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Grammar School Educational Society's Group of Institutions
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ENGINEERING MECHANICS

D.K.JAWAD's Class Note Book

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ENGINEERING MECHANICS

Basic concepts:- It the branch of science that deals the action of force on bodies is called mechanics.

Engineering mechanics:- Also known as applied mechanics.

Is a branch of science concerned with the application of mechanics in Engineering field

Types of Engineering mechanics:-

1) static

2) Dynamic

1) static:- In the branch of science that deals with the effect of forces on bodies are at rest is called static.

2. Dynamic:- In the branch of study that considering the effect of forces on bodies that are in motion is called Dynamic.

Types of Dynamics:-

> Kinematics:- Is the study of the motion of the bodies without consideration of mass & force is called Kinematics.

Kinetics:- Is the study of the motion of the bodies with consideration of mass and force is called kinetics.

Engineering mechanics

- Static [body is at rest
- Dynamics [body is in motion]
  - Kinematics [mass and force are not considered]
  - Kinetics [mass and force considered]

Body (or) substance (or) matter :- Anything in a space having (or) possesing mass

physical Quantities :-

scalar and vector :-

Scalar :- scalar quantities having magnetude but no direction Ex :- are length, mass, Time, wait etc.

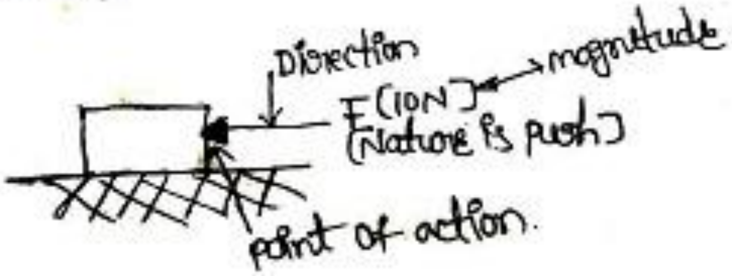
vektor :- vetor Quantities having magnetude and direction.

Ex :- are velocity, Acceleration, force, displacement etc.

Force : force is an external agent that causes to change the position of body is at rest or in motion.

Specification of force :-

- magnetude
- Direction
- point of action
- Nature



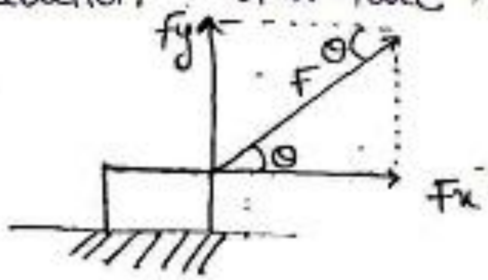
Resaluation of forces:-

is the process of resaluing the force in to two components they are X & Y. seach that it doesn't produce any effect on body

a) Resalution of a force in to rectangular component

b) Resalution of a force, the body is an inclined plane.

a)
sol:-



$$\sin\theta = \frac{Fy}{F}$$

$$fy = f\sin\theta$$

$$\cos\theta = \frac{Fx}{F}$$

$$Fx = F\cos\theta$$

$$F = \sqrt{(fx)^2 + (fy)^2}$$

F is resultant force

$$\tan\theta = \frac{Fy}{f(x)}$$

$$\theta = \tan^{-1}\left(\frac{fy}{f(x)}\right)$$

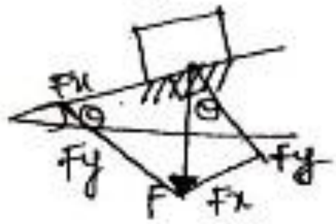
1) Resolve a force of 100N which makes an angle 30° to the horizontal along two mutually perpendicular direction.

$$Fx = F\cos\theta$$
$$= 100 \times \cos 30$$
$$= 100 \times \sqrt{3}/2$$
$$= 86.6\,N$$

$$Fy = F\sin\theta$$
$$= 100 \times \sin 30$$
$$= 100 \times 1/2$$
$$= 100 \times 0.5$$
$$= 50\,N$$

Resolving the force components the body is an inclin -ed plane:-

sol:- a) vertical force:-



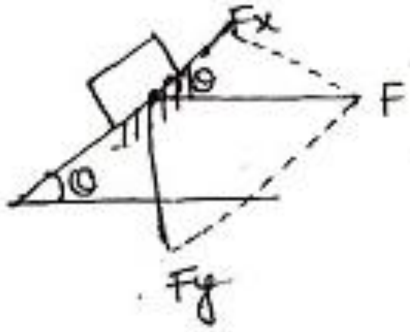
$\cos\theta = \frac{F_y}{F}$

$F_y = F\cos\theta$

$\sin\theta = \frac{F_x}{F}$

$F_x = F\sin\theta$

b) Horizontal force:-



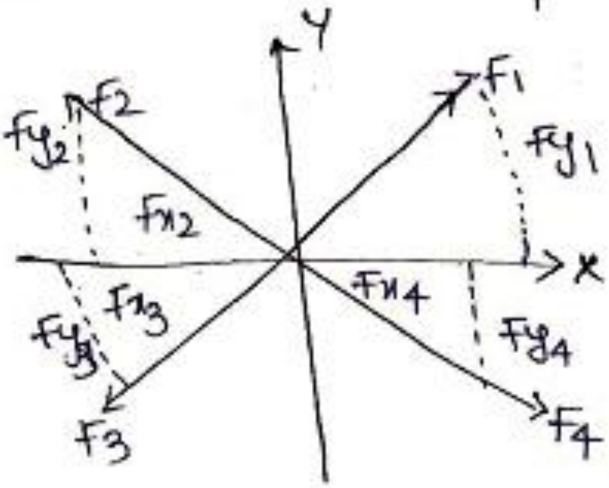
$\cos\theta = \frac{F_x}{F}$

$F_x = F\cos\theta$

$\sin\theta = \frac{F_y}{F}$

$F_y = F\sin\theta$

General method of composite forces:-



Step:-1:-

Find the components of all the forces in x and y direction such as F_{x1}, F_{x2}, F_{x3} & F_{x4} and also F_{y1}, F_{y2}, F_{y3} & F_{y4}.

Step 2:-

Find the algebric sum of components forces in x and y directions.

$$\Sigma f_x = F_{x_1} + F_{x_2} + F_{x_3} + F_{x_4}$$

$$\Sigma f_y = F_{y_1} + F_{y_2} + F_{y_3} + F_{y_4}$$

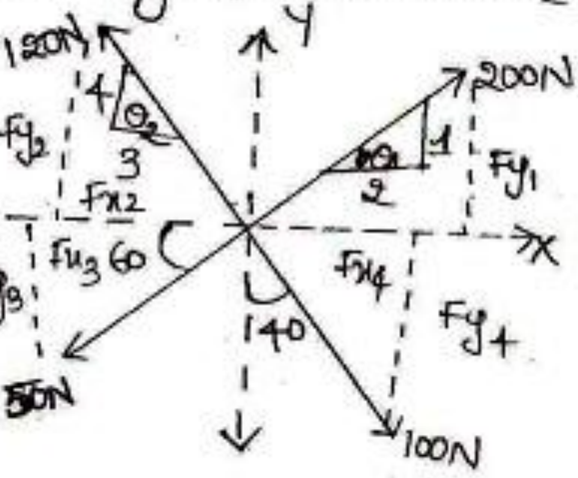
Step 3:-

system of forces along x & y directions are mutually perpendicular to each other.

$$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

Q:- A system of four forces acting at a point on a body shown as in figure, Determine the resultant force.



Step-1:-

$$F_{x_1} = F_1 \cos\theta_1$$

$$= 200 \cos(\tan^{-1}\tfrac{1}{2})$$

$$= 178.88\ N$$

$$F_{x_2} = F_2 \cos\theta_2$$

$$= 120 \cos[\tan^{-1}\tfrac{4}{3}]$$

$$= 72N$$

$$F_{x_3} = F_3 \cos\theta_3$$

$$F_{x_3} = F_3 \cos\theta_3$$

$$= 50 \cos 60$$

$$= 25N.$$

$$F_{x_4} = 100 \cos 40$$

$$= 76.6N$$

$$Fy_1 = F_1 \sin\theta_1$$
$$= 200 \sin\left[\tan^{-1}\frac{1}{2}\right]$$
$$= 89.7\ N$$

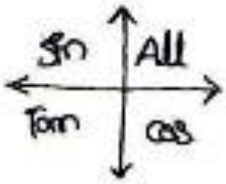
$$Fy_2 = F_2 \sin\theta_2$$
$$= 120 \sin\left[\tan^{-1}\frac{4}{3}\right]$$
$$= 95.9N$$

$$Fy_3 = F_3 \sin\theta_3$$
$$= 50 \sin 60$$
$$= 43.3N$$

$$Fy_4 = F_4 \sin\theta_4$$
$$= 100 \sin 40$$
$$= 64.2\ N.$$

step:-2

$$\Sigma Fx = Fx_1 + Fx_2 + Fx_3 + Fx_4$$
$$= 178.88 + (-72) + (-25) + 76.6$$
$$= 158.48\ N$$



$$\Sigma Fy = Fy_1 + Fy_2 + Fy_3 + Fy_4$$
$$= 89.7 + (95.9) + (-43.3) + (-64.2)$$
$$= 78.1\ N$$

step:-3. $R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$

$$= \sqrt{(158.48)^2 + (78.1)^2}$$
$$= 176.6N$$

$$\theta = \tan^{-1}\left(\frac{\Sigma Fy}{\Sigma Fx}\right)$$

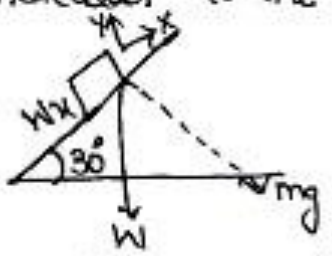
$$= \tan^{-1}\left[\frac{78.1}{158.48}\right]$$

$$= 26.3°$$

* A small block of weight 50N is placed on inclined plane which makes an angle θ=30° with the horizontal what is the component of the weight

(i) parallel to the inclined plane.

(ii) perpendicular to the inclined plane.



(i) parallel to the inclined plane.

x - component

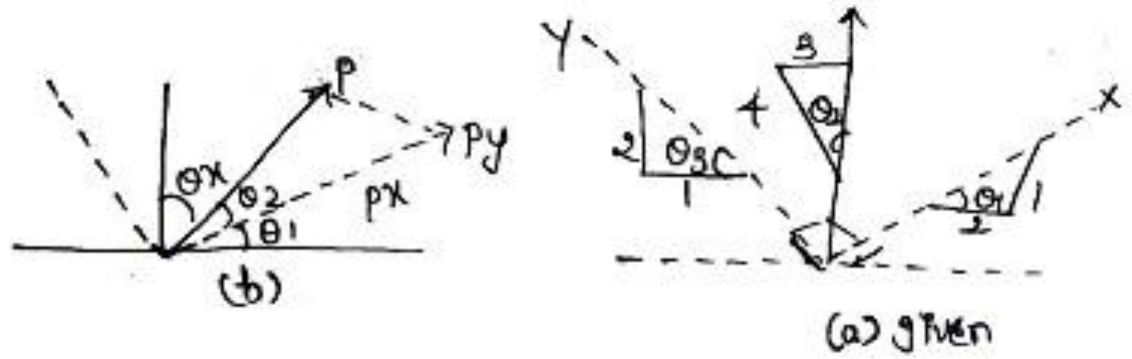
$$W_x = W\sin\theta$$
$$= 50\sin 30$$
$$= 25N$$

(ii) perpendicular to the inclined plane.

Y - component

$$W_y = W\cos\theta$$
$$= 50\cos 30$$
$$= 50\times 0.866$$
$$= 43.3.$$

Q* the x-component of P is 893N. Determine P and y component



$\theta_1 = \tan^{-1}(1/2) = 26.56°$

$\theta_2 = \tan^{-1}(3/4) = 36.8°$

$\theta = (90 - (\theta_1 + \theta_2))$

$= [90 - (26.56 + 36.8)]$

$= 26.64$

x - Component

$px = 893N$

$px = P\cos\theta$

$P = \dfrac{px}{\cos\theta}$

$P = \dfrac{893}{\cos 26.64}$

$= 999.058 N.$

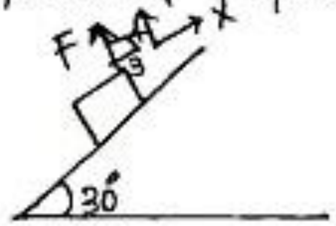
Y- component

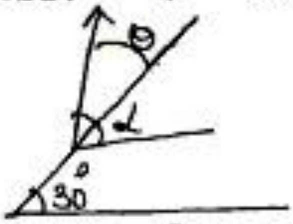
$py = P\sin\theta$

$= 999.058 \times \sin 26.64$

$= 447.96 N$

* A body is subjected to a force F as shown in fig. If x- component of force is 600N, find the component ⊥ perpendicular to the plane.





$fx = 600N$

$\alpha = \tan^{-1}(4/3)$

$= 53.130°$

$\theta = (\alpha - 30)$

$= (53.13 - 30)$

$= (23.13°)$

$fx = F\cos\theta$

$F = \dfrac{fx}{\cos\theta}$

$$= \frac{600}{\cos 23.13}$$

$$= 652.446\ N.$$

The component perpendicular to the plane

$$F_y = F \sin\theta$$

$$= 652.446 \times \sin 23.13$$

$$= 256.29\ N.$$

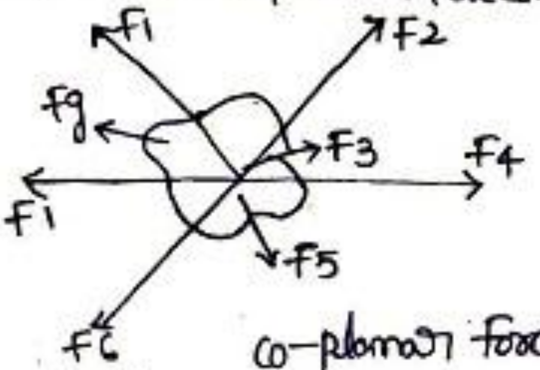
System of forces:-

→ several forces acting on a body are called a system of forces.

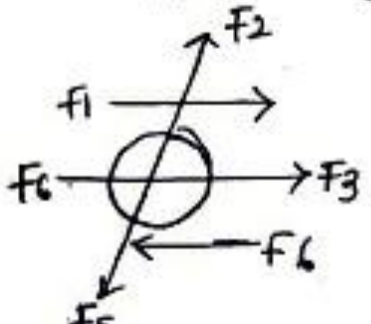
There are two types.

(i) Co-planar forces

(ii) Non-coplanar Forces.



co-planar forces.

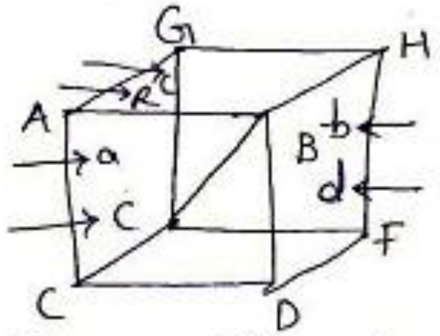


Non-coplanar Forces.

several forces acting on a same plane (or) same body are called co-planar forces.

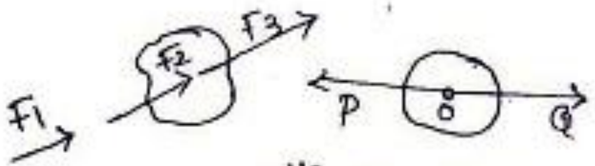
→ several forces are not acting on same plane

plane (or) same body are called non coplanar -r forces. (10)



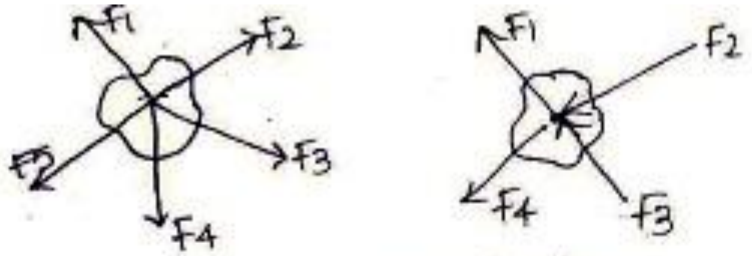
the Forces . a&c1 , b&d1 ,C&F . Co planar forces.
a&d , C&b, E&d are non - coplanar Forces.
Forces again divided Into
a) Collinear forces
b) Concurrent forces
c) paraller forces
d) Non - concurrent and non paraller Forces.

a) collinear Forces:-
A system of Forces acting on a body line a Same line are called as collinear Forces.



collinear forces.

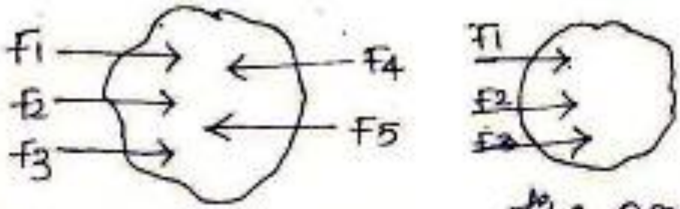
b) Concurrent Forces:-
A system of forces acting on a body at Same point Called as concurrent Forces.



Concurrent forces.

c) paralleo Forces:-

A system of forces acting on a body and all forces parallel to each other. then system of forces called as paralleo forces.

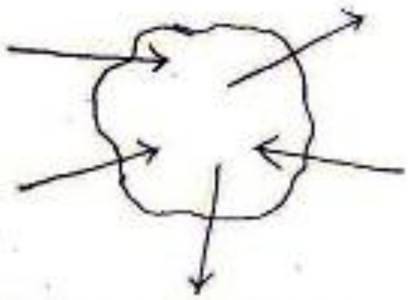


unlike paralleo Forces. Like paralleo Forces.

paralleo Forces.

d) Non concurrent and Non paralleo Forces:-

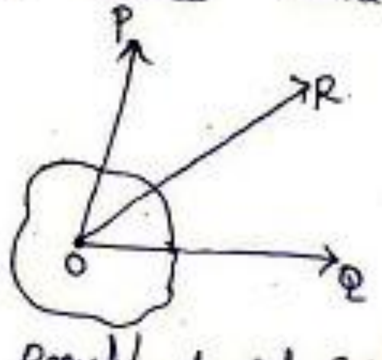
A system of forces acting on a body not on same point and not paralleo to each other. the system of Forces are called as non concurrent and non paralleo forces.



Non concurrent and non paralleo Forces.

Resultant of system of forces:-

A single force produce the same effect as that of several forces produces on the body then a single force is called resultant of system of forces.

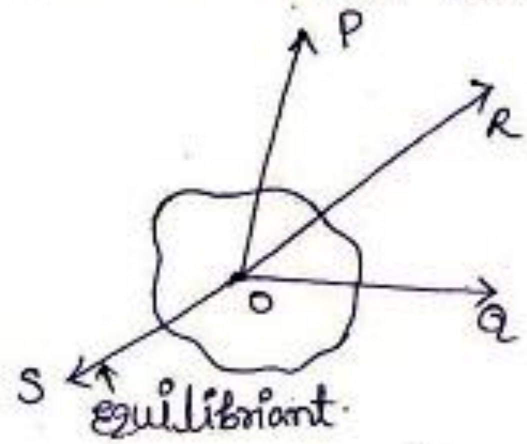


Resultant of system of forces.

Equilibrium and equilibriant:-

A body is in at rest under the action of forces then the body is known as equilibrium.

A force which tends to body is in equilibrium then force is called equilibriant.



Resultant and equilibrium both are same and opposite in direction.

Resultant of coplanar system of forces:-

when two forces P and Q acting on a body at a point 'O' Then resultant of system of forces may be found out by the following methods.

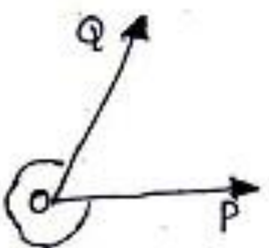
(i) parallelogram law of system of forces.

(ii) Resolution of system of forces.

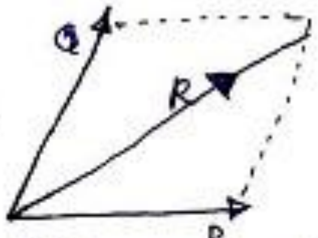
Parallelogram law of system of forces:- *

Two forces ~~represen~~ represtend

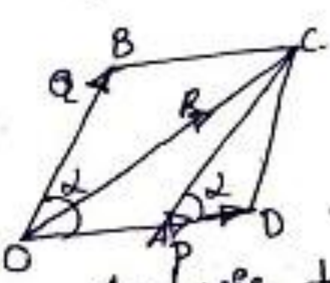
Two forces represented in magnetude and direction be the sides of parallelogram, Then diagonal of parallelogram represent in magnetude and direction find out resultant of two forces.



P & Q acting on 'O'



P, Q are the sides of parallelogram



Analysis to find R.

P represent by OA, Q represent by OB and R is represent by OC

To find out, OC is carried out from right angle triangle OCD.

$\angle BOD = \angle CAD = \alpha$

$OC^2 = OD^2 + CD^2$

$= [(OA)^2 + (AD)^2] + CD^2 \Rightarrow [OA + AD]^2 + CD^2$

$CD = AC \sin\alpha$

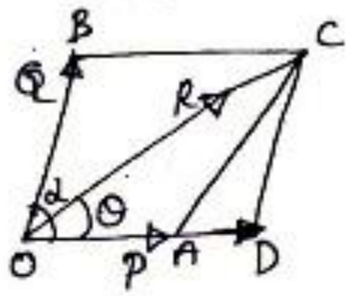
$AD = AC \cos\alpha \Rightarrow R^2 = (P + Q\cos\alpha)^2 + Q^2\sin^2\alpha$

$R^2 = P^2 + Q^2\cos^2\alpha + Q^2\sin^2\alpha + 2PQ\cos\alpha$

$R^2 = P^2 + Q^2(\cos^2\alpha + \sin^2\alpha) + 2PQ\cos\alpha$

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$



$\tan\theta = \frac{CD}{OD}$

$$\boxed{\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)}$$

Special cases

1 P, Q are Collinear and angle $\alpha = 0$

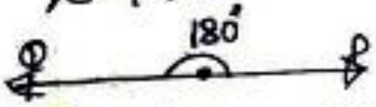
$R = P + Q$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

$= \sqrt{P^2 + Q^2 + 2PQ}$

$= \sqrt{(P+Q)^2}$

$R = P+Q$

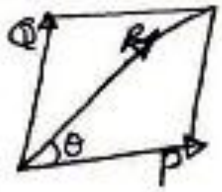


ii) Angle between P & Q is 180°

$P > Q$

$R = P - Q$

iii) Angle between two forces is 90°



$R = \sqrt{P^2+Q^2}$

$\theta = \tan^{-1}(Q/P)$

iv) If two forces P, Q are equal

$P = Q$

$R = \sqrt{P^2+P^2+2P^2\cos\alpha}$

$= \sqrt{2P^2(1+\cos\alpha)}$

$= \sqrt{2P^2 \times 2\cos^2\alpha/2}$

$= \sqrt{4P^2\cos^2\alpha/2}$

$= 2P\cos\alpha/2$

$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right)$

$= \tan^{-1}\left(\frac{P\sin\alpha}{P+P\cos\alpha}\right)$

$= \tan^{-1}\left(\frac{2\sin\alpha/2 \cdot \cos\alpha/2}{1+\cos\alpha}\right)$

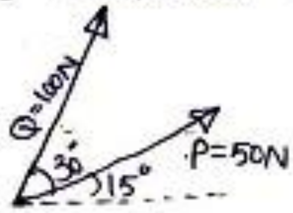
$= \tan^{-1}\left(\frac{\cancel{2}\sin\alpha/2 \cdot \cancel{\cos\alpha/2}}{\cancel{2}\cos^{\cancel{2}}\alpha/2}\right)$

$= \tan^{-1}(\tan\alpha/2) \Rightarrow \theta = \alpha/2$

*imp Q

Two forces are acting at a point 'o' as shown in figure Determine magnitude and direction of the resultant

$P = 50N$, $Q = 100N$ $\alpha = 30°$



$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

$= \sqrt{(50)^2 + (100)^2 + 2 \times 50 \times 100 \times \cos 30}$

$= 145.46N$

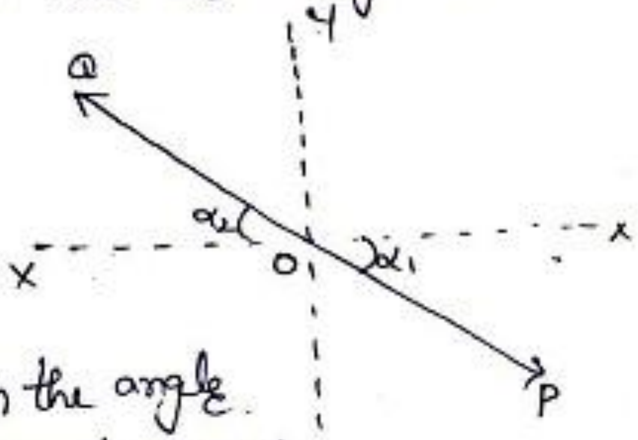
$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$

$= \tan^{-1}\left(\frac{100 \times \sin 30}{50 + 100\cos 30}\right)$

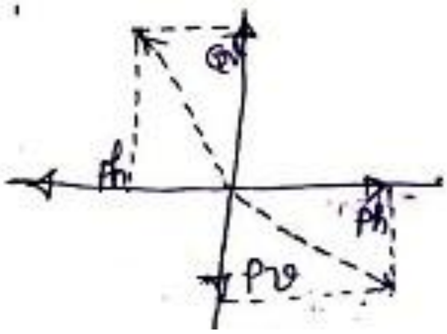
$= 20.103°$

Resolution of system of forces:-

Two forces P, Q are acting at 'o' as shown in figure

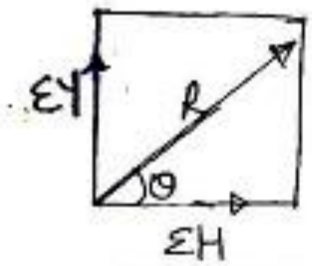


P & Q maken the angle α_1, α_2 to the horizental Axis X-X



ΣH = Algebraic sum of X-components

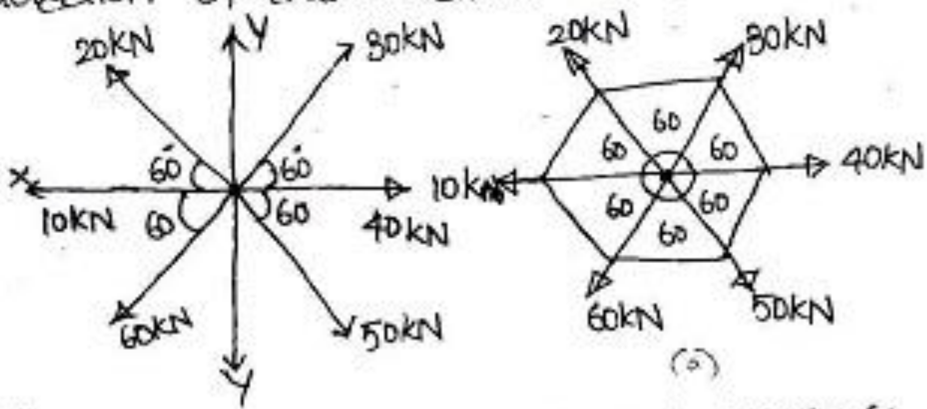
ΣY = Algebraic sum of Y-components



$$R = \sqrt{(\Sigma Y)^2 + (\Sigma H)^2}$$

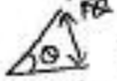
$$\theta = \tan^{-1}\left(\frac{\Sigma Y}{\Sigma H}\right)$$

*Forces of magnitude 10, 20, 30, 40, 50 and 60 kN respectively act from the centre of regular hexagon towards its six angular points. Find the magnitude & direction of the resultant force.



$$\Sigma H = 40 + 30\cos 60 - 20\cos 60 - 10 - 60\cos 60 + 50\cos 60$$

$$\Sigma H = 30\,kN$$



$\cos\theta = \frac{Fn}{F}$

$Fn = F\cos\theta$

$Fv = F\sin\theta$

$$\Sigma Y = 30\sin 60 + 20\sin 60 - 60\sin 60 - 50\sin 60$$

$$= -51.96\,kN.$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma Y)^2}$$

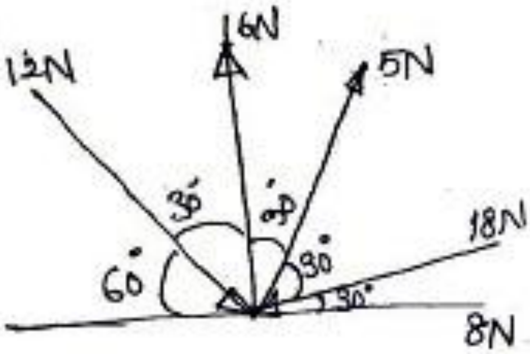
$$= \sqrt{(30)^2 + (-51.96)^2}$$

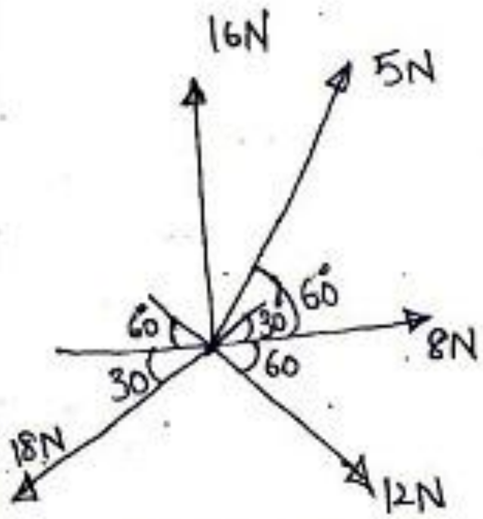
$$= 59.99\ kN$$

$$\theta = \tan^{-1}\left(\frac{\Sigma Y}{\Sigma H}\right) = \tan^{-1}\left(\frac{51.96}{30}\right)$$

$$= 59.99$$

The magnitude of following forces which are in equilibrium as show in figure





$$\Sigma H = 8\cos 0 + 5\cos 60 - 16\cos 90 - 18\cos 30 + 12\cos 60$$

$$= 8 + 5\cos 60 - 18\cos 30 + 12\cos 60$$

$$= 0.911\,N$$

$$\Sigma Y = 8\sin 0 + 5\sin 60 + 16\sin 90 - 18\sin 30 - 12\sin 60$$

$$= 5\sin 60 + 16 - 18\sin 30 - 12\sin 60$$

$$= 0.937\,N$$

$$R = \sqrt{(0.937)^2 + (10.46)^2}$$

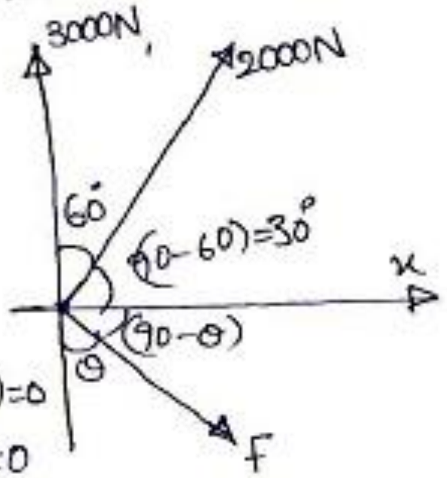
$$= 1.30\,N.$$

$$\theta = \tan^{-1}\left(\frac{\Sigma Y}{\Sigma H}\right) = \tan^{-1}\left(\frac{0.937}{0.911}\right) = 45.80^\circ$$

A body is subjected to the three forces as shown in figure, determine the direction of forces F so that the resultant in x-direction when

(i) F = 10000N

(ii) F = 6000 N



ΣX = x direction

ΣY = 0

(i) = 2000 sin 30 + 3000 − 10000 sin(90−θ) = 0

= 2000 sin 30 + 3000 − 10000 cos θ = 0

θ = 66.42

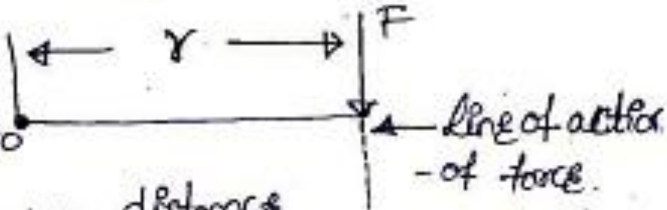
(ii) 2000 sin 30 + 3000 − 6000 cos θ = 0

2000 sin 30 + 3000 = 6000 cos θ

θ = 48.70°

Moment of force:-

The turning moment of the body produced by the application of force is called as moment of force



It is measured by product of force and perpendicular distance from the point 'o' to the line of action of force.

mf = moment of force

F = Force activity in a body

r = perpendicular Distance.

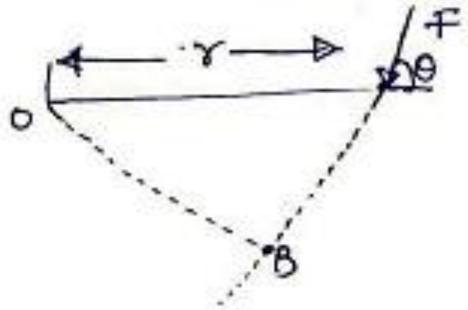
mf = F × r

$\sin\theta = \frac{OB}{r}$

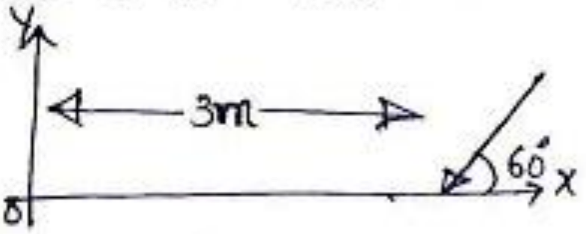
$OB = r\sin\theta$

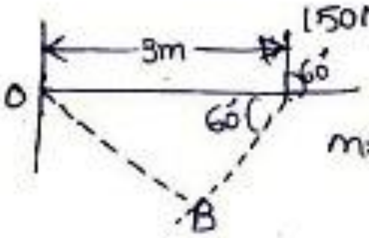
$mf = F \times OB$

$= F \times r\sin\theta$



A force of 150N acting at a point A as shown in figure. Determine the moment of this force about 'O'





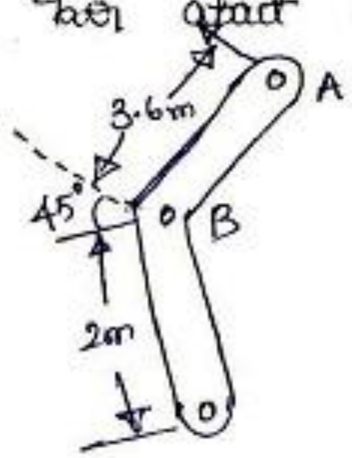
$mf = F \times OB$

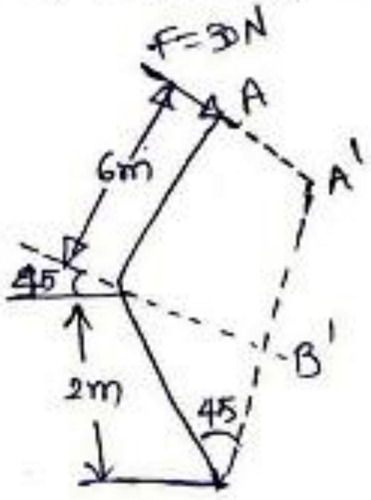
$= F \times r\sin 60°$

$= 150 \times 3 \sin 60°$

$= 389.7$ N-m

1) Determine moment of 30N force acting at a of bent bar about 'O'?





$mf = F \times OA'$

$= F \times (OB' + B'A')$

$= 30 \times (r\sin 45 + 3.6)$

$= 30 \times (2\sin 45 + 3.6)$

$= 150.42$ N.

Types of moment :- depending up on direction of rota-tions the moments are two typs.

a.) Clock wise moment :-
Turning effect produceed to a body in clockwise direction is called clock wise moment.

b) Anti clock wise moment :-
Turning effect produceed to a body in anticlock wise direction then it is called anti clockwise moment.

fig clock wise moment.

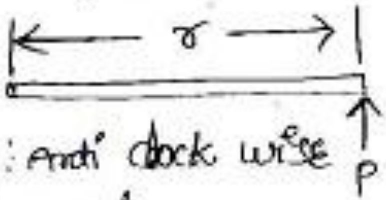
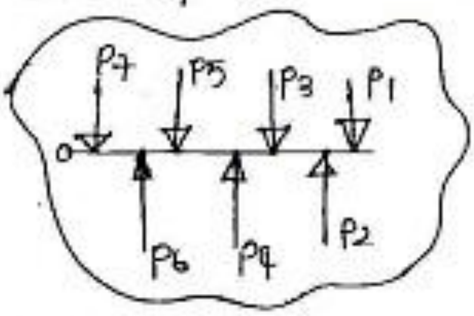


fig: Anti clock wise moment.

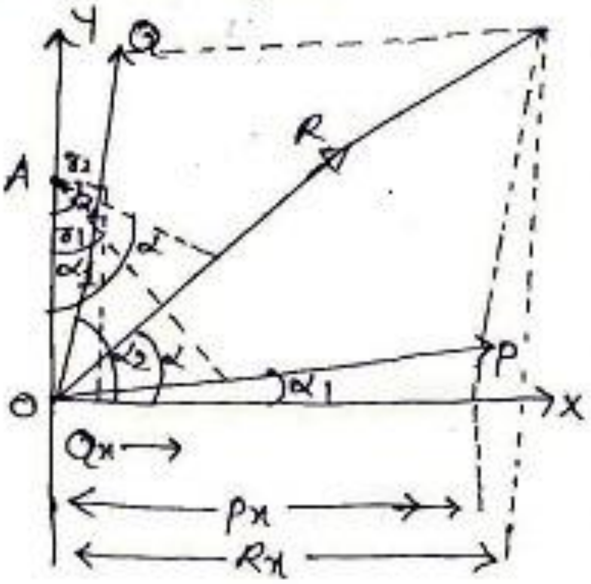
I
Law of moment :-
If a body is in equilibrium under the action of system of forces then the sum of clockwise moments is equal to sum of anticlockwise moments, and algebraic sum of at moments are 0



Clock wise moments = Anti clock wise moments.

Varingnon's Principle:-

It states that algebraic sum of moment of system of forces about any point is equal to moment of their resultant at the same point.



moment of P about A

$= P \times r_1$

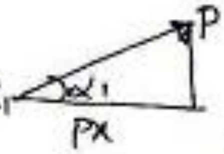
$= P \times OA \cos\alpha_1$

=

$\cos\alpha_1 = \frac{OA}{r_1}$

$r_1 = OA \cos\alpha_1$

$= \frac{P_x}{\cos\alpha_1} \times OA \cos\alpha_1$



$\cos\alpha_1 = \frac{P_x}{P}$

$P = \frac{P_x}{\cos\alpha_1}$

$= P_x \times OA$

moment of Q About A

$= Q \times r_2$

$= Q \times OA \cos\alpha_2$

$= Q_x \times OA$

moment of R about A

$= R \times r$

$= R \times OA \cos\alpha$

$= R_x \times OA$

$(\because R_x = P_x + Q_x)$

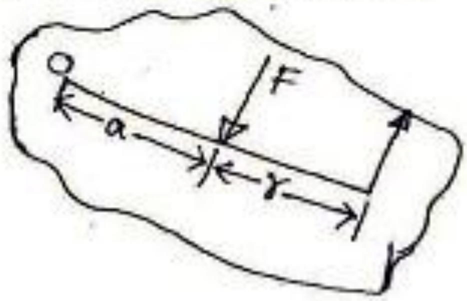
$(\because R_y = P_y + Q_y)$

the sum of moment of P & Q

$(P_x + Q_x) OA = R_x OA$

$= R \cdot r$

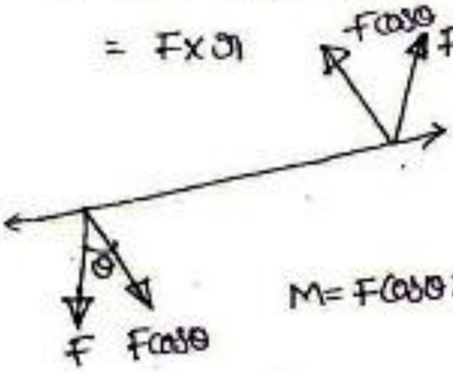
Couple:- * Two equal unlike parallel forces whose line of action are not same (i.e., not collinear) form a couple. The effect of couple cause rotation of body.



$M_f = F(a+r) - F \times a$

$= F \times a + F \times r - F \times a$

$= F \times r$

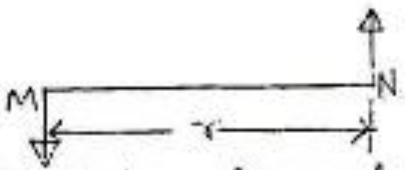


$M = F\cos\theta \times r$

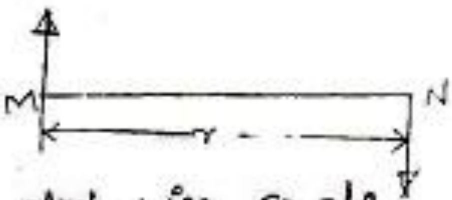
Classification of couple:-

① clock wise couple

② Anti clock wise couple.



Anti clock wise couple



clock wise couple

① Four parallel forces of magnitude 100N, 150N, 25N and 200N are as shown in the figure.
Determine the magnitude of the resultant and also the distance of resultant from point A.

$\Sigma f_x = 0$

$\Sigma f_y = R = 100 + 150 - 25 + 200$

$= 425\,N$

position of Resultant

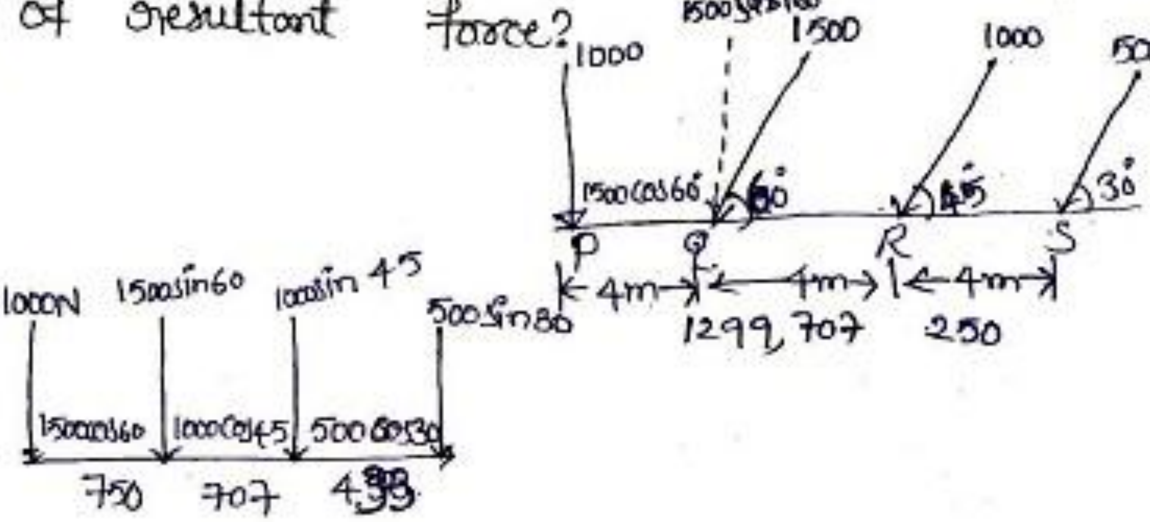
$R \times x = 150 \times 0.9 - 25 \times 2.1 + 200 \times 2.85$

$= 652.5$

$x = \frac{652.5}{425} = 1.535\,m$

*(1)

(2) A Horizontal line PQRS is 12m long. where PQ = QR = RS = 4m forces of 1000, 1500, 1000 and 500N act, at P, Q, R and S respectively and action of these forces make angle 90°, 60°, 45° and 30° respectively with PS Find the magnitude, direction and position of resultant force?



Resolving horizontal components

$\Sigma H = 750 + 707 + 433$

$= 1890\,N$

$\Sigma V = 1000 + 1299 + 707 + 250$

$= 3256\,N$

Resultant

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(1890)^2 + (3256)^2}$

$= 3764.79N$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$= \tan^{-1}\left(\frac{3256}{1890}\right)$

$= 59.86°$

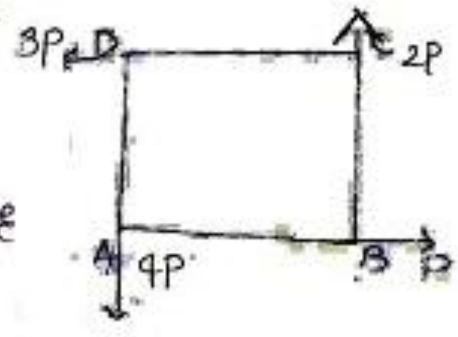
Position of resultant force

$R \times x = (1299 \times 4) + (707 \times 8) + (250 \times 12)$

$x = \frac{13852}{3764.79}$

$x = 3.67$ m

*(I). (3) (a) Four forces P, 2P, 3P and 4P are acting along the four sides of the square ABCD respectively, taken in order. side = 40mm.
Find the magnitude, direction and position of resultant force.



Resolving horizontal component of force $\Sigma H = -3P + P = -2P$

Resolving vertical component of force

$\Sigma V = -4P + 2P = -2P$

Resultant

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(2P)^2 + (2P)^2}$

2.828 P

Direction $\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$= \tan^{-1}\left(\frac{2P}{2P}\right)$

$= 45°$

position of the Resultant.

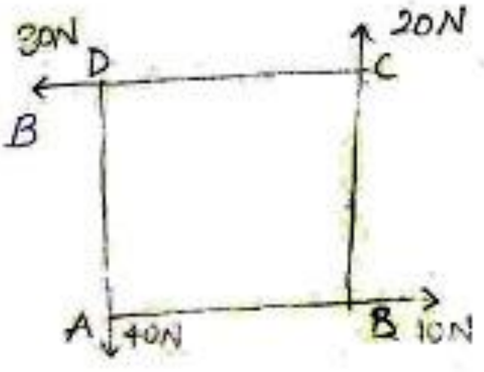
$R \times x = 2P \times 40$

$x = \frac{80P}{2.828P}$

$= 28.28$ mm

b) 4 forces are magnetude 10N, 20N, 30N and 40N are acting respectily along the four side of square ABCD as shown in figure. Determine the resultant moment about point A each side of the square is given as 2m

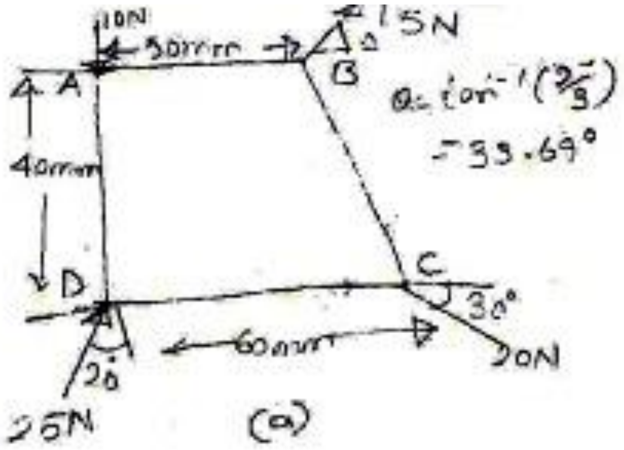
Resaltant moment about point B

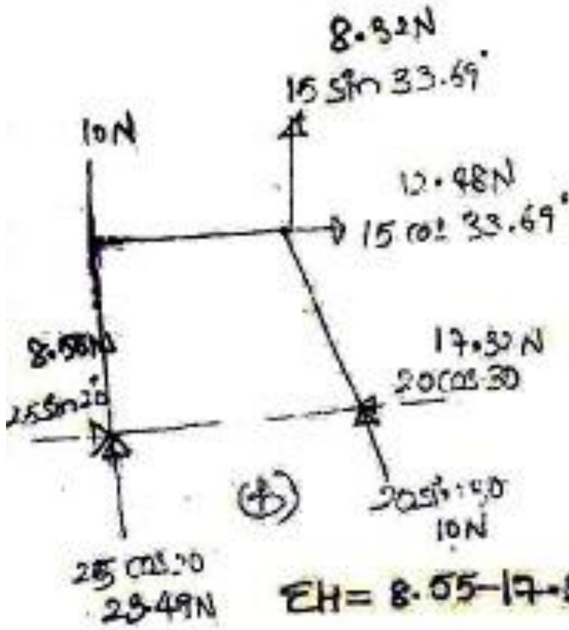


$R \times x = 30 \times 2 + 20 \times 2$

$= 100$ N·m.

④* Replace the given system of forces acting on a body by a single force and couple acting at a point A as shown in figure?





$\Sigma H = 8.55 - 17.32 + 12.48$

$= 3.71\ N$

$\Sigma V = 23.49 + 10 + 8.32 - 10$

$= 31.81N$

Resultant

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(3.71)^2 + (31.81)^2}$

$= 32.02\ N$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$\theta = \tan^{-1}\left(\frac{31.81}{3.71}\right)$

$= 83.34°$

couple at A

$R \times x = 8.55 \times 40 + 8.32 \times 30 - 17.32 \times 40 + 10 \times 60$

$= 665.2\ N \cdot mm.$

Equilibrium of forces:-

Equilibrium

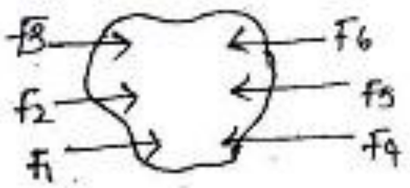
Equilibriant

conditions of equilibrium:-

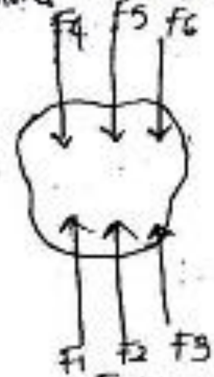
i) $\Sigma H = 0$. algebraic sum of horizontal components of all forces must be zero.

ii) $\Sigma V = 0$, algebraic sum of vertical components of all forces must be zero.

iii) $\Sigma M = 0$, algebraic sum of moments of all forces about any point in plane must be 0.



$F_1 + F_2 + F_3 = F_4 + F_5 + F_6$



$F_1 + F_2 + F_3 = F_4 + F_5 + F_6$

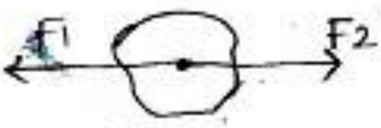
Types of equilibrium:-

1) Two force system

2) Three force system.

1) Two force system:-

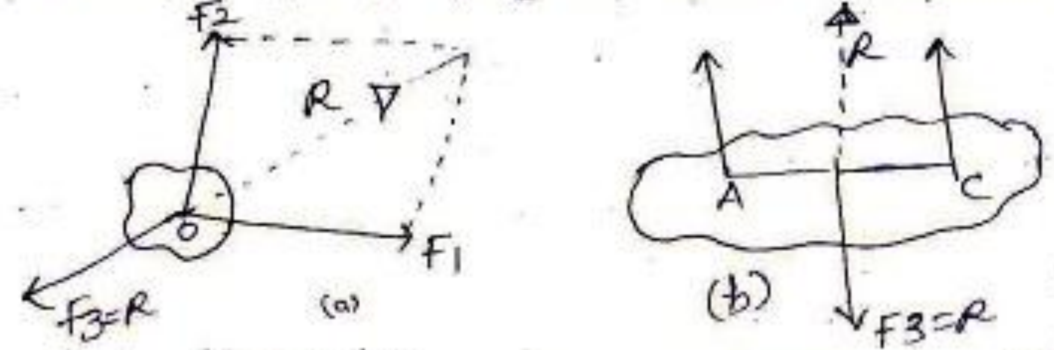
Body is in equilibrium, if two forces are collinear equal and at opposite direction



$F_1 = F_2$

② Three force system :-

A body will be in equilibrium under the action of three forces if the resultant of two forces is collinear, equal and opposite to the third force. the third force is called equilibrant.

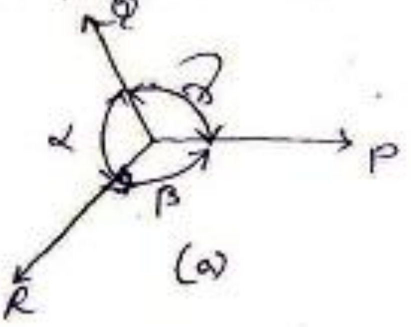


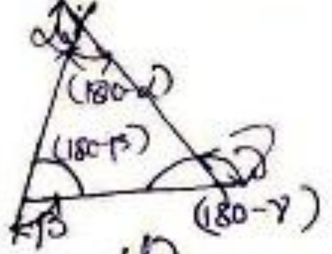
Lami's theorem :-

It states, "If three forces acting at a point are in equilibrium each force will be proportional to the sine of angle between the other two forces.

Let P, Q, R be the three forces actions at a point O and α, β and γ be the angles as shown in figure (a) then Lami's theorem expressed by the equation.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



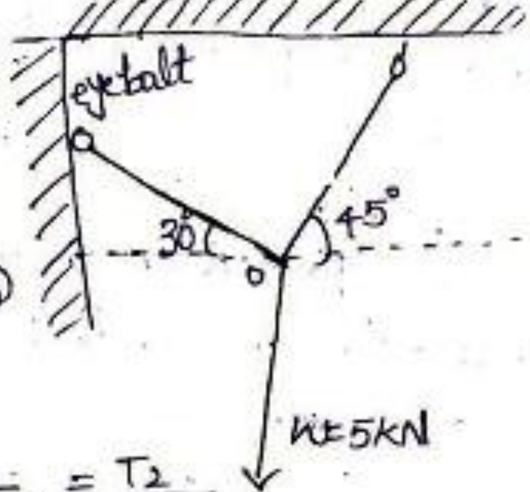


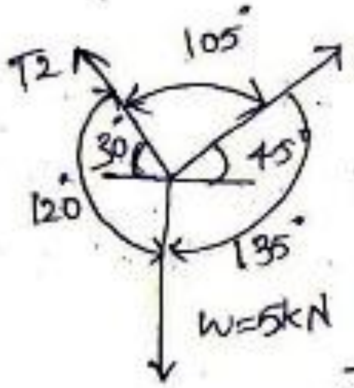
proof :- the three forces acting at a point are in equilibrium and can be represented by three sides of triangle taken in the same order as shown in fig (b) from the sine value

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

hence, the theorem proved.

* ① A machine weighing 5kN is supported by two chai-ns attached to same point on the machine one chain goes to hook in the ceiling and has inclination of 45° with the horizontal. the other chain goes to the eye bolt in the wall and is inclined at 30° to the horizontal make ~~sktux~~ calculation for the tension induced in the chain?





$= 360-(135+120)$
$= 105$

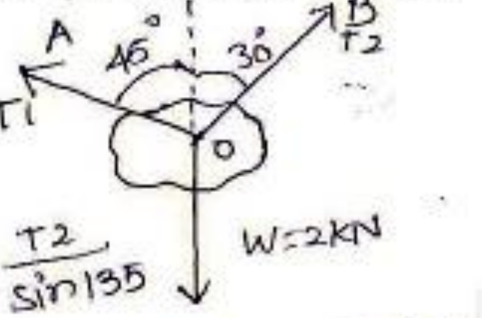
$$\frac{W}{\sin 105} = \frac{T_1}{\sin 120°} = \frac{T_2}{\sin 135}$$

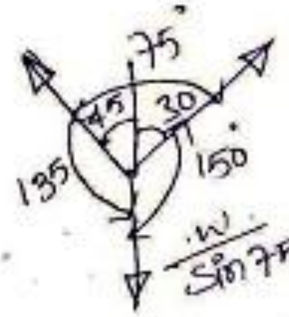
$$T_1 = \frac{W \times \sin 120}{\sin 105} = \frac{5 \times 0.866}{0.966}$$

$\Rightarrow 4.482$ kN

$$T_2 = \frac{W \sin 135}{\sin 105} = \frac{5 \times 0.707}{0.966} \Rightarrow 3.659 \text{ kN}$$

(2) Two men carry a weight of 2kN by means of two ropes fixed to the weight one rope is inclined at 45° and the other at 30° with their vertices. Find the tension in each rope.





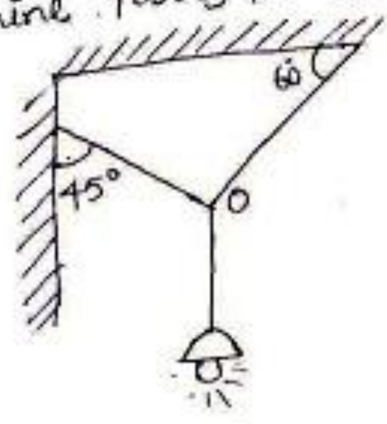
$$\frac{W}{\sin 75} = \frac{T1}{\sin 150} = \frac{T2}{\sin 135}$$

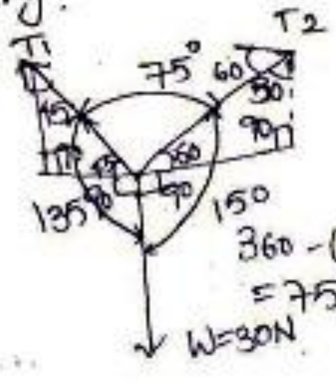
$$T_1 = \frac{W \times \sin 150}{\sin 75} = \frac{2 \times 0.5}{0.9659} = 1.03\,kN$$

$$T2 = \frac{W \times \sin 135}{\sin 75} = \frac{2 \times 0.707}{0.9659} \Rightarrow 1.469\,kN$$

(3)

An electric light fixture weight 30 N hangs from a point 'O' by two strings OA and OB as shown in fig (a) Determine forces in the string?





360 − (135+150)
= 75°

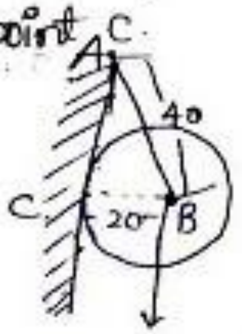
$$\frac{W}{\sin 75} = \frac{T_1}{\sin 150} = \frac{T2}{\sin 135}$$

$$T_1 = \frac{W \times \sin 150}{\sin 75} = \frac{30 \times 0.5}{0.9659} = 15.52\,N$$

$$T_2 = \frac{W \times \sin 135}{\sin 75} = \frac{30 \times 0.707}{0.9659} = 21.95\,N$$

*Imp

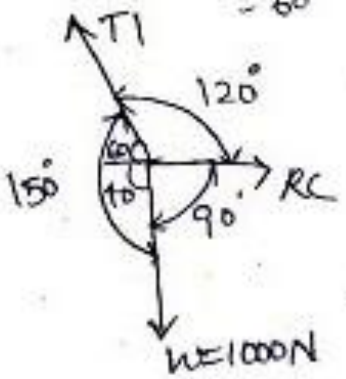
A cricular roller of weight 1000N and radiotly 20 cm hangs by a tie rod AB= 40cm and rest against a smooth vertical wall at C as shown in figure. Determine tension in the rod and reaction Rc at point C.



$$\cos\theta = \frac{20}{40}$$

$$\theta = \cos^{-1}\left(\frac{20}{40}\right)$$

$$= 60^\circ$$

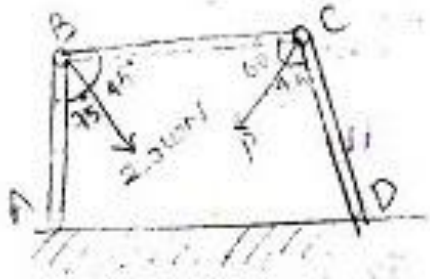


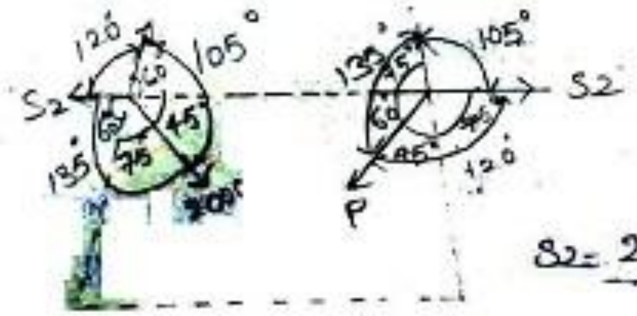
$$\frac{W}{\sin 120} = \frac{T_1}{\sin 90} = \frac{R_C}{\sin 150}$$

$$T_1 = \frac{W \times \sin 90}{\sin 120} = \frac{1000\times 1}{0.866} = 1154.7N$$

$$R_C = \frac{W\times \sin 150}{\sin 120} = \frac{1000\times 0.5}{0.860} = 577.35N$$

three bars pinned together at B and C and hinged at A and D as shown in figure from a four link mechanison. Determine the value of P that will prevent motion.





$$\frac{2000}{\sin 120} = \frac{S_2}{\sin 105}$$

$$S_2 = \frac{2000 \times 0.966}{\ldots} = 2230.94\,N$$

$$\frac{P}{\sin 105} = \frac{S_2}{\sin 135}$$

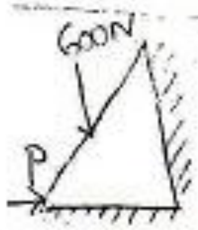
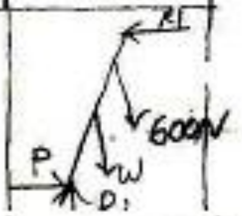
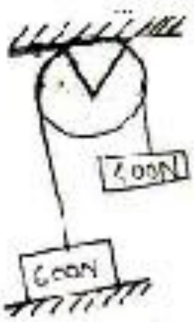
$$P = \frac{2230.94 \times \sin 105}{\sin 135}$$

$$= \frac{2230.94 \times 0.966}{0.707}$$

$$= 3047.5\,N$$

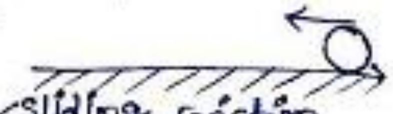
* Free body Diagram :-

for analysis of equilibrium condition it is a necessary to isolate the body under consid-erations from the other bodies in contact and draw all the forces acting on the body. for this first body is drawn and then all the applied forces acting on the body for this self weight reactions from the other bodies in contact are drawn. such diagram of the body in which the body under consideration is free-d from all contact surfaces and is shown with all the forces on it. is called the free body diagr-am (FBD)

Existing bodies	FBD required for	FBD
	Ball	
	Ball	
	Ladder	
	Block weighing 600N	

unit 2:- Friction.

when a body moves or tends to move over another body a force opposing the motion develops at the contact surfaces. This force which opposes the movement or tendency of movement is called Friction force (or) Simple Friction.



Types of Friction:-

1) solid (or) Dry Friction < sliding friction, Rolling Friction
2) viscous (or) fluid Friction
3) Non - viscous (or) Greases friction (boundary Friction).

Dry (or) solid Friction:- the Friction that exist between perfectly clean and dry (unlubricated) solid surfaces is called solid or dry dry friction. The two surfaces may be at rest (static friction) or one surface is moving on other surface is at rest (dynamic or kinetic Friction).

The Friction between two contacting surfaces at the point of sliding is called limiting Fric the Friction that exist when one surface sides over the other is called sliding friction.

the Friction that exist when one surface rolls over the other surface is called rolling Friction.

ii) viscous (or) fluid Friction:-

If a thick layer of oil or lubricant is introduced between the two surfaces a film lubricant form on both the surfaces and there is no direct contract between the surfaces. the friction between two surfaces separated completely by a film of lubricant is called viscous (or) fluid friction.

iii) Non viscous (or) Greasy (or) boundary Friction:-

the thin layer of an oil (or) lubricant introduced between the two surfaces prevents the metal to contact and reduce the Friction the Friction exist between the two surfaces separated by an extremely thin layer of oil is called non-viscous (or) Greasy friction.

Laws of solid (or) dry friction:-

(i) Frictional force always opposes the motion.

(ii) Frictional force is independed of area in contact and depends upon nature and rough-ness of the surface.

(iii) Frictional force is propotional to the normal reaction between the surfaces

(iv) The static frictional force at any instant is equal to the force applied to the body

(v) For low speeds the kinetic frictional force is independed of the speed but increases slighty with increasing speed.

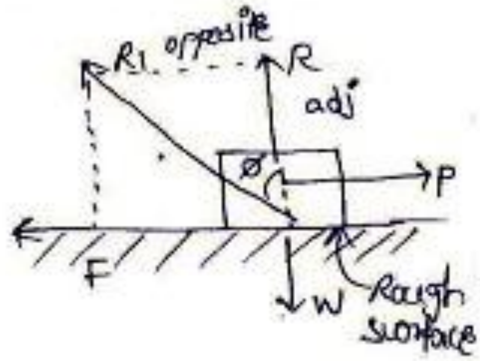
*co-efficient of friction:- The ratio of limiting force of friction to the normal reaction between two bodies is called co-efficient of friction it is denoted by a letter μ

Co-efficient of friction μ.

$$\mu = \frac{\text{Limiting force of frictional force} (F)}{\text{Normal Reaction} (R)}$$

$$\boxed{F = \mu R}$$

Angle of Friction:- The angle between the resultant reaction (resultant of normal reaction and limiting force of friction) and the normal reaction. It is denoted by ϕ



Let R_1 = Resultant Reaction

R = Normal Reaction

F = Limiting force of friction.

ϕ = Angle of friction

$\mu = F/R$

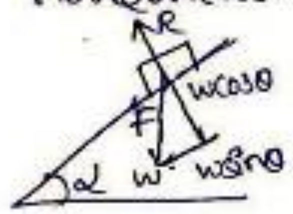
$F = \mu R$

$$\tan\phi = \frac{\text{opposite}}{\text{adj}} = F/R$$

$$= \frac{\mu R}{R}$$

$$\boxed{\tan\phi = \mu}$$

Angle of Repose:- Let a body is resting on a rough inclined plane making an angle α with horizontal



If the body on the point of sliding the angle is θ.

$F = w\sin\theta$

$R = w\cos\theta$

$$\mu = F/R = \mu = \frac{w\sin\theta}{w\cos\theta}$$

$$= \tan\theta$$

$$\boxed{\mu = \tan\phi}$$

$\tan\phi = \tan\theta$ θ is called angle of repose
$\theta = \phi$ and defined as maximum angle of the inclined plane at which a body.

*Equilibrium of a body on horizontal plane:-

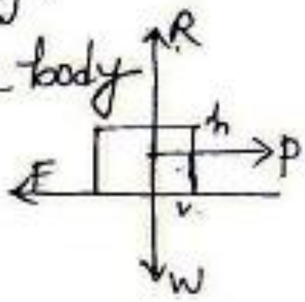
A body of weight 'w' rest on a horizental sur-face, let μ be the co-efficient of friction between the body and horizental surface. the least force p required to move the body in its directions

(i) Force is horizontal.

(ii) Force acts at given angle θ

(i)* Force is applied horizontally:-

least force required to move the body



$P = F$ horizontal

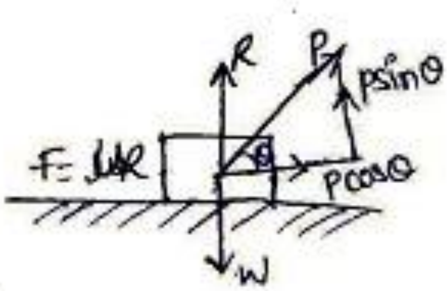
$F = \mu R$

$P = \mu R$.

vertical $R = W$

Force is applied act on θ with the horizontal:-

resolving forces horizentally



$F = P\cos\theta$

$\mu R = P\cos\theta \rightarrow (1)$

Resloving forces vertically.

$W = R + P\sin\theta$

$R = W - p\sin\theta \longrightarrow (2)$

By eqn (1) & (2)

$\mu(W - p\sin\theta) = p\cos\theta$

$\mu W - \mu p\sin\theta = p\cos\theta$

$P(\mu\sin\theta + \cos\theta) = \mu W$

$$P = \frac{\mu W}{\mu\sin\theta + \cos\theta}$$

$\tan\phi = \mu$

$$= \frac{\tan\phi\, W}{\tan\phi\sin\theta + \cos\theta}$$

multiply numerated & denominator with $\cos\phi$

$$= \frac{\sin\phi\, W}{\cos\phi\sin\theta + \cos\theta\cos\phi}$$

$$= \frac{\sin\phi\, W}{\cos(\theta - \phi)}$$

If the value p is less

$\tan\theta = \phi$

$P = \sin\phi\, W$

$\boxed{P = \sin\theta W}$

(1) A small block of weight 50N is resting on a rough horizontal surface. The co-efficient of friction between the block and surface being 0.6. Find the least force which act as on the block at an angle of 60° with the horizontal will ~~come~~ cause the block to side

weight (W) = 50N

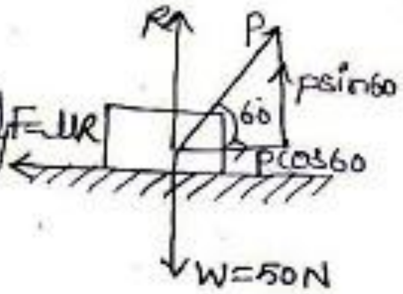
weight (W)= 50N

coefficent of friction (μ)= 0.6

$\theta = 60^\circ$

$P = ?$

Resolving forces horizentally



$F = P\cos 60$

$0.6 \times R = P\cos 60$

$0.6 \times R = P \times 0.5 \rightarrow (1)$

Resolving the forces vertically

$W = R + P\sin 60$

$50 = R + P\sin 60 \longrightarrow (2)$

$R = 50 - P\sin 60 \longrightarrow (3)$

$0.6 \times (50 - P\sin 60) = P\cos 60$

$0.6 \times 50 = P(0.6\sin 60 + \cos 60)$

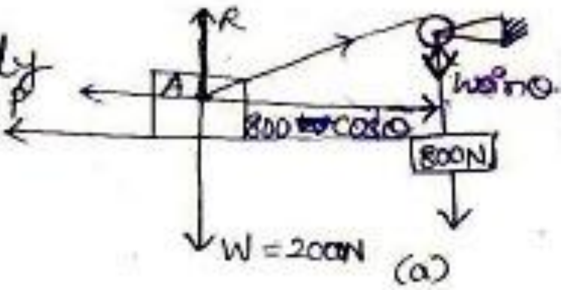
$$P = \frac{0.6 \times 50}{(0.6\sin 60 + \cos 60)} = 57.44N$$

*(1) (2) A block 'A' shown in figure weighs 2000N The cord attached to 'A' pass through a frictionles pulley and supports a weight equal to 800N. The value of coefficient of friction is 0.35 Slove the horizental force P.

(i) If the motion is impending towards the left?

(ii) If the motion is impending towards right?

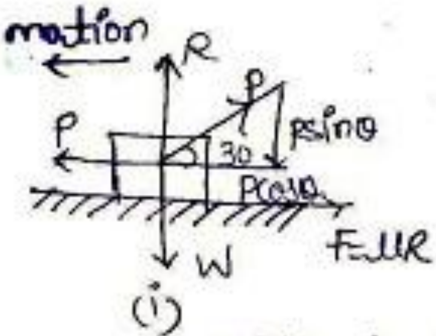
Resolving the force vertically

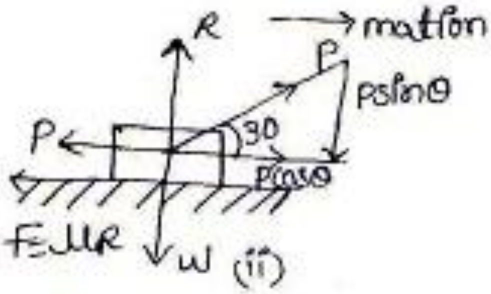


$R = W + 800\sin 30$

$= 2000 + 400$

$= 2400\,N$





Frictional force $(F) = \mu R$

$= 0.35 \times 2400$

$= 840\,N$

(i) motion impending towards left.

$P = F + \cos\theta \times 800$

$= 840 + 800 \times \cos 30$

$= 1532.82\,N$

(ii) motion impending towards right

$P + F = 800\cos\theta$

$P = 800\cos\theta - F$

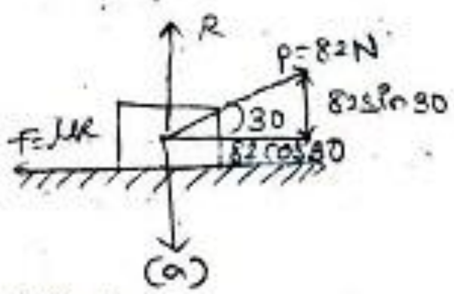
$= 800\cos 30 - 840$

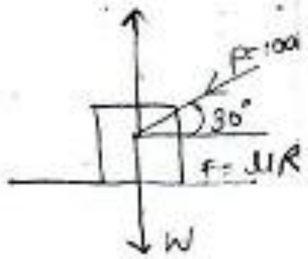
$= -147.17\,N$

$= 147.17\,N$ (toward at left)

A body is resting on rough horizontal plane required a pull of 82N inclined at 30° to the plane just move it. It was found that push of 100N inclined at 30° to the plane just moved the body. Determine weight of the body and the coefficient of friction

sol:-





(i) force $(P)=82$

$\theta=30°$

Resolving forces horizontally

$F=82\cos30$

$\mu\times R=82\cos30 \rightarrow (1)$

$=71.012$

Resolving forces vertically

$W=R+82\sin30$

$R=W-82.\sin30.$

$R=W-41$

$\mu\times(W-41)=71.02 \rightarrow (3)$

(ii) Force $(P)=100N$

$\theta=30°$

Resolving forces horizontally

$F=100\cos30$

$=100\times0.866$

$\mu R=86.6 \rightarrow (4)$

Resolving forces vertically

$R=W+100\sin30.$

$R=W+50 \rightarrow (5)$

$\mu(W+50) = 86.6 \rightarrow ⑥$

$\mu(W-41) = 71.02 \rightarrow ③$

$\mu(W+50) = 86.6 \rightarrow ⑥$

$\mu W + 50\mu = 86.6$

$\mu W - 41\mu = 71.02$

$91\mu = 15.58$

$\mu = \frac{15.58}{91} = 0.171$

$0.171(W-41) = 71.02$

$W = \frac{71.02}{0.171} + 41 = 455.5N.$

4a) A wooden block weighing 30N is placed on horizontal plane, A horizontal force of 12N is applied and the block is on the point of moving?
(i) co-efficient of friction.
(ii) Angle of friction.
(iii) The Resultant reactions.

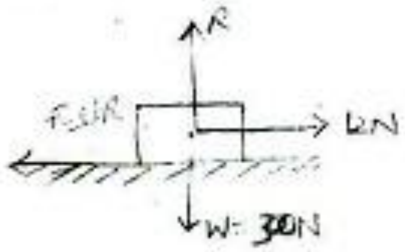
$W = 30N$

$P = 12N$

$\mu = ?$

$\phi\ ?$

$R = ?$



Resolving forces horizontally

$F = 12N$

$\mu R = 12N \rightarrow ①$

Resolving vertical forces

$R = W$

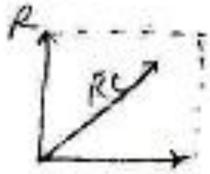
$= 30 \rightarrow ②$

$\mu = 12/R$

$= 12/30$

$= 0.4$

$\tan\phi = \mu$



(ii) $\phi = \tan^{-1}(0.4)$

$= 21.8°$

(iii) $R_C = \sqrt{R^2 + P^2}$

$= \sqrt{30^2 + 12^2}$

$= 32.31\,N$

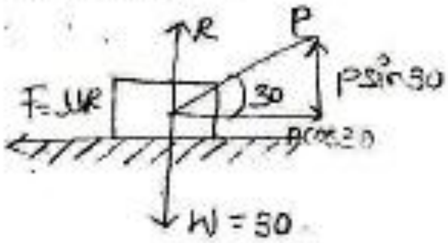
b) a wooden block weight 80N is placed a horizontal plane where the coefficient of friction is 0.25 Find the force that show of 30° with the horizontal to in the condition of equilibrium ?

$W = 80N$

$\mu = 0.25$

$\theta = 30°$

$P = ?$



Reslovíng forces horizontally

$F = P\cos 30$

$\mu \times R = P\cos 30 \rightarrow (1)$

Resolving forces vertically

$W = R + P\sin 30$

$R = W - P\sin 30 \rightarrow (2)$

$0.25(W - P\sin 30) = P\cos 30$

$\mu W - \mu P\sin 30 = P\cos 30$

$\mu W = P(\cos 30 + 0.25 \times \sin 30)$

$$P = \frac{0.25 \times 80}{(\cos 30 + 0.25 + \sin 30)}$$

$$= 23.21$$

Equilibrium of a body on a rough inclined surface:

If the inclination of a plane is more than angle of repose i.e. body will need extreme force to maintain equilibrium the magnitude of force in the following directions will be considered.

(i) The force p is parallel to the plane

(ii) The force p is horizontal

(iii) The force p is inclined at angle θ with the plane.

(i)(a) The force is parallel to the plane and the body tends to slide down:-

Consider a body of weight W lying on rough inclined plane θ be the inclination of the plane $(\theta > \alpha)$ and p be the force applied along the plane.

The forces keeping the body under the equilibrium as shown in figure.

Resolving forces normal to the inclined plane.

$$R = W\cos\alpha$$

Resolving forces along the inclined plane.

$$P = W\sin\alpha + F$$

$$= W\sin\alpha + \mu R \longrightarrow (2)$$

from eqn (1) & (2)

$$P = W\sin\alpha + \mu (W\cos\alpha)$$

$= W \sin\alpha + \tan\phi (W \cos\alpha)$

$= W (\sin\alpha + \tan\phi \cos\alpha)$

$= W (\sin\alpha + \frac{\sin\phi}{\cos\phi} \cos\alpha)$

$= W \frac{(\sin\alpha \cos\phi + \sin\phi \cos\alpha)}{\cos\phi}$

$$\boxed{P = W \left[\frac{\sin(\alpha+\phi)}{\cos\phi}\right]}$$

b) The force is parallel to the plane, the body tends to move up the plane.

When the body tends to move up the plane the force of friction is acting down the plane i.e. in the direction opposite to the applied force. The various forces keeping the body under equilibrium as shown in the figure.

⇒ Resolving forces normal to the plane $R = W \cos\alpha \rightarrow (1)$

Resolving forces along inclined plane

$P = W \sin\alpha + F$

$= W \sin\alpha + \mu R \rightarrow (2)$

from Eq (1) & (2)

$P = W \sin\alpha + \mu (W \cos\alpha)$

$= W \sin\alpha + \tan\phi (W \cos\alpha)$

$= W (\sin\alpha + \tan\phi \cos\alpha)$

$= W (\sin\alpha + \frac{\sin\phi}{\cos\phi} \cos\alpha)$

$= \frac{W (\sin\alpha \cos\phi + \sin\phi \cos\alpha)}{\cos\phi}$ $\Rightarrow P = W \left(\frac{\sin(\alpha+\phi)}{\cos\phi}\right)$

ii)a) The force is horizontal (i.e. parallel to the base) and body tends to slide down:-

consider a body weight w lying on a rough incli-ned plane. Let the force 'p' be applied horizon-tal to prevent the body sliding down. since, the body tends to salide down a frictional force act up on the plane.

The various forces keeping the body under equi-llibrium.

Resalving the forces normal to the plane.

$R = W\cos\alpha + P\sin\alpha \longrightarrow (1)$

Resalving forces along the plane.

$P\cos\alpha + F = W\sin\alpha \longrightarrow$

$P\cos\alpha = W\sin\alpha - \mu R \longrightarrow (2)$

From eqn (1) & (2)

$P\cos\alpha = W\sin\alpha - \mu(W\cos\alpha + P\sin\alpha)$

$P(\cos\alpha + \mu\sin\alpha) = W(\sin\alpha - \mu\cos\alpha)$

$P(\cos\alpha + \tan\phi\sin\alpha) = W(\sin\alpha - \tan\phi\cos\alpha)$

$P\left(\cos\alpha + \frac{\sin\phi}{\cos\phi}\sin\alpha\right) = W\left(\sin\alpha - \frac{\sin\phi}{\cos\phi} \times \cos\alpha\right)$

$P\left(\frac{\cos\alpha\cos\phi + \sin\phi\sin\alpha}{\cos\phi}\right) = W\left[\frac{\sin\alpha\cos\phi - \sin\phi\cos\alpha}{\cos\phi}\right]$

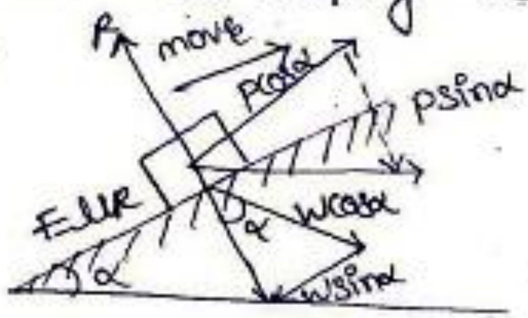
$P\cos(\alpha - \phi) = W\sin(\alpha - \phi)$

$\boxed{P = W\tan(\alpha - \phi)}$

(b) The force is horizontal the body tends to move up the plane:-

when the body tends to move up the plane, the force of friction acts down the plane. Let force P applied horizontally to move the body up the plane.

The various forces keeping the body under equilibrium.



Resolving the forces Normal to the plane.

$R = W\cos\alpha + P\sin\alpha \rightarrow (1)$

Resolving forces along the plane.

$P\cos\alpha = W\sin\alpha + F$

$P\cos\alpha = W\sin\alpha + \mu R \rightarrow (2)$

from Eqn (1) & (2)

$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha)$

$P(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$

$P(\cos\alpha - \tan\phi\sin\alpha) = W(\sin\alpha + \tan\phi\cos\alpha)$

$P\left(\cos\alpha - \frac{\sin\phi}{\cos\phi}\sin\alpha\right) = \left[W\left(\sin\alpha + \frac{\sin\phi}{\cos\phi}\cos\alpha\right)\right]$

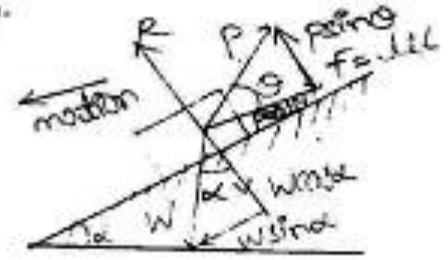
$P\left[\frac{\cos\alpha\cos\phi - \sin\phi\sin\alpha}{\cos\phi}\right] = W\left[\frac{\sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\phi}\right]$

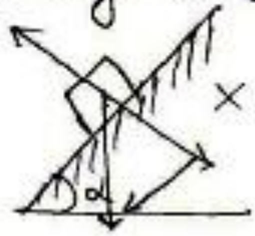
$P\cos(\alpha+\phi) = W\sin(\alpha+\phi)$

$$\boxed{P = W\tan(\alpha+\phi)}$$

(iii) a) the force make an angle θ with the plane and the body tends to slide down:-

Consider a body weigh 'w' lying on a rough inclined plane. Let the force 'p' applied at an angle 'θ' with inclined plane when the body tends to slide down the frictional force act up on the plane the various forces keeping the body under equilibrium.





Resolving forces Normal to the plane.

$$R = W\cos\alpha - p\sin\theta \rightarrow ①$$

Resolving forces along the plane

$$P\cos\theta + F = W\sin\alpha \rightarrow ②$$

$$P\cos\theta = W\sin\alpha - \mu R$$

from ① & ②

$$P\cos\theta = W\sin\alpha - \mu(W\cos\alpha - p\sin\theta)$$

$$P(\cos\theta - \mu\sin\theta) = W(\sin\alpha - \mu\cos\alpha)$$

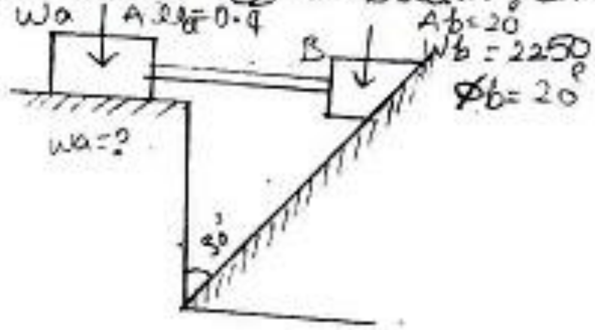
$$P(\cos\theta - \tan\phi\sin\theta) = W(\sin\alpha - \tan\phi\cos\alpha)$$

$$P\left(\frac{\cos\theta\cos\phi - \sin\phi\sin\theta}{\cos\phi}\right) = W\left[\frac{\sin\alpha\cos\phi - \sin\phi\cos\alpha}{\cos\phi}\right]$$

$$P\cos(\theta+\phi) = W\sin(\alpha-\phi)$$

$$P = \frac{W(\sin(\alpha - \phi))}{\cos(\theta + \phi)}.$$

Two blocks A and B are connected by a horizontal rod and supported on two rough planes shown in fig. The coefficient of friction for block A is 0.4 the angle of friction for the blocks on the inclined plane E. is $\phi = 20°$ Find the smallest weight of the block A for which equilibrium can exist?

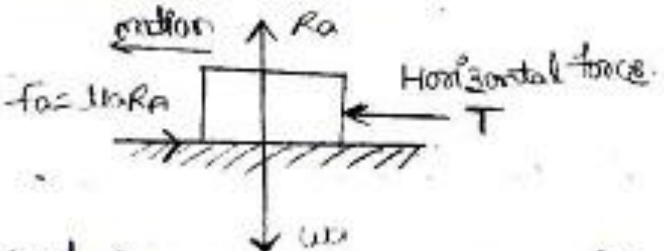


soln:-

$\mu_a = 0.4$ $W_b = 2250$ N

$\phi_b = 20°$ $W_a = ?$

$\mu_b = \tan 20 = 0.36$



Resolving vertical components.

$R_a = W_a \rightarrow ①$

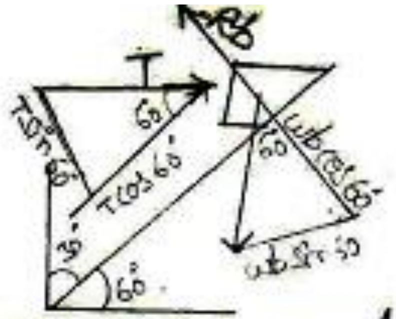
Resolving horizontal components.

$F_a = T$

$\mu_a R_a = T$

$0.4 \times R_a = T \rightarrow ②$

$0.4 \times W_a = T$



Resolving force normal to the inclined plane.

$R_b = w_b \cos 60 + T \sin 60$

$= 2250 \times 0.5 + T \times 0.866$

$= 1125 + 0.866T \rightarrow (3)$

Resolving forces along the plane.

$F_b + T\cos 60 = w_b \sin 60$

$F_b = w_b \sin 60 - T\cos 60$

$= 2250 \times 0.866 - T \times 0.5$

$= 1948.55 - 0.5T$

$\mu_b R_b = 1948.55 - 0.5T$

$0.36 R_b = 1948.55 - 0.5T \rightarrow (4)$

eqn (3) multiply with 0.36

$0.36 R_b = 350.73 + 0.311$

$0.36 R_b = 1948.55 - 0.5T$

$0.36 R_b = 405 - 0.311$

$1543.55 - 0.811T$

$T = \frac{1543.55}{0.811}$

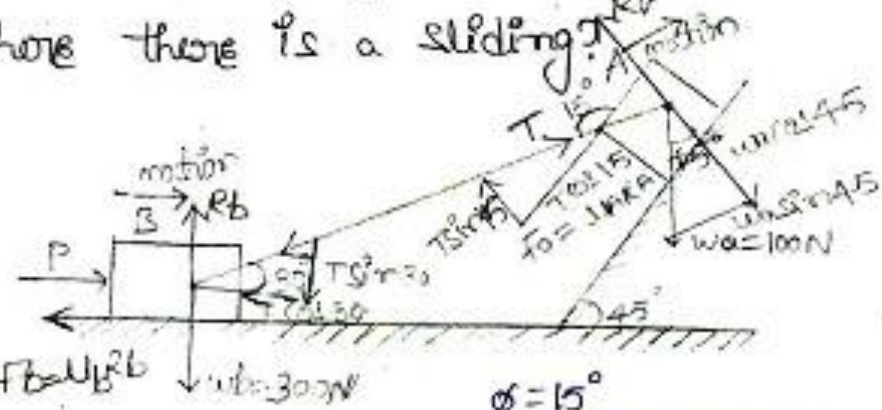
$= 1903.26N$,,

$0.4 \times w_a = T$

$w_a = \frac{1903.26}{0.4}$

$= 4758.15N$

A block A weighing 100N rests on a rough inclined plane whose inclination to the horizontal 40°. The block is connected to another block B weighing 300N resting on a rough horizontal plane, by a weightless rigid bars inclined at 30° to the horizontal find the horizontal force required to be applied to the block B to just move the block A in upward direction.
Assume angle of limiting friction as 15° at all surfaces where there is a sliding?



$\phi = 15°$
$\mu = \tan 15 = 0.25$

considering block B
Resolving forces horizontally

$P = T\cos 30 + Fb$
$= T \times 0.866 + 0.26 \times Rb \rightarrow (1)$

Resolving forces vertically

$Rb = T\sin 30 + wb$
$= T \times 0.5 + 300N \rightarrow (2)$

from Eqn (1) & (2)

$P = T \times 0.866 + 0.26 \times [T \times 0.5 + 300]$
$P = [T \times 0.866 + 0.26 \times T 0.5] + [0.26 [\times T \times 0.5 + 300]$
$P = 78 + 0.996\,T$
$P \simeq = 78 + T$

consider block A

Resolving forces along the plane

$T\cos 15 = W_a \sin 45 + F_a$

$0.96T = 100 \times 0.707 + 0.26 R_a \longrightarrow (3)$

Resolving forces normal to the inclined plane.

$R_a + T\sin 45 = W_a \cos 45$

$R_a = 100 \times 0.707 - 0.259\,T \longrightarrow (4)$

from Eqn (3) & (4)

$0.268 R_a = 18.94 - 0.069T$

$(-)\ 0.26 R_a = -70.7 + 0.96T$

$\cancel{0.26R} = 89.64 - 1.02T$

$T = \dfrac{89.64}{1.02}$

$= 87.88N$

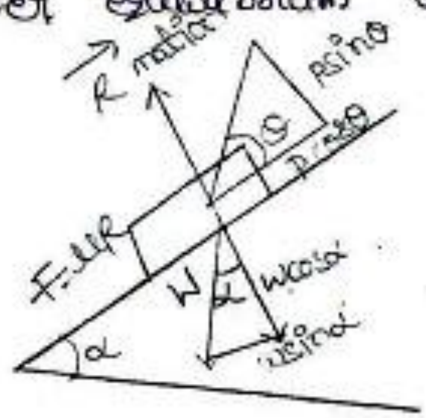
$P = 80.9 + T$

$P = 78 + T$

$= 78 + 87.88$

$= 165.88N.$

the force is applied at an angle θ with the pll-ne and the body tends to move up the plane. When the body is on the point of sliding up the plane. the frictional force acts down the plane. The system of force keeping the body under equilibrium as shown in fig.



Resolving the forces ⊥ to plane.

$R = W\cos\alpha - P\sin\theta \rightarrow (1)$

Resolving the forces along the plane.

$P\cos\theta = W\sin\alpha + F$

$P\cos\theta = W\sin\alpha + \mu R \rightarrow (2)$

from Eqn (1) & (2)

$P\cos\theta = W\sin\alpha + \mu(W\cos\alpha + P\sin\theta)$

$P(\cos\theta + \mu\sin\theta) = W(\sin + \mu\cos\alpha)$

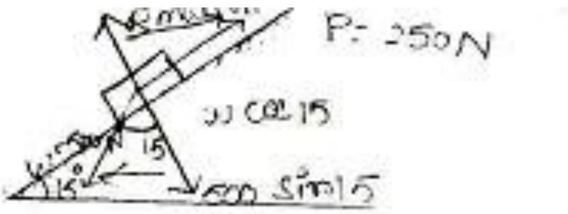
$P(\cos\theta + \tan\phi\sin\theta) = W(\sin\alpha + \tan\phi\cos\alpha)$

$P(\cos\theta\cos\phi + \sin\phi\sin\theta] = W(\sin\alpha\cos\phi + \sin\phi\cos\alpha)$

$P(\cos(\theta-\phi) = W(\sin(\alpha+\phi)$

$\therefore P = W = \dfrac{\sin(\alpha+\phi)}{\cos(\theta-\phi)}$

A force of 250 N pulls a body of wt 500 N up on inclined plane, the force being applied ||el to is find the coefficient of friction



$P = 250N$

$W = 500N$

$\alpha = 15$

$\mu = ?$

||er to 15 fig

Resolving force normal to the plane

$R = W\cos 15$

$= 500\cos 15$

$= 500 \times 0.966$

$R = 483N$

$F = \mu R$

$\mu = F/R$

$= \frac{120.5}{483}$

$= 0.249$

Resloving forces along the plane.

$P = 500 \sin 15 + F$

$F = P - 500 \sin 15$

$= 250 - 500 \times 0.259 = 120.5N$

A body of wt 1000N is to be pulled up an inclined plane of angle 20. Coefficient of friction body & plane is 0.28 find the effect required

a) when it is ||el to plane.

b) " " " ||el to base

c) " " inclined to plane at 10.

sol: $W = 1000N$

$\alpha = 20$

$\mu = 0.28$

$P = ?$

$\tan\phi = \mu$

$\phi = \tan^{-1}\mu$

$= \tan^{-1}(0.28)$

$= 15.64$

$P = W\tan(\alpha + \phi)$

$= 1000 + \tan(35.64)$

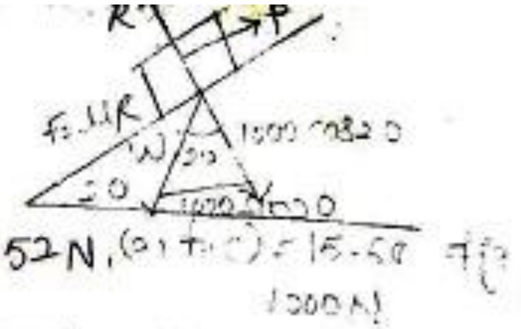
$= 716.98N$

c) $\tan\phi = \mu$

$\phi = \tan^{-1}\mu$

$= \tan^{-1}(0.28)$

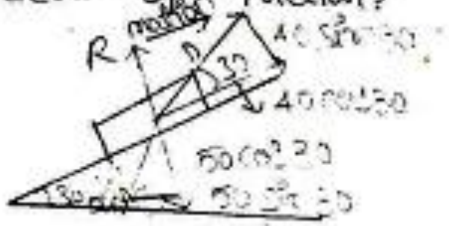
$= 15.64.$



$$P = \frac{W \sin(\alpha+\phi)}{\cos(\theta-\phi)}$$

$$= 1000 \frac{\sin 35.64}{\cos 5.64} = 585.52 N$$

(7) A body weighing 50N is just pulled up on inclined plane of 30° by a force of 40N 30° above the plane. Find the co-efficient of friction?



$W = 50N$
$\alpha = 30°$
$P = 40N$
$\theta = 30°$
$\mu = ?$

Resolving forces normal to the plane

$R = 50\cos 30 - 40\sin 30 = 43.3 - 20 = 23.3N$

Resolving forces along

$40\cos 30 = 50\sin 30 + F$

$= 50\sin 30 + \mu \times 23.3$

$\mu = 40\cos 30 - 50\sin 30 / 23.3$

$= \frac{9.64}{23.3} = 0.418$

(8) Find the least horizontal force 'p' to start motion of any part of system of 3 blocks resting up on one another as in fig. The weight of the blocks are A=300N, B=1000N, C=2000N, B/w A & B $\mu = 0.3$ b/w B & C $\mu = 0.2$ & b/w C & the ground $\mu = 0.1$?

$W_A = 3000N$

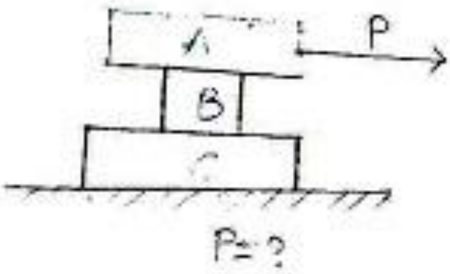
$W_B = 1000N$

$W_C = 2000N$

$\mu_{AB} = 0.3$

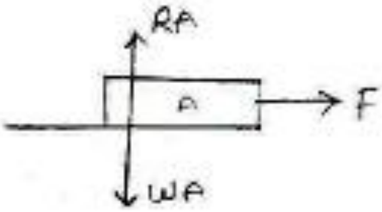
$\mu_{BC} = 0.2$

$\mu_{Ca} = 0.1$



Reslowing forces vertically.

$R_A = W_A = 3000N$

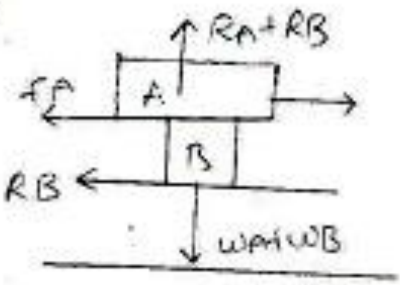


Resloving forces horizontally

$P = F_a$

$= \mu_A R_A$

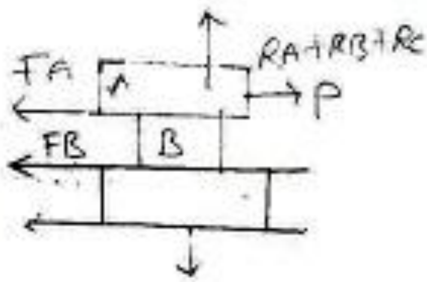
$= 0.3 \times 3000 = 900N$



Resloving forces vertically

$R_A + R_B = 3000 + 1000$

$= 4000$

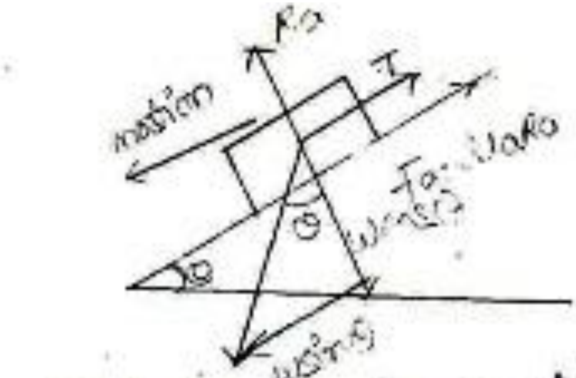


Reslving forces horizontally

$= F_A + F_B + F_C$

$= 0.3 \times 3000 \times 0.2 \times 1000 + 0.1 \times 2000$

Two equal bodies A and B of weight W each are placed on rough inclined plane. the bodies are connected by light string. If $\mu_a = 1/2$ and $\mu_b = 1/3$ show that the bodies both on the point of motion when the plane is inclined at $\tan^{-1}(\frac{5}{12})$



Resolving the forces along the plane.

$T + F_a = W\sin\theta$

$T + \mu_a R_a - W\sin\theta = 0 \rightarrow (1)$

Resolving the force Normal to the plane

$R_a = W\cos\theta$

$R_a - W\cos\theta = 0 \rightarrow (2)$

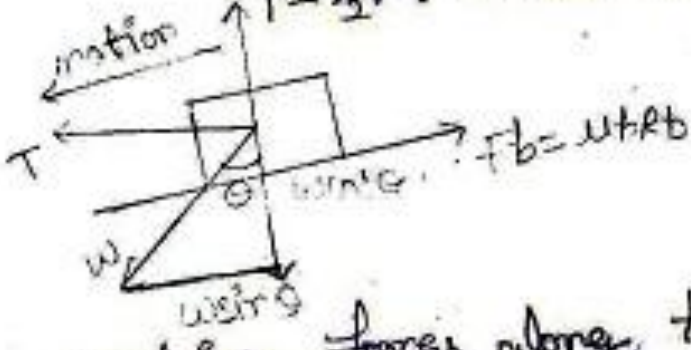
from eqn (1) & (2)

$T + \mu_a R_a - W\sin\theta = R_a - W\cos\theta$

$T + \mu_a R_a - R_a - W\sin\theta + W\cos\theta = 0$

$T + \frac{1}{2} R_a - R_a - W\sin\theta + W\cos\theta = 0$

$T - \frac{1}{2} R_a - W\sin\theta + W\cos\theta = 0 \rightarrow (3)$



Resolving forces along the plane.

$T + W\sin\theta = F_b$

$T + w\sin\theta - \mu_b R_b = 0 \longrightarrow (4)$

Resolving Normal to the plane

$R_b = W\cos\theta$

$R_b - w\cos\theta = 0$ — (5)

from (5) & (4)

$T + w\sin\theta - \mu_b R_b = R_b - w\cos\theta$

$T + w\sin\theta - \frac{1}{3}R_b - R_b + w\cos\theta = 0$

$T + w\sin\theta - \frac{4R_b}{3} + w\cos\theta = 0 \longrightarrow (6)$

from (3) & (6)

$T - \frac{1}{2}R_a - w\sin\theta + w\cos\theta = T + w\sin\theta - \frac{4R_b}{3} + w\cos\theta$

$(R_a = R_b)$

$-\frac{1}{2}R_a + \frac{4R_a}{3} - w\sin\theta - w\sin\theta = 0$

$\frac{-3R_a + 8R - 12\,w\sin\theta}{6} = 0$

$5R_a - 12\,w\sin\theta = 0$

$5(w\cos\theta) - 12\,w\sin\theta = 0$

$5w\cos\theta = 12\,w\sin\theta$

$\frac{5}{12} = \frac{\sin\theta}{\cos\theta}$

$\frac{5}{12} = \tan\theta$

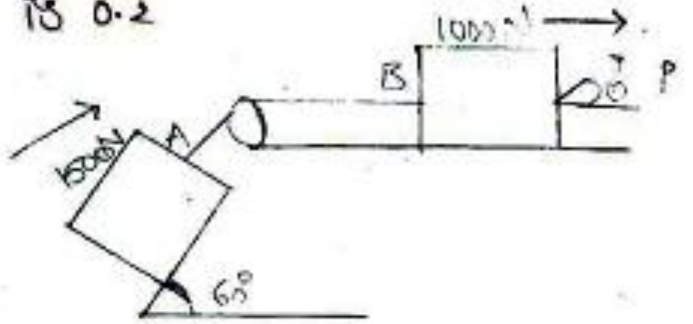
$\theta = \tan^{-1}\left(\frac{5}{12}\right)$.

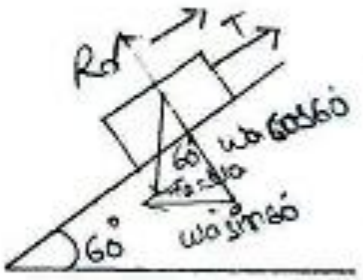
* Referring the figure below determine least value of force p to cause the motion to impend right word Assume co-efficient of friction under the blocks to be 0.2 and the pulley to be frictions?

μ for blocks is 0.2

$w_a = 1500N$

$w_b = 1000N$





Resolving the forces normal to the plane.

$R_a = w_a \cos 60$

$= 1500 \times 0.5$

$R_a = 750N$

Resolving the forces along the plane.

$T = f_a + w_a \sin 60$

$= \mu_a R_a + 1500 \times 0.866$

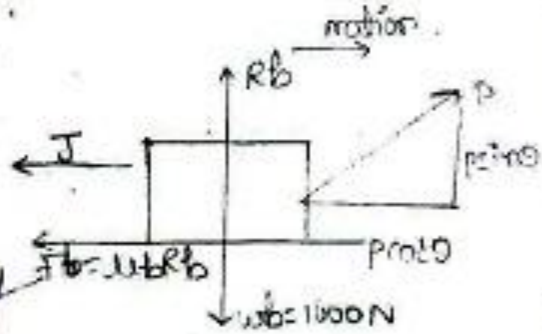
$= 0.2 \times 750 + 1500 \times 0.866$

$= 1449 N$

Resolving forces vertically.

$R_b + p\sin\theta = w_b$

$R_b = 1000 - p\sin\theta$



Resolving forces horizontally

$T + F_b = P\cos\theta$

$\mu_b R_b = 8\cos\theta$

$T + 0.2 \times (1000 + p\sin\theta] = P\cos\theta$

$T + 200 - 0.2 p\sin\theta = P\cos\theta$

$T + 200 = p\cos\theta + 0.2 p\sin\theta$

$T + 200 = P(\cos\theta + 0.2\sin\theta)$

$P = \frac{200 + 1449}{(\cos\theta + 0.2\sin\theta)} = \frac{1649}{\cos\theta + 0.2\sin\theta}$

To get the values of denominator & different

$w \cdot r + \theta$

$\frac{d}{d\theta}(\cos\theta + 0.2\sin\theta) = 0$

$-\sin\theta + 0.2\cos\theta = 0$

$0.2\cos\theta = \sin\theta$

$0.2 = \frac{\sin\theta}{\cos\theta}$

$0.2 = \tan\theta$

$\theta = \tan^{-1}(0.2)$

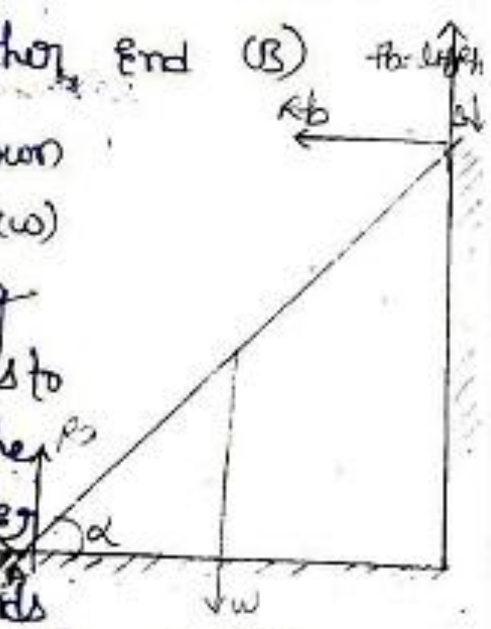
$\theta = 11.31$

$P = \frac{1649}{\cos 11.37 + 0.2\sin 11.31}$

$\boxed{P = 1681.88N}$

Ladder friction:-

Consider a ladder AB one, end of the ladder (A) is lying on the ground and other end (B) is leaning against wall as shown in figure. Due to the self-weight (w) weight of the woman standing on the ladder the End B tends to slip down words and hence the friction force of b/w the ladder and wall surface acts towards right as the End A tends to slid to left



For Equilibrium of ladder the algebric sum of vertical and horizontal forces be zero and al[illegible] algebric sum of moments about a point must be zero.

A ladder 5m long and 250N weight is placed again -a vertical wall in a position where its inclination to the vertical is 30°. A man weighing 800N claim the ladder. At what position will be induced slipping? The coefficient of friction for both contact surfaces of the ladder i.e, with the wall and the floor is 0.2

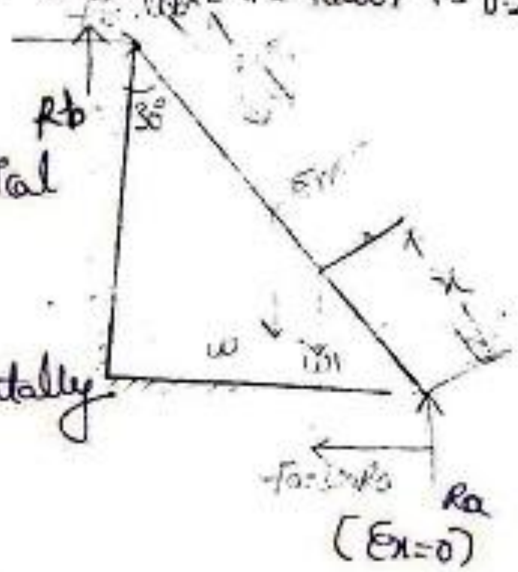
$L = 5m$ Rb

$W = 250N$

30° inclination to the vertical

$W_1 = 800N$

$\mu = 0.2$



Resolving the forces horizontally

$R_b = \mu_a R_a$

$= 0.2 \times R_a \rightarrow (1)$

Resolving forces vertically.

$R_a + F_b = w + w_1$

$R_a + 0.2 R_b = 250 + 800 \rightarrow (2)$

from (1) & (2)

$R_a + 0.2 \times 0.2 R_a = 1050$

$R_a(1 + 0.04) = 1050.$

$R_a = \frac{1050}{1.04}$

$= 1009.6N$

$R_b = 0.2 \times R_a$

$= 0.2 \times 1009.6$

$= 201.9N$

Taking moments about A.

$W \times 2.5\cos 60 + w \times x \cos 60 = Fb\, 5\cos 60 + Rb\, 5\sin 60$

$250 \times 2.5 \times 0.5 + 800x \times 0.5 = 0.2 \times 201.9 \times 0.5 + 201.9 \times 5 \times 0.866$

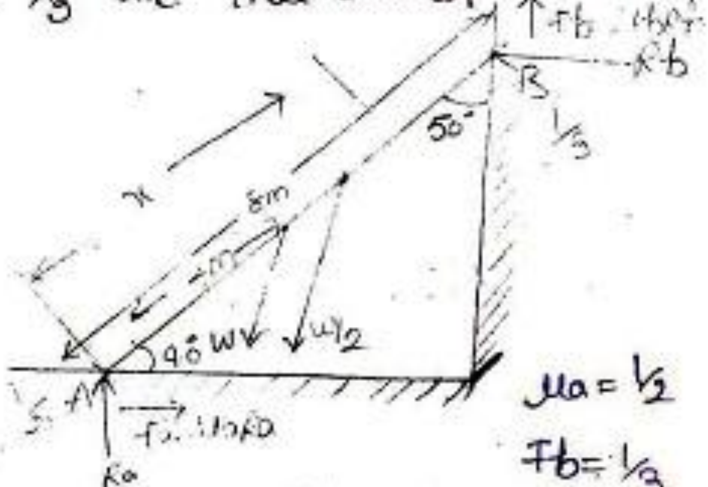
$400x = (0.2 \times 201.9 \times 0.5 + 201.9 \times 0.5 + 201.9 \times 5 \times 0.866 - 250 \times 2.5 \times 0.5$

$400x = 975.25 - 3125$

$x = 1.45m.$

*. V.V.I -

AB in long ladder rest against a vertical wall mak-ing an angle of 50° with the wall and resting on a floor. If a body whose weight is one half with the ladder climbs it, at whose distance along the ladder will be when the ladder is about to slip? The coefficient of friction b/w the ladder and wall is 1/3 and that ladder and floor is 1/2.



$\mu a = 1/2$

$Fb = 1/3$

Resolving forces vertically.

$Ra + Fb = W + W/2$

$Ra + \mu b Rb = W + W/2$

$Ra + \frac{1}{3}Rb = \frac{3w}{2} \rightarrow (1)$

Resolving forces horizontally

$RB = Fa$

$RB = \mu_a Ra$

$= \frac{1}{2}Ra \rightarrow (2)$

from (1) & (2) eq^n.

$Ra + \frac{1}{3} \times \frac{1}{2}Ra = \frac{3w}{2}$.

$\frac{6Ra + Ra}{6} = \frac{3w}{2}$

$7Ra = \frac{6 \times 3w}{2}$

$Ra = \frac{9 \times w}{7}$

$Fa = \mu_a Ra$

$= \frac{1}{2} \times \frac{9}{7}w$

$= \frac{9}{14}w$

Taking moments from 'A'

$w4\cos 40 + \frac{w}{2} x \cos 40$.

$= RB\ 8\cos 40 + Fb\ 8\ \overset{\sin}{\cancel{\cos}} 40$.

$w(4\cos 40 + \frac{1}{2}x\cos 40) = \frac{9}{14}w \times 8\cos 40 + \frac{1}{3} \times \frac{9w}{14}$ ...

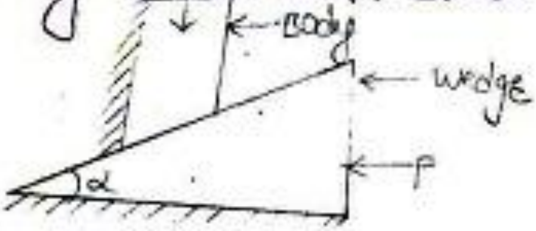
$\cancel{w}(4\cos 40 + \frac{1}{2} \times x\cos 40] = \cancel{w}[\frac{9}{14} \times 8\cos 40 + \frac{9}{42} \times 8\sin 40]$

$4 \times 0.766 + x \times 0.383 = 5 \times 0.6428 + 1.714 \times 0.642$

$x = \frac{[3.3056 + 1.313]}{0.383}$

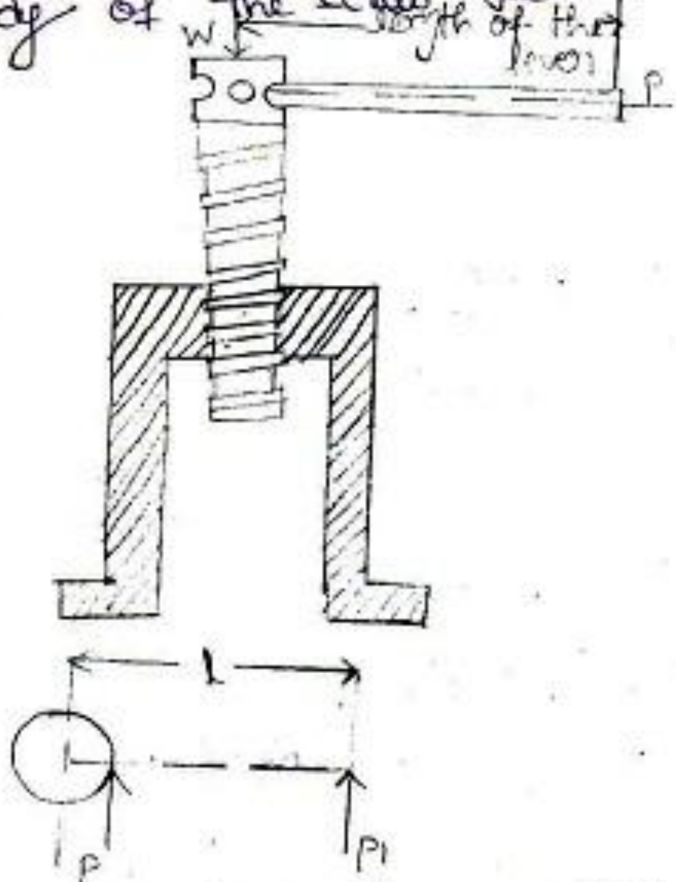
$x = 8.26$

Wedge Friction:– the wedge is a piece of metal or wood with triangular or trapezoidal cross ~~friction~~ section. It is used splitting devices and lift or adjust the heavy load with small displacement.



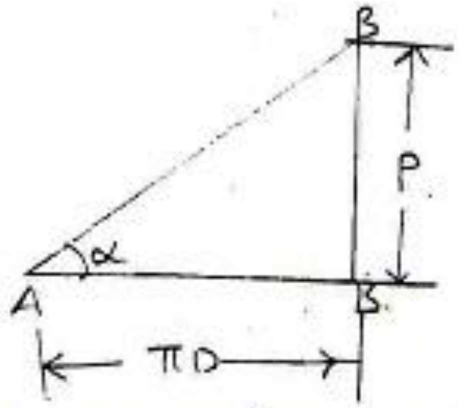
Screw friction:–

the general form of simple screw jack is shown in figure. It consists of vertical screw and nut. the load rest on the screw head and the nut forms the body of the screw jack.



P = effort at mean radius

P1 = effort at the end of the lever.



the hight of the plane BC is the distance moved axially in one revelution of the screw in its nut. i,e. pitch (P) the base of the plane (AB). is circumference of the thread at the mean radius i,e, πD, where D is mean thead diameter.

the angle α of the plane is given by

$\tan\alpha = P/\pi D$

for double start thread.

$$\tan\alpha = \frac{2P}{\pi D}$$

for n-start

$$\tan\alpha = \frac{nP}{\pi D}$$

Let

W = axial load on screw

P = Tangental force required at mean radius to turn the screw.

consider the effect p in two cases

(i) Load being raised.

(ii) Load being raised.

Effort required at radius to lift the load. $P = W \tan(\alpha + \phi)$

Torque required to rotate the screw against the load $T = P \times R_m$

$$\boxed{T = W \times \tan(\alpha + \phi) \times R_m}$$

where R_m = mean radius $D/2$

Effect required at the end of the lever.

$P_1 \times l = P \times R_m$

$$\boxed{P_1 = \frac{W \times R_m \times \tan(\alpha + \phi)}{l}}$$

(ii) Load being lowered effort required lowering a load at mean radius.

$$\boxed{P = W \times \tan(\alpha - \phi)} \text{ if } (\alpha > \phi)$$

$$\boxed{= W \times \tan(\phi - \alpha)} \text{ if } (\phi > \alpha)$$

Effort required at the end of the lever

$$\boxed{P_1 = \frac{P \times R_m}{l}}$$

Efficiency of screw jack :-

$$\eta = \frac{W \times P}{P \times \pi D}$$

$$\frac{W}{P} = \frac{1}{\tan(\alpha + \phi)}, \quad \frac{P}{\pi D} = \tan\alpha$$

$$\boxed{\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)}}$$

when load falls reverse efficiency used

$$\boxed{\eta = \frac{\tan(\alpha - \phi)}{\tan\alpha}}$$

maximum efficiency

$$\boxed{\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}}$$

① *① A screw jack has a square thread of mean diameter 6cm and pitch 0.8cm. The coefficient of friction at the screw thread is 0.09. A load of 14 kN is to be lifted through 15cm. Determine the torque required and work done in lifting the load through 15cm. Find the efficiency of the jack also?

$D_m = 6cm$

$R_m = 6/2 = 3cm$

$0.03m$

$P = 0.8cm$

$\mu = 0.09$

$W = 14 KN$

~~torque~~ $P = w\tan(\alpha+\phi)$

$$\tan\alpha = \frac{P}{\pi D}$$

$$= \frac{0.8}{\pi \times 6} = 0.0424$$

$= \tan^{-1} 0.0424$

$\alpha = 2.43°$

$= w\tan(\alpha+\phi)$

$= 14000 \tan(2.43+5.14)$

$= 1860.53$

$T = P \times R_m$

$= 1860.53 \times 0.03$

$= 55.816$ N-m.

$\mu = \tan\phi$

$\phi = \tan^{-1}\mu$

$= \tan^{-1}(0.09)$

$= 5.14$.

work done $= 2\pi N T$

$N = \frac{15}{P} = \frac{15}{0.8} = 18.75$ revolution.

$W = 2 \times \pi \times 18.75 \times 55.816$

$= 6575.6$ N-m

efficiency $(\eta) = \frac{\tan\alpha}{\tan(\alpha+\phi)}$

$= \frac{\tan 2.43}{\tan(2.43+5.14)} \times 100$

$= 31.932\%$

② A single threaded screw jack has a pitch of 12mm and mean radius of 45mm. The coefficient of static friction is 0.15, and kinetic friction is 0.1

(a) Determine the force P applied at the end of the lever 600mm long which will start lifting a weight of 20,000N.

b) what value of P will keep the screw jack turning

g. $R_m = 45$mm

$P = 12$mm $D_m = 2 \times R_m$

$= 90$mm

$\mu_s = 0.15$

$\mu_k = 0.1$

$P_1 = ?$

$l = 600$mm

$W = 20000$N

$$\tan\alpha = \frac{P}{\pi D} = \frac{12}{\pi \times 90} = 0.092$$

$$\alpha = \tan^{-1} 0.092$$
$$= 2.4$$

$$\mu s = 0.15$$
$$\phi = \tan^{-1} 0.15$$
$$= 8.53^\circ$$

a) $P = W \tan(\alpha + \phi)$

$$= 20000 \tan(2.4 + 8.53)$$
$$= 3862.25 N$$

$$P_1 = \frac{P \times Rm}{l}$$

$$= \frac{3862.25 \times 45}{600}$$

$$P_1 = 289.66 N$$

b) $\mu K = 0.1$

$$\alpha K = \tan^{-1} \mu K$$
$$= \tan^{-1} 0.1$$
$$= 5.71$$

$$P = W \tan(\alpha + \phi K)$$
$$= 20000 \times \tan(2.4 + 5.71)$$
$$= 2849.98 N$$

$$P_1 = \frac{P \times Rm}{l} = \frac{2849.98 \times 45}{600}$$

$$= 213.74 N$$

Centre of gravity:-

Centre of gravity of a body is defined as the point through which resultant of the gravitational force (weight) acts. consider that the gravitational forces acting on the various particles of the body repres -ent a system of parallel forces the resultant of these parallel forces. is called resultant gravitati -onal force or weight of the body (ω) and acts through a point this point is called centre of gravity. The position of centre of gravity can be determined by applying the principle of moments let the weight of indivual particles are w_1 and w_2 and their coordinates are $(x_1 \cdot y_1)$ and (x_2, y_2) respectively. the coordinates of resultant gravi tational force (W) are $(\bar{X} \cdot \bar{Y})$

Taking moments about Y-axis

$W\bar{x} = w_1x_1 + w_2x_2$

the position of C.G from Y-axis

$$\bar{x} = \frac{w_1x_1 + w_2x_2}{W} = \frac{\Sigma wx}{\Sigma W}$$

The position of G.G from x-axis

$$\bar{Y} = \frac{w_1y_1 + w_2y_2}{W} = \frac{\Sigma x}{\Sigma W}$$

Centroid:-

the centre of gravity of thin plate of uniform thickness and homogeneous material is replaced with centroid or centre of area. centroid is defi-nd as a point where a whole area of a plane figure assumed to be concentrated.

$W =$ specific weight $\times$ volume

$= w \times v$

$= (\rho g) \times A \times t$

for homogenous material of uniform thickness, t and ρ are constants.

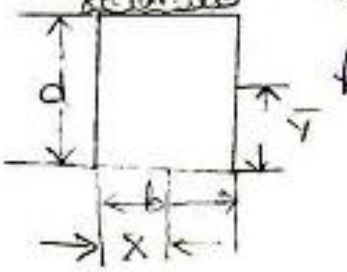
position of centroid from y-axis.

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a} = \frac{\Sigma a x}{a}$$

Position of centroid from x-axis

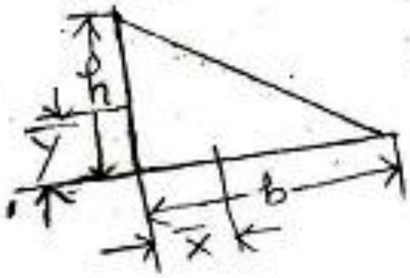
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a} = \frac{\Sigma a y}{\Sigma a}$$

Centroids of plane geometrical shapes:-

shape:-	Area	$\bar{X}$	$\bar{Y}$
Rectangle 	$b \times d$	$b/2$	$d/2$
square 	b^2	$b/2$	$b/2$

Triangle:-

Area	$\bar{x}$	$\bar{y}$
$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{h}{3}$

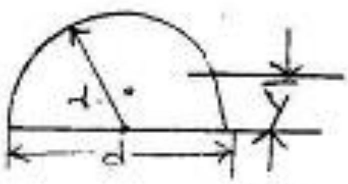


Circle:-

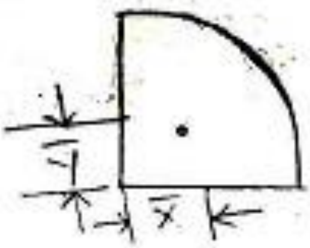
πr^2 $\quad \frac{d}{2}$ $\quad \frac{d}{2}$

Semicircle:-

Area	$\bar{x}$	$\bar{y}$
$\frac{\pi r^2}{2}$	$\frac{d}{2} = r$	$\frac{4r}{3\pi}$



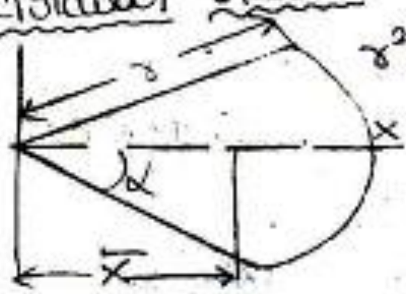
Quadrant circle:-



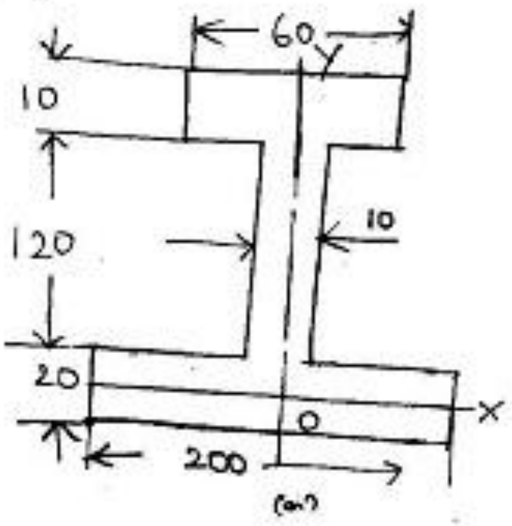
$\frac{\pi r^2}{4}$ $\quad \frac{4r}{3\pi}$ $\quad \frac{4r}{3\pi}$

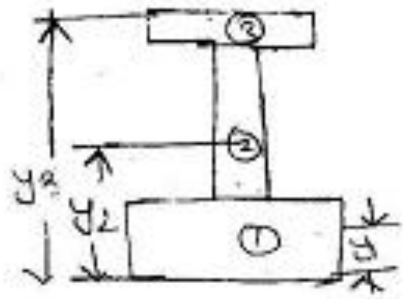
Circular sector:-

$r^2\alpha$ $\quad \frac{2}{3}\frac{\sin\alpha}{\alpha}$ $\quad 0$



Determine the centroid of I- section of the Dimensions in mm Bottom flange = 200X20 Top flange = 60X10 web = 120X10.





$a_1 = 200 \times 20 = 4000 \text{ mm}^2$

$y_1 = d/2 = 20/2 = 10 \text{mm}$

$a_2 = 120 \times 10 = 1200 \text{ mm}^2$

$y_2 = d/2 + 20$

$y_2 = \frac{120}{2} + 20 = 80 \text{mm}$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$

$a_3 = 10 \times 60 = 600 \text{mm}^2$

$y_3 = d/2 + 120 + 20$

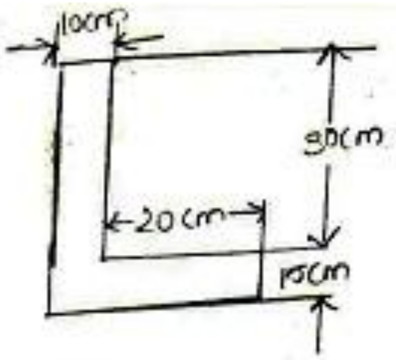
$= 10/2 + 120 + 20$

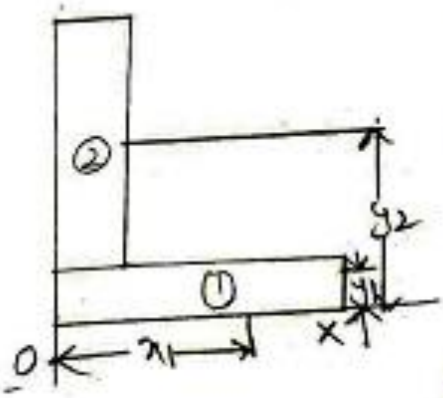
$= 145 \text{mm}$

$= \frac{(4000 \times 10) + (1200 \times 80) + (600 \times 145)}{4000 + 1200 + 600}$

$= 38.44 \text{mm}$

Find the Centroid of Plane lamina





$a_1 = 15 \times 30 = 450\ cm^2$

$(x_1 = 30/2 = 15,\ y_1 = 15/2 = 7.5)$

$a_2 = 10 \times 30 = 300\ cm^2$

$x_2 = 10/2 = 5,\ y_2 = 30/2 + 15 = 30$

Centroid from OY

$$\bar{X} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{(450 \times 15) + (300 \times 5)}{(450 + 300)}$$

$= 11\ cm.$

Centriod from OX

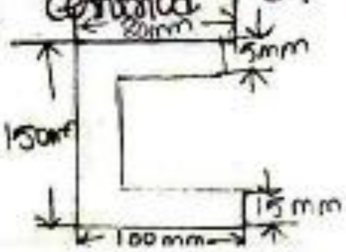
$$\bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

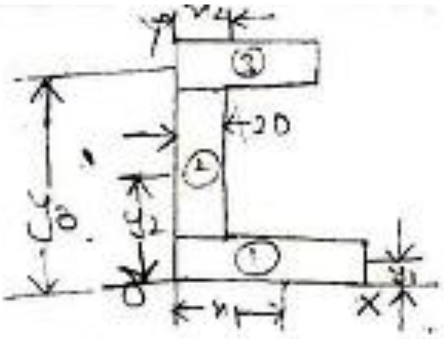
$$= \frac{(450 \times 7.5) + (300 \times 30)}{(450 + 300)}$$

$= 16.5$

$(\bar{X}, \bar{Y}) = (11, 16.5).$

Find the Centriod of plane lamina.





$a_1 = 100 \times 15 = 1500\ mm^2$

$x_1 = \frac{100}{2} = 50,\ y_1 = \frac{15}{2} = 7.5$

$a_2 = 120 \times 20 = 2400\ mm^2$

$x_2 = \frac{20}{2} = 10,\ y_2 = \frac{120}{2} + 15 = 75$

$a_3 = 80 \times 15 = 1200\ mm^2$

$x_3 = \frac{80}{2} = 40,\ y_3 = \frac{15}{2} + 135 = 142.5$

Centroid from oy

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$\frac{(1500 \times 50) + (2400 \times 10) + (1200 \times 40)}{(1500 + 2400 + 1200)}$$

$$= 28.82\ mm.$$

Centroid from ox

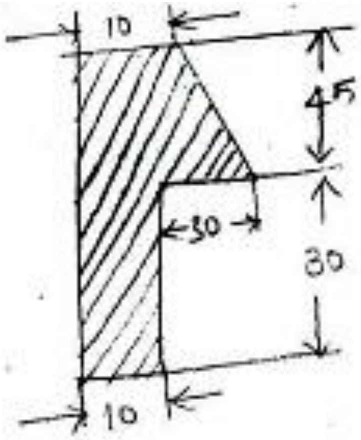
$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

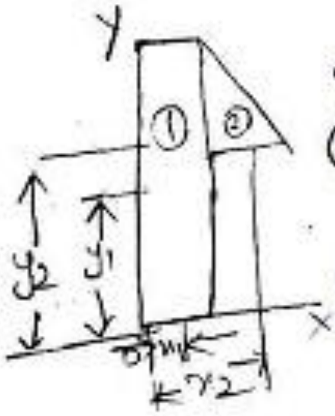
$$= \frac{(1500 \times 7.5) + (2400 \times 75) + (1200 \times 142.5)}{(1500 + 2400 + 1200)}$$

$$= 71.02\ mm.$$

$(\bar{X}, \bar{Y}) = (28.82, 71.02)$

For the shaded area shown in figure determine the coordinates of centroid w.r.t X and Y axis. All dimensions are in cm.





$a_1 = 75 \times 10 = 750\,cm^2$

$(x_1 = \frac{10}{2} = 5,\ y = \frac{75}{2} = 37.5$

$a_2 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 30 \times 45$

$= 675\,cm^2$

$x_2 = \frac{b}{3} + 10 = \frac{30}{3} + 10 = 20$

$y_2 = \frac{d}{3} + 30 =$

$= \frac{45}{3} + 30 = 45$

centroid from OY

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{(750 \times 5) + (675 \times 20)}{(750 + 675)}$$

$= 12.1\,cm$

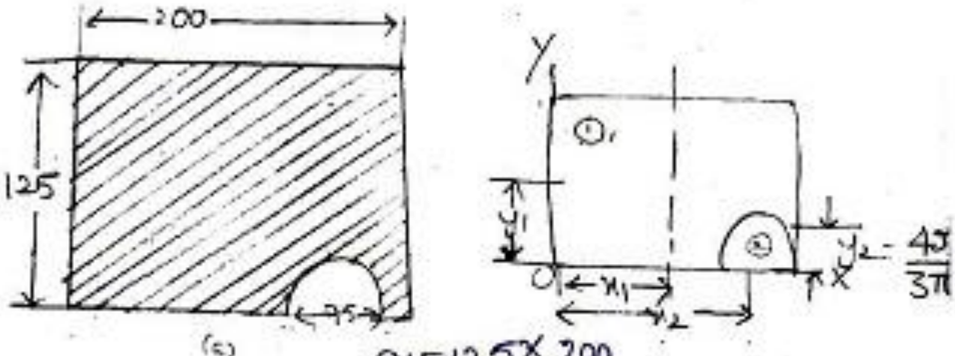
centroid from OX

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(750 \times 37.5) + (675 \times 45)}{(750 + 675)} = 41.05\,cm.$$

$(\bar{X}, \bar{Y}) = (12.1, 41.05)$

Determine the coordinate of the centroid of the shade area as shown in figure. If the area removed is semi circular. All dimensions are in mm.



$a_1 = 125 \times 200$

$= 25000\ mm^2$

$x_1 = \frac{200}{2} = 100mm$

$y_1 = \frac{125}{2} = 62.5$

$a_2 = \frac{\pi r^2}{2} = \frac{\pi \times (37.5)^2}{2}$

$= 2208.93\ mm^2$

$x_2 = \frac{75}{2} + 125 = 162.5,$

$y_2 = \frac{4r}{3\pi} = \frac{4 \times 37.5}{3 \times \pi} = 15.4$

centroid from OY

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{(25000 \times 100) - (2208.93 \times 162.5)}{25000 - 2208.93}$$

$= 93.94$ mm.

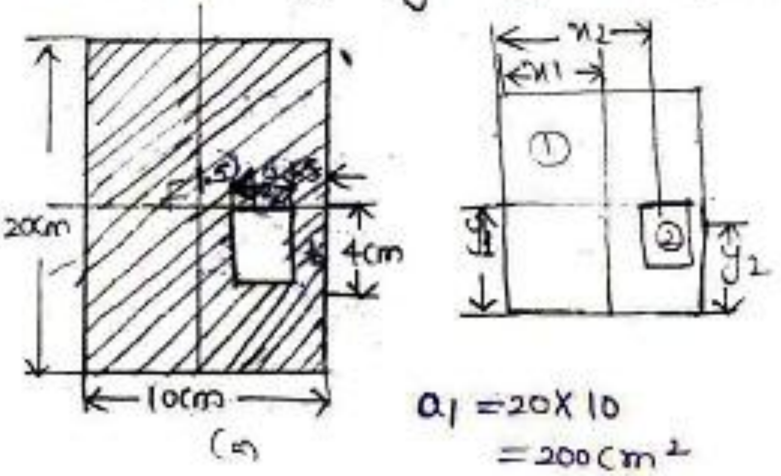
Centroid from OX

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$= \frac{(25000 \times 62.5) - (2208.93 \times 15.91)}{(25000 - 2208.93}$$

$= 67.01 mm.$

From a rectangular lamina shown in figure of dimensions 10x20cm, a rectangular hole of 2cm X 4cm is cut find the centre of gravity of the remainder.



$a_1 = 20 \times 10$
$= 200\, cm^2$
$x_1 = 10/2 = 5cm$
$y_1 = 20/2 = 10\, cm$

$a_2 = 4 \times 2$
$= 8\, cm^2$
$x_2 = 2/2 + 1.5 + 10/2$
$= 7.5\, cm$
$y_2 = 20/2 - 4/2$
$= 10 - 2 = 8cm$

Centroid from oy

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}$$

$$= \frac{(200 \times 5) - (8 \times 7.5)}{200 - 8}$$

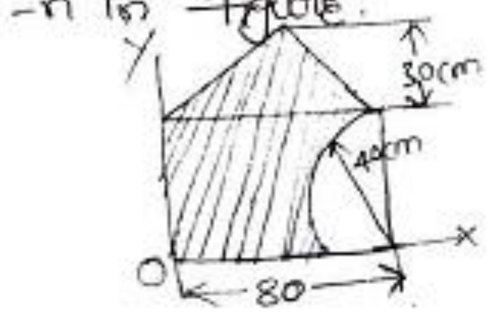
$= 4.89cm$

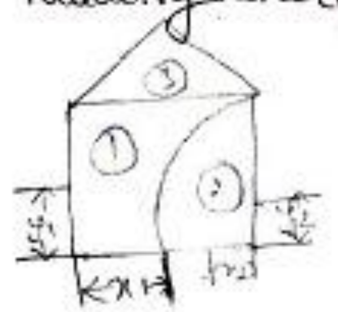
Centroid from ox

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

$$= \frac{(200 \times 10) - (8 \times 8)}{200 - 8} = 10.08\, cm$$

Find the Centroid of the following shaded area shown in figure.





$a_1 = 80 \times 40$

$= 3200 \text{ cm}^2$

$(x_1 = 80/2 = 40, \; y_1 = 40/2 = 20)$

$a_2 = \frac{\pi r^2}{4}$

$= \frac{\pi \times (40)^2}{4}$

$= 1254.64 \text{ cm}^2$

$x_2 = 80 - \frac{4r}{3\pi}$

$= 80 - \frac{4 \times 40}{3 \times \pi}$

$= 63.02 \text{ cm}^2.$

$y_2 = \frac{4r}{3\pi}$

$= \frac{4 \times 40}{3 \times \pi}$

$= 16.97 \text{ cm}.$

$a_3 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 80 \times 30$

$= 1200 \text{ cm}^2$

$x_3 = \frac{b}{2} = \frac{80}{2} = 40$

$y_3 = \frac{h}{3} + 40$

$= 30/3 + 40 = 50$

Centroid from OY

$$\bar{X} = \frac{a_1x_1 - a_2x_2 + a_3x_3}{a_1 - a_2 + a_3}$$

$$= \frac{(3200\times40) - (1254.64\times63.02) + (1200\times40)}{(3200 - 1254.64 + 1200)}$$

$= 30.81$ cm.

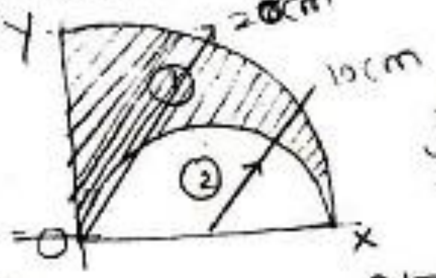
Centroid from OX

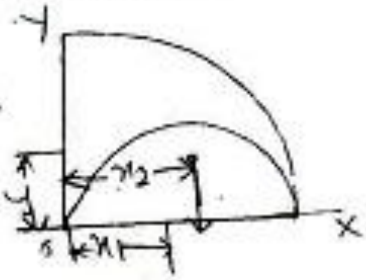
$$\bar{Y} = \frac{a_1y_1 - a_2y_2 + a_3y_3}{a_1 - a_2 + a_3}$$

$$= \frac{(3200\times20) - (1254.64\times16.97) + (1200\times50)}{(3200 - 1254.64 + 1200)}$$

$= 32.65$ cm

Locate the centroid of the shaded area as shown in the figure.





$a_1 = \frac{\pi r^2}{4}$

$= \frac{\pi \times (20)^2}{4}$

$= 314.15$ cm^2

$x_1 = \frac{4\times r}{3\pi}$

$= \frac{4\times20}{3\times\pi}$

$= 8.48$ mm

$y_1 = \frac{4r}{3\pi}$

$y_1 = 8.48$ mm

$a_2 = \frac{\pi r^2}{2}$

$= \frac{\pi\times10^2}{2}$

$= 157.07$ cm^2

$x_2 = \frac{d}{2} = r$

$= 10$cm

$y_2 = \frac{4r}{3\pi}$

$= \frac{4\times10}{3\times\pi}$

$= 4.24$ cm.

centriod from oy

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}$$

$$= \frac{(314.15 \times 8.48) - (157.07 \times 10)}{(314.15 - 157.07)}$$

$$= 6.96 \text{ cm.}$$

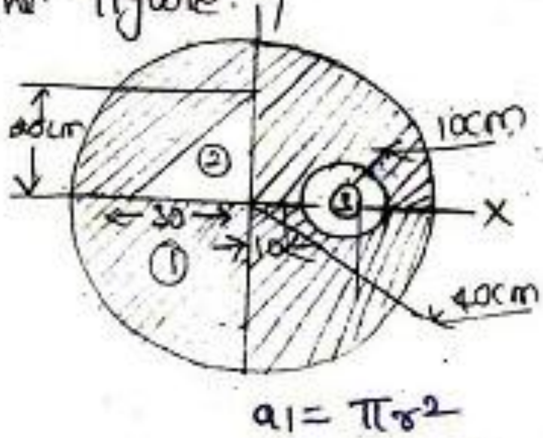
Centriod from ox

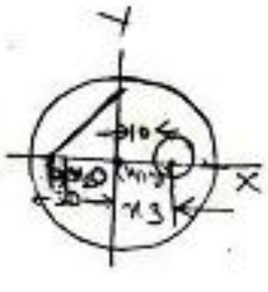
$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

$$= \frac{(314.15 \times 8.48) - (157.07 \times 4.24)}{(314.15 - 157.07)}$$

$$= 12.71 \text{ Cm}$$

Find the Centroid of the shaded area shown in the figure.





$a_1 = \pi r^2$

$= \pi \times (40)^2$

$= 5026.55 \text{cm}^2$

$(x_1 = 0, y_1 = 0)$

$a_2 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 30 \times 25$

$= 375 \text{ cm}^2$

$x_2 = -b/3 = 30/3 = (10\text{cm})$

$y_2 = h/3 = 25/3 = 8.33$

$a_3 = \pi r^2$

$= \pi \times (10)^2$

$= 314.15 \, cm^2$

$x_3 = \frac{d}{2} + 10$

$= \frac{20}{2} + 10$

$= 20 cm$

$y_3 = 0$

centriod from oy

$$\bar{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3}$$

$$= \frac{(5026.55 \times 0) - (375 \times (10)) - (314.15 \times 20)}{(5026.55) - (375) - (314.15)}$$

$= 8.584 \, cm.$

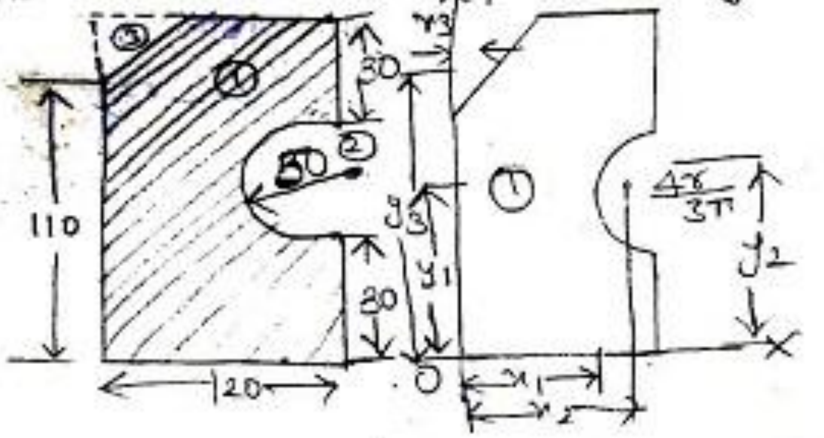
centroid from ox

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3}$$

$$= \frac{(5026.55 \times 0) - (375 \times 8.33) - (314.15 \times 0)}{(5026.55 - 375 - 314.15}$$

$= -0.72 \, cm.$

Locate the centroid of the following figure.



$a_1 = 120 \times 160$

$= 19200\,mm^2$

$x_1 = \frac{b}{2} = \frac{120}{2} = 60.$

$y_1 = \frac{d}{2} = \frac{160}{2} = 80.$

$a_2 = \frac{\pi \times r^2}{2}$

$= \frac{\pi \times (50)^2}{2}$

$= 3926.99\,mm^2$

$x_2 = 120 - \frac{4r}{3\pi}$

$= 120 - \frac{4 \times 50}{3\pi}$

$= 98.78\,mm.$

$y_2 = 30 + d/2$

$= 30 + 50$

$= 80mm.$

$a_3 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 40 \times 50$

$= 1000\,mm^2$

$x_3 = \frac{b}{3} = \frac{40}{3}$

$= 13.33\,mm$

$y_3 = 160 - \frac{d}{3}$

$= 160 - \frac{50}{3}$

$= 143.33\,mm.$

Centroid from OY.

$\bar{x} = \frac{a_1 x_1 - a_2 x_2 - a_3 x_3}{a_1 - a_2 - a_3}$

$= \frac{(19200 \times 60) - (3926.99 \times 98.78) - (1000 \times 13.33)}{19200 - 3926.99 - 1000}$

$= 52.6\,mm$

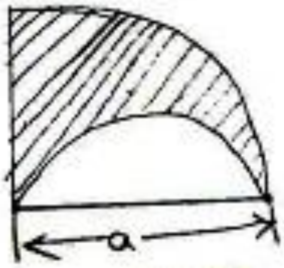
centroid from OX

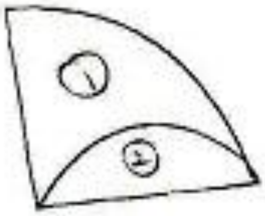
$$\bar{y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3}$$

$$= \frac{(19200 \times 80) - (3926.99 \times 80) - (1000 \times 143.33)}{19200 - 3926.99 - 1000}$$

$= 75.56\,mm.$

Locate the centroid of the shaded area obtained by removing a semi-circle of a diameter 'a' from a quadrant circle of radius 'a' as shown in figure





$$a_1 = \frac{\pi r^2}{4} = \frac{\pi \times a^2}{4}$$

$$x_1 = \frac{4r}{3\pi} = \frac{4a}{3\pi}$$

$$y_1 = \frac{4r}{3\pi} = \frac{4 \times a}{3\pi}$$

$$a_2 = \frac{\pi r^2}{2} = \frac{\pi (a/2)^2}{2}$$

$$\frac{\pi \times a^2}{8}$$

$$x_2 = d/2 = a/2$$

$$y_2 = \frac{4r}{3\pi} = \frac{4 \times a/2}{3 \times \pi} = \frac{2a}{3\pi}.$$

Centroid from OY

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}$$

$$= \frac{\pi a^2}{4} \times \frac{4a}{3\pi} - \frac{\pi \times a^2}{8} \times a/2 \Big/ \left(\frac{\pi a^2}{4} - \frac{\pi a^2}{8}.\right)$$

$$= \frac{\cancel{\pi a^2/4}\left(\frac{4a}{3\pi} - a/4\right)}{\cancel{\pi a^2/4}\left(1 - \frac{1}{2}\right)}$$

$$= \frac{\frac{4a}{3\pi} - a/4}{\frac{1}{2}}$$

$$= 2 \times a\left(\frac{4}{3\pi} - \frac{1}{4}\right)$$

$$= 0.59\,a.$$

centroid ~~from~~ from OX

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$= \frac{\frac{\pi \times a^2}{4} \times \frac{4 \times a}{3\pi} - \frac{\pi \times a^2}{8} \times \frac{2a}{3\pi}}{\left(\frac{\pi a^2}{4} - \frac{\pi a^2}{8}\right)}$$

$$= \frac{\cancel{\pi a^2/4}\left[\frac{4a}{3\pi} - 2a/6\pi\right)}{\cancel{\pi a^2/4}\left(1 - \frac{1}{2}\right)}$$

$$= \frac{\left(\frac{4a}{3\pi} - 2a/6\pi\right)}{\frac{1}{2}}$$

$$= 2 \times a\left(4/3\pi - 2/6\pi\right)$$

$$= 0.63a$$

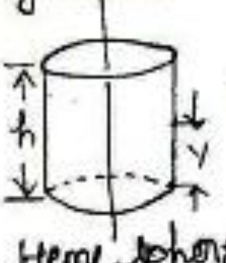
A solid hemisphere of 20mm radius suppose centre of gravity for solids:—

$$\bar{X} = \frac{V_1 x_1 + V_2 x_2}{V_1 + V_2}$$

$$\bar{Y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$$

solid :—	value	$\bar{X}$	$\bar{Y}$
cone:	$\frac{\pi r^2 h}{3}$	–	$h/4$

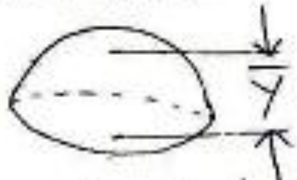
cylinder:



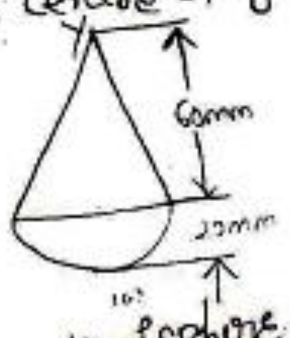
$\pi r^2 h$ – $h/2$

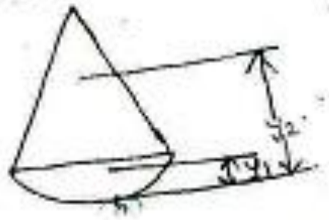
Hemisphere:—

	volume	$\bar{X}$	$\bar{Y}$
	$\frac{2}{3}\pi r^3$	—	$\frac{3}{8}r$



A solid hemisphere of 20mm radius supports a solid cone of same base and height 60mm in figure. locate the centre of gravity of composite section.





Hemisphere

$V_1 = \frac{2}{3}\pi \times r^3$

$= \frac{2}{3} \times \pi \times (20)^3$

$V_1 = 16755.16\ mm^3$

$y_1 = 20 - \bar{Y}$

$= 20 - \frac{3}{8} \times r$

$= 20 - \frac{3}{8} \times 20$

$y_1 = 12.5mm$

cone:- $V = \frac{\pi r^2}{3} \times h$

$= \frac{\pi \times (20)^2 \times 60}{3}$

$V_2 = 25132.74\ mm^3.$

$y_2 = 20 + h/4$

$= 20 + 60/4$

$= 35mm.$

$$\bar{Y} = \frac{v_1 y_1 + v_2 y_2}{}$$

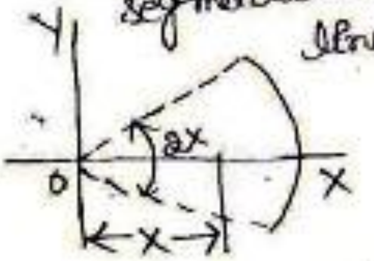
$$= \frac{(16755.16 \times 12.5) + (25132.74 \times 35)}{(16755.16 + 25132.74)}$$

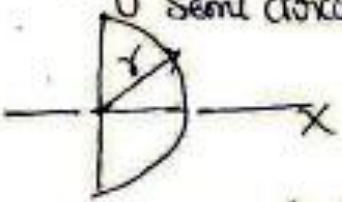
$$= 26 mm.$$

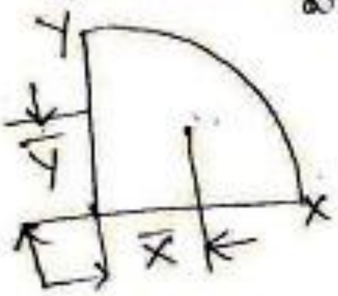
Centroid of lines:-

$$\bar{X} = \frac{l_1 x_1 + l_2 x_2}{l_1 + l_2}$$

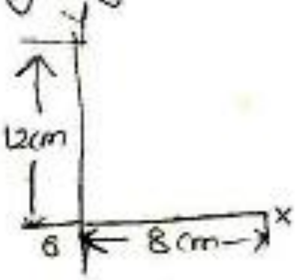
$$= \frac{l_1 y_1 + l_2 y_2}{l_1 + l_2}$$

Line segmentation of line	length	$\bar{X}$	$\bar{Y}$
	$2\alpha r$	$\frac{r \sin\alpha}{\alpha}$	0

Segmentation of line. Semi circular arc	length	$\bar{X}$	$\bar{Y}$
	πr	$\frac{2r}{\pi}$	0

Quadrant circular arc.			
	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$2r/\pi$

A wire of length 20cm is bent in the form of L the length of short leg 8cm and long leg 12cm locate the centroid



$l_1 = 8cm$

$x_1 = 8/2 = 4cm$

$y_1 = 0cm$

$l_2 = 12cm$

$x_2 = 0$

$y_2 = 6cm$

centroid from OY

$$\bar{x} = \frac{l_1x_1 + l_2x_2}{l_1+l_2} = \frac{(8\times4)+(12\times0)}{(8+12)}$$

$$= 1.6cm.$$

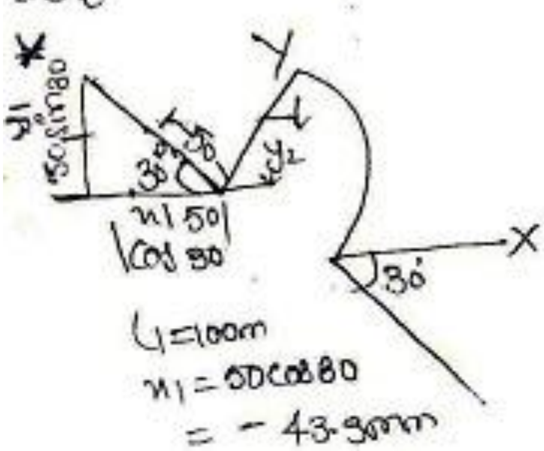
centroid from OX

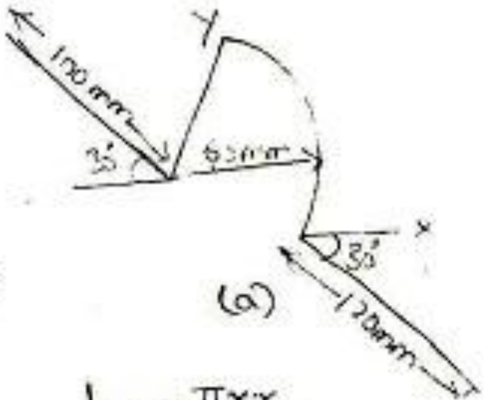
$$\bar{y} = \frac{l_1y_1 + l_2y_2}{l_1+l_2}$$

$$= \frac{(8\times0)+(12\times6)}{(8+12)} = 3.6\ cm$$

Locate the centroid of the wire bent as shown in figure.

Sol.





$L_1 = 100mm$

$x_1 = 50\cos30$

$= -43.3mm$

$y_1 = 50\sin30$

$= 25$

$L_2 = 60mm$

$x_2 = 0mm$

$y_2 = 30mm$

$$L_3 = \frac{\pi\times r}{2} = \frac{\pi\times60}{2} = 94.24mm$$

$$x_3 = \frac{2\times r}{\pi} = 38.18mm$$

$$y_3 = \frac{2r}{\pi} = \frac{2\times60}{\pi} = 38.18mm.$$

$l_4 = 120mm$

$x_4 = 60 \cos 30$

$= 51.96$

$y_4 = 60 \sin 30$

$= 30mm$

Centroid from OY

$$\bar{X} = \frac{l_1x_1 + l_2x_2 + l_3x_3 + l_4x_4}{l_1 + l_2 + l_3 + l_4}$$

$$= \frac{(100 \times -43.5) + (60 \times 0) + (94.24) \times 38.18) + (120 \times 51.96)}{100 + 60 + 94.24 + 120}$$

$= 26.42 mm$

Centroid from OX

$$\bar{y} = \frac{l_1y_1 + l_2y_2 + l_3y_3 + l_4y_4}{l_1 + l_2 + l_3 + l_4}$$

$$= \frac{(100 \times 25) + (60 \times 30) + (94.24)(58.18) + (120 \times 30)}{100 + 60 + 94.24 + 120}$$

$= 30.72 mm.$

Determine the centroid of the para-bolic spandel as shown in figure

Eqn of parabola

$y = kx^2$

$x = a,\ y = b$

$b = ka^2$

$K = b/a^2$

$y = \frac{b}{a^2} \times x^2$

Area of the strip

$dA = y\,dx$

Total Area $(A) = \int y\,dx$

$\int_0^a \frac{b}{a^2} \times x^2 \times dx$

$\frac{b}{a^2} \times \frac{x^3}{3}$

$\left[\frac{b}{a^2} \times \frac{a^3}{3}\right]$

$\frac{ba}{3}$

centroid of strip oy

from $oy = x$

$\int x\,dA = \int x \cdot y\,dx.$

$\int x \cdot \frac{b}{a^2} \times x^2 dx$

$\left[\frac{b}{a^2}\ \frac{x^4}{4}\right]_0^a$

$= \frac{ba^2}{4}$

$\bar{x} = \frac{\int_0^a x\,dA}{\int_0^a dA}$

$\frac{\left[\frac{ba^2}{4}\right]}{\left[\frac{ba}{3}\right]} = 3a/4$

centroid of strip

from $ox = y/2$

$\int y/2\ dA$

$= \int y/2 \times y \times dx$

$$\int_0^a \left[\frac{b^2}{a^4} \times x^4\right] \times \frac{dx}{2}$$

$$= \left[\frac{b^2}{a^4} \times \frac{x^5}{5}\right]_0^a \times \frac{1}{2}$$

$$= \left[\frac{b^2 a}{10}\right]$$

$$\bar{y} = \frac{\int y/2\, dx}{\int dA} \Rightarrow \frac{\left[\frac{b^2 a}{10}\right]}{\left[\frac{ba}{3}\right]} = \frac{3b}{10}$$

centroid of sparctrel

$$(\bar{x}, \bar{y}) = \left(\frac{3a}{4}, \frac{3b}{10}\right)$$

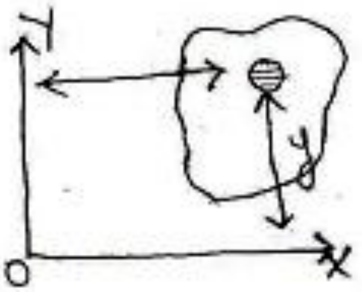
Unit - 3

moment of Inertia

moment of inertia :- moment of inertia is a geometric characteristics of a cross section of members strength and stiffness of bending member depends on the moment of inertia of its section. the moment of inertia a of its section about axis is defined as the sum of products of element area (dA) and square of its distance from its axis

consider a plane fig. of Area A in the xy plane and let (dA) be the element area situated at a distance.

x and y from oy and ox respectively as shown in figure



the m.I of area A about x-axis (ox)

$I_{xx} = \int y^2 dA$

the m.I of area about Y-axis (oy)

$I_y = \int x^2 dA$

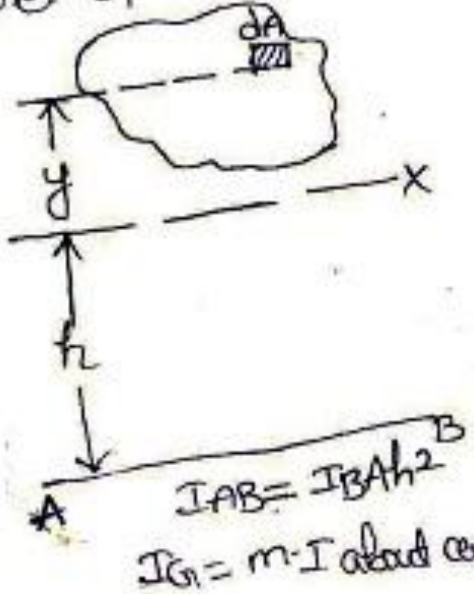
$I = AK^2$

$k = \sqrt{I/A}$ where k is radius gyration.

Radius of gyration :- consider the entire area concentrated at a point on the lamina. The distance of the point from the given axis of reference is called radius gyration.

parallel axis theorem :-

It states that the m.I from area from any axis is equal to the m.I about paraller axis passing through centroid plus and area multiplied by the square of the distance between the axis.



$I_{AB} = I_B A h^2$

h = Distance between centroid axis and given axis.

I_G = m.I about centroid axis.

A = Area of given figure

perpendicular axis theorem:—

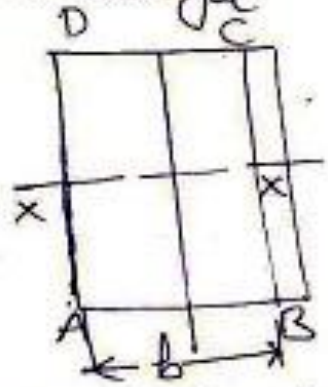
If Ixx and Iyy be the moment of inertia of a lamina about mutually perpendicular to each other about x-axis and y-axis Then moment of inertia about z-axis normal to the lamina.

$$I_{zz} = I_{xx} + I_{yy}$$

formulas:—

moment of Inertia of plane figures:-

a) Rectangle

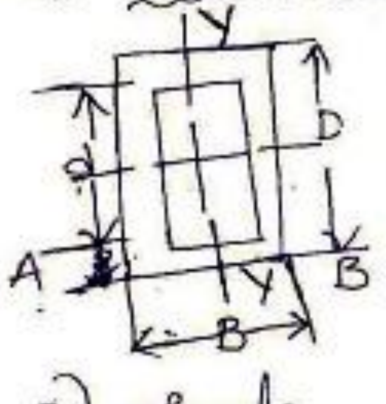


$$I_{xx} = \frac{bd^3}{12}$$

$$I_{yy} = \frac{db^3}{12}$$

$$I_{AB} = \frac{bd^3}{3}$$

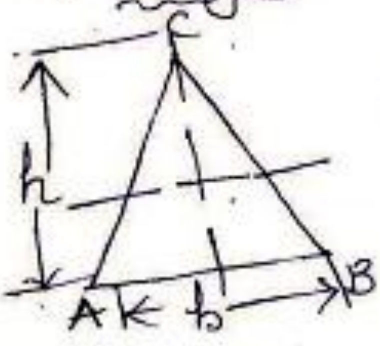
b) Hollow rectangle:—



$$I_{xx} = \frac{BD^3 - bd^3}{12}$$

$$I_{yy} = \frac{DB^3 - db^3}{12}$$

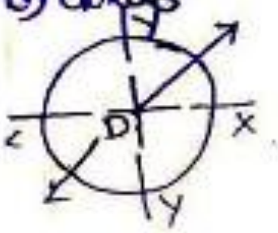
c) triangle.



$$I_{xx} = \frac{bh^3}{36}$$

$$I_{AB} = \frac{bh^3}{12}$$

d) circle



$$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$$

$$= \frac{\pi R^4}{4}$$

e) Hollow circle.

$$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$$

$$= \frac{\pi R^4}{4}$$

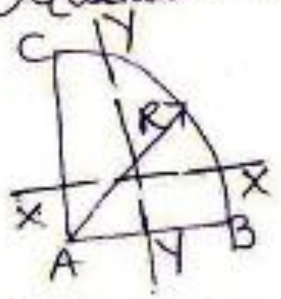
f) semicircular lamina

$$I_{AB} = I_{YY}$$

$$= \frac{\pi D^4}{128} = \frac{\pi R^4}{8}$$

$$I_{XX} = 0.11 R^4$$

g) Quadrant circular lamina



$$I_{AB} = I_{AC}$$

$$= \frac{\pi R^4}{16}$$

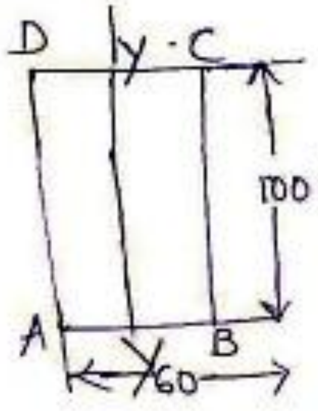
$$I_{XX} = I_{YY}$$

$$= 0.055 R^4$$

1) Find the m.I of a rectangle as shown in figure about

a) centroid axis (I_{xx} and I_{yy})

b) its base AB.



$$I_{xx} = \frac{bd^3}{12}$$

$$= \frac{60 \times 100^3}{12}$$

$$= 5 \times 10^6$$

$$= 5000000 \text{ mm}^4$$

$$= 5 \times 10^6 \text{ mm}^4$$

$$I_{YY} = \frac{db^3}{12}$$

$$= \frac{100 \times 60^3}{12}$$

$$= 18 \times 10^5 \, mm^4$$

$$I_{AB} = \frac{bd^3}{3}$$

$$= \frac{60 \times 100^3}{3}$$

$$= 2 \times 10^7 \, mm^4.$$

$$I_{AB} = I_G + Ah^2$$

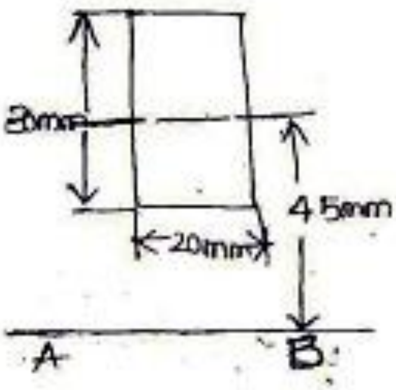
$$= I_{XX} + AX$$

$$= 5 \times 10^6 + (60 \times 100) \times 50^2$$

$$= 2 \times 10^7 \, mm^4.$$

② Find the moment of Inertia of a rectangle 20mm wide, and 30mm deep about a given axis AB, which is the distance of 45mm from its centroid.

$$I_{AB} = I_G + Ah^2$$



$$I_{XX} = \frac{bd^3}{12}$$

$$= \frac{20 \times 30^3}{12}$$

$$= 45000 \, mm^4$$

$$I_{AB} = I_{XX} + A \times h^2$$

$$= 45 \times 10^3 + (20 \times 30) \times 45^2$$

$$= 126 \times 10^4 \, mm^4$$

moment of inertia of composite section:-

the method for finding m.I of a composite section with components having centroidal axis differ from that of the entire section, is outlined below-

① Divide the composite section into components.

② locate the centroid for component section. 97

③ Determine the distances $h_1, h_2, h_3 \ldots,$

④ compute m.I of each component $I_{G_1}, I_{G_2} \ldots,$

⑤ compute the values $A_1h_1^2$, $A_2h_2^2$.

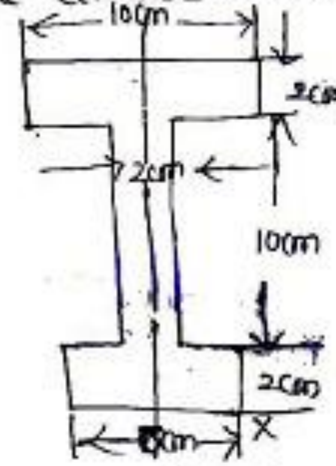
⑥ compute m.I of given section

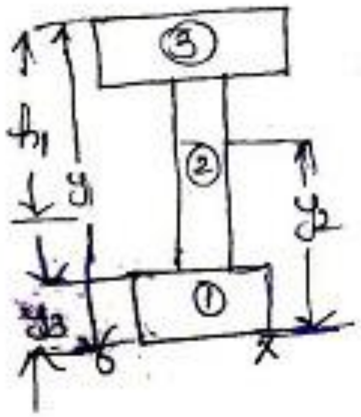
$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}.$$

problem:—

Find the m.I (moment of Inertia) of the section shown in figure about the centroidal axis x-x perpendicular to web.





$a_1 = 10 \times 2$
$= 20 cm^2$

$y_1 = 2 + 10 + 2/2$
$= 13 mm.$

$a_2 = 2 \times 10$
$= 20 cm^2$

$y_2 = 2 + 10/2$
$= 7 cm$

$a_3 = 10/2 \times 2$
$= 10 cm^2.$

$y_3 = 2/1 = 1 cm$

centroid from OX.

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(20 \times 13) + (20 \times 7) + (10 \times 1)}{(20 + 20 + 10)}$$

$$= 8.2\,cm$$

$$I_{XX_1} = I_{G_1} + a_1h_1^2$$

$$= \frac{b_1d_1^3}{12} + a \times h_1^2$$

$$= \frac{10 \times 2^3}{12} + (10 \times 2) \times (y_1 - \bar{Y})^2$$

$$= \frac{10 \times 2^3}{12} + (10 \times 2) \times (13 - 8.2)^2$$

$$= 467.46\,cm^4$$

$$I_{XX_2} = \frac{b_2d_2^3}{12} + a_2h_2^2$$

$$= \frac{2 \times 10^3}{12} + (2 \times 10) \times (\bar{Y} - y_2)^2$$

$$= \frac{2 \times 10^3}{12} \times 20 \times (1.2)^2$$

$$= 195.47\,cm^4$$

$$I_{XX_3} = \frac{b_3d_3^3}{12} + a_3h_3^2$$

$$= \frac{5 \times 2^3}{12} + (5 \times 2) \times (\bar{Y} - y_3)$$

$$= \frac{5 \times 2^3}{12} + (10) \times (7.2)^2$$

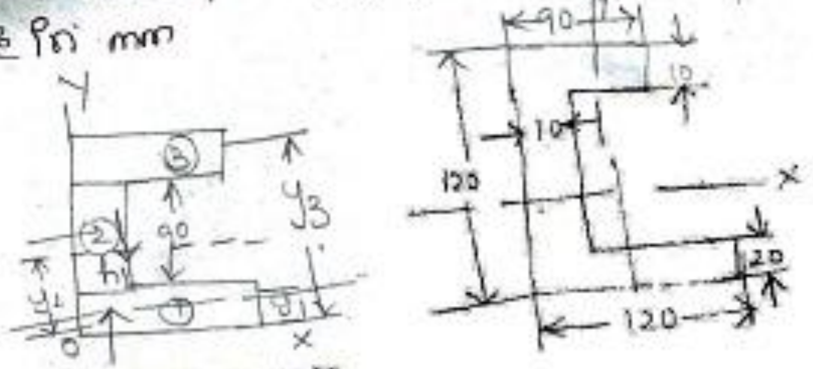
$$= 521.73\,cm^4$$

$$I_{XX} = I_{XX_1} + I_{XX_2} + I_{XX_3}$$

$$= 467.46 + 195.47 + 521.73$$

$$= 1184.67\,cm^4$$

Find the M.I of section shown in figure about its centroidal axis parallel to the base. All dimensions are in mm



$a_1 = 120 \times 20$

$= 2400 mm^2$

$y_1 = 20/2 = 10mm$

$a_2 = 90 \times 10$ $\quad 120-10-20 = 90$

$= 90 \times 10 = 900 mm^2$

$y_2 = \frac{90}{2} + 20 = 65mm$

$a_3 = 90 \times 10$

$= 900mm^2$

$y_3 = \frac{10}{2} + 90 + 20$

$= 115 mm.$

Centroid from OX

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(2400 \times 10) + (900 \times 65) + (900 \times 115)}{2400 + 900 + 900}$$

$= 44.28mm$

$h_1 = (\bar{Y} - y_1)$

$= (44.28 - 10) = 34.28 mm$

$h_2 = (y_2 - \bar{Y})$

$= (65 - 44.28)$

$= 20.72 mm$

$h_3 = (y_3 - \bar{Y})$
$= (115 - 44.28)$
$= 70.72\,mm.$

$$I_{XX1} = \frac{b_1 d_1^3}{12} + A_1 \times h_1^2$$
$$= \frac{120 \times 20^3}{12} + (120 \times 20) \times (34.28)^2$$
$$= 29 \times 10^5\ mm^4$$

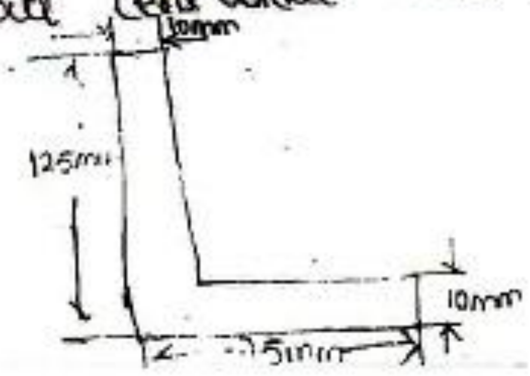
$$I_{XX2} = \frac{b_2 d_2^3}{12} + A_2 \times h_2^2$$
$$= \frac{(10 \times (90)^3}{12} + (10 \times 90) \times (20.72)^2$$
$$= 99.38 \times 10^4\ mm^4.$$

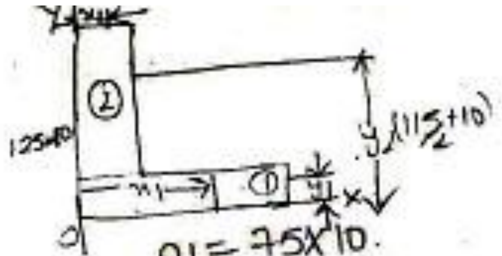
$$I_{XX3} = \frac{b_3 d_3^3}{12} + A \times h_3^2$$
$$= \frac{90 \times 10^3}{12} + (90 \times 10) \times (70.72)^2$$
$$= 45.086 \times 10^5\ mm^4.$$

$$I_{XX} = I_{XX1} + I_{XX2} + I_{XX3}$$
$$= 29 \times 10^5 + 99.38 \times 10^4 + 45.086 \times 10^5$$
$$= 84.02 \times 10^5\ mm^4.$$

*10m

for an unequal section 125X75X10mm, find the m.I. about centroidal axes (IXX and IYY)





$a_1 = 75 \times 10$

$= 750 mm^2$

$x_1 = \frac{75}{2} = 37.5 mm$

$y_1 = 10/2 = 5mm$

$a_2 = 115 \times 10$

$= 1150 mm^2$

$x_2 = 10/2 = 5mm$

$y_2 = 115/2 + 10$

$= 67.5mm$

Centroid from ox

$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2}$

$= \frac{(750 \times 5) + (1150 \times 67.5)}{(750+1150)}$

$= 42.83cm$

centroid from oy

$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2}$

$= \frac{(750 \times 37.5) + (1150 \times 5)}{(750+1150)}$

$= 17.83mm.$

$h_{x1} = (x_1 - \bar{x})$

$= (37.5 - 17.83)$

$h_{x1} = 19.67cm$

$hx_2 = (\bar{x} - x_2)$

$= (17.83 - 5) = 12.83 cm$

$hy_1 = (\bar{y} - y_2)$

$= (42.83 - 5)$

$= 37.83 cm$

$hy_2 = (y_2 - \bar{y})$

$= (67.5 - 42.83)$

$= 24.67 cm$

$$I_{xx1} = \frac{b_1 d_1^3}{12} + A_1 hy_1^2$$

$$= \frac{75 \times 10^3}{12} + (75 \times 10) \times (37.83)^2$$

$= 10.79 \times 10^5 mm^4$

$$I_{xx2} = \frac{b_2 d_2^3}{12} + A_2 hy_2^2$$

$$= \frac{10 \times 115^3}{12} + (10 \times 115) \times (24.67)^2$$

$= 19.67 \times 10^5 mm^4$

$I_{xx} = I_{xx1} + I_{xx2}$

$= 10.79 \times 10^5 + 19.67 \times 10^5$

$= 30.46 \times 10^5 mm^4$

$$I_{yy1} = \frac{d_1 b_1^3}{12} + A_1 \times h^2 x_1$$

$$= \frac{10 \times 75^3}{12} + (10 \times 75) \times 19.67$$

$= 64.17 \times 10^4 mm^4$

$$I_{yy_2}' = \frac{d_2 b_2^3}{12} + A_2 \times h x_2^2$$

$$= \frac{115 \times 10^3}{12} + (115 \times 10) \times (12.83)^2$$

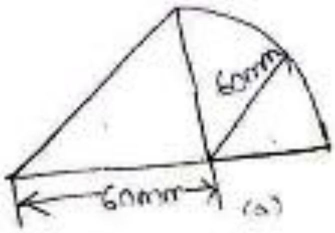
$$= 19.88 \times 10^4 \, mm^4$$

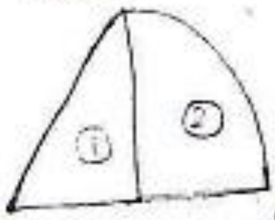
$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$= 64.17 \times 10^4 + 19.88 \times 10^4$$

$$= 84.06 \times 10^4 \, mm^4$$

Find the m.I of the shaded area as shown in figure below about its centroidal axis parallel to x-axis





$$a_1 = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 60 \times 60$$

$$= 1800 \, mm^2$$

$$y_1 = h/3 = 60/3$$

$$= 20 \, mm$$

$$a_2 = \frac{\pi r^2}{4}$$

$$= \frac{\pi \times (60)^2}{4}$$

$$= 2827.4 \, mm^2$$

$$y_2 = \frac{4r}{3\pi}$$

$$= \frac{4 \times 60}{3 \times \pi}$$

$$= 25.46 \, mm$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(1800 \times 20) + (2827.4 \times 25.46)}{(1800 + 2827.4)}$$

$$= 23.33mm$$

$$h_1 = (\bar{y} - y_1)$$

$$= (23.33 - 20)$$

$$= 3.33mm$$

$$h_2 = (y_2 - \bar{y})$$

$$= (25.46 - 23.33)$$

$$= 2.13mm$$

$$I_{XX1} = I_{G1} + a_1 h_1^2$$

$$= \frac{b_1 d_1^3}{36} + a_1 h_1^2$$

$$= \frac{60 \times 60^3}{36} + (1800 \times (3.33)^2)$$

$$= 37.99 \times 10^4 mm^4$$

$$I_{XX2} = I_{G2} + a_2 h_2^2$$

$$= 0.055 \times R^4 \times a_2 h_2^2$$

$$= 0.055 \times (60)^4) + (2827.4 \times (2.13)^2)$$

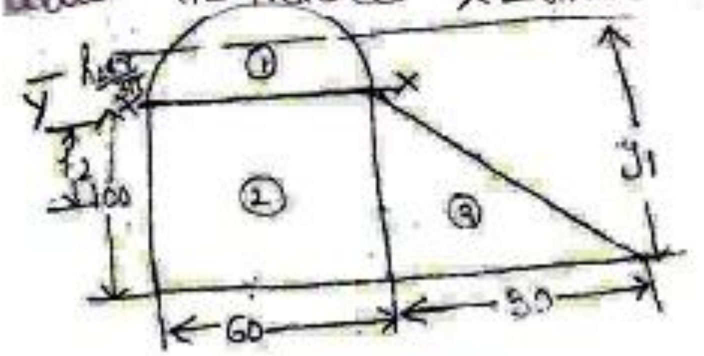
$$= 72.56 \times 10^4 mm^4$$

$$I_{XX} = I_{XX1} + I_{XX2}$$

$$= 37.99 \times 10^4 + 72.56 \times 10^4$$

$$= 11.05 \times 10^5 mm^4$$

calculate m.I of a composite area shown in figure about the indicated x-axis.



$y_1 = \frac{4r}{3\pi} + 100$

$= \frac{4 \times 30}{3 \times \pi} + 100$

$= 112.7mm \quad (\bar{y} - y_1)$

$h_1 = (112.7 - 100)$

$= 12.7 \text{ mm}$

$y_2 = \frac{100}{2} = 50mm$

$h_2 = (\bar{y} - y_2)$

$= (100 - 50)$

$= 50mm.$

$y_3 = \frac{h}{3} = \frac{100}{3}$. $\quad h_3 = (\bar{y} - y_3)$

$= 33.33mm \quad\quad = (100 - 33.33)$

$\quad\quad = 66.67 \text{ mm}$

$a_1 = \frac{\pi \times r^2}{2}$

$= \frac{\pi \times (30)^2}{2}$

$= 1413.71mm^2$

$a_2 = b \times d$

$= 60 \times 100$

$= 6000mm^2$

$a_3 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 30 \times 100$

$= 1500 mm^2$

$I_{XX1} = I_{G1} + a_1 h_1^2$

$= 0.11 R^4 + a_1 h_1^2$

$= 0.11 \times (30)^4 + (1413.7) \times (62.7)^2$

$= 31.71 \times 10^6 \ mm^4$

$I_{XX2} = I_{G2} + a_2 h_2^2$

$= \frac{b_2 d_2^3}{12} + a_2 h_2^2$

$= \frac{60 \times (100)^3}{12} + 6000 \times (50)^2$

$= 20 \times 10^6 \ mm^4$

$I_{XX3} = I_{G3} + a_3 h_3^2$

$= \frac{b_3 d_3^3}{36} + a_3 h_3^2$

$= \frac{80 \times (100)^3}{36} + 1000 \times (66.67)^2$

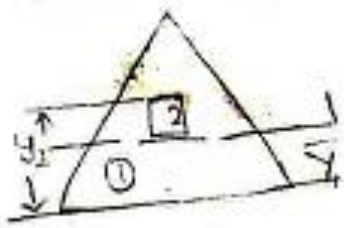
$= 7.5 \times 10^6 m^4$

$I_{XX} = I_{XX1} + I_{XX2} + I_{XX3}$

$= 31.71 \times 10^6 + 20 \times 10^6 + 7.5 \times 10^6$

$= 27.8 \times 10^6 \ mm^4$.

Find the moment of Inertia and radius of gyration about horizontal centroidal axis. shaded area



$a_1 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 60 \times 60$

$= 1800 mm^2$

$y_1 = h/3 = 60/3 = 20mm$

$a_2 = b \times d$

$= 10 \times 20$

$= 200 mm^2$

$y_2 = \frac{20}{2} + 20$

$= 30mm$

$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$

$= \frac{(1800 \times 10) - (200 \times 30)}{1800 - 200}$

$= 18.75 mm$

$h_1 = (y_1 - \bar{y})$

$= (20 - 18.75)$

$= 1.25mm$

$h_2 = (y_2 - \bar{y})$

$= (30 - 18.75)$

$= 11.25 mm$

$I_{XX_1} = I_{G_1} + a_1 h_1^2$

$= \frac{b_1 d_1^3}{12} + a_1 h_1^2$

$= \frac{60 \times (60)^3}{36} + (1800) \times (1.25)^2$

$= 36.28 \times 10^4 mm^4$

$$I_{XX_2} = I_{G_2} + a_2 h_2^2$$

$$= \frac{b_2 d_2^3}{12} + a_2 h_2^2$$

$$= \frac{10 \times (20)^3}{12} + 200 \times (11.25)^2$$

$$= 31.97 \times 10^3 \text{ mm}^4$$

$$I_{XX} = I_{XX_1} - I_{XX_2}$$

$$= 36.28 \times 10^4 - 31.97 \times 10^3$$

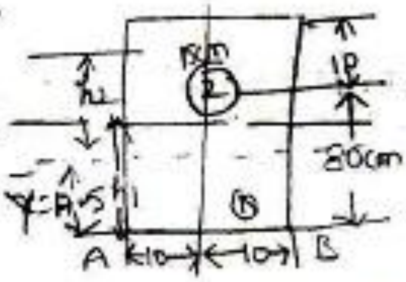
$$= 33.08 \times 10^4 \text{ mm}^4$$

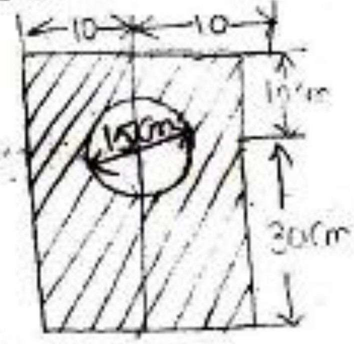
$$K = \sqrt{I/A}$$

$$= \sqrt{\frac{33.08 \times 10^4}{1800 - 200}}$$

$$= 14.5 \text{ mm}$$

Find the m.I of a plate with a circular hole about the centroidal axis and about its base.





$$a_1 = 20 \times 40$$

$$= 800 \text{cm}^2$$

$$y_1 = \frac{d}{2} = \frac{40}{2}$$

$$= 20 \text{cm}$$

$$a_2 = \pi R^2$$

$\pi(7.5)^2$

$= 176.7 cm^2$

$y_2 = 30cm$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

$$= \frac{(800 \times 20) - (176.7 \times 30)}{(800 - 176.7)}$$

$= 17.16cm$

$h_1 = (y_1 - \bar{Y})$

$= (20 - 17.16)$

$= 2.84\ cm$

$h_2 = (y_2 - \bar{Y}) = (30 - 17.16)$

$= 12.84\ cm$

$I_{XX_1} = I_{G_1} + a_1h_1^2$

$$= \frac{b_1d_1^3}{12} + a_1h_1^2$$

$$= \frac{20 \times (40)^3}{12} + 800 \times (2.84)^2$$

$= 11.31 \times 10^4\ cm^4$

$I_{XX_2} = I_{G_2} + a_2h_2^2$

$$= \frac{\pi R^4}{4} + \pi R^2 \quad 176.7 \times (12.84)$$

$$= \frac{\pi(7.5)^4}{4} + 176.7 \times (12.84)^2 = 3.16 \times 10^4$$

$3.16 \times 10^4\ cm^4$

$I_{XX} = I_{XX_1} - I_{XX_2}$

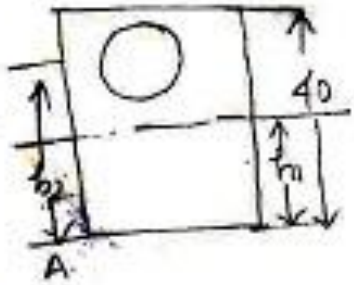
$= 11.31 \times 10^4 - 3.16 \times 10^4$

$= 81.48 \times 10^3\ cm^4$

$$I_{AB_1} = \frac{b_1d_1^3}{3}$$

$= \frac{20 \times 40^3}{3}$

$= 42.66 \times 10^4 \, cm^4$



$I_{AB1} = I_{G1} + a_1 h_1^2$

$= \frac{b_1 d_1^3}{12} + a_1 h_1^2$

$= \frac{20 \times 40^3}{12} + 800 \times \left(\frac{40}{2}\right)^2$

$= 42.66 \times 10^4 \, cm^4$

$I_{AB2} = I_{G2} + a_2 h_2^2$

$= \frac{\pi \times r^4}{4} + a_2 h_2^2$

$= \frac{\pi \times (7.5)^4}{4} + 176.7 \times (30)^2$

$= 16.15 \times 10^4 \, cm^2$

$I_{AB} = I_{AB1} - I_{AB2}$

$= 42.66 \times 10^4 - 16.15 \times 10^4$

$= 26.51 \times 10^4 \, cm^4.$

4 unit

MASS moment of inertia

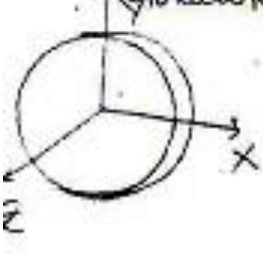
Consider a body of mass 'm' and 'dm' is the mass of the element at a distance 'r' from the axis AA the moment of inertia of body with respect to AA is defined by the equation

$$I = \int r^2 dm \ \ kg\text{-}m^2$$

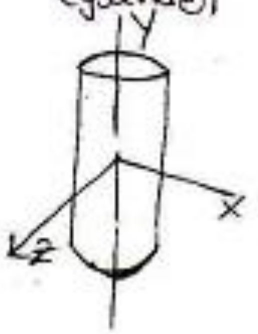
Also the radius of gyration (k) of body with respect to AA is given as

$$k = \sqrt{\frac{I}{m}}$$

Mass moment of inertia of bodies:-

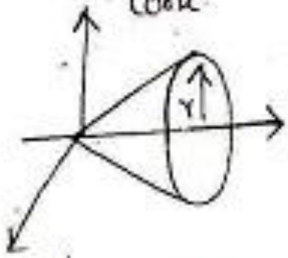
Body (mass, m)	mass m.I		
	I_x	I_y	I_z
Circular disc 	$\frac{mr^2}{4}$	$\frac{mr^2}{4}$	$\frac{mr^2}{2}$
Sphere 	$\frac{2}{5}mr^2$	$\frac{2}{5}mr^2$	$\frac{2}{5}mr^2$

Right circular cylinder



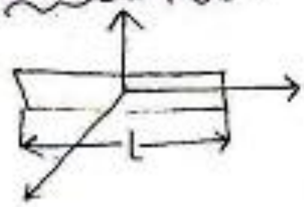
I_x	I_y	I_z
$\frac{m}{12}(3r^2+L^2)$	$\frac{mr^2}{2}$	$\frac{m}{12}(3r^2+L^2)$

Right circular cone



I_x	I_y	I_z
$\frac{3m}{10} \times r^2$	$\frac{3m}{5}\left(\frac{r^2}{4}+h^2\right)$	$\frac{3m}{5}\left(\frac{r^2}{4}+h^2\right)$

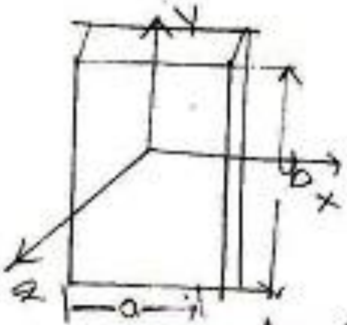
Slender bar



I_x	I_y	I_z
0	$\frac{mL^2}{12}$	$\frac{mL^2}{12}$

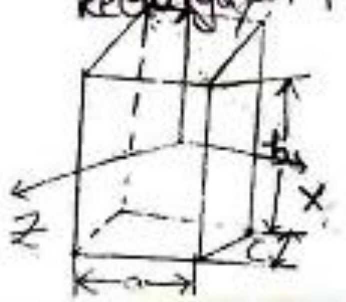
Note:- $I_A = \frac{mL^2}{3}$

Rectangular plate:-



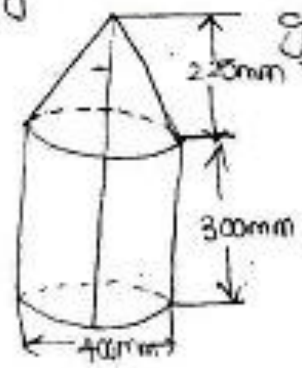
I_x	I_y	I_z
$\frac{mb^2}{12}$	$\frac{ma^2}{12}$	$\frac{m(a^2+b^2)}{12}$

Rectangular prism:-



I_x	I_y	I_z
$\frac{m(b^2+L^2)}{12}$	$\frac{m(a^2+c^2)}{12}$	$\frac{m(a^2+b}{12}$

A brass cone with base diameter of 400mm and height of 225mm is placed on a vertical aluminium cylinder of height 300mm and diameter 400mm. Density of brass is 83 kN/m³ and density of Aluminium is 25.6 kN/m³ Determine the mass moment of inertia of the composite body about the vertical geometrical axis:-



$P_{brass} = 83 \text{ kN/m}^3$

$P_{aluminium} = 25.6 \text{ kN/m}^3$

$\underset{kg}{mass} = \text{Density} \times \underset{m^3}{\text{volume}}$

$P_{brass} = \dfrac{83 \times 1000}{9.81} \text{ kg/m}^3$

$= 8460.75 \text{ kg/m}^3$

$P_{aluminium}$

$= \dfrac{25.6 \times 1000}{9.81}$

$= 2609.58 \text{ kg/m}^3$

$mass_{brass} = 8460.75 \times \left(\dfrac{\pi r^2}{3} \times h\right)$

$= 8460.75 \times \left(\dfrac{\pi \times (0.2)^2}{3} \times 0.225\right)$

79.74

$mass_{aluminium} = 2609.58 \times (\pi \times r^2 \times h)$

$$= 2609.58 \times \pi \times (0.2)^2 \times (0.3)]$$

$$= 98.378 \text{ kg}$$

$$I_{y1} = \frac{3 \times m}{10} \times r^2$$

$$= \frac{3 \times 79.79}{10} \times (0.2)^2$$

$$= 0.95 \text{ kg m}^2$$

$$I_{y2} = \frac{mr^2}{2}$$

$$= \frac{98.37 \times (0.2)^2}{2}$$

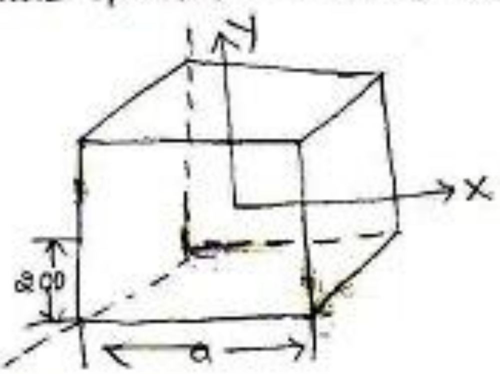
$$= 1.9 \text{ kg-m}^2$$

$$I_y = I_{y1} + I_{y2}$$

$$= 0.95 + 1.9$$

$$= 2.85 \text{ kg-m}^2.$$

A cube of side 400mm has a mass density of 2000 kg/ find out the mass moment of inertia of the cube about its centroidal axis parallel to one of the side and on of its edges.



$$\text{Side} = 400\text{mm}$$
$$= 0.4\text{m}$$

$$\rho = 2000 \text{ kg/m}^3$$

mass = $\rho \times$ volume

$= 2000 \times (0.4)^3$

$= 128$ kg

m.I about centroidal axis

$= \frac{m(a^2+a^2)}{12}$

$= \frac{128\,(2\times(0.4)^2)}{12}$

$= 3.413$ kg-m^2

m.I about edge

$I_{AA} = I_G + mh^2$

$\boxed{= \frac{bd^3}{12} + 128 \times (0.2)^2}$

$= 3.413 + 128 \times (0.2)^2$

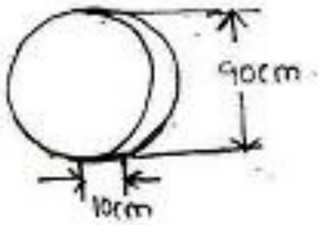
$= 8.53$ kg-m^2

calculate the moment of inertia and radius of gyration of a grinding stone 90cm in diameter and 10cm thick with respect to its axis of rotation stone weights 0.026 kg/cm^3.

D = 90cm

t = 10cm

$\rho = 0.026$ kg/cm^3



volume $= \left(\frac{\pi D^2}{4}\right) \times t$

$= \left[\frac{\pi \times (90)^2}{4}\right] \times 10$

$= 63617.25$ cm^3

mass $= \rho \times v$

$= 0.026 \times 63617.25$

$= 1654.04$ kg

$I_x = \frac{mr^2}{2}$

$= \frac{1654.04 \times (0.45)^2}{2}$

$= 83.73$ kg-m^2

Radius of gyration

$I = mK^2$

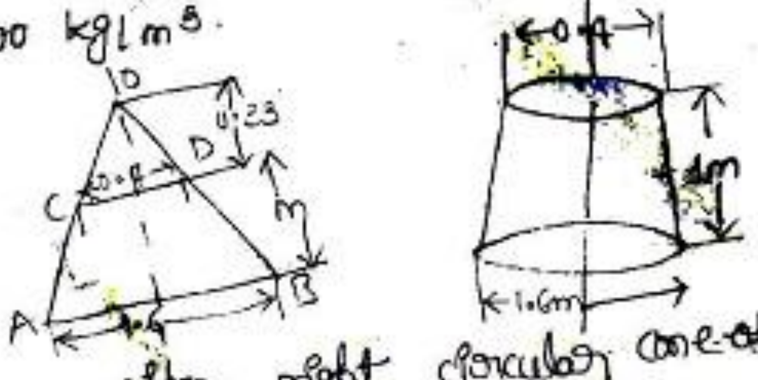
$k = \sqrt{I/m}$

$= \sqrt{\frac{83.73}{1654.04}}$

$= 0.22$m

$\Rightarrow 22$cm

Calculate the mass moment of inertia of the frustum of the cone shown in fig. with respect to the axis ZZ assuming the density of the cone as 2500 kg/m^3.



consider right circular cone of OCD from which right circular cone OAB is cut off - from the geometry of cone OAB = 0.35m

mass = $\rho \times$ volume

$= 2500 \times \frac{\pi r^2}{3} \times h$

$= 2500 \times \frac{\pi \times (0.8)^2}{3} \times 1.33$

$= 2228.43$ kg

m.I about zz

$I_{z_1} = \frac{3m}{10} \times r^2$

$= \frac{3 \times 2228.43}{10} \times (0.8)^2$

$= \cancel{488.275}$ kg-m²

$= 427.85$ kg-m²

mass of $= \rho \times$ volume

$2500 \times \frac{\pi r^2}{3} \times h$

$= 2500 \times \frac{\pi \times (0.2)^2}{3} \times 0.83$

$= 34.55$ kg

$I_{z_2} = \frac{3m}{10} \times r^2$

$= \frac{3 \times 34.55}{10} \times (0.2)^2$

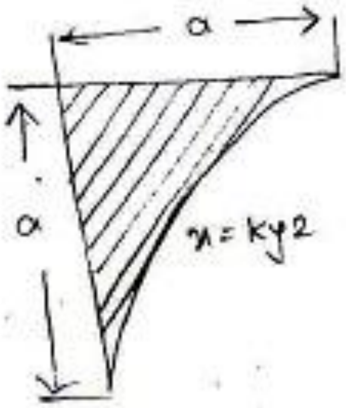
$= 0.414$ kg-m²

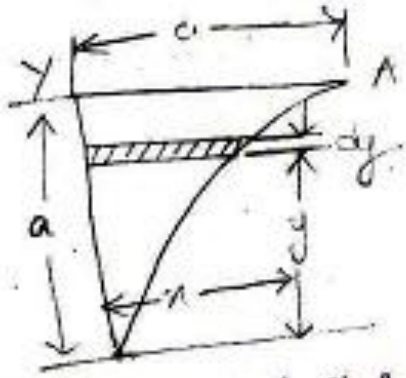
$I_z = I_{z_1} - I_{z_2}$

$= 427.85 - 0.414$

$= 427.43$ kg-m²

(1) calculate the moment of Inertia of a shaded area. The x-axis as shown in figure





Consider horizontal strip Ay at a distance y from OX as shown in fig.

$x = ky^2$ —①

$x = a,\ y = a$

$a = ka^2$

$k = \frac{1}{a}$

$x = \frac{1}{a}y^2$

Element area

$dA = x.dy$

$I = \int dA \cdot y^2$

$= \int x \cdot dy \cdot y^2$

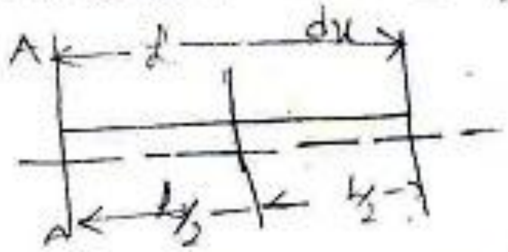
$= \int_0^a \frac{1}{a} \times y^2 \times y^2 \cdot dy$

$= \frac{1}{a}\left[\frac{y^5}{5}\right]_0^a$

$= \frac{1}{\cancel{a}}\left[\frac{a^5}{5}\right]$

$= \frac{a^4}{5}$

* Determine the mass m.I of a Slender rod of length 'L' and a mass 'm' with respect to a centroidal axis perpendicular to the rod and axis passing through one end of the rod perpendicular to it?



mass m.I about centroidal axis perpendicular to rod consider an element length dx at a distance rod consider an element length dx at a distance x as shown in fig.
(Eq x = from centroidal axis y
m_x = mass of the Element length
= mass for unit length

$m_x = m/L \Rightarrow m_x \times L = m$

$dm = m_x \times dx$

m.I of inertia mass of the element

$\therefore I.dm = dm \times x^2$

m.I of slender rod.

$$I_y = \int_{-L/2}^{+L/2} dm \times x^2$$

$$= \int_{-L/2}^{+L/2} m_x dx \times x^2$$

$$= m_x \left[\frac{x^3}{3}\right]_{-L/2}^{+L/2}$$

$$= m_x \left[\frac{\left(\frac{L^3}{8}\right) - \left(-\frac{L^3}{8}\right)}{3}\right]$$

$$m_x \times \frac{2l^3}{8\times 3} = m_x \times \frac{l^3}{12} = m_x \times l \times \frac{l^2}{12}$$

$$= \frac{ml^2}{12}$$

case(ii). Determine the mass m.I of a slender rod of length 'l' and a mass 'm' with respect to a centroidal axis perpendicularly to the rod and perpendicular to it.

$m_x = m/l$

$dm = m_x \cdot dx$

m.I of element length.

$I_{dx} = dm \times x^2$

m.I of slender rod.

$$I_A = \int_0^l dm \times x^2$$

$$= m_x \int_0^l x^2 dx$$

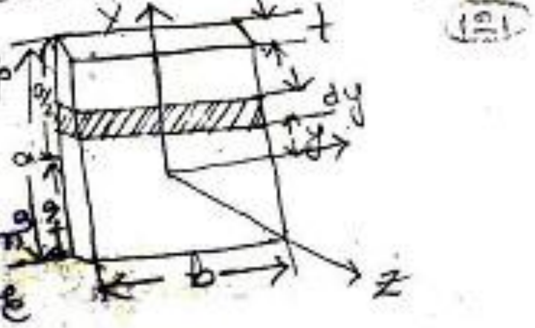
$$= m_x \left[\frac{x^3}{3}\right]_0^l$$

$$\Rightarrow m_x \left[\frac{l^3}{3}\right]$$

$$= m_x \times l \times \frac{l^2}{3}$$

$$= \frac{m \times l^2}{3}$$

Determine the mass m.I. of a rectangular plate of size a×b and thickness 't' about its centroidal axis?



mass of the element strip
$dm = \rho(bt\, dy)$

where ρ is density kg/m^3

m.I of rectangular plate about x-axis

$$I_{xx} = \int_{-a/2}^{a/2} \rho(bt\, dy)y^2$$

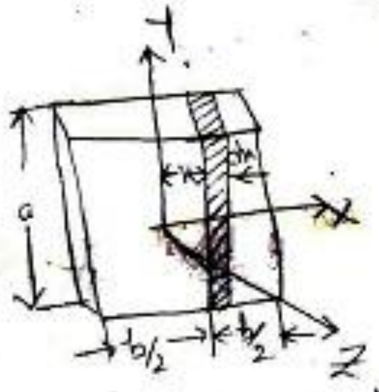
$$= \rho bt\left[\frac{y^3}{3}\right]_{-a/2}^{+a/2}$$

$$= \rho bt\left[\frac{\left(\frac{a^3}{8}\right)-\left(-\frac{a^3}{8}\right)}{3}\right]$$

$$= \frac{\rho bt\, 2a^3}{8\times 3}$$

$$= \frac{\rho abt \times a^2}{12}$$

$$= \frac{ma^2}{12} \quad (\because \rho abt = m)$$



m.I of a rectangle about Y-axis

$$I_{YY} = \int_{-b/2}^{+b/2} \rho at\, dx\, x^2$$

$$= \rho a t \left[\frac{x^3}{3}\right]_{-b/2}^{+b/2}$$

$$= \rho a t \left[\frac{\left[\frac{b^3}{8}\right] - \left[-\frac{b^3}{8}\right]}{3}\right]$$

$$= \rho a t \, \frac{2b^3}{8 \times 3}$$

$$= \rho a b t \times b^2/12$$

$$= \frac{mb^2}{12} \quad (\because \rho b t = m)$$

M.I about z-axis.

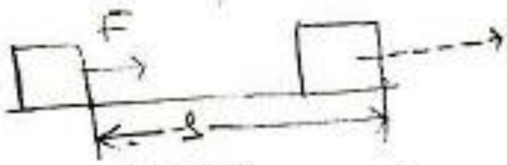
$$I_{zz} = I_{xx} + I_{yy}$$

$$= \frac{ma^2}{12} + \frac{mb^2}{12}$$

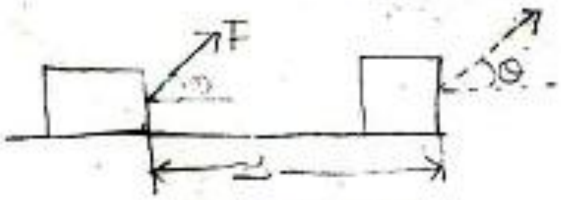
$$= \frac{m}{12}(a^2+b^2)$$

Virtual work:-

Work done:- work is done when a force applied to body and the body moves in the direction of force, the work done by a force is given as



$W = F \times s$.



$W = F\cos\theta \times s$

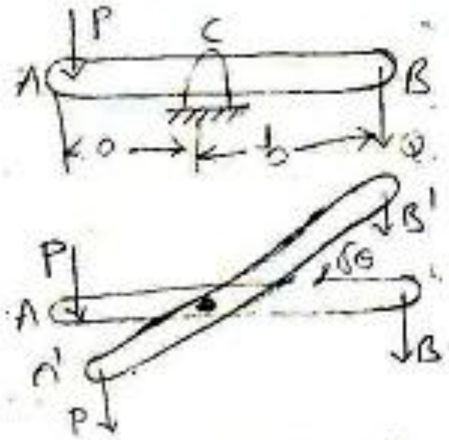
concept of viortual work:- If a body is in equilibrium under the action of system forces then the workdone by the system of forces is zero. Now consider a body undergoes extreamely small displacement which is consistent with the geometrical conditions under which the body exist. This imaginary displacement which doesn't actually take place is called viortual displacement. The work done on a system of forces on a body during a viortual displacement is called viortual work.

principle of viortual work:-
the principle of viortual work states that if a system of forces acting on a body be in equilibrium and body undergoes a slight displacement, the algebraic sum of virtual work is zero. mathematically principle of viortual work is expressed as ΣF

$$\delta x = 0$$

where δx = viortual displacement in the direction of force. The viortual displacement is expressed in terms of δx and δy.

Application of principle of viortual work:-
consider a lever AB hinged at C and is subjected to forces P and Q as shown in fig.



Let an extreamly small angular displacement $\delta\theta$ is given at C.

for small angular displacement

$$AA' = a \times \delta\theta$$

$$BB' = b \times \delta\theta$$

Application of principle of virtual work

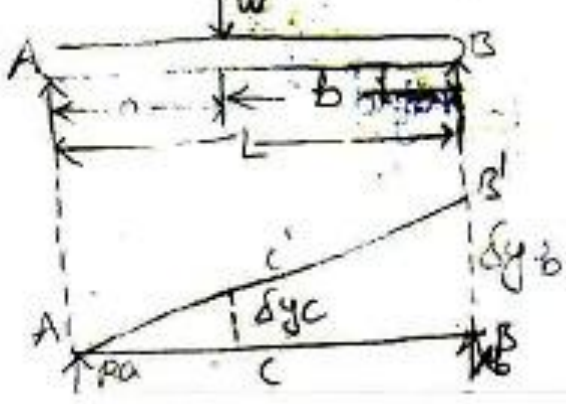
$$P \cdot AA' - Q \cdot BB' = 0$$

-ve sign for displacement at B is due to its direction opposite to the direction of Q.

$$P \times a \times \delta\theta - Q \times b \times \delta\theta = 0.$$

Application to simply supported beam:-

consider a simply supported beam AB of span L is subjected to a point load w as shown in fig. Let A kept stationary, δy_b is virtual displacement at B, and displacement at C is δc



from the geometry of the fig.

$$\frac{\delta y_b}{L} = \frac{\delta y_c}{a}$$

$$\delta y_b = L/a \times \delta y_c \quad \text{--- (1)}$$

Applying the principle of virtual work

$$R_b \times \delta y_b - w \delta y_c = 0 \quad \text{--- (2)}$$

from Eqn (1) & (2)

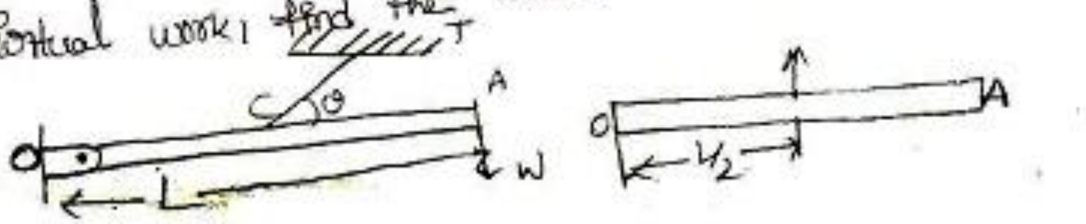
$$R_b \times (L/a \times \delta y_c) - w(\delta y_c) = 0$$

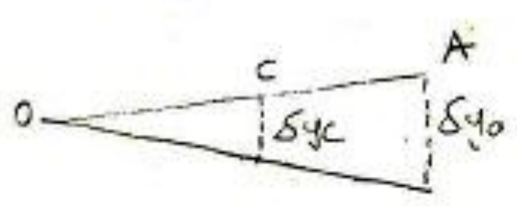
$$R_b \cdot L/a - w = 0$$

$$R_b = a/L \, W$$

~~(1) Two beams AB and CD~~ negligible weight is

(1) A beam OA of length L and has a load w at free end A hinged at O and this beam at the mid point A wire BC supports of beam as shown in fig below. By the method of virtual work, find the tension in the wire?





$$\frac{\delta y_a}{L} = \frac{\delta y_c}{L/2}$$

$$\delta y_a = 2\delta y_c$$

Applying principle of virtual work

$$T\cos 60 \times \delta y_C - w\delta y_C = 0$$

$$T\cos 60 \times \delta y_C - w \times 2\delta y_C = 0$$

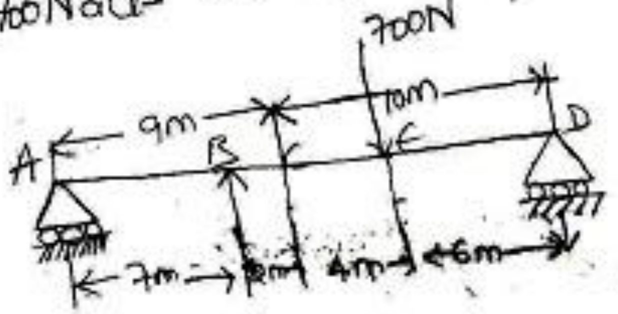
$$\delta y_C (T \times \tfrac{1}{2} - w \times 2) = 0$$

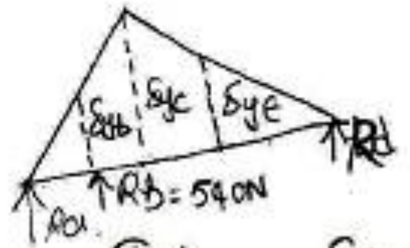
$$T \times \tfrac{1}{2} = 2w$$

$$T = 4w$$

*m

(2) Two beams AC and BD of lengths 9m and 10m respectively are hinged at C. these are supported on rollers at the left and right ends (A and D) A hinged support is pivoted at B. 7m from A. Using the principle of virtual work determine the reaction at the hinge C and the support B, when a load of 700N acts at a point 6m from D





$$\frac{\delta y_b}{7} = \frac{\delta y_c}{9}$$

$$\delta y_c = 1.2857\,\delta y_b$$

$$\frac{\delta y_e}{6} = \frac{\Delta y_c}{10}$$

$$\delta y_e = \frac{6}{10} \times \delta y_c$$

$$= \frac{6 \times 1.2857}{10} \delta y_b$$

$$= 0.7714 \delta y_b.$$

applying principle of virtual ~~force~~ work

$$R_b \times \delta y_b - 700 \times \delta y_e = 0$$

$$R_b = \frac{700 \times 0.7714 \delta y_b}{\delta y_b}$$

$$= 540N$$

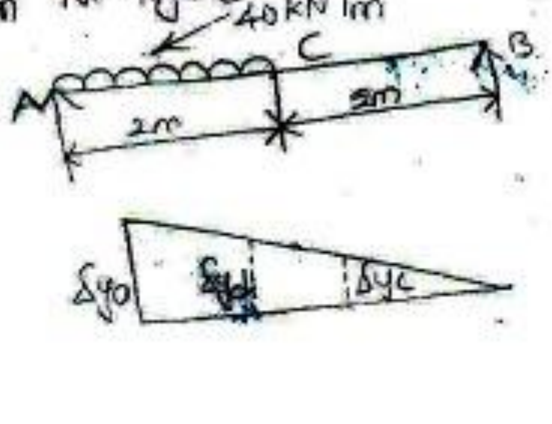
consider the beam AC

$$R_b \times \delta y_b - R_c \delta y_c = 0$$

$$\delta R_c = \frac{540 \times \delta y_b}{1.2857 \times \delta y_b}$$

$$= 420N.$$

Determine the reaction at A in simply supported beam shown in figure.



$$\frac{\delta y_a}{4} = \frac{\delta y_c}{2}$$

$$\delta y_c = \frac{\delta y_a}{2}$$

$$\delta y_d = \frac{\delta y_a + \delta y_c}{2}$$

$$= \frac{\delta y_a + \delta y_a/2}{2} = \frac{3\delta y_a}{4}$$

$$= 0.75 \delta y_a$$

Applying principle of virtual work.

$$R_a \times \delta y_a - 40 \times 2 \times \delta y_d = 0$$

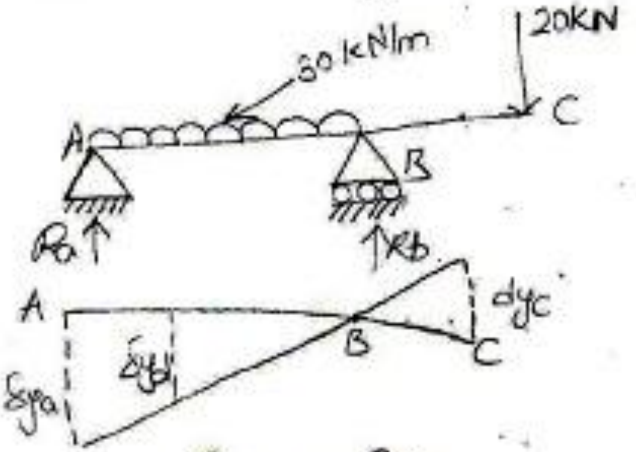
$$R_a \times \delta y_a - 80 \times 0.75\, \delta y_a = 0.$$

$$\delta y_a (R_a - 80 \times 0.75) = 0$$

$$R_a = 80 \times 0.75$$

$$= 60N$$

Determine the reaction at A in the overhanging beam AB as shown in fig.



$$\frac{\delta y_a}{6} = \frac{\delta y_c}{3}$$

$$\delta y_c = \frac{\delta y_a}{2}$$

$$\delta y_d = \frac{\delta y_a + \delta y_B}{2}$$

$$= \frac{\delta y_a + 0}{2} = \delta y_a/2$$

Applying principle of virtual work.

$$R_a \times \delta y_a - (30 \times 6) \times \delta y_d - 20 \times (-\delta y_c)$$

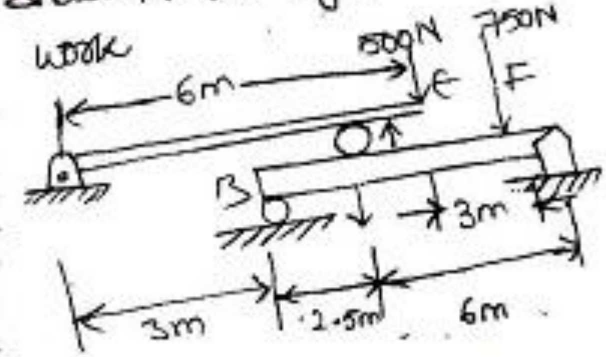
$$R_a \times \delta y_a - (30 \times 6) \times \frac{\delta y_a}{2} + 20 \times \frac{\delta y_a}{2} = 0.$$

$R_a \delta y_a - 90\delta y_a + 10\delta y_a = 0$

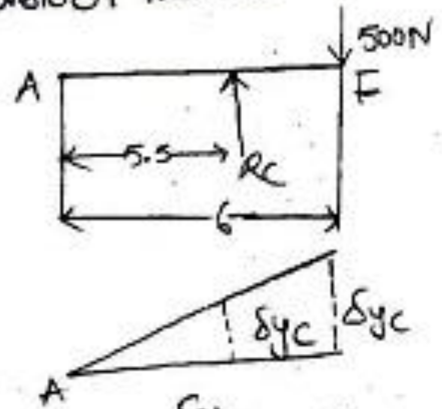
$\delta y_a [R_a - 80] = 0$

$R_a = 80$

* Determine the reactions at rollers B and C of the beam shown in the fig. below using the method of virtual work



consider the beam AE



$$\frac{\delta y_C}{6} = \frac{\delta y_c}{5.5}$$

$$\delta y_C = \frac{5.5}{6} \times \delta y_C$$

from the principle of virtual work

$$R_C \times \delta y_c - 500 \times \delta y_C = 0$$

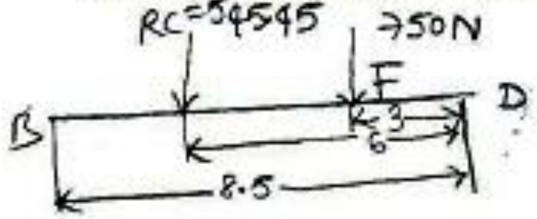
$$R_C \times \frac{5.5}{6} \times \delta y_C - 500 \times \delta y_C = 0$$

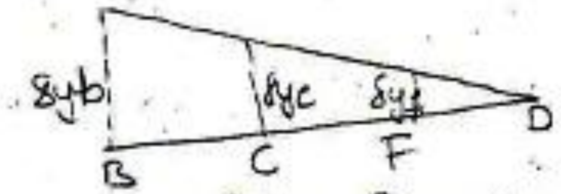
$$\delta y_C \left(R_C \times \frac{5.5}{6} - 500 \right) = 0$$

$$R_C = 500 \times 6/5.5$$

$$= 545.45\,N$$

consider the beam BD





$$\frac{\delta y_b}{8.5} = \frac{\delta y_c}{6} = \frac{\delta y_f}{3}$$

$$\delta y_c = \frac{6}{8.5} \times \delta y_b$$

$$\delta y_f = \delta y_c \times \frac{1}{2}$$

$$= \frac{6}{8.5} \times \delta y_b \times \frac{1}{2}$$

$$= \frac{3}{8.5} \delta y_b$$

from the principles of virtual work

$$R_B \times \delta y_b - R_C \times \delta y_c - 750 \times \delta y_f = 0.$$

$$R_b \times \delta y_b - R_C \times \frac{6}{8.5} \delta y_b - 750 \times \frac{3}{8.5} \times \delta y_b = 0.$$

$$\delta y_b \left(R_b - 545.45 \times \frac{6}{8.5} - 750 \times \frac{3}{8.5}\right) = 0$$

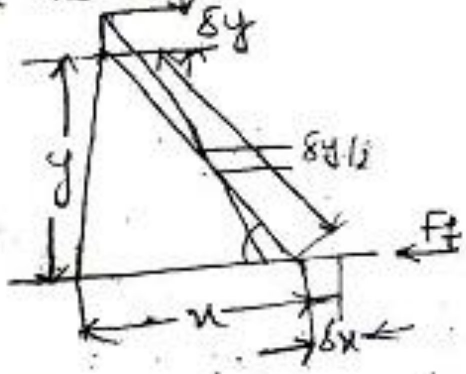
$$R_b = 545.45 \times \frac{6}{8.5} + 750 \times \frac{3}{8.5}.$$

$$R_b = 649.73N$$

Application to ladders:-

consider a ladder of length 'l' and weight 'w' leans against a smooth vertical wall as shown fig. by the principle of virtual work the

-nal force at the foot of the ladder can be dete-
-rmined.



$\cos\theta = x/l$

$x = l\cos\theta$

$\sin\theta = y/l$

$y = l\sin\theta$.

$\delta x = -l\sin\theta\,\delta\theta$

$\delta y = l\cos\theta.\delta\theta$

AD θ increases,

x decreases and y increases

thus δx is −ve and δy is +ve from the principle of virtual work.

$$F\times\delta x - w\times\frac{\delta y}{2} = 0$$

$$F = \frac{w}{2}\times\frac{\delta y}{\delta x}$$

$$F = w/2\times\frac{l\cos\theta\,\delta\theta}{l\sin\theta\,\delta\theta}$$

$$= \frac{w}{2}\cot\theta$$

A uniform ladder of weight 300N rest against a smo-oth vertical wall and rough horizontal floor making an angle of 60° with the horizontal fin-d the force of friction at the floor using the method of virtual work.

l = length of ladder

$\theta = 60^\circ$

By the principle of virtual work

$$F\delta x - \frac{W\delta y}{2} = 0$$

$$\delta x = l\sin\theta\,\delta\theta$$

$$\delta y = l\cos\theta\,\delta\theta$$

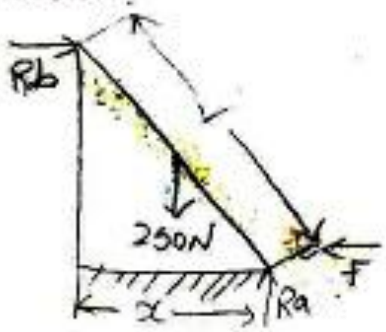
$$F = \frac{W}{2}\cot\theta$$

$$= \frac{300}{2}\cot 60^\circ$$

$$= \frac{300}{2} \times \frac{1}{\tan 60}$$

$$= 86.6N$$

A uniform ladder of weight 250N as shown in fig rests with its upper end against a smooth verti-cal wall and its foot on a rough horizontal ground making an angle of 45° with the ground find the force of friction of the ground using the method of virtual work?



$W = 250N$

$\theta = 45°$

from the pancipie of virtual work.

$F \times \delta x - W \frac{\delta y}{2} = 0$

$\delta x = L \sin\theta \, \delta\theta$

$\delta y = L \cos\theta \, \delta\theta$

$F = W/2 \cot\theta$

$= \frac{230}{2} \times \cot 45$

$= 125 \times \frac{1}{\tan 45}$

$= 125 N$

System of pulley:-

consider asystem of pulleys as shown in fig.

Giving virtual downward displacement, δy to effect p. the load w moves up through a distance, $\frac{\delta y}{2}$

from the principle of virtual work

$p \times \delta y - w \times \frac{\delta y}{2} = 0$

$p = w/2$

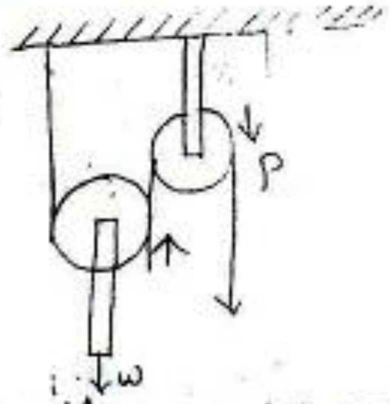
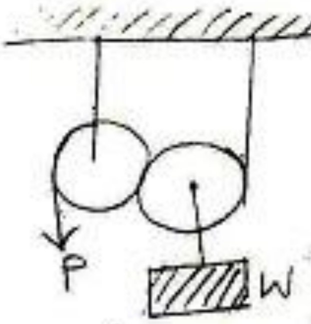


figure shows asystem of pulleys used to raise a weight of 250N. using the method of virtual work find the force required to hold the weight in equilibrium?



δy = virtual displacement of p in down word direction

then displacement of w in upword direction is $\left(\frac{\delta y}{2}\right)$

total virtual work.

$$P \times \delta y - w\frac{\delta y}{2} = 0$$

$$P = \frac{W}{2} = \frac{250}{2} = 125\,N.$$

Inclined plane:-

consider an inclined plane as shown in fig. Giving virtual down word displacement δy to lift p, the block weighing W moves up the plane.

The principle of virtual work can be expressed as. $P \times \delta y - F\delta y - w\delta y = 0$

$$F = \mu R$$

$$R = W\cos\theta$$

$$P \times \delta y - W\sin\theta\, \delta y - W\cos\theta \times \mu \times \delta y = 0$$

$$P = W(\sin\theta + \mu\cos\theta)$$

efficiency of inclined plane.

$$\eta = \frac{\text{useful work}}{\text{Extended work}}$$

$$= \frac{W\sin\theta\, \delta y}{P \cdot \delta y}$$

$$= \frac{W \sin\theta\, \delta y}{W(\sin\theta\, \delta y + \mu \cos\theta\, \delta y)}$$

$$= \frac{\sin\theta}{\sin\theta + \mu\cos\theta}$$

kinetics

the study of forces that causes the motion(ex: torque, gravity, friction etc) and classified into two groups Linear and angular motions.

Kinematics:- The study of describing movements (ex:- are displacement, time, velocity, etc)

formula:-

motion under uniform acceleration:-

(i) velocity of a particle $v = u + at$

(ii) Displacement $s = ut + \frac{1}{2}at^2$

(iii) Relation between velocity, acceleration and displacem

$$v^2 - u^2 = 2as$$

where u = initial velocity, m/sec

v = final velocity, m/sec

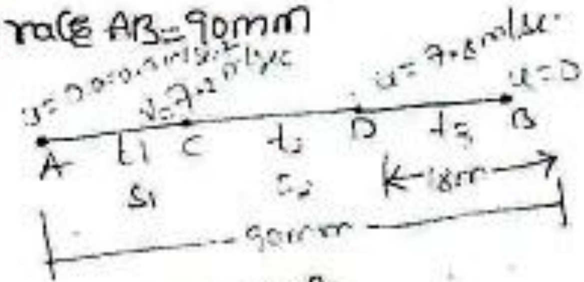
a = acceleration, m/sec^2

s = displacement, m.

problem:-

A bus starts from rest at a point 'A' and acceleration at the rate of 0.9 m/sec^2 until it reaches a speed of 7.2m/sec. it is then proceed with the same speed until brakes are applied. It comes to rest at a point B, 18m beyond the point where the brakes are applied

Assuming uniform acceleration determine the time required for the bus travelled from A to B. Distance AB = 90mm



$a_1 = 0.9$ m/sec^2

$v_1 = 7.2$ m/sec

$u_1 = 7.2$ m/sec

$v_1 = u_1 + a t_1$

$$t_1 = \frac{v_1 - u_1}{a_1}$$

$$= \frac{7.2-0}{0.9} = 8 \text{ sec}$$

$S_1 = u_1 t_1 + \frac{1}{2} a_1 t_1^2$

$= 0 \times 8 + \frac{1}{2} \times 0.9 \times (8)^2$

$= 28.8$m

Consider the motion from C to D

$S_2 = S - (S_1 + S_3)$

$= 90 - (28.8 + 18)$

$= 43.2$m.

for uniform velocity

$S_2 = u_2 \times t_2$

$$t_2 = \frac{S_2}{u_2}$$

$$= \frac{43.2}{7.2} = 6 \text{ sec.}$$

Consider motion from D to B

$$v_3^2 - u_3^2 = 2a_3 S_3$$

$$a_3 = \frac{v_3^2 - u_3^2}{2 \times s_3}$$

$$= \frac{0 - (7.2)^2}{2 \times 18}$$

$$= -1.44 \text{ m/sec}^2$$

$$t_3 = \frac{v_3 - u_3}{a_3}$$

$$= \frac{0 - 7.2}{-1.44}$$

$$= 5 \text{ sec}$$

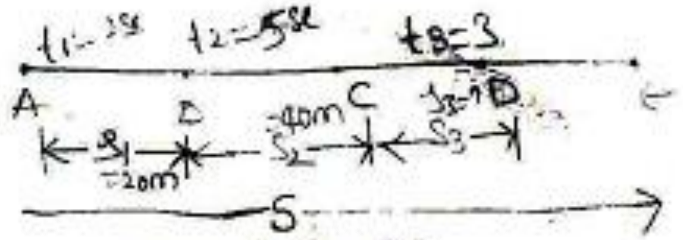
Total time from A to B.

$$T = t_1 + t_2 + t_3$$

$$= 8 + 6 + 5$$

$$T = 19 \text{ sec}.$$

* ⑵ A particle under a const decleration is moving in a straight line and cover a distance of 20m in first 2sec. and 40m in next 5 sec. calculate the distance it covers in the subsequent 3 sec. and total distance covered before it comes to rest



$$s = ut + \tfrac{1}{2} at^2$$

foor the motion from A to B

$$20 = u \times 2 + \frac{1}{2} a \times (2)^2$$

$$10 = u + a \quad \text{---} ① \qquad 70 = 7u + 7a$$

for the motion from A to C

$$60 = 7 \times u + \frac{1}{2} \times a \times 7^2$$

$$60 = 7u + 24.5a \quad \text{---} ②$$

$$7u + 24.5 \times a - 60 = 0$$

$$\underline{7u + 7a \quad - 70 = 0}$$

$$17.5a + 10 = 0$$

$$a = \frac{-10}{17.5}$$

$$= -0.57 \, m/sec^2$$

$$u = 10 + 0.57$$

$$u = 10.51 \, m/sec$$

foor the motion from A to D.

$$(60 + S_3) = 10.51 \times 10 - \frac{1}{2} \times 0.571 \times (10)^2$$

$$S_3 = (77.16 - 60) = 17.16 \, m$$

$$v^2 - u^2 = 2as$$

$$s = \frac{v^2 - u^2}{2 \times a}$$

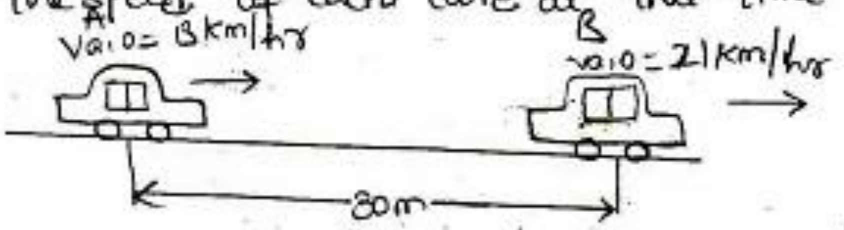
$$= \frac{(0 - 10.51)^2}{2 \times (-0.571)}$$

$$= 97.851 \, m.$$

Two cars A and B are travelling in adjecent highway lanes and at $t=0$ have the position and speed shown in fig. the car A has a constant acceleration 0.8 m/sec² and a car B has a constant deceleration of 0.6 m/sec², determine.

a) when and where car A will over take car B car B

b) the speed of each care at that time



sol:- car A overtakes car B after t seconds at a distance x meters ~~from its start x.~~ meters from its start

$a_a = 0.8\ m/sec$ $\qquad a_b = -0.6\ m/sec^2$

$v_{a,0} = 18\ km/hr$ $\qquad v_{b,0} = 21\ km/hr$

$= 3.61\ m/sec$

$v_{a,0} = 18\ km/hr$

$= \frac{18 \times 1000}{60 \times 60}$

$= 3.61\ m/sec$

$v_{b,0} = 21\ m/hr$

$= \frac{21 \times 1000}{3600}$

$= 5.83\ m/sec$

consider the motion of car A

$$S_A = u_{a0} t + \frac{1}{2} a_a t^2$$

$$= 3.61 \times t + \frac{1}{2} 0.8 \times t^2$$

$$x = 3.61t + 0.4t^2 \quad \text{--- (1)}$$

consider the motion of car B

$$S_{B20} = v_B t + \frac{1}{2} a_B t^2$$

$$(x-30) = 5.83 \times t + \frac{1}{2} \times (-0.6) \times t^2$$

$$(x-30) = 5.83t - 0.3t^2 \quad \text{--- (2)}$$

$$(x-30) = 5.83t - 0.3t^2$$

$$(x-30) = 3.61t + 0.4t^2 - 30$$

$$2.22t - 0.7t^2 + 30$$

$$0.7t^2 - 2.22t - 30 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2.22 \pm \sqrt{(-2.22)^2 - 4 \times 0.7 \times -30}}{2 \times 0.7}$$

$$= 8.32$$

$$x = 3.61 \times 8.82 + 0.4 \times (8.82)^2$$

$$= 57.92 m$$

velocity of cars after 8.32 sec

$$v_{a,t} = v_{a,0} + a t$$

$$= 3.61 + 0.8 \times (8.82)$$

$$= 10.26 m/sec$$

$$= 36.936 m/sec^2$$

velocity of car B

$v_{B,t} = v_{B,0} + at$

$= 5.83 + (-0.6) \times (8.32)$

$= 0.84$ m/sec

$= 3.017$ km/hr

motion of lift:-

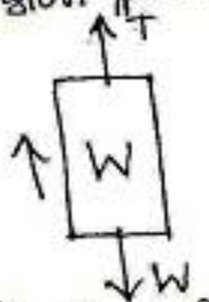
consider a lift moving with uniform acceleration. and carrying some weight consider two cases

a) lift moving upward.

b) lift moving downward.

a) lift moving upward:-

Let 'T' be the tension ~~ball~~ in the cable supporting the lift



lift is moving upward.

w be the total weight carried by the lift the net acceleration force.

$(T - w) = m \times a$

$T = w + m \times a$

$= w + \frac{mg \times a}{g}$

$w + w \frac{a}{g}$

$= w\left(1 + \frac{a}{g}\right)$

where

m= mass carried by the lift

a= acceleration of the lift

lift moving down words:-

As the lift moving downwords the weight is greater than the tension in the cable.

the net accelerating force

$$(w-T) = m \times a$$

$$-T = -w + ma$$

$$T = w - \frac{m \times g \times a}{g}$$

$$= w - w \times a/g$$

$$= w(1 - a/g)$$

(1) A lift has an upword direction of 1.225 m/sec²

a) what pressure will a man weighing 500 N exert on the floor of the lift?

b) what pressure would he exert if the lift had an acceleration of 1.225 m/sec² down words.

c) what upword acceleration would cause his weight to exert a pressure of 600N on the floor

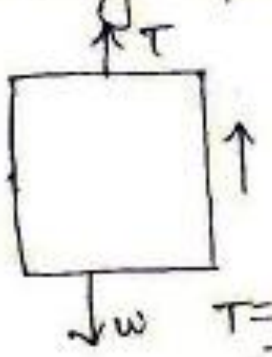
$$W = 500N$$

$$a = 1.225 \text{ m/sec}^2$$

$$m = \frac{w}{g} = \frac{500}{9.81}$$

$$= 50.97 \text{ kg}$$

a) Lift is moving upward

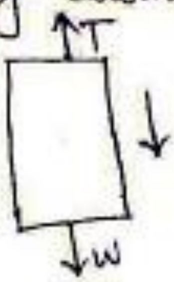


$T = w + ma$

$= 500 + 50.97 \times 1.225$

$= 562.436N$

b) lift moving downward



$(w - T) = ma$

$T = w - ma$

$= 500 - 50.97 \times 1.225$

$= 437.56N$

c) $T = 600N$

a = upward acceleration

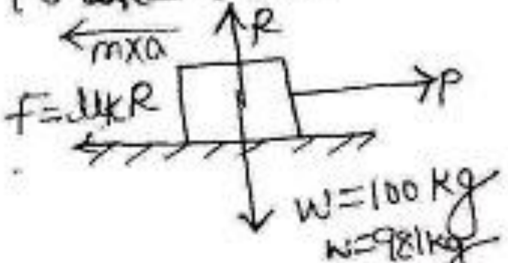
$(T - w) = m \times a$

$a = \frac{(T - w)}{m}$

$= \frac{(600 - 500)}{50.97}$

$= 1.962 \ m/sec^2$

* 100 kg block rest on a horizontal plane, find the magnitude of force required to give the block an acceleration of 2.5 m/sec² to the right the co-efficient of friction b/w the block and plane is $\mu_k = 0.25$.



$m = 100kg$

$W = m \times g$

$= 100 \times 9.81$

$= 981N$

$F = \mu_k R$

$= 0.25 \times 981$

$= 245.25N$

magnitude of force

$P = F + m \times a$

$= 245.25 + 100 \times 2.5$

$= 495.25$

Kinetics of rigid body:—

(i) Force and translation :— when a rigid body is constrained to move in translation (motion in a straight path) then its angular acceleration is zero.

$F = m \times a$

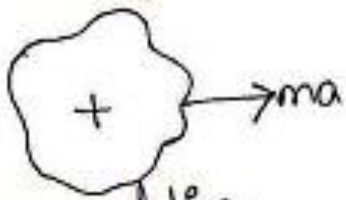
(ii) moment of couple (TORQUE) and rotation:— when a body is constrained to rotate about a fixed axis perpendicular to the reference plane and passing through its centre it is said to be centroidal rotation (rotation about centroidal axis)

$$\text{Torque} = I\alpha$$

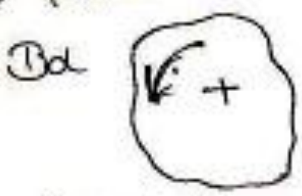
I is mass moment of inertia.

iii) motion of translation and rotation:-

In most cases of kinetics the body moves in a general plane motion which is equal to the sum of transla-tion and centroidal motions. In this case the body simultaneously subjected accelerating force and moment of couple.



a) Translation.



b) Rotation

c) Translation & rotation.

Relation b/w torque and moment of inertia:-

$$T = I\alpha \ \text{N-m}$$

where I = mass M.I. kg-m^2

α = Angular Acceleration rad/sec^2

motion of a body tied to a string passing over a pulley:-

consider a body of 'mass m' connected to a string passing over a pulley as shown in fig.

consider the motion of body

$(W-P) = ma$ —①

consider the motion of pulley

Torque, $T = P \times r$

$T = I\alpha$

$P \times r = I\alpha$

$P = \frac{I\alpha}{r}$

Linear acceleration

$a = \alpha \times r$

$\alpha = a/r$

$\therefore P = \frac{I \times a}{r^2}$ —②

from eqn ① & ②

$a = \frac{mg}{\left[\frac{I}{r^2} + m\right]}$ and

$P = \frac{mIg}{I + mr^2}$

If pulley is solid disc of mass m

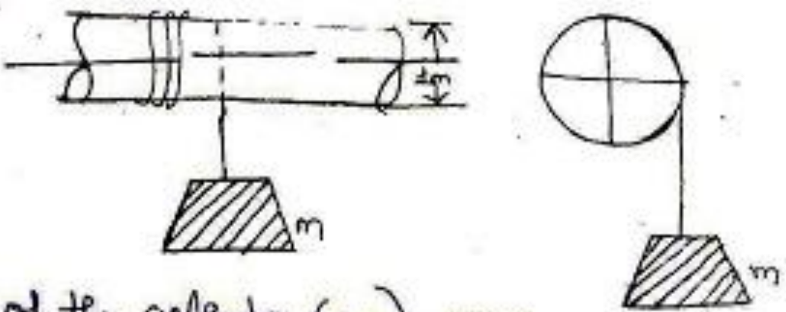
$I = \frac{mr^2}{2}$

$$a = \frac{m \times g}{\left(m + \frac{M}{2}\right)} = \frac{2mg}{2m+M}$$

$$P = \frac{mMg}{M+2m}.$$

y.m

① A homogeneous solid cylinder of weight 100N whose axis horizontally rotates about x-axis in frictionless bearing under the action of a 10N block which is carried by a rope wrapped around the cylinder what will be the angular velocity of the cylinder 2seconds after the motion starts. Assume diameter of the cylinder as 100cm?



Weight of the cylinder $(w_1) = 100N$

mass of the cylinder $(m) = \frac{100}{9.81}$

$= 10.194 kg$

weight of the block $(w) = 10N$

mass of the block $(m) = \frac{10}{9.81} = 1.019$.

Consider the motion of block

$(w - P) = m \times a$

$(10 - P) = 1.019 \times a$ —— ①

Consider the motion of cylinder.

$$P = \frac{I \times a}{r^2}$$

where I is moment of Inertia

$$= \frac{mr^2}{2}$$

$$P = \frac{mr^2}{2} \times \frac{a}{r^2} = \frac{m \times a}{2}$$

$$= \frac{10.194 \times a}{2}$$

$$P = 5.097 \times a \quad \text{--- (2)}$$

from eqn (1) & (2)

$$(10 - 5.097 \times a) = 1.019 \times a$$

$$10 = a(1.019 + 5.09)$$

$$a = \frac{10}{(1.019 + 5.09)}$$

$$a = 1.635 \text{ m/sec}^2$$

Angular acceleration.

$$\alpha = a/r$$

$$= \frac{1.635}{0.5}$$

$$= 3.27 \text{ rad/sec}^2$$

Angular velocity after 2 seconds

$$\omega = \omega_0 + \alpha t$$

$$= 0 + 3.27 \times 2$$

$$\omega = 6.54 \text{ rad/sec.}$$

D'Alembort's principle:-

Consider a mass "m" moving with uniform acceleration under the influence of external force, F. Then according to newtons second law of motion

$$F = ma \quad \text{--- (1)}$$

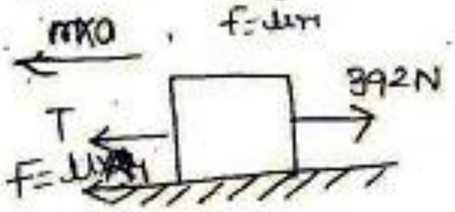
Jean . D 'Alembort introduce the concept of dynamic equilibrium to slove the problem related to motion of bodies D' Alembert principle states that the body will be in dynamic equilibrium under the action of external force(F) and inertia force (ma). Based on D'Alembert principle eq (1) can be expressed as $F - ma = 0$

problems:

(1) Two weights 785N and 196N are connected by a thread and moves along a rough horizontal plane under the action of a force 392N applied to the first weight of 785N as shown in the figure. the co-efficient of friction between the sliding surfaces of the weights and the plane is 0.3 Determine the acceleration of weights and tension in the thread using 'D' Alembert principle?

$W_1 = 785N$, $W_2 = 196N$, $P = 392N$, $\mu = 0.3$, $a = ?$

Consider body (1)



$$392 = T + m\times a + F$$

$$392 - T - m\times a + \mu R = 0$$

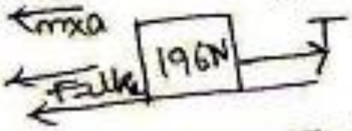
$$392 - T - \left(\frac{785}{9.81}\right)\times a + 0.3\times 785 = 0$$

$$392 - T - \left(\frac{785}{9.81}\right)\times a + 0.3\times 785 = 0$$

$$627.5 - T - 80.02\times a = 0$$

$$T + 80.02\times a - 627.5 = 0 \quad -(1)$$

Consider the body (2)



$$T - m\times a - F = 0$$

$$T - \left(\frac{196}{9.81}\right)\times a - 0.3\times R_2 = 0$$

$$T - 19.98a - 58.8 = 0 \quad -(2)$$

$$T + 80.02\times a - 627.5 = 0$$

$$T - 19.98\times a - 58.5 = 0$$

$$100\times a - 569 = 0$$

$$100\times a = 569$$

$$a = \frac{569}{100} = 5.69\ m/sec^2$$

$$T + 80.02\times a - 156.5 = 0$$

$$T - 19.98\times a - 58.5 = 0$$

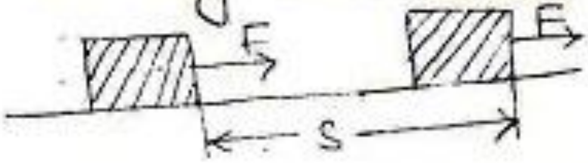
$$100\times a - 97.7 = 0$$

$$a = \frac{97.7}{100} = 0.977\ m/sec^2$$

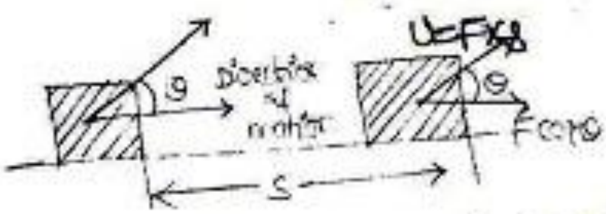
$$T = 58.5 + 19.98\times 0.977 = 78.82\ N$$

Work, power and Energy

Work is done when a force is applied to a body and body moves in the direction of force, it is de-fined as the product of force and displacement of the body in the direction of force.



Work done = Force in the direction of motion x displacement



Workdone = Component of force in the direction of motion x displacement

$U = F\cos\theta \, S$

The unit of work is Joule (J)

$1J = 1 N\text{-}m$

power:-

power is the rate of doing work. The unit of power is watt (w) which is defined as the rate of work equal to 1 J/sec. It is equal to work done by force of one neuton in moving through a distance of one metre in one second.

$$power = \frac{work\,done}{time}$$

The instantaneous power developed by a force F moving at a speed is given by

power = force × speed.

$= F \times \theta \quad \frac{N\text{-}m}{s}, \frac{J}{s}$, watt.

power developed by torque:-

consider a torque, T applied to rotate and angle, θ in time, t then the work done by torque work done = torque × angle turned

$= T \times \theta$

Rate of doing work

$P = T\theta / t$

But $\theta / t = \omega$

Rate of doing work (power)

$P = \frac{T \times \theta}{t}$

$P = T \times \omega$

where T = Torque, N-m

ω = angular speed, rad/sec

If N = No of revolutions made by an axle per second then

$\omega = 2\pi N$

∴ power developed

$P = 2\pi N T$ watts

Efficiency:- The mechanical efficiency of a machine or engine is defined as ratio of useful work output to the actual work input in a given time

$$\text{Efficiency} = \frac{\text{work output}}{\text{work input}}$$

$$= \frac{\text{power output}}{\text{power input}}.$$

Energy:-

The Energy of a body is its capacity to do work there are many different forms of energy such as mechanical energy, heat energy, chemical energy etc.

there two forms of mechaniced energys.

1) Kinetic energy
2) potential energy.

1. kinectic energy:- The energy pocessed by a body by virtual of its motion is called kinetic energy. The kinetic energy of a body of mass 'm' kg moving with velocity of 'v' m/sec is given as

$$\text{kinetic Energy} = \frac{1}{2}mv^2 \quad N\text{-}m \text{ (or) } J.$$

2. Work - energy equation:- work done by a body is equal to change in kinectic energy of the same body.

$$\text{force} \times \text{displacement} = \text{change in } K.E$$

KE of body in rotation, work done by torque

= change in K.E of body

$$W \cdot D = \frac{I \times \omega^2}{2}$$

If ω_1 is initial angular velocity, rad/sec

ω_2 is final angular velocity rad/sec.

W.D by torque = change in K.E

$$W \cdot D = \frac{1}{2} I \left[\omega_2^2 - \omega_1^2\right]$$

② potential Energy:- It is the energy possed by a body by virtue of its position.

consider a mass of 'm' kg is raised through a higt 'h' above the ground level. Then

$$P.E = W.D = \text{weight} \times \text{height}$$

$$= mgh \quad N\text{-}m \text{ (or) } J.$$

P.E=0 $K.E = \frac{1}{2}mv^2$

$$v = \sqrt{2gh}$$

potential energy at B

= kinetic energy at A

$$mgh = \frac{1}{2}mv^2$$

principle of conservation of energy:—

the law of conservation of energy states that can neither be created nor destroyed, but it can only be transformed from one form to another

Law conservation of energy applied to freely fallin-g body:-

from the principle of conservation of energy it is clear that sum of potential and kinetic energies of a freely falling body is constant throughout its motion. To prove this

consider a body of mass 'm' at height 'h' (i.e, at position 'A' from the ground level 'c')

let 'B' be the another position (mid point b/w C and A) of the body

velocity at 'A' is zero. and let v_b and v_c be the velocity at 'B' and 'C' respectively

$$v_b = \sqrt{2g\,h/2} = \sqrt{gh}$$

$$v_c = \sqrt{2gh}$$

$$\text{Energy at } A = P.E + K.E$$
$$= mgh + 0$$
$$= mgh$$

$$\text{Energy at } B = P.E + K.E$$
$$= mg\,h/2 + \tfrac{1}{2} m v_b^2$$
$$= mg\,h/2 + \tfrac{1}{2} m (\sqrt{gh})^2$$
$$= mg\,h/2 + \tfrac{1}{2} mgh$$
$$= mgh$$

$$\text{energy at } C = P.E + K.E$$
$$= 0 + \tfrac{1}{2} m v^2 \quad (\because h = 0)$$

$= \frac{1}{2} m (\sqrt{2gh})^2$

$= \frac{1}{2} \times m \times 2gh$

$= mgh$

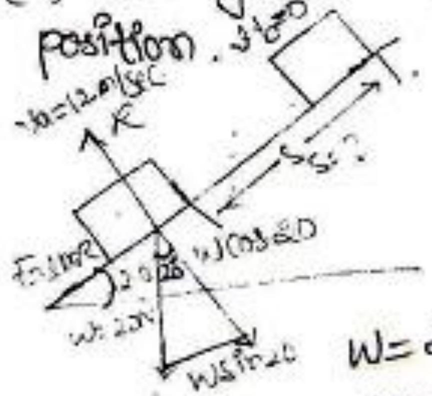
from the above it is clear that the sum of energ-ies of the body remains const.

*m

① A body weighing 20N is projected up a 20° inclined plane with a velocity of 12m/sec co-efficient of friction is 0.15, find

(i) the maximum distance 'S' the body will move up the inclined plane.

(ii) velocity of a body when it returns to original position.



$W = 20N$

$m = \frac{20}{9.81} = 2.039$ kg

K.E at position on A

$K.E = \frac{1}{2} m v_a^2$

$= \frac{1}{2} \times 2.039 \times (12)^2$

$= 146.81$ N-m.

Total Resistance, R

$R = (F + W\sin 20)$

$R = (\mu R + W\sin 20)$

$= [0.15 \times W\cos 20 + W\sin 20]$

$= [0.15 \times 20 \times \cos 20 + 20 \cdot \sin 20]$

$= 9.65N$

By principle of work energy

w.D by resistance = change in k.E

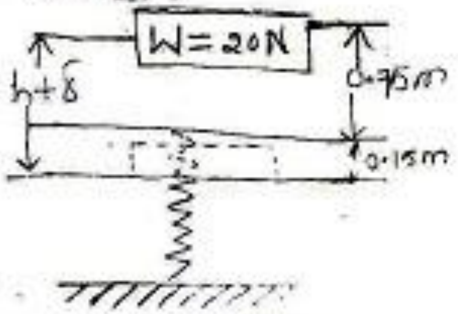
$R \times s = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$

$= \frac{1}{2} \times 2.039 \times (12)^2$

$s = \frac{0.5 \times 2.039 \times (12)^2}{9.65}$

$= 15.198m.$

A block of weight 20N falls at a distance of 0.75m on top of the spring. Determine the spring stiffness if it is compressed by 150mm to bring the weight momentarily to rest?



$W = 20N$

$h = 0.75m$

$\delta = 150mm$

$= 0.15m$

from work energy principle

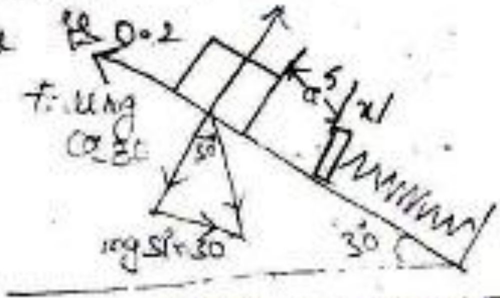
$W(h+\delta) = \frac{1}{2} k \delta^2$

$20(0.75 + 0.15) = \frac{1}{2} \times k [0.15]^2$

$k = \frac{2 \times 20 \times 0.9}{0.0225}$

$= 1600 N/m$

A block of mass 5kg resting on a 30° inclined plane is released. The block travelling of 0.5m along inclined plane hits a spring stiffness 15N/cm shown fig. Find the maximum compression of spring. Assume co-efficient of friction b/w the block and inclined plane is 0.2



Stiffness $(k) = 15\,N/cm$

$= 1500\,N/m$

$\mu = 0.2$

$F = \mu R$

$= 0.2 \times W\cos30$

$= 0.2 \times mg\cos30$

$= 0.2 \times 5 \times 9.81 \times 0.86$

$F = 8.49N$

work done on the block

$= (mg\sin30 - F)(0.5 + x)$

$= (5 \times 9.81 \times 0.5 - 8.49)(0.5 + x)$

$= (16.085) \times (0.5 + x)$

$= 18.015 + 16.085x)$

Energy absorbed by the spring

$= \frac{1}{2} \times k \times x^2$

$= \frac{1}{2} \times 1500 \times x^2$

$= 750x^2\ N\text{-}m$

from the work-energy

from the work-energy principle

w.D = change K.E

$(8.015 + 16.035x) = 750x^2$

$750x^2 - 16.035x - 8.15 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

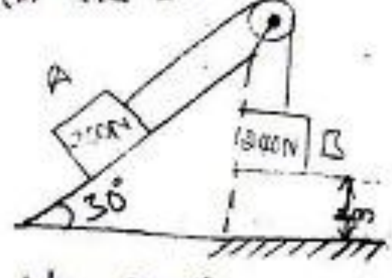
$x = 0.115m$

$x = 115mm.$

Two blocks of A (200N) and B (240N) are connected a shown in fig. when the motion begins the block B is fr 1m above the floor. Assuming the pulley to be fricti less and weightless Determine

a) The velocity of block A, when the block B touches the floor.

b) How fix the block A move, up the plane?

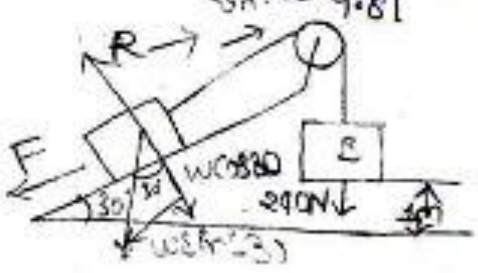


$W_A = 200N$

$$m_A = \frac{200}{9.81} = 20.38kg$$

$W_B = 240N$

$$m_B = \frac{240}{9.81} = 24.46kg$$



$F = \mu \times R_a$

$= 0.2 \times W \cos 30$

$= 0.2 \times 200 \times 0.866$

$= 34.64N$

Apply work energy principle

$(240 - W\sin 30 - F) \times 1$

$= \frac{1}{2} \times m_A \times v_a^2 + \frac{1}{2} \times m_B \times v_b^2$

$240 - 200 \times 0.5 - 34.64 = \frac{1}{2} \times 20.38 \times v^2 + \frac{1}{2} \times 24.46 \times v^2$

$105.36 = v^2 [12.23 + 10.19)$

$v = \sqrt{\frac{105.36}{22.42}}$

$= 2.168\ m/sec.$

$(W\sin 30 + F) \times x = \frac{1}{2} \times m_A \times v^2$

$(200 \times 0.5 + 34.64) \times x = \frac{1}{2} \times 20.38 \times (2.168)^2$

$x = \frac{0.5 \times 20.38 \times (2.168)^2}{134.64}$

$= 0.35\ m.$

A flywheel 50KN and having radius of gyration 1m, loss its speed from 400rpm to 280rpm in 2min calculate.

(i) Torque acting on it (ii) change in kinetic energy (iii) change in angular momentum.

$W = 50 \times 10^3\ N$

$m = W/g$

$= \frac{50000}{9.81}$

$= 5096.83kg$

$K = 1m$

mass m. $I = mk^2$

$= 5096.83 \times 1$

$= 5096.83 \, kg \, m^2$

$\omega_1 = 2\pi N_1$

$= \frac{2 \times \pi \times 400}{60}$

$= 41.88$ rad/sec

$\omega_2 = 2\pi N_2 / 60$

$= \frac{2 \times \pi \times 280}{60}$

$= 29.32$ rad/sec

α = angular acceleration

$\alpha = \frac{(\omega_2 - \omega_1)}{t}$

$= \frac{(29.32 - 41.88)}{2 \times 60}$

$= -0.1047$ rad/sec^2

(i) Torque $T = I\alpha$

$= 5096.83 \times -0.1047$

$= -533.63$ N-m

(ii) Change in k.E

$= \frac{I}{2}\left[\omega_2^2 - \omega_1^2\right]$

$= \frac{5096.83}{2}\left((29.32)^2 - (41.88)^2\right)$

$= -2281115.4$ N-m

(decreasing in k.E)

(iii) change in angular momentum

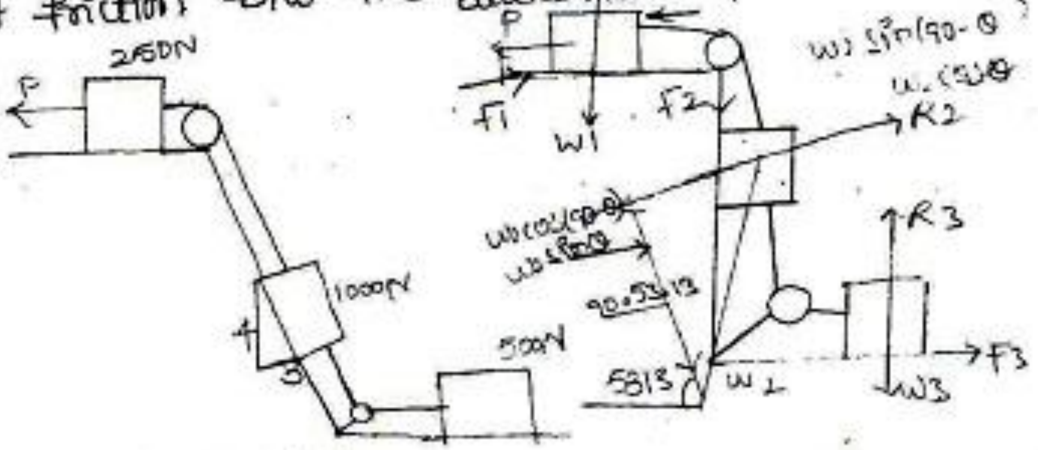
$= I(\omega_2 - \omega_1)$

$= 5096.83\,(29.42 - 41.88)$

$= -63557.6\ N\text{-}m$

(decreasing)

Determine the constant force 'p' that will give the system of bodies shown in fig. a velocity of 3 m/sec after moving 4.5m from rest coefficient of friction b/w the blocks and plane is 0.3.



$W_1 = R_1 = 250$

$\theta = \tan^{-1} 4/3$

$= 53.13°$

$F = \mu R_1$

$= 0.3 \times 250$

$= 75N$

$F_2 = \mu \times w_2 \sin(90-53.13)$

$= 0.3 \times 100 \cos 53.13$

$= 18.08N$

$F_3 = \mu \times R_3$

$= 0.3 \times 500$

$= 150N$

from principle of work energy

W.D = K.E

Total Resistence (R)

$= F_1 + F_2 + w_2 \sin 53.13 + F_3$

$= 75 + 18.03 + 1000 \sin 53.13 + 150$

$= 1043.02N$

$(P - R) \times 5 = \frac{1}{2} m v^2$

$(P - 1043.02) \times 4.5 = 0.5 \times \left(\frac{250 \times 1000 + 500}{9.81}\right) \times 3^2$

$4.5 \times P - 1043.02 \times 4.5 = 0.5 \times 178.39 \times 9$

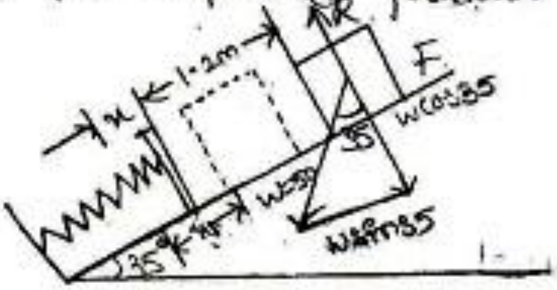
$$P = \frac{0.5 \times 178.39 \times 3 + 1043.02 \times 4.5}{4.5}$$

$= 1310.6N$

(2) A 50N block is released from rest on an inclined plane which makes an angle 35° to the horizontal. The block starts from 'A' slides down a distance of 1.2m and strike a spring with a stiffness of 8 kN/m. The coefficient of friction b/w the inclined plane and the block is 0.25. Determine

(i) The amount of spring get compressed.

(ii) Distance the block will rebound up the plane from the compressed position.



from work energy principle

$$(W \sin 35 - F) \times (1.2 + x) = \frac{1}{2} \times 8000 \times x^2$$

$$(50 \times \sin 35 - 0.25 \times W\cos 35)(1.2 + x) = 4000x^2$$

$$18.439(1.2 + x) = 4000x^2$$

$$4000x^2 - 18.439x - 22.13 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{18.439 + \sqrt{(18.439)^2 - 4 \times 4000 \times (-22.13}}{2 \times 4000}$$

$= 0.076\,m$

x_1 = The distance the block will rebound.

$(f_1 + w \sin 35) \times x_1 = \frac{1}{2} \times 8000 \times (0.076)^2$

$(0.25 \times 50 \cos 35 + 50 \sin 35) \times x_1$

$= \frac{1}{2} \times 8000 \times (0.076)^2$

$(28.679 + 10.24) x_1 = 4000 \times (0.076)^2$

$$x_1 = \frac{4000 \times (0.076)^2}{(28.679 + 10.24)}$$

$= 0.59\,m.$

mechanical vibration :-

vibration is a periodic motion which repeat itself up the definite interval of time

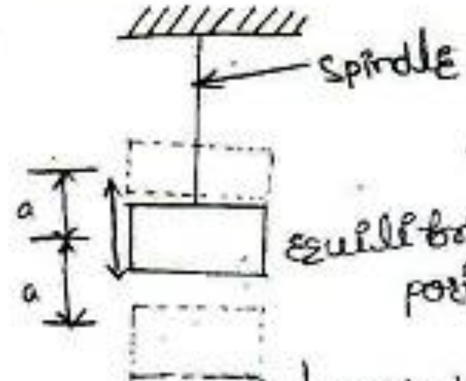
Free vibration :-

The periodic motion in the absence of external force is called free vibration.

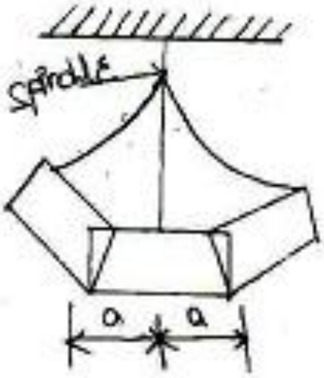
Free vibrations are classified as

(i) Longitudinal vibration.

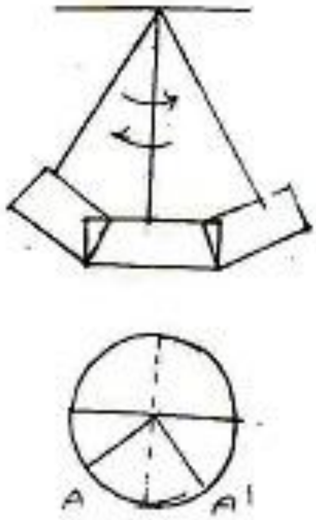
(ii) Torsional vibration.



Longitudinal vibration



Transverse vibration.



Equilibrium position

Torsional vibration

In longitudinal vibration the body moves up and down from its equilibrium position. If a weight moves up and down and the spindle is subjected compression and tension respectively. In transverse vibration the wei-ght is moved from one side to other side from its mean (or) equilibrium position. In this case spindl-e is subjected to bending.

In torsional vibration the body has an oscilla-tory motion the body has or about the vertical axis. under torsional vibration the spindle is subjected to compression and tension respectively.

Definitions:-

(i) Amplitude:- The maximum displacement of a body from its mean position is called amplitude.

(ii) oscillation:-
one complete vibration (i.e, to and fro motion) of a body is called oscillation.

(iii) periodic Time (T):-
It is the time taken by a particle for one complete oscillation.

periodic time $T = \frac{2\pi}{\omega}$

iv) Frequency:- the number of oscillations or vibra-tions per second is called frequency and denoted by 'n'

frequency, $n = \frac{\omega}{2\pi} = \frac{1}{T}$

Thus frequency is reciprocal of periodic time
The frequency of periodic motion (vibration) in terms of static defelction (δ)

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

where δ = static deflection

$= \frac{\sigma}{E} \times l$ for longitudinal vibration.

$\delta = \frac{wt^3}{3EI}$ for transverse vibrations

for torsional vibration.

$$n = \frac{1}{2\pi}\sqrt{\frac{Kt}{I}}$$

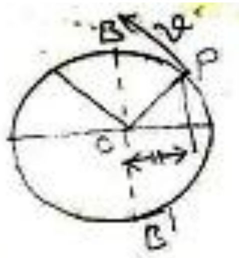
where Kt = torsional stiffness

I = mass moment of Inertia.

Simple harmonic motion:-
simple harmonic motion is a particular form of periodic motion in which acceleration of a particle or body is always directed towards fixed point in its path and is proportional to its distance from fixed point.

Equation of simple harmonic motion:-
velocity of a particle moving with SHM.

$$v = \omega\sqrt{r^2 - x^2}$$



$v_{x, max} = \omega \times r$

Acceleration of particle moving with SHM

$a_x = \omega^2 x$

$a_{x, max} = \omega^2 \times r$.

In a mechanism a cross head moves in a straight guide with simple harmonic motion at a distance of 125mm and 200mm from its mean position. It has velocity of 6m/sec and 3m/sec respectively. find the amplitude, maximum velocity and period of vibration. If the cross head weighs 20N, what is maximum force on it in the direction of motion?

6m/sec — 125 mm

3m/sec — 200mm

$x_1 = 125mm$

$= 0.125m$

$v_1 = 6 m/sec$

$x_2 = 200mm$

$= 0.2 m$

$v_2 = 3 m/sec$

$W = 20N$

$m = \frac{20}{9.81}$

$= 2.08 Kg$

velocity $v = \omega\sqrt{r^2 - x^2}$

$$6 = \omega\sqrt{r^2 - 0.125^2} \quad \text{--- (1)}$$

$$3 = \omega\sqrt{r^2 - (0.2)^2} \quad \text{--- (2)}$$

Dividing (1) by (2)

$$\frac{6}{3} = \frac{\sqrt{r^2 - 0.015}}{\sqrt{r^2 - 0.04}} = 2$$

Squaring on both sides

$$4 = \frac{r^2 - 0.0156}{r^2 - 0.04}$$

$$4(r^2 - 0.04) = r^2 - 0.0156$$

$$8r^2 = 0.16 - 0.0156$$

i) amplitude $(r) = 0.219\,m$

$$6 = \omega\sqrt{(0.219)^2 - 0.125^2}$$

$$\omega = \frac{6}{\sqrt{0.1798}} = 33.37 \text{ rad/sec}$$

(ii) maximum velocity

$$v_{max} = \omega \times r$$

$$= 33.37 \times 0.219$$

$$= 7.089 \text{ m/sec}$$

iii) period of vibration

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2 \times \pi}{33.37} = 0.19 \text{ sec}$$

maximum acceleration

$$a_{max} = \omega^2 \times r$$

$$= 33.37^2 \times 0.219$$

$$243.87 \text{ m/sec}^2$$

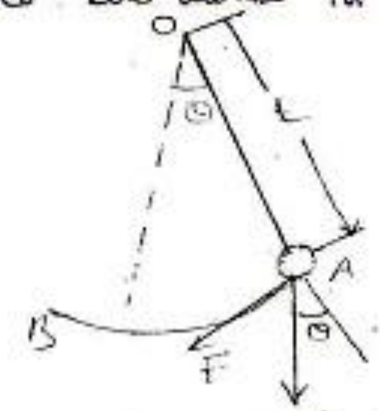
iv) maximum force.

$F_{max} = m \times a_{max}$

$= 2.038 \times 243.87$

$= 49N.$

simple pendulum:— A simple pendulum consisted of a heavy bob of certain mass (m) tied to one end of an inextensible string, the other end of which is fixed to a rigid support. The bob swings forwards and backwards in harmonic motion.



let θ = angle made by the string to the vertical

l = length of string

Time period $(T) = 2\pi\sqrt{l/g}$

② A pendulum having a time period of 1 second is installed in a lift. Determine its time period when

(a) the lift moves upward with an acceleration of g/10 m/sec²

b) The lift is moving downwards with an acceleration of g/10 m/sec²

$T = 1$ sec

$T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{l}{g} = \left(\frac{1}{2\pi}\right)^2$

$= 0.02533$

$l = 0.02533 \times 9.81$

a) $= 0.248$ m

when lift is moving upwards

$T = 2\pi\sqrt{\frac{l}{(g+a)}}$

$= 2\pi\sqrt{\frac{0.248}{\left(9.81 + \frac{9.81}{10}\right)}}$

$= 0.95$ sec

b) when lift moving down woods

$T = 2\pi\sqrt{\frac{l}{(g-a)}}$

$= 2\pi\sqrt{\frac{0.248}{\left(9.81 - \frac{9.81}{10}\right)}}$

$= 1.05$ sec.

Torsional pendulum:—

Periodic Time $(T) = 2\pi\sqrt{\frac{LI}{GJ}}$

where I= mass moment of Inertia

θ= Angle of rotation

G= modules of rigidity

J= Area moment of inertia.

A vertical shaft 5mm in diamiter and 1.2m in length has its upper end fixed to celling. At the lower end is carried a rotor is $0.85 \times 10^5 N/mm^2$ calculate the frequency of torsional vibration for the system? if diameter of rotor 180mm.

sol:- $d = 5mm$

$= 0.05m$

$J = \frac{\pi (0.05)^4}{32}$

$= 6.135 \times 10^{-11} m^4$

$W = 30N$

$m = \frac{30}{9.81} = 3.058 kg$

$r = \frac{180}{2} = 90mm$

$= 0.09m$

$I = \frac{mr^2}{2} = \frac{3.058 \times 0.09^2}{2}$

$= 0.0124 \; kg\text{-}m^2$

$G = 0.85 \times 10^5 N/mm^2$

periodic time

$T = 2\pi \sqrt{\frac{lI}{G \cdot J}}$

$= 2\pi \sqrt{\frac{1.2 \times 0.0124}{0.85 \times 10^{11} \times 6.136 \times 10^{-11}}}$

$= 0.335 \; sec$

$\therefore$ Frequency $n = \frac{1}{T}$

$= \frac{1}{0.335}$

$= 2.985$ cycles/sec