RRB JE CBT 2

Marathon Class

Complete Fluid Mechanics

LIVE 10AM | 4^{th} Aug (SUN)

Civil Engineering by Sandeep Jyani

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Fluid Mechanics

Mechanics

1. Engineering Mechanics
2. Strength of Materials
3. Fluid Mechanics

Mechanics (Branch of Science which deals with study of Forces and their Effects on bodies)

1. Engineering Mechanics

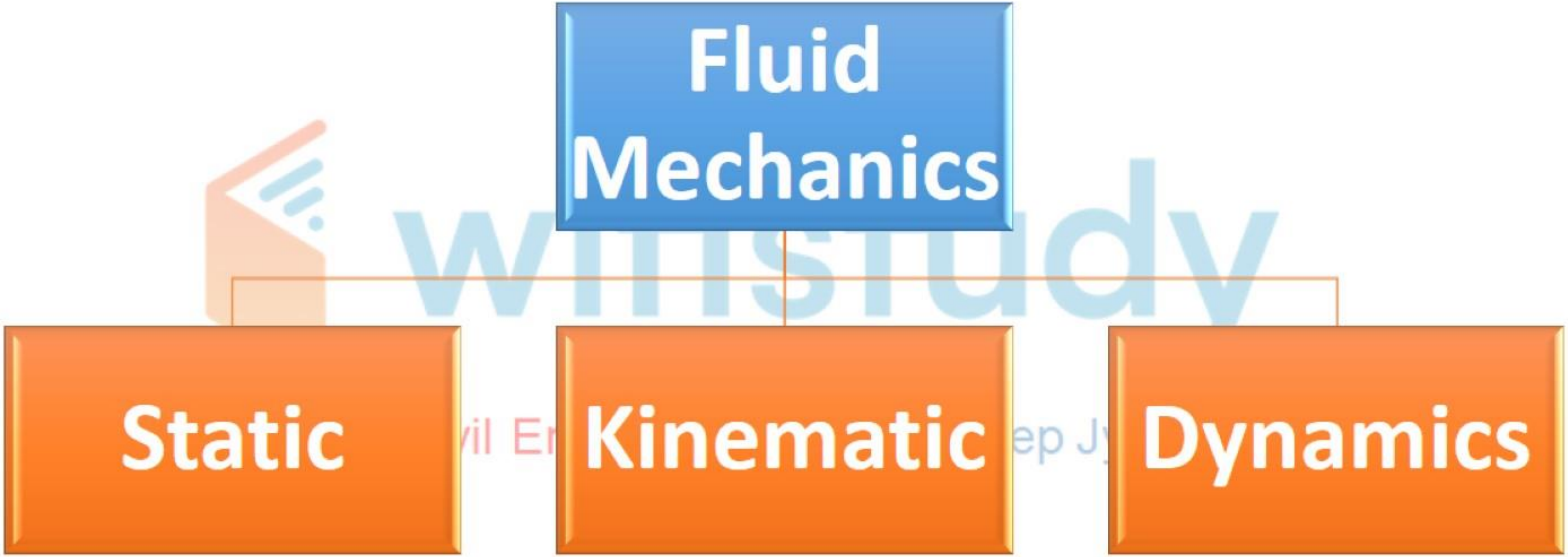
- Branch of Science that deals with study of forces and their effects on **RIGID BODIES**
- Displacement, Velocity, Acceleration, etc

2. Strength of Materials

- Branch of Science that deals with study of forces and their effects on **DEFORMABLE BODIES**
- Stress, Strain, etc

3. Fluid Mechanics

- Branch of science that deals with study of forces and their effects on **Fluids**
- Flow, continuous deformation, viscosity, etc


Fluid Mechanics
Static
Kinematic
Dynamics

Fluid Mechanics

Branch of science that deals with behavior of fluid(liquid or gases) at rest as well as in motion

1. Static

- Study of fluids at rest is called as Fluid Statics

2. Kinematics

- Study of Fluids in motion where ***pressure forces are not considered*** is called Fluid Kinematics

3. Dynamic

- Study of Fluids in motion, where ***pressure forces are also considered*** for the fluids in motion

FLUID

Liquid *Gas*

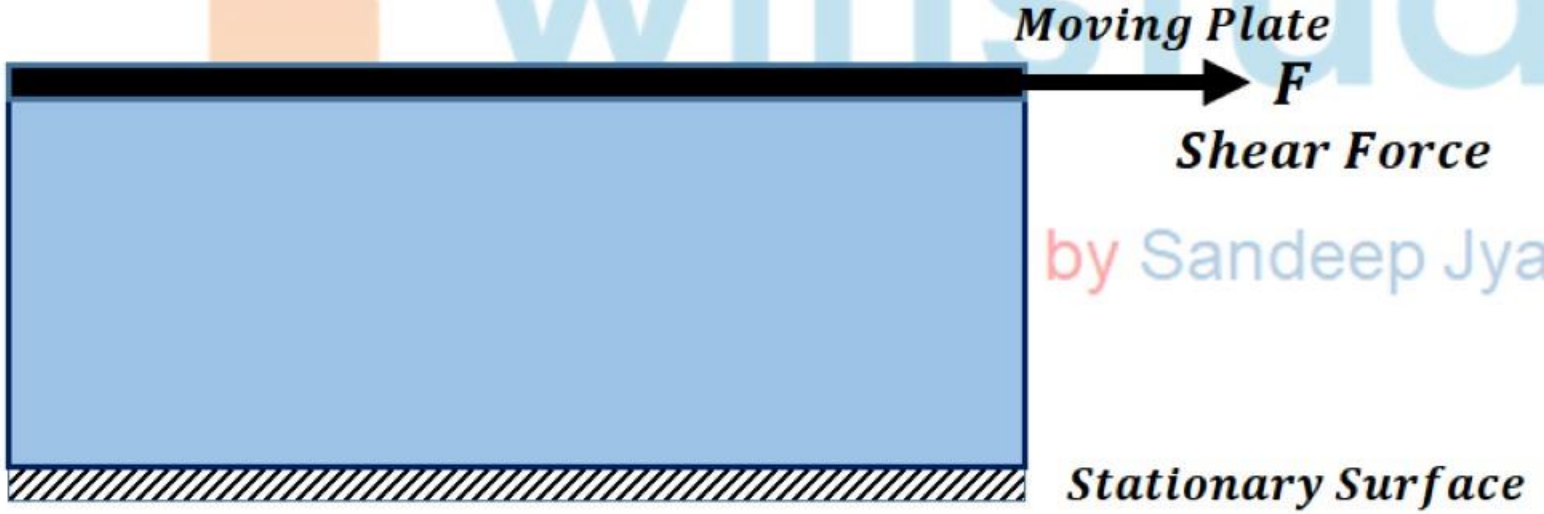
- *Liquid*→ Visible under atmospheric pressure and temperature
- *Gas*→ Not visible under atmospheric temperature and pressure

Some Basic Terms

- **FLUID: A fluid is a substance which is capable of flowing under the action of Shear Force**
- **In this connection fluid can also be defined as the state of matter that cannot sustain any shear stress.**
- **However smaller the force may be, i.e. under the action of even small shear force also, the fluid is capable to flow**

Some Basic Terms

- **<u>FLUID</u>: A fluid is a substance which is capable of flowing under the action of Shear Force**



Some Basic Terms

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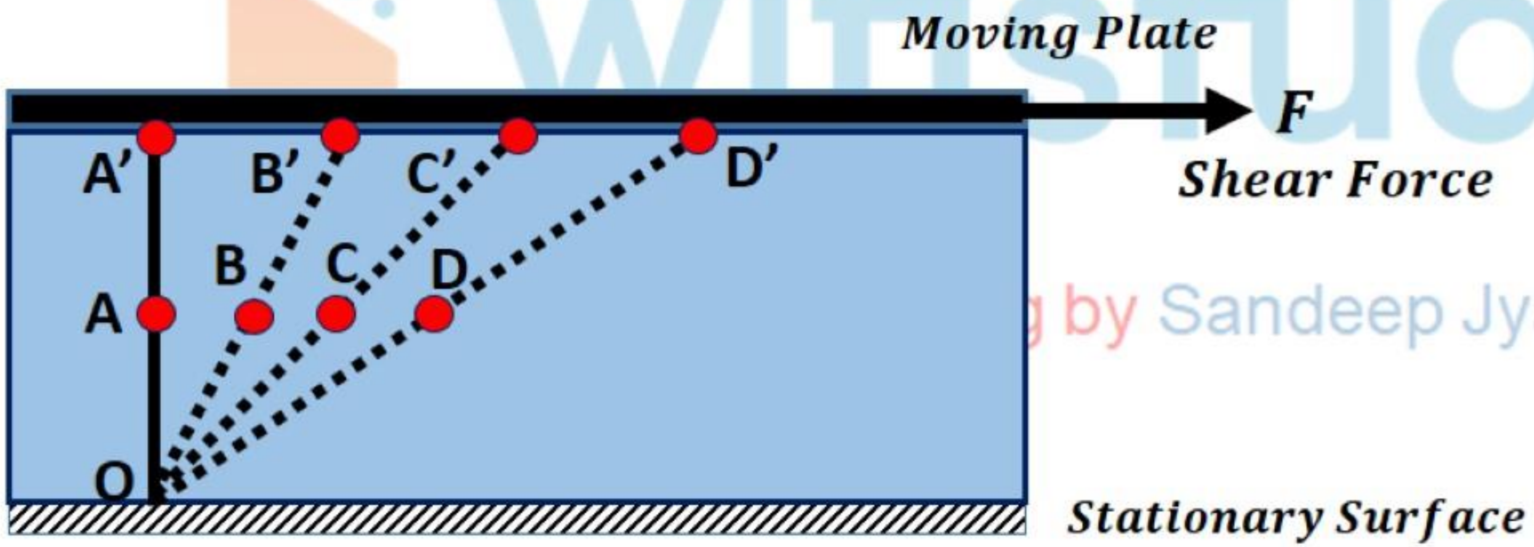

If a shear stress τ is applied at any location in a fluid, the element OAA' which is initially at rest, will move to OBB', then to OCC'. Further, it moves to ODD' and continues to move in a similar fashion.

Some Basic Terms

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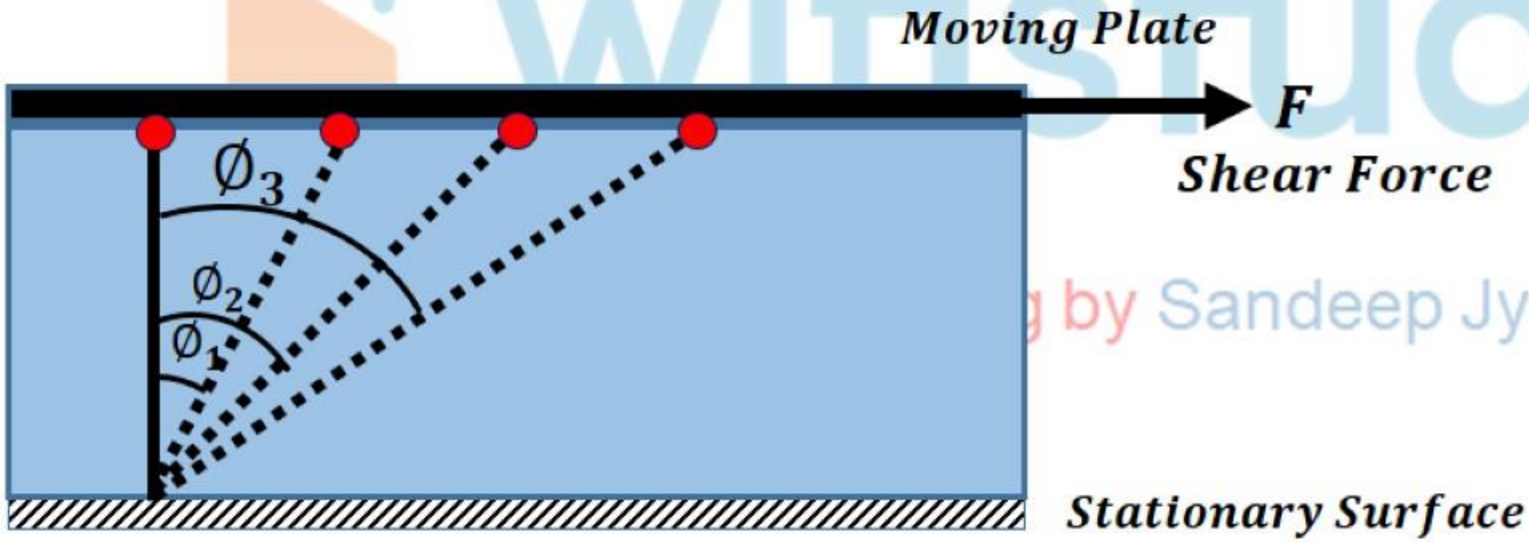


$$\emptyset_1 < \emptyset_2 < \emptyset_3$$

$$\tau \propto \frac{d\emptyset}{dt}$$

$$\frac{d\emptyset}{dt} = \text{continuous deformation is called as Flow.}$$

UNITS and DIMENSIONS

- **All physical quantities are given by a few fundamental quantities or their combinations. The units of such fundamental quantities are called Base Units.**
- **Combinations of them being called Derived units**
- **The system in which length, mass and time are adopted as the basic quantities, and from which the units of other quantities are derived, is called the <u>Absolute System of Units</u>**

Absolute System of Units

1. **MKS system of units**
   - **This is the system of units where**
     - **metre (m) is used for the unit of length,**
     - **kilogram (kg) for the unit of mass, and**
     - **second (s) for the unit of time as the base units**
2. **CGS system of units**
   - **This is the system of units where**
     - **Centimetre (cm) is used for length,**
     - **Gram (g) for mass, and**
     - **second (s) for time as the base units.**
3. **International system of units (SI)**

Absolute System of Units

3. International system of units (SI)

- **Important fundamental SI units:**
  - metre (m) for length,
  - kilogram (kg) for mass,
  - second (s) for time,
  - ampere (A) for electric current,
  - kelvin (K) for thermodynamic temperature,
  - mole (mol) for mass quantity and
  - candela (cd) for intensity of light.
- **Derived units consist of these units.**

Some Basic Units

- Density (Mass density) — kg / m^3
- Force — N (Newton)
- Pressure — N/mm^2 or N/m^2
- Work done (in joules) — J = N-m
- Power in watts — W = J/s

Properties of Fluid

Characteristics of a continuous fluid which are independent of the motion of the fluid are called Basic properties of the fluid. Some of the basic properties are :

1. Density

- **Density or MASS DENSITY of fluid is the ratio of mass of a fluid to its volume.**
- **So, mass per unit volume is called as Density**

$$\rho = \frac{Mass\ of\ fluid}{Volume\ of\ fluid}$$

$$SI\ unit \Rightarrow \frac{kg}{m^3}$$

- **Dimensional Formula** $\Rightarrow M^1\ L^{-3}$
- **Density of AIR at atmospheric pressure at 20°C is 1.24** $\frac{kg}{m^3}$
- **Density of Water is** $1000\ \frac{kg}{m^3}$ **or** $1\ \frac{g}{cm^3}$
- **With increase in TEMPERATURE, density DECREASES and vice versa**
- **With increase in PRESSURE, density INCREASES and vice versa**

Properties of Fluid

2. Specific Weight / Weight Density

- **Specific Density or Weight DENSITY of fluid is the ratio of weight of a fluid to its volume.**
- **So, Weight per unit volume is called as Specific Density**

$$w = \frac{Weight\ of\ fluid}{Volume\ of\ fluid}$$

$$\Rightarrow w = \frac{mg}{V} \Rightarrow \ w = \rho g$$

$$SI\ unit \Rightarrow \frac{N}{m^3}$$

- **Dimensional formula** $M^1 L^{-2} T^{-2}$
- **Unit Weight or Specific Weight of Water is** $9.81 \times 1000\ \mathrm{N}/m^3$ **or** $9.81\ kN/m^3$

Properties of Fluid

3. Specific Volume

- Specific volume of a fluid is defined as the volume of fluid occupied by a unit mass
- So, Volume per unit mass is called as Specific Volume

$$specific\ volume = \frac{Volume\ of\ fluid}{Mass\ of\ fluid}$$

$$\Rightarrow specific\ volume = \frac{V}{m} = \frac{1}{\rho}$$

$$SI\ unit \Rightarrow m^3/kg$$

- Specific Volume is reciprocal of Mass Density
- Commonly used for gases
- Specific volume of air = 0.78 m^3/kg

Properties of Fluid

4. Specific Gravity

- Specific Gravity is the ratio of weight density or density of a fluid to the weight density or density of a standard fluid

$$Specific\ Gravity\ (s) = \frac{Weight\ density\ of\ fluid}{Weight\ density\ of\ standard\ fluid}$$

- For liquids, standard fluid is taken as Water at 4°C
- For gases, standard fluid is taken as Air
- Weight density of any liquid = $s \times Weight\ density\ of\ water$
- Density of any liquid = $s \times Density\ of\ water$

Que 1. If specific gravity of mercury is 13.6, the density of mercury is..

a) $13600\ N/m^3$

b) $13600\ kg/m^3$

c) $13.6\ kg/m^3$

d) $13.6\ N/m^3$

Que 1. If specific gravity of mercury is 13.6, the density of mercury is..

a) $13600\ N/m^3$

b) $13600\ kg/m^3$ ✓

c) $13.6\ kg/m^3$

d) $13.6\ N/m^3$

Density of any liquid = $s \times Density\ of\ water$

Density of mercury= $13.6 \times 1000\ kg/m^3$

=> **Density of mercury=** $13600\ kg/m^3$

Que 2. Calculate specific weight, density and specific gravity of one litre of liquid that weighs 7N (take $g = 10 m/sec^2$).

Que 2. Calculate specific weight, density and specific gravity of one litre of liquid that weighs 7N (take $g = 10m/sec^2$).

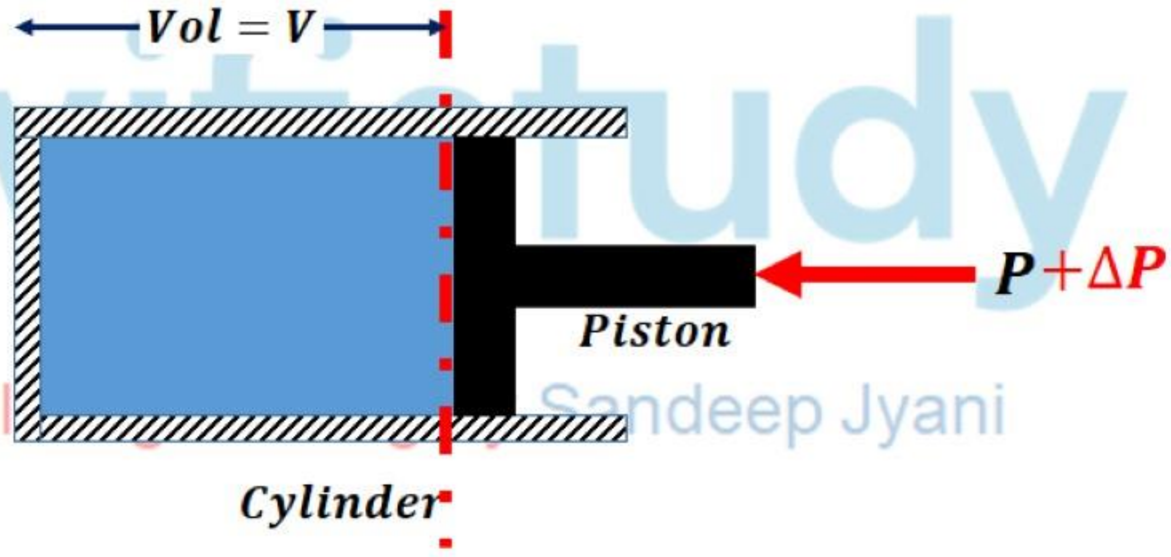
Answer:

$$w = 7000N/m^3$$
$$\rho = 700\ kg/m^3$$
$$s = 0.7$$

Properties of Fluid

4. Compressibility (β) and Bulk Modulus (K)

- **Compressibility is the reciprocal of Bulk Modulus**
- $\beta = \frac{1}{K}$



Properties of Fluid

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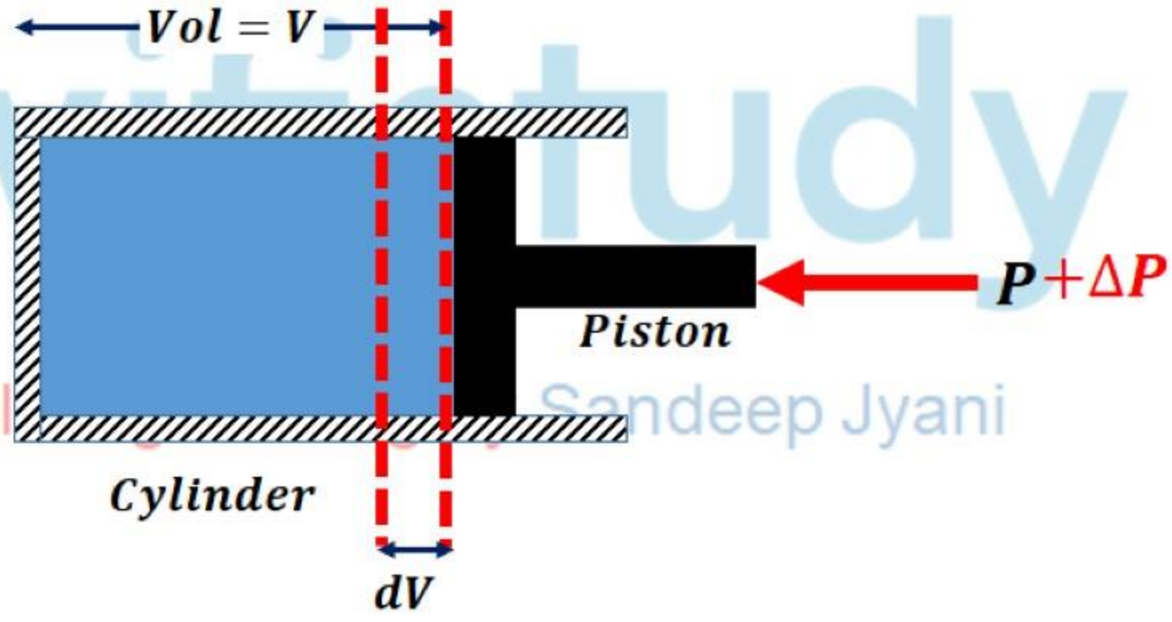


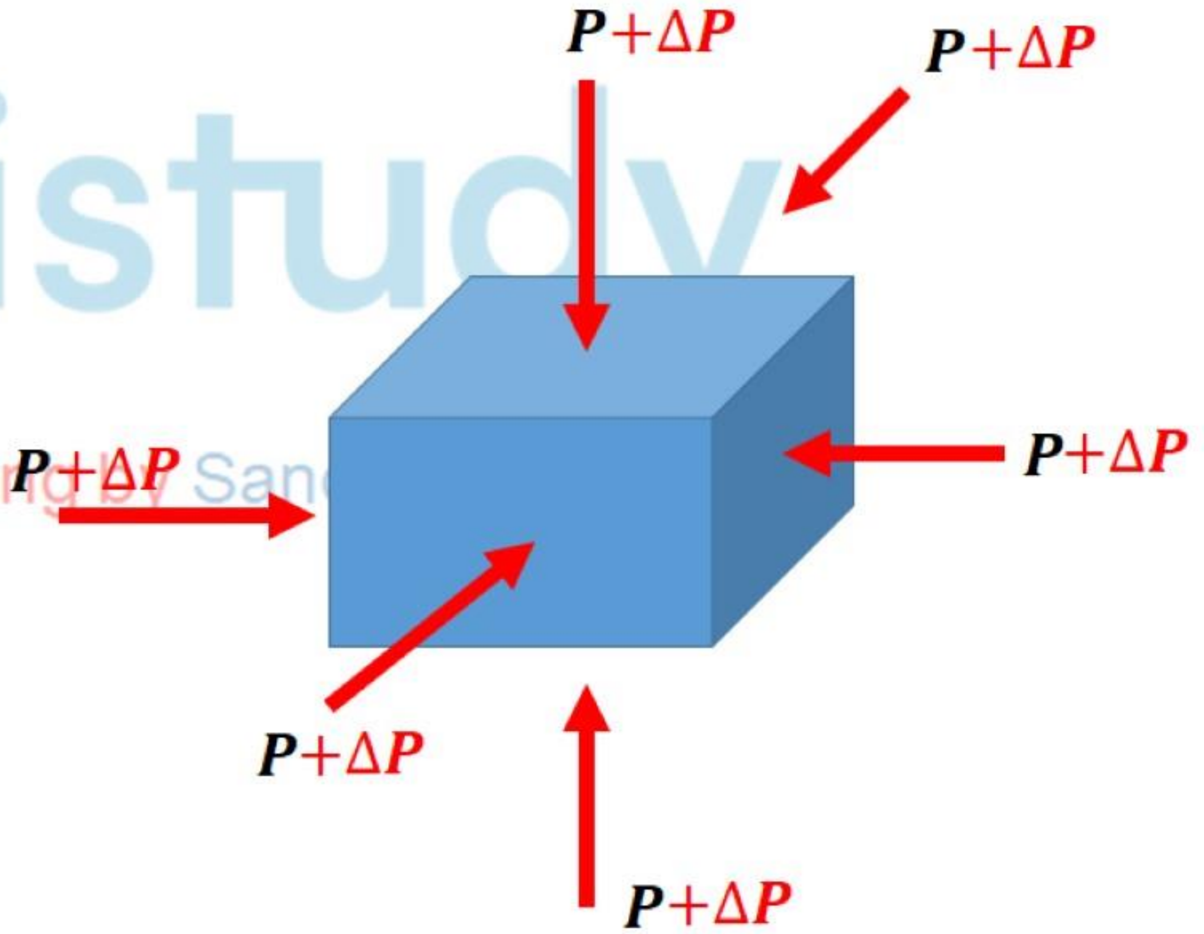
Properties of Fluid

4. Compressibility (β) and Bulk Modulus (K)

- **Compressibility is the reciprocal of Bulk Modulus**
- $\beta = \frac{1}{K}$

$$Bulk\ Modulus\quad K = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\Rightarrow K = -\frac{P_2 - P_1}{\frac{V_2 - V_1}{V_1}}$$



4. Compressibility and Bulk Modulus

- For perfect liquid or incompressible liquid, β=0

$$K = -\frac{dP}{\frac{dV}{V}} \qquad \Rightarrow \beta = -\frac{dV}{V\,dP}$$

$$since\ \rho = \frac{m}{V} \qquad hence\ m = \rho \times V$$

If mass is constant, differentiating both sides,

$$\Rightarrow 0 = \rho\, dV + V\, d\rho$$

$$\Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V}$$

$$\Rightarrow K = -\rho \frac{dP}{d\rho}$$

4. Compressibility and Bulk Modulus

- **Bulk Modulus *for ideal gas*:**
- $PV = nR'T$
  - R'=universal gas constant= 8314 J/(kg-mole °K)
  - Mass/mole of air m'= 29

$$\Rightarrow PV = n \times m' \times \frac{R'}{m'} \times T$$

$$\Rightarrow \frac{R'}{m'} = \frac{8314}{29} = 287 = R = gas\ constant\ J/(kg - K)$$

$$\Rightarrow PV = m \times R \times T$$

$$\Rightarrow P = \rho RT$$

4. Compressibility and Bulk Modulus

- **Compressibility of ideal fluid for isothermal compression:**

$$\Rightarrow P = \rho RT \quad \dots(1)$$

$$\Rightarrow \frac{dP}{d\rho} = \frac{d}{d\rho}(\rho RT)$$

$$\Rightarrow \frac{dP}{d\rho} = RT$$

$$\Rightarrow K = \rho \frac{dP}{d\rho}$$

$$\Rightarrow K = \rho\,(RT) \dots (2)$$

From (1) and (2),

$$K = P$$

$$\beta = \frac{1}{P}$$

For ideal gas, unit of Bulk Modulus is N/m^2

4. Compressibility and Bulk Modulus

- **Bulk Modulus for Adiabatic compression:**

$$\Rightarrow PV^{\gamma} = constant$$

$$where\ \gamma = adiabatic\ constant = 1.4$$

$$\Rightarrow PV^{\gamma} = c$$

An adiabatic process occurs without transfer of heat or mass of substances between a thermodynamic system and its surroundings.

$$\Rightarrow V^{\gamma}\frac{dP}{dV} + P(\gamma V^{\gamma-1}dV) = 0$$

$$\Rightarrow \frac{dP}{dV} = \frac{-P(\gamma V^{\gamma-1}dV)}{V^{\gamma}}$$

$$\Rightarrow \frac{dP}{dV} = \frac{-\gamma P}{V}$$

$$=> K = -V\frac{dP}{dV} \qquad => K = -V \times \frac{-\gamma P}{V}$$

$$=> K = \gamma P$$

$$=> K = 1.4P$$

Que 3. In MKS, gravitational system of unit, the unit for mass is

a) kilogram

b) Newton

c) Metric Slug

d) Gram

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b) Newton
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1. MKS system of units
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Que 4. Fluid is a substance that offers no resistance to change of

a) Pressure
b) Volume
c) Shape
d) Flow

Que 4. Fluid is a substance that offers no resistance to change of

a) Pressure

b) Volume

c) **Shape**

d) Flow

Que 5. The increase in pressure required to decrease unit volume of mercury by 0.1 % is … ? Given K=28.5 Mpa

$a)\ -28.5\ MPa$

$b)\ -28.5\ kPa$

$c)\ -28500\ kPa$

$d)\ -28500\ MPa$

Que 5. The increase in pressure required to decrease unit volume of mercury by 0.1 % is … ? Given K=28.5 Mpa

a) $-28.5\ MPa$

b) $\underline{-28.5\ kPa}$

c) $-28500\ kPa$

d) $-28500\ MPa$

$$\Rightarrow K = -\frac{dP}{\frac{dV}{V}}$$

$$\Rightarrow 28.5 \times 10^6 = -\frac{dP}{\frac{0.1}{100}}$$

$$\Rightarrow dP = -28.5\ kPa$$

Que 6. If an increase in pressure of 2 bar decrease the volume of liquid by 0.01%, then the bulk modulus in "*Pa*" is

a) -2×10^4

b) -2×10^9

c) -2×10^6

d) -2×10^2

1 **atm** = 1.01325 **bar**

1 **bar** = 1,00,000 **Pa**

Que 6. If an increase in pressure of 2 bar decrease the volume of liquid by 0.01%, then the bulk modulus in "*Pa*" is

a) -2×10^4

b) $\underline{-2 \times 10^9}$

c) -2×10^6

d) -2×10^2

=> $K = -\frac{dP}{\frac{dV}{V}}$

1 atm = 1.01325 bar

1 bar = 1,00,000 Pa

=> $K = -\frac{2}{\frac{0.01}{100}}$

=> $K = -2 \times 10^4\ bars$

=> $K = -2 \times 10^9\ Pa$

Properties of Fluid

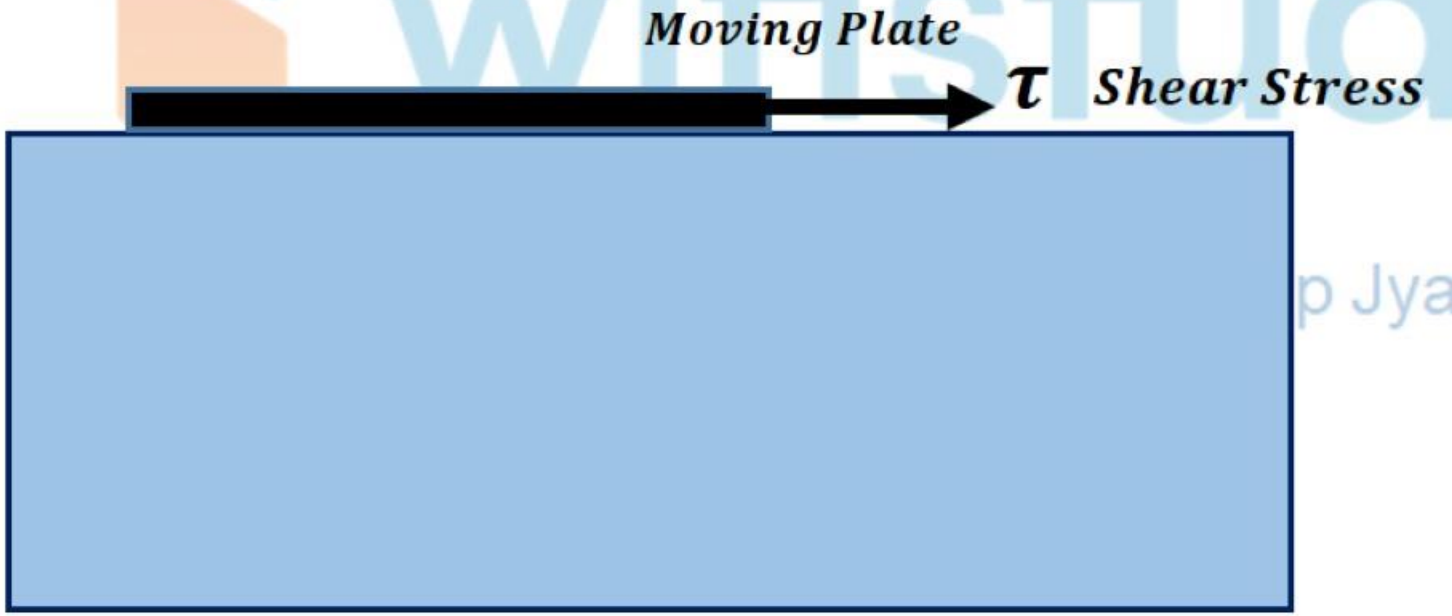
5. Viscosity:

- **It is the internal resistance offered by one layer of fluid to the other layer of fluid**
- **Viscosity is a fluid property whose effect is understood when the fluid is in motion.**
- **In a flow of fluid, when the fluid elements move with different velocities, each element will feel some resistance due to fluid friction within the elements.**
- **Therefore, shear stresses can be identified between the fluid elements with different velocities.**
- **The relationship between the shear stress and the velocity field was given by Sir Isaac Newton.**

Properties of Fluid

5. Viscosity:

- **Let us observe a flow in which all fluid particles are moving in the same direction in such a way that the fluid layers move parallel with different velocities**



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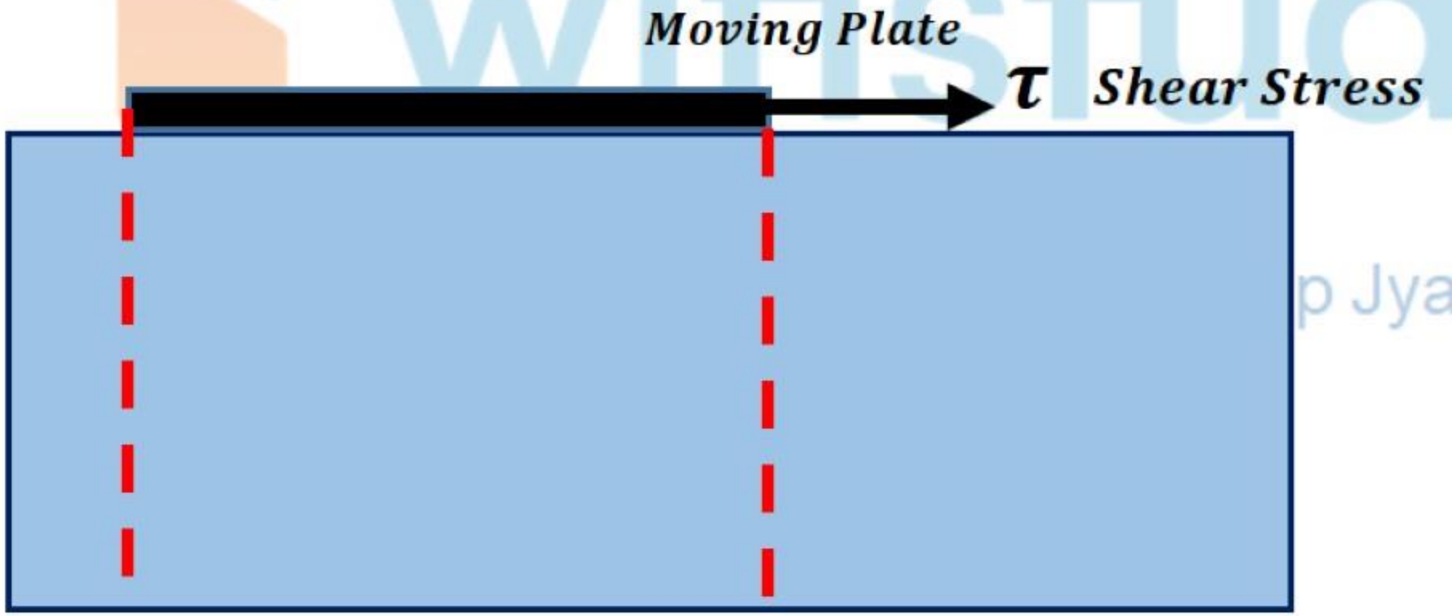
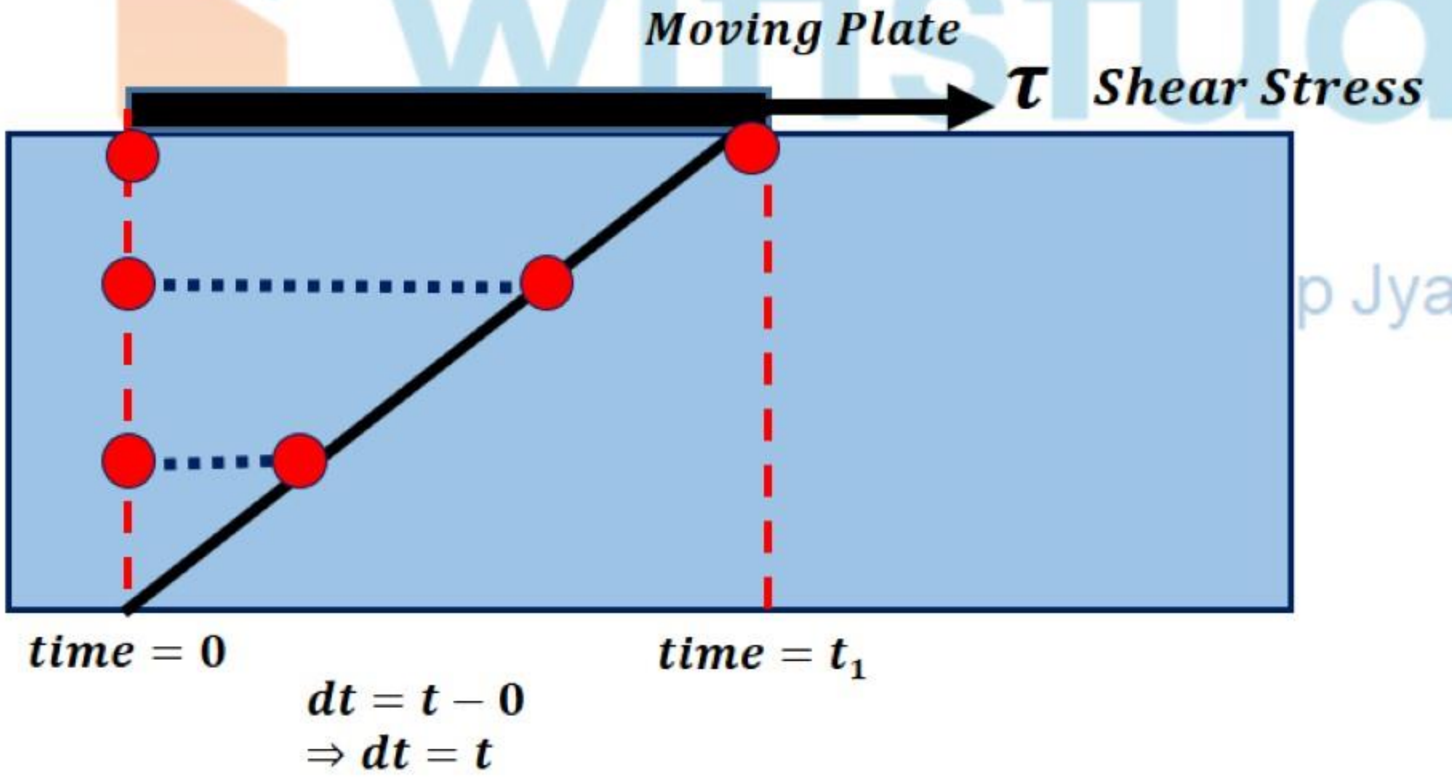


$time = 0$ $time = t_1$

$dt = t - 0$
$\Rightarrow dt = t$

5. Viscosity:

- **Let us observe a flow in which all fluid particles are moving in the same direction in such a way that the fluid layers move parallel with different velocities**



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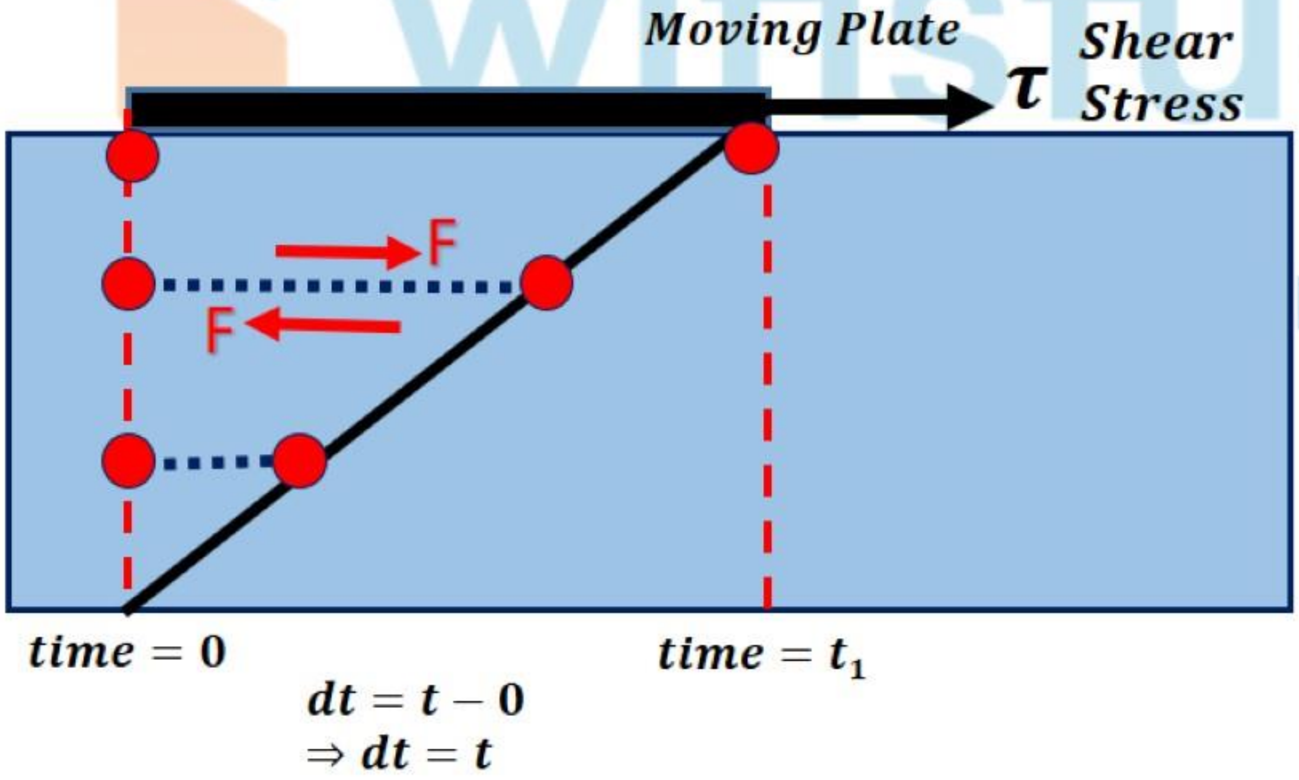
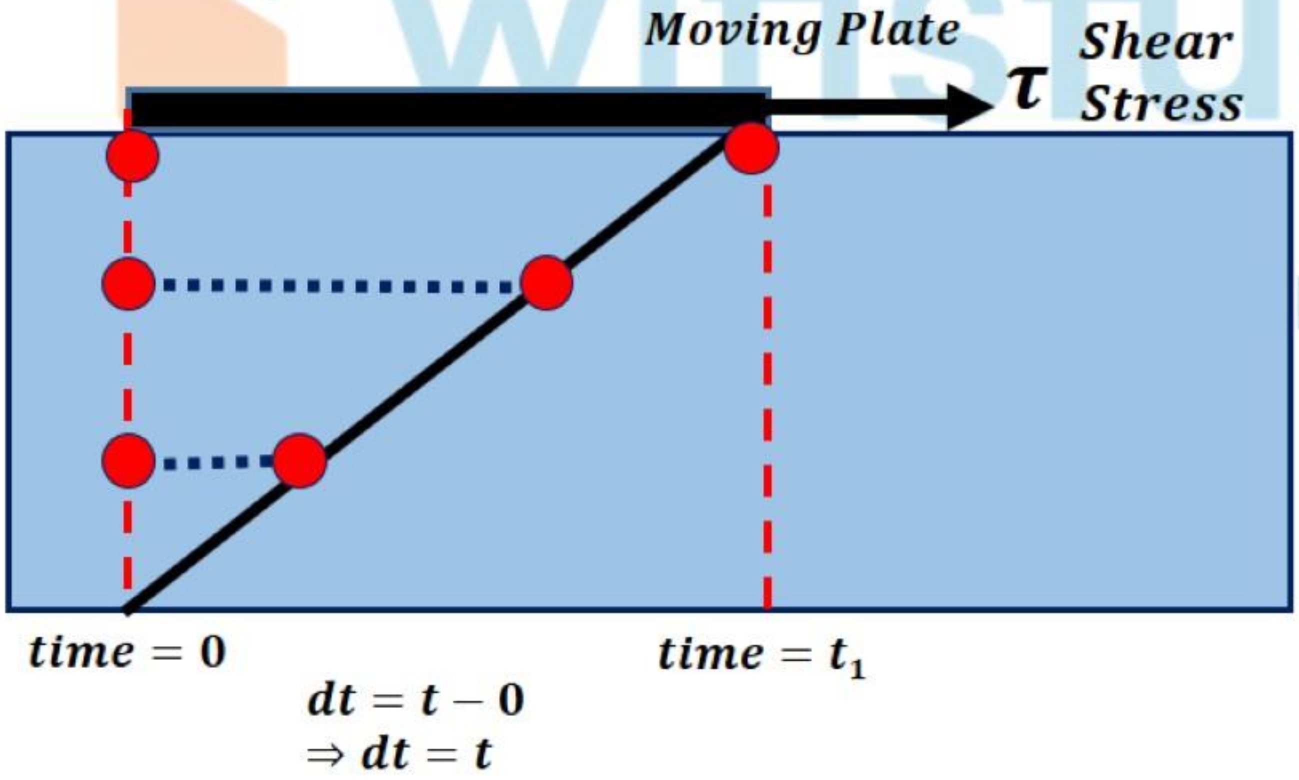


The upper layer, which is moving faster, tries to draw the lower slowly moving layer along with it by means of a force F along the direction of flow on this layer. Similarly, the lower layer tries to retard the upper one, according to Newton's third law, with an equal and opposite force F

Properties of Fluid

5. Viscosity:

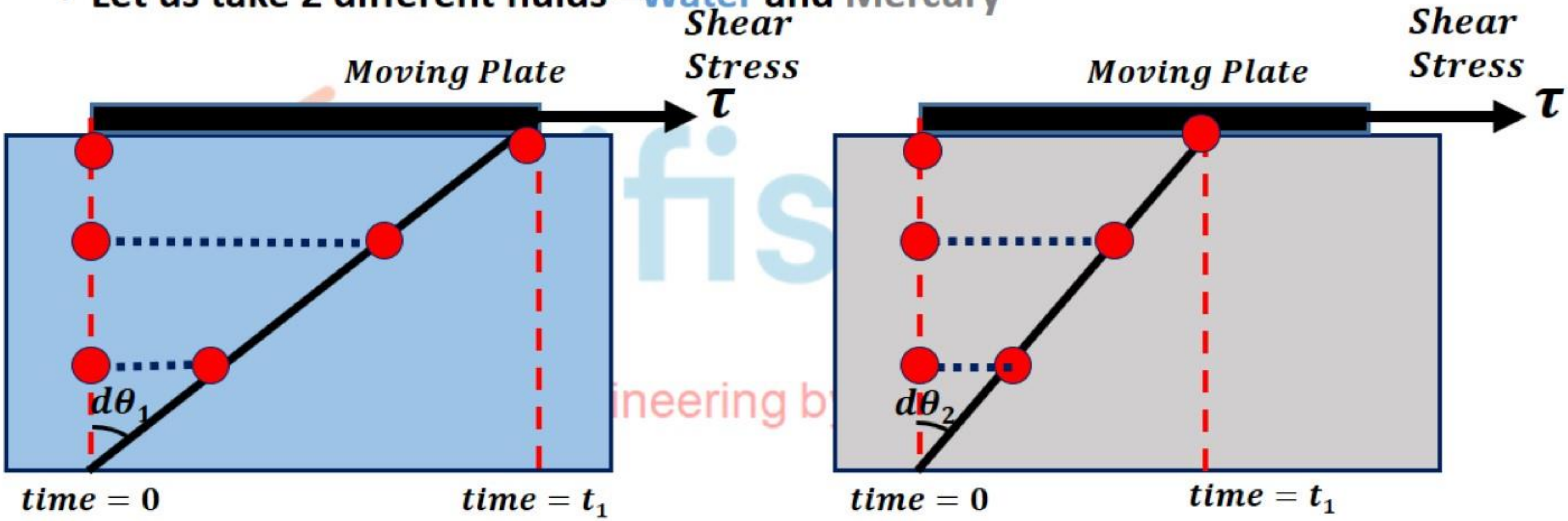
- **Let us observe a flow in which all fluid particles are moving in the same direction in such a way that the fluid layers move parallel with different velocities**



Such a fluid flow where x-direction velocities, change with y-coordinate is called Shear Flow of the fluid.

5. Viscosity:

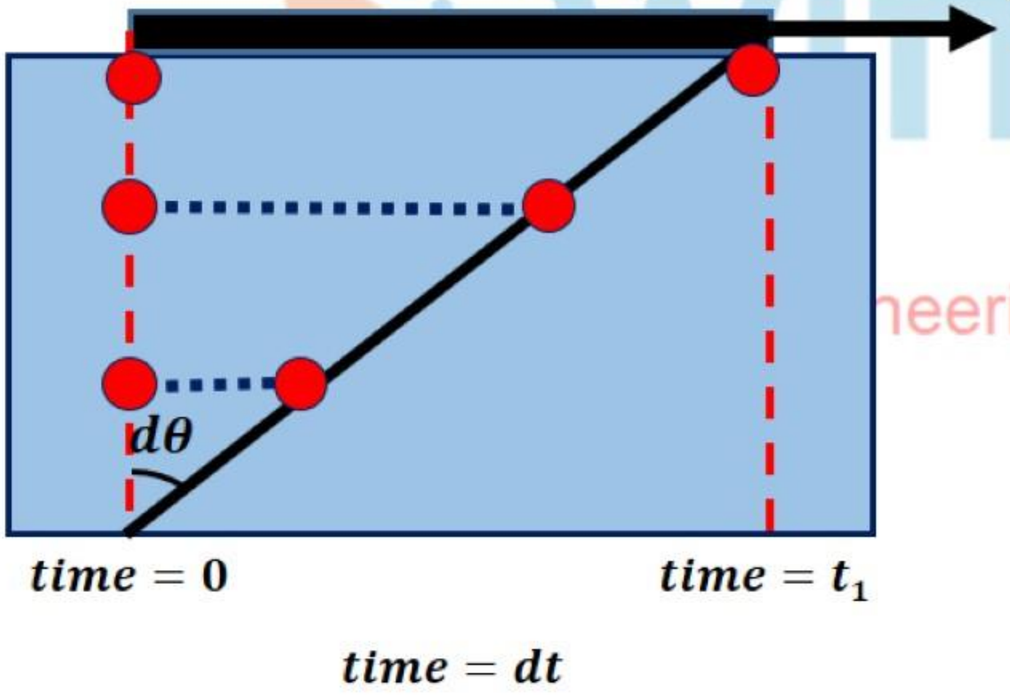
- Let us take 2 different fluids – Water and Mercury



$d\theta_1$ *is large it means easy flow* $d\theta_1 > d\theta_2$ $d\theta_2$ *is smaller it means difficult to flow or more resistance to flow*

Properties of Fluid

5. Viscosity:

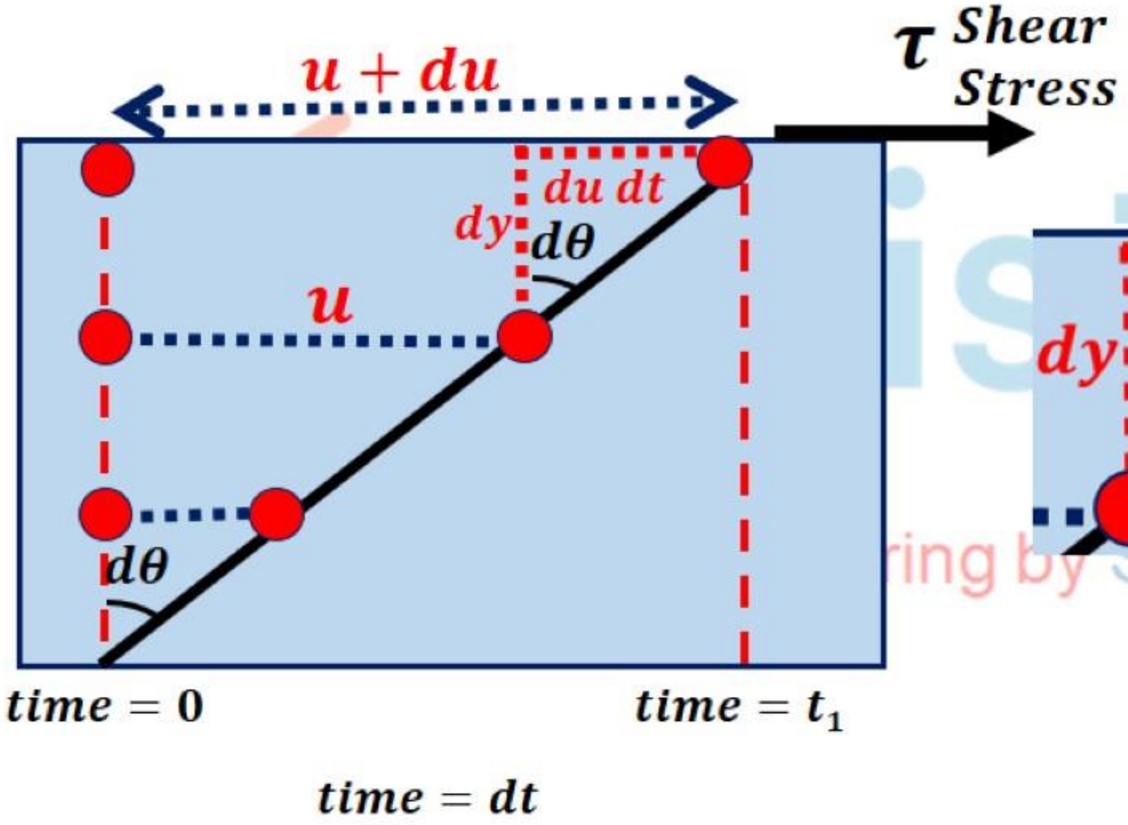


$$\tau \propto \frac{d\theta}{dt}$$

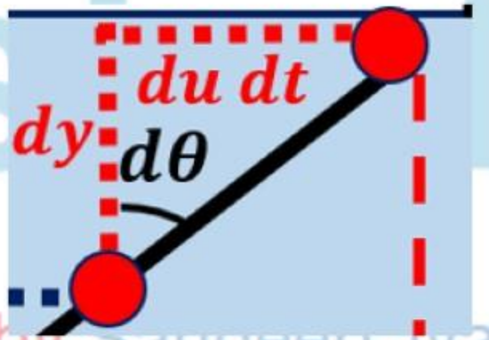
$$\Rightarrow \tau = \mu \frac{d\theta}{dt}$$

$$\Rightarrow \mu = \frac{\tau}{\frac{d\theta}{dt}}$$

5. Viscosity:



$$\Rightarrow \mu = \frac{\tau}{\frac{d\theta}{dt}}$$



$$\tan d\theta = \frac{du\, dt}{dy}$$

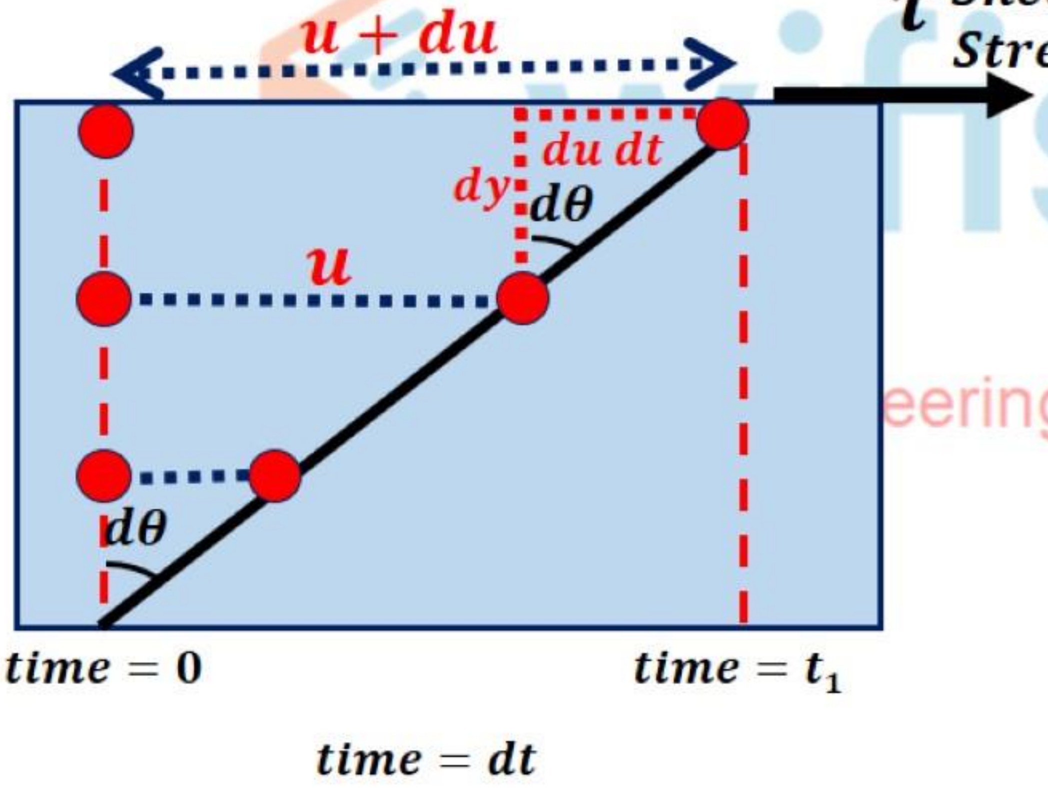
$$\Rightarrow d\theta = \frac{du\, dt}{dy}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{du}{dy}$$

$$\Rightarrow \mu = \frac{\tau}{\frac{du}{dy}}$$

Properties of Fluid

5. Viscosity:



Shear stress caused by top layer on adjacent layer or lower layer causes a shear stress on adjacent top layer is directly proportional to the rate of change of Velocity with respect to y

$$\tau \propto \frac{du}{dy}$$

$$\frac{du}{dy} = \text{velocity gradient}$$

$$\tau = \mu \frac{du}{dy} \qquad \tau = \mu \frac{u}{y}$$

$$\Rightarrow \mu = \frac{\tau}{\frac{du}{dy}} \qquad \Rightarrow \mu = \frac{\tau}{\frac{u}{y}}$$

Properties of Fluid

5. Viscosity:

$$\tau = \mu \frac{du}{dy} \qquad \tau = \mu \frac{d\theta}{dt}$$

$$\tau = \frac{F}{A} = \mu \frac{u}{y}$$

$$F = \frac{\mu A u}{y}$$

$$\Rightarrow \mu = \frac{\tau}{\frac{du}{dy}} \qquad \Rightarrow \mu = \frac{\tau}{\frac{d\theta}{dt}}$$

Units of Viscosity:

$$\mu = \frac{N/m^2}{\frac{m/sec}{m}} = Ns/m^2$$

$$\underline{\mu = Pa.s}$$

- ***Units of Viscosity***:
- $\mu = \dfrac{N/m^2}{\frac{m/sec}{m}} = Ns/m^2$

$$\mu = Pa.s$$

$$= \frac{N}{m^2} sec$$

$$= \frac{kg\,\cancel{m}}{sec^{\cancel{2}}\,m^{\cancel{2}}}\cancel{sec}$$

$$= \frac{1000g}{sec \times 100\ cm}$$

$$= \frac{10g}{cm - sec}$$

$$1\ Pa.s = 10 Poise$$

6. Kinematic Viscosity

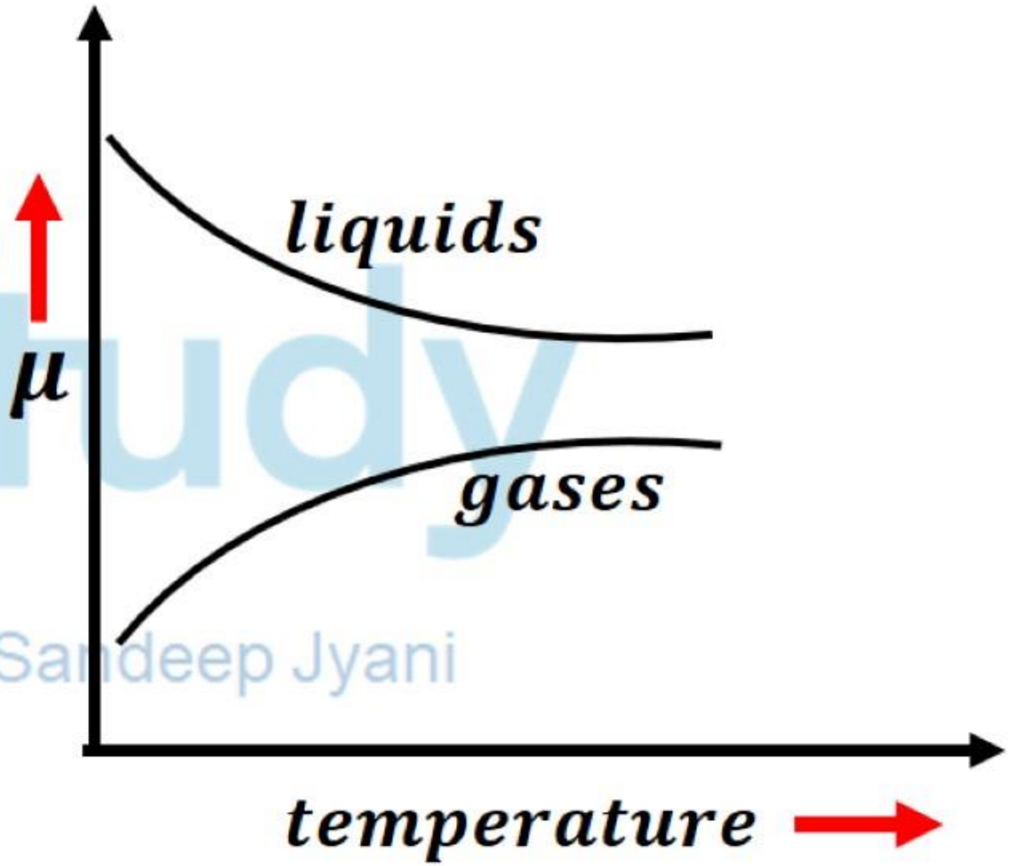
$$\nu = \frac{\mu}{\rho}$$

- SI Unit = m^2/sec

- In CGS, $\frac{m^2}{sec} = \frac{100 \times 100 cm^2}{sec} = 10^4 \frac{cm^2}{sec} = 10^4$ stokes

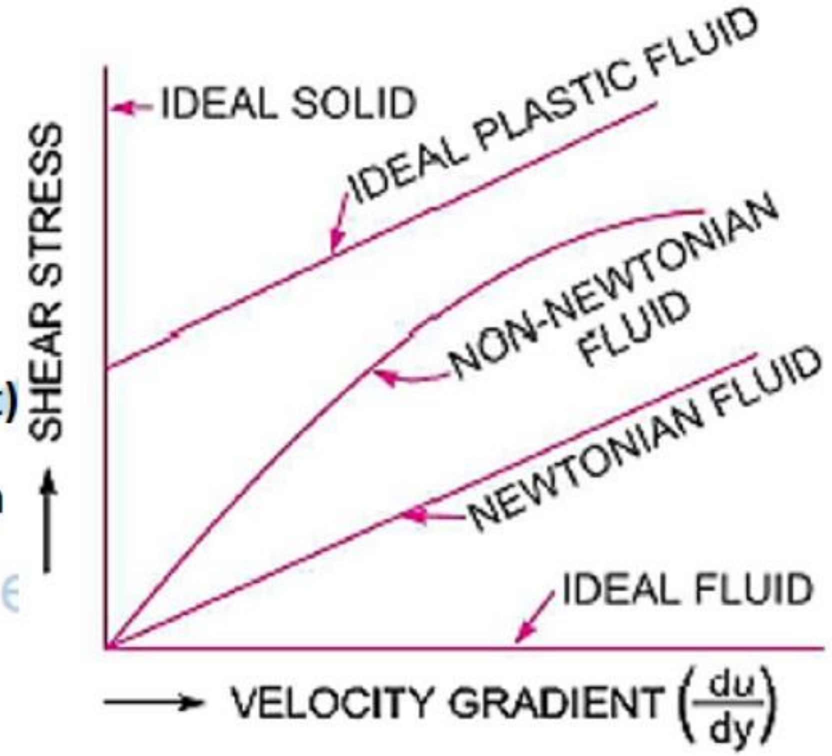
Que 7. A flat plate 0.1 m^2 of area is pulled at 30cm/sec relative to another plate located at a distance of 0.01 cm from it. The viscosity of fluid is 0.001 Pa-s., then find the power required to maintain the velocity.

- **Variation of Viscosity with temperature**
  - In case of liquids, the intermolecular distance is small and hence buoyancy forces are large.
  - With increase in temperature, the buoyancy forces decreases and hence the resistance to flow also decreases
  - In case of liquid, with increase in temperature, viscosity of fluid decreases.
  - In case of Gases, intermolecular distance is very large and hence buoyancy force are negligible, so with increase in temperature molecular disturbance increases and hence resistance to flow also increases with increase in temperature, viscosity increases.
  - Note:
    - Viscosity of water at 20°C is $10^{-3}\ Pa - s$
    - Viscosity of air is 50-55 times less than viscosity of water



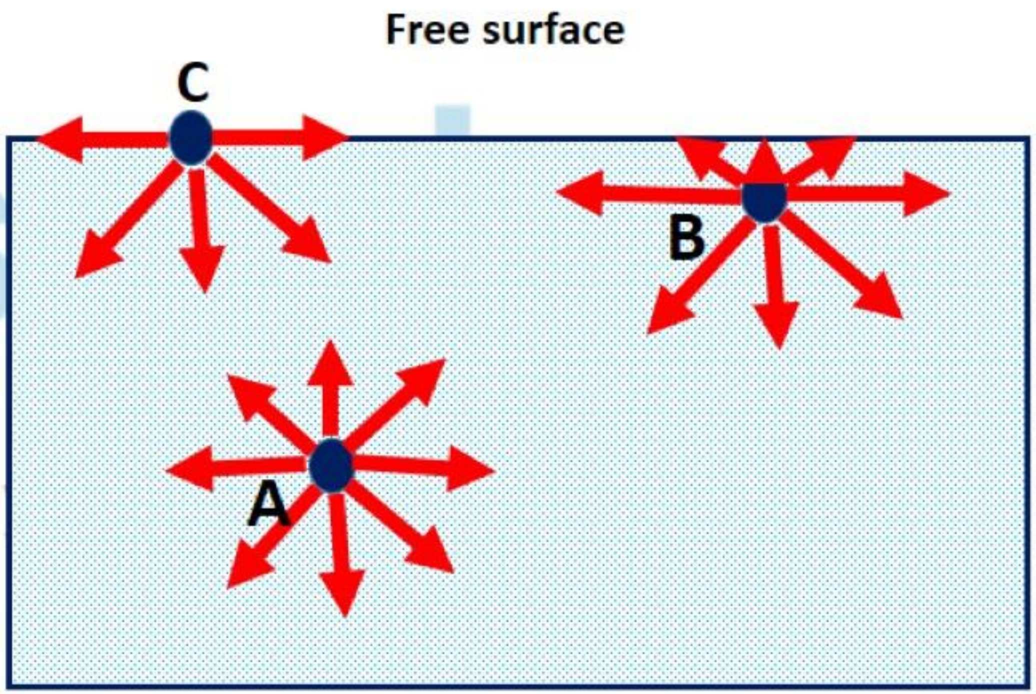
Types of Fluids

1. **Ideal Fluid**
   - A fluid that is incompressible and is having no viscosity, is known as Ideal Fluid
   - Ideal fluid is only an imaginary fluid
2. **Real Fluid**
   - A fluid that possess viscosity, is known as Real Fluid.
   - All the fluids in practise are real fluids
3. **Newtonian Fluid**
   - A REAL Fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient) is called as Newtonian Fluid
   - Air, water, kerosene, oil, mercury, diesel, etc are Newtonian Fluid
4. **Non Newtonian Fluid**
   - A real fluid in which shear stress is not proportional to the rate of shear strain (or velocity gradient)
5. **Ideal Plastic Fluid (Bingham Plastic)**
   - A fluid in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient)
   - Exp: Cold cream, toothpaste



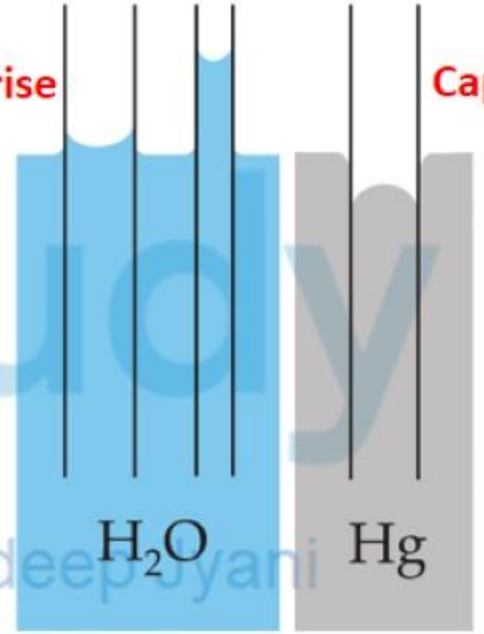
Surface Tension and Capillarity

- **Surface tension:**
  - It is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.
  - The magnitude of this force per unit length of free surface
  - $\sigma = \frac{F}{l}$



Capillarity

- The rise or fall of liquid when small dia tube is immersed in it, is known as Capillarity
- **Capillary rise** is due to adhesive force is more as compared to cohesive force
- **Capillary Fall** happens when cohesive force is stronger than adhesive force.



PRESSURE INSIDE LIQUID DROPLET:

PRESSURE INSIDE LIQUID DROPLET:

$$F_s = \sigma L$$

$$F_s = \sigma \pi d$$

$$F_p = P \times \frac{\pi}{4} d^2$$

For equilibrium,

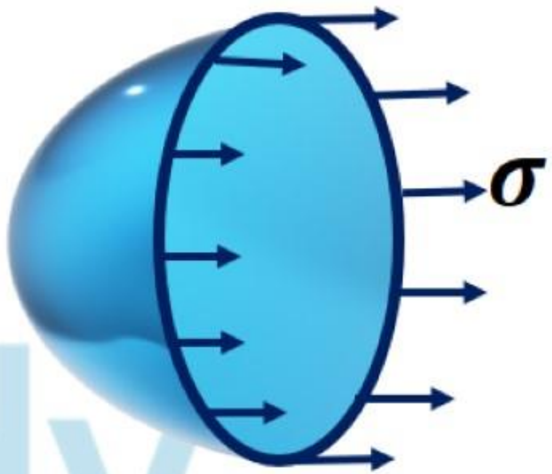
=> surface tension = pressure force

$$F_s = F_p$$

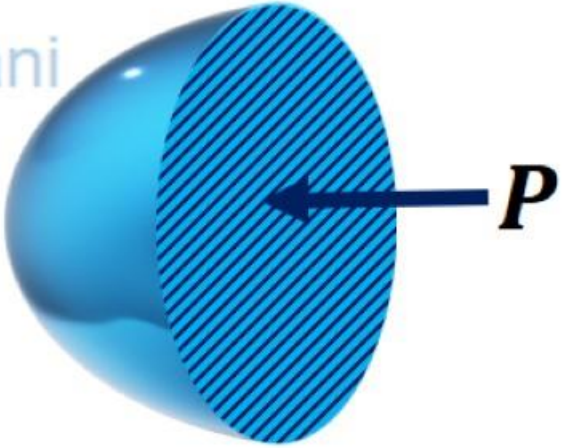
$$\sigma \pi d = P \times \frac{\pi}{4} d^2$$

$$P = \frac{4\sigma}{d}$$

LIQUID DROPLET



Surface Tension



Pressure Forces

PRESSURE INSIDE BUBBLE:

PRESSURE INSIDE BUBBLE:

$$F_s = \sigma(2 \times \pi d)$$

$$F_s = 2\sigma\pi d$$

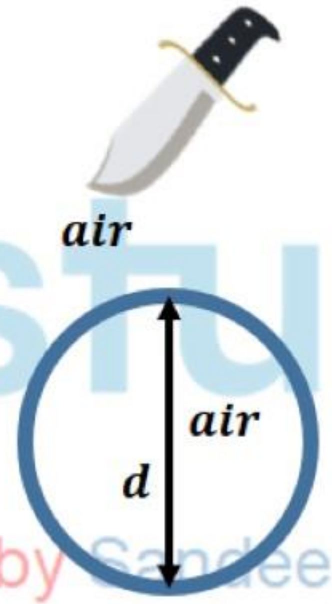
$$F_p = P \times \frac{\pi}{4} d^2$$

For equilibrium,

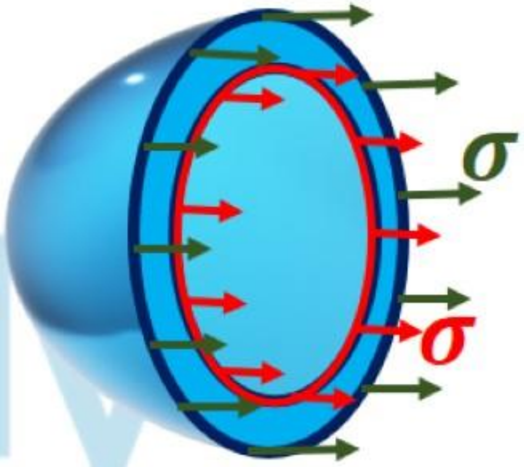
surface tension = pressure force

$$2\sigma\pi d = P \times \frac{\pi}{4} d^2$$

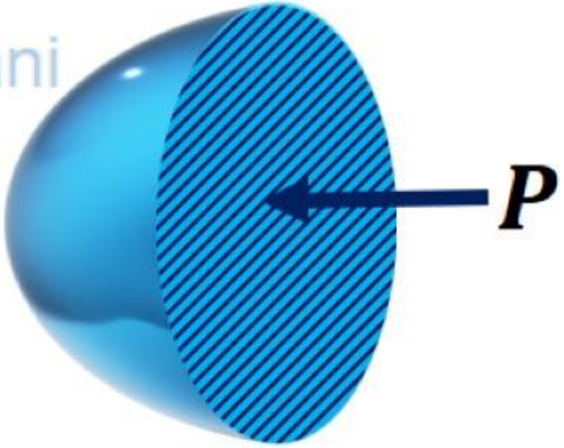
$$P = \frac{8\sigma}{d}$$



LIQUID BUBBLE



Surface Tension



Pressure Forces

PRESSURE INSIDE LIQUID JET

PRESSURE INSIDE LIQUID JET:

$$F_s = \sigma(2L)$$

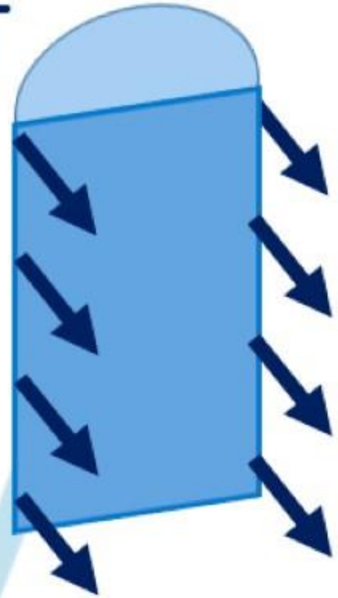
$$F_p = P \times L \times d$$

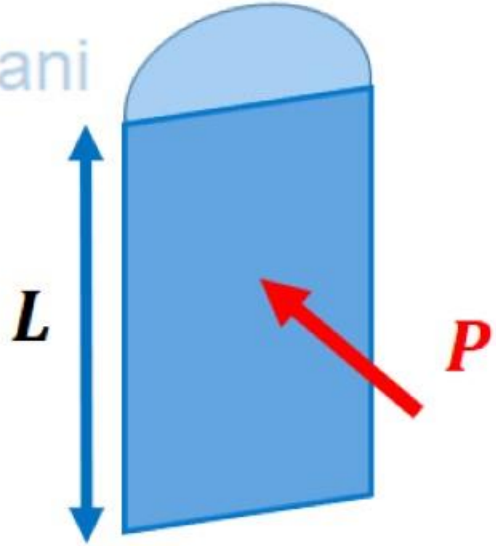
For equilibrium,

surface tension = pressure force

$$P \times L \times d = \sigma(2L)$$

$$P = \frac{2\sigma}{d}$$





$$\textit{Bubble} : \textit{Droplet} : \textit{liquid jet}$$

$$\frac{8\sigma}{d} \qquad \frac{4\sigma}{d} \qquad \frac{2\sigma}{d}$$

$$4 : 2 : 1$$

WETTING AND NON WETTING LIQUID:

WETTING AND NON WETTING LIQUID:

Angle of contact = θ

If $\theta < 90^{O}$, then fluid is wetting fluid.

If $cohesive\ forces < adhesive\ forces$, then fluid is wetting fluid.

If $\theta > 90^{O}$, then fluid is non wetting fluid

If $cohesive\ forces > adhesive\ forces$, then fluid is non wetting fluid.

Expression for Capillary Rise and Capillary Fall

- **For equilibrium condition,**

$$\Rightarrow F_s \cos\theta = wt\ (weight\ of\ fluid\ column)$$

$$\Rightarrow \sigma \times perimeter \times \cos\theta = \rho \times A \times h \times g$$

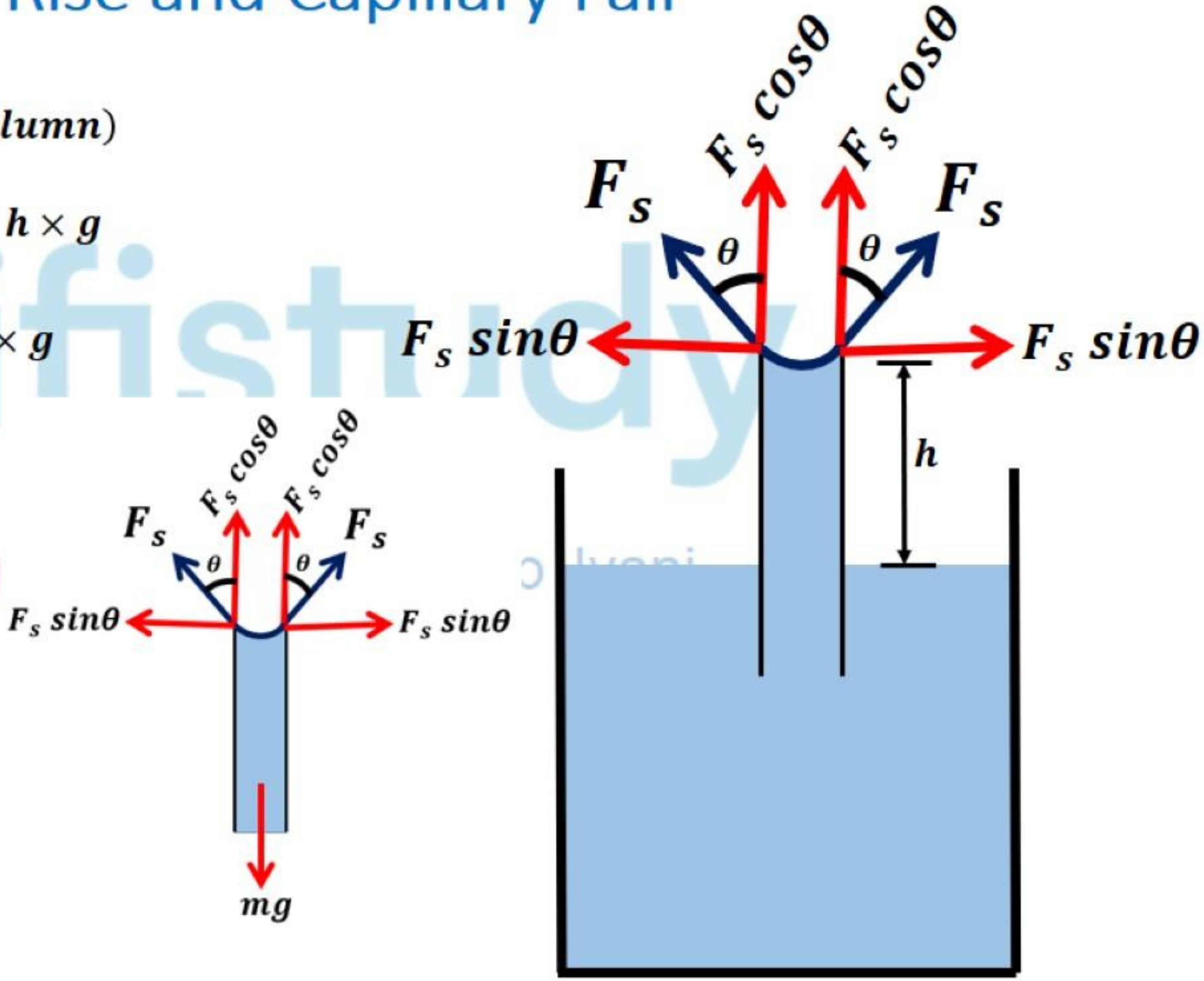
$$\Rightarrow \sigma \times \pi d \times \cos\theta = \rho \times \frac{\pi}{4} d^2 \times h \times g$$

$$\boxed{\Rightarrow h = \frac{4\sigma \cos\theta}{\rho g d}}$$

Since specific weight, $w = \rho g$

- **For capillary rise,**

$$h = \frac{4\sigma \cos\theta}{wd}$$



Expression for Capillary Rise and Capillary Fall

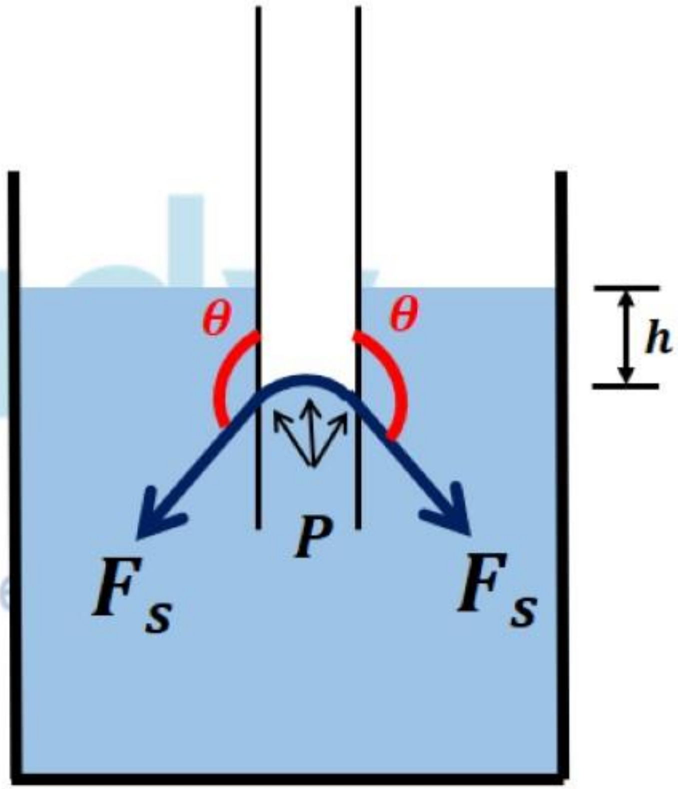
For capillary fall,

$$\Rightarrow F_s \cos\theta = P \times \frac{\pi}{4} \times d^2$$

$$\Rightarrow \sigma \times perimeter \times \cos\theta = \rho g h \times A$$

$$\Rightarrow \sigma \times \pi d \times \cos\theta = \rho \times \frac{\pi}{4} d^2 \times h \times g$$

$$\Rightarrow h = \frac{4\sigma\cos\theta}{\rho g d}$$



CAPILLARY RISE FOR PARALLEL PLATE

For equilibrium,

$$weight\ of\ fluid\ element = F_s \cos\theta$$

$$F_s = \sigma \times 2b \cos\theta$$

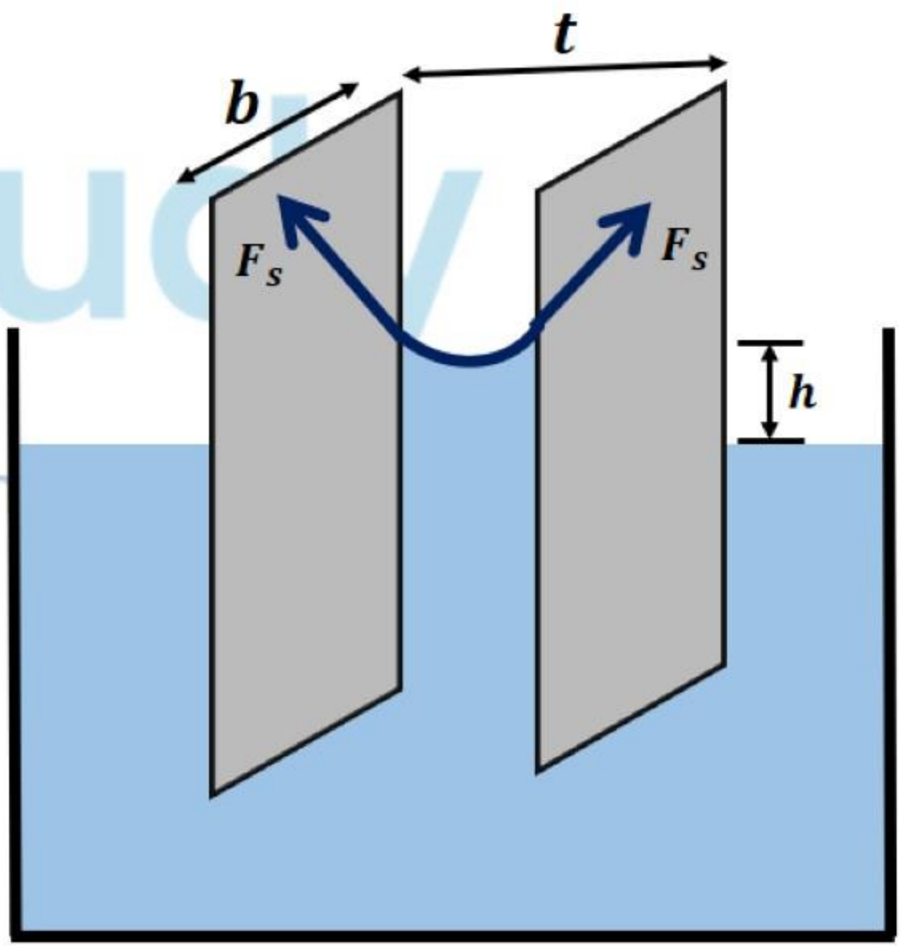
$$\Rightarrow W = \sigma \times 2b \cos\theta$$

$$\Rightarrow \rho \times A \times h \times g = \sigma \times 2b\cos\theta$$

$$\Rightarrow \rho \times b \times t \times h \times g = \sigma \times 2b\cos\theta$$

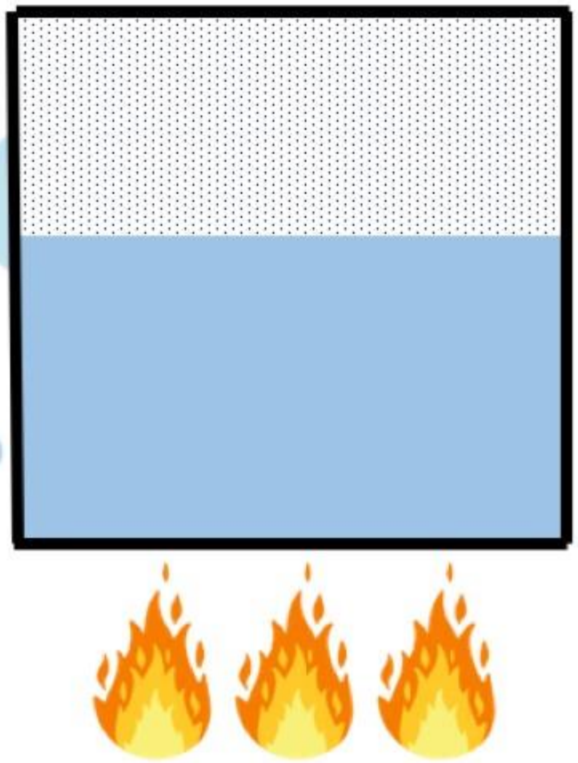
$$\Rightarrow h = \frac{2\sigma\cos\theta}{Wt}$$

$$\Rightarrow \boldsymbol{h} = \frac{\boldsymbol{2\sigma\cos\theta}}{\boldsymbol{\rho g t}}$$



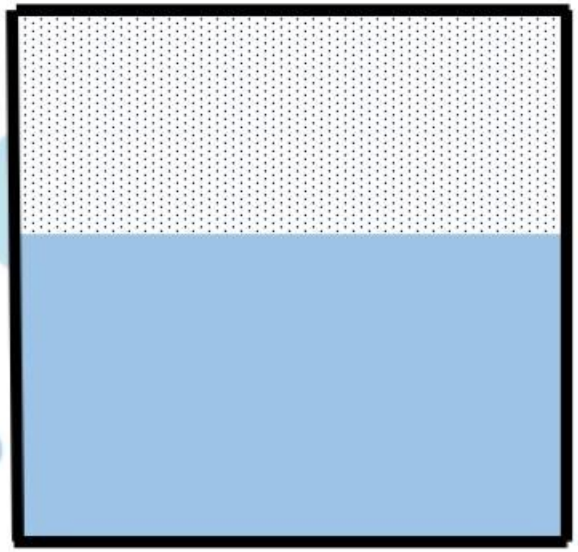
Vapour Pressure and Cavitation

- **Vaporization:**
  - **Depends upon temperature and pressure conditions**
  - **Occurs because of escaping of molecules through the free liquid surface**
  - **Consider water at 20°C and atmospheric pressure in a closed vessel, it will vaporise at 100°C**
  - **When vaporizing occurs, the molecules escape from free surface and gets accumulated in the free space between the liquid surface and top of the vessel**
  - **These accumulated vapours exert a pressure on the liquid surface, which is known as Vapour Pressure**
  - **This is the pressure at which liquid is converted to Vapours**



Vapour Pressure and Cavitation

- **Vaporization:**
  - Again if same liquid at 20°C is at atmospheric pressure in a closed vessel.
  - If pressure above liquid surface is reduced is reduced by some means, the boiling temperature will also be reduced.
  - If pressure is reduced to such an extent that it becomes equal to or less than vapour pressure, boiling of liquid will start even at 20°C
  - Thus the liquid can boil at any temperature if the pressure above the liquid surface is reduced to vaporization pressure



Vapour Pressure and Cavitation

- **Cavitation:**
  - **Liquid in a flowing system, when subjected to low pressure which becomes equal to or less than vaporisation pressure, starts to vaporize or form bubbles.**
  - **These bubbles are carried away by flowing water into the region of high pressure, where they collapse giving rise to high impact pressure.**
  - **This impact pressure developed by collapsing bubbles is so high that material from adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is called Cavitation.**

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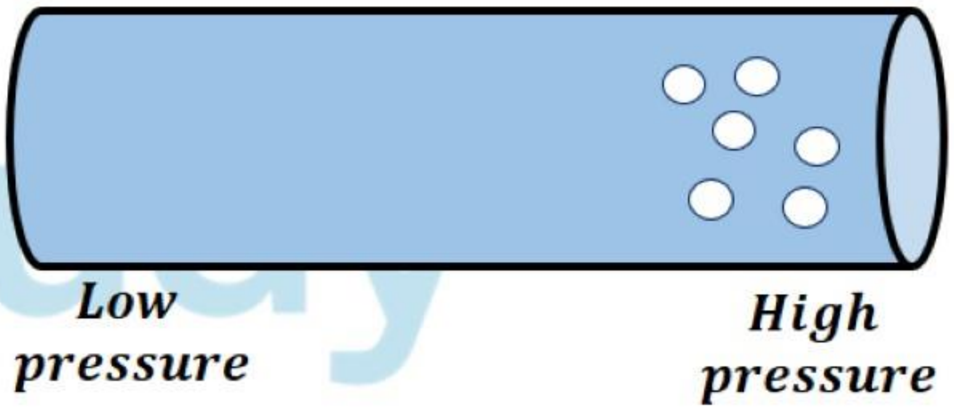

8. An ideal fluid is
   a) One which obeys Newton's law of viscosity
   b) Frictionless and incompressible
   c) Very viscous.
   d) Frictionless and viscous

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9. Newton's law of viscosity relates to
   a) Intensity of pressure and rate of angular deformation.
   b) Viscosity and rate of angular deformation
   c) Among shear stress, viscosity and temperature
   d) None of these

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10. Poise is the unit of-

a) Mass density
b) Kinematic viscosity
c) Viscosity
d) Velocity gradient

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11. If the diameter of a capillary tube is doubled, the capillary rise will be

a) Unaffected
b) Doubled
c) Halved
d) None of the above

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a) Unaffected
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d) None of the above

$$\Rightarrow h = \frac{4\sigma \cos\theta}{\rho g d}$$

12. Flow of fluid takes place due to its

a) Viscosity
b) Compressibility
c) Surface tension
d) Deformation under shear stress

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a) Viscosity
b) Compressibility
c) Surface tension
d) Deformation under shear stress

13. The pressure intensity in kg/cm^2 at any point in liquid is

a) w

b) $\frac{w}{h}$

c) $\frac{h}{w}$

d) wh

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a) w
b) $\frac{w}{h}$
c) $\frac{h}{w}$
d) $\underline{wh}$

$$P = \rho g h$$

$$P = wh$$

14. Bulk modulus of a fluid is the ratio of

a) Shear stress to shear strain
b) Increase in volume to the viscosity of fluid
c) Increase in pressure to the volumetric strain
d) Critical velocity to the velocity of fluid

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15. Capillary is the phenomenon that is attributed to the following property of fluid

a) Vapour pressure
b) Viscosity
c) Density
d) Surface tension

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a) Vapour pressure
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16. Specific gravity has a unit

a) g/cc
b) Kg/m^3
c) N/m
d) No unit - dimensionless

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a) g/cc
b) Kg/m^3
c) N/m
d) **No unit - dimensionless**

17. A fluid which is incompressible and is having no viscosity is

a) Ideal fluid
b) Real fluid
c) Newtonian fluid
d) Non Newtonian fluid

17. A fluid which is incompressible and is having no viscosity is

a) **Ideal fluid**
b) Real fluid
c) Newtonian fluid
d) Non Newtonian fluid

18. Capillarity is due to

a) Surface tension
b) Cohesion
c) Viscosity
d) Vapour pressure
e) Weight density of liquid

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a) **Surface tension**

b) Cohesion

c) Viscosity

d) Vapour pressure

e) Weight density of liquid

$$\Rightarrow h = \frac{4\sigma \cos\theta}{\rho g d}$$

19. For a fluid, shear stress was found to be directly proportional to the rate of angular deformation. The fluid is classified as

a) Non Newtonian fluid
b) Ideal fluid
c) Newtonian fluid
d) Thixotropic fluid

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b) Ideal fluid
c) **Newtonian fluid**
d) Thixotropic fluid

20. With the increase in temperature the viscosity of air and water varies as

a) Viscosity of air increases and viscosity of water decreases.
b) Viscosity of air increases and viscosity of water increases.
c) Viscosity of air decreases and viscosity of water decreases.
d) Viscosity of air decreases and viscosity of water decreases.

20. With the increase in temperature the viscosity of air and water varies as

a) **Viscosity of air increases and viscosity of water decreases.**
b) Viscosity of air increases and viscosity of water increases.
c) Viscosity of air decreases and viscosity of water decreases.
d) Viscosity of air decreases and viscosity of water decreases.

21. Viscosity of a fluid with specific gravity 1.3 is measured to be $0.0034 Ns/m^2$. Its kinematic viscosity in m^2/s is

a) 2.6×10^{-6}
b) 4.4×10^{-6}
c) 5.8×10^{-6}
d) 7.2×10^{-6}

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a) $\mathbf{2.6 \times 10^{-6}}$
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c) 5.8×10^{-6}
d) 7.2×10^{-6}

22. When the adhesion between the molecules of fluid is greater than adhesion between fluid and the glass, then the free level of fluid in glass tube dipped in the glass vessel will be

a) Same as the surface of the fluid
b) Lower than the surface of fluid
c) Higher than the surface of fluid
d) Dependent upon atmospheric pressure

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a) Same as the surface of the fluid
b) **Lower than the surface of fluid**
c) Higher than the surface of fluid
d) Dependent upon atmospheric pressure

23. Newton's law of viscosity is a relationship between

a) Pressure, velocity and temperature
b) Shear stress and rate of shear strain
c) Shear stress and velocity
d) Rate of shear strain and temperature

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a) Pressure, velocity and temperature

b) **Shear stress and rate of shear strain**

c) Shear stress and velocity

d) Rate of shear strain and temperature

Chapter 2:

Pressure and Its Measurement

PRESSURE

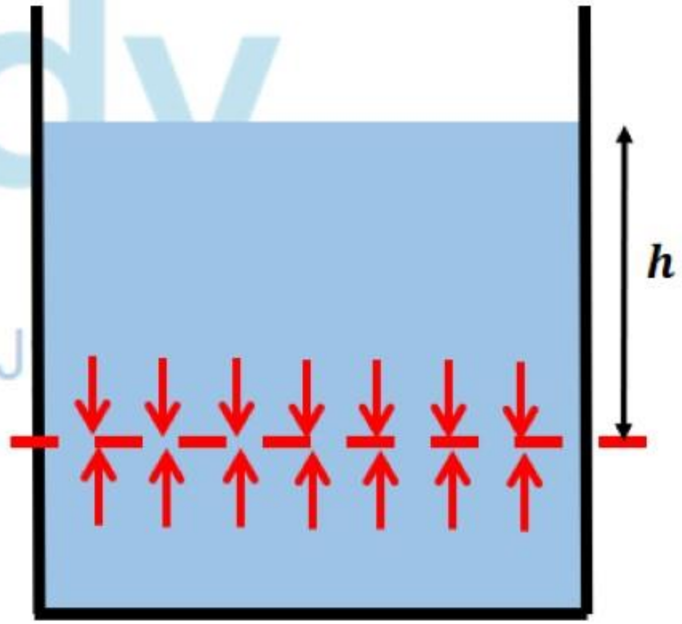
- **Pressure is defined as normal force exerted by fluid per unit area.**

$$\Rightarrow w = mg$$

$$\Rightarrow w = A \times h \times \rho \times g$$

$$P = \frac{w}{A} = \frac{A \times h \times \rho \times g}{A}$$

$$P = \rho g h$$



Note: The standard atmosphere (atm) is a unit of pressure defined as 1.01325 bar (101325 Pa), equivalent to 760 mm Hg (torr)

For equilibrium of fluid element

$$PdA + W = (P + dP)dA$$

$\Rightarrow W = (dP)dA$

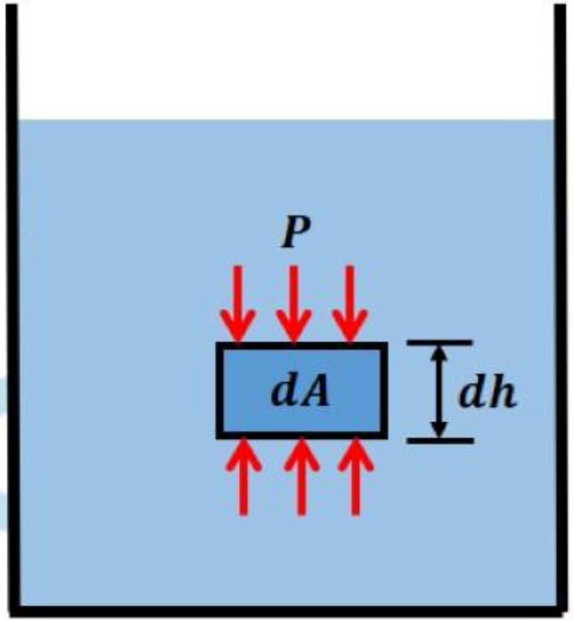
And $W = \rho \times g(dA \times dh)$

$\Rightarrow W = w(dA \times dh)$ (w =sp. wt)

$\Rightarrow (dP)dA = w(dA \times dh)$

$\Rightarrow w = \frac{dP}{dh}$ (specific weight)

*Rate of increase of pressure in vertical direction is equal to the weight density of the fluid at that point. This is **Hydrostatic Law***



Integrating both sides

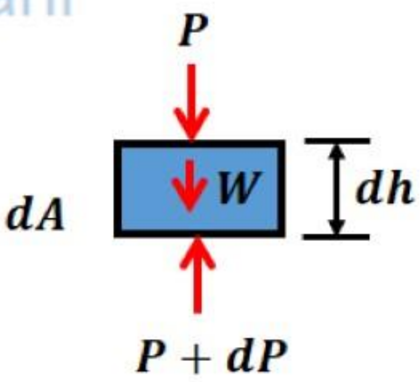
$\Rightarrow P = wh + c$

We assume at h=0 and $P = P_{atm}$

$\Rightarrow c = P_{atm}$

$\Rightarrow P = wh + P_{atm}$

$\Rightarrow P - P_{atm} = wh$



Here, for gauge reading we assume $P_{atm}=0$

$\Rightarrow P = wh = \rho gh$ ***where h is called as Pressure head***

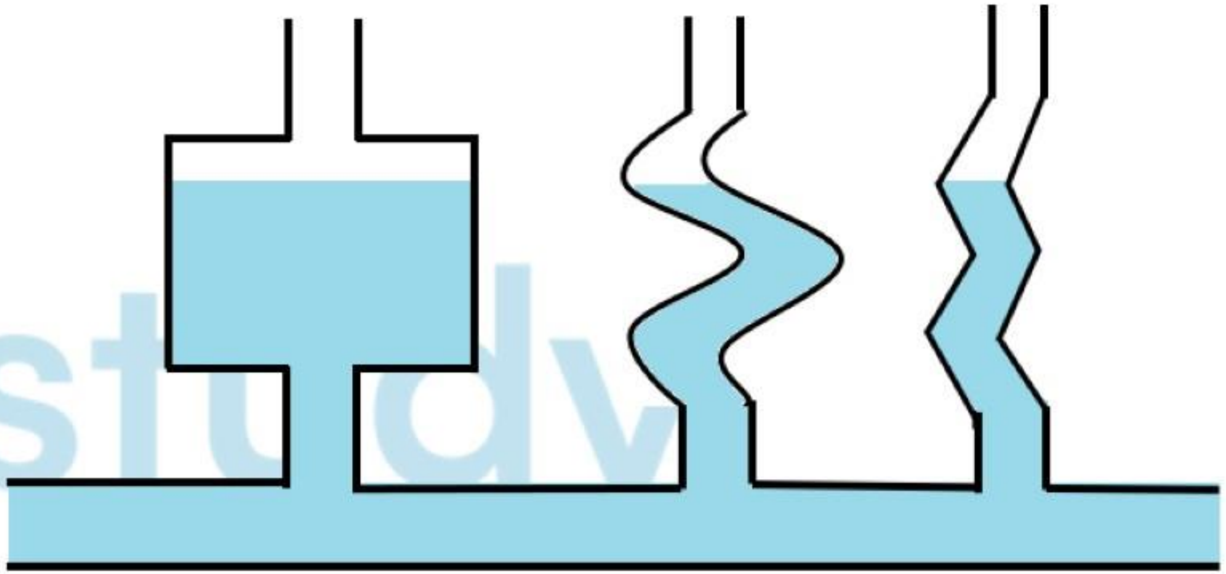
Que 24. Relation between pressure at A, B and C is ...?

a) $P_A = P_B = P_C$

b) $P_A > P_B = P_C$

c) $P_A < P_B = P_C$

d) $P_A > PB > P_C$



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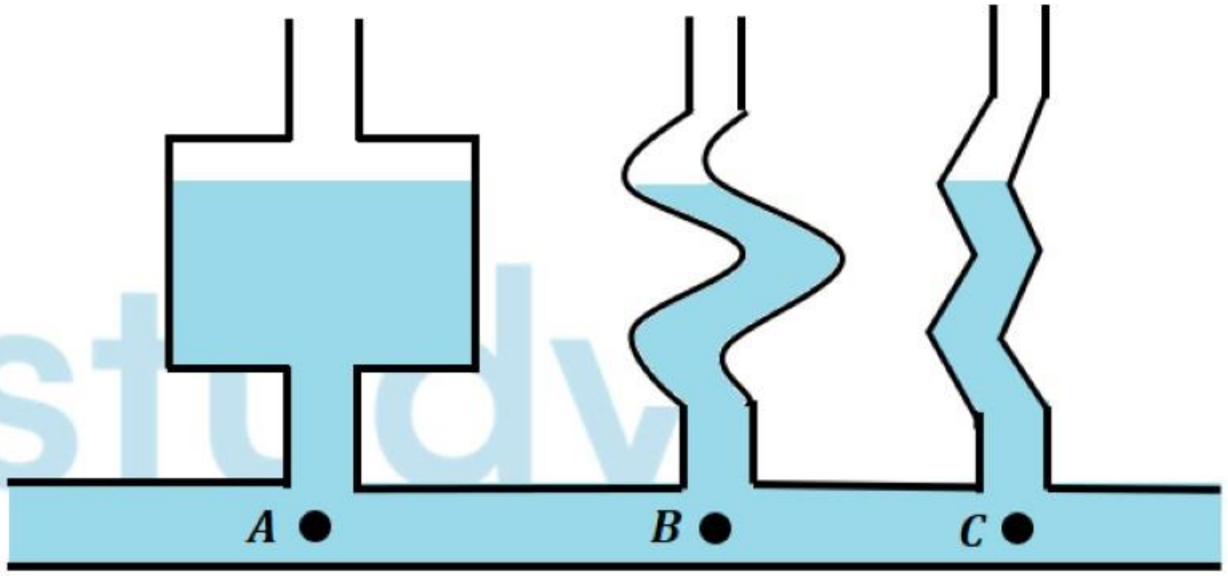


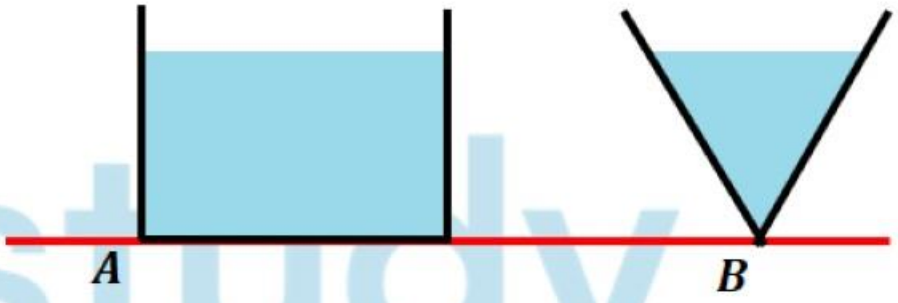
Que 25. Relation between pressure at A, B is …?

a) $P_A = P_B$

b) $P_A > P_B$

c) $P_A < P_B$

d) $None$



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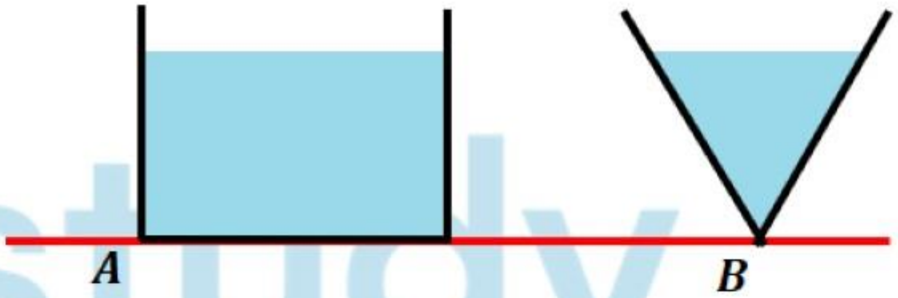
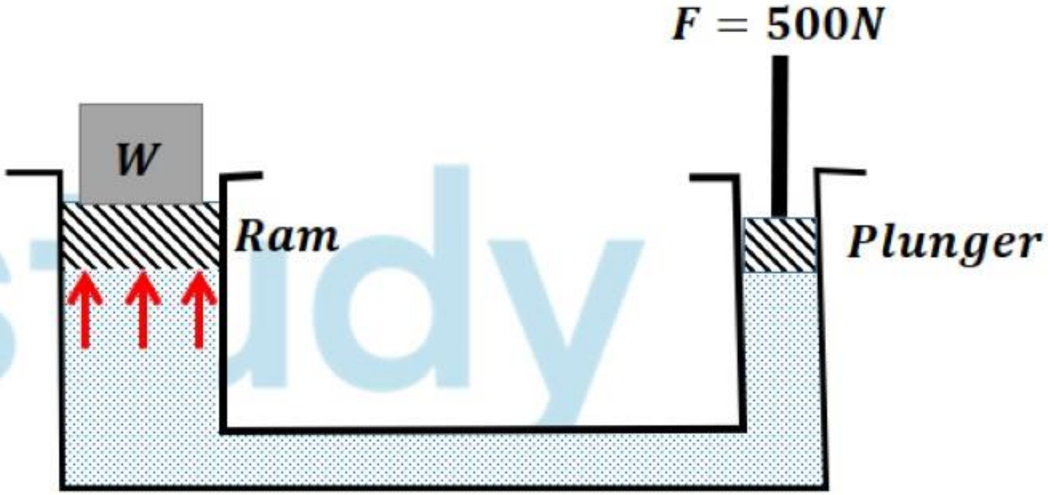


Pascal's Law

- **It states that pressure or intensity of pressure at a point in a static fluid is equal in all directions**

Que 26. A hydraulic press has a ram of 30 cm dia and a plunger of 4.5 cm dia. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500N



Que 27. A hydraulic press has a ram of 20cm dia and a plunger of 3 cm diameter. It is used for lifting a weight of 30kN. Find the force required at the plunger.

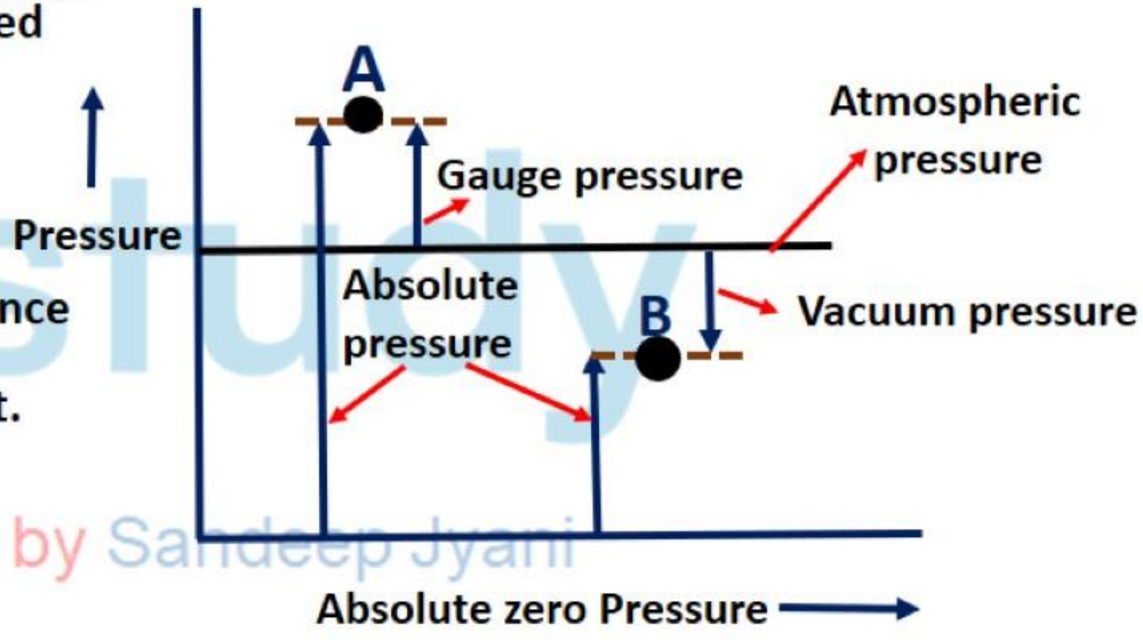
Homework!

TYPES OF PRESSURE

1. **Atmospheric pressure**
   - Atmospheric air exerts a normal pressure upon all surface with which it is in contact and it is called atmospheric pressure.
   - Measured by barometer.

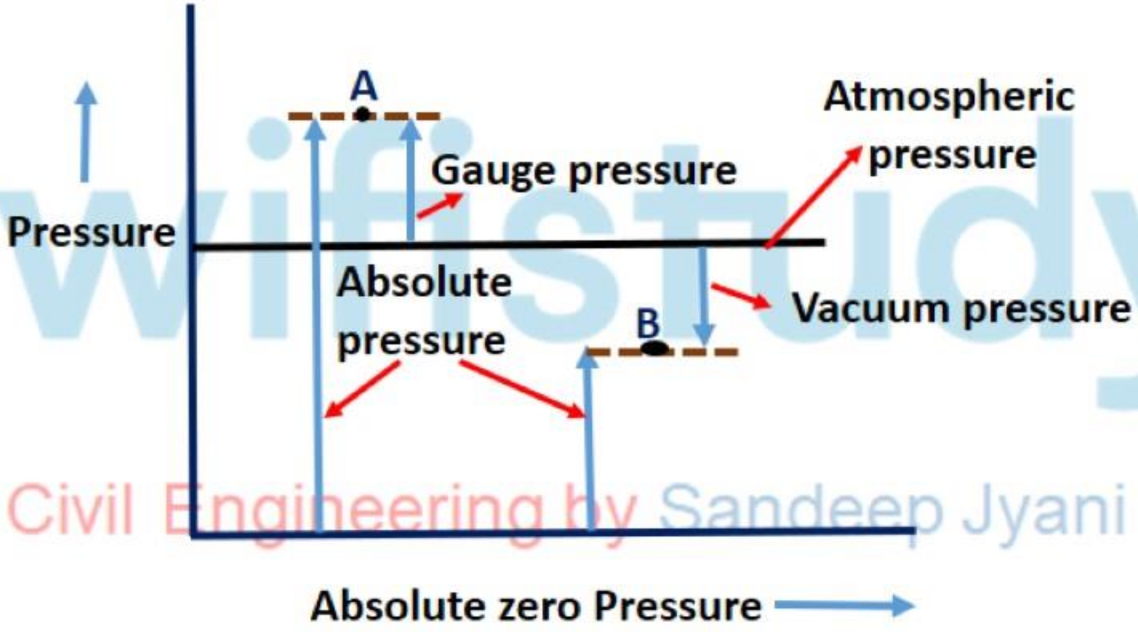
2. **Absolute pressure**
   - Pressure which is measured about with reference to complete vacuum pressure.
   - It is also called actual pressure at a given point.

3. **Gauge and Vacuum pressure**
   - Vacuum pressure is the pressure below atmospheric pressure.
   - Gauge pressure is the pressure measured above atmospheric pressure.
   - Gauge pressure is measured with the help of pressure measuring instrument in which atmospheric pressure is taken as datum i.e. atmosphere pressure on the scale is marked as 0.



$$P_{abs} = P_{atm} + P_{gauge}$$

RELATIONSHIP BETWEEN PRESSURES



$$Absolute\ pressure = atmospheric\ pressure + gauge\ pressure$$

$$Vacuum\ \ pressure = atmospheric\ pressure - absolute\ pressure$$

$$Absolute\ pressure\ at\ sea\ level\ at\ 15°C\ is\ 101.3kN/m^2$$

$$Absolute\ pressure\ head\ is\ 760mm\ of\ Hg\ or\ 10.33m\ of\ water$$

Que 28. The standard atmospheric pressure is 762mm of mercury. It is a specific location. The local atmospheric pressure is 700mm of mercury at this place. What is vacuum pressure w.r.t. reading of 380mm absolute pressure?

a) 320mm of Hg vacuum

b) 62mm of Hg vacuum

c) 382mm of Hg vacuum

d) 300mm of Hg vacuum

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a) <u>320mm of Hg vacuum</u>

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c) 382mm of Hg vacuum

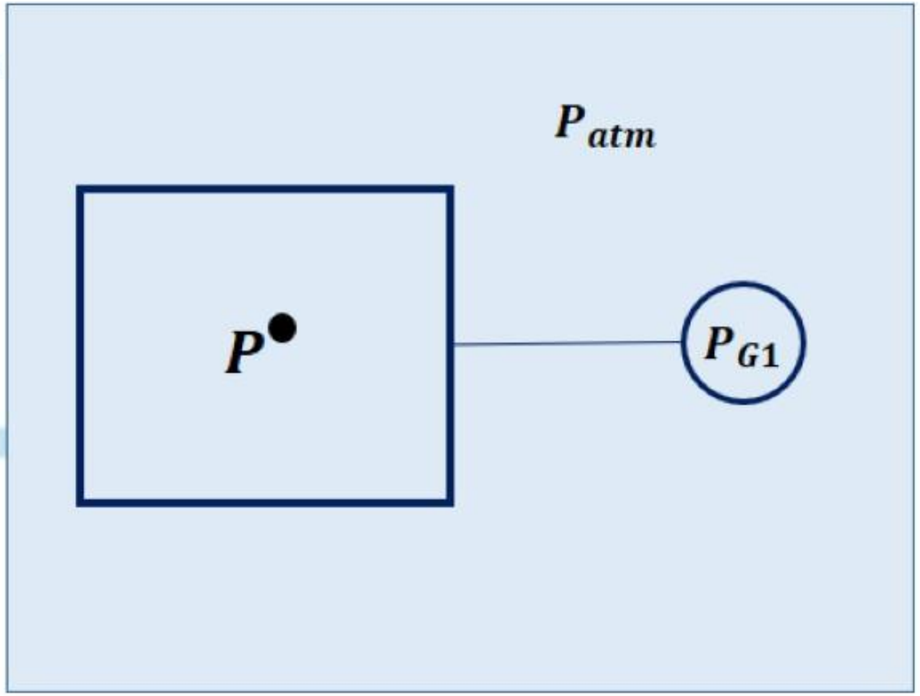
d) 300mm of Hg vacuum

$$\textit{Absolute pressure} = \textit{atmospheric pressure} + \textit{gauge pressure}$$

Que 29. Find out the absolute pressure.

$P_{G1} = 5$ bar

$P_{atm} = 1.01$ bar



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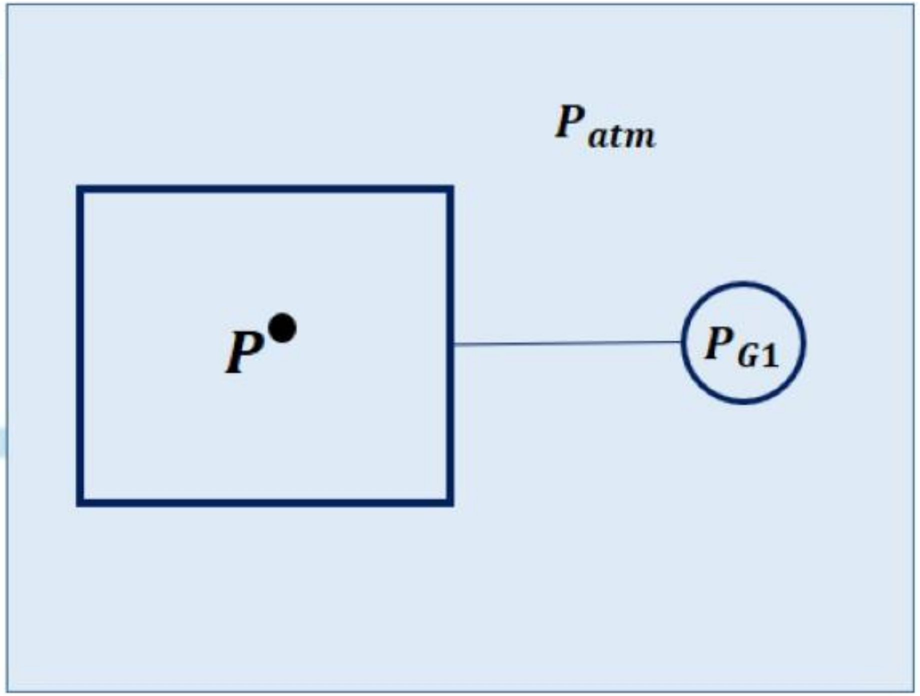


$$P_{abs} = P_{atm} + P_{gauge}$$

$$P_{abs} = 1.01 + 5$$

$$P_{abs} = 6.01 \; bar$$

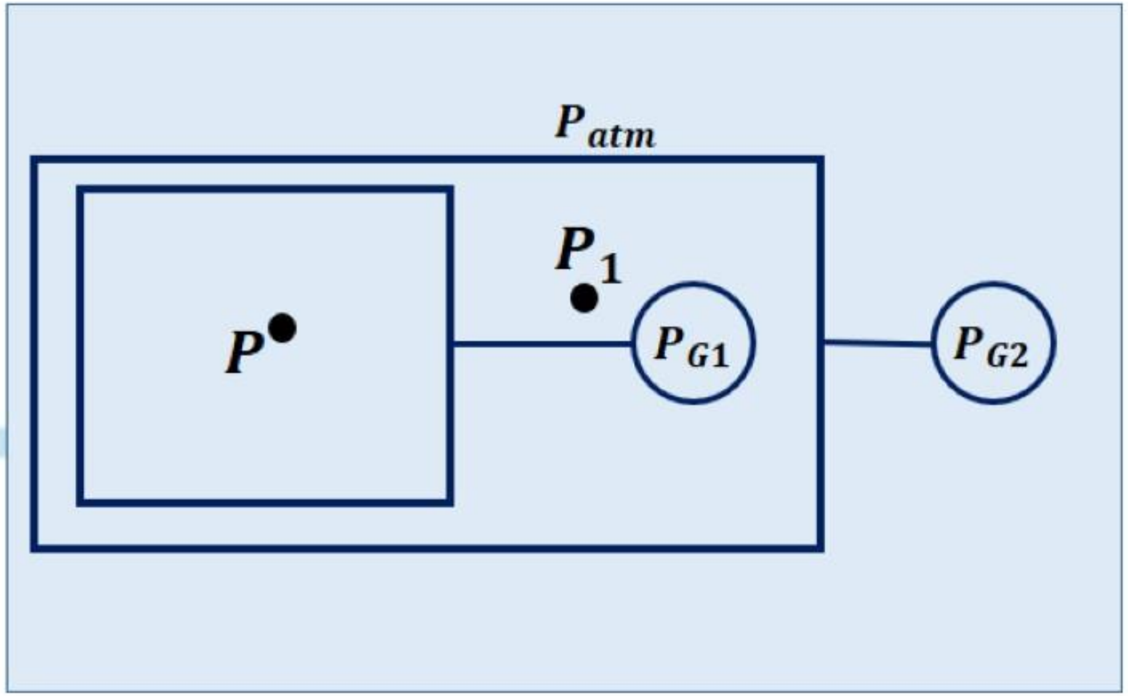
Que 30. Find out the absolute pressure "P".

$P_{G1} = 5 \text{ bar}$

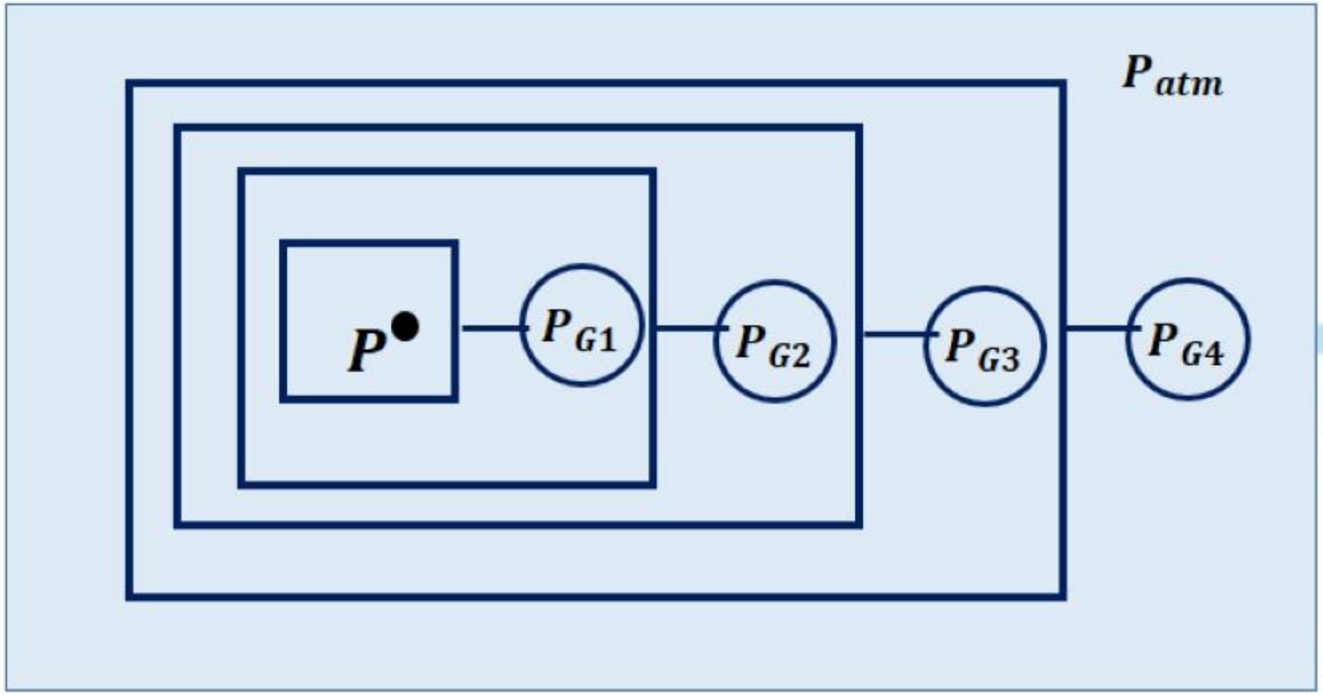
$P_{G2} = 10 \text{ bar}$

$P_{atm} = 1.01 \text{ bar}$

$$P_{abs} = P_{atm} + P_{gauge}$$



Que 31. Find out the absolute pressure “P”.



$P_{G1} = 1$ bar

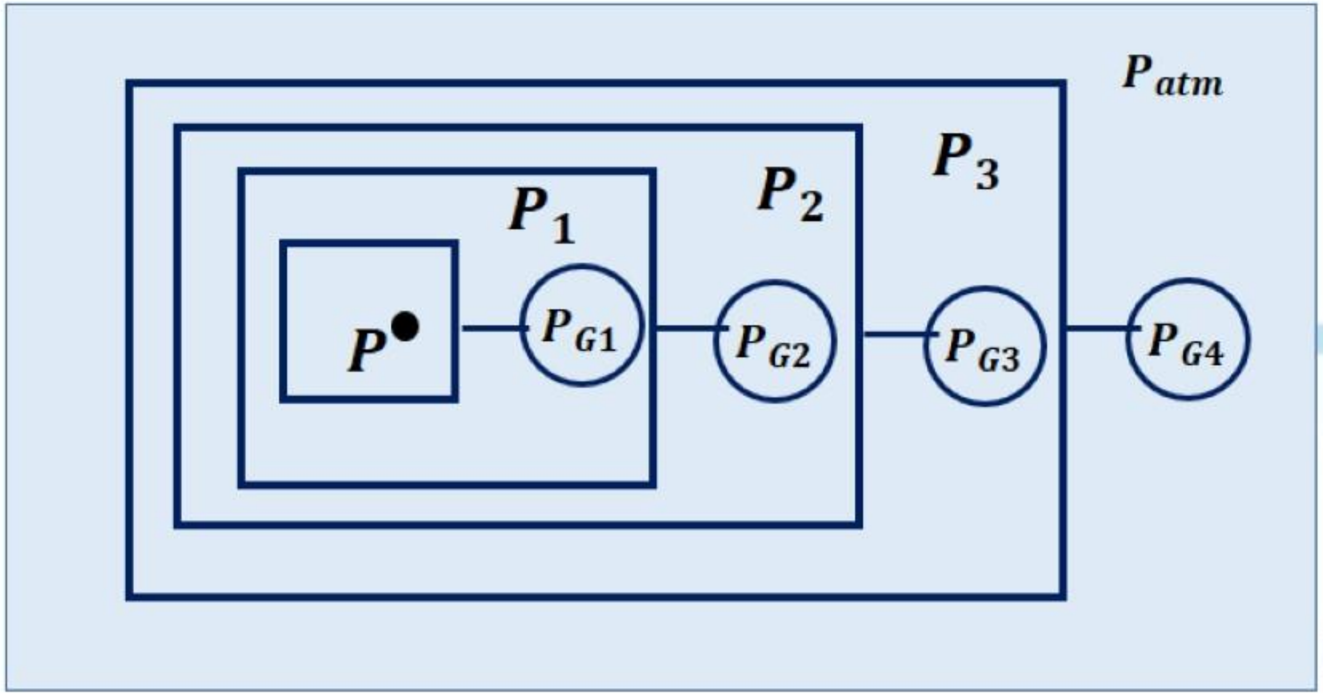
$P_{G2} = 2$ bar

$P_{G3} = 3$ bar

$P_{G4} = 4$ bar

$P_{atm} = 1.01$ bar

Que 31. Find out the absolute pressure "P".



$P_{G1} = 1$ bar

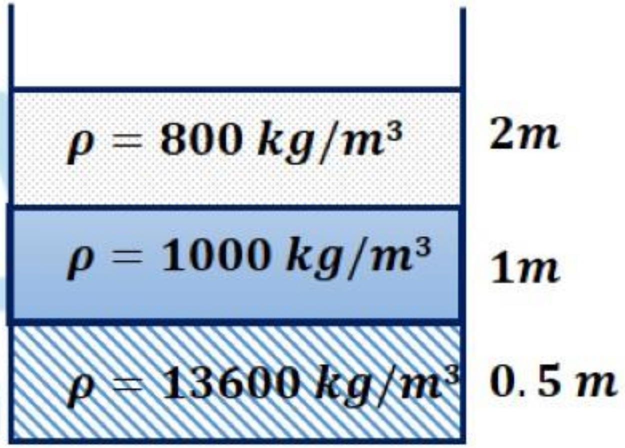
$P_{G2} = 2$ bar

$P_{G3} = 3$ bar

$P_{G4} = 4$ bar

$P_{atm} = 1.01$ bar

Que 32. An open tank contains mercury to a depth of 0.5m, water above it up to a height of 1 m and oil above the water to a height of 2m. Find the gauge pressure at water mercury interface. (Take $g=10m/sec^2$)



MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices:

1. **Manometers (used for liquids)**
2. **Mechanical Gauges (used for gases)**

1. MANOMETER GAUGE:

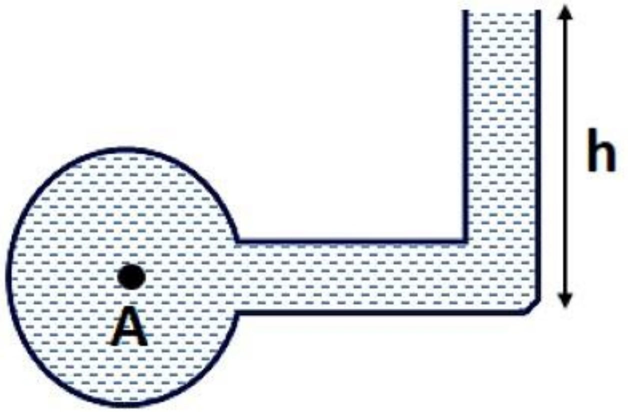
- **Manometers are those pressure measuring devices which are based on the principle of balancing the column of liquid by same or other column of liquid. (where pressure is to be formed).**
- **Types of manometers:**
  - **Simple manometers**
  - **Differential manometers**
- **Note: Mercury is ideal manometric fluid, because it has some properties:**
  a) **No vapour pressure**
  b) **High density**
  c) **Immiscible with other fluids**

MEASUREMENT OF PRESSURE

1. MANOMETER GAUGE:

1. SIMPLE MANOMETERS

- **They consist of a glass tube having one of its end connected to the point where pressure is to be measured and other remains open to atmosphere.**
  - a) Piezometers,
  - b) U-tube manometers,
  - c) Single column manometers



a) **PIEZOMETERS – It is the tube open at both ends where one is connected to a point, where the pressure is to be formed and the other is open to atmosphere.**

- $P - P_{atm} = \rho g h$
- Since $P_{atm} = 0$
- $P = \rho g h$

- **Piezometers are not used for finding out large pressure because at large pressure, manometric height would be very large which is difficult to handle.**
- **In order to notify the effect of capillary, the dia of tube must be large that is greater than 1cm.**

MEASUREMENT OF PRESSURE

1. MANOMETER GAUGE:

1. SIMPLE MANOMETERS

b) U-tube manometers (used for high pressure measurement)

- It is the tube open at both ends where one is connected to a point, where the pressure is to be formed and the other is open to atmosphere.
  - For gauge pressure – Let 'B' be the point at which pressure is to be determined, whose value is 'p'.
  - The datum line is A-A
  - As the pressure is same for the horizontal surface, hence pressure above the horizontal datum line A-A in the left column is same to that of the right column of U-tube manometer
  - Pressure above A-A in left column = $p + \rho_1 g h_1$
  - Pressure above A-A in right column = $\rho_2 g h_2$
  - Hence equating both

    $p = \rho_2 g h_2 - \rho_1 g h_1$

h_1 =Height of light liquid above the datum line

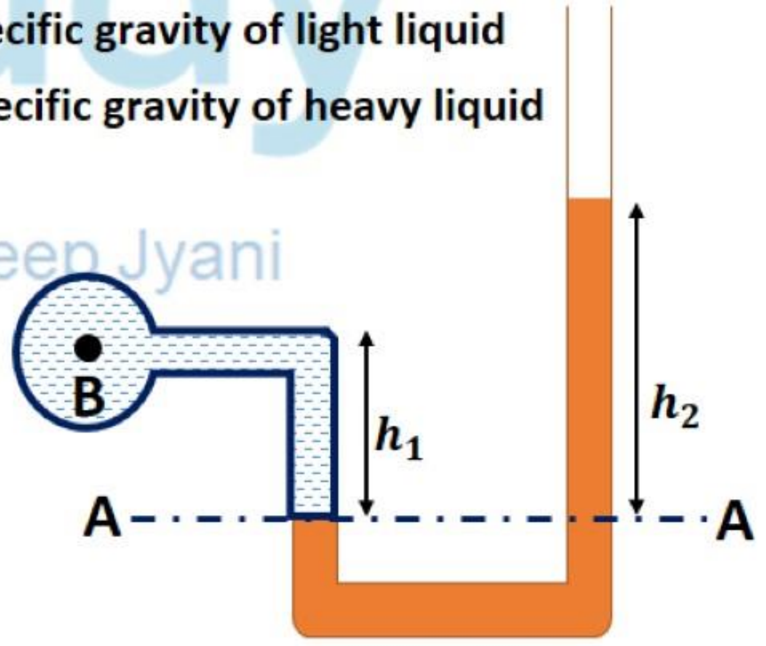
h_2 =Height of heavy liquid above the datum line

ρ_1= density of light liquid = $1000 \times S_1$

ρ_2= density of heavy liquid = $1000 \times S_2$

S_1=specific gravity of light liquid

S_2=specific gravity of heavy liquid



For gauge pressure

MEASUREMENT OF PRESSURE

1. MANOMETER GAUGE:

1. SIMPLE MANOMETERS

b) U-tube manometers (used for high pressure measurement)

- For vacuum pressure – For measuring vacuum pressure,
- Pressure above A-A in left column = $p + \rho_2 g h_2 + \rho_1 g h_1$
- Pressure head in the right column = 0
- Hence equating both
  $p = -(\rho_2 g h_2 + \rho_1 g h_1)$

h_1 =Height of light liquid above the datum line

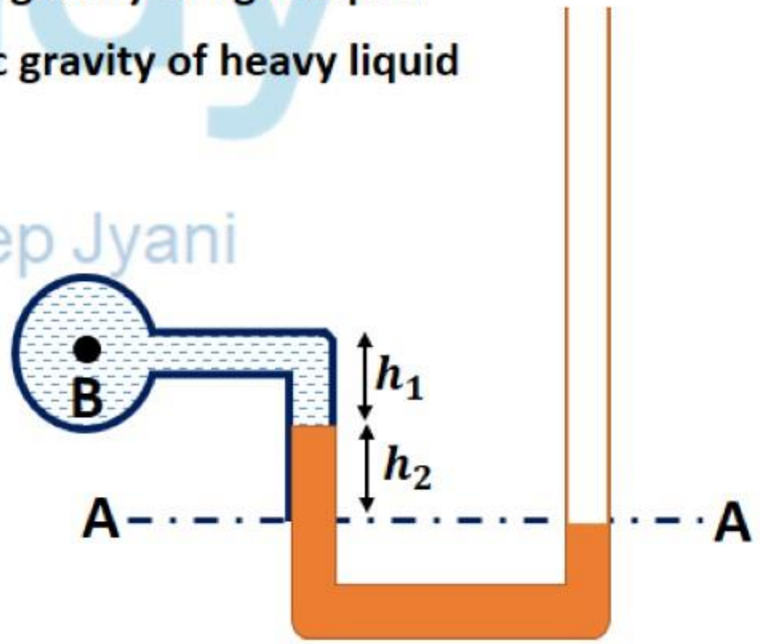
h_2 =Height of heavy liquid above the datum line

ρ_1= density of light liquid = $1000 \times S_1$

ρ_2= density of heavy liquid = $1000 \times S_2$

S_1=specific gravity of light liquid

S_2=specific gravity of heavy liquid



MEASUREMENT OF PRESSURE

1. MANOMETER GAUGE:

1. SIMPLE MANOMETERS

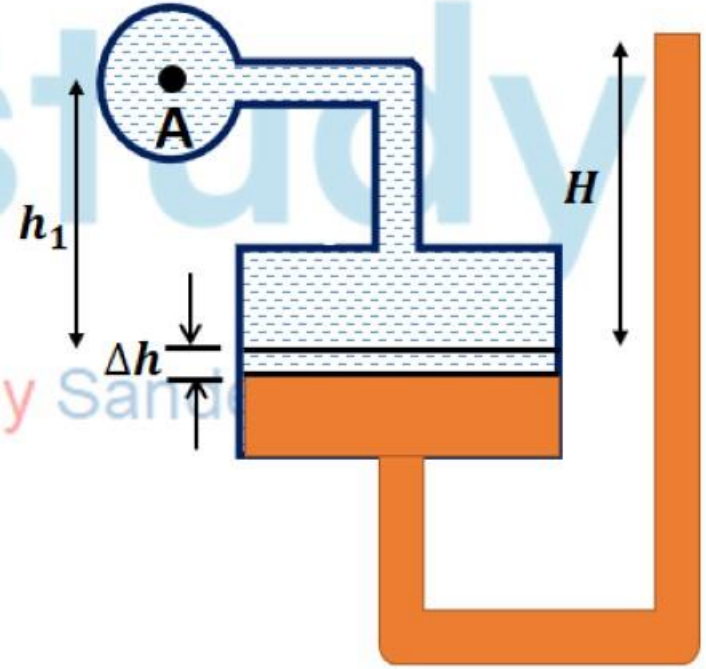
c) Single column manometers

- Modified form of U tube manometer in which a reservoir having large cross-sectional area is connected to one of the limbs
- Fluid leaving = Fluid entering

$\Rightarrow A \times \Delta h = a \times H$

$\Rightarrow \Delta h = \frac{a \times H}{A}$

$\Rightarrow \frac{a}{A} = \%error$



MEASUREMENT OF PRESSURE

1. MANOMETER GAUGE:

2. DIFFERENTIAL MANOMETERS

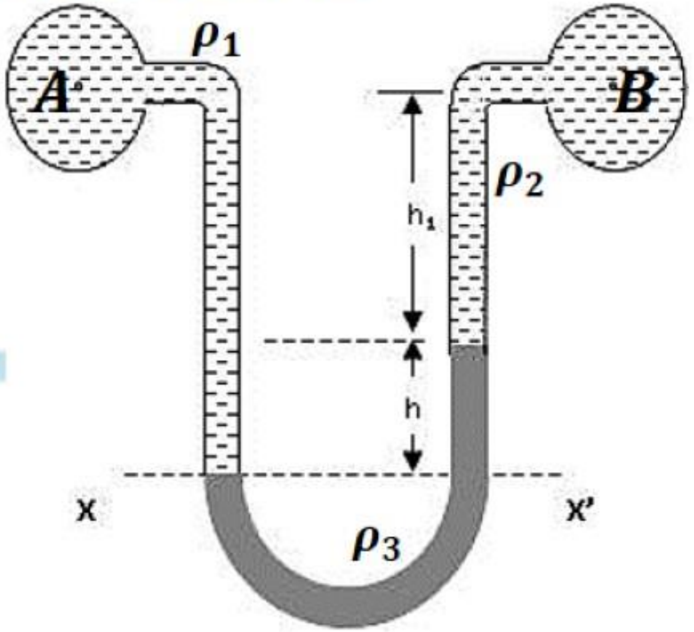
- Differential Manometers are devices used to measure pressure difference between two points in a pipe or in two different pipes
- It consists of U-tube containing a heavy liquid whose two ends are connected to the points, whose difference pressure is to be measured
- Commonly used differential manometers are
  1. U-Tube differential manometer and
  2. Inverted U-Tube differential manometer

MEASUREMENT OF PRESSURE

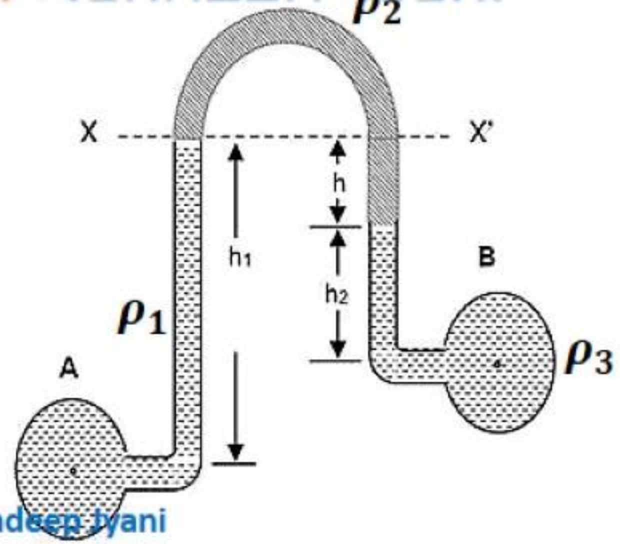
1. MANOMETER GAUGE:

2. DIFFERENTIAL MANOMETERS

1. U-Tube differential manometer
   - $P_A + \rho_1(h_1 + h)g = P_B + \rho_2 h_1 g + \rho_3 hg$

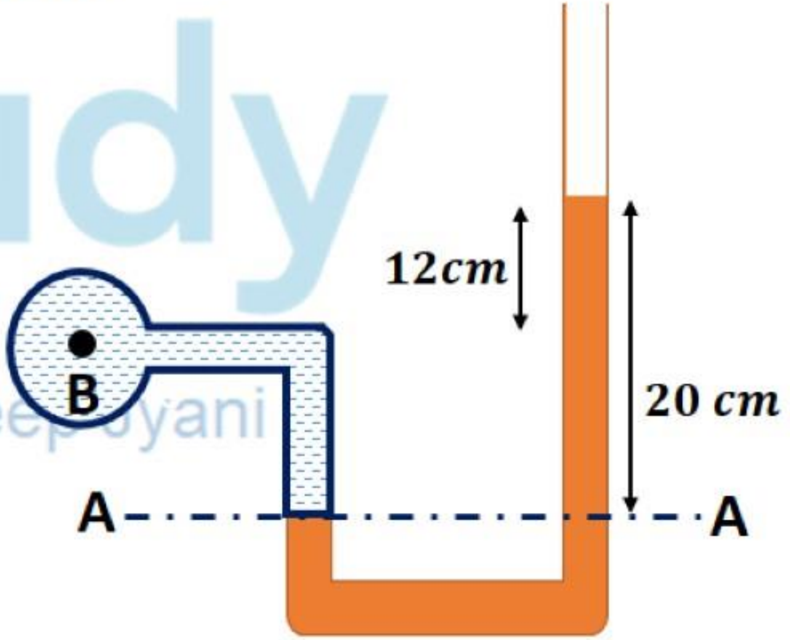


2. Inverted U-Tube differential manometer
   - $P_A - \rho_1 g h_1 = P_B - \rho_2 hg - \rho_3 h_2 g$



Que 33. The right limb of a simple U tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which fluid of specific gravity 0.9 is flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20cm.

Que 33. The right limb of a simple U tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which fluid of specific gravity 0.9 is flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20cm.



Que 34. A simple U tube manometer containing mercury is connected to a pipe in which fluid of specific gravity 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40cm and the height of fluid in the left from centre of pipe is 15 cm below.

HOMEWORK

Que. 35 Fluid is a substance which offers no resistance to

a) Volume

b) Pressure

c) Shape

d) Flow

Que. 35 Fluid is a substance which offers no resistance to

a) Volume

b) Pressure

c) Shape

d) Flow

Que. 36 Surface tension is due to

a) Viscous forces

b) Cohesion

c) Adhesion

d) Difference between cohesion and adhesion

Que. 36 Surface tension is due to

a) Viscous forces

b) <u>Cohesion</u>

c) Adhesion

d) Difference between cohesion and adhesion

Que. 37 If the angle of contact of a drop of liquid is acute than

a) Adhesion is more than cohesion
b) Cohesion is more than Adhesion
c) Adhesion is equal to cohesion
d) Adhesion and cohesion have no bearing with angle of contact

Que. 37 If the angle of contact of a drop of liquid is acute than

a) Adhesion is more than cohesion
b) Cohesion is more than Adhesion
c) Adhesion is equal to cohesion
d) Adhesion and cohesion have no bearing with angle of contact

Que. 38 What is unit of dynamic viscosity of a fluid termed as ‘poise’ equivalent to

a) Dyne/cm^2
b) gm-s/cm^2
c) Dyne-s/cm^2
d) gm-cm/s^2

Que. 38 What is unit of dynamic viscosity of a fluid termed as 'poise' equivalent to

a) Dyne/cm^2

b) gm-s/cm^2

c) <u>Dyne-s/cm^2</u>

d) gm-cm/s^2

Que. 39 The mercury is known as non wetting liquid because it does not wet the surface. The mercury possess this property due to

a) Cohesion

b) Adhesion

c) Viscosity

d) Surface tension

Que. 39 The mercury is known as non wetting liquid because it does not wet the surface. The mercury possess this property due to

a) Cohesion
b) Adhesion
c) Viscosity
d) Surface tension

Que. 40 The example of Bigham Plastic is

a) Air

b) Blood

c) Tooth paste

d) Printing ink

Que. 40 The example of Bigham Plastic is

a) Air

b) Blood

c) <u>Tooth paste</u>

d) Printing ink

Que. 41 The example of Non-Newtonian fluid of Pseudo plastic variety is

a) Milk

b) Air

c) Water

d) Printing ink

Que. 41 The example of Non-Newtonian fluid of Pseudo plastic variety is

a) Milk

b) Air

c) Water

d) Printing ink

Que. 42 The fluid which obeys the Newton's law of viscosity is

a) Newtonian fluid

b) Real fluid

c) Bingham plastic

d) Perfect fluid

Que. 42 The fluid which obeys the Newton's law of viscosity is

a) Newtonian fluid

b) Real fluid

c) Bingham plastic

d) Perfect fluid

Que. 43 The dimensions of kinematic viscosity is Newtonian fluid

a) $M^0 L^1 T^{-2}$

b) $M^0 L^2 T^{-1}$

c) $M^1 L^1 T^{-2}$

d) $M^1 L^{-2} T^{-2}$

Que. 43 The dimensions of kinematic viscosity is Newtonian fluid

a) $M^0L^1T^{-2}$

b) $\underline{M^0L^2T^{-1}}$

c) $M^1L^1T^{-2}$

d) $M^1L^{-2}T^{-2}$

Que. 44 The property by virtue of which a liquid opposes relative motion between its different layers is

a) Surface tension

b) Viscosity

c) Cohesion

d) Capillarity

Que. 44 The property by virtue of which a liquid opposes relative motion between its different layers is

a) Surface tension

b) Viscosity

c) Cohesion

d) Capillarity

Que. 45 Which one of the following is true for Newtonian fluid

a) Viscosity shear stress is independent of viscosity gradient

b) Viscosity shear stress depends linearly on viscosity gradient

c) Viscosity shear stress is zero at all viscosity gradient

d) Viscosity shear stress decreases with viscosity gradient

Que. 45 Which one of the following is true for Newtonian fluid

a) Viscosity shear stress is independent of viscosity gradient

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c) Viscosity shear stress is zero at all viscosity gradient

d) Viscosity shear stress decreases with viscosity gradient

Que. 46 Viscosity of which one of the following fluid does not change with the rate of deformation

a) Ideal

b) Real

c) Newtonian

d) Non-Newtonian

Que. 46 Viscosity of which one of the following fluid does not change with the rate of deformation

a) Ideal

b) Real

c) Newtonian

d) Non-Newtonian

Que. 47 The value of 1 poise is equal to

a) $\frac{1}{20} N.s/m^2$

b) $\frac{1}{30} N.s/m^2$

c) $\frac{1}{10} N.s/m^2$

d) $\frac{1}{10} N.s/cm^2$

Que. 47 The value of 1 poise is equal to

a) $\frac{1}{20} N.s/m^2$

b) $\frac{1}{30} N.s/m^2$

c) $\frac{1}{10} N.s/m^2$

d) $\frac{1}{10} N.s/cm^2$

Que. 48 The weight per unit volume of a liquid at a standard temperature and pressure is called

a) Specific weight
b) Mass density
c) Specific gravity
d) None of these

Que. 48 The weight per unit volume of a liquid at a standard temperature and pressure is called

a) Specific weight

b) Mass density

c) Specific gravity

d) None of these

FLUID STATIC

Net force on body = $(P_2 - P_1) \times Area$

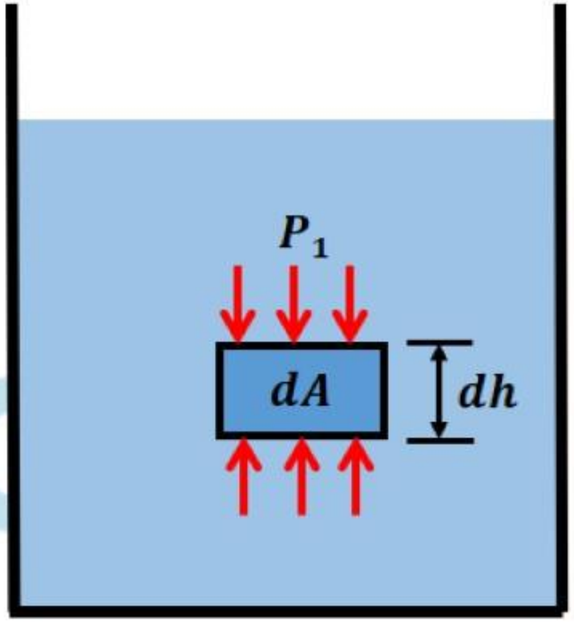
= $(\rho g h_2 - \rho g h_1) \times Area$

= $\{\rho g(h_2 - h_1)\}Area$

= $(\rho g \, dh)Area$

= $\rho g(dh \times A)$

F= $\rho g V$



ARCHIMEDES PRINCIPLE

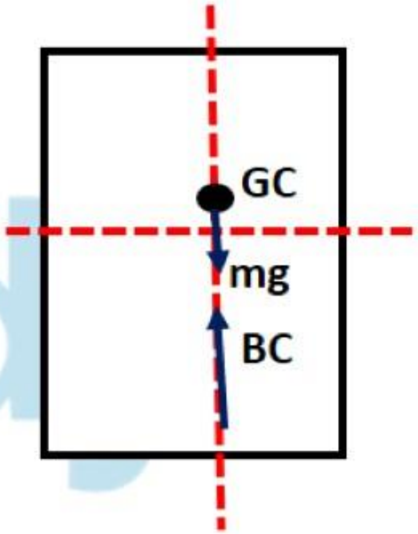
- **When a body is immersed either partially or completely, then the net vertical force exerted by this fluid on the body is known as Buoyancy Force.**
- **This buoyancy force is equal to the weight of the fluid displaced.**
- **In a constant density fluid, buoyancy force is basically due to pressure difference.**
- **CENTRE OF BUOYANCY : It is the point from which buoyancy force is supposed to be acting, centre of buoyancy will be at the centre of displaced volume.**

Que 49. A man's weight in water was found 30 kg. What is the actual weight of man if volume of displaced water by man is $0.07m^3$

Que 50. A metallic body is placed at interface of mercury and water, in such a way that 40% of volume submerged in mercury and 60% is submerged in water. Then find the density of the body.

PRINCIPLE OF FLOATATION

- **For a floating surface, equilibrium weight of a body is equal to buoyancy force and line of action of these two forces will be same.**



BC – buoyancy centre

GC – gravitational centre

- **TYPES OF EQUILIBRIUM**

Stable equilibrium

Unstable equilibrium

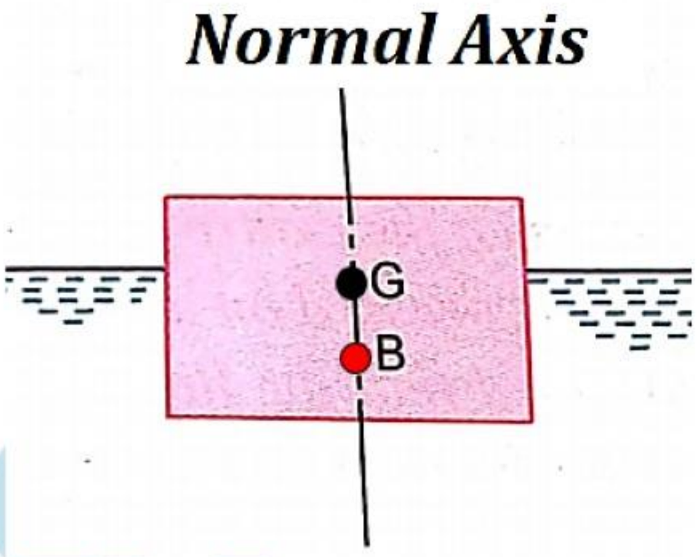
METACENTRE

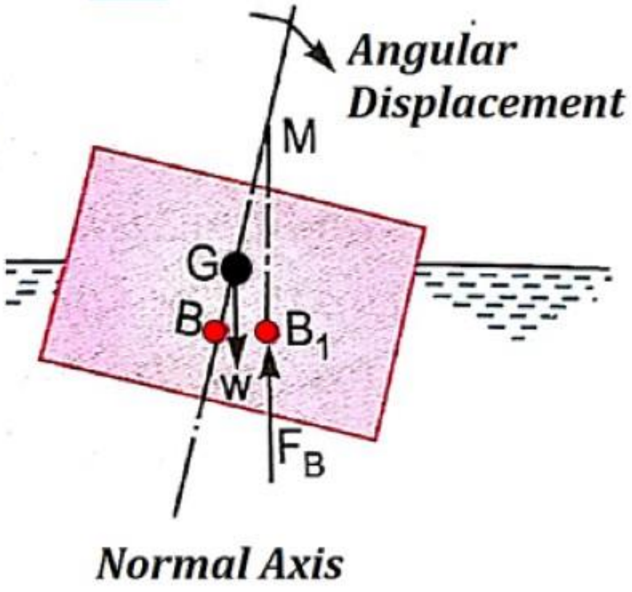
- **It is the point of intersection of normal axis with the new line of action of force of buoyancy, when a small displacement is given.**
- **It is a point about which a body starts to oscillate when it is tilted by small angle.**

$$G.M. = B.M - B.G$$

$$GM = \frac{I}{V} - BG$$

- Where $I_{xx} = \frac{bd^3}{12}$; $I_{yy} = \frac{db^3}{12}$; $\frac{I_{min}}{V}$ = metacentric radius
- **For stability G.M. should be positive, i.e., B.M>B.G**





- **From designing point of view,**
  - **the moment of inertia (I) is taken about the top view at the surface of liquid.**
  - **Oscillation about longitudinal axis is called rolling.**
  - **Oscillation about transverse axis is called pitching.**

- **TIME PERIOD FOR OSCILLATION:**

$$T = 2\pi \sqrt{\frac{K^2}{GM.g}}$$

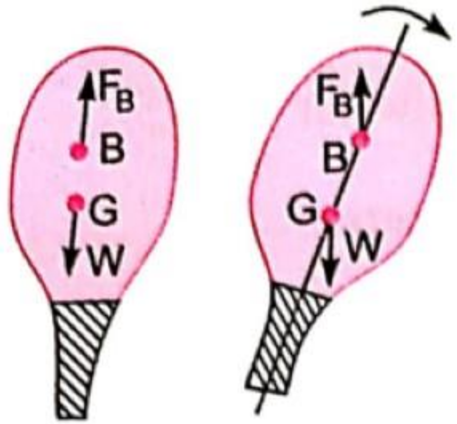
  - **Where K is radius of gyration**
  - **GM is metacentric height**
  - **T is time of one complete oscillation**

- **STABILITY CONDITION FOR FULLY SUBMERGED BODY:**

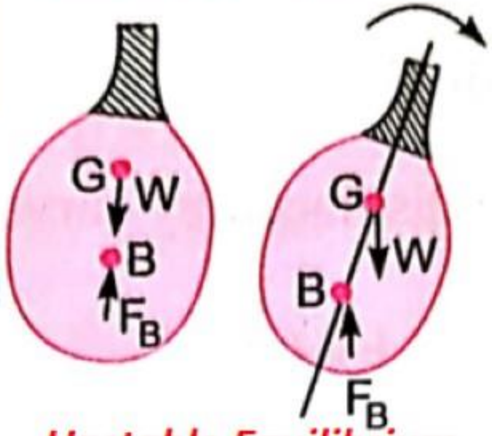
  - **A completely submerged body will be in *Stable Equilibrium* when centre of buoyancy is above centre of gravity.**

  - **If centre of buoyancy is below centre of gravity, then completely submerged body will be in *Unstable Equilibrium*.**

  - **If centre of buoyancy and centre of gravity coincides, then fully submerged body will be in *Neutral Equilibrium.***



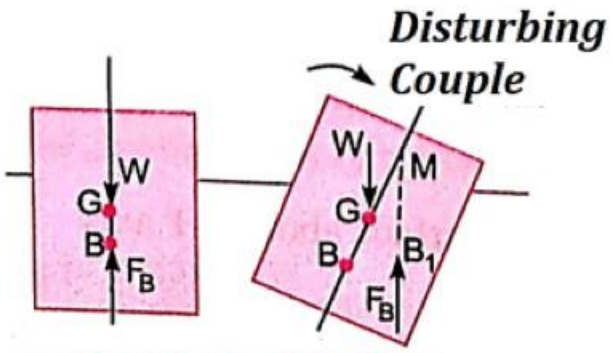
Stable Equilibrium



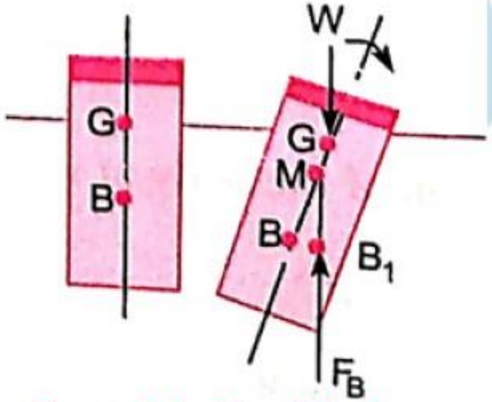
Unstable Equilibrium

Neutral Equilibrium

- **STABILITY CONDITION FOR PARTIALLY SUBMERGED BODY:**
  - **A partially submerged body will be in *Stable Equilibrium* when metacentre is above centre of gravity.**
  - **If metacentre is below centre of gravity, then partially submerged body will be in *Unstable Equilibrium.***
  - **If metacentre and centre of gravity coincides, then partially submerged body will be in neutral equilibrium.**
- **The distance between C.G. and metacenter is called metacentric height.**



Stable Equilibrium



Unstable Equilibrium.

Stable equilibrium

Unstable equilibrium

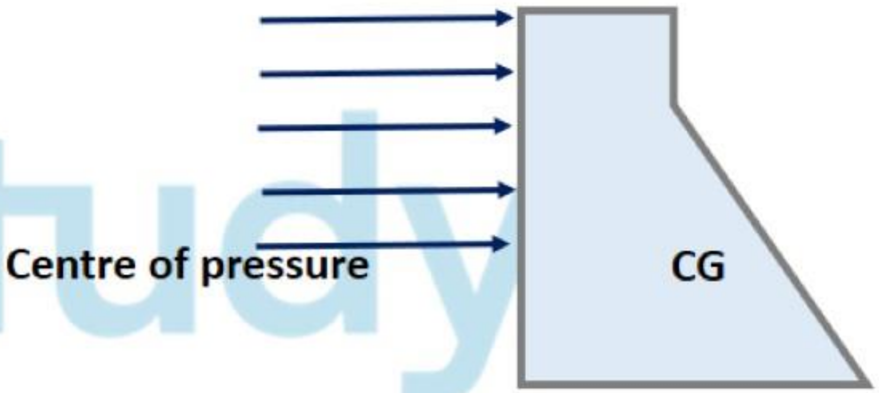
Neutral equilibrium

HYDROSTATIC FORCES ON PLANE SURFACES

- This chapter deals about fluids at rest
- No relative motion between adjacent or neighboring layers
- Velocity gradient(rate of change of velocity between two adjacent fluid layers divided by distance between the two layers will be zero $\frac{du}{dy} = 0$)
- Hence Shear Stress $\tau = \mu \frac{du}{dy} = 0$

HYDROSTATIC FORCES ON PLANE SURFACES

- **HYDROSTATIC FORCE:**
  - **The force exerted by the static fluid, when a body is exposed to it.**



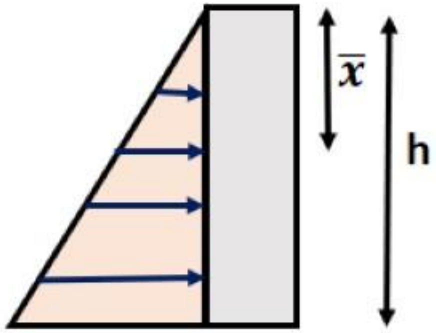
- **CENTRE OF PRESSURE:**
  - **It is the point from which the total hydrostatic force is supposed to be acting.**

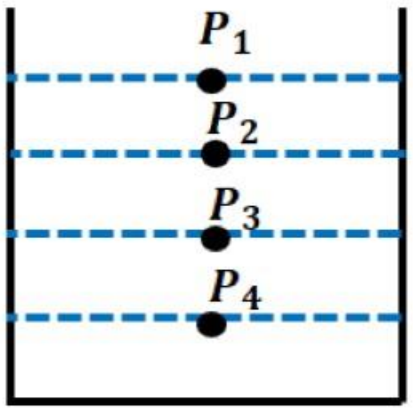
$$F = \left(\frac{0+\rho g h}{2}\right) \times Area$$

$$F = \left(\frac{\rho g h}{2}\right) \times Area$$

$$F_{net} = wA\bar{x}$$

$\bar{x}$ = disance of CG from free water surface





$P_4 > P_3 > P_2 > P_1$

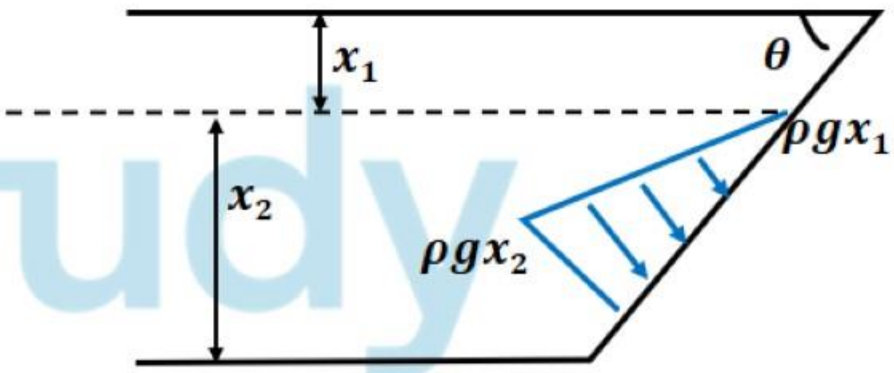
Centre of pressure = $\bar{x} + \frac{I_G}{A\bar{x}}$ **(for vertical body)**

HYDROSTATIC FORCES ON PLANE SURFACES

- **CENTRE OF PRESSURE FOR INCLINED SURFACE:**

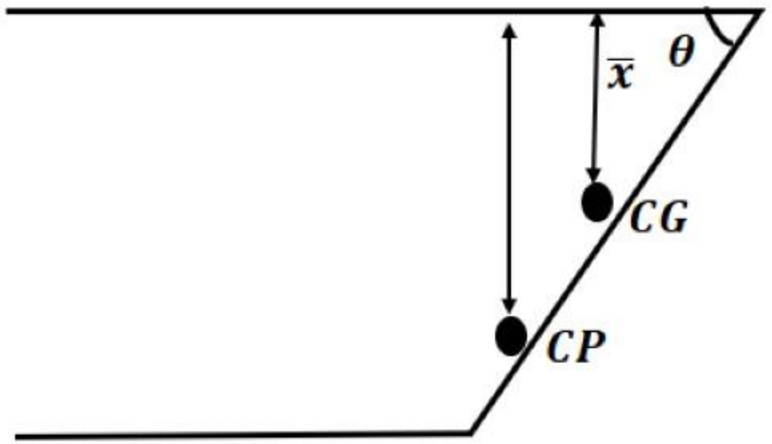
Centre of pressure = $\overline{x} + \frac{I_G}{A\overline{x}} \sin^2 \theta$

$$F_{net} = wA\overline{x}$$



For horizontal body, θ = 0

h=$\overline{x}$



I_G is moment of inertia about C-G axis

h is distance of centre of pressure from free surface

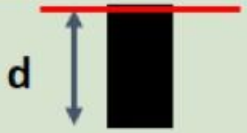
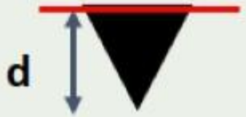
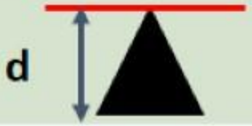
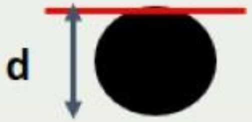
$\overline{x}$ is distance of centre of gravity from free surface

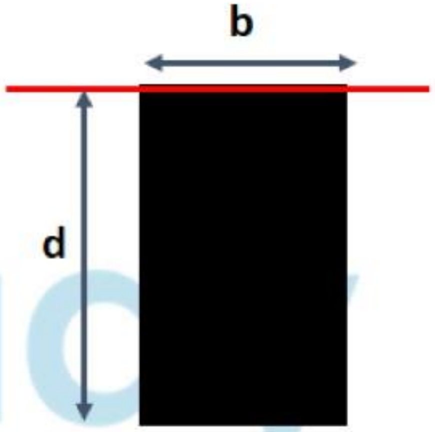
HYDROSTATIC FORCES ON PLANE SURFACES

NOTE:

1. **Centre of pressure will be below the centre of gravity because pressure intensity is more below the centre of gravity.**
2. **In case of plane vertical surface, centre of gravity and centre of pressure are almost same when depth of immersion is very large.**

CENTRE OF PRESSURE

BODY	CENTRE OF PRESSURE (h)
	
	
	
	



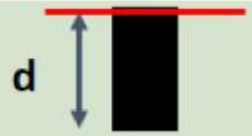
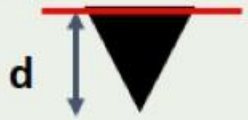
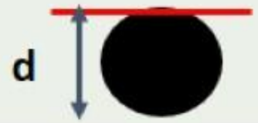
Centre of pressure h = $\bar{x} + \frac{I_G}{A\bar{x}}$

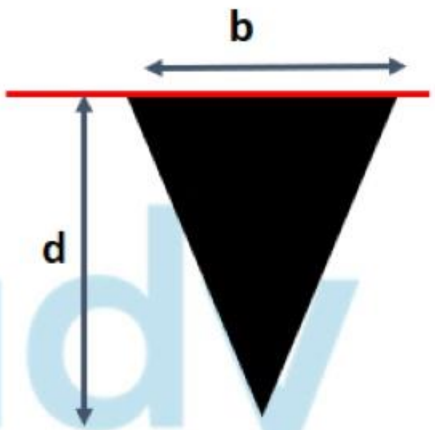
$$\Rightarrow \mathbf{h} = \frac{d}{2} + \frac{\left(\frac{bd^3}{12}\right)}{(d\times b)\left(\frac{d}{2}\right)}$$

$$\Rightarrow \mathbf{h} = \frac{d}{2} + \frac{d}{6}$$

$$\Rightarrow \mathbf{h} = \frac{2d}{3}$$

CENTRE OF PRESSURE

BODY	CENTRE OF PRESSURE (h)
	$\frac{2}{3}d$
	
	
	



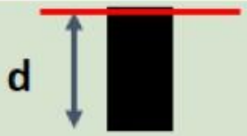
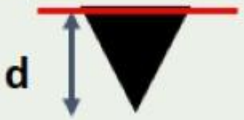
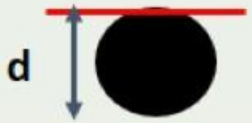
Centre of pressure h = $\bar{x} + \frac{I_G}{A\bar{x}}$

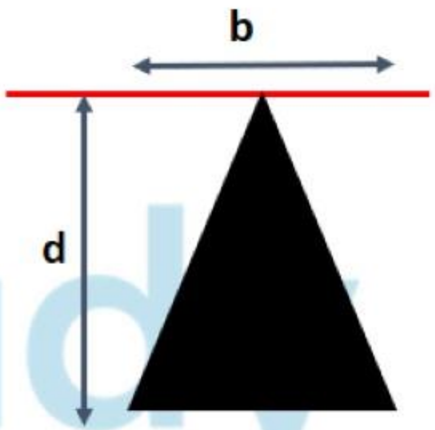
$$\Rightarrow \mathbf{h} = \frac{d}{3} + \frac{\left(\frac{bd^3}{36}\right)}{\left(\frac{1}{2}\times d\times b\right)\left(\frac{d}{3}\right)}$$

$$\Rightarrow \mathbf{h} = \frac{d}{3} + \frac{d}{6}$$

$$\Rightarrow \mathbf{h} = \frac{d}{2}$$

CENTRE OF PRESSURE

BODY	CENTRE OF PRESSURE (h)
	$\frac{2}{3}d$
	$\frac{1}{2}d$
	
	



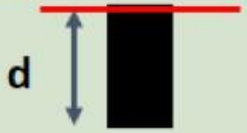
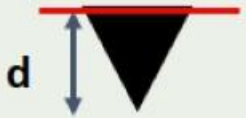
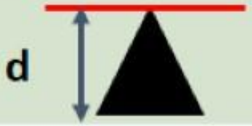
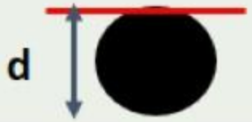
Centre of pressure h = $\bar{x} + \frac{I_G}{A\bar{x}}$

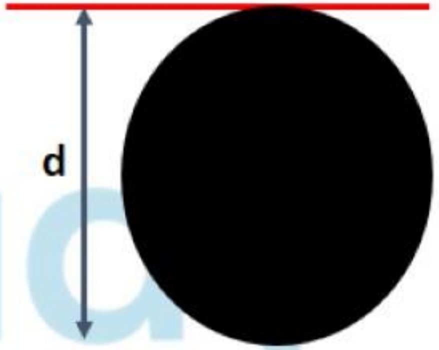
$$\Rightarrow \mathbf{h} = \frac{2d}{3} + \frac{\left(\frac{bd^3}{36}\right)}{\left(\frac{1}{2}\times d\times b\right)\left(\frac{2d}{3}\right)}$$

$$\Rightarrow \mathbf{h} = \frac{2d}{3} + \frac{d}{12}$$

$$\Rightarrow \mathbf{h} = \frac{3d}{4}$$

CENTRE OF PRESSURE

BODY	CENTRE OF PRESSURE (h)
	$\frac{2}{3}d$
	$\frac{1}{2}d$
	$\frac{3}{4}d$
	



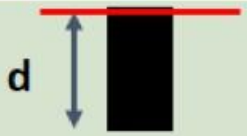
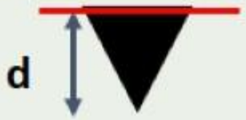
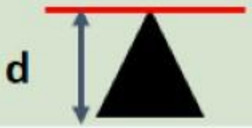
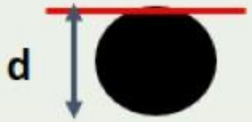
Centre of pressure h = $\bar{x} + \frac{I_G}{A\bar{x}}$

$$\Rightarrow \mathbf{h} = \frac{d}{2} + \frac{\left(\frac{\pi d^4}{64}\right)}{\left(\frac{\pi d^2}{4}\right)\left(\frac{d}{2}\right)}$$

$$\Rightarrow \mathbf{h} = \frac{d}{2} + \frac{d}{8}$$

$$\Rightarrow \mathbf{h} = \frac{5d}{8}$$

CENTRE OF PRESSURE

BODY	CENTRE OF PRESSURE (h)
	$\frac{2}{3}d$
	$\frac{1}{2}d$
	$\frac{3}{4}d$
	$\frac{5}{8}d$

Que 51. A rectangular plane surface is 2m wide and 3m deep. It lies in a vertical plane in water. Determine the total pressure force and position of centre of pressure on the plane surface when its upper edge is horizontal and *coincides with water surface*

Que 52. A rectangular plane surface is 2m wide and 3m deep. It lies in a vertical plane in water. Determine the total pressure force and position of centre of pressure on the plane surface when its upper edge is horizontal and *2.5m below the water surface.*

Que 53. Determine the total pressure on a circular plate of diameter 1.5m which is placed vertically in water in such a way that the centre of the plate is 3m below the free surface of water. Find the position of centre of pressure also.

Que 54. A vertical gate $6m \times 6m$ holds water on one side with free surface on its top, the moment about the bottom edge of gate due to water force is (w=specific weight)

a) *36w*

b) *72w*

c) *108w*

d) *216w*

Que 54. A vertical gate $6m \times 6m$ holds water on one side with free surface on its top, the moment about the bottom edge of gate due to water force is (w=specific weight)

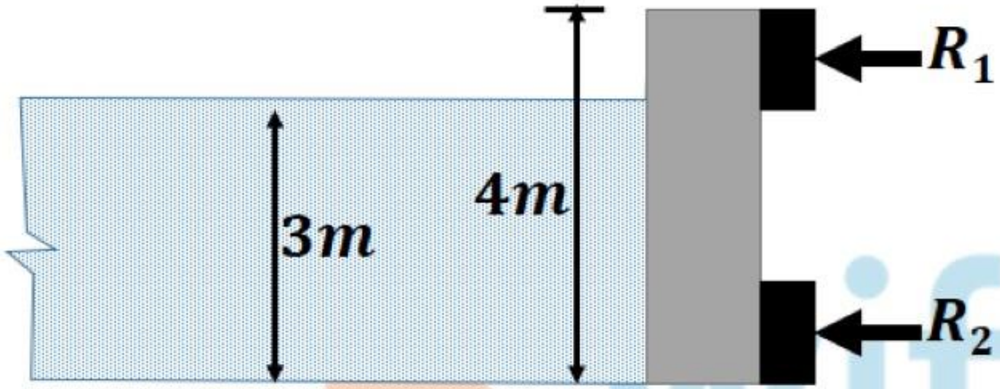
a) *36w*

b) *72w*

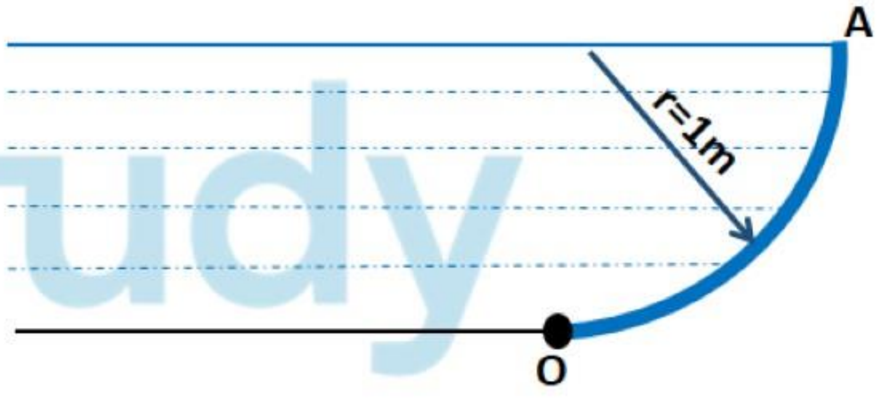
c) *108w*

d) ***216w***

Que 55. Find reaction on supports of the gate.



Que 56. A gate OA as shown in figure is hinged at O and is in the form of ¼ of a circle of radius. It supports water on one side as shown in figure. The width of gate is 3 m. Find horizontal, vertical force.



FLUID KINEMATICS

- **Kinematics deal with motion of fluid without any reference to force.**
- **Fluid flow is analyzed by using**
  1. **Lagrangian Approach and**
  2. **Eulerian Approach.**
- **In Lagrangian approach, the behavior of single fluid particle is analysed whereas in the Eulerian Approach, certain section or point is taken and at this section, fluid flow is analyzed.**
- **Due to simplicity, Eulerian Approach is used mostly in fluid mechanics.**

TYPES OF FLUID FLOW

1. **Steady and Unsteady Flow**
2. **Uniform and Non Uniform Flow**
3. **Laminar and Turbulent Flow**
4. **Compressible and Incompressible Flow**
5. **Rotational and Irrotational Flow**

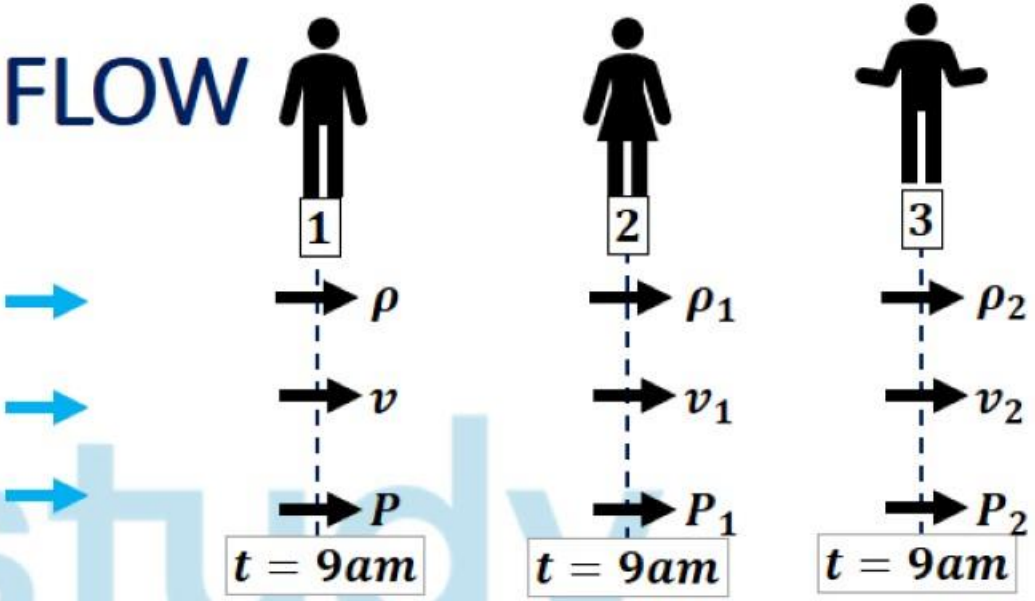
1. STEADY AND UNSTEADY FLOW

1. A flow is said to be a steady flow, if the fluid properties (velocity, pressure, density,temperature etc.) do not vary with respect to time at any given section, otherwise flow is unsteady.
    - For steady Flow,

$$\frac{dV}{dt} = 0$$

$$\frac{d\rho}{dt} = 0$$

$$\frac{dP}{dt} = 0$$



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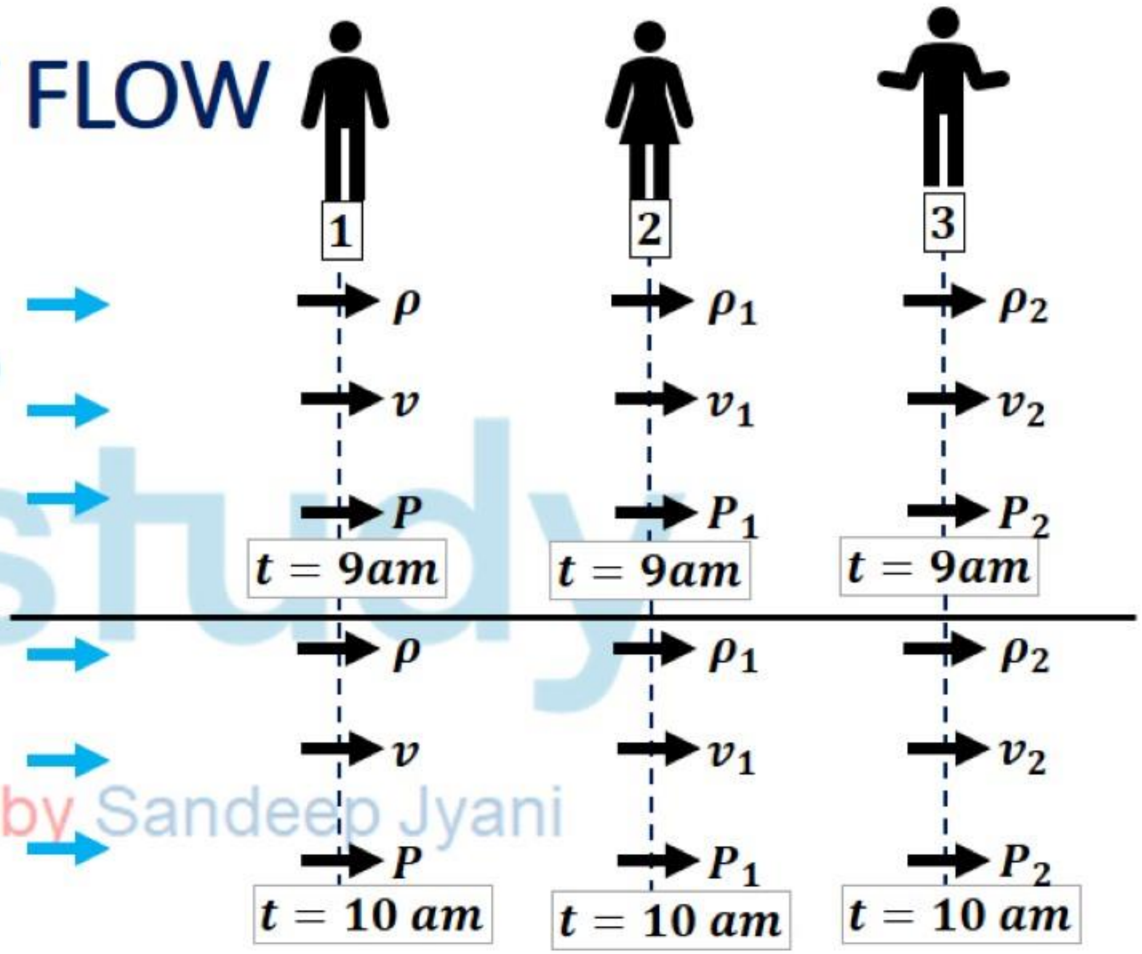

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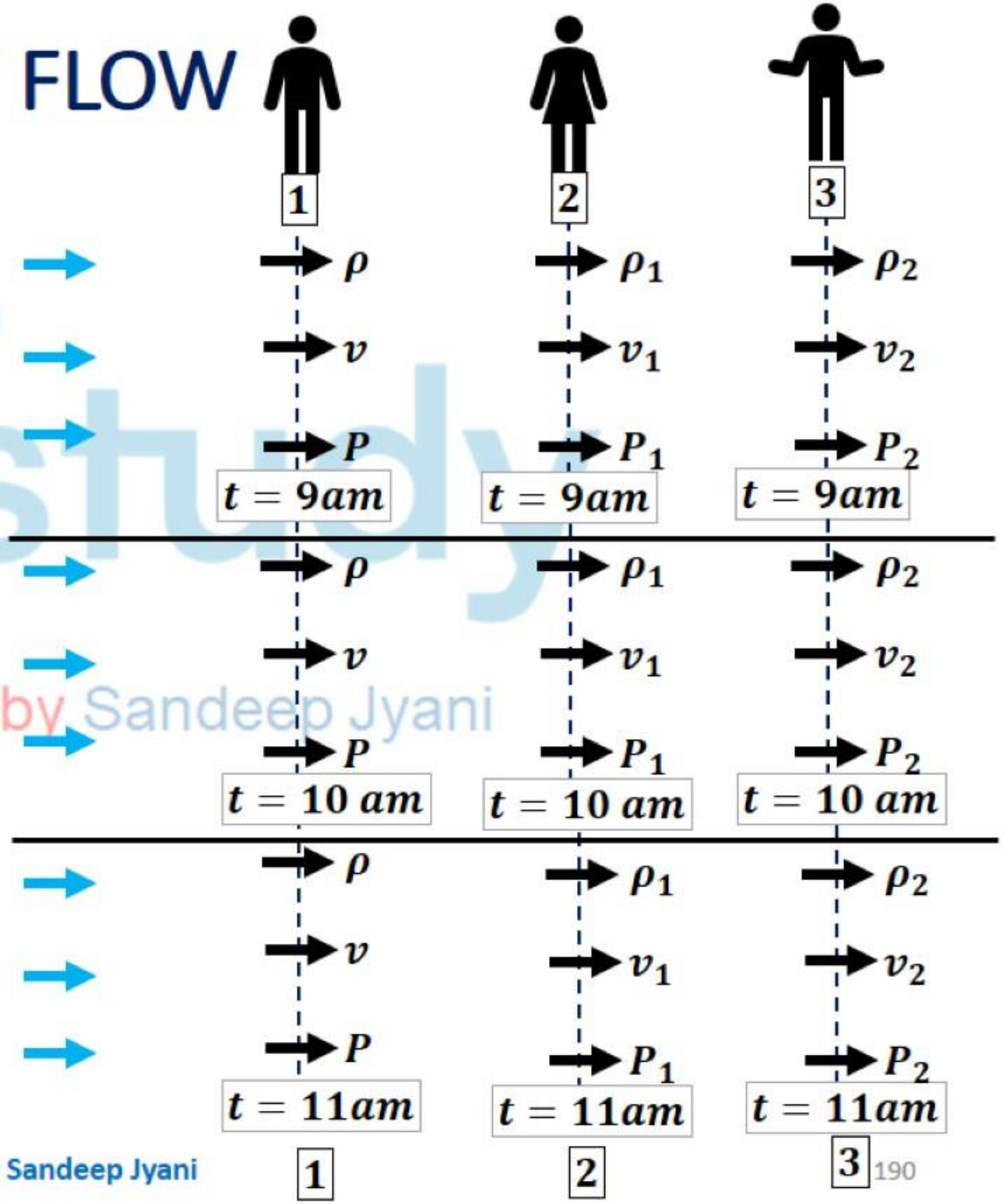
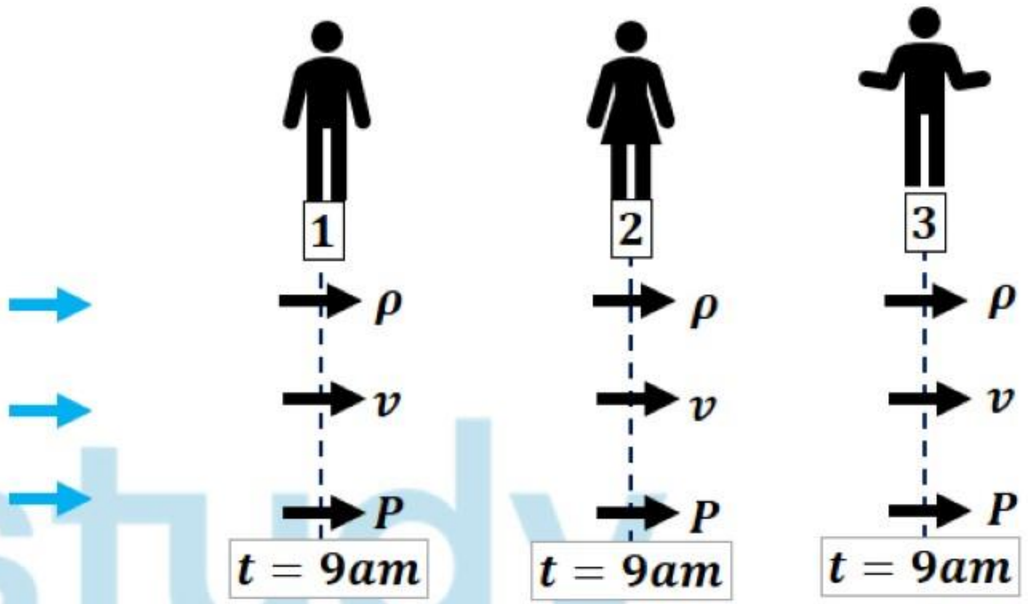


2. UNIFORM AND NON-UNIFORM FLOW

- **A flow is said to be a uniform flow, if the velocity remains constant at a different section at any given instant of time, otherwise flow is non uniform.**

$$\left(\frac{\partial v}{\partial s}\right)_{t=constant} = 0$$



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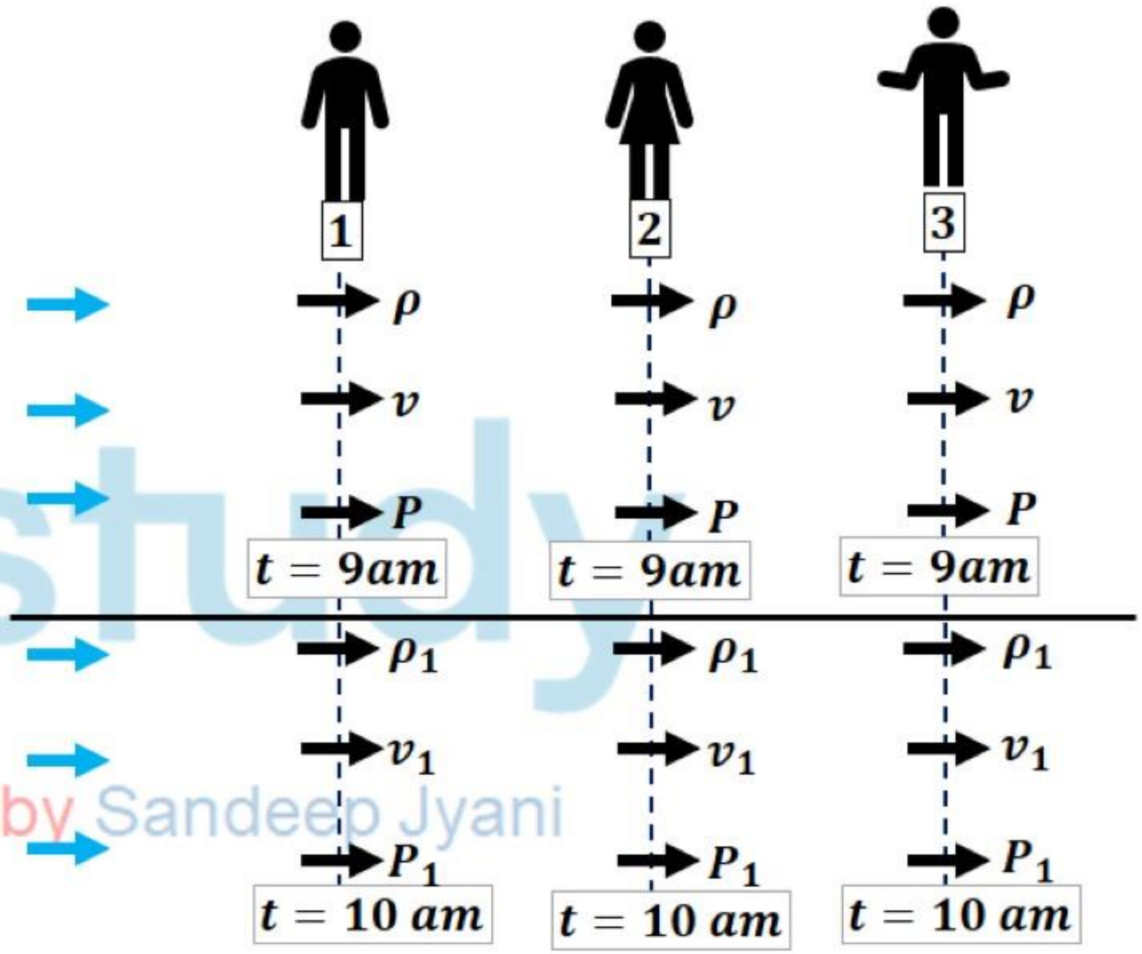

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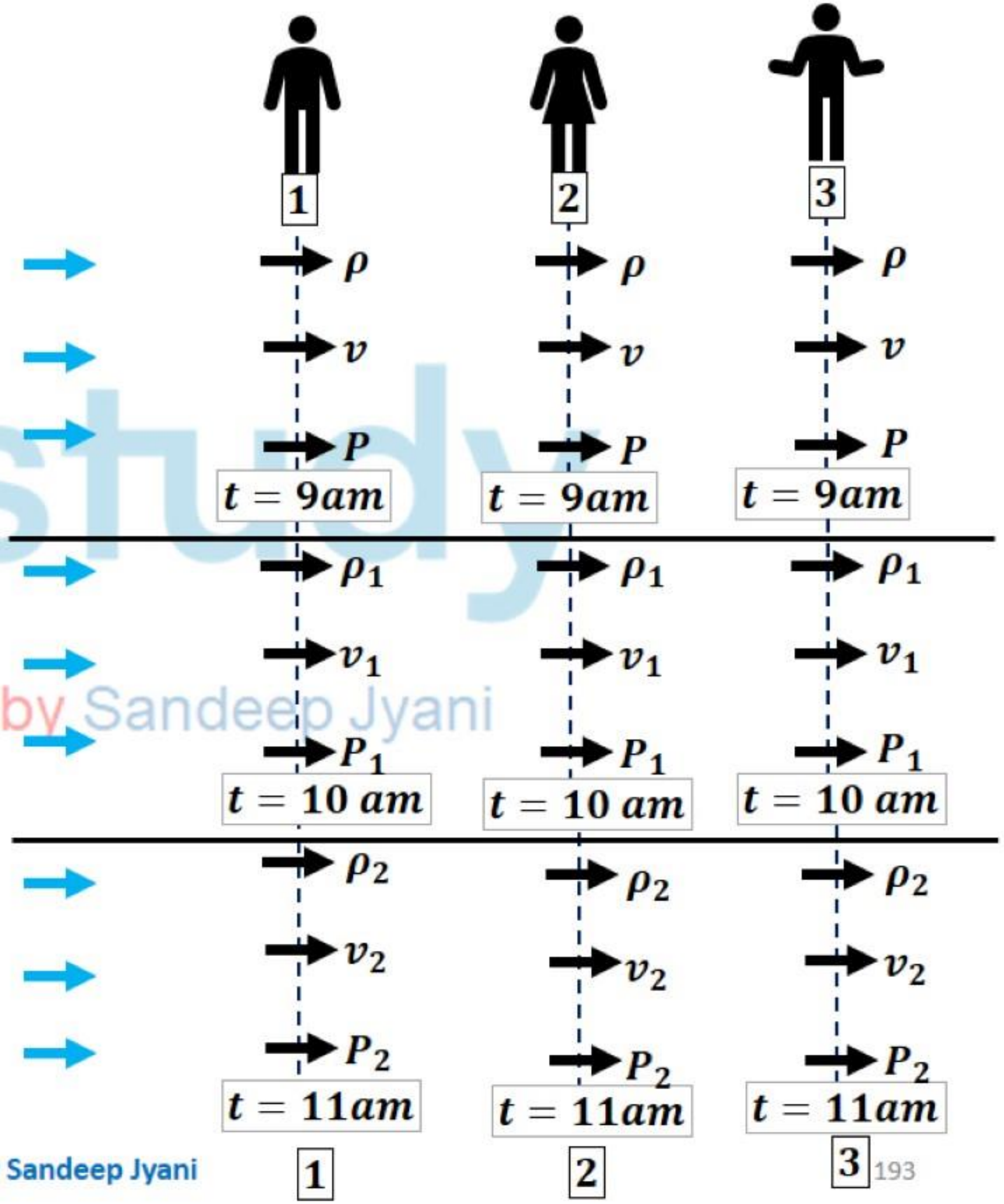
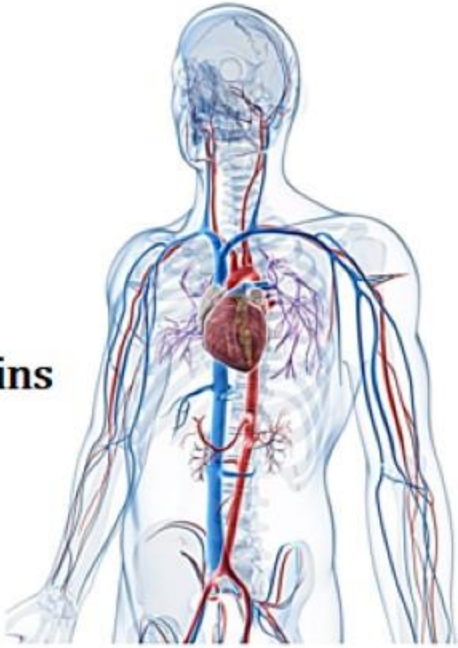


3. LAMINAR AND TURBULENT FLOW

- **When fluid particles move in the form of layer with one layer sliding over the other layer, then that flow is known as Laminar Flow.**
- **In laminar flow, fluid particles move along well-defined paths or stream line and all stream lines are straight and parallel.**
- **This kind of flow is also known as Stream Line flow or Viscous flow.**
- **Laminar Flow generally occurs at low velocity.**
- **When fluid flows in highly disorganized manner leading to rapid mixing of fluid particles, then that flow is known as Turbulent flow.**
- **Turbulent Flow generally occurs at high velocity.**



Laminar flow – flow of blood in veins

Turbulent flow

flow of gases coming out of chimney

flow of water in rivers

3. LAMINAR AND TURBULENT FLOW

- **For a pipe flow, type of flow is determined by a non dimensional number called Reynold's number:**

$$R_e = \frac{VD}{\upsilon}$$

- $V = mean\ velocity\ of\ flow\ in\ pipe$
- $D = dia\ of\ pipe$
- $\upsilon = kinematic\ viscosity\ of\ fluid$

- $R_e < 2000$ ***Laminar Flow***
- $R_e > 4000$ ***Turbulent Flow***
- $2000 < R_e < 4000$ ***May be laminar or Turbulent***

4. COMPRESSIBLE AND INCOMPRESSIBLE FLOW

- **When the density of fluid changes from point to point, i.e., density is not constant, then it is called compressible flow.**
- **For example, gases.**

$$\rho \neq constant$$

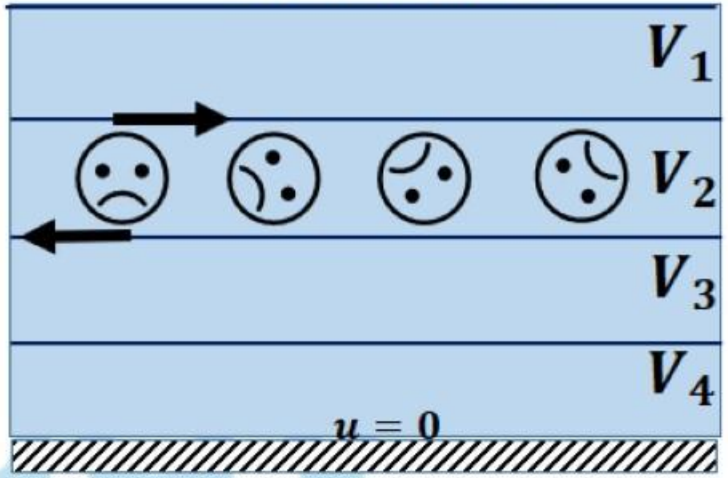
- **When the density of fluid does not change from point to point, i.e., density is constant, then it is called Incompressible flow.**
- **For example, liquids.**

$$\rho = constant$$

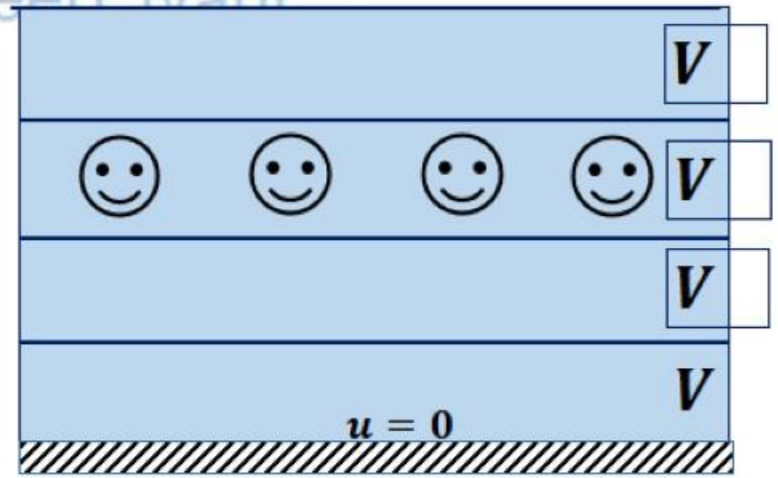
5. ROTATIONAL AND IRROTATIONAL FLOW

- When fluid particles while flowing along the stream lines, also rotate about their own axis, then it is termed as Rotational flow.
- When fluid particles while flowing along the stream lines, do not rotate about their own axis, then it is termed as Irrotational flow.
- In case of Irrotational flow, there is no rotation and hence there is no torque, i.e., there is no tangential force(shear force) and it usually occurs in Non-Newtonian Fluid.

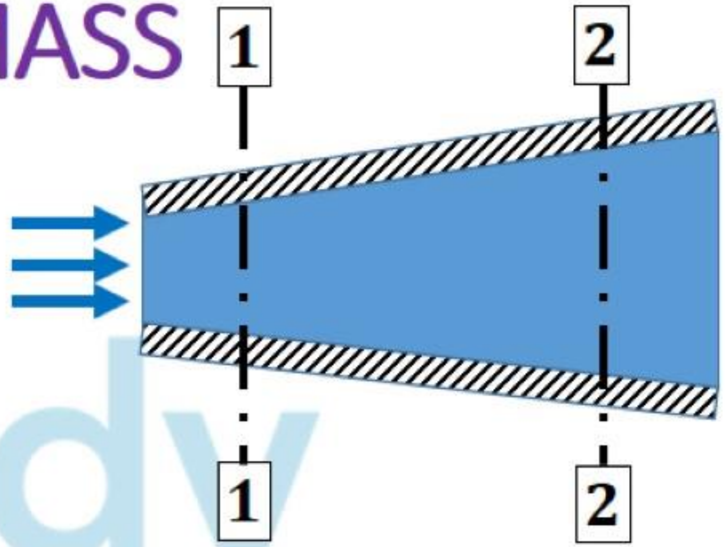
Rotational flow



Irrotational flow



CONSERVATION OF MASS



CASE – 1 : Steady and one dimensional flow

Mass entering = mass exit

$$(\dot{m}_1) = (\dot{m}_2)$$

$(\dot{m}_1) = \textit{mass passing per unit time}$

$$\dot{m} = \frac{\rho V}{t} = \frac{\rho AL}{t}$$

$$\dot{m} = \rho A v$$

$$\dot{m}_1 = \rho A_1 v_1$$

$$\dot{m}_2 = \rho A_2 v_2$$

density	ρ_1	ρ_2
average velocity	V_1	V_2
Area of Cross section	A_1	A_2

$$A_2 v_2 = A_1 v_1$$ *for incompressible flow* $\rho_1 = \rho_2$

$$Q = A_1 v_1$$ *discharge* m^3/sec

CONSERVATION OF MASS

CASE – 2: GENERALISED CONTINUITY EQUATION

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

- This equation is valid for all types of flow.
  - Steady as well as unsteady
  - Uniform as well as non uniform
  - Compressible as well as incompressible

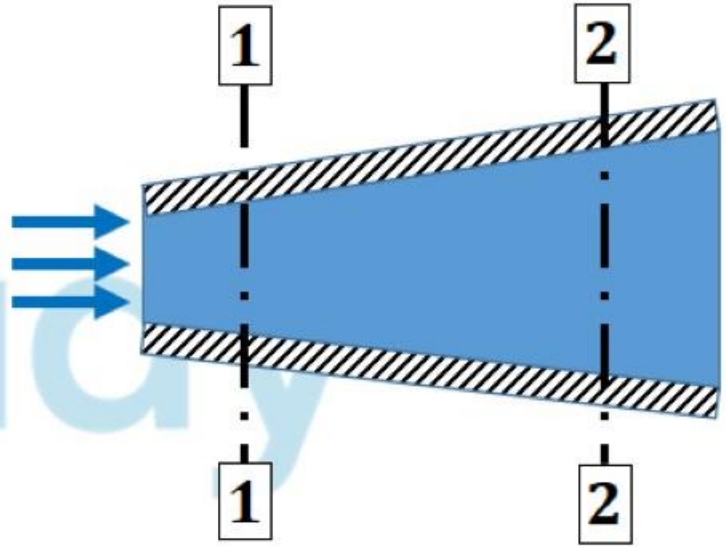
- If flow is incompressible (ρ=constant)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Continuity Equation

Case 1: Steady and 1-Dimensional Flow

- This equation is based on the principle of **conservation of mass**
- So For a fluid flowing in pipe at all cross sections, the quantity of fluid per second is constant.
- Let us take 2 cross sections



density	ρ_1	ρ_2
average velocity	V_1	V_2
Area of Cross section	A_1	A_2

$$rate\ of\ flow\ at\ section\ 1-1 = \rho_1 A_1 V_1$$

$$rate\ of\ flow\ at\ section\ 2-2 = \rho_2 A_2 V_2$$

According to law of cnoservation of mass,

$$rate\ of\ flow\ at\ section\ 1-1 = rate\ of\ flow\ at\ section\ 2-2$$

$$\boxed{\Rightarrow \rho_1 A_1 V_1 = \rho_1 A_1 V_1}$$

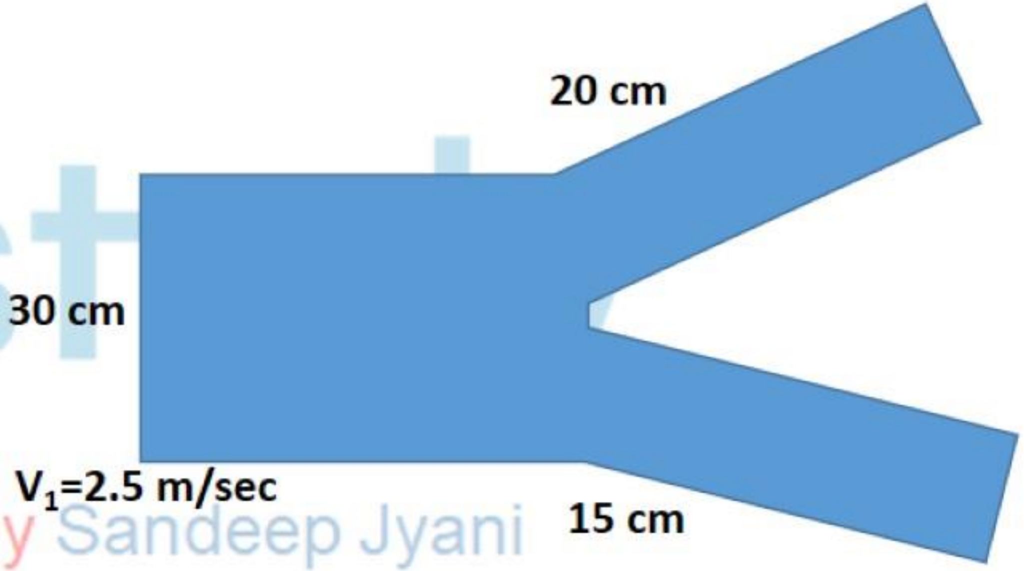
This equation is valid for Compressible as well as incompressible fluids

Que 57. The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5m/sec.

Also find velocity at section 2.

Que 58. A 30 cm dia pipe conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm dia pipe. Is 2.5 m/sec, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/sec.

Que 58. A 30 cm dia pipe conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm dia pipe. Is 2.5 m/sec, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/sec.



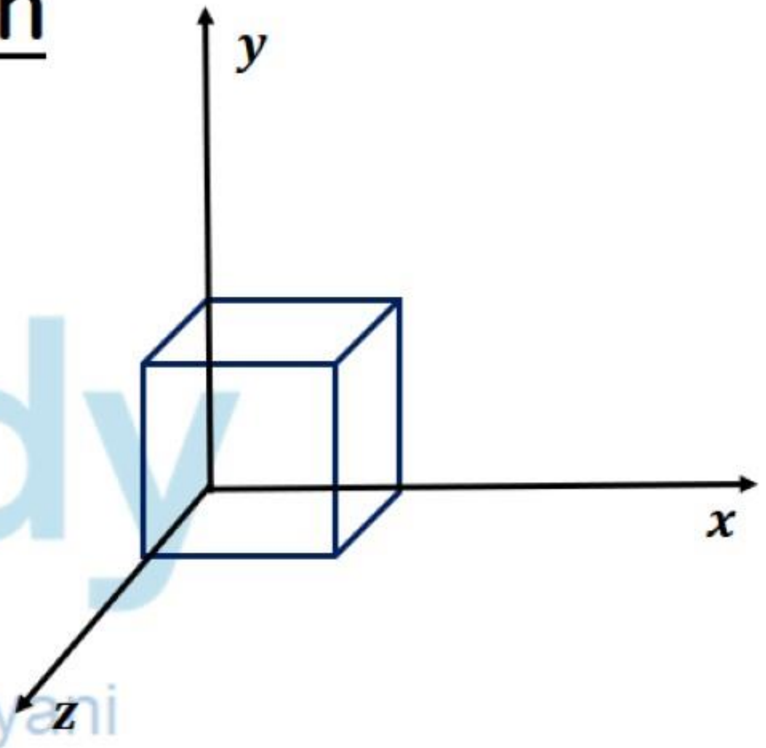
Velocity and Acceleration

Let V is the resultant velocity at any point in a fluid flow.

Let u, v *and* w are components in x, y *and* z *direction respectively*

Velocity components are functions of space coordinates and time

$$\boldsymbol{u = f_1(x, y, z, t)}$$
$$\boldsymbol{v = f_2(x, y, z, t)}$$
$$\boldsymbol{w = f_3(x, y, z, t)}$$



Velocity Vector $\boldsymbol{V = u\hat{i} + v\hat{j} + w\hat{k}}$

Resultant Velocity $|\boldsymbol{V}| = \sqrt{u^2 + v^2 + w^2}$

Let a_x, a_y *and* a_z are **total** acceleration in x, y *and* z *direction respectively*

$$a_x = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$
$$a_y = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$
$$a_z = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For steady flow, $\frac{\partial V}{\partial t} = 0$

Acceleration Vector $\boldsymbol{a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$

Resultant Acceleration $|\boldsymbol{a}| = \sqrt{\boldsymbol{a_x}^2 + \boldsymbol{a_y}^2 + \boldsymbol{a_z}^2}$

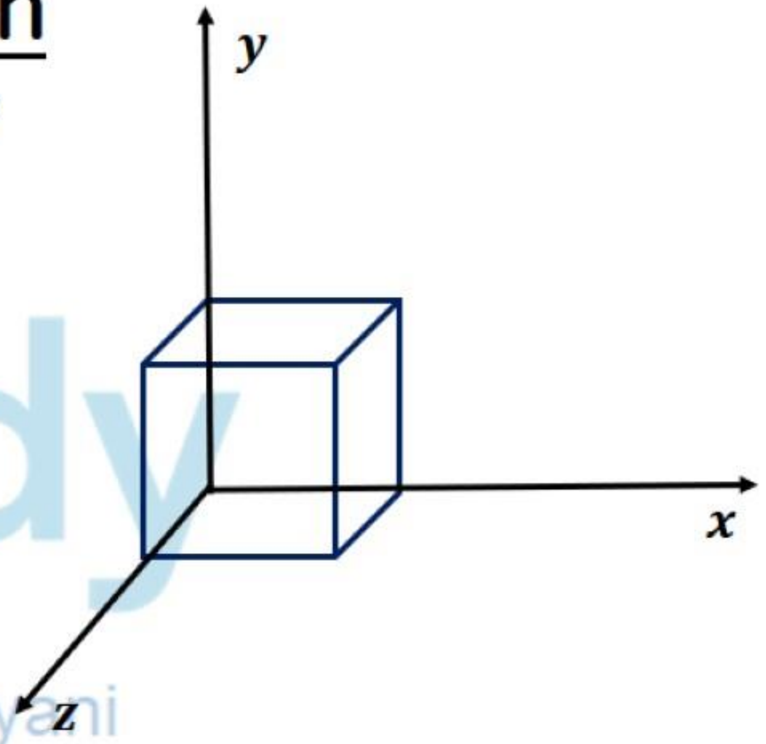
Velocity and Acceleration

Let a_x, a_y and a_z are acceleration in x, y and z $direction$ $respectively$

$$a_x = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$



Acceleration Vector $\boldsymbol{a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$

Resultant Acceleration $|\boldsymbol{a}| = \sqrt{\boldsymbol{a_x}^2 + \boldsymbol{a_y}^2 + \boldsymbol{a_z}^2}$

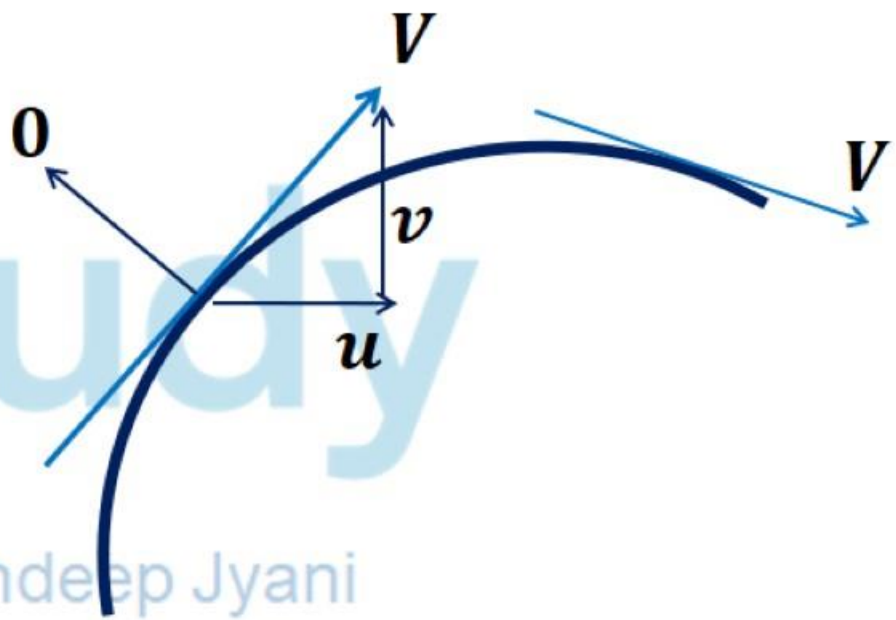
These all are known as CONVECTIVE ACCELERATION.

LOCAL ACCELERATION:

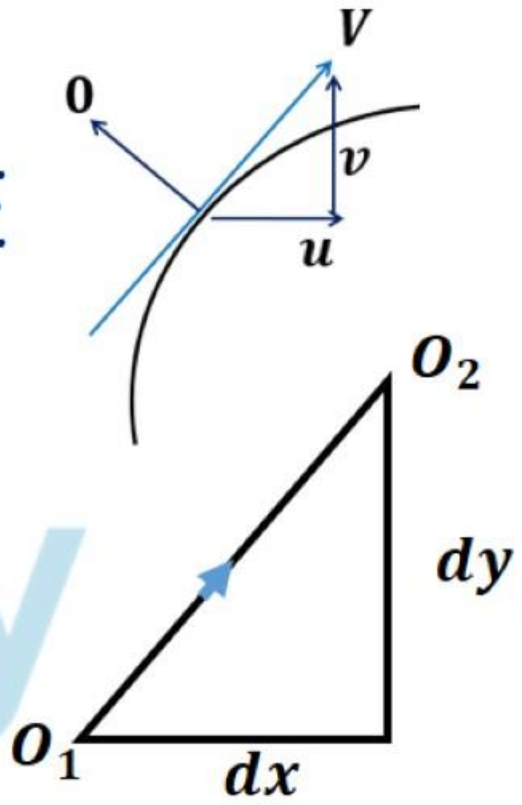
Rate of increase of velocity with respect to time at a given point in flow field is called as Local Acceleration $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$

STREAM LINE

- **It is an imaginary line or curve drawn in space such that a tangent drawn to it at any point gives the velocity vector at that instant.**
- **Stream lines can be observed experimentally as 'path line' and stream lines are same in a steady flow.**
- **Two stream lines can never intersect or a single stream line will never intersect itself because at any given instant at a point velocity must be unique.**



EQUATION OF STREAM LINE



$$v = \frac{dy}{dt} \qquad u = \frac{dx}{dt}$$

$$dt = \frac{dy}{v} \qquad dt = \frac{dx}{u}$$

$$\vec{v} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$$

$$\boxed{\frac{dx}{u} = \frac{dy}{v}}$$

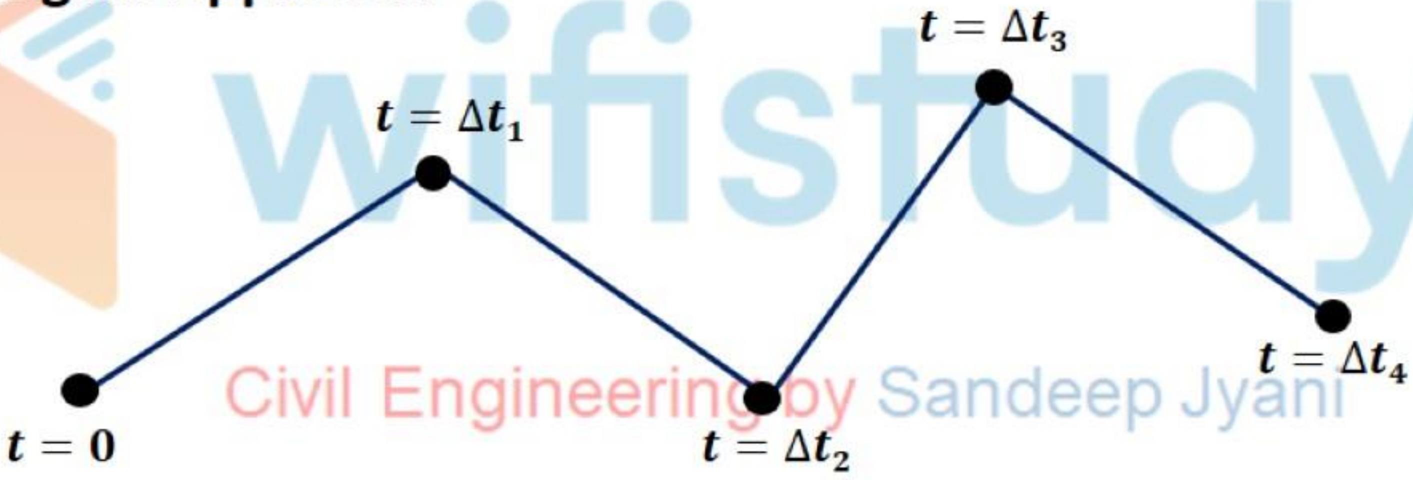
Equation of stream line for $2 - D$

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

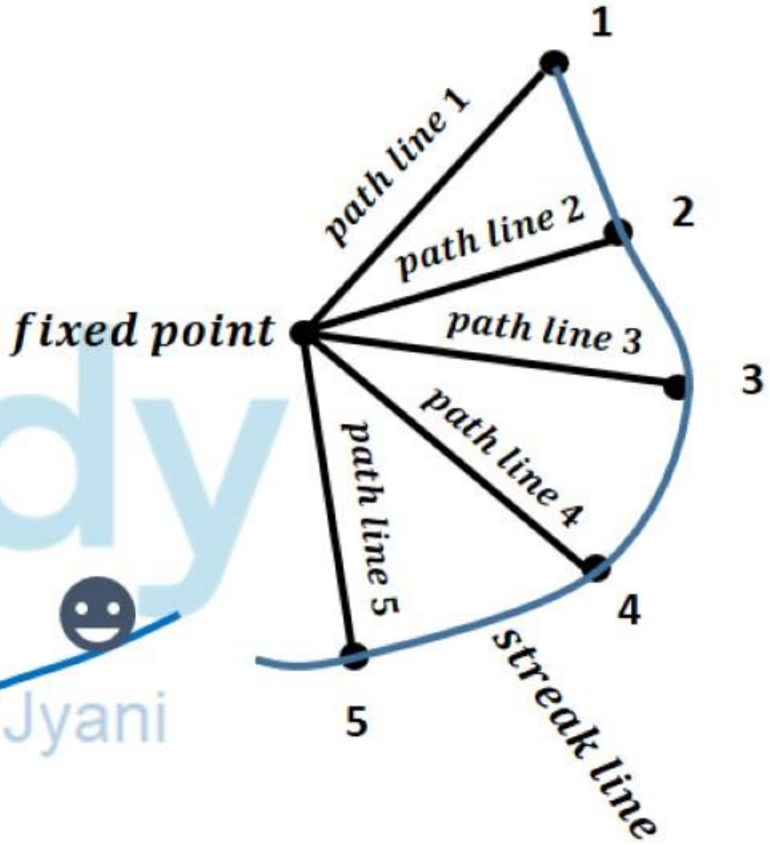
Equation of stream line for $3 - D$

Que 59. A flow is represented by $\boldsymbol{V} = \boldsymbol{ax\hat{\imath}} + \boldsymbol{ay\hat{\jmath}}$ where a=constant. Find the equation of stream line passing through (1,2).

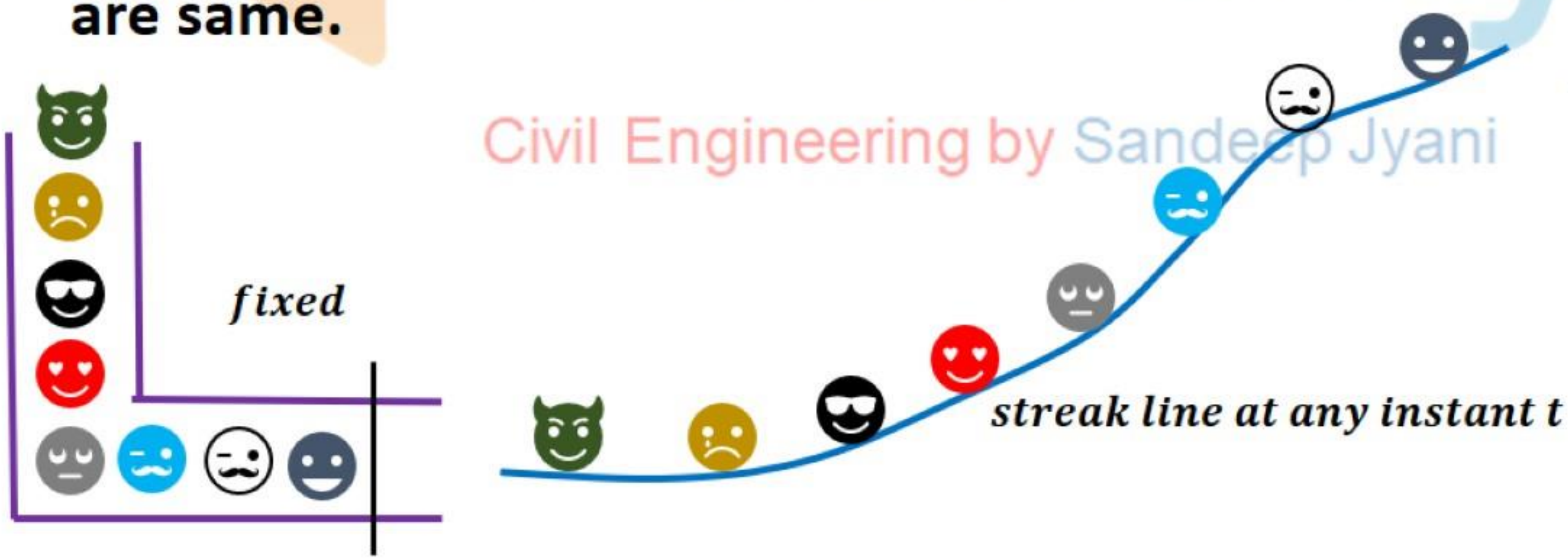
- **PATH LINE**
  - **It's a path traced by single fluid particle at different instant of time. It follows Lagrangian Approach.**



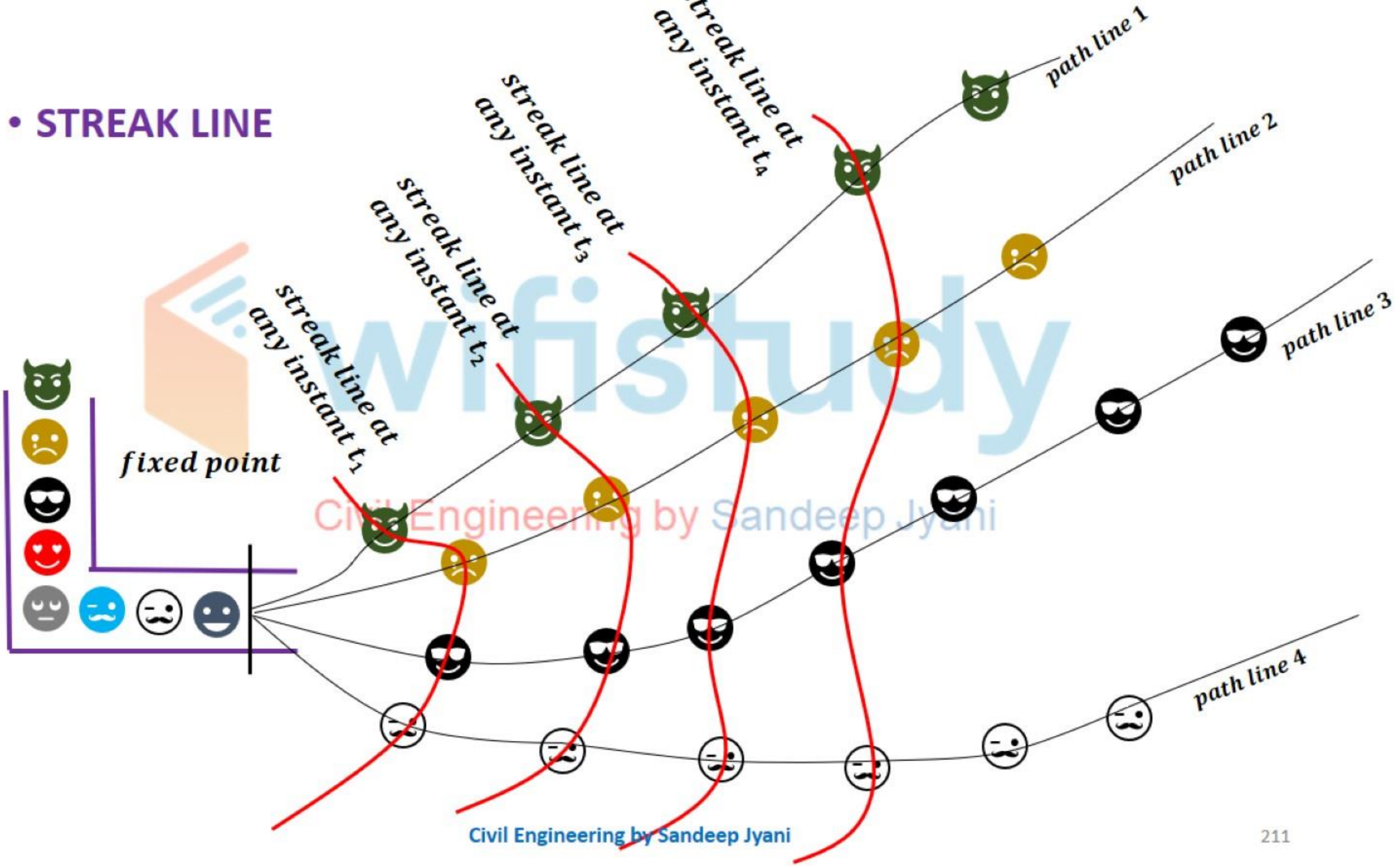
- **STREAK LINE**
  - It is locus of various fluid particles passing through a fixed point.



- In a steady flow, as stream line are fixed, path lines, stream lines and streak lines are same.



- **STREAK LINE**


fixed point
streak line at any instant t_1
streak line at any instant t_2
streak line at any instant t_3
streak line at any instant t_4
path line 1
path line 2
path line 3
path line 4

Velocity potential Function

- Velocity potential Function is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction.
- It is another way of representing velocity component
- -ve sign is taken because the flow is in the direction of decreasing potential.
- It is defined by Phi Ø
- Mathematically $\emptyset = f\ (x, y, z)$ for steady flow,

$$u = -\frac{\partial \emptyset}{\partial x}$$

$$v = -\frac{\partial \emptyset}{\partial y}$$

u, v and w are components of velocities in x, y and z directions

$$w = -\frac{\partial \emptyset}{\partial z}$$

Velocity potential Function

- From continuity equation,

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \frac{\partial(-\frac{\partial \emptyset}{\partial x})}{\partial x} + \frac{\partial(-\frac{\partial \emptyset}{\partial y})}{\partial y} + \frac{\partial(-\frac{\partial \emptyset}{\partial z})}{\partial z} = 0$$

$$\Rightarrow \frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} + \frac{\partial^2 \emptyset}{\partial z^2} = 0 \quad \text{which is A Laplace Equation}$$

Velocity potential Function

- Case 1: If For Certain value of $\emptyset$,

$$\Rightarrow \frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} + \frac{\partial^2 \emptyset}{\partial z^2} = 0 \quad \text{Flow is possible}$$

- Case 2: If For Certain value of $\emptyset$,

$$\Rightarrow \frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} + \frac{\partial^2 \emptyset}{\partial z^2} \neq 0 \quad \text{Flow is not possible}$$

Velocity potential Function

Properties of Velocity Potential function are:

1. If velocity potential exists $\emptyset$, the flow should be irrotational
2. If velocity potential $\emptyset$ satisfies the Laplace equation, it represents the possible steady incompressible irrotational flow

ROTATION

- It is defined as the movement of a fluid element in such a way that both of its axes rotate in the same direction.
- Note: In fluid mechanics, angular velocity ω is defined as the average of rate of change of angle of initially two perpendicular line segment
- It is equal to $\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ for two-dimensional element in $x - y$ plane.

$rotation\ in\ x - y$ plane $\quad \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$rotation\ in\ y - z$ plane $\quad \omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$

$rotation\ in\ z - x$ plane $\quad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$

$$\omega = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

Rotational components

VORTICITY

- It is defined as the value twice of the rotation and hence it is given as 2ω

$$Vorticity = 2\omega = \frac{2}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$Vorticity = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

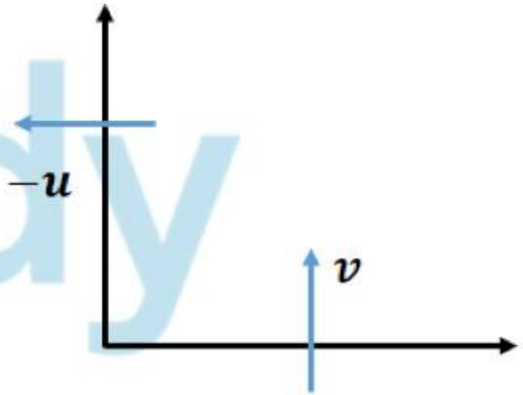
Que 60. A fluid flow is given by $V = 8x^3\, i - 10x^2y\, j$. Is the flow rotational?

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Stream Function

- It is a scalar function of space and time such that its partial derivative with respect to any direction gives velocity component at right angles to that direction.

$$\boxed{v = \frac{\partial \Psi}{\partial x}} \qquad \boxed{u = -\frac{\partial \Psi}{\partial y}}$$



- We know that continuity equation for 2-D flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- Put the values of u and v,

$$\Rightarrow \frac{\partial(-\frac{\partial \Psi}{\partial y})}{\partial x} + \frac{\partial(\frac{\partial \Psi}{\partial x})}{\partial y} = 0$$

$$\Rightarrow -\frac{\partial^2 \Psi}{\partial x \partial y} + \frac{\partial^2 \Psi}{\partial x \partial y} = 0 \quad \textit{i.e. it satisifies Laplace equation}$$

$$v = \frac{\partial \Psi}{\partial x}$$

$$u = -\frac{\partial \Psi}{\partial y}$$

- $\boldsymbol{Rotation\ in\ x - y}$ **plane** $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$
- Putting the values of u and v,

$$\omega_z = \frac{1}{2}\left(\frac{\partial(\frac{\partial \Psi}{\partial x})}{\partial x} - \frac{\partial(-\frac{\partial \Psi}{\partial y})}{\partial y}\right)$$

$$\Rightarrow \omega_z = \frac{1}{2}\left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2}\right)$$

We know that for irrotational flow, ω_z=0, hence above equation becomes

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0 \quad \text{which is a Laplace equation}$$

Properties of Stream Function

1. If stream function Ψ exits, flow is always possible (rotational as well as irrotational flow)
2. If stream function satisfies laplace equation, flow will be IRROTATIONAL.
3. Difference between any two stream functions gives discharge per unit width

$$Q = \boldsymbol{\Psi}_2 - \boldsymbol{\Psi}_1$$

$$\boxed{\boldsymbol{v} = \frac{\boldsymbol{\partial \Psi}}{\boldsymbol{\partial x}}} \quad \boxed{\boldsymbol{u} = -\frac{\boldsymbol{\partial \Psi}}{\boldsymbol{\partial y}}}$$

$$\boldsymbol{v} = -\frac{\boldsymbol{\partial \emptyset}}{\boldsymbol{\partial y}} \quad \boldsymbol{u} = -\frac{\boldsymbol{\partial \emptyset}}{\boldsymbol{\partial x}}$$

Equipotential Line

- A line along which the velocity potential ϕ is constant, is called Equipotential line

Flow Net

- A grid obtained by drawing a series of equipotential lines and stream lines is called as a Flow Net.

Relationship between Equipotential lines and Constant Stream Function Lines

- For Equipotential line ϕ is constant,

$$\phi = f(x, y)$$

$$\Rightarrow d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$$

$$\Rightarrow \frac{\partial \phi}{\partial x} dx = -\frac{\partial \phi}{\partial y} dy$$

$$\Rightarrow u dx = -v dy$$

$$\Rightarrow \frac{dy}{dx} = -\frac{u}{v} = \mathrm{m}_1$$

It is slope of velocity potential line

- For constant stream function, $\boldsymbol{\Psi}$ is constant,

$$\boldsymbol{\Psi} = f(x, y)$$

$$\Rightarrow d\boldsymbol{\Psi} = \frac{\partial \boldsymbol{\Psi}}{\partial x} dx + \frac{\partial \boldsymbol{\Psi}}{\partial y} dy = 0$$

$$\Rightarrow \frac{\partial \boldsymbol{\Psi}}{\partial x} dx = -\frac{\partial \boldsymbol{\Psi}}{\partial y} dy$$

$$\Rightarrow v dx = u dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{v}{u} = m_2$$

It is slope of stream function line

$m_1 \times m_2 = -1$ *i.e both lines are perpendicular to each other*

Cauchy Reiman Equation

$$u = -\frac{\partial \Psi}{\partial y}$$

$$u = -\frac{\partial \phi}{\partial x}$$

$$-\frac{\partial \Psi}{\partial y} = -\frac{\partial \phi}{\partial x}$$

$$\Rightarrow \frac{\partial \Psi}{\partial y} = \frac{\partial \phi}{\partial x}$$ Cauchy Reiman Equation

$$v = \frac{\partial \Psi}{\partial x}$$

$$v = -\frac{\partial \phi}{\partial y}$$

$$\frac{\partial \Psi}{\partial x} = -\frac{\partial \phi}{\partial y}$$

SUMMARY

$$\frac{dx}{u} = \frac{dy}{v}$$ *Equation of STREAM LINE*

$$rotation\ in\ x - y\ plane\ \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$u = -\frac{\partial \emptyset}{\partial x}$$

$$Vorticity = 2\omega$$

$$v = -\frac{\partial \emptyset}{\partial y}$$ Velocity potential Function

$$w = -\frac{\partial \emptyset}{\partial z}$$

Stream Function

$$v = \frac{\partial \Psi}{\partial x} \qquad u = -\frac{\partial \Psi}{\partial y}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$ CONTINUITY EQUATION

$$\Rightarrow \frac{\partial \Psi}{\partial y} = \frac{\partial \emptyset}{\partial x}$$

$$\frac{\partial \Psi}{\partial x} = -\frac{\partial \emptyset}{\partial y}$$

Cauchy Reiman Equation

Que 61. Velocity potential function is given by

$$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2$$

Find the velocity in x and y directions at (3,3).

Que 62. Two dimensional flow is given by $\boldsymbol{V} = (\boldsymbol{x} + \boldsymbol{2y} + \boldsymbol{2})\hat{\boldsymbol{\imath}} + (\boldsymbol{4} - \boldsymbol{y})\hat{\boldsymbol{\jmath}}$. The flow is

a) Incompressible and Irrotational
b) Incompressible and Rotational
c) Compressible and Irrotational
d) Compressible and Rotational

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \quad \textit{rotation}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \textit{continuity equation}$$

Que 62. Two dimensional flow is given by $\boldsymbol{V} = (\boldsymbol{x} + 2\boldsymbol{y} + 2)\hat{\boldsymbol{\imath}} + (4 - \boldsymbol{y})\hat{\boldsymbol{\jmath}}$. The flow is

a) Incompressible and Irrotational

b) <u>Incompressible and Rotational</u>

c) Compressible and Irrotational

d) Compressible and Rotational

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \quad \textit{rotation}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \textit{continuity equation}$$

Que 63. A stream function is given by $\boldsymbol{\Psi} = 3xy$, then find velocity at (2,3).

VORTEX FLOW

- **Vortex Flow is defined as the flow of fluid along a curved path or the flow of rotating mass of a fluid**
- **It is of two types:**
  1. **Forced Vortex**
  2. **Free Vortex**

VORTEX FLOW

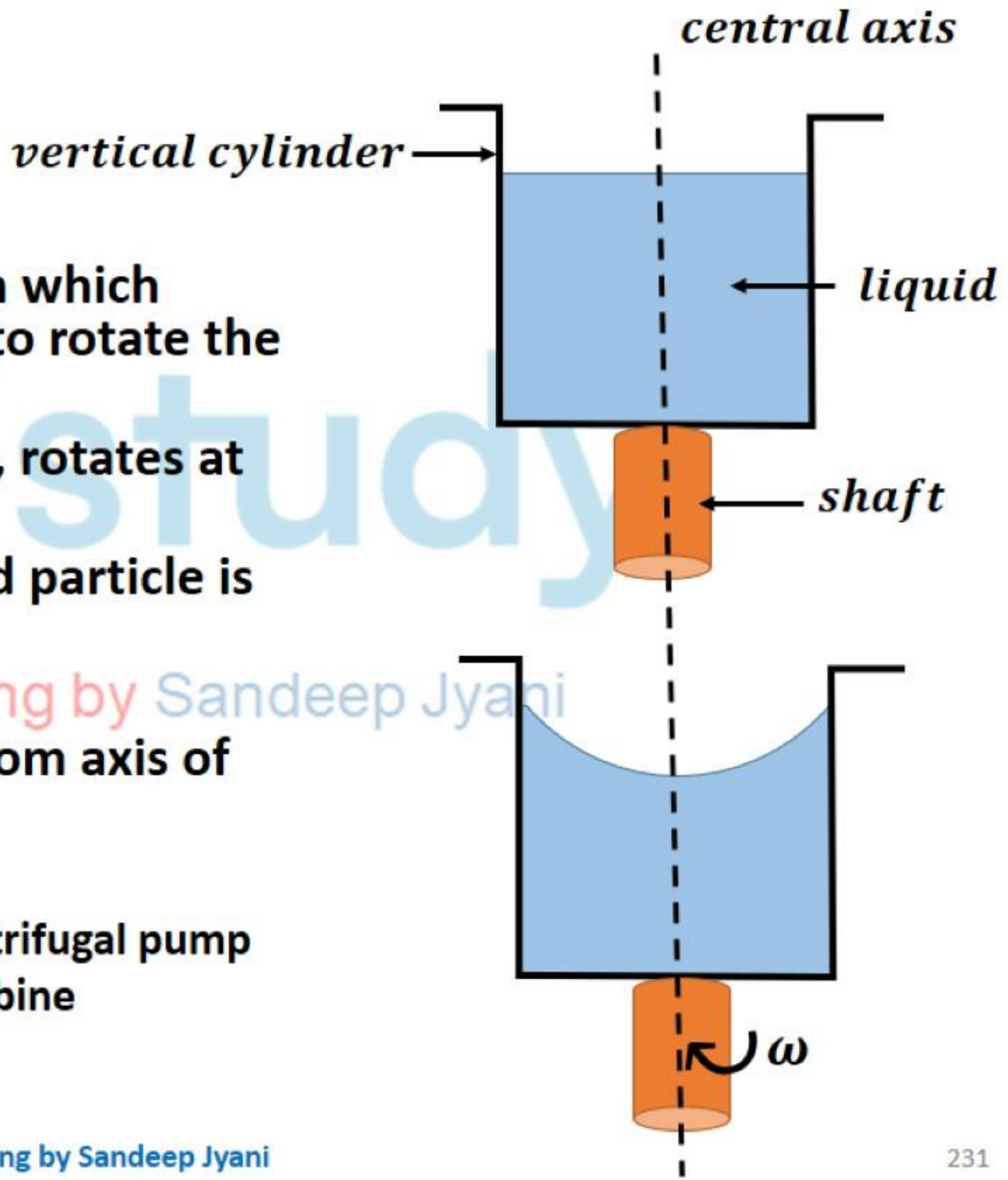
1. Forced Vortex Flow

- **It is defined as that type of flow in which some external torque is required to rotate the fluid mass.**
- **The fluid mass in this type of flow, rotates at constant angular velocity ω.**
- **The tangential velocity of any fluid particle is given by**

$$v = \omega r$$

Where r is radius of fluid particle from axis of rotation

- **Example:**
  - **Flow of liquid inside impeller of centrifugal pump**
  - **Flow of water through runner of turbine**

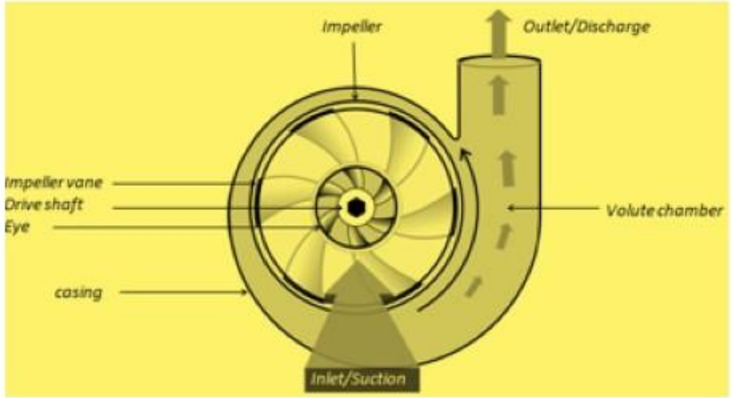


VORTEX FLOW

2. Free Vortex Flow

- **When no external torque is required to rotate the fluid mass, that type of flow is called as Free Vortex Flow.**
- **SO the liquid in case of free vortex is rotating due to rotation provided to it previously**

- **Examples:**
  - **Flow of fluid through a hole provided at the bottom of container**
  - **Flow of liquid around a bend in pipe**
  - **A whirlpool in river**
  - **Flow of fluid in centrifugal pump casing**



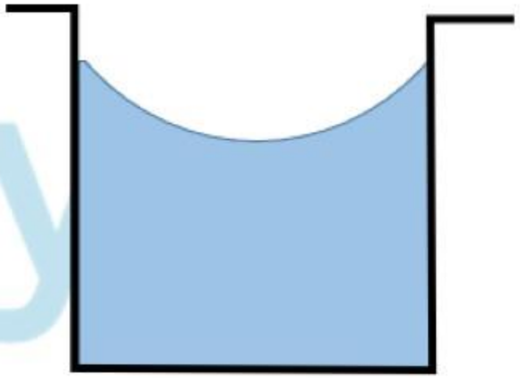
VORTEX FLOW

$$\boldsymbol{Height\ of\ paraboloid\ formed\ z = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}}$$

- **Free surface of liquid is paraboloid.**
- **Volume of paraboloid formed**$= \frac{\pi r^2 z}{2}$

VORTEX FLOW

Que 64. A tank of dia 20cm and height 100 cm contains water upto a height of 60cm. It is rotated about its vertical axis at 300 rpm. Find the depth of parabola formed at free surface of water



$$z = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$

$$\omega = \frac{2\pi N}{60}$$

$$\omega = \frac{2\pi \times 300}{60}$$

$$\omega = 31.41\ rad/sec$$

$$r = \frac{20}{2} = 10cm$$

$$z = \frac{\omega^2 r^2}{2g}$$

$$\Rightarrow z = \frac{(31.41)^2(10)^2}{2(981)}$$

$$\Rightarrow z = 50.68cm$$

Dynamics of Fluid Flow

- Dynamics include study of forces causing fluid flow
- Dynamic behavior of the fluid flow is analyzed by Newton's Second law of motion
- Fluid is assumed to be incompressible and viscous

Dynamics of Fluid Flow

- On any fluid element, net force F_x acting on it in direction of x is equal to

$$F_x = ma_x$$

- In a fluid flow, following forces are present,
  1. $F_g = gravity\ force$
  2. $F_p = pressure\ force$
  3. $F_v = viscous\ force$
  4. $F_t = due\ to\ turbulence$
  5. $F_c = due\ to\ compressibility$ *(negligible)*

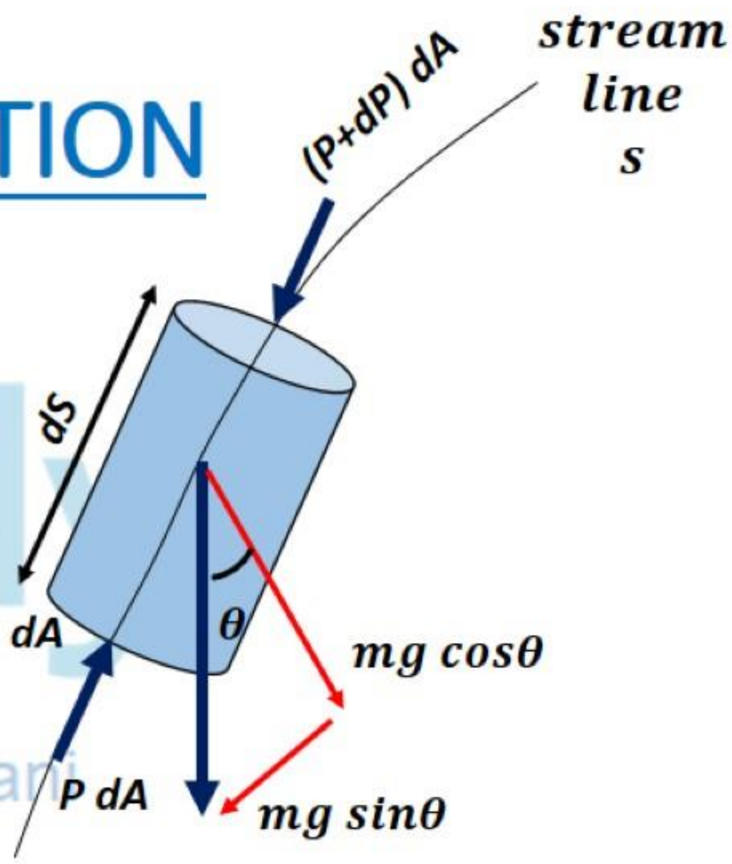
Dynamics of Fluid Flow

- **In a fluid flow, following forces are present,**
  1. $F_g = gravity\ force$
  2. $F_p = pressure\ force$
  3. $F_v = viscous\ force$
  4. $F_t = due\ to\ turbulence$
  5. $F_c = due\ to\ compressibility$ ***(negligible)***

  - ***If force due to compressibility is neglected , then***
    - ***Net force*** $F = F_g + F_p + F_v + F_t$.
    - ***Such equations of motion are called as Reynold's Equation of motion***
  - ***For flow where force due to turbulence is also neglected***
    - ***Net force*** $F = F_g + F_p + F_v\ (gravity, pressure\ and\ viscous)$
    - ***Such equations of motion are called as Navier Stokes Equation***
  - ***If the flow is assumed to be ideal, viscous force is also zero,***
    - ***Net force*** $F = F_g + F_p\ (gravity, and\ pressure\ )$
    - ***Equation of motions are known as Euler's Equation of motion***

EULER's EQUATION OF MOTION

- *Net force* $F = F_g + F_p$ $(gravity, and\ pressure)$
- *Consider a stream line "s" in which flow is taking place and a cylindrical element of cross section "dA" and length "dS" is taken*
- *Forces acting on Fluid element are*
  a) Pressure force $\boldsymbol{F_{p1} = PdA}$ in direction of flow
  b) Pressure force force $\boldsymbol{F_{p2} = (P + dP)dA}$ in direction opposite to flow
  c) Weight of the element $\boldsymbol{F_g = mg = \rho dAdsg}$



$\boldsymbol{m = \rho dAds}$

Resultant force $\boldsymbol{F = ma}$

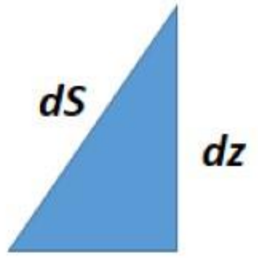
$\Rightarrow \boldsymbol{PdA - (P + dP)dA - \rho dAdsgsin\theta = ma}$

EULER's EQUATION OF MOTION

- **Resultant force** $F = ma$
- $a_s = \frac{dv}{dt} = v\frac{\partial v}{\partial s}$ *concective* $+ \frac{\partial v}{\partial t}$ *local*

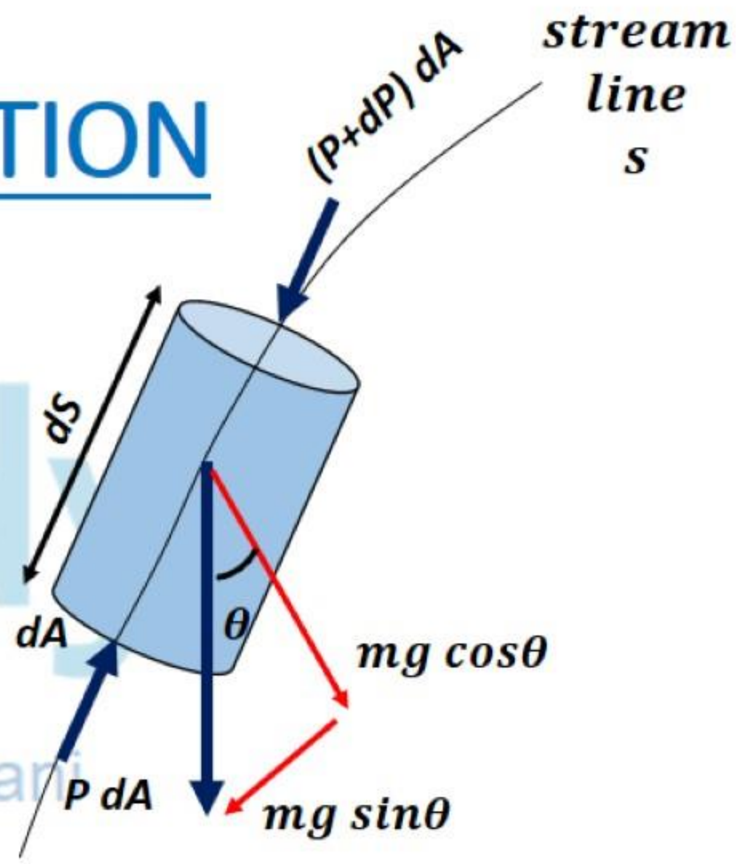
$$\Rightarrow PdA - (P + dP)dA - \rho dAds g sin\theta = \rho dAds(v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t})$$

$$\Rightarrow PdA - PdA - dPdA - \rho dAds g sin\theta = \rho dAds\ (v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t})$$



$$sin\theta = \frac{dz}{ds}$$

$$dz = ds\ sin\theta$$



$m = \rho dAds$

$$\Rightarrow -dP - \rho g\ dz = \rho ds\left(v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right)$$ *"Euler's Equation"*

- $\Rightarrow -dP - \rho g\, dz = \rho ds \left(v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \right)$ "*Euler's Equation*"
- In a steady flow, $\frac{\partial v}{\partial t} = 0$,

$$\Rightarrow \frac{dP}{\rho} + g\, dz + vdv = 0$$

Integrating whole equation,

$$\Rightarrow \int_0^P \frac{dP}{\rho} + g\int_0^z dz + \int_0^v vdv = 0$$

$$\Rightarrow \frac{P}{\rho} + gz + \frac{v^2}{2} = constant$$

$$\Rightarrow \frac{P}{\rho g} + z + \frac{v^2}{2g} = constant$$

Bernoulli's equation

Bernoulli′s equation

Pressure head → $\Rightarrow \frac{P}{\rho g} + z + \frac{v^2}{2g} = constant$ ← Kinetic Head

↑ Potential Head

- $\frac{P}{\rho g}$ = Pressure energy per unit weight of fluid or ***Pressure head***
- z = potential energy per unit weight or ***Potential Head***
- $\frac{v^2}{2g}$ = ***kinetic energy per unit weight or Kinetic Head***
- **Bernoulli′s Theorem:**
  - It states that in a STEADY, IDEAL flow of an incompressible fluid, the total energy at any point of the fluid is constant (pressure, kinetic, and potential energy)

Bernoulli's equation

Pressure head → $\frac{P}{\rho g}$; *Kinetic Head* → $\frac{v^2}{2g}$; *Potential Head* → z

$$\Rightarrow \frac{P}{\rho g} + z + \frac{v^2}{2g} = constant$$

- **Bernoulli's Theorem**:
  - It states that in a STEADY, IDEAL flow of an incompressible fluid, the total energy at any point of the fluid is constant (pressure, kinetic, and potential energy)
- Assumptions of Bernoulli's Equation
  1. Fluid is ideal (viscosity is zero)
  2. The flow is incompressible
  3. Flow is steady
  4. The flow is irrotational

Bernoulli's equation for Real Fluid

$$\Rightarrow \frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2 + h_L$$

- All real fluids are viscous and hence offer resistance to flow
- There are always some losses in fluid flows and hence these losses are taken into consideration
- h_L is loss of energy or head from point 1 to 2 in a flow.

-

Que 65. Water flows through a pipe of 5cm dia under pressure of 50 kN/m^2 and velocity 2 m/sec. Find total head per unit weight of water at cross section which is 10 m above datum line.

PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

1. Venturimeter
2. Orifice Meter
3. Pitot Tube

PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

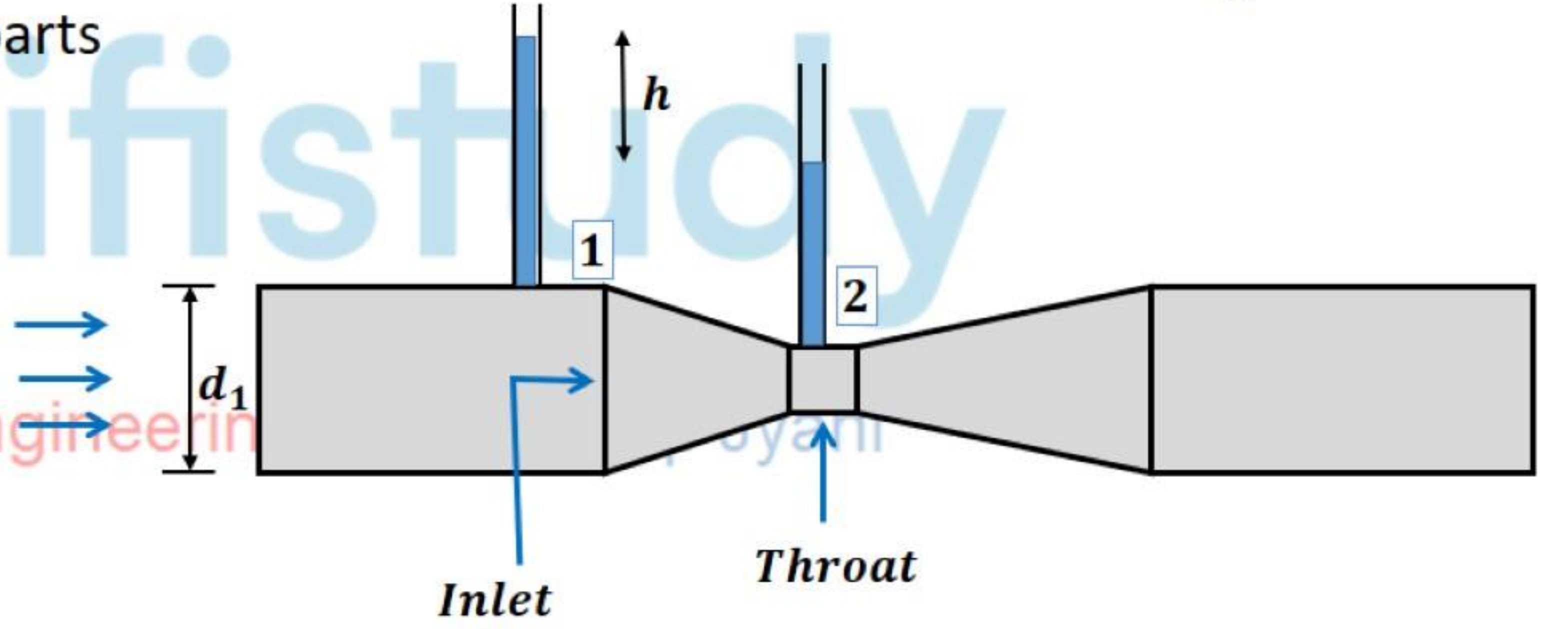
1. Venturimeter

- Venturimeter is a device used to measure the rate of flow of a fluid through a pipe. It consists of three parts

a) Short Converging part

b) Throat

c) Diverging part



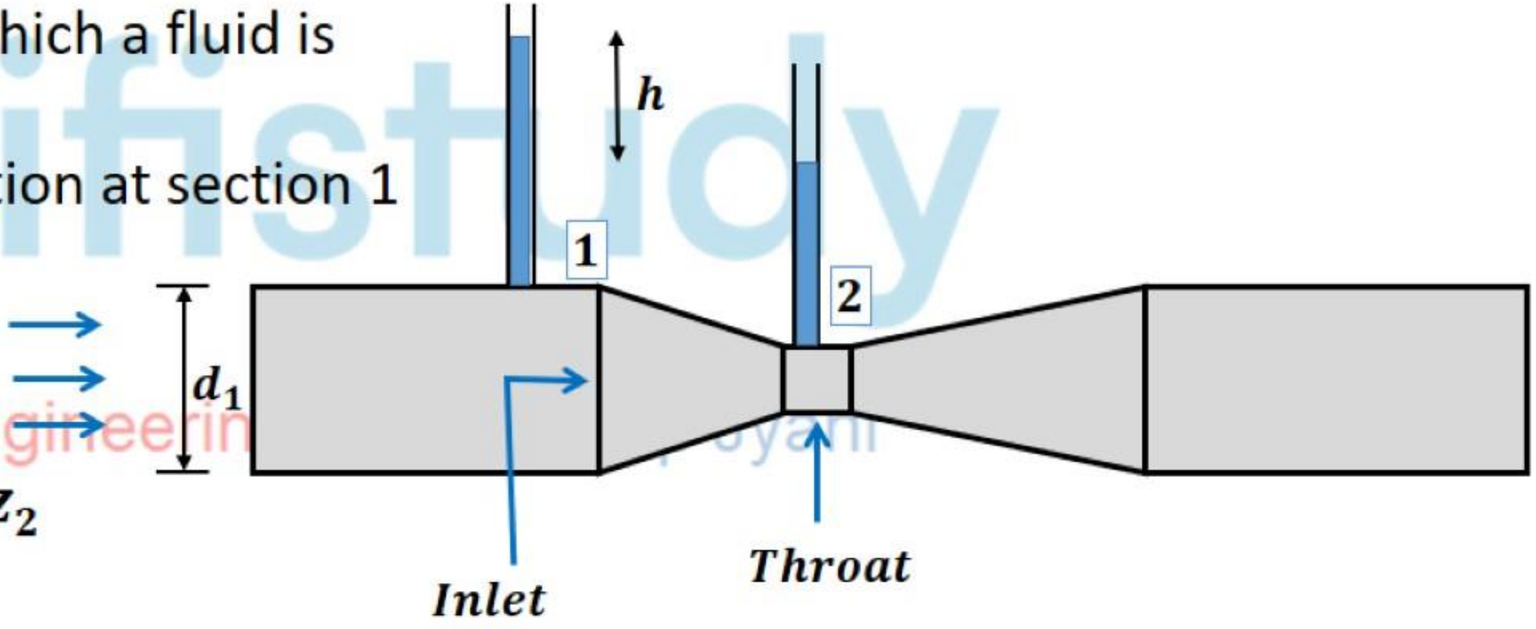
$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

1. Venturimeter

- Consider a Venturimeter fitted in horizontal pipe through which a fluid is flowing
- Applying Bernoulli's equation at section 1 and 2

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$



- As pipe is horizontal, $z_1 = z_2$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

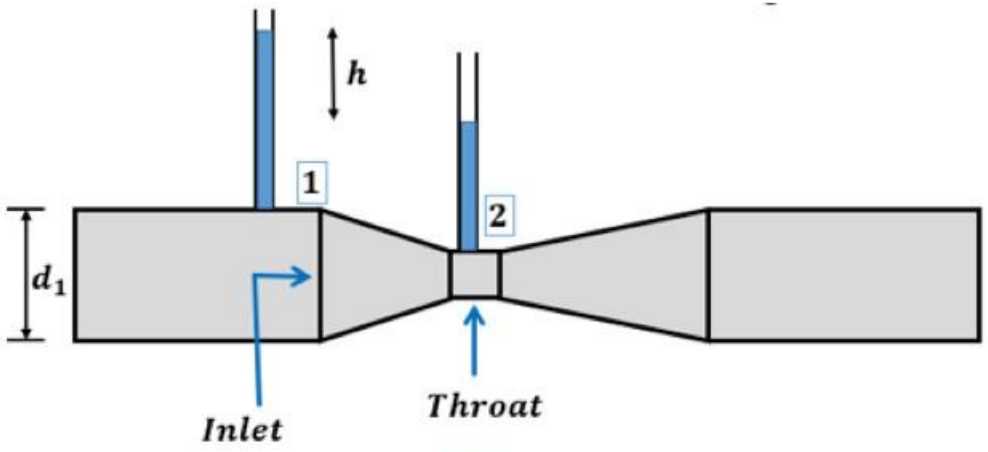
$\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads at section 1 and 2 and it is equal to $h = \frac{P_1 - P_2}{\rho g}$

PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

1. Venturimeter

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$

$$\Rightarrow h = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$



Applying continuity equation at section 1 and 2

$$a_1 v_1 = a_2 v_2 \qquad \Rightarrow v_1 = \frac{a_2 v_2}{a_1}$$

$$\Rightarrow h = \frac{{v_2}^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g}$$

$$\Rightarrow h = \frac{{v_2}^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

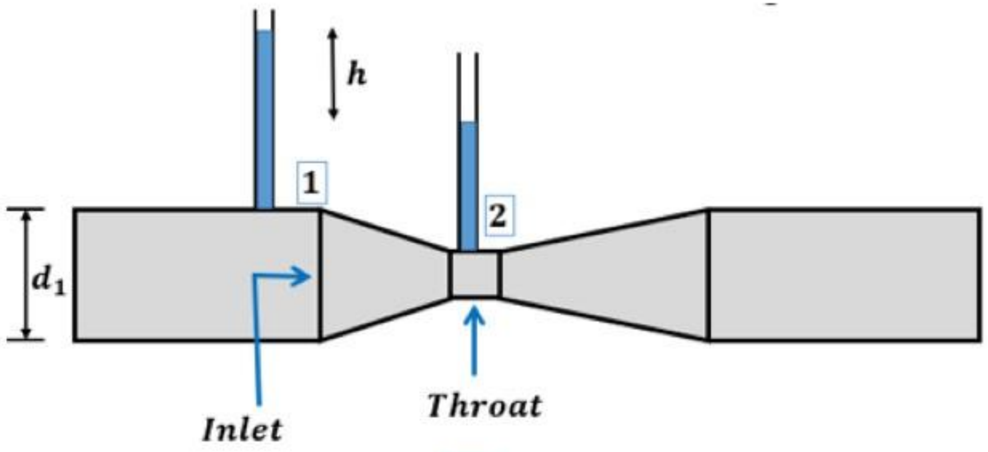
$$\Rightarrow v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}}$$

PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

1. Venturimeter

$$\Rightarrow Q = a_2 v_2 = \sqrt{2gh\frac{a_1^2}{a_1^2 - a_2^2}}$$

$$\Rightarrow Q = a_2 v_2 = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$



$$\Rightarrow Q_{th} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

Theoretical discharge

This equation gives value of discharge under ideal conditions, so it is called as Theoretical Discharge. Actual discharge will be less than Theoretical discharge

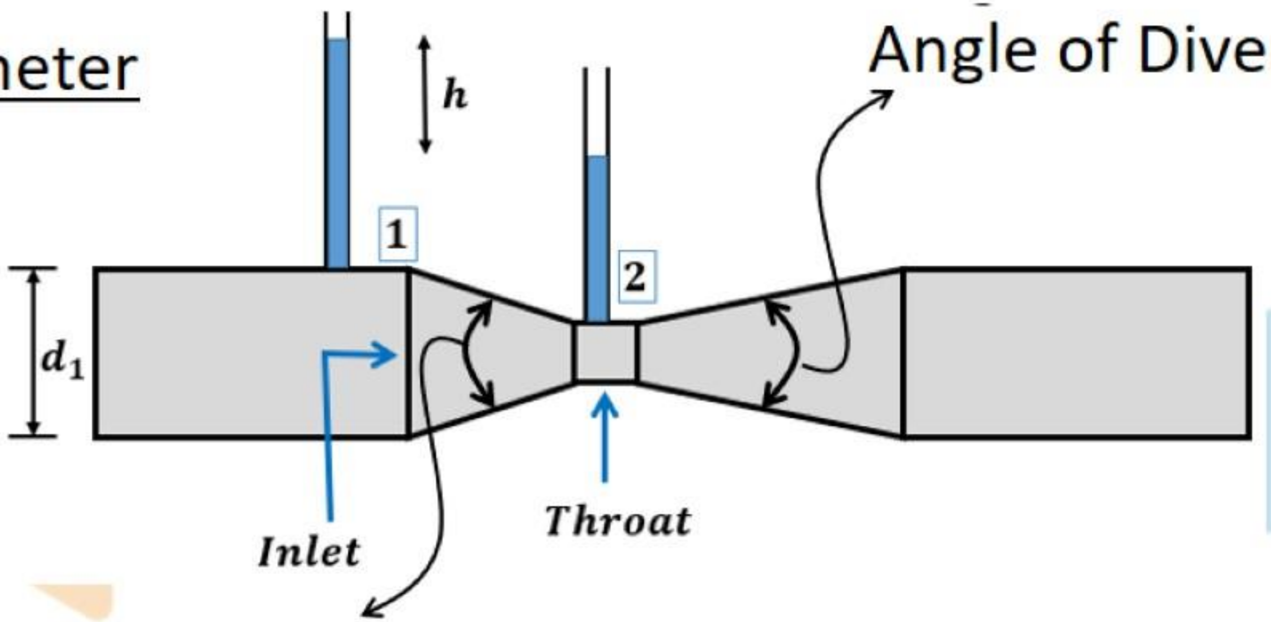
$$\Rightarrow Q_{actual} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

Actual discharge

C_d = coefficient of discharge and its value is less than 1 $(0.94 - 0.98)$

$$C_d = \frac{Actual\ Discharge}{Theoretical\ Discharge}$$

1. <u>Venturimeter</u>

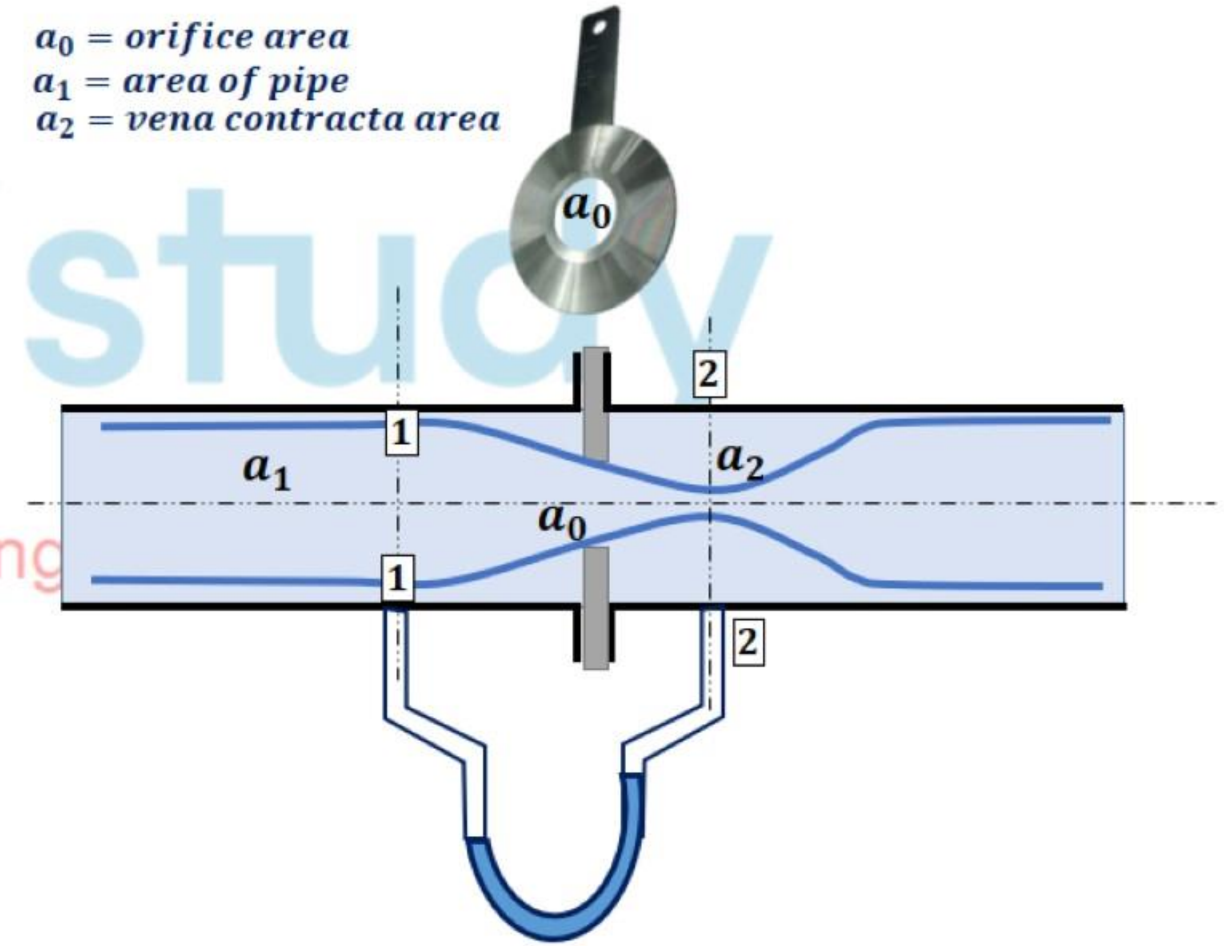


- Angle of Convergence= 21°- 22°

PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

2. Orificemeter

- It is used for finding out discharge or rate of flow of fluid through pipe and is based on same principle as that of Venturimeter.
- It is the cheapest instrument for finding out discharge
- It consists of a flat circular plate which has a circular sharp edge hole known as Orifice
- Orifice is generally kept as 0.5 times or (0.4 to 0.8) times pipe diameter
- A differential manometer is connected at section **1** which is at a distance of 1.5 to 2 times pipe dia upstream and at section **2** which is at a distance of half the diameter of orifice downstream side from orifice plate



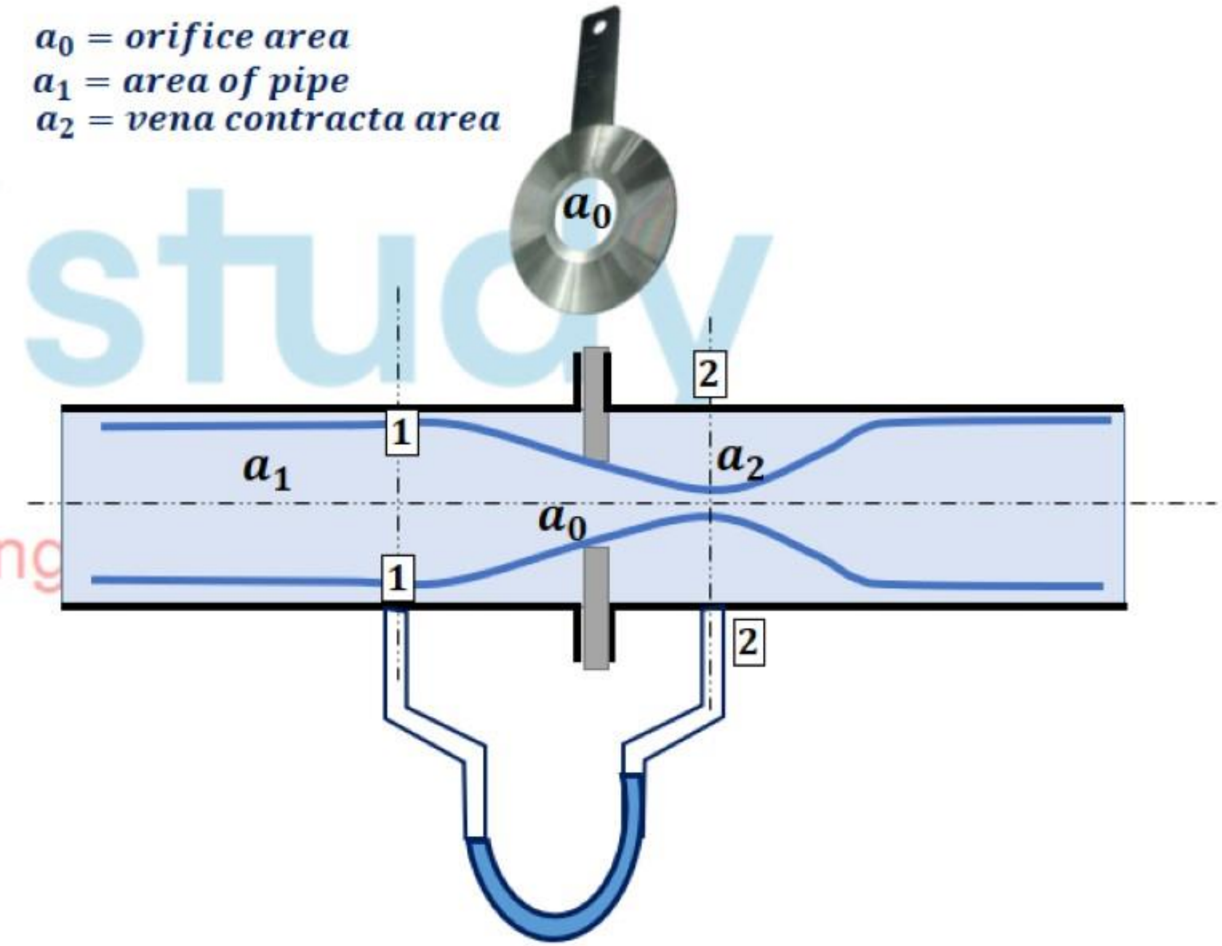
PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

2. Orificemeter

- Coefficient of Contraction

$$C_c = \frac{a_2}{a_0}$$

- Coefficient of discharge for Orificemeter ***is much smaller*** than that for a Venturimeter as the area reduction is sudden in Orificemeter, the losses are more, hence the C_d of Orificemeter is less than C_d of Venturimeter
- $C_d = 0.68 - 0.74$
- $\boldsymbol{Q_{actual} = \frac{C_d a_1 a_0}{\sqrt{a_1^2 - a_0^2}} \sqrt{2gh}}$

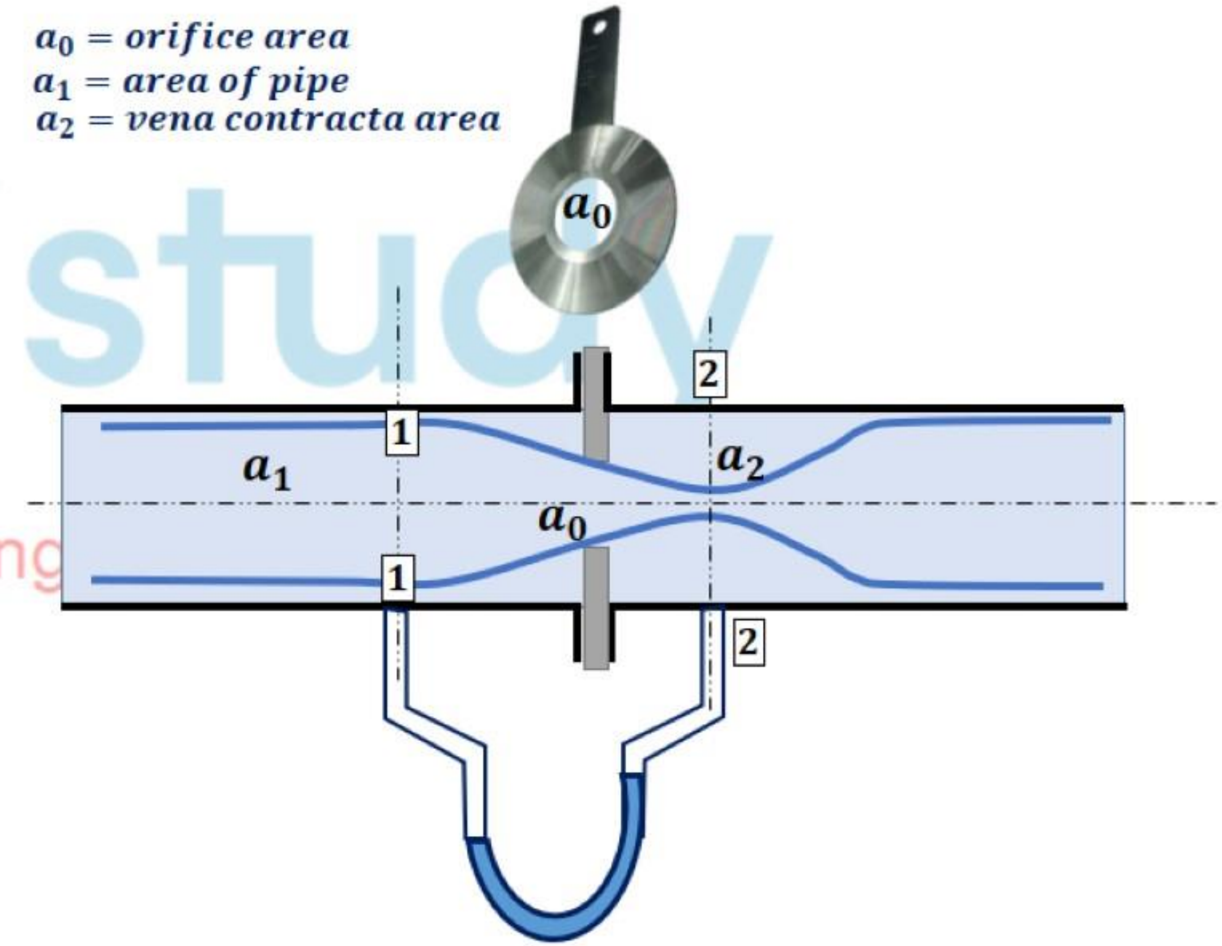


PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

2. Orificemeter

- $C_d = 0.68 - 0.74$
- $Q_{th} = \frac{C_d a_1 a_0}{\sqrt{a_1^2 - a_0^2}}\sqrt{2gh}$
- $\frac{Q_{actual}}{Q_{theoretical}} = \frac{A_{actual} \times V_{actual}}{A_{theoretical} \times V_{theoretical}}$
- $C_d = C_c \times C_v$
- $C_v = coefficient\ of\ velocity$

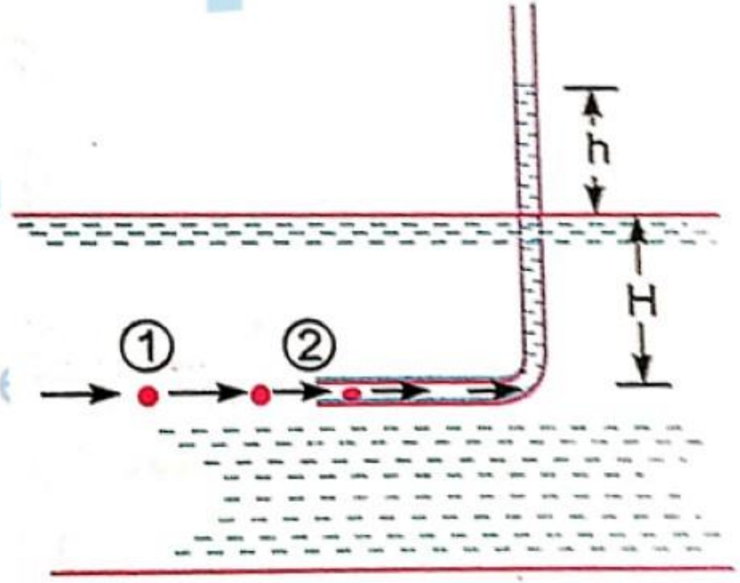
$a_0 = orifice\ area$
$a_1 = area\ of\ pipe$
$a_2 = vena\ contracta\ area$



PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

3. Pitot Tube ***It is device used for measuring the velocity of flow at any point in a pipe or channel***

- It is based on principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the kinetic energy into pressure energy
- Pitot tube consists of a glass tube, bent at right angles
- Lower end is bent at 90° is directed in the upstream direction
- Liquid rises up in the tube due to the conversion of Kinetic Energy into pressure energy
- Velocity is measured by rise of liquid in the tube

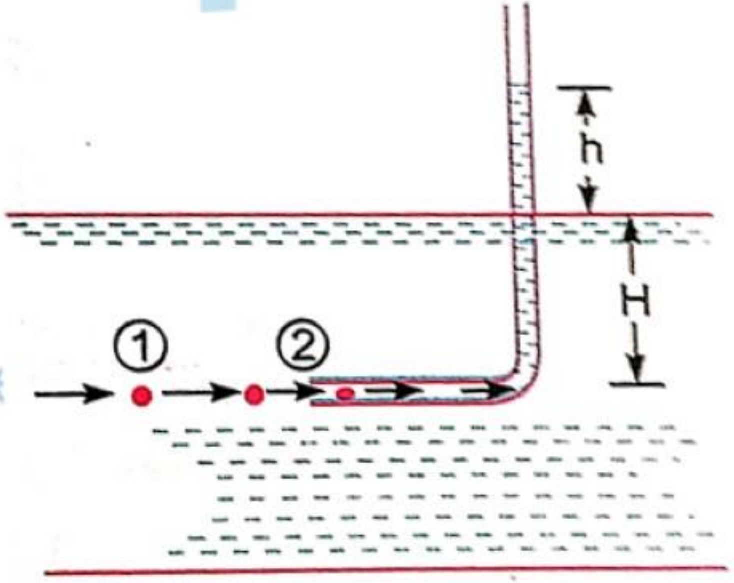


PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

3. Pitot Tube

It is device used for measuring the velocity of flow at any point in a pipe or channel

$H = \text{due to pressure}$
$h = \text{due to velocity}$



Using Bernoulli's equation

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As points 1 and 2 are on same line, $z_1 = z_2$ and $v_2 = 0$

$$\Rightarrow \frac{P_1}{\rho g} = \text{pressure head at } 1 = H$$

$$\Rightarrow \frac{P_2}{\rho g} = \text{pressure head at } 2 = h + H$$

Putting these values

$$\Rightarrow H + \frac{v_1^2}{2g} = \text{h} + \text{H} \qquad \Rightarrow h = \frac{v_1^2}{2g}$$

$$\Rightarrow v_1 = \sqrt{2gh}$$

This is theoretical velocity

PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

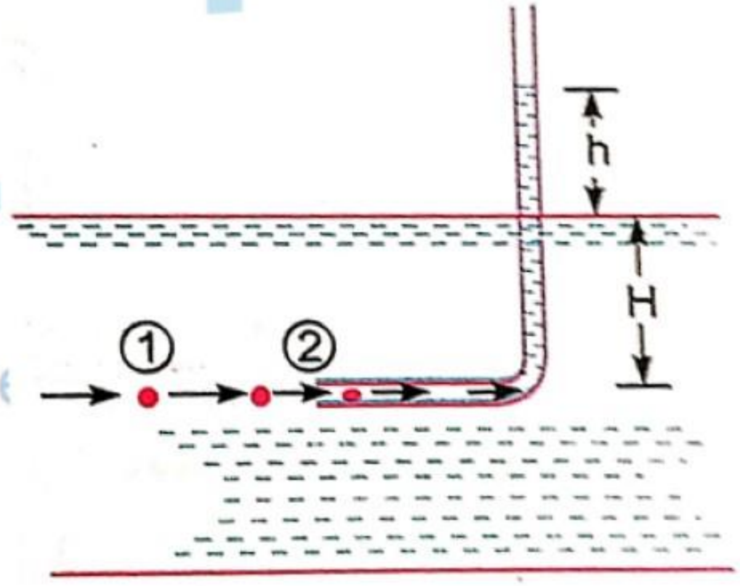
3. Pitot Tube *It is device used for measuring the velocity of flow at any point in a pipe or channel*

$$\Rightarrow v_1 = \sqrt{2gh}$$

This is theoretical velocity

$$\Rightarrow v_{actual} = C_v\sqrt{2gh}$$

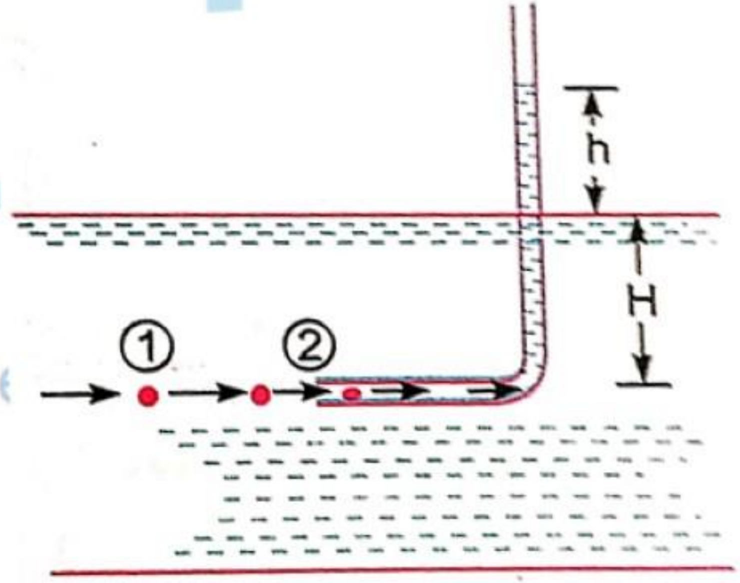
C_v = coefficient of pitot tube



$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g}$$ ***Stagnation pressure head***

Static pressure head
H

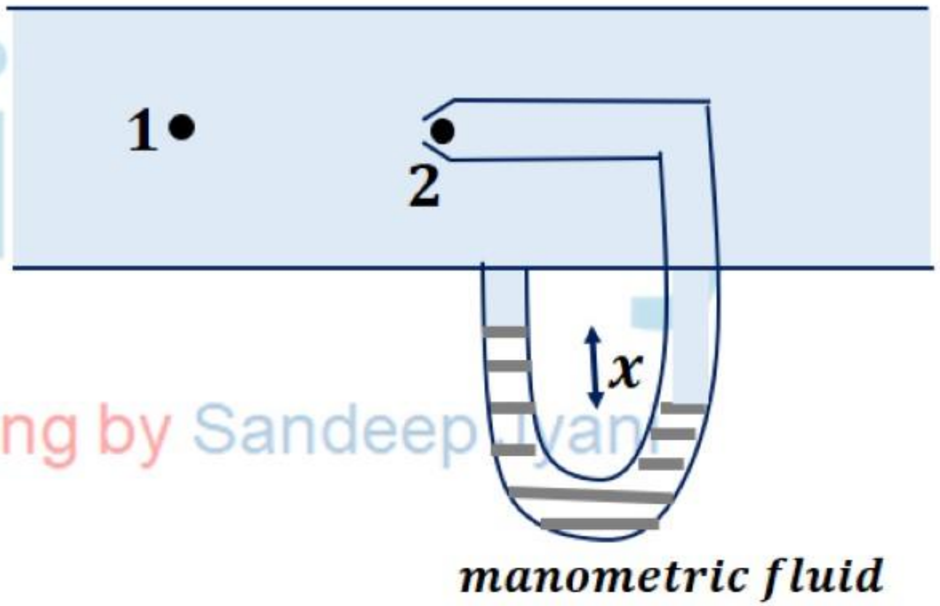
Dynamic pressure head
h



Dynamic pressure head = Stagnation pressure head − Static pressure head

PRACTICAL APPLICATIONS OF BERNOULLI's EQUATION

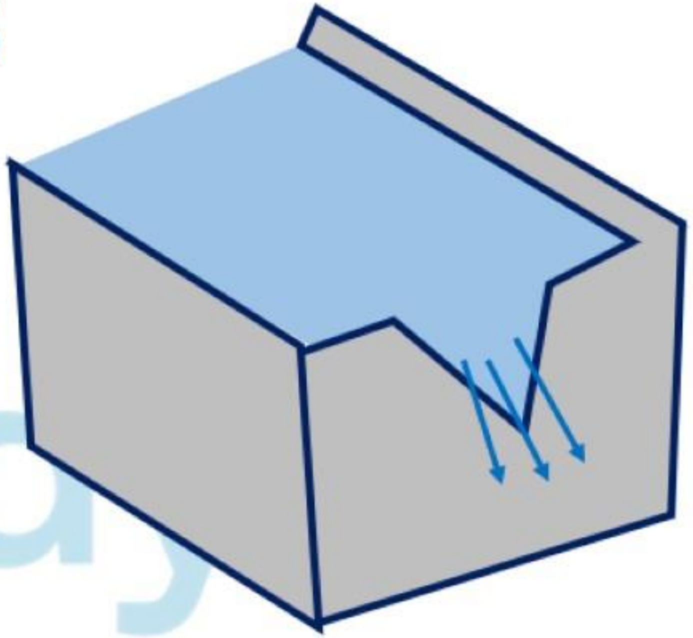
$$\Rightarrow v_1 = \sqrt{2gx\left[\frac{s_{manometric\,fluid}}{s_{fluid\,in\,pipe}} - 1\right]}$$



Que 66. A pitot static tube is used to measure the velocity of water in pipe. The stagnation pressure head is 6m and static pressure head is 5m. Calculate the velocity of flow assuming the coefficient of tube equal to 0.98

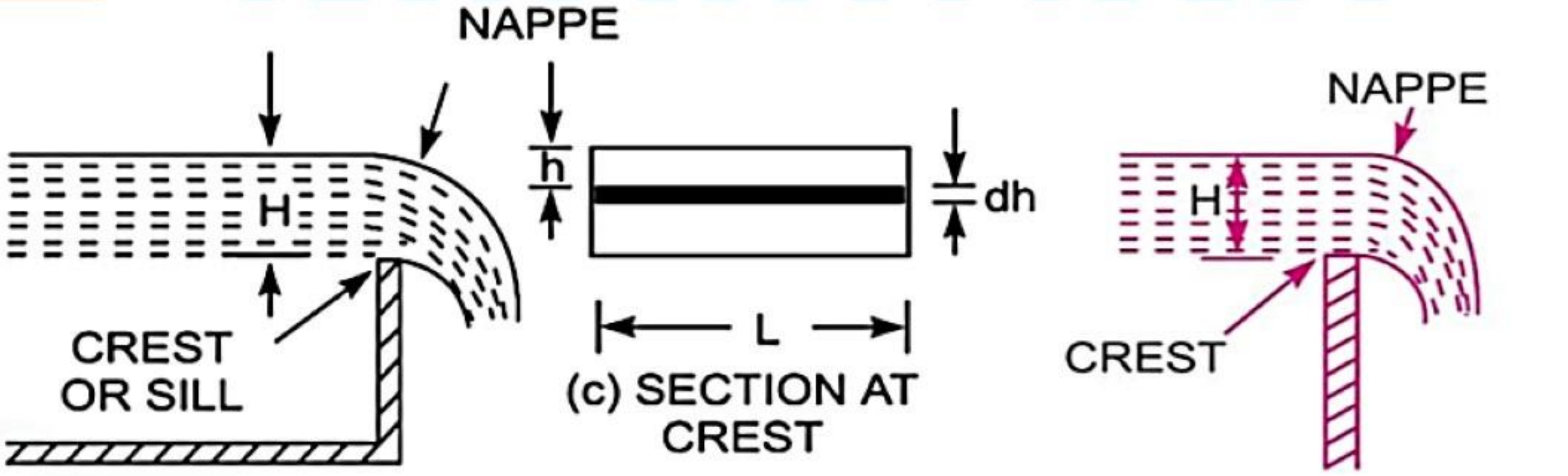
$$\textit{Dynamic pressure head} = \textit{Stagnation pressure head} - \textit{Static pressure head}$$

NOTCHES AND WEIRS



- **NOTCH:**
  - **It is used to measure the discharge in open channel flow.**
  - **It is used for measuring the rate of flow of a liquid through a small channel or tank.**
  - **It is an opening in the side of tank in such a way that liquid surface in the channel is below the top edge of opening.**
- **WEIR:**
  - **It is a concrete or masonry structure, placed in an open channel over which the flow occurs.**
  - **It is in the form of vertical wall, with sharp edge at the top, running all way across the open channel.**
- **Notch is of small size while weir is of larger size.**

- ***Nappe***: The layer of fluid flowing above the weir or below the notch is known as Nappe (or Vein).
- The bottom edge of notch or the top edge of weir is known as ***Sill or Crest***



(a) RECTANGULAR NOTCH (b) RECTANGULAR WEIR

WEIRS

- **WEIR:**
  - **It is a concrete or masonry structure, placed in an open channel over which the flow occurs.**
  - **It is in the form of vertical wall, with sharp edge at the top, running all way across the open channel.**
  - **TYPES OF WEIRS:**
    1. **Rectangular weir**
    2. **Triangular weir**
    3. **Trapezoidal weir**
    4. **Sharp crested weir**
    5. **Broad crested weir**
    6. **Narrow crested weir**
    7. **Ogee shaped weir**
    8. **Weir with end contraction**
    9. **Weir without end contraction**

NOTCHES

- **NOTCH:**
  - **It is used to measure the discharge in open channel flow.**
  - **It is used for measuring the rate of flow of a liquid through a small channel or tank.**
  - **It is an opening in the side of tank in such a way that liquid surface in the channel is below the top edge of opening.**
  - **TYPES OF NOTCHES:**
    1. **Rectangular notch**
    2. **Triangular notch (V-Notch)**
    3. **Trapezoidal notch**
    4. **Parabolic notch**
    5. **Stepped notch**
    6. **Notch with end contraction**
    7. **Notch without end contraction**

COMPARISON OF WEIR AND NOTCH

WEIR

- Used to regulate discharge of large channels, like canals and rivers.
- They are bigger in size.
- They are usually made of concrete.
- Fluid flows above the weir.

NOTCH

- Used to regulate discharge of small channels.
- They are smaller in size.
- They are usually made of metallic plate
- Fluid flows below the notch.

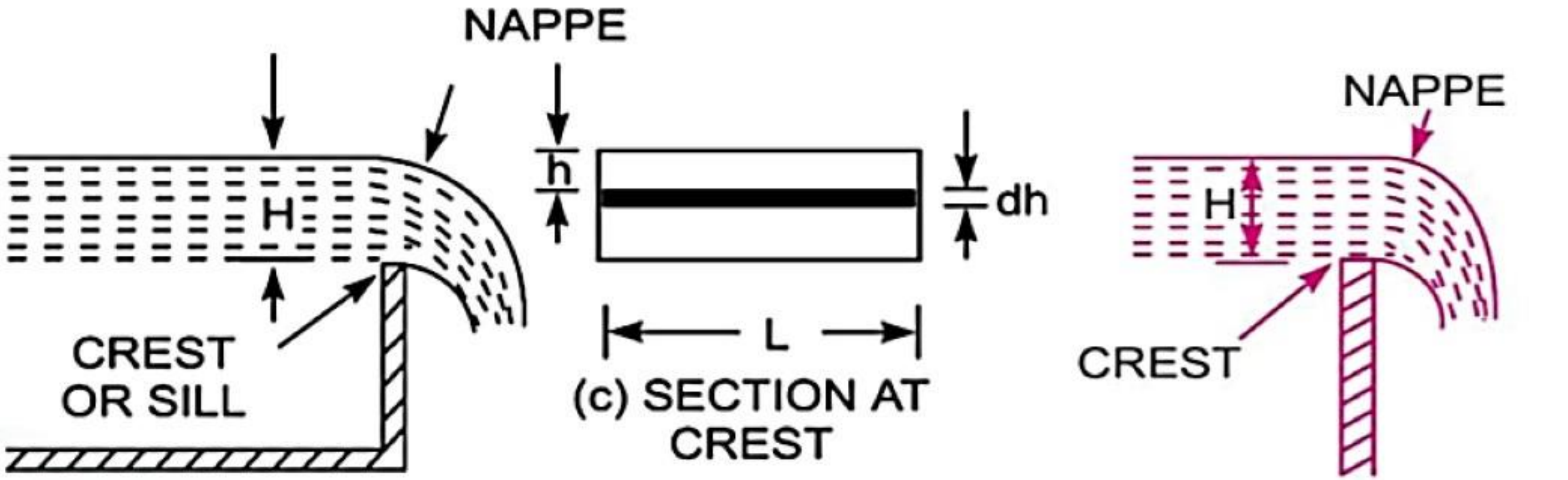
WEIRS AND NOTCHES

- **Rectangular notch or weir:**

$$Q_{theoretical} = \frac{2}{3}\sqrt{2g} \times LH^{3/2}$$

$$Q_{actual} = \frac{2}{3}C_d\sqrt{2g} \times LH^{3/2}$$

Where -
Q is the total discharge
C_d is coefficient of discharge
L is length of notch or weir
H is head of water over the crest



(a) RECTANGULAR NOTCH

(b) RECTANGULAR WEIR

Que 67. Find the discharge of water flowing over a rectangular notch of 4m length when the constant head is 500mm. (take C_d= 0.70)

Que 67. Find the discharge of water flowing over a rectangular notch of 4m length when the constant head is 500mm. (take C_d= 0.70)

Sol. $$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$Q = \frac{2}{3} \times 0.70 \times 4 \times \sqrt{2 \times 9.81} \times (0.5)^{3/2} = 2.92 m^3/sec$$

Que 68. The head water over a rectangular notch is 700mm. The discharge is 200litres/sec. Find the length of notch. (take C_d= 0.62)

Que 68. The head water over a rectangular notch is 700mm. The discharge is 200litres/sec. Find the length of notch. (take C_d= 0.62)

Sol. $Q = \frac{2}{3} C_d \sqrt{2g}\, L\, H^{3/2}$

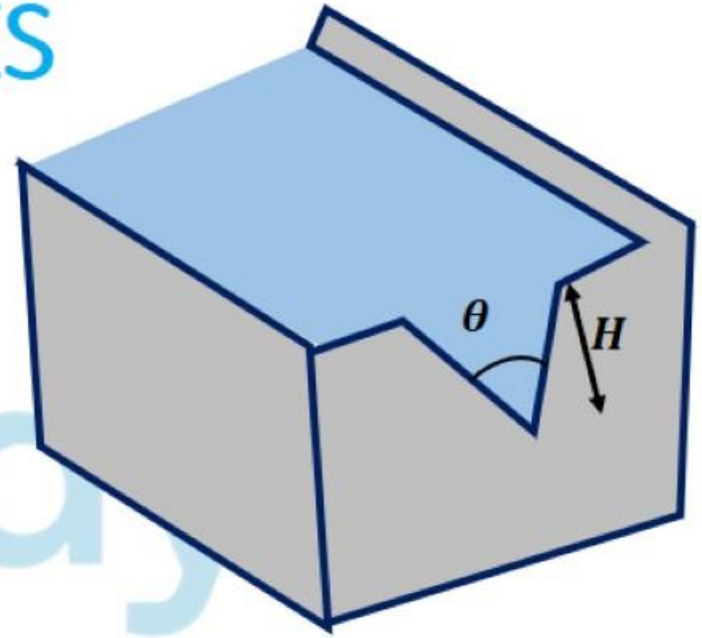
$\Rightarrow$**0.2 =** $\frac{2}{3}(0.62) \times L \times \sqrt{2 \times 9.81}(0.7)^{3/2}$

$\Rightarrow L$ **= 0.186m = 186mm**

WEIRS AND NOTCHES

- **Triangular notch or weir:**

$$Q_{theoretical} = \frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}\,H^{5/2}$$



$$Q_{actual} = \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2}\,H^{5/2}$$

Where -

Q is the total discharge

C_d is coefficient of discharge

θ is the angle of notch

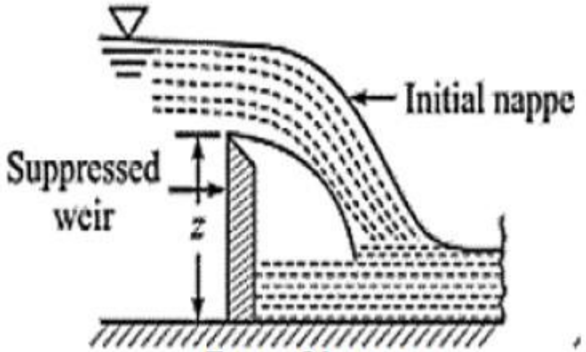
H is head of water above the V-notch

VENTILATION OF WEIR

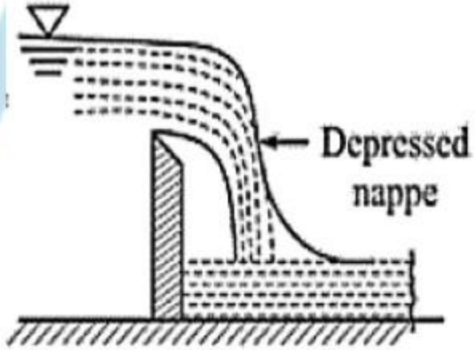
- **Air is trapped in the space between the side walls, channels and falling nappe.**
- **It is gradually carried away with the flowing water, thereby reducing the pressure in space between the nappe which may even become negative and negative pressure draws more water thereby increasing the actual discharge.**
- **But the various formulae indicated earlier for computing the discharge are based on the assumption that in the space below the emerging nappe, there is atmospheric pressure, such nappe is known as free nappe.**

VENTILATION OF WEIR

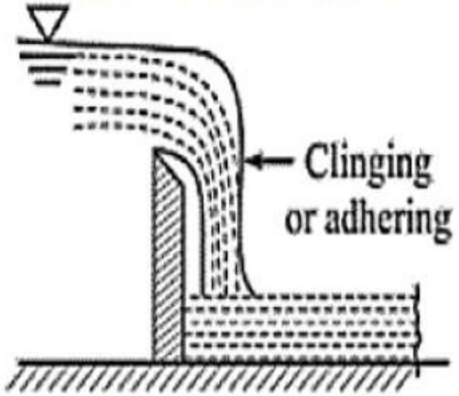
- **Free Nappe**
  - **If the atmospheric pressure exists beneath the nappe, it is known as a free nappe.**
  - **A free nappe is obtained by ventilating a weir.**
- **Depressed Nappe**
  - **Sometimes a weir is not fully ventilated, but is partially ventilated.**
  - **If the pressure below the nappe is negative, it is called a depressed nappe.**
  - **The discharge of the nappe, in this case, depends upon the amount of ventilation and the negative pressure.**
  - **Generally, the discharge of a depressed nappe is 6% to 7% more than that of a free nappe.**
- **Clinging Nappe**
  - **Sometimes, no air is left below the water, and the nappe adheres or clings to the downstream side of the weir. Such a nappe is called clinging nappe or an adhering nappe.**
  - **The discharge of a clinging nappe is 25% to 30% more than that of a free nappe.**



Free Nappe



Depressed Nappe



Clinging Nappe

ADVANTAGES OF TRIANGULAR WEIR OVER RECTANGULAR WEIR

1. **The nappe emerging from a triangular weir or notch has the same shape for all the heads and hence the value of coefficient of discharge for triangular weir is constant for all heads. While on the other hand, for rectangular weir, the shape of the nappe is affected by the head and therefore the coefficient of discharge varies with the heads.**
2. **For measuring low discharge, a triangular weir is very useful. This is because of low crest in triangular notch.**
3. **For right angle notch, the expression for computation is very simple.**
   $\boldsymbol{Q = Q = \frac{8}{15}\sqrt{2g}H^{5/2}}$
4. **In triangular weir, ventilation is not required.**

Que 58. Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.5m. (take C_d= 0.65)

Que 69. Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.5m. (take C_d = 0.65)

Sol. $Q_{actual} = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{5/2}$

$\Rightarrow Q = \frac{8}{15} \times 0.65 \times \tan\frac{60^\circ}{2} \times \sqrt{2 \times 9.81} \times (0.5)^{5/2}$

$\Rightarrow Q = 0.157 m3/sec$

WEIRS AND NOTCHES

- **Trapezoidal notch or weir:**

$$Q_{actual} = \left(C_{d1}\frac{2}{3}\sqrt{2g} \times LH^{3/2} + C_{d2}\frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}\, H^{5/2} \right)$$

where -

Q **is the total discharge.**

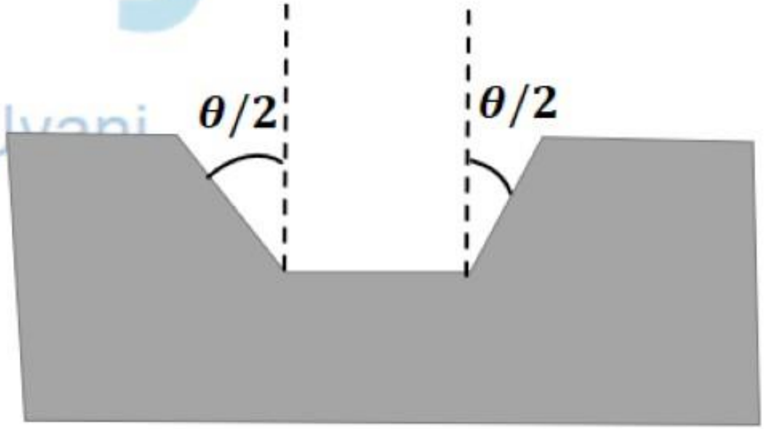
C_{d_1} **is coefficient of discharge over rectangular portion**

C_{d_2} **is coefficient of discharge over triangular portion**

θ **is the angle of notch**

L **is length of crest of the notch or weir**

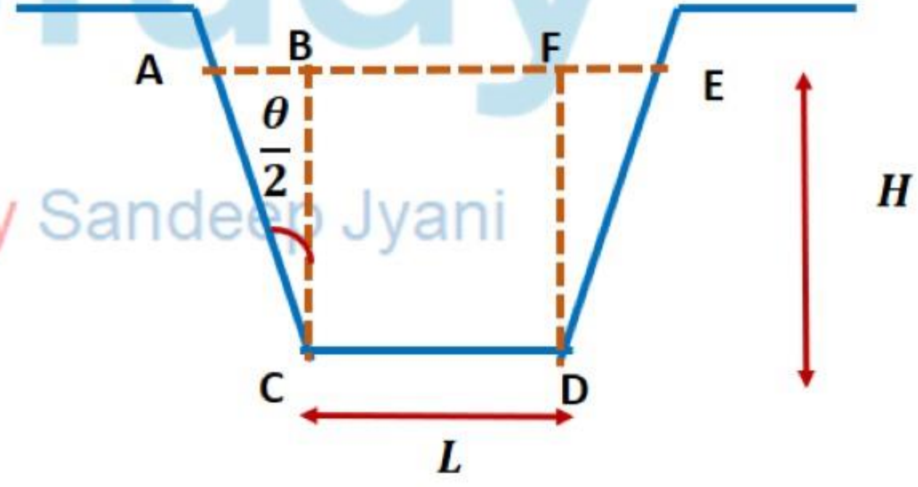
H **is height of water over the notch**



Que 70. Find the discharge through a trapezoidal notch which is 2m wide at the top and 0.80m at the bottom and is 50cm in height. The head of water of the notch is 30cm. Assume C_d= 0.62 for rectangular portion and for triangular portion C_d= 0.60.

Que 70. Find the discharge through a trapezoidal notch which is 2m wide at the top and 0.80m at the bottom and is 50cm in height. The head of water of the notch is 30cm. Assume C_d= 0.62 for rectangular portion and for triangular portion C_d= 0.60.

Sol. $$Q = \frac{2}{3}C_{d_1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15}C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$



$$\tan\frac{\theta}{2} = \frac{(AE - CD)/2}{BC}$$

$$\tan\frac{\theta}{2} = \frac{(2.0 - 0.8)/2}{0.60}$$

$$\tan\frac{\theta}{2} = 1$$

Que 59. Find the discharge through a trapezoidal notch which is 2m wide at the top and 0.80m at the bottom and is 60cm in height. The head of water of the notch is 30cm. Assume C_d= 0.62 for rectangular portion and for triangular portion C_d= 0.60.

Sol. $Q = C_{d1}\frac{2}{3}\sqrt{2g} \times LH^{3/2} + C_{d2}\frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}\ H^{5/2}$

$$Q = \frac{2}{3} \times 0.62 \times 0.8 \times \sqrt{2 \times 9.81} \times (0.3)^{3/2} + \frac{8}{15} \times 0.60 \times 1 \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$Q = 0.310 m^3/sec$

WEIRS AND NOTCHES

- **Stepped notch or weir:**

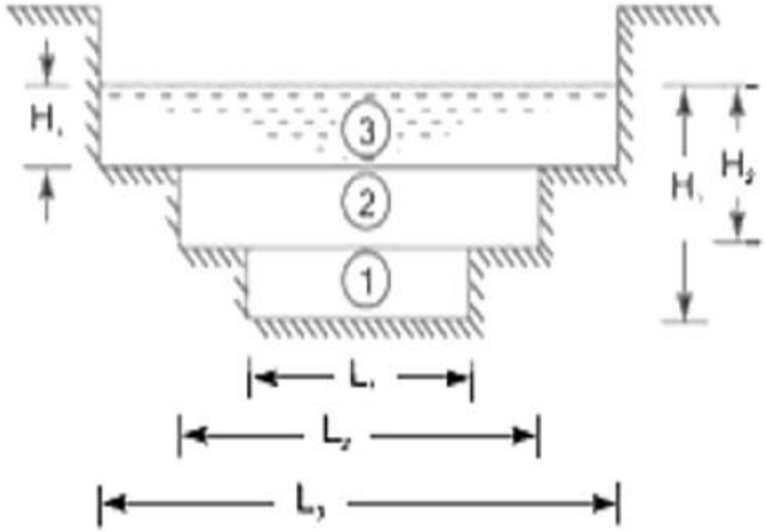
$$Q = \frac{2}{3} C_d \times L_1 \times \sqrt{2g} \times \left[H_1^{3/2} - H_2^{3/2}\right] + \frac{2}{3} C_d \times L_2 \times \sqrt{2g} \times \left[H_2^{3/2} - H_3^{3/2}\right] + \frac{2}{3} C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}$$

where -

Q is the total discharge

L is length of crest of the notch or weir

H is height of water over the notch



VELOCITY OF APPROACH

- **Velocity of approach is defined as the velocity with which the water approaches or reaches the weir or notch before it flows over it.**

$$V_a = \frac{Q}{area\ of\ channel}$$

- **With the help of this velocity of approach, an additional head is determined which is acting on the water flowing over the notch.**

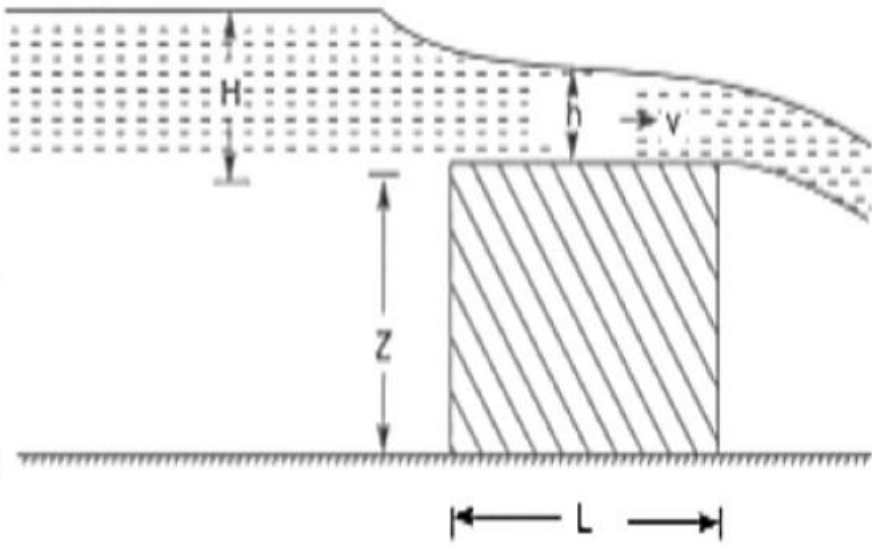
$$h_a = \frac{V_a^2}{2g}$$

- **Discharge over a rectangular weir with velocity of approach**

$$Q = \frac{2}{3} C_d \sqrt{2g}\ L\left[(H_1 + h_a)^{3/2} - (h_a)^{3/2}\right]$$

WEIRS AND NOTCHES

- **Broad crested weir:**
  - A weir having a wide crest is called broad crested weir.



$$Q = C_d \, L \, h\sqrt{2g(H - h)}$$

Where

H = height of water above the crest

L = length of crest

h = head of water at the middle of the weir which is constant

The discharge will be maximum if $h = \frac{2}{3}H$

$$Q_{max} = 1.705 \times C_d \times L \times H^{3/2}$$

If $2L > H$; *then it is broad crested weir*

If $2L < H$; *then it is narrow crested weir*

Que 70. Determine the maximum discharge for a broad crested weir of length 60m, and height of water 60m above its crest. Assume C_d= 0.60

Que 70. Determine the maximum discharge for a broad crested weir of length 60m, and height of water 60cm above its crest. Assume C_d= 0.62

Sol. $Q_{max} = 1.705 \times C_d \times L \times H^{3/2}$

$\Rightarrow Q_{max} = 1.705 \times 0.62 \times 60 \times (0.60)^{3/2}$

$\Rightarrow Q_{max} = 29.47\ m^3/sec$

WEIRS AND NOTCHES

- **Narrow crested weir:**
  - **This weir behaves same as that of rectangular weir as $2L < H$**

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2}$$

Where -

Q is the total discharge

C_d is coefficient of discharge

L is length of notch or weir

H is head of water over the crest

WEIRS AND NOTCHES

- **Ogee weir:**
  - **In this weir, the crest of the weir rises upto a maximum height of $0.115H$ $(where\ H\ is\ height\ of\ water\ above\ the\ inlet\ of\ the\ weir)$ and then falls.**

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2}$$

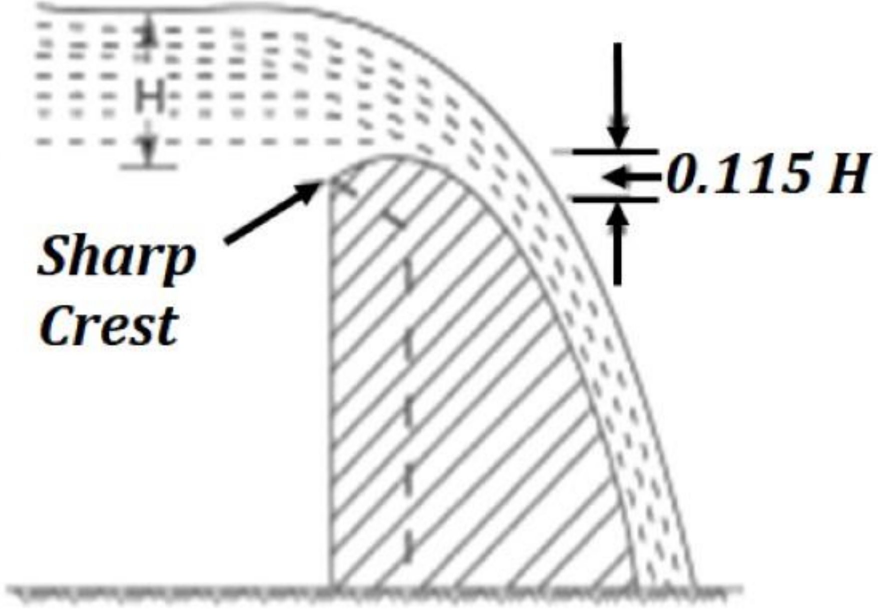
Where -

Q is the total discharge

C_d is coefficient of discharge

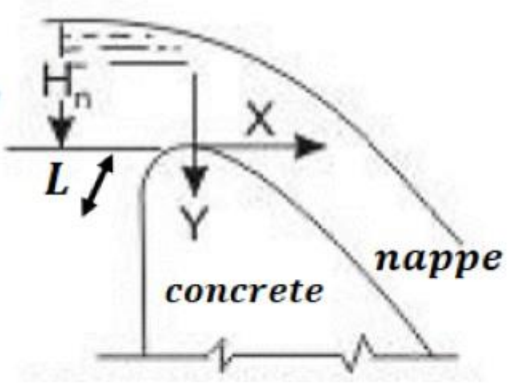
L is length of notch or weir

H is head of water over the crest



SPILLWAY

- **A spillway is a portion of dam over which the excess water which cannot be stored in the reservoir is drained out.**
- **The shape of spillway profile is made to follow the profile of the lower nappe of a well ventilated sharp crested weir.**



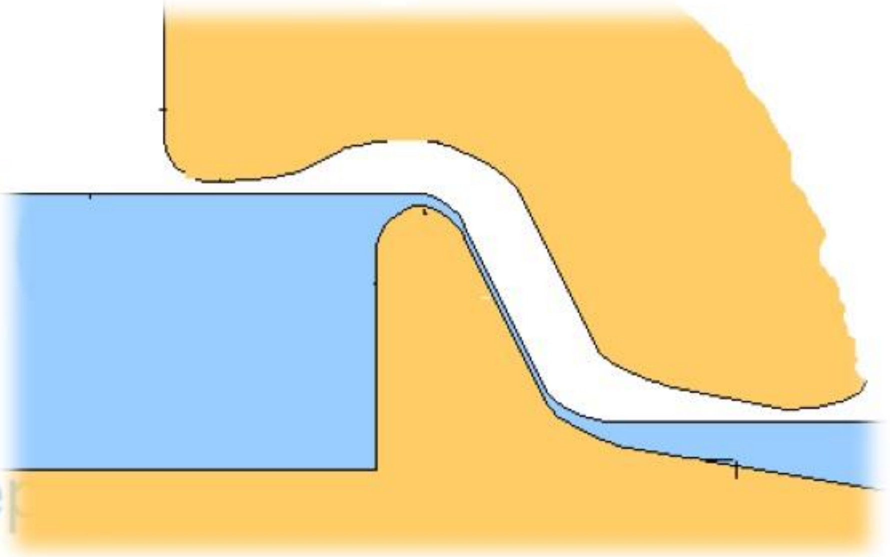
- **The main advantage of providing of providing such a shape for the spillway is that the flowing sheet of water remains in contact with the surface of the spillway.**

$$Discharge\ of\ spillway = CLH^{3/2}$$

  - Where C – coefficient of spillway
  - L – length of spillway

SYPHON SPILLWAY

- **A syphon spillway consists of ogee weir which is provided with air tight cover and a large rectangular section pipe.**
- **The working of syphon spillway is automatic.**
- **When the water level in the upstream rises above the crest of spillway, water begins to flow over the crest through the syphon.**
- **The jet of the flowing water strikes the inside of cover in which air is trapped, as more water flows down, it sucks the trapped air from the kink. Thus creating a partial vacuum in this portion which sucks up the water from the reservoir and completely filling the pipe and thus a syphonic action occurs.**



Viscous Flow

- This chapter deals with fluids that are viscous and flowing at very low velocity
- When velocity is low, fluid flow in layers
- Each layer slides over the adjacent layer
- Due to relative velocity between two layers, the velocity gradient $\frac{du}{dy}$ exists and hence shear stress $\tau = \mu \frac{du}{dy}$ acts on the layers

REYNOLD'S NUMBER

- It is the ratio of inertia force to viscous force.

$$R_e = \frac{F_i}{F_v}$$

$R_e < 2000$; *Laminar Flow*

$R_e > 4000$; *Turbulent Flow*

$2000 < R_e < 4000$; *Transition Flow*

} *for pipe flow*

- For flat plate or open flow,

$$R_e = \frac{\rho V L}{\mu}$$

L – *length of plate*

- For duct/pipe or closed flow,

$$R_e = \frac{\rho V D}{\mu}$$

D – *diameter of pipe*

DARCY WEISBACH EQUATION/FORMULA

- It is used to find out head loss due to friction, this equation is applicable for laminar as well as turbulent flow but the flow must be steady.

Experimental Formula for h_l

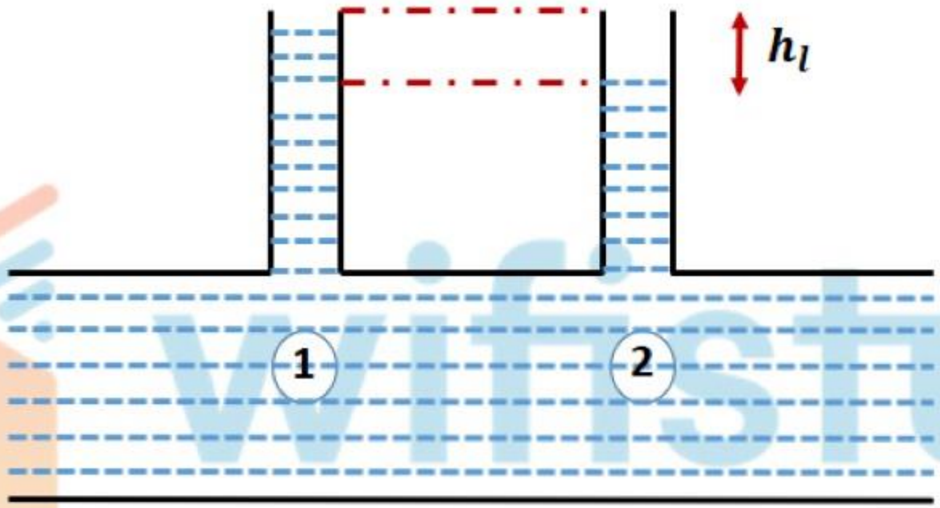
$$h_l = \frac{4f'Lv^2}{2gD}$$

f' is friction coefficient or Fanning's coefficient

$$h_l = \frac{fLv^2}{2gD}$$

f is friction factor or Darcy Weisbach coefficient

PIPE FLOW



h_l - head loss

$$\frac{P_1}{w} + \frac{v_1^2}{2g} = \frac{P_2}{w} + \frac{v_2^2}{2g} + h_l$$

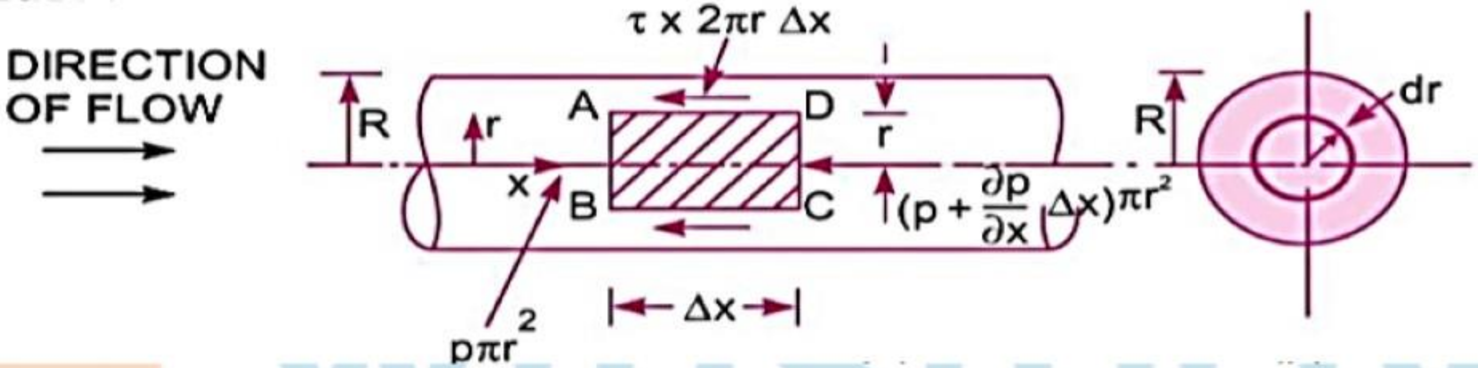
As $A_1 v_1 = A_2 v_2$

$\Rightarrow A_1 = A_2$

$\Rightarrow v_1 = v_2$

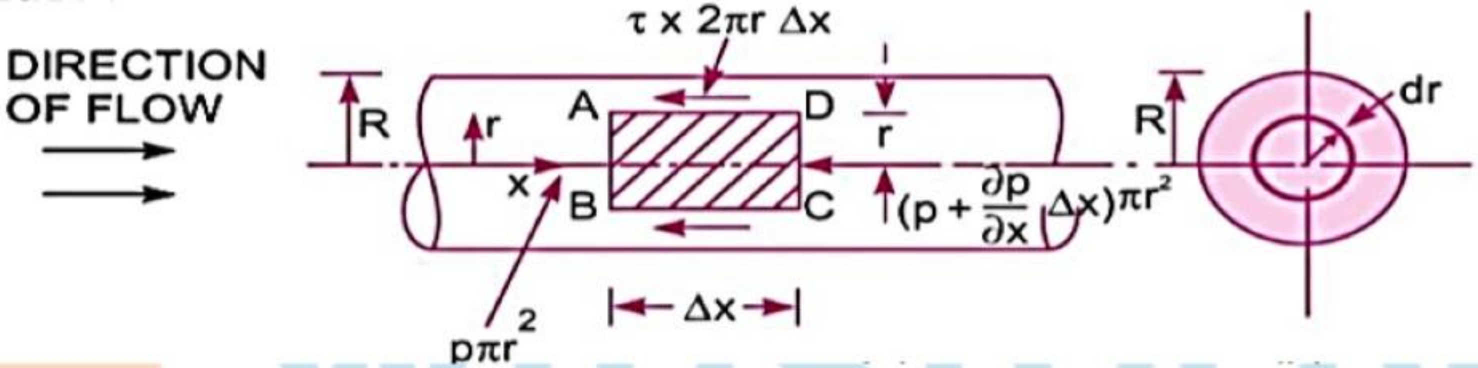
$$\frac{P_1}{w} - \frac{P_2}{w} = h_l$$ (Losses in pipe)

LAMINAR FLOW THROUGH CIRCULAR PIPE



- **HAGEN POISEUILLE LAW:**
  - **Consider a horizontal pipe of radius 'R'. The viscous fluid is flowing from left to right in the pipe.**
  - **Consider a fluid element of radius 'r', sliding in a cylindrical fluid element of radius $(r + dr)$.**
  - **Let the length of the fluid element be Δx.**
  - **If 'p' is the intensity of pressure on the face AB, then the intensity of pressure on the face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$**

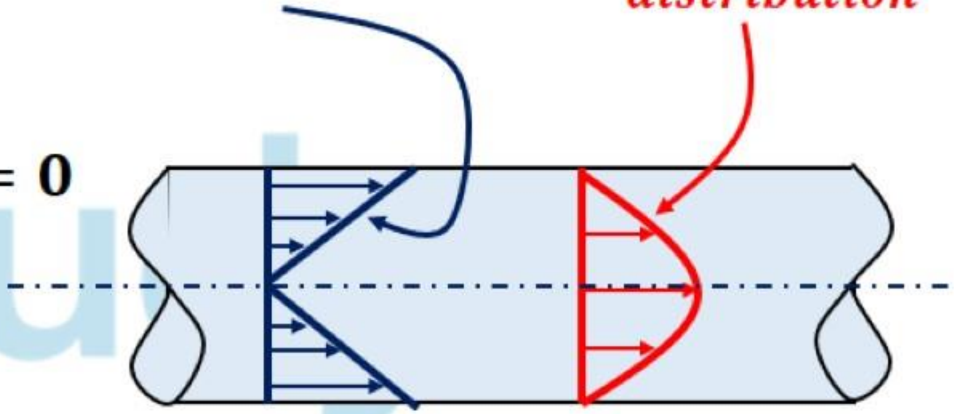
LAMINAR FLOW THROUGH CIRCULAR PIPE



- **HAGEN POISEUILLE LAW:**
  - **Forces acting on fluid element-**
    1. **Pressure force =** $p \times \pi r^2$ **on face AB.**
    2. **Pressure force =** $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$ **on face CD.**
    3. **Shear force =** $\tau \times 2\pi r \Delta x$ **on the surface of fluid element.**
  - **As there is no acceleration, hence summation of all forces in direction of flow=0 (in case of steady state)**

LAMINAR FLOW THROUGH CIRCULAR PIPE

- **HAGEN POISEUILLE LAW:**



1. Shear Stress

$$\Rightarrow p \times \pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \Delta x = 0$$

$$\Rightarrow -\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \Delta x = 0$$

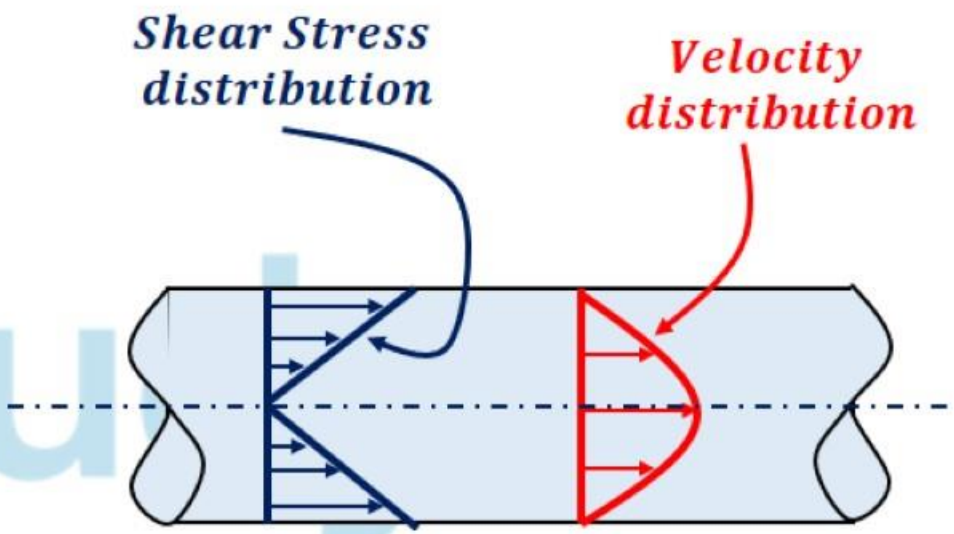
$$\Rightarrow -\left(\frac{\partial p}{\partial x}\right) \times \mathrm{r} - 2\tau = 0$$

$$\Rightarrow \tau = -\left(\frac{\partial p}{\partial x}\right) . \frac{\mathrm{r}}{2}$$

- **The shear stress τ across a section varies with 'r'.**
- **Hence shear stress distribution across a section is linear.**

LAMINAR FLOW THROUGH CIRCULAR PIPE

$$\tau = -\left(\frac{\partial p}{\partial x}\right)\cdot\frac{r}{2}$$



2. Velocity Distribution

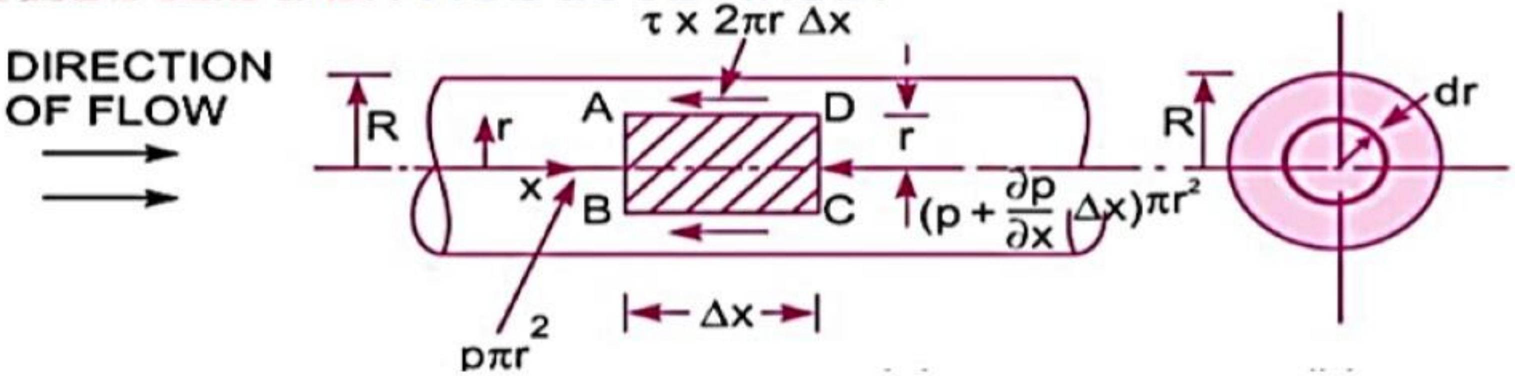
To obtain the velocity distribution across a section substituting the value of shear stress $\tau = \mu\frac{du}{dy}$

$$\Rightarrow y = R - r \quad and \quad dy = -dr$$

$$\Rightarrow \tau = -\left(\frac{\partial p}{\partial x}\right)\cdot\frac{r}{2}$$

$$\Rightarrow -\mu\frac{du}{dr} = -\left(\frac{\partial p}{\partial x}\right)\cdot\frac{r}{2}$$

$$\Rightarrow \frac{du}{dr} = \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)\cdot\frac{r}{2}$$



Integrating w.r.t. r

$$\Rightarrow u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)r^2 + C$$

LAMINAR FLOW THROUGH CIRCULAR PIPE

- **HAGEN POISEUILLE LAW:**

$$\Rightarrow u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) r^2 + C$$

Here C is the constant of integration and its value is obtained from boundary condition r=R and u=0

$$\Rightarrow 0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2 + C$$

$$\Rightarrow C = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2$$

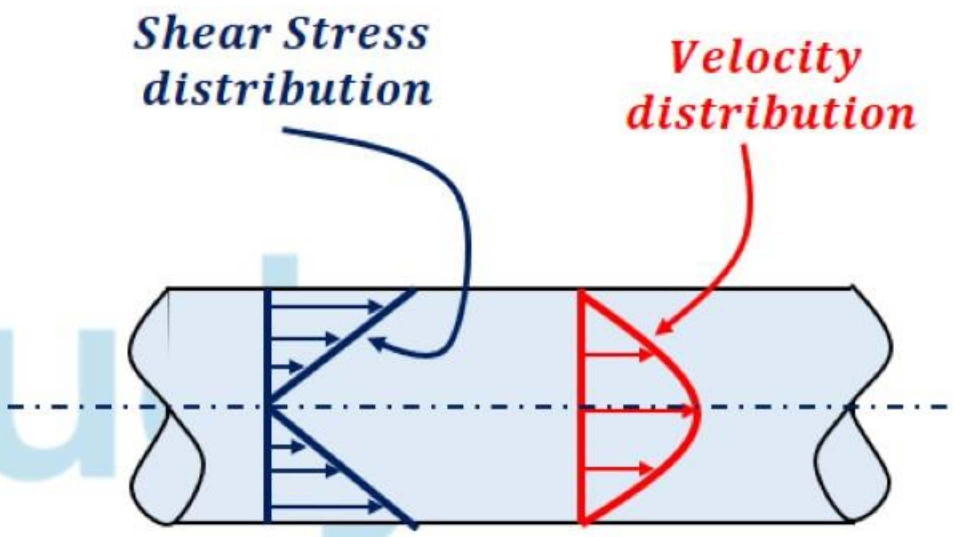
Now substituting value of C in the equation for u

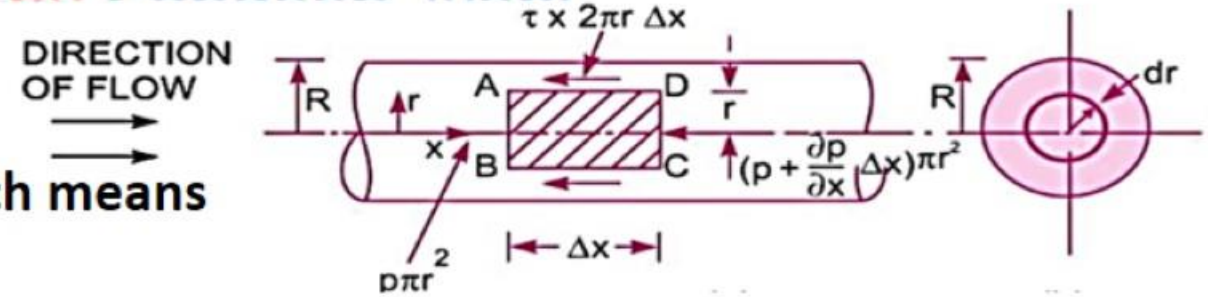
$$\Rightarrow u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) r^2 - \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2$$

$$\Rightarrow u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2)$$

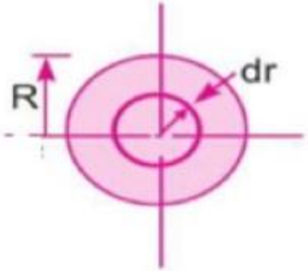
Values of μ, $\left(\frac{\partial p}{\partial x}\right)$ and R are constant, which means the velocity u, varies with the square of r.

Hence Velocity distribution across the section of pipe is parabolic.





LAMINAR FLOW THROUGH CIRCULAR PIPE



3. Maximum velocity

- The velocity is maximum when r=0

$$\Rightarrow u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2)$$

$$\Rightarrow U_{max} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)R^2$$

4. Discharge

It is calculated by considering the flow through a circular ring element of radius 'r' and thickness 'dr'. The fluid flowing per second through this elementary ring-

$$\Rightarrow dQ = velocity\ at\ radius\ r \times area\ of\ ring\ element$$

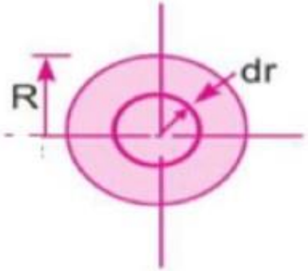
$$\Rightarrow dQ = u \times 2\pi r dr$$

$$\Rightarrow dQ = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2) \times 2\pi r dr$$

On integrating both sides, we get-

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4$$

LAMINAR FLOW THROUGH CIRCULAR PIPE



5. Average velocity

$$\Rightarrow \bar{u} = \frac{Q}{Area}$$

$$\Rightarrow \bar{u} = \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4}{\pi R^2}$$

$$\Rightarrow \bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

6. Hence ratio of maximum and average velocity is-

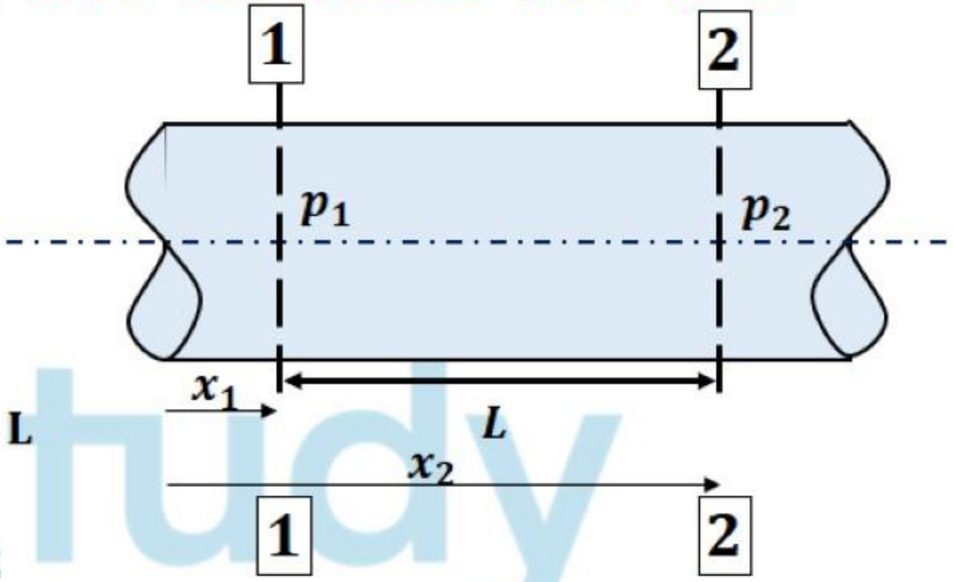
$$\Rightarrow \frac{U_{max}}{\bar{u}} = \frac{\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2} = 2.0$$

LAMINAR FLOW THROUGH CIRCULAR PIPE

7. Drop of pressure

$$\Rightarrow \bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$$

$$\Rightarrow \left(-\frac{\partial p}{\partial x}\right) = \frac{8\bar{u}\mu}{R^2}$$



$(x_2 - x_1) = \mathrm{L}$

$\mathrm{R} = \mathrm{D}/2$

Integrating both sides, we get-

$$\Rightarrow (p_1 - p_2) = \frac{32\bar{u}\mu L}{D^2}$$

$(p_1 - p_2) = drop\ of\ pressure$

$$\Rightarrow \frac{(p_1 - p_2)}{\rho g} = \frac{32\bar{u}\mu L}{\rho g D^2}$$

$\frac{(p_1 - p_2)}{\rho g} =$ loss of pressure head

HAGEN POISEUILLE FORMULA

$$\frac{(p_1 - p_2)}{\rho g} = \frac{32\,\mu\,V_{avg}\,L}{\rho g D^2}$$

As per Darcy Weisbach - $h_l = \frac{fLv^2}{2gD}$

And as per Hagen - $h_l = \frac{32\bar{u}\mu L}{\rho g D^2}$

Hence equating both the head loss

$$\Rightarrow \frac{fL\bar{u}^2}{2gD} = \frac{32\bar{u}\mu L}{\rho g D^2}$$

$v = \bar{u}\ (average\ velocity)$

$$\Rightarrow f = \frac{64\mu}{\rho\bar{u}D}$$

$$\Rightarrow f = \frac{64}{\rho\bar{u}D/\mu}$$

$$\Rightarrow f = \frac{64}{R_e} \qquad friction\ factor$$

$$\Rightarrow f' = \frac{16}{R_e} \qquad friction\ coefficient$$

Experimental Formula for h_l:

- $h_l = \frac{4f'Lv^2}{2gD}$ — f' is friction coefficient or Fanning's coefficient
- $h_l = \frac{fLv^2}{2gD}$ — f is friction factor or Darcy Weisbach coefficient

Que 71. Find the minimum value of friction factor than can occur in laminar flow through a circular pipe.

Que 71. Find the minimum value of friction factor than can occur in laminar flow through a circular pipe.

Sol. $f = \frac{64}{R_e} \Rightarrow f = \frac{64}{2000}$ **= 0.032**

Que 72. In a laminar flow through a pipe of 10 cm radius, the average velocity is 5m/sec, then find the velocity at 5cm radius.

Que 73. For a pipe of 10cm dia, find distance from pipe wall where local velocity is equal to average velocity in pipe

KINETIC ENERGY CORRECTION AND MOMENTUM CORRECTION FACTORS

- **KINETIC ENERGY CORRECTION FACTOR (α):**
  - **The ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy.**

$$(\alpha) = \frac{KE_{actual}}{KE_{theoretical}}$$

(α) for turbulent flow $= 1.33$

(α) for laminar flow $= 2.0$

- **MOMENTUM CORRECTION FACTOR (β):**
  - **The ratio of the momentum of the flow per second based on actual velocity across a section to the momentum of the flow per second based on average velocity across the section.**

$$(\beta) = \frac{Momentum_{actual}}{Momentum_{theoretical}}$$

(β) for laminar flow $= 1.33$

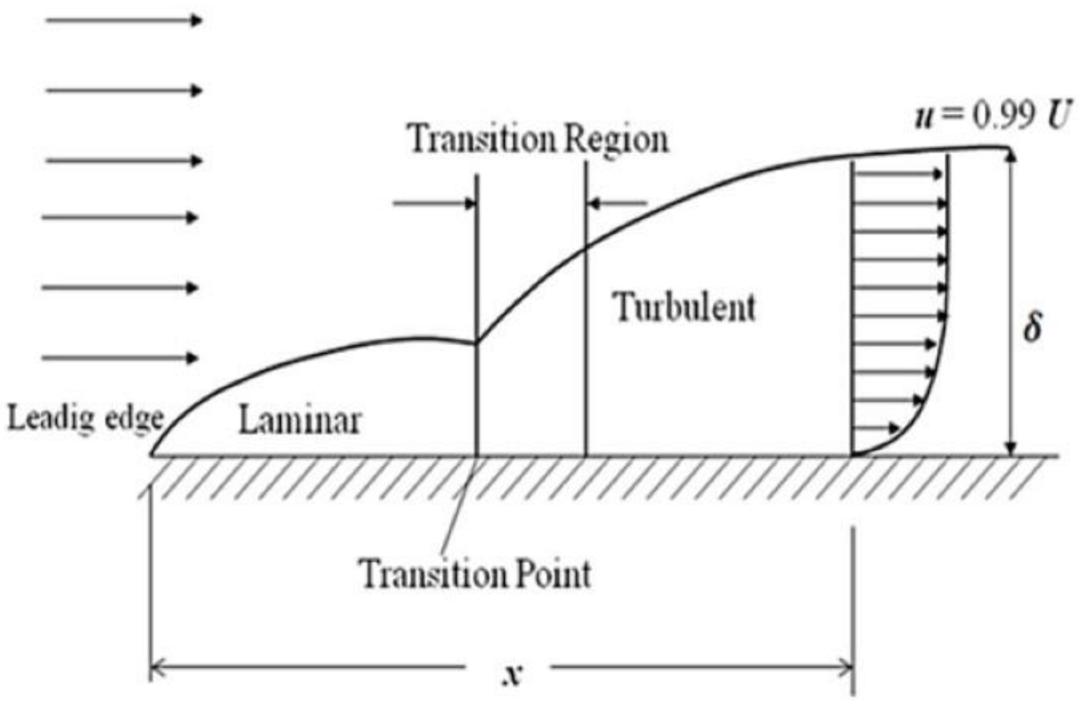
(β) for turbulent flow $= 1.20$

KINETIC ENERGY CORRECTION AND MOMENTUM CORRECTION FACTORS

- Lower the values of correction factor, the flow will be more turbulent and velocity distribution is more uniform but in the laminar flow, velocity distribution is not uniform, hence momentum correction factor and kinetic energy correction factor are different.

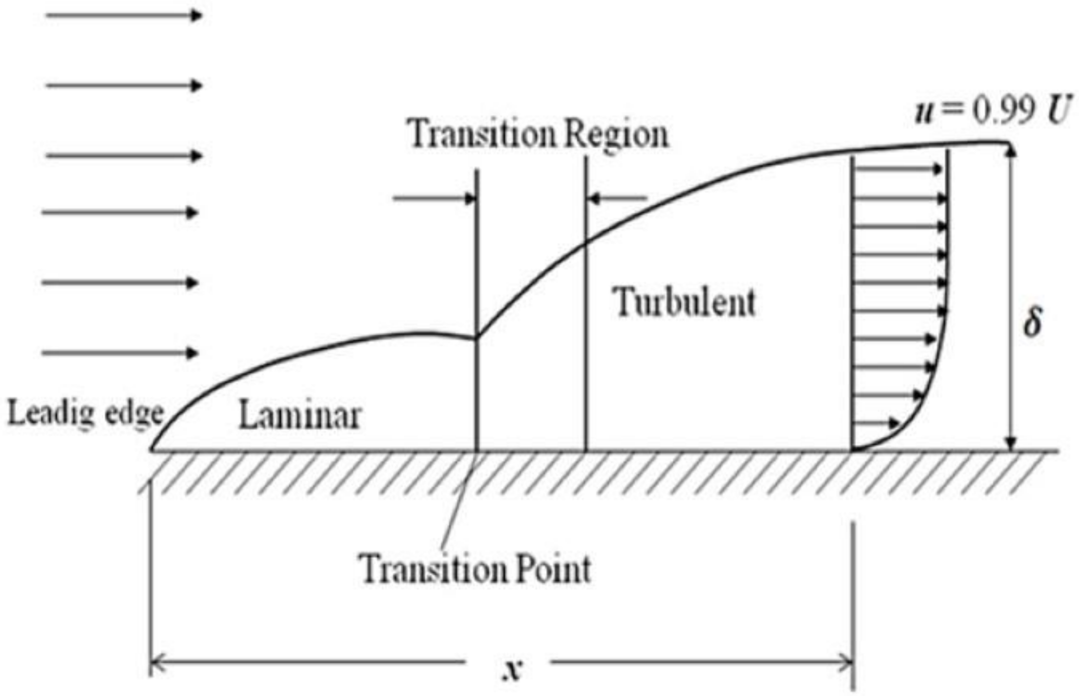
BOUNDARY LAYER

- When a real fluid flows past a solid body or solid wall, the fluid particles adhere to the boundary and condition of no slip occurs.
- Hence the velocity of fluid close to the boundary will same as that of the boundary.
- For stationary body, the velocity of fluid at the boundary will be zero.
- Farther away from the boundary, the velocity will be higher and velocity gradient $(\frac{du}{dy})$ will exist.
- The velocity of fluid increases from zero to free stream velocity (U) of the fluid in the direction normal to the boundary, in a narrow region in the vicinity of boundary, this region is termed as boundary layer.
- The flow in boundary layer region is viscous in nature.
- Bernoulli's equation is not applicable in boundary layer region



BOUNDARY LAYER

- When a real fluid passes a flat plate, the velocity at the leading edge is zero and the retardation of particles increases when more area of plate is exposed to the flow and hence the boundary layer thickness increases as the distance from the leading edge increases upto certain distance from leading edge.
- The flow in the boundary layer is laminar and as the laminar boundary layer grows, instability occurs and flow changes from laminar to turbulent through transition.
- Even in the turbulent boundary layer region, close to the plate, the flow is laminar. Hence this region is known as laminar sub region. It is only defined for turbulent flow.

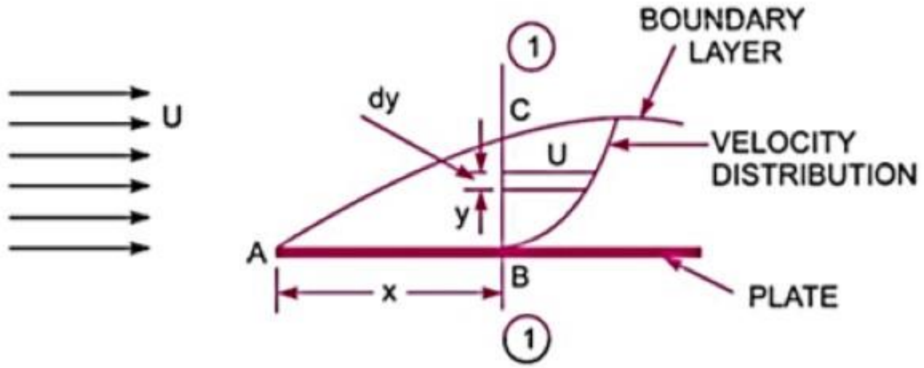


BOUNDARY LAYER

- **BOUNDARY LAYER THICKNESS (δ):**
  - **It is the distance from the boundary of the solid body measured in the y-direction to the point, where the velocity of fluid is approximately equal to 0.99times the free stream velocity (U) of the fluid.**

- **DISPLACEMENT THICKNESS (δ^*):**
  - **It is the thickness by which solid boundary must be displaced in order to compensate for the reduction in mass flow rate due to boundary layer growth.**

$$\delta^* = \int_0^{\delta}\left(1 - \frac{u}{U}\right) dy$$



u=velocity of fluid at elementary strip

U= free stream velocity of fluid

BOUNDARY LAYER

- **MOMENTUM THICKNESS (θ):**
  - **It is the distance by which boundary should be displaced in order to compensate for the reduction in momentum due to boundary layer growth.**

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

- **KINETIC ENERGY THICKNESS (δ^{**}):**
  - **It is the thickness by which solid boundary must be displaced in order to compensate for the reduction in kinetic energy due to boundary layer growth.**

$$\delta^{**} = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

u=velocity of fluid at elementary strip

U= free stream velocity of fluid

$$H(Shape\ Factor) = \frac{\delta^{*}}{\theta}$$

VON KARMAN'S MOMENTUM INTEGRAL EQUATION

- It is applicable to laminar, transition and turbulent boundary layers flows.

$$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}$$

$$\tau_0 = \frac{1}{2} C_d \rho U^2$$

τ_0=shear stress at surface

θ=momentum thickness

U= free stream velocity of fluid

x=distance from leading edge

C_d=drag coefficient

FLOW THROUGH PIPES

- **LOSS OF ENERGY IN PIPES:**
  - **When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.**
  - **Classification:**
    1. **Major energy losses (80-90%)**
       - **This is due to friction and it is calculated by using –**
         - **a) Darcy – Weisbach Formula**
         - **b) Chezy's Formula**
    2. **Minor energy losses (10-20%)**
       - **This is due to**
         - **a) Sudden expansion of pipe**
         - **b) Sudden contraction of pipe**
         - **c) Bend in pipe**
         - **d) Pipe fittings**
         - **e) An obstruction in pipe**

MAJOR ENERGY LOSSES

a) Darcy – Weisbach Formula

- It is used to find out head loss due to friction, this equation is applicable for laminar as well as turbulent flow but the flow must be steady.

$$h_f = \frac{4f'LV^2}{2gd}$$

f=coefficient of friction

h_f= loss of head due to friction

L=length of pipe

V=mean velocity of flow

d=diameter of pipe

$f' = \frac{16}{R_e}$ (for R_e<2000)

$f' = \frac{0.079}{R_e^{1/4}}$ (for R_e varying 4000 to 10^6)

f= friction factor

$$h_f = \frac{fLV^2}{2gd}$$

$$Q = AV \Rightarrow V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2}$$

$$\Rightarrow h_L = \frac{16fLQ^2}{\pi^2 2gd^5}$$

$$\Rightarrow h_L = \frac{fLQ^2}{12d^5}$$

MAJOR ENERGY LOSSES

b) Chezy's Formula

$$V = C\sqrt{mi}$$

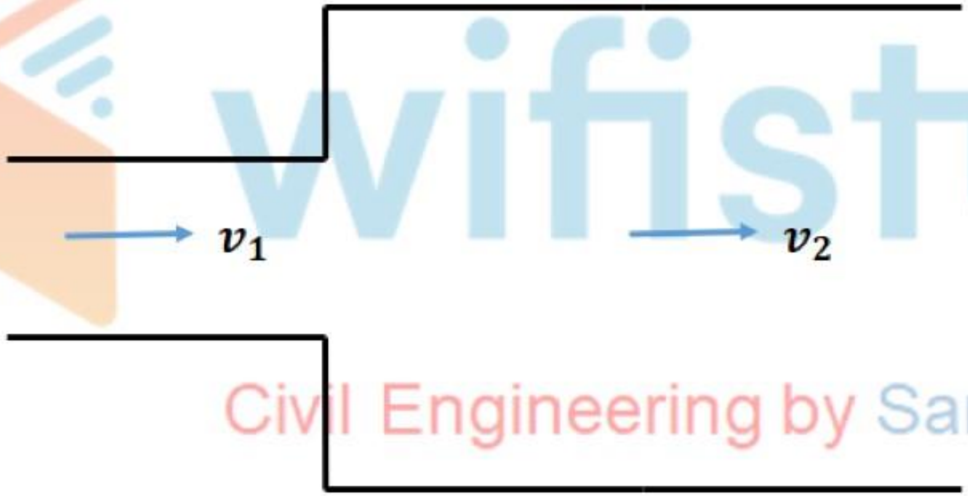
$$m = \text{hydraulic mean depth} = \frac{Area\ of\ flow}{Wetted\ Perimeter} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

$$C = Chezy's\ Constant$$

$$i = \text{loss of head per unit length of pipe} = \frac{h_f}{L}$$

MINOR ENERGY LOSSES

1. LOSS OF HEAD DUE TO SUDDEN EXPANSION:



$$h_{l(expansion)} = \frac{(v_1 - v_2)^2}{2g}$$

$$Q = A_1 v_1 = A_2 v_2$$

$$\frac{v_1}{v_2} = \frac{A_2}{A_1}$$

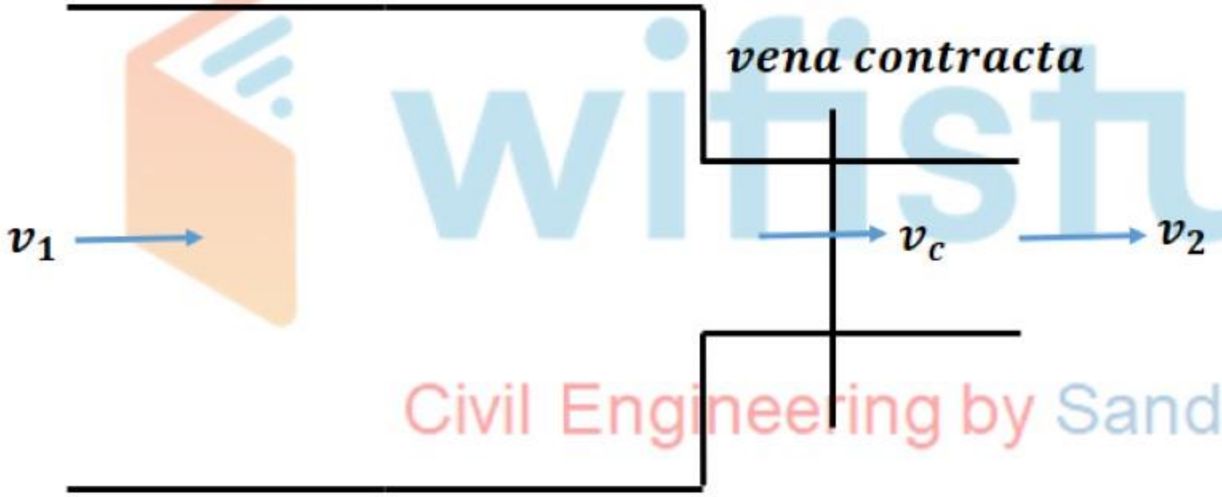
$$h_{l(expansion)} = \frac{{v_1}^2}{2g}\left(1 - \frac{v_2}{v_1}\right)^2$$

$$h_{l(expansion)} = \frac{{v_1}^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

Loss from exit of pipe $h_l = \frac{{v_1}^2}{2g}$ (here A=infinity)

MINOR ENERGY LOSSES

2. LOSS OF HEAD DUE TO SUDDEN CONTRACTION:



$$h_{l(contraction)} = \frac{(v_C - v_2)^2}{2g}$$

$$Q = A_c v_c = A_2 v_2$$

$$\frac{v_c}{v_2} = \frac{A_2}{A_c} \qquad \frac{A_c}{A_2} = C_c$$

$$h_{l(contraction)} = \frac{{v_2}^2}{2g}\left(\frac{v_c}{v_2} - 1\right)^2$$

$$h_{l(contraction)} = \frac{{v_2}^2}{2g}\left(\frac{A_2}{A_c} - 1\right)^2$$

$$h_{l(contraction)} = \frac{{v_2}^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

If C_c is not given, take $h_{l(contraction)} = \frac{0.5{v_2}^2}{2g}$ (where v_2 is smaller pipe velocity)

MINOR ENERGY LOSSES

3. LOSS AT ENTRANCE OF PIPE

$$h_l = \frac{0.5{v_2}^2}{2g}$$

v_2 = velocity in pipe

4. LOSS AT EXIT OF PIPE

$$h_l = \frac{v^2}{2g}$$

v = velocity at outlet of pipe

5. LOSS DUE TO BEND IN PIPE

$$h_l = \frac{kV^2}{2g}$$

V=velocity of flow

k=coefficient of bend (constant)

Depends upon angle of bend or radius of curvature of bend

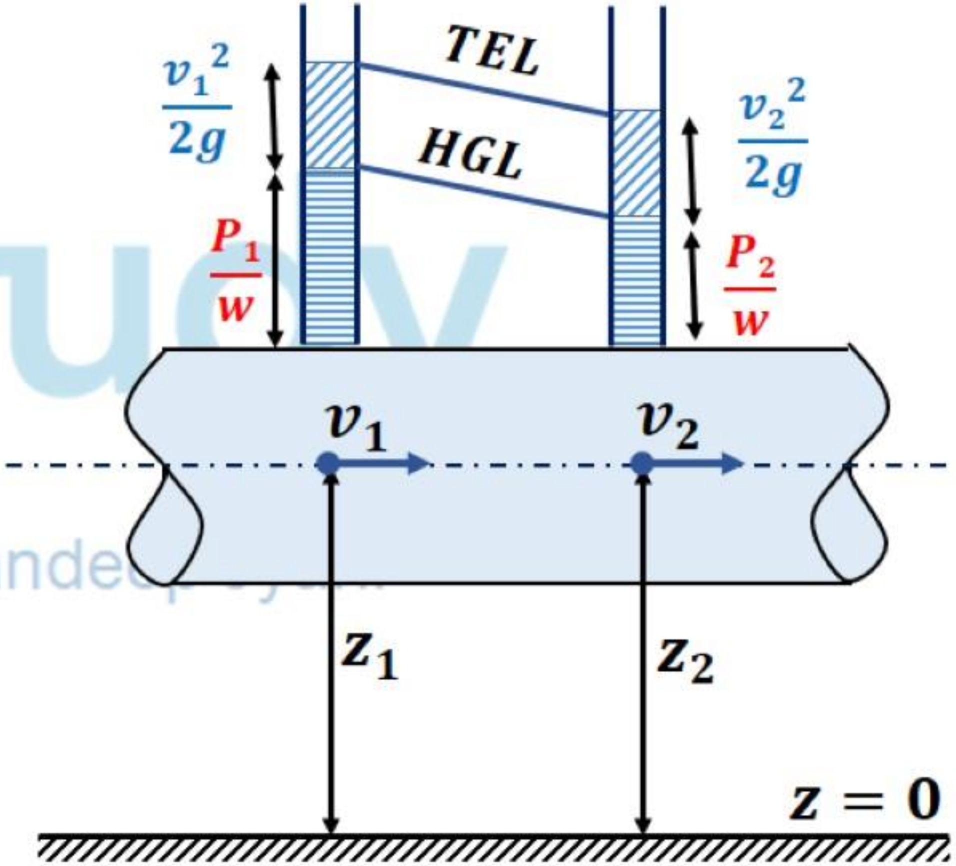
HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

- **HYDRAULIC GRADIENT LINE:**
  - **It is defined as the line which gives the sum of pressure head $\left(\frac{p}{w}\right)$ and datum head(z) of a fluid flowing in a pipe w.r.t some reference line.**

$$HGL = \frac{P_1}{w} + z_1$$

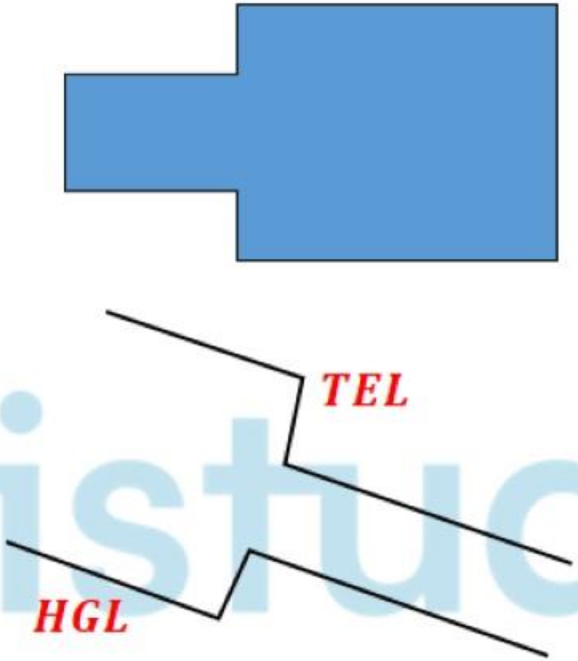
- **TOTAL ENERGY LINE:**
  - **It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a fluid flowing in a pipe w.r.t some reference line.**

$$TEL = \frac{P_1}{w} + \frac{v_1^2}{2g} + z_1$$



HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

- **In Sudden Expansion**



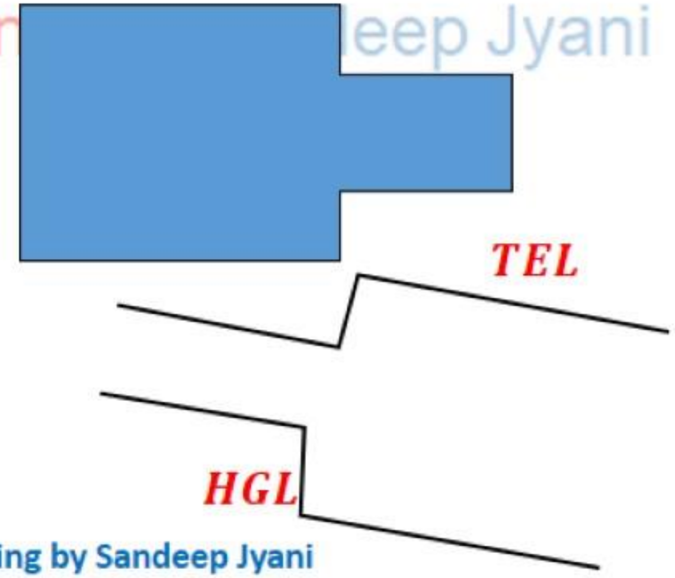
Pressure increases, so HGL increases

Velocity decreases, so TEL decreases

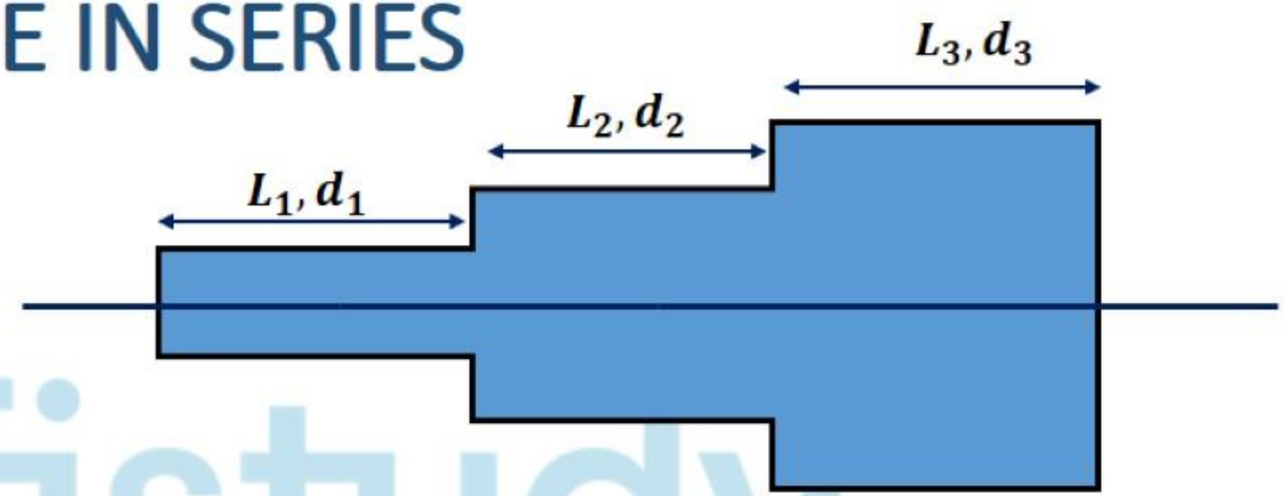
$$HGL = \frac{P_1}{w} + z_1$$

$$TEL = \frac{P_1}{w} + \frac{v_1^2}{2g} + z_1$$

- **In Sudden Contraction**

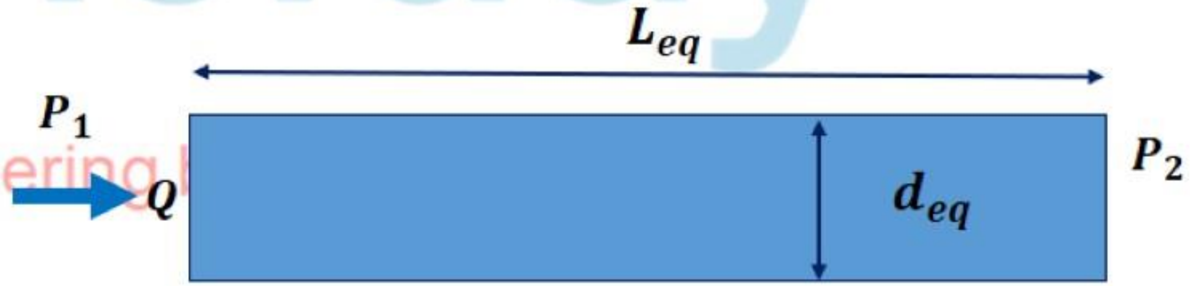


PIPE IN SERIES



$$h_L = h_{L1} + h_{L2} + h_{L3}$$

$$\Rightarrow h_L = \frac{fLQ^2}{12d^5}$$



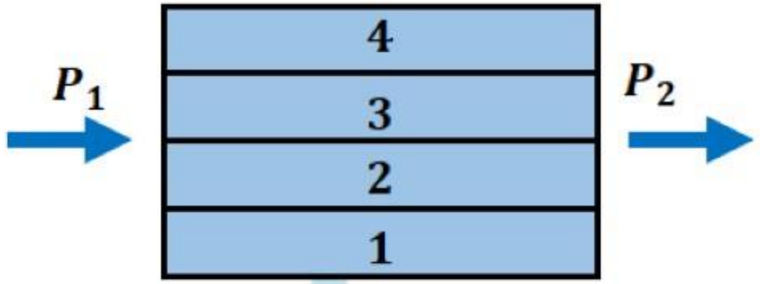
$$\Rightarrow \frac{fL_{eq}Q^2}{12d_{eq}^5} = \frac{fL_1Q^2}{12d_1^5} + \frac{fL_2Q^2}{12d_2^5} + \frac{fL_3Q^2}{12d_3^5}$$

$$\Rightarrow \frac{L_{eq}}{d_{eq}^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

Duputis Equation

PIPE IN PARALLEL

For same pressure difference, head loss should be same



$$h_{eq} = h_{L1} = h_{L2} = h_{L3} = h_{L4}$$

And

$$Q_{eq} = Q_1 + Q_2 + Q_3 + Q_4$$

$$\Rightarrow \frac{fL_{eq}Q^2}{12d_{eq}^5} = \frac{fL\left(\frac{Q}{n}\right)^2}{12d^5}$$

$$\Rightarrow \frac{L_{eq}}{d_{eq}^5} = \frac{L}{n^2d^5}$$

$If\ L = Leq$

$$n^{\frac{2}{5}}d = deq$$

Turbulent Flow

- Viscosity is a property of fluid but eddy viscosity is not.
- Eddy Viscosity is 'flow' characteristic of fluid
- In a turbulent flow, there is a continuous mixing of fluid particles and hence the velocity fluctuate with respect to time and position.
- Boussinesq developed the equation for turbulent Shear Stress

$$\boxed{\tau_{turb} = \mu \frac{du}{dy} + \eta \frac{du}{dy}}$$

μ = ***viscosity*** **(fluid characteristic)**

η = ***eddy viscosity*** **(flow characteristic)**

eddy viscosity **is flow characteristic and depends on flow condition and it is not present in Laminar Flow**

- As it is difficult to find eddy viscosity, this equation is not used in practice
- Reynold developed the equation for turbulent shear Stress

Turbulent Flow

- Reynold developed the equation for turbulent shear Stress

$$\tau = \rho U'V'$$

$$U' = fluctuating\ velocity\ in\ x - direction$$
$$V' = fluctuating\ velocity\ in\ y - direction$$

Pradalt Mixing Length Theory

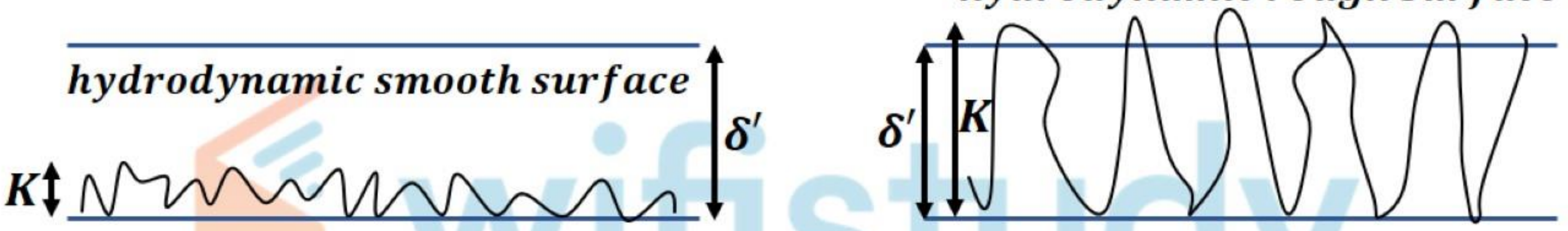
- According to Pradalt mixing length theory, Pradalt mixing length is that length in transverse direction where in fluid particles, after colliding, loses excess momentum and reach the same momentum as Local Environment.
- It is similar to mean free path in gases

$$\textit{Pradalt assumed } U' = V' = l\frac{du}{dy}$$

$$\tau = \rho U'V'$$

$$\boxed{\tau = \rho l^2 \left(\frac{du}{dy}\right)^2}$$

Hydrodynamic Smooth and Rough Surface



$\delta' > K$ $\delta' < K$

$K = average\ height\ of\ roughness$

$\delta' = laminar\ sub\ layer\ thickness$

- $\frac{K}{\delta'} < 0.25 \quad \Rightarrow Smooth$
- $\frac{K}{\delta'} > 6 \quad \Rightarrow Rough$
- $0.25 < \frac{K}{\delta'} < 6 \quad \Rightarrow Transition$

FLOW IN OPEN CHANNEL

- **Open-channel flow is a flow of liquid (basically water) in a conduit with a free surface.**
- **On this a surface on which pressure is equal to local atmospheric pressure.**
- **Classification :**
  1. **Steady flow and unsteady flow**
  2. **Uniform flow and non uniform flow**
  3. **Laminar flow and turbulent flow**
  4. **Sub critical, critical and super critical flow**

CLASSIFICATION

1. **Steady flow and unsteady flow-**
   - If the flow characteristics such as velocity of flow, depth of flow, rate of flow at any point in an open channel remains constant, then it is steady flow.
   - If the flow characteristics such as velocity of flow, depth of flow, rate of flow at any point in an open channel changes w.r.t time, then it is unsteady flow

velocity	*rate of flow*	*depth of flow*
$\frac{\partial V}{\partial t} = 0$	$\frac{\partial Q}{\partial t} = 0$	
$\frac{\partial V}{\partial t} \neq 0$	$\frac{\partial Q}{\partial t} \neq 0$	$\frac{\partial y}{\partial t} \neq 0$

2. **Uniform flow and non uniform flow**
   - If for a given length of channel, the velocity of flow, depth of flow, slope of channel and cross section remains constant, then it is uniform flow.
   - If for a given length of channel, the velocity of flow, depth of flow, slope of channel and cross section do not remain constant, then it is non uniform flow.

$$\frac{\partial V}{\partial s} = 0 \qquad \frac{\partial y}{\partial s} = 0$$

$$\frac{\partial V}{\partial s} \neq 0 \qquad \frac{\partial y}{\partial s} \neq 0$$

CLASSIFICATION

3. Laminar flow and turbulent flow –

- If Reynolds no. is less than 500, in an open channel then flow is laminar.
- If Reynolds no. is more than 2000, in an open channel then flow is turbulent in open channel flow.
- If R_e lies between 500 and 2000, it is in transition state

$$R_e = \frac{\rho V R}{\mu}$$

$\rho = density$

$R = hydraulic\ radius\ or\ hydraulic\ mean\ depth$

$$R = \frac{Cross\ Section\ area\ of\ flow\ normal\ to\ the\ direction\ of\ flow}{Wetted\ perimeter}$$

CLASSIFICATION

4. Sub critical, critical and super critical flow

$$F_e = \frac{V}{\sqrt{gD}}$$

$F_e < 1; sub\ critical\ flow$

$F_e = 1; critical\ flow$

$F_e > 1; super\ critical\ flow$

$$D - hydraulic\ depth = \frac{area}{top\ width}$$

DISCHARGE THROUGH OPEN CHANNEL

- **By Chezy's Formula-**

$$Q = AC\sqrt{mi}$$

$$m = \text{hydraulic mean depth} = \frac{Area\ of\ flow}{Wetted\ Perimeter}$$

$$C = Chezy's\ Constant$$

$$i = bed\ slope$$

$$A = area\ of\ channel$$

Dimension of C = $M^0L^{1/2}T^{-1}$

EMPERICAL FORMULA FOR CHEZY'S CONSTANT

1. **Bazin Formula** $\Rightarrow C = \frac{157.6}{1.81+\frac{K}{\sqrt{m}}}$

m = hydraulic mean depth

K – Bazin's constant depending upon the roughness of surface of channel

2. **Ganguillet-Kutter Formula** $\Rightarrow C = \frac{23+\frac{0.00155}{i}+\frac{1}{N}}{1+\left(23+\frac{0.00155}{i}\right)\frac{N}{\sqrt{m}}}$

N – Kutter's constant (roughness coefficient)

m = hydraulic mean depth

$i = bed\ slope$

3. **Manning Formula** $\Rightarrow C = \frac{1}{N} m^{1/6}$

N – Manning's constant m = hydraulic mean depth

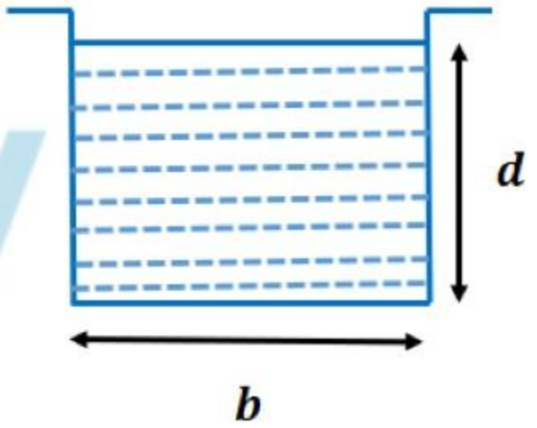
MOST ECONOMICAL SECTIONS OF OPEN CHANNELS

1. Rectangular:

- The condition for most economical section is for a given area, the perimeter should be minimum.

a) $b = 2d$ (*width is twice the depth*)

b) $m = \frac{d}{2}$ (*hydraulic depth is half the depth of flow*)



2. Trapezoidal:

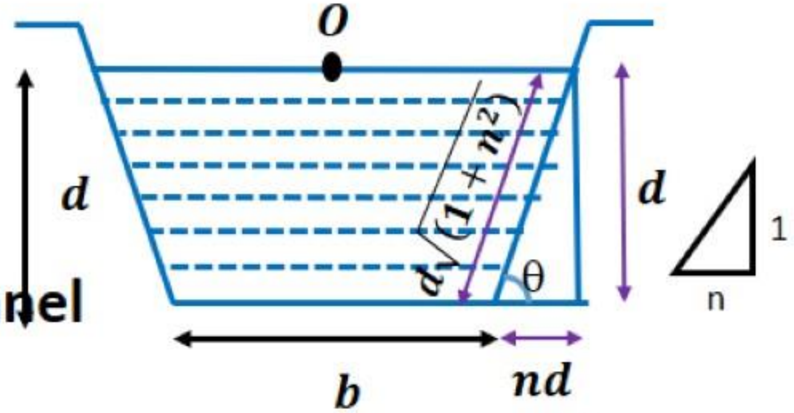
- The condition for most economical section

a) $\frac{b+2nd}{2} = d\sqrt{(1+n^2)}$

b) $m = \frac{d}{2}$

c) Best side slope $\theta = 60^\circ$

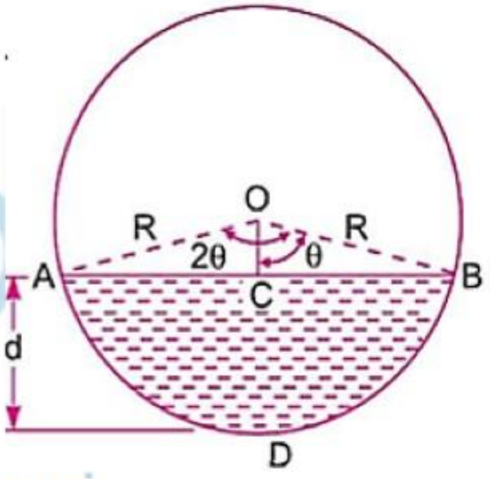
d) A semi circle drawn with O as centre and radius equal to depth of flow will touch the three sides of channel



MOST ECONOMICAL SECTIONS OF OPEN CHANNELS

3. Circular:

1. **Condition for Maximum Velocity –**
   - **For maximum velocity, the depth of water in circular channel should be equal to 0.81times the diameter of circular channel.**
   - **For maximum velocity, the hydraulic mean depth is 0.3times the diameter of circular channel.**
2. **Condition for Maximum Discharge –**
   - **For maximum discharge, the depth of flow in circular channel should be equal to 0.95times the diameter of circular channel.**



SPECIFIC ENERGY

The total energy of a flowing liquid per unit weight is given by

$$total\ energy = z + h + \frac{V^2}{2g}$$

$V = mean\ velocity\ of\ flow$

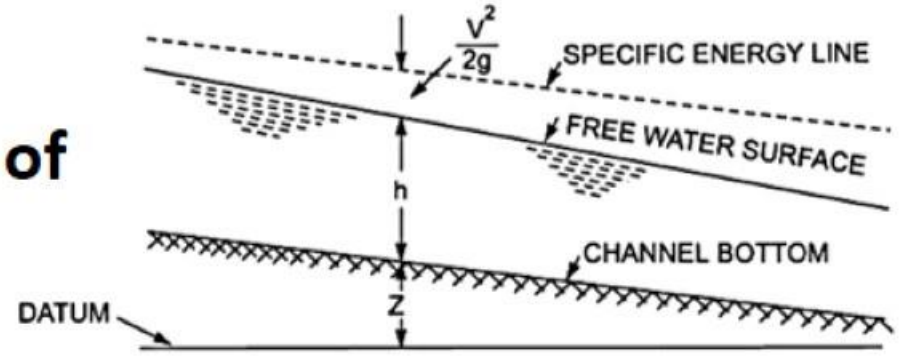
$h = height\ of\ channel$

z = height of bottom of channel above datum

If the channel bottom is taken as datum, total energy of a flowing liquid per unit weight is

$$specific\ energy = h + \frac{V^2}{2g}$$

SPECIFIC ENERGY – energy per unit weight of the liquid w.r.t bottom of the channel



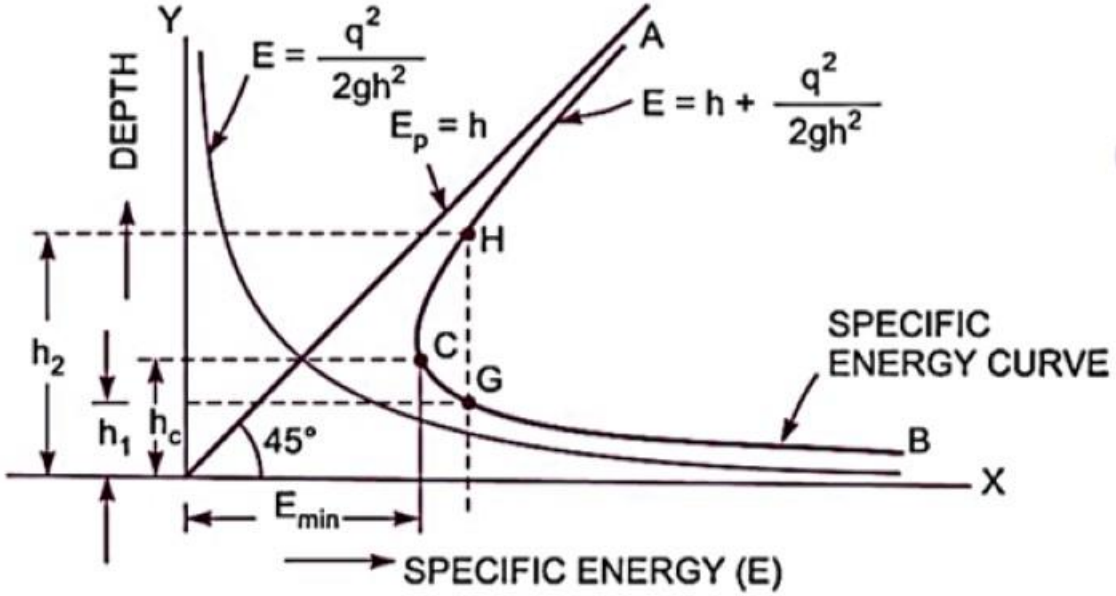
SPECIFIC ENERGY CURVE

SPECIFIC ENERGY CURVE – The curve which shows the variation of specific energy with depth of flow.

$$E = h + \frac{q^2}{2gh^2} = E_p + E_k$$

q = *discharge per unit width*

h = *depth of flow*



CRITICAL DEPTH (h_c):

- The depth of flow of water at which the specific energy is minimum.

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

CRITICAL VELOCITY (V_c):

- The velocity of flow at critical depth.

$$V_c = \sqrt{gh_c}$$

Sub critical, critical and super critical flow

$$Froude\ No. = \frac{V_c}{\sqrt{gh_c}}$$

$F_e < 1; sub\ critical\ flow$

$F_e = 1; critical\ flow$

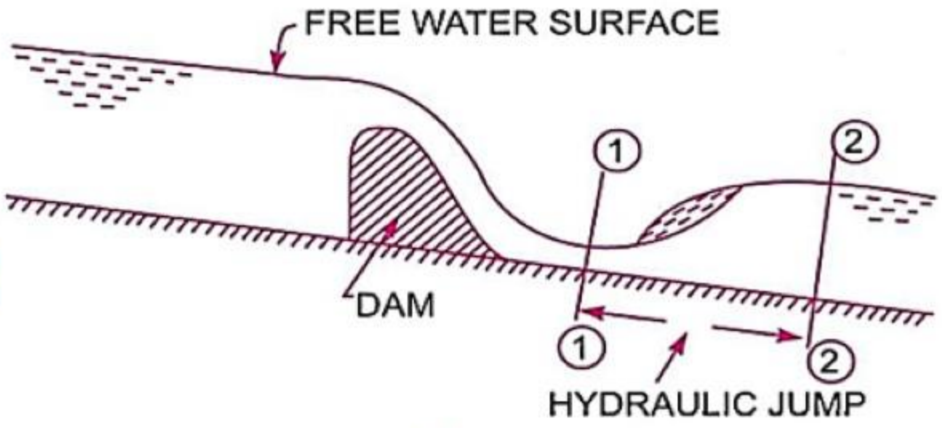
$F_e > 1; super\ critical\ flow$

CONDITION FOR MAXIMUM DISCHARGE FOR A GIVEN SPECIFIC ENERGY:

$$E = \frac{3h}{2}$$

HYDRAULIC JUMP

- HYDRAULIC JUMP is the rise of water which takes place due to transformation of unstable shooting flow (super critical flow) to stable shooting flow (sub critical flow)
- When a hydraulic jump takes place, loss of energy due to eddy formation and turbulence occurs.



d_1 = depth of flow before hydraulic jump

d_2 = depth of flow after hydraulic jump

$$depth\ of\ hydraulic\ jump = d_2 - d_1$$

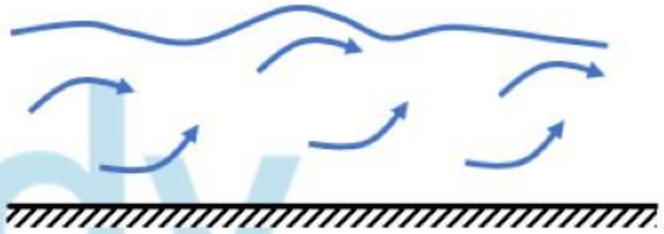
$$depth\ of\ hydraulic\ jump(in\ terms\ of\ froude\ no.) = \frac{d_1}{2}\left(\sqrt{1 + 8(F_e)_1^2} - 1\right)$$

$$energy\ for\ loss\ due\ to\ hydraulic\ jump = \frac{(d_2 - d_1)^3}{4d_1d_2}$$

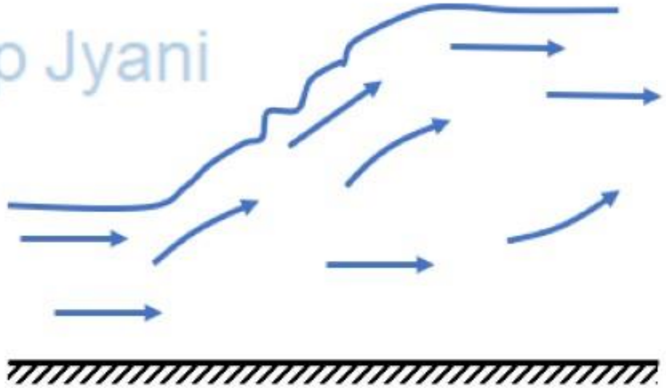
HYDRAULIC JUMP

- Froude's number of incoming flow should be greater than 1 for Hydraulic jump to occur

1. Undular Jump $(1.0 < F_1 \leq 1.7)$
    - Water surface is undulating with small ripples on the surface

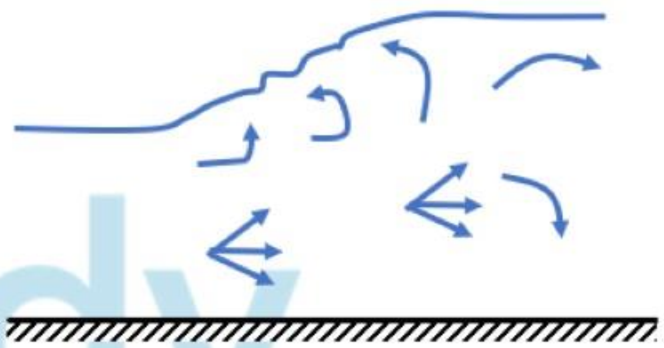


2. Weak Jump$(1.7 < F_1 \leq 2.5)$
    - A series of small rollers forms on the jump surface but the downstream water surface remains smooth

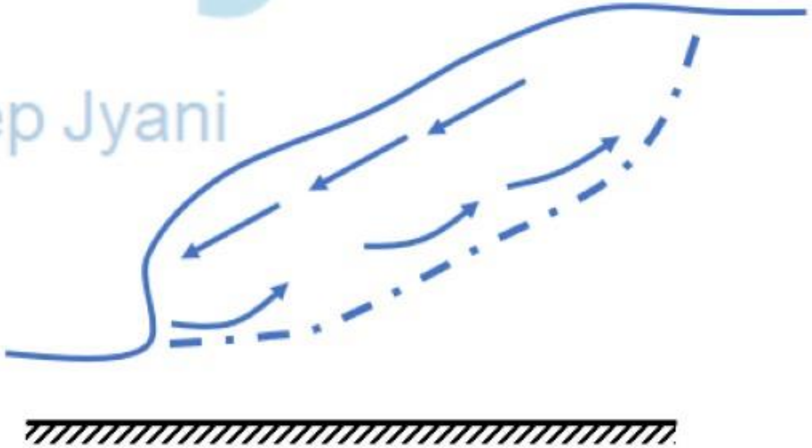


HYDRAULIC JUMP

3. Oscillating Jump $(2.5 < F_1 \leq 4.5)$
   - Entering jet of water oscillates in a random manner between bed and surface.
   - These oscillations are very common in canals and can travel considerable distances and damaging earthen banks

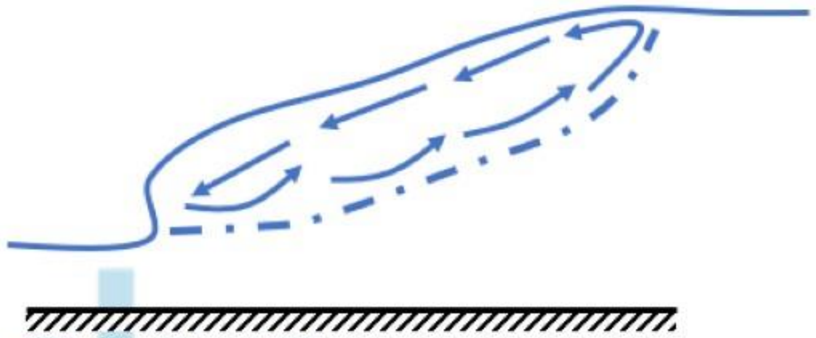


4. Steady Jump$(4.5 < F_1 \leq 9)$
   - The jump is well established, the roller and jump action is fully developed to cause appreciable energy loss (downstream surface smooth)
   - It is always preferred to have steady jump



HYDRAULIC JUMP

5. Strong or Choppy Jump ($9 < F_1$)
   - During this jump, water surface is very rough and choppy, which continues downstream for a long distance
   - Energy dissipation is very efficient



Hydraulic Machinery

- Hydraulic Turbine:
  - Hydraulic turbine is a prime mover that uses the energy of flowing water and converts it into the mechanical energy in the form of rotation of the runner
- Note:- Hydraulic turbine has very high efficiency as compared to steam turbine and other turbines.
- Dam – above 30 m height
- Barrage – below 30m height
- Barrage is used for irrigation purpose and water supply and Dam is used for Electricity generation

Advantage of Hydropower

1. Uses Renewable energy, water as a fuel.
2. Running cost is very low.
3. No pollution, no ash, no greenhouse gases
4. Can be switched on and off in a very short time, and can be used as Peak-Load plant.
5. Simple in operation and system reliability is high
6. Equipment has a greater life expectancy and lasts 50 year and more.
7. Higher efficiency over a large range of load.
8. Do not require high skilled worker and manpower requirement is low
9. Other benefits like irrigation, flood control, afforestation, navigation and aquaculture

Disadvantage of Hydropower plant

1. Capital is intensive and low rate of return
2. Gestation period is large
3. Power generation is dependent on quantity of water available which may very season to season & year to year.
4. Plants are far away from the load centre and require long transmission line
5. Large hydropower plants disturb the ecology of area.
6. Seismic vibrations with large dam like Koyna dam
7. Pollutant content in Indian river is too high & that creates lot of problem to hydrostation

Selection of Site for Hydropowerplant

1. Availability of water
2. Availability of water head
3. Availability of water storage capacity
4. Accessibility of site
5. Distance from load centre
6. Type of land site.

Hydraulic Turbines

- Hydraulic turbines convert hydraulic energy into mechanical energy
- Mechanical energy is converted into Electrical energy by electric generator coupled by shaft of turbine

- An impulse turbine is a turbine that is driven by high velocity jets of water or steam from a nozzle directed on to vanes or buckets attached to a wheel.
- The resulting impulse (as described by Newton's second law of motion) spins the turbine and removes kinetic energy from the fluid flow
- Energy available is only kinetic energy

- A reaction turbine is a type of turbine that develops torque by reacting to the pressure or weight of a fluid.
- Energy available is kinetic as well as Pressure

Hydraulic Turbines

1. Impulse Turbines
   a) Pelton Wheel Turbine
   b) Turbo
   c) Cross Flow
2. Reaction Turbine
   a) Propeller
      i. Kaplan
      ii. Fixed Blade
      iii. Tubler
      iv. Bulb Turbine
   b) Francis
   c) Deriaz

Turbine	Turbine Type	Head	Specific Speed Rpm	Flow type	Discharge	Speed Ratio
Pelton Single Jet	Impulse	High Head 300-2000m	10-35	Tangential Flow	Low	0.43 to 0.48
Pelton Multiple Jet			35-60			
Francis	Reaction	Medium Head 60-300mm	60-300	Mixed Flow (radially in and axially out)	Medium	0.6 to 0.9
Kaplan (adjustable blades, mostly used in India)	Reaction	Low Head Upto 60m	300-1000	Axial Flow	high	1.4 to 2

Relationship between Speed, pole and Frequency

$$N = \frac{120f}{P}$$

N= rotational speed

f =frequency

P=number of poles in generator

For very low rpm, no of poles is very large and it is not feasible from design point of view, so at low head in should not install a turbine

Types of Hydroelectric Power Scheme

1. Reservoir Scheme (large reservoir)/ storage scheme.
2. Run off River scheme (Very low reservoir, used only for water supply purpose)
3. Pump storage scheme
4. Pump storage scheme means when electricity has no need then we reverse the turbine and pump the water to restore the head.

Hydraulic Efficiency

$$\eta_h = \frac{Runner\ power}{water\ power\ (wQH)}$$

Mechanical Efficiency

$$\eta_{mechanical} = \frac{shaft\ power}{Bucket\ power}$$

Volumetric efficiency

$$\eta_{volumetric} = \frac{Volume\ of\ water\ actualy\ striking\ bucket}{volume\ of\ water\ supplied\ by\ jet.}$$

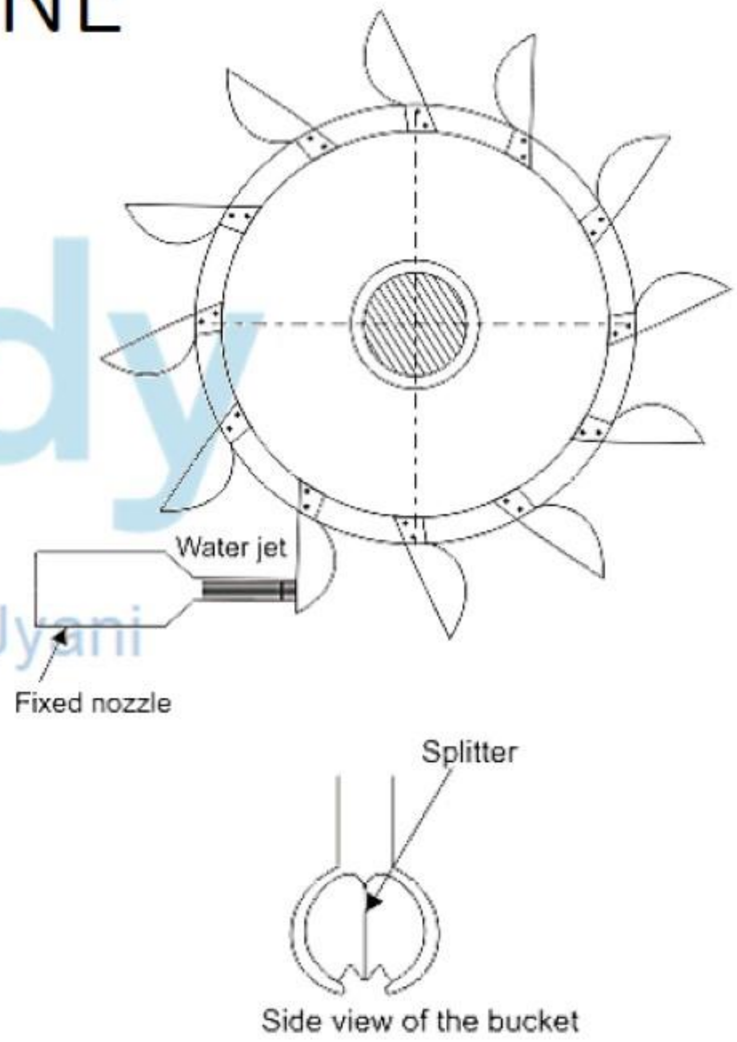
Overall efficiency

$$(n_o) = \frac{shaft\ power}{Hydraulic\ power}$$

$$\eta_{all} = \eta_{hydraulic} \times \eta_{mechanical} \times \eta_{volumetric}$$

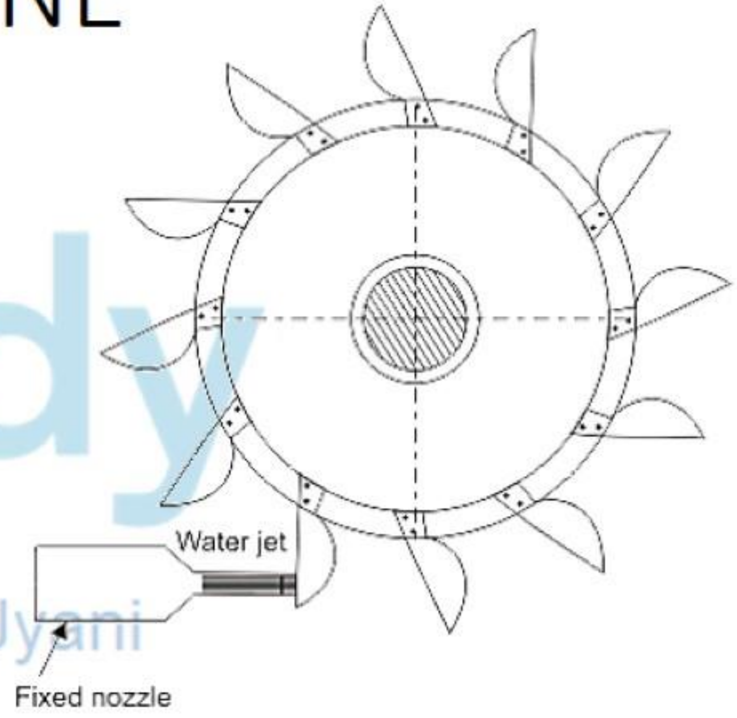
PELTON WHEEL TURBINE

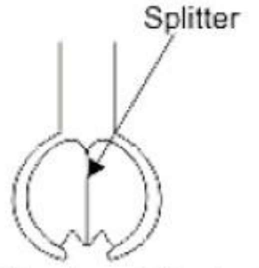
- The Pelton wheel or Pelton Turbine is a tangential flow impulse turbine and is used for high head and low discharge
- In an impulse turbine, the pressure energy of water is converted into kinetic Energy when passed through nozzle, and from the high velocity jet of water the water jet is used for driving the wheel.
- The rotor consists of a large circular disk or wheel on which a number of spoon shaped bucket are spaced uniformly round its periphery. The wheel is driven by jets of water being discharged at atmosphere pressure from pressure nozzle.



PELTON WHEEL TURBINE

- The nozzle are mounted so that each direct jet hits along a tangent to circle through the centre of bucket.
- Down the centre of each bucket, there is splitter which divides the jet into 2 equal stream which flow round the smooth inner surface of bucket, and leaves the bucket with a relative velocity almost opposite in the direction to original jet.
- For maximum change in momentum of the fluid & have for maximum driving force on the wheel, the direction of water jet should be 180°.
- Note: Splitter is provided for smooth flow on the bucket and for removing eddy formation.

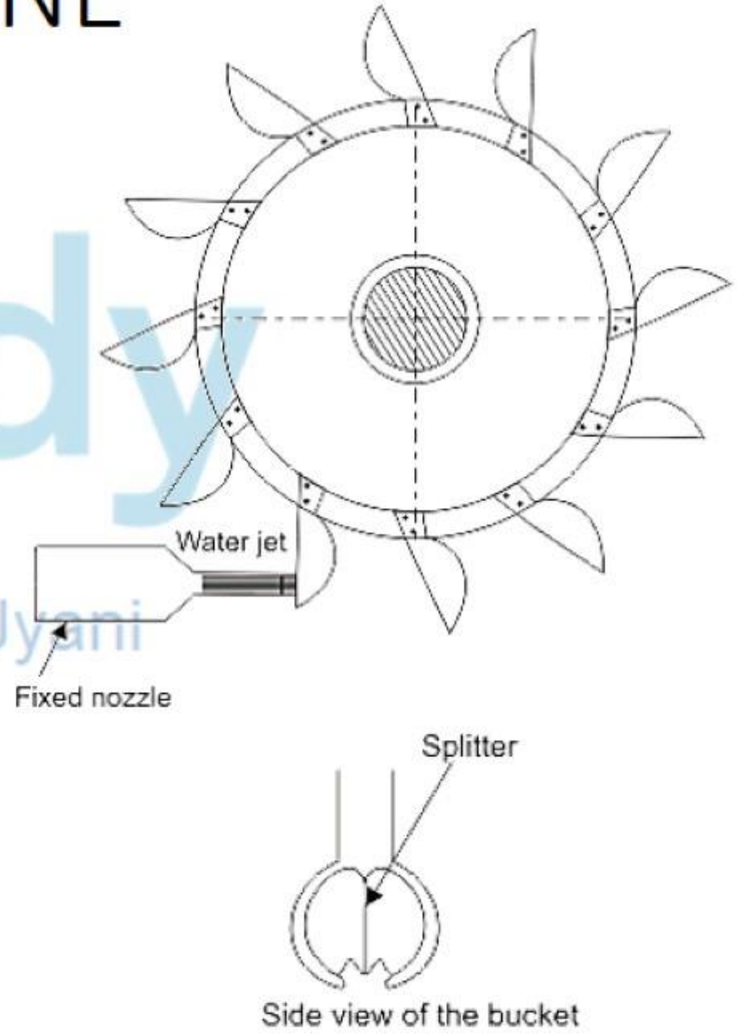




Side view of the bucket

PELTON WHEEL TURBINE

- In practice, however the deflection is limited to about 165° so that the water leaving a bucket may not hit back the following bucket
- The number of jets is not more than 2 for horizontal shaft turbine and is limited to 6 for vertical shaft turbine.



Design Aspects of Pelton Wheel

- Velocity of jet $V_1 = \sqrt{2g\,H}$, If losses are not considered
- $V_1 = C_v\sqrt{2g\,H}$, if losses are considered
- $u = \phi\sqrt{2g\,H}$ velocity of vane
- $u = \frac{\pi DN}{60}$ $where\ D\ is\ mean\ diameter$
- Speed ratio = $\frac{u}{v_1} = 0.43\ to\ 0.48$
- Jet ratio $m = \frac{D(peripheral\ dia\ of\ impeller)}{d\ (dia\ of\ jet)}$ (between 11 to 16)
- Number of buckets= $15 + 0.5m$

$$= 15 + 0.5\frac{D}{d}$$

GOVERNING OF PELTON WHEEL

- We cannot immediately stop the water in the nozzle because in that case water hammering will be induced and pipe will burst, so we use nozzle spear system to close water flow gradually.
- But sometime when we want to immediately stop pelton wheel in case of emergency we use DEFLECTOR PLATE to deflect the water which is going to strike pelton wheel.

- **SPECIFIC SPEED:**
  - **The speed of a geometrically similar turbine which would develop unit power when working under a unit head.**

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

H=head P=power N=rpm

  - **It plays an important role in the selection of the type of turbine.**
  - **The suitability of turbine for a particular project depends on:**
    - **Head of water**
    - **Rotational speed**
    - **Power development**
  - **Lesser the specific speed, less will be the size of runner.**
  - **Higher specific speed, lower efficiency of turbine**

UNIT QUANTITIES

- **For a single unit, when head is changed, the speed of ungoverned turbine changes.**
- **If various quantities are reduced to a theoretical 1m head, the comparison of performance data and computation of experimental values on a single unit are considerably simplified.**
- **Unit quantities are required for preventing the performance of geometrically similar turbine, independent of the actual head, discharge and power output. The unit characteristic prove quite helpful.**
- **Geometrically similar turbine will have the same characteristics.**

UNIT SPEED

$$u = \frac{\pi D N}{60}$$

$$u \propto N$$

Take D= constant for diameter of impeller

$$u \propto V = \sqrt{2gH}$$

$$\frac{u}{V} = \emptyset$$

$$u \propto \sqrt{H}$$

$$N \propto \sqrt{H}$$

$$N = K\sqrt{H}$$

$$for\ H = 1;\ N = N_u$$

$$K = N_u$$

$$N = N_u\sqrt{H}$$

$$N_u = \frac{N}{\sqrt{H}}\ (unit\ speed)$$

UNIT DISCHARGE

$$Q = AV$$

$$Q \propto V \propto \sqrt{H}$$

$$Q \propto \sqrt{H}$$

$$Q = K\sqrt{H}$$

$$for\ H = 1;\ Q = Q_u$$

$$Q = Q_u\sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}}\ (unit\ discharge)$$

UNIT POWER

$$P = w\,QH$$

$$P \propto QH$$

$$P \propto H\sqrt{H}$$

$$P \propto H^{3/2}$$

$$P = KH^{3/2}$$

$$for\ H = 1;\ P = P_u and\ K = P_u$$

$$P = P_u H^{3/2}$$

$$P_u = \frac{P}{H^{3/2}}\ \ (unit\ power)$$

MODULUS LAW AND MODULUS ANALYSIS OF TURBINE

$$\sqrt{H} \propto \frac{\pi D N}{60}$$

$$\sqrt{H} \propto DN$$

$$H \propto D^2 N^2$$

$$H = K D^2 N^2$$

$$K = \frac{H}{D^2 N^2}$$ constant

$$\left(\frac{H}{D^2 N^2}\right)_{model} = \left(\frac{H}{D^2 N^2}\right)_{prototype} = C_H (head\ coefficient)$$

MODULUS LAW AND MODULUS ANALYSIS OF TURBINE

$$Q = AV$$
$$Q = d^2V$$
$$V \propto u = \frac{\pi DN}{60}$$
$$V \propto D^3N$$
$$Q = kD^3N$$
$$k = \frac{Q}{D^3N}$$

$$\left(\frac{Q}{D^3N}\right)_{model} = \left(\frac{Q}{D^3N}\right)_{prototype} = flow\ coefficient$$

MODULUS LAW AND MODULUS ANALYSIS OF TURBINE

$$P = h_0 w Q H$$
$$P \propto QH$$
$$P \propto D^3 N H$$
$$u \propto V = \sqrt{2gH} \propto \frac{\pi D N}{60}$$
$$H \propto D^2 N^2$$
$$P \propto D^3 N \quad D^2 N^2$$
$$P \propto D^5 N^3$$

$$\left(\frac{P}{D^5 N^3}\right)_{model} = \left(\frac{P}{D^5 N^3}\right)_{prototype} = power\ coefficient$$

Que 74. Pascal's law states that pressure at any point in a fluid at rest has

a) Different magnitude in all directions
b) Same magnitude, in all directions
c) Zero magnitude in all directions
d) None of the above

Que 74. Pascal's law states that pressure at any point in a fluid at rest has

a) Different magnitude in all directions

b) Same magnitude, in all directions

c) Zero magnitude in all directions

d) None of the above

Pascal's Law

- **It states that pressure or intensity of pressure at a point in a static fluid is equal in all directions**

Que 75. The magnitude of the buoyant force can be determined by :

a) Newton’s law of viscosity
b) Archimedes principle.
c) Principles of moments
d) None of the above

Que 75. The magnitude of the buoyant force can be determined by :

a) Newton's law of viscosity
b) Archimedes principle.
c) Principles of moments
d) None of the above

Que 76. Flow of fluid takes place due to its:

a) Viscosity

b) Compressibility

c) Surface tension

d) Deformation under shear force

Que 76. Flow of fluid takes place due to its:

a) Viscosity

b) Compressibility

c) Surface tension

d) Deformation under shear force

Que 77. The characteristic of an ideal fluid is

a) One which satisfies continuity equation

b) One which flows with least friction

c) One which obeys Newton’s law of Viscosity

d) Frictionless and incompressible

Que 77. The characteristic of an ideal fluid is

a) One which satisfies continuity equation

b) One which flows with least friction

c) One which obeys Newton's law of Viscosity

d) <u>Frictionless and incompressible</u>

Que 78. The pressure intensity at a point in a fluid is 3.9 N/cm^2. Find the corresponding height of fluid when the fluid is water

Que 79. The pressure intensity at a point in a fluid is 3.9 N/cm^2. Find the corresponding height of fluid when the fluid is oil of specific gravity 0.9.

80. The ratio of specific weight of a liquid to the specific weight of pure water at a standard temperature is called

a) Compressibility of liquid
b) Surface tension of liquid
c) Density of liquid
d) Specific gravity of liquid

80. The ratio of specific weight of a liquid to the specific weight of pure water at a standard temperature is called

a) Compressibility of liquid

b) Surface tension of liquid

c) Density of liquid

d) Specific gravity of liquid

81. Bulk modulus of fluid is the ratio of

a) Shear stress to shear strain

b) Increase in volume to the viscosity of fluid

c) Increase in pressure to the volumetric strain

d) Critical velocity to the velocity of fluid

81. Bulk modulus of fluid is the ratio of

a) Shear stress to shear strain

b) Increase in volume to the viscosity of fluid

c) Increase in pressure to the volumetric strain

d) Critical velocity to the velocity of fluid

82. The buoyancy depends upon the

a) Pressure with which the liquid is displaced

b) Weight of the liquid displaced

c) Viscosity of the liquid

d) Compressibility of the liquid

82. The buoyancy depends upon the

a) Pressure with which the liquid is displaced

b) Weight of the liquid displaced

c) Viscosity of the liquid

d) Compressibility of the liquid

83. Manometer is a device used for measuring

a) Velocity

b) Pressure

c) Density

d) Discharge

83. Manometer is a device used for measuring

a) Velocity

b) Pressure

c) Density

d) Discharge

84. Capillarity rise is due to

I. Surface tension

II. Cohesion

III. Viscosity

IV. Weight density of liquid

a) II, III

b) III

c) I

d) II, III, V

84. Capillarity rise is due to
I. Surface tension
II. Cohesion
III. Viscosity
IV. Weight density of liquid
a) II, III
b) III
c) I
d) II, III, V

85. Pressure in terms of metres of oil (specific gravity = 0.9) equivalent to 4.5 m of water is

a) 4.05

b) 5.0

c) 3.6

d) 0.298

86. Pressure in terms of metres of oil (specific gravity = 0.9) equivalent to 4.5 m of water is

a) 4.05

b) 5.0

c) 3.6

d) 0.298

86. Capillary rise is a phenomenon that is attributed to the following properly of fluid

a) Vapour pressure
b) Viscosity
c) Density
d) Surface tension

86. Capillary rise is a phenomenon that is attributed to the following properly of fluid

a) Vapour pressure
b) Viscosity
c) Density
d) Surface tension

87. Specific gravity has a unit ;

a) g/cc

b) kg/m^3

c) N/m^3

d) No unit i.e., dimensionless

87. Specific gravity has a unit ;

a) g/cc

b) kg/m^3

c) N/m^3

d) No unit i.e., dimensionless

88. A fluid, which is incompressible and is having no viscosity is

a) Ideal fluid
b) Real fluid
c) Newtonian fluid
d) Non Newtonian fluid

72. A fluid, which is incompressible and is having no viscosity is

a) Ideal fluid
b) Real fluid
c) Newtonian fluid
d) Non Newtonian fluid

Que 89. The relationship between atmospheric pressure (P_{atm}), gauge pressure (P_{gauge}) and absolute pressure (P_{abs}) is given by:

a) $P_{atm} = P_{abs} - P_{gauge}$

b) $P_{abs} = P_{atm} + P_{gauge}$

c) $P_{abs} = P_{atm} - P_{gauge}$

d) $P_{atm} = P_{abs} + P_{gauge}$

Que 89. The relationship between atmospheric pressure (P_{atm}), gauge pressure (P_{gauge}) and absolute pressure (P_{abs}) is given by:

a) $P_{atm} = P_{abs} - P_{gauge}$

b) $P_{abs} = P_{atm} + P_{gauge}$

c) $P_{abs} = P_{atm} - P_{gauge}$

d) $P_{atm} = P_{abs} + P_{gauge}$

Que 90. For stability of floating bodies, the metacentre should be

a) Above the centre of gravity
b) Below the centre of gravity
c) Above the centre of buoyancy
d) Below the centre of buoyancy

Que 90. For stability of floating bodies, the metacentre should be

a) Above the centre of gravity

b) Below the centre of gravity

c) Above the centre of buoyancy

d) Below the centre of buoyancy

METACENTRE

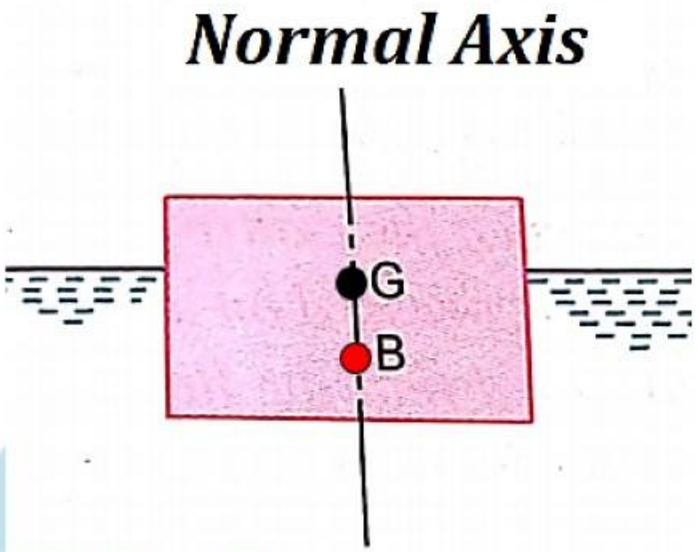
- **It is the point of intersection of normal axis with the new line of action of force of buoyancy, when a small displacement is given.**
- **It is a point about which a body starts to oscillate when it is tilted by small angle.**

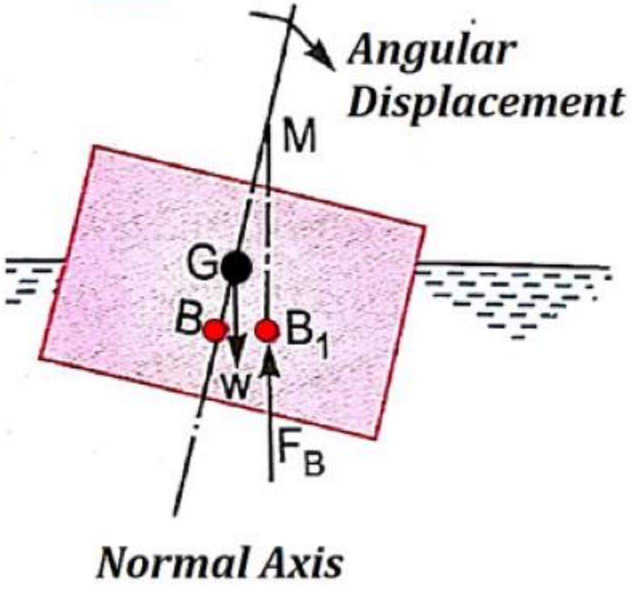
$$G.M. = B.M - B.G$$

$$GM = \frac{I}{V} - BG$$

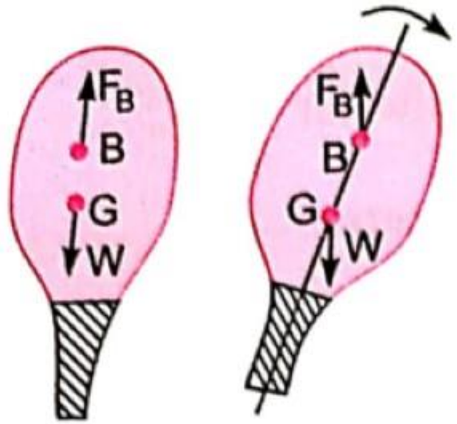
- **Where** $I_{xx} = \frac{bd^3}{12}$; $I_{yy} = \frac{db^3}{12}$; $\frac{I_{min}}{V}$ **= metacentric radius**

- **For stability G.M. should be positive, i.e., B.M>B.G**

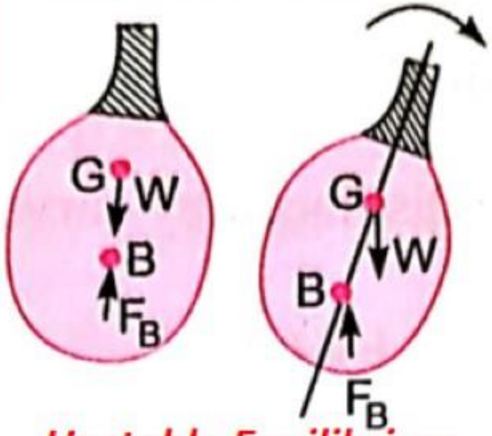




- **STABILITY CONDITION FOR FULLY SUBMERGED BODY:**
  - **A completely submerged body will be in *Stable Equilibrium* when centre of buoyancy is above centre of gravity.**
  - **If centre of buoyancy is below centre of gravity, then completely submerged body will be in *Unstable Equilibrium*.**
  - **If centre of buoyancy and centre of gravity coincides, then fully submerged body will be in *Neutral Equilibrium.***



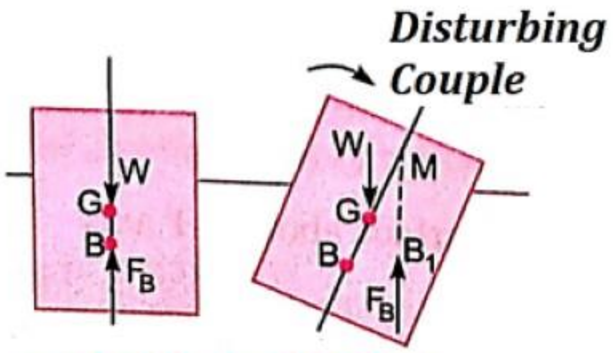
Stable Equilibrium



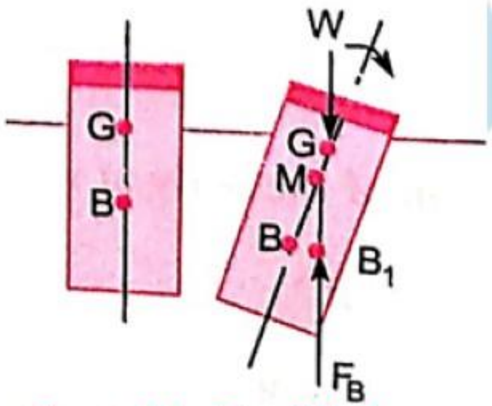
Unstable Equilibrium

Neutral Equilibrium

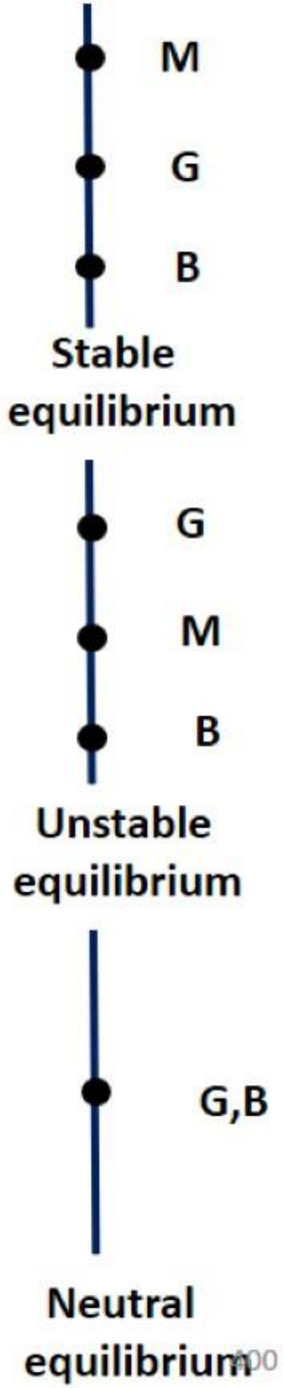
- **STABILITY CONDITION FOR PARTIALLY SUBMERGED BODY:**
  - **A partially submerged body will be in *Stable Equilibrium* when metacentre is above centre of gravity.**
  - **If metacentre is below centre of gravity, then partially submerged body will be in *Unstable Equilibrium.***
  - **If metacentre and centre of gravity coincides, then partially submerged body will be in neutral equilibrium.**
- **The distance between C.G. and metacenter is called metacentric height.**



Stable Equilibrium



Unstable Equilibrium.



Que 91. The point in the immersed body through which the resultant pressure of the liquid may be taken to act is known as

a) Metacentre
b) Centre of pressure
c) Centre of buoyancy
d) Centre of gravity

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a) Metacentre
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Que 92. Surface tension

a) Acts in the plane of interface normal to any line in the surface
b) Is also known as capillarity
c) Is a function of the curvature of the interface
d) Decreases with fall in temperature

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Que 93. The resultant upward pressure of the fluid on an immersed body is called

a) Upthrust
b) Buoyancy
c) Centre of pressure
d) All options are correct

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a) Upthrust
b) Buoyancy
c) Centre of pressure
d) All options are correct

Que 94. Center of pressure on an inclined plane is________.

a) At the centroid
b) Above the centroid
c) Below the centroid
d) At metacentre

Que 94. Center of pressure on an inclined plane is_______.

a) At the centroid
b) Above the centroid
c) Below the centroid
d) At metacentre

Que 95. A body floats in stable equilibrium_____.

a) When its metacentric height is zero

b) When metacentre is above centre of gravity

c) When its center of gravity is below the center of buoyancy.

d) None of these

Que 95. A body floats in stable equilibrium_____.

a) When its metacentric height is zero

b) When metacentre is above centre of gravity

c) When its center of gravity is below the center of buoyancy.

d) None of these

Que 96. When the adhesion between molecules of a fluid is greater than adhesion between fluid and the glass, then the free level of fluid in glass tube dipped in the glass vessel will be__________.

a) Same as the surface of the fluid

b) Lower than the surface of the fluid

c) Higher than the surface of the fluid

d) Dependent on atmospheric pressure

Que 96. When the adhesion between molecules of a fluid is greater than adhesion between fluid and the glass, then the free level of fluid in glass tube dipped in the glass vessel will be__________.

a) Same as the surface of the fluid

b) Lower than the surface of the fluid

c) Higher than the surface of the fluid

d) Dependent on atmospheric pressure

Que 97. Metacentric height is given as the distance between______.

a) The centre of gravity of the body and the metacentre

b) The centre of gravity of the body and the centre of buoyancy

c) The centre of gravity of the body and the centre of pressure

d) Centre of buoyancy and metacentre

Que 97. A U-tube manometer measures

a) Local atmosphere pressure

b) Difference in pressure between two points

c) Difference in total energy between two points

d) Absolute pressure at a point

Que 97. A U-tube manometer measures

a) Local atmosphere pressure

b) <u>Difference in pressure between two points</u>

c) Difference in total energy between two points

d) Absolute pressure at a point

Que 99. Gauge pressure at a point is equal to

a) absolute pressure plus atmospheric pressure

b) absolute pressure minus atmospheric pressure

c) vacuum pressure plus absolute pressure

d) one of the above

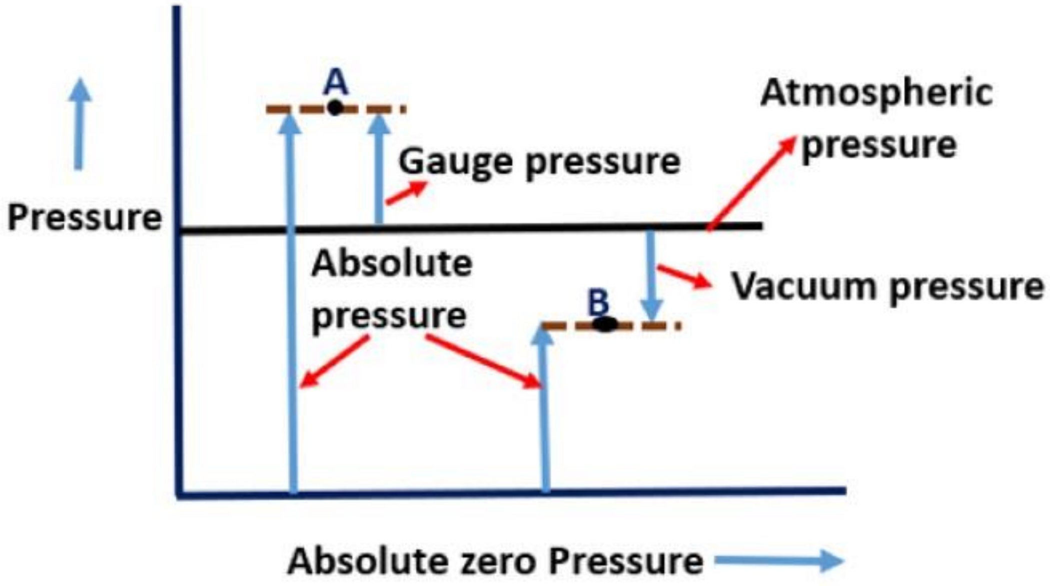
Que 99. Gauge pressure at a point is equal to

a) absolute pressure plus atmospheric pressure

b) <u>absolute pressure minus atmospheric pressure</u>

c) vacuum pressure plus absolute pressure

d) one of the above



Absolute pressure = atmospheric pressure + gauge pressure

Que 100. The difference in pressure head, measured by a mercury water differential manometer for a 20 cm difference of mercury level will be

a) 2.72 m
b) 2.52 m
c) 2.0 m
d) 0. 2 m

Que 100. The difference in pressure head, measured by a mercury water differential manometer for a 20 cm difference of mercury level will be

a) 2.72 m

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THANK YOU