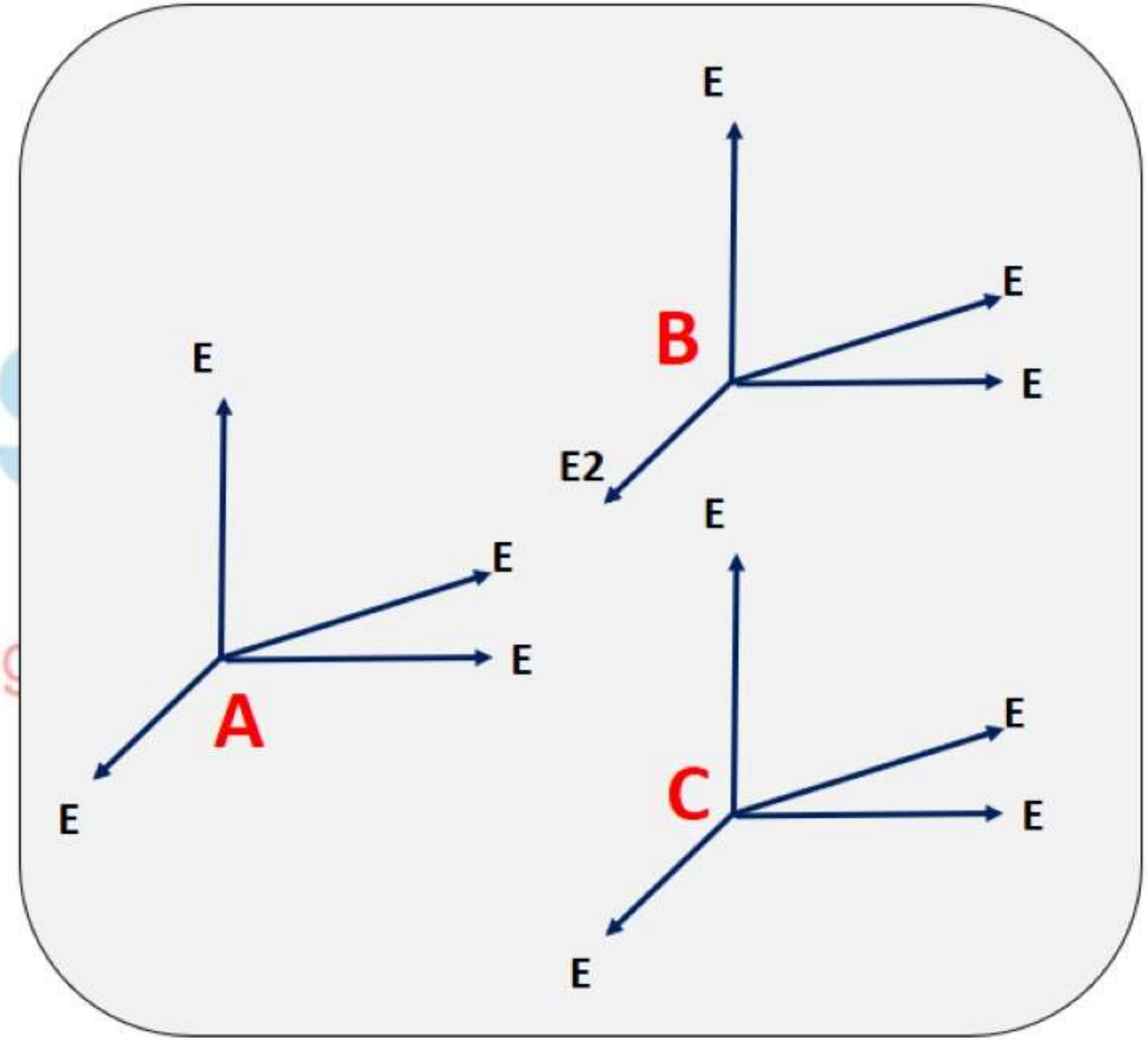
2. Isotropic Material

- A material is said to be isotropic when it shows same elastic properties IN ANY GIVEN DIRECTION AT A GIVEN POINT



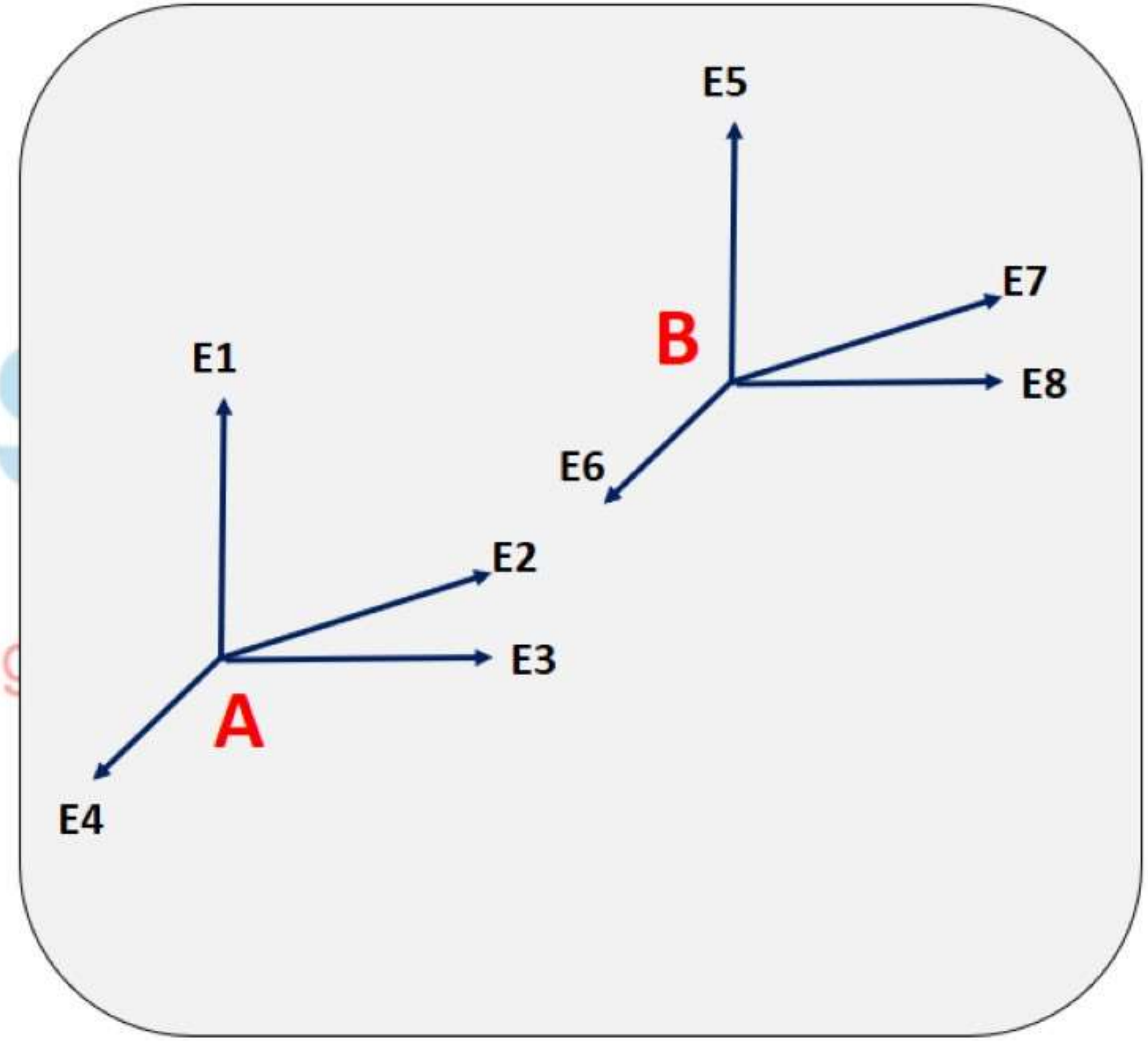
3. Homogenous and Isotropic Material

- A material is said to be homogenous and isotropic when it shows same elastic properties **IN ANY GIVEN DIRECTION** and **AT ANY GIVEN POINT**



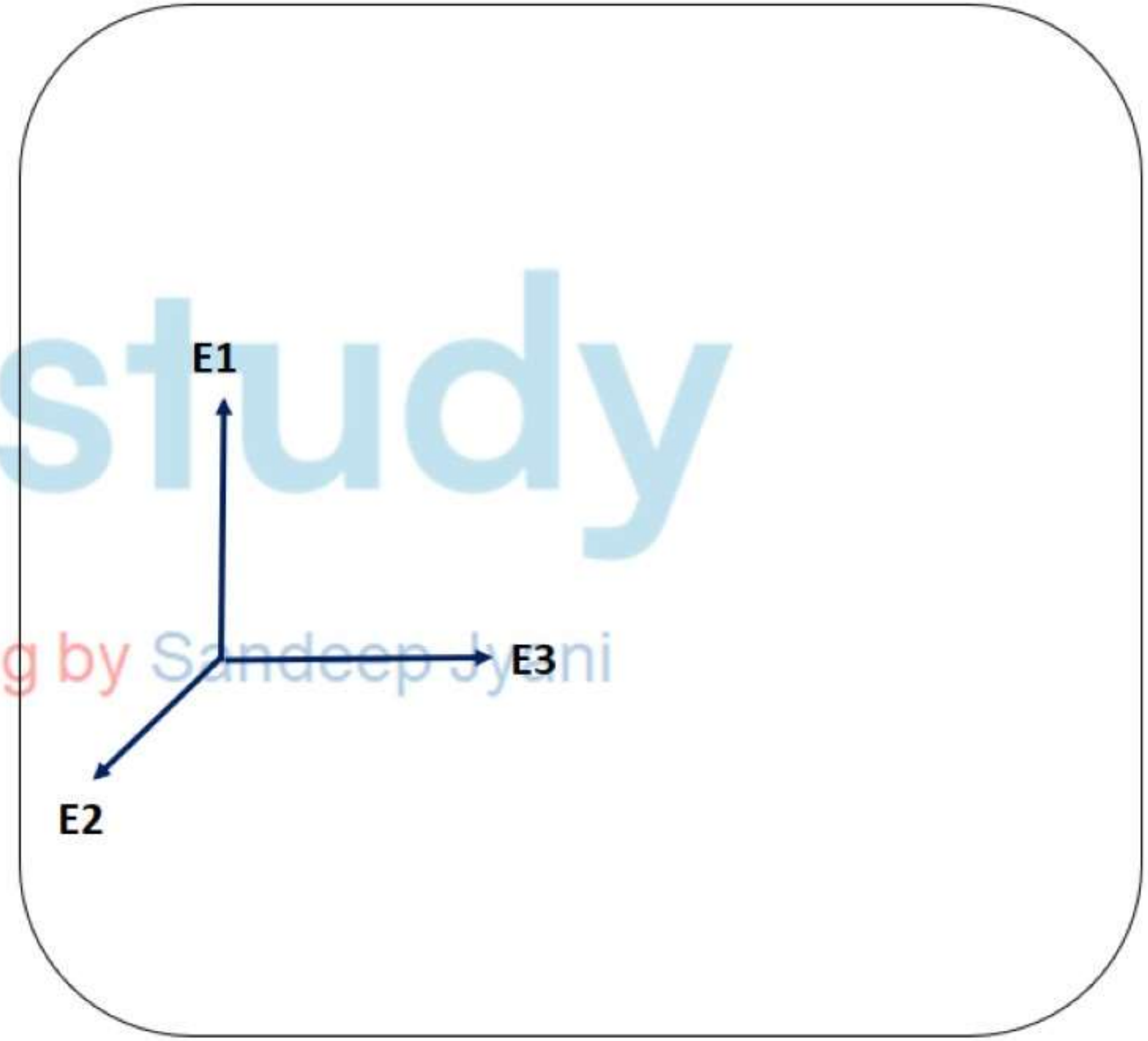
4. Anisotropic Material

- A material is said to be anisotropic when it shows *different* elastic properties IN ANY GIVEN DIRECTION AT ANY GIVEN POINT



5. Orthotropic Material

- **A material is said to be orthotropic when it shows different elastic properties IN 3 ORTHOGONAL DIRECTION AT A GIVEN POINT**
- **Total number of elastic constants = 3 x 4 =12**



Sr. No	Material	Total No. of Elastic Constants	Total Number of Independent Elastic Constant
1	Homogenous and Isotropic	4	2
2	Anisotropic	infinite	21
3	Orthotropic	12	9

Que. 17 The number of independent Elastic constants are .....for a homogenous and Isotropic material

a) (E, μ, K)

b) (E, G, μ,)

c) (E, μ,)

d) (G, μ, K)

Que. 17 The number of independent Elastic constants are …..for a homogenous and Isotropic material

Answer: two

a) (E, μ, K)
b) (E, G, μ,)
c) (E, μ,)
d) (G, μ, K)

Shear Modulus/ Modulus of Rigidity

$$Shear\ Modulus = \frac{Shear\ Stress}{Shear\ Strain}$$

$$G = \frac{\tau}{y}$$

Bulk Modulus

- $$Bulk\ Modulus = \frac{direct\ Stress}{Volumetric\ Strain}$$

$$K = \frac{\sigma}{\frac{\Delta V}{V}}$$

IMPORTANT RELATIONS
(E, G, μ, K)

$E = 2G(1+\mu)$

$E = 3K(1-2\mu)$

$$E = \frac{9KG}{3K+G}$$

For metals

$E > K > G$

Questions: Strength of Materials

Q. 18

Every material obeys. Hooke's law within its

a) Elastic limit

b) Plastic limit

c) Limit of proportionality

d) None of the above

Questions: Strength of Materials

Q. 18

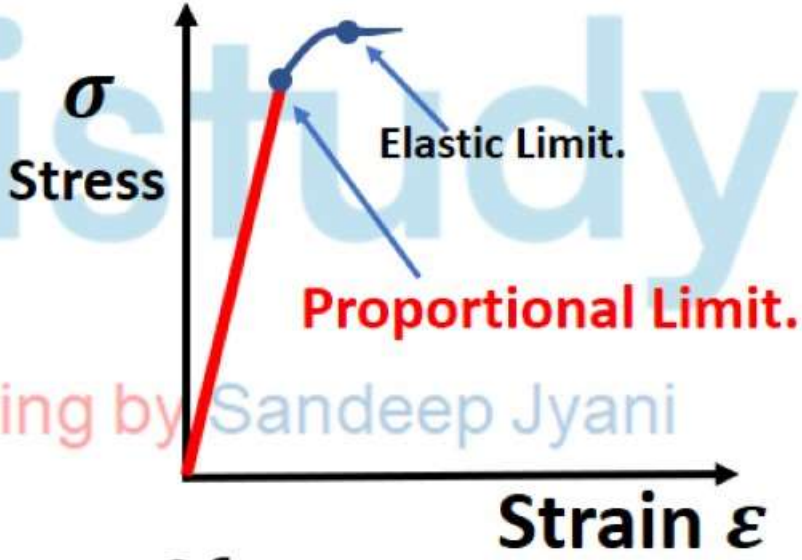
Every material obeys. Hooke's law within its

a) Elastic limit

b) Plastic limit

c) Limit of proportionality

d) None of the above



$$\sigma \propto \varepsilon$$

Questions: Strength of Materials

Q. 19

The limit of Poisson's ratio is:

a) 0.25

b) 0.15

c) 0.50

d) 0.65

Questions: Strength of Materials

Q. 19

The limit of Poisson's ratio is:

a) 0.25

b) 0.15

c) 0.50

d) 0.65

- $\mu_{generally}$ **0<= μ<=0.5**
- μ_{ideal} **-1<= μ<=0.5**

Q20.

Ductility of which of the following is the maximum?

a) Mild steel

b) Cast iron

c) Carbon Steel

d) Pig iron

Q20.

Ductility of which of the following is the maximum?

a) Mild steel

b) Cast iron

c) Carbon Steel

d) Pig iron

- More the carbon content, more compressive strength and less tensile strength
- Mild steel has least carbon content 0.05% to 0.25%

Doubt

Ductility of which of the following is the maximum?

a) Mild steel

b) Cast iron

c) Wrought iron

d) Pig iron

Which of the following has least carbon content?

a) Wrought iron (less than 0.1%)

b) Cast iron

c) Mild steel

d) Pig iron

✓More the carbon content, more compressive strength and less tensile strength

- Wrought iron is an iron alloy containing very little carbon (less than 0.1%)
- Steel-metal alloy of iron, carbon, manganese, sulphur, tungsten, etc

✓Mild steel has more tensile strength than wrought iron

Questions: Strength of Materials

Q21.

Relation between Young's modulus (E) and modulus of rigidity (G) is given as

a) E = 3G $(1+\mu)$

b) E = 2G $(1 - \mu)$

c) E = 2G $(1 + \mu)$

d) E = 3G $(1 - 2\mu)$

Questions: Strength of Materials

Q21.

Relation between Young's modulus (E) and modulus of rigidity (G) is given as

a) E = 3G $(1+\mu)$

b) E = 2G $(1 - \mu)$

c) E = 2G $(1 + \mu)$

d) E = 3G $(1 - 2\mu)$

E $= 2G(1 + \mu)$

E $= 3K(1 - 2\mu)$

$$E = \frac{9KG}{3K + G}$$

Questions: Strength of Materials

Q22.

The ratio of normal stress to volumetric strain in defined as

a) Young's modulus

b) Bulk Modulus

c) Rigidity Modulus

d) Tangent modulus

Questions: Strength of Materials

Q22.

The ratio of normal stress to volumetric strain in defined as

a) Young's modulus

b) Bulk Modulus

c) Rigidity Modulus

d) Tangent modulus

$$K = \frac{\sigma}{\frac{\Delta V}{V}}$$

Questions: Strength of Materials

Q23.

A material is called ductile if it

a) Has little plastic elongation range

b) Has long plastic elongation range

c) Could be hammered into a very thin sheet

d) Shows large elastic strain

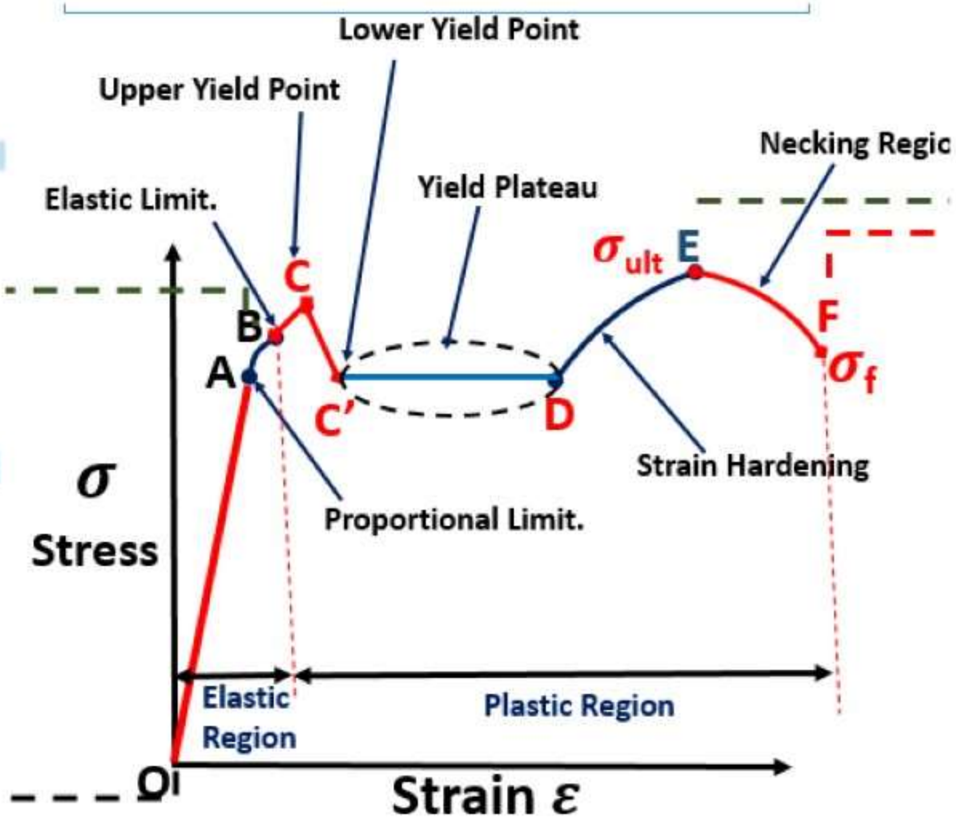
Questions: Strength of Materials

Q23.

A material is called ductile if it

a) Has little plastic elongation range

b) Has long plastic elongation range



c) Could be hammered into a very thin sheet

d) Shows large elastic strain

Questions: Strength of Materials

Q. 24

Poisson's ratio is defined as

a) Longitudinal strain/lateral strain

b) Lateral strain/longitudinal strain

c) Lateral strain × longitudinal strain

d) ½ (lateral strain) × (Longitudinal strain)

Questions: Strength of Materials

Q. 24

Poisson's ratio is defined as

a) Longitudinal strain/lateral strain

b) Lateral strain/longitudinal strain

c) Lateral strain × longitudinal strain

d) ½ (lateral strain) × (Longitudinal strain)

$$\mu = \frac{-\,lateral\ strain}{Longitudinal\ Strain}$$

Questions: Strength of Materials

Q. 25

Modulus of rigidity is expressed as

a) Compressive stress/compressive strain

b) Tensile stress/tensile strain

c) Shear stress/shear strain

d) Stress/volumetric strain

Questions: Strength of Materials

Q. 25

Modulus of rigidity is expressed as

a) Compressive stress/compressive strain

b) Tensile stress/tensile strain

c) Shear stress/shear strain

d) Stress/volumetric strain

$$G = \frac{\tau}{y}$$

$$\textit{Modulus of rigidity or Shear Modulus} = \frac{\textit{Shear Stress}}{\textit{Shear Strain}}$$

Questions: Strength of Materials

Q. 26

The ability of a material to absorb energy till the elastic limit is known as

a) Resilience

b) Ductility

c) Elasticity

d) Malleability

Questions: Strength of Materials

Q. 26

The ability of a material to absorb energy till the elastic limit is known as

a) Resilience

b) Ductility

c) Elasticity

d) Malleability

Questions: Strength of Materials

Q. 27

Out of the following, which is least elastic?

a) Silver

b) Rubber

c) Iron

d) Copper

Questions: Strength of Materials

Q. 27

Out of the following, which is least elastic?

a) Silver

b) Rubber

c) Iron

d) Copper

The more difficult it is to stretch, the more elastic a material is called to be because elasticity is defined by the ratio stress to strain and not vice versa.

$E_{steel} = 200\ GPa \quad E_{rubber} = 50\ Gpa$

$E_{rigid} = infinte$

Questions: Strength of Materials

Q. 28

The ability of a material to absorb energy till the breaking or rupture take place is known as

a) Hardness

b) Toughness

c) Brittleness

d) Softness

Questions: Strength of Materials

Q. 28

The ability of a material to absorb energy till the breaking or rupture take place is known as

a) Hardness

b) Toughness

c) Brittleness

d) Softness

Questions: Strength of Materials

Q. 29

The property of a material by which it gets permanent deformation under a load which is not recovered after removal of load is called

a) Elasticity

b) Brittleness

c) Ductility

d) Plasticity

Questions: Strength of Materials

Q. 29

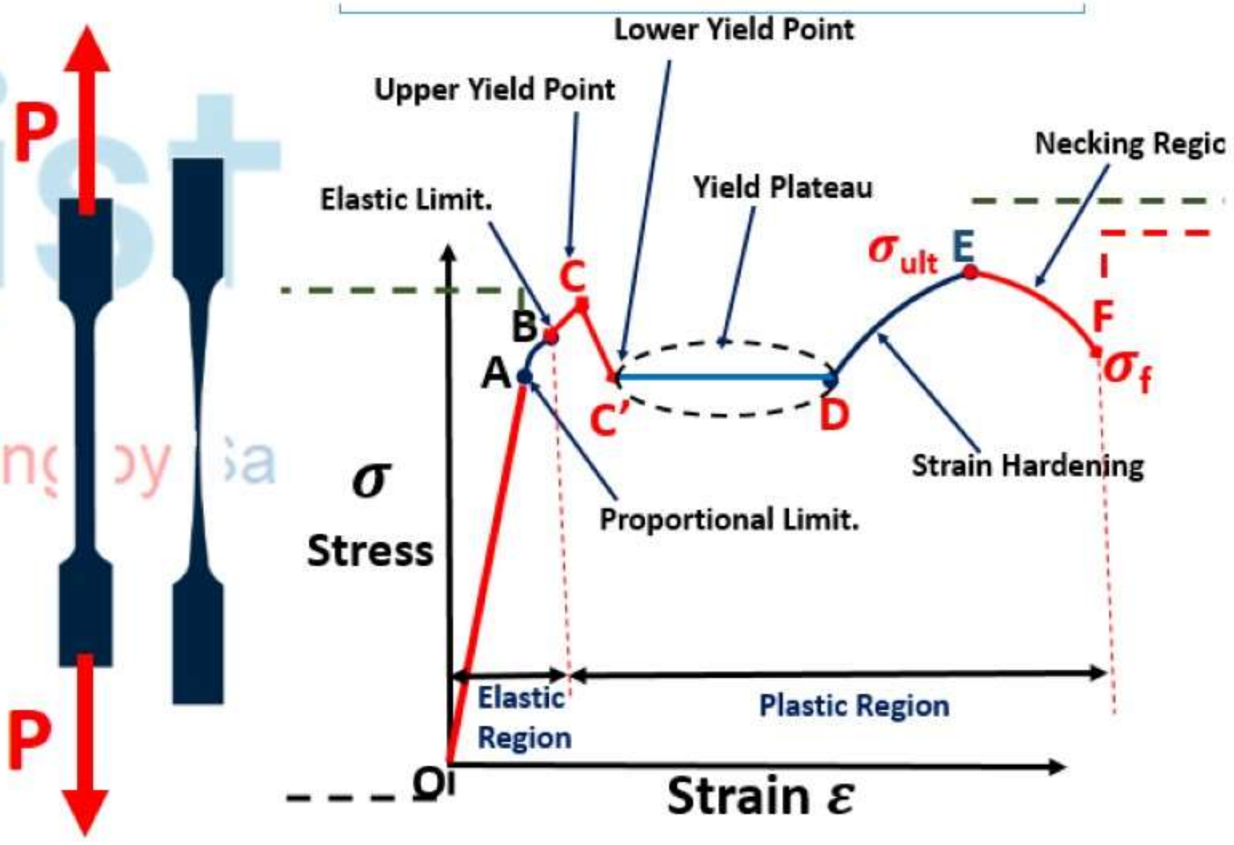
The property of a material by which it gets permanent deformation under a load which is not recovered after removal of load is called

a) Elasticity

b) Brittleness

c) Ductility

d) Plasticity



Questions: Strength of Materials

Q. 30

Which of the following has least carbon content?

a) Wrought iron

b) Cast iron

c) Mild steel

d) Pig iron

Questions: Strength of Materials

Q. 30

Which of the following has least carbon content?

a) <u>Wrought iron (less than 0.1%)</u>

b) Cast iron

c) Mild steel

d) Pig iron

Pig iron **(4–5%)**
> *Cast Iron*(2–4.5%)
> *Cast Steel* (>2%)
> *Carbon steel (less than* 2%)
> *High carbon steel* (0.6–1.4%)
>*Medium carbon*(0.25–0.6%)
>*low carbon steel (less than* 0.25%)
> *Wrought Iron (less than* 0.1%)
>*Pure iron* (0%)

Questions: Strength of Materials

Q. 31

Total number of elastic constant for isotropic material are

a) 2

b) 3

c) 4

d) 5

Questions: Strength of Materials

Q. 31

Total number of elastic constant for isotropic material are

a) 2

b) 3

c) 4

d) 5

An isotropic material has two elastic components (Young's Modulus, Poisson's ratio)

Questions: Strength of Materials

Q. 32

Creep of a material is

a) Not being ductile

b) To become brittle

c) Disappearance of deformation on removal of load

d) Continued deformation with time under sustained loading

Questions: Strength of Materials

Q. 32

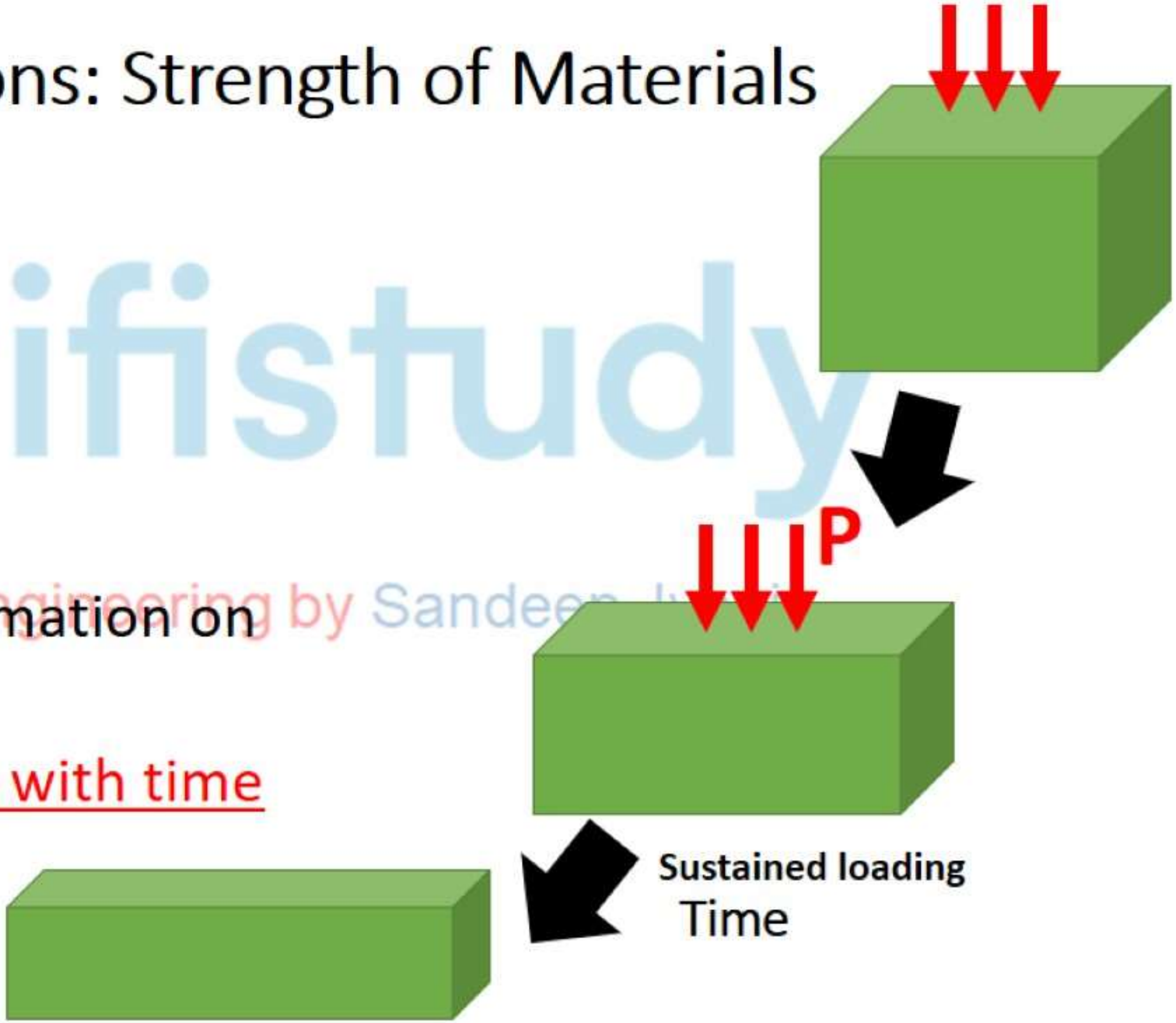
Creep of a material is

a) Not being ductile

b) To become brittle

c) Disappearance of deformation on removal of load

<u>d) Continued deformation with time under sustained loading</u>



Questions: Strength of Materials

Q. 33

Which of the following is a relatively ductile material?

a) High carbon steel

b) Bronze

c) Mild steel

d) Cast iron

Questions: Strength of Materials

Q. 33

Which of the following is a relatively ductile material?

a) High carbon steel

b) Bronze

c) Mild steel

d) Cast iron

Wrought iron (less than 0.08%)
Cast iron (2.1% to 4%)
Mild steel (less than 0.25%)
Pig iron (3.5 percent)

Questions: Strength of Materials

Q. 34

One cubic metre of mild steel weighs about

a) 1000 kg

b) 3625 kg

c) 7850 kg

d) 12560 kg

Questions: Strength of Materials

Q. 34

One cubic metre of mild steel weighs about

a) 1000 kg

b) 3625 kg

c) 7850 kg

d) 12560 kg

Questions: Strength of Materials

Q. 35

In Brunel Hardness test, the type of indenter used is

a) Hard steel ball

b) Diamond cone

c) Mild steel ball

d) Hard steel cone

Questions: Strength of Materials

Q. 35

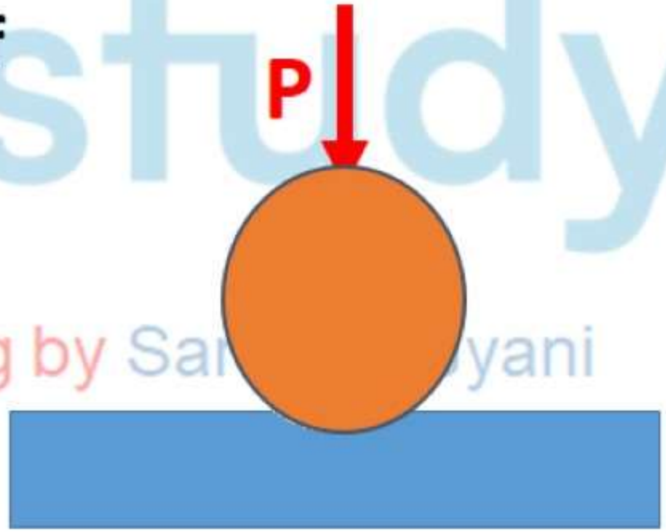
In Brunel Hardness test, the type of indenter used is

a) Hard steel ball

b) Diamond cone

c) Mild steel ball

d) Hard steel cone



Questions: Strength of Materials

Q. 36

Percentage increase of carbon in steel, decreases its

a) Hardness

b) Ductility

c) Strength

d) Brittleness

Questions: Strength of Materials

Q. 36

Percentage increase of carbon in steel, decreases its

a) Hardness

b) Ductility

c) Strength

d) Brittleness

Questions: Strength of Materials

Q. 37

Which of the following materials is expected to have the least value of Young's modulus of elasticity?

a) Wood

b) Copper

c) Glass

d) Aluminum

Questions: Strength of Materials

Q. 37

Which of the following materials is expected to have the least value of Young's modulus of elasticity?

a) Wood

b) Copper

c) Glass

d) Aluminum

Q. 38

The weight of 10mm diameter mild steel rod per metre length is equal to

a) 0.22 kg

b) 0.32 kg

c) 0.42 kg

d) 0.62 kg

Questions: Strength of Materials

Q. 38

The weight of 10mm diameter mild steel rod per metre length is equal to

a) 0.22 kg

b) 0.32 kg

c) 0.42 kg

d) 0.62 kg

Density of mild steel = 7850 kg/m^3

Weight of 1 metre long rod = vol $\times$ $density$

$$= \left(\frac{\pi d^2}{4} \times 1m\right) \times 7850$$

$$= \left(\frac{\pi (0.01)^2}{4} \times 1\right)$$

= 0.62kg

Questions: Strength of Materials

Q. 39

Match List – 1 with List 2

List – 1	List – 2
1. Young Modulus	a) Lateral Strain to linear strain within elastic unit
2. Poisson's Ratio	b) Direct stress to normal strain within elastic limit.
3. Bulk Modulus	c) Shear stress to shear strain within elastic limit.
4. Rigidity Modulus	d) Direct stress to corresponding volumetric strain.

Questions: Strength of Materials

Q. 39 **1B, 2A, 3D, 4C**

Match List – 1 with List 2

List – 1	List – 2
1. Young Modulus	a) Lateral Strain to linear strain within elastic unit
2. Poisson's Ratio	b) Direct stress to normal strain within elastic limit.
3. Bulk Modulus	c) Shear stress to shear strain within elastic limit.
4. Rigidity Modulus	d) Direct stress to corresponding volumetric strain.

Questions: Strength of Materials

Q. 40

The ratio of young's modulus to modulus of rigidity for a material having Poisson's ratio 0.2 is

a) 2.4

b) 0.416

c) 0.357

d) 2.8

Questions: Strength of Materials

Q. 40

The ratio of young's modulus to modulus of rigidity for a material having Poisson's ratio 0.2 is

a) 2.4

b) 0.416

c) 0.357

d) 2.8

$$E = 2G(1 + \mu)$$

$$E/G = 2(1 + \mu)$$

$$= 2(1+0.2)$$

$$= 2.4$$

Questions: Strength of Materials

Q.41 What will be the relation between E (Young's modulus of Elasticity) and K (Bulk Modulus), when Poisson's ratio is 0.25?

a) E =K

b) E = 2K

c) E = 1.5K

d) E = K = 0

Questions: Strength of Materials

Q.41 What will be the relation between E (Young's modulus of Elasticity) and K (Bulk Modulus), when Poisson's ratio is 0.25?

a) E =K

b) E = 2K

c) E = 1.5K

d) E = K = 0

E $= 3K(1 - 2\mu)$

E $= 3K(1 - 2 \times 0.25)$

E $= 3K(0.5)$

E $= 1.5K$

Questions: Strength of Materials

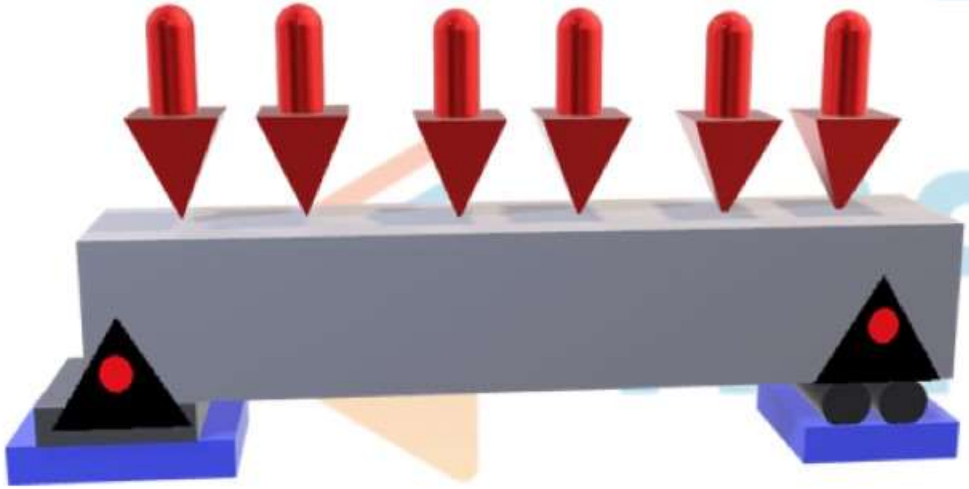
Important question:

Is Strength of Materials easy ?

a) **Yes!**
b) **Surely!**
c) **Definitely!**
d) **Of course!**

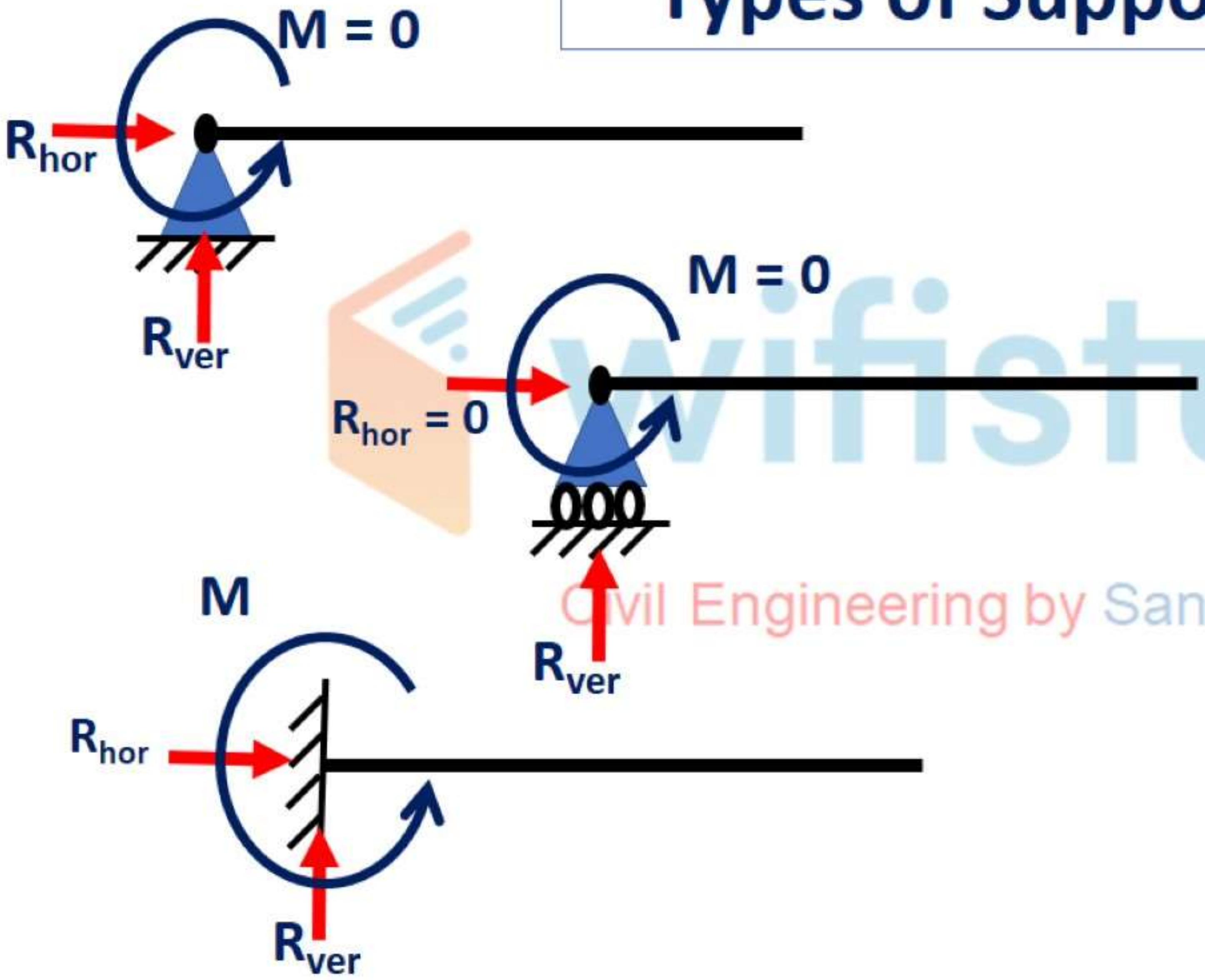
BEAM

BEAM



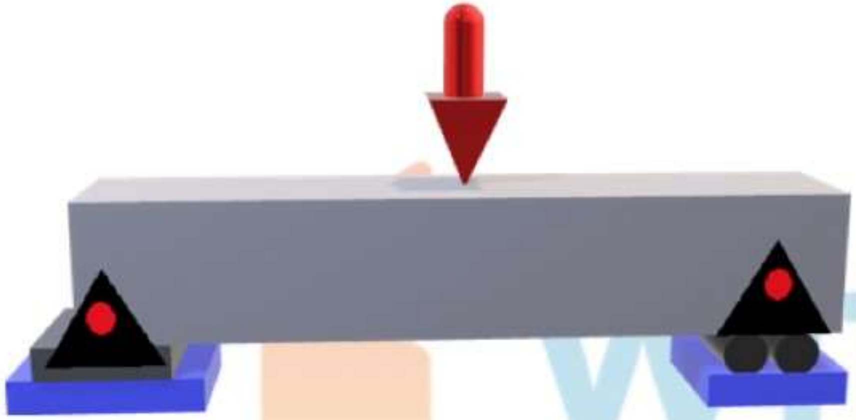
- **Beam is defined as the structural member which is subjected to *transverse shear load*, due to this *transverse shear load*, beams are subjected to variable shear force and variable bending moment over the length of the beam.**
- **Hence to know types of variation and maximum value of Shear Force and Bending Moment , SFD and BMD are drawn.**

Types of Support

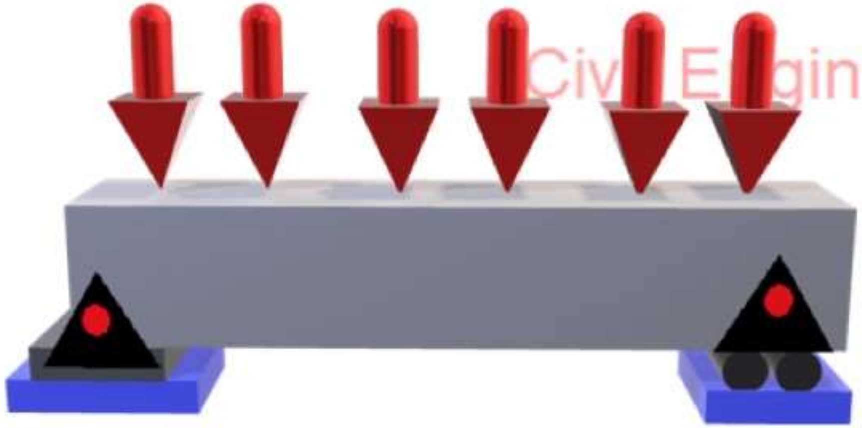


1. **Hinge Support**
   - **2 reactions**
2. **Roller Support**
   - **1 reaction**
3. **Fixed Support**
   - **2 Reactions, 1 Moment**

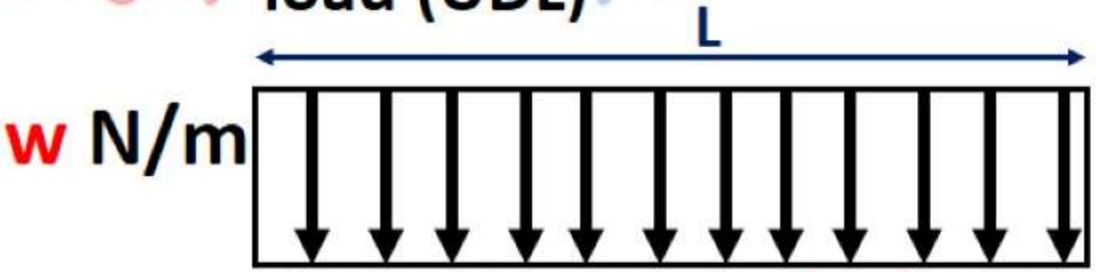
Types of Loading



1. **Concentrated or Point Load**
   - **Load that acts over a point or very small area**



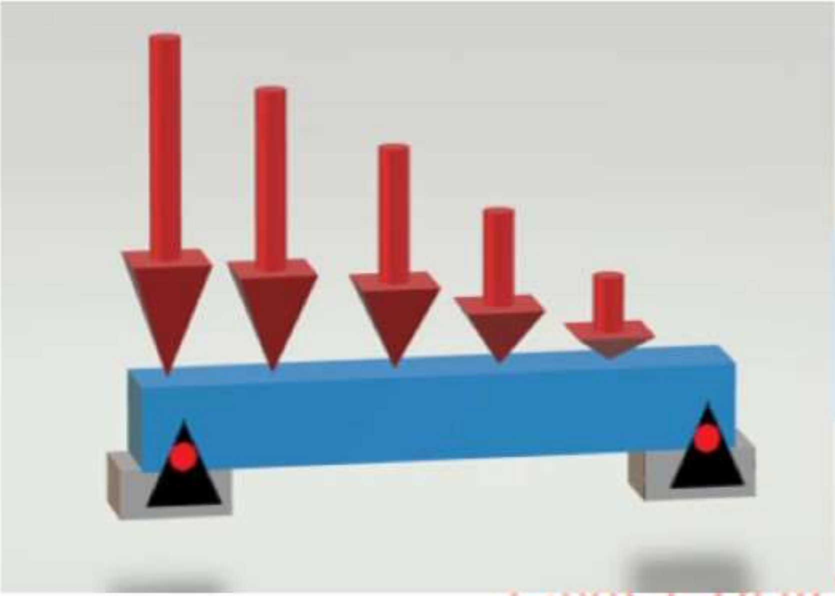
2. **Uniformly distributed load (UDL)**



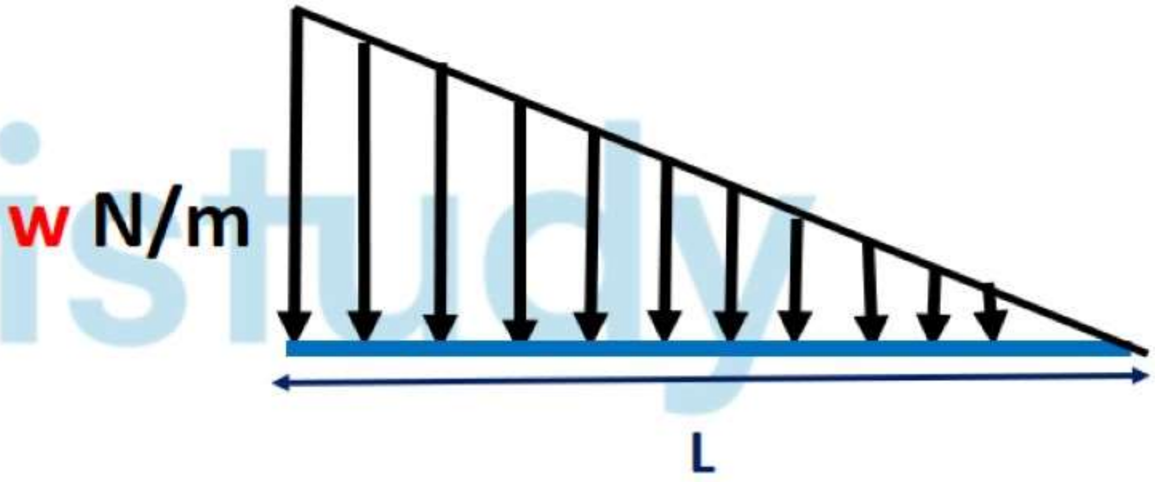
Total Load= area of loading diagram

$W = wL$

Types of Loading



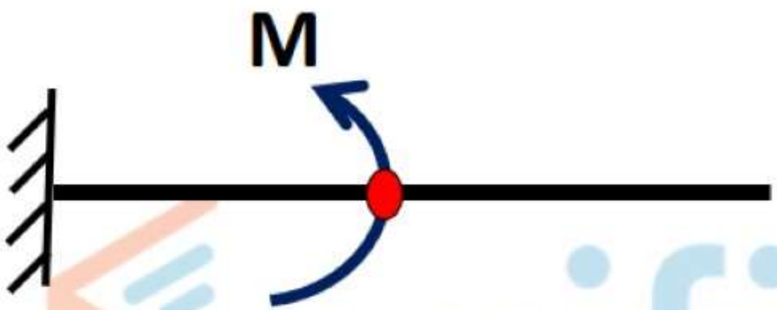
3. Uniformly Varying Load (UVL)



Total Load= area of loading diagram

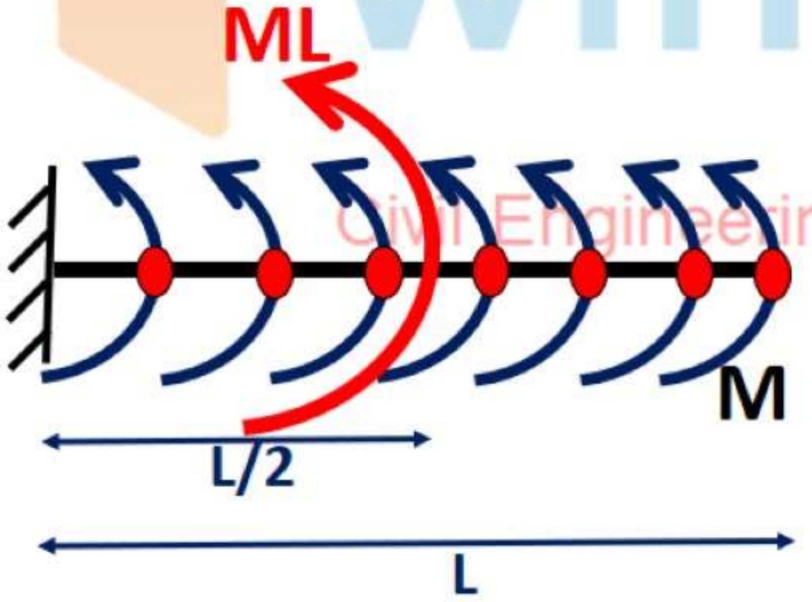
$$W = \frac{1}{2} \times w \times L$$

Types of Loading



4. Concentrated Moment

- A moment at a point is called as Concentrated Moment



$M\ (\frac{N-m}{m})$

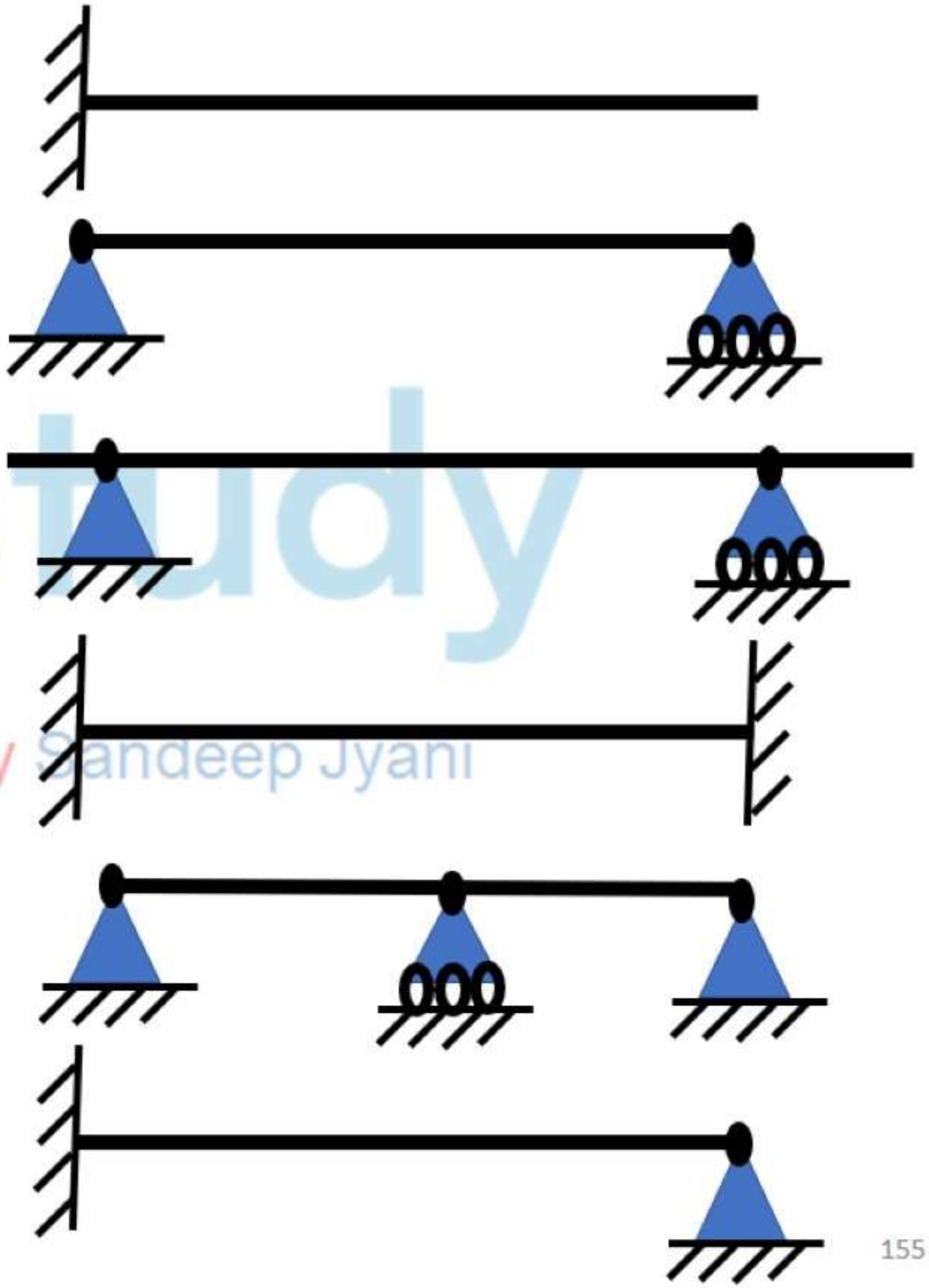
5. Uniformly Distributed Moment (UDM)

Total Moment= UDM x Length

$W = M \times L$

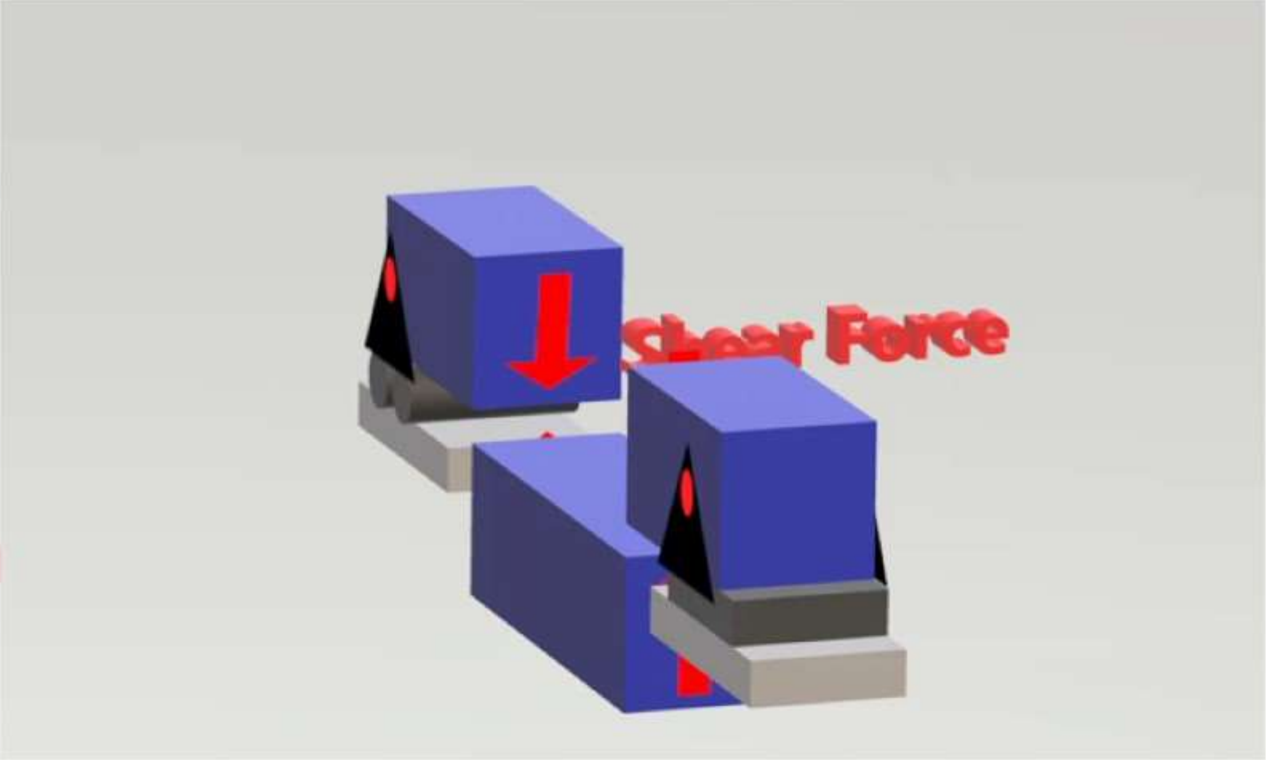
Types of Beams

- **Statically Determinate**
  - Cantilever
  - Simply Supported
  - Overhang Beam
- **Statically Indeterminate Beams**
  - Fixed Beam
  - Continuous Beam
  - Propped Beam

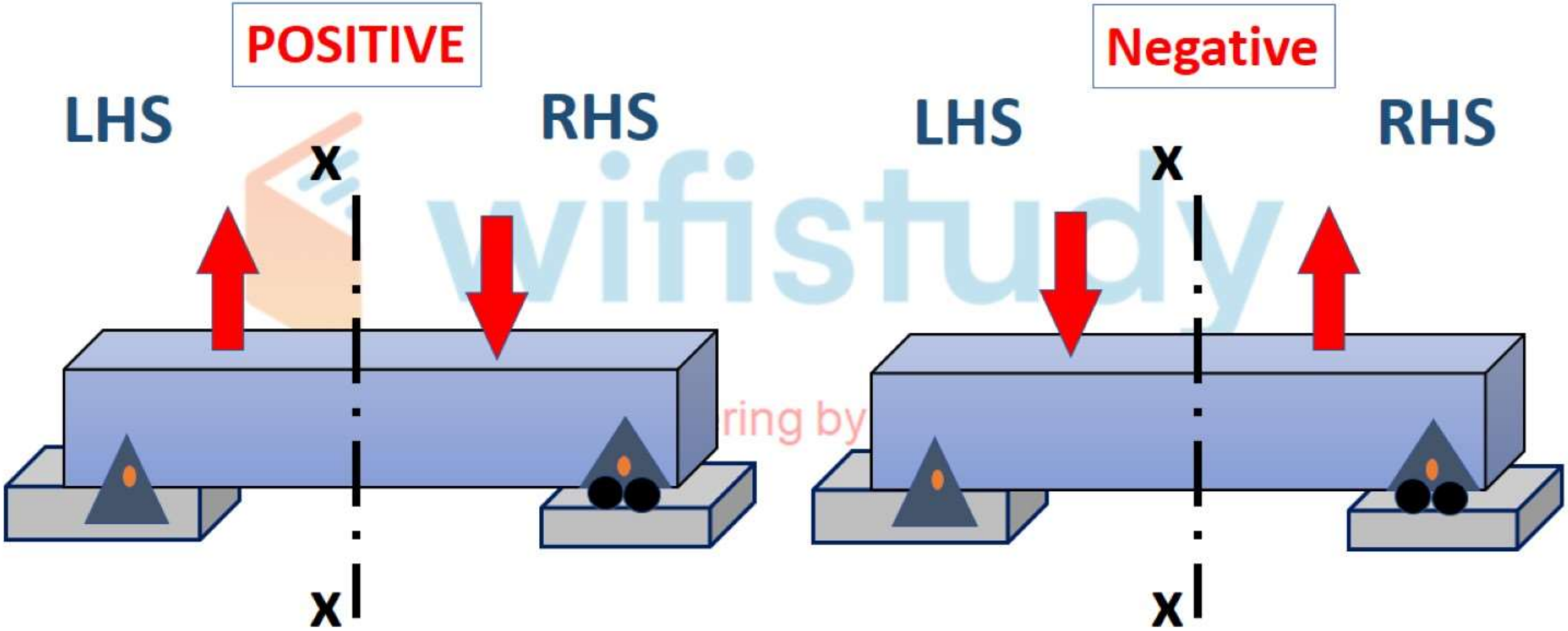


Shear Force

- **Algebraic sum of all the vertical forces at any section of the beam, to the right or to the left of the section Is known as Shear Force.**
- **It is shortly written as <u>SF</u>**



Sign Convention for Shear Force:

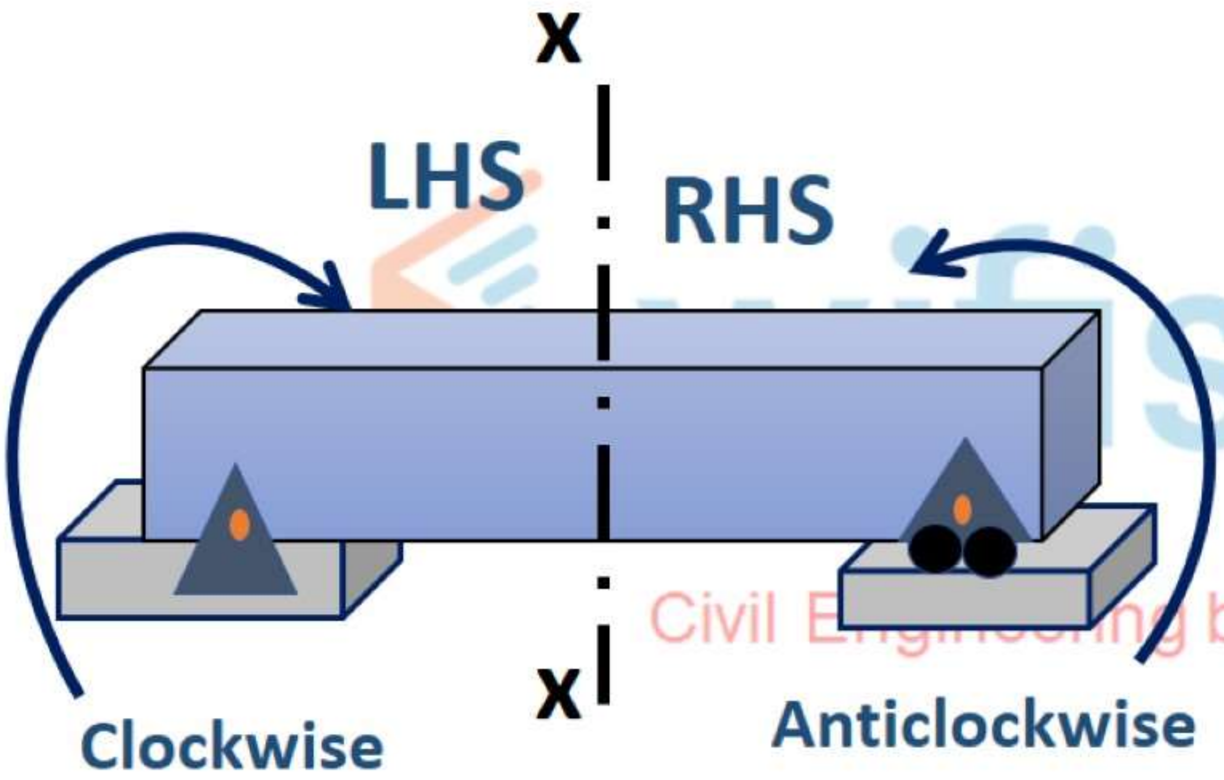


*RUN- Right side Upward force is Negative

Bending Moment

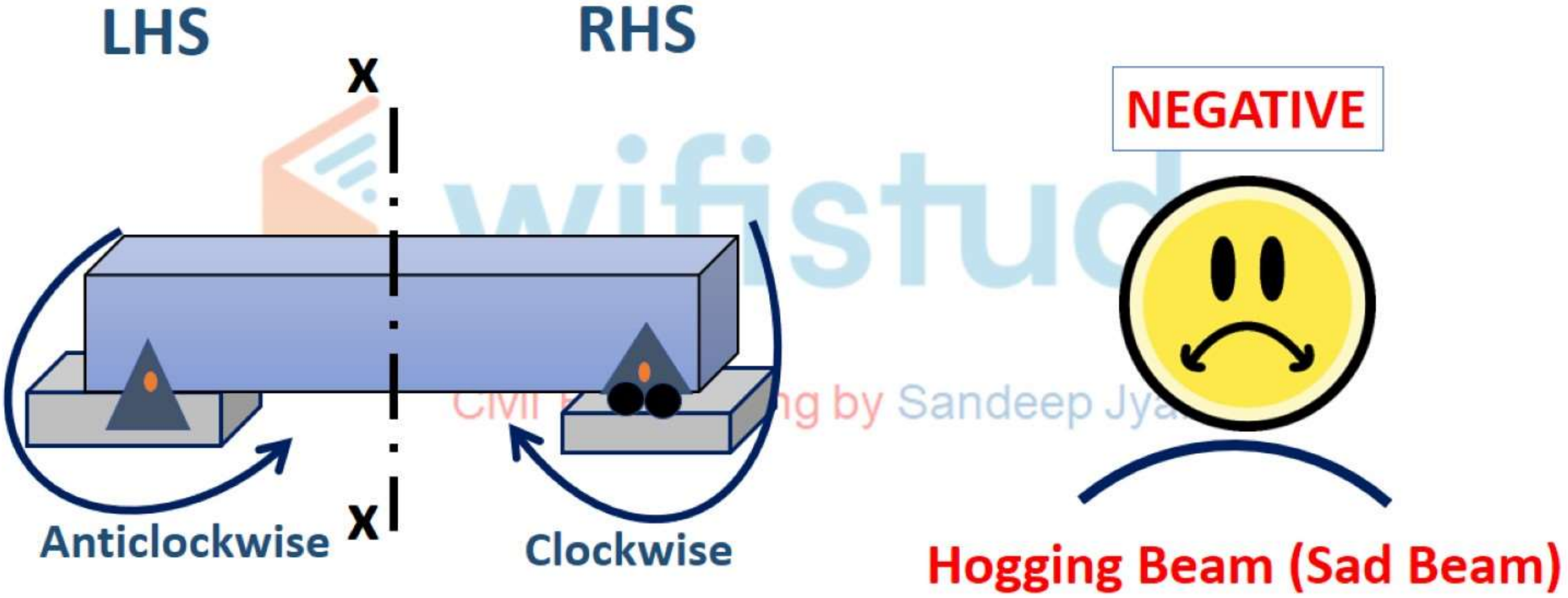
- **Algebraic Sum of the moments of all the forces acting to the left or right of the section is known as Bending Moment**
- **It is shortly written as <u>BM</u>**

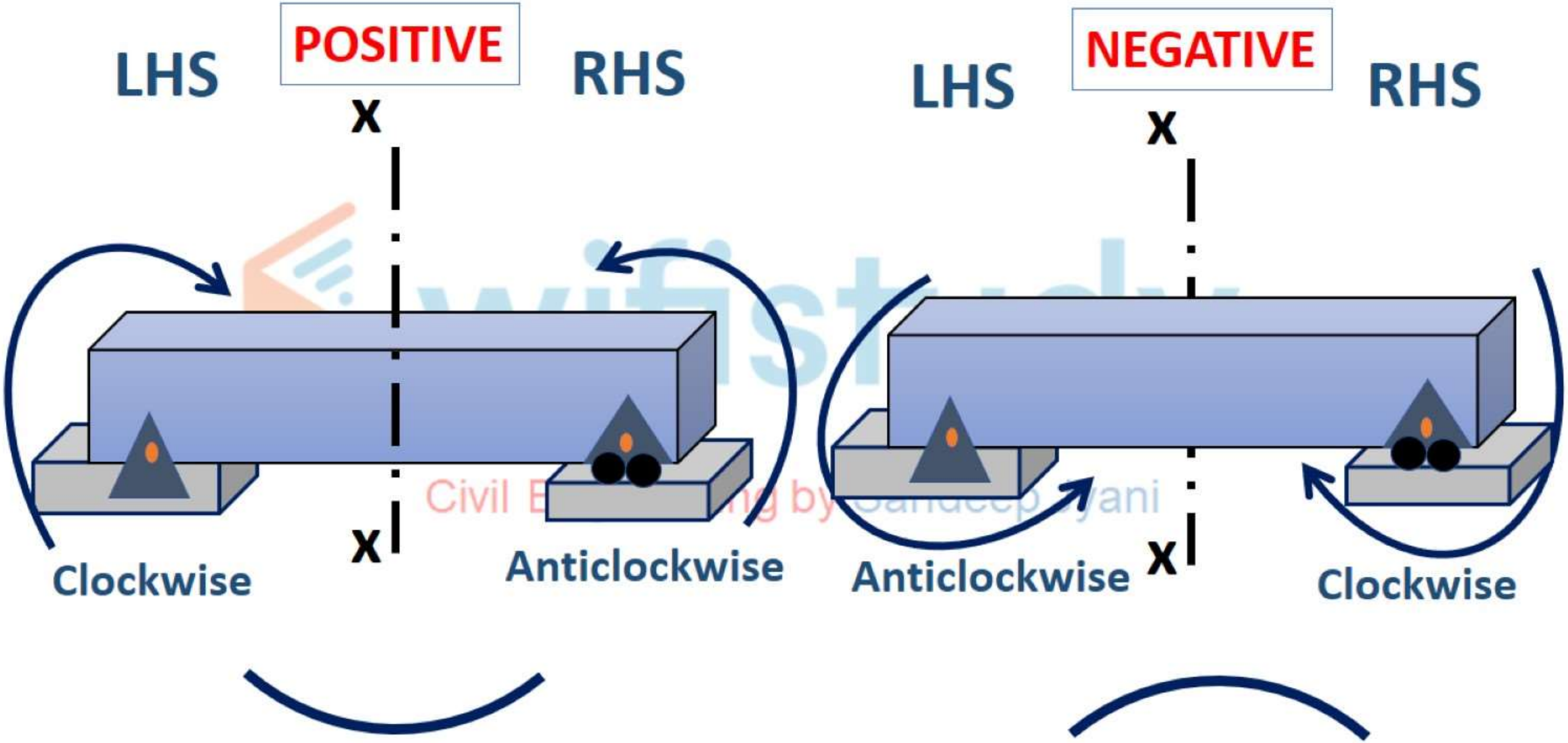
Sign Convention for Bending Moment



Sagging Beam (Happy Beam)

Sign Convention for Bending Moment



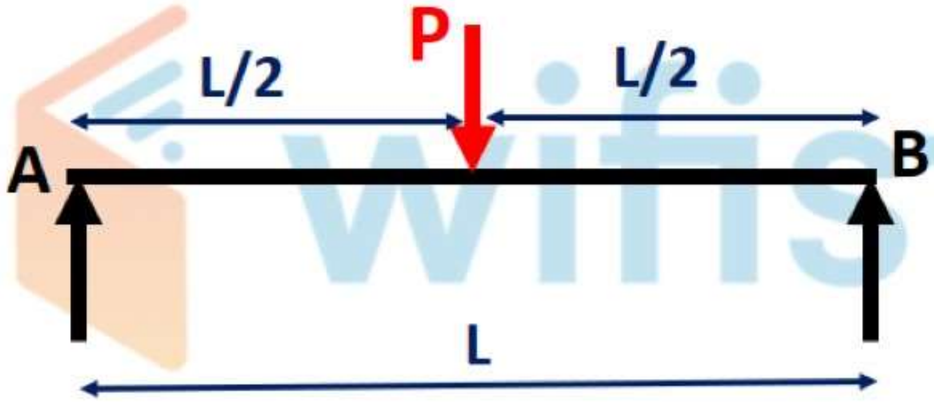


Sagging Beam (Happy Beam)

Hogging Beam (Sad Beam)

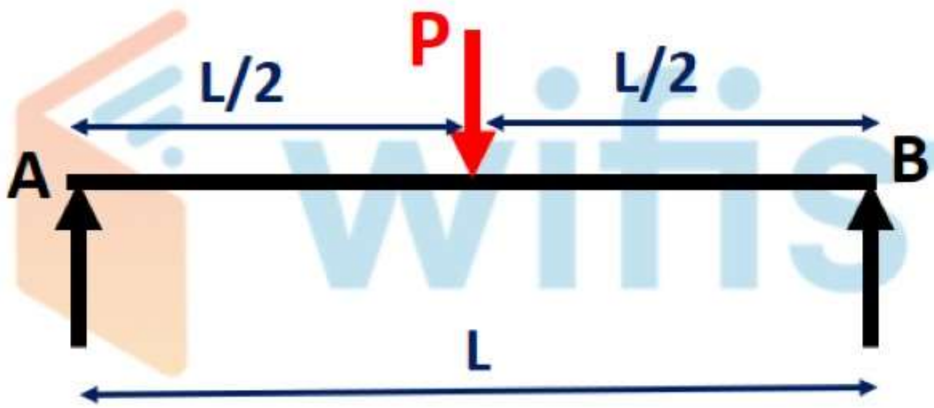
Calculation of Reactions

Que. 42 Calculate reactions at A and B



Calculation of Reactions

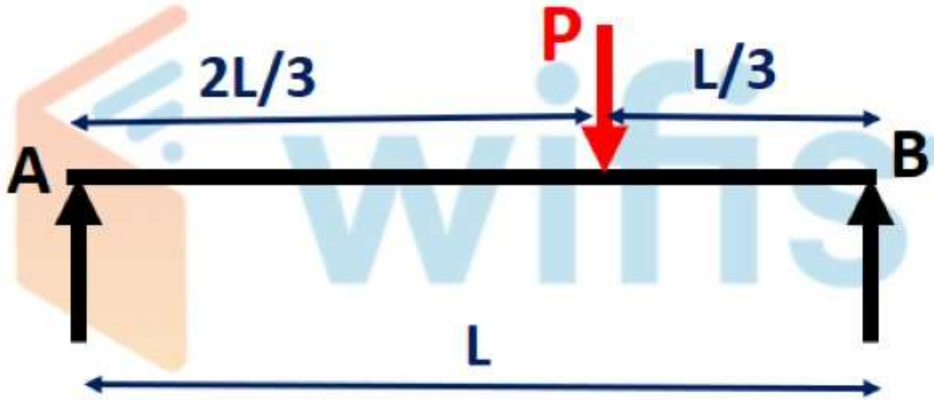
Que. 42 Calculate reactions at A and B



Short Trick: $R_a = \frac{Total\ Load}{Total\ Distance} \times distance\ of\ the\ opposite\ side$

Calculation of Reactions

Que. 43 Calculate reactions at A and B

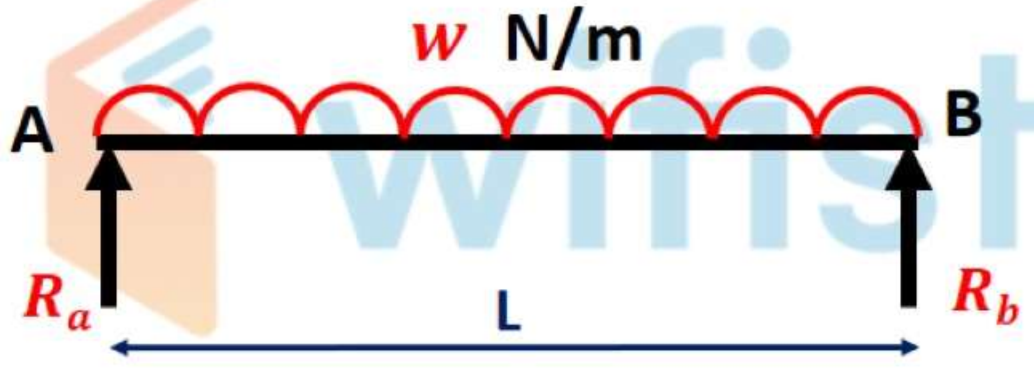


Short Trick: $R_a = \frac{Total\ Load}{Total\ Distance} \times distance\ of\ the\ opposite\ side$

$R_a = \frac{P}{L} \times \frac{L}{3}$ $\qquad$ $R_b = \frac{P}{L} \times \frac{2L}{3}$

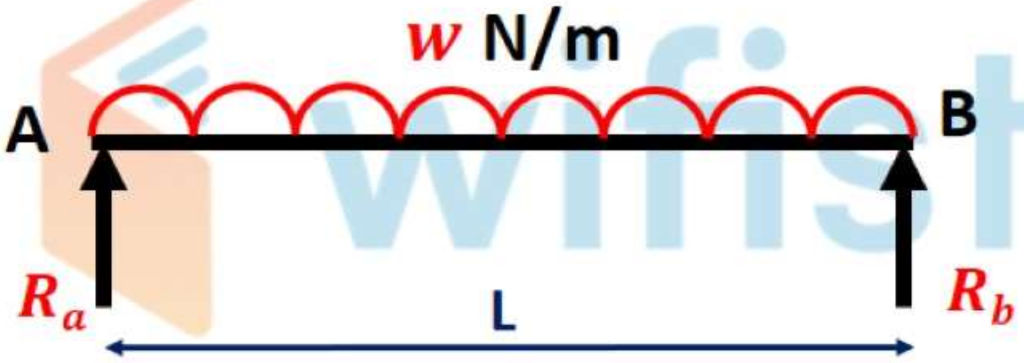
Calculation of Reactions

Que. 44 Calculate reactions at A and B



Calculation of Reactions

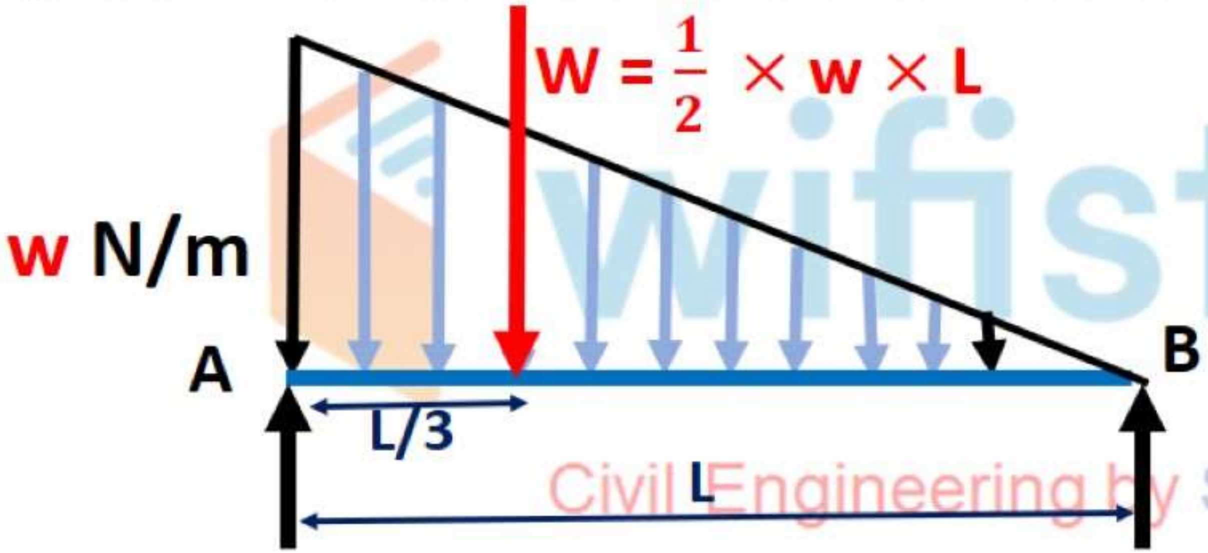
Que. 44 Calculate reactions at A and B



$$R_a = R_b = \frac{wL}{2}$$

Calculation of Reactions

Que. 45 Calculate reactions at A and B

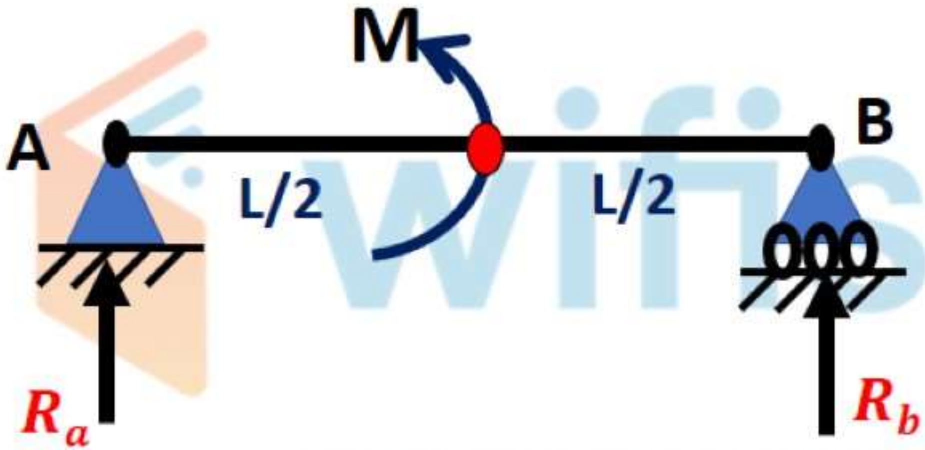


Total Load= area of loading diagram

$$W = \frac{1}{2} \times w \times L$$

Calculation of Reactions

Que. 46 Calculate reactions at A and B



$$R_a + R_b = 0$$

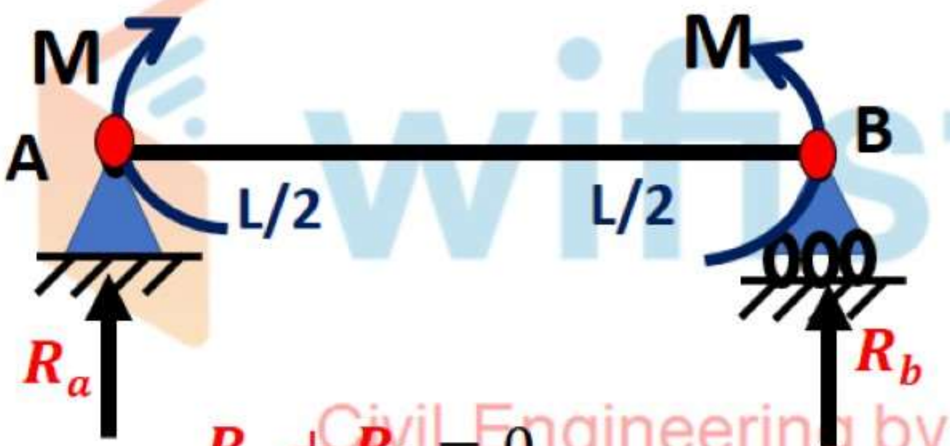
$$\Sigma M_A = 0$$

$$-R_b \times L - M = 0$$

$$R_b = -\frac{M}{L} \qquad R_a = +\frac{M}{L}$$

Calculation of Reactions

Que. 47 Calculate reactions at A and B



$$R_a + R_b = 0$$

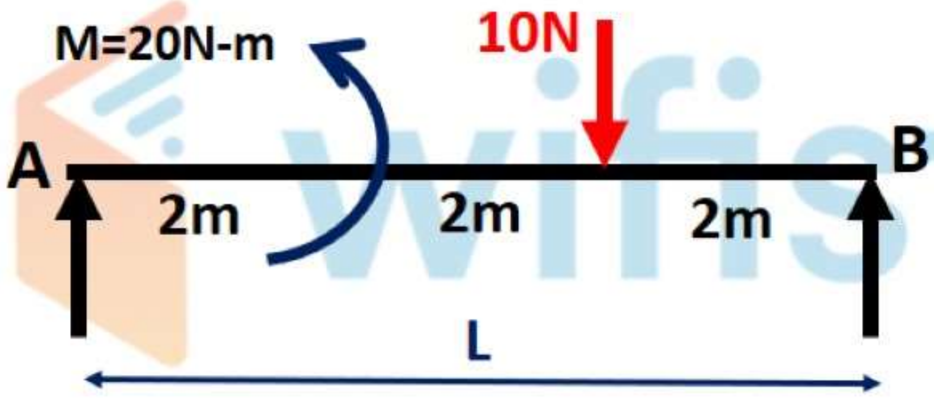
$$\Sigma M_A = 0$$

$$-R_b \times L - M + M = 0$$

$$R_b = 0 \qquad R_a = 0$$

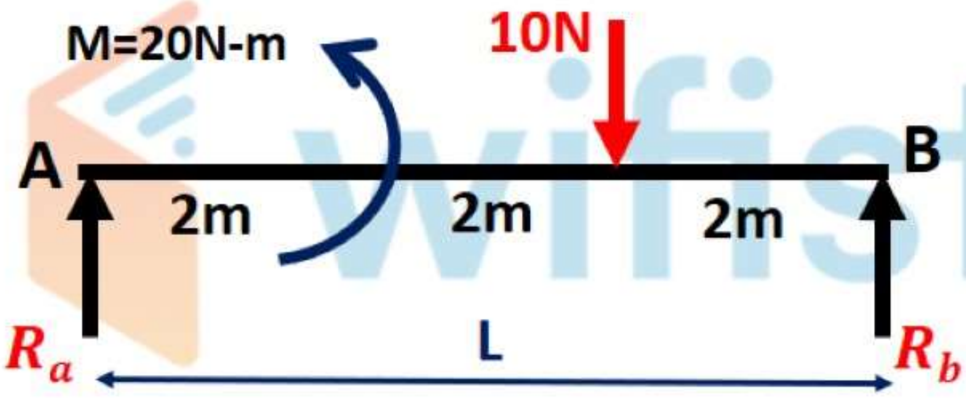
Calculation of Reactions

Que. 48 Calculate reactions at A and B



Calculation of Reactions

Que. 48 Calculate reactions at A and B

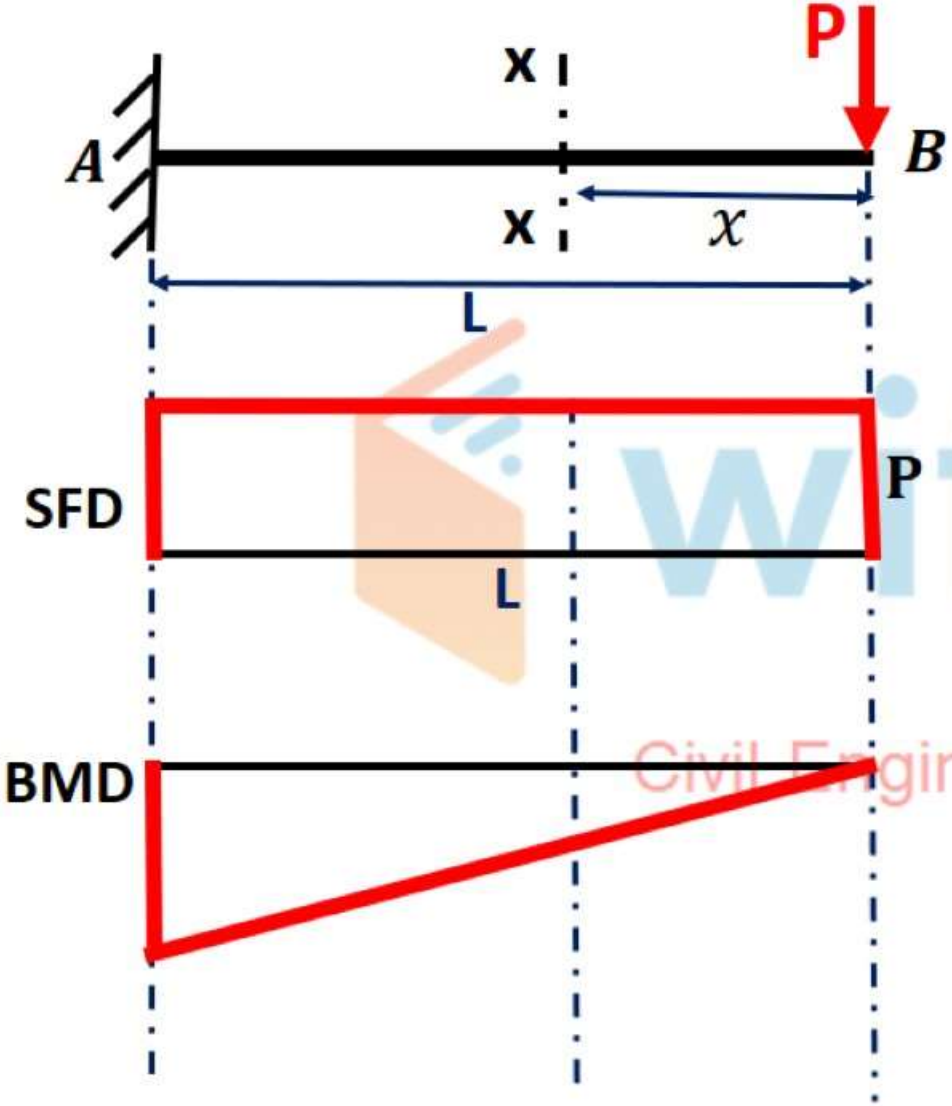


$$R_a + R_b = 10$$

$$\Sigma M_A = 0$$

$$-R_b \times 6 + 10 \times 4 - 20 = 0$$

$$R_b = \frac{10}{3} N \qquad R_a = \frac{20}{3} N$$



CASE 1: CANTILEVER BEAM

a) Cantilever beam subjected to Point load at the free end

$(SF)_{x-x} = \mathrm{P}$

$(M)_{x-x} = -\mathrm{P} \times x$

$(SF)_A = \mathrm{P}$

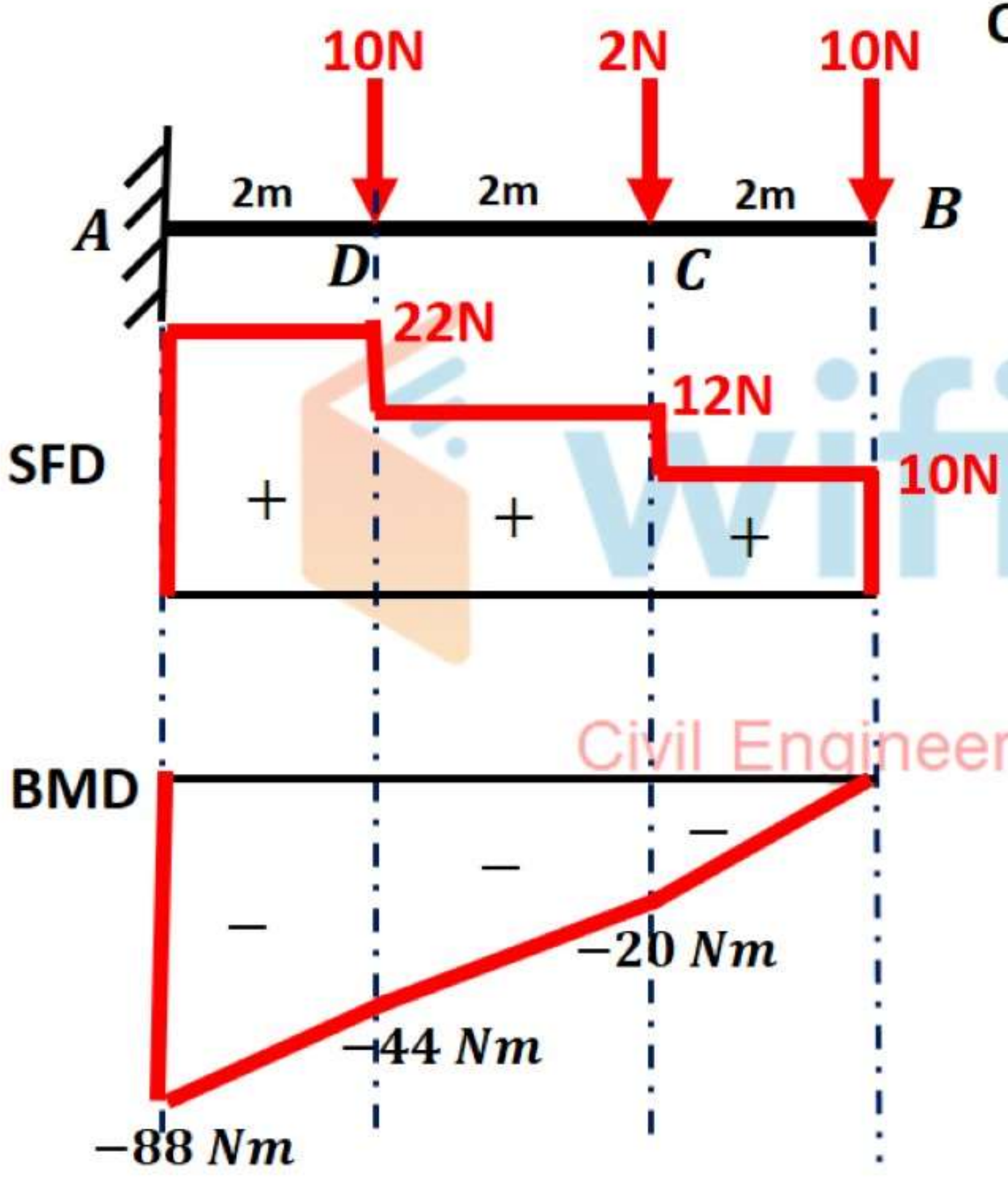
$(M)_B = -\mathrm{P} \times 0 = 0$

$(SF)_B = \mathrm{P}$

$(M)_A = -\mathrm{P} \times L = -PL$

$(M)_B - (M)_A = area\ of\ SFD\ btw\ B\ and\ A$

Que. 49 Draw SFD and BMD for the following



$$(M)_{x-x} = -P \times x$$

$(M)_B - (M)_C =$ *area of SFD btw B and C*

$\Rightarrow (M)_B - (M)_C = 10 \times 2$

$\Rightarrow 0 - (M)_C = 20$

$\Rightarrow (M)_C = -20\ Nm$

$(M)_C - (M)_D =$ *area of SFD btw C and D*

$(M)_C - (M)_D = 12 \times 2$

$\Rightarrow -20 - (M)_D = 12 \times 2$

$\Rightarrow -(M)_D = 24 + 20$

$\Rightarrow (M)_D = -44$ Nm

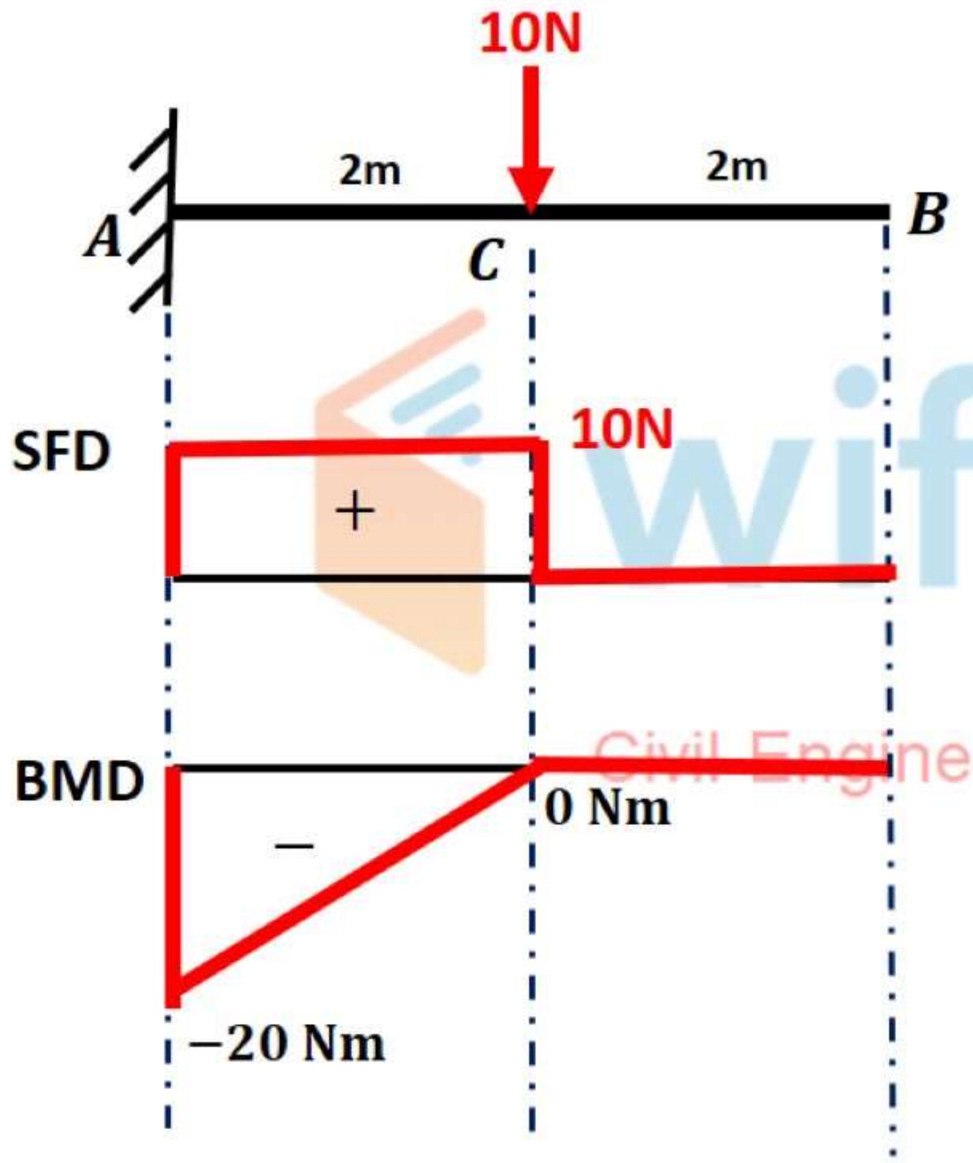
$(M)_D - (M)_E = 22 \times 2 = 44$

$\Rightarrow -44 - (M)_E = 22 \times 2 = 44$

$\Rightarrow -(M)_E = 88$

$\Rightarrow (M)_E = -88\ Nm$

Que. 50 Draw SFD and BMD for the following



$(M)_B - (M)_C = area\ of\ SFD\ btw\ B\ and\ C$

$\Rightarrow 0 - (M)_C = 0$

$\Rightarrow 0 - (M)_C = 0$

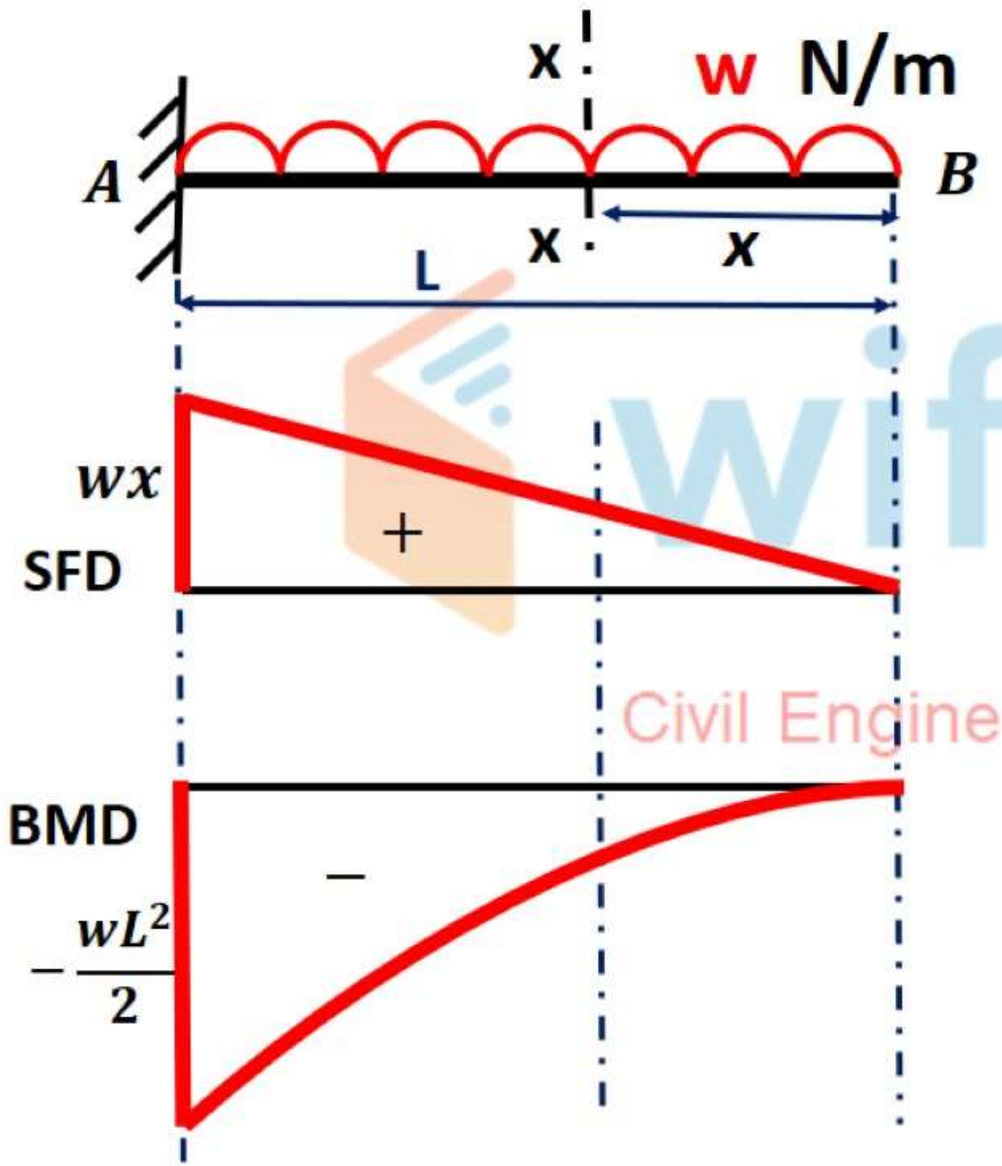
$\Rightarrow (M)_C = 0\ Nm$

$(M)_C - (M)_A = area\ of\ SFD\ btw\ C\ and\ A$

$\Rightarrow 0 - (M)_A = 10 \times 2$

$\Rightarrow -(M)_A = 20$

$\Rightarrow (M)_A = -20\ \text{Nm}$



CASE 1: CANTILEVER BEAM

b) Cantilever beam subjected to Uniformly Distributed Load (UDL)

$$(SF)_{x-x} = wx$$

$$(SF)_A = wL$$

$$(SF)_B = 0$$

$$(M)_{x-x} = -w \times x \times \frac{x}{2}$$

$$(M)_{x-x} = -\frac{wx^2}{2}$$

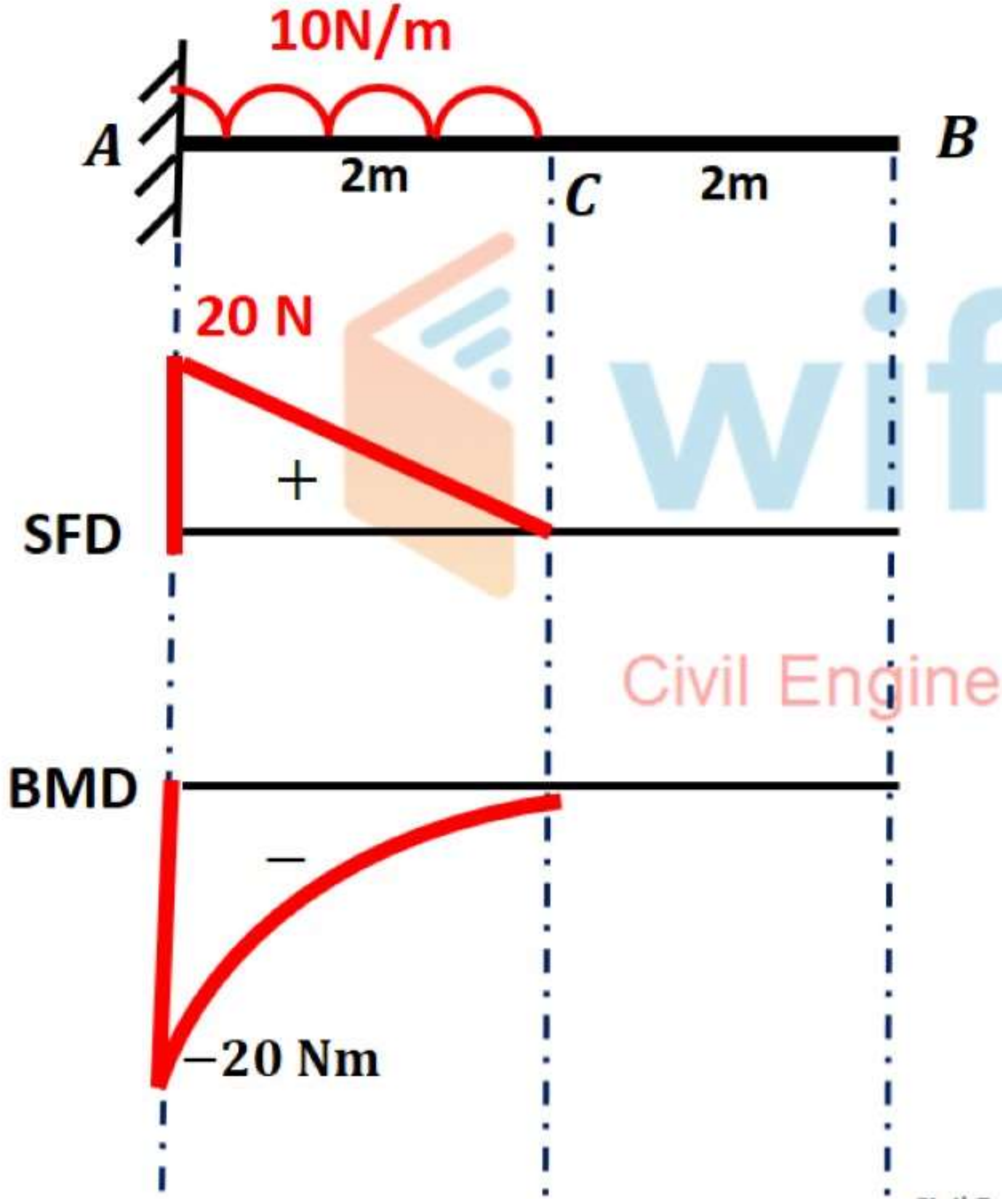
$$(M)_B = 0 \ \ at\ x = 0$$

$$(M)_A = -\frac{wL^2}{2} \ \ at\ x = L$$

When SFD is rectangular, BMD triangular

When SFD is trianglular, BMD is parabolic

Que. 50 Draw SFD and BMD for the following



$$(M)_C = 0$$

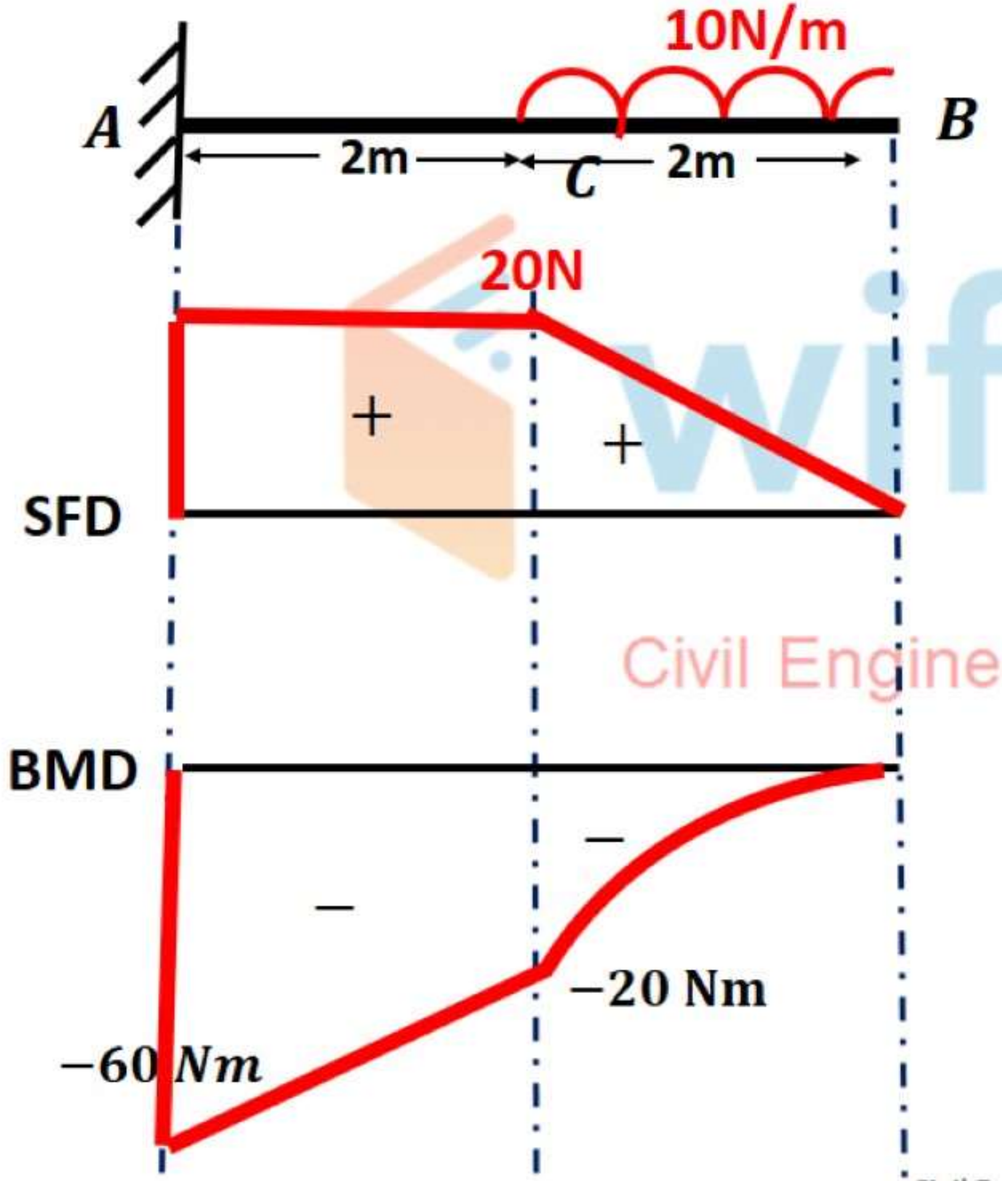
$$(M)_C - (M)_A = area\ of\ SFD\ btw\ C\ and\ A$$

$$(M)_C - (M)_A = \frac{1}{2} \times 20 \times 2$$

$$\Rightarrow 0 - (M)_D = 20$$

$$\Rightarrow (M)_D = -20\ \text{Nm}$$

Que. 51 Draw SFD and BMD for the following



$$(M)_B = 0$$

$$(M)_B - (M)_C = \textit{area of SFD btw B and C}$$

$$\Rightarrow 0 - (M)_C = \frac{1}{2} \times 20 \times 2$$

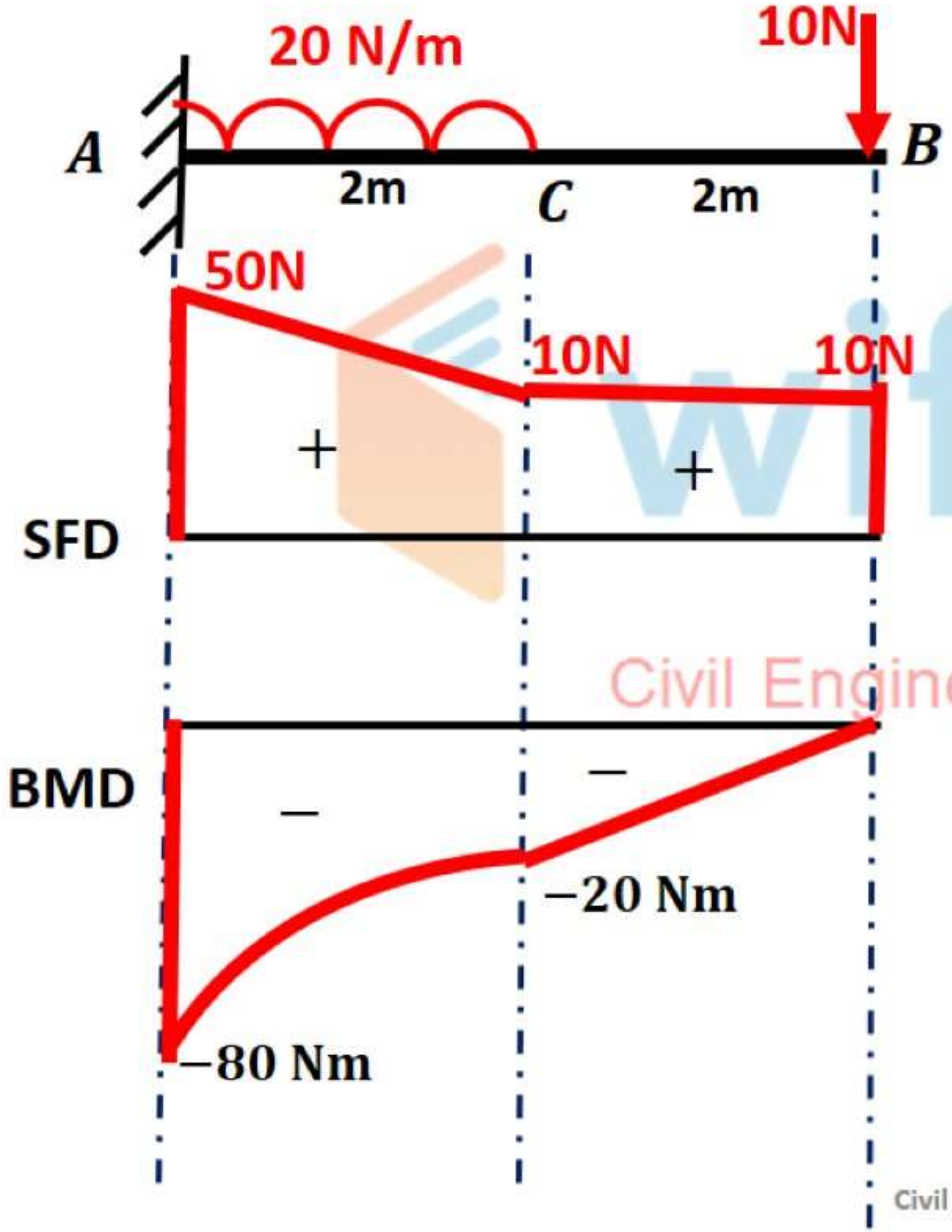
$$\Rightarrow (M)_C = -20 \text{ Nm}$$

$$(M)_C - (M)_D = \textit{area of SFD btw C and D}$$

$$\Rightarrow -20 - (M)_D = 20 \times 2$$

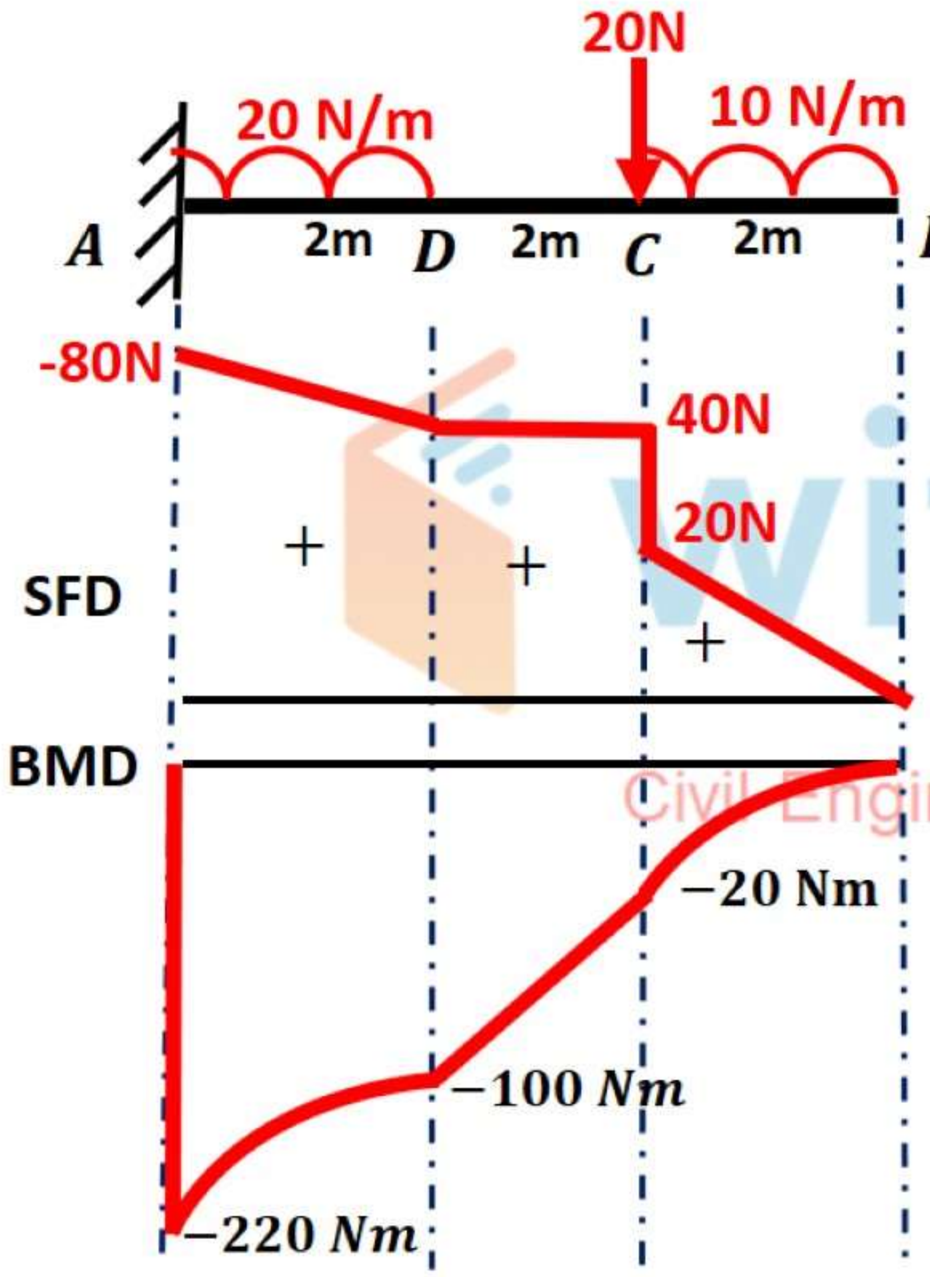
$$\Rightarrow (M)_D = -60\ Nm$$

Que. 52 Draw SFD and BMD for the following



DO YOURSELF ....!

Que. 53 Draw SFD and BMD for the following



$(M)_B = 0$

$(M)_B - (M)_C = area\ of\ SFD\ btw\ B\ and\ C$

$\Rightarrow 0 - (M)_C = \frac{1}{2} \times 20 \times 2$

$\Rightarrow (M)_C = -20\ \text{Nm}$

$(M)_C - (M)_D = area\ of\ SFD\ btw\ C\ and\ D$

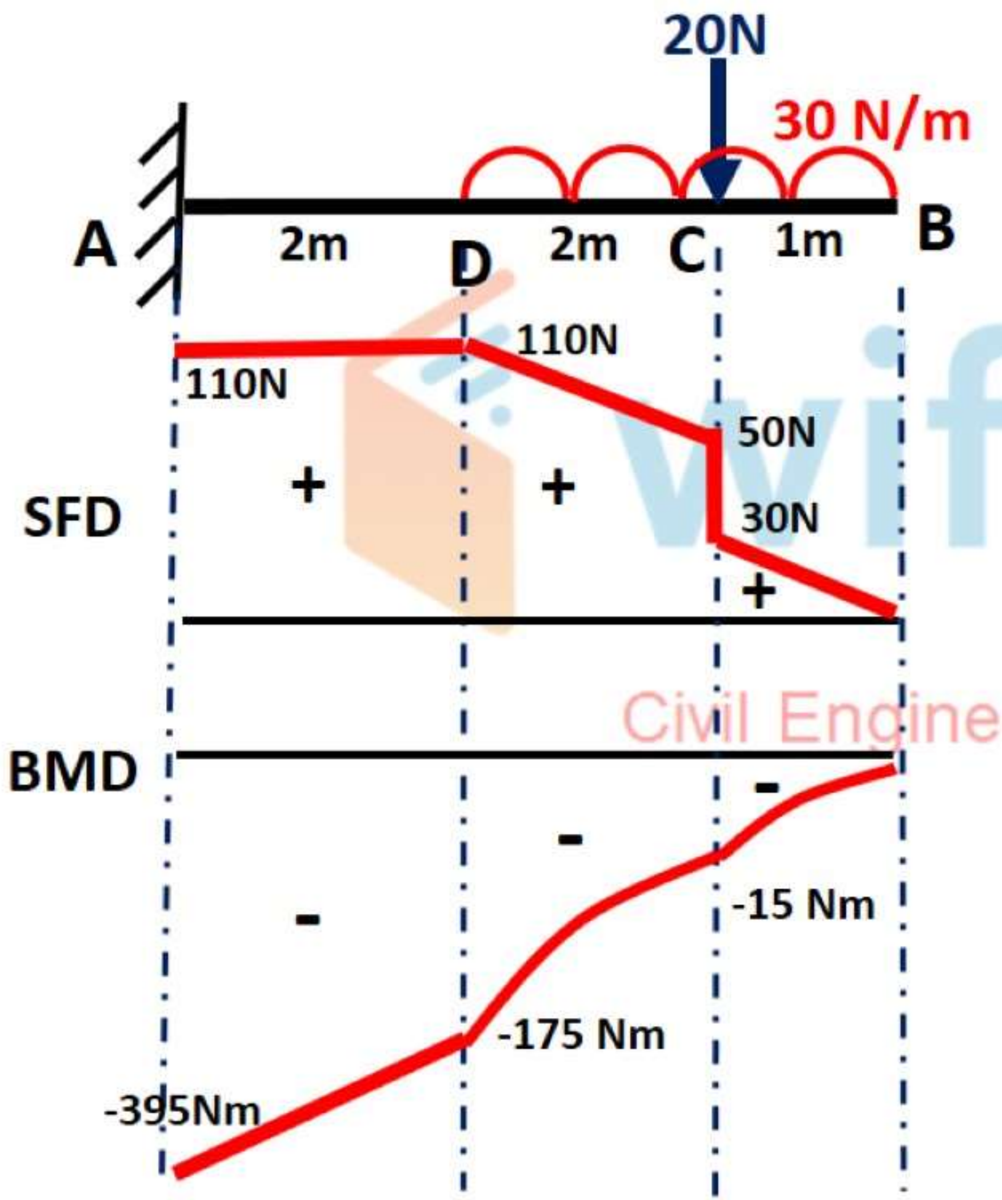
$\Rightarrow -20 - (M)_D = 40 \times 2$

$\Rightarrow (M)_D = -100\ Nm$

$(M)_D - (M)_A = area\ of\ SFD\ btw\ D\ and\ A$

$\Rightarrow -100 - (M)_A = \frac{1}{2} \times (80 + 40) \times 2$

$\Rightarrow (M)_A = -220\ Nm$



Que. 54 Draw SFD and BMD for the following

Solution:

$(SF)_B = 0$

$(SF)_C = (30 \times 1) = 30N$ and $(SF)_{C'} = 30+20=50N$

$(SF)_D = 50 + 30 \times 2 = 110\ N$

$(SF)_A = 110N$

$M_B = 0$

$M_B - M_C =$ area of SFD between C and B

$M_B - M_C = ½ \times 30 \times 1$

$M_C = -15\ Nm$

$M_C - M_D =$ area of SFD between C and D

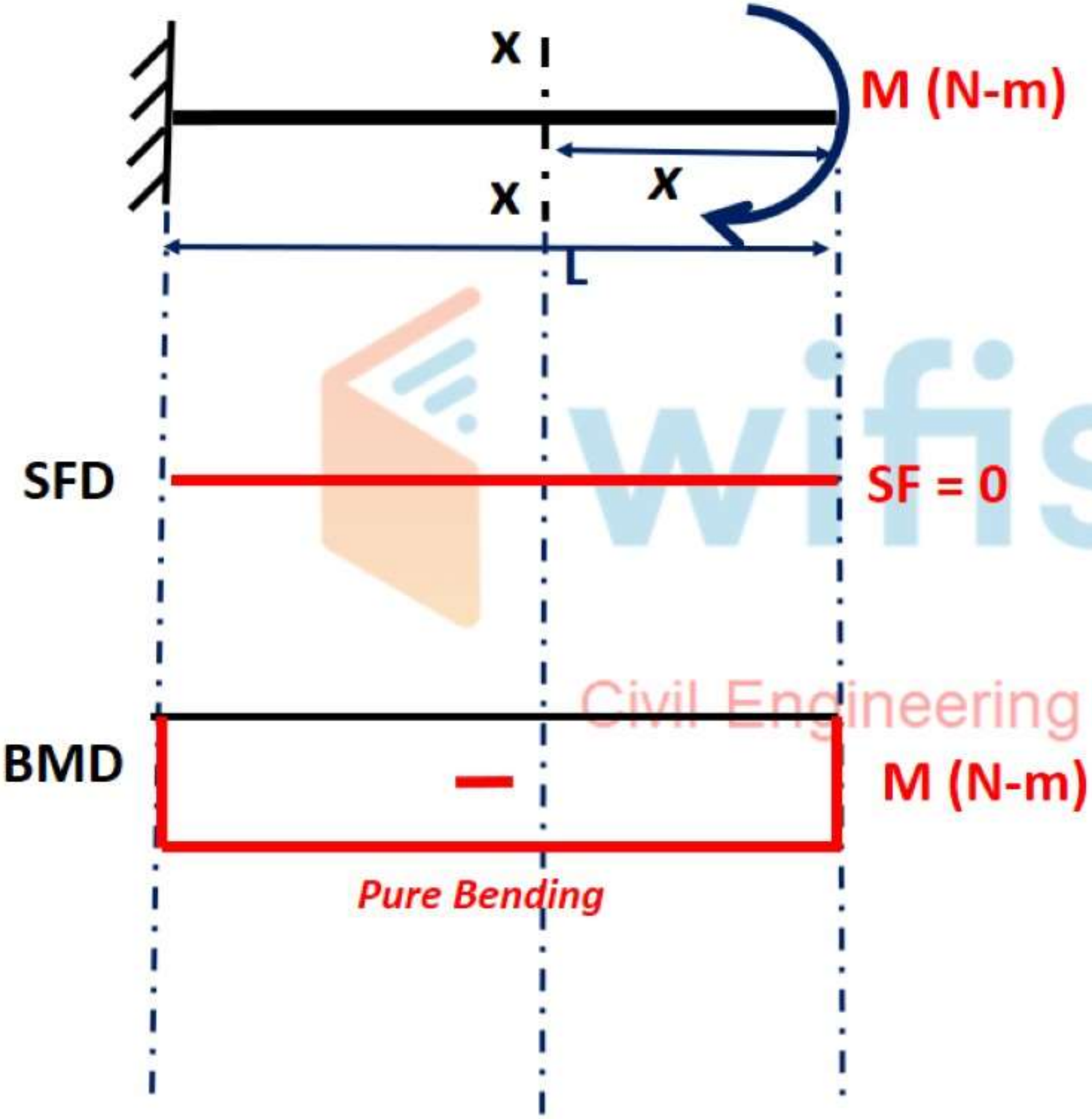
$M_C - M_D = (50 \times 2) + (1/2 \times 60 \times 2)$

$\Rightarrow -15 - M_D = 160$

$\Rightarrow M_D = -175\ Nm$ Similarly $M_A = -395Nm$

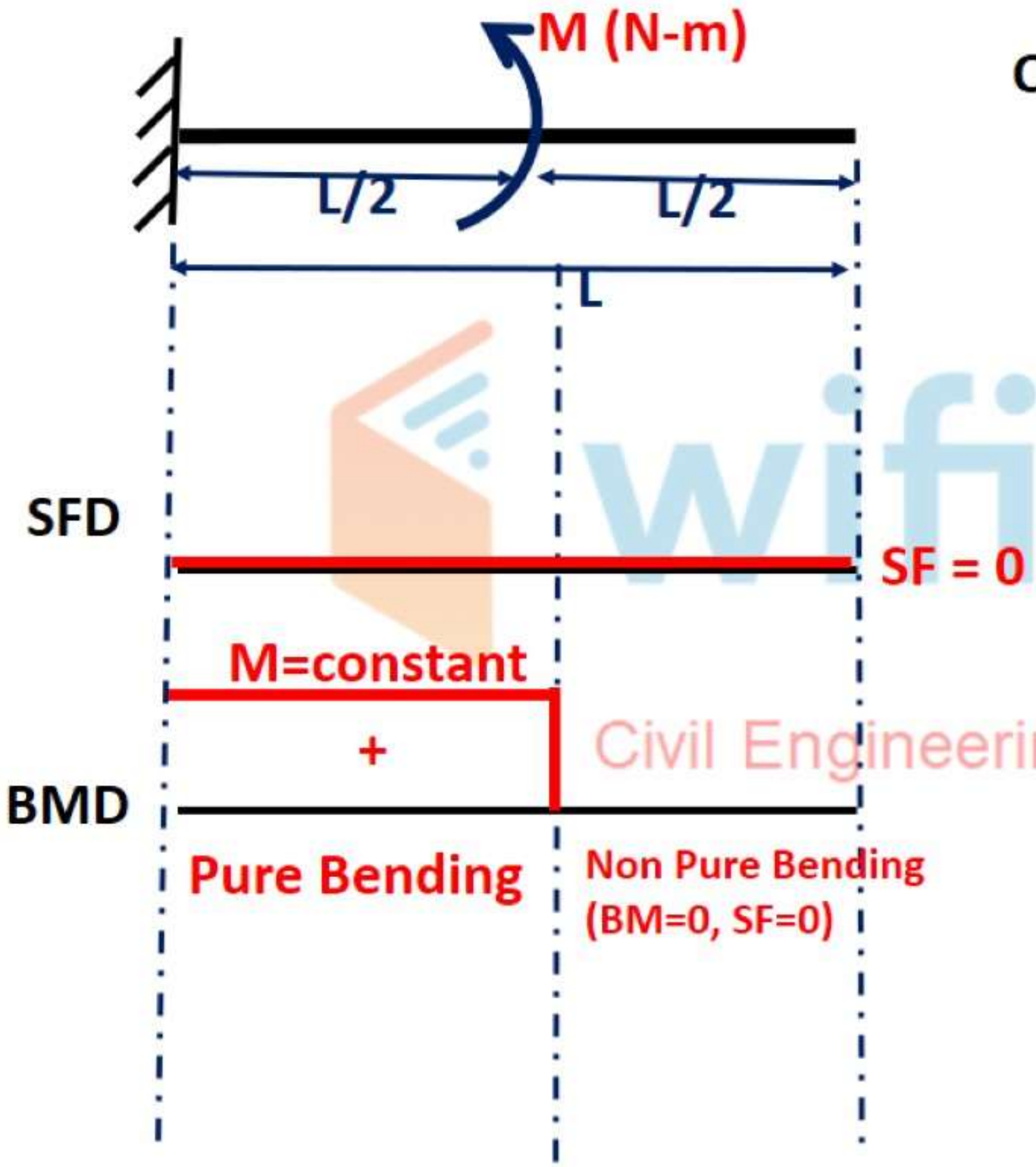
CASE 1: CANTILEVER BEAM

c) Cantilever beam subjected to Concentrated Point Moment

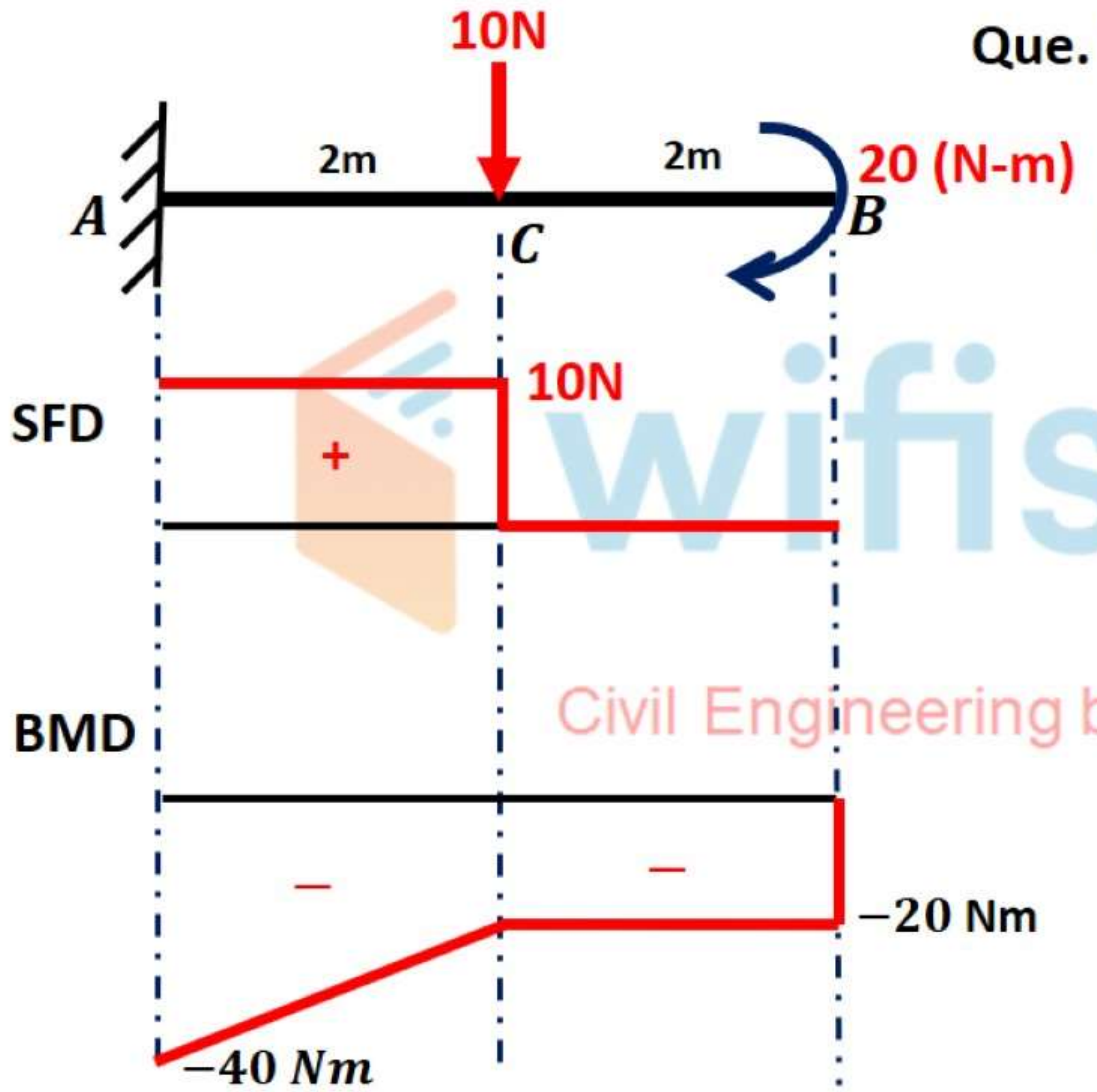


When SF = 0 and Bending Moment is constant, this state is said to be *Pure Bending*

Que. 55 Draw SFD and BMD for the following



Que. 56 Draw SFD and BMD for the following



$(M)_B = -20$ Nm

$(M)_B - (M)_C = area\ of\ SFD\ btw\ B\ and\ C$

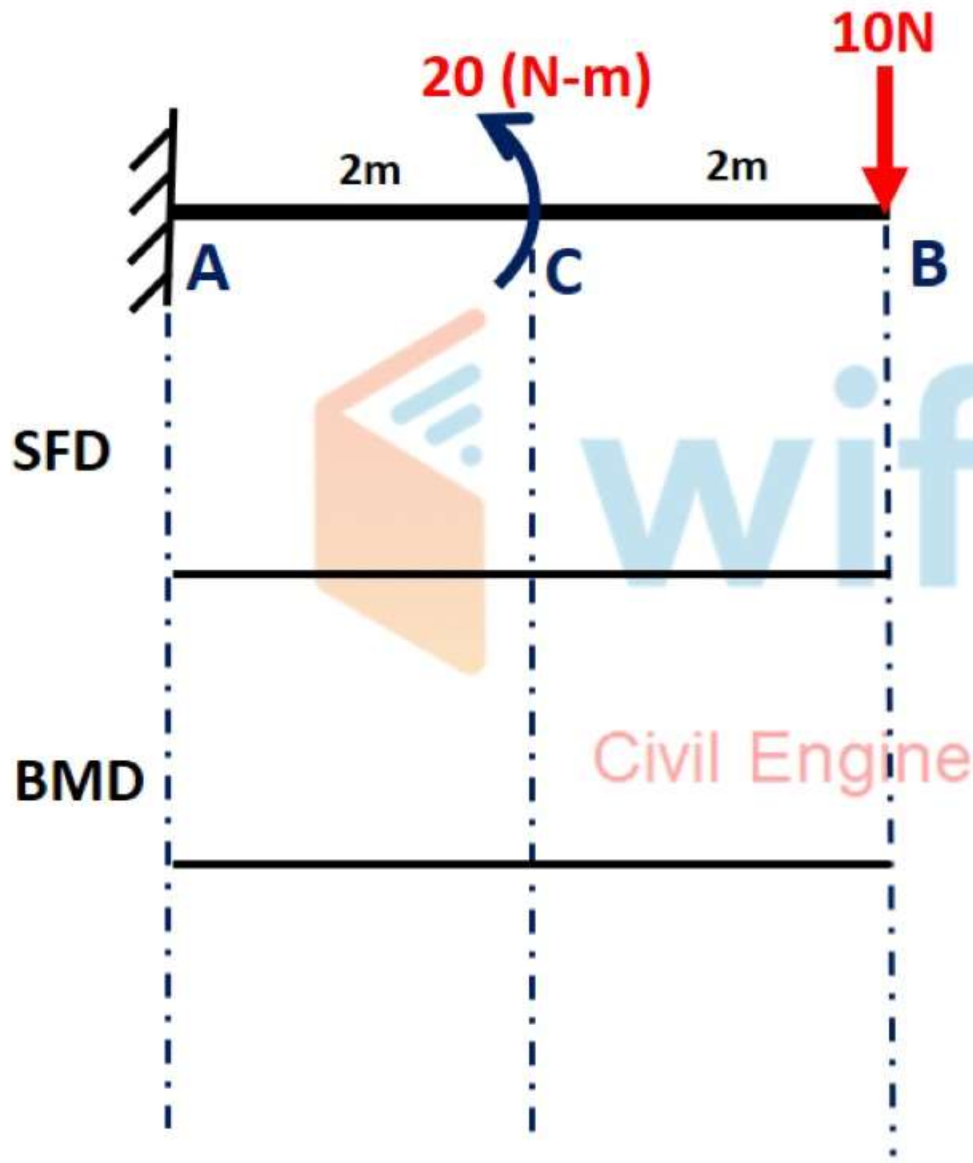
$\Rightarrow -20 - (M)_C = 0$

$\Rightarrow (M)_C = -20$ Nm

$(M)_C - (M)_A = area\ of\ SFD\ btw\ B\ and\ C$

$\Rightarrow -20 - (M)_A = 10 \times 2$

$\Rightarrow (M)_A = -40\ Nm$

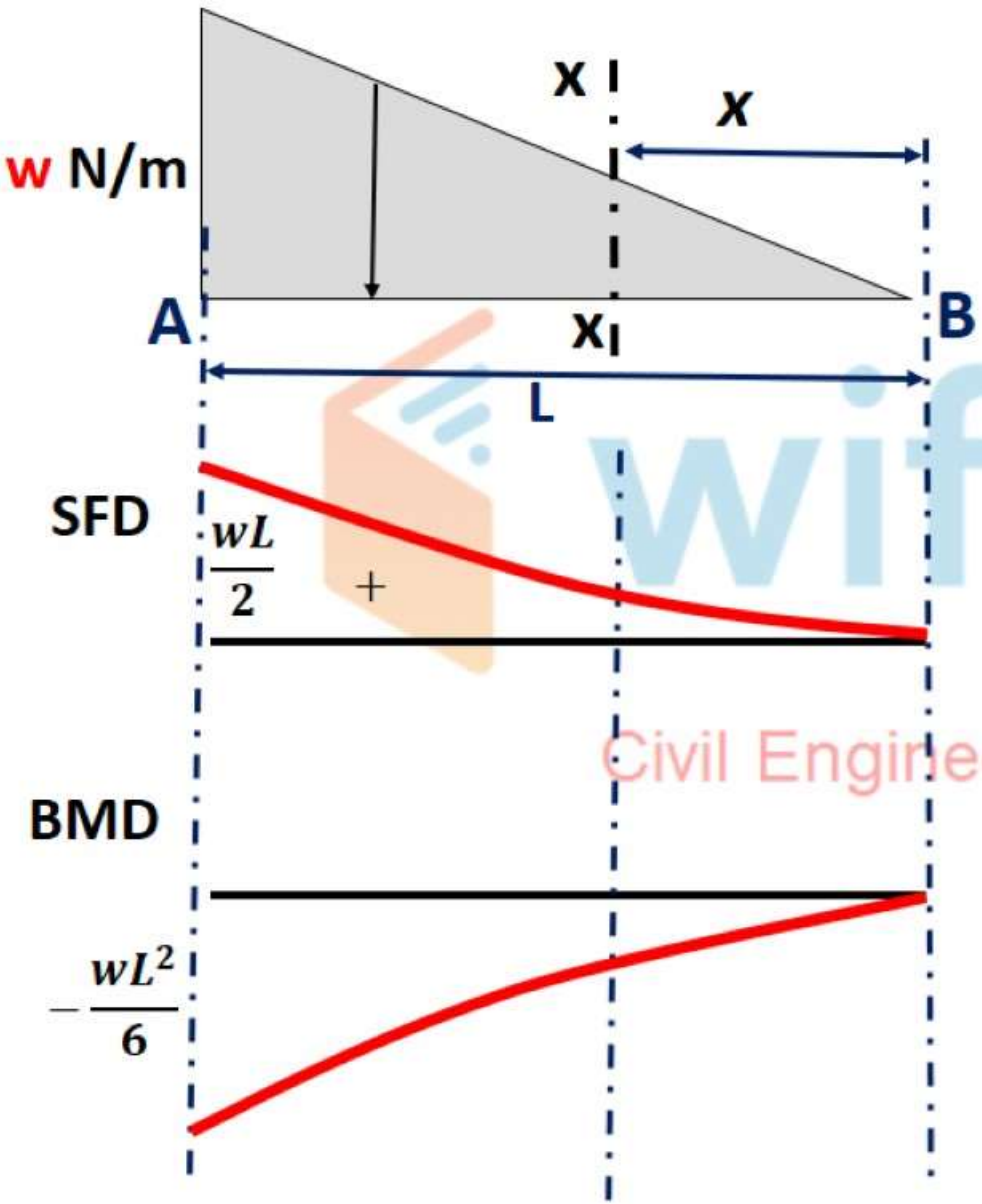


Que. 57 Draw SFD and BMD for the following and comment values of

SF at A,B,C

BM at A,B,C

HOMEWORK !



CASE 1: CANTILEVER BEAM

d) Cantilever beam subjected to Uniformly Varying Load

$$(SF)_{x-x} = \frac{1}{2} \times x \times \frac{w}{L}x$$

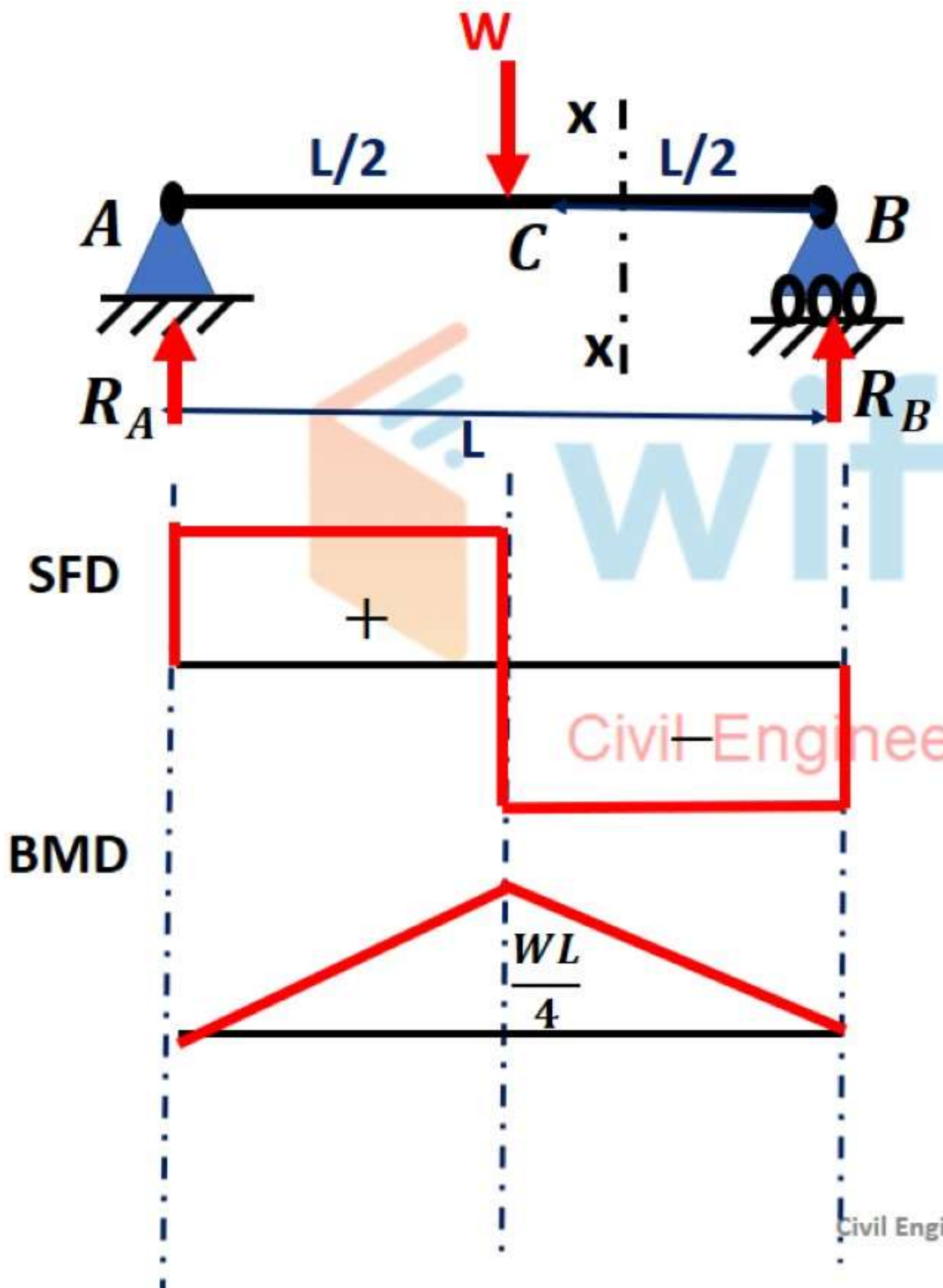
$$(SF)_{x-x} = \frac{wx^2}{2L} \quad parabolic$$

$$(SF)_B = 0 \qquad (SF)_A = \frac{wL^2}{2L} = \frac{wL}{2}$$

$$(BM)_{x-x} = -\frac{wx^2}{2L} \times \frac{x}{3}$$

$$(BM)_{x-x} = -\frac{wx^3}{6L} \quad cubic$$

$$(BM)_B = 0 \qquad (BM)_A = -\frac{wL^2}{6}$$



CASE 2: Simply Supported BEAM

a) Simply Supported beam subjected to Point Load

When x lies between B and C

$(SF)_{x-x} = -RB$

$\Rightarrow (SF)_{x-x} = -\frac{W}{2}$

$\Rightarrow (SF)_{C-} = -\frac{W}{2}$ $\quad \Rightarrow (SF)_{C+} = \frac{W}{2}$

$(M)_{x-x} = +R_B \times x = \frac{Wx}{2}$ (+due to sagging)

$(M)_B = 0 \; at \; x = 0$ $\quad (M)_A = \frac{WL}{4} \; at \; x = \frac{L}{2}$

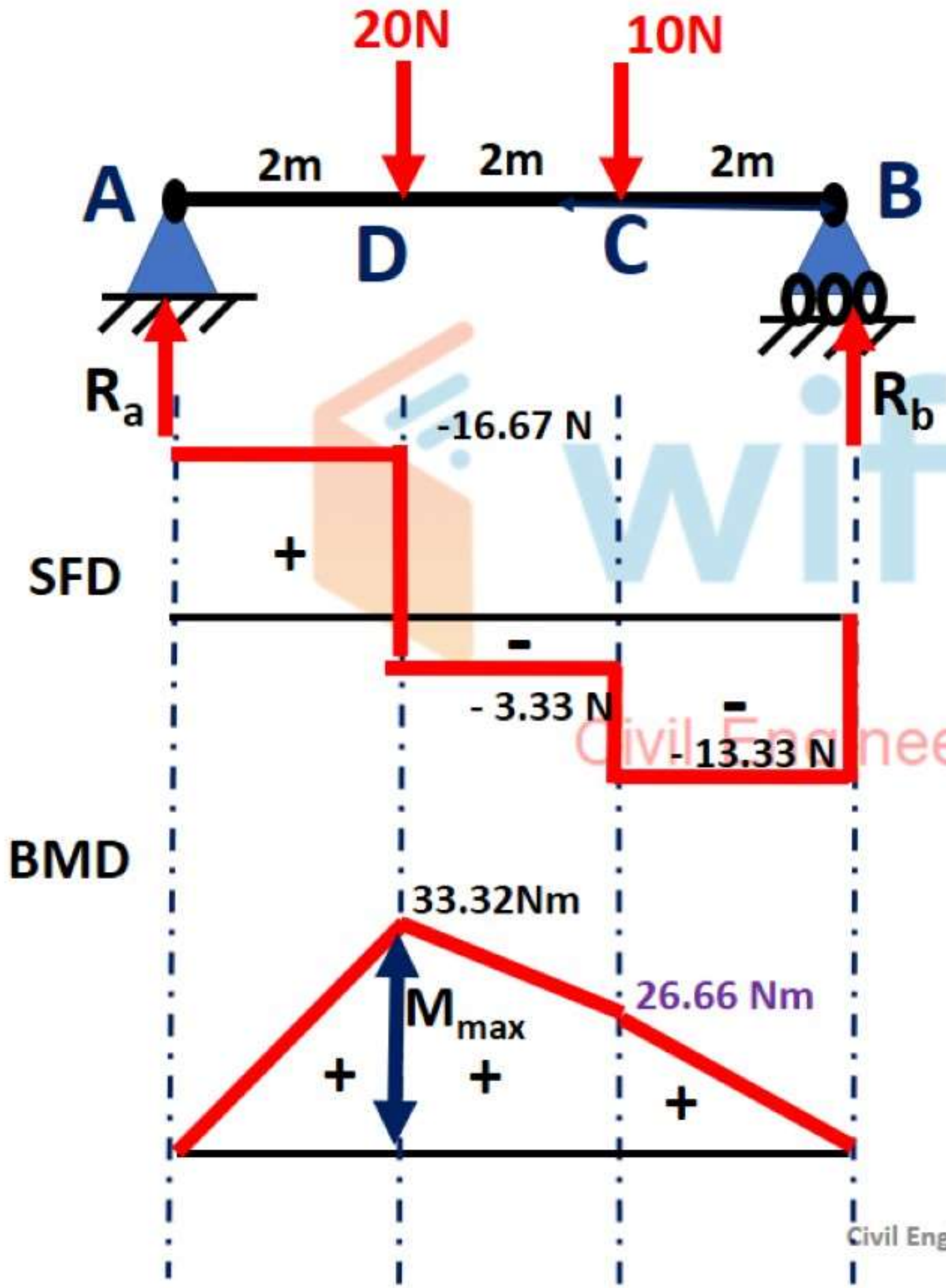
When x lies between C and A

$(SF)_{x-x} = W - RB$

$(SF)_{x-x} = W - \frac{W}{2} = \frac{W}{2}$

$(M)_{x-x} = +R_B \times y - W(y - \frac{L}{2})$

$(M)_{C+} = +R_B \times \frac{L}{2} - W\left(\frac{L}{2} - \frac{L}{2}\right) = \frac{WL}{4}$



Que 58. Draw the SFD and BMD for the following.

Solution:

$R_a + R_b = 20+10 = 30N$ ....(1)

Taking moment about A = 0

$\Rightarrow (+R_b)x6 - 10x4 - 20x2 = 0$

$\Rightarrow R_b = 13.33$ N and Hence

$\Rightarrow R_a = 16.67N$ (from eqn 1)

$M_B = 0$

$M_B - M_C$ = area of SFD between C and B

=> $M_B - M_C = -(13.33 \text{ x } 2)$

=> $M_C = 26.66$ Nm

$M_C - M_D$ = area of SFD between C and D

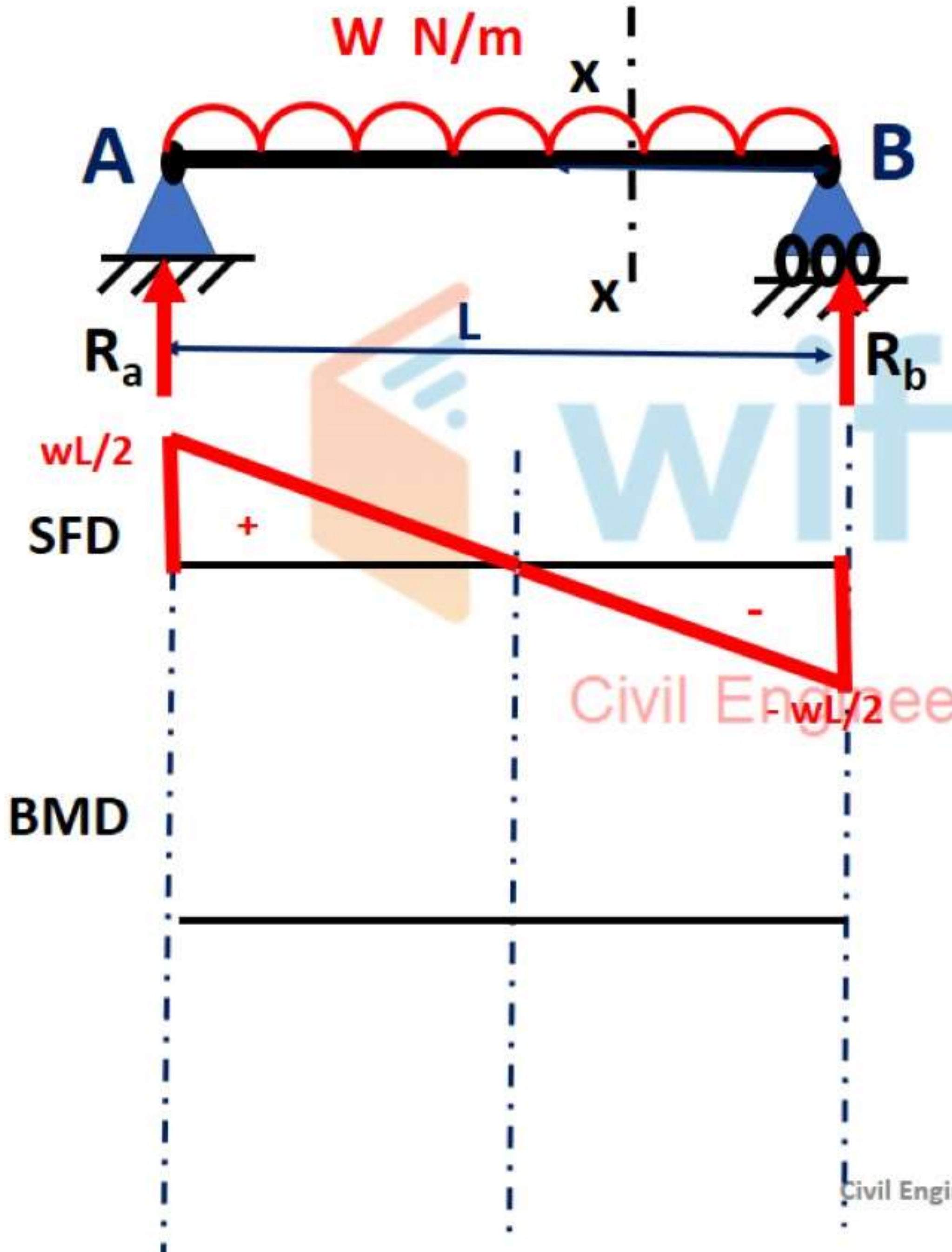
$M_C - M_D = -3.33 \text{ x } 2$

$\Rightarrow 26.66 - M_D = -6.66$

$\Rightarrow M_D = 33.32Nm$ Similarly $M_A = 0$

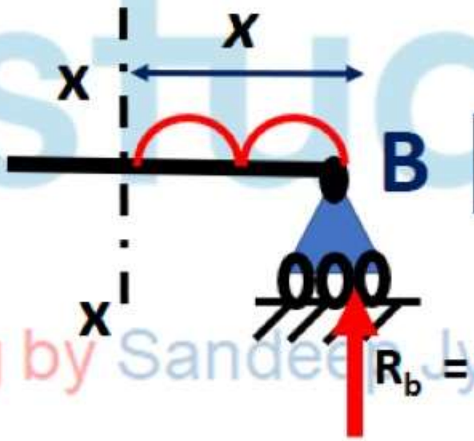
CASE 2: Simply Supported BEAM

b) Simply Supported beam subjected to Uniformly Distributed Load (UDL)



Total load = w x L

$R_a = R_b = \frac{wL}{2}$



R_b = wL/2

$(SF)_{xx}$ = - wL/2 + wx

$(SF)_B$ = - wL/2 + wx *(Put x =0)*

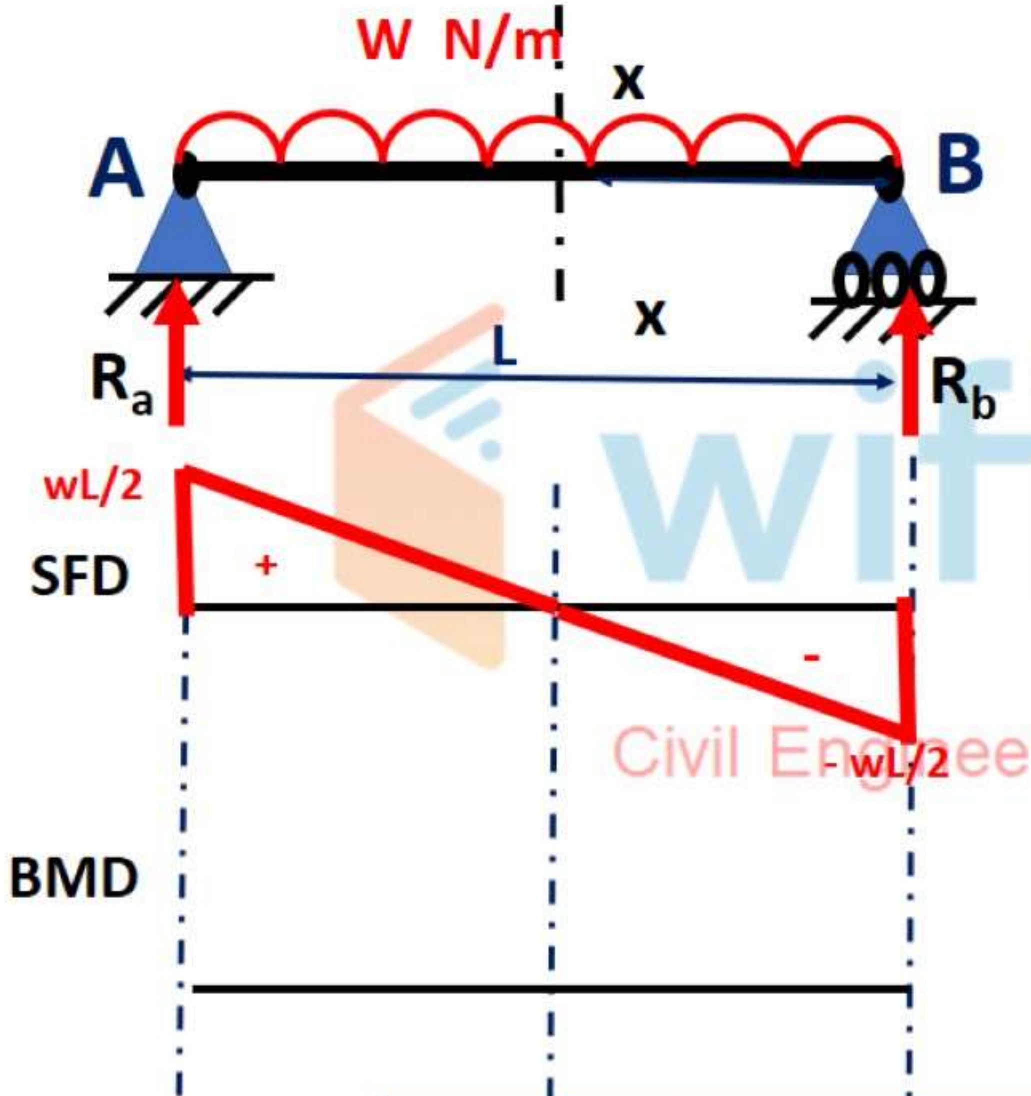
$(SF)_B$ = - wL/2

$(SF)_A$ = - wL/2+ wx *(Put x =L)*

$(SF)_A$ = wL/2

CASE 2: Simply Supported BEAM

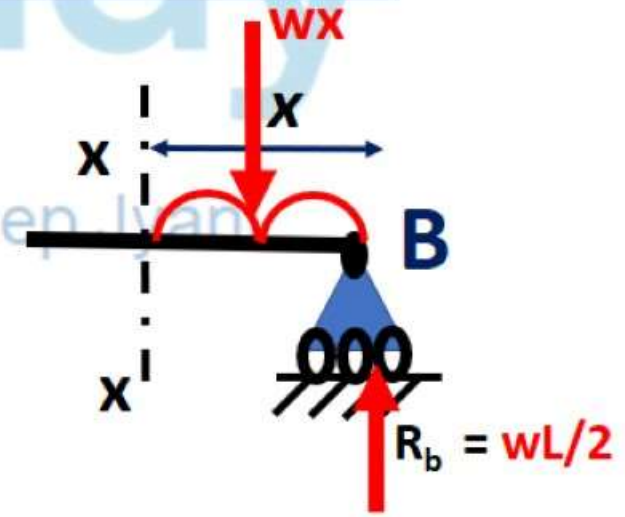
b) Simply Supported beam subjected to Uniformly Distributed Load (UDL)



Point where shear force is 0

$(SF)_{xx} = -wL/2 + wx = 0$

=> $x = L/2$

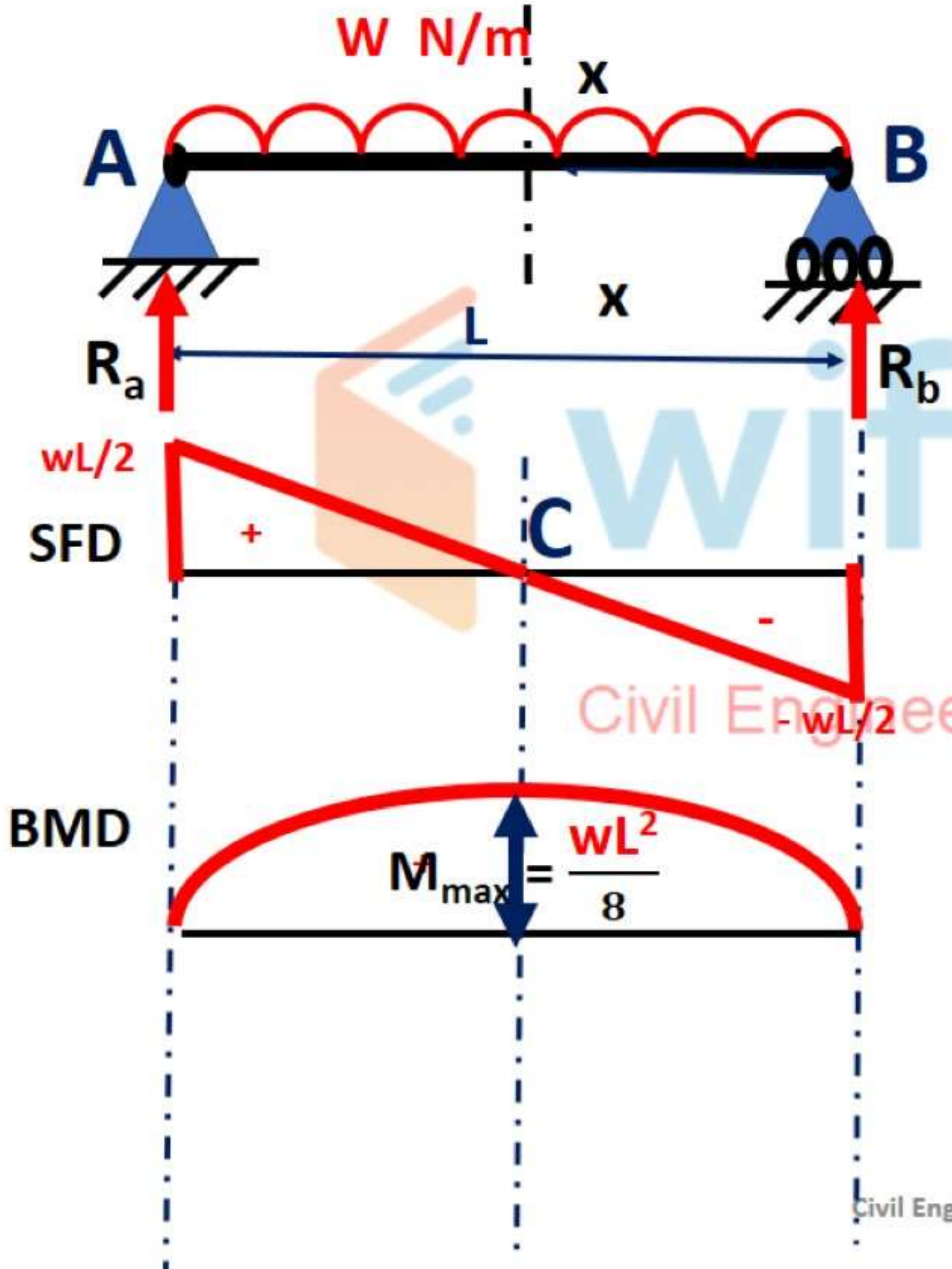


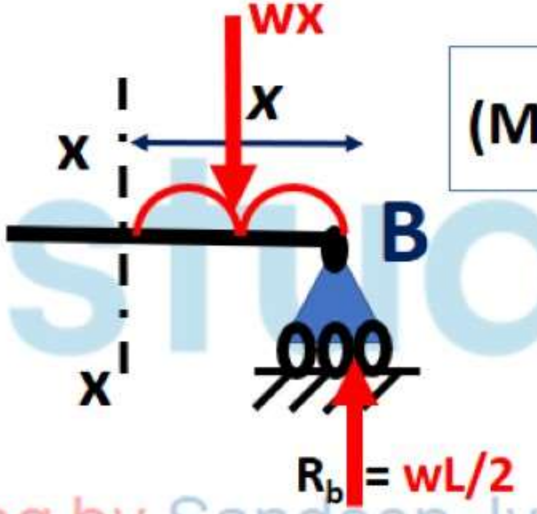
$(SF)_B = -wL/2 + wx$ *(Put x =0)*

$(SF)_B = -wL/2$

CASE 2: Simply Supported BEAM

b) Simply Supported beam subjected to Uniformly Distributed Load (UDL)





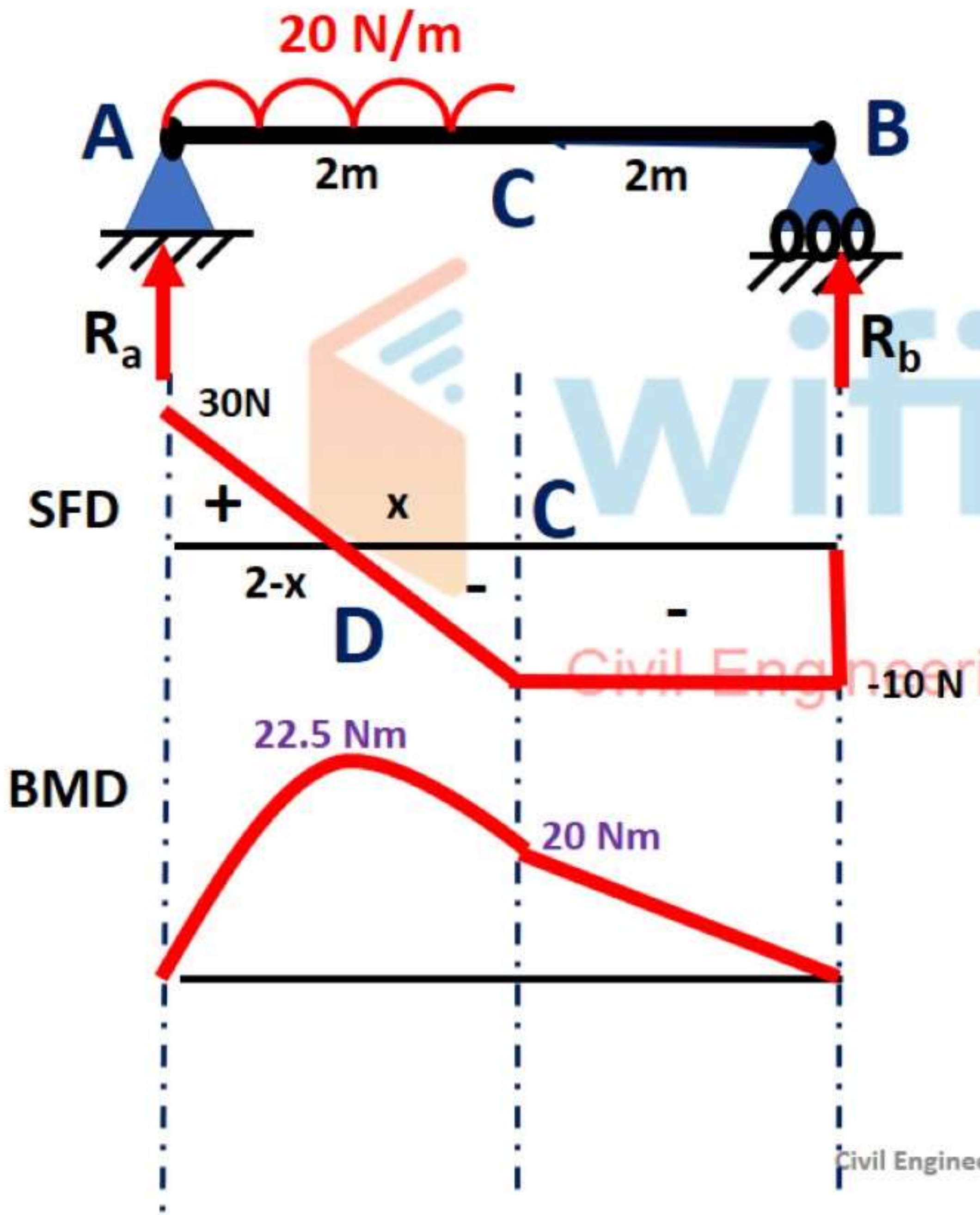
$$(M)_{xx} = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$(M)_{xx} = +R_b \times x - (wx) \times \frac{x}{2}$$

(put x=0) $(M)_B = 0$

(put x=L/2) $(M)_C = \frac{wL^2}{8}$

(put x=L) $(M)_A = 0$



Que 58. Draw the SFD and BMD for the following.

Solution:

$R_a + R_b = 20 \times 2 = 40N$ ….(1)

Taking moment about A = 0

$\Rightarrow (+R_b) x4 - (20x2) \times 1 = 0$

$\Rightarrow R_b = 10$ N and Hence

$\Rightarrow R_a = 30N$ (from eqn 1)

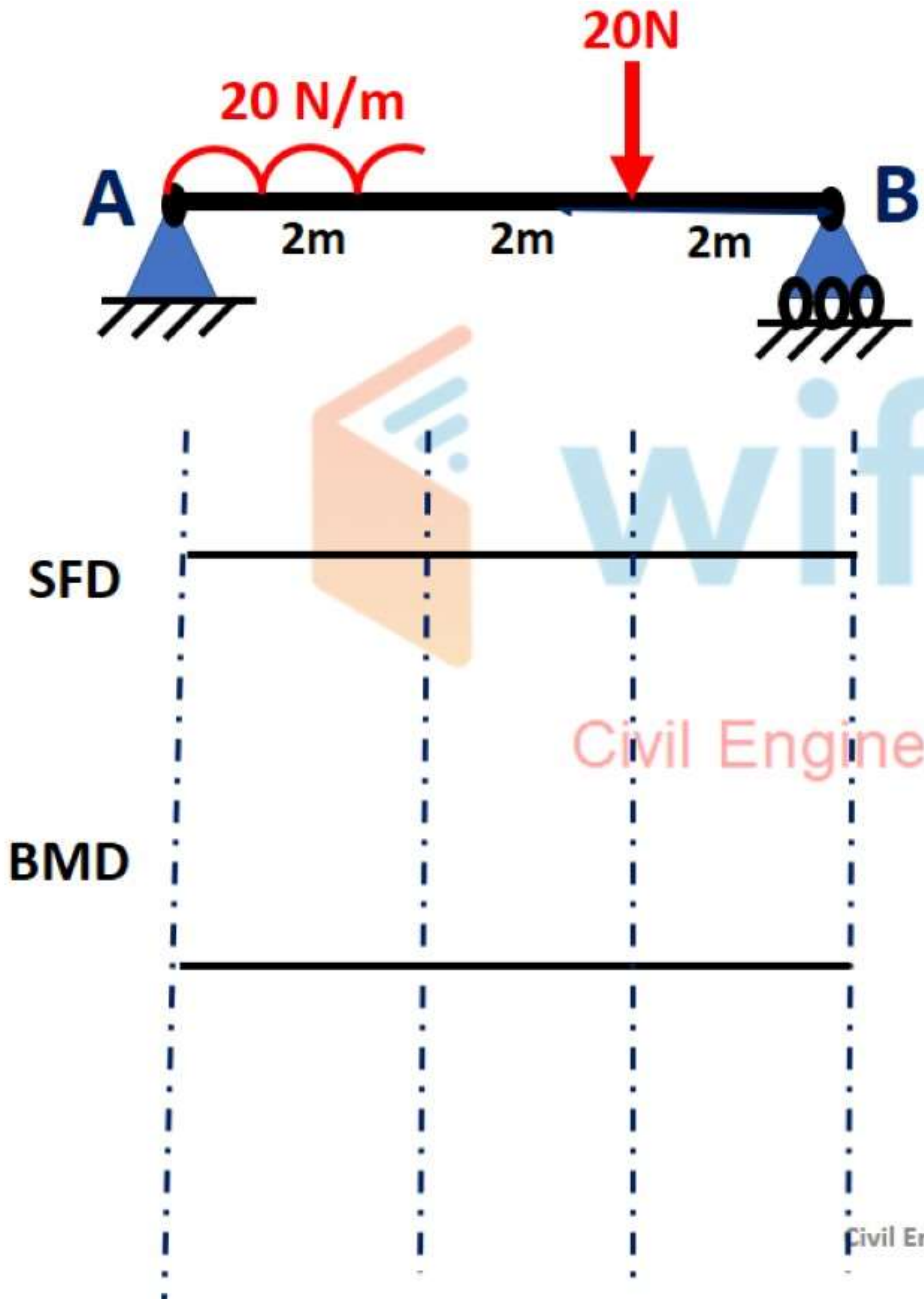
$M_B = 0$

$M_B - M_C$ = area of SFD between C and B

$\Rightarrow M_B - M_C = -(10 \times 2)$

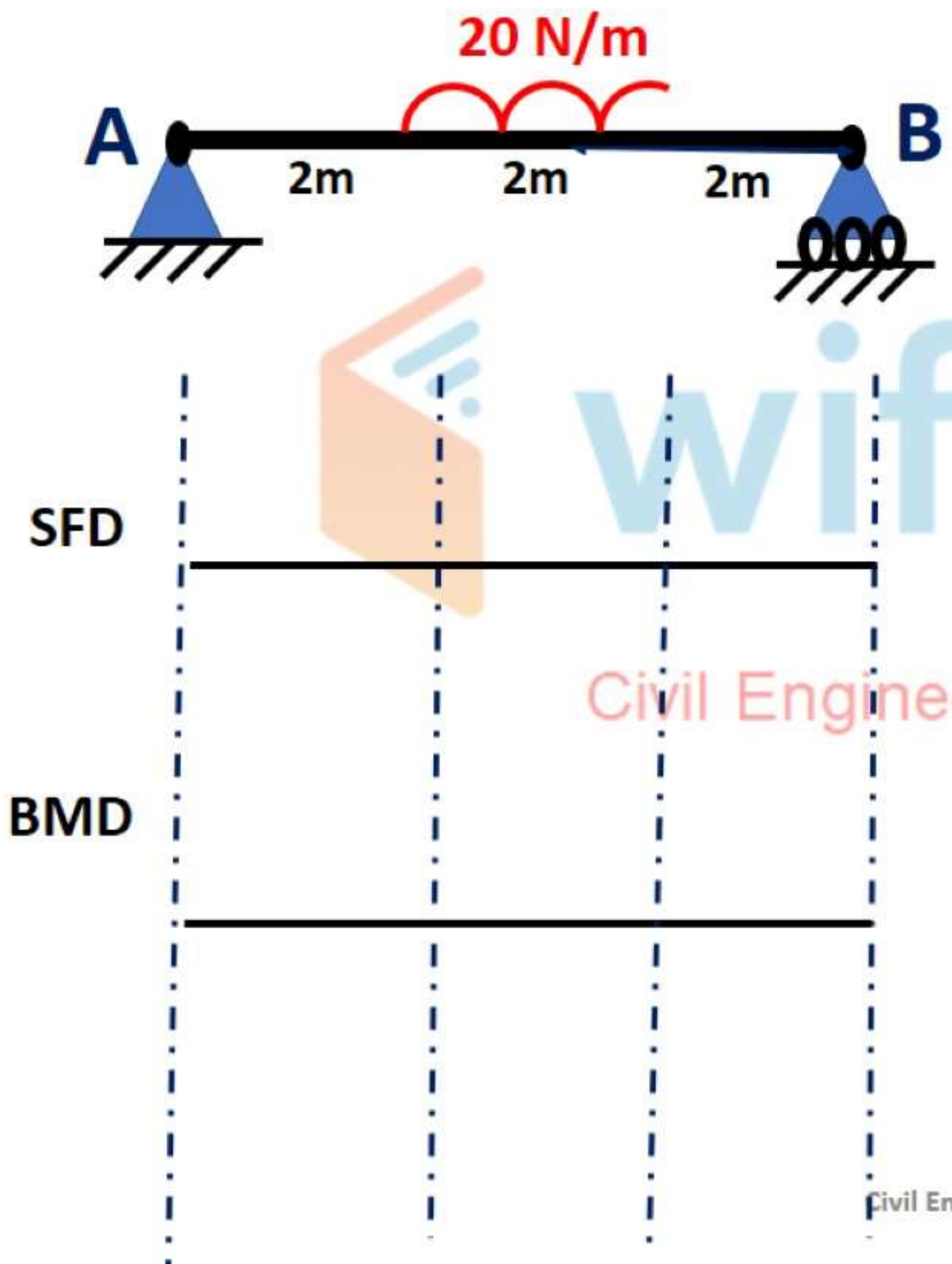
$\Rightarrow M_C = 20$ Nm

$\Rightarrow$ Similarly $M_D = 22.5$ Nm



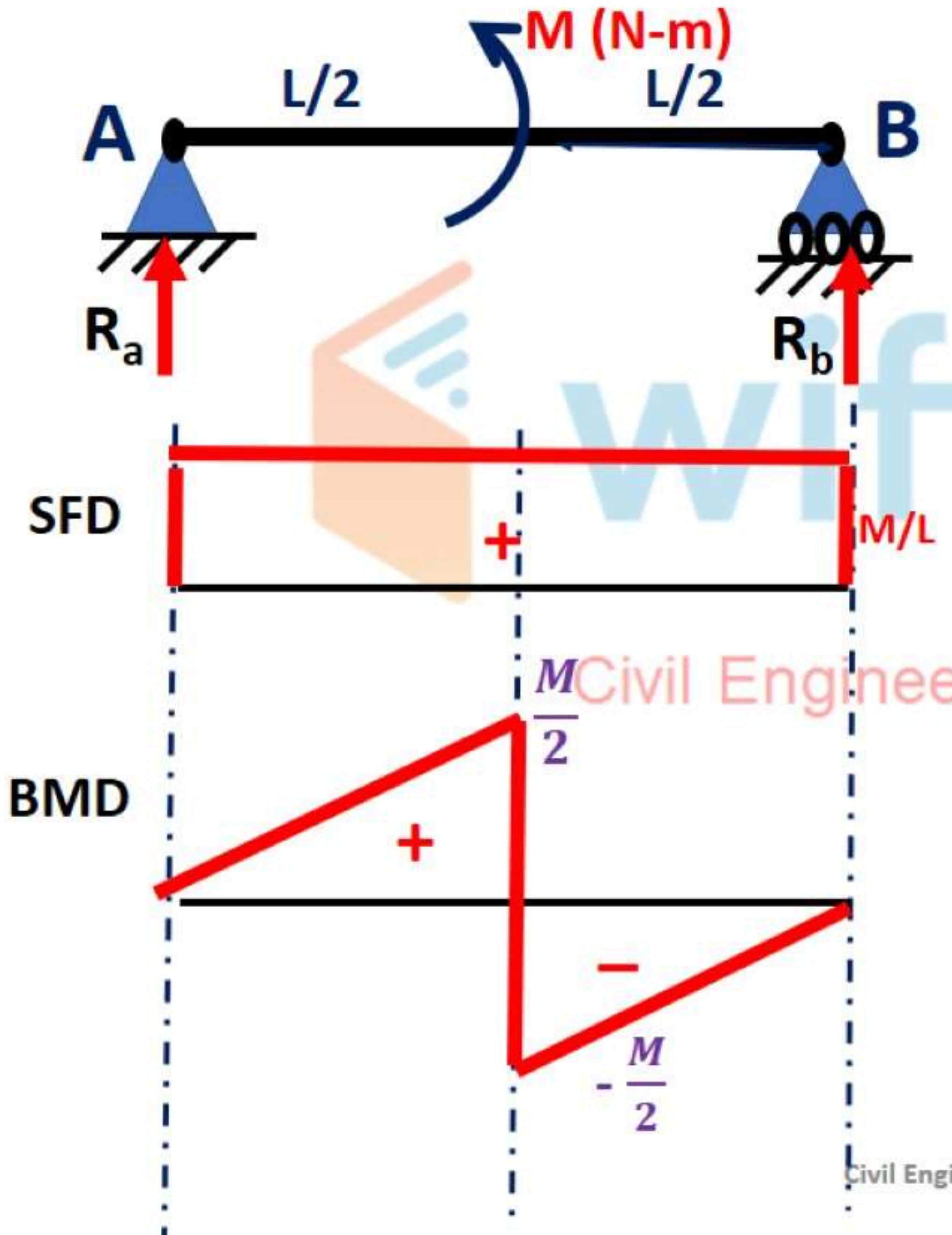
Que 59. Draw the SFD and BMD for the following.

HOMEWORK!



Que 60. Draw the SFD and BMD for the following.

HOMEWORK!



CASE 2: Simply Supported BEAM

c) Simply Supported beam subjected to Concentrated Moment

$R_a + R_b = 0$ ....(1)

Taking moment about A = 0

$\Rightarrow (+R_b) \times L + M = 0$

$\Rightarrow R_b = -M/L$ (or we say M/L downward)

$\Rightarrow R_a = +M/L$ (from eqn 1)

$M_B = 0$

$M_B - M_C$ = area of SFD between C and B

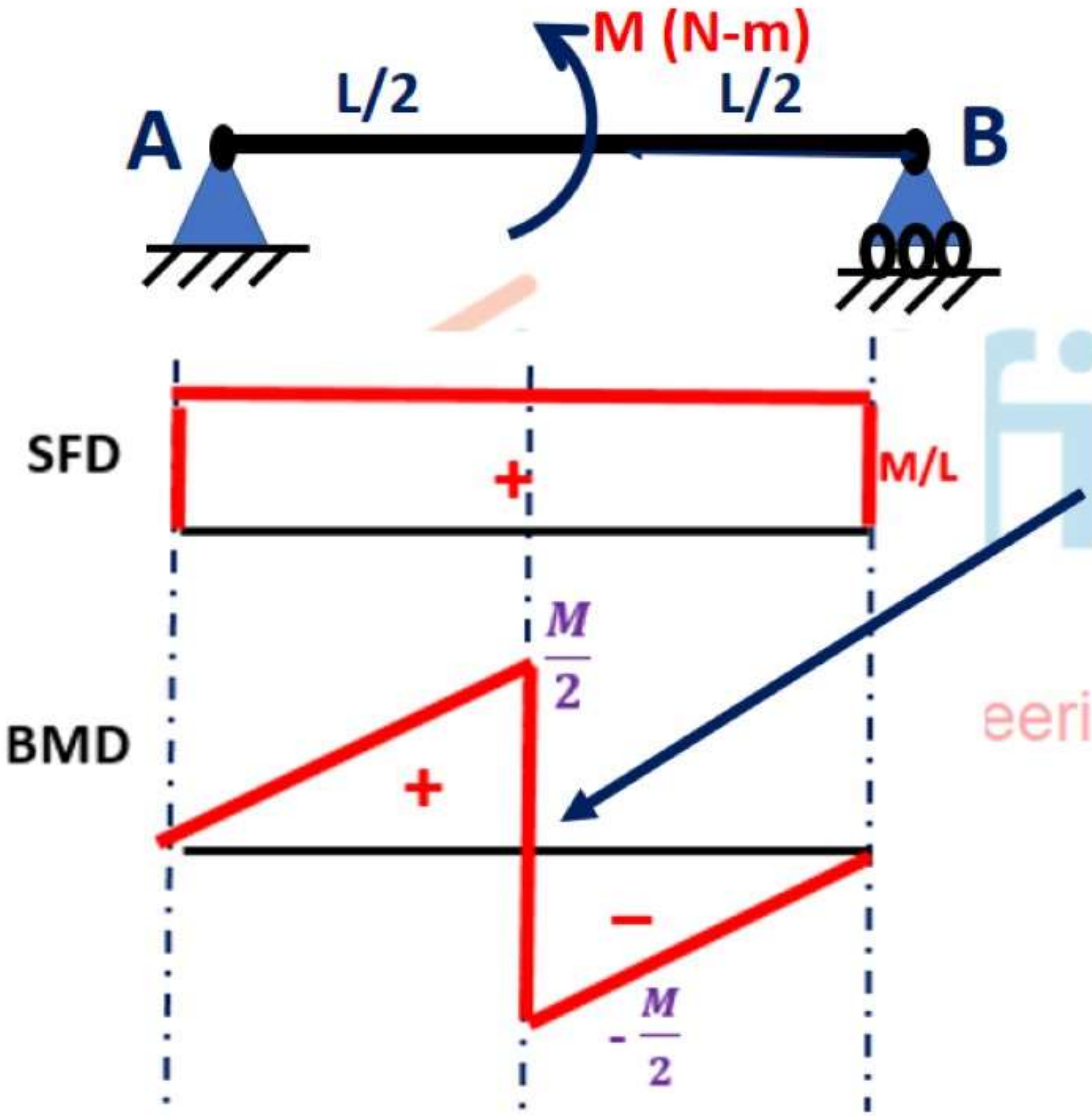
$\Rightarrow M_B - M_C = (\frac{M}{L} \times \frac{L}{2})$

$\Rightarrow M_C = -\frac{M}{2}$ (considering upto length L/2 from B)

$\Rightarrow$ Now at Length just beyond C

$\Rightarrow \Rightarrow M_{C'} = -\frac{M}{2} + M = \frac{M}{2}$

And $M_A = 0$



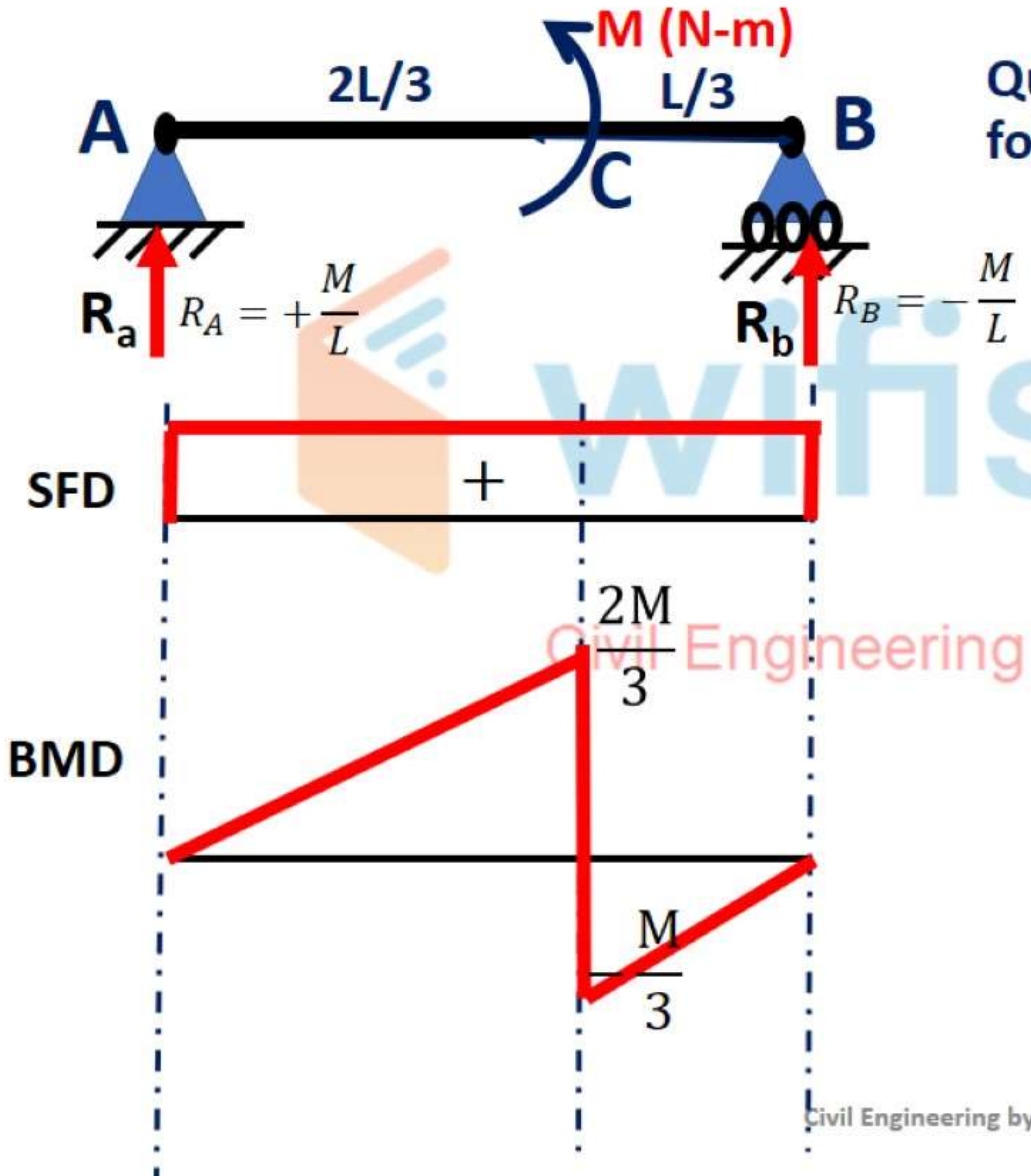
CASE 2: Simply Supported BEAM

c) Simply Supported beam subjected to Concentrated Moment

POINT OF CONTRAFLEXURE

1. It is the point where bending moment changes its sign or changes the nature from hogging to sagging or sagging to hogging
2. In this case, two similar shape and size of triangle are obtained in BMD.
3. The point of contraflexure occur at the application of Concentrated Moment
4. In this case, SFD is a rectangle with height equal to M/L

Que 61. Draw the SFD and BMD for the following.



$$M_B - M_C = \frac{M}{L} \times \frac{L}{3}$$

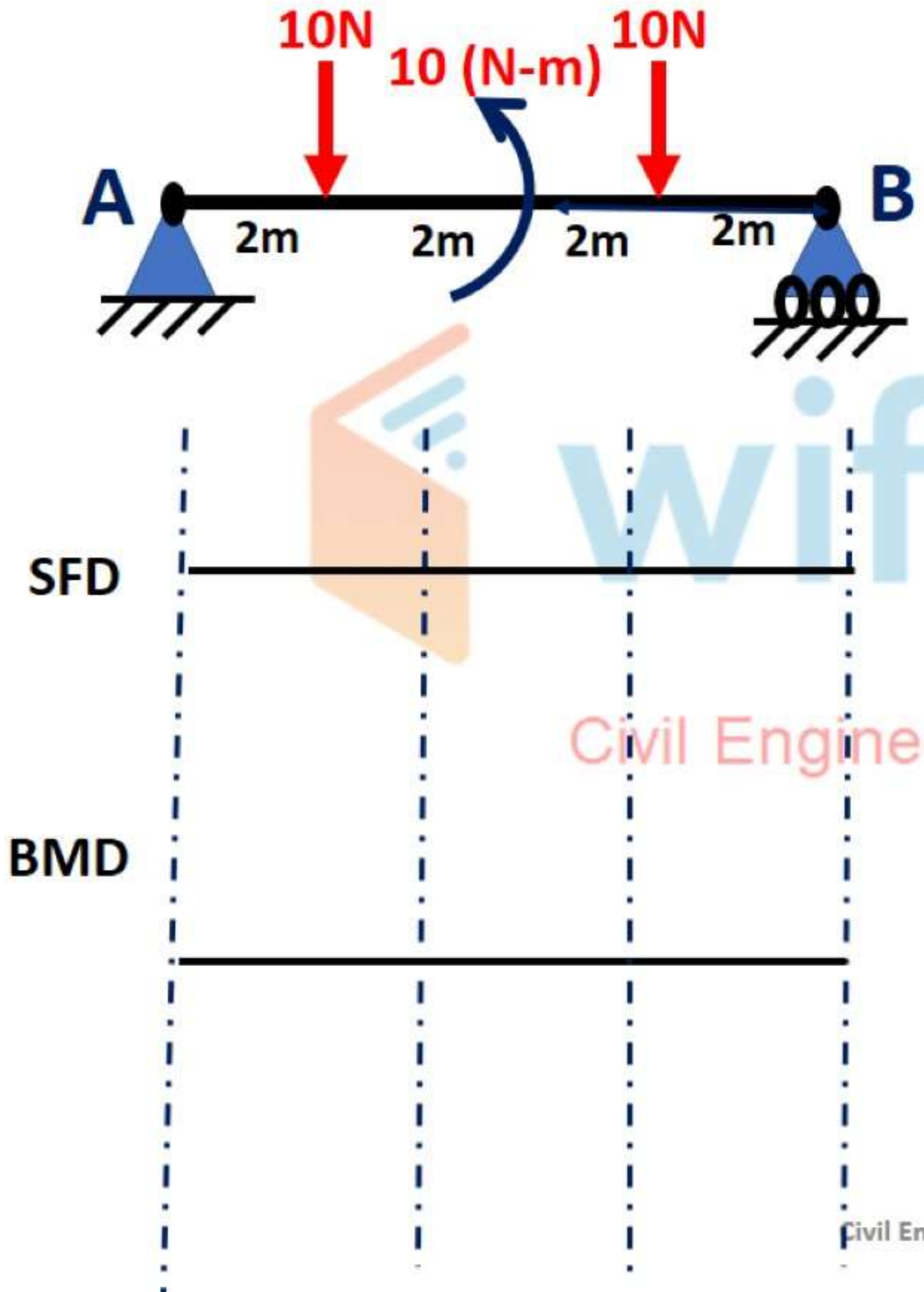
$$\Rightarrow 0 - M_C = \frac{M}{3}$$

$$\Rightarrow M_C = -\frac{M}{3}$$

$$\Rightarrow M_{C+} = -\frac{M}{3} + M$$

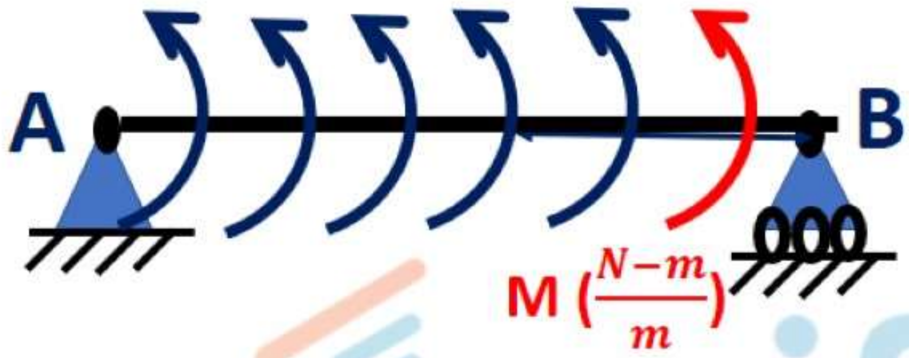
$$\Rightarrow M_{C+} = \frac{2M}{3}$$

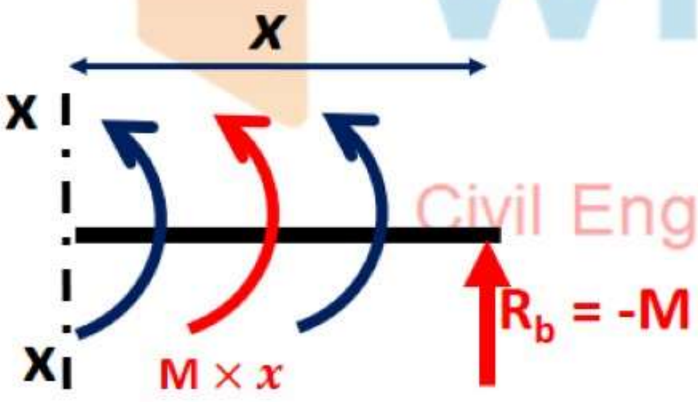
Que 62. Draw the SFD and BMD for the following.



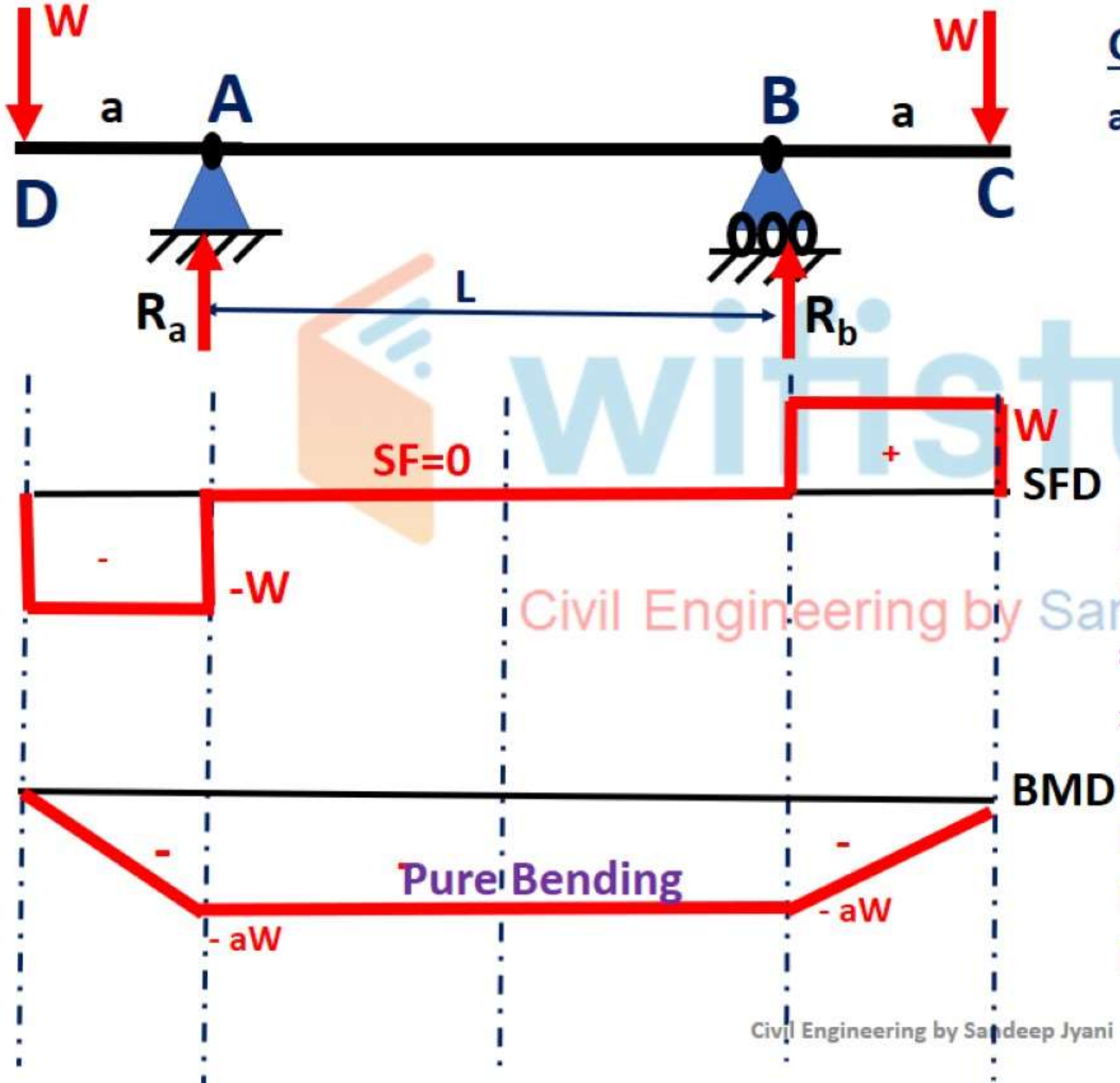
CASE 2: Simply Supported BEAM

d) Simply Supported beam subjected to Uniformly Distributed Moment





$$M_{xx} = M \times x + R_b \times x$$
$$= M \times x + (-M) \times x$$
$$= 0$$



CASE 3: OVERHANG BEAM

a) Overhang beam subjected to point load

$R_a + R_b = W+W = 2W$ ....(1)

Taking moment about A = 0

$\Rightarrow (+R_b) x L - W x (L+a) - W x a = 0$

$\Rightarrow R_b = W$ and Hence

$\Rightarrow R_a = W$ (from eqn 1)

$M_C = 0$

$M_C - M_B$ = area of SFD between C and B

=> $M_C - M_B = a \times W$

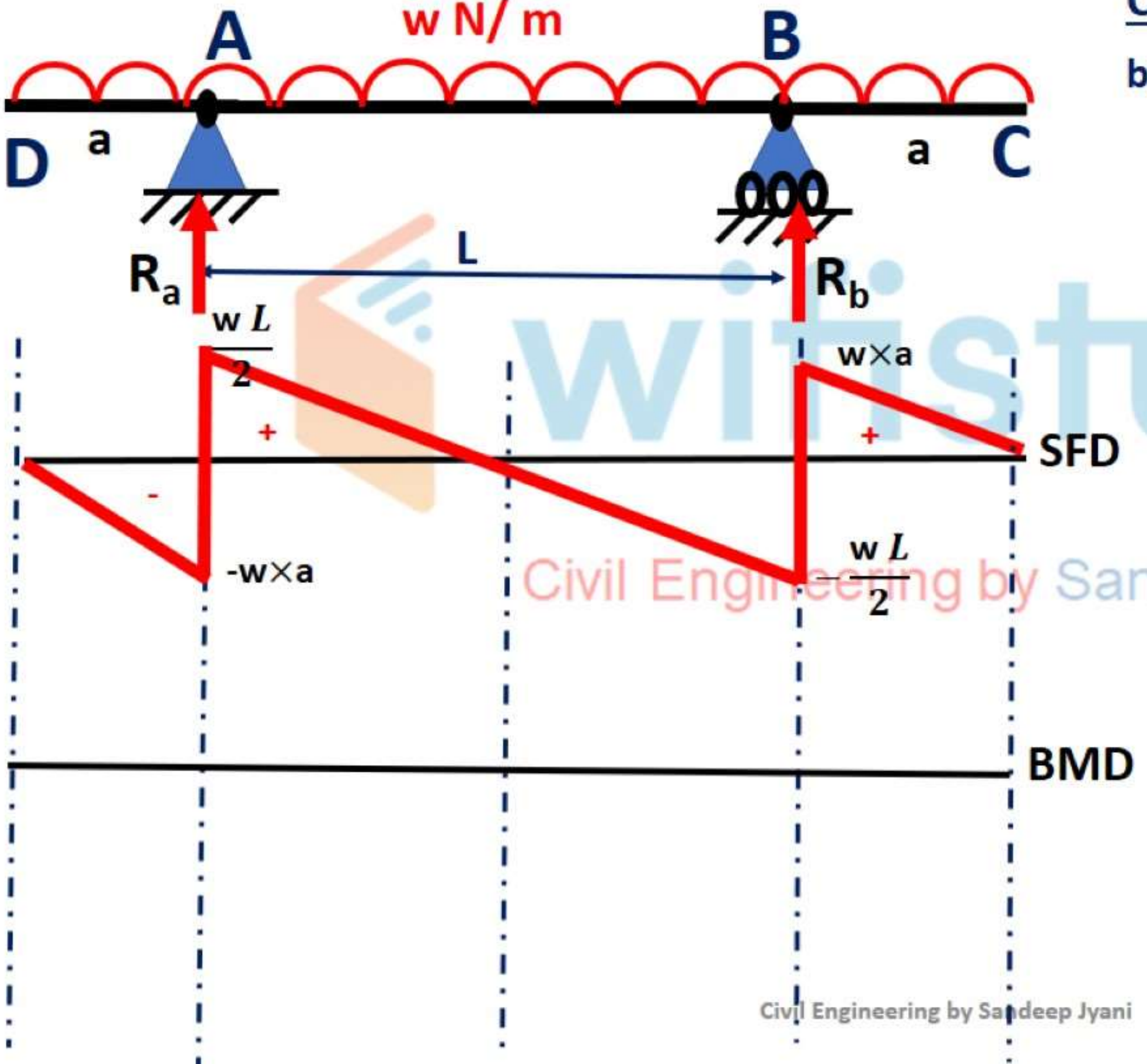
$\Rightarrow M_B = -aW$

$M_B - M_A$ = area of SFD between A and B

$M_B - M_A = 0$ => $M_A = -aW$

$M_A - M_D$ = area of SFD between A and D

$M_D = 0$



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

Total Load = $w \times (L + 2a)$

$R_a + R_b = \{w \times (L + 2a)\}$ or

$R_a = R_b = \frac{w \times (L+2a)}{2}$

$(SF)_C = 0$

$(SF)_{B+} = w \times a$

$(SF)_{B-} = (w \times a) - \frac{w \times (L+2a)}{2}$

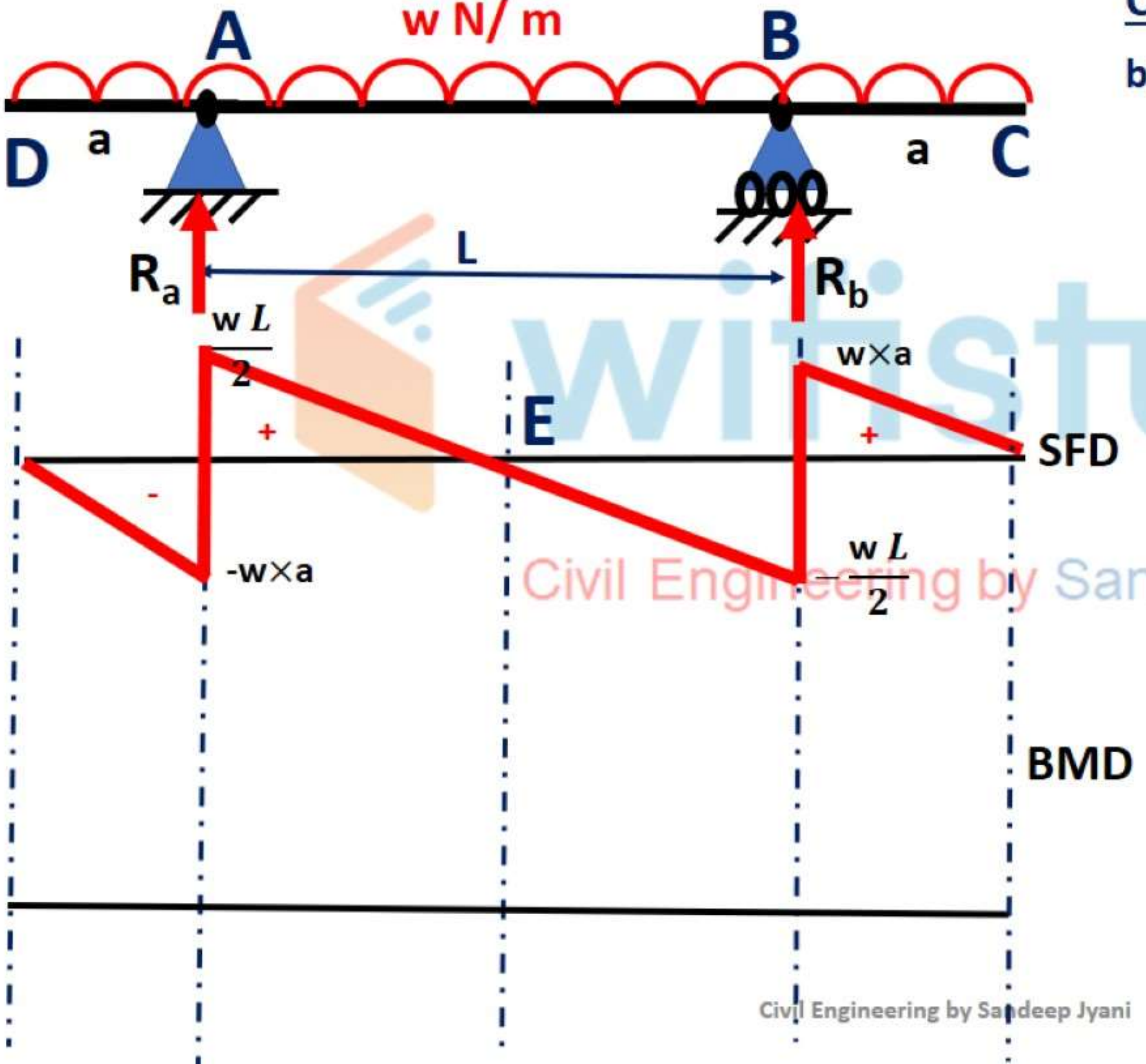
$= - \frac{wL}{2}$

$(SF)_{A+} = (w \times a) - \frac{w \times (L+2a)}{2} + wL$

$= \frac{wL}{2}$

$(SF)_{A-} = \frac{wL}{2} - \frac{w \times (L+2a)}{2}$

$= - \frac{wa}{L}$



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

$M_C = 0$

$M_C - M_B$ = area of SFD between C and B

$\Rightarrow M_C - M_B = \frac{1}{2} \times a \times (wa)$

$\Rightarrow M_B = -\frac{wa^2}{2}$

$M_B - M_E$ = area of SFD between E and B

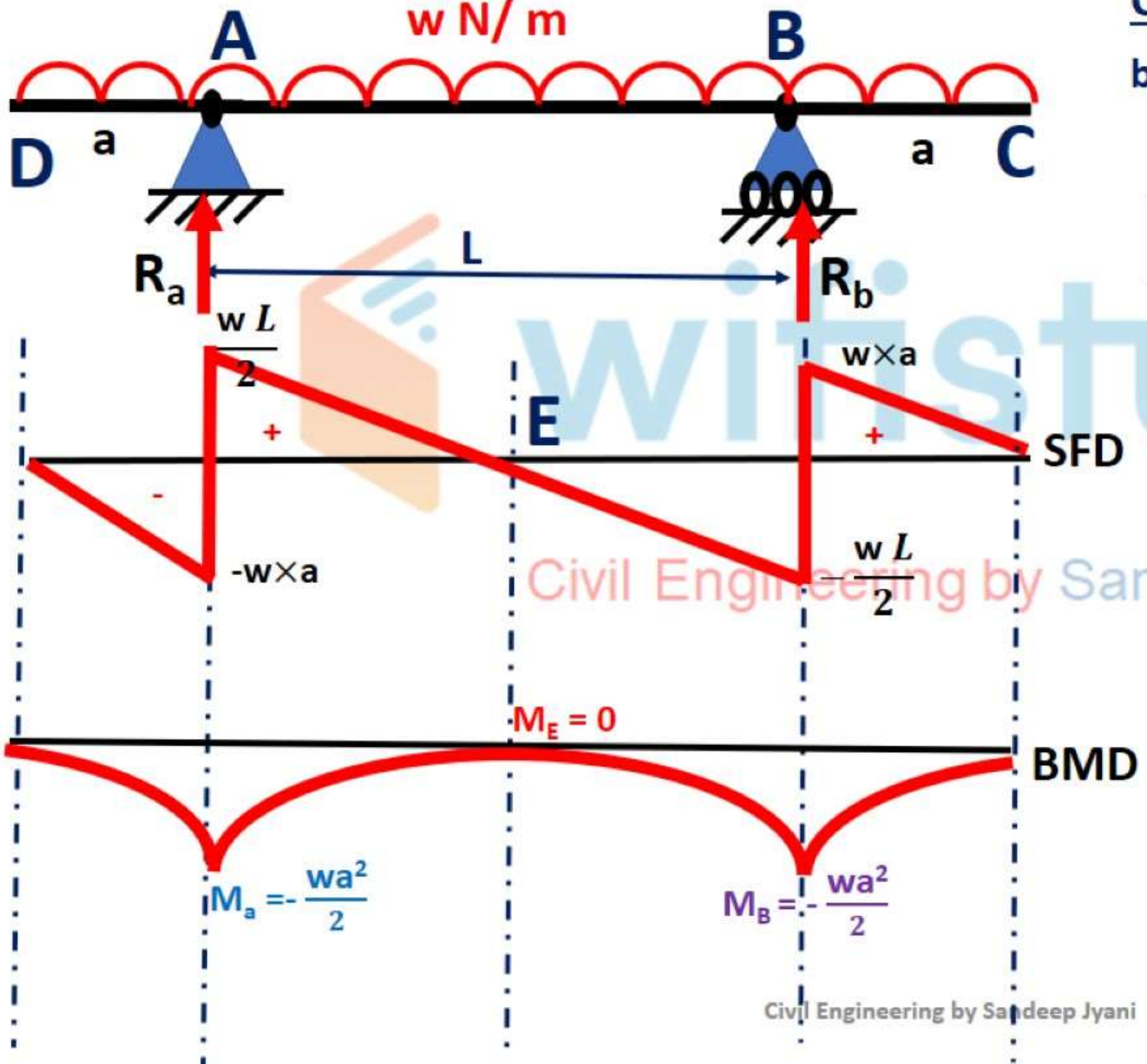
$\Rightarrow M_B - M_E = \frac{1}{2} \times \left(-\frac{wa^2}{2}\right) \times L/2$

$\Rightarrow -\frac{wa^2}{2} - M_E = -\frac{wL^2}{8}$

$\Rightarrow M_E = \frac{wL^2}{8} - \frac{wa^2}{2}$

$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$

$\Rightarrow$Similarly $M_a = -\frac{wa^2}{2}$



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

$\Rightarrow M_B = -\frac{wa^2}{2}$

$\Rightarrow M_a = -\frac{wa^2}{2}$

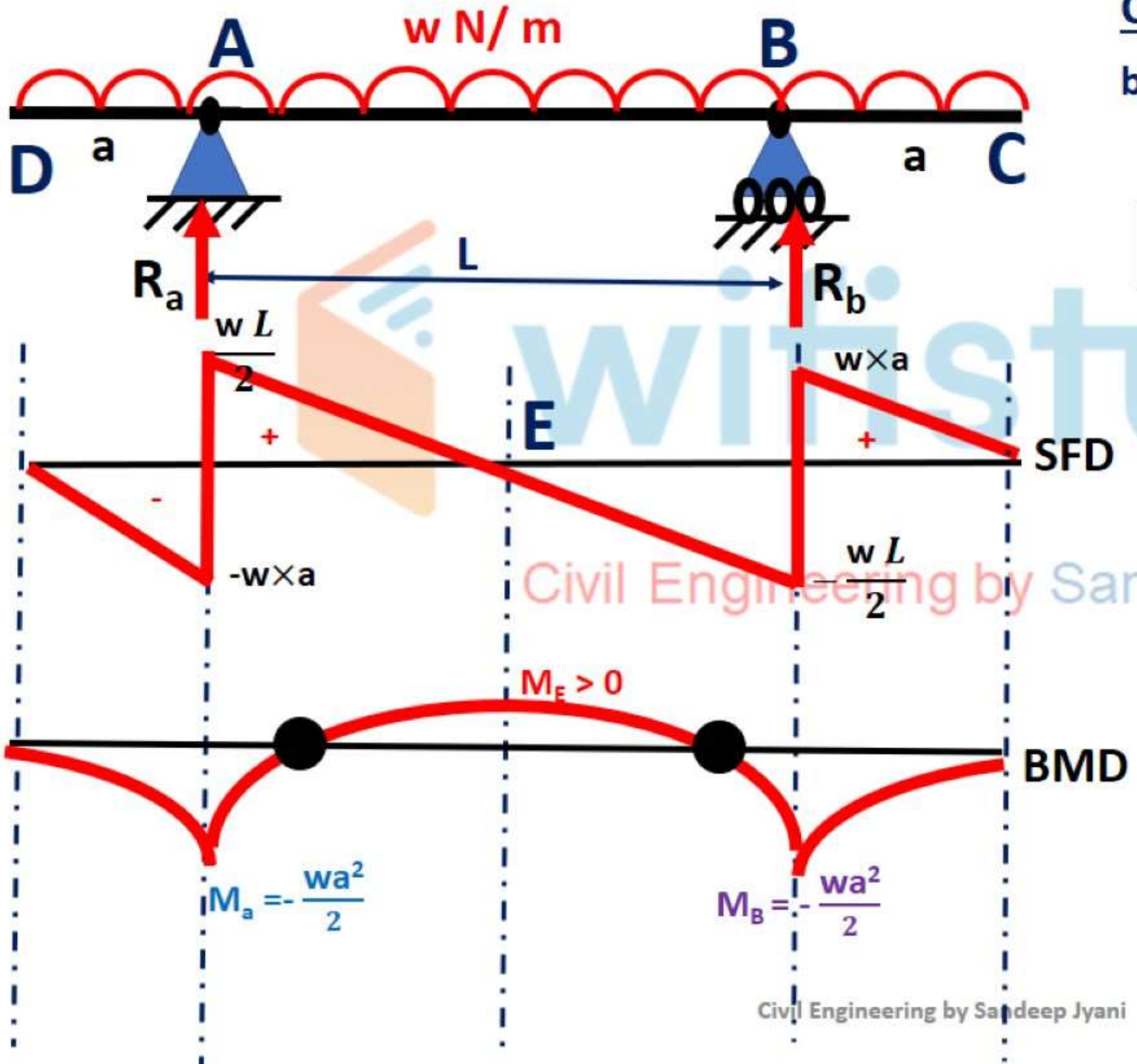
$$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$$

Case i)

If L= 2a

$$M_E = \frac{w}{8}((2a)^2 - 4a^2)$$

$$M_E = 0$$



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

$\Rightarrow M_B = -\frac{wa^2}{2}$ $\Rightarrow M_a = -\frac{wa^2}{2}$

$$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$$

Case ii)

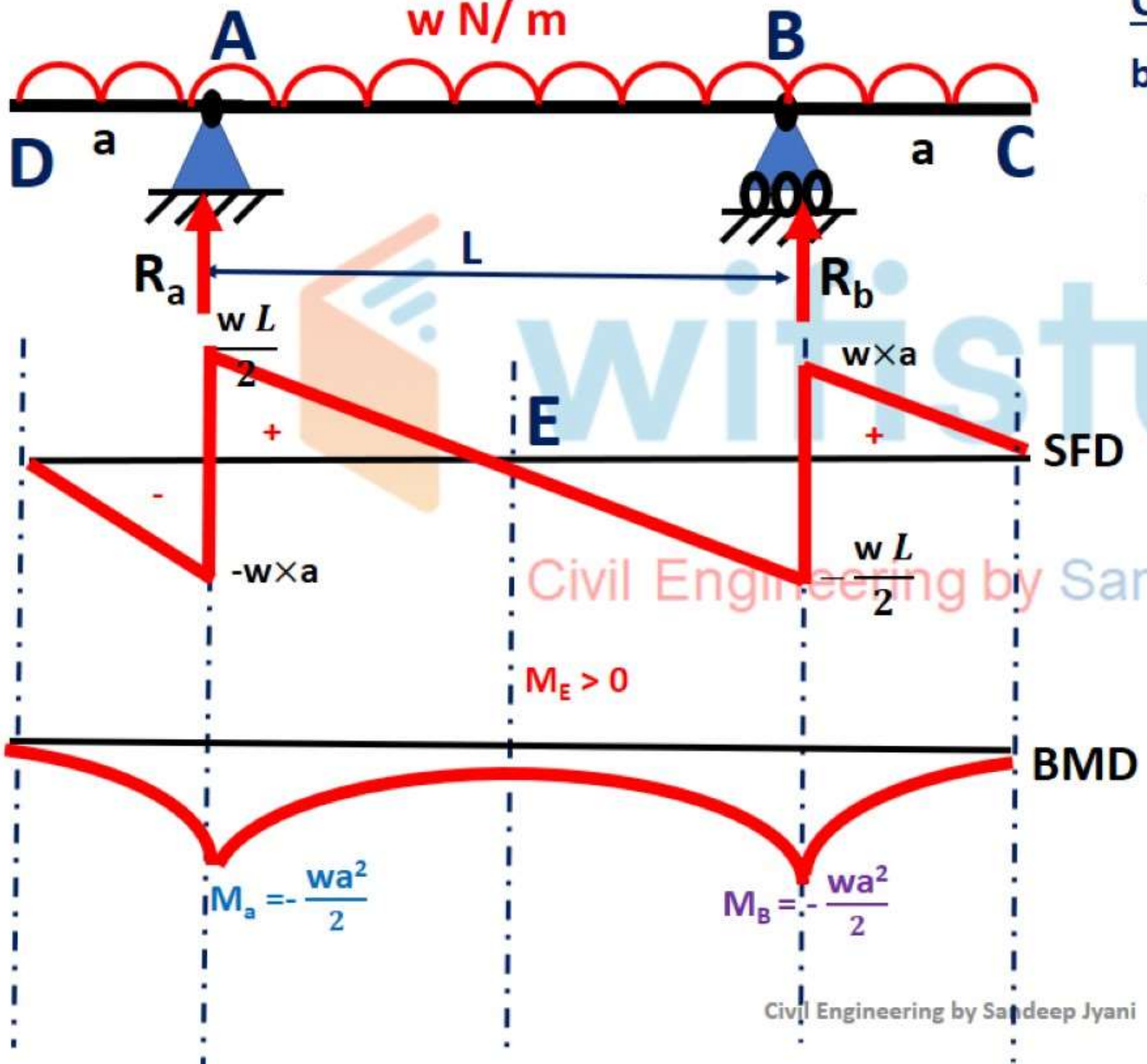
If L> 2a

$M_E = positive$

We get two points of contraflexure

CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load



$\Rightarrow M_B = -\frac{wa^2}{2}$

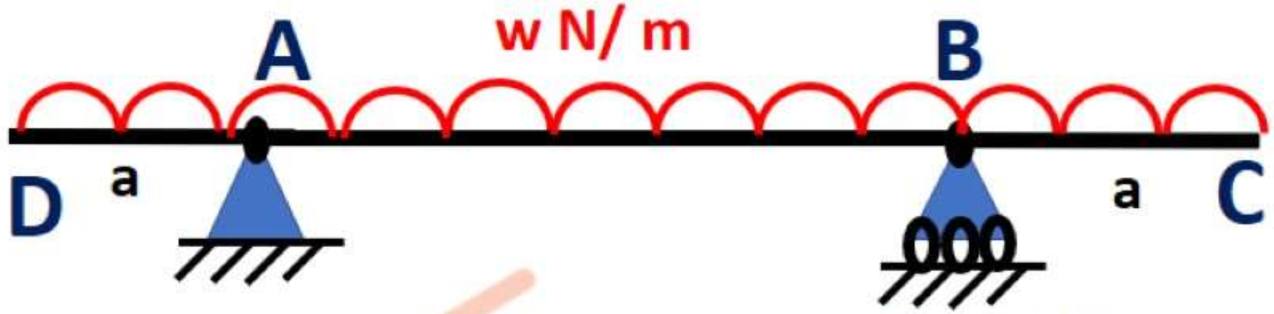
$\Rightarrow M_a = -\frac{wa^2}{2}$

$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$

Case ii)

If $L < 2a$

$M_E = Negative$



Que. 63 What percentage of total length should either overhang be so that BM= 0 at the centre ?

$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$

$M_E = 0$

$M_E = \frac{w}{8}(L^2 - 4a^2) = 0$

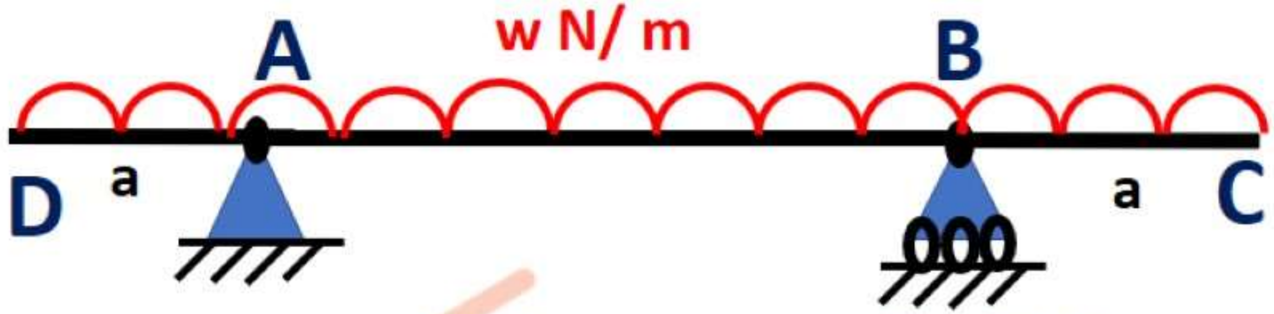
So L= 2a or

a = L/2

Total length = L+2a = 4a

Therefore, $\frac{a}{4a} \times 100$

$= 25\% \; of \; (L + 2a))$



Que. 64 If magnitude of bending moment at support is equal to Bending moment at centre, what is the relation between a and L?

$$\Rightarrow M_E = M_B$$

$$M_E = \frac{w}{8}(L^2 - 4a^2) = \left|-\frac{wa^2}{2}\right|$$

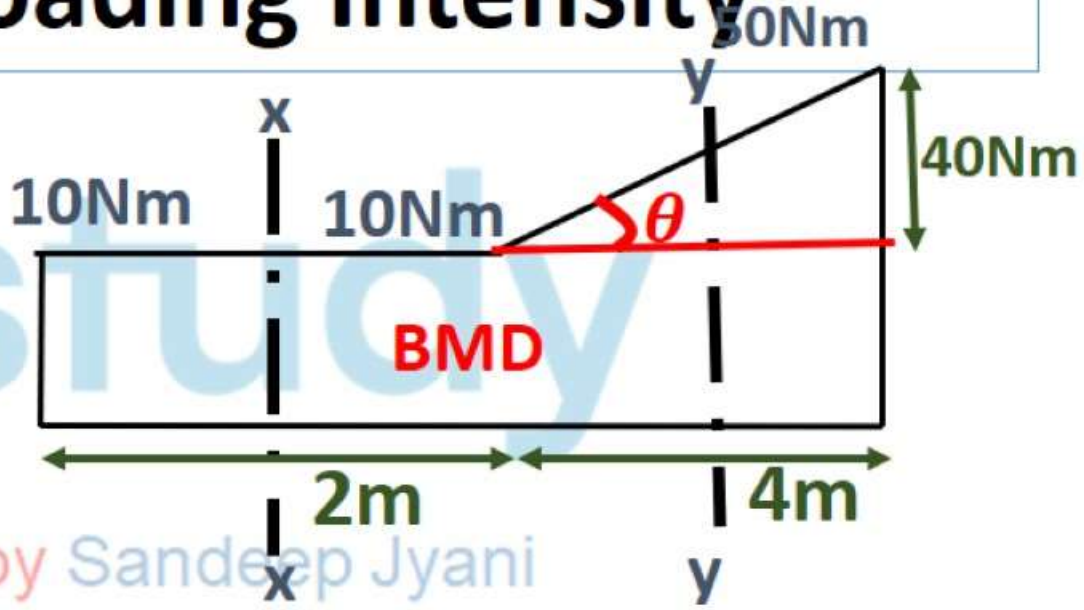
$$L^2/4 - a^2 = a^2$$

$$L = 2\sqrt{2}a$$

Relationship between Bending Moment, Shear Force and Loading Intensity

1. The slope of the BMD curve at a given section gives the value of Shear Force at that section

$(SF)_{xx}$ = ? And $(SF)_{yy}$ = ?



$(SF)_{xx}$ = slope of BMD at xx

$= 0$

$(SF)_{yy}$ = slope of BMD at yy

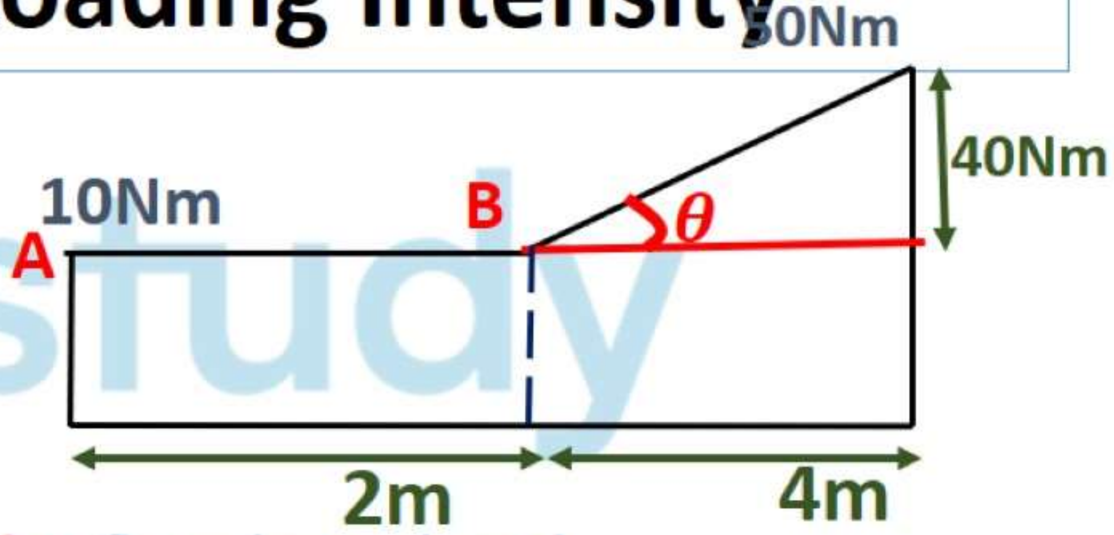
$= \tan\theta$

$= \frac{40}{4}$

$= 10$ N

Relationship between Bending Moment, Shear Force and Loading Intensity

1. **The slope of the BMD curve at a given section gives the value of Shear Force at that section**



$\frac{dM}{dx}$ = slope of BMD = F

$\Rightarrow dM = F\,dx$

$\Rightarrow \int_A^B dM = \int_A^B F dx$

$\Rightarrow M_B - M_A$ = Area of SFD between A and B

Relationship between Bending Moment, Shear Force and Loading Intensity

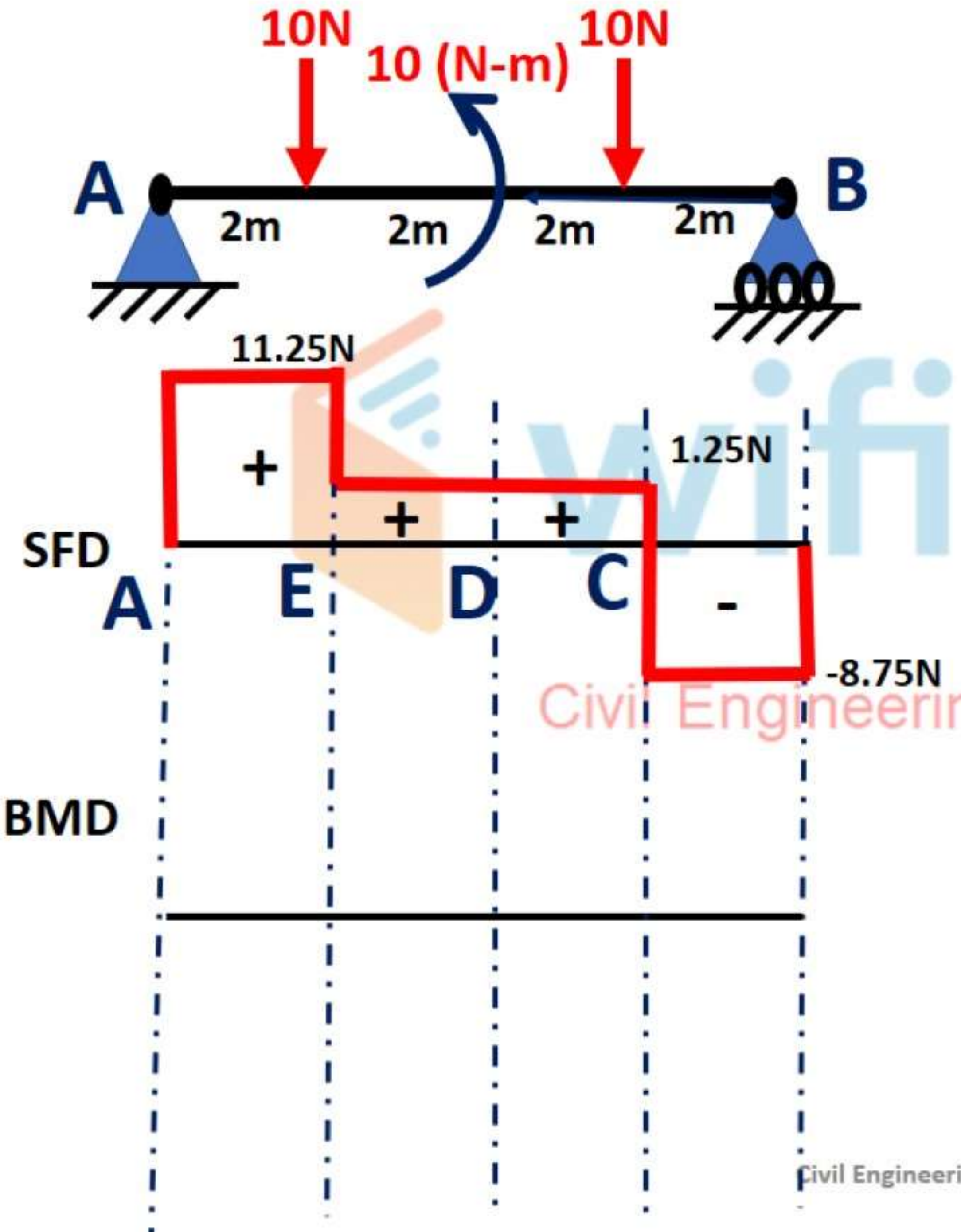
2. The slope of the SFD curve gives the value of Downward loading intensity on the member

$\frac{dF}{dx}$ = slope of SFD = - w (negative represents downward direction)

$\Rightarrow dF = -w\, dx$

$\Rightarrow \int_A^B dF = \int_A^B -w dx$

$\Rightarrow F_B - F_A$ = Area of Loading diagram between A and B



Que 63. Draw the SFD and BMD for the following.

Solution:

$R_a + R_b = 10 + 10 = 20N$ ....(1)

Taking moment about A = 0

$\Rightarrow (+R_b) \times 8 - (10 \times 6) + 10 - 20 = 0$

$\Rightarrow R_b = 70/8 = 8.75N$ and Hence

$\Rightarrow R_a = 11.25N$ (from eqn 1)

$M_B = 0$

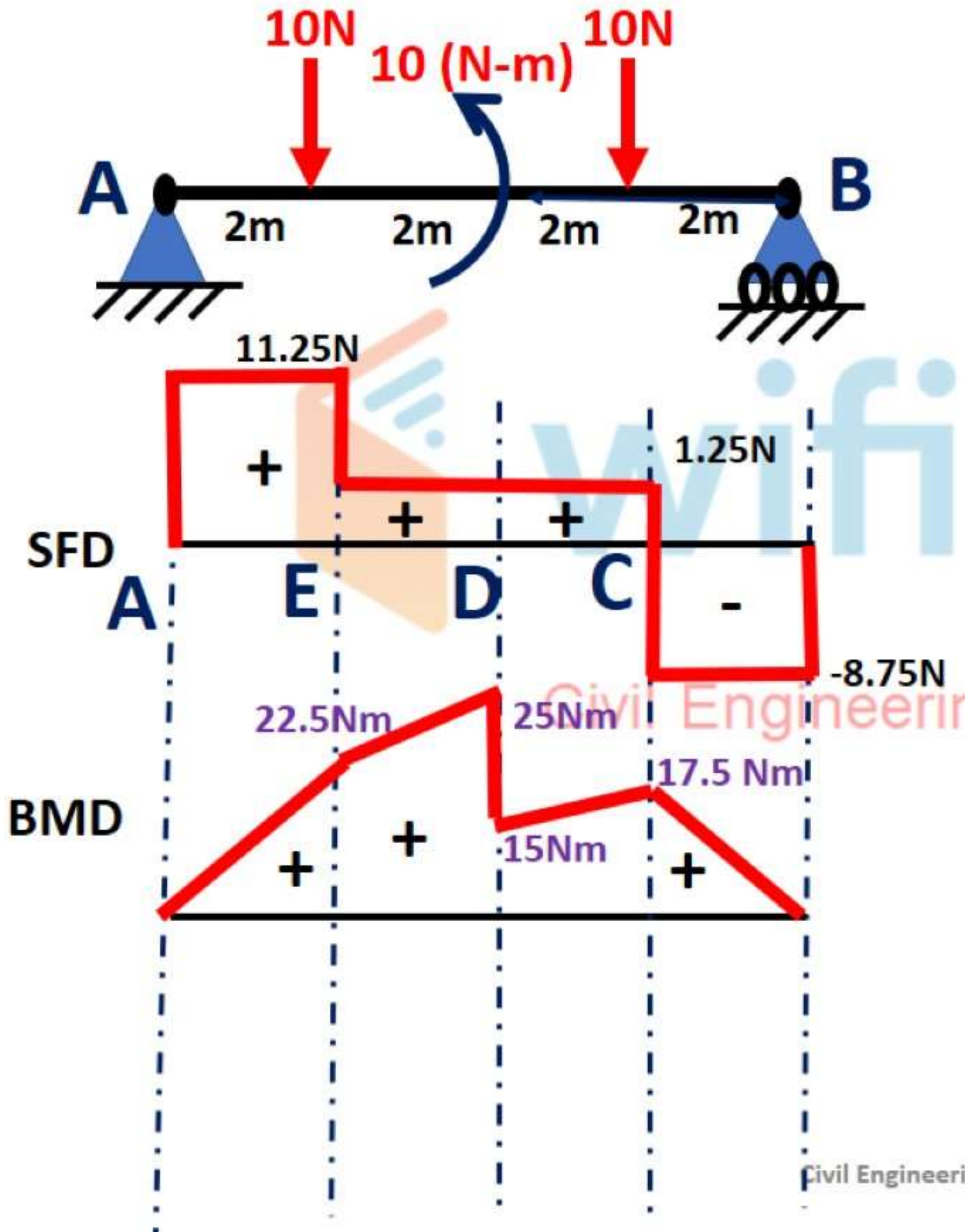
$M_B - M_C$ = area of SFD between C and B

$\Rightarrow M_B - M_C = - 8.75 \times 2$

$\Rightarrow M_C = 17.5$ Nm

$\Rightarrow$ Similarly $M_D = 15$Nm

$\Rightarrow M_{D\ left}$



Que 63. Draw the SFD and BMD for the following.

$M_B = 0$

$M_B - M_C$ = area of SFD between C and B

=> $M_B - M_C = -8.75 \times 2$

$\Rightarrow M_C = 17.5$ Nm

$\Rightarrow$Similarly $M_D = 15$Nm

$\Rightarrow M_{D\ left} = 15\text{Nm} + 10\text{Nm} = 25\text{Nm}$

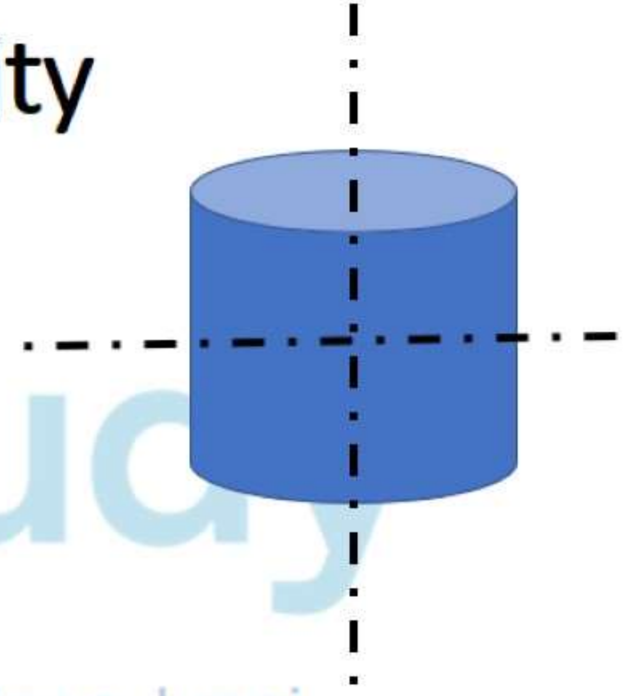
$\Rightarrow M_{D\ left} - M_E = 1.25 \times 2$

$\Rightarrow 25 - M_E = 2.5$

$\Rightarrow M_E = 22.5$Nm

Centre of Gravity

- A point through which the whole weight of the body acts, irrespective of its position, is known as centre of gravity

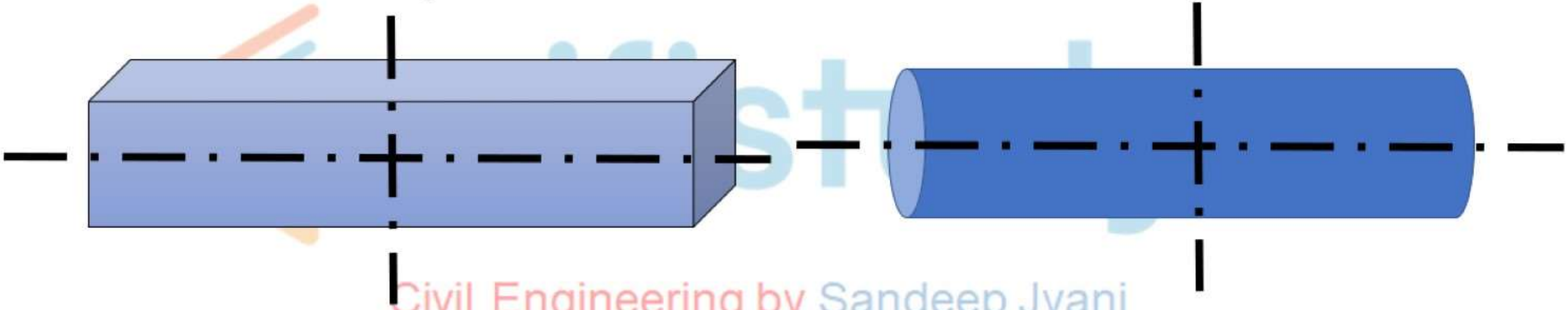


Centroid

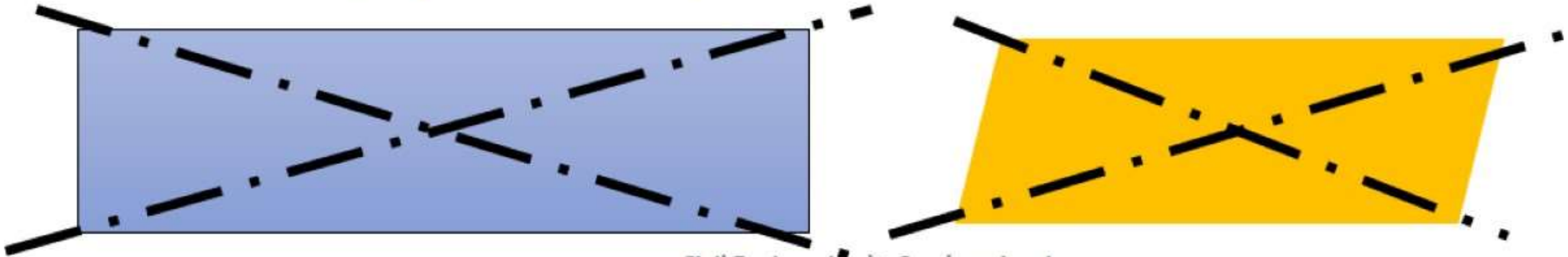
- The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre of area of such figures is known as *centroid*.

Centre of Gravity

1. **Uniform Rod/ Bar**

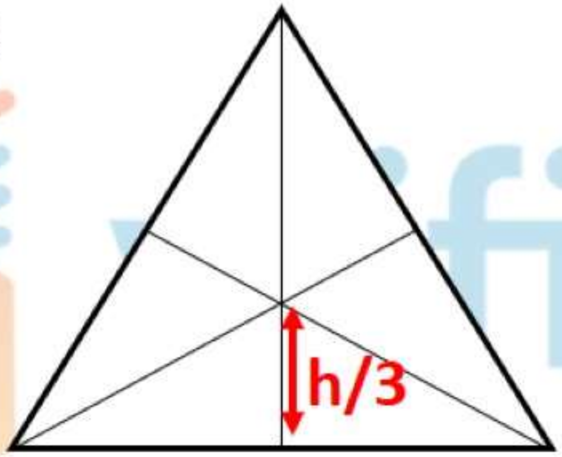


2. **Rectangle/ Parallelogram**

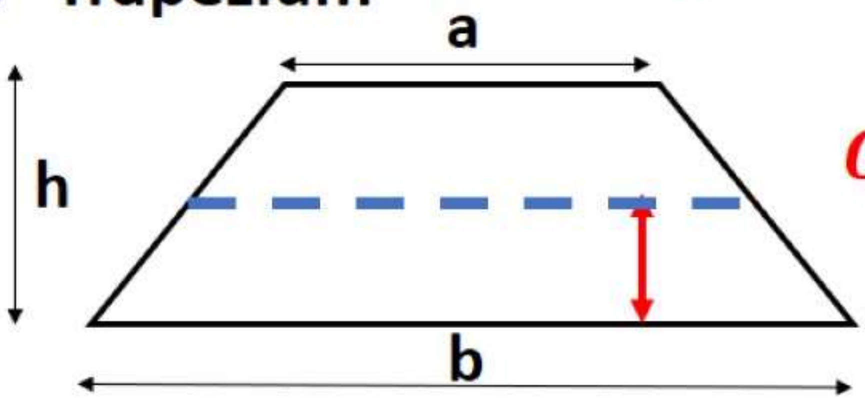


Centre of Gravity

3. Triangle



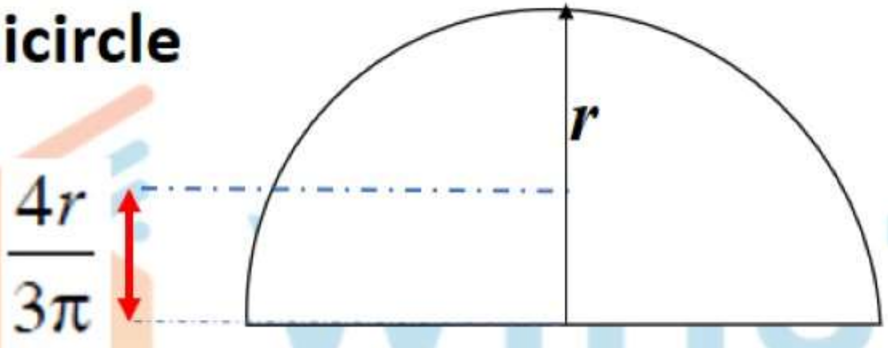
4. Trapezium



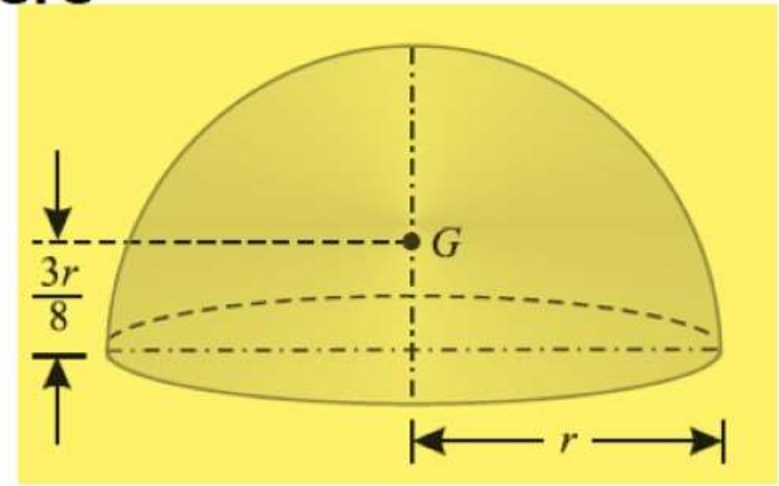
$CG = \frac{h}{3} \times \frac{2a+b}{a+b}$ **from Base b**

Centre of Gravity

4. Semicircle

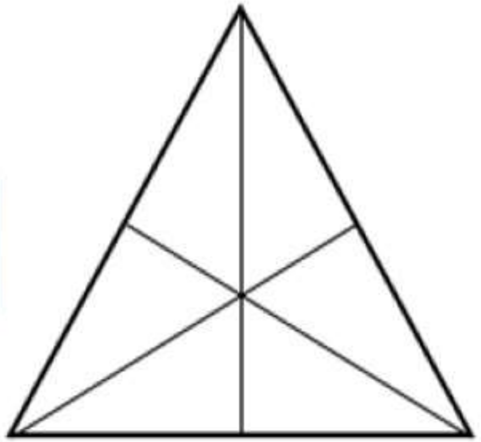


5. Hemisphere



Centre of Gravity

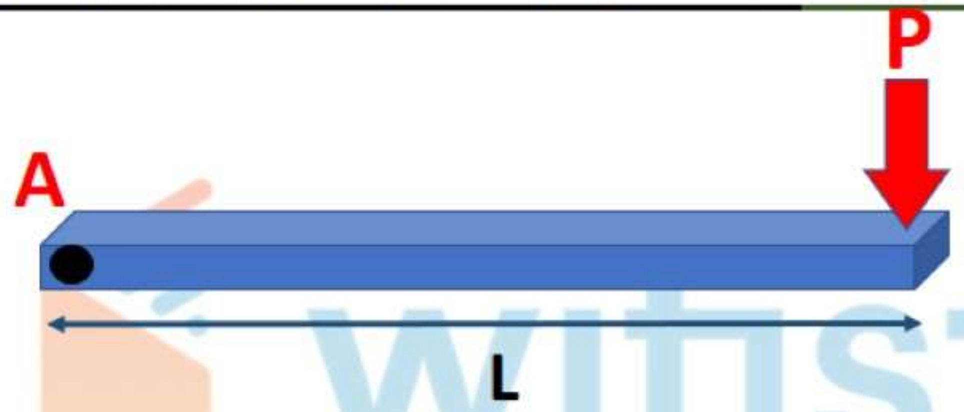
6. Right circular *solid* cone



MOMENT OF INERTIA

- FORCE
- AREA
- MASS

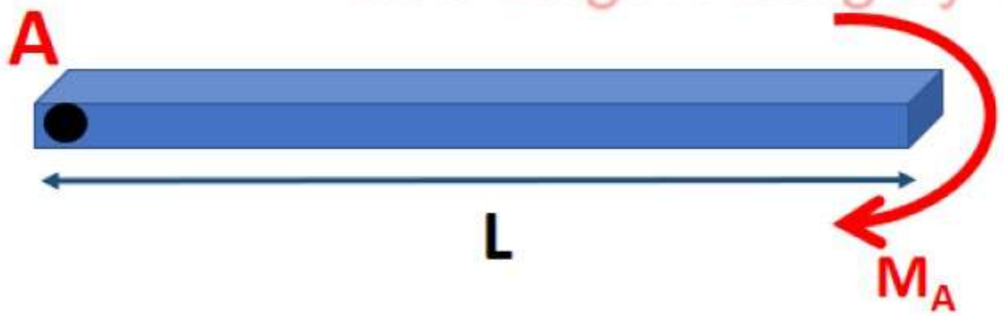
Moment of Force or First Moment of Force



$M_A = P \times L$

Moment of Moment of Force / Second Moment of Force

Moment of Inertia



$= M_A \times L$

$= P \times L \times L$

$I_A = PL^2$

MOMENT OF INERTIA OF A PLANE AREA

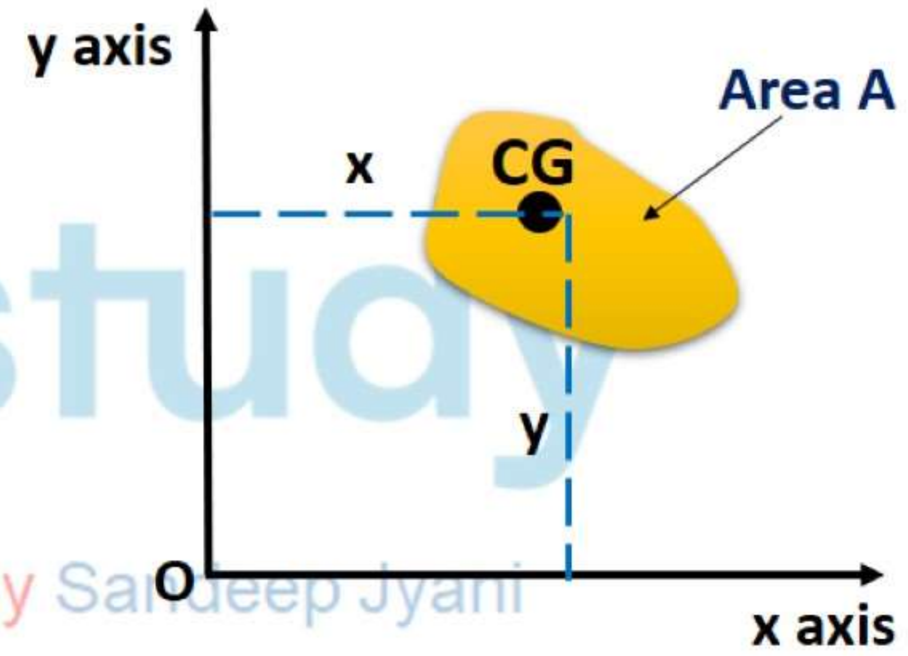
1. Moment of Area

$M_{y\ axis}$ = Area x Perpendicular distance of CG from OY

= Ax

$M_{x\ axis}$ = Area x Perpendicular distance of CG from OX

= Ay



2. Area Moment of Inertia

$I_{y\ axis} = M_{y\ axis}$ x Perpendicular distance of CG from OY

= Ax^2

$I_{x\ axis} = M_{x\ axis}$ x Perpendicular distance of CG from OX

= Ay^2

UNITS OF MOMENT OF INERTIA

= area x (per. distance)^2

= m^4 or mm^4

MOMENT OF INERTIA OF A PLANE AREA

3. **Polar Moment of Inertia (Resistance against torsion)**

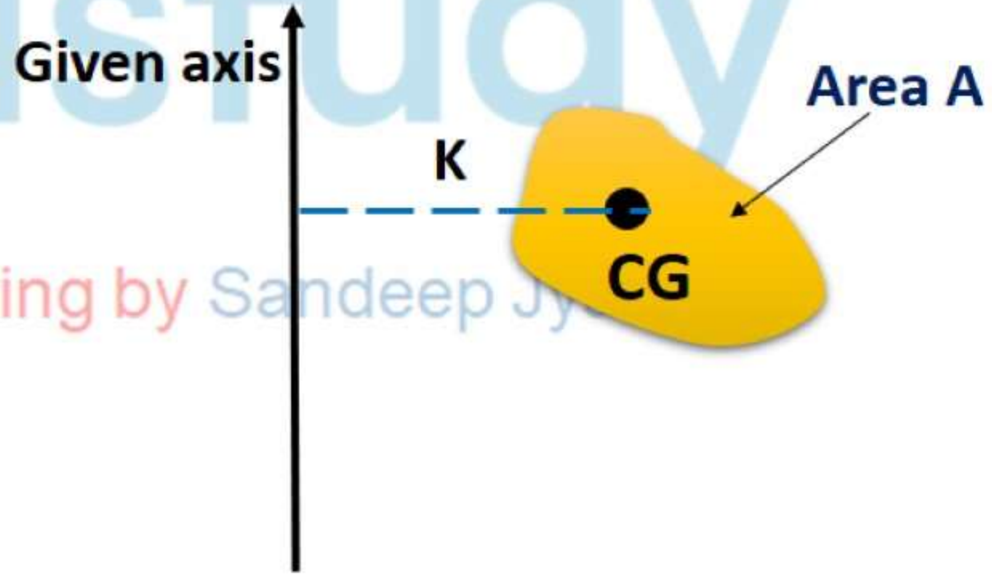
$$I_P = I_{x\ axis} + I_{y\ axis}$$

4. **Radius of Gyration**

- It is distance such that its square multiplied by area gives Moment of inertia about the given axis

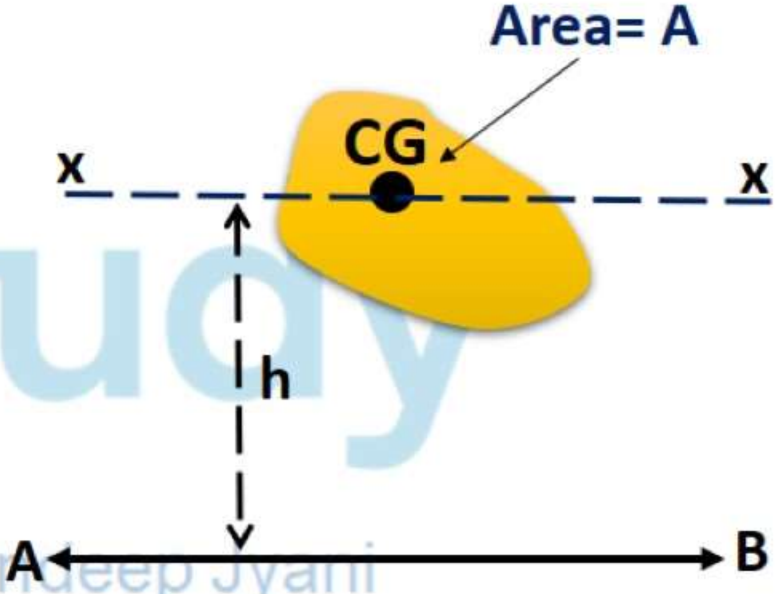
$$K^2 \times A = I$$

$$K = \sqrt{\frac{I}{A}}$$



THEOREM OF PARALLEL AXIS

Statement: **If moment of Inertia of a plane area about an axis through Centre of Gravity is I_G ($I_{xx} = I_G$), then**



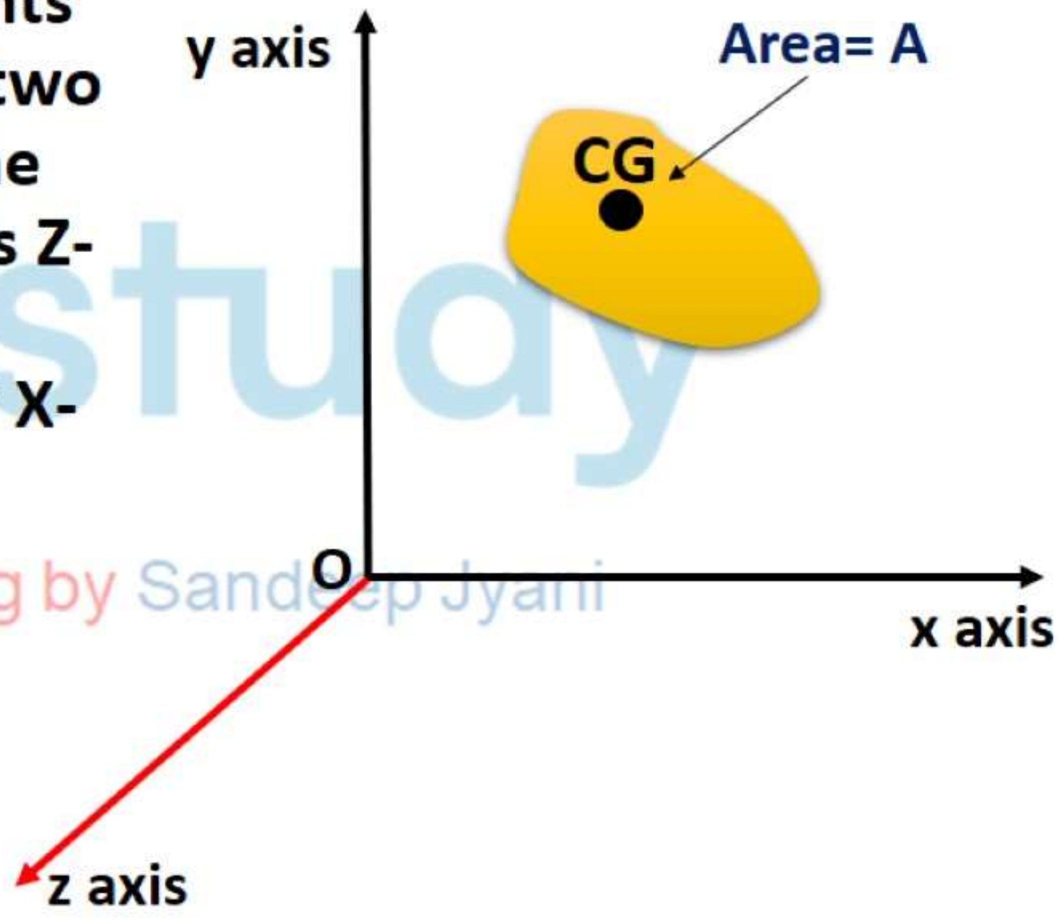
Moment of Inertia of the given plane area about a parallel axis AB in the plane of area at distance *h* from CG is given by:

$$I_{AB} = \text{MOI at CG} + \text{Area} \times (\text{distance})^2$$

$$I_{AB} = I_G + Ah^2$$

THEOREM OF PERPENDICULAR AXIS

It states, If I_{xx} and I_{yy} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{zz} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:



$$I_{zz} = I_{xx} + I_{yy}$$

MOMENT OF INERTIA OF A MASS

1. Moment of Mass

$M_{y\ axis}$ = Mass x Perpendicular distance of CG from OY

= mx

$M_{x\ axis}$ = Mass x Perpendicular distance of CG from OX

= my

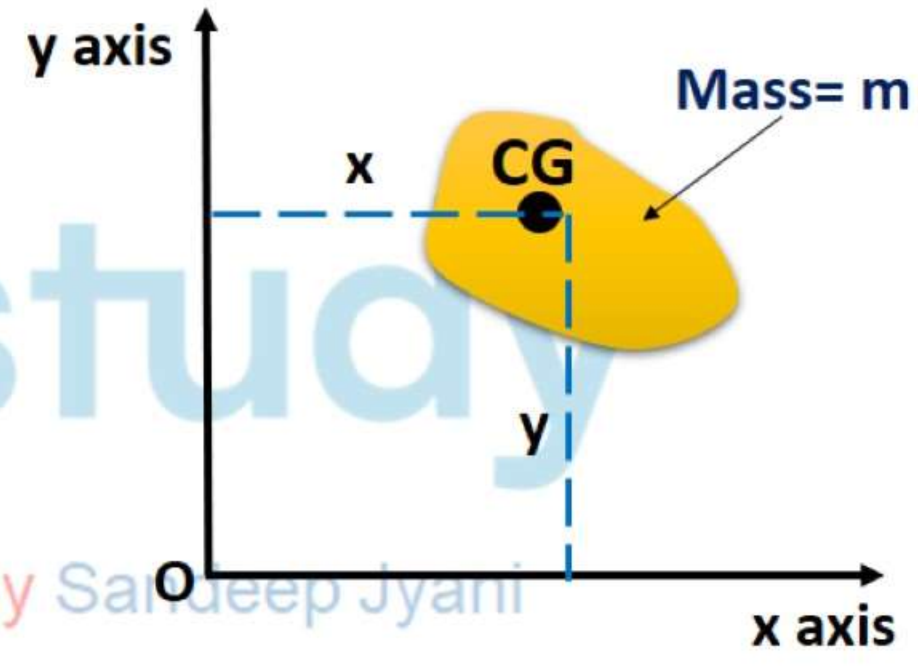
2. Mass Moment of Inertia

$I_{y\ axis} = M_{y\ axis}$ x Perpendicular distance of CG from OY

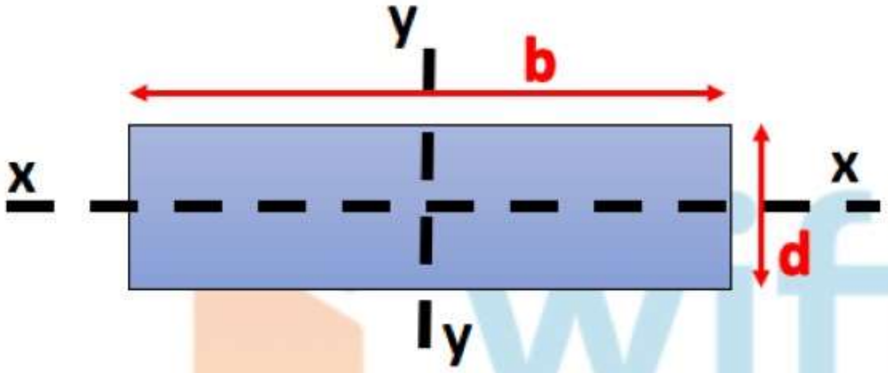
= mx^2

$I_{x\ axis} = M_{x\ axis}$ x Perpendicular distance of CG from OX

= my^2

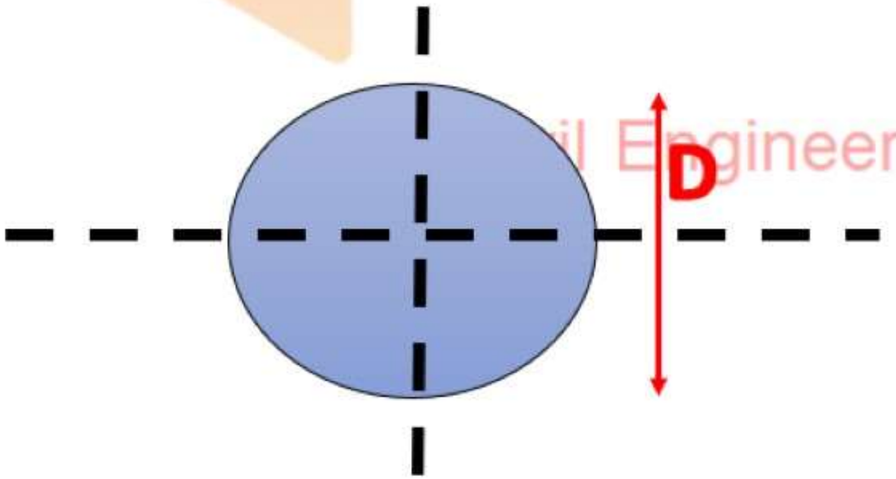


Moment of Inertia of Some Important Sections



1. Rectangle

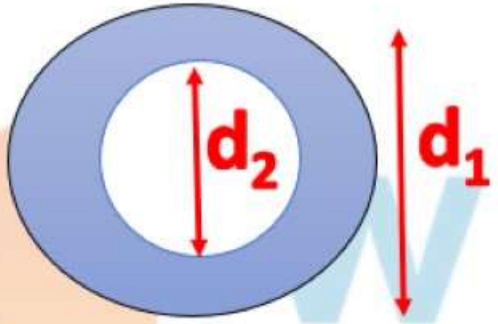
$I_{xx} = \frac{bd^3}{12}$ $\quad I_{yy} = \frac{db^3}{12}$



2. Circle

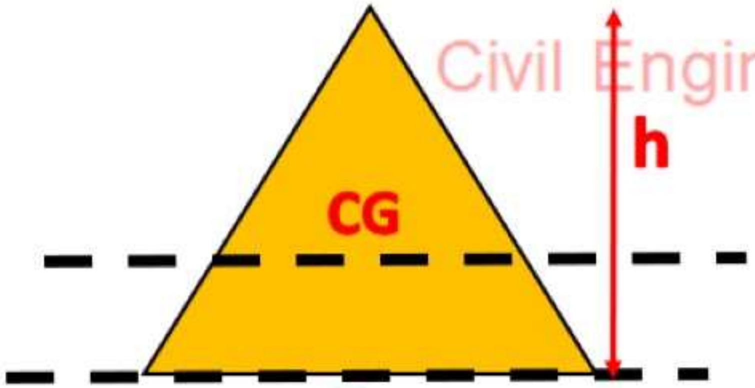
$I_{xx} = I_{yy} = \frac{\pi}{64} d^4$

Moment of Inertia of Some Important Sections



3. Concentric Circles

$$I_{xx} = I_{yy} = \frac{\pi}{64}(d_1^4 - d_2^4)$$

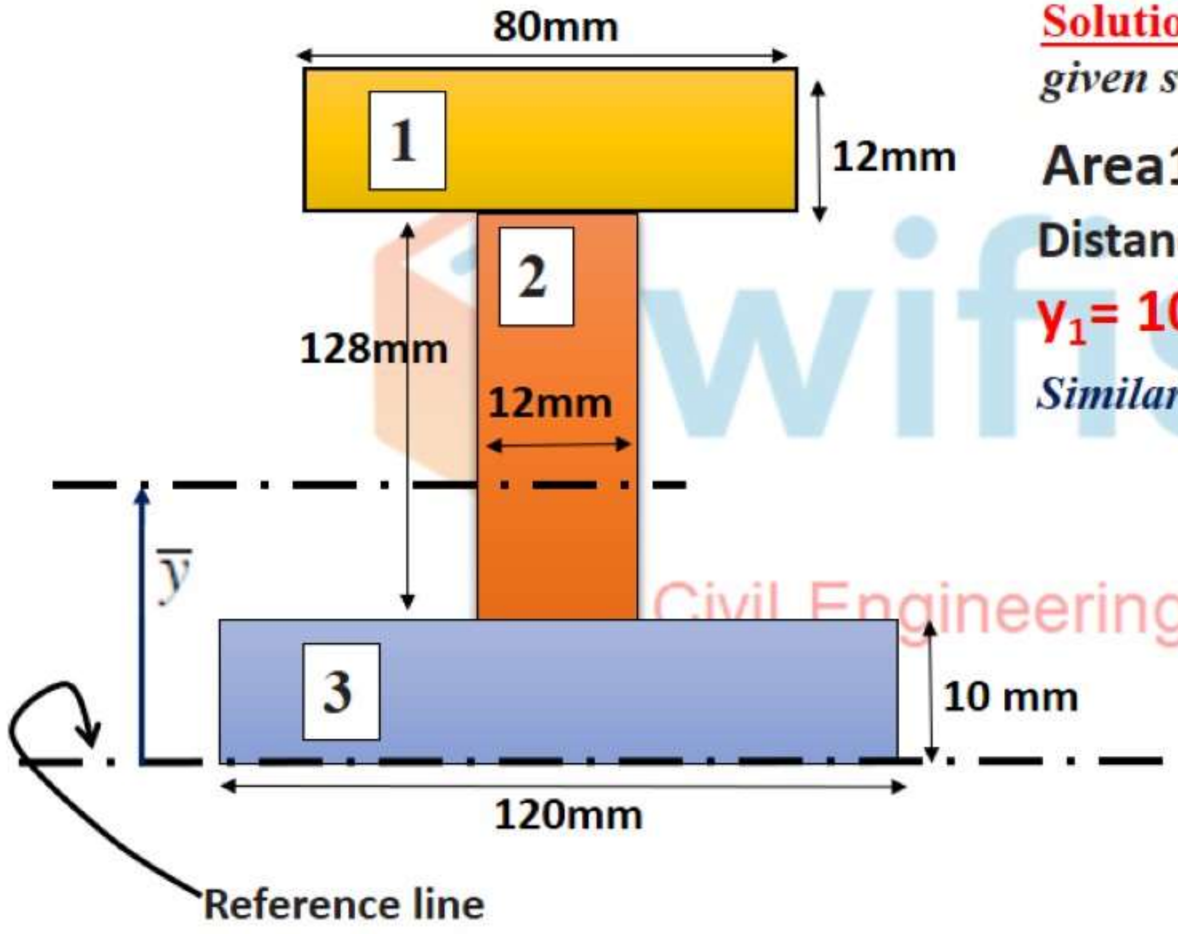


4. Triangle

$$I_{CG} = \frac{1}{36} bh^3$$

$$I_{base} = \frac{1}{12} bh^3$$

Q. 65 Find out the Polar Moment of Inertia of given I section.



Solution: *Let us first find out the Centre of Gravity of given section*

Area1 (a_1) = 80×12 = **960 mm²**

Distance of CG of rectangle 1 from Bottom reference line

$y_1 = 10 + 128 + 12/2 = 144\text{mm}$

Similarly $a_2 = 128 \times 12 = 1536\ \text{mm}^2$

$y_2 = 10 + 128/2 = 74\ \text{mm}$

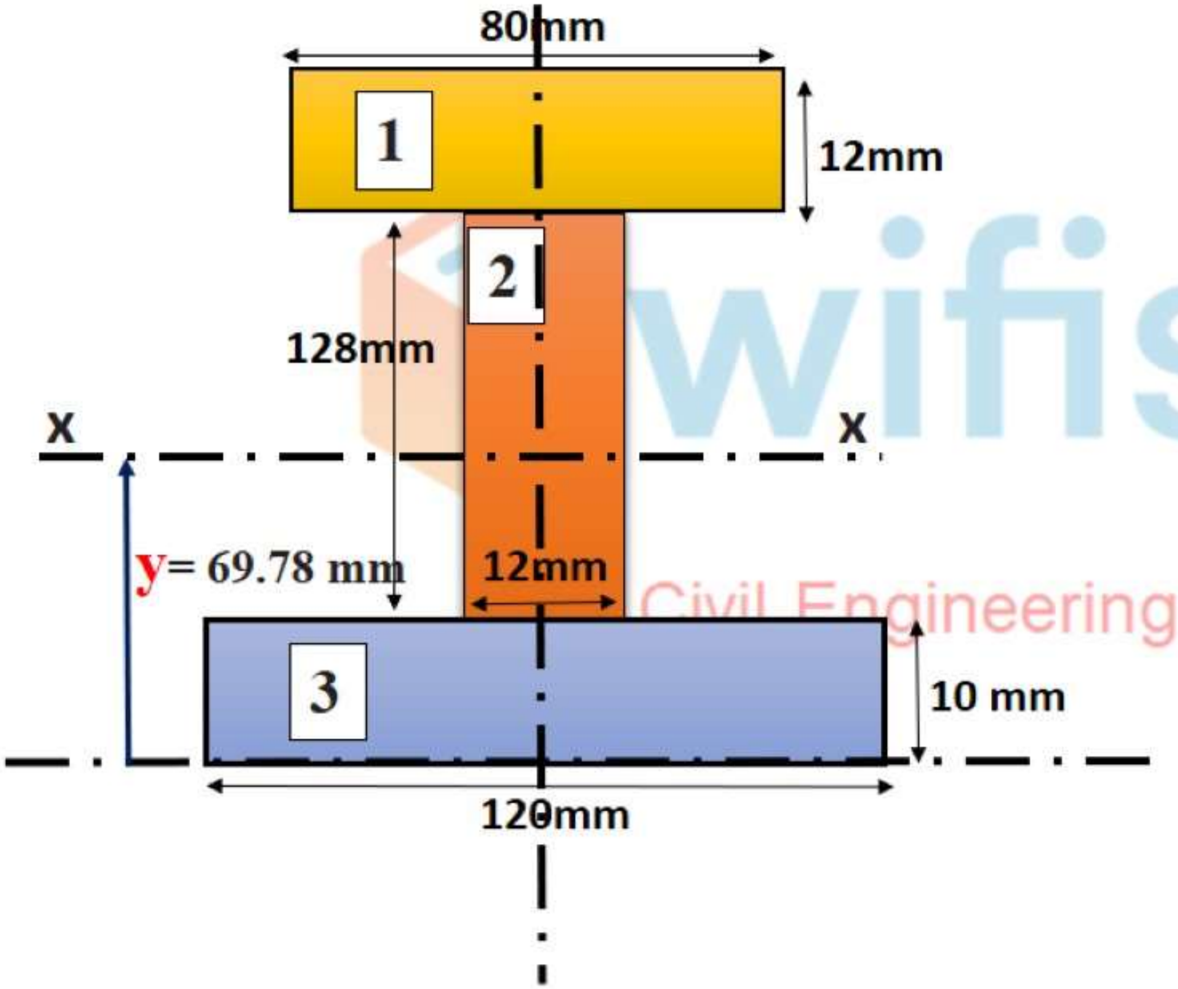
$a_3 = 120 \times 10 = 1200\ \text{mm}^2$

$y_3 = 10/2 = 5\ \text{mm}$

$$\overline{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(960\text{x}144)+(1536\text{x}74)+(1200\text{x}5)}{960 + 1536 + 1200} = 69.78\ \text{mm}$$

Q. 65 Find out the Polar Moment of Inertia of given I section.



Moment of Inertia about xx axis:

$$I_{xx} = I_G + Ah^2$$

$$I_{xx\,1} = I_{G1} + A_1 \times h_1^2$$

$$= \frac{80 \times 12^3}{12} + 960(144 - 69.78)^2$$

$$= 5.3 \times 10^6 \text{ mm}^4$$

$$I_{xx\,2} = I_{G2} + A_2 \times h_2^2$$

$$= \frac{12 \times 128^3}{12} + 1536(74 - 69.78)^2$$

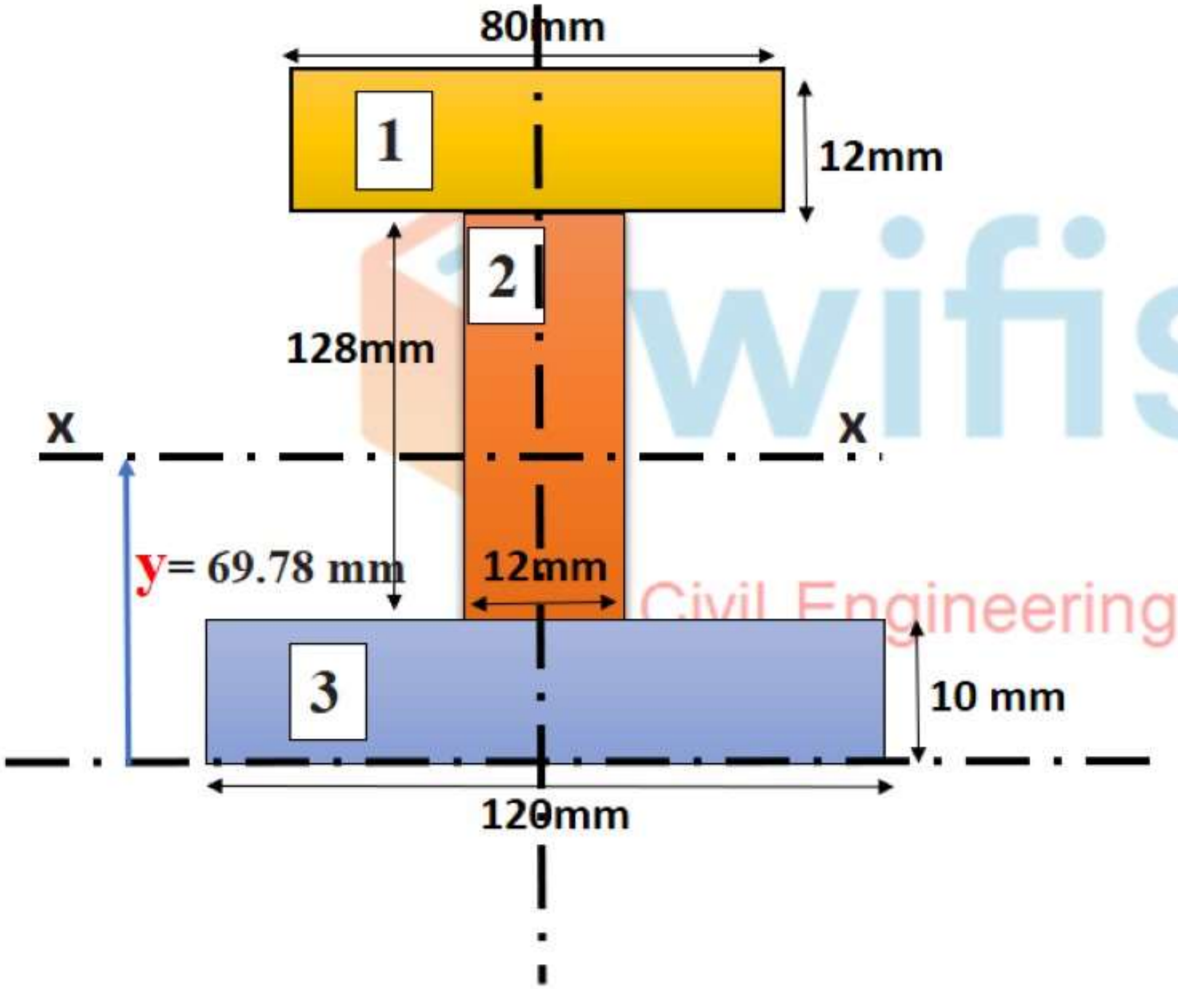
$$= 2.12 \times 10^6 \text{ mm}^4$$

$$I_{xx\,3} = I_{G3} + A_3 \times h_3^2$$

$$= \frac{120 \times 10^3}{12} + 1200\,(5 - 69.78)^2$$

$$= 5.04 \times 10^6 \text{ mm}^4$$

Q. 65 Find out the Polar Moment of Inertia of given I section.



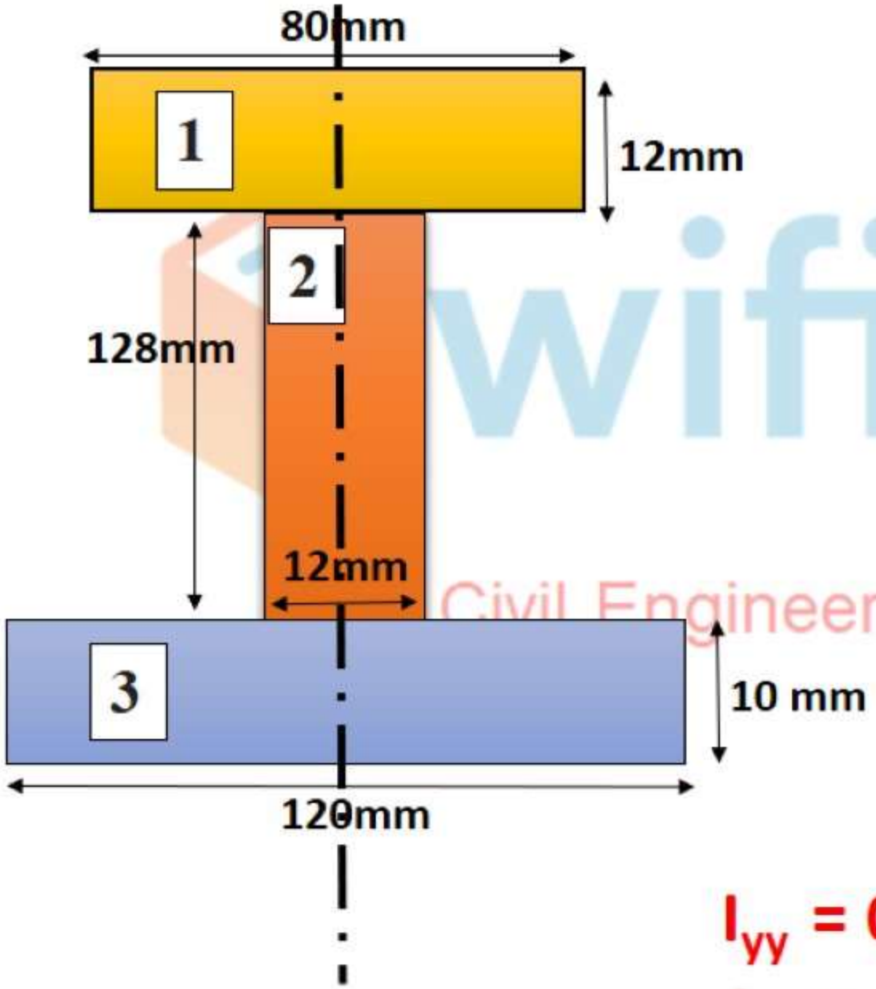
Moment of Inertia about xx axis:

$$I_{xx} = I_{xx\,1} + I_{xx\,2} + I_{xx\,3}$$

$$I_{xx} = 5.3 \times 10^6 + 2.12 \times 10^6 + 5.04 \times 10^6$$

$$I_{xx} = 12.46 \times 10^6\ mm^4$$

Q. 66 Find out the Polar Moment of Inertia of given I section.



Moment of Inertia about yy axis:

$$I_{yy\,1} = \frac{12 \times 80^3}{12}$$
$$= 0.521 \times 10^6 \text{ mm}^4$$

$$I_{yy\,2} = I_{G2}$$
$$= \frac{128 \times 12^3}{12}$$
$$= 0.0184 \times 10^6 \text{ mm}^4$$

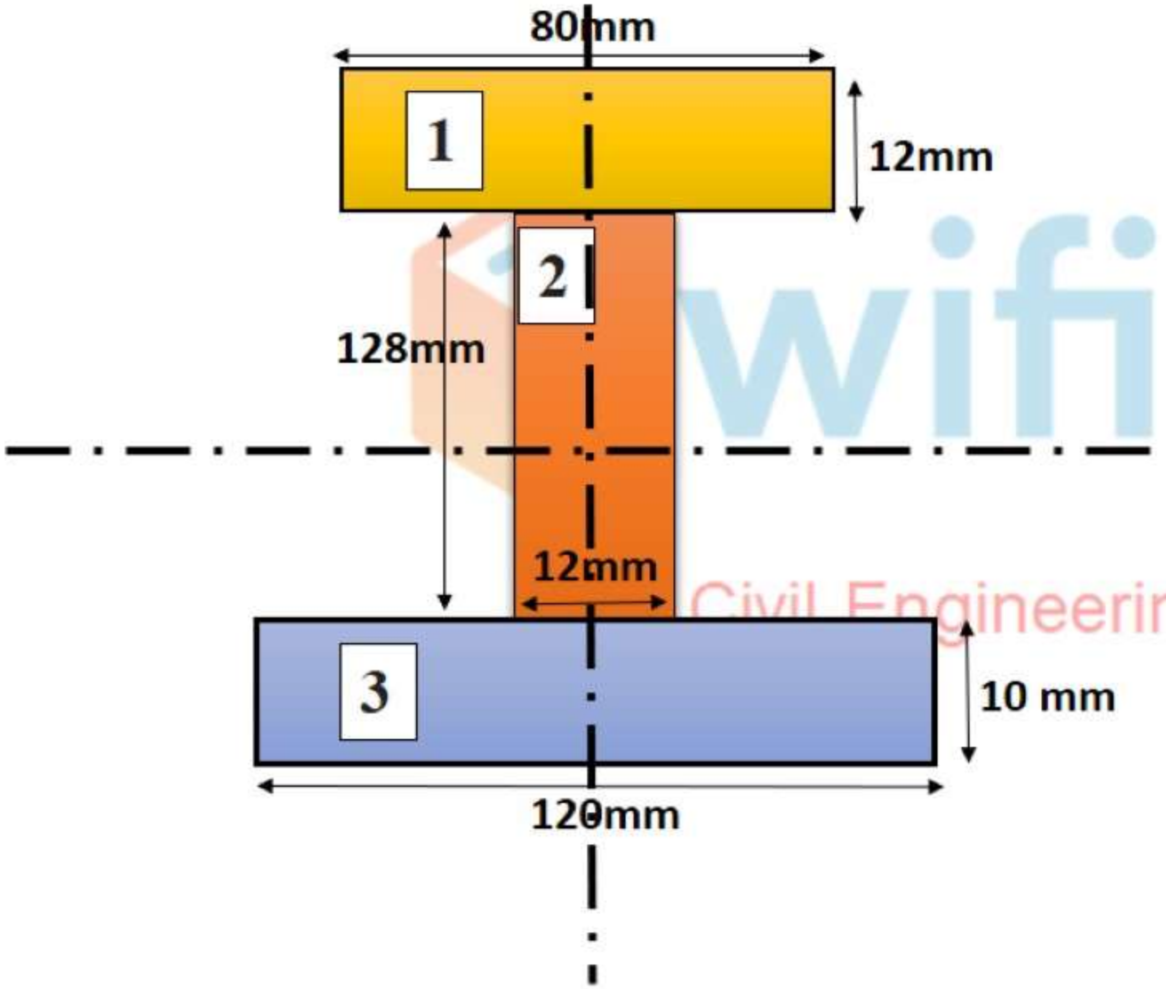
$$I_{yy\,3} = \frac{10 \times 120^3}{12}$$
$$= 1.44 \times 10^6 \text{ mm}^4$$

$$I_{yy} = I_{yy\,1} + I_{yy\,2} + I_{yy\,3}$$

$$I_{yy} = 0.521 \times 10^6 + 0.0184 \times 10^6 + 1.44 \times 10^6$$

$$I_{yy} = 1.979 \times 10^6 \text{ mm}^4$$

Q. 66 Find out the Polar Moment of Inertia of given I section.



POLAR MOMENT OF INERTIA

$$I_P = I_{xx} + I_{yy}$$

$$I_P = 12.46 \times 10^6 + 1.979 \times 10^6$$

$$I_P = 14.439 \times 10^6 \text{ mm}^4$$

Que. 67 If the area of a section is in mm2 and the distance of the centre of area from a lines is in mm, then units of the moment of inertia of the section about the line is expressed in

(a) mm^2

(b) mm^3

(c) mm^4

(d) mm^5

Que. 67 If the area of a section is in mm2 and the distance of the centre of area from a lines is in mm, then units of the moment of inertia of the section about the line is expressed in

(a) mm^2

(b) mm^3

(c) $\underline{mm^4}$

(d) mm^5

Que 68 Theorem of perpendicular axis is used in obtaining the moment of inertia of a

(a) triangular lamina

(b) square lamina

(*c*) circular lamina

(*d*) semicircular lamina

Que 68 Theorem of perpendicular axis is used in obtaining the moment of inertia of a

(a) triangular lamina

(b) <u>square lamina</u>

(*c*) circular lamina

(*d*) semicircular lamina

Que 69 The moment of inertia of a circular section of diameter (*d*) about diameter is given by the relation

a) $\frac{\pi}{16}d^4$

b) $\frac{\pi}{32}d^4$

c) $\frac{\pi}{64}d^4$

d) $\frac{\pi}{96}d^4$

Que 69 The moment of inertia of a circular section of diameter (*d*) about diameter is given by the relation

a) $\frac{\pi}{16}d^4$

b) $\frac{\pi}{32}d^4$

c) $\frac{\pi}{64}d^4$

d) $\frac{\pi}{96}d^4$

Que. 70 The moment of inertia of a triangular section of base (*b*) and height (*h*) about an axis through its *c.g.* and parallel to the base is given by the relation:

a) $\frac{1}{12}bh^3$

b) $\frac{1}{24}bh^3$

c) $\frac{1}{36}bh^3$

d) $\frac{1}{48}bh^3$

Que. 70 The moment of inertia of a triangular section of base (*b*) and height (*h*) about an axis through its *c.g.* and parallel to the base is given by the relation:

a) $\frac{1}{12}bh^3$

b) $\frac{1}{24}bh^3$

c) $\frac{1}{36}bh^3$

d) $\frac{1}{48}bh^3$

Que 71 The moment of inertia of a triangular section of base (*b*) and height (*h*) about an axis passing through its vertex and parallel to the base is ..... as that passing through its C.G. and parallel to the base:

(a) twelve times

(b) nine times

(*c*) six times

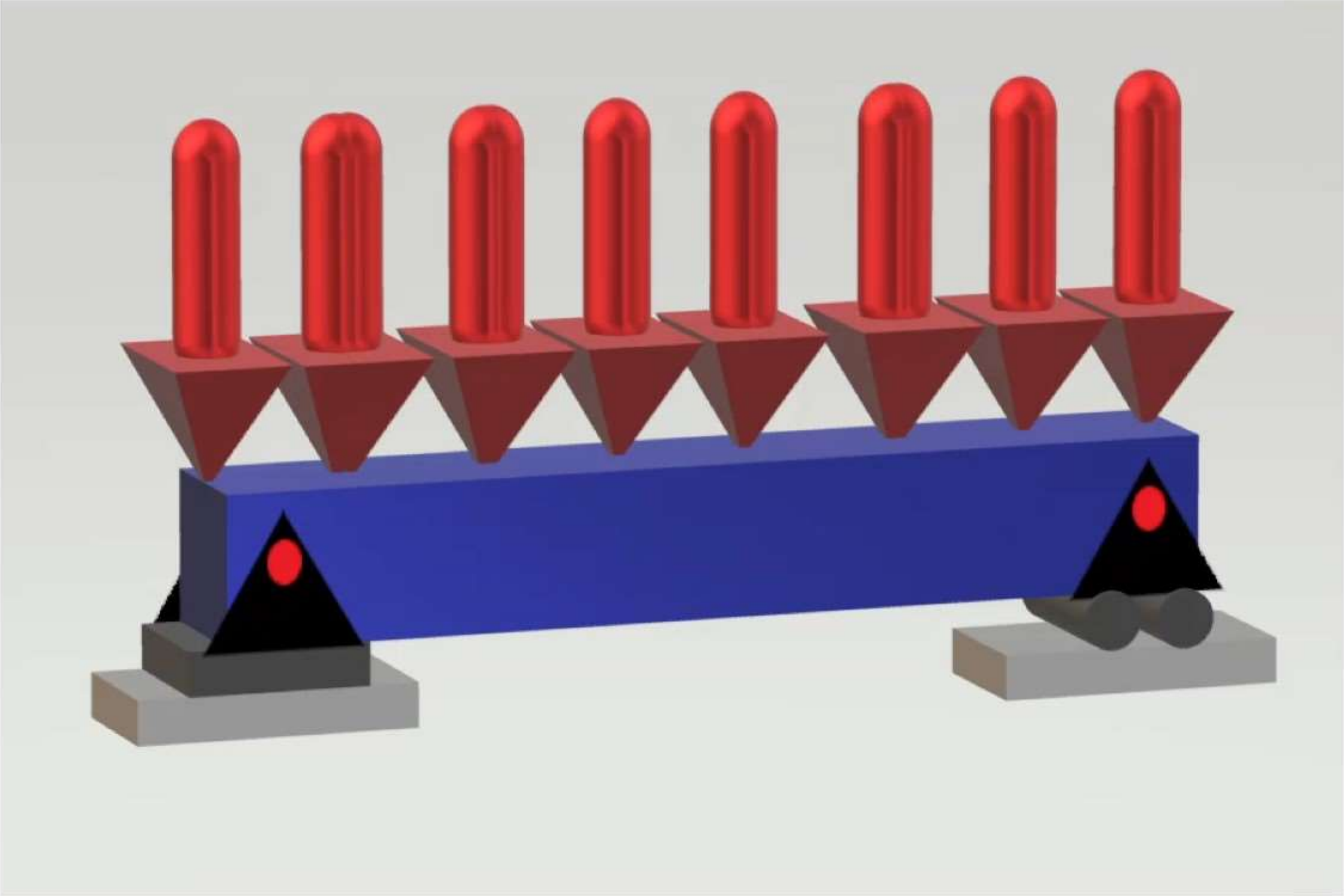
(*d*) four times

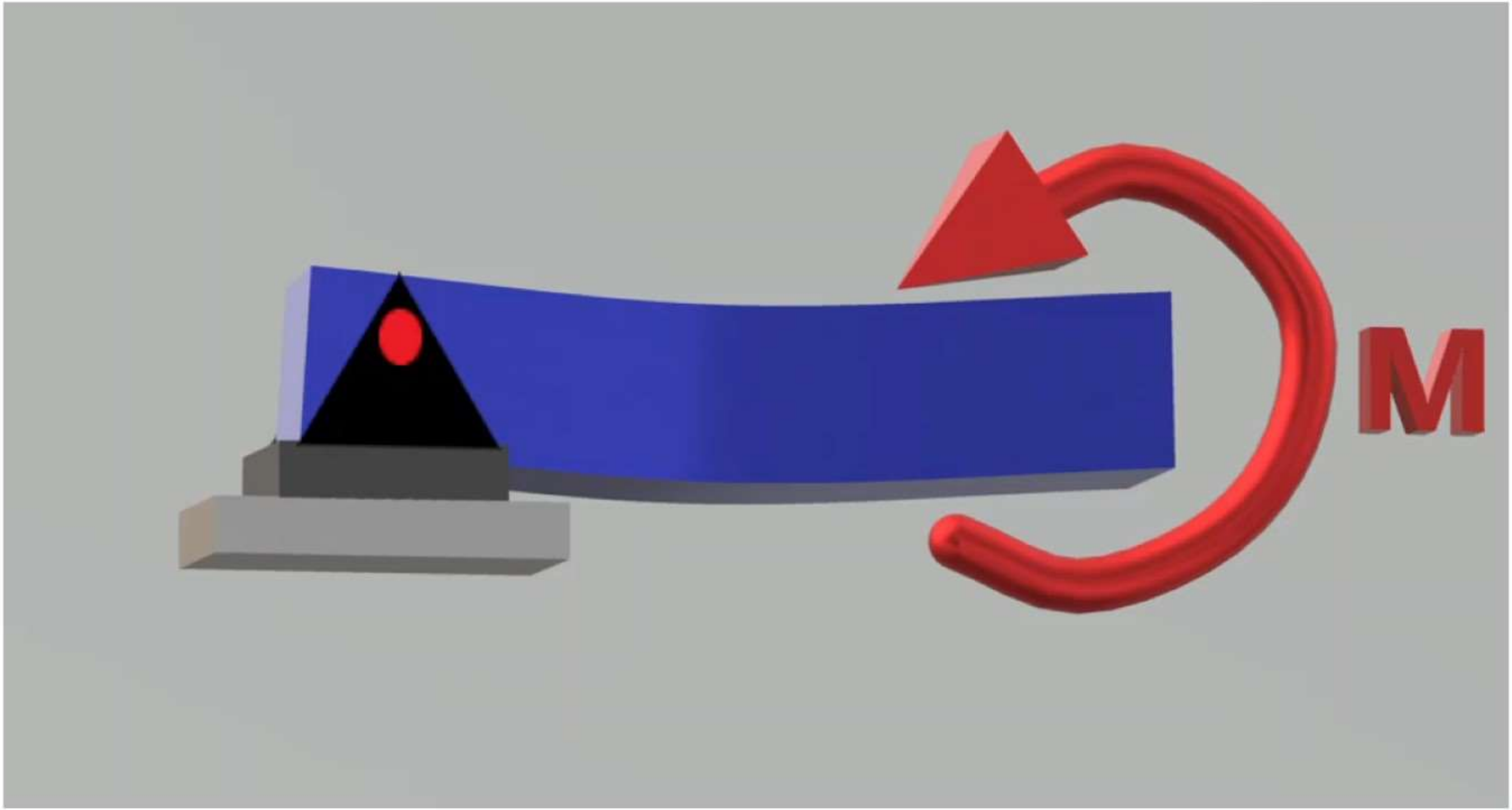
Homework!!

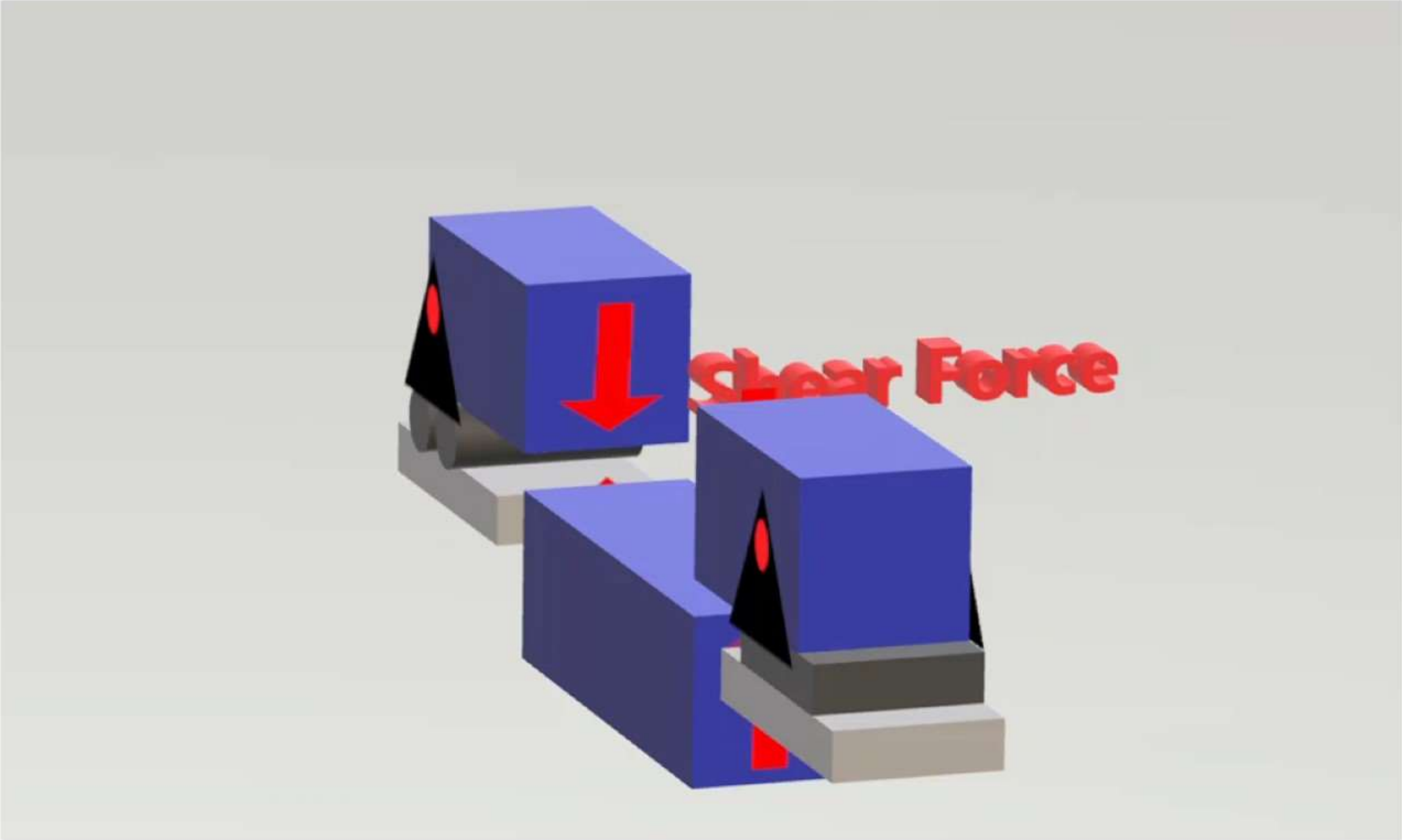
Types of Stresses

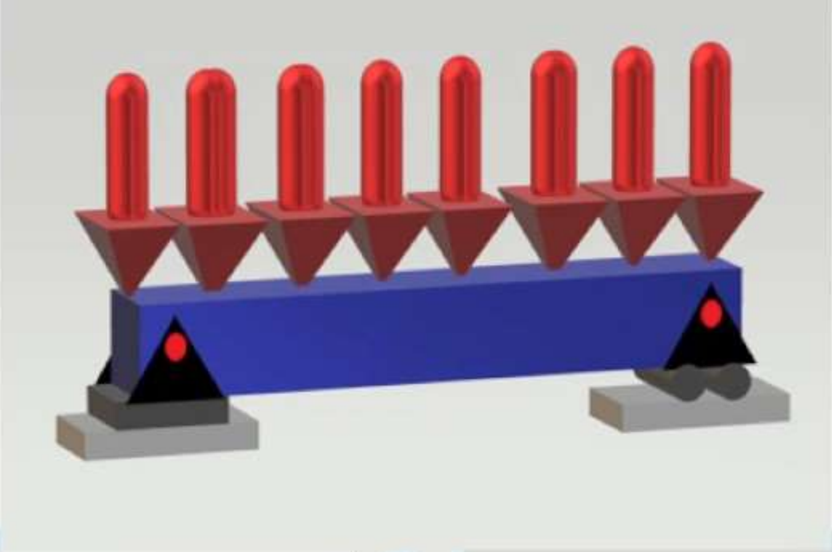
Stress

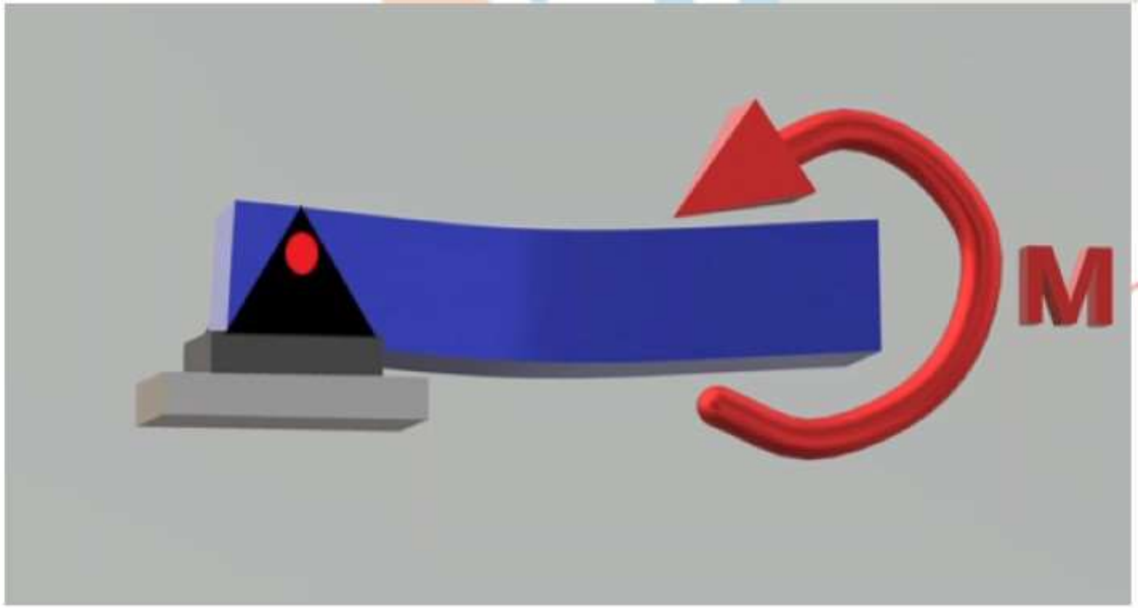
- **Normal Stress**
  - Direct Normal Stress: $\sigma_d = \pm\frac{P}{A}$
  - Bending Stress: $\sigma_b = \pm\frac{M}{I}y$
- **Shear Stress**
  - Primary Shear Stress: $\tau = \pm\frac{fa\bar{y}}{IB}$
  - Torsional Shear Stress: $\tau = \frac{T}{J} \times r$




M


Shear Force

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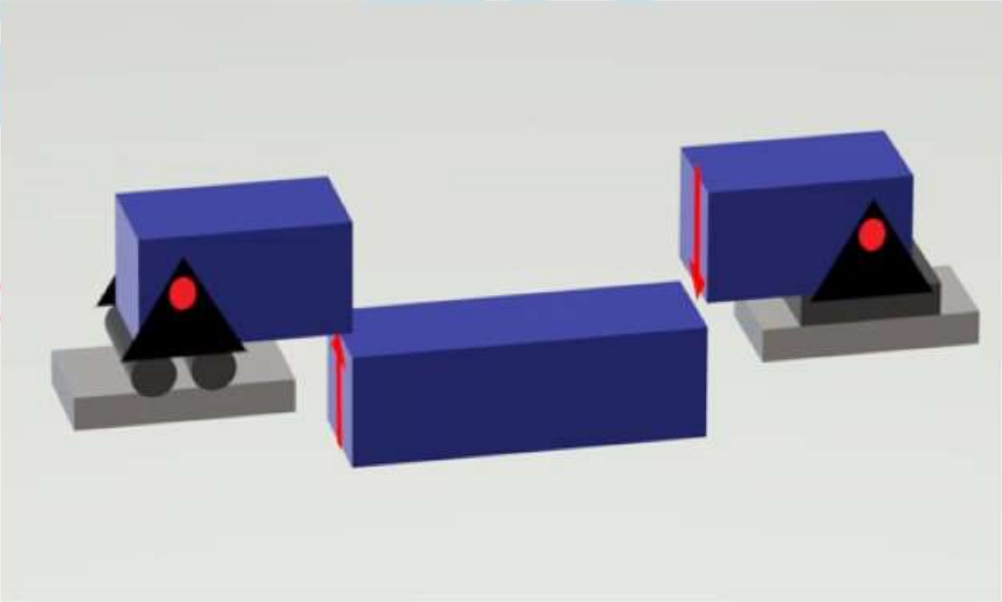
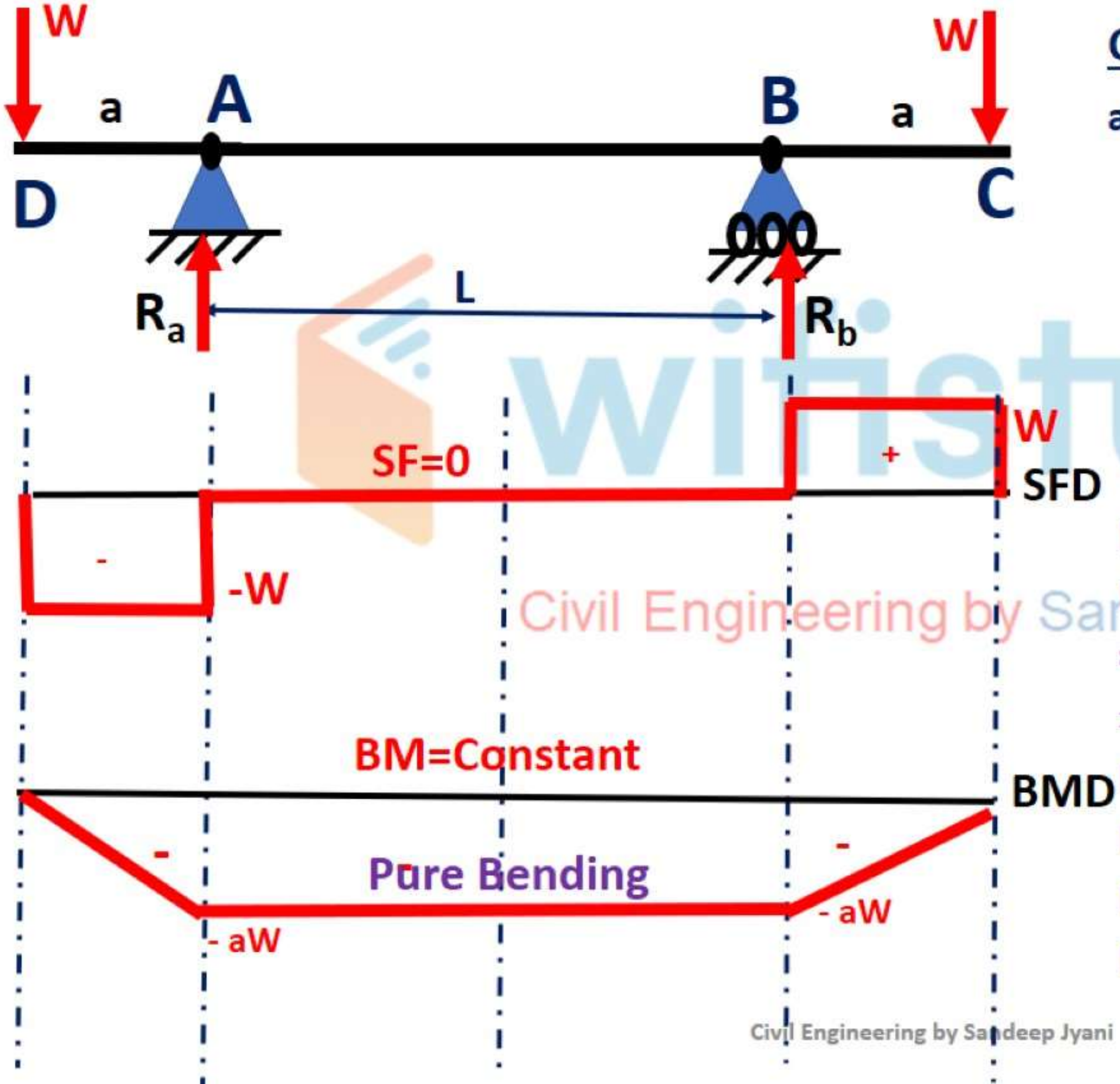


PURE BENDING / BENDING STRESSES



CASE 3: OVERHANG BEAM

a) Overhang beam subjected to point load

$R_a + R_b = W + W = 2W$ ....(1)

Taking moment about A = 0

$\Rightarrow (+R_b) x L - W x (L+a) - W x a = 0$

$\Rightarrow R_b = W$ and Hence

$\Rightarrow R_a = W$ (from eqn 1)

$M_C = 0$

$M_C - M_B$ = area of SFD between C and B

=> $M_C - M_B = a \times W$

$\Rightarrow M_B = -aW$

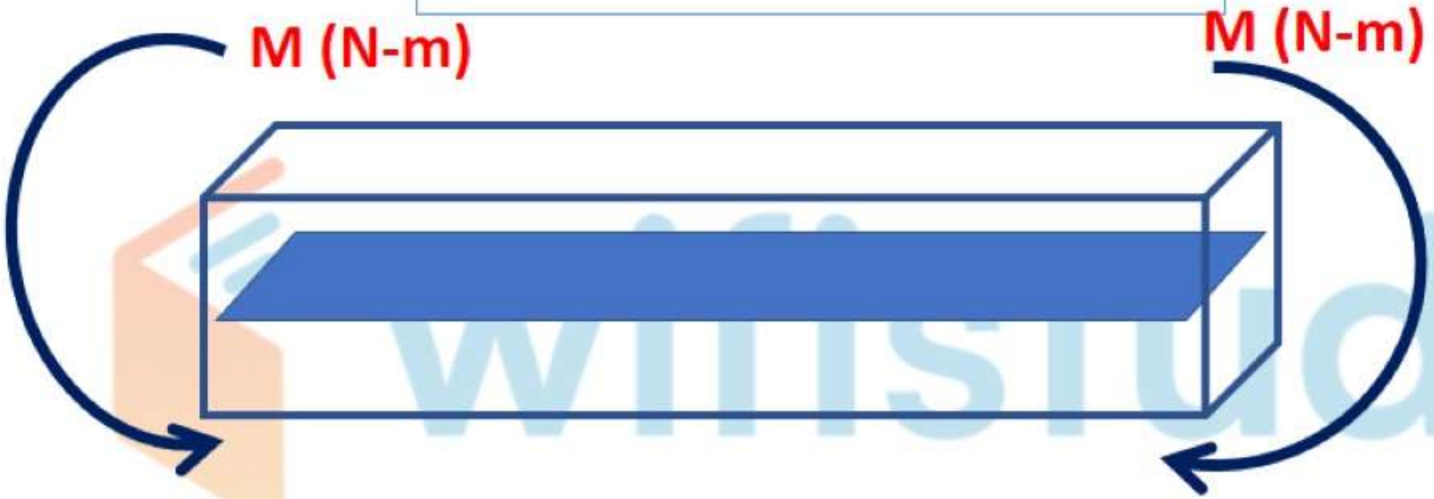
$M_B - M_A$ = area of SFD between A and B

$M_B - M_A = 0$ => $M_A = -aW$

$M_A - M_D$ = area of SFD between A and D

$M_D = 0$

PURE BENDING



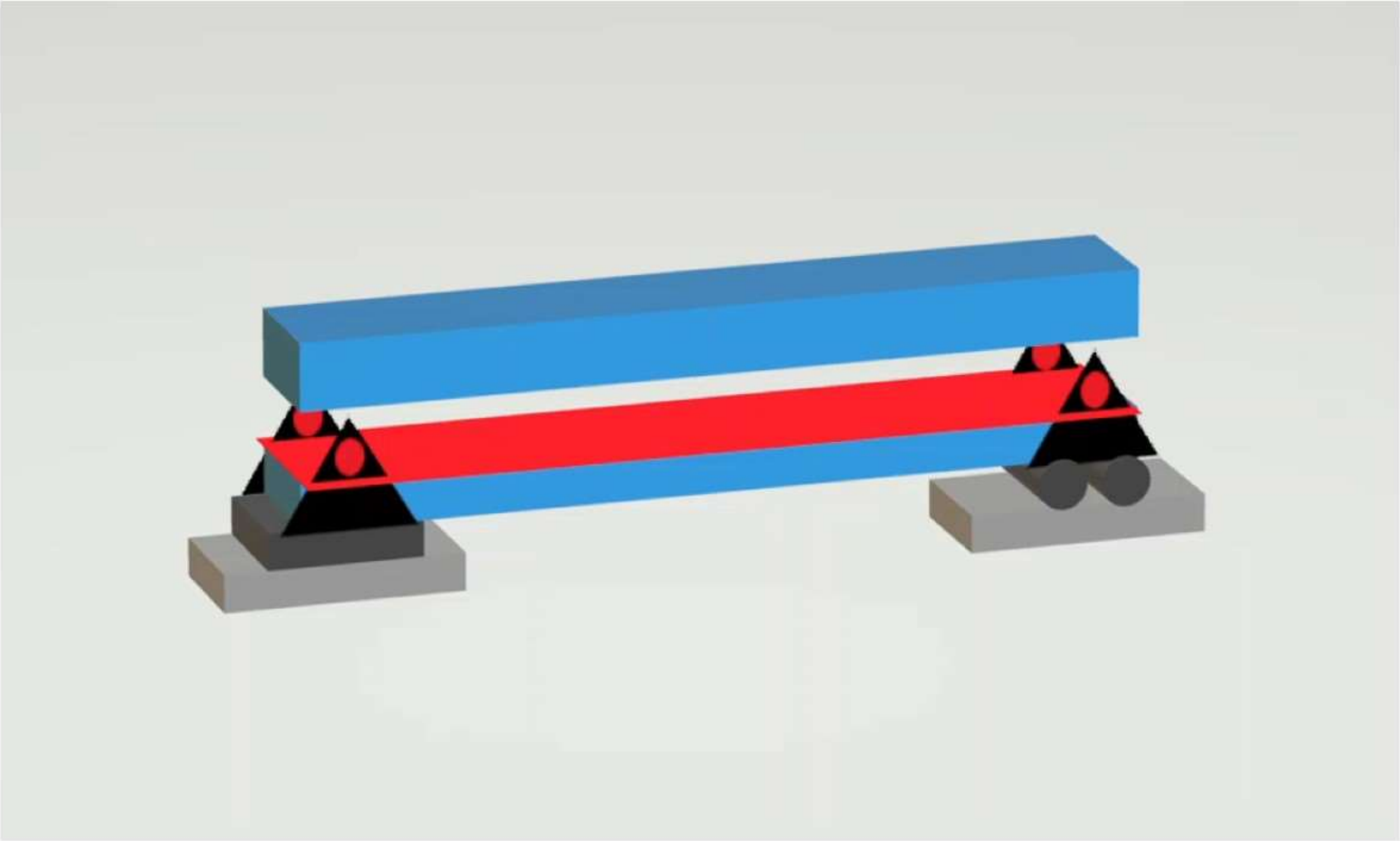
A beam is said to be under Pure Bending if it is subjected to equal and opposite couple in a longitudinal plane in such a way that the magnitude of bending moment remains constant throughout the length, i.e.

BM= Constant

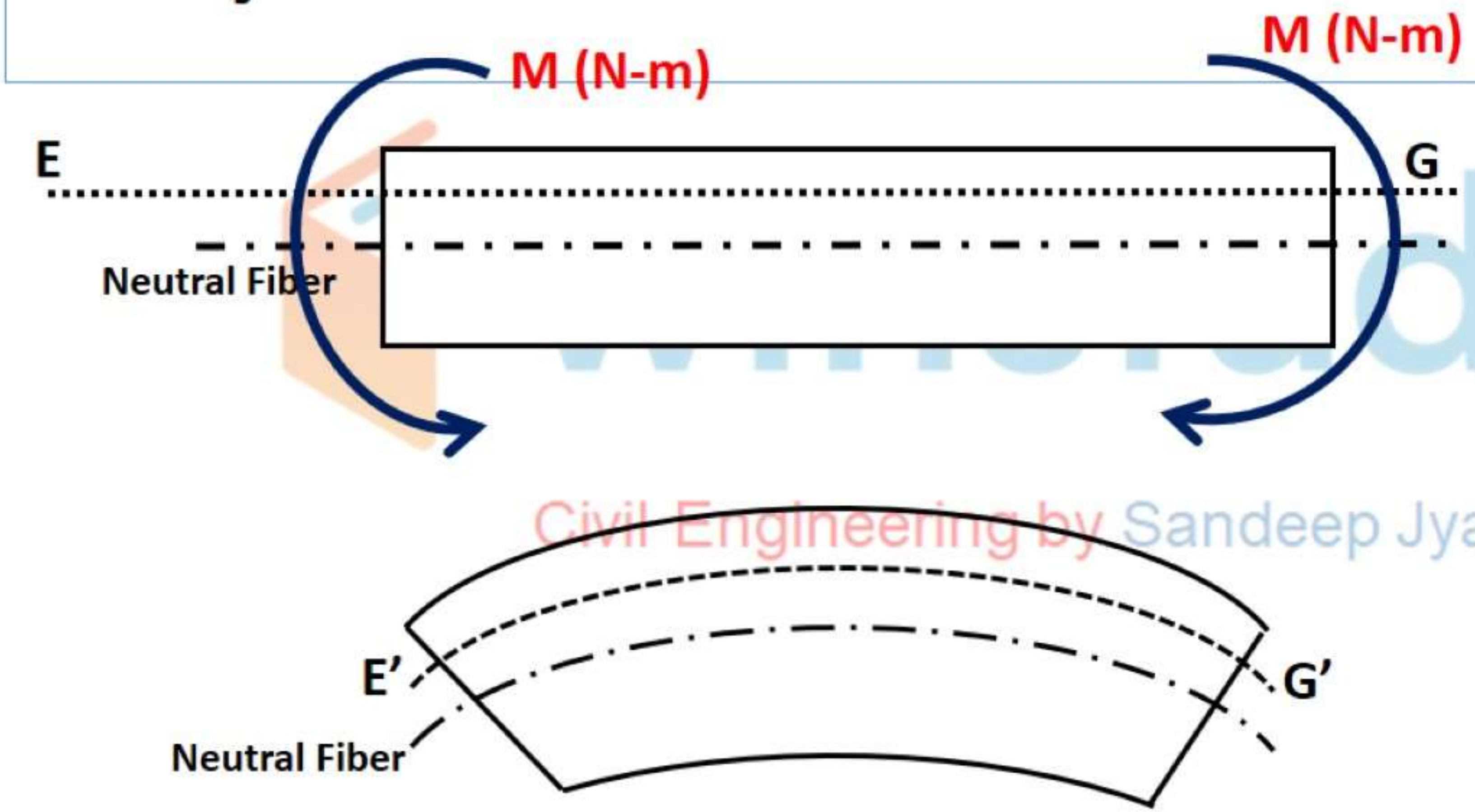
SF= 0

PURE BENDING

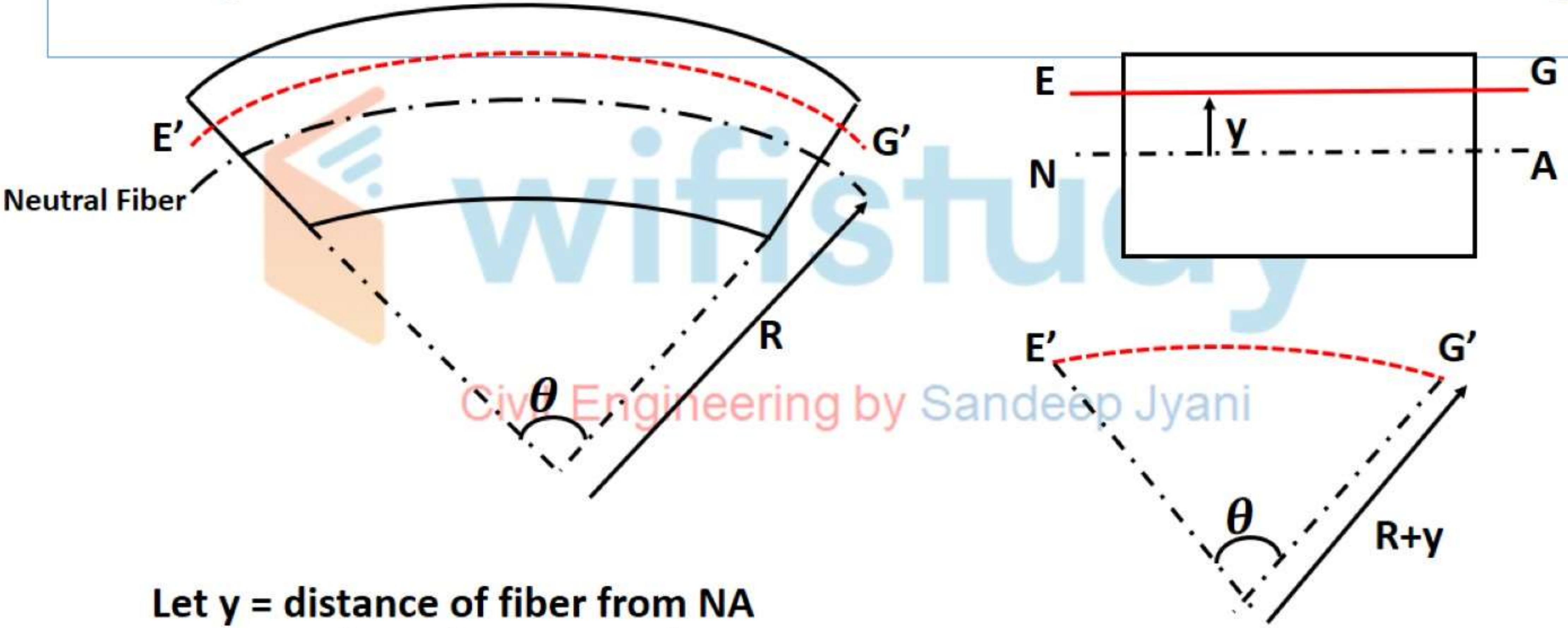
1. **Neutral Fiber:** It is the fiber at which net force is zero, hence elongation of fiber is zero, so stress and strain are also zero.
2. **Neutral Axis (NA):** It is defined as the line of Intersection between plane of cross section and neutral fiber



Analysis of Stress and Strain in Pure Bending



Analysis of Stress and Strain in Pure Bending

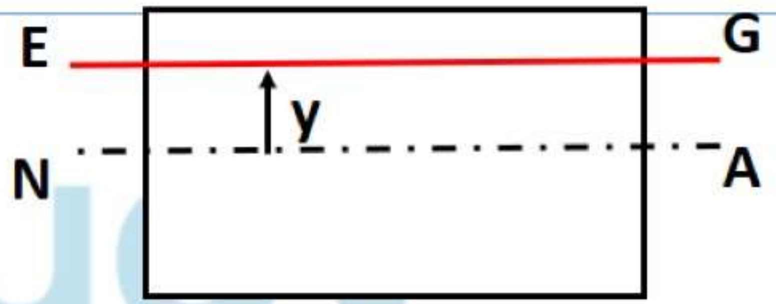


Let y = distance of fiber from NA
Let R = Radius of Curvature from NA

Analysis of Stress and Strain in Pure Bending

Let y = distance of fiber from NA

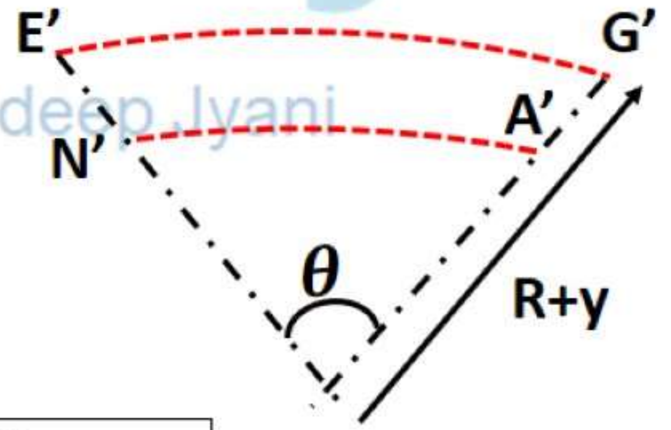
Let R = Radius of Curvature from NA



Initially EG = NA

But NA = N'A'

Strain in Fiber EG $\in_{EG} = \dfrac{change\ in\ length}{original\ length}$



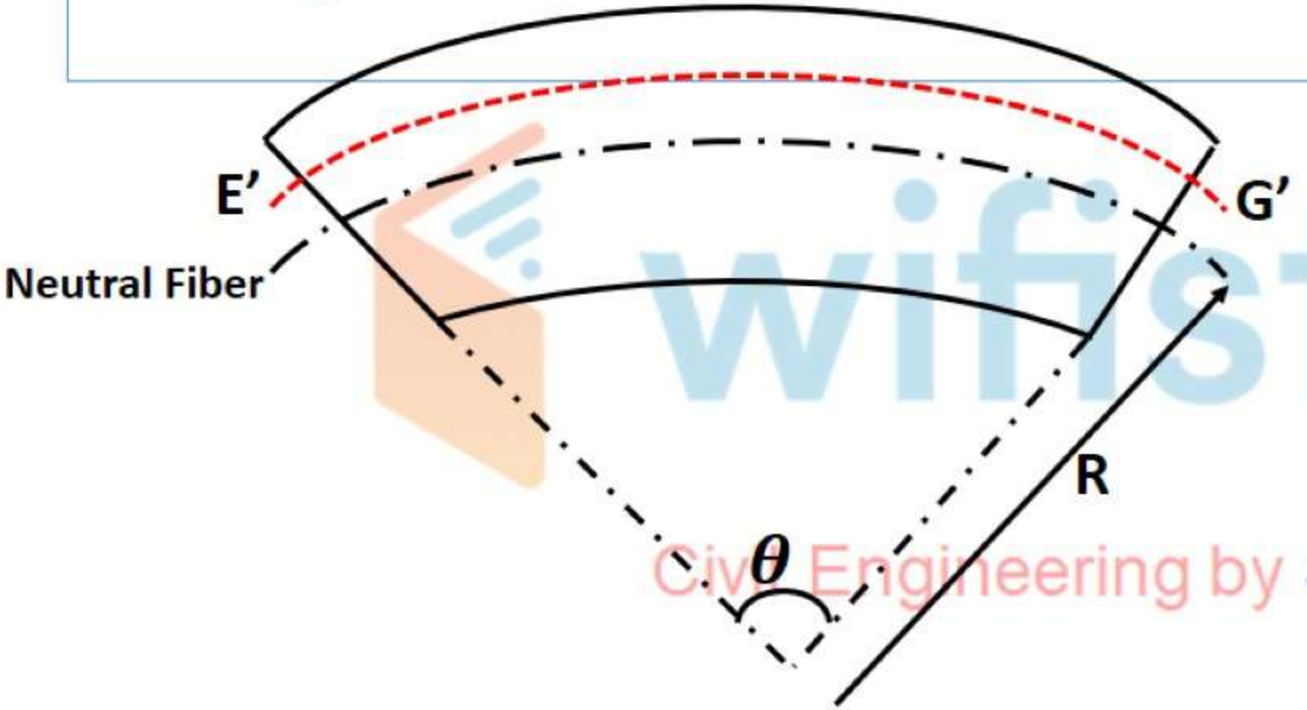
$$\in_{EG} = \frac{E'G' - EG}{EG}$$

$$\in_{EG} = \frac{(R+y)\theta - R\theta}{R\theta}$$

$$\in_{EG} = \frac{y\theta}{R\theta}$$

$$\in_{EG} = \pm\frac{y}{R}$$

Analysis of Stress and Strain in Pure Bending

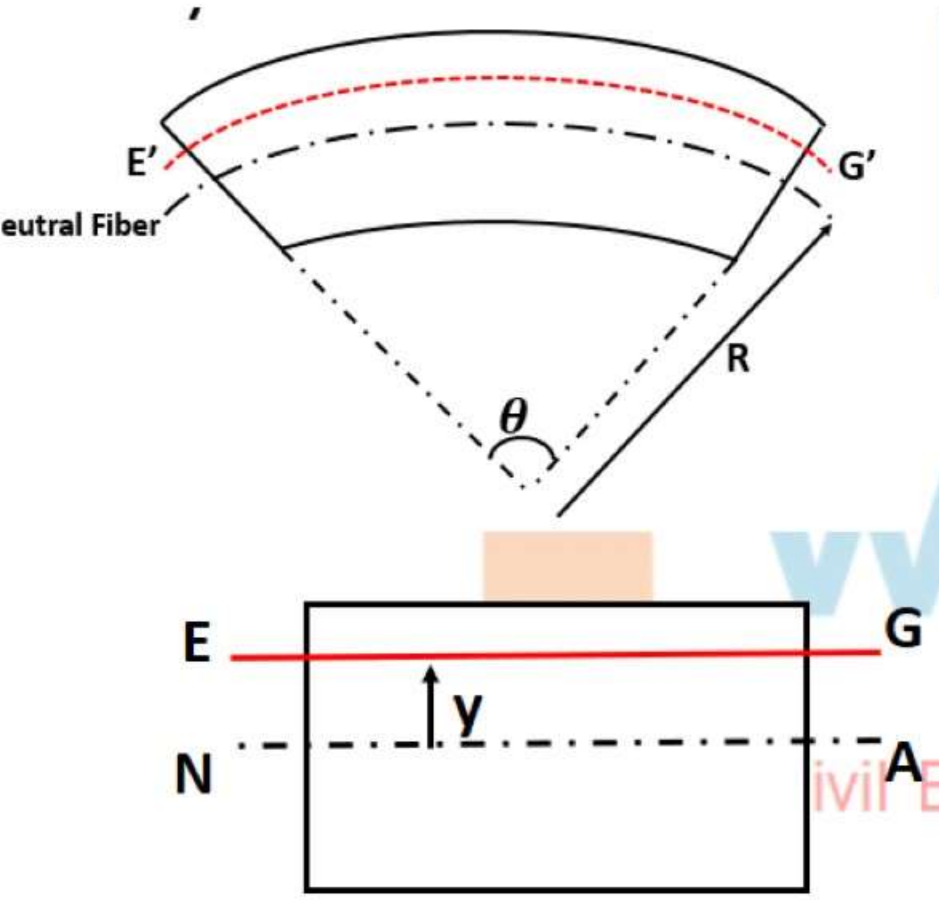


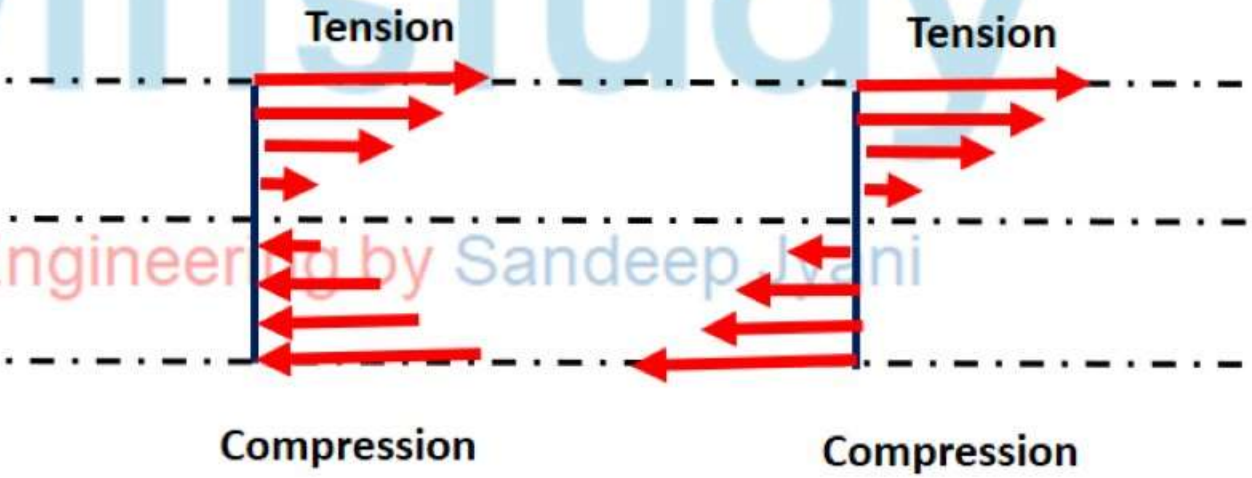
$$\in_{EG} = \pm \frac{y}{R}$$

+ => above neutral axis

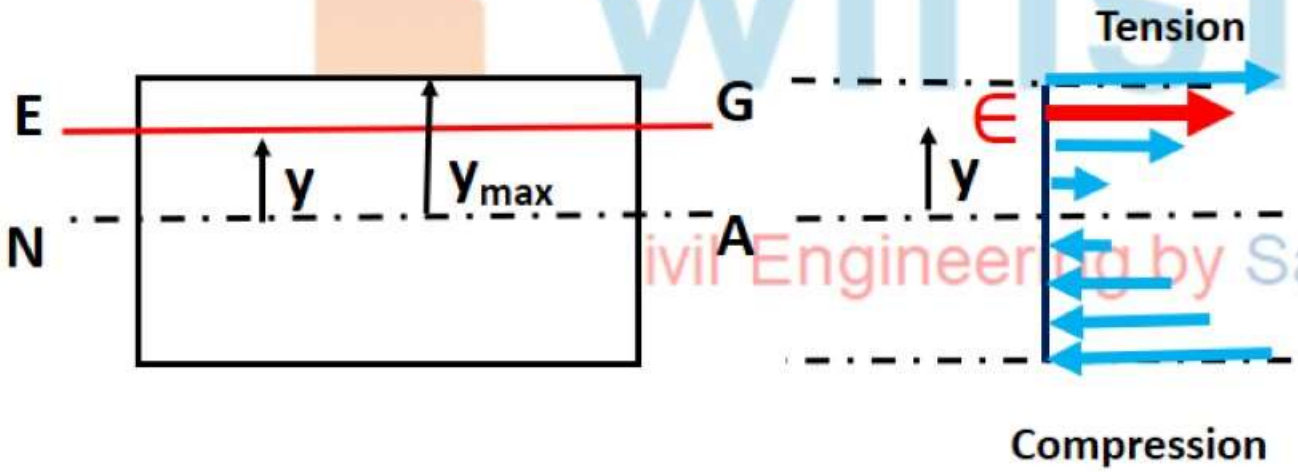
- => below neutral axis

Strain Distribution





Strain Distribution

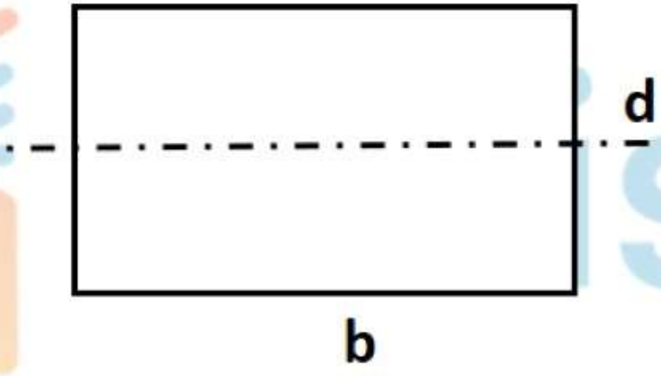


$$\frac{\epsilon_{max}}{y_{max}} = \frac{\epsilon}{y}$$

$$\epsilon = \frac{\epsilon_{max}}{y_{max}} \times y$$

Que 72

$$\frac{\in_{top}}{\in_{bottom}} = ?$$

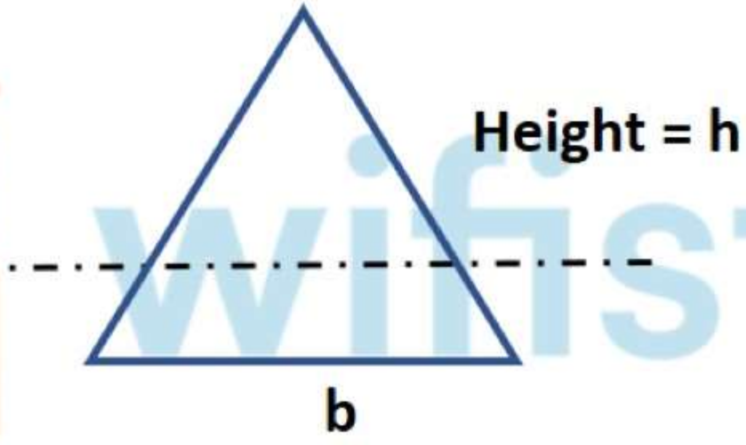


$$\frac{\in_{top}}{\in_{bottom}} = \frac{+y_{top}/R}{-ybo_{ttom}/\boldsymbol{R}}$$

$$\frac{\in_{top}}{\in_{bottom}} = -1$$

Que 73

$$\frac{\epsilon_{\ top}}{\epsilon_{\ bottom}} = ?$$



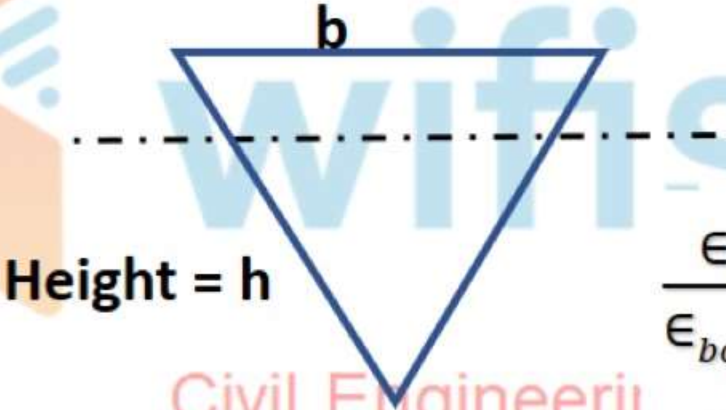
$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+\boldsymbol{y}_{top}/R}{-ybo_{ttom}/\boldsymbol{R}}$$

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+2h/3}{-h/3}$$

$$\frac{\epsilon_{\ top}}{\epsilon_{\ bottom}} = -2$$

Que 74

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = ?$$



$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+y_{top}/R}{-ybo_{ttom}/\boldsymbol{R}}$$

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+h/3}{-2h/3}$$

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = -\frac{1}{2}$$

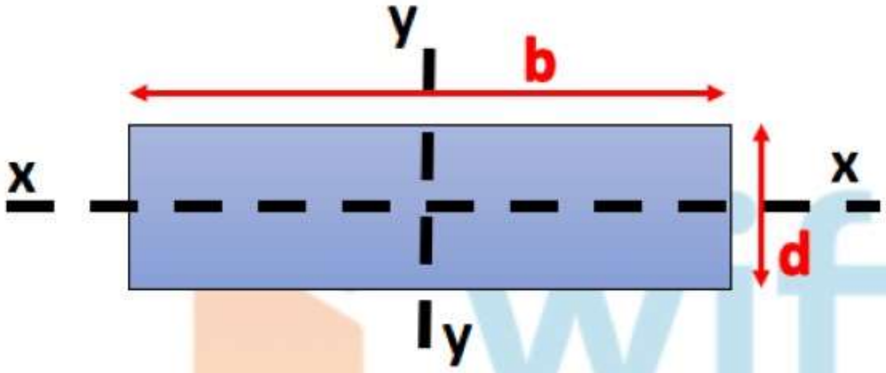
Section Modulus (z)

- It is the ratio of Moment of Inertia about the neutral axis to y_{max} (fiber which is at maximum distance from Neutral Axis) i.e.

$$z = \frac{I_{NA}}{y_{max}}$$

- Section Modulus represents Bending strength of the Section
- Greater the value of z, greater the bending strength.
- The value of z depends upon Moment of Inertia and Distribution of Area

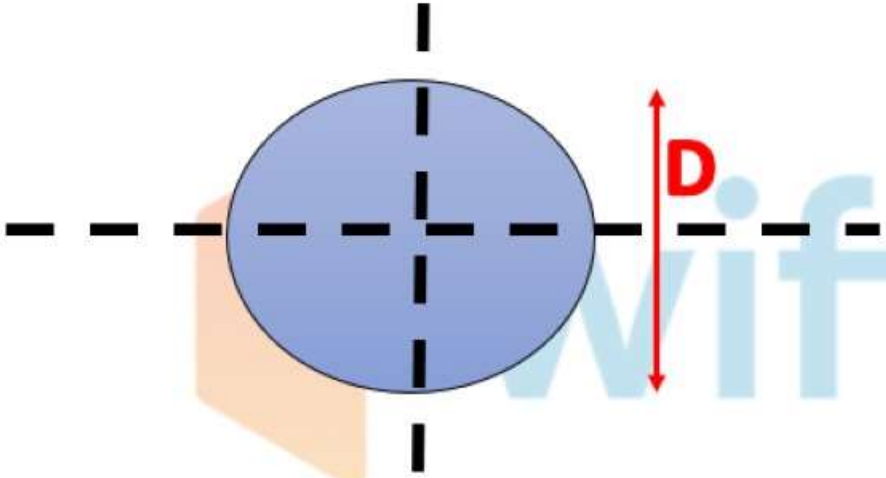
Moment of Inertia of Some Important Sections



1. Rectangle

$I_{xx} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$

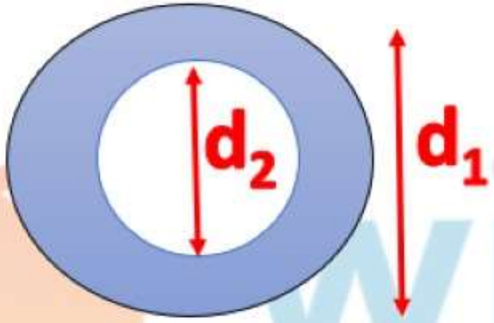
Moment of Inertia of Some Important Sections



2. Circle

$I_{xx} = I_{yy} = \frac{\pi}{64} d^4$

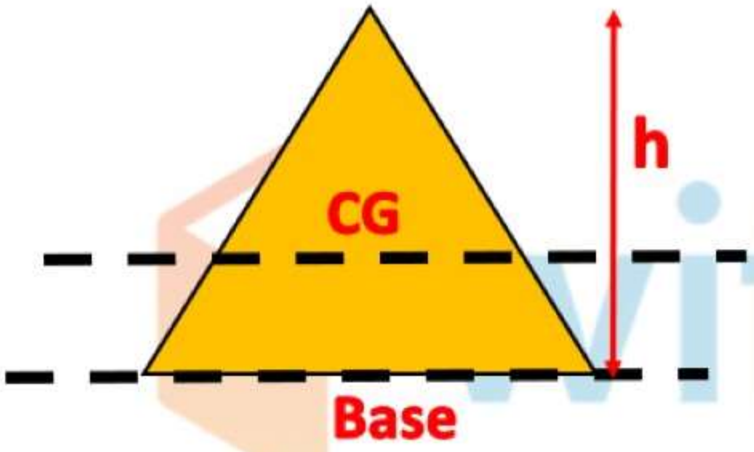
Moment of Inertia of Some Important Sections



3. Concentric Circles

$$I_{xx} = I_{yy} = \frac{\pi}{64}(d_1^4 - d_2^4)$$

Moment of Inertia of Some Important Sections



4. Triangle

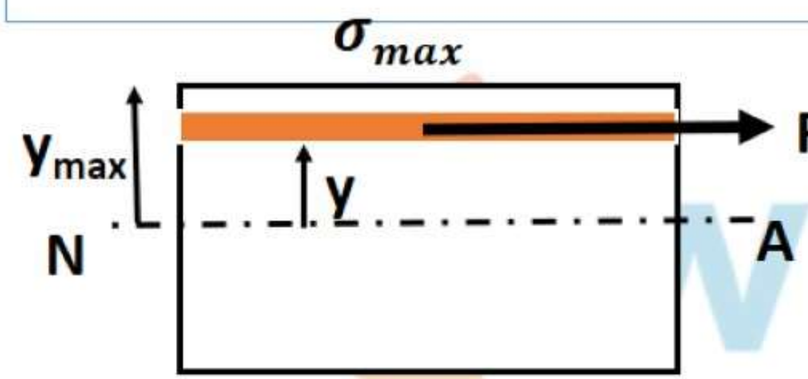
$$I_{CG} = \frac{1}{36}bh^3$$

$$I_{base} = \frac{1}{12}bh^3$$

Moment of Resistance

- It is defined as the internal resisting Bending Couple by the plane of cross section of the member
- For safe condition, $M_R \geq M$(externally applied)
- If $M_R < M$(externally applied), so plastic deformation occurs

Moment of Resistance



$$\sigma = \frac{F}{A}$$

$$or \;\; \sigma = \frac{dF}{dA}$$

$$\sigma\, dA = dF \quad ....(1)$$

$$Since \;\; \frac{\epsilon_{max}}{y_{max}} = \frac{\epsilon}{y}$$

$$so \;\; \frac{\sigma_{max}}{y_{max}} y = \sigma$$

Putting value of stress in (1)⇒

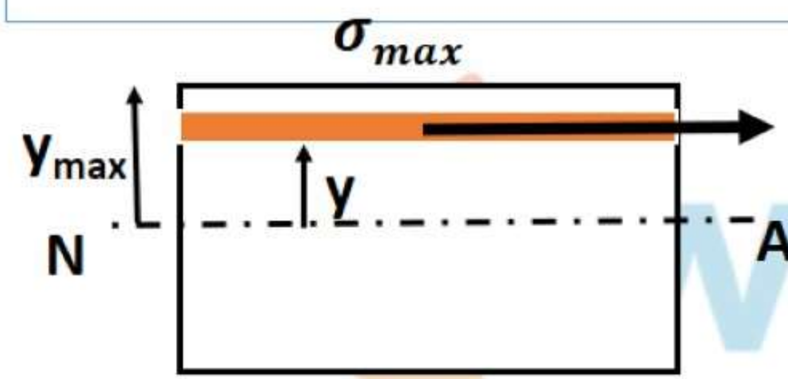
$$\frac{\sigma_{max}}{y_{max}} \times y \times dA = dF$$

Resisting Moment

$$dM_R = dF \times y$$

$$dM_R = \frac{\sigma_{max}}{y_{max}} \times y \times dA \times y$$

Moment of Resistance



Resisting Moment

$$dM_R = \frac{\sigma_{max}}{y_{max}} \times y \times dA \times y$$

For total resisting moment on this cross section

$$\int dM_R = \int \frac{\sigma_{max}}{y_{max}} \times y^2 \times dA$$

$$M_R = \frac{\sigma_{max}}{y_{max}} \times \int y^2 \times dA$$

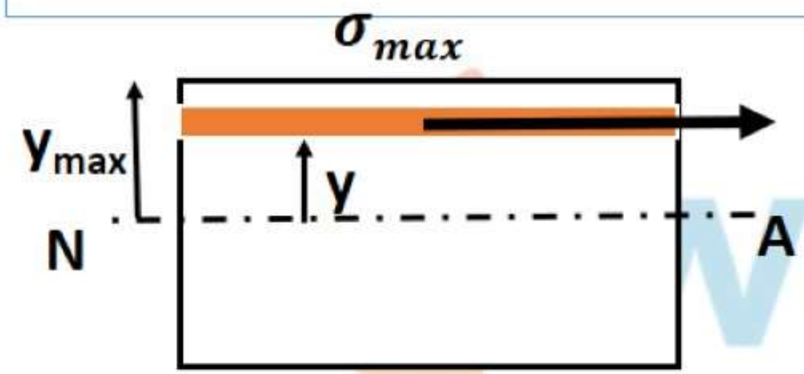
second moment of *Area*

$$M_R = \frac{\sigma_{max}}{y_{max}} \times I_{NA}$$

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{max}}{y_{max}}$$

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{max}}{y_{max}} = \frac{E}{R}$$

Moment of Resistance



$$\frac{M_R}{I_{NA}} = \frac{\sigma_{max}}{y_{max}}$$

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{BENDING}}{y} = \frac{E}{R}$$

Analysis of Bending Equations

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{BENDING}}{y} = \frac{E}{R}$$

- **CASE 1:** If $\frac{\sigma_B}{y} = \frac{M_R}{I_{NA}}$, $\Rightarrow \frac{(\sigma_B)_{max}}{y_{max}} = \frac{M_R}{I_{NA}}$

$$\Rightarrow (\sigma_B)_{max} = \frac{M_R \times y_{max}}{I_{NA}}$$

$$\Rightarrow (\sigma_B)_{max} = \frac{M_R}{\frac{I_{NA}}{y_{max}}}$$

$$\Rightarrow (\sigma_B)_{max} = \frac{M_R}{Z}$$

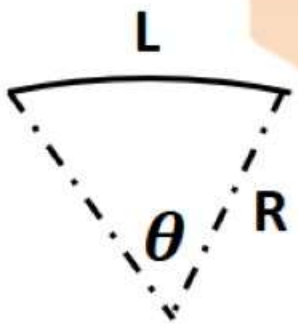
- More the value of Z, more is the Bending strength and less is the bending stress
- More is the section modulus, more will be the Moment of Resistance for given bending stress

Analysis of Bending Equations

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{BENDING}}{y} = \frac{E}{R}$$

- **CASE 2:** If $\frac{M_R}{I_{NA}} = \frac{E}{R}$,

$$=> \frac{M_R}{I_{NA}} = \frac{E\theta}{L}$$



$$=> \frac{M_R}{\theta} = \frac{EI_{NA}}{L}$$

$$=> K = \frac{M_R}{\theta} = \frac{EI_{NA}}{L}$$

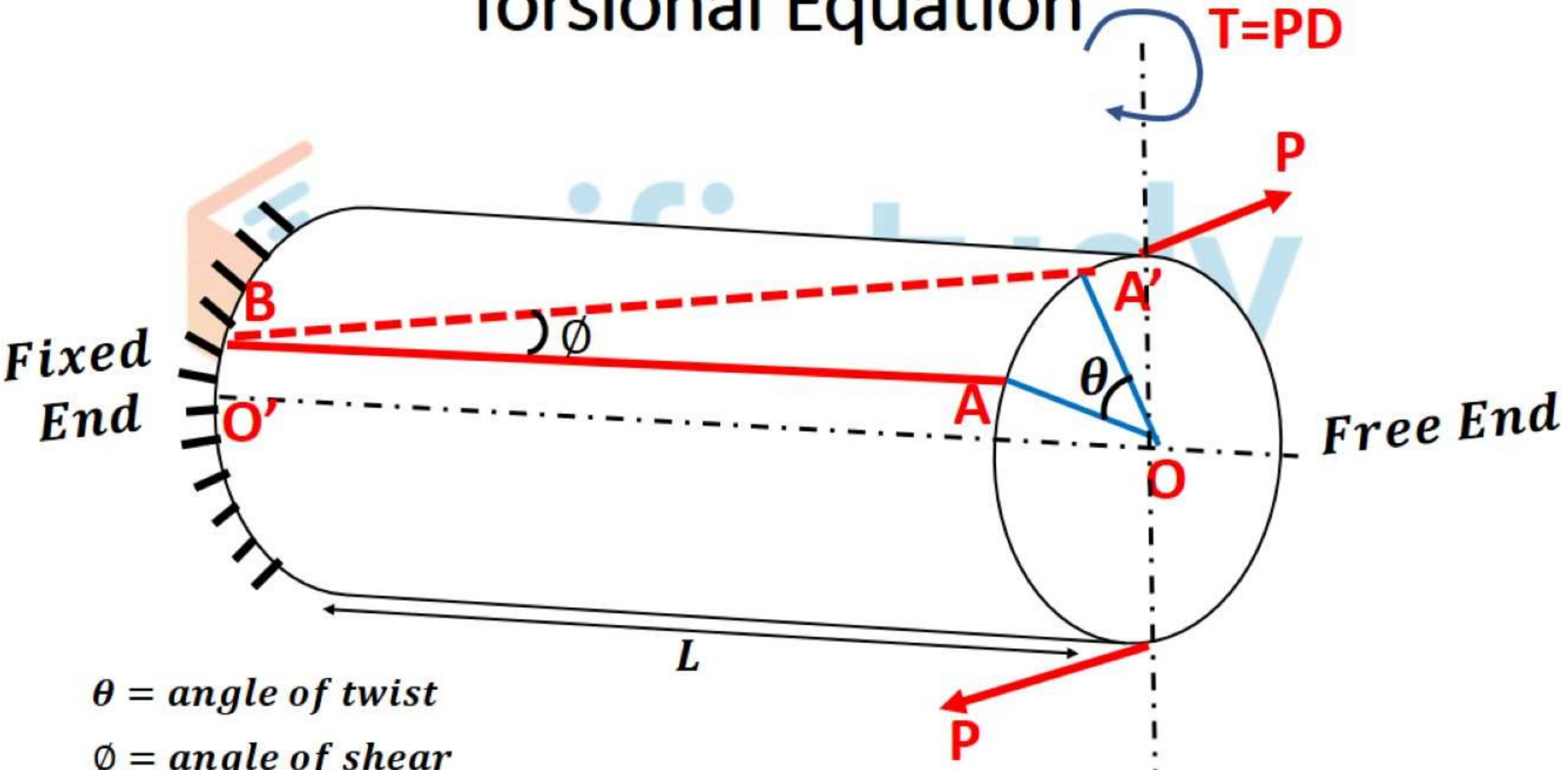
K= Stiffness in Bending

EI= Flexural Rigidity

MOMENT vs COUPLE vs TORQUE vs TORSION

- **Moment** refers to the tendency of a force to move or rotate an object at an axis through a point
- A **couple** is a pair of **forces**, equal in magnitude, oppositely directed, and displaced by perpendicular distance or moment
- **Torque** causes an angular acceleration of rotation of a body about its axis
- **Torsion** occurs when it is twisted causing twisting force acting on the member, known as torque, and the resulting stress is known as shear stress

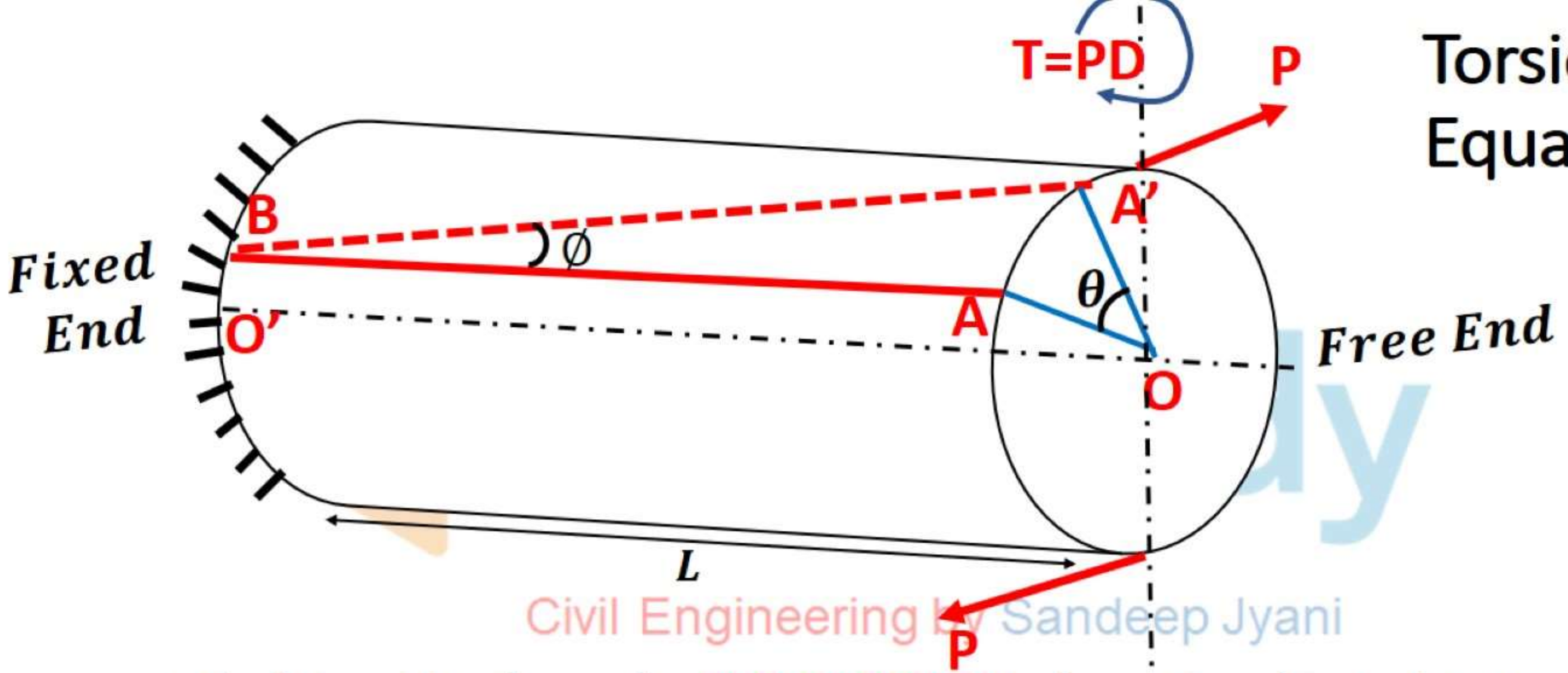
Torsional Equation



$\theta = angle\ of\ twist$

$\emptyset = angle\ of\ shear$

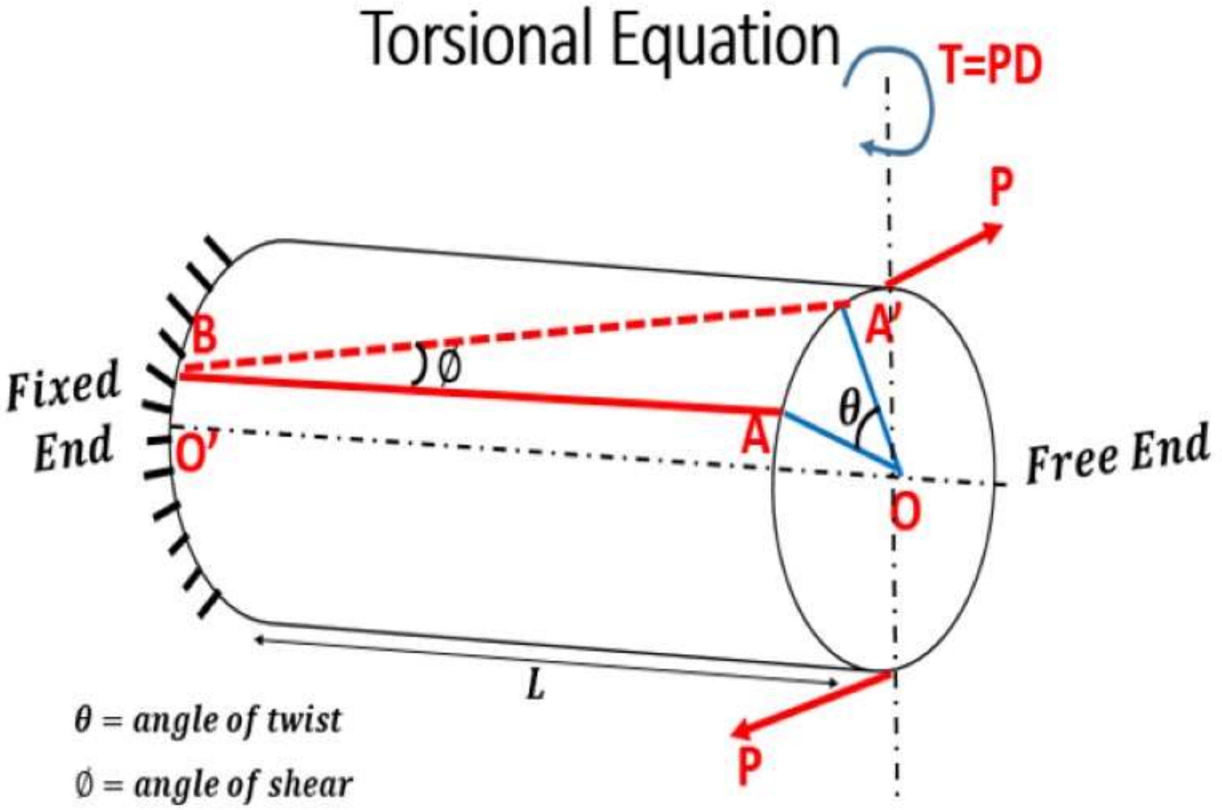
Torsional Equation



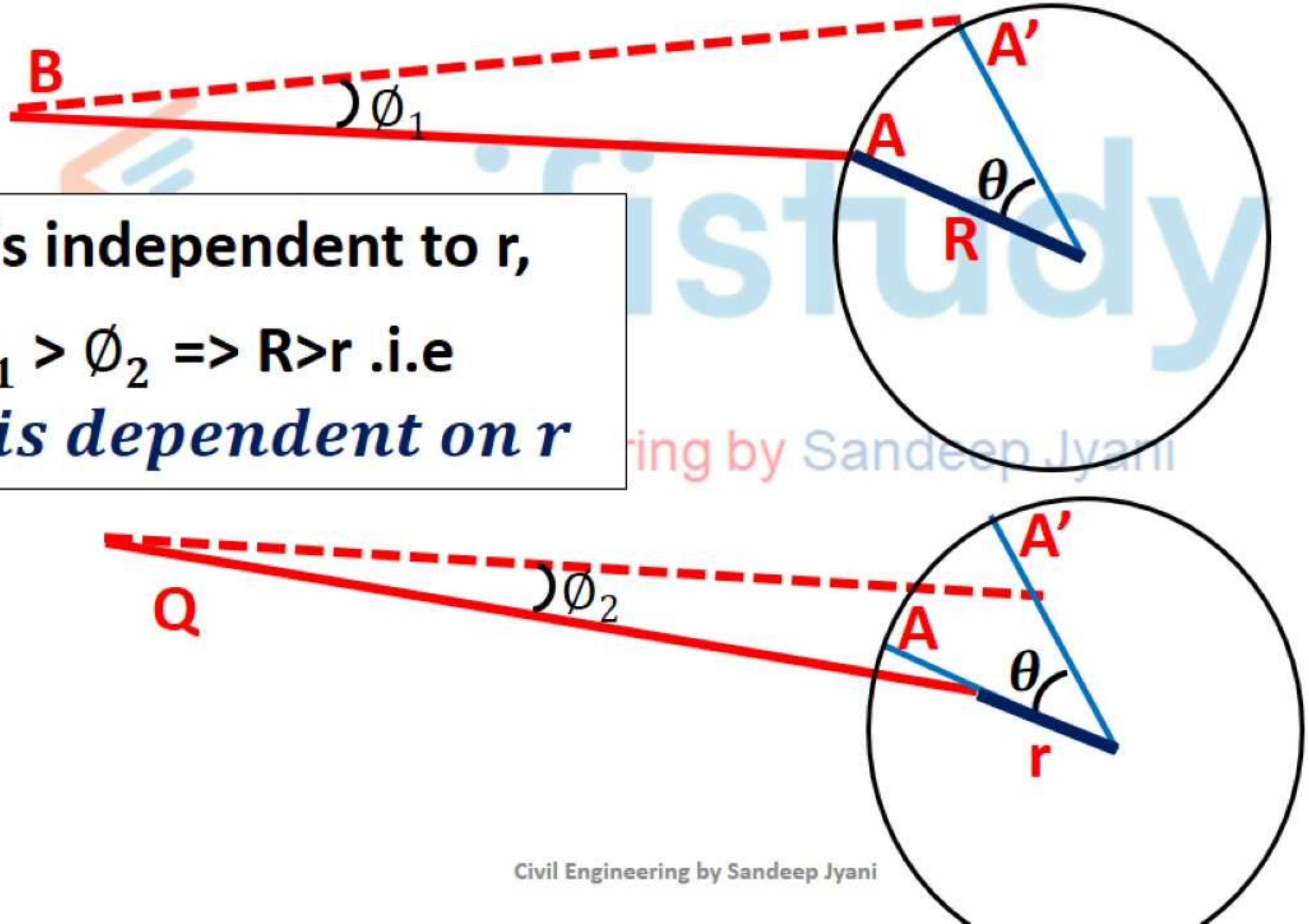
- A Shaft is said to be under PURE TORSION when It is subjected to Equal and Opposite Couple in a plane perpendicular to the longitudinal axis of the member in such a way that the magnitude of twisting moment remains the constant throughout the length of member i.e.

Twisting Moment = constant

- **θ = Angle of Twist** It represents how much angle the radial line which is present on the cross section at the free end gets twisted
- **Ø=Angle of Shear**: It represents how much angle the line on surface of shaft gets distorted
- **L =** distance of cross section from the fixed end
- **r=** distance of a point from the centre of shaft



Case1: Effect of r on θ and $\emptyset$.



θ is independent to r,

$\emptyset_1 > \emptyset_2$ => R>r .i.e

$\emptyset$ ***is dependent on r***

Case1: Effect of r on θ and $\emptyset$.

In ΔABA'

$\tan \emptyset_1 = \frac{AA'}{AB} = \frac{R\theta}{L}$ .....(1)

In ΔPBP'

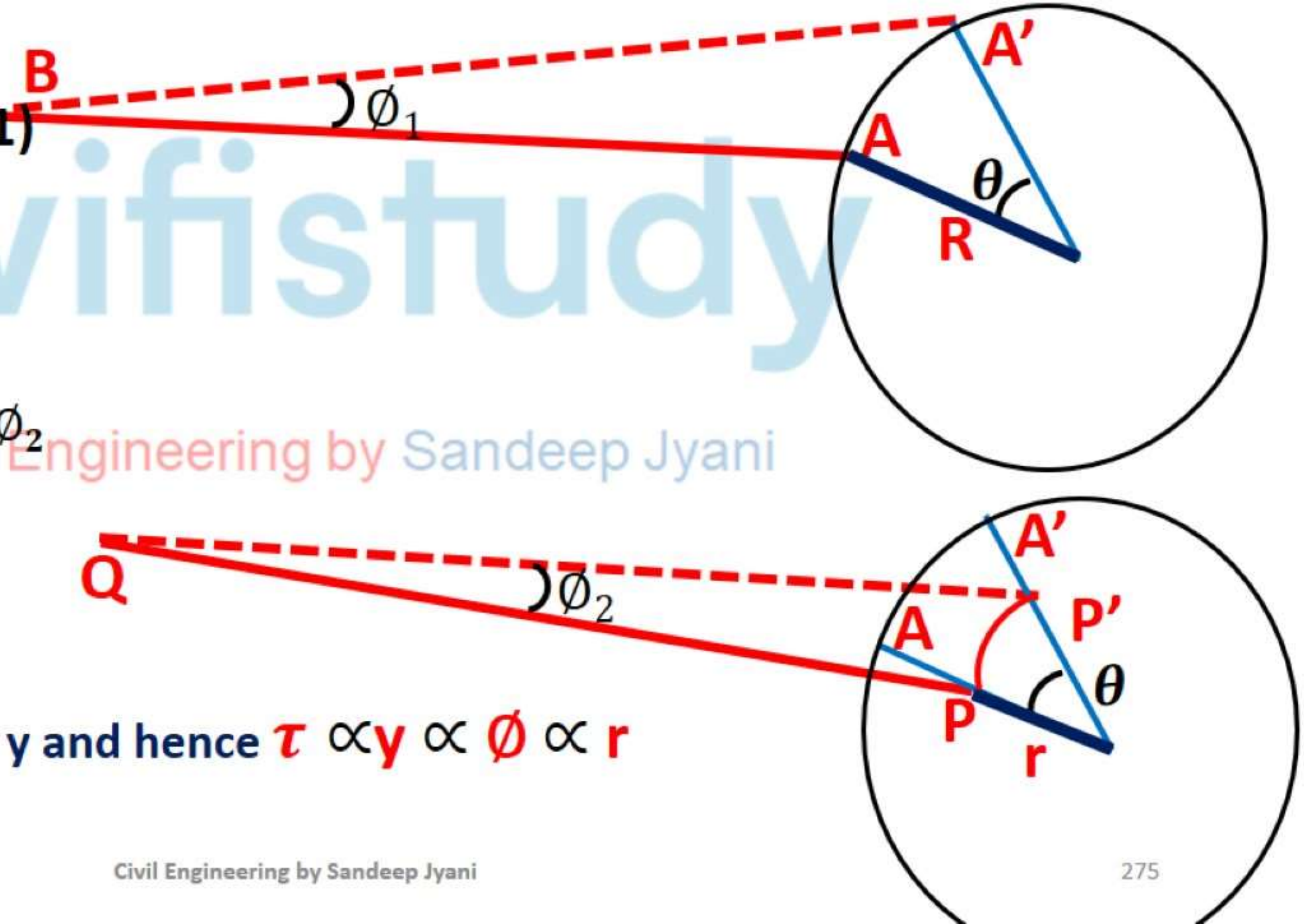
$\tan \emptyset_2 = \frac{PP'}{AB} = \frac{r\theta}{L}$

Since R>r, therefore $\emptyset_1 > \emptyset_2$

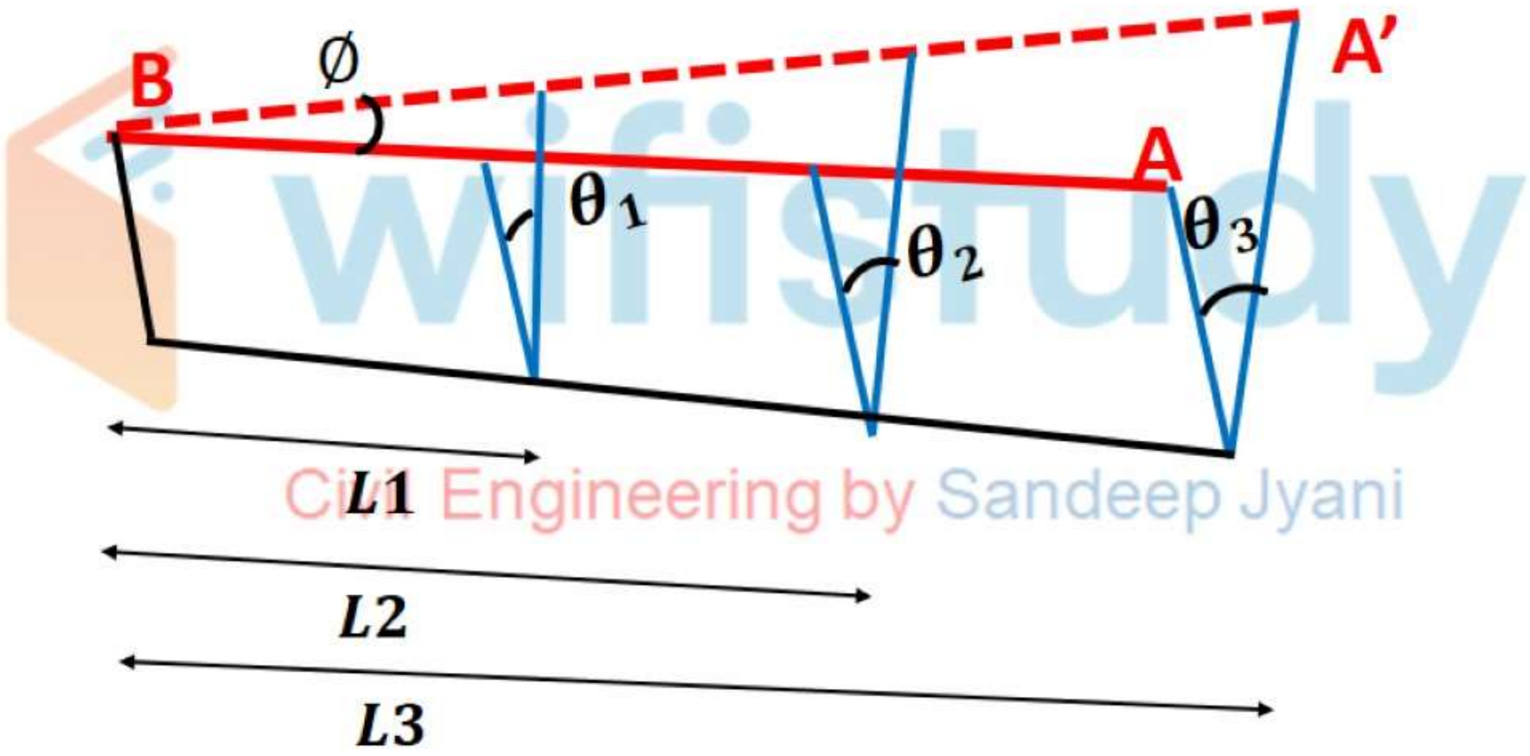
If y= Shear Strain

$$y = \frac{\Delta L}{L} = tan\emptyset$$

Using Hooke's law, $\tau \propto$ y and hence $\tau \propto y \propto \emptyset \propto r$



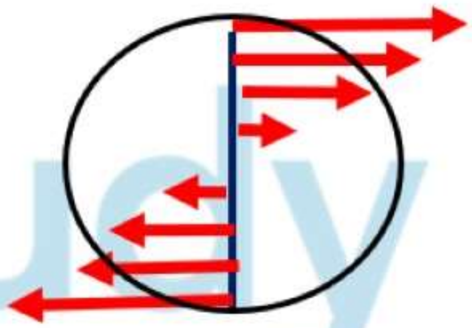
Case2: Effect of L on θ and $\emptyset$.



Conclusion

1. **Shear Angle ($\emptyset$) or Shear Strain and Shear Stress (τ) is directly proportional to r but independent to L**

   Which means Shear Strain and Shear Stress are maximum at a point far away from the center of the shaft (it means surface of the shaft)

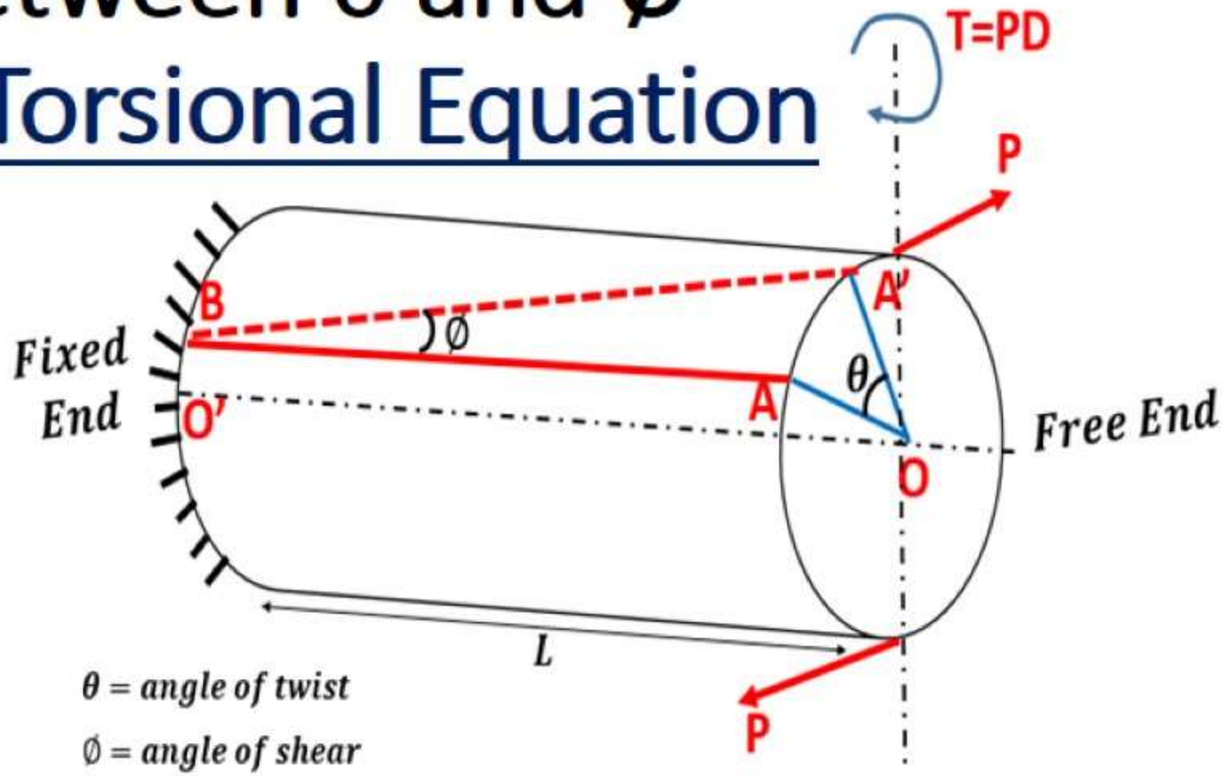


Conclusion

2. **Angle of Twist is directly proportional to L but independent to r and hence we can conclude that angle of twist is maximum on a cross section which is far away from the fixed end.**

Relationship between θ and ∅

Derivation of the Torsional Equation



In ΔABA' , $\tan \phi = \frac{AA'}{AB}$

$$\tan \phi = \frac{R\theta}{L}$$

If ∅ *is very small*,

$$\phi = \frac{R\theta}{L} \dots \dots (1)$$

Using Hooke's law, $\tau \propto y$

$$\tau = Gy$$

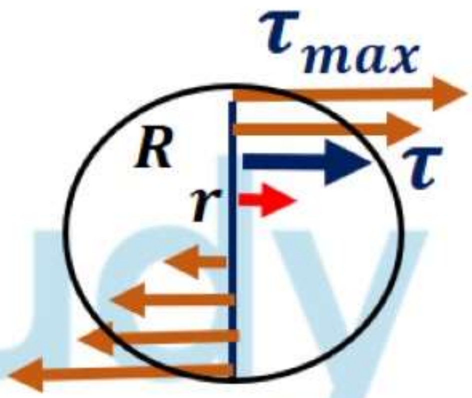
$$\frac{\tau}{G} = y$$

$$\frac{\tau}{G} = y = \phi$$

$$\frac{\tau}{G} = \frac{R\theta}{L}$$

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

$$\frac{\tau}{r} = \frac{\tau_{max}}{R}$$



Resisting Torque (T_R)

- T_R (Resisted by cross section) > T (Externally applied)

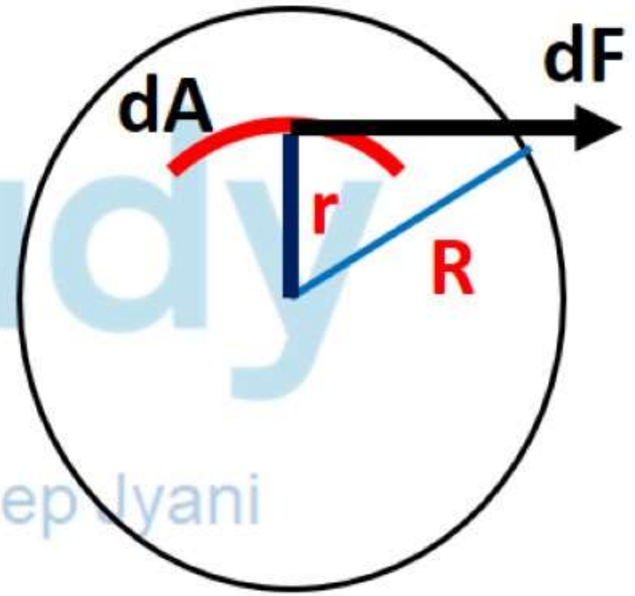
- It is defined as the resisting twisting couple offered by the plane of cross section.
- For the safe condition $T_R \geq T$

Resisting Torque (T_R)

Stress located at a fiber which is located at a distance r

$$\frac{\tau}{r} = \frac{\tau_{max}}{R}$$

dF is acting parallel to the plane



Force on Element Area dA

$dF = \tau \times dA$

$dF = \frac{\tau_{max}}{R} \times r \times dA$

Resisting Torque on the element area $dT_R = dF \times r$

$$dT_R = \frac{\tau_{max}}{R} \times r \times dA \times r$$

$$dT_R = \frac{\tau_{max}}{R} \times r^2 \times dA$$

$$dT_R = \frac{\tau_{max}}{R} \times r^2 \times dA$$

For total Resisting Moment,

$$\int dT_R = \int \frac{\tau_{max}}{R} \times r^2 \times dA$$

Second moment of area or Polar moment of Inertia

$$T_R = \frac{\tau_{max}}{R} \times J$$

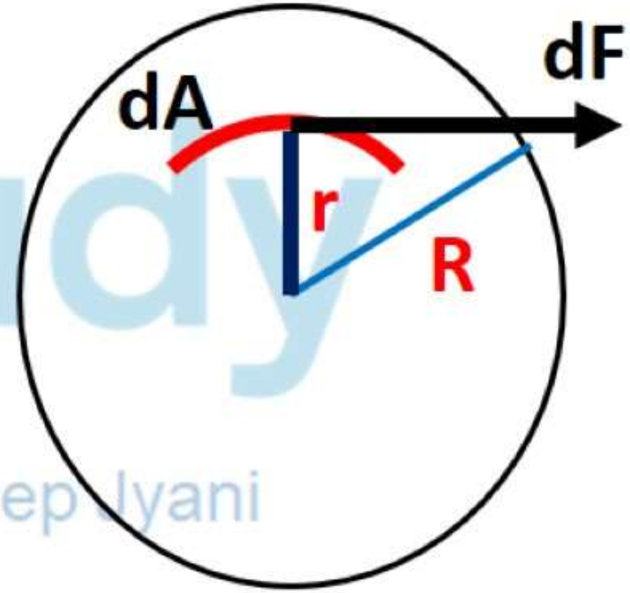
We know that

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

Therefore,

$$\frac{\tau}{R} = \frac{G\theta}{L} = \frac{T_R}{J}$$

dF is acting parallel to the plane

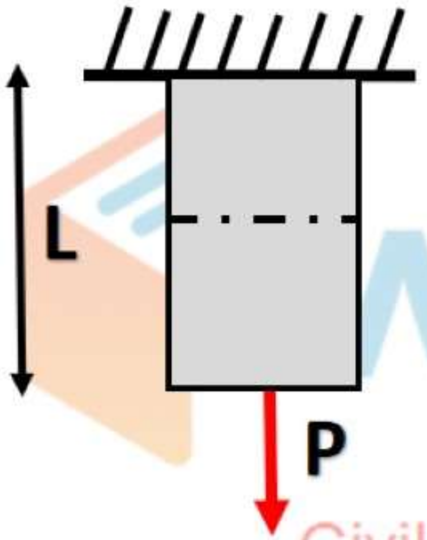


Power

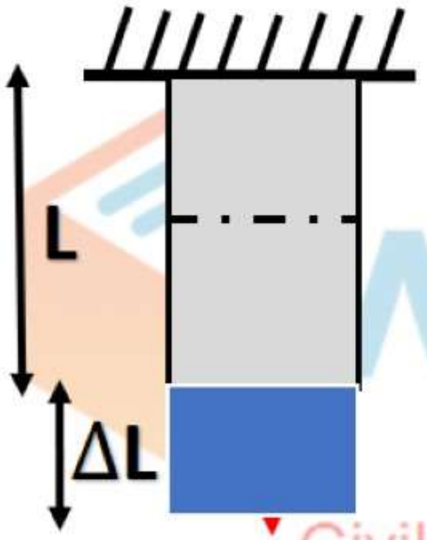
- Power is rate of doing work
- Work = force x displacement (linear)
- Work= torque x angular displacement
  - =T x $d\theta$
- $P = \frac{Work}{Time}$ = T x $\frac{d\theta}{dt}$
- $P = \frac{Work}{Time}$ = T x ω

- $P = \frac{Work}{Time}$ = T x $\frac{d\theta}{dt}$
- $P = \frac{Work}{Time}$ = T x ω
- $\omega = \frac{2\pi N(rpm)}{60}$
- $P = \frac{2\pi NT}{60}$ **Watt**
- **N=rpm, T=N-m**

Strain Energy



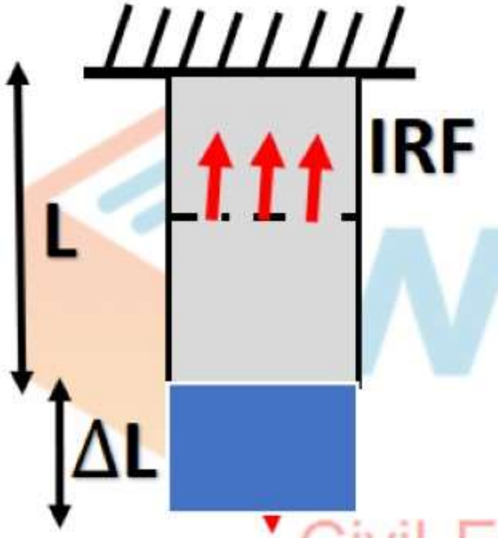
Strain Energy



Work Done =Net Force x Displacement

- Whenever a body is strained, energy is absorbed in the body. The energy which is absorbed in the body due to straining effect is known as Strain Energy.
- The energy stored in the body is equal to the work done by the applied load in stretching the body

Strain Energy



Work Done =Net Force x Displacement

Case1 : When Load is applied gradually:

Work Done =Net Force x Displacement

Work Done =$(\frac{P}{2})$x ΔL

Case2 : When Load is applied suddenly:

Work Done =Net Force x Displacement

Work Done =(P)x ΔL

Resilience

1. Resilience:

- Total strain Energy stored in body is commonly known as Resilience.
- Whenever straining force is removed from strained body, the body is capable of doing work
- Resilience is also defined as capacity of a strained body for doing work on removal of straining force

U = Work done by IRF

= Work done by force P

$$U = \left(\frac{P}{2}\right) \times \Delta L$$

$$U = \frac{1}{2} P \times \frac{PL}{AE}$$

$$\boxed{U = \frac{P^2 L}{2AE}}$$

Resilience

1. **Resilience:**

U = Work done by IRF

= Work done by force P

$U=(\frac{P}{2})\times \Delta L$

$U=\frac{1}{2} P \times \frac{PL}{AE}$

$$U= \frac{P^2 L}{2AE}$$

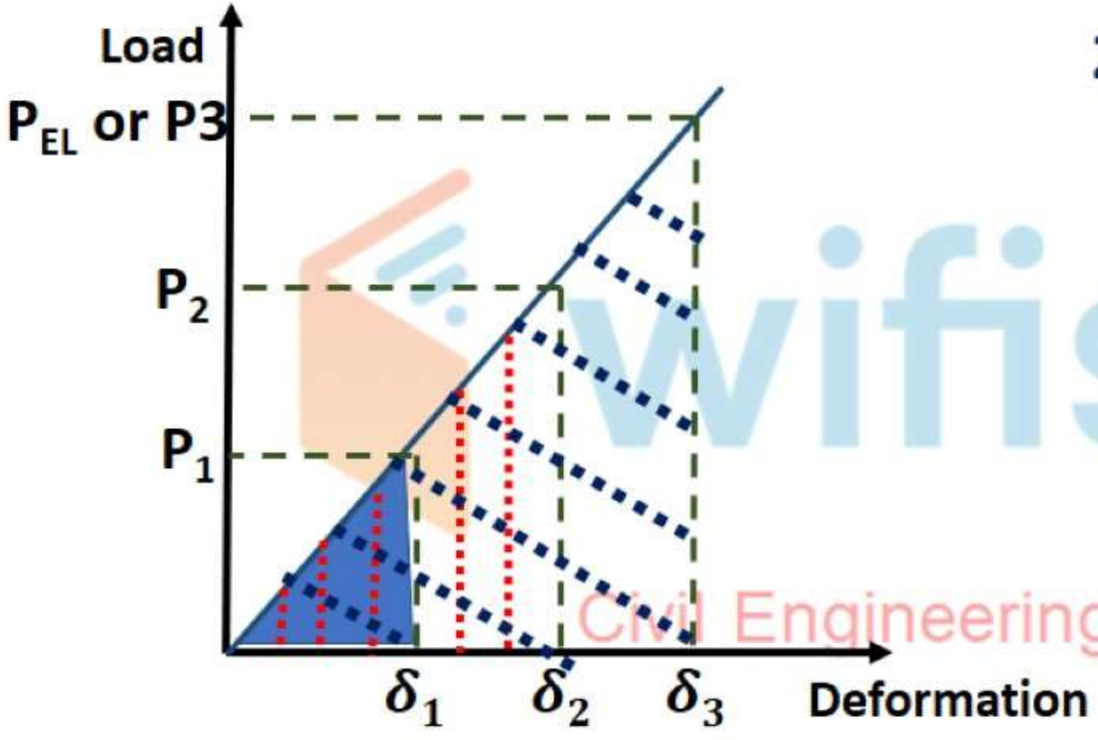
$$U= \frac{1}{2} \times \frac{P}{A} \times \frac{\Delta L}{L} \times A \times L$$

$$U= \frac{1}{2} \times Stress \times Strain \times Volume$$

$$U= \frac{1}{2} \times \sigma \times \varepsilon \times V$$

$$U= \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times V$$

$$U= \frac{\sigma^2}{2E} \times V$$



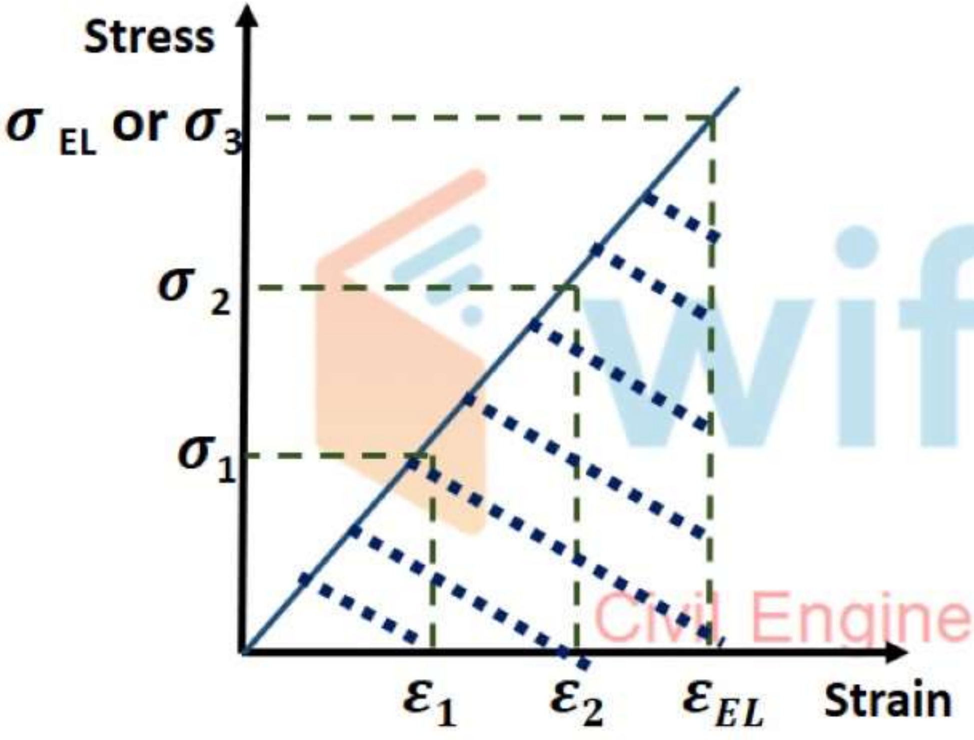
2. Proof Resilience

- Maximum strain energy, stored in body in called as Proof Resilience
- Strain Energy stored in body will be maximum when body is stressed upto Elastic Limit
- Area of Load vs Deformation curve upto Elastic Limit gives us the value of Proof Resilience


Work done by P_1
Resilience upto P_1


Work done by P_2
Resilience upto P_2


Max strain energy in elastic region



3. Modulus of Resilience

- It is defined as Proof Resilience of a material per unit volume

$$MR = \frac{Proof\ Resilience}{Volume}$$

$$MR = \frac{Maximum\ SE\ upto\ elastic\ limit}{Volume}$$

$$MR = \frac{\frac{\sigma^2}{2E} \times V}{V}$$

$$MR = \frac{\sigma^2_{Elastic\ Limit}}{2E}$$

Some Definitions

1. **Resilience:**
   - Total strain Energy stored in body is commonly known as Resilience.
   - Whenever straining force is removed from strained body, the body is capable of doing work
   - Resilience is also defined as capacity of a strained body for doing work on removal of straining force

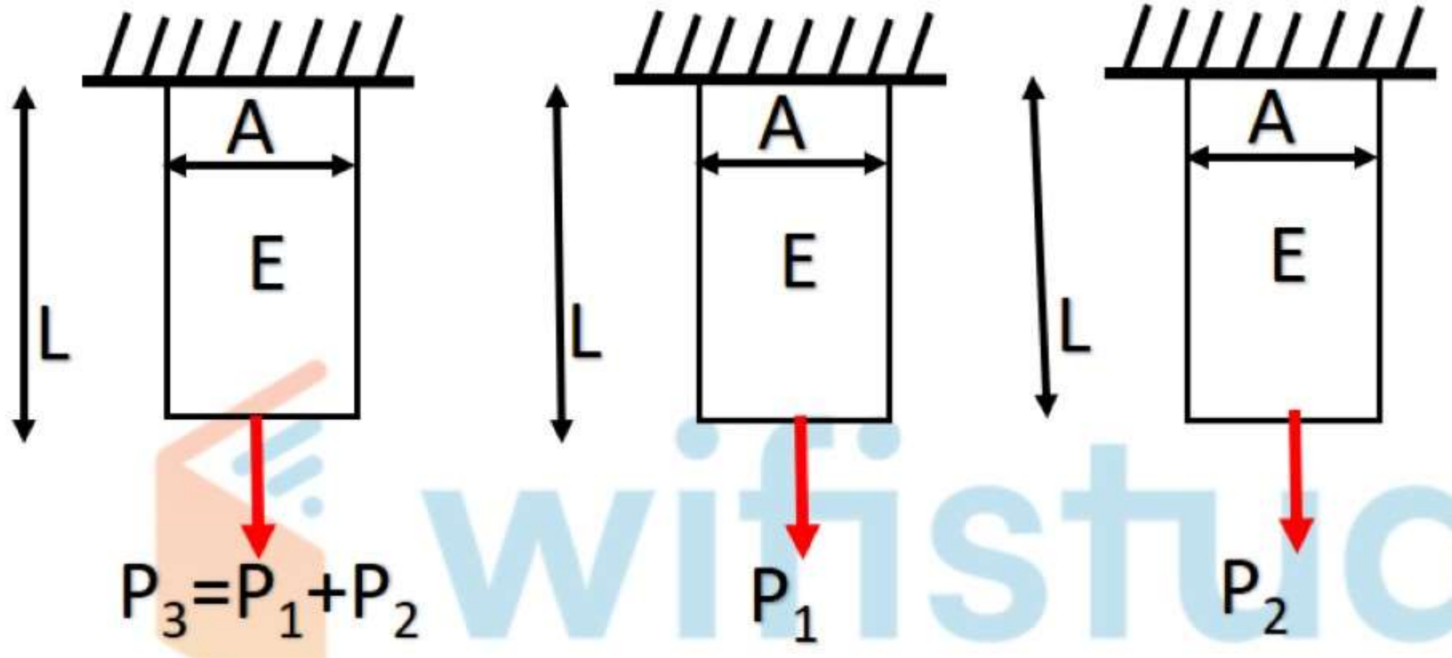
2. **Proof Resilience**

   Maximum strain energy, stored in body in called as Proof Resilience

   Strain Energy stored in body will be maximum when body is stressed upto Elastic Limit

3. **Modulus of Resilience**

   It is defined as Proof Resilience of a material per unit volume



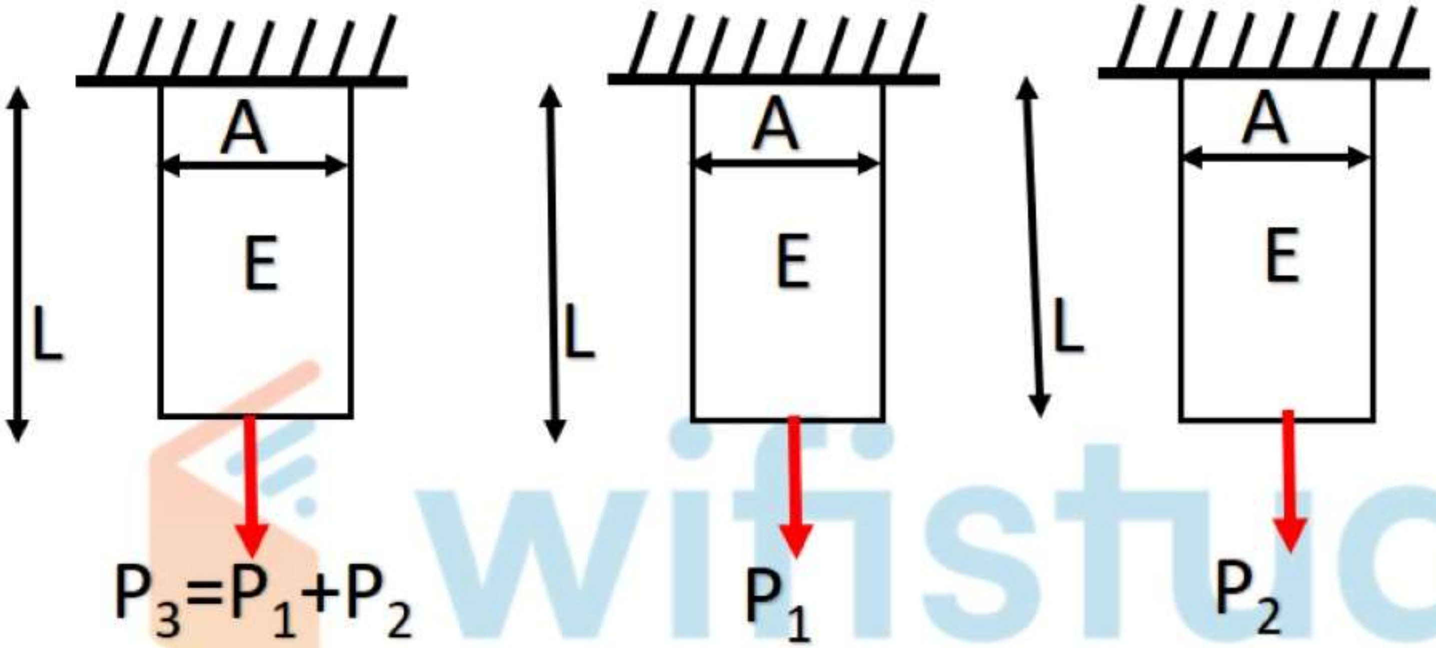
Que 74 Relation between Strain Energy of these three is…

a) $U_3 = U_1 + U_2$

b) $U_3 > U_1 + U_2$

c) $U_3 < U_1 + U_2$

d) None of the above



Que 74 Relation between Strain Energy of these three is…

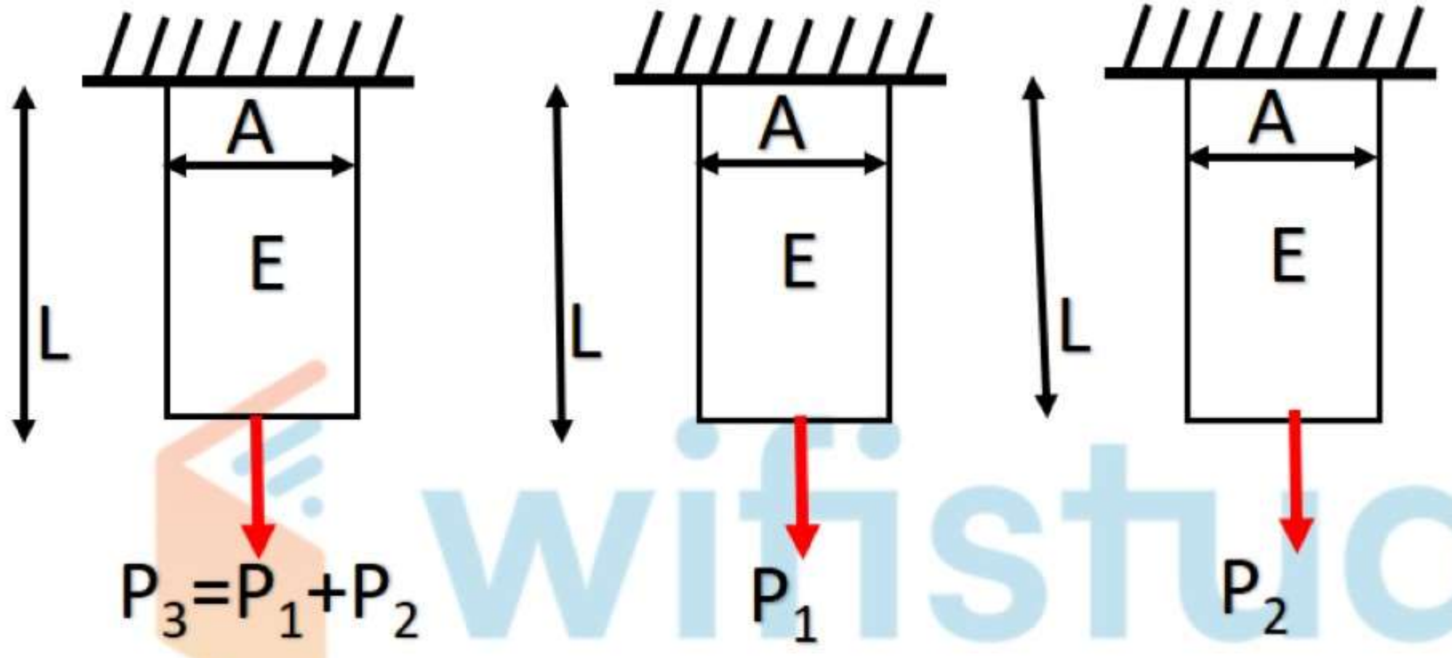
$$U = \frac{P^2 L}{2AE}$$

a) $U_3 = U_1 + U_2$

b) $U_3 > U_1 + U_2$

c) $U_3 < U_1 + U_2$

d) None of the above



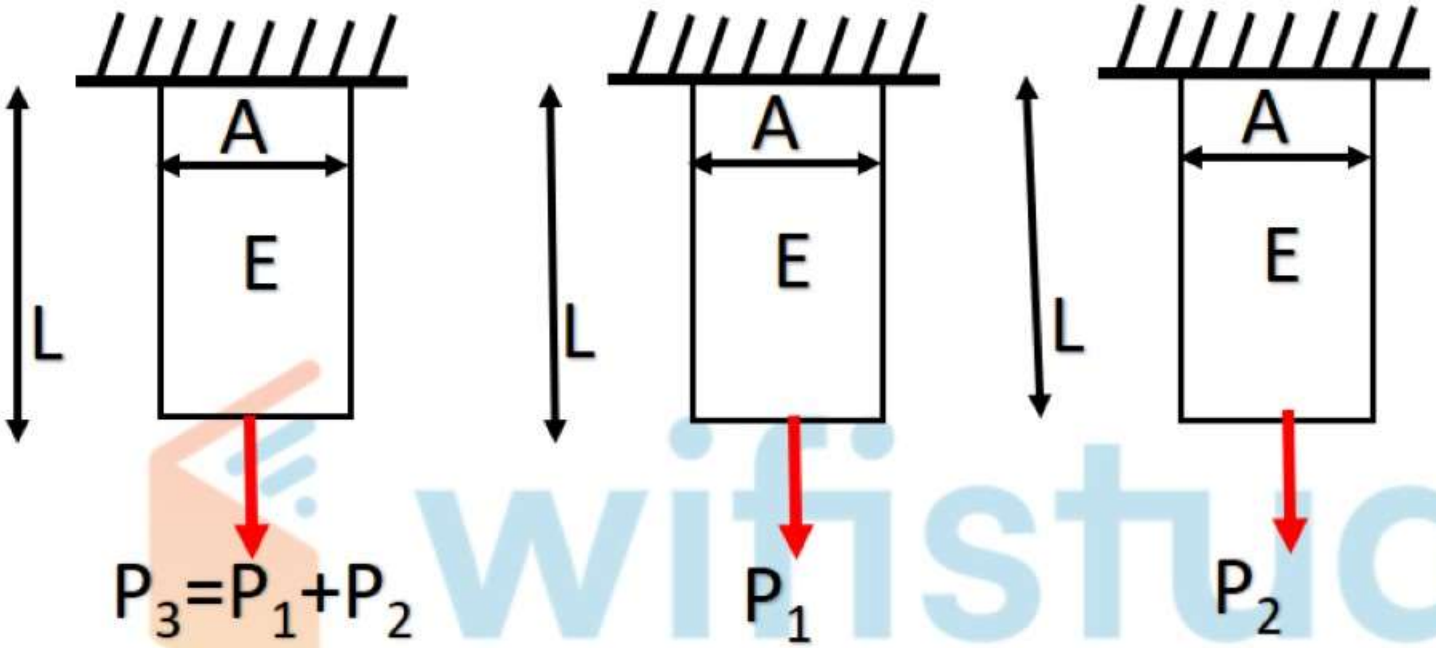
Que 75 Relation between Elongation of these three is…

a) $\Delta L_3 = \Delta L_1 + \Delta L_2$

b) $\Delta L_3 > \Delta L_1 + \Delta L_2$

c) $\Delta L_3 < \Delta L_1 + \Delta L_2$

d) None of the above



Que 75 Relation between Elongation of these three is...

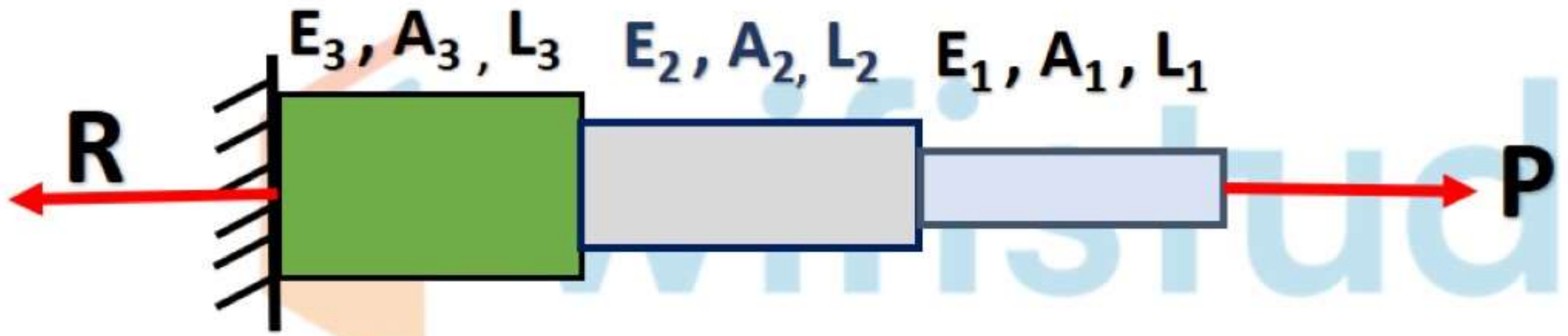
a) $\Delta L_3 = \Delta L_1 + \Delta L_2$

b) $\Delta L_3 > \Delta L_1 + \Delta L_2$

c) $\Delta L_3 < \Delta L_1 + \Delta L_2$

d) None of the above

Question: **Bar in Series**



$U_{Total} = U_1 + U_2 + U_3$

$$U = \frac{P^2 L}{2AE}$$

$P_1 = P_2 = P_3 = P$

Strain Energy due to Shear

$U = \frac{\sigma^2}{2E} \times V$

$U = \frac{\tau^2}{2G} \times V$

Strain Energy due to Torque

$U = \frac{1}{2} \times P \times \Delta L$

$U = \frac{1}{2} \times T \times \theta$

$U = \frac{1}{2} \times T \times \frac{TL}{GJ}$

$$U = \frac{T^2 L}{2GJ}$$

Strain Energy due to Moment

$$U = \frac{1}{2} \times P \times \Delta L$$

$$U = \frac{1}{2} \times M \times \theta$$

$$U = \frac{1}{2} \times M \times \frac{ML}{EI}$$

$$U = \frac{M^2 L}{2EI}$$

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{BENDING}}{y} = \frac{E}{R}$$

$$\frac{M_R}{I_{NA}} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E\theta}{L}$$

Strain Energy of a Prismatic Bar due to Self Weight

$$U = \frac{P^2 L}{2AE}$$

$$\delta U_x = \frac{(P_{xx})^2 dx}{2AE}$$

$$\delta U_x = \frac{(\lambda A x)^2 dx}{2AE}$$

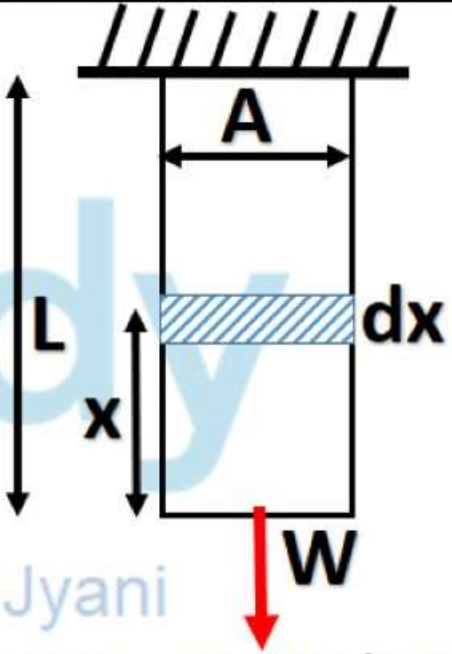
$$\lambda = \frac{W}{V}$$

$$\lambda = \frac{W}{A \times L}$$

$$W = A \times L \times \lambda$$

$$W_{xx} = A \times x \times \lambda$$

$$P_{xx} = A \times x \times \lambda$$



Weight Density (Wt/Volume)= λ

For total Strain Energy Stored,

$$\int \delta U_x = \int_0^L \frac{\lambda^2 A^2 x^2 dx}{2AE}$$

$$U_x = \frac{\lambda^2 A^2}{2AE} \frac{x^3}{3}$$

$$U_x = \frac{A\lambda^2 x^3}{6E}$$

For Strain Energy Stored if Total length, is considered

$$U_L = \frac{A\lambda^2 L^3}{6E}$$

Question:

- **Strain Energy depends upon area (when bar is subjected to Self weight) but elongation due to self weight does not depend upon area.**

$$U_L = \frac{A\lambda^2 L^3}{6E}$$

$$\Delta L = \frac{\lambda L^2}{2E}$$

Thermal Stresses

1. **Mechanical Stresses** $(\sigma_{mechanical})$
   - **These stresses are produced in the body due to external load**

2. **Thermal Stresses** $(\sigma_{Thermal})$

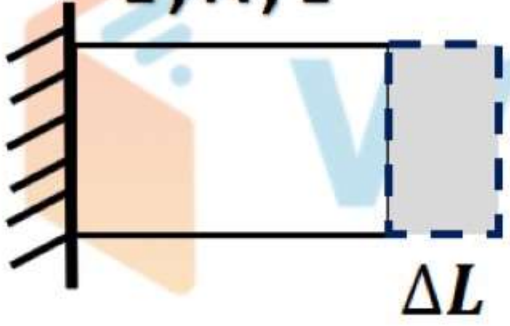
   If Following two conditions are satisfied, then thermal stresses are produced:

   A. **There should be temperature variation or temperature difference**

   B. **Due to this temperature difference, the material expands or contracts. If this expansion or contraction is prevented by completely or partially, thermal stresses are produced**

Thermal Stresses

Case 1: Bar is free to Expand



- ΔL = Elongation on temperature variation
- α = Thermal Coefficient of expansion i.e.

$$\alpha = \frac{\frac{\Delta L}{L}}{\Delta T}$$

$$\alpha = \frac{\varepsilon_T}{\Delta T}$$

$$\boxed{\varepsilon_T = \alpha \Delta T}$$

Elongation on temperature variation

$$\Delta L = \alpha\, \Delta T\, L$$

Thermal Stresses in free expansion are zero

Thermal Stresses

Case 2: Completely Prevented Case

Elongation due to temp variation = Elongation due to reaction

$$\frac{-RL}{AE} = \alpha \Delta T L \quad \Rightarrow \frac{\sigma_{thermal}}{E} = -\alpha \Delta T$$

$$\Rightarrow \sigma_{thermal} = -\alpha \Delta T E$$

Thermal Stresses

Case 3: Partially Prevented Case

$$\frac{\alpha \Delta T L - \lambda}{L} = \varepsilon_{th}$$

$$\Rightarrow \frac{\sigma_{thermal}}{E} = \frac{\alpha \Delta T L - \lambda}{L}$$

Thermal Stresses

$$\sigma_{thermal} = \pm\, \alpha \Delta T E$$

1. **If the temperature is increased, the nature of thermal stresses is compressive**
2. **If the temperature is decreased, the nature of thermal stresses is Tensile**
3. **In case of combined bar, the material having more value of thermal coefficient will experience Compressive stress and the material having less value of thermal coefficient will experience Tensile strength**

Que 76 A steel rod 10mm in dia and 1m long is heated from 20℃ to 120℃. If E=200GPa and $\alpha = 12 \times 10^{-6}$ per ℃. If the rod is not free to expand, what is the thermal stress?

240 MPa comp

Que 77 A bar of length 1m, diameter 50mm, fixed between two rigid supports, the initial tensile stress in the bar is 10MPa at a temperature of 10℃. Determine the stress induced in the bar if the temperature is rising to 15℃. E=200GPa and $\alpha = 10 \times 10^{-6}$ per ℃.

a) **0 MPa**

b) **10MPa tensile**

c) **10MPa Compressive**

d) **None of these**

Que 77 A bar of length 1m, diameter 50mm, fixed between two rigid supports, the initial tensile stress in the bar is 10MPa at a temperature of 10℃. Determine the stress induced in the bar if the temperature is rising to 15℃. E=200GPa and $\alpha = 10 \times 10^{-6}$ per ℃.

a) <u>**0 MPa**</u>

b) **10MPa tensile**

c) **10MPa Compressive**

d) **None of these**

$$\sigma_{thermal} = \pm\, \alpha \Delta T E$$

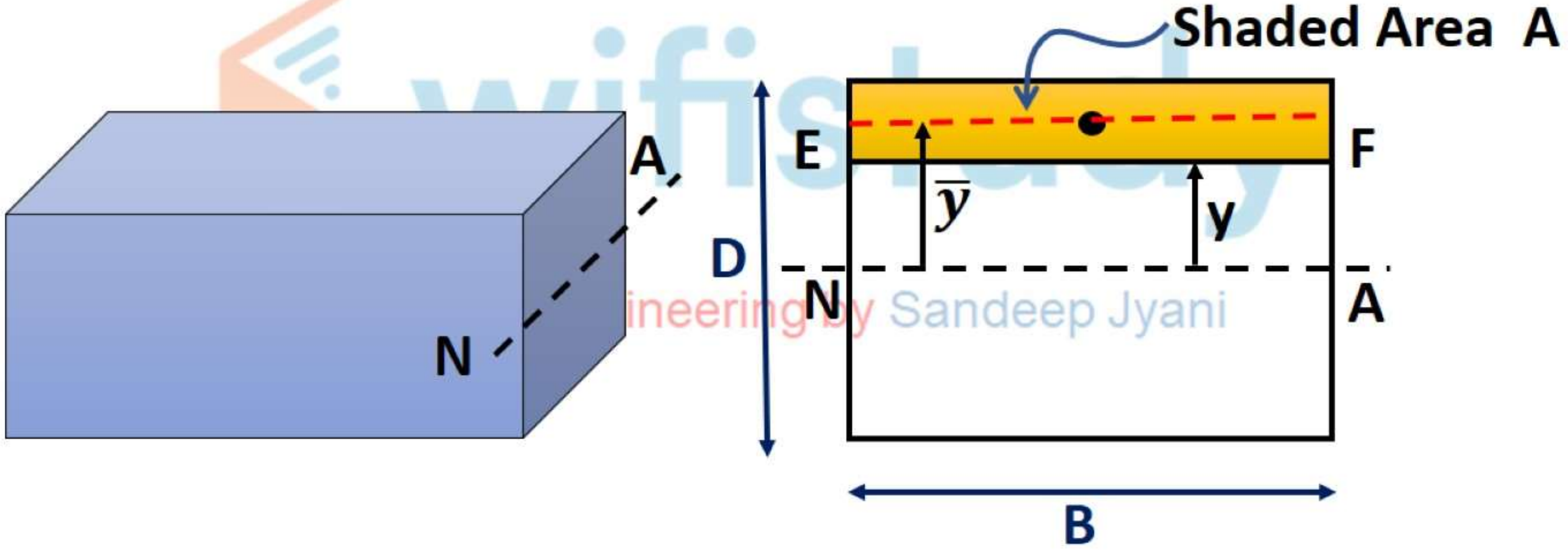
$$\sigma_{thermal} = (200 \times 10^3) \times (10 \times 10^{-6}) \times 5$$

$$\sigma_{thermal} = (200 \times 10^3) \times (10 \times 10^{-6}) \times 5$$

$$\sigma_{thermal} = 10\ MPa\ (comp)$$

Since it is already in tensile stress of 10Mpa,
so net force = 10 MPa − 10MPa = 0

Shear Stress Distribution

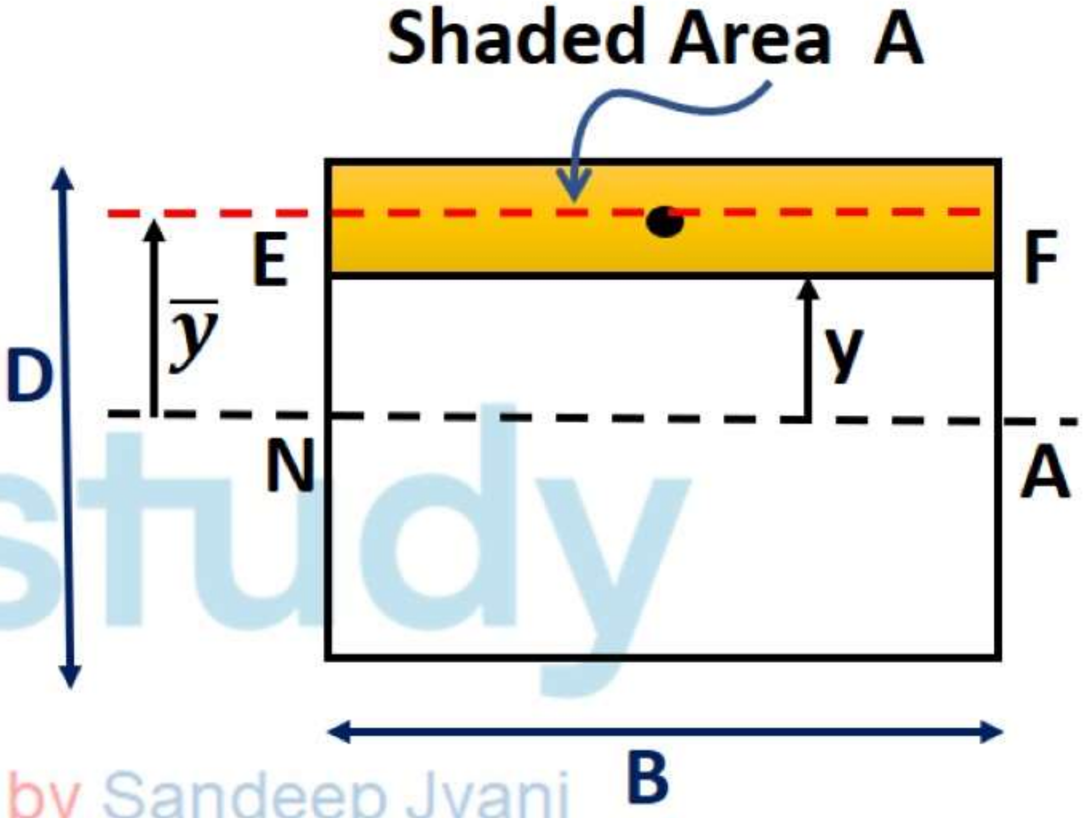


Shear Stress Distribution

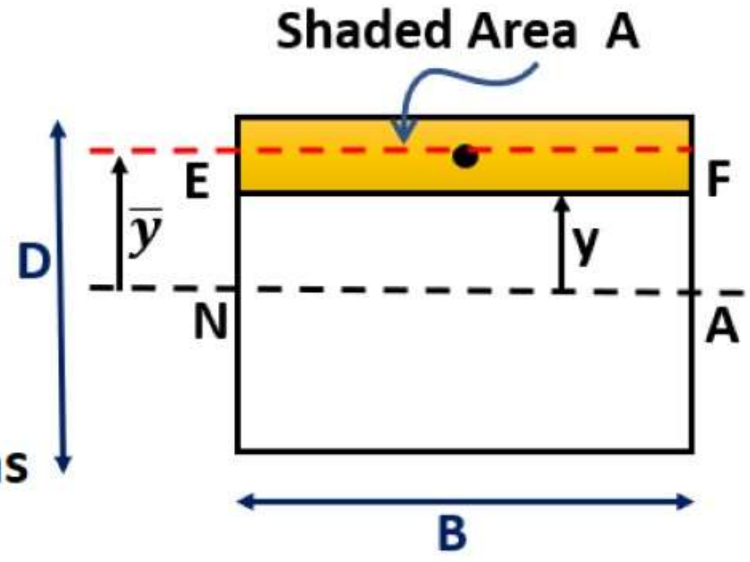
Let us consider a section x-x on which shear force is F

Shear Stress on a fiber EF which is located at a distance y from the NA is given by

$$\tau = \frac{FA\bar{y}}{IB}$$



$$\tau = \frac{FA\bar{y}}{IB}$$



Where

F = Shear Force at that section

A = Area of given cross section beyond the level EF as shaded in the figure

$\bar{y}$ = distance of CENTROID of shaded area A from the neutral axis

A$\bar{y}$ = Moment of shaded region A about the neutral axis

I = Moment of inertia of TOTAL CROSS SECTION AREA

B = width of the section at the level of EF

Shear Stress Distribution

Assumptions:

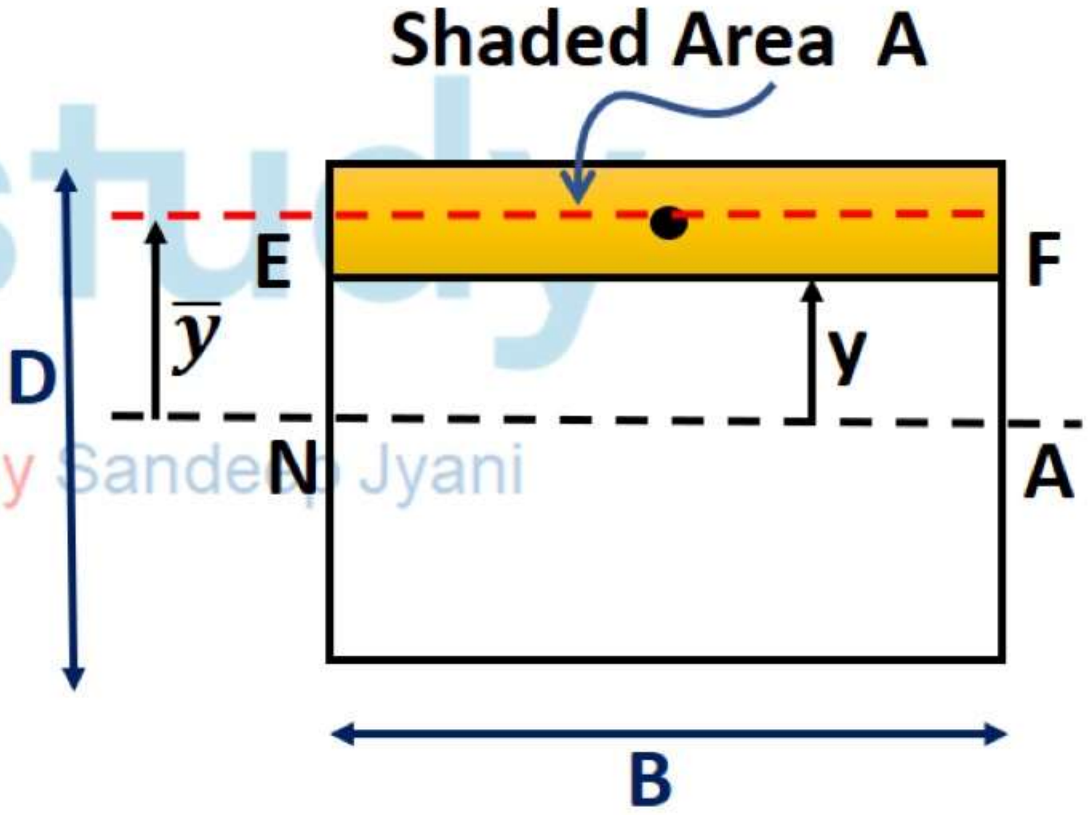
1. **Material should be homogenous and Isotropic, and it must obey the Hooke's law**
2. **The shear stress is constant along the WIDTH (It means Shear Stress is constant from E to F but it varies along the DEPTH)**

Note: For all shapes of cross section, Shear Stress distribution is parabolic which is zero at the top and Bottom

Shear Stress Distribution in Rectangular Section

$$\tau = \frac{F A \bar{y}}{I B}$$

- Where Force = F
- Shaded Area A = $B \times \left(\frac{D}{2} - y\right)$
- $\bar{y} = \frac{\frac{D}{2} - y}{2} + y = \frac{\frac{D}{2} + y}{2}$
- $I = \frac{1}{12} B D^3$
- Width at EF = B



Shear Stress Distribution in Rectangular Section

$$\boxed{\tau = \frac{FA\bar{y}}{IB}}$$

- Where Force = F
- Shaded Area A = $B \times \left(\frac{D}{2} - y\right)$
- $\bar{y} = \frac{\frac{D}{2} - y}{2} + y = \frac{\frac{D}{2} + y}{2}$
- $I = \frac{1}{12} BD^3$
- Width at EF = B

Putting the values in formula,

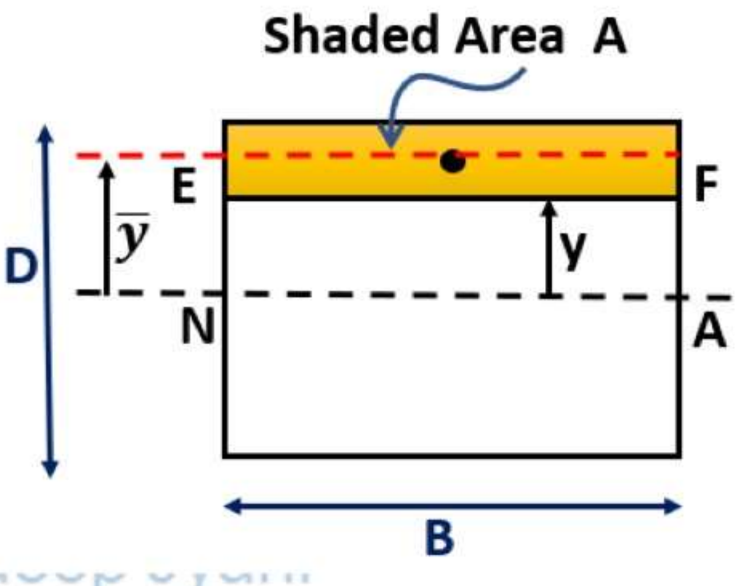
$$\tau = \frac{F \times \left(B \times \left(\frac{D}{2} - y\right)\right)\left(\frac{\frac{D}{2} + y}{2}\right)}{IB}$$

$$\tau = \frac{F \times \left(B \times \left(\frac{D}{2} - y\right)\right)\left(\frac{\frac{D}{2} + y}{2}\right)}{IB}$$

$$\tau = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$$

Shear Stress Distribution in Rectangular Section

$$\tau = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$$



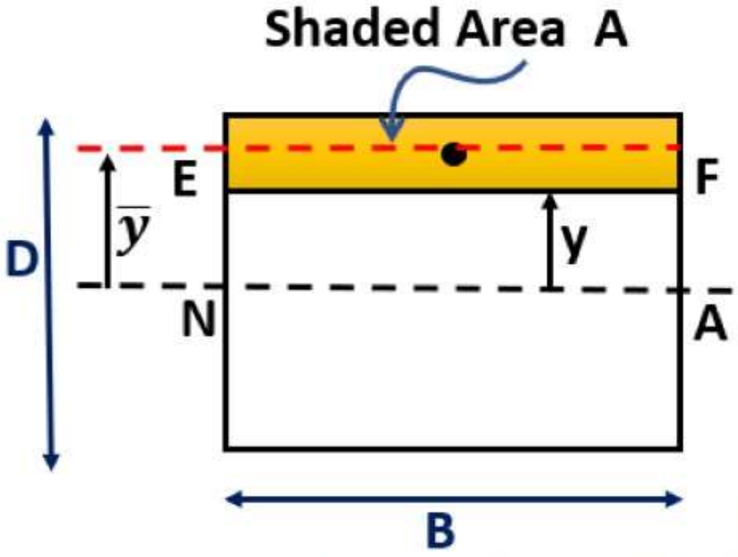
1. **Shear Stress on the top or bottom fiber**

$$y = \pm\frac{D}{2}$$

$$\therefore \tau = \frac{F}{2I}\left(\frac{D^2}{4} - (D/2)^2\right)$$

$$\tau_{top} = 0$$

$$\tau_{bottom} = 0$$



Shear Stress Distribution in Rectangular Section

2. Location of $\tau_{max} = ?$

$Since\ \tau = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$, for τ_{max},

Put $\frac{d\tau}{dy} = 0$

$$\frac{F}{2I}(-2y) = 0$$

$$y = 0$$

Hence τ_{max} is located at Neutral Axis

3. Value of $\tau_{max} = ?$

$Since\ \tau = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$, for τ_{max},

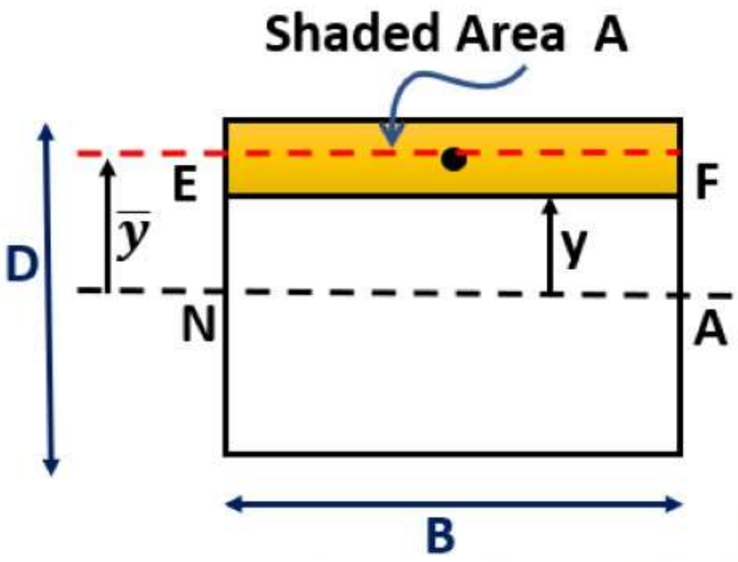
Put $y = 0$

$$\tau_{max} = \frac{F}{2I}\left(\frac{D^2}{4} - 0^2\right)$$

Hence $\tau_{max} = \frac{FD^2}{8I}$

$$\tau_{max} = \frac{FD^2}{8\left(\frac{1}{12}BD^3\right)}$$

$$\tau_{max} = \frac{3F}{2BD}$$



Shear Stress Distribution in Rectangular Section

4. Value of $\tau_{average} = ?$

$$\tau_{average} = \frac{F}{A}$$

$$\tau_{average} = \frac{F}{BD}$$

5. Relation between τ_{max} and $\tau_{average} = ?$

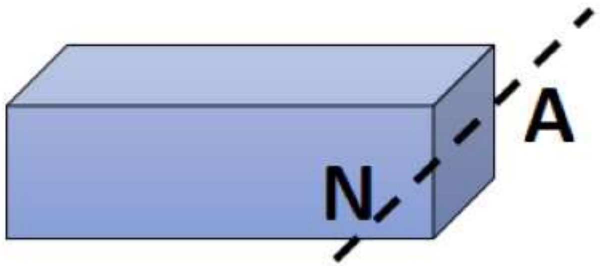
$$\tau_{max} = \frac{3F}{2BD}$$

$$\tau_{average} = \frac{F}{BD}$$

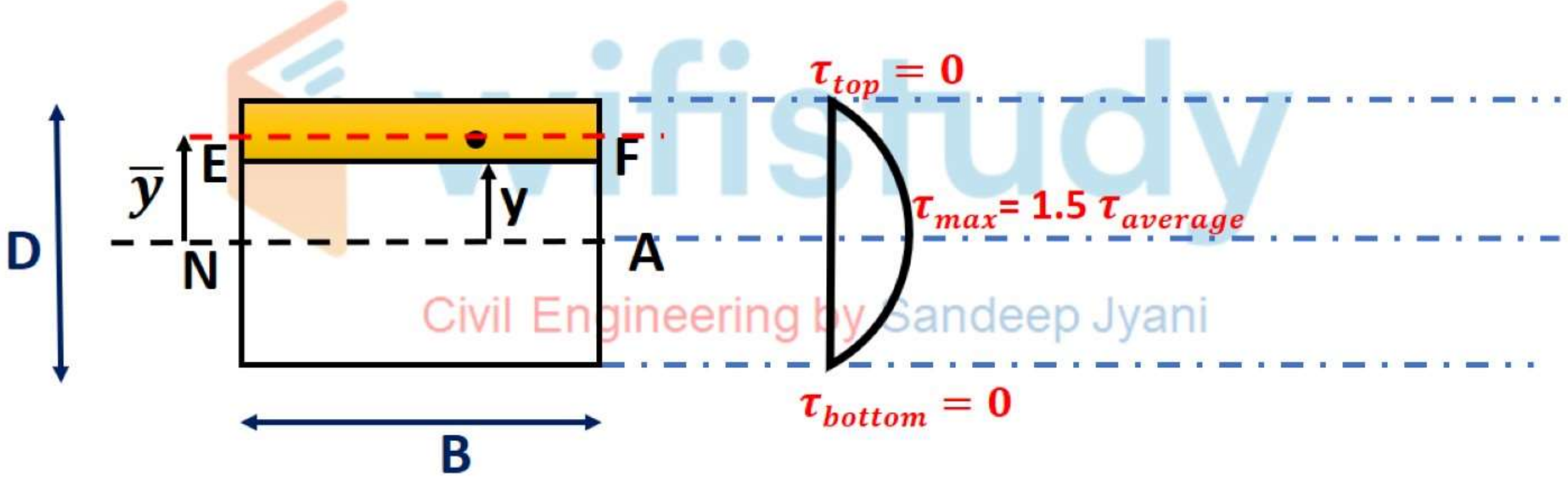
$$\therefore \frac{\tau_{max}}{\tau_{average}} = \frac{\frac{3F}{2BD}}{\frac{F}{BD}}$$

$$\frac{\tau_{max}}{\tau_{average}} = 1.5$$

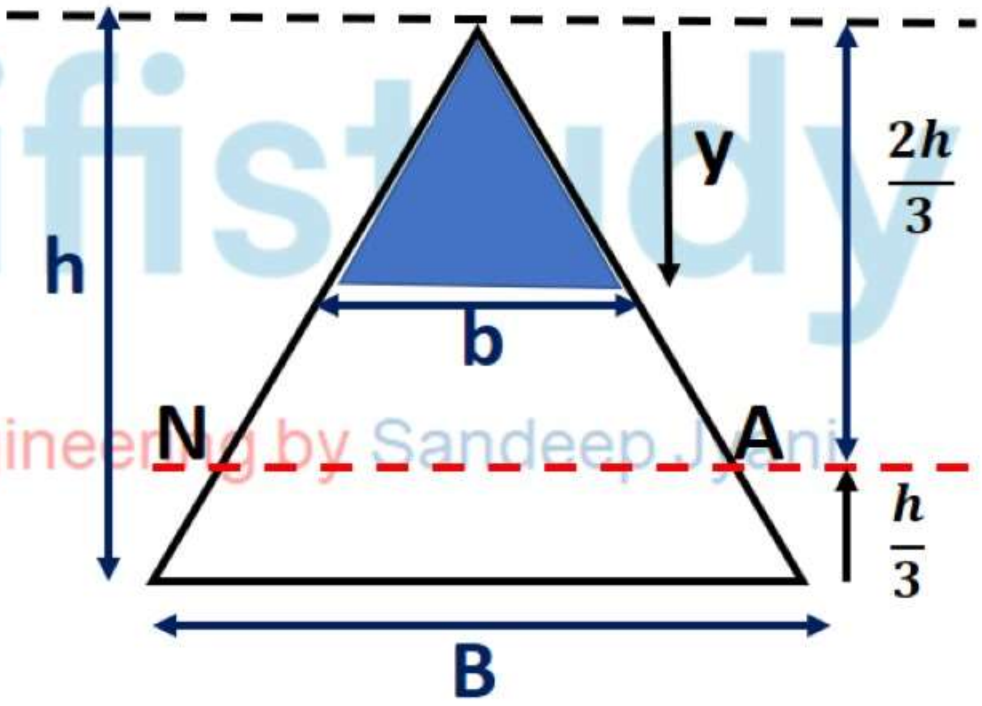
$$\tau_{max} = 1.5\ \tau_{average}$$



Shear Stress Distribution in Rectangular Section



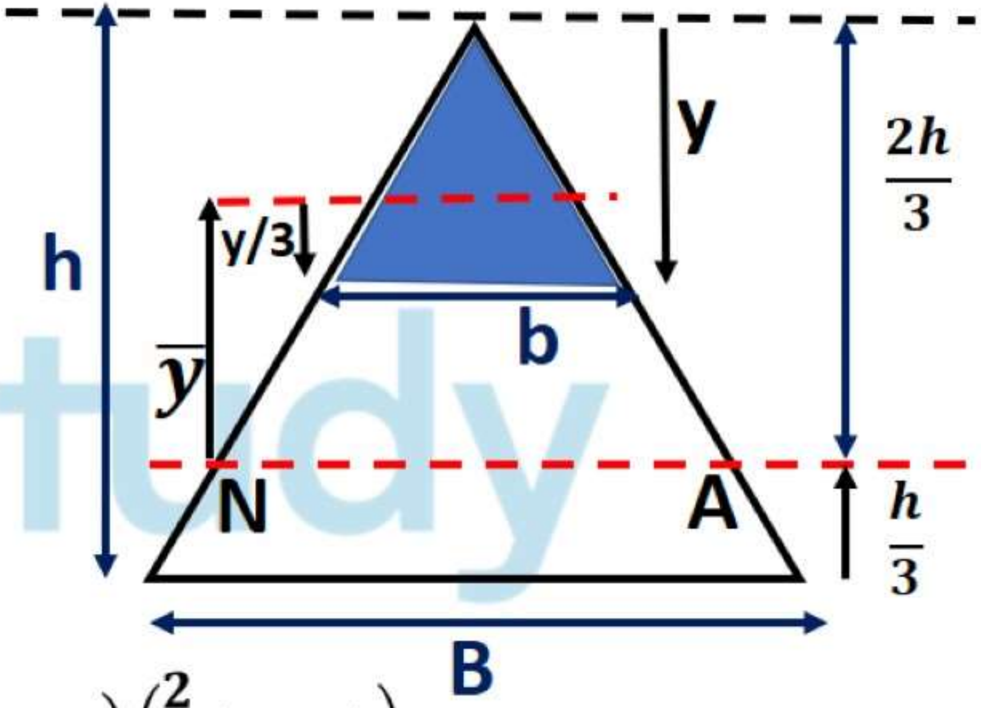
Shear Stress Distribution in Triangular Section



Shear Stress Distribution in Triangular Section

$$\tau = \frac{FA\overline{y}}{IB}$$

- Where Force = F
- Shaded Area A $= \frac{1}{2} \times b \times y$
- $\overline{y} = \frac{2}{3}(h - y)$
- $I = I$
- Width = b



$$\tau = \frac{F\left(\frac{1}{2} \times b \times y\right)\left(\frac{2}{3}(h - y)\right)}{Ib}$$

$$\tau = \frac{F}{3I}(hy - y^2)$$

Shear Stress Distribution in Triangular Section

$$\tau = \frac{F}{3I}(hy - y^2)$$

1. Shear Stress on the top or bottom fiber

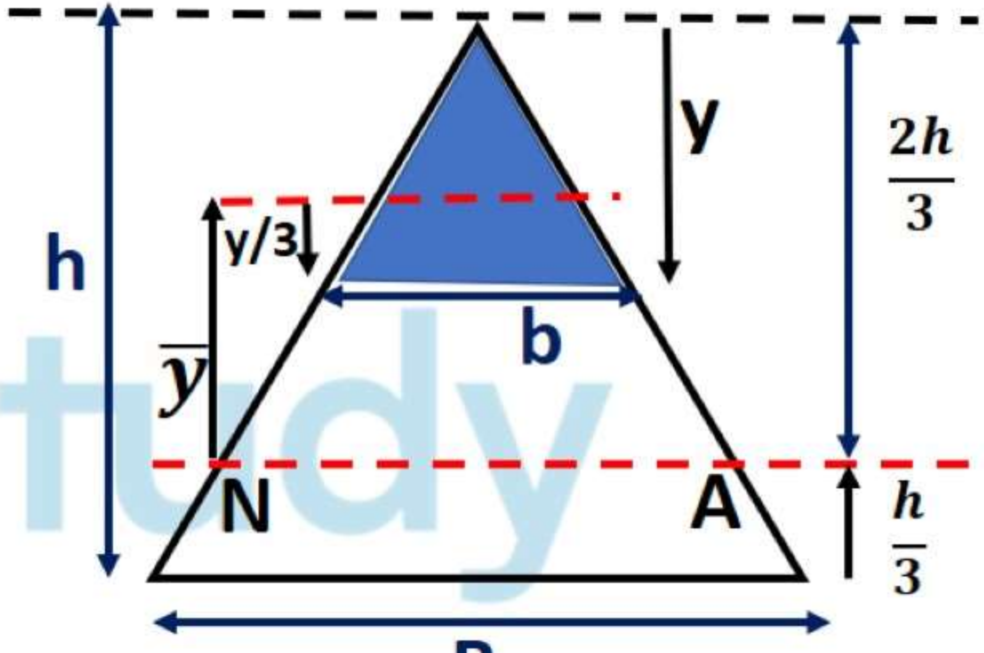
$y(top)$ =0

y(bottom)=h

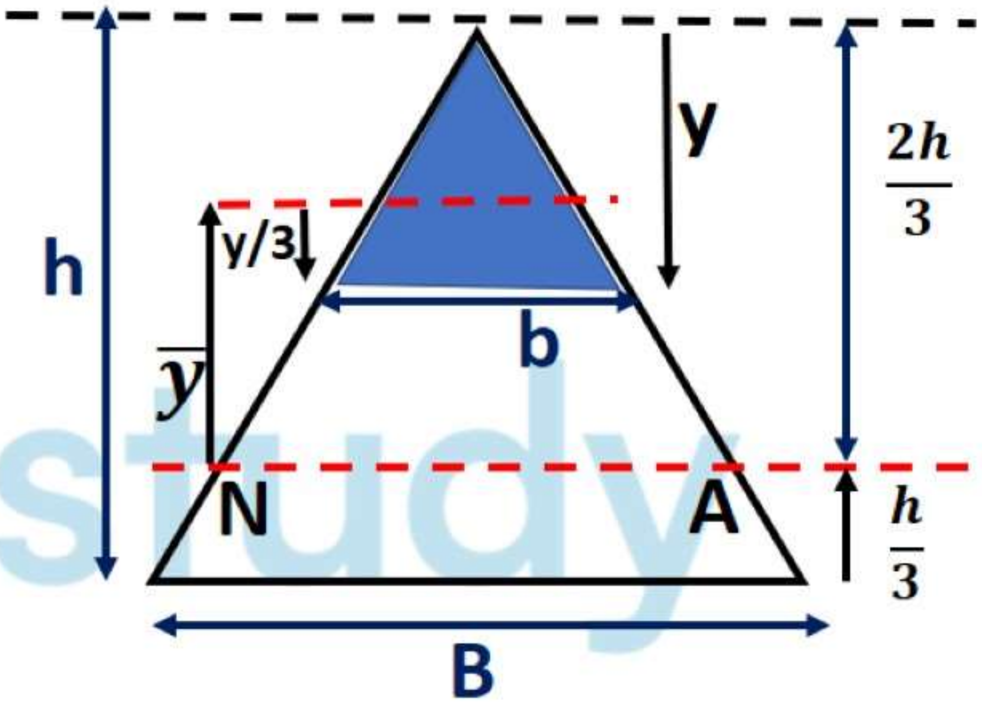
$$\tau = \frac{F}{3I}(hy - y^2)$$

$$\tau_{top} = 0$$

$$\tau_{bottom} = 0$$



Shear Stress Distribution in Triangular Section



2. Location of $\tau_{max} = ?$

Since $\tau = \frac{F}{3I}(hy - y^2)$, for τ_{max},

Put $\frac{d\tau}{dy} = 0$

$$(h - 2y) = 0$$

$$y = h/2$$

Hence τ_{max} is located at h/6 from Neutral Axis

Shear Stress Distribution in Triangular Section

3. Value of $\tau_{max} = ?$

Since $\tau = \frac{F}{3I}(hy - y^2)$, for τ_{max},

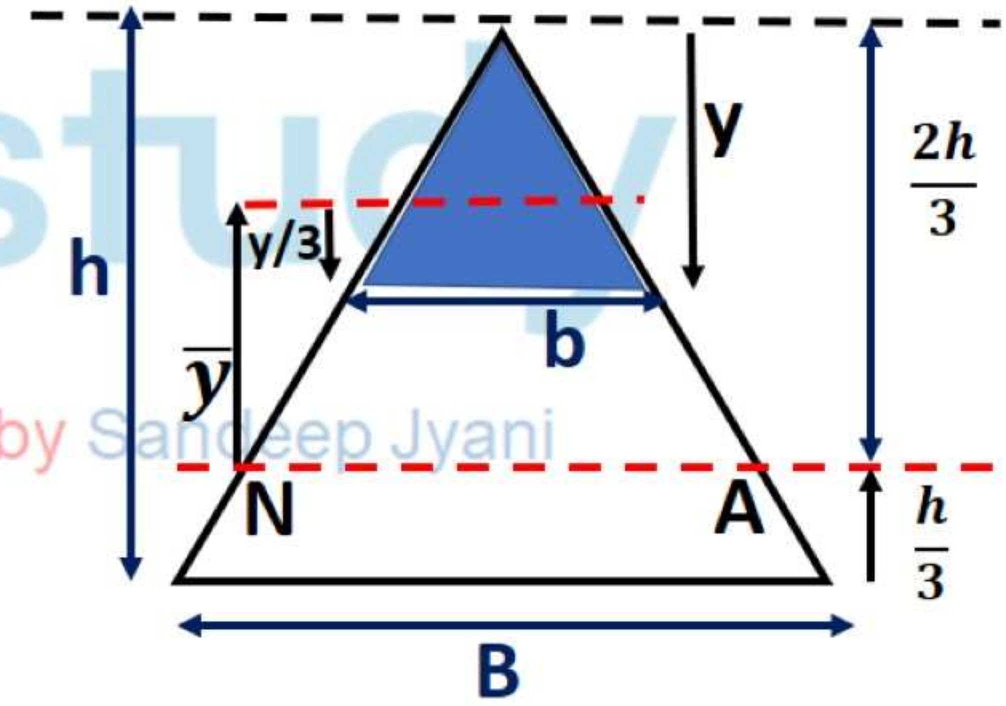
Put $y = h/2$

$$\tau_{max} = \frac{F}{3I}(h(h/2) - (h/2)^2)$$

Hence $\tau_{max} = \frac{Fh^2}{12I}$

$$\tau_{max} = \frac{Fh^2}{12\left(\frac{1}{36}Bh^3\right)}$$

$$\tau_{max} = \frac{3F}{2A}$$



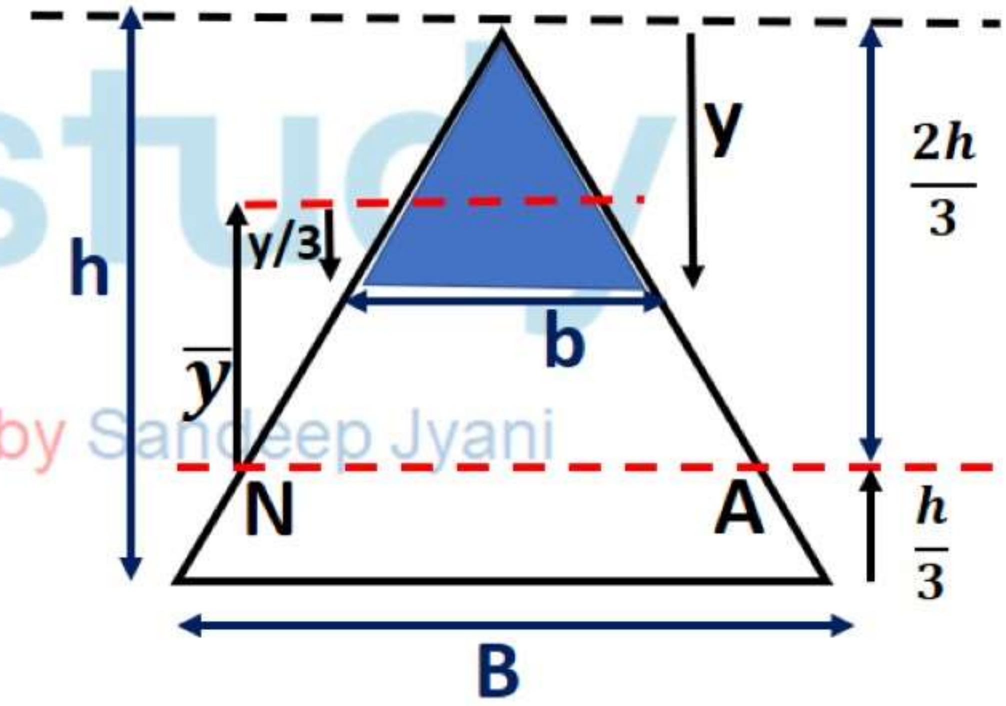
Shear Stress Distribution in Triangular Section

4. Value of $\tau_{average}$ = ?

$$\tau_{average} = \frac{F}{A}$$

$$\tau_{max} = \frac{3F}{2A}$$

$$\frac{\tau_{max}}{\tau_{average}} = \frac{\frac{3F}{2A}}{\frac{F}{A}}$$



τ_{max}= 1.5 $\tau_{average}$

4. Value of τ at neutral axis = ?

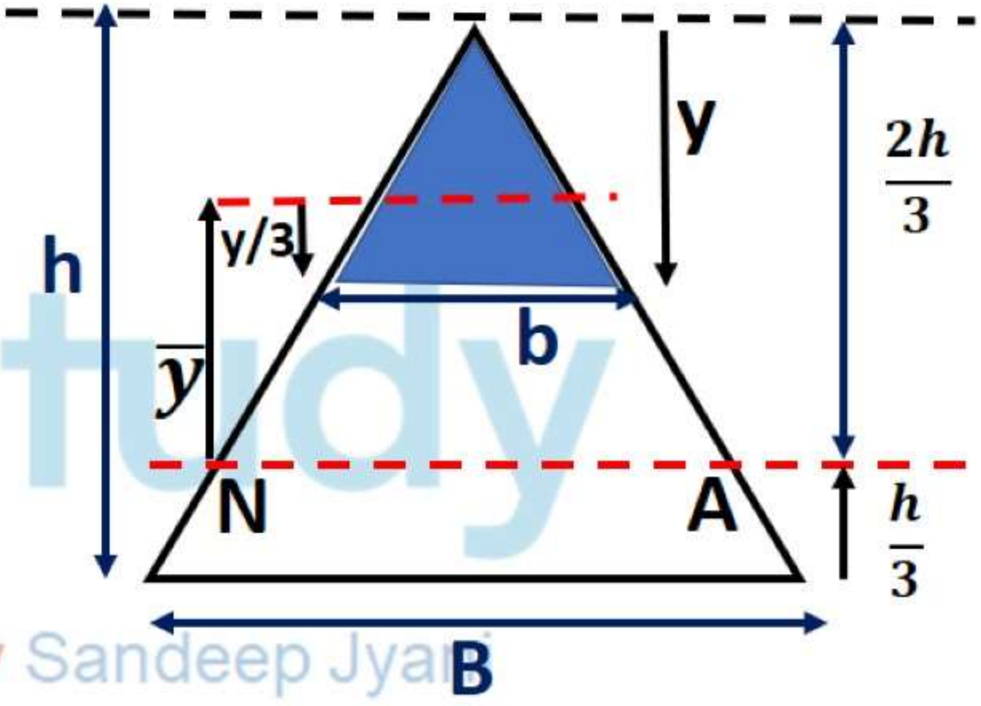
$$\tau = \frac{F}{3I}(hy - y^2)$$

Put $y = 2h/3$

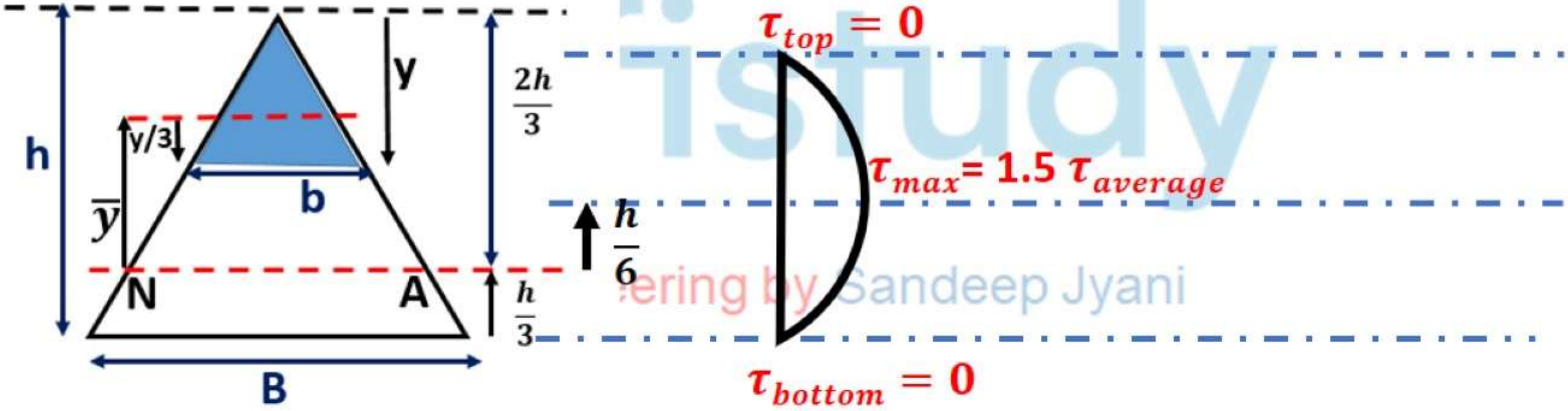
$$\tau = \frac{F}{3\left(\frac{1}{36}Bh^3\right)}(h(2h/3) - (2h/3)^2)$$

$$\tau_{NA} = \frac{12 \times 2 \times F}{9 \times B \times h}$$

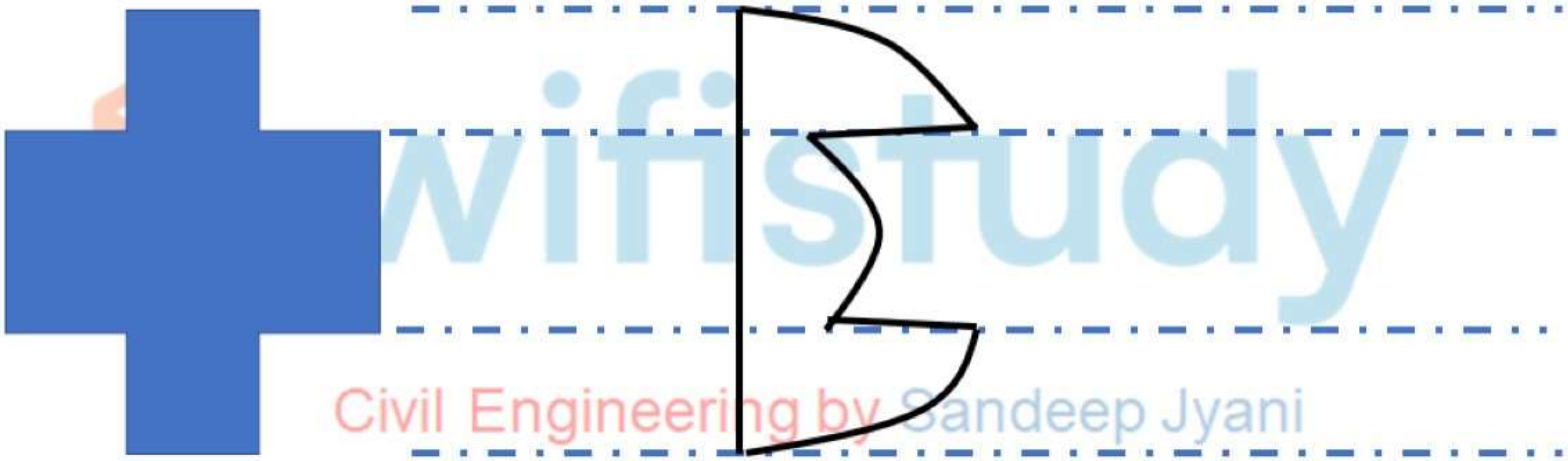
$$\tau_{NA} = \frac{4}{3}\tau_{Avg}$$



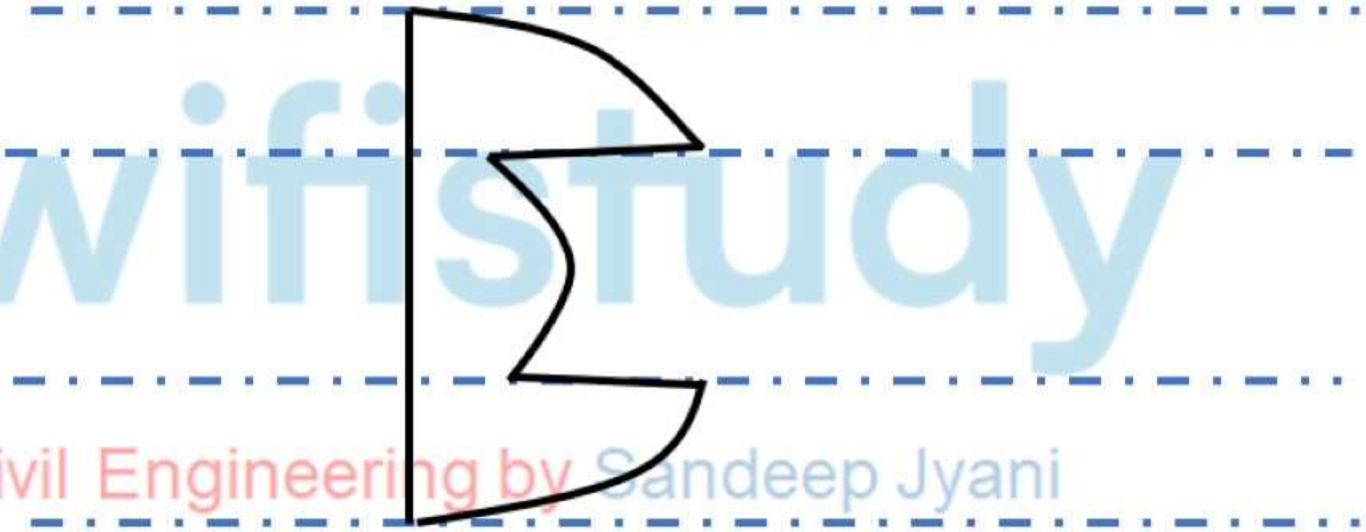
Shear Stress Distribution in Triangular Section



Que 77



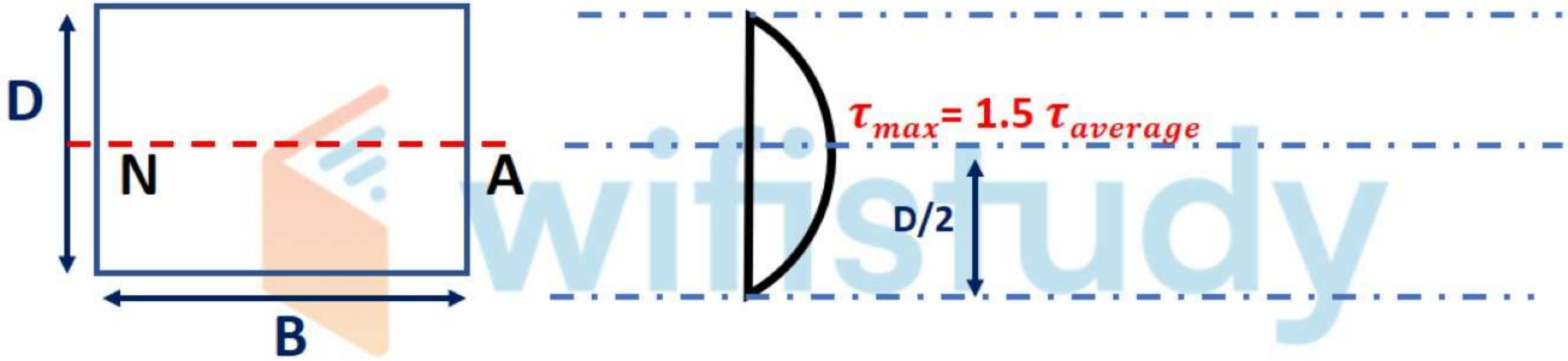
Que 78

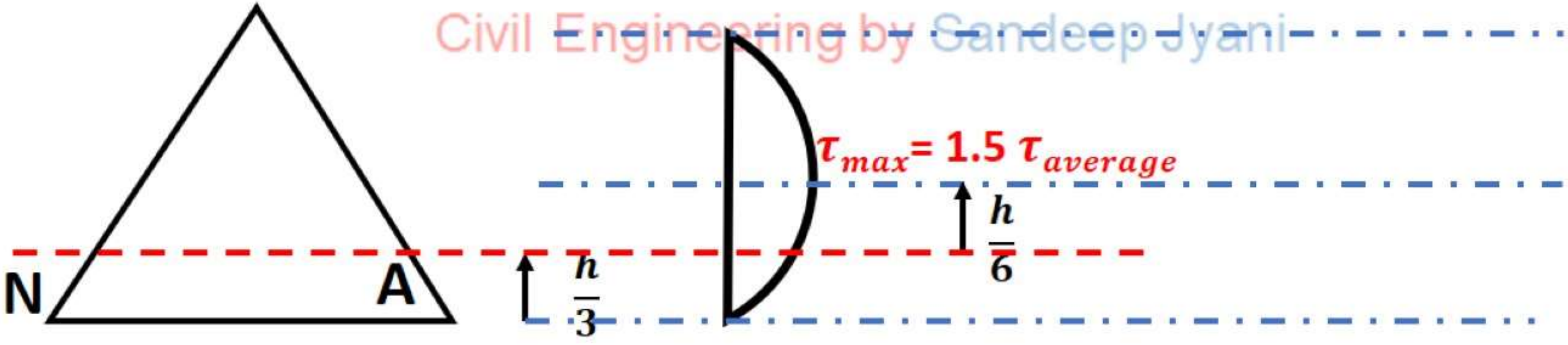


Que 79

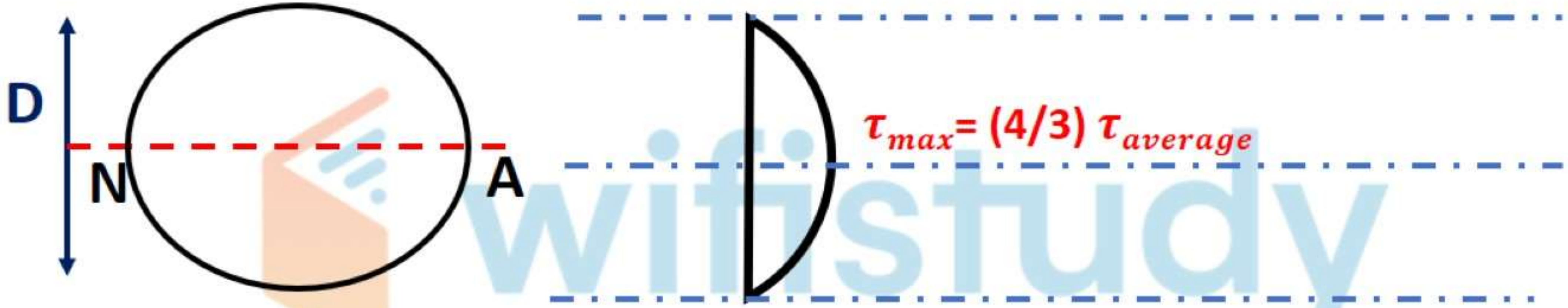
Que 80

Important Relations

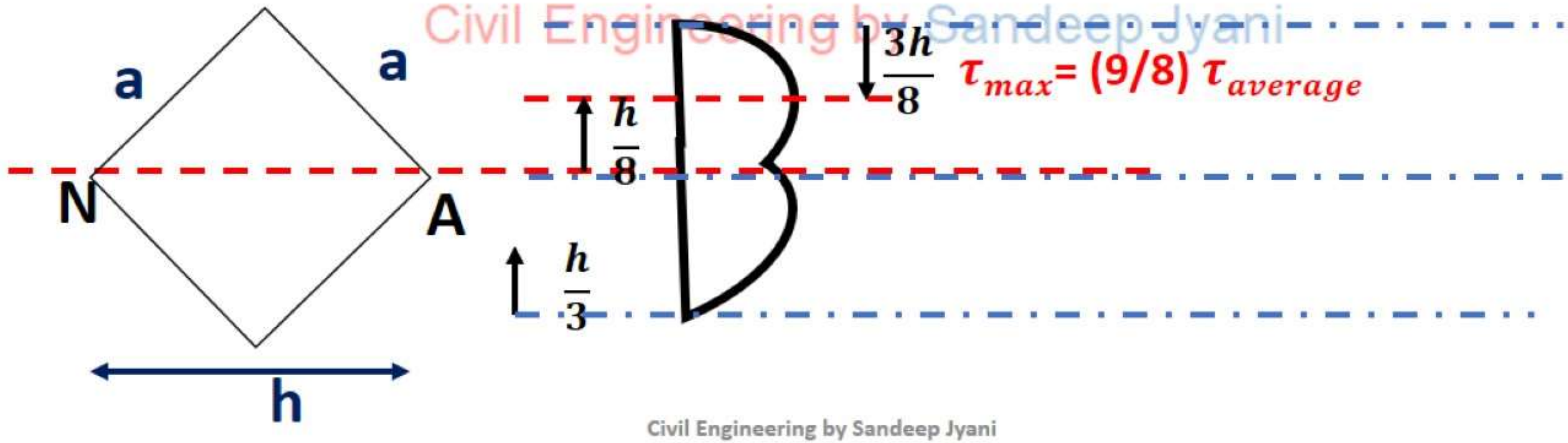




Important Relations

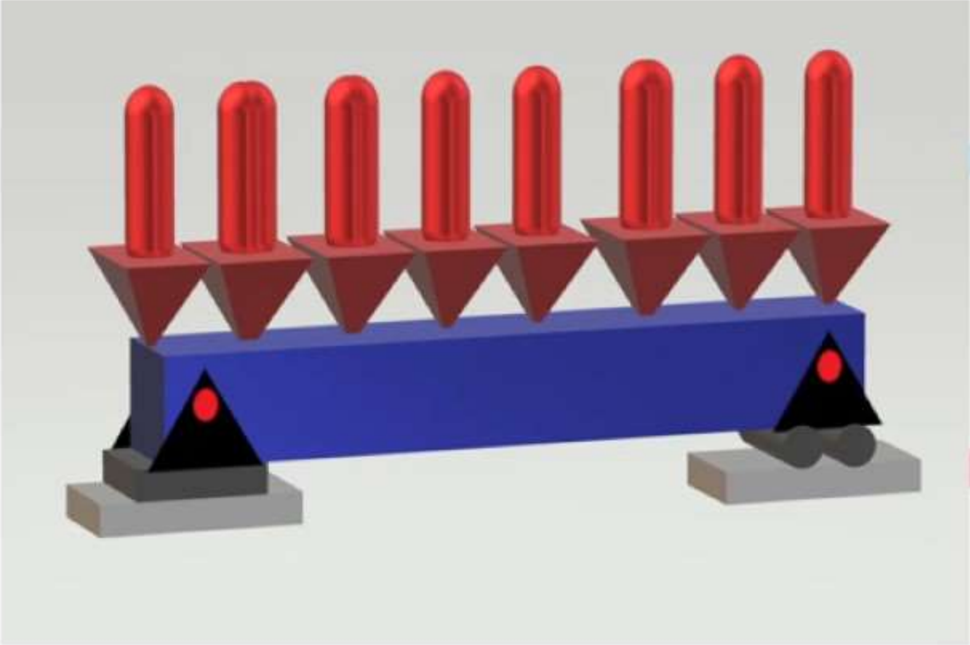


$\tau_{max} = (4/3)\ \tau_{average}$



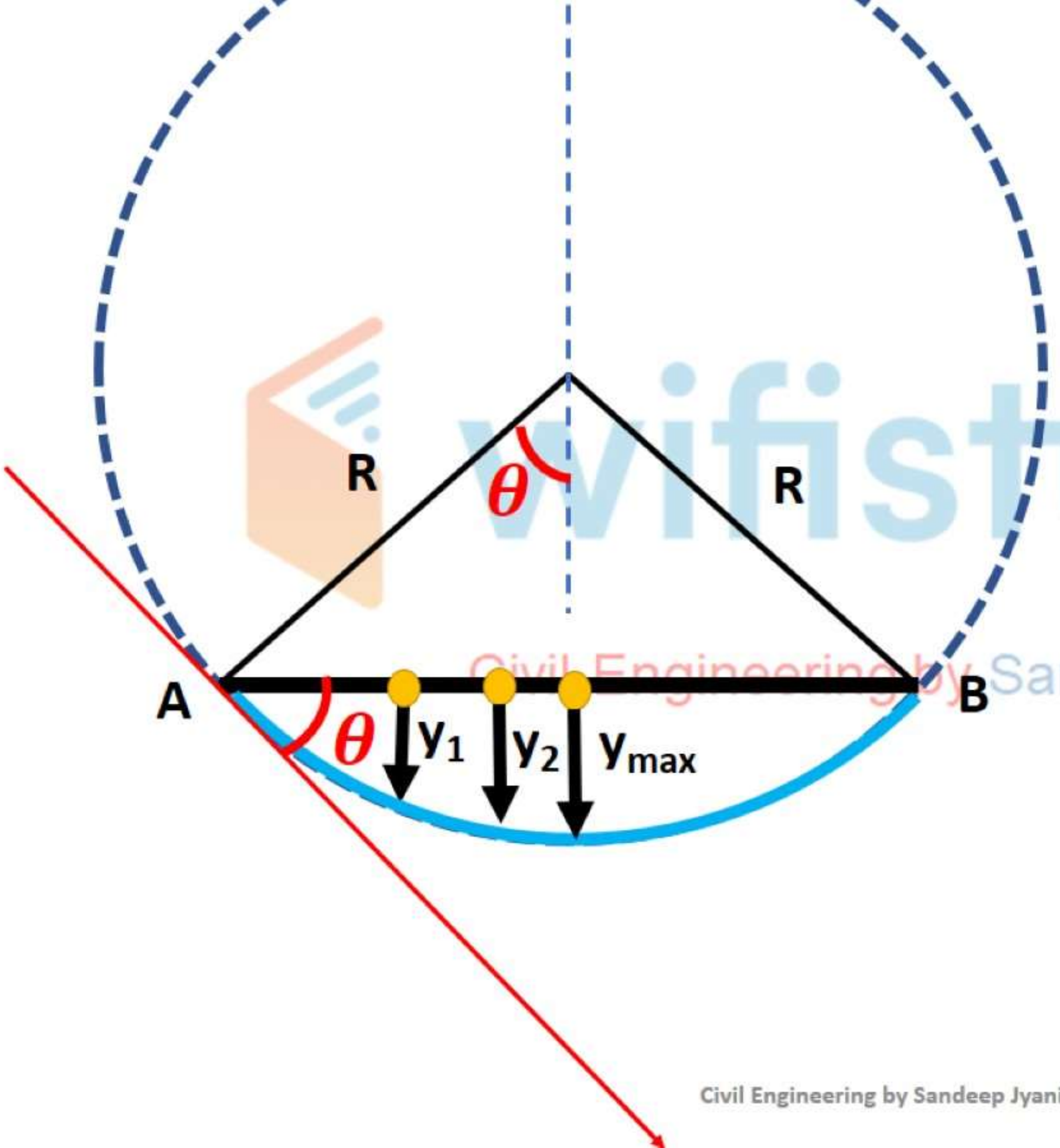
$\tau_{max} = (9/8)\ \tau_{average}$

Slope and Deflection



A beam carrying a load is deflected from its original Position

The maximum slope and Deflection equations are used in the design of beams and in determination of Natural Frequencies under transverse vibrations



Beam Subjected to Uniform Bending Moment

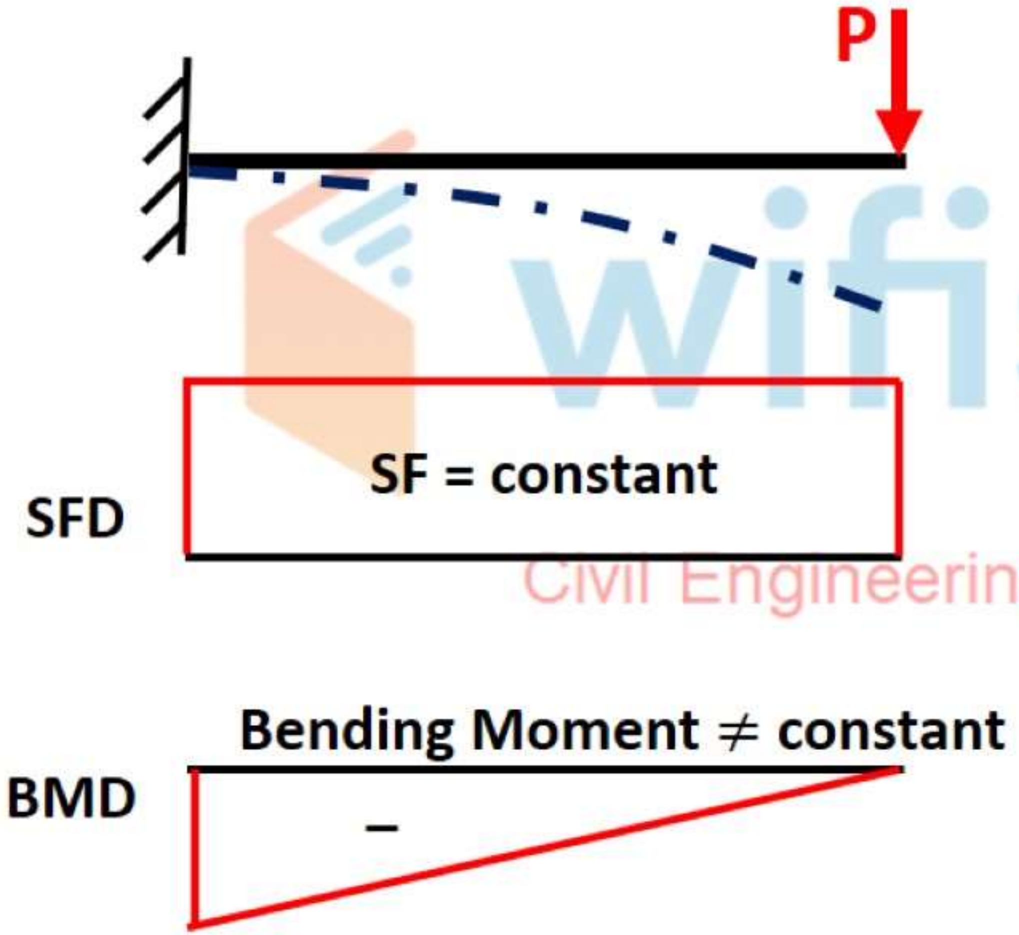
- R= radius of Curvature of deflected beam
- y = deflection of the beam
- θ = slope of the beam at the end A (i.e angle made by tangent A with the beam AB)

Shape of Elastic Curve

- We know that $\frac{M}{I} = \frac{E}{R}$
- Or **Radius of Curvature** $R = \frac{EI}{M}$

1. SF = 0, BM = 0
   - R = infinite or STRAIGHT LINE
2. SF = 0, BM =const,
   - R=constant or CIRCULAR ARC
3. SF $\neq$ const, BM $\neq$ const
   - R $\neq$ const or Non Circular arc

Que 80.

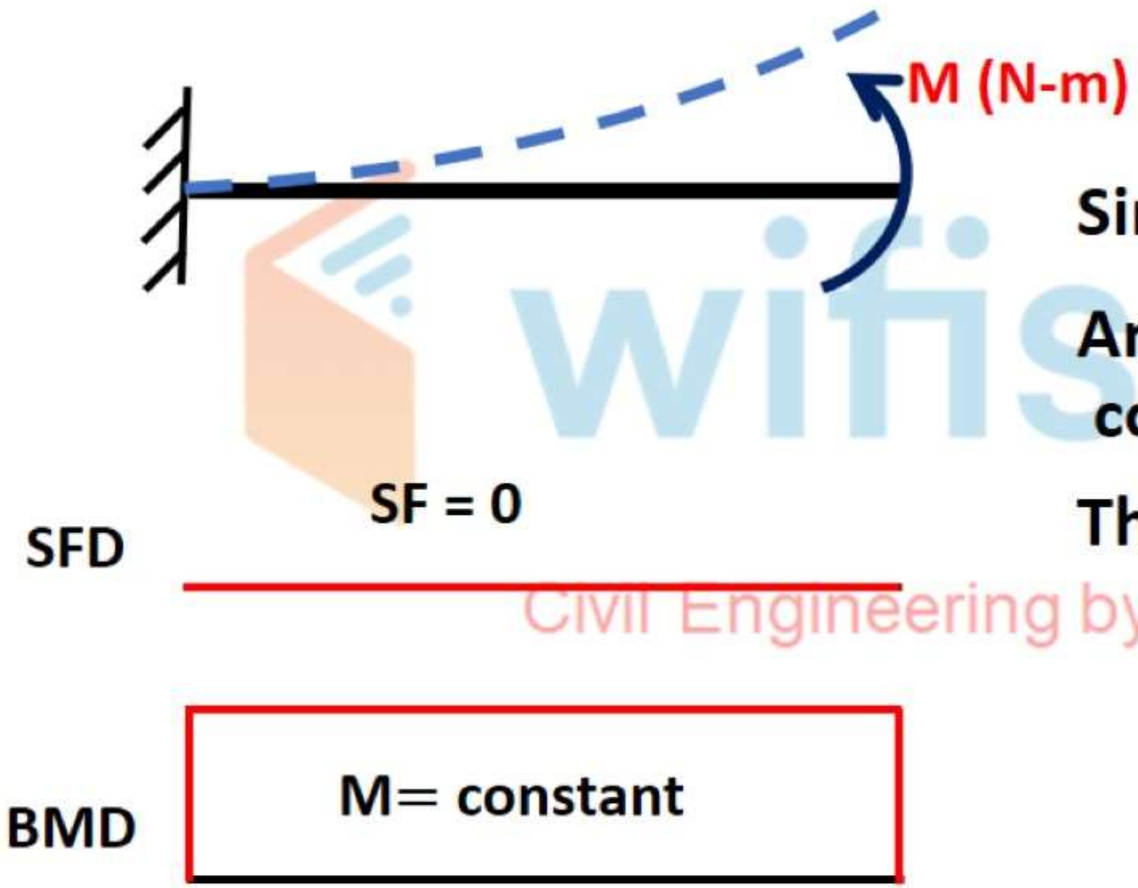


Since $\mathbf{R} = \frac{EI}{M}$,

And BM ≠ constant, so R ≠ constant

Therefore, **Non circular arc**

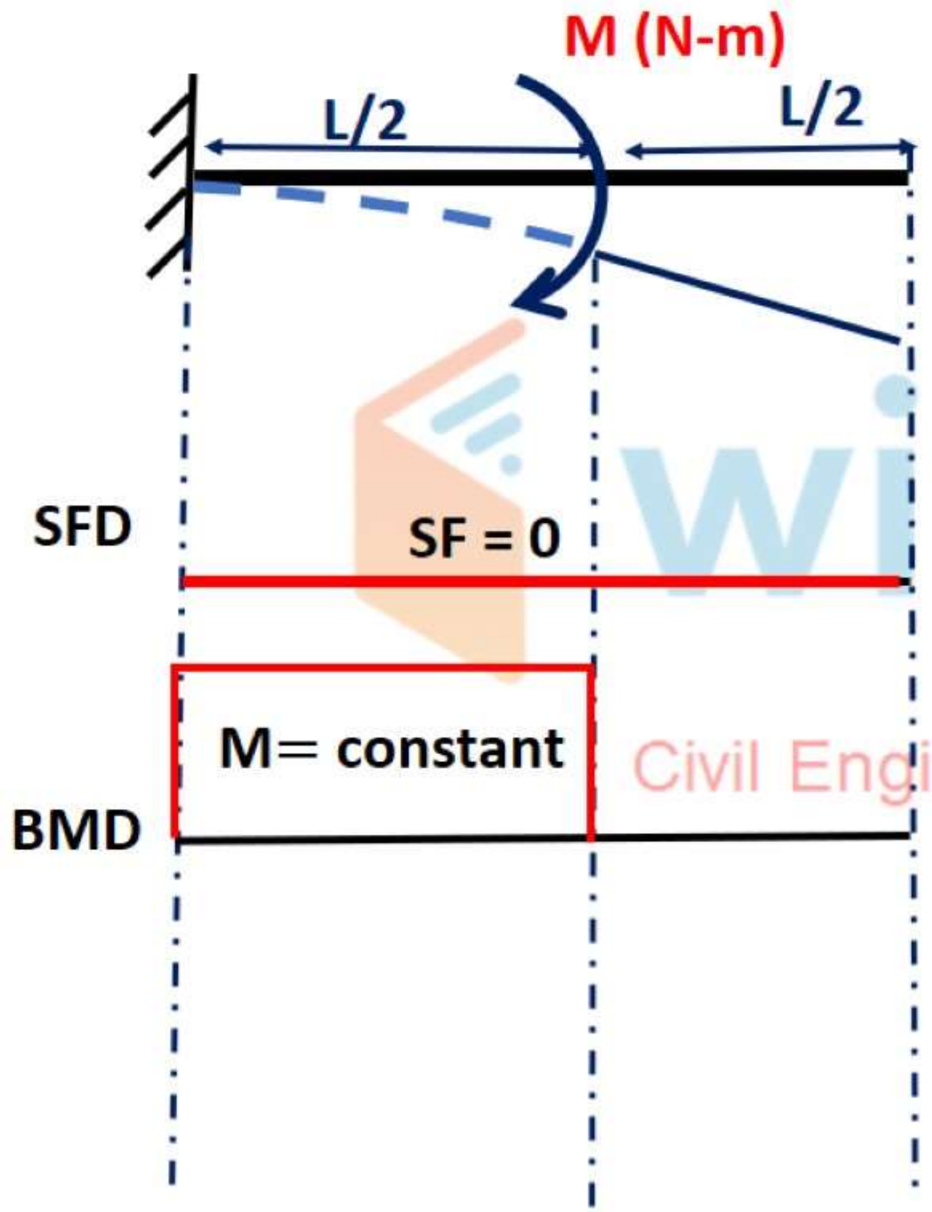
Que 81.



Since $\mathbf{R} = \frac{EI}{M}$,

And BM = constant, so R = constant

Therefore, **Circular arc**



Que 82.

Since $\mathbf{R} = \frac{EI}{M}$,

And BM = constant, so R = constant

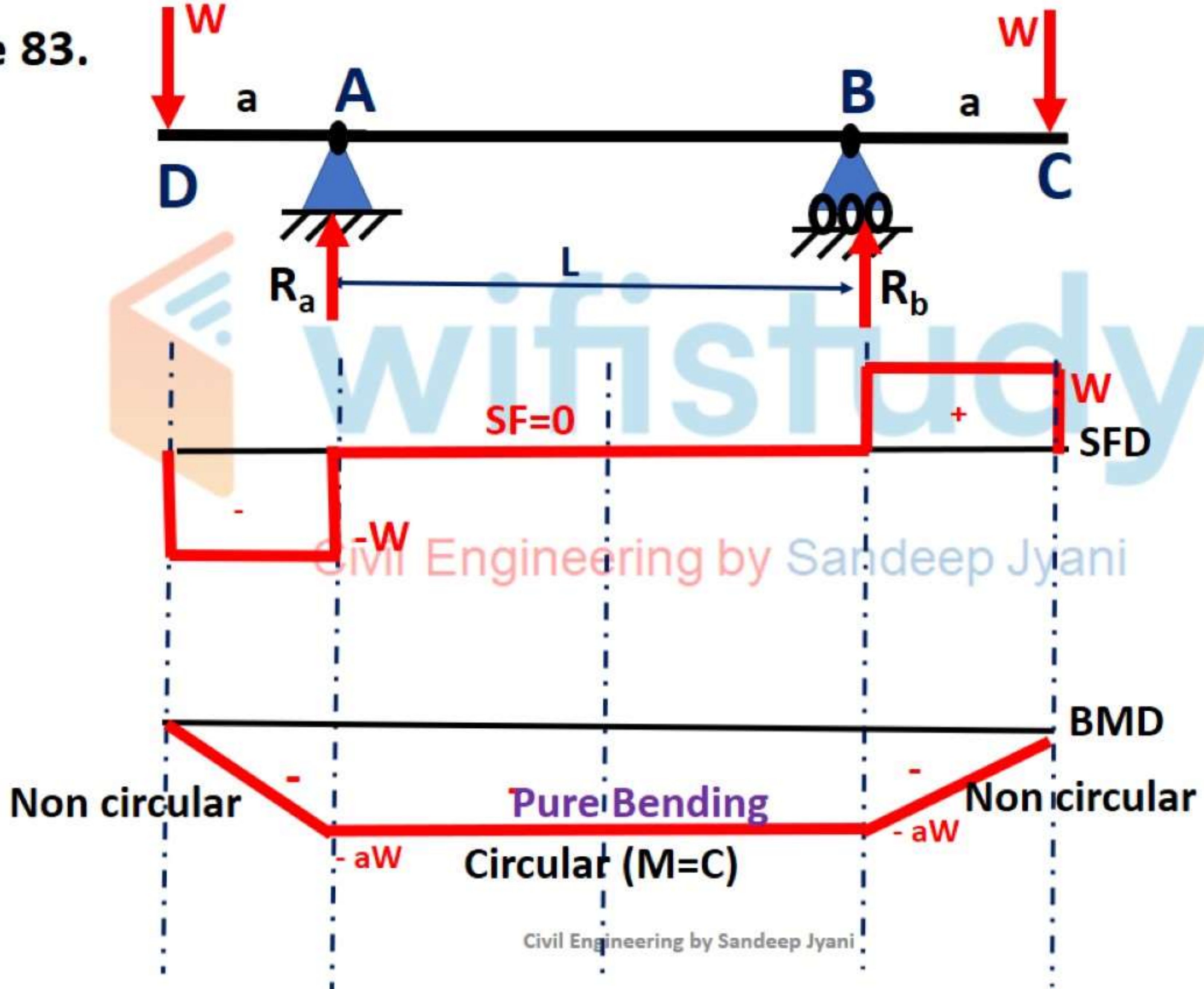
Therefore, Circular arc upto L/2

After L/2,

BM = 0, R = infinite,

so Straight Line

Que 83.

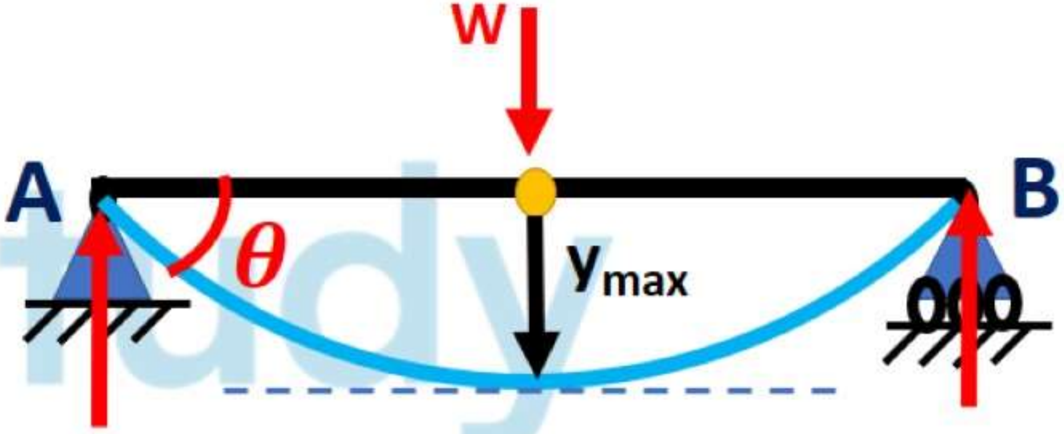

W
W
a
A
B
a
D
C
L
R_a
R_b
SF=0
W
+
SFD
-
-W
BMD
-
-
Non circular
Pure Bending
Non circular
- aW
- aW
Circular (M=C)

Relation between Deflection Slope, BM, SF, Loading

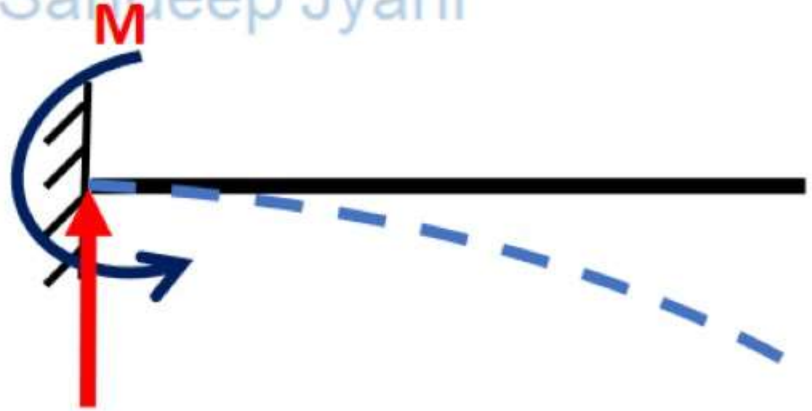
- **If Deflection = y**
- **Slope =** $\frac{dy}{dx}$
- **Curvature** $\frac{1}{R} = \frac{d^2y}{dx^2}$
- **We know that** $\frac{\sigma_b}{y} = \frac{M}{I} = \frac{E}{R}$ **hence** $\frac{M}{EI} = \frac{1}{R}$
- $\therefore \frac{M}{EI} = \frac{d^2y}{dx^2}$ **or Bending Moment** $M = EI\frac{d^2y}{dx^2}$
- **Shear Force** $= EI\frac{d^3y}{dx^3}$
- **Rate of Loading =** $EI\frac{d^4y}{dx^4}$

Boundary Conditions of Simply Supported and Cantilever Beam

1. **When Simply supported beam is subjected to Symmetrical Loading condition:**
   a) At support Deflection y=0, because presence of Reaction and
   b) $\theta = 0$ at the point where y_{max} occurs

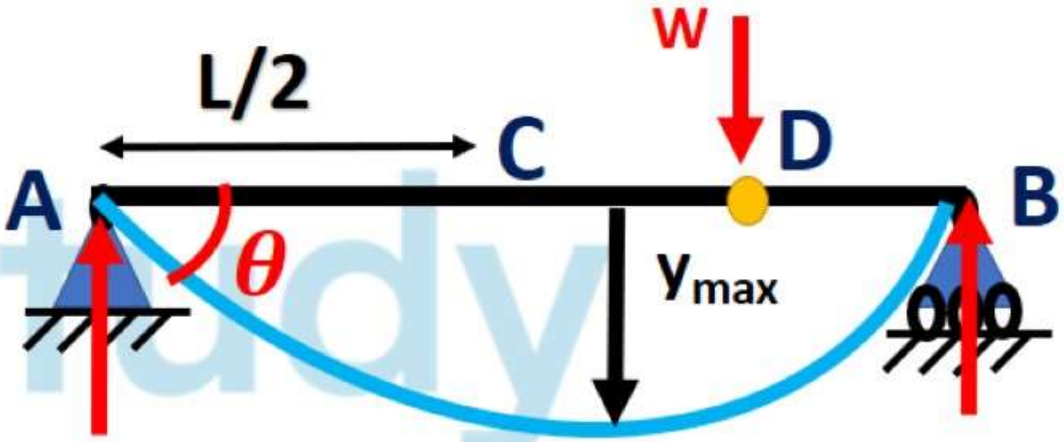


2. **When cantilever is subjected to symmetrical loading,**
   a) Deflection at support y = 0 (due to reaction)
   b) Slope at Support $\theta = 0$ (due to resisting moment)



Boundary Conditions of Simply Supported and Cantilever Beam

3. **In case of Unsymmetrical Loading condition, point of maximum deflection or point of zero slope is present between the point of Application of the load and mid length of the beam:**



Methods of Determination of Slope and Deflection

1. **Double Integration Method**
2. **Area Moment Method**
3. **Strain Energy Method**
4. **Conjugate Beam Method**
5. **Super Position Theorem**
6. **Load Transfer Method**

Methods of Determination of Slope and Deflection

1. Double Integration Method

We know that **Curvature** $\frac{1}{R} = \frac{d^2y}{dx^2}$

Also, $\frac{\sigma_b}{y} = \frac{M}{I} = \frac{E}{R}$ **hence** $\frac{M}{EI} = \frac{1}{R}$

$$M = EI\frac{d^2y}{dx^2} \qquad SF = EI\frac{d^3y}{dx^3}$$

Rate of Loading = -w = $EI\frac{d^4y}{dx^4}$

Methods of Determination of Slope and Deflection

1. Double Integration Method

a) **Step 1:** Calculate M_{xx}

b) **Step 2:** $EI \frac{d^2y}{dx^2} = \mathbf{Mxx}$ then Integrate

c) **Step 3:** $EI \frac{dy}{dx} = \int \mathbf{M_{xx}} + C_1$ (C_1 can be found out using boundary condition of slope) , then again integrate

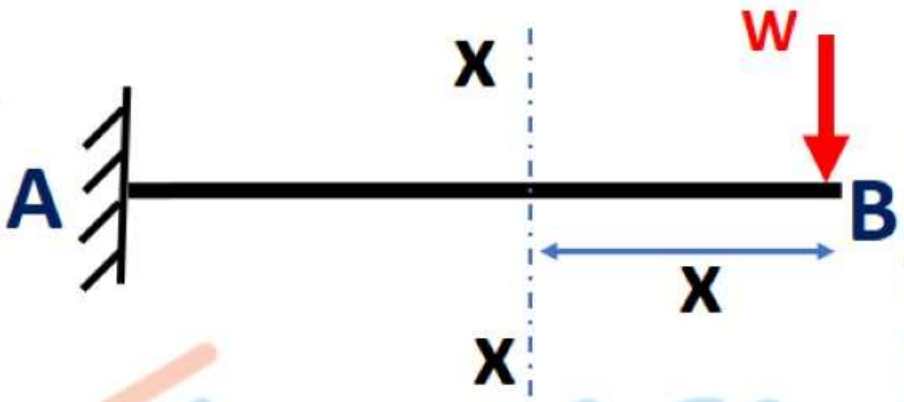
d) **Step 4:** $EI\ dy = \iint \mathbf{M_{xx}} + C_1x + C_2$ (C_2 can be found out using boundary condition of deflection)

Limitations of Double Integration method:

a) Used only for Prismatic beam having E and I constant

b) Equation of bending remains same throughout the length

Que 84



M_{xx} = -wx

$$M_{xx} = EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \int -wx = \int EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{-wx^2}{2} + C_1 = EI\frac{dy}{dx} \quad ....(1)$$

⇒Again, double integration with respect to x,

$$\Rightarrow \frac{-wx^3}{6} + xC_1 + C_2 = EI(y) \; ... (2)$$

To find out C1 and C2, we use boundary conditions

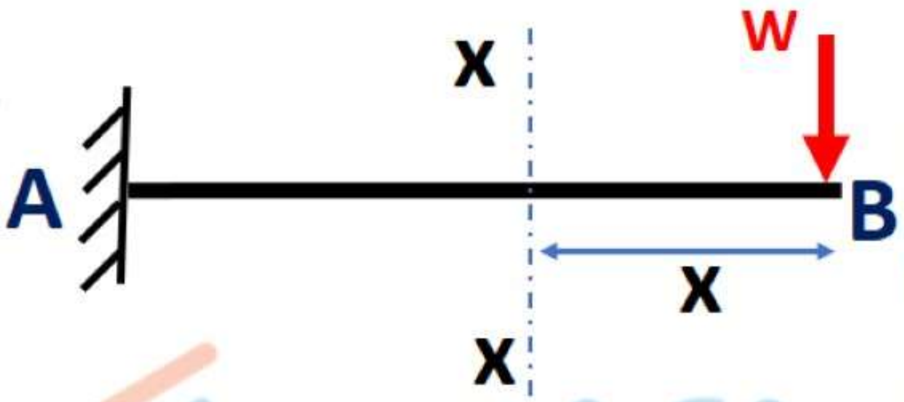
⇒At x=L, $\left(\frac{dy}{dx}\right)_{x=L} = \theta_A = 0$, *putting in eqn (1)*

$$\Rightarrow \frac{-w(L)^2}{2} + C_1 = EI\,(0)$$

$$\Rightarrow C_1 = \frac{w(L)^2}{2}$$

⇒Eqn of Slope $\frac{-wx^2}{2} + \frac{w(L)^2}{2} = EI\frac{dy}{dx}$

Que 84



$M_{xx} = -wx$

$$M_{xx} = EI\,\frac{d^2y}{dx^2}$$

$$\Rightarrow \int -wx = \int EI\,\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{-wx^2}{2} + C_1 = EI\,\frac{dy}{dx} \quad ....(1)$$

$\Rightarrow$Again, double integration with respect to x,

$$\Rightarrow \frac{-wx^3}{6} + xC_1 + C_2 = EI(y) \;...\;(2)$$

From 2[nd] boundary condition,

$\Rightarrow$At x=L, $y = 0$, *putting in eqn (2)*

$$\Rightarrow \frac{-w(L)^3}{6} + (L)\left(\frac{w(L)^2}{2}\right) + C_2 = EI(0)$$

$$\Rightarrow C_2 = \frac{-w(L)^3}{3}$$

$\Rightarrow$**Equation of Deflection**

$$\Rightarrow \frac{-wx^3}{6} + x\left(\frac{w(L)^2}{2}\right) - \frac{w(L)^3}{3} = EIy$$

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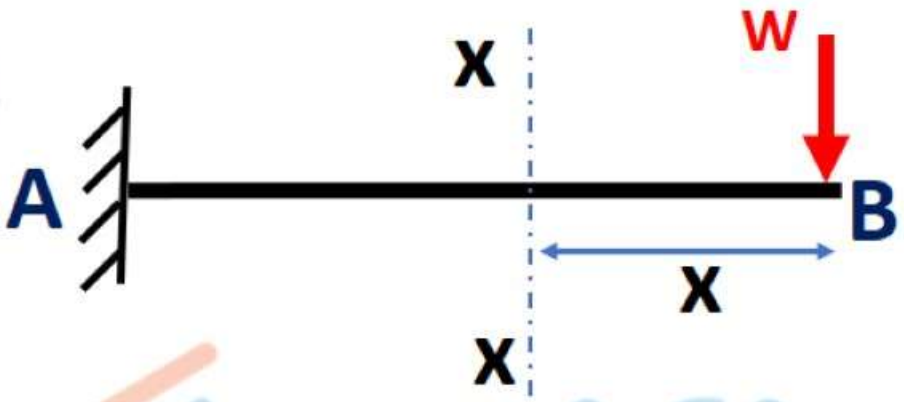

Equation of Slope

$$\frac{-wx^2}{2} + \frac{w(L)^2}{2} = EI\frac{dy}{dx}$$

Equation of Deflection

$$\Rightarrow \frac{-wx^3}{6} + x\left(\frac{w(L)^2}{2}\right) - \frac{w(L)^3}{3} = EIy$$

$\Rightarrow$ For $\theta_B = \left(\frac{dy}{dx}\right)_{x=0} = ?$

$\Rightarrow$ Put x=0, *in slope eqn*

$$\Rightarrow \frac{-w(0)^2}{2} + \frac{w(L)^2}{2} = EI\frac{dy}{dx}$$

$$\Rightarrow \frac{-w(0)^2}{2} + \frac{w(L)^2}{2} = EI\theta_B$$

$$\Rightarrow \theta_B = \frac{wL^2}{2EI}$$ (due to point load)

$\Rightarrow$ For $y_B = ?$

$\Rightarrow$ Put x=0, *in deflection eqn*

$$\Rightarrow \frac{-w(0)^3}{6} + (0)\left(\frac{w(L)^2}{2}\right) - \frac{w(L)^3}{3} = EIy$$

$$\Rightarrow -\frac{w(L)^3}{3} = EIy$$

$$\Rightarrow y = -\frac{w(L)^3}{3EI}$$ (due to point load)

Methods of Determination of Slope and Deflection

2. Area Moment Method

Theorem 1: **Difference of slope of any two points of a beam is equal to** $\frac{1}{EI}$ ***of the area of BMD between those points*** **i.e.**

$$\theta_B - \theta_A = \frac{1}{EI}\ (\textit{area of BMD between}\ \text{A and B})$$

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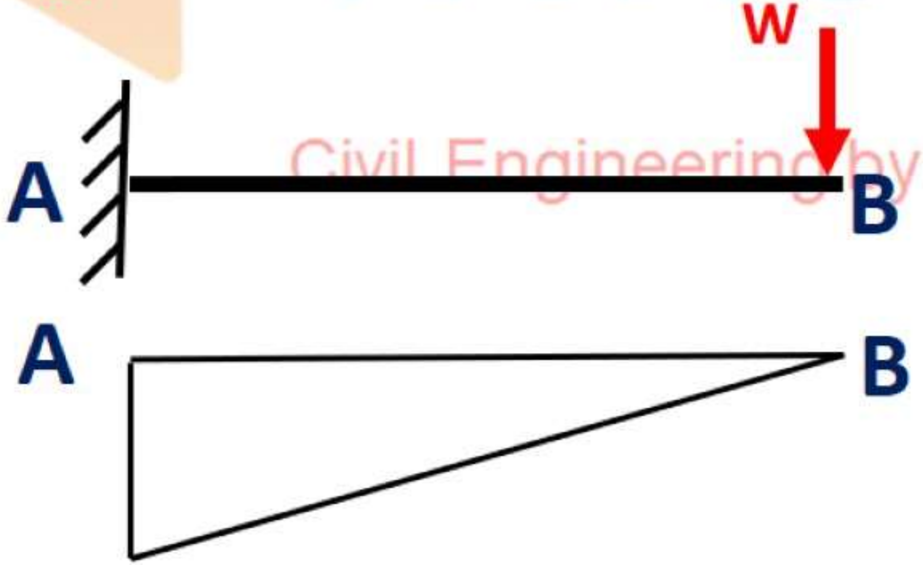


$$\theta_B - \theta_A = \frac{1}{EI}\ (\frac{-1}{2} \times W \times L)$$

Since $\theta_A = 0$

$$\theta_B = \frac{WL^2}{2EI}$$

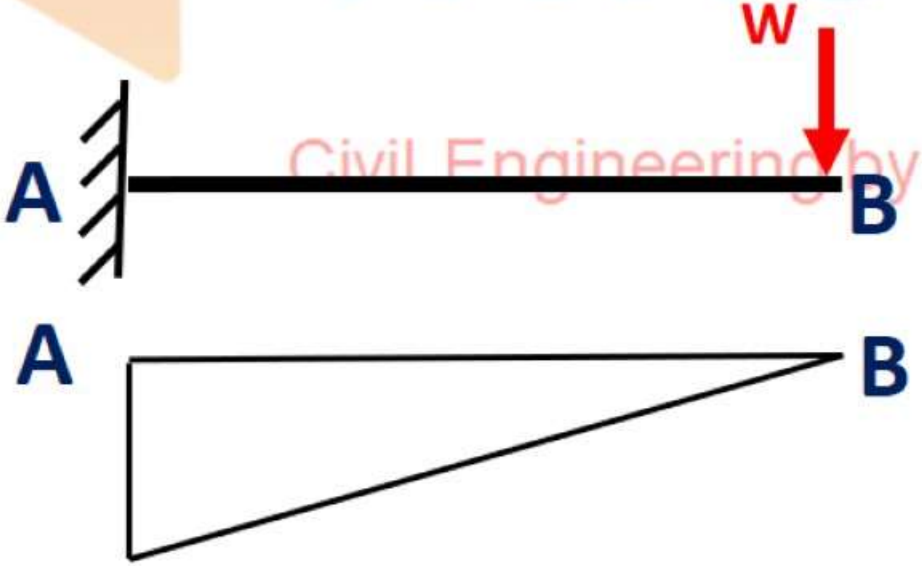
Methods of Determination of Slope and Deflection

2. Area Moment Method

Theorem 2: **Difference of DEFLECTION of any two points of a beam is equal to $\frac{1}{EI}$ *of the MOMENT of area of BMD between those points* i.e.**

$$y_B - y_A = \frac{1}{EI}\ (\textit{moment of area of BMD between}\ \text{A and B})$$

Que 86



$$y_B - y_A = \frac{1}{EI}\ (A\bar{x})$$

Since $yA = 0$

$$y_B = \frac{1}{EI}\left(\left(\frac{-1}{2} \times W \times L\right) \times \frac{2L}{3}\right)$$

$$y_B = \frac{-WL^3}{3EI}$$

Methods of Determination of Slope and Deflection

2. Area Moment Method

- ✓ **Always select the two points such that one point should be of Non Zero slope (where slope is to be determined). This point is called Origin Point**
- ✓ **Another point should be point of zero slope, such type of point is called Reference Point**
- ✓ **Always measure $\bar{x}$ from Point of non zero slope or the origin point**

3. Strain Energy Method

$$U = \frac{M^2 L}{2EI} \quad \text{Or} \quad U = \int \frac{M^2 dx}{2EI}$$

To find out Slope and Deflection using the Strain Energy Method, we use following steps:

a) According to Strain Energy Method, deflection at any section in direction of load is equal to derivatives of total strain energy w.r.to that load

$$y_B = \frac{\delta U}{\delta P} = \int \frac{2M \times \delta M \, dx}{2EI \times \delta P}$$ (Since M is function of P i.e. M= -Px)

$$y_B = \int \frac{M}{EI} \frac{\delta M}{\delta P} dx$$

Methods of Determination of Slope and Deflection

3. Strain Energy Method

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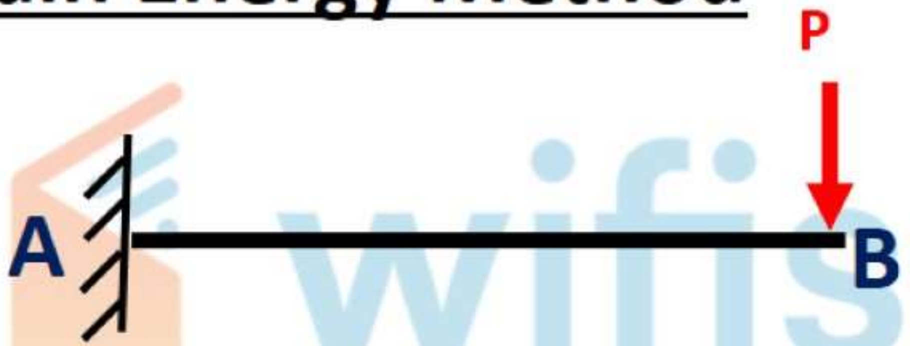


M= -Px

$$\frac{\delta M}{\delta P} = -x \quad \ldots.(1)$$

$$y_B = \int \frac{M}{EI}\frac{\delta M}{\delta P}dx$$

$$y_B = \int \frac{(-Px)}{EI}(-x)dx$$

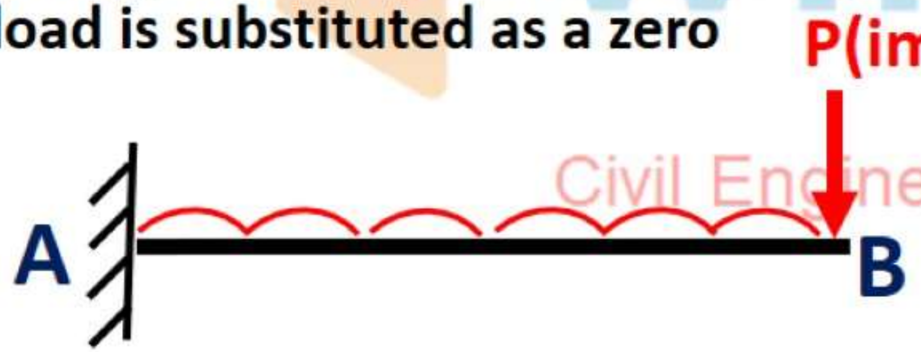
$$y_B = \int_0^L \frac{(-Px^2)}{EI}dx \qquad y_B = \frac{-Px^3}{3EI}$$

Methods of Determination of Slope and Deflection

3. Strain Energy Method

If deflection is required at a section where there is no point load, then we have to apply an imaginary load in the direction of Deflection.

In this case deflection will be equal to partial derivatives of total strain energy with respect to imaginary load and in the final solution imaginary load is substituted as a zero



$$M = -\left(Px + \frac{wx^2}{2}\right)$$

$$\frac{\delta M}{\delta P} = -x$$

$$y_B = \frac{\delta U}{\delta P}$$

$$y_B = \int \frac{-\left(Px + \frac{wx^2}{2}\right)}{EI}(-x)dx$$

$$y_B = \frac{wL^4}{8EI}$$

Methods of Determination of Slope and Deflection

3. Strain Energy Method

According to Strain Energy Method, the slope at any section is Partial Derivatives of Total Strain Energy with respect to concentrated moment at that section.

If the slope is required at a section, there is no concentrated moment, then we have to apply an imaginary moment in the direction of load

$$\theta_B = \frac{\delta U}{\delta M}$$

$$y_B = \int \frac{M}{EI}\frac{\delta M}{\delta M}\,dx$$

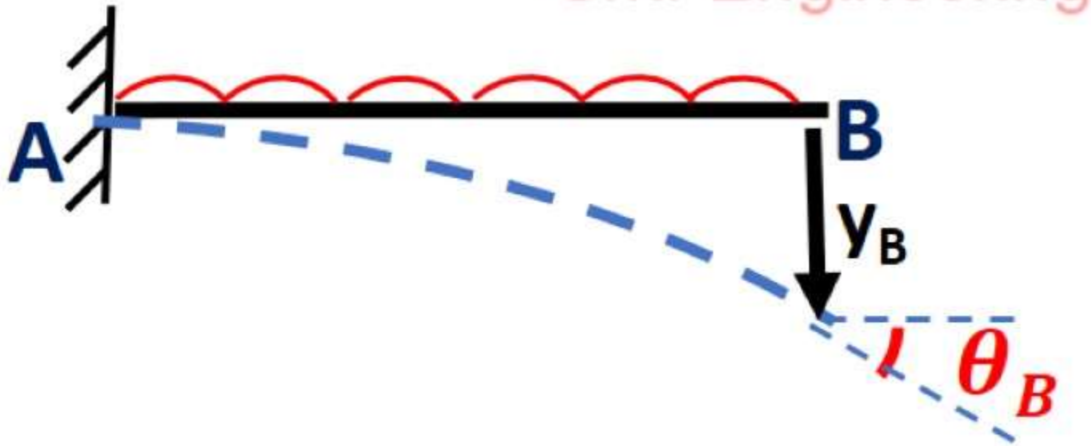
4. Super Position Theorem

Slope | **Deflection**

1.

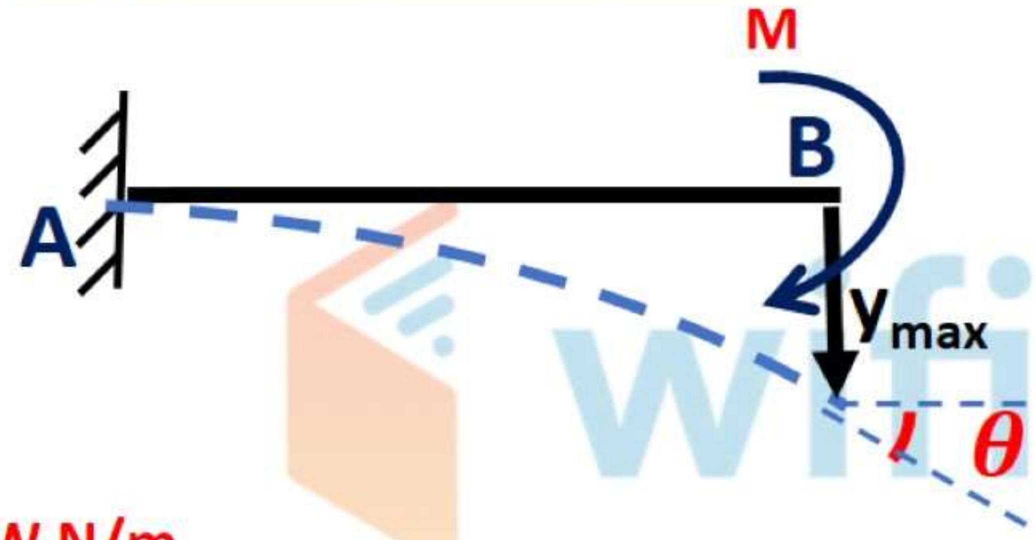
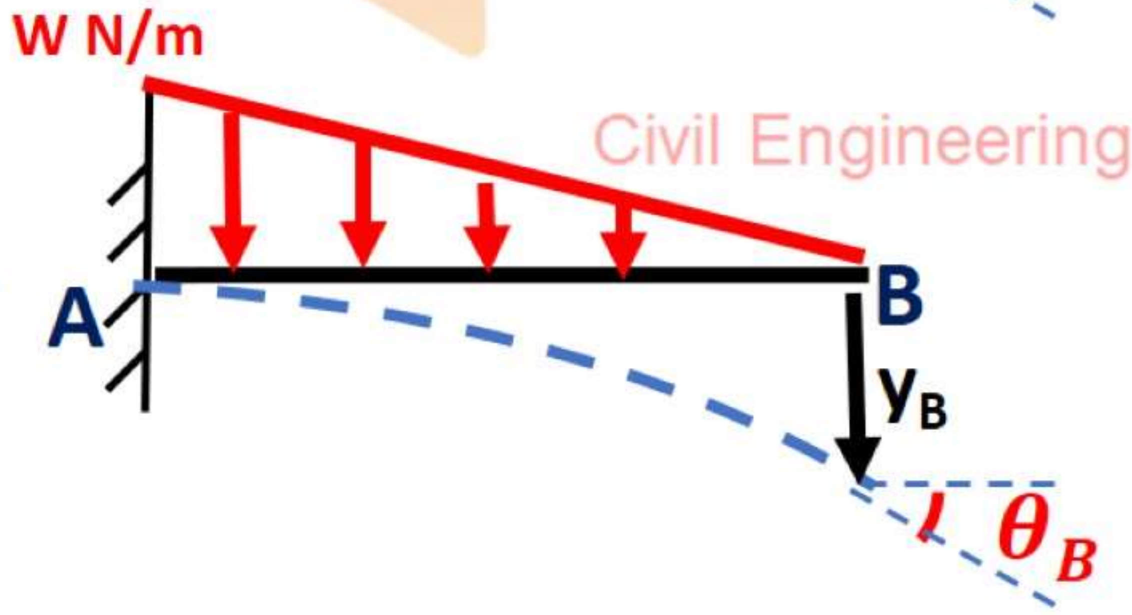
$$\theta_{max} = \frac{WL^2}{2EI} \qquad y_{max} = \frac{WL^3}{3EI}$$

2.



$$\theta_{max} = \frac{WL^3}{6EI} \qquad y_{max} = \frac{WL^4}{8EI}$$

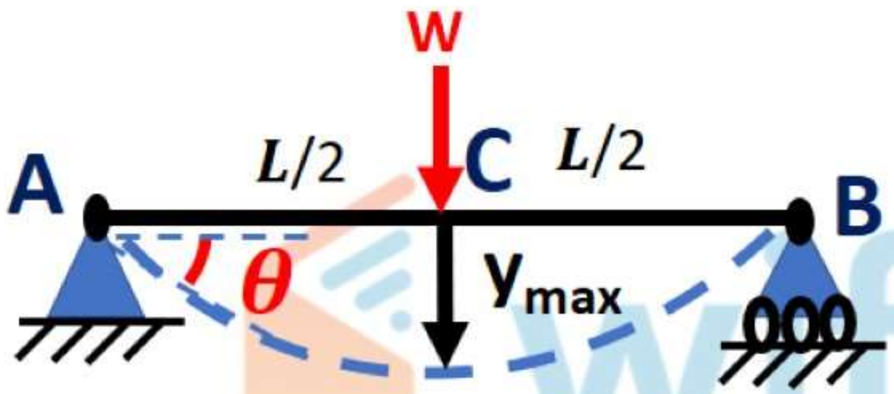
4. Super Position Theorem

	Slope	Deflection
3. 	$\theta_{max} = \frac{ML}{EI}$	$y_{max} = \frac{ML^2}{2EI}$
4. 	$\theta_{max} = \frac{WL^3}{24EI}$	$y_{max} = \frac{WL^4}{30EI}$

4. Super Position Theorem

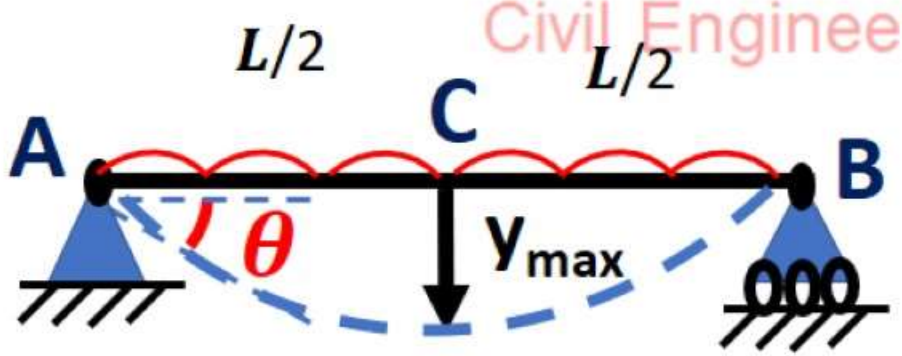
Slope | **Deflection**

5.



$$\theta_A = \theta_C = \frac{WL^2}{16EI} \qquad y_C = \frac{WL^3}{48EI}$$

6.



$$\theta_A = \theta_C = \frac{WL^3}{24EI} \qquad y_{max} = \frac{5WL^4}{384EI}$$

Objective Questions

85. For a given shear force across a symmetrical '*I*' section, the intensity of shear stress is maximum at the

(a) At the junction of the flange and the web, but on the web

(b) At the junction of the flange and the web, but on the flange

(c) Extreme fibre

(d) Centroid of the section

Objective Questions

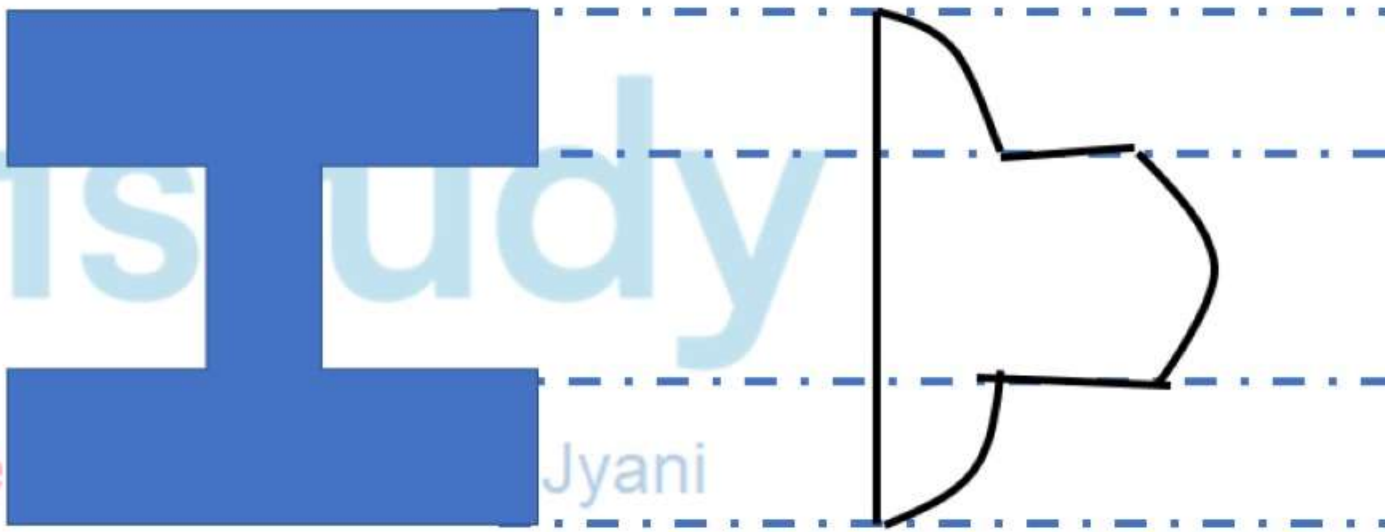
85. For a given shear force across a symmetrical '*I*' section, the intensity of shear stress is maximum at the

(a) At the junction of the flange and the web, but on the web

(b) At the junction of the flange and the web, but on the flange

(c) Extreme fibre

(d) <u>Centroid of the section</u>



Objective Questions

86. For a given stress, the ratio of moment of resistance of a beam of square cross-section when placed with its two sides horizontal to the moment of resistance with its its one of the diagonal horizontal is given by

(a) $\frac{1}{\sqrt{2}}$

(b) $\sqrt{2}$

(c) $\frac{1}{2}$

(d) 1

Objective Questions

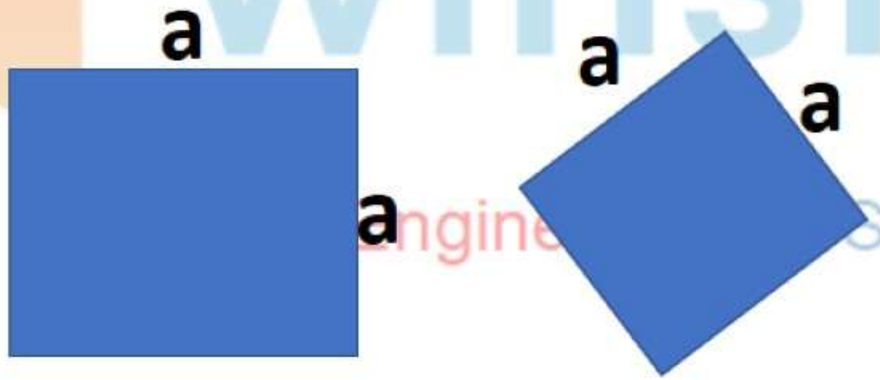
86. **For a given stress, the ratio of moment of resistance of a beam of square cross-section when placed with its two sides horizontal to the moment of resistance with its its one of the diagonal horizontal is given by**

(a) $\frac{1}{\sqrt{2}}$

(b) $\sqrt{2}$

(c) $\frac{1}{2}$

(d) 1



$$I_1 = \frac{a^4}{12} \qquad I_2 = \frac{a^4}{12}$$

$$\frac{\sigma_b}{y} = \frac{M}{I}$$

$$\frac{\sigma_b}{y_1} = \frac{M_1}{I} \qquad \frac{\sigma_b}{y_2} = \frac{M_2}{I}$$

$$\frac{M_1}{M_2} = \frac{y_2}{y_1}$$

$$\frac{M_1}{M_2} = \frac{\sqrt{2}a/2}{a/2}$$

$$\frac{M_1}{M_2} = \sqrt{2}$$

Objective Questions

87. Two beams, one of circular cross-section and the other of square cross section, have equal areas of cross section. If subjected to bending, then

(a) Both sections are equally economical

(b) Both sections are equally stiff

(c) Circular cross section is more economical square cross section is more economical

(d) Square cross section is more economical

Objective Questions

87. Two beams, one of circular cross-section and the other of square cross section, have equal areas of cross section. If subjected to bending, then

(a) Both sections are equally economical

(b) Both sections are equally stiff

(c) Circular cross section is more economical square cross section is more economical

(d) Square cross section is more economical

$$area(square) = d^2$$

$$area(circle) = \frac{\pi D^2}{4}$$

$$area(circle) = area(square)$$

$$d^2 = \frac{\pi D^2}{4}$$

$$z(square) = \frac{\frac{d^4}{12}}{\frac{d}{2}} = \frac{d^3}{6} = \frac{(0.886D)^3}{6} = 0.116\ D^3$$

$$z(circle) = \frac{\frac{\pi D^4}{64}}{\frac{D}{2}} = \frac{\pi D^3}{32} = 0.098\ D^3$$

Objective Questions

88. In a beam at a section carrying a shear force F, the shear stress is maximum at

(a) Bottommost fibre

(b) Mid depth

(c) Neutral surface

(d) Topmost fibre

Objective Questions

88. In a beam at a section carrying a shear force F, the shear stress is maximum at

(a) Bottommost fibre

(b) Mid depth

(c) <u>Neutral surface</u>

(d) Topmost fibre

Objective Questions

89. The ratio of maximum shear stress to average shear stress of a circular beam is

$(a)\frac{2}{3}$

$(b)\frac{3}{2}$

$(c)\frac{3}{4}$

$(d)\frac{4}{3}$

Objective Questions

89. The ratio of maximum shear stress to average shear stress of a circular beam is

(a) $\frac{2}{3}$

(b) $\frac{3}{2}$

(c) $\frac{3}{4}$

(d) $\frac{4}{3}$

Objective Questions

90. In a section undergoing bending, the neutral surface is subjected to

(a) Compression strain

(b) tensile strain

(c) zero strain

(d) None of the above

Objective Questions

90. In a section undergoing bending, the neutral surface is subjected to

(a) Compression strain

(b) tensile strain

(c) zero strain

(d) None of the above

Objective Questions

91. A structure which offers negligible or zero resistance on bending at any point is known as

(a) Beam

(b) Girder

(c) Lintel

(d) Cable

Objective Questions

91. A structure which offers negligible or zero resistance on bending at any point is known as

(a) Beam

(b) Girder

(c) Lintel

(d) Cable

Objective Questions

92. The assumption in the theory of bending of beams is ______.

(a) material is homogeneous

(b) material is isotropic

(c) Young's modulus is same in tension as well as in compression

(d) All option are correct

Objective Questions

92. The assumption in the theory of bending of beams is ______.

(a) material is homogeneous

(b) material is isotropic

(c) Young's modulus is same in tension as well as in compression

(d) All option are correct

Objective Questions

93. **The ratio of moment of inertia about the neutral axis to the distance of the most distant point of the section from the neutral axis is called**

(a) Polar module

(b) Section modulus

(c) Modulus of rupture

(d) Flexural rigidity

Objective Questions

93. The ratio of moment of inertia about the neutral axis to the distance of the most distant point of the section from the neutral axis is called

(a) Polar module

(b) Section modulus

(c) Modulus of rupture

(d) Flexural rigidity

Objective Questions

94. The maximum bending stress in an I-beam occurs at the ______.

(a) Neutral axis

(b) Outermost fiber

(c) Joint of wedge and flange

(d) Section where shear stress is maximum

Objective Questions

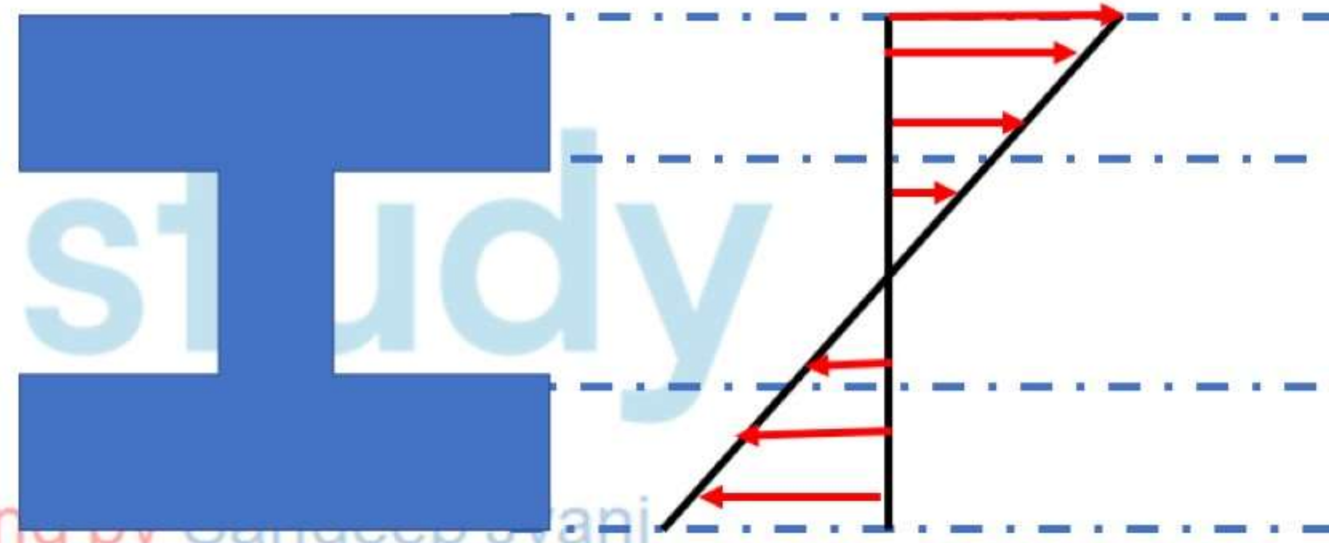
95. The maximum bending stress in an I-beam occurs at the ______.

(a) Neutral axis

(b) Outermost fiber

(c) Joint of wedge and flange

(d) Section where shear stress is maximum



$$\frac{\sigma_b}{y} = \frac{M}{I} \qquad \sigma_b = \frac{M}{I} y$$

Objective Questions

96. Two beam of equal cross-sectional area are subject to equal bending moment. If one beam has square cross-section and the other has circular section, then_____.

(a) both beams will be equally strong

(b) circular section beam will be stronger

(c) square section beam will be stronger

(d) the strength of the beam will depend on the nature of aiding

Objective Questions

96. Two beam of equal cross-sectional area are subject to equal bending moment. If one beam has square cross-section and the other has circular section, then_____.

(a) both beams will be equally strong

(b) circular section beam will be stronger

(c) square section beam will be stronger

(d) the strength of the beam will depend on the nature of aiding

Objective Questions

97. Most efficient and economical section used as a beam is

(a) I section

(b) Circular section

(c) Angles

(d) H section

Objective Questions

97. Most efficient and economical section used as a beam is

(a) I section

(b) Circular section

(c) Angles

(d) H section

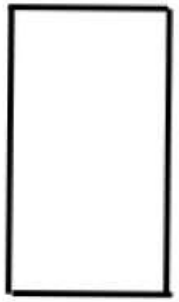
I section has more width at the flange than web. Also, bending stress is maximum at outer fiber so I section can resist higher value of Benidng stress as compared to other sections

Objective Questions

98. Shear stress distribution of a beam of rectangular cross-section. Subjected to transverse loading will be

(i) 

(ii) 

(iii) 

(iv) 

(a) (ii)

(b) (i)

(c) (iii)

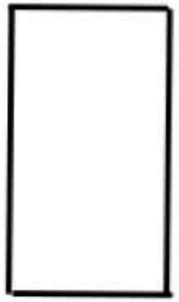
(d) (iv)

Objective Questions

98. Shear stress distribution of a beam of rectangular cross-section. Subjected to transverse loading will be

(i)


(ii)


(iii)


(iv)


(a) (ii)

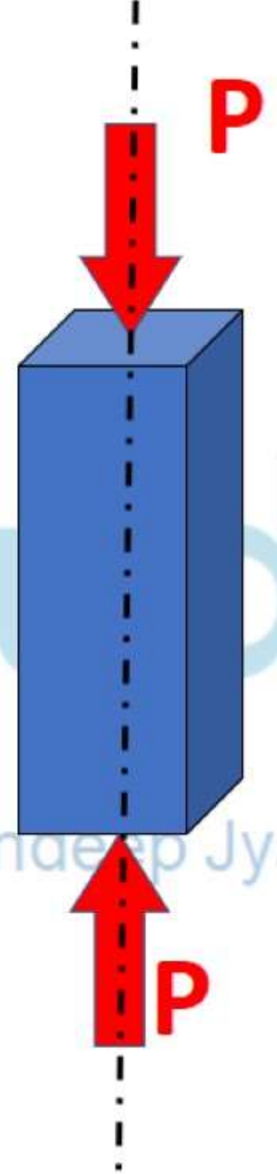
(b) (i)

(c) (iii)

(d) (iv)

Column

Column or Strut is a structural member subjected to Axial Compressive forces

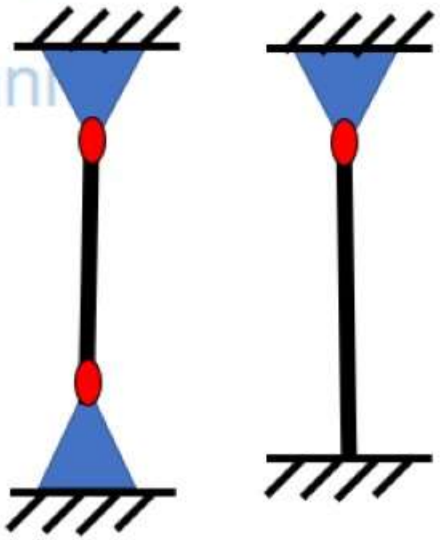


Column vs Strut

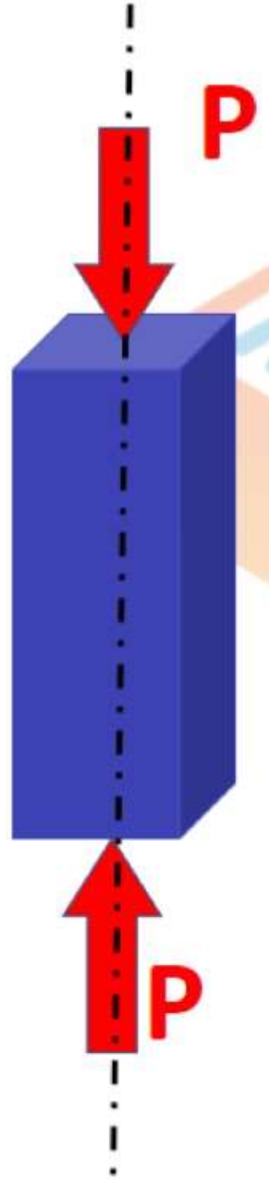
If the member is vertical and both ends are fixed rigidly while subjected to axial compressive load, it is known as **COLUMN** (exp. Vertical pillar between roof and floor)

If the member of Structure is not vertical or one or both ends are hinged or pin jointed, the bar is known as Strut

(Connecting rods, piston rods, etc)

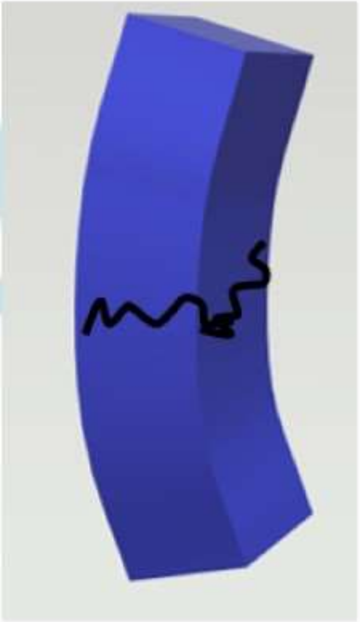


Types of Failure of Column



Crushing

Buckling



Crushing and Buckling

Types of Failure of Column

1. **Crushing Failure:**
   - **Normally it occurs in short column due to direct compressive stresses**
2. **Buckling Failure**
   - **It occurs in Long column due to buckling stresses**
3. **Combined Failure**
   - **It has been observed in case of intermediate column due to combined compressive and buckling stresses**

Euler's Theory of Buckling Failure

Assumptions

1. The material should be homogenous and isotropic
2. The slender should be long prismatic
3. The material should obey Hooke's law
4. Plane section should remain plane before and after buckling
5. Column always fail in Buckling i.e long column is considered

Applying double integration Method,

$$M_{xx} = EI\,\frac{d^2y}{dx^2}$$

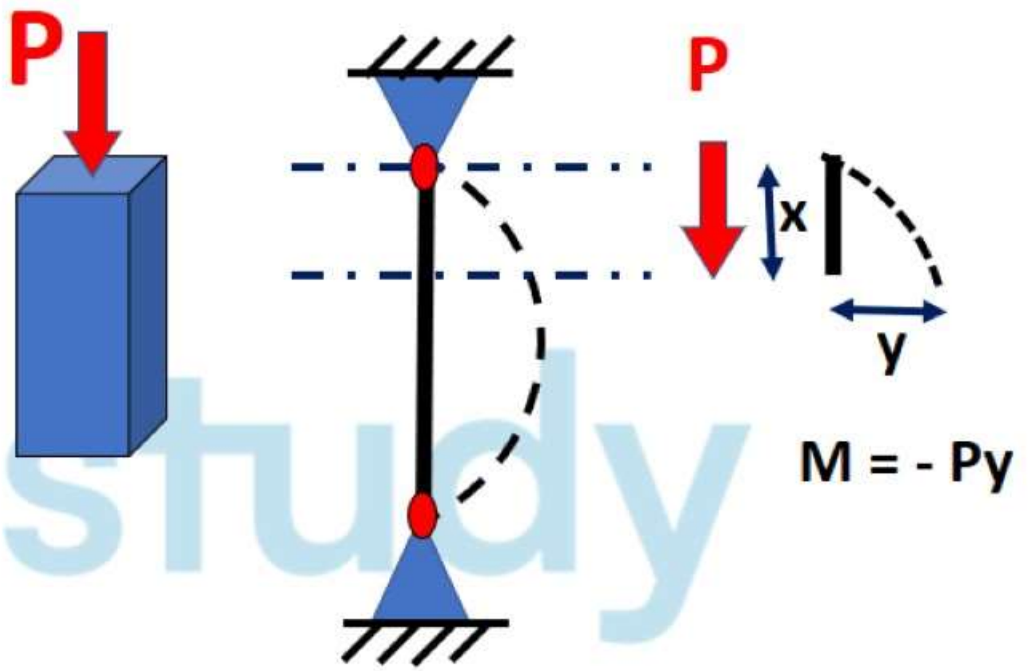
$$-Py = EI\,\frac{d^2y}{dx^2}$$

$$EI\,\frac{d^2y}{dx^2} + Py = 0$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0$$

It is second order differential equation, solution of this equation is

$$y = C_1 \cos\sqrt{\frac{P}{EI}}\,x + C_2 \sin\sqrt{\frac{P}{EI}}\,x$$



C1 and C2 are unknown and are determined by using Boundary conditions

When x=0, y=0

$$0 = C_1 \cos\sqrt{\frac{P}{EI}}\,(0) + C_2 \sin\sqrt{\frac{P}{EI}}\,(0)$$

$$0 = C_1 \cos\sqrt{\frac{P}{EI}}\,(0) + C_2 \sin\sqrt{\frac{P}{EI}}\,(0)$$

C1 = 0

If C1 is 0, C2 can not be zero else the equation will be y=0 which is not possible

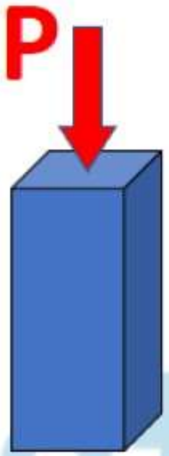
When x=L, y=0

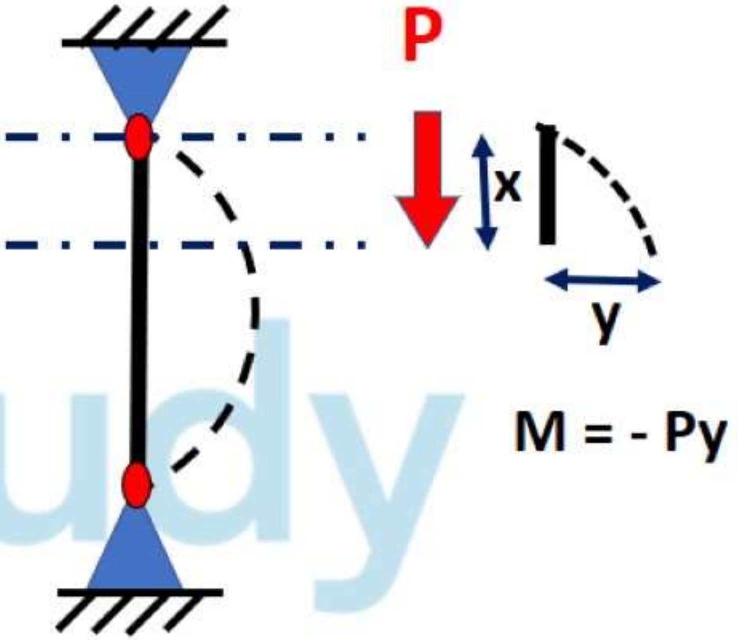
$$\Rightarrow C_2 \sin\sqrt{\frac{P}{EI}}\,(L) = 0$$

$$\Rightarrow \sin\sqrt{\frac{P}{EI}}\,(L) = \sin(n\pi)$$

$$\Rightarrow \sqrt{\frac{P}{EI}}\,(L) = (n\pi)$$

$$\Rightarrow \frac{P}{EI} L^2 = (n^2\pi^2)$$





$$\Rightarrow P = \frac{\pi^2 n^2 EI}{L^2}$$

$$\Rightarrow P_1 = \frac{\pi^2 EI}{L^2}$$

$$\Rightarrow P_2 = \frac{4\pi^2 EI}{L^2}$$

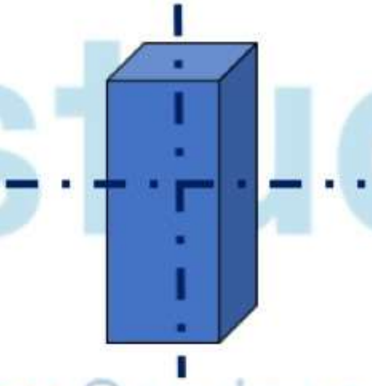
Buckling Load or Critical Load or Euler Load or Crippling Load or Failure load or Bending load :

Minimum load at which Crippling starts

$$P_{cl} = \frac{\pi^2 EI}{L^2}$$

$$(P_{cl})_{xx} = \frac{\pi^2 EI_{xx}}{L^2}$$

$$(P_{cl})_{yy} = \frac{\pi^2 EI_{yy}}{L^2}$$



Case1: $I_{xx} > I_{yy}$ $\quad (P_{cl})_{xx} > (Pcl)_{yy}$

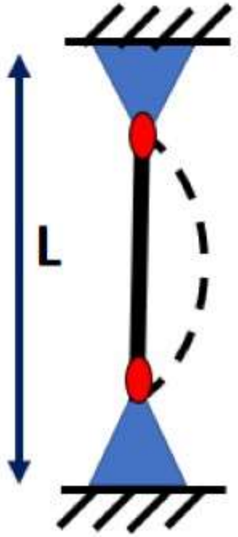
Case2: $I_{xx} < I_{yy}$ $\quad (P_{cl})_{xx} < (Pcl)_{yy}$

General Formula for Buckling Load

$$P_{cl} = \frac{\pi^2 EI_{min}}{L^2_{effective}}$$

Effective Length as Per End Conditions

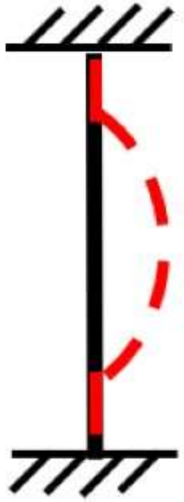
$$P_{cl} = \frac{\pi^2 EI_{min}}{L^2_{effective}}$$



1. Both End Hinged

$$l_{eff} = lac_{tual} = L$$

$$P_{cl} = \frac{\pi^2 EI}{L^2}$$



2. Both End Fixed

$$l_{eff} = L/2$$

$$P_{cl} = \frac{4\pi^2 EI}{L^2}$$

3. One End Fixed and other is free

$$l_{eff} = 2L$$

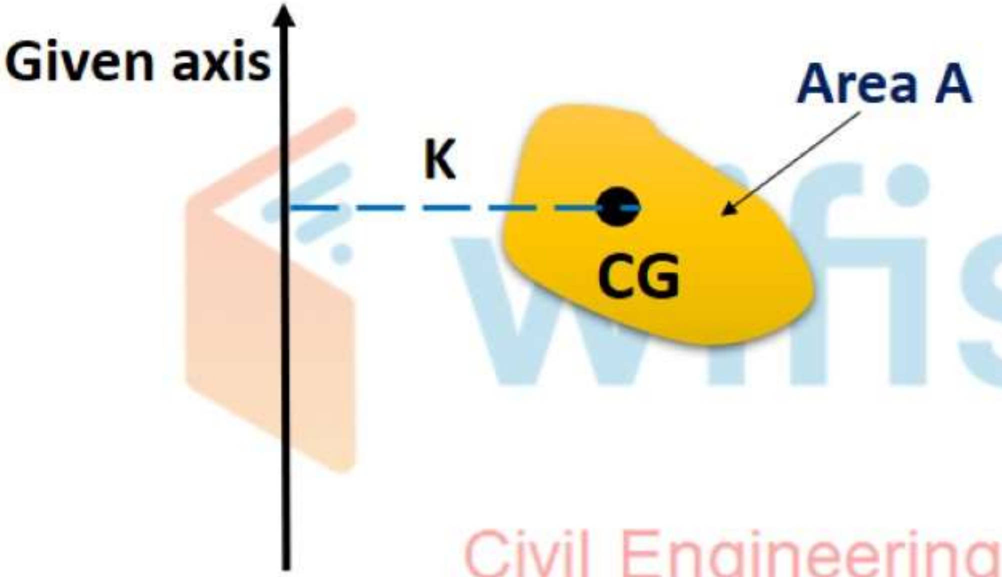
$$P_{cl} = \frac{\pi^2 EI}{4L^2}$$

4. One End Fixed and other is Hinged

$$l_{eff} = L/\sqrt{2}$$

$$P_{cl} = \frac{2\pi^2 EI}{L^2}$$

Slenderness Ratio



Radius of Gyration

- It is distance such that its square multiplied by area gives Moment of inertia about the given axis

$K^2 \times A = I$

$$K = \sqrt{\frac{I}{A}}$$

Slenderness ratio $\lambda = \frac{Effective\ length}{radius\ of\ Gyration}$

$$\lambda = \frac{L_{Effective}}{K_{minimum}}$$

Slenderness Ratio

Slenderness ratio $\lambda = \frac{Effective\ length}{radius\ of\ Gyration}$

$$P_{cl} = \frac{\pi^2 E I_{min}}{L^2_{effective}}$$

$$\lambda = \frac{L_{Effective}}{K_{minimum}}$$

$$\Rightarrow Pcl = \frac{\pi^2 E(AKmi_n^{\ 2})}{L^2_{effective}}$$

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$$\Rightarrow \frac{P_{cl}}{A} = \frac{\pi^2 E\ Kmi_n^{\ 2}}{L^2_{effective}}$$

$$\Rightarrow \sigma_b = \frac{\pi^2 E}{\lambda^2}$$

Limitations of Euler's Theory

- **Euler's theory is valid only for Long Column**
- **Euler's theory is not valid for Short column because short column fails in Crushing before buckling occurs in it**

Validity of Euler's Theory

σ_c = **crushing strength** $= \frac{P_c}{A}$

σ_b = **buckling strength** $= \frac{P_b}{A}$

$\sigma_b = buckling\ load = \frac{\pi^2 E}{\lambda^2}$

- For validity of Euler's Theory, Buckling should occur before crushing, i.e

$$\sigma_b < \sigma_c$$

$$\frac{\pi^2 E}{\lambda^2} < \sigma_c$$

$$\frac{\pi^2 E}{\sigma_c} < \lambda^2$$

$$\sqrt{\frac{\pi^2 E}{\sigma_c}} < \lambda$$

- **For validity of Euler's Theory, Value of Critical Slenderness ratio is 88.85%**

Rankine's Theory

- Rankine's Theory is applicable for both Long and Short column

  P_R = Rankine's load

  P_b = Buckling Load

  P_c = Crushing Load

- Rankine's load is given by:

$$\frac{1}{P_R} = \frac{1}{P_b} + \frac{1}{P_c}$$

$$\frac{1}{P_R} = \frac{P_b + P_c}{P_c P_b}$$

$$\sigma_b = \frac{\pi^2 E}{\lambda^2}$$

$$P_R = \frac{P_c P_b}{P_b + Pc}$$

$$P_R = \frac{P_c}{1 + Pc/P_b}$$

$$P_R = \frac{P_c}{1 + \frac{\sigma_c \times A}{\sigma_b \times A}}$$

$$P_R = \frac{P_c}{1 + \frac{\sigma_c \times A}{\frac{\pi^2 E}{\lambda^2} \times A}}$$

$$P_R = \frac{P_c}{1 + \frac{\sigma_c}{\pi^2 E} \times \lambda^2}$$

Rankine's Theory

$$P_R = \frac{P_c}{1 + \frac{\sigma_c}{\pi^2 E} \times \lambda^2}$$

$$P_R = \frac{P_c}{1 + \alpha\lambda^2}$$

$$\alpha = Rankine's\ Constant$$

$$P_R = \frac{\sigma_c \times A}{1 + \alpha\lambda^2}$$