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Topic-wise Test

GATE

(Graduate Aptitude Test in Engineering)

Civil Engineering

TRISHNA KNOWLEDGE SYSTEMS

Pearson

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Preface

Graduate Aptitude Test in Engineering (GATE) is one of the primarily tests for various undergraduate subjects—Engineering/Technology/Architecture and postgraduate level for Science. The GATE examination pattern has undergone several changes over the years—sometimes apparent and sometimes subtle. It is bound to continue to do so with changing technological environment. Apart from giving the aspirant a chance to pursue M.Tech. from institutions like the IITs /NITs, a good GATE score can be highly instrumental in landing the candidate a plush public sector job since many PSUs are recruiting graduate engineers on the basis of their performance in GATE.

Topic-wise Practice Tests GATE Civil Engineering acts as a one-stop assessment tool for all GATE aspirants. The book consists of topic-wise tests on *(1) General Aptitude, (2) Engineering Mathematics, and (3) Civil Engineering*. Five full-length mock tests based on latest GATE pattern are provide at the end of the book, which will help students to check their level of preparation for GATE exam. As a result, this book would serve as an effective tool for GATE aspirant to crack the examination.

Highlights

- Includes (2000+) problems for practice
- Includes (51) topic-wise tests based on latest GATE pattern.
- Detailed solutions given for each test
- Includes tests on General Aptitude and Engineering Mathematics
- Includes 5 full-length mock tests based on latest GATE pattern

Despite of our best efforts, some errors may have inadvertently crept into the book. Constructive comments and suggestions to further improve the book are welcome and shall be acknowledged gratefully.

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PART I GENERAL APTITUDE

PART A
VERBAL ABILITY TEST

PART B
NUMERICAL ABILITY TEST

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PART A
VERBAL ABILITY TEST

Verbal Ability Test 1..... 1.5
Verbal Ability Test 2..... 1.9
Verbal Ability Test 3..... 1.12
Verbal Ability Test 4..... 1.17
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Verbal Ability Test I

Number of Questions: 40 **Time: 30 min**

Directions for questions 1 to 5: Each of the given sentences has four underlined parts. One of them has a mistake. Mark the number of the wrong part as answer.

1. In the Sub-Saharan countries incidences of (A)
Sunstrokes correlates positively withthe level of (B) (C)
Solar radiation. (D)

2. Either you transfer the data which was demanded (A) (B)
nor file a report explaining why you did not (C)
submit the overall annual figures. (D)

3. Neither the judge nor I am ready toannounce (A) (B) (C)
who the winner is. (D)

4. He went about the bad phase in his career with (A) (B) (C)
philosophical clam (D)

5. A score of apple is purchased by him (A) (B) (C)
for his consumption. (D)

Directions for questions 6 to 10: Read each sentence to find out whether there is any error in it. The error, if any, will be in one part of the sentence. Identify the error and mark the number of the erroneous part as your answer.

6. All the members / of the club / was present / (A) (B) (C)
at the special meeting. (D)

7. Over the course of the twentieth century/ (A)
the internal combustion engine / (B)
has replaced the horse / to the basic means of transport. (C) (D)

8. We must / never give up with /trying to enhance/ (A) (B) (C)
the quality of life. (D)

9. It will profit a man nothing / if he was / (A) (B)
to gain the world / and lose his own soul. (C) (D)

10. Injustice and discrimination / can never be / (A) (B)
tolerated by / no one. (C) (D)

11. Alas! / How lovely / and fragrant / (A) (B) (C)
these flowers are! (D)

12. Your story is / so ridiculous that / it could not/ (A) (B) (C)
be believed. (D)

13. Government schools / has too many students / (A) (B)
in a class / for a teacher to control. (C) (D)

14. Either of / the methods / lead to the / (A) (B) (C)
same result. (D)

15. The doctor has / advised him / to avoid the sugar / (A) (B) (C)
in his milk. (D)

Directions for questions 16 to 20: Each sentence given below is divided into four parts. One of them has an error. Mark the number of the incorrect part as your answer.

16. The church accepts this popular sentiment / (A)
gives it a religious significance /and crystallizes / (B) (C)
in a system (D)

17. The government of the Tudors / (A)
were masters in the art of disguising / (B)
common place, and sometimes sordid, motives / (C)
beneath a glittering façade of imposing principles. (D)

18. The increasing reluctance of the sun to rise,/ (A)
the extra nip in the breeze / (B)
the patten of shed leaves dropping - all the evidences of fall / (C)
drifting in winter were clearer each day. (D)

19. Because of it's hardness / this steel is / used principally/
(A) (B) (C)
for making razors.
(D)

20. Laying aside all hindrance /
(A)
thrusting away all private aims /
(B)
devote yourself unswerving and unflinchingly /
(C)
to the vigorous and successful prosecution of this war
(D)

Directions for questions 21 to 25: In each question below, two sentences are given. These two sentences are to be combined into a single sentence without changing their meaning. Three probable starters of the combined sentence are given which are denoted by (A), (B) and (C). Any one or more or none of them may be correct. Find out the correct starters(s) and accordingly select your answer from among the given five answer choices.

21. I am a layman. I do not know how a computer works.
(A) Being a layman, I ……..
(B) As I am a layman, I …….
(C) While I am a layman, I……..
(A) A and B (B) B and C
(C) A and C (D) B only
(E) A only

22. He always listened to good advice. He rose to a good position in life.
(A) Though he listened to good advice ……..
(B) As he rose to good position in life ……..
(C) Since he listened to good advice …….
(A) A and B (B) B and C
(C) A only (D) B only
(E) C only

23. The soldiers saw the camp of the enemy. They started attacking the enemy.
(A) While the soldiers saw the enemy camp ……
(B) When the soldiers saw the camp …….
(C) Because the soldiers saw the camp …….
(A) B and C (B) A and C
(C) B only (D) A only
(E) C only

24. The weather is warm. I like to go for swimming now.
(A) If the weather is ……
(B) Whenever the weather is ……
(C) As the weather is …….
(A) A and B (B) A only
(C) B only (D) C only
(E) None of these

25. We cannot achieve our goals. We lack confidence in our abilities.
(A) When we lack confidence ……..
(B) However we can achieve our goals if we lack …..
(C) Whenever we achieve our goals ……
(A) A and B (B) A only
(C) B only (D) C only
(E) B and C

Directions for questions 26 to 30: Each sentence has a missing part. Choose the best option from those given below the statement to make up the missing part.

26. Given the long and porous border between the two countries and, more important, the links _____.
(A) among Nepalese and Indian rebel groups, New Delhi cannot afford ignoring the Maoists threat of Nepal
(B) between Nepalese and Indian rebel groups, New Delhi cannot afford the ignorance of the Maoists threat in Nepal
(C) among Nepalese and Indian rebel groups, New Delhi cannot afford to ignore the Maoists threat of Nepal
(D) between Nepalese and Indian rebel groups, New Delhi cannot afford to ignore the Maoists threat in Nepal

27. At a time when Beijing's officially scripted anti-Japanese protests are bound to prompt a rethink in Japan about the advisability of continued investment in China, India should be _____.
(A) persuading aggressively Japanese business to shifting at least some of their mammoth investments to its secure location
(B) aggressively persuading Japanese business to shifting at least some of their mammoth investments to its secure location
(C) aggressively persuading Japanese business to shift at least some of their mammoth investments to its secure location
(D) persuading aggressively Japanese business to shift at least some of their mammoth investments to its secure location

28. _____, that they could compete successfully, even with the higher techniques of production, which were being established in England.
(A) So efficient and highly organized were Indian methods of production, and such was the skill of India's artisans and craftsmen
(B) So efficiently and highly organized were Indian methods of production, and such were the skill of India's artisans and craftsmen
(C) So efficient and highly organized were Indian methods of production, and such was the skill of India's artisan and craftsman
(D) So efficiently and highly organized were Indian methods of production, and such were the skill of India's artisan and craftsman

Directions for questions 29 to 33: In the following questions, two sentences are given. There may be an error in the sentence(s). Mark as your answer

(A) if there is an error only in the first sentence;
(B) if there is an error only in the second sentence;
(C) if there are errors in both the sentences and
(D) if there is no error in either of the two sentences.

29. I. He said that he will come, but he didn't.
II. I admit, 'No news is good news now - a -days'.

30. I. I and my friend like to play tennis in grass court.
II. Each of the mistakes have to be corrected before printing.

31. I. I cannot see anything wrong with the plan.
II. You may read the book if you have enough time.

32. I. There is a little truth in what we have heard.
II. You are not going to the theatre, isn't it?

33. I. He informed me before he had posted the letter yesterday.
II. No one is as happy as he.

Directions for questions 34 to 40: A sentence is given in four different forms. Only one of them is correct grammatically. Mark the number of the correct one as the answer.

34. (A) My sister likes painting, dancing and to cooking.
(B) My sister likes painting, dancing and to cook.
(C) My sister like painting, dancing and cooking.
(D) My sister likes painting, dancing and cooking.

35. (A) If you want to play well, you must practise.
(B) If you want to play well, one must practise.
(C) If one want to play well, you must practise.
(D) If one wants to play well, he must practise.

36. (A) Einstein was more cleverer than any other scientist.
(B) Einstein was more cleverer than any scientist.
(C) Einstein was cleverer than any other scientist.
(D) Einstein was cleverer than any scientist.

37. (A) An argument developed among his sister and him.
(B) An argument developed between his sister and he.
(C) An argument developed between his sister and his.
(D) An argument developed between his sister and him.

38. (A) The youngster will benefit from the experience.
(B) The youngster will benefit by the experience.
(C) The youngster will benefit of the experience.
(D) The youngster will benefit out of the experience.

39. (A) There were lesser children in the class than expected.
(B) There were fewer children in the class than expected.
(C) There were a little children in the class than expected.
(D) There were a small children in the class than expected.

40. (A) The child can't hardly wait till its birthday.
(B) The child can wait till it's birthday.
(C) The child can hardly wait till its birthday.
(D) The child can wait hardly till its birthday.

Answer Keys

1. B	**2.** C	**3.** D	**4.** A	**5.** A	**6.** C	**7.** D	**8.** B	**9.** B	**10.** D
11. A	**12.** C	**13.** B	**14.** C	**15.** C	**16.** C	**17.** B	**18.** D	**19.** A	**20.** C
21. A	**22.** D	**23.** C	**24.** D	**25.** B	**26.** D	**27.** C	**28.** A	**29.** A	**30.** C
31. D	**32.** B	**33.** A	**34.** D	**35.** A	**36.** C	**37.** D	**38.** A	**39.** B	**40.** C

Hints and Explanations

1. Indences correlate. Choice (B)

2. 'Nor' must be replaced by 'or'. Choice (C)

3. 'Who the winner is' is redundeant. Use only 'the winner'. Choice (D)

4. 'Went about' is the wrong phrase in the context of the given sentence. Choice (A)

5. "A score of apples' is the correct phrase, as 'score' means twenty or a set of twenty. Choice (A)

6. Choice (C)

7. Choice (D)

8. Choice (B)

9. Choice (B)

10. Choice (D)

11. Choice (A)

12. Choice (C)

13. Choice (B)

14. Choice (C)

15. Choice (C)

16. The third part of the sentence should be 'and crystallizes it'. Crystallizes is a verb which takes an object. Choice (C)

17. Part 2 of the sentence is faulty as the preposition used should be 'of' and not 'in'. One is the 'master of the art' not in the art. Choice (B)

18. The fourth part of the sentence is faulty, as 'drifting into winter' is correct - it indicates motion. 'In' does not indicate motion. Choice (D)

19. In the first part of the sentence 'its' is in the genitive case, so there need not be an apostrophe, 'Because of its hardness' is correct. Choice (A)

20. The third part of the sentence should read 'devote yourself unswervingly and unflinchingly' to maintain parallelism in construction. Choice (C)

21. The two sentences can be combined into a simple sentence by changing the verb 'am' into '*v* + ing' (i.e.) 'being'. Hence 'A' is possible. The conjunction 'as' also can be used, as it shows reason. But 'C' cannot be used as the sentence cannot be combined with the conjunction 'while'. Choice (A)

22. The sentence donates a positive meaning. Hence it cannot begin with 'though'. The conjunction 'as' and 'since' can begin the sentence. But here 'B' begins with the second sentence which changes the meaning of the given sentence. Choice (E)

23. The sentences cannot give the same meaning if the conjunctions 'while' or 'because' are used. It shows a particular time and hence 'when' would be the right way to begin the sentence. Choice (C)

24. 'If' or 'whenever' can also begin the sentence. But here it is not a general statement. It specifies that particular time as 'now' is used. Hence 'as' is the most appropriate way to begin this sentence. Choice (D)

25. The sentence is about 'not achieving'. 'B' and 'C' talk about 'achieving' and hence cannot be appropriate. Choice (B)

Solutions for questions 26 to 28:

26. 'Between' is better than 'among' since only two – the Indian and Nepalese – rebel groups are mentioned. (Between is used for two or more, among for three or more). We are talking of the threat in Nepal not of Nepal. New Delhi cannot ignore (disregard intentionally) not ignorance (lacking knowledge). Choice (D)

27. The adverb 'aggressively' qualifies 'persuading' and hence must precede it (rules out choices 1 and 4). Choice 2 is wrong because 'to shifting' is incorrect. Choice (C)

28. Since the verb at the beginning of the sentence is 'were' (plural) it must be 'efficient and highly organized'. If it were 'efficiently' then both (efficiently and highly) qualify 'organized' and the verb would be was (rules out 2 and 4). We are talking of artisans and craftsmen (plural again not singular) Choice (A)

29. The past tense should be used. The first sentence should be "He said that he would come, but he didn't". Choice (A)

30. The second person must come first.
Statement 1 should be "My friend and I like to play tennis on grass court."
Statement 2 Each of the mistakes has to be corrected before printing. Choice (C)

31. No error in both the sentences. Choice (D)

32. You are not going to the theatre, are you? Choice (B)

33. He informed me before he posted the letter yesterday. Choice (A)

34. The gerund form should be maintained throughout a sentence. Choice (D)

35. 'You' must be followed by 'you'. 'One' must be followed 'one'. Therefore, Choice (A) is right and the other choices are incorrect. Choice (A)

36. ……. Cleverer than any other…… Choice (C)

37. 'Between' should be followed by the objective case 'him' and not the subjective case 'he'. Choice (D)

38. You benefit from something. Correct preposition. Choice (A)

39. In case of numbers we use 'fewer'. 'Less/Lesser' are used in the case of weight and 'little/small' in the case of size. Choice (B)

40. 'Hardly' indicates 'cannot'. So, 'the child cannot wait for its birthday' is being intended. Choice (C)

Verbal Ability Test 2

Number of Questions: 40 **Time: 30 min**

Directions for questions 1 to 5: In each of the following questions, 3 words are related in some way. Find the "odd man" out.

1. (A) Expert (B) Professional (C) Civilized (D) Maestro

2. (A) Blue (B) Crimson (C) Ruby (D) Scarlet

3. (A) Commentary (B) Critique (C) Authority (D) Review

4. (A) Decahedron (B) Decade (C) Decagon (D) Decibel

5. (A) Grave (B) Coffin (C) Tomb (D) Monument

Directions for questions 6 to 15: In each question given below, identify the word which is similar in meaning (synonym) to the question word.

6. ABATE
(A) improve (B) decrease (C) subside (D) sharpen (E) sweep

7. RENOUNCE
(A) relinquish (B) withdraw (C) forgive (D) punish (E) accept

8. ABDUCT
(A) ransack (B) surround (C) induce (D) destroy (E) kidnap

9. DETEST
(A) hate (B) rebel (C) neglect (D) pretend (E) captivate

10. CLEANSE
(A) polish (B) flow (C) absolve (D) reveal (E) revolve

11. ABODE
(A) sanctuary (B) residence (C) reformatory (D) dwelling (E) shelter

12. RESCIND
(A) withhold (B) countermand (C) hamper (D) suppress (E) encroach

13. EPITOMIZE
(A) disappoint (B) distend (C) exemplify (D) generate (E) lengthen

14. REPEAL
(A) continue (B) prolong (C) promote (D) reject (E) abrogate

15. ABSCOND
(A) run away (B) give away (C) move away (D) forbid (E) waste away

Directions for questions 16 to 25: In each of the questions given below, identify the word which is opposite (antonym) in meaning to the question word.

16. FUSION
(A) union (B) participation (C) isolation (D) marriage (E) gloom

17. SOMNOLENT
(A) drowsy (B) lively (C) cheerful (D) joyous (E) active

18. MELLIFLUOUS
(A) harmonious (B) pitchable (C) discordant (D) internal (E) external

19. PRODIGAL
(A) generous (B) revisable (C) frugal (D) pauper (E) rich

20. DISASTER
(A) puzzle (B) success (C) omen (D) festival (E) fiasco

21. ANIMATE
(A) truthful (B) false (C) active (D) dull (E) lazy

22. WRETCHED
(A) filthy (B) tidy (C) neat (D) clear (E) scenic

23. SQUANDER
(A) waste (B) liberate (C) presume (D) donate (E) economize

24. PERTURBED
(A) servable (B) controllable (C) composed (D) decided (E) resolved

25. SLUGGISH
(A) lethargic (B) indolent
(C) apathetic (D) intelligent
(E) exuberant

Directions for questions 26 to 29: In each of the following questions, an idiomatic expression and its five possible meanings are given. Pick out the correct meaning of the idiomatic expression and mark the number of that meaning as your answer.

26. To be on the fiddle
(A) to work on something important
(B) to be doing something dishonest to get money
(C) to constantly find fault with others
(D) to have a less important position than somebody or something else alive
(E) to always keep oneself busy

27. As the crow flies
(A) in a very swift manner
(B) in a straight line
(C) in a very precise manner
(D) very short distance away
(E) in a clumsy way

28. In a melting pot
(A) in a very difficult situation
(B) to take an important decision
(C) to be prone to bad influences
(D) in a helpless situation
(E) in the process of changing

29. To open somebody's eyes
(A) to keep a watch on someone
(B) to be more observant and quick to notice things
(C) to refuse to listen to others
(D) to become close or friendly with someone
(E) to make someone realize or understand something

Directions for questions 30 to 34: In each question, a sentence is given with an idiom (underlined). Four possible meanings of the idiom are also given. Identify the correct meaning and mark the number of the correct choice as answer.

30. The officer was guilty of sharp practices and so he was dismissed.
(A) angry behaviour
(B) disobeying
(C) questionable dealings.
(D) constant quarrel

31. We were kept on tenterhooks while the judges were deciding the winners.
(A) in anxiety (B) in trouble
(C) on the bridge (D) in the witness box

32. The dispute among the students came to a head and the principal declared a holiday.
(A) came to one man's decision
(B) reached a crisis
(C) did not stop
(D) started all over again

33. After getting a job he had no difficulty in keeping the wolf out of the door.
(A) being physically safe
(B) getting the door clear
(C) avoiding starvation
(D) keeping the job intact

34. He asked me not to thrust my nose into his affairs.
(A) smell anything wrong
(B) meddle officiously
(C) forget
(D) combine

Directions for questions 35 to 40: In each of the following questions four numbered choices are given. Three of them belong to the same category. Mark the number of the 'odd man' as your answer.

35. (A) Parents (B) Love
(C) Guidance (D) Punishment

36. (A) Periphery (B) Perfunctory
(C) Cursory (D) Superficial

37. (A) Prune (B) Abridge
(C) Trim (D) Spruce

38. (A) Overfly (B) Overeat
(C) Overdressed (D) Overdose

39. (A) Serendipity (B) Fortuity
(C) Guess (D) Coincidence

40. (A) Vague (B) Abstract
(C) Intangible (D) Empirical

Answer Keys

1. C	**2.** A	**3.** C	**4.** D	**5.** D	**6.** B	**7.** A	**8.** D	**9.** A	**10.** C
11. D	**12.** B	**13.** C	**14.** D	**15.** A	**16.** C	**17.** D	**18.** C	**19.** C	**20.** B
21. D	**22.** B	**23.** D	**24.** C	**25.** D	**26.** B	**27.** B	**28.** D	**29.** A	**30.** C
31. A	**32.** B	**33.** C	**34.** B	**35.** A	**36.** A	**37.** D	**38.** A	**39.** C	**40.** D

Hints and Explanations

1. Expert, professional and maestro talk about the skill or knowledge of a person; 'civilised' talks about the behaviour of a person. Choice (C)

2. Crimson, ruby and scarlet are shades of red. Choice (A)

3. Commentary, critique, review refer to the expression of opinion. Choice (C)

4. Choices 1, 2 and 3 are related to 'ten' (ie) 'deca'. In 'decibel', 'deci' means one-tenth. Choice (D)

5. Grave, tomb and coffin are related to death. Choice (D)

6. The words abate and decrease are synonymous. Choice (B)

7. The word relinquish is synonymous with renounce (give up, discard). Choice (A)

8. Abduct and kidnap are synonymous. Choice (E)

9. Detest means to loathe or hate. Choice (A)

10. Cleanse (rid of something unpleasant or unwanted) is synonymous with absolve (declare free from guilt, blame or sin). Choice (C)

11. The word abode which is a formal or literary term means a house or home, 'dwelling' is its closest synonym. The word 'residence' can be ruled out because it refers only to a person's home, whereas the word 'abode' has a wider connotation. For eg. the abode of animals, the abode of god but it is absurd to say the residence of god or the residence of animals. Choice (D)

12. Rescind means to cancel (a law, order or agreement) the word countermand, which also the means the same, is its synonym. Choice (B)

13. The word epitomize means to be a perfect example of. The word 'exemplify' is its closest synonym. Choice (C)

14. Repeal means to officially cancel (a law of act of parliament) the word 'abrogate' also means the same. Choice (E)

15. Abscond means to leave quickly and secretly to escape from custody or avoid arrest. Choice (A)

16. Fusion (the process of joining two or more things to form a whole) and isolation are antonymous. Choice (C)

17. Active (alert) is an antonym of somnolent (sleepy; drowsy). Choice (E)

18. The word mellifluous means pleasingly smooth or musical to hear. The word discordant (cacophonous, harsh) is its antonym. Choice (C)

19. Prodigal (wasteful, extravagant) and frugal (thrifty, economical) are antonyms. Choice (C)

20. Success is an antonym of the word disaster which also means failure. Choice (B)

21. Animate (alive) and dull are antonyms. Choice (D)

22. The words wretched (miserable) and tidy are antonyms. Choice (B)

23. Squander means to waste (money, time, etc) in a reckless or foolish way. Economize is its antonym. Choice (E)

24. Composed (calm) is an antonym of the word perturbed (alarmed). Choice (C)

25. Exuberant (lively, cheerful) and sluggish (inactive) are antonyms. Choice (E)

26. "To be on the fiddle" means doing something dishonest to get money. Choice (B)

27. The idiom, 'as the crow flies' means 'in a straight line'. Choice (B)

28. The idiom, 'in a melting pot' means 'in the process of changing'. Choice (E)

29. 'To open somebody's eyes' is to cause or make someone realize or understand something. Choice (E)

30. Choice (C)

31. Choice (A)

32. Choice (B)

33. Choice (C)

34. Choice (B)

35. Parents are not concepts as the other three are. Choice (A)

36. Periphery. The other three are associated with casual attitude. Choice (A)

37. The first three refer to cutting short. Choice (D)

38. Overfly is to fly above a place. In all the other three over has the meaning of excessive. Choice (A)

39. Guess. The others refers to a favourable chance. Choice (C)

40. Empirical is verifiable the others are not. Choice (D)

Verbal Ability Test 3

Number of Questions: 40 **Time: 30 min**

Directions for questions 1 to 10: In each question the word at the top is used in four different ways. Select the option in which the usage of the word is INCORRECT or INAPPROPRIATE.

1. COMMUNICATE

A.	The deaf and dumb communicate by means of sign language.
B.	The excitement was palpable and communicated itself to the crowd.
C.	People living in the suburbs have to communicate a long distance every day.
D.	A contagious disease is communicated through physical contact.

2. OPPOSITE

A.	Being an adolescent, Mrinal felt shy talking to members of the opposite sex.
B.	I expected the bride to be shy and quiet, but she was just the opposite.
C.	The Raos live further down, on the opposite side of the road.
D.	The bank is opposite to the supermarket.

3. RAW

A.	Dostovsky's novels often portray life in the raw.
B.	Women labourers often get raw deal from contractors.
C.	Her own experiences provided the raw material for her first novel.
D.	Being marooned on the island forced them to eat raw meat.

4. PICTURE

A.	There have been a number of changes recently - let me put you in picture.
B.	Atticus always told the old lady that she looked a picture.
C.	Ever since he went into pictures the couple have been drifting apart.
D.	From the reports, the picture for the service sector is encouraging.

5. MASTER

A.	Vivek realised that he was expected to master Japanese before leaving for Japan.
B.	Tilak has a masters in Business Administration.
C.	The master bedroom was spacious and comfortable.
D.	"Don't be led by others, be your own master," Sampath said severely.

6. CLOSE

A.	The soldiers advanced in a close formation.
B.	Over the next few months we have to keep a close eye on sales.
C.	Alind closed down in the nineties.
D.	The police often close ranks when one of their officers is accused.

7. BACKGROUND

A.	The name 'TIME' is written in red on a white background.
B.	The film has good background music by Rehman.
C.	The elections in Sri Lanka took place on a background of violence.
D.	The Director asked for more background on the company's financial position.

8. DEFENCE

A.	Whenever Roja was criticized, her brother leapt to her defence.
B.	Why don't you give a chance for the body's natural defence mechanism to protect it?
C.	No cost is too high when it is for the defence of the country.
D.	News of an imminent attack forced the troops onto the defence.

9. CHANGE

A.	The property changed hands several times in the last decade.
B.	Expecting a change of heart from that stubborn mule is useless.
C.	Some of my old dresses will have to be changed to fit me now.
D.	Marriage has changed Sania for better.

10. EARTH

A.	Yasho was the happiest person on earth when she won the gold medal.
B.	Aditya flung his bike on the earth and rushed inside when he saw smoke emanating form the house.
C.	The good earth always gives back several times what you put in.
D.	Be sure to earth household electrical gadgets so that they are safe to handle.

Directions for questions 11 to 20: In each of the following questions, a paragraph with a 'blank' is given. From the four choices, select the sentence, which can go into the blank to make the paragraph logically coherent.

11. India has the distinction of becoming a country with a billion people. Thus, it becomes the second largest populated country in the world. (____.)
(A) The global population has almost touched the six billion mark.
(B) Statistically speaking, every sixth person in the world is an Indian.
(C) Forced population control is not desirable.
(D) It is difficult to judge how many forests have been encroached upon.

12. In the armed forces, before independence, Muslims constituted around 35 percent of the total strength. (____.) Why so few Muslims?
(A) Most of them were recruited from Punjab and nearby areas.
(B) Today, it has gone down to just a mere two percent out of a total close to a million.
(C) This is basically due to a lack of lobby.
(D) This can be attributed to a change in human behaviour.

13. It would be a Herculean task to remodel our settlements to keep them clean through proper waste disposal systems. As such we have already realized that, recycling of waste liquids and solids would be a saner approach. (____.)
(A) Now we are very used to human interference with nature.
(B) We may not be able to survive utilising the available resources.
(C) But organising such measures will involve considerable time, effort, management and education.
(D) Many of our organisation lack this foresight.

14. When a bird hits an aircraft, it can cause potentially catastrophic damage. (____.) So a team at Britain's Defence Evaluation and Research Agency plans to use crystals that glow when fractured to warn of such unseen damage.
(A) This makes visual inspection of damage unreliable.
(B) This is one of the greatest dangers of information technology.
(C) This is the ease with which communication goes on these days.
(D) But in planes made of carbon composites, such damage may be impossible to spot.

15. The General Electric Company is setting up India's first multi-disciplinary research centre. (____.) It will contribute to the development of multi-disciplinary engineering capabilities in India.
(A) Named the GE India Technology centre, it is also the largest of its kind.
(B) It plans to recruit 500 research scientists.
(C) This will help develop GE's global business.
(D) The project will be over by December, 2000.

16. One major change in careers is that one can work from home. (_____.) So far, only work relating to Information Technology has been thus affected. It is expected that many other careers will afford this flexibility in the future.
(A) Therefore one should develop a confident, outgoing personality.
(B) There is no such thing as a permanent job.
(C) New technologies ensure that geographical distance is not a hindrance to one's work.
(D) While it is true that people will switch jobs faster than ever before, one must be loyal to one's organisation.

17. (____.) There are several cave paintings, stone engravings and carved figures which bear this out. The Neanderthal man attempted this too, but his drawings of the tools he used show that they were rather crude.
(A) Prehistoric man used sophisticated tools for drawing and carving figures.
(B) The Cro-Magnon man, who was the forerunner of modern man, earned his daily bread through paintings.
(C) The Cro-Magnon man, who was the forerunner of modern man, was the first fine artist in the history of man's evolution.
(D) Prehistoric man pursued painting and carving figures as a hobby.

18. The natural atmosphere which man has inherited from the past, has been deteriorating under the impact of industrialization. Factories pump millions of tons of dust into the air, vehicles spread fumes and sprays are used to kill agricultural pests – all combine to change the ideal picture. (____.) The situation near big cities and heavily industrialized areas has become particularly bad, and the air is not fit for breathing.
(A) Movement of vehicular traffic on the roads should be restricted.
(B) The pollution of air has become a matter of great concern because it continues to increase as civilization spreads.
(C) Society will have to move towards stricter pollution control.
(D) The atmosphere should be protected as it is a great and irreplaceable resource for living.

19. In recent times, the number of working women has increased considerably in urban areas. With more and more women opting for career-oriented courses, offices and business establishments are flooded with applications from qualified women. (____.) Women are working side by side with men in all walks of life.
(A) A working woman's life is not a bed of roses.
(B) In fact, there are very few workplaces today which do not have single women.

(C) It is possible to maintain a good standard of living only if the woman contributes to the family income.
(D) Even in small towns and villages, most women are employed.

20. For several thousands of years, the moon has been the only satellite of the earth. Today, however, the earth has many other satellites – all made by man. (____.) However, some of them will still be going around the earth thousands of years from now.
(A) Artificial satellites do not fall because they are not affected by earth's gravity.
(B) They travel in an orbit around the earth.
(C) As they speed along, they tend to go straight off into space.
(D) These artificial satellites are very much smaller than the moon.

Directions for questions 21 to 30: In the following passage there are blanks, each of which has been numbered. These numbers are printed below the passage and against each, five words are suggested, one of which fits the blank appropriately. Find the appropriate word in each case and mark its number as your answer.

In most developed and developing nations, the illicit trade in live wild animals is (21) . Each year millions of (22) are wrenched from their natural habitats by people (23) to make quick money, then routed through a ragtag chain of middlemen and international dealers to meet the (24) demand of private collectors in Saudi Arabia, pet shops in Germany, Japan and the U.S.; zoos and circuses in Eastern Europe and folk healers in Asia. According to a wildlife expert, it is the third biggest (25) business, after drugs and arms.

Though many exotic species can be purchased (26) trade in animals and birds that are in (27) danger of extinction is (28) under the United Nations Convention on International Trade in Endangered Species (CITES), which has been signed by 120 nations. The treaty also regulates trade in other species that are seriously threatened, but its provisions are widely (29) , even in signatory countries. Tigers have all but (30) from China and are fast disappearing from India and Siberia.

21. (A) diminishing (B) unfavourable (C) miserable (D) flourishing

22. (A) creatures (B) categories (C) people (D) characters

23. (A) hopeless (B) agreeable (C) desperate (D) susceptible

24. (A) fulfilled (B) imperative (C) unavoidable (D) insatiable

25. (A) licensed (B) virtual (C) authentic (D) illegal

26. (A) inequitably (B) immorally (C) legally (D) profitably

27. (A) imminent (B) preliminary (C) precursory (D) terrible

28. (A) authorized (B) recommended (C) sanctioned (D) banned

29. (A) defended (B) ignored (C) cherished (D) maintained

30. (A) departed (B) depleted (C) withdrawn (D) vanished

Directions for questions 31 to 40: In each of the following questions a pair of words in capitals is given followed by four numbered pairs of words. Select from the choices the pair which exhibits the same relationship as the capitalised pair of words and mark the number as your answer.

31. COGENT : CONVINCING
(A) Insane : Distinguished
(B) Laconic : Pithy
(C) Illogical : Reasonable
(D) Jovial : Abstruse

32. RETROSPECTION : PAST
(A) Syllogism : Logic
(B) Idiosyncrasy : Coherence
(C) Prognostication : Future
(D) Transience : Rigidity

33. EULOGISE : LAMBAST
(A) Mystify : Narrate
(B) Dissemble : Besmirch
(C) Invigorate : Debilitate
(D) Malinger : Adhere

34. LION : PRIDE
(A) Rabbit : Burrow (B) Pup : Litter (C) Whale : Consort (D) Sow : Sty

35. PENURIOUS : AFFLUENCE
(A) Interrogation : Accusation
(B) Garnishment : Command
(C) Taciturn : Verbosity
(D) Condemnation : Mischief

36. MACHIAVELLIAN : DECEIT
(A) Amphibious : Plants
(B) Acquisition : Assumption
(C) Acquittal : Suit
(D) Naïve : Gullibility

37. BENEVOLENT : GRASPING
(A) Repulsive : Pushing
(B) Euphonious : Discordant
(C) Churlish : Impolite
(D) Rebellious : Disorderly

38. FRIGHTEN : PETRIFY
(A) Enamour : Protect
(B) Sneer : Appreciate
(C) Abbreviate : Interest
(D) Humiliate : Mortify

39. MUNIFICENT : STINGY
(A) Inclement : Merciless
(B) Incorrigible : Recalcitrant
(C) Articulate : Obscure
(D) Egregious : Outstanding

40. EUPHORIC : ECSTASY
(A) Modified : Version
(B) Redundant : Relevant
(C) Licentious : Sentiment
(D) Cryptic : Enigma

Answer Keys

1. C	**2.** D	**3.** B	**4.** A	**5.** B	**6.** A	**7.** C	**8.** D	**9.** C	**10.** B
11. B	**12.** B	**13.** C	**14.** D	**15.** A	**16.** C	**17.** A	**18.** B	**19.** D	**20.** D
21. D	**22.** A	**23.** C	**24.** D	**25.** D	**26.** C	**27.** A	**28.** D	**29.** B	**30.** D
31. B	**32.** C	**33.** C	**34.** B	**35.** C	**36.** D	**37.** B	**38.** D	**39.** C	**40.** D

Hints and Explanations

1. In sentence 3 the intended word is 'commute' (travel regularly by bus, train etc between your place of work and home) and not "communicate" (to exchange information, ideas etc). Choice (C)

2. In sentence 4 opposite is a preposition meaning "on the other side of a particular area from something" and does not require 'to'. Choice (D)

3. In sentence 2 it should be 'a raw deal'. The idiom means 'the fact of somebody being treated unfairly'. Choice (B)

4. Sentence 1 should read … in the picture. The idiomatic expression 'to put somebody in the picture' means to give somebody the information they need in order to understand a situation. In sentence 2 'to look a picture' means to look very beautiful or special. Choice (A)

5. When 'masters' refers to a university degree it takes the apostrophe, hence 'Master's in Business Administration'. Choice (B)

6. In sentence 1 'close' means 'without space'. Hence it is 'close formation' not 'a close formation'. To "close ranks" (sentence 4) means to work closely together to defend themselves. Choice (A)

7. In sentence 3, background refers to the past. It should be '. . . against a background of violence' but not 'on'. Choice (C)

8. Sentence 4 should read ' . . . onto the defensive' - an idiom that means acting in a way that shows that you expect to be attacked or criticized. Choice (D)

9. 'Change' is a very general term that is used to describe any act of making something different. In sentence 3 the right word is 'alter' not 'change'. You 'alter' something by making a difference in its appearance, character or use. You cannot use 'change' here. Choice (C)

10. To talk about our planet we use 'earth'. Earth is also used to refer to the soil. As a verb it means making an electrical equipment safe by connecting it to the ground by a wire. But when we refer to the hard surface we walk on, we use 'ground' for outside and 'floor' for inside. In sentence 2 it should be 'ground' not earth. Choice (B)

11. The topic is about India. Global population, population, forests are irrelevant. Hence, option B is the only relevant choice. Choice (B)

12. Option B is the only statement that leads to the question, "Why so few Muslims?" Choice (B)

13. The first line has the words "Herculean task" indicating that the task is not easy. Option C brings out the aspect that such measures will require a lot in terms of time, effort, management and education. Option C is the right answer. Choice (C)

14. Choice D is the only relevant statement. Since, the damage could be impossible to spot, an agency is coming up with a new alternative (as suggested by the last line). Choice (D)

15. Stylistically, the flow of the sentences is as follows: GE is going to set up a research center. Named . . ., it is . . . largest of its kind. It will (this research center will) contribute to . . . Hence, option A is the right answer, as it fits the blank perfectly. Choice (A)

16. Sentence 3 is the ideal choice because it supports the previous sentence and explains the subsequent sentences. Choice (C)

17. The second line states, "There were . . . Figures which bear this out. He attempted to create his daily life through this. The Neanderthal man attempted this too, but his drawings of the tools . . . So, we need a statement in the blank which is a general statement on the pre-historic man and drawing/painting. Our choice narrows down to (A) and (D). Out of these two choices, option A is better because it contrasts the sophisticated tools used by pre historic man with crude ones used by Neanderthal man. Choice (A)

18. Sentence 2 explains how air pollution has increased with the spread of civilization. This sentence explains the last sentence of the paragraph. Choice (B)

19. Choice (D) best supports the argument put forth in the paragraph. Choice (D)

20. Choice (D) speaks about artificial satellites, which though smaller than the moon will revolve around the earth even after several years. The sentence emphasizes the power of artificial satellites.

Choice (D)

21. The illicit trade in wild animals is prospering or growing or 'flourishing' in most developed and developing nations. The passage goes on to explain how it is 'flourishing'. All the other choices are negative terms which are inappropriate in this context.

Choice (D)

22. Animals live in their natural habitat. These 'creatures' are wrenched or harshly pulled away from their natural habitat. Choice (A)

23. Who are the people who wrench these creatures away from their natural habitat? They are those who are 'desperately' looking for quick money. The 'desperate' desire to make quick money drives these people to get themselves involved in this illicit trade. Choice (C)

24. The paragraph goes on to give a list of people who are interested in this trade – from private collectors to folk healers. Hence there is always a heavy demand or an 'insatiable' (impossible to satisfy) demand for these animals. Choice (D)

25. But this trade in live wild animals is an 'illegal' business and is rated next to drugs and arms business at the global level. It cannot be 'authentic' or 'licensed' or 'legal' as the second para goes on to explain the directives of the CITES treaty made by various nations in order to protect the endangered species from extinction. Choice (D)

26. Some of these exotic species can be purchased. The use of 'though' indicates that this purchase is considered 'legal' with reference to certain species of animals and birds which are available in plenty. Further the sentence goes on to say that trade in some species of birds and animals that are near extinction is not allowed under the UN convention. Hence 'legally' is the most appropriate word. Choice (C)

27. There are certain species of animals and birds that are in 'imminent' (impending) danger of extinction. Trading of such animals and birds are considered illegal. Choice (A)

28. It is considered illegal because it has been 'banned' by the CITES, a world organisation established in order to protect endangered species of birds and animals globally. Choice (D)

29. The treaty not only 'bans' the trade of certain endangered species but also regulates trade in other species that are threatened. But still the illegal trade of all these animals continues to flourish even in the 120 countries that have signed the treaty. This means that the treaty and its directives are 'ignored' even by these countries. All the other choices are positive and conveys an opposite meaning and hence are incorrect. Choice (B)

30. The last sentence says that tigers are fast disappearing from India and Siberia. Preceding this, it says tigers have already disappeared from China. Hence 'vanished' is the most appropriate choice. 'Deplete' or 'withdrawn' or 'depart' shows that they are decreasing. But the sentence suggests that tigers have already disappeared from China. Hence choice D is most appropriate. Choice (D)

31. Cogent and convincing are synonyms as are laconic and pithy. Choice (B)

32. Retrospection is analysis of something related to the past. Prognostication means prediction, which is related to the future. Choice (C)

33. Eulogise means praise, whereas lambast indicates criticism. Hence, option (C) is the answer as this choice also has a pair of antonyms. 'Invigorate' means 'to energise' and 'debilitate' means to 'weaken'.

Choice (C)

34. A group of lions is referred to as a 'pride', whereas a group of pups (puppies) is called 'litter'. Choice (B)

35. Antonyms Choice (C)

36. A machiavellian person indulges in deceit. A naïve person shows gullibility. Choice (D)

37. Antonymous relationship Choice (B)

38. Relationship of degree. Petrify indicates extreme fright. Mortify indicates extreme humiliation. Choice (D)

39. A munificent person is generous (not stingy) Likewise, any argument which is articulate is very clear (not obscure). Choice (C)

40. Cryptic and enigma are also synonymous.
In both the first is an adjective and the second a noun.

Choice (D)

VERBAL ABILITY TEST 4

Number of Questions: 40 **Time: 30 min**

Directions for questions 1 to 8: Four alternative summaries are given below each text. Choose the option that best captures the essence of the text.

1. An experiment is an observation that can be repeated, isolated and varied. The more frequently you can repeat an observation, the more likely are you to see clearly what is there and to describe accurately what you have seen. The more strictly you can isolate an observation, the easier does your task of observation become, and the less danger is there of being led astray by irrelevant circumstances, or of placing emphasis on the wrong point. The more widely you can vary an observation, the more clearly will the uniformity of experience stand out and the better is your chance of discovering laws.

(A) It is essential that scientific experiments be repeated, isolated and varied because repetition increases accuracy, isolation facilitates observation and variety increases the chance of discovering laws.

(B) Isolation and repetition of experiments help in reducing the possibility of error in observation. The lesser the margin of error, the better is the chance of discovering new laws.

(C) Scientific experiments should be isolated, repeated and varied. Repetition helps to see clearly and isolation reduces the chance of being led astray. But it is uniformity in the variety that gives a better chance of discovering laws.

(D) An experiment should be isolated, repeated and varied. They help in observing things clearly and accurately. They also increase the chance of discovering new laws.

2. Aristotle in his little treatise on 'Definitions' suggests that every good definition has two parts, stands on two solid feet: first it assigns the object in question to a class or group whose general characteristics are also its own - so man is, first of all, an animal : and secondly, it indicates wherein the object differs from all the other members in its class - so man, in Aristotelian system, is a rational animal, his 'specific difference' is that unlike all other animals he is rational. Aristotle drops an object into the ocean of its class, then takes it out all dripping with generic meaning with the marks of its kind and group; while its individuality and difference shine out all the more clearly for this juxtaposition with other objects that resemble it so much and are so different.

(A) A good definition is like a man standing on two solid feet. It not only helps in assigning an object to a generic group but also shows the differences and dissimilarities to make it shine out of the group.

(B) Aristotle suggests that man as an object has to be defined first as an animal and then shown to be different from other animals in his rationality. Such an interpretation helps in revealing the unique characteristic of an object.

(C) In order to define an object, Aristotle first assigns it to its class and then points out its dissimilarities. Such a comparison makes its individuality more conspicuous.

(D) According to Aristotle, any good definition of an object has two parts, one that identifies it with its group and the other that shows its uniqueness. A definition that includes both these will help in identifying the individuality of the object.

3. Sociologists are often criticized for their use of jargon, their apparent predilection to develop new words while at the same time giving new and often strange meanings to old and familiar terms. The charges are often justified. Equally often they go beyond reason. Systematic discussion is impossible if one does not work with more or less precisely defined terms. Without a technical language, scientific communication becomes cumbersome and inefficient. In the humanities too, the desire to be more precise in analysis leads to the elaboration of technical terms.

(A) Sociologists are often criticized for the use of strange jargon and for giving extended meaning to familiar terms. But they are justified in doing so as such usage makes their analysis more precise and systematic.

(B) Critics are justified in their charge against the sociologists for their use of strange jargon or for giving extended meaning to familiar terms. But the sociologists give the example of the scientists to justify their stand.

(C) In a desire to be more precise in their analysis sociologists develop new words or give strange meaning to old terms. Their justification is that only such extension of meaning can help them in systematic discussions.

(D) While critics are justified in accusing sociologists of using strange jargons and for giving extended meaning to familiar terms, sociologist use such terms in their desire to be more precise and systematic in their discussions and analysis.

4. How big is your footprint? We're not talking about whether you walk around in dainty Cinderalla's slippers or size 18 Wellington boots, but rather the size of the damaging dent that you leave in the Earth's environment

and its resources while going about your daily life. One way of finding out if your impact on the world is like teetering on tiptoes or stomping about in steel-capped boots is to measure your personal ecological and carbon footprints. A carbon foot print is a measurement of the effect you have on the climate in terms of the total amount of greenhouse gases that your actions cause to be produced, while an ecological footprint is a measure of the amount of productive land required to support your resource demands and to absorb the waste you produce.

(A) Is your footprint on the environment like walking on tiptoe or stomping around in steel boots? That is what is measured by ecological footprint - the land you require for resources to meet your needs, the waste you dump and the greenhouse gases that you cause.

(B) How much we impact our environment is measured by carbon footprint and ecological footprint - the former a measure of the greenhouse gases we are responsible for and the latter the land we need for resources and waste.

(C) If one walks around daintily like Cinderella, one harms the environment less than if one tramps around in boots. The greenhouse gases one emits, the waste one generates and the resources one needs, all measure the damage one inflicts on one's surrounding.

(D) The effect our actions have on our environment is measured by carbon footprint, the amount of land we need to sustain ourselves, the waste and greenhouse gases generated by us are noted to arrive at this figure.

5. It's clear that in many countries protected areas are seriously compromised. The dramatic rise of hunting for bushmeat in west and central Africa over the past 30 years has been well documented, as has the explosion in the past decade of illegal logging in southeast Asia. But elsewhere, protected areas face more insidious threats. The Dong Hua Sao National Biodiversity Conservation Area in Laos, for example, has suffered a gradual erosion of its boundaries at the hands of coffee growers keen to take advantage of its rich volcanic soil. The reasons for such pressures are varied and complex, but they often include population growth, land pressure, poverty, corruption and poor law enforcement.

(A) While hunting and logging are generally considered the main threat to protected areas, a greater evil is the gradual occupation of land by people living around who are attracted by its fertile land.

(B) Population growth, poverty, and corruption have led people in Laos to gradually occupy the land under conservation for biodiversity. Thus it is a greater challenge than the rise in hunting for bushmeat in west and central Africa or logging in southeast Asia.

(C) Protected areas are threatened by hunting and logging as also by subtle actions like poaching on its land for cultivation. The main reason for the threat are population pressure on land, poverty and corruption.

(D) Poaching by coffee growers of protected land is as much a threat as hunting and logging in Asia. The villain of the piece is, however, corruption and poor law enforcement.

6. Two decades after the event, and the word 'Chernobyl' still carries a lot of baggage but then, we're still 25,000 years away from being clean. The meltdown at Reactor 4 of the Soviet nuclear power station was caused by an unnecessary low-power test, an experiment to determine whether the reactor could restart itself with all external power shut off. The result was a black fireball that blew the reactor's roof off and spread radiation across much of the Northern Hemisphere. Equally toxic was the Soviet government's initial response : a three-day silence as to the dangers of the situation. Protective foam was sprayed around the nearby towns, Chernobyl and Pripyat, whose combined population was 135,000, but otherwise, life proceeded as normal - children played in the foam, marched in the May Day parade and breathed in the contaminated air. While the official death toll remains at 41, who knows how many fatalities resulted from the delay in evacuating these towns?

(A) Twenty years after Chernobyl, caused by an experiment, we are still emotionally involved and haven't been able to wipe out the radiation that spread over the Northern Hemisphere. No one knows how many deaths have resulted from the Soviet government's refusal to evacuate the people in the region.

(B) Chernobyl was caused by an unnecessary test that affected millions. The Soviet government's low-keyed response led to many more deaths though officially it is only 41. We are still emotionally swayed by the memory and are far from cleaning up the mess.

(C) The Soviet government's reaction to Chernobyl was as poisonous as the gas it emitted. The government's apathy led to millions of avertable deaths and we still do not know how to stop the radiation from continuing and spreading.

(D) Chernobyl spread radiation across the Northern Hemisphere and affected so many that two decades later we are still emotionally affected by the memory. The Soviet government put up a brave front by not evacuating the people and spraying foam to contain the radiation.

7. Down syndrome babies are generally born to older mothers. The probability of having a Down syndrome baby grows rapidly and exponentially as the age of the

mother increases, from 1 in 2,300 at the age of twenty to 1 in 100 at forty. It is for this reason alone that Down embryos are the principal victims or their mothers the principal users, of genetic screening. In most countries amniocentesis is now offered to - perhaps even imposed on - all older mothers to check whether the foetus carries an extra chromosome. If it does, the mother is offered or cajoled into an abortion. The reason given is that despite the happy demeanor of these children, most people would rather not be parent of a Down child. If you are of one opinion, you see this as a manifestation of benign science, miraculously preventing the birth of cruelly incapacitated people at no suffering. If you are of another opinion you see the officially encouraged murder of a sacred human life in the dubious name of human perfection and to the disrespect of disability. You see, in effect, eugenics still in action, more than fifty years after it was grotesquely discredited by Nazi atrocities.

(A) Since Down syndrome babies are born to women over forty, they are often forced to have amniocentesis. This has been controversial. Some see the prevention of the birth of a disabled child as kindness while others think the state is indulging in eugenics as did the Nazis.

(B) The probability of having a child with Down syndrome increases as the age of the mother increases. So older women take the amniocentesis test and abort a Down embryo even though this amounts to a murder. The killing of the disabled is similar to the practices of the Nazis.

(C) The amniocentesis has been a controversial test since it is used to identify and do away with a Down embryo in older women. While some think it is kindness to prevent the birth and unnecessary suffering that a Down child suffers, others think they are murdering the disabled to selectively breed a healthy generation.

(D) Mothers over forty are more likely to have a Down child. They are persuaded or forced to have amniocentesis. The abortion of Down embryo can be seen as an expression of benevolent science or as officially sanctioned murder of disabled, that is eugenics fifty years after the Nazis.

8. "I place economy among the first and most important of Republican virtues and public debt as the greatest of the dangers to be feared". So wrote Thomas Jefferson in 1816 in a letter to William Plumer, the governor of New Hampshire. However, contrary to the wisdom of the great man, national debt is by no means an inherently bad thing. Borrowing money is a historically tried-and-tested method of expanding the productive capacity of an economy, if not a pre-requisite. The USA would go on to amass an enormous debt, but this was used relatively sensibly to bring about a more-than-proportionate growth in the size of its economy.

(A) Though Jefferson warned against public debt, the USA accumulated enormous debt as it considered it to be an essential prerequisite for an economy to grow.

(B) As the USA amassed an enormous debt, Jefferson felt public debt to be an evil that should be avoided. But it is essential for an economy to grow.

(C) Though Jefferson called public debt 'the greatest of dangers' yet the example of the USA reaffirms it to be a proven method of bringing about growth in the economy.

(D) Jefferson said public debt would lead to the weakening of an economy but his own country showed that it is one of the means of achieving a more than proportionate progress.

Directions for questions 9 to 18: A number of sentences are given below which, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a letter. Choose the most logical order of sentences from among the four given choices to construct a coherent paragraph.

9. (a) Here is the world's newest temple of modern art, an artistic triumph in itself, and combinig grandeur, originality and stunning power.

(b) It can be compared to a cathedral, in that it occupies such a vast space as inspires awe.

(c) One of the unique elements of this edifice is a monstrous steel spider that must surely have been inspired by a horror movie.

(d) The place is a huge physical entity, vast, and stupendous.

(A) abdc (B) cdba
(C) adbc (D) cadb

10. (a) In the South, freshly grated coconut is the usual gamish.

(b) In other parts, fresh coriander leaves provide the mandatory finishing touch.

(c) Besides the tempering exercise, the fat-laden coconut helps to release fat-soluble carotence, from a carrot for example.

(d) Traditional gamishes are another characteristic feature of our cooking.

(A) cabd (B) dacb
(C) acdb (D) cbda

11. (a) Every ceramic object is a sculpture in miniature, and constitutes a study in several kinds of paradox.

(b) The fragility of material culture is nowhere more manifest than it is in the art of pottery.

(c) At the formal level, the solidity of the ceramic object conveys a sense of permanence and enduring value.

(d) At the same time, the baked earth, from which it is made, renders it fragile, frangible, a hostage to chance and threat of damage.

(A) bacd (B) abcd
(C) bcda (D) acdb

12. (a) It is more than 200 years since Tipu Sultan's mysterious death occurred while defending his fort in Srirangapattana near Mysore in India.
(b) As a warrior, he was a formidable and implacable enemy.
(c) His reputation in the annals of British history puts him in the same class as Chenghis Khan, Attila the Hun and Emperor Napoleon.
(d) But his legend in India and in the West, is still going stron.

(A) abcd (B) adcb
(C) acbd (D) adbc

13. (a) Puranic literature describes the progression of the cosmos which passes through various yugas, each of which is measured in precise numerical terms.
(b) But in the cosmic scheme of Time, it does not amount to much.
(c) Ten centuries may be a substantial time-span in human reckoning.
(d) In that calculation, our millennium, just over, does not form even a small fraction of the Kaliyuga, the last of the four cosmic cycles.

(A) cdba (B) abcd
(C) adbc (D) cbad

14. (a) My boss asked me to call on one of the Ministers he knew there.
(b) I was leaving for Delhi on work as usual.
(c) This meeting was supposed to sort out the mess he was in, created by a new amendment to excise duty provisions on export of cotton.
(d) At Delhi I sought an appointment with the Minister concerned.

(A) dabc (B) bdca
(C) bacd (D) cdab

15. (a) Since then sponsorship and endorsements started playing a big role, with the profit motive becoming all pervasive.
(b) Recent reports trace the corruption of Olympic movement to the total commercialisation of games in 1984.
(c) Sordid behind the scenes going on in the IOC have been finally exposed.
(d) Salt Lake city which is bidding for Winter Olympics bent all the norms and even bribed IOC members.

(A) cdba (B) cabd
(C) cdab (D) dcba

16. (a) Which pump out an enormous quantity of smoke, poisonous gases and other wastes in the surrounding environs.
(b) The land to accommodate huge population was obtained by cutting down trees.
(c) Area cleared by chopping down forests was used to set up industries, thermal plants etc.
(d) Acre after acre of forest is cleared in one day leaving the atmosphere susceptible to pollution and the earth poorer.
(e) The speed of cutting down forest has increased ever since.

(A) cabde (B) ecdba
(C) bcaed (D) dcbae

17. (a) No place in the house seemed secure.
(b) I remembered the agonies of my own childhood when my sister discovered I was writing poems and began to tease me by chanting them in public.
(c) When my daughter began to write her memories, at the age of four, I decided that she must have a place to keep them, if only a section of a bureau drawer or as it happened, a box with a key.
(d) It seems to me that we can't learn too early in life to respect the privacy of the individual.
(e) I tried desperately to hide the notebook of poems.

(A) bcdae (B) dcbea
(C) cedba (D) aecbd

18. (a) My father and mother were the complements of each other.
(b) Her face was responsive, my father's impassive.
(c) My mother was fragile, my father robust.
(d) My mother was not intellectual her natural propensity was intuitive. Her face illustrated that saying "Appearances are deceptive: for it did not show the immense strength of her moral convictions.
(e) My mother's face rippled to emotions as waters to the wind.

(A) acbed (B) cabde
(C) ecbad (D) dbcea

Directions for questions 19 to 23: In each question below, a paragraph or a statement is given followed by four statements. Classify each of the four statements as per the following categories and from among the answer choices, select the one that gives the sequence of letters that matches with your categorization. Categorise the statement as

(A) if it is a CONCLUDING ASSERTION
(B) if it is a SUPPORTING REASON
(C) if it is a STATEMENT OF ARGUMENT
(D) if it is a COUNTER ARGUMENT

19. With globalisation making its presence felt, a number of foreign influences are gradually seeping into our culture. And these are felt in food as well. Moreover, with the break-up of the joint family system and more and more working couples setting up homes on their own, people prefer eating out to returning home to cook after a hectic day's work. Capitalising on this new trend,

established as well as upcoming restaurateurs feel that constant improvisation and innovation is the key to a successful restaurant.

(a) Most restaurateurs feel that sticking to the cuisine of the 60's evokes feelings of nostalgia and improves sales.
(b) Customers want to experiment and wish to tryout cuisine from other countries.
(c) Similar changes have been noticed in the entertainment industry where viewers now give preference to comedies and action parked adventure movies.
(d) Restaurants which improvise and innovate will definitely be successful.

(A) cbbd (B) bcda
(C) cbca (D) bbcb

20. Organization of the Petroleum Exporting Countries (OPEC) has used output curbs to maintain average oil prices over the past three years in its $ 22 to $ 28 target range. OPEC was worried that rising supplies from rival non-OPEC nations and another year of modest demand growth could cause a downward price spiral. It is particularly vulnerable to a price fall during the second quarter when demand eases. Forecasts from the Paris based International Energy Agency backed this outlook. It is estimated that if OPEC keeps pumping unchecked it will overwhelm world demand next year by 1.8 million BPb causing a huge stock build-up.

(a) The existence of a similar cartel in the global coal market would have definitely pushed up the price of coal.
(b) A price range of $ 22 - $ 28 ensures that the producers make the minimum possible profit out of their operations required to keep their economy in shape.
(c) Any excess production by OPEC would be mopped up in an energy hungry global market and thus OPEC need not worry about excess production.
(d) OPEC countries should concentrate on developing their non oil based economy so as to be able to absorb any adverse shocks as a result of decreasing oil reverses.

(A) cbdc (B) bcca
(C) abcd (D) bcda

21. The recent elections have thrown up so many critical weaknesses in the American electoral system - inconsistent voting procedures from state to state, early declaration of the winner by the media before the polls closed and partisanship of breathtaking proportions that intensified after the balloting. Surely a set of statutory rules about the counting and recounting of votes could have saved the US all this embarrassment.

(a) The adoption of a set of rules to be uniformly applied all over the country would most certainly eradicate the weakness in the electoral system.
(b) Countries which had adopted similar procedures have now started reaping the benefits.
(c) The media should be banned from pronouncing the winner before the polls close so as to prevent any sort of malpractice from taking place.
(d) Different voting procedures in various states and the massive media involvement ensures free and fair polls and thus are invaluable.

(A) abbb (B) abcd
(C) abdd (D) abbd

22. Memory may have been found to be affected by genes, but no gene has been found to have even a small causative relationship with normal 'intelligence'. Intelligence has never been successfully defined or measured, unless one refers to highly specialized, arbitrary, narrowly defined and largely learned skills measured by standard IQ tests. Why should genetic intelligence have evolved along the lines of IQ tests when better measures could have been evolved is something inexplicable.

(a) Genes that influence 'intelligence' are masked by chemicals produced in the brain and are thus difficult to detect.
(b) IQ tests are the most accurate way of measuring intelligence exhibited by humans and no other measure is required.
(c) IQ tests are not the best measure of intelligence as these tests can be taken quite successfully by people who have practiced hard enough.
(d) IQ levels in a human being are function of the nutrition that one receives in childhood.

(A) ddbc (B) ddda
(C) dbaa (D) dbad

23. After Rwanda and Kosovo, another dark page of modern history is being written in East Timor by the Indonesian army. It's a shame the international community did not react faster in condemning Indonesia and preventing the genocide, which has been on-going since the mid 70's in East Timor. Above all political and economic considerations, the international community should never forget that the brave people of East Timor are struggling and dying for something that is so dear to all of us - freedom.

(a) The international community has not found any evidence of wrong doing by the Indonesian army and hence has been averse to condemning it.
(b) The East Timorians are waging a battle not for freedom but because of the fact that most of them are being forced to convert to Christianity.
(c) Genocide in any place under any pretext is abominable and it is the responsibility of the international forum to put a stop to it.
(d) History has proved that freedom is a concept for which people have sacrificed their lives and hold it in high esteem.

(A) dcbb (B) dcaa
(C) dcab (D) abcc

Directions for questions 24 to 28: Each of the following questions has a paragraph from which a sentence has been deleted. From the given options, choose the sentence that completes the paragraph in the most appropriate way.

24. This is a rare moment in the history of economic globalisation. Policy making across the developed and developing world has never been so uniform and homogenised. Whether it is G-7 or G-20 economies the only buzz word among policy makers is 'inject more liquidity". President George Bush himself made the unusal gesture of walking into the meeting of G-20 countries, seeking their active support in mitigating the impact of the financial meltdown. _____

(A) At a macro level, the attempt by the G-7 governments is to ensure that the big asset bubble built over the past six years must not be allowed to burst.
(B) There are clear pointers that the global financial crises will result in loss of demand across the board.
(C) The thirst for more money seems unending.
(D) Never before have the heads of state met only to discuss how to put in extra cash in the global financial system.

25. At the heart of the wellness concept is the most romantic notion of all: the suggestion that we might postpone, halt and even reverse the aging process itself. While some scientists are doing research on ageing and on human mortality, many wellness buffs-and even some reasonably accredited scientists-are promising that we may soon penetrate the secret of the ageing process and with this knowledge, actually extend Homo Sapiens' normal life span. _____

(A) While some may search for a magic elixir, others stress on diet and exercise as the keys.
(B) The death barrier may actually be broken.
(C) Wellness is a kind of superstate of mental and physical well being.
(D) Those who remain impervious to these romantic hopes have other incentives or coercions.

26. To test the role of cognition in hypocrisy, scientists had volunteers assign themselves an easy task and a stranger an onerous one. But before judging the fairness of their actions they had to memorize seven numbers. This play keeps the brain's thinking regions too tied up to think about anything else, and it worked: hypocrisy vanished. People judged their own behaviour as harshly as they did others'-strong evidence that moral hypocrisy requires a high-order cognitive process. _____

(A) When 'people like us' torture, it is justified; when people unlike us do, it is an atrocity.
(B) When the thinking part of the brain is otherwise engaged, we are left with gut-level reaction and instinctively condemn bad behaviour.
(C) Scientists have long wondered whether hypocrisy is driven by emotion or reason.
(D) When we judge our own transgressions less harshly than we judge the same transgressions in others it may be because we have this instinct to preserve our self-image.

27. Celebrity has become the primary commodity of popular culture. Fans used to fall for a specific album or film, but now the public tends to base its consumption on the aura of celebrity attached to any given product. Singers can act in films and actors can record albums, not thanks to any special talent but because their brand is big enough to transcend categories. _____

(A) Witness the birth of the celebrity luxury fashion brand.
(B) Fashion magazines have all but abandoned the practice of putting models on the cover of their magazines.
(C) Celebrities have wised up to their incredibly powerful market potential, moving from endorsing someone else's high end products to producing their own.
(D) The most successful start-ups have been those by celebrities with iconic personal style.

28. The growth stories of China and India have always been different–China is well known for being the world's factory, while India's new wealth has been built on services. But the result is the same. Over the next twenty years 213 million Chinese households and 123 million Indian ones will begin to have discretionary income. _____

(A) If both countries continue roughly on their current growth paths we will witness the creation of massive new consumer markets.
(B) The speed of the change will rival Japan's economic miracle of the 1950s.
(C) That will lead to an Asian shopping spree of historic proportions.
(D) The projection is that incomes will grow eight fold cutting China's poverty rate to just 16 percent.

Directions for questions 29 to 40: Select the correct alternative from the given choices.

29. Manufacturers of a food drink claim that their product is a complete planned food containing all the nutrients necessary for the health and growth of a child. However, it is better that children are given a well-balanced diet consisting of a variety of foods for health and growth. Which of the following, if true would best support the position above?

(A) The flavour of the food drink does not appeal to many children.

(B) Children who are used to taking the food drink do not take natural foods rich in these nutrients.
(C) The nutrients contained in a well-balanced diet are more easily absorbed by the body for effective use.
(D) The contention of the manufacturers that the food drinks are complete planned foods is yet to be proved right.

30. A recent study conducted with school children aged ten to fifteen showed that a plan where these children were provided with a proper meal at school ensured that they were less likely to remain absent from the school than other children. Thus providing a proper meal at school plays a role in reducing student absenteeism. So such a plan must be introduced in schools to reduce absenteeism.
Under which of the following conditions will the breakfast plan work best?
(A) In a locality where student absenteeism is significant.
(B) In a school where there are more boys, (who are more likely to remain absent), than girls.
(C) In schools, where a majority of students have both parents working.
(D) In schools where the students are wards of those who cannot afford a square meal a day.

31. It is generally said that radiations emitted by cell phones cause immediate damage to the membrane in the ear and results in hearing impairment. Hence one is advised to avoid using a cell phone to prevent total loss of hearing.
Which of the following, if true, seriously weakens the above argument?
(A) Use of electronic gadgets such as computers also impairs hearing besides affecting eyesight.
(B) Electronic gadgets such as i pods do not damage hearing in users.
(C) A majority of those who use hearing-aids are found to have never used cell phones in their lives.
(D) It is found in a survey that ninety per cent of employees working as telephone operators have been regular users of cell phones for years.

32. Last year, Alpha University implemented a procedure that encouraged students to evaluate the teaching effectiveness of their professors.
Which of the following principles can the above plan be taken to be based upon?
(A) The effectiveness of any service depends on how useful it actually is to the receiver of the service.
(B) The effectiveness of any service is measured by the perception of the receiver of the service regarding its utility.
(C) The receiver of a service should have the freedom to choose the mode of delivery.
(D) The service provided should take care of the ultimate beneficiary of the services.

33. A recent study indicates that Venadial, a new medicine currently produced in the country *X* only, actually reduces cholesterol levels. Venadial, derived from the resin of pine trees, works by activating a metabolic response that is not yet well understood. However, cholesterol levels fell fifteen to twenty per cent among participants in the study who consumed Venadial daily and reduced the risk of heart attack by one-third. Therefore company *A* in country *Y* should obtain the exclusive right to sell Venadial in *Y*. The profits are sure to increase within a short span of time.
Which of the following, if possible must first be done in order to translate the above optimism into reality?
(A) A publicity campaign informing the public of cholesterol and its permissible levels.
(B) A publicity campaign advertising the role of Venadial in reducing heart attacks.
(C) Conducting research on the metabolic response to Venadial which is not yet well understood.
(D) Convince the government to allow import without restraint.

34. Software companies have grown by leaps and bounds. The employees required to man these companies far outnumber the available candidates with academic skills relevant to the jobs that they are required to perform.
Which of the following plans, if possible, can allow such companies to meet their manpower requirements?
(A) The institutions can enhance the pay of the existing employees to encourage them to take up additional work.
(B) The institutions can scale down their operations to the extent where the requirements can be met with the number of available persons with the needed academic skills.
(C) The institutions can recruit even those who do not have the necessary skills and give them training to enable them to perform their jobs.
(D) The institutions can enter into agreements with the academic institutions like colleges and universities to absorb their students in toto.

35. Smoking is a serious health concern as it affects not only those who smoke, but also others who don't, as they inhale the polluted air. The Government is concerned about this. So it has decided to rope in film celebrities for its campaign against smoking, through documentary films. Which of the following, if true, most strengthens the plan of the Government in its campaign against smoking?
(A) The general public abides by the advice of film celebrities on the screen.

(B) The film celebrities are role models for the common man.
(C) Film celebrities earn huge sums of money and it is in the fitness of things that they contribute to a social cause.
(D) It is easier to produce such films with film personalities as they are already well versed in acting.

36. It is true that there are international laws against drug traffic. But if the Government takes stern action to plug its entry in one place, drugs will enter the country through other sources.
Which of the following is most like the argument above in its logical structure?
(A) It is true that the officials of the police department are expected to be incorruptible. But if they are paid poorly, they may resort to graft to sustain a decent living.
(B) It is true that most of the terrorists are religious fundamentalists. But there are several persons who are deeply religious but they are not terrorists.
(C) It is true that there are severe laws against violation of copy rights. But some company would have done it if the defendant had not done it first.
(D) It is true that adoption of third degree methods against prisoners to extract confessions is against law. But it cannot be helped in certain cases where the prisoner does not reveal all the truth.

37. Balance of trade refers to the difference between the exports and imports of a country. It is said to be positive when exports are more than imports and negative if imports are more than exports. The exports of a country '*C*' increased over the previous years' exports. However the balance of trade has become negative though it was positive last year.
Which of the following cannot be true if the above statements are true?
(A) The imports during this year are more than last year's.
(B) Exports during the previous year were more than imports.
(C) Last year's imports were more than this year's exports.
(D) The imports during this year are more than the exports.

38. The town administration of a hill station where tourists far outnumber the local residents, found that the tourists were using non biodegradable plastic cups, plates and polyethylene bags in the place of substitutes made out of paper which are environmentally preferable. In order to reduce the use of non biodegradable goods but at the same time not to annoy the tourists the administration encouraged the sale of both plastic and paper made goods, but the paper-made goods were sold at substantially very low rates compared to the plastic goods. Which of the following, if true, would undermine the purpose for which paper - made goods are sold at very low rates by the administration?
(A) The shop keepers were found to sell the plastic goods at a price higher than that fixed by the administration.
(B) The tourists were found to possess a higher value for aestheticism (which they find in plastic goods compared to the paper goods) than for costs.
(C) The tourists themselves were well aware of the desirability of using biodegradable goods as against non biodegradable ones.
(D) A similar strategy adopted in another hill station in the same country did not produce desired results immediately.

39. Company "Trendy" is engaged in the manufacture of products which face stiff competition from others in the market. Constant change in the products, their design, packaging, etc is the key to success in the industry. Trendy has been allocating a significant portion of the resources over the years to research and development which takes care of devolvement of products and innovations in designs etc. However, the allocation has fallen from a whopping ten percent to a mere 6 percent of the total resources, in the current year, because of which it is feared that the sales might drop significantly this year. Which of the following is an assumption made in the conclusion drawn above?
(A) Ten per cent of the total resources is enough allocation for research development activities, to for achieve the desired levels of sales.
(B) The allocation of resources to research and development efforts determines the extent of innovation in products and product designs.
(C) Sales of the company will also fall from ten per cent to six per cent.
(D) The production of goods by the company would also drop significantly in the current year.

40. Which of the following, if the dictum 'more the resources for R&D, more will be the innovations' be true, would most weaken the conclusion drawn in question 5 above?
(A) The total resources allocated by the company to all the activities put together were double those of the previous year.
(B) The sales of the same products by the other companies also drop in the year.
(C) The market share of the company for the products it manufactures increases during the current year.
(D) The resources allocated to marketing during this year increased substantially over the previous year.

Answer Keys

1. A	**2.** C	**3.** D	**4.** B	**5.** C	**6.** B	**7.** D	**8.** C	**9.** C	**10.** B
11. D	**12.** D	**13.** D	**14.** C	**15.** A	**16.** C	**17.** B	**18.** A	**19.** C	**20.** A
21. B	**22.** A	**23.** C	**24.** D	**25.** B	**26.** B	**27.** C	**28.** C	**29.** D	**30.** D
31. D	**32.** B	**33.** B	**34.** C	**35.** A	**36.** C	**37.** C	**38.** B	**39.** B	**40.** A

Hints and Explanations

1. The main points are:
(1) An experiment is repeated to increase accuracy.
(2) It is isolated to make the task of observation easier.
(3) Variety in experiments gives a better chance of discovering laws.

Choice A covers all the important points and hence can be called its summary. Choice B does not say anything about 'variety'. Choice C is not as clear or comprehensible as choice A. Choice D does not specify what helps in what aspect. Choice (A)

2. The main points are:
(1) Aristotle says any good definition of an object has two parts.
(2) The first is to assign the object to its class or genre with which it has common features.
(3) The second is to indicate where and how it is unique.
(4) Juxtaposing it with objects of the same kind helps in identifying its individuality.

Choice A does not mention 'Aristotle' and Choice B focuses on the definition of 'man' in particular and then says 'object'. Hence A and B can be ruled out. Choice C summarises the passage. The idea given in the last sentence of choice D is incorrect. Choice (C)

3. The main points are:
(1) Sociologists use strange jargon or give extended meaning to familiar terms.
(2) They are criticized for doing this.
(3) But they have their reasons.
(4) They do it in a desire to be more precise and systematic in their analysis.

Choice A is incorrect as it says' . . . They are justified to do so . . .' which is not stated in the passage. It is the critics 'who are justified'. The second sentence in choice B makes it incorrect. 'The charges/criticism . . .' is not mentioned in choice C. Choice D precisely summarizes the passage. Choice (D)

4. The main points in the text are:
(1) The damage we inflict on our surrounding is measured by carbon footprint and ecological footprint.
(2) Carbon footprint measures the greenhouse gases that is emitted because of us.
(3) Ecological footprint measures the land we need for resources and for dumping waste.

Choice A misses carbon footprint.
Choice B is the right answer.
The first sentence in choice C makes an absurd suggestion.
Choice D mixes up the concept of ecological footprint and carbon footprint. Choice (B)

5. The main points in the text are:
(1) Hunting and logging are a threat to protected areas.
(2) A more subtle threat is illegal occupation of forest land by cultivators.
(3) The reason for all the above is pressure of population on land, poverty, corruption and poor law enforcement.

Choice A is wrong because it calls poaching 'a greater evil', not stated in the passage.
Choice B becomes too specific by mentioning Laos, west and central Africa and southeast Asia. Choice D is again specific as it talks of coffee growers. Choice C is appropriate. Choice (C)

6. The main points in the text are:
(1) Chernobyl still evokes poignant memories after 20 years and we are far from cleaning up the mess.
(2) It was caused by an unnecessary experiment.
(3) The Soviet government did not act immediately and we do not know how many deaths were caused by the failure of the government to evacuate the people.

The original para does not mention that the Soviet government refused to evacuate people. It was reluctant and delayed the process. So, choice A is a distortion. Choice C distorts in calling the radiation 'a gas', and it leaves out many important points. Choice D distorts in saying 'the Soviet government put up a brave front' – not stated in the text. Choice B is concise and correct.
Choice (B)

7. The main points in the text are:
(1) Older women, those over forty, are more likely to have a child with Down syndrome.
(2) As such they are offered, or forced to have, amniocentesis.
(3) The abortion of a Down embryo can be seen as a benign aspect of science or as officially encouraged murder.

(4) It is eugenics, fifty years after the Nazis.
Choice A is not the best summary because it is not amniocentesis but abortion that is controversial. Choice B is not apt since it says older women take the test and abort, as if the initiative is theirs. Choice C makes the test controversial. Choice D is apt. Choice (D)

8. The main points in the text are :
(1) Jefferson warned against public debt.
(2) But it is a tried-and-tested method. The USA being an example.
Choice A distorts the meaning when it says the USA considered it to be a prerequisite. Choice B distorts in saying that because the USA amassed debt Jefferson warned against it. Choice C is an apt summary. Choice D appears to be right but Jefferson never said (as per the passage) that public debt would weaken economy. Choice (C)

9. A and C are our possible starting parts. 'One of the' in C indicates that it is a continuation of 'a them' rather than a starting part and hence C is rejected as our starting part which leaves A as the only possible starting part. "The place' in D relates to 'the newest temple' in A and hence D follows A. 'Vast space' in B connects with 'huge physical entity' in D and hence B follows D.
Choice (C)

10. As per the choices A, C and D are our possible starting parts. 'Besides' in C rules it out as our starting part and out of A and D, D is a better starting part as it is a more generalized statement than A. 'Usal garnish' in A follows 'garnishes' in D. 'Fat-laden coconut' in C links up with 'freshly greated cocounut' in A and hence C follows A. B provides a contrast to A and hence follows C. Choice (B)

11. The 'paradoz' associated with ceramic sculptures is elaborated in C and D. C says that the ceramic object's solidity gives a sense of permanence but this is contrasted by D which states that the baked earth form which this sculpture is made makes it more prone to damage on account of its fragile nature. "fragility in B connets with 'fragile, frangible' in D and hence B follows D. Choice (D)

12. A is our starting part 'but' in D offers a contrast by emphasizing the fact that though 200 years have elapsed since Tipu's death yet his fame is growing and hence D follows A. B describes Tipu and hence follows D. 'Reputation' in C is a logical extension of 'formidable and implacable' in B. Choice (D)

13. A and C are our possible starting part of which C is better as it is a very general statement. 'It' in B refers to the 'ten centuries' in C and hence B follows C. 'In that calculation' in D refers to the 'yugas' in A and hence D follows A. Choice (D)

14. B is the opening sentence (it makes a generalized statement). A follows - it explains what the work (mentioned in B) is. C follows A – 'This meeting' in C linking it 'to call on….' In A. D concludes with what he did.
Choice (C)

15. C is a better opening sentence than D, as it is a generalized statement. D follows C – it explains what the 'sordid' going on mentioned in C are. A follows B – 'sponsorship and endorsements' in A linking it to 'commercialization' in B. Choice (A)

16. B is the opening sentence. C follows B – 'Areas cleared' in C linking it to 'land..... obtained by cutting down trees' in B. A follows C – 'set up industries ….. which pump out …..'. E and D follow – The speed of cutting down has increased and its consequences.
Choice (C)

17. D is the opening sentence – 'privacy' is the central idea of the para. C follow D giving an example to elaborate the need for privacy. B follows C – the author's child hood experience compared to his daughter's. E and A follow to complete the author's reminiscence.
Choice (B)

18. A is the opening sentence – it sets out the basic idea of the para. C follows A – explaining (very briefly) how his parents were the complements of each other. B follows C – one more instance of complementing. E and D follow elaborating the qualities of his mother.
Choice (A)

19. The passage states that globalisation has made its impact on the food tastes of the masses, it goes on to state that people prefer innovation in cooking.
A - states a point that runs counter to the argument given in the passage regarding the changing tastes of the masses hence it is a COUNTER ARGUMENT.
B - gives one more reason for the changing states and hence it is a SUPPORTING ARGUMENT.
C - states a point that is not mentioned in the passage and hence this is a STATEMENT OF ARGUMENT.
D - categorically states that restaurants which cater to the changed tastes will be definitely successful and hence it is a CONCLUDING ASSERTION.
Choice (C)

20. The passage talks about the efforts made by OPEC to maintain he level of prices at a certain level and also the scenario that emerges as a result of the efforts that it makes to achieve this objective.
A - talks about a hypothetical case of how the coal prices would have benefited from the existence of such a cartel. Hence this is a STATEMENT OF ARGUMENT.

B - gives a reason as to why the OPEC nations would want to maintain the oil prices in a specific range and hence it is a SUPPORTING REASON.

C - This point runs against the view of the passage that excess production by Opec would result in a decrease of the general price of oil and hence it is a COUNTER ARGUMENT.

D - talks about a possible remedy to the problem and as such it is a new argument that has been proposed. Hence, it is a STATEMENT OF ARGUMENT.

Choice (A)

21. The passage talks about the weakness by the American electoral system and gives a few examples to substantiate this.

A - states categorically that the adoption of a system of rules across the country would correct the system and stress the root hence, this is an ASSERTION.

B - gives a SUPPORTING REASON as to how other countries have benefited by implementing the rules.

C - the role played by the media has been specified in the passage but the question of banning it has not been discussed and as such C which proposes that the media be banned from announcing results is a STATEMENT OF ARGUMENT.

D - gives a reason in support of the measures being proposed in the passage to remedy the situation and as such it is a SUPPORTING REASON.

Choice (B)

22. The passage states the relation between memory and genes and says that intelligence has not been defined clearly and talks about how ineffective IQ tests are in measuring intelligence.

A - this contradicts the first line of the passage and hence this is a COUNTER ARGUMENT.

B - also contradicts what is stated in the passage and hence it is a COUNTER ARGUMENT.

C - provides support to the view given in the passage and hence it is a SUPPORTING REASON.

D - states a new point not mentioned hitherto in the passage and hence it is a STATEMENT OF ARGUMENT. Choice (A)

23. The passage laments the apathy shown by the international community towards the genocide being perpetrated in East Timor by the Indonesian army in order to the quell the demand for freedom.

A - contradicts what is stated in the passage and hence it is a COUNTER ARGUMENT.

B - introduces a new angle to the problem stated and hence it is a STATEMENT OF ARGUMENT.

C - firmly states that genocide is an aberration or civil society and must be stopped and hence this is an assertion.

D - substantiates what is given in the passage and hence it is a supporting reason. Choice (C)

24. The central idea of the paragraph is the notion that the meeting of G-20 nations to discuss how to inject more liquidity was a rare or unusual event. This idea reaches its natural conclusion in 4. Choice (D)

25. The paragraph harps on the fact that wellness can reverse the ageing process. The penultimate sentence refers to 'extending life span'. This leads to the conclusion that death may be conquered which is presented in option B. Choice D begins another idea and so cannot conclude this paragraph. Choice (B)

26. The focus of the paragraph is on the role of cognition in hypocrisy and the revelation that when the mind is busy thinking about other things morality becomes a gut-reaction. This conclusion is brought out in option (B) which summarizes the result of the experiment and is hence a good conclusion. Choice (B)

27. The paragraph focuses on the idea that celebrity status sells. Awareness of celebrities regarding their brand value has led to the stars producing their own products instead of endorsing those of others. This is the logical conclusion found in option C. Option A can continue the para rather than conclude it. Option D can begin a new paragraph. Choice (3)

28. The central idea of the paragraph is the growth track of India and China and the fact that this will lead to greater disposable incomes in both countries. Sentence 3 which states the consequence of such incomes is the logical conclusion of the idea. Sentence 1 can be ruled out because the 'if' in the sentences raises doubts about the growth pattern and thus contradicts what has been stated earlier. Choice (3)

29. Argument evaluation:

Situation: Manufacturers claim that their food drinks contain all the nutrients necessary for a child. But a balanced diet containing variety of foods is better.

Reasoning: A well balanced diet is preferable to a food drink because it acts better for the child.

(A) 'Flavour' is irrelevant to the argument.

(B) Children's preference is not the issue.

(C) Also outside the scope of the argument.

(D) Correct. It properly identifies the statement which supports the argument.

D is correct. Choice (D)

30. Evaluation of plan:

Situation: Providing a meal to school going children is likely to reduce absenteeism.

Reasoning: A proper meal provided to school going children reduces absenteeism. This plan will best work only where the absenteeism has been due to non-availability of a good meal to the children.

(A) The absenteeism may be due to other reasons and hence the plan to provide as usual may not work.

(B) The composition of the students whether they are boys or girls is beyond the scope of the argument.
(C) 'Working parents' doesn't provide ground for providing breakfast and hence reducing absenteeism.
(D) Correct. It properly identifies the condition under which the breakfast plan will succeed. Choice (D)

31. Argument evaluation:
Situation: Radiations from cell phones cause hearing defects. Hence cell phones must be avoided.
Reasoning: The argument is that cell phones should not be used as they cause heaving defects. Hence the statement which tells us that cell phones do not affect the ears weakens the argument.
(A) The effect of using 'computers' is beyond the scope of the argument.
(B) Use of a `Ipod' is not the question.
(C) It is not necessary that all those who are defective in hearing must use hearing aids.
(D) Correct. It properly identifies the statement which weakens the argument. Since the telephone operators' job involves having, the fact that their hearing is not impaired despite using cell phones regularly weakens the argument. Choice (D)

32. Evaluation of a plan:
Situation: The students of Alpha University were asked to evaluate the teaching effectiveness of their professors.
Reasoning: The university asks the students, the recipients of the service, to evaluate the effectiveness of teaching of the professors, the providers of services. So the plan is based on the principle that the effectiveness of a service rendered is measured by how the receiver views it.
(A) The plan doesn't measure the actual effectiveness of teaching which must have been done by evaluating the students not the teachers.
(B) Correct. The statement properly identifies the principle on which the plan is based.
(C) It is irrelevant to the argument.
(D) This may be the purpose of the plan but not the principle on which it is based.
B is correct. Choice (B)

33. Evaluation of a plan:
Situation: Venadial is effective in reducing cholesterol levels and the risk of heart attack according to a study. Company *A* in country *Y* should sell the medicine, as its sole selling agent, to earn profit in a short span of time.
Reasoning: To realise the optimism expressed above people must be made aware of the effectiveness of Venadial in reducing heart attacks.
(A) Knowing the permissible cholesterol level will not induce the public to go for the medicine.
(B) Correct. It properly identifies the course to be adopted.
(C) Research on the metabolic response to Venadial is irrelevant to the issue.
(D) This is also irrelevant to the argument.
B is correct. Choice (B)

34. Evaluation of a plan:
Situation: Software companies find the persons required outnumbering the availability. The plan is to meet the requirement.
Reasoning: The available candidates with necessary academic skills is insufficient to meet the requirement. So the only way is to recruit even those without the necessary academic skills, but impart them training to make them fit for the jobs.
(A) It is irrelevant to the discussion as 'pay' is the issue.
(B) The question is of a plan to meet the enhanced demand. So scaling down operations is not the answer.
(C) Correct. It properly identifies the correct plan to meet the increased need.
(D) As it is already stated that the number of candidates with necessary skills is insufficient, entering into agreement to absorb the students in to will not help.
C is correct. Choice (C)

35. Evaluation of plan:
Situation: The Government wants to involve film celebrities in anti smoking campaigns as it is concerned about the health hazards that smoking causes to the public.
Reasoning: The Government plans to involve film celebrities in its anti smoking campaign. This plan will be successful if the general public heeds the film celebrities words on the screen.
(A) Correct. It properly identifies the situation when the Government's plans will be successful.
(B) The film celebrities may be smokers in real life and hence if the public emulates their lives, the plan will not be successful.
(C) 'Why the film celebrities need to be involved in social causes' is beyond the scope of the argument.
(D) It may be helpful in production of the film but it does not strengthens the achievement of the purpose of the plan. Choice (A)

36. Argument constructions:
Situation: Though laws exist against drug trafficking, it is bound to exist. Even if the Government plugs one source, as it will enter through others.
Reasoning: The situation is one where there is a law against something. But even if it is curbed at one end, it is bound to be committed by someone else.
(A) It is not similar. It provides a reason for why the police may be corrupt despite laws.
(B) It only says that all religious persons need not be terrorists.

(C) Correct. It properly identifies the situation which is a similar argument to the one given.
(D) It gives an explanation as to why third degree methods become inevitable. Choice (C)

37. Argument construction:

Situation: Last year exports were more than imports. This year imports were more than exports. Also exports during this year were more than last year's exports.

Reasoning: Last year E, > I, (E – Exports and I – Imports). This year $I_2 > E_2$. Also $E_2 > E_1$. So $I_2 > E_2 > E_1 > I_1$. Any statement contradicting the cannot be an inference.

(A) It is correct because $I_2 > I_1$ so it is not the answer.
(B) It is correct as $E_1 > I_1$, so it is not the correct choice.
(C) Correct. It properly identifies a situation which cannot be inferred. This statement implies $I_1 > E_2$. This is not possible as $E_2 > I_1$.
(D) It can be inferred as $I_2 > E_2$.

C is correct. Choice (C)

38. Evaluation of a plan:

Situation: The Administration wants to encourage the use of biodegradable paper products by encouraging their sale at very low prices.

Reasoning: The strategy will fail to work if the users do not take the bait, i.e. they don't give importance to price differentials.

(A) It should in effect actually help the administration in realising it s purpose.
(B) Correct. It properly identifies a situation where the tourists may not give any importance to the price differentials.
(C) It should actually strengthen not undermine the administrations purpose.
(D) It is said that the results were not produced only in the short run.

B is correct. Choice (B)

39. Argument construction:

Situation: Trendy' is a company engaged in the manufacture of goods which need constant innovation in products and their designs. It is feared that a drop in the percentage of allocation of resources of R&D will reduce sales.

Reasoning: Unless the proportion of resources allocated to R&D is going to determine the extent of innovation, the conclusion cannot be drawn as above.

(A) It cannot be an assumption as there is no mention from which it can be assumed that a particular percentage of resources has to be allocated for research and development to achieve the desired results.
(B) Correct. It correctly identifies the assumption that unless the proportion of resources allocated to the research and development to the total resources determines the extent of innovation, it cannot be held that sales will be affected.
(C) It is not an assumption.
(D) It is outside the scope of the argument.

B is correct. Choice (B)

40. Argument evaluation:

Situation: As above

Reasoning: Allocation of resources are held to directly govern the sales. Then the argument will be weakened if there had been no real decrease in allocation of resources to R&D.

(A) Correct. If the total resources are doubled, then six percent of it will be more than ten percent of the pervious years total resources. Hence there has been a real increase in allocation to R&D which can if anything only increase the sales.
(B) It is outside the scope of the argument.
(C) It may be true but it does not mean that the sales have improved, as the sales of other companies also might have dropped.
(D) Resources allocated to marketing is not the crux the argument.

A is correct. Choice (A)

Verbal Ability Test 5

Number of Questions: 20 **Time: 30 min**

Directions for question 1: The question has a sentence with three blanks followed by four pairs of words as choices. From among the choices, select the pair of words that can best complete the given sentence

1. It is now recognized in many countries that family and social environment of the children have a _____ impact on their performance than the quality of teaching at school and children from poor families score significantly _____ in vocabulary, communication skills, simple arithmetic and the ability to concentrate compared to children from _____ income households.
(A) meagre...higher...higher
(B) lower.....lower....greater
(C) prime.....faster.....lower
(D) greater....lower.....higher

Directions for questions 2 to 11: Fill in the blanks in the given sentences so as to make sense. Select the correct word from the answer choices and mark its number as the answer.

2. If the items in a diary are reliably listed and the list is _____ reviewed then certainly nothing should be forgotten.
(A) perversely (B) phlegmatically
(C) languidly (D) conscientiously

3. The child had a _____ birth defect in the form of a cleft palate.
(A) hereditary (B) compulsive
(C) congenial (D) congenital

4. The people of South Asia had been left behind in economic development _____ of tensions and conflicts.
(A) in view of (B) because
(C) want (D) full

5. Financial difficulties _____ him to discontinue his studies, and take up a job.
(A) enchanted (B) compelled
(C) entreated (D) dictated

6. Sixty houses were totally gutted _____the fire accident.
(A) at (B) beyond
(C) in (D) off

7. The Kaveri community in Bihar, which has for long _____ with poverty and backwardness, is yet to join the mainstream because of its nomadic lifestyle.
(A) suffered (B) exhausted
(C) struggled (D) figured

8. Alzheimer's disease is not usually _____ until significant memory loss is evident.
(A) detected (B) observed
(C) revealed (D) disclosed

9. When the dilapidated structure collapsed, the _____ of the people could be heard from a distance.
(A) gossip (B) wails
(C) eviction (D) bombardment

10. He does not _____ the ugly aspects of human nature in his picture of life.
(A) eliminate (B) include
(C) exclude (D) extricate

11. The airline has increased its frequency _____ nine flights a week to eleven flights a week.
(A) between (B) of
(C) from (D) for

Directions for question 12 to 17: The question has a sentence with two blanks followed by four pairs of words as choices. From among the choices, select the pair of words that can best complete the given sentence.

12. I am not a _____, I am a _____.
(A) 'have been' . . . ''will be'
(B) 'has been' . . . 'will be'
(C) 'was been' . . . 'will being'
(D) 'had been' . . . 'will be'

13. During his childhood, Williams _____ his parents _____ teachers.
(A) never obeyed . . . . or
(B) never had obeyed . . . . or
(C) never obeyed . . . .nor
(D) never had obeyed . . . . nor

14. The two men were very _____ and rather looked the same but had _____ views on important aspects of life.
(A) close . . . wide (B) similar . . . diverse
(C) different . . . complex (D) familiar . . . similar

15. I am not a ____, I am a ____.
(A) 'have been' . . . 'will be'
(B) 'had been' . . . 'will be'
(C) 'was been' . . . 'will being'
(D) 'has been' . . . 'will be'

16. The two thieves _____ arrested on the charge of stealing from _____ house
(A) was . . . an (B) am . . . a
(C) were. . . an (D) were. . . a

17. The country has no need to test its nuclear arsenal in the absence of an external _____ which is precisely why we have a voluntary _____ on testing.
(A) provocation . . . regression
(B) collaboration . . . relapse
(C) wherewithal . . . setback
(D) aggression . . . moratorium

Directions for question 18 to 20: Read the following paragraph and answer the question that follows:

18. People are throwing themselves off the Golden Gate Bridge at the rate of two a month, which makes it the most popular place in the world for suicide. Many San Franciscans think that the solution is to put up a simple barrier. The Psychiatric Foundation of California, which has proposed to construct a barrier to act as a deterrent, has heard several arguments against a barrier; the most persistent has been that people would simply kill themselves somewhere else, so why bother?
Here the word 'deterrent' is closest in meaning to?
(A) Impediment (B) Assistance
(C) Indemnity (D) Restitution

19. In "ambush marketing" campaigns, companies try to promote their brands at sporting events without paying sponsorship fees. Ambush marketers have replaced hooligans as the villains of sporting events, because they undermine official sponsors, who are the main source of revenue in some sports. The IPL organizers have responded by taking control of all prominent advertising sites in the city, including those at train stations and airports, and their use will be limited to official sponsors only. Nevertheless, preventing ambushes is difficult.
Which of the following is closest in meaning to 'ambush'?
(A) Deception (B) Integrity
(C) Virtue (D) Equity

20. Studies of promising young prodigies frequently reveal that most turned out to be failures, in both their professional and personal lives, in later years. Researchers also found that most of the prodigies were diagnosed with symptoms of paranoia and schizophrenia at a later stage. The researchers concluded that mental illnesses inhibited the prodigies from realizing their full potential in later life.
Here, the word 'prodigies' is closest in meaning to
(A) genius (B) Maestros
(C) Adept (D) Apprentice

Answer Keys

1. D	**2.** D	**3.** D	**4.** B	**5.** B	**6.** C	**7.** C	**8.** A	**9.** B	**10.** B
11. C	**12.** B	**13.** A	**14.** B	**15.** D	**16.** D	**17.** D	**18.** A	**19.** A	**20.** A

Hints and Explanations

Explanatory notes for question 1:

1. The sentence compares the social environment and the quality of teaching on the performance of children. Hence the comparative degree of an adjective has to be used in the first blank. 'Lower', and 'greater' are comparative degree adjectives and ore apt. Hence choices A and C can be ignored. According to the passage, children from low income social background are likely to score '*lower'* than their affluent counterparts with *higher* income background. All these words are found in choice D making it the right answer. Choice (D)

Explanatory notes for questions 2 to 11:

2. The context demands 'proper listing' and 'meticulous review' of the list. Among all the options only option D conveys this meaning. Options B and C refer to 'laziness' and 'perversely' refers to 'contrary to what is accepted'. The correct option is D 'conscientiously' which refers to 'being careful and thorough in one's work'. Choice (D)

3. Birth defects are congenital (present from birth) and not hereditary (genetic), compulsive (uncontrollable), or congenial (affable; friendly). Choice (D)

4. The reason the people of South Asia have been left behind in tensions and conflicts. Choice (B)

5. Compel means to force. Financial difficulties forced him to drop his studies and take up a job. Choice (B)

6. **'**In' is the correct preposition to be used as it expresses a period of time during which an event happens. Choice (C)

7. The sentence is about a community in Bihar which has not progressed. The reason for this is its poverty and backwardness. Exhausted means extremely tired. This is a wrong choice. 'Figured' means to understand. 'Sustained' means continued for a long time. The word 'suffered' means to experience physical or mental pain. So these words are not suitable when used in this particular context. 'Struggled' means to use a lot of effort to defeat someone, prevent something, or achieve something. The community has been struggling for a long time to join the mainstream. Choice (C)

8. Detected, which means to discover or identify the presence or existence of something, fits the blank appropriately. Choice (A)

9. Wails are prolonged high-pitched cries of pain or grief thus the word is apt here. Choice (B)

10. The preposition with choices (A), (B), and (D) is 'from' (exclude "from"). Only 'include' will collocate with the preposition 'in'. Choice (B)

11. The frequency is increased from something to something. Hence 'from' is the appropriate preposition in the blank. Choice (C)

Explanatory notes for questions 12 to 17:

12. The sentence implies that the focus should be on what the speaker 'will be' in the future and not what he was in the past. I has - been is a person considered to be outmoded, past his/her prime or no longer of any importance. Choice (B)

13. The correct option is 1. "Never……or" is more apt then "Never……nor". Similarly, there are no two actions taking place here. Therefore, the sentence should be in simple past. Choice (A)

14. The words apt in the context are 'similar' and diverse; note the use of the conjunction 'but' to link these opposite words. Choice (B)

15. A 'has been' is a person or thing considered to be no longer of any importance. The speaker say that he/she does have it him/her to remain relevant. Choice (D)

16. Since 'thieves' is plural, 'very' should be 'were'. Since the word 'house' begins with a consonant, the article before it should be '*a*' Choice (D)

17. The word 'voluntary' rules out 'regression' 'relapse' and 'setback'. Only 'moratorium' makes sense. Choice (D)

Explanatory notes for questions 18 to 20:

18. The word 'deterrent' means an obstacle. This is closest in meaning to 'impediment'. The rest of the options are antonyms or inapt in the given concept. Choice (A)

19. The word 'ambush' means to be in a concealed position, waiting to make a move. Choice (A)

20. The meaning of the word 'prodigy' is a genius. Choice (A)

PART B
NUMERICAL ABILITY

UNIT I
QUANTITATIVE APTITUDE

Quantitative Ability Test 1 ..... **1.35**

Quantitative Ability Test 2 ..... **1.40**

Quantitative Ability Test 3 ..... **1.48**

(Indices, Surds and Logarithms) 1.48

Quantitative Ability Test 4 ..... **1.55**

(Data Interpretation)..... 1.55

Quantitative Ability Test 5 ..... **1.62**

(Permutations and Combinations)..... 1.62

Quantitative Aptitude Test 6 ..... **1.68**

(ERPV, Numbers)..... 1.68

Quantitative Aptitude Test 7 ..... **1.74**

(Quadratic equations and Inequalities)..... 1.74

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Quantitative Ability Test I

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. What is the percentage change in the volume of a cylinder if its height increases by 20% and radius remains the same?
(A) No change
(B) 10% increase
(C) 20% decrease
(D) 20% increase

2. 30% of a number when subtracted from twice the number equals to 33 less than five times the same number. What is the number?
(A) 15 (B) 10
(C) 20 (D) 5

3. In the second and the third match of a series Dravid's score increases by 10% and $21^{7}/_{33}\%$ respectively over the previous match. By what percentage did his score increase in the third match as compared to the first match?
(A) $33^{1}/_{3}\%$ (B) $31^{7}/_{33}\%$
(C) $35^{15}/_{33}\%$ (D) $37^{9}/_{33}\%$

4. Two tests are given with maximum marks of 300 and 200 respectively. A student scores an overall percentage of 60%. If he has scored 40% in the second test, then how many marks did he score in the first test?
(A) 200 (B) 180
(C) 220 (D) 240

5. If a kerosene dealer sells kerosene at ₹8 per litre, he loses ₹400, but if he sells at ₹10 per litre he makes a profit of ₹400. Then, how many litres of kerosene did the dealer sell?
(A) 250 (B) 200
(C) 350 (D) 400

6. By selling 50 metres of cloth, a merchant gains the cost of 10 metres. Find the gain percentage.
(A) 25% (B) $33^{1}/_{3}\%$
(C) 20% (D) 30%

7. By selling 60 metres of cloth, a merchant gains the selling price of 10 metres. The gain percentage is
(A) 25% (B) 20%
(C) 15% (D) $16^{2}/_{3}\%$

8. If $3/5^{th}$ of *B*'s income is 25% less than *A*'s income, then what percentage of *B*'s income is *A*'s income?
(A) 60% (B) 80%
(C) 120% (D) 125%

9. If a dealer gets a commission of 10% on the list price from the company, then the profit made by the company is 50% of its manufacturing cost. If the dealer's commission is increased to 25%, then what will be the profit percentage on the cost of manufacturing for the company?
(A) 10% (B) 25%
(C) 30% (D) 20%

10. Chetan started a cable TV service with an investment of ₹2,00,000. After a few months David joined him with an investment of ₹1,50,000. If at the end of the year David's share was ₹3,00,000 out of a total profit of ₹7,80,000 after how many months did David join Chetan?
(A) 2 (B) 4
(C) 8 (D) 10

11. A man sells an article at a profit of 25%. Had he bought it at 25% less and sold for ₹25 less, he would still have gained 25%. Find the cost of the article.
(A) ₹80 (B) ₹40
(C) ₹60 (D) ₹50

12. A man sold a Doberman and a German Shepherd for ₹5,750 each. On the Doberman he made a profit of 25%, and on the German Shepherd he made a profit of 15%. Find the approximate profit percentage on the whole transaction.
(A) 23% (B) 16%
(C) 18% (D) 20%

13. *A*, *B*, and *C* invested capitals in the ratio 4 : 5 : 6. At the end of the year, they received the profits in the ratio 6 : 5 : 4. Find the ratio of time periods for which they invested their capitals.
(A) 9 : 6 : 2 (B) 9 : 8 : 6
(C) 9 : 3 : 4 (D) 9 : 6 : 4

14. A man purchases a certain number of chocolates at 2 per rupee and the same number of pepper-mints at 5 per rupee. He mixes them together and sells them at 3 per rupee. What is his approximate profit or loss percentage?
(A) 5% profit (B) 5% loss
(C) 4% profit (D) 4% loss

15. A shopkeeper bought a table marked at ₹600 at successive discounts of 10% and 20% respectively. He spent ₹8 on transportation and sold the table for ₹450. Find his profit percentage.
(A) $2^{7}/_{11}\%$ (B) $2^{3}/_{11}\%$
(C) $2^{5}/_{11}\%$ (D) $2^{8}/_{11}\%$

16. A sum of money is invested at a certain rate of simple interest. Find the annual rate of interest if the sum becomes 84% more in 6 years.
(A) 12% (B) 14%
(C) 16% (D) 18%

17. A sum of money becomes $1^{91}/_{125}$ times itself, when invested at compound interest at 20% p.a. Find the duration of the investment.
(A) 1 year (B) 2 years
(C) 3 years (D) 4 years

18. Two sums of money in the ratio 3:4 are lent for a period of 5 years. The rates of interest on the two sums are in the ratio 1:2. The difference in the simple interest on the two sums is ₹1000. Find the total simple interest on the two sums.
(A) ₹2000 (B) ₹2200
(C) ₹2400 (D) ₹2500

19. Two litres of 20% H_2SO_4 solution, three litres of water and five litres of 10% H_2SO_4 are mixed together. How many litres of the resulting solution must be mixed with thirty litres of 18% H_2SO_4 solution so as to get a 15% H_2SO_4 solution?
(A) 30 (B) 10
(C) 15 (D) 20

20. In which of the following ratios by weight should three alloys of gold of purity 18, 20 and 22 carats be mixed to form a fourth alloy whose purity is $20^1/_3$ carats?
(A) 3 : 4 : 5 (B) 4 : 3 : 5
(C) 4 : 5 : 6 (D) 6 : 5 : 4

21. A family consists of a grandfather, a grandmother, father, mother and three children. The average age of father, mother and the 3 children is 25 years. The average age of the three children is 15 years. The average age of parents and grandparents is 50 years, then find the average age of the grand parents (in years).
(A) 70 (B) 68
(C) 58 (D) 60

22. Groundnuts contain 70% oil by weight. Oil is partially extracted and what is left behind is groundnut cake which contains 17% oil by weight. What is the weight of the groundnuts which yield 1 kilogram of cake?
(A) 3.77 kg (B) 2.67 kg
(C) 3.58 kg (D) 2.77 kg

23. One-third of a bottle full of pure listerene is replaced with water. Find the ratio of water and listerene if the above process is carried out for a total of four times.
(A) 16 : 65 (B) 16 : 89
(C) 89 : 16 (D) 65 : 16

24. The number of matches played by a cricketer in the year 2003 is one-fourth of the total number of matches played by him upto the end of 2002. His average score (*i*.e., runs scored per match played) upto the end of the year 2003 is four-thirds of his average upto the end of the year 2002. What is the ratio of the runs scored in the year 2003 to that of the total scored upto the end of the year 2002?
(A) 2 : 3 (B) 3 : 2
(C) 3 : 4 (D) 15 : 16

25. The weighted average of the scores of all the students of three sections *X*, *Y* and *Z* of a class is $33^1/_3$% more than the average of the section *X*. The weighted average of sections *Y* and *Z* is $65^5/_{11}$. If section *X* has a strength of 40 and an average score of 45, what is the combined strength of the section *Y* and Z?
(A) 60 (B) 75
(C) 110 (D) 85

26. Find the expression for the sum of *n* terms of an arithmetic progression, if the tenth term is 40 and the 12^{th} term is 44.
(A) $10n + 25n^2$ (B) $20n + 20n^2$
(C) $25n + 15n^2$ (D) $n^2 + 21n$

27. The first term of an arithmetic progression consisting of 30 terms is 10 and the common difference is 5. Find the ratio of the sum of the 30 terms of the arithmetic progression. to the sum of the last 20 terms of the A.P.
(A) 99 : 13 (B) 96 : 17
(C) 99 : 86 (D) 99 : 68

28. If the sum of the fifth, thirteenth and eighteenth terms of an A.P is zero, find the 12th term of the arithmetic progression.
(A) –2 (B) –1
(C) 0 (D) 1

29. Find the sum of the first 10 terms of the series: $3\ (2^2) + 4\ (3^2) + 5\ (4^2) + \ldots\ldots$
(A) 3009 (B) 4860
(C) 3408 (D) 3608

30. The sum of the first eight terms of a geometric progression. is 510 and the sum of the first four terms of the geometric progression. is 30. Find the first term of the geometric progression, given that it is positive.
(A) 2 (B) 4
(C) 6 (D) 8

31. The terms of an arithmetic progression are all positive. The square of fourth term equals the sum of the squares of the previous two terms. The sum of the first four terms is 14. Find the common difference.
(A) 1
(B) 2
(C) 2
(D) Cannot be determined

32. The first, second and third terms of a geometric progression are equal to the first, seventh and twelfth terms of an arithmetic progression. If the first term and common difference have opposite signs, find the 37^{th} term of the arithmetic progression.
(A) 1 (B) 0
(C) 1 (D) 2

33. Find the sum of the terms of the series
$(1) \times (20), (2) \times (19), (3) \times (18), \ldots (20) \times (1)$.
(A) 1750 (B) 1645
(C) 1540 (D) 1435

34. In a geometric progression, each term is the sum of all the terms following it. The sum to infinity of the terms is 32. If all the terms are positive, then find the first term.

(A) 16 (B) $16\sqrt{2}$

(C) 64 (D) $8\sqrt{2}$

35. The sum of five numbers in geometric progression is 62. The sum of their reciprocals is $\frac{31}{32}$. Find the square of the third number.

(A) 100 (B) 121

(C) 36 (D) 64

Answers Key

1. D	**2.** B	**3.** A	**4.** C	**5.** D	**6.** C	**7.** B	**8.** B	**9.** B	**10.** A
11. A	**12.** D	**13.** D	**14.** B	**15.** B	**16.** B	**17.** C	**18.** B	**19.** C	**20.** A
21. D	**22.** D	**23.** D	**24.** A	**25.** C	**26.** D	**27.** C	**28.** C	**29.** B	**30.** A
31. A	**32.** B	**33.** C	**34.** A	**35.** D					

Hints and Explanations

1. Volume of a cylinder = $\pi r^2 h$
As π is constant and there is no change in radius, volume varies only with height. New height = $(1.2)h$
$\therefore$ New volume = $1.2\pi r^2 h$ i.e., the volume increases by 20%. Choice (D)

2. Let the number be = x
According to the problem, $2x - 0.3x = 5x - 33$
$\Rightarrow$ $3.3x = 33 \Rightarrow x = 10$ Choice (B)

3. Let Dravid's score in the first match be x.
It increases by 10% in the second to $\frac{110}{100}x = \frac{11x}{10}$.
In the third match it increases by
$21\frac{7}{33}\% = \frac{700}{33}\%$ to $\left(100 + \frac{700}{33}\right)\% = \frac{4000}{33}\%$
i.e., score in the third match is
$\left(\frac{1}{100}\right)\left(\frac{4000}{33}\right)\left(\frac{11x}{10}\right) = \frac{4x}{3}$
Over the two year it increases by $\frac{4x}{3} - x = \frac{x}{3}$.

Fractional increase = $\frac{\frac{x}{3}}{x} = \frac{1}{3} = 33\frac{1}{3}\%$. Choice (A)

4. The two tests together had 300 + 200 = 500 marks
The student scored an overall percentage of 60%
$\therefore$ Total marks scored by the student = 500 × 0.6 = 300
Marks scored by the student in the second test = 200 × 0.4 = 80
Hence marks scored in the first test = 300 – 80 = 220.
Choice (C)

5. Let the quantity of kerosene sold by the dealer be x litres
$\therefore$ CP – $8x$ = 400 and $10x$ – CP = 400
$\Rightarrow$ $8x + 400 = 10x - 400 \Rightarrow 5x = 400$ Choice (D)

6. Let the cost price of 1m = CP
Let the sale price of 1m = SP
Given 50SP = 50CP + 10 CP i.e., 60CP = 50SP
$\therefore$ % Profit = (60 – 50) × 100/50 = 20%. Choice (C)

7. Let the cost price of 1m = CP
Let the sale price of 1m = SP
Given 60SP = 60CP + 10SP i.e., 60CP = 50SP
$\therefore$ %Profit = (60 – 50) × 100/50 = 20%. Choice (B)

8. Let A's and B's increase be a and b
$\frac{3b}{5} = a - \frac{25a}{100} = a\left(\frac{3}{4}\right)$
$\therefore a = \frac{4}{5}b = \frac{80}{100}b$
a in 80% of b. Choice (B)

9. Let the list price be = LP
If 10% of LP is given as commission to the dealer the profit for the company is 50% $\Rightarrow$ 0.9 LP = 1.5 CP
If 25% commission is given then
0.75 LP = 0.75 × (1.5 CP / 0.9) = 1.25 CP
% of profit = 25% Choice (B)

10. Chetan's total investment = 2,00,000 X 12
David's total investment = 1,50,000 X x
(where x is the period of investment of David)
Chetan's share in total profit
= 7,80,000 – 3,00,000 = Rs.4,80,000
Now, $\frac{2,00,000 \times 12}{1,50,000 \times x} = \frac{4,80,000}{3,00,000} \Rightarrow x = 10.$
$\therefore$ David joined after 2 months. Choice (A)

11. SP = 1.25 CP
Also given SP - 25 = 1.25 × 0.75 CP
Solving the above equations CP = 80 Choice (A)

12. Total selling price = 5750 × 2 = ₹11,500
Total cost price =
$\frac{5750}{1.25} + \frac{5750}{1.15} = 4600 + 5000 = ₹9600$
$\therefore$ Overall profit percentage = $\frac{1900}{9600} \times 100 \cong 20$
Choice (D)

13. Ratio of time is 6/4 : 5/5 : 4/6

$$\frac{6\times 60}{4}:\frac{5\times 60}{5}:\frac{4\times 60}{6}=9:6:4$$ Choice (D)

14. Let the number of éclairs be x

There CP = $x/2$

CP of peppermint = $x/5$

Therefore total CP = $\frac{x}{2}+\frac{x}{5}=\frac{7x}{10}$

Total SP = $\frac{2x}{3}$

P% = $\frac{\frac{2x}{3}-\frac{7x}{10}}{\frac{7x}{10}}\times 100$. Therefore loss = 5% Choice (B)

15. CP = 600 (.9) (.8) = 432

Total CP = 432 + 8 = 440

SP = 450

P% = $\frac{10}{440}$ x 100 = $2^{3}/_{11}$% Choice (B)

16. If a sum of money invested under simple interest at R% p.a. becomes N times after t years, R is given by $\frac{100(N-1)}{t}$.

As $N = 1 \times 84$ and $t = 6$ years, $R=\frac{100(1\cdot 84-1)}{6}=14$

∴ the rate is 14% p.a. Choice (B)

17. Let the sum of money be ₹ p and duration of the investment be n years.

$$1\frac{91}{125}p=\frac{216}{125}p=p\left(1+\frac{20}{100}\right)^{n}$$

$$\left(\frac{6}{5}\right)^{3}=\left(\frac{6}{5}\right)^{n}\Rightarrow n=3.$$ Choice (C)

18. Let the two sums of money be $3x$ and $4x$ and the rates of interest on the two sums be R% p.a. and $2R$% p.a. respectively. Difference in the simple interest on the

two sums = $\frac{(4x)(2R)(5)}{100}-\frac{(3x)(R)(5)}{100}=1000$

$\Rightarrow \frac{xR}{4}=1000$

$\Rightarrow xR = 4000$

Total simple interest on the two sums

$=\frac{(4x)(2R)(5)}{100}+\frac{(3x)(R)(5)}{100}=\frac{55xR}{100}=₹2200$.

($\because$ xR = 4000) Choice (B)

19. Concentration of H_2SO_4 in the resulting solution

$=\frac{2(0.2)+3(0)+5(0.1)}{2+3+5}=9\%$

Let the amount of the resulting solution that is being mixed with 30 lt of 0.18 H_2SO_4 be x.

$(0.09)x + 0.18(30) = 0.15\ (x + 30)$

$0.9 = 0.06x \Rightarrow x = 15$ Choice (C)

20. Let the weight of 18, 20 and 22 carat gold be x, y and z respectively.

$$\frac{18x+20y+22z}{x+y+z}=\frac{61}{3}$$

Only option (1) satisfies this condition. Choice (A)

21. Average age of children and parents 5 members = 25

∴ Sum of ages = 125 years.

Average age of 3 children = 15

Sum of ages of 3 children = 45

∴ Sum of ages of parents = 80

Sum of the ages of parents and grand parents = 50(4) = 200.

Sum of the ages of grand parents = 120

Average = 120/2 = 60 Choice (D)

22. Since groundnut contains 70% of oil, it means in 1kg of groundnut there is 30% of (or 0.3kg) solid material. From this 1kg of groundnut, 0.3kg is solid which translates to 83% of cake.

∴ The cake in 1kg of groundnut is $\frac{0.3}{0.83}$ kg.

Groundnut cake		Groundnut
$\frac{0.3}{0.83}$ kg	–	1 kg
1kg	–	?

$\Rightarrow \frac{1(1\text{kg})}{\left(\frac{0.3}{0.83}\right)}=2.77\text{kg}$ Choice (D)

23. Let the capacity of the bottle be 1 litre.

The fraction of listerine left after repeating the process 4 times = $\left(\frac{3-1}{3}\right)^{4}=\left(\frac{2}{3}\right)^{4}=\left(\frac{16}{81}\right)$

$\Rightarrow$ For every 81 lt of mixture, listerine is 16 lt and water is 81 – 16 = 65 lt.

∴ The ratio of water and listerine is 65 : 16.

Choice (D)

24. The number of matches and average score are tabulated below.

	Upto end of 2002	In 2003	Upto end of 2003
Average	3x	—	4x
No. of matches	4n	n	5n

Score upto end of 2002 $(3x)(4n) = 12\ xn$.

Score upto end of 2003 $(4x)(5n) = 20\ xn$.

Score in 2003 = $8\ xn$.

Required ratio = $8\ xn : 12\ xn = 2 : 3$ Choice (A)

25. Let $A_{(x+y+z)}$ be the average score of the three section combined. It is given that $A_{(x+y+z)}$

= (1 + 33 1/3%) of Average of x.

= (4/3)45 = 60….. (1).

Let n be the combined strength of sections Y and Z.
Average of the two sections Y and Z together = $65^{5}/_{11}$
Total strength of X + Total strength of (Y and Z)
= Total score of ($X + Y + Z$)

$\Rightarrow$ $(45)(40) + n(65\frac{5}{11}) = (n + 40)(60)$

$\Rightarrow$ $1800 + \frac{720n}{11} = 60n + 2400 \Rightarrow \frac{60n}{11} = 600$

$\Rightarrow$ $n = 110$.

$\therefore$ The combined strength of section Y and Z = 110. Choice (C)

26. If the first term of the A.P is a and the common difference is d, we have the tenth term as
$a + 9d = 40$ ….. (1)
and the twelfth term as a + 11d = 44 …. (2)
Subtracting (1) from (2), we have $2d = 44 - 40$,
$d = 2$
Substituting the value of d in (1) we get, $a = 22$.
The sum of n terms of the A.P.

$= \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2(22) + (n-1)2]$

$= \frac{n}{2}[44 + 2n - 2] = \frac{n}{2}[42 + 2n] = 21n + n^2$ Choice (D)

27. Sum of the first 30 terms of the A.P $= \frac{30}{2}[2(10) + 29(5)]$
$= 15[20 + 145] = 15[165] = 2475$.
Sum of the first 10 terms of the A.P $= \frac{10}{2}[2(10) + 9(5)]$
$= 5[20 + 45] = 5[65] = 325$. Ratio of the sum of the first 30 terms of the A.P. to the sum of the last 20 terms of the A.P = (2475) : (2475 – 325) = 2475 : 2150
= 5[495] : 5[430] = 495 : 430 = 99 : 86. Choice (C)

28. If the first term is a and the common difference is d, we have the sum of the fifth, thirteenth and eighteenth terms as
$(a + 4d) + (a + 12d) + (a + 17d) \Rightarrow 3a + 33d = 0$.
Dividing by 3, we have $a + 11d = 0$. Hence the 12^{th} term of the A.P is 0. Choice (C)

29. $S = 3(2)^2 + 4(3)^2 + 5(4)^2 + \ldots\ldots$ 10 terms.
$= (2 + 1)2^2 + (3 + 1)3^2 + (4 + 1)4^2 + \ldots..$ 10 terms
$= (2^3 + 2^2) + (3^3 + 3^2) + (4^3 + 4^2) + \ldots..$ 10 terms
$= (2^3 + 3^3 + 4^3 + \ldots\ldots + 11^3) + (2^2 + 3^2 + 4^2 \ldots\ldots + 11^2)$
$= (1^3 + 2^3 + 3^3 \ldots + 11^3) + (1^2 + 2^2 + 3^2 + \ldots + 11^2) - 1^3 - 1^2$
= 4355 + 505 = 4860. Choice (B)

30. If the first term of the G.P is a and the common ratio is r, we have $\frac{a(r^8 - 1)}{r - 1} = 510$ … (1)

and $\frac{a(r^4 - 1)}{r - 1} = 30$ …. (2)

Dividing (1) by (2) we have $\frac{\frac{a(r^8 - 1)}{r - 1}}{\frac{a(r^4 - 1)}{r - 1}}$

$= r^4 + 1 = \frac{510}{30} = 17$.

$r^4 = 17 - 1 = 16$

$r = \pm\sqrt[4]{16} = \pm 2$

First term of the G. P, a $= \frac{510(r-1)}{r^8 - 1}$

As first term is positive, $r = 2$ is taken.

$a = \frac{510(2-1)}{2^8 - 1} = \frac{510(1)}{255} = 2$. Choice (A)

31. Let the first term and the common difference be a and d respectively.
$(a + 3d)^2 = (a + 2d)^2 + (a + d)^2$
$a^2 + 6ad + 9d^2 = 2a^2 + 6ad + 5d^2 \Rightarrow a = \pm 2d$
As all the terms are positive, $a = 2d$
$a + a + d + a + 2d + a + 3d = 14 \Rightarrow d = 1$.
Choice(A)

32. Let the first term of either progression be a.
Let the common difference of the arithmetic progression be d.d $(a + 36d) = 0$
as a and d have opposite signs, $d \neq 0$.
$a + 36d = 0$
37th term = 0 Choice (B)

33. The terms of the series are in the form $x(21 - x)$
Required sum $= \sum_{x=1}^{20} x(21 - x)$

$= \frac{(21)(20)(21)}{2} - \frac{1}{6}(20)(21)(41) = 1540$ Choice (C)

34. Let the first term and the common ratio be a and r respectively
First term = sum of all the terms following it
$a = \frac{a}{1-r} - a$
$a(1 - 2r) = 0$
As all the terms are positive, $a \neq 0$.
$1 - 2r = 0$
$r = 1/2$
$\frac{a}{1-r} = 16$ Choice (A)

35. Let the first number and the common ratio be $\frac{a}{r^2}$ and r respectively.

$\frac{a}{r^2} + \frac{a}{r} + a + ar + ar^2 = 62$ …. (1)

$\frac{r^2}{a} + \frac{r}{a} + \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} = \frac{31}{32}$

$\Rightarrow$ $\frac{1}{a^2}\left(ar^2 + ar + a + \frac{a}{r} + \frac{a}{r^2}\right) = \frac{31}{32}$

(1) ÷ (2) $\Rightarrow a^2 = \frac{62}{\frac{31}{32}} = 64$ Choice (D)

QUANTITATIVE ABILITY TEST 2

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 16: Select the correct alternative from the given choices.

1. Two men, two women and six part-time workers take 12 days to complete a job. The same job can be completed by 10 men and 18 part-timers in 4 days. If two men and three women take 16 days to complete that job, find the time taken by one woman to complete that job (in days).
(A) 96 (B) 60
(C) 75 (D) 100

2. *A* can complete a job in 20 days. *B* works twice as fast as *A*. They both work together for 5 days. On the 6th day, they complete the job with the help of *C*. Find the time taken by *C* alone to complete the job (in days).
(A) 5 (B) 6
(C) 10 (D) 12

3. Four men take ten days to complete one-third of a work. How many more men are required to complete the remaining work in five days?
(A) 16 (B) 14
(C) 15 (D) 12

4. *A* and *B* working separately can do a piece of work in 5 days and 10 days respectively. They work on alternate days starting with *B* on the first day. In how many days will the work be completed?
(A) 6 (B) 7
(C) 8 (D) 9

5. A tank is fitted with three pipes *A*, *B* and *C*. The three pipes can be used as inlet or outlet pipes with the same flow rates. When one among *A*, *B* and *C* in turns works as outlet pipe and the other two as inlet pipes, it takes 30, 40 and 24 minutes to fill the tank respectively. Find the time taken by *A* to fill the empty tank (in minutes).
(A) 30 (B) 40
(C) 25 (D) 20

6. *A* and *B* take respectively 12 days and 27 days more time to complete a piece of work, working alone, than when they work together. Find the time taken by them to complete the work working together.
(A) 15 days (B) 20 days
(C) 24 days (D) 18 days

7. *A*, *B* and *C* work together to complete a job. *A* gets ₹600 out of the total share of ₹2400. If *A* works twice as fast as *B*, find the share of *C*.
(A) ₹1200 (B) ₹1500
(C) ₹1000 (D) ₹1600

8. *A*, *B* and *C* work at the same rate. *A* starts the job and after 25% of the work is completed, he leaves. *B* and *C* take over and complete the remaining work together in 18 more days. Find the time for which *A* worked (in days).
(A) 6 (B) 20
(C) 12 (D) 24

9. Six taps working together take 12 minutes to fill a tank. Find the time taken (in minutes) by 24 taps working together to fill a tank twice as big.
(A) 6 (B) 8
(C) 12 (D) 24

10. *P* can do a piece of work in 12 days working 6 hrs a day. *Q* can do the same work in 18 days working 5 hrs a day. If *P* and *Q* work together 4 hrs a day, then in how many days can they complete the work?
(A) 10 (B) 11
(C) 12 (D) 14

11. *A*, *B* and *C* can complete a piece of work in 20, 30 and 20 days respectively. They start the work together but *A* leaves after 5 days. After some more days *C* leaves. *B* completes the remaining work in 5/3 more days. For how many days does *B* work?
(A) $\frac{10}{3}$ (B) 10
(C) $\frac{14}{3}$ (D) 14

12. *A*, *B* and *C* take 20, 30 and 60 days to complete a job. *A* works along with *B* on the 1st day and with *C* on the 2nd day. If they continue in this manner, then find the time taken (in days) to complete the work.
(A) $12\frac{5}{6}$ (B) $18\frac{4}{5}$
(C) $14\frac{1}{4}$ (D) $13\frac{1}{4}$

13. Amar can complete a job in 15 days, while Bhavan can complete it in 10 days. They start working together and two days before the work was expected to be completed, Bhavan left. Find the time taken by Amar to complete the remaining work (in days).
(A) 4 (B) 5
(C) 6 (D) 8

14. A pipe can fill a 1000 litre tank in 10 minutes while another pipe can empty a 600 litre tank in 8 minutes. If they work together, then how long will they take (in minutes) to fill a 500 litre tank?
(A) 10 (B) 15
(C) 20 (D) 25

15. *A* and *B* can complete a job in 25 days and 20 days respectively, working alone. With the help of *C*, they can complete the job in $6^2/_3$ days. Find the

percentage of work completed by the fastest worker of the three.

(A) $20\frac{1}{3}\%$ (B) 25%

(C) 40% (D) $16\frac{2}{3}\%$

16. A tank has three inlet pipes I, II and III fitted to it whose flow rates are in the ratio 2 : 5 : 6. Pipe III takes 1 hour less than pipe II to fill the tank. Find the time (in hours) taken by pipe I to fill the tank.

(A) 6 (B) 10

(C) 5 (D) 15

Directions for questions 17 and 18: These questions are based on the following data.

In a city there are 5 major traffic junctions – *A*, *B*, *C*, *D* and *E*. There are no direct roads connecting *AC*,*BE* or *CE* but for every other pair of junctions, there are direct connecting roads, which all happen to be of equal length. Traffic moves at recommended uniform speeds on each road – at 20 km/hr on *BD* and *AD*, at 30 km/hr on *AE*, at 40 km/hr on *BC* and *CD*, and at 60 km /hr on *AB* and *DE* (Assume any direct connecting road is straight line).

17. A traffic inspector wants to visit any three traffic junctions in the shortest possible time, starting from *A*. What are the three points that he can visit (other than *A*) in order if he drives at the recommended speed on each road?

(A) *BCD* (B) *EDB*

(C) *BDC* (D) *BDE*

18. A new direct connecting road is constructed joining *A* and *C* with its recommended speed fixed at 50 km/hr. If *AB* = 10 km, find the time (in minutes) in which the traffic inspector can complete the round trip *ABCA*.

(A) $12 + 25\sqrt{3}$

(B) $25 + 12\sqrt{3}$

(C) $24 + 12.5\sqrt{3}$

(D) Cannot be determined

Directions for questions 19 and 20: These questions are based on the following data.

Cities *P*, *Q* and *A* are in different time zones. *P* and *Q* are located at 4500 km, east and west of *A* respectively. The table below describes the schedule of an airline operating non-stop flights between *A* and *P*, *A* and *Q*. All the times indicated are local and on the same day.

Departure		Arrival	
City	Time	City	Time
A	7 : 00 am	P	3 : 00 pm
A	9 : 00 am	Q	12 : 00 noon

Planes cruise at the same speed to both the cities but effective speed is influenced by a steady wind blowing from east to west at 75 kmph.

19. What is the plane's cruising speed (in kmph)?

(A) 825

(B) 900

(C) 875

(D) Cannot be determined

20. What is the time difference between cities *A* and *Q*?

(A) 1 hour

(B) $2\frac{1}{2}$ hours

(C) 2 hours

(D) Cannot be determined

Directions for questions 21 to 35: Select the correct alternative from the given choices.

21. Amar covered the first one-fourth of a certain distance at 2 km/hr, half of the remaining distance at 3 km/hr and the remaining distance at 4 km/hr. Find his average speed (in km/hr) for the entire journey.

(A) $2\frac{7}{11}$ (B) $2\frac{8}{11}$

(C) $2\frac{9}{11}$ (D) $2\frac{10}{11}$

22. A man starts from *P* at 8 a.m. and reaches *Q* by 9 : 30 a.m. At what time should he start from *Q* to reach *R* at 11 : 30 a.m., where *PQ* : *QR* = 10 : 11?

(A) 10 : 01 a.m. (B) 9 : 59 a.m.

(C) 9 : 50 a.m. (D) 9 : 51 a.m.

23. A boat covered a certain distance upstream and returned to the starting point. If the speed of the boat in still water is doubled and the speed of the stream is tripled, it would have taken the same time for the round trip. Find the ratio of the speed of the boat in still water to the speed of the stream.

(A) $\sqrt{5} : \sqrt{2}$ (B) $\sqrt{3} : \sqrt{2}$

(C) $\sqrt{7} : \sqrt{2}$ (D) 3 : 2

24. A boat started travelling downstream from a point *A* on a river. After it had travelled 12 km, a log started floating from *A*. The boat travels for 2 more hours in the same direction and then turns around and meets the log at a point 12 km from *A*. If the speed of the boat in still water is thrice the speed of the stream, find the speed of the stream (in km/hr).

(A) 1 km/hr (B) 2 km/hr

(C) $\frac{3}{2}$ km/hr (D) 4 km/hr

25. In a race, *A* gives *B* a start of 25 m and *C* a start of 50 m. If *B* runs 50% faster than *C* and all the three reach the finishing point simultaneously, then find the ratio of the speeds of *A* and *C*.

(A) 2 : 1 (B) 4 : 3

(C) 5 : 4 (D) 3 : 1

26. In a 200 m race, A gives B a start of 10 m and beats him by 10 m or 2 seconds. Find the speed of A (in m/s).

(A) $\frac{200}{17}$ (B) $\frac{150}{17}$

(C) $\frac{125}{17}$ (D) $\frac{50}{9}$

27. On a 900 m long circular track, A, B and C start running from the same point simultaneously. A runs in the clockwise direction at 2 m/s while B and C run in the anti-clockwise direction at 3 m/s and 4 m/s respectively. Find the time interval (in seconds) between A and C meeting for the first time at the starting point and B and C meeting for the first time.

(A) 600 (B) 450

(C) 750 (D) 800

28. A, B and C run along a circular track with speeds in the ratio 1 : 2 : 4 starting from the same point simultaneously. If A takes 3 minutes to complete one round of the track, find the time taken (in minutes) by the three to meet at the starting point for the first time.

(A) 6 (B) $\frac{3}{2}$

(C) 3 (D) Cannot be determined

29. Two men A and B start from two points P and Q simultaneously towards each other. They meet after two hours of their starting, B takes 3 hours less to reach P than A takes to reach Q. Find the ratio of the speeds of A and B.

(A) 3 : 2 (B) 2 : 1

(C) 1 : 2 (D) 3 : 1

30. A man travels 51 km in 61 minutes and 30 seconds with an usual speed of 50 km/hr. There are some speed breakers on the road. Each speed breaker reduces his speed to 80% of his usual speed for a distance of 50 m about the speed breaker. Find the number of speed breakers that he crossed.

(A) 20 (B) 25

(C) 30 (D) 35

31. When the speed of a train is increased by 5 m/s, it would take 40 seconds to cross a 200 m long platform. If it crosses a 300 m long platform in 50 seconds, at its original speed, then find the original speed of the train (in m/s).

(A) 35 (B) 15

(C) 20 (D) 30

32. Two trains take 80 seconds to cross each other, when travelling in the same direction. They take 60 seconds to cross each other, when travelling in opposite directions. Find the ratio of the speeds of the faster and the slower train.

(A) 4 : 1 (B) 7 : 1

(C) 3 : 1 (D) 6 : 1

33. There are two cars 80 km apart. When they travel in the same direction, they would take twice the time to meet, compared to the time they would take to meet while travelling towards each other. Find the ratio of their speeds.

(A) 3 : 1 (B) 2 : 1

(C) 4 : 3 (D) 3 : 2

34. By travelling 20% faster than his usual speed, a person reaches his office from home 10 minutes earlier than his usual time. By how many minutes would he be delayed as compared with his usual time, if he travels 25% slower than his usual speed?

(A) 10 (B) 15

(C) 20 (D) 25

35. A frog spots a snake 30 m behind it. It starts moving away from it at 12 m/s. After 5 seconds, it sees that the snake has just begun to move towards it at 20 m/s and increases its speed by 3 m/s. Find the time taken by the snake (in seconds) to catch the frog.

(A) 15 (B) 18

(C) 21 (D) 24

Answer Key

1. A	**2.** C	**3.** D	**4.** B	**5.** A	**6.** D	**7.** B	**8.** C	**9.** A	**10.** A
11. B	**12.** D	**13.** B	**14.** C	**15.** C	**16.** D	**17.** A	**18.** B	**19.** A	**20.** C
21. D	**22.** D	**23.** C	**24.** B	**25.** A	**26.** D	**27.** B	**28.** C	**29.** C	**30.** A
31. D	**32.** B	**33.** A	**34.** C	**35.** B					

Hints and Explanations

1. Let the work done by 1 man, 1 woman and part-timer in a day be m, w and p units respectively.

Given $(2m + 2w + 6p)\ 12 = (10m + 18p)\ 4$

$24w = 16m \Rightarrow m = \frac{3}{2}w$

Work done by 2 men and 3 women in 16 days

$= [[3\ w] + 3\ w]\ 16 = 96w$

Time taken by 1 woman to complete that job

$= \frac{96w}{w} = 96$ days Choice (A)

2. A can do $\frac{1}{20}th$ of the job in a day. B can do $\frac{1}{10}th$ of the job in a day. In 6 days they will together complete $\frac{9}{10}th$ of the job. The balance $\frac{1}{10}th$ of the job is done by C on 6th day. Hence C would take 10 days to complete the job independently. Choice (C)

3. Four men can do one-third of the work in 10 days. In 5 days, one-third of the work can be completed by 8 men. Two-thirds of the work can be completed by 16 men. As there are 4 men, 12 men are required additionally. Choice (D)

4. A can do the work in 5 days. The part of the work done by A in one day $= \frac{1}{5}$

B can do the work in 10 days.

The part of the work done by B in one day $= \frac{1}{10}$

both A and B in two days $= \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

The part of the work done in 6 days $= 3 \times \frac{3}{10} = \frac{9}{10}$

The part of the remaining work $= 1 - \frac{9}{10} = \frac{1}{10}$

As B starts the work, $\frac{1}{10}$th of the work can be done by B on 7th day.

$\therefore$ In 7 days, the work will be completed. Choice (B)

5. Let the time (in minutes) taken by A, B and C to either fill or empty the tank be a, b and c respectively.

$\frac{1}{b} + \frac{1}{c} - \frac{1}{a} = \frac{1}{30}$ ..... (1)

$\frac{1}{c} + \frac{1}{a} - \frac{1}{b} = \frac{1}{40}$ ..... (2)

$\frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{1}{24}$ ..... (3)

Adding the above three equations, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{10}$ --- (4)

Subtracting equation (1) from equation (4),

we get $\frac{2}{a} = \frac{2}{30}$

$\Rightarrow \quad a = 30.$ Choice (A)

6. Let the time taken by A and B working together to complete the work be t days.Time taken by A alone and B alone to complete the work is $(t + 12)$ days and $(t + 27)$ days respectively.

Work done by A and B working together in a day

$= \frac{1}{t+12} + \frac{1}{t+27}$ which is equal to $\frac{1}{t}$.

$\Rightarrow \quad \frac{1}{t+12} + \frac{1}{t+27} = \frac{1}{t}$

$\Rightarrow \quad \frac{t+27+t+12}{(t+12)(t+27)} = \frac{1}{t}$

$\Rightarrow \quad 2t^2 + 39t = t^2 + 39t + 324$

$\Rightarrow \quad t^2 = 324 \Rightarrow t = 18.$ Choice (D)

7. As A gets $\frac{1}{4}$th of the total share, he completes $\frac{1}{4}$th of the total work. B whose rate is half that of A, completes $\frac{1}{8}$th of the total work, for which he gets ₹300.

$\therefore$ Share of $C = 2400 - 600 - 3000$

$= ₹1500.$ Choice (B)

8. The time taken by B and C together to complete $\left(\frac{3}{4}\right)$th of the work is 18 days

$\therefore$ The time taken to complete 1 unit of work

$= 18 \times \left(\frac{4}{3}\right) = 24$ days

$\therefore$ The time taken by each alone to complete the work $= (24)\ (2) = 48$ days.

$\Rightarrow$ The time taken by A to complete (1/4)th of the work $= \left(\frac{1}{4}\right)(48) = 12$ days Choice (C)

9. Time taken by 24 taps working together to fill the tank would be $\frac{1}{4}th$ of the time taken by 6 taps working together i.e. 3 minutes. To fill a tank twice as big, 24 taps would take 6 minutes working together Choice (A)

10. Time taken by P to complete the work in 12×6 i.e., 72 man hours.

Time taken by Q to complete the work in 18×5 i.e., 90 man hours

The part of the work done by P and Q in

1 hr $= \frac{1}{72} + \frac{1}{90} = \frac{90+72}{72(90)} = \frac{1}{40}$

$\therefore$ They complete the work in 40 hours.

By working 4 hrs per day, they can complete in $\frac{40}{4}$ i.e., 10 days Choice (A)

11. Let the total work be 1 unit

$$\frac{5}{20}+\frac{x+5}{20}+\frac{x+5+\frac{5}{3}}{30}=1$$

$\Rightarrow \quad x = 10/3$

Time for which B worked $= x+5+\frac{5}{3}=10$ days.

Choice (B)

12. A and B complete $\frac{1}{20}+\frac{1}{30}$ *i.e.*, $\frac{1}{12}$th of the work on 1st day. A and C complete $\frac{1}{20}+\frac{1}{60}$ *i.e.*, $\frac{1}{15}$th of the work on the 2nd day. In 2 days, $\frac{3}{20}$th of the work would be completed. Working in this way, $\frac{9}{10}$th of the work would be completed in 12 days. Of the balance $\frac{1}{10}$th of the work, A and B would complete $\frac{1}{12}$th of the work the next day. A and C would complete the balance $\frac{1}{60}$th of the work in another $\frac{1}{4}$th of a day. Hence a total of $13\frac{1}{4}$ days would be taken to complete the work.

Alternate method:

Assume the work (in units) to be the LCM of the individual time taken by A, B and C to complete the job i.e., 60 units. Capacities of A, B and C would be 6 units a day. A and B would complete 5 units the first day. A and C would complete 4 units the second day. Hence 9 units would be completed in two days. In 12 days, 54 units would be completed. Of the balance 6 units, 5 units would be completed the 13th day. The balance 1 unit would be completed in $\frac{1}{4}$th of the 14th day. Hence a total of $13^1/_4$ days would be taken to complete the work. Choice (D)

13. Amar and Bhavan would have taken 6 days to complete the job working together. Bhavan left after 4 days of the start of the work. In 4 days, Bhavan would have completed $\frac{4}{10}$ *i.e.* $\frac{2}{5}$th of the work. Amar completes the remaining $\frac{3}{5}$th of the work for which he would have taken $\frac{3}{5}(15)$ i.e., 9 days. Hence Amar completes the remaining work in 5 days. Choice (B)

14. Filling rate of first pipe $=\frac{1000\text{ lit}}{10\text{ min}}=100$ lit/min.

Emptying rate of second pipe $=\frac{600\text{ lit}}{8\text{ min}}=75$lit/min.

Working together they can fill 25 lit/min. Time taken by them working together to fill a 500 litre tank $=\frac{500}{25}$ or 20 minutes. Choice (C)

15. Let us assume that the third person takes c days to complete the work independently. Total work completed by the three in a day, working together $=\frac{1}{25}+\frac{1}{20}+\frac{1}{c}=\frac{1}{6\frac{2}{3}}$

$$\frac{1}{c}=\frac{3}{20}-\frac{1}{20}-\frac{1}{25}\Rightarrow\frac{1}{c}=\frac{3}{50}$$

As the third person completes most of the work in a day, he is the fastest.

Ratio of work completed by the three persons $=\frac{1}{25}:\frac{1}{20}:\frac{3}{50}=4:5:6$

Percentage of work done by the third person $=\frac{6}{(4+5+6)}(100\%)=40\%$. Choice (C)

16. Let the volume of the tank be V litres. Let the filling rates of pipes I, II and III be $2x$, $5x$ and $6x$ respectively (in litres/hour)

$$\frac{V}{6x}=\frac{V}{5x}-1\Rightarrow V=30x$$

Time taken by pipe I to fill the tank $=\frac{30x}{2x}=15$ hours.

Choice (D)

If the 10 possible pairs of points, for 7 pairs, the distances between the points equal. This is possible if of the 5 points, 4 are consecutive vertices of a regular hexagon and the 5th is the centre of the circum circle. We can think of the following figure.

Now we have to final which of these 5 points in A, which is B etc., AC, BE, CE have no direct roads connecting them. In the figure 13, 25 and also 35 have no direct roads connecting them (35 is connected through 4 not directly)

$\therefore$ We get the following possibilities. For either questions that follow it does not matter, which of the two figures we use.

17. The routes and recommended speeds are shows in the figure below.

The routes and the time taken are tabulated below.

We can take

$AB = BC = CD = DB = AD = DE = EA = r$ km

Route time Route Time

(1) $ABCD$ $\frac{r}{60}+\frac{r}{40}+\frac{r}{40}$

(2) $AEDC$ $\frac{r}{30}+\frac{r}{60}+\frac{r}{40}$

(3) $AEDB$ $\frac{r}{30}+\frac{r}{60}+\frac{r}{20}$

(4) $ABDE$ $\frac{r}{60}+\frac{r}{20}+\frac{r}{60}$

Multiplying all the time by LCM (60, 40, 30, 20) or 120, we get $8r$, $12r$, $9r$, $10r$ respectively

$\therefore$ For $ABCD$ the time is the least. Choice (A)

18. If AC and BE are also connected, each dist each distance would be $\sqrt{3}\,r$. The time for the round trip $ABCD$ is

$$\frac{r}{60}+\frac{r}{40}+\frac{\sqrt{3}r}{50}=\frac{10r+15r+12\sqrt{3}r}{600}=\frac{\left(25+12\sqrt{3}\right)(10)}{600}$$

($\because r = 10$ km) $= 25 + 12\sqrt{3}$ min. Choice (B)

Solutions for questions 19 and 20:

Let the speed of the plane be x km

Let the time difference between A and P be t hours (i.e., P is t hours ahead of A).

$\therefore$ The time difference between A and Q is t hours (Q is t hours behind A)

$\therefore$ we have the following equations.

(from A to P) $\frac{4500}{x-75} = 8 - t$ ..... (1)

(from A to Q) $\frac{4500}{x+75} = 3 + t$ ..... (2)

(1) + (2) $\Rightarrow 4500\left[\frac{2x}{x^2-75^2}\right] = 11$

$\Rightarrow 11x^2 - 11(75)^2 = 9000x$

$\therefore x = \frac{9000\pm\sqrt{(9000)^2-4(11)(-11)75^2}}{2(11)}$

$= \frac{9000\pm(150)\sqrt{60^2+121}}{22} = \frac{9000\pm150(61)}{22}$

As x is positive, $x = \frac{9000+9150}{22} = 825$

From (1), $\frac{4500}{825-75} = 8 - t$

$\Rightarrow 6 = 8 - t \Rightarrow t = 2$

19. Choice (A)

20. Choice (C)

Solutions for questions 21 to 35:

21. Let the total distance covered by Amar be d km. Amar covered $\frac{d}{4}$ km at 2 km/hr, $\frac{3d}{8}$ km at 3 km/hr and $\frac{3d}{8}$ km at 4 km/hr.

Total travel time of Amar $= \frac{\frac{d}{4}}{2}+\frac{\frac{3d}{8}}{3}+\frac{\frac{3d}{8}}{4}=\frac{11d}{32}$ hours

Average speed of Amar

$= \frac{\text{Total distance he travelled}}{\text{His total travel time}}$

$= \frac{d}{\frac{11d}{3}11d/3} = 2\frac{10}{11}$ hours

Alternative method:

It can be seen from the normal method, as d cancels finally, any value of d can be taken. Taking $d = 8$,

total travel time of Amar $= \frac{2}{2}+\frac{3}{3}+\frac{3}{4}=\frac{11}{4}$ hours

Average speed of Amar $= \frac{8}{\frac{11}{4}} = 2\frac{10}{11}$ km/hr.

Choice (D)

22. From Q to R, he has to cover $\frac{11}{10}$ of the distance from P to Q.

$\therefore$ Time taken by him to reach R from Q will be $\frac{11}{10}$ times the time taken by him to reach R from Q i.e., 90 minutes $\times \frac{11}{10}$.

$\therefore$ He needs 99 minutes.

To reach R by 11 : 30 a.m. he should start from Q at 9 : 51 a.m. Choice (D)

23. Let the distance travelled in each direction (upstream as well as downstream) be d km. Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr. Total travel time of the boat

= upstream travel time + downstream travel time

$= \frac{d}{x+y}+\frac{d}{x-y}$

Now the speed of the boat in still water and the speed of the stream are $2x$ km/hr and $3y$ km/hr respectively,

Total travel time $= \frac{d}{2x+3y}+\frac{d}{2x-3y}$

Given, $\frac{d}{x+y}+\frac{d}{x-y}=\frac{d}{2x+3y}+\frac{d}{2x-3y}$

$\frac{2dx}{x^2-y^2}=\frac{4dx}{4x^2-9y^2}$; $dx\,(2x^2-7y^2)=0$

$dx=0$ or $x=\pm\sqrt{\frac{7}{2}y}$

As dx cannot be 0 ($\because d>0$ and $x>0$) and x and y are both positive,

$x=\frac{\sqrt{7}}{\sqrt{2}}y \Rightarrow \frac{x}{y}=\frac{\sqrt{7}}{\sqrt{2}}$. Choice (C)

24. Let the point 12 km from A and the point where the boat turns back be B and C respectively.
If the speed of the stream is y km/hr, speed of the boat in still water = $3y$ km/hr. It travels for 2 hrs to cover BC.
$\therefore$ $BC = (3y + y)\,2 = 8y$ km
Time taken by the boat to travel from C to B
$= \frac{8y}{3y-y} = 4$ hr.
As the boat takes 6 hours to travel from B to C and back,
$\frac{12}{y}=6$ or $y=2$ Choice (B)

25. Let the length of the race be x m.
By the time A finish the race, B and C would have run $(x-25)$ and $(x-50)$ m respectively.
As B is 50% faster than C, $\frac{x-25}{x-50}=\frac{3}{2} \Rightarrow x=100$
Ratio of the speeds of A and $C = x : (x-50) = 2 : 1$.
Choice (A)

26. Speed of $B = 10/2 = 5$ m/s
Time for which B would have run when A finishes the race $= \frac{200-(10+10)}{5} = 36$ seconds.
So, A takes 36 seconds to run the race.
$\therefore$ Speed of $A = \frac{200}{36}=\frac{50}{9}$ m/s. Choice (D)

27. Time taken by A and C to meet for the first time at the starting point = LCM $\left(\frac{900}{2},\frac{900}{4}\right)=450$ sec
Time taken by B and C to meet for the first time
$= \frac{900}{\text{Difference of the speeds of } B \text{ and } C} = 900$ seconds
Required time interval is 450 seconds.
Choice (B)

28. Let the speeds (in m/min) of A, B and C be x, $2x$ and $4x$ respectively. Let the length of the track be L m. Given that the time taken by A to complete one round = L/X = 3 minutes. Time taken by all the three to meet for the first time = LCM $\left(\frac{L}{x},\frac{L}{2x},\frac{L}{4x}\right)=\frac{L}{x}=3$ minutes.
Choice (C)

29. Let the speeds of A and B be x km/hr and y km/hr respectively. Distance from P and Q to their first meeting point are $2x$ km and $2y$ km respectively. Times taken by A and B to reach Q and P from their first meeting point are $\frac{2y}{x}$ hours and $\frac{2x}{y}$ hours respectively.

$2\frac{y}{x}=2\frac{x}{y}+3$

Substituting the choices in the above equation, only $\frac{x}{y}=\frac{1}{2}$, satisfies the condition. Choice (C)

30. If the man covers the entire distance at the usual speed, he takes $\frac{51}{50}$ hr or 1.02 hr = 1 hr $\frac{2(60)}{100}$ min = 1 hr / min 12 s.
But he actually takes 1 hr 1 min 30 s, i.e., 18 s more. For one speed breaker, he takes a certain extra time, which is $\left(\frac{0.05}{40}-\frac{0.05}{50}\right)\text{hr}=\frac{0.05}{10}\left(\frac{1}{20}\right)\text{hr}=\frac{5(36)}{200}\text{s}$
$= 0.9$s
$\therefore$ He has to cross $\frac{18}{0.9}$ or 20 speed breakers.
Choice (A)

31. Let the length of the train be L m and speed of the train be s m/sec. Time taken by the train to cross a 200 m long platform at increased speed = $\frac{L+200}{s+5}=40$
$\Rightarrow$ $L = 40s$
Time taken by the train to cross a 300 m long platform (in seconds) = $\frac{L+300}{s}=50$
As $L = 40s$,
$40s + 300 = 50s$
$s = 30$. Choice (D)

32. Let the lengths of the two trains be L_1 and L_2
Let the speeds of the faster and slower trains be S_1 and S_2 respectively.
$\frac{L_1+L_2}{S_1-S_2}=80 \Rightarrow L_1+L_2=80\,(S_1-S_2)$
$\frac{L_1+L_2}{S_1-S_2}=60 \Rightarrow L_1+L_2=60\,(S_1-S_2)$
$80\,(S_1-S_2)=60\,(S_1+S_2)$
$20S_1=140S_2 \Rightarrow \frac{S_1}{S_2}=\frac{7}{1}$. Choice (B)

33. Let the speeds (in km/hr) of the faster and slower cars be x and y respectively.Time taken by the cars to meet when they travel in the same direction and in the opposite direction are $\frac{80}{x-y}$ hours and $\frac{80}{x+y}$ hours respectively.

$\frac{80}{x-y} = 2\left(\frac{80}{x+y}\right)$

$80(x+y) = 160(x-y)$ $240y = 80x$

$\frac{x}{y} = \frac{3}{1}$. Choice (A)

34. Let the usual speed of the man be S km/hr. If he travels 20% faster, he would travel at $S + \frac{20}{100}S = \frac{6}{5}S$ km/hr.

As his speed is $\frac{6}{5}th$ of his usual speed, he would take $\frac{5}{6}th$ of the usual time to travel to office. He saves one sixth of his usual time = 10 minutes

$\Rightarrow$ His usual time = 60 minutes

If he travels 25% slower than his usual speed, he would travel at $S - \frac{25}{100}S = \frac{3}{4}S$

If his speed is $\frac{3}{4}ths$ of his usual speed, he would take $\frac{4}{3}rds$ of the usual time to travel to office. He would be late by $\frac{1}{3}rd$ of the usual time i.e., 20 minutes. Choice (C)

35. In 5 seconds, the frog would move 60 m. When the snake is spotted by the frog, the frog would be 90 m ahead of the snake. Time, the snake would take to catch the frog (in seconds)

$= \frac{90}{\text{DIfference of speeds of snake and frog}}$

$= \frac{90}{20-15} = 18$. Choice (B)

QUANTITATIVE ABILITY TEST 3

(INDICES, SURDS AND LOGARITHMS)

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Find the value of x, if $2^x = 8^y$ and $6^{4y} = 216^{x+y-2}$.

(A) $2\frac{1}{4}$ (B) $2\frac{1}{2}$

(C) $3\frac{1}{2}$ (D) $3\frac{1}{3}$

2. If $x = \sqrt[3]{9\ \sqrt[3]{9\ \sqrt[3]{9\ \sqrt[3]{9\ldots\ldots\ldots\ldots}}}}$, what is the value of x?

(A) 3 (B) –3

(C) $\sqrt{3}$ (D) 9

3. If $9^{3x-4} = 6561.\sqrt{27^{x-2}}$, then find the value of x.

(A) 1 (B) 2

(C) 3 (D) None of these

4. If $p^\ell = q^m = r^n$ and $\frac{1}{\ell} + \frac{1}{n} = \frac{2}{m}$, then which of the following is valid? ($p > 1$, $q > 1$, $r > 1$)

I. $p\ell = q^2$ II. $pq = \ell^2$ III. $p^2 = qr$

(A) Both I and I (B) Both II and III

(C) Both III and I (D) None of these

5. Find the value of x if $(125)^{2x-3} = (25)^{3^{(-1)^{2^{3^4}}}}$.

(A) 4.5 (B) 2.5

(C) 1.5 (D) None of these

6. If $2^{2x^2+3} = 8^{2x+1}$ and x is positive, then what is the value of x?

(A) 2 (B) 3

(C) 1 (D) 4

7. If $t_1 = \sqrt{5}$, $t_2 = \sqrt{5\sqrt{5}}$, $t_3 = \sqrt{5\sqrt{5\sqrt{5}}}$ and so on, then the product of the first ten terms $(t_1)(t_2)(t_3)(t_4)\ldots.(t_{10})$ is equal to

(A) $\sqrt[512]{5^{4609}}$ (B) $\sqrt[2048]{5^{18431}}$

(C) $\sqrt[1024]{5^{9217}}$ (D) $\sqrt[512]{5^{4607}}$

8. If $x^2 \neq x$, then

$$\frac{x^{4b} + x^{2(a+b)} + x^{4a}}{\left(x^{2a} + x^{a+b} + x^{2b}\right)\left(x^{2a} - x^{a+b} + x^{2b}\right)}$$ is equal to

(A) $\frac{x^a}{x^b}$ (B) $\frac{x^{2a}}{x^{2b}}$

(C) x^{a+b} (D) None of these

9. If $3^{x+3} - 3^{x-3} = 6552$, then find x^2.

(A) 5 (B) 25

(C) 3 (D) 9

10. If x, y, z are real numbers such that $xyz = 1$, then the expression $\frac{1}{1+x+y^{-1}} + \frac{1}{1+y+z^{-1}} + \frac{1}{1+z+x^{-1}}$ is equal to

(A) 1 (B) $\frac{3}{x+y+z}$

(C) $\frac{3}{x^{-1}+y^{-1}+z^{-1}}$ (D) $\frac{x+y+z}{3}$

11. If $\frac{(81^a)^a (81^b)^b (81^c)^c}{(6561^b)^{-c}(6561^c)^{-a}(6561^a)^{-b}} = 3$.

Then $a + b + c$ could be

(A) 2 (B) $\frac{1}{3}$

(C) $-\frac{1}{2}$ (D) $-\frac{1}{3}$

12. If $x = \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots.\infty}}}}$ find x.

(A) $\sqrt{3} + \frac{3}{2}$ (B) $\sqrt{3} - \frac{3}{2}$

(C) $\sqrt{3} + \frac{1}{2}$ (D) $\sqrt{3} - \frac{1}{2}$

13. If $A = 8^{8^{8^8}}$, $B = 8^{8^{88}}$, $C = 8^{888}$ and $D = 8^{8^{8^8}}$, which of the following represents the ascending order of the values of A, B, C, D?

(A) $CDAB$ (B) $CABD$

(C) $CBAD$ (D) $ACBD$

14. Solve for x: $\sqrt{x} + \sqrt{x - \sqrt{1-x}} = 1$.

(A) 1 (B) $\frac{16}{25}$

(C) $\frac{4}{5}$ (D) 0

15. The arithmetic mean of two surds is $5 + 9\sqrt{2}$, and one of the surds is $1 + 12\sqrt{2}$

What is the square root of the other surd?

(A) $6 - 21\sqrt{2}$ (B) $4 - 3\sqrt{2}$

(C) $\sqrt{3}\left(\sqrt{2} + 1\right)$ (D) $\sqrt{2}\left(2 - \sqrt{3}\right)$

16. $\frac{1}{\sqrt{6}+\sqrt{7}-\sqrt{13}}+\frac{1}{\sqrt{6}-\sqrt{7}-\sqrt{13}}$ =

(A) $\sqrt{6}$ (B) $\frac{1}{\sqrt{6}}$

(C) 6 (D) $\frac{1}{6}$

17. Find the square root of

$$\left[1+\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots\ldots\ldots+\frac{1}{\sqrt{324}+\sqrt{323}}\right]$$

(A) $3\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $2\sqrt{3}$ (D) $\frac{\sqrt{3}-1}{2}$

18. If $x^y = y^z = z^x$ and $(x, y, z) > 0$, then

(A) $3\left[\frac{xy+yz+zx}{xyz}\right]$ (B) $\frac{xy+yz+zx}{xyz}$

(C) $\frac{x+y+z}{xyz}$ (D) $\frac{xy+yz+zx}{x+y+z}$

19. If $\log_4 3$, $\log_4(3^m-2)$ and $\log_4\left(3^m-\frac{8}{3}\right)$ are in arithmetic progression, then the number of possible values of m are

(A) 1 (B) 2

(C) 4 (D) 5

20. If $\log_x 162 = m$ and $\log_x 72 = n$, then what is the value of $\log_x 7776$ in terms of m and n?

(A) $\frac{m+3n}{m+5n}$ (B) $\frac{3m-5n}{m+2n}$

(C) $\frac{m+3n}{2}$ (D) $\frac{3m-5n}{2}$

21. Which of the following is a possible value of x if $\log_3 x^2 - \log_3 x\sqrt{x} = 8\log_x 3$?

(A) $\frac{1}{81}$ (B) $\frac{1}{243}$

(C) 243 (D) 9

22. If $a = a = \log_4 31$, then _____ .

(A) $a < 2$ (B) $2 < a < 2.5$

(C) $2.5 < a < 2.8$ (D) $2.8 < a$

23. If $\log_{10}(2x+3) - 1 = \log_{10} x$, then find x.

(A) $\frac{2}{7}$ (B) $\frac{3}{4}$

(C) $\frac{7}{8}$ (D) $\frac{3}{8}$

24. If $a^2 + 4b^2 = 12ab$, what is the value of $\log(a+2b)$?

(A) $\log\left(\frac{a}{2}\right)+\log\left(\frac{b}{2}\right)+\log 2$

(B) $(\log a+\log b-\log 2)\frac{1}{2}$

(C) $\frac{1}{2}(\log a+\log b+4\log 2)$

(D) $\frac{1}{2}(\log a-\log b+4\log 2)$

25. Simplify $\frac{\log_m p.\log_n p}{\log_m p+\log_n p}$.

(A) 1 (B) $\log_p(m+n)$

(C) $\log_p mn$ (D) $\log_{mn} p$

26. If $a > 1$, $\log_a a + \log_{\frac{1}{a^2}} a + \log_{\frac{1}{a^3}} a + \ldots.. + \log_{\frac{1}{a^{20}}} a$ =

(A) 420 (B) 210

(C) 380 (D) 190

27. If $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}$, then find the value of $\log\left[\left(x^{y^2+yz+z^2+z}\right)\left(y^{z^2+zx+x^2+z}\right)\left(z^{x^2+xy+y^2+z}\right)\right]$

(A) 2 (B) 0

(C) 3 (D) None of these

28. If $abc = 1$, then find the value of $\frac{1}{\log_{bc} a^3}+\frac{1}{\log_{ac} b^3}+\frac{1}{\log_{ab} c^3}$

(A) $\frac{-1}{3}$ (B) $\frac{1}{3}\log abc$

(C) -1 (D) $\log_{a+b+c} abc$

29. If $\log_6 27 = t$, then find the value of $\log_{18} 4$ in terms of t.

(A) $3\left(\frac{2-t}{2+t}\right)$ (B) $2\left(\frac{3-t}{3+t}\right)$

(C) $\frac{6+t}{3+t}$ (D) $\frac{4-t}{3+t}$

30. For $a \geq b$, $b > 1$ the value of the expression $\log_a\left(\frac{a}{b}\right)+\log_b\left(\frac{b}{a}\right)$ can never be

(A) 0 (B) 1

(C) -2 (D) -0.5

31. If $\log_4(x^2+x) - \log_4(x+1) = 2$, then $\sqrt{x}$ =

(A) 2 (B) 4

(C) 8 (D) 16

32. If $\log_{10} 3 = 0.4771$, then find the number of digits in $(243)^{50}$.

(A) 200 (B) 205

(C) 120 (D) 210

33. If (log 16) (log 27) = (log x) (log y) and (log 4096) [log x – log 27] = [log16 – log y] (log 512), which of the following can be the value of $(x - y)$?
(A) –11
(B) 73
(C) –73
(D) More than one options

34. What is the value of?

$$\log_{64}\sqrt{512\sqrt{512\sqrt{512\sqrt{512.....\infty}}}}.$$

(A) 2.5 (B) 3
(C) 1.5 (D) 1

35. If $\log_{bc} a = \frac{1}{p}$, $\log_{ca} b = \frac{1}{q}$ and $\log_{ab} c = \frac{1}{r}$, find the value of $\frac{1}{p+1}+\frac{1}{q+1}+\frac{1}{r+1}$

(A) 1 (B) $\frac{3}{2}$
(C) 2 (D) None of these

Answer Keys

1. A	**2.** A	**3.** D	**4.** D	**5.** B	**6.** B	**7.** C	**8.** D	**9.** B	**10.** A
11. C	**12.** B	**13.** B	**14.** B	**15.** C	**16.** B	**17.** A	**18.** A	**19.** B	**20.** C
21. A	**22.** B	**23.** D	**24.** C	**25.** D	**26.** B	**27.** B	**28.** C	**29.** B	**30.** B
31. B	**32.** C	**33.** D	**34.** C	**35.** A					

Hints and Explanations

1. Given $2^x = 8^y \Rightarrow 2^x = (2^3)^y$
$\Rightarrow \quad x = 3y$
Also $6^{4y} = 216^{x+y-2}$
$\Rightarrow \quad 6^{4y} = 6^{3(3y+y-2)}$
$\Rightarrow \quad 6^{4y} = 6^{12y-6}$
$\Rightarrow \quad 4y = 12y - 6$
$\Rightarrow \quad 6 = 8y \Rightarrow y = \frac{3}{4}$
$\therefore \quad x = 3y = \frac{9}{4} = 2\frac{1}{4}$
Choice (A)

2. Given $x = \sqrt[3]{9\ \sqrt[3]{9\ \sqrt[3]{9\ \sqrt[3]{9.....}}}}$

$\Rightarrow \quad x^3 = 9\sqrt[3]{9\ \sqrt[3]{9\ \sqrt[3]{9\ \sqrt[3]{9.....}}}} = 9x$

$\Rightarrow x = 0, 3, -3$. Since x is positive, $x = 3$. Choice (A)

3. $9^{3x-4} = 3^8.(27)^{\frac{(x-2)}{2}}$

$9^{3x-4} = 3^8.\left(3^3\right)^{\frac{(x-2)}{2}}$

$3^{2(3x-4)} = 3^8 \cdot 3^{3\frac{(x-2)}{2}}$

$\frac{3x}{2} = 8 + 8 - 3$

$6x - 8 = 8 + \frac{(3x-6)}{2}$

$\frac{9x}{2} = 13 \Rightarrow x = \frac{26}{9}$
Choice (D)

4. Given that $p^\ell = q^m = r^n$
Let each be equal to k.
Hence $p^\ell = k$; $\log_p k = \ell$

$\log_q k = m$; $\log_r k = n$ given that

$\frac{1}{\ell} + \frac{1}{n} = \frac{2}{m}$

$\Rightarrow \quad \frac{1}{\log_p k} + \frac{1}{\log_r k} = \frac{2}{\log_q k}$

$\Rightarrow \quad \log_k p + \log_k r = 2 \cdot \log_k q$

$\Rightarrow \quad \log_k (pr) = \log_k q^2 \quad pr = q^2$
Choice (D)

5. 2^{3^4} is always even, as 2 raised to any power is even.
Hence $(-1)^{2^{3^4}} = (-1)^{\text{even number}} = +1$

Hence, $(25)^{3(-1)^{2^{3^4}}} = (25)^{3^1} = 5^6$.

$(125)^{2x-3} = 5^6$. $\Rightarrow 5^{3(2x-3)} = 5^6$. $\Rightarrow 2x - 3 = 2$; $x = 2.5$
Choice (B)

6. By equating the index of 2 on both sides we get
$2x^2 + 3 = 6x + 3 \Rightarrow x = 3$ (as $x > 0$) Choice (B)

7. $t_1 = 5^{\frac{1}{2}} = 5^{1-\frac{1}{2}}$

$t_2 = 5^{\frac{3}{4}} = 5^{1-\frac{1}{2^2}}$

$t_3 = 5^{\frac{7}{8}} = 5^{1-\frac{1}{2^3}}$

$t_{10} = 5^{1-\frac{1}{2^{10}}}$

$= (t_1)(t_2)(t_3).....(t_{10}) = \left(5^{1-\frac{1}{2}}\right)\left(5^{1-\frac{1}{2^2}}\right)\left(5^{1-\frac{1}{2^3}}\right)$

$.....\left(5^{1-\frac{1}{2^{10}}}\right)$

$= 5^{10-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....+\frac{1}{2^{10}}\right)}$

$= 5^{10-\frac{1}{2}\left[1+\frac{1}{2}+\frac{1}{2^2}+....+\frac{1}{2^9}\right]}$

$= 5^{10-\frac{1}{2}\frac{\left[1-\frac{1}{2^{10}}\right]}{1-\frac{1}{2}}}$

$= 5^{10-1+\frac{1}{2^{10}}} = \sqrt[1024]{5^{9217}}$

Choice (C)

8. Let $E =$ $E = \dfrac{x^{4b}+x^{2(a+b)}+x^{4a}}{\left(x^{2a}+x^{a+b}+x^{2b}\right)\left(x^{2a}-x^{a+b}+x^{2b}\right)}$.

Den $(E) = x^{2b}\left(x^{2a-2b}+x^{a-b}+1\right).\ x^{2b}\left(x^{2a-2b}-x^{a-b}+1\right)$

$E = \dfrac{x^{4b}\left(1+x^{2(a-b)}+x^{4(a-b)}\right)}{x^{4b}\left[1-x^{(a-b)}+x^{2(a-b)}\right]\left[1+x^{a-b}+x^{2(a-b)}\right]}$

Considering $x^{a-b} = t$, we get

$\dfrac{1+t^2+t^4}{\left(1-t+t^2\right)\left(1+t+t^2\right)} = \dfrac{1+t^2+t^4}{1+t^2+t^4} = 1$

$(1 - t + t^2)(1 + t + t^2) = (1 + t^2 - t)(1 + t^2 + t)$

$= ((1 + t^2)^2 - t^2)$

Note: The condition $x^2 \neq x$ means $x \neq 0$, $x \neq 1$. If $x = 0$, E is not defined. If $x = 1$, $E = 1$.

$\therefore$ This condition need not be imposed. But imposing the condition does not make the statement (that $E = 1$) false. Choice (D)

9. $3^{x+3} - 3^{x-3} = 6552$

$3^x\left[3^3 - \dfrac{1}{3^3}\right] = 6552$

$3^x\left[\dfrac{728}{27}\right] = 6552$

$3^x = 243 = 3^5$

$\Rightarrow \quad x = 5$

$\therefore \quad x^2 = 5^2 = 25$ Choice (B)

10. Given $xyz = 1$

$\Rightarrow \quad qxy = \dfrac{1}{z}, \dfrac{1}{xy} = z$ -- (1)

Given expression,

$\dfrac{1}{1+x+y^{-1}} + \dfrac{1}{1+y+z^{-1}} + \dfrac{1}{1+z+x^{-1}}$

$= \dfrac{y}{y+xy+1} + \dfrac{1}{1+y+xy} + \dfrac{1}{1+\frac{1}{xy}+\frac{1}{x}}$ (from (1))

$= \dfrac{y}{y+xy+1} + \dfrac{1}{1+y+xy} + \dfrac{xy}{xy+1+y}$

$= \dfrac{y+1+xy}{1+xy+y} = 1$ Choice (A)

11. $\dfrac{81^{a^2+b^2+c^2}}{81^{[-2bc-2ca-2ab]}} = 81^{a^2+b^2+c^2+2ab+2bc+2ca}$

$= 81^{(a+b+c)^2} = 3 = 81^{\frac{1}{4}}$

$\Rightarrow \quad a+b+c = \pm\dfrac{1}{2}$ Choice (C)

12. Given $x = \dfrac{1}{4+\dfrac{1}{3+\dfrac{1}{4+\dfrac{1}{3+.....\infty}}}}$

$\Rightarrow \quad x = \dfrac{1}{4+\dfrac{1}{3+x}} \Rightarrow x = \dfrac{3+x}{4(3+x)+1}$

$\Rightarrow \quad x = \dfrac{3+x}{4x+13}$

$\Rightarrow \quad 4x^2 + 12x - 3 = 0$

$\Rightarrow \quad x = \dfrac{-12 \pm \sqrt{144+48}}{8}$

$\Rightarrow \quad x = \dfrac{4\left(-3 \pm \sqrt{12}\right)}{8}$

$\Rightarrow \quad x = \dfrac{-3 \pm 2\sqrt{3}}{2}$

Since $x > 0$,

$x = \ x = \dfrac{-3}{2} + \sqrt{3}$ Choice (B)

13. $A = 8^{88^8}$ $B = 8^{8^{88}}$ $C = 8^{888}$ $D = 8^{8^{8^8}}$

Since the base of all the numbers is 8, the number power with highest index is the greatest number. Clearly 'C' has the lowest value.

Consider $A = 8^{88^8}$

and $B = 8^{888}$.

Consider the indices is 88^8

and 8^{88}

$(88)^8$ and $(8^{11})^8$

Since $8^{11} > 88$

$8^{88} > 88^8$

$\therefore \quad B > A$

Also, among the four powers the greatest power is 8^{8^8}. Hence D is the largest number.

$\therefore$ the ascending order is $CABD$. Choice (B)

14. $\sqrt{x} + \sqrt{x - \sqrt{1-x}} = 1$

Squaring both sides we get, $x - \sqrt{1-x} = 1 + x - 2\sqrt{x}$

Squaring again, we get, $1 - x = 1 + 4x - 4\sqrt{x}$

$16x = 25x^2$

$x = \dfrac{16}{25}$ Choice (B)

15. Let the other surd be "a".

$a + 1 + 12\sqrt{2}$

$= 10 + 18\sqrt{2}$

$a = 9 + 6\sqrt{2}$

$= 9 + 2\sqrt{18} = \left(\sqrt{6} + \sqrt{3}\right)^2$

$\therefore \quad \sqrt{a} = \left(\sqrt{6} + \sqrt{3}\right)$ Choice (C)

16. $$\frac{1}{\sqrt{6}+\sqrt{7}-\sqrt{13}} = \frac{\left(\sqrt{6}+\sqrt{7}+\sqrt{13}\right)}{\left(\sqrt{6}+\sqrt{7}+\sqrt{13}\right)} \frac{(1)}{\left(\sqrt{6}+\sqrt{7}-\sqrt{13}\right)}$$

$$= \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{\left(\sqrt{6}+\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2} = \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{13+2\sqrt{42}-13}$$

$$= \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{2\sqrt{42}}$$

$$\frac{1}{\sqrt{6}-\sqrt{7}-\sqrt{13}} = \frac{1\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{\left(\sqrt{6}-\sqrt{7}-\sqrt{13}\right)\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}$$

$$= \frac{\sqrt{6}-\sqrt{7}+\sqrt{13}}{\left(\sqrt{6}-\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2} = \frac{\sqrt{6}-\sqrt{7}+\sqrt{13}}{13-2\sqrt{42}-13}$$

$$= \frac{-\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{2\sqrt{42}}$$

$$\text{Required value} = \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{2\sqrt{42}} + \frac{-\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{2\sqrt{42}}$$

$$= \frac{2\sqrt{7}}{2\sqrt{42}} = \frac{1}{\sqrt{6}}$$

Choice (B)

17. The given function is $1 + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} +$

$$= 1 + \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \ldots . +$$

(on rationalizing the denominator of each term)

$= 1 + \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + ... + \sqrt{324} - \sqrt{323}$

$= \sqrt{324} = 18$ ($\because$ all terms cancel off except $\sqrt{324}$)

Hence, the square root of the given expression is $\sqrt{18}$

$= 3\sqrt{2}$. Choice (A)

18. Let $x^y = y^z = z^x = k$

$\Rightarrow \quad x = k^{\frac{1}{y}}, y = k^{\frac{1}{z}}, z = k^{\frac{1}{x}}$

consider $\frac{1}{x}\log_z xyz$

$$= \frac{1}{x}\log_{k^{\frac{1}{x}}}\left(k^{\frac{1}{y}}.k^{\frac{1}{z}}.k^{\frac{1}{x}}\right)$$

$$= \frac{1}{x}\log_{k^{\frac{1}{x}}}\left(k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\right) = \frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

Similarly $\frac{1}{y}\log_x xyz = \frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ and $\frac{1}{z}\log_y xyz$

$$= \frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

Hence the given expression is equal to

$$3\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right] = 3\left(\frac{xy+yz+zx}{xyz}\right)$$

Choice (A)

19. Given $\log_4 3 + \log_4\left(3^m - \frac{8}{3}\right) = 2\log_4\left(3^m - 2\right)$

$$\Rightarrow \quad 3\left(3^m - \frac{8}{3}\right) = \left(3^m - 2\right)^2$$

$\Rightarrow \quad 3^{m+1} - 8 = 3^{2m} + 4 - 4\,(3^m)$

$\Rightarrow \quad 3^{2m} - 7\,(3^m) + 12 = 0$

Let $3^m = x$

$\Rightarrow \quad x^2 - 7x + 12 = 0$

$x^2 - 4x - 3x + 12 = 0$

$x\,(x-4) - 3\,(x-4) = 0$

$\Rightarrow \quad (x-4)\,(x-3) = 0$

$\Rightarrow \quad x = 4$ or 3

$\Rightarrow \quad 3^m = 4$ or $3^m = 3$

$\Rightarrow \quad m = \log_3 4$ (or) $m = 1$

Hence m can take two values. Choice (B)

20. Given $\log_x 162 = m$

$\Rightarrow \quad \log_x 3^4 \;(2) = m$

$\therefore \quad m = 4\log_x 3 + \log_x 2$

Given $\log_x 72 = n$

$\Rightarrow \quad \log_x 3^2\, 2^3 = n$

$\therefore \quad n = 2\log_x 3 + 3\log_x 2$

Let $\log_x 3 = 1$ and $\log_x 2 = b$

$\Rightarrow m = 4a + b$ --- (1)

$n = 2a + 3b$ --- (2)

2 (2) – (1) gives

$5b = 2n - m \Rightarrow b = \frac{2n-m}{5}$

similarly $a = \frac{3m-n}{10}$

Now consider $\log_x 7776$

$= \log_x 3^5.2^5$

$= 5[\log_x 3 + \log_x 2]$

$$= 5\left[\frac{3m-n}{10} + \frac{2n-m}{5}\right]$$

$$= 5\left[\frac{m+3n}{10}\right] = \frac{m+3n}{2}$$

Choice (C)

21. $\log_3 x^2 - \log_3 x\sqrt{x}$

$= 8 \log_x 3$

$\Rightarrow \quad \log_3 \frac{x^2}{x\sqrt{x}} = 8\log_x 3$

$\Rightarrow \quad \log_3 \sqrt{x} = 8\log_x 3$

$\Rightarrow \quad \frac{1}{2}\log_3 x = \frac{8}{\log_3 x}$

$\Rightarrow \quad (\log_3 x)^2 = 16$

$\Rightarrow \quad \log_3 x = 4$

$\Rightarrow \quad x = 3^4 = 81$ or $x = 3^{-4} = \frac{1}{81}$

Choice (A)

22. $\log_4 31 = \log_{2^2} 31 = \frac{1}{2}\log_2 31$

$2^4 < 31 < 2^5$

$\Rightarrow \quad \log_2 2^4 < \log_2 31 < \log_2 2^5$

$\Rightarrow \quad 4 \log_2 2 < \log_2 31 < 5 \log_2 2$

$\Rightarrow \quad \frac{4}{2} < \frac{1}{2}\log_2 31 < \frac{5}{2}$

$\Rightarrow \quad 2 < \frac{1}{2} \log_2 31 < 2.5.$

Choice (B)

23. $\log (2x + 3) - 1 = \log x$

$\log (2x + 3) - \log 10 = \log x$

$\Rightarrow \quad \log\left(\frac{2x+3}{10}\right) = \log x$

$\Rightarrow \quad \frac{2x+3}{10} = x$

$\Rightarrow \quad 2x + 3 = 10x$

$\Rightarrow \quad x = \frac{3}{8}$

Choice (D)

24. $a^2 + 4b^2 = 12ab$; adding $4ab$ to both sides of the equation, we get $(a + 2b)^2 = 16ab$

$2 \log (a + 2b) = 4 \log 2 + \log a + \log b$

$\log (a + 2b) = \frac{1}{2}$

$[\log a + \log b + 4 \log 2]$

Choice (C)

25. $\frac{\log_m p . \log_n p}{\log_m p + \log_n p}$

$= \frac{1}{\frac{\log_m p + \log_n p}{\log_m p . \log_n p}} = \frac{1}{\frac{1}{\log_n p} + \frac{1}{\log_m p}}$

$= \frac{1}{\log_p n + \log_p m} = \frac{1}{\log_p mn}$

$= \log_{mn} p$

Choice (D)

26. $\log_a a + \log_a \frac{1}{2}a + \log_a \frac{1}{3}a + \ldots + \log_a \frac{1}{20}$ a

$= \frac{\log a}{\log a} + \frac{\log a}{\log a^{\frac{1}{2}}} + \frac{\log a}{\log a^{\frac{1}{3}}} + \ldots + \frac{\log a}{\log a^{\frac{1}{20}}}$

$= 1 + 2 + 3 + \ldots + 20 = \frac{20 \times 21}{2} = 210$

Choice (B)

27. Let $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$

$\log x = k(y - z)$

$\log y = k(z - x)$

$\log z = k(x - y)$

$\log x^{y^2+yz+z^2+z} = (y^2 + yz + z^2 + z)(\log x)$

$= k(y-z)(y^2 + yz + z^2 + z)$

$= k(y^3 - z^3) + k(y - z)z$ --- (1)

$\log y^{z^2+xz+x^2+z} = k(z-x)(z^2 + xz + x^2 + z)$

$= k(z^3 - x^3) + k(z - x)z$ --- (2)

$\log z^{x^2+xy+y^2+2} = k(x-y)(x^2 + xy + y^2 + z)$

$= k(x^3 - y^3) + k(x y) z$ --- (3)

Adding (1), (2) and (3),

$\log x^{y^2-yz+z^2+z} + \log y^{z^2+xz+x^2+z} + \log z^{x^2+xy+y^2+z}$

$= k(y^3 - z^3) + kz(y - z) + k(z^3 - x^3) + kz(z - x) + k(x^3 - y^3) + kz(x - y)$

$= k(y^3 - z^3 + z^3 - x^3 + x^3 - y^3) + kz(y - z + z - x + x - y)$

$= 0 + 0 = 0$

Choice (B)

28. $\frac{1}{\log_{bc} a^3} + \frac{1}{\log_{ac} b^3} + \frac{1}{\log_{ab} c^3}$

$= \frac{1}{\log_{1/a} a^3} + \frac{1}{\log_{1/b} b^3} + \frac{1}{\log_{1/c} c^3}$

$\left[\because bc = \frac{1}{a}, ac = \frac{1}{b} \text{ and } ab = \frac{1}{c}\right]$

$= \frac{1}{\log_{a^{-1}} a^3} + \frac{1}{\log_{b^{-1}} b^3} + \frac{1}{\log_{c^{-1}} c^3}$

$= \frac{1}{-3} + \frac{1}{-3} + \frac{1}{-3} = -1$

Choice (C)

29. $t = \log_6 27 = \frac{\log_{10} 27}{\log_{10} 6} = \frac{3\log_{10} 3}{\log_{10} 2 + \log_{10} 3}$

Now $3 - t = 3 - \frac{3\log 3}{\log 2 + \log 3} = \frac{3\log 2}{\log 2 + \log 3}$

$3 + t = 3 + \frac{3\log 3}{\log 2 + \log 3} = \frac{3\log 2 + 6\log 3}{\log 2 + \log 3}$

$\frac{3-t}{3+t} = \frac{(3\log 2)/(\log 2 + \log 3)}{(3\log 2 + 6\log 3)/(\log 2 + \log 3)}$

$$= \frac{\log 2}{\log 2 + 2\log 3} = \log_{18} 2$$

$$\therefore \quad 2\frac{(3-t)}{3+t} = 2\log_{18} 2 = \log_{18} 4$$

Choice (B)

30. Given: $\log_a\left(\frac{a}{b}\right) + \log_b\left(\frac{b}{a}\right)$

$= \log_a a - \log_a b + \log_b b - \log_b a = 2 - (\log_a b + \log_b a)$

But $\log_a b + \log_b a$ is in the form of $x + \frac{1}{x}$ which is ≥ 2

$\therefore \log_a b + \log_b a \geq 2$

$\therefore$ The given expression can not be positive

$\therefore$ It can not be 1

Choice (B)

31. $\log_4\left[\frac{(x^2+x)}{(x+1)}\right] = 2$

$\frac{(x^2+x)}{(x+1)} = 4^2$

$\Rightarrow \quad x^2 + x = 16x + 16$

$= x^2 - 15x - 16 = 0$

$x^2 - 16x + x - 16 = 0$

$x(x-16) + 1(x-16) = 0$

$\Rightarrow \quad (x+1)(x-16) = 0$

$x = -1$ or 16

$\sqrt{x} = \sqrt{16} = 4$

Choice (B)

32. Let $k = (243)^{50} = 3^{250}$

Taking log on both sides, we get

$\log k = 250 \log 3$

$= 250\ (0.4771)$

$= 25(4.771) = \frac{100}{4}\ (4.771) = 119.4$

The characteristic of $\log k$ is 119.

Number of digits in $(243)^{50}$ are $119 + 1 = 120$

Choice (C)

33. Given $(\log 16)(\log 27) = (\log x)(\log (y)$

let $\log x = X$ and $\log y = Y$ then $(\log 4096)(\log x - \log 27)$

$= \log 512\ (\log 16 - \log y)$

becomes

$12 \log 2\ (X - 3\log 3) = 9\log 2(4\log 2 - Y)$

$4(X - 3\log 3) = 3(4\log 2 - Y)$ ——— (1)

and $(\log 16)(\log 27) = (\log x)(\log y)$ becomes $12 \log 2 \log 3 = XY$ ——— (2)

eliminating Y using (1) and (2) we have

$4(X - 3\log 3) = 3(4\log 2 - 12(\log 2)(\log 3) \div X)$

$4X(X - 3\log 3) = 3(4X \log 2 - 12 \log 2.\log 3)$

$4X^2 - 12X \log 3 = 12X \log 2 - 36 \log 2.\log 3$

$X^2 - 3X \log 3 - 3X \log 2 + 9 \log 2 \log 3 = 0$

$(X - 3\log 3)(X - 3\log 2) = 0$

$\Rightarrow \quad X = \log 27$ or $X = \log 8$

i.e., $\log x = \log 27$ or $\log x = \log 8$

$\Rightarrow \quad x = 27$ or 8

when $x = 27$ then $y = 16$ and $x - y = 11$ and when $x = 8$, then $y = 81$ and $x - y = -73$

Choice (D)

34. Let $\sqrt{512\sqrt{512\sqrt{512\sqrt{512.....\propto}}}} = x$

$x^2 = 512\sqrt{512\sqrt{512\sqrt{512.....\propto}}}$

$x^2 = 512x, \Rightarrow x = 0$ or 512.

As x is clearly not zero, $x = 512$.

Hence the required quantity is $\log_{64} 512$.

$= \log_{8^2} 8^3 = \frac{3}{2} = 1.5$

Choice (C)

35. $\log_{bc} a = 1/p \Rightarrow p = \log_a bc \Rightarrow p + 1 = \log_a abc$

Similarly $\log_{ca} b = 1/q \Rightarrow q + 1 = \log_b abc$

and $\log_{ab} c = 1/r \Rightarrow r + 1 = \log_c abc$

$\therefore \quad \frac{1}{p+1} + \frac{1}{q+1} + \frac{1}{r+1} = \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc = 1$

Choice (A)

QUANTITATIVE ABILITY TEST 4

(DATA INTERPRETATION)

Number of Questions: 30 **Section Marks: 30**

Directions for questions 1 to 4: These questions are based on the information given below.

Vinod Melkote, HR Manager of *XYZ* Ltd. received partial information from his subordinates about the employees for annual appraisal.

	Performance			Total
	Below Average	Average	Above Average	
Males				96
Females		30		
Total	48			

Vinod also received the following information.

(1) One-third of the total employees are above average performers.

(2) 25% of the males are below average performers.

(3) The total number of females is twice the number of average performing males.

1. How many people are above average performers?

(A) 48 (B) 60

(C) 66 (D) 54

2. How many females are there?

(A) 54 (B) 72

(C) 90 (D) 84

3. What is the difference between the total number of average performers and that of males?

(A) 18 (B) 12

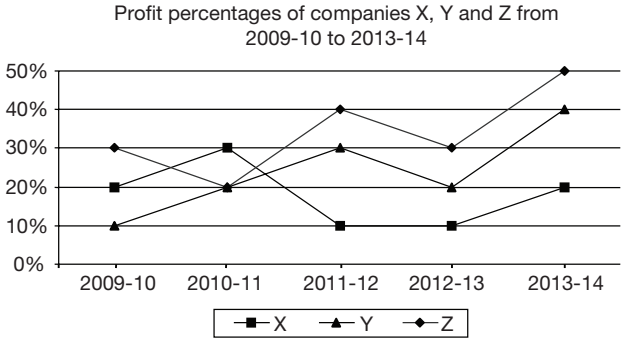
(C) 24 (D) 20

4. What is the ratio of the number of average performers and the number of females?

(A) 6 : 7 (B) 11 : 12

(C) 3 : 4 (D) 2 : 3

***Directions for questions 5 to 8*:** These questions are based on the following line graph.



Note: Profit = Income – Expenditure.; Profit % = $\frac{\text{Profit}}{\text{Expenditure}} \times 100$

5. If the profit of *X* in 2010-11 is ₹120 lakhs, then what is its income in 2010-11? (in ₹lakhs)

(A) 520 (B) 200

(C) 320 (D) 140

6. If the income of *Y* in 2013-14 is thrice that of *X* in 2009-10, then what is the ratio of profits of *X* and *Y* in 2013-14?

(A) 12 : 1 (B) 12 : 13

(C) 1 : 13 (D) None of these

7. If in 2009-10, the ratio of profits of *X*, *Y* and *Z* is 4 : 2 : 3, then what is the ratio of their expenditures in that year?

(A) 3 : 6 : 4 (B) 2 : 2 : 1

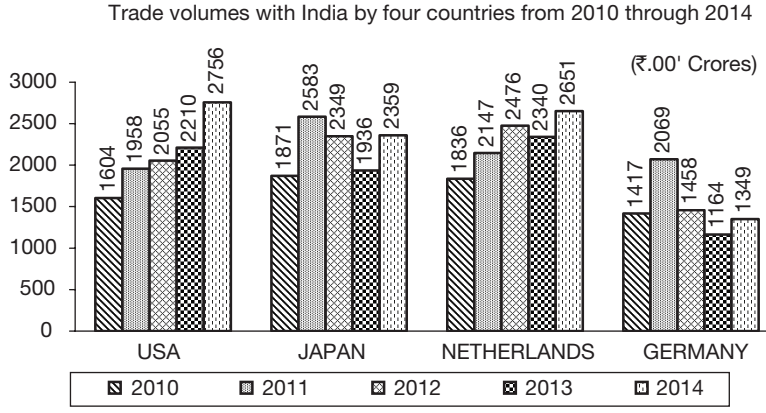
(C) 3 : 2 : 4 (D) None of these

8. If both *Y* and *Z* had the same profit in 2011-12, then what is the ratio of their incomes in that year?

(A) 4 : 3 (B) 1 : 1

(C) 26 : 21 (D) 2 : 3

Directions for questions 9 to 13: These questions are based on the following bar graph.



9. From the year 2010 to 2011, the trade with India by which of the following countries showed the highest percentage increase?
(A) USA (B) Japan
(C) U.K (D) Germany

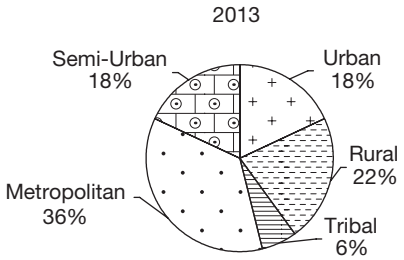
10. In the year 2013, the trade volume by the given countries constitutes 62.5% of the total trade volume of India. What is the trade volume (in ₹'00 crores) by all other countries with India in that year?
(A) 4590 (B) 5430
(C) 6140 (D) 7250

11. During which year is the total trade volume by the given countries with India the highest?
(A) 2010 (B) 2011
(C) 2012 (D) 2014

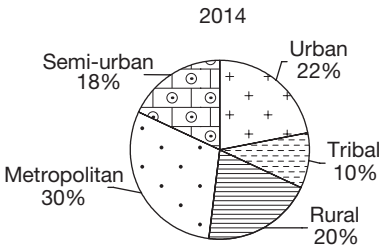
12. In how many of the given years is the trade volume of Netherlands with India greater than the average trade volume per year by the Netherlands in India in the given period?
(A) 1 (B) 2
(C) 3 (D) 4

13. For which country is the percentage increase in the trade volume with India in any year when compared to that in the previous year, the greatest?
(A) USA (B) Japan
(C) UK (D) Germany

Directions for questions 14 to 18: These questions are based on the following pie-charts which show the percentage distribution of births in different areas, during the two years 2013 and 2014.



Total number of births = 3.6 lakh



Total number of births = 3.0 lakh

14. What is the difference in the number of births in the year 2013 and 2014 in rural areas?
(A) 16,400 (B) 19,200
(C) 18,600 (D) 17,600

15. The percentage increase/decrease in the number of births in semi-urban areas from 2013 to 2014 is _____.
(A) 16.66% decrease
(B) 20% increase
(C) 16.66% increase
(D) 20% decrease

16. What is the ratio of the number of births in the metropolitans in 2013 to that in 2014?
(A) 36 : 25 (B) 15 : 17
(C) 3 : 7 (D) 7 : 3

17. The number of births in tribal areas in the year 2013 is what percentage of that in 2014?
(A) 56% (B) 64%
(C) 72% (D) 84%

18. If 18% of the children born in 2013 and 16% of the children born in 2014 lacked good medical facilities in the same year, then what is the total number of children (in lakhs) who doesn't lacked good medical facilities in these two years?
(A) 6.329 (B) 5.472
(C) 4.289 (D) 5.689

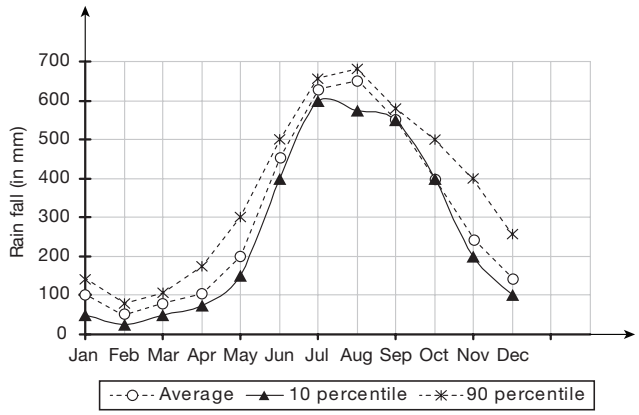
Directions for questions 19 to 20: Select the correct alternative from the given Choices.

19. A total of 600 persons participated in a survey. Each respondent was asked whether he/she owned a vehicle. Any respondent owning a vehicle was asked to mention whether he/she owned a two-wheeler or four wheeler or both.
The results of the survey are tabulated below.

		men	Women
Number of persons having own vehicle	Only Four-wheeler	80	68
	Only Two-wheeler	60	40
	Both	120	92
Number of persons not owning a vehicle		40	100

What percent of the respondents do not own a four wheeler?
(A) 60% (B) 40%
(C) 30% (D) 80%

20. The monthly rainfall chart for a certain city was prepared, based on 40 years of data. The graph below shows the x (x percentile means that for x% of the 40 years, the rainfall was less than the indicated value.



Which of following statements can be concluded?

(i) The average rainfall in December is less than that in June.

(ii) Each year, the amount of rainfall in July is more than that in February.

(iii) In May, there is at least 250mm of rainfall each year.

(iv) The rainfall in August can be estimated with better certainity than the rainfall in November.

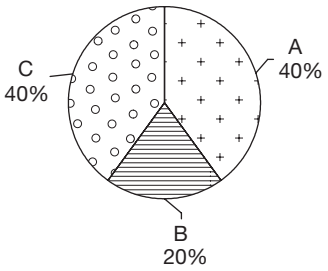
(A) (i) and (iii) (B) (i) and (iv)

(C) (ii) and (iv) (D) (ii) and (iii)

Directions for questions 21 and 22: Study the following table and pie chart carefully to the answer the questions.

Percentage of two wheelers of the total vehicles manufactured in a year by a certain company.

Type	Percentage of total vehicles	Out of which	
		100 cc	150 cc
Scooters (without gear)	25%	70%	30%
Scooter with gear	35%	40%	60%
Bikes	40%	65%	35%



The two wheelers are manufactured in three models as shown in the above pie chart.

Note: The above percentages mentioned in the table are applicable for all models. The total number of vehicles produced in that year is 70,000.

21. What is the total number of 150 cc bikes produced by the company in that year?

(A) 9000 (B) 9600

(C) 9500 (D) 9800

22. Which of the following numbers is the least?

(A) 100 cc bikes

(B) 150 cc bikes

(C) 100 cc scooters with gear

(D) 150 cc scooters without gear

Directions for questions 23 and 24: These questions are based on the following table which gives the percentage by weight of proteins, carbohydrates, minerals and fats in four formulations – *A*, *B*, *C* and *D*.

Formulation	Proteins	Carbohydrates	Minerals	Fats
A	10	20	25	45
B	25	35	20	20
C	30	10	40	20
D	15	50	30	5

	Proteins	Carbohydrates	Minerals	Fats
Cost (in ₹) per 10 g	4	3	2	1

23. What is the cost of a mixture containing 100 g each of *A*, *B*, *C* and *D*?

(A) ₹96.5 (B) ₹98

(C) ₹98.5 (D) ₹99

24. Which of the following would cost the least?

(A) 400 g of *B*

(B) 400 g of *C*

(C) 200 g of *A* and 200 g of *D*

(D) 300 g of *A* and 150 g of *D*

Directions for question 25: Select the correct alternative from the given choices.

25. The table below shows the test batting averages of 5 cricket players from 2010 to 2014. The test batting average of any batsman in any number of matches is his total score in those matches divided by the number of those matches.

Year	P	Q	R	S	T
2010	44	46	41	42	19
2011	55	52	44	48	22
2012	50	55	36	52	28
2013	46	51	40	46	34
2014	48	52	35	40	39

Who among the five players had the least percentage increase in the test batting average from 2010 to 2014?

(A) P (B) Q

(C) T (D) S

Directions for questions 26 and 27: These questions are based on the data given below.

A survey was conducted among 100 students in a hostel to find their favourite breakfast dish. Five students liked cutlet and sandwich only. 21 students liked omlette only. 25 students liked sandwich and 30 liked cutlet. There are 3 students who liked both sandwich and omlette. 48 students liked exactly one dish among the three. 2 students liked all the three dishes.

26. How many students like at least one dish?
(A) 69 (B) 100
(C) 2 (D) 21

27. How many students like cutlet or sandwich but not both?
(A) 48 (B) 41
(C) 31 (D) 62

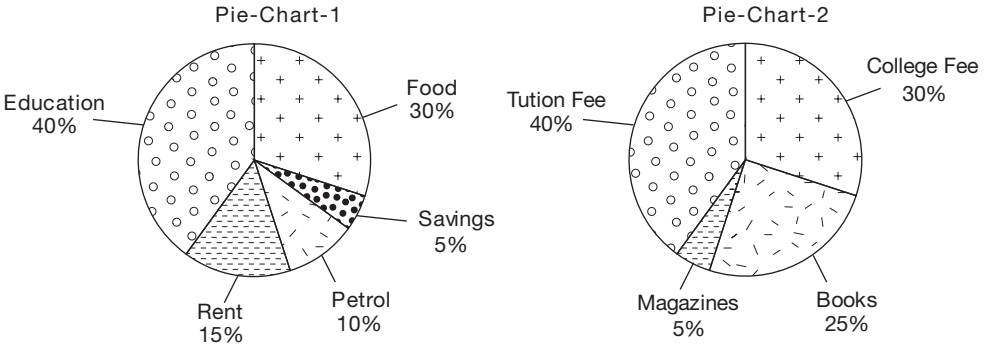
Directions for question 28: Select the correct alternative from the given choices.

28. Ten companies produce the same tool. Each of those companies rejects all the defective units of the tool produced. The table below gives the percentage of accepted units and the number of rejected units among all the units produced by the companies in month *M*.

Company	Percentage of accepted units	Number of rejected units
C1	80%	32
C2	90%	29
C3	85%	27
C4	81%	38
C5	76%	36
C6	77%	46
C7	84%	20
C8	88%	21
C9	93%	28
C10	95%	30

The company which produced the greatest total number of units in month *M* is
(A) *C*10 (B) *C*9
(C) *C*2 (D) *C*4

Directions for questions 29 and 30: These questions are based on the pie charts given below. Pie chart – 1 represents the distribution of income of Manohar and pie chart – 2 represents the split up of expenditure on education.



Note: Total income of Manohar is ₹15000

29. Which of the following statement/s is/are true?
(A) The expenditure on food is ₹2800 more than the expenditure towards college fee.
(B) The expenditure on education is ₹2,250 less than the remaining expenditure.
(C) The tuition fee is ₹2,460.
(D) Both (B) and (C).

30. The expenditure on books is how much more/less than the expenditure on food?
(A) ₹750 more (B) ₹750 less
(C) ₹3,000 more (D) ₹3,000 less

Answer Keys

1. B	**2.** D	**3.** C	**4.** A	**5.** A	**6.** D	**7.** B	**8.** C	**9.** D	**10.** A
11. D	**12.** C	**13.** D	**4.** B	**15.** A	**16.** A	**17.** C	**18.** B	**19.** B	**20.** B
21. D	**22.** D	**23.** C	**24.** C	**25.** A	**26.** A	**27.** B	**28.** A	**29.** B	**30.** D

Hints and Explanations

1. Given that 25% of the males are below average performers. Hence their number is 25% of 96 = 24.
As the total number of below average performers is 48 and 24 of them are males, remaining 24 are females.
Given, number of females = 2 (Number of male average performers)
Let the number of male average performers be x. Then, we get the following table.

	Below Average	Average	Above Average	Total
Males	24	x	72 – x	96
Females	24	30	2x – 54	2x
Total	48	30 + x	x + 18	2x + 96

Given, above average performers = $\frac{1}{3}$ (total number of employees)

i.e., $x + 18 = \frac{1}{3}(2x + 96)$

$\Rightarrow \quad 3x + 54 = 2x + 96 \Rightarrow x = 42$

Number of above average performers = $x + 18$ i.e., 60. Choice (B)

2. Given that 25% of the males are below average performers. Hence their number is 25% of 96 = 24.
As the total number of below average performers is 48 and 24 of them are males, remaining 24 are females.
Given, number of females = 2 (Number of male average performers)
Let the number of male average performers be x. Then, we get the following table.

	Below Average	Average	Above Average	Total
Males	24	x	72 – x	96
Females	24	30	2x – 54	2x
Total	48	30 + x	x + 18	2x + 96

Given, above average performers = $\frac{1}{3}$ (total number of employees)

i.e., $x + 18 = \frac{1}{3}(2x + 96)$

$\Rightarrow \quad 3x + 54 = 2x + 96$

$\Rightarrow \quad x = 42$

Number of females = $2x$

i.e., 2(42) = 84. Choice (D)

3. Given that 25% of the males are below average performers. Hence their number is 25% of 96 = 24.
As the total number of below average performers is 48 and 24 of them are males, remaining 24 are females.
Given, number of females = 2 (Number of male average performers)
Let the number of male average performers be x. Then, we get the following table.

	Below Average	Average	Above Average	Total
Males	24	x	72 – x	96
Females	24	30	2x – 54	2x
Total	48	30 + x	x + 18	2x + 96

Given, above average performers = $\frac{1}{3}$ (total number of employees)

i.e., $x + 18 = \frac{1}{3}(2x + 96)$

$\Rightarrow \quad 3x + 54 = 2x + 96$

$\Rightarrow \quad x = 42$

Total number of average performers = $x + 30$ i.e., 72
Also as the number of males = 96,
the required difference is 96 – 72 = 24. Choice (C)

4. Given that 25% of the males are below average performers. Hence their number is 25% of 96 = 24.
As the total number of below average performers is 48 and 24 of them are males, remaining 24 are females.
Given, number of females = 2 (Number of male average performers)
Let the number of male average performers be x. Then, we get the following table.

	Below Average	Average	Above Average	Total
Males	24	x	72 – x	96
Females	24	30	2x – 54	2x
Total	48	30 + x	x + 18	2x + 96

Given, above average performers = $\frac{1}{3}$ (total number of employees)

i.e., $x + 18 = \frac{1}{3}(2x + 96)$

$\Rightarrow \quad 3x + 54 = 2x + 96$

$\Rightarrow \quad x = 42$

Required ratio of above average performers : Number of females = $x + 30 : 2x$ i.e., 72 : 84 = 6 : 7. Choice (A)

5. Given profit of X in 2010-11 = ₹120 lakhs
Profit percentage of X in 2010-11 = 30%

As profit % = $\frac{\text{Profit}}{\text{Expenditure}} \times 100$

$30 = \frac{120\,\text{lakhs}}{\text{expenditure}} \times 100 = ₹400$ lakhs.

$\therefore$ Expenditure of X in 2010–11 is ₹400 lakhs
Hence, its income in 2010–11 = ₹520 lakhs. Choice (A)

6. As the income of X in 2013–14 is not known, the required ratio cannot be determined. Choice (D)

7. Given, 20% of X; 10% of Y : 30% of Z = 4 : 2 : 3
i.e., $X : Y : Z = 20 : 20 : 10 = 2 : 2 : 1$. Choice (B)

8. Let the profit of Y and R in 2010-11 be ₹x. Then, using the graph, we have $\frac{x}{\exp Q} = 30$ and $\frac{x}{\exp R} = 40$

Let $\exp_Q = 400x$ and $\exp_R = 300x$

$\therefore$ $30(\exp_Q) = 40(\exp_R)$

$\Rightarrow$ Expenditures of X and Y are in the ratio 4 : 3.
Incomes ratio = $520x : 420x = 26 : 21$. Choice (C)

9. The percentage increase in the trade volume from 2010 to 2011

for USA = $\frac{354}{1604} \times 100 < 25\%$

for Japan = $\frac{711}{1871} \times 100 < 40\%$

for UK = $\frac{311}{1836} \times 100 < 20\%$

for Germany = $\frac{652}{1417} \times 100 > 40\%$

The percentage increase in trade volume is the greatest for Germany. Choice (D)

10. Total trade volume by the given countries in 2013
= 2210 + 1936 + 2340 + 1164 = 7650
Given that 62.5% of the total trade volume = 7650
Then, 37.5% of the total trade volume = $\frac{37.5}{62.5} \times 7650$
= $\frac{3}{5} \times 7650$ = ₹459,000 Crores. Choice (A)

11. By observation, we find that the total trade volume is the least in 2010 and 2013.
By comparing the total trade volume in 2011 and that in 2012, we find that the trade volume in 2012 is lower.
Now, by comparing the total trade volume in 2011 and that in 2014 we find that the total trade volume in 2014 is the highest. Choice (D)

12. We can observe that the trade volume by the Netherlands in India is less than 2000 in one year and more than that in four years. By assuming the average to be 2000, we find the average of the deviations
= $\frac{-164+147+476+340+651}{5} = \frac{1450}{5} = 290$
The average = 2290. Three values are more than the average. Choice (C)

13. The trade volume from Germany registered more than 40% growth. By observation, we find that for no other country there is more than 40% growth in the trade volume in any two successive years. Choice (D)

14. No. of births in rural area in 2013 = 22% of 3.6 lakh = 79,200
No. of births in rural area in 2014 = 20% of 3 lakh = 60,000
Difference = 79,200 − 60,000 = 19,200. Choice (B)

15. The number of births in semi-urban area in 2013 = 18% of 3.6 lakh = 64,800
The number of births in semi-urban area in 2014 = 18% of 3 lakh = 54,000
Percentage decrease
= $\frac{64,800-54,000}{64,800} \times 100 = 16.66\%$. Choice (A)

16. The number of births in metropolitan area in 2013 = 36% of 3.6 lakh = 1,29,600
The number of births in metropolitan area in 2014 = 30% of 3 lakh = 90,000
Required ratio = 129600 : 90000 = 36 : 25. Choice (A)

17. The number of births in tribal area in 2013 = 6% of 3.6 lakh = 21,600
The number of births in tribal area in 2014 = 10% of 3 lakh = 30,000
$\therefore$ Required percentage = $\frac{21600}{30000} \times 100 = 72\%$.
Choice (C)

18. The number of children lacking good medical facilities in 2013 = 82% of 3.6 lakh = 2,95,200
The number of children lacking good medical facilities in 2014 = 84% of 3 lakh = 2,52,000
$\therefore$ Total number of children who does not lack good medical facilities = 2,95,200 + 2,52,000
= 5,47,200 = 5.472 lakhs. Choice (B)

19. A total of 80 + 68 + 120 + 92, viz., 360 persons own a four wheeler.
$\therefore$ The remaining 60 + 40 + 40 + 100, viz., 240 of the respondents do not own a four wheeler, i.e., 40% of the respondents do not own a four-wheeler.
Choice (B)

20. (i) The average rainfall in December is 150mm. The average rainfall in June is between 400mm and 500mm. $\therefore$ (i) is true.
(ii) For each month, we know the average rainfall, the 10 percentile value (*i*.e., the value below which 10 percent of the data falls) and the 90 percentile value. In a particular year, the rainfall in July may be less than that in February. (ii) cannot be concluded
(iii) This statement does not follow (From the explanation in (ii) above)
(iv) We see a narrow gap between any two of the three curves in August and a significant gap between any two of the three curves in November. $\therefore$ (iv) follows. Only (i) and (iv) follow.
Choice (B)

Solutions for questions 21 and 22:

Total number of vehicles produced = 70,000
Scooters without gear = 25% of 70,000.
= $70,000 \times \frac{25}{100} = 17,500$
100 cc scooters without gear = $17,500 \times \frac{70}{100} = 12,250$

150 cc scooters without gear = 17,500 – 12,250 = 5,250

Number of scooters with gear = 35% of 70,000

i.e., $70{,}000 \times \frac{35}{100} = 24{,}500$

100 cc scooters with gear = $24500 \times \frac{40}{100} = 9{,}800$

150 cc scoters with gear = $24{,}500 \times \frac{60}{100} = 14{,}700$

Number of bikes = $70{,}000 \times \frac{40}{100} = 28{,}000$

100 cc bikes = $28{,}000 \times \frac{65}{100} = 18{,}200$

150 cc bikes = $28{,}000 \times \frac{35}{100} = 9{,}800$

21. Number of 150 cc bikes produced in that year = 9,800. Choice (D)

22. Of the four options the number of 150 cc scooters without gear is the least in number i.e., 5,250. Choice (D)

Solutions for questions 23 and 24:

The cost of 100 g of A = 4 (1) + 3 (2) + (in ₹) 2 (2.5) + (4.5) = 19.5

100 g of B (in ₹) = 26.5

100 g of C (in ₹) = 25

100 g of D (in ₹) = 27.5

23. The cost of the required mixture = 19.5 + 26.5 + 25 + 27.5 = ₹98.5. Choice (C)

24. Cost of 400 g of B = ₹106

400 g of C = ₹100

200 g of A and 200 g of D = ₹94

300 g of A and 150 g of D = ₹99.75. Choice (C)

25. Among the five players, only P, Q and T had an increase in the test batting average from 2010 to 2014. Percentage increase in the test batting average from 2010 to 2014 of

$P = \frac{48-44}{44} \times 100\%$

$= \frac{100\%}{11} = 9\frac{1}{11}\%$

$Q = \frac{52-46}{46} \times 100\%$

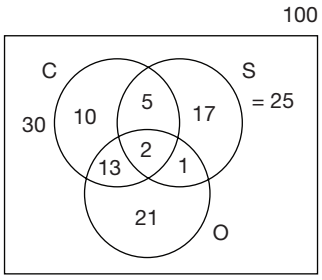
$= \frac{300}{23}\% = 13\frac{1}{23}\%$

$T = \frac{39-19}{19} \times 100\% = \frac{20}{19} \times 100\%$ which is > 100%.

P had the least percentage increase in the test batting average. Choice (A)

Solutions for questions 26 and 27:

From the given data we get the following diagram.



(i) Only cutlet = 10
(ii) Only omlette = 21
(iii) Only sandwich = 17
(iv) All the three = 2
(v) Exactly two dishes = 13 + 5 + 1 = 19
(vi) At least two dishes = 19 + 2 = 21
(vii) Cutlet = 10 + 5 + 2 + 13 = 30
(viii) Sandwich = 5 + 2 + 1 + 17 = 25
(ix) Omlette = 13 + 2 + 1 + 21 = 37
(x) None = 100 – (10 + 17 + 21 + 5 + 13 + 1 + 2) = 31

26. The number of students who like at least one dish = sum of the elements = 69. Choice (A)

27. The number of students who like cutlet or sandwich but not both is given by 10 + 13 + 17 + 1 = 41. Choice (B)

Solutions for question 28:

28. The number of units produced by C_1, C_2, C_3 ……C_{10}

Are $\frac{32\times100}{100-80}, \frac{29\times100}{100-90}, \frac{27\times100}{100-85}, \frac{38\times100}{100-81}, \frac{36\times100}{100-76}$

$\frac{46\times100}{100-77}, \frac{20\times100}{100-84}, \frac{21\times100}{100-88}, \frac{28\times100}{100-93}$, and

$\frac{30\times100}{100-95}$

Respectively i.e.160, 290, 180, 200, 150, 200, 125, 175, 400 and 600

∴ $C10$ produced the greatest total number of units in month M. Choice (A)

Solutions for questions 29 and 30:

29. Statement A is not true as (30% – 40% of 40%) of $15000 \neq 2800$

Statement B is true, since the expenditure on education $= \left(\frac{40}{100}\right)(15000) = 6000$.

The remaining expenditure $= \frac{(15+10+30)}{100}(15000) = 8250$.

∴ Difference is ₹2250. Choice (B)

30. Expenditure on books = $15000 \left(\frac{40}{100}\right)\left(\frac{25}{100}\right) = 1500$.

The expenditure on food = $\left(\frac{30}{100}\right)(15000) = 4500$.

Expenditure on books is less than the expenditure on food by 4500 – 1500 = ₹3000. Choice (D)

QUANTITATIVE ABILITY TEST 5

(PERMUTATIONS AND COMBINATIONS)

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. In how many ways can five boys and five girls be arranged around a circular table such that on either side of every boy, a girl must sit?
(A) 2560 (B) 2880
(C) 120 (D) 1440

2. Twelve boys have to be seated in a row such that two particular boys occupy the middle two positions. In how many ways can they be seated?
(A) $\frac{12!}{2!}$ (B) 10! 2!
(C) 12! (D) $\frac{10!}{2!}$

3. How many five letter words can be formed using the letters of the word "QUESTION" so that the word contains 2 vowels and 3 consonants?
(A) 5670 (B) 120
(C) 2880 (D) 1440

4. There are seven pairs of shoes. In how many ways can one select 4 shoes from them such that no complete pair is included among them?
(A) 840 (B) 420
(C) 13440 (D) 560

5. There are 4 apples, 3 oranges and 6 mangoes in a basket. In how many ways can one select one or more fruits from the basket?
(A) 139 (B) 140
(C) 71 (D) 72

6. There are 10 questions in a paper each with four options, of which only one is correct. In how many ways can a student get exactly 7 questions correct given that he attempted all the questions?
(A) 2400 (B) 2880
(C) 3240 (D) 3600

7. A number is formed using the digits 5, 8, 1, 4 and 3. When we arrange the numbers in ascending order find the rank of the number 58413. (Each digit occurs at most once in each number)
(A) 228 (B) 290
(C) 299 (D) 300

8. A team of 11 is to be selected from two groups *A* and *B*, which consist of 10 and 8 persons respectively. In how many ways can this team be selected such that exactly five members are selected from the first eight persons of group *A*?
(A) 11,760 (B) 10,760
(C) 12,760 (D) 11,670

9. In how many arrangements of the word MATHEMATICS, the two *A*'s are separated?
(A) $\frac{10!}{2!2!2!}$ (B) $\frac{9!}{2!2!2!}$
(C) $9 \times 10!$ (D) $\frac{9 \times 10!}{2!2!2!}$

10. Find the number of ways of arranging the letters of the word CALENDAR in such a way that exactly two letters are present in between *L* and *D*?
(A) 2640 (B) 3600
(C) 2600 (D) 7200

11. There are 12 men and 7 women. In how many ways can a team of six members be formed such that there are atmost two women?
(A) 16,683 (B) 16,386
(C) 16,863 (D) 16,638

12. Three bags *X*, *Y* and *Z* contain six, five and four marbles respectively. A person has to choose 11 marbles at random. In how many ways can this be done such that at least 3 marbles are to be chosen from each bag.
(A) 860 (B) 840
(C) 960 (D) 870

13. In a factory there are three class I employees, two class II employees, three class III employees and four class IV employees. A team of five members is to be formed. In how many ways can this be done if the team must have at least one class I and at most two class IV employees?
(A) 590 (B) 491
(C) 600 (D) 591

14. There are two groups *X* and *Y* in a colony. *X* consists of five boys and four girls, *Y* consists of four boys and five girls. They plan an educational tour of four boys and four girls such that exactly four persons are selected from each of the two groups. In how many ways can this be done?
(A) 5626 (B) 5026
(C) 15876 (D) 43758

15. There are 20 players and 6 of them are from Hyderabad. In how many ways can a team of 12 players be formed so that exactly three persons of the team are from Hyderabad?
(A) 4004 (B) 20020
(C) 40040 (D) None of these

16. In how many ways can five men and five women be seated in a row, so that all men are sitting together and all women are sitting together?

(A) $(5!)^2$ (B) $(5!)^2 2!$

(C) $\frac{(5!)^2}{2!}$ (D) $(5!)^2 4!$

17. For a company board meeting, eight directors and a chairperson have to be seated around a circle. If two particular directors are seated on either sides of the chairperson, in how many ways can they be seated?
(A) 1440 (B) 1200
(C) 10080 (D) 1080

18. In how many ways can seven persons be selected from 14 persons such that two particular persons are selected and three other particular persons are not selected?
(A) 120 (B) 126
(C) 300 (D) 240

19. There are six different consonants and three different vowels of the English alphabet. The number of words that can be formed using them such that each word contains two vowels and three consonants is
(A) 3600 (B) 1800
(C) 2400 (D) 7200

20. Find the number of sides of a regular polygon in which the number of diagonals is equal to one and half times the number of its sides.
(A) 5 (B) 6
(C) 8 (D) 10

21. Find the number of ways of arranging the letters of the word RAINBOW such that the vowels occupy odd places.
(A) 7! (B) 720
(C) 576 (D) 120

22. On Sports Day in a school, some competitions are held. Every student has to play exactly one game with every other student. It was found that in 36 games both players were girls and in 126 games one player was a boy and the other was a girl. Find the number of games played in which both players were boys.
(A) 56 (B) 72
(C) 91 (D) 45

23. A box contains coins of denominations 50 paisa, ₹1, ₹2 and ₹5. Given that there are unlimited coins of each type, find the number of ways of selecting the coins so that any such selection gives a total amount of ₹10.
(A) 49 (B) 12
(C) 48 (D) 46

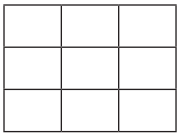
24. Find the number of ways in which the letters of the word THURSDAY can be arranged such that no word starts with *T* or ends with *Y*.
(A) 9360 (B) 31680
(C) 29520 (D) 30960

25. How many natural numbers are there from 10000 to 1000000 for which the sum of the digits is 3?
(A) 16 (B) 36
(C) 27 (D) 35

26. The number of ways of posting seven different letters into three post boxes so that at least one letter is posted in each post box is
(A) 1806 (B) 1803
(C) 2184 (D) 2059

27. A palindrome is a word, which spells the same when read from left to right or from right to left. How many palindromes of length 8 can be formed using the operative symbols +, –, ÷ and ×?
(A) 8! (B) 2^{16}
(C) 256 (D) 243

28. Six identical balls have to be placed in the square cells of the given figure such that each row contains at least one ball. In how many ways can this be done? (Given that each square can take at the most one ball)



(A) 84 (B) 76
(C) 77 (D) 81

29. The number of two digit codes that can be formed using the digits 0 to 9 with '0' not taking the tens place and an odd number taking the units place is
(A) 40 (B) 50
(C) 45 (D) 36

30. There are 5 balls of different colours and 5 boxes of colours the same as those of the balls. The number of ways in which the balls, one in each box can be placed such that a ball does not go to a box of its own colour is
(A) 40 (B) 44
(C) 42 (D) 36

31. A question paper consists of 5 problems, each problem having 3 internal choices. In how many ways can a candidate attempt one or more problems?
(A) 63 (B) 511
(C) 1023 (D) 15

32. Six points are marked on a straight line and five points marked on another line which is parallel to the first line. How many straight lines, including the first two, can be formed with these points?
(A) 29 (B) 33
(C) 55 (D) 32

33. Sixteen guests have to be seated around two circular tables, each accommodating 8 members. 3 particular guests desire to sit at one particular table and 4 others at the other table. The number of ways of arranging these guests is
(A) ${}^9C_5$ (B) $\frac{9!(7!)}{4!5!}$
(C) $\frac{9!(7!)^2}{4!5!}$ (D) $(7!)^2$

34. In how many ways can one or more of 5 letters be posted into 4 mail boxes, if any letter can be posted into any of the boxes?
(A) 5^4 (B) 4^5
(C) $5^5 - 1$ (D) $4^5 - 1$

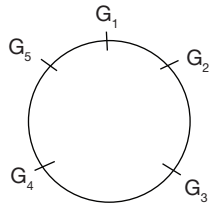
35. The number of non negative integral solutions to the equation $a + b + c = 14$ is
(A) 78 (B) 45
(C) 120 (D) 110

Answer Keys

1. B	**2.** B	**3.** C	**4.** D	**5.** A	**6.** C	**7.** D	**8.** A	**9.** D	**10.** B
11. C	**12.** D	**13.** D	**14.** A	**15.** C	**16.** B	**17.** A	**18.** B	**19.** D	**20.** B
21. C	**22.** C	**23.** A	**24.** D	**25.** B	**26.** A	**27.** C	**28.** D	**29.** C	**30.** B
31. C	**32.** D	**33.** C	**34.** C	**35.** C					

Hints and Explanations

1. First arrange the girls in the circle as shown in the diagram. They can be arranged in (5 – 1)! ways.



Then there are 5 gaps and the 5 boys can be seated in these gaps in 5! Ways.
∴ The total number of arrangements is 4! × 5! = 2880. Choice (B)

2. There are two middle positions (6th and 7th). The two particular boys can occupy the middle two positions in 2! ways. The remaining 10 boys can be seated in 10! ways.
∴ The total number of ways = (10!) (2!). Choice (B)

3. The word QUESTION, contains 4 vowels and 4 consonants; 2 vowels and 3 consonants can be selected from 4 vowels and 4 consonants in $^4C_2 . {}^4C_3$ ways. = 6 × 4 = 24
Again these 5 letters can be rearranged among themselves in 5! ways.
∴ The total number of 5 letter words formed = 5! × 24 = 120 × 24 = 2880. Choice (C)

4. Since there are 7 pairs of shoes and 4 shoes are to be selected without any complete pair being included in it, the 4 shoes must be from 4 different pairs.
This can be done in $^7C_4$ ways (choosing 4 pairs from the 7 pairs)
Now from each of these 4 pairs, one can select a shoe in 2 ways
(*i*.e., any one of the two shoes present in each pair)
Therefore the total number of ways in which one can select 4 shoes from 7 pairs of shoes such that no complete pair is included among them = $^7C_4(2)(2)(2)(2)$ = 35(16) = 560 ways. Choice (D)

5. We can choose the apples in 5 ways i.e., either choosing 0 apples or 1 apple or 2 apples or 3 apples or 4 apples.
Similarly the oranges can be chosen in 4 ways and the mangoes can be chosen in 7 ways.
Therefore the total number of ways in which one can select one or more fruits from 4 apples, 3 oranges and 6 mangoes = 5 × 4 × 7 – 1 = 139 ways.
(we subtract 1 for the case where we select 0 apples, 0 oranges and 0 mangoes i.e., no fruit at all). Choice (A)

6. In each question, there are 3 wrong options and exactly one correct option.
The number of ways in which a student gets exactly 7 questions correct = $^{10}C_7(3)^3$
$^{10}C_7$ gives us the number of ways in which the student selects seven questions (in which the student marks correct option) from the given 10 questions.
3^3 gives us the number of ways in which the student answers the 3 questions which he gets wrong.
Therefore the required number of ways = 3240.
Choice (C)

7. The number of single digit numbers = 5.
Given digits are {5, 8, 1, 3, 7}.
The number of two digit numbers formed with the digits is $^5P_2$ = 20. The number of three digit numbers formed with the digits is $^5P_2$ = 60.
The number of four digit numbers formed with digits is $^5P_4$ = 120.
The number of five digit numbers formed which begin with either 1 or 3 or 4 is 4!
The number of five digit numbers formed which begin with 51 or 53 or 54 is 3!
The number of 5 digit numbers which begin with 581 or 583 is 21.
The next number 58413
The rank of the number 58413 is 5 + 20 + 60 + 120 + 3 (24) + 3 (6) + 2(2) + 1 = 300. Choice (D)

8. In group *A*, there are 10 persons.
From first 8 persons 5 can be selected in $^8C_5$ ways and remaining 6 must be selected from the remaining 2

persons of group A and 8 persons from group B.
This can be done in ${}^{10}C_6$.
$\therefore$ The required number of ways = ${}^{8}C_5 \ {}^{10}C_6 = 11760$.
Choice (A)

9. The total number of words that can be formed is $\frac{11!}{2!2!2!}$

Number of arrangements in which the 2 A's are together $= \frac{10!}{2!2!}$

Total number of arrangements in which the A's are separated = Total number of words – number of words, in which the two A's together.

$= \frac{10!}{2!2!}\left(\frac{11}{2}-1\right) = \frac{9(10!)}{2!2!2!}$ Choice (D)

10. C A L E N D AR
$\overline{1}\,\overline{2}\,\overline{3}\,\overline{4}\,\overline{5}\,\overline{6}\,\overline{7}\,\overline{8}$

There are 5 positions to fix the L and D i.e., (1, 4), (2, 5), (3, 6), (4, 7) and (5, 8) and L and D can be interchanged. The remaining 6 letters can be arranged in $\frac{6!}{2!}$ ways.

$\therefore$ Required number of ways = $\frac{6!}{2!} \times 5 \times 2 = 360 \times 10$
$= 3600$. Choice (B)

11. The following table gives the complete possibilities of selecting the team as per the given conditions

Men (12)	Women (7)	Number of ways
4	2	${}^{12}C_4 \times {}^{7}C_2$
5	1	${}^{12}C_5 \times {}^{7}C_1$
6	–	${}^{12}C_6$

Total number of ways of selecting the team
$= {}^{12}C_4 \ {}^{7}C_2 + {}^{12}C_5 \ {}^{7}C_1 + {}^{12}C_6$
$= 10395 + 5544 + 924 = 16863$. Choice (C)

12. The following table will give the number of marbles selected from each bag and the number of ways of selecting them.

X(6)	Y(5)	Z(4)	Number of ways of selecting them
3	4	4	${}^{6}C_3 \ {}^{5}C_4 \ {}^{4}C_4 = 100$
3	5	5	${}^{6}C_3 \ {}^{5}C_5 \ {}^{4}C_3 = 80$
4	3	4	${}^{6}C_4 \ {}^{5}C_3 \ {}^{4}C_4 = 150$
4	4	3	${}^{6}C_4 \ {}^{5}C_4 \ {}^{4}C_3 = 300$
5	3	3	${}^{6}C_5 \ {}^{5}C_3 \ {}^{4}C_3 = 240$
			Total number of ways = 870

Choice (D)

13. The following table gives the number of persons of each category and the number of ways of selecting them as per the given conditions

Class I (3) (3)	Class II + Class III (5)	Class IV (4)	Number of ways of selecting them
1	4	0	${}^{3}C_1 \ {}^{5}C_4 \ {}^{4}C_0 = 15$
1	3	1	${}^{3}C_1 \ {}^{5}C_3 \ {}^{4}C_1 = 120$
1	2	2	${}^{3}C_1 \ {}^{5}C_2 \ {}^{4}C_2 = 180$
2	3	0	${}^{3}C_2 \ {}^{5}C_3 \ {}^{4}C_0 = 30$
2	2	1	${}^{3}C_2 \ {}^{5}C_2 \ {}^{4}C_1 = 120$
2	1	2	${}^{3}C_2 \ {}^{5}C_1 \ {}^{4}C_2 = 90$
3	2	0	${}^{3}C_3 \ {}^{5}C_2 \ {}^{4}C_0 = 10$
3	1	1	${}^{3}C_3 \ {}^{5}C_1 \ {}^{4}C_1 = 20$
3	0	2	${}^{3}C_3 \ {}^{5}C_0 \ {}^{4}C_2 = 6$
			Total = 591

Choice (D)

14. The following table shows the number of persons and the number of ways of selecting them as per the given conditions.

Boys		Girls		Number of ways of selecting them
X(5)	Y(4)	X(4)	Y(5)	
0	4	4	0	${}^{5}C_0 {}^{4}C_4 \ {}^{4}C_0 \ {}^{5}C_0$ (or) = 1
1	3	3	1	${}^{5}C_1 \ {}^{4}C_3 \ {}^{4}C_3 \ {}^{5}C_1$ (or) = 400
2	2	2	2	${}^{5}C_2 \ {}^{4}C_2 \ {}^{4}C_2 \ {}^{5}C_2$ (or) = 3600
3	1	1	3	${}^{5}C_3 \ {}^{4}C_1 \ {}^{4}C_1 \ {}^{5}C_3$ (or) = 1600
4	0	0	4	${}^{5}C_4 \ {}^{4}C_0 \ {}^{4}C_0 \ {}^{5}C_1$ (or) = 25
				Total = 5626

Choice (A)

15. There are 20 players, of them 6 are from Hyderabad. Exactly 3 players can be selected from 6 players in ${}^{6}C_3$ ways and the remaining 9 players are to be selected from the remaining 14 players.
This can be done in ${}^{14}C_9$ ways.
$\therefore$ Total number of ways of selecting the team
$= {}^{6}C_3 \ {}^{14}C_9 = 40040$. Choice (C)

16. There are 5 men and 5 women.
As all men and all women are to sit together, treat all men as one unit and all women as one unit.
The two units can be arranged in 2! ways.
But five men and five women can arrange among themselves in 5! 5! ways.
$\therefore$ Total number of ways they can sit is given by $(5!)^2 \ 2!$. Choice (B)

17. As two particular directors are to sit on either sides of the chairperson we treat these three as one unit. The remaining six directors and this one unit of three persons can sit around a circular table in $(7 - 1)! = 6!$ ways.

But the two directors sitting on either side of the chairperson can arrange themselves in 2! ways.
∴ The required number of arrangements possible
= 6! 2! = 1440 ways. Choice (A)

18. Total there are 14 persons, of them two particular persons are always selected.
We need to select only 5 persons from 14 – 2 = 12 persons. As three other particular persons are not to be selected, keeping them away there are only nine persons left from which we have to select any five persons.
This is possible in ${}^{9}C_{5} = 126$ ways. Choice (B)

19. There are 6 consonants , of them 3 can be selected in ${}^{6}C_{3}$ ways. Similarly, 2 vowels can be selected in ${}^{3}C_{2}$ ways.
These 3 consonants and 2 vowels can be arranged in 5! Ways.
∴ Total number of words that can be formed
$= {}^{6}C_{3} \times {}^{3}C_{2} \times 5! = 7200$ Choice (D)

20. The number of diagonals of a regular polygon of n sides $= \frac{n(n-3)}{2}$
It is given that, $\frac{n(n-3)}{2} = \frac{3}{2}n \Rightarrow n = 6$. Choice (B)

21. There are seven letters in the word RAINBOW, of them three are vowels and four are consonants.
There are four odd places, three vowels can occupy four places in ${}^{4}P_{3}$ ways and the remaining 4 places can be occupied by the remaining 4 consonants in 4! ways.
∴ Total number of arrangements possible in given by
${}^{4}P_{3} \times 4! = 576$ Choice (C)

22. Let m boys and n girls participate in the competitions.
Given the number of games in which both players are girls = 36
$\Rightarrow \quad {}^{n}C_{2} = 36$
$\Rightarrow \quad \frac{n(n-1)}{2} = 36 \Rightarrow n = 9$
∴ Number of girls = $n = 9$
The number of games in which one boy and one girl participated = 126
${}^{m}C_{1}\ {}^{n}C_{1} = 126$
$m(9) = 126 \Rightarrow m = 14$
Number of boys, $m = 14$
Number of games, in which both players were boys
$= {}^{m}C_{2} = {}^{14}C_{2} = \frac{14(13)}{1(2)} = 91.$ Choice (C)

23. The number of ₹5 coins that can be selected is 0, 1 or 2.
Let x, y and z represent the number of 50 paise, ₹1 and ₹2 coins respectively to be selected.
Case 1 : When the number of ₹5 coins selected is zero, then
$\frac{x}{2} + y + 2z = 10$ i.e., $x + 2y + 4z = 20$
If $z = 0$, then $y = 0, 1, 2, \ldots 10$; the number of solutions = 11
$z = 1$, then $y = 0, 1, 2, .. 8$; the number of solutions = 9
- - - -

Combinations	Internal arrangements	Number of ways
1, 1, 5	$\frac{3!}{2!}=3$	$3\times\frac{7!}{5!}=126$
1, 2, 4	3! = 6	$6\times\frac{7!}{4!\,2!}=630$
1, 3, 3	$\frac{3!}{2!}=3$	$3\times\frac{7!}{3!\,3!}=420$
2, 2, 3	$\frac{3!}{2!}=3$	$3\times3\times\frac{7!}{2!\,2!\,3!}=630$
	Total	1806

Choice (A)

27. The first and eighth places must be filled with the same symbol. This can be done in 4 ways.
Similarly the second and seventh place must be filled with same symbol. This can be done in 4 ways.



Similarly, the other entries can be filled.
∴ The number of ways for forming a palindrome of length 8 symbols is $4^4 = 256$. Choice (C)

28. There are 9 boxes in total, so the 6 identical balls can be placed in these boxes in ${}^9C_6$ ways. These include 3 ways in which the balls are placed in exactly two rows.
∴ Required number of ways $= {}^9C_6 - 3$
$= 84 - 3 = 81$. Choice (D)

29. Units place can be filled with 1, 3, 5, 7 or 9 and the tens can be filled with any of the 9 non-zero digits.
∴ The number of two digit codes that can be formed $= 9 \times 5 = 45$. Choice (C)

30. ∴ Required number of ways

$$= n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+----+(-1)^n\frac{1}{n!}\right)$$

$$= 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$$= 5!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$= 60 - 20 + 5 - 1 = 44$. Choice (B)

31. Given that, the question paper consists of 5 problems. For each problem, one or two or three or none of the choices can be attempted.
∴ Hence, the required number of ways $= 4^5 - 1$.
$= 2^{10} - 1 = 1024 - 1 = 1023$.
Choice (C)

32. We know that, the number of straight lines that can be formed by the 'n' points in which r points are collinear and no other set of three points, except those that can be selected out of these r points are collinear) is ${}^nC_2 - {}^rC_2 + 1$.
∴ Hence, the required number of straight lines
$= {}^{11}C_2 - {}^6C_2 - {}^5C_2 + 1 + 1$
$= 55 - 15 - 10 + 2 = 32$.
Choice (D)

33. After arranging 3 and 4 particular guests, the remaining number of people is 9.
To arrange on first table we require 5 members. They can be selected in ${}^9C_5$ ways.
To arrange on the second table, we require 4 members. They can be selected in ${}^4C_4$ ways.
∴ Hence, required arrangements is $= {}^9C_5\,(7!)\,(7!)$
$= {}^9C_5\,(7!)^2$.
Choice (C)

34. Let the letters be L_1, L_2, L_3, L_4 and L_5 and the mail boxes be B_1, B_2, B_3, B_4. Now L_1 can be dealt in 5 ways i.e., either post it into B_1 or B_2 or B_3 or B_4 or do not post it at all (since one or more letters have to be posted, there is a possibility of not posting L_1 at all). Similarly each of L_2, L_3, L_4 and L_5 can be dealt in 5 ways, giving us a total of 5^5 possibilities which includes the case of not posting any of the letters, which has to be ruled out. Hence the required ways are $5^5 - 1$.
Choice (C)

35. We know that, the number of non negative integral solutions of $a_1 + a_2 + a_3 + \text{-------} + a_r = n$ is ${}^{n+r-1}C_{r-1}$ here $n = 14$ $r = 3$
∴ Required answer is $(14 + 3 - 1)\,C_{3-1} = {}^{16}C_2 = 120$.
Choice (C)

QUANTITATIVE APTITUDE TEST 6

Number of Questions: 35 **Section Marks: 30**

(ERPV, NUMBERS)

Directions for questions 1 to 6: Select the correct alternative from the given choices.

1. A three-digit number when reversed and subtracted from the original number gives the result as 792. If the sum of the digits of the number is 18, find the tens digit of the number.
(A) 6 (B) 7
(C) 5 (D) 8

2. A sum of ₹209 was intended to be divided among A, B and C in the ratio 2 : 4 : 5. By mistake, it got divided in the reciprocal of the intended ratio. Find the gain of A due to this mistake.
(A) ₹48 (B) ₹72
(C) ₹56 (D) ₹60

3. In a fraction, the numerator exceeds the denominator by 7. If twice the numerator exceeds thrice the denominator by 2, find the fraction.
(A) $\frac{8}{15}$ (B) $\frac{19}{12}$
(C) $\frac{20}{3}$ (D) $\frac{17}{24}$

4. If $a : b = 3 : 4$, $c : b = 2 : 7$ and $c : d = 5 : 7$, find $a : b : c : d$.
(A) 105 : 140 : 50 : 70 (B) 105 : 140 : 40 : 49
(C) 105 : 140 : 40 : 56 (D) 105 : 140 : 50 : 49

5. In a parking place, there are a total of 20 vehicles (bikes and cars) are parked. If the total number of tyres of all these vehicles is 70, find the ratio of the number of bikes and cars. (No vehicle has spare tyres).
(A) 3 : 1 (B) 1 : 2
(C) 1 : 3 (D) 2 : 1

6. Ajay, Balu, Chetan and Dinesh have a total of ₹240 with them. The amount with Ajay is half of the total amount with the others. The amount with Balu is one-fourth of the total amount with the others. The amount with Chetan is one-fifth of the total amount with the others. Find the amount with Dinesh.
(A) ₹30 (B) ₹72
(C) ₹40 (D) ₹48

Directions for questions 7 and 8: These questions are based on the following information.
Siddharth has two landlines of a particular telephone service provider. For each line,he has to pay a monthly rent of ₹250. This entitles him to a certain fixed number of free calls per line. Calls over and above this allowance are charged at the rate of ₹1 per call. One month he received bills of ₹450 and ₹350 for the two lines. Had he made all his calls on a single line, his bill for that line would have been ₹700.

7. What was the total number of calls on the first line?
(A) 375 (B) 300
(C) 350 (D) 400

8. What is the number of free calls per line?
(A) 150 (B) 125
(C) 180 (D) 200

Directions for questions 9 to 35: Select the correct alternative from the given choices.

9. The cost of 2 pens, 4 erasers and 5 sharpeners is ₹36. The cost of 3 pens,7 erasers and 9 sharpeners is ₹63. Find the total cost of one pen, one eraser and one sharpener.
(A) ₹9 (B) ₹8
(C) ₹10 (D) ₹11

10. If $a + b - c : b + c - a : a + c - b = 3 : 4 : 5$, find $a : b : c$.
(A) 1 : 2 : 3 (B) 8 : 7 : 9
(C) 5 : 7 : 8 (D) 1 : 3 : 2

11. For which of the following values of k does the system of equations $2x + 5y = 1$ and $6x + 15y = k/2$ have infinite solutions?
(A) 6 (B) 3
(C) –6 (D) Any value except 6

12. If $a : b = 3 : 4$, find the value of $\frac{3a^2 + 4b^2}{4a^2 + 3b^2}$.
(A) $\frac{11}{12}$ (B) $\frac{12}{11}$
(C) $\frac{12}{13}$ (D) $\frac{13}{12}$

13. The distance a stone falls under free fall varies directly with the square of the time for which it falls. If a stone falls at a distance of 35 m in the fourth second, find the total distance it falls in the first 5 seconds.
(A) 100 m (B) 140 m
(C) 135 m (D) 125 m

14. A sum of ₹750 is divided among P, Q and R. If ₹30, ₹20 and ₹10 is added to their respective shares then the ratio of amounts of P, Q and R becomes 10 : 8 : 9. What is the share of P?
(A) 280 (B) ₹250
(C) ₹260 (D) ₹270

15. Ajay told Bharat, "I am four times as old as you were when I was as old as you are". Bharat told Ajay "Ten

years ago, I was nine years younger to you". Find the sum of present ages of Ajay and Bharat.
(A) 39 years (B) 36 years
(C) 42 years (D) 45 years

16. The ratio of the incomes of A and B is 3 : 2. The ratio of their expenditures is 5 : 4. If the savings of A is twice that of B, find the ratio of the income and expenditure of B.
(A) 3 : 16 (B) 3 : 4
(C) 3 : 2 (D) 5 : 4

17. A bag has a total of 40 coins in denominations of ₹1, ₹2 and ₹5. If the total value of the coins is ₹130, find the maximum number of ₹5 coins.
(A) 24 (B) 21
(C) 23 (D) 22

18. The age of Harish is 8 years more than twice the age of Ganesh. After how many years will the age of Harish be twice the age of Ganesh?
(A) 4
(B) 6
(C) 8
(D) Cannot be determined

19. Rohan made 13 calls from a public booth. Each was either a local call or STD call or ISD call. The average costs of his local calls, STD calls and ISD calls were ₹6, ₹11 and ₹13, respectively. The total amount spent by Ram on the calls was ₹119. Find the number of ISD calls he made
(A) 3 (B) 4
(C) 2 (D) 5

20. Ram has a certain number of notes of the denominations ₹5, ₹10 and ₹20. The total amount he has is ₹540. If he has a total of 24 currency notes in the denominations of ₹5 and ₹20, then what is the greatest number of notes of ₹10 notes he could have?
(A) 42 (B) 40
(C) 39 (D) 38

21. Three vessels are filled to their capacities with mixtures of milk and water. The ratioof their capacities is 2:3:4. The ratio of the quantities of milk and water in the first,the second and the third vessels is 1:3, 5:1 and 3:5 respectively. Find the ratio of the total quantity of milk in the vessels to the total quantity of water in the vessels.
(A) 1 : 1 (B) 2 : 3
(C) 3 : 2 (D) 3 : 4

22. a, b, c are positive integers such that $a : b = 4 : 3$ and $b : c = 4 : 3$. If the sum of the squares of c and b is less than square of the sum of b and a by 2236, then what is the value of the number which is neither the greatest nor the least?
(A) 24 (B) 32
(C) 18 (D) 36

23. What is the least value of x, if the nine-digit number $23x4567y4$ is divisible by 44?
(A) 1 (B) 0
(C) 5 (D) 7

24. A number when divided by 259 leaves a remainder of 161. If one-seventh of the same number is divided by 37, the remainder will be
(A) 14
(B) 23
(C) 7
(D) Cannot be determined

25. $(36^3 + 36^2 - 37)$ is not divisible by, which of the following?
(A) 185
(B) 37
(C) 36
(D) More than one of the above

26. Mohit has 290 toffees with him. He distributes all his toffees among his friends, such that each of his friends gets a different number of toffees, which is at least 5 and at most 29. What could be the least number of friends?
(A) 19 (B) 20
(C) 13 (D) 14

27. When the King of Patiala distributed 33,274 gold coins equally among his subjects, the Maharaja of Mysore also distributed 30,905 gold coins equally to his subjects, each giving the same number of coins to his subjects. Surprisingly, both the kings were left with the same number of gold coins.If the number of coins that each subject received is a 2-digit number, what is the difference in the number of subjects of the two kings?
(A) 309
(B) 103
(C) 23
(D) Cannot be determined

28. The units digit of $(2^{4n})(6^{7n}) + (5^{3n})(7^{9n})$, (where n is a natural number) is
(A) 3
(B) 1
(C) 5
(D) Cannot be determined

29. Three bells X, Y and Z ring at regular intervals and ring simultaneously 24 times in a day. If Y rings less frequently than X but more frequently than Z, what could be the minimum number of times for which Y rings in a day?
(A) 48 (B) 36
(C) 54 (D) 72

30. If a, b and c are three consecutive positive integers, then which of the following is not necessarily true?
I. $(4a + 5b + 3c)$ is an odd number.
II. $(2a + 3b + 4c)$ is an even number.
III. $a^2 b^3 c^4$ is an even number.

IV. $(3a + 2b + 4c)$ is an odd number.

(A) Only I (B) Only II and III
(C) Only III and IV (D) Only II and IV

31. Some students have to be seated in some rows, such that equal number of students is seated in each row. If there are 13 rows 3 students will be left out and if there are 21 rows, 11 students will be left out. How many students will be left out if there are 19 rows and less than 300 students?

(A) 7 (B) 9
(C) 11 (D) 16

32. After the division of a number successively by 4, 5 and 3 the remainders obtained are 1, 2 and 2 respectively. What will be the remainder if the least of such numbers is divided by 37?

(A) 12 (B) 14
(C) 17 (D) 31

33. The sum and the difference of the LCM and the HCF of two numbers is 784 and 756 respectively. What is the least possible difference between the two numbers?

(A) 112 (B) 70
(C) 84 (D) 98

34. A number when divided by 30 and 36 leaves a remainder of 15 and 21 respectively. When such a number is divided by 35, it leaves a remainder of 10. What is such least number?

(A) 885 (B) 895
(C) 985 (D) 15

35. What will be the remainder when $223 \times 226 \times 228$ is divided by 11?

(A) 10 (B) 7
(C) 6 (D) 1

Answer Key

1. D	**2.** B	**3.** B	**4.** C	**5.** C	**6.** B	**7.** C	**8.** A	**9.** A	**10.** B
11. A	**12.** D	**13.** D	**14.** D	**15.** A	**16.** C	**17.** D	**18.** C	**19.** A	**20.** C
21. A	**22.** A	**23.** A	**24.** B	**25.** C	**26.** C	**27.** B	**28.** B	**29.** A	**30.** D
31. D	**32.** A	**33.** C	**34.** A	**35.** D					

Hints and Explanations

1. Let the three-digit number be abc.
Its value is $1000a + 10b + c$.
On reversing it becomes cba whose values are $100c + 10b + a$
$abc - cba = 99(a - c) = 792$
$\Rightarrow$ $a - c = 8$
(a, c) can be (9, 1) or (8, 0)
$a + b + c = 18$
when $a = 9$, $c = 1$, then $b = 8$
when $a = 8$, $c = 0$, then
$b = 10$, which is not possible.
Hence $b = 8$

Choice (D)

2. A should have got 2 out of the total 11 parts i.e., 2/11 (209) = ₹38
Ratio of actual division of ₹209 among A, B and C is 1/2 : 1/4 : 1/5 = 10 : 5 : 4
A actually got 10 out of 19 parts i.e., 10/19(209) = ₹110
Gain of A is 110 – 38 = ₹72. Choice (B)

3. Let the numerator and denominator of the fraction be n and d.
$2n - 3d = 2$ ..... (1)
$n - d = 7$ ..... (2)
Solving the two equations we get $n = 19$ and $d = 12$.

Choice (B)

4. $a = \frac{3}{4}b, b = \frac{7}{2}c, c = \frac{5}{7}d$

Hence $a = \frac{3}{4}\left(\frac{7}{2}\left(\frac{5}{7}d\right)\right) = \frac{15}{8}d$

$b = \frac{7}{2}\left(\frac{5}{7}\right)d = \frac{5}{2}d$

Hence $a : b : c : d = \frac{15}{8}d : \frac{5}{2}d : \frac{5}{7}d : d$

Multiplying each term of the ratio by LCM of denominators of the coefficients of d i.e. 56,
We got, $a : b : c : d = 105 : 140 : 40 : 56$
Alternate method:
Going by the choices, $a : b = 3 : 4$, $b : c = 7 : 2$ and $c : d = 5 : 7$ is satisfied only in Choice (C).

Choice (C)

5. Let the number of bikes sand cars parked be S and C respectively.
$S + C = 20$ ..... (1)
Each bike has 2 tyres and each car has 4 tyres.
Total number of tyres = $2S + 4C = 70$
$\Rightarrow$ $S + 2C = 35$ ..... (2)
Subtracting the second equation from the first equation, we get $C = 15$,
so $S = 20 - C = 5$ $S : C = 1 : 3$ Choice (C)

6. Let the amounts with Ajay, Balu, Chetan and Dinesh be a, b, c and d respectively.

$a + b + c + d = 240$ $a = \frac{1}{2}(b + c + d) = \frac{1}{2}(240 - a)$

$\Rightarrow \quad 2a = 240 - a$

$\Rightarrow \quad 3a = 240 \Rightarrow a = \frac{240}{3} = ₹80$

It can be seen above that half of the total amount with Balu, Chetan and Dinesh becomes one-third of the total amount.

Similarly amounts with Balu and Chetan which are one-fourth and one-fifth of the total amount with the others become one-fifth and one-sixth of the total amounts.

Hence $b = \frac{240}{5} = 48$

$c = \frac{240}{6} = 40$

$d = 240 - (a + b + c) = 72$ Choice (B)

Solutions for questions 7 and 8:

On line 1, ₹250 was rent and ₹200 was call charges. On line 2, ₹250 was rent and ₹100 was call charges. The total chargeable calls was 300 ….. (1)

On a single line, ₹250 is the rent and ₹450 is the call charges, i.e. there are 450 chargeable calls ……. (2)

Comparing (1), (2) we conclude that the number of free calls is 150. The total number of calls (free + chargeable) on the first line is 150 + 200 = 350

7. 350 Choice (C)

8. 150 Choice (A)

Solutions for questions 9 to 35:

9. Let the cost of each pen, eraser and sharpener be p, e and s respectively.

$2p + 4e + 5s = 36$

$3p + 7e + 9s = 63$

Multiplying the first equation by 2 and subtracting the second equation from it, $p + e + s = 9$ Choice (A)

10. Let $a + b - c = 3x$ $\rightarrow (1)$

$b + c - a = 4x$ $\rightarrow (2)$

$a + c - b = 5x$ $\rightarrow (3)$

Adding these three equations, $a + b + c = 12x$ $\rightarrow (4)$

$2c = 9x \rightarrow (4) - (1)$

so $c = 4.5x$

$2a = 8x \rightarrow (4) - (2)$

so $a = 4x$

$2b = 7x \rightarrow (4) - (3)$

so $b = 3.5x$

$a : b : c = 4x : 3.5x : 4.5x = 8 : 7 : 9$ Choice (B)

11. If two equations should be in the form

$a_1x + b_1y = k_1$ and

$a_2x + b_2y = k_2$ to have infinite solutions,

$a_1/a_2 = b_1/b_2 = k_1/k_2$ must be satisfied.

Hence for the given equations, $2/6 = 1/k/2$ so $k = 6$

Choice (A)

12. Given that $a : b = 3 : 4$

so $\frac{a}{b} = \frac{3}{4}$

Dividing both numerator and denominator of $\frac{3a^2 + 4b^2}{4a^2 + 3b^2}$

by b^2, it becomes

$$\frac{3\left(\frac{a}{b}\right)^2 + 4}{4\left(\frac{a}{b}\right)^2 + 3} = \frac{3\left(\frac{9}{16}\right) + 4}{4\left(\frac{9}{16}\right) + 3} = \frac{13}{12}$$ Choice (D)

13. Let the distance travelled by the stone and the time of travel of the stone be denoted by d and t respectively. $d = ct^2$ where c is a constant.

Distance travelled in the fourth second by the stone = Total distance travelled in first four seconds – the total distance it travelled in first three seconds

$= c(4^2 - 3^2) = 35.$

$7c = 35 \Rightarrow c = 5$

Total distance it falls in the first 5 seconds

$= c(5^2) = 125$ m. Choice (D)

14. The shares of P, Q and R be ₹x, ₹y and ₹z respectively.

So $x + y + z = 750$

$(x + 30) : (y + 20) : (z + 10) = 10 : 8 : 9$

$x + y + z + 30 + 20 + 10 = 750 + 60 = 810.$

So $x + 30 = \frac{810 \times 10}{(10 + 8 + 9)} = 300$. So $x = 270$

Share of p = ₹270 Choice (D)

15. Let the present age of Ajay be x years.

Some time in the past, Bharat was $x/4$ years.

The age of Ajay at that time is Bharat's present age.

Bharat's present age is $(x - 9)$ years.

As their difference of ages is constant,

$x - (x - 9) = (x - 9) - x/4$. $x = 24$

Sum of their present ages $= 2x - 9 = 39$ years

Choice (A)

16. Let the incomes of A and B be $3x$ and $2x$, respectively. Let the expenditures of A and B be $5y$ and $4y$, respectively. Savings of A and B are $3x - 5y$ and $2x - 4y$ respectively. Given that, $3x - 5y = 2(2x - 4y) \Rightarrow x = 3y$

Ratio of income and expenditure of $B = 2x : 4y$

$= 2(3y) : 4y = 3 : 2$ Choice (C)

17. Let the number of ₹1, ₹2 and ₹5 coins be x, y and z respectively.

$x + y + z = 40$ ……. (1)

$x + 2y + 5z = 130$ …… (2)

Subtracting the equation (1) from (2), $y + 4z = 90$

In order to satisfy the above equation, y must be divisible by 2.

As z must be maximum, y must be minimum.
Hence when y has the least value, then z is obtained as 22 from the above equation. Choice (D)

18. Let the present age of Ganesh be G.
Present age of Harish = $2G + 8$
Let Harish become twice the age of Ganesh k years from now. Hence, $2G + 8 + k = 2(G + k) \Rightarrow k = 8$.
Choice (C)

19. Let the number of local calls, STD calls and ISD calls made by Rohan be l, s and i respectively.
$l + s + i = 13$ and $6l + 11s + 13i = 119$
$6l + 11s + 13i - 6(l + s + i) = 119 - 6(13)$
$5s + 7i = 41$
$5s$ ends with 0 or 5. Also, R.H.S ends with 1. $\therefore 7i$ must end with 1 or 6. Also $7i < 41$.
$\therefore$ $7i = 21$ ($\therefore$ No value of $7i$ ends with 6)
$i = 3$ Choice (A)

20. Let Ram have f, t and w notes of denomination ₹5, ₹10 and ₹20 respectively. Given that
$5f + 10t + 20w = 540$ ------ (1) (f, t, w are integers)
and $f + w = 24$ ---------- (2)
(1) – 5 × (2) gives $10t + 15w = 420$

$$\Rightarrow \quad t = \frac{420 - 15w}{10}$$

$$\text{As } w > 0,\ t_{max} = \frac{420 - 15 \times 2}{10} = 39$$

Choice (C)

21. The vessels are filled to their capacities
Let the capacities of the first, the second and the third vessels be $2x$, $3x$ and $4x$ respectively

$$\text{Required ratio} = \frac{1}{1+3}(2x) + \frac{5}{6}(3x) + \frac{3}{8}(4x) : 3/4(2x)$$

$$+ \frac{1}{6}(3x) + \frac{5}{8}(4x) = \frac{x}{2} + \frac{5x}{2} + \frac{3x}{2} : \frac{3x}{2} + \frac{x}{2} + \frac{5x}{2}$$

= 1 : 1. Choice (A)

22. Given that $a : b = 4 : 3$ and $b : c = 4 : 3$
$\Rightarrow$ $a : b = 16 : 12$ and $b : c = 12 : 9$
$\therefore$ $a : b : c = 16 : 12 : 9$
Let $a = 16k$, $b = 12k$ and $c = 9k$
Also given $(a + b)^2 - (b^2 + c^2) = 2236$
$(28k)^2 + (144k^2 + 81k^2) = 784k^2 - 225k^2 = 2236$
$\Rightarrow$ $k^2 = 4 \Rightarrow k = 2$
$\therefore$ $b = 12k = 12 \times 2 = 24$. Choice (A)

23. The number $23x\ 456\ 7y\ 4$ is divisible by 4 and by 11.
$\therefore$ $y = 0, 2, 4, 6$ or 8 and $(2 + x + 5 + 7 + 4) - (3 + 4 + 6 + y)$,
i.e., $(18 + x) - (13 + y)$ or $(5 + x - y)$ is also a multiple of 11.
As the least value of $5 + x - y$ is $5 + 0 - 9$ or -4 and the greatest value is $5 + 9 - 0$ or 14, it could be 0 or 11.
$\therefore$ $x = y - 5$ or $x + y + 6$. The least value of x is $6 - 5$ or 1. Choice (A)

24. Let the number be N
$\therefore$ $N = 259K + 161$.
$\Rightarrow$ $N/7 = 37K + 23$
$\therefore$ $N/7$ leaves a remainder of 23, when divided by 37.
Choice (B)

25. The given number N is $36^2(36 + 1) - 37$
$= 36^2(37) - 37 = 37(36^2 - 1) = (37)(35)(37)$
$\therefore$ N is not divisible by 36. Choice (C)

26. Since the number of friends has to be the least, the number of toffees should be as great as possible. Let the toffees distributed be 29, 28, 27, 26 …..
Given that, $29 + 28 + 27 + 26 + \ldots$. up to n terms ≤ 290.
If $n = 12$, the number of toffees that can be distributed is $29 + 28 + \ldots\ldots + 19 + 18 = 6(29 + 18) = 282$
The last person gets 8 toffees. The least number of friends is $12 + 1 = 13$. Choice (C)

27. Let there be m subjects in Patiala and n in Mysore. Let each subject get p coins and say the number of coins left with either king is r.
$\therefore$ $mp + r = 33,274$ and $np + r = 30,905$
$\therefore$ $(m - n)p = 2369 = (23)(103)$
As p is a 2–digit number $p = 23$ and $m - n = 103$.
Choice (B)

28. $2^{4n} \times 6^{7n} + 5^{3n} \times 7^{9n}$ can be written as $16^n \times 6 + 5 \times 7^{9n}$.
As 16^n can be written as 6^n, the unit's digit of the sum becomes $6 \times 6 + 5 \times$ (odd number) $= 6 + 5 = 1$.
Choice (B)

29. In a day, the three bells toll together = 24 times, or once in every hour or 60 minutes. Let X, Y, Z ring once in every x, y, z minutes respectively. The maximum value of z is 60 and $y < 60$. Since y is a factor of 60, its greatest value is 30. Hence if Y rings once every 30 minutes, the number of times it tolls in a day will be the minimum, which is $= \frac{(24)(60)}{30} = 48$ Choice (A)

30. The numbers a, b, c are consecutive. Therefore there are only 2 possibilities for the types (odd / even) of a, b, c. Either a, b, c are o, e, o or they are e, o, e. We tabulate the expressions, and the truth values (definitely true, could be false, definitely false, i.e., dt, cf, df).

				dt	cf	df
I	4a + 5b + 3c	4(o) + 5(e) + 3(o) = o	4(e) + 5(o) + 3(e) = o	✓		
II	2a + 3b + 4c	2(o) + 5(e) + 4(o) = e	2(e) + 5(o) + 4(e) = o		✓	
III	a2 b3 c4	(o) (e) (o) = e	(e) (o) (e) = e	✓		
IV	3a + 2b + 4c	3(o) + 2(e) + 4(o) = o	3(e) + 2(o) + 4(e) = e		✓	

We see that II, IV are not definitely true
Choice (D)

31. When the students are arranged in 13 rows, let the number of students in each row be x. The total number of students is $(13x + 3)$. Similarly in the second case, the total number of students is $21y + 11$.
This is LCM model II
Number of students: = (LCM of 13 and 21) – 10 = 263
Required remainder 263 ÷ 19 = 16. Choice (D)

32. Divisors are 4, 5, 3. Remainders are 1, 2, 2.
The least number is [(2) (5) + 2] (4) + 1 = 49.
When 49 is divided by 37, the remainder is 12.
Choice (A)

33. Let the two numbers be ha and hb respectively,where a and b are co-primes. Then LCM of the two numbers is hab.
Given $hab + h = 784$ and $hab – h = 756$
$\Rightarrow$ $hab = 770$ and $h = 14$ Þ $ab = 55$.
The possible values of a, b are (1, 55) and (5, 11).
When the numbers are close, the difference will bethe least, i.e., $\{a, b\} = \{5, 11\}$.
The least possible difference is 14 (6) = 84.
Choice (C)

34. The given conditions represent the problem as LCM model-2. The general form of such numbers is (LCM of 30, 36)K – [common difference i.e., 15 in this case] = $180K – 15$.
Now, when $180K – 15$ is divided by 35, ($175K$ is a already divisible by 35), the remainder is given to be 10. Hence $180K – 25$ i.e., $5K – 25$ is exactly divisible by 35. This will be possible when $K = 5$.
Hence the required number is 180(5) – 15 = 885.
Choice (A)

35. Remainder required

$$= \text{Rem}\left(\frac{(220+3)(220+6)(220+8)}{11}\right)$$

(220 + 3) (220 + 6) (220 + 8) = (A multiple of 220 + 3.6) (220 + 8) = A multiple of 220 + (3) (6) (8). As 220 is divisible by 11, reminder required = $\text{Rem}\left(\frac{(3)(6)(8)}{11}\right)$

$= \text{Rem}\left(\frac{144}{1}\right) = 1.$ Choice (D)

QUANTITATIVE APTITUDE TEST 7

Number of Questions: 35 **Section Marks: 30**

(QUADRATIC EQUATIONS AND INEQUALITIES)

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Two students independently attempted to solve a quadratic equation in x. One of them copied the constant term wrong and obtained roots as –15 and 16. The other student copied the coefficient of x wrong and obtained his roots as –10 and 21. Find the correct roots of the quadratic equation?

(A) (–15, 14) (B) (–14, 15)
(C) (–25, 7) (D) (–7, 25)

2. $\sqrt{132+\sqrt{132+\sqrt{132+.....}}}$ = _____

(A) 11.11 (B) 12.32
(C) 11 (D) 12

3. The range of k for which the sum as wells as the product of the roots of $6x^2 - kx + 9 - k^2 = 0$ are negative is _____

(A) (–3, 3) (B) (–∞, 3)
(C) (–∞ , –3) (D) (3, ∞)

4. Find the range of k, for which $-x^2 + 4kx + 3k - 1$, is always negative.

(A) $\left(-\frac{1}{4},1\right)$ (B) $(1, \infty)$

(C) $\left(-\infty,-\frac{1}{4}\right)$ (D) $\left(-1,\frac{1}{4}\right)$

5. a and b are the roots of the equation $2x^2 - 15x + k = 0$. Find the value of k if $a^2 - b^2 = 45$.

(A) $5\frac{1}{16}$ (B) $10\frac{1}{8}$

(C) $13\frac{1}{2}$ (D) $12\frac{1}{2}$

6. If the sum of the roots of the quadratic equation $3x^2 + (2k + 1)\,x - k - 5 = 0$ is equal to the product of the roots, which of the following is true?

(A) $k^2 - 4 = 0$ (B) $k^2 - 9 = 0$
(C) $k^2 - 16 = 0$ (D) $k^2 - 25 = 0$

7. If the roots of the equation $x^2 - 7x - 12 = 0$ are diminished by one and then multiplied by two, which of the following equations is formed with those values as the roots?

(A) $x^2 - 10x + 24 = 0$ (B) $x^2 - 6x - 76 = 0$
(C) $x^2 - 2x - 48 = 0$ (D) $x^2 - 10x - 72 = 0$

8. Which of the following statements is true about the roots of the equation $k^2 x^2 - k x + (1 + 2x^2) = 0$, where k is a real number?

I. Roots are equal
II. Roots are complex
III. Roots are rational
IV. Roots are real

(A) I and IV (B) III only
(C) I and II (D) II only

9. All the roots of two quadratic equations are positive integers. The sum of the squares of the roots of the first quadratic equation is equal to that of the second quadratic equation. If the sum of the roots of the two equations are 10 and 8 respectively, then what is the greatest possible root of these quadratic equations?

(A) 7 (B) 6
(C) 8 (D) 5

10. If a and b are positive numbers, what is the nature of the roots of the equation $(a + b)\,x^2 + 2\,abx + \frac{(a+b)^3}{16} = 0$?

(A) Real and distinct. (B) Real and equal.
(C) Non-real and distinct. (D) Either (*B*) or (*C*)

11. If a positive number is increased by three and then squared, the result is 23 more than the original number. Find the original number.

(A) 1 (B) 2
(C) 3 (D) 4

12. Find the value of R, so that one of the roots of $x^2 + 6Rx + 64 = 0$ is the square of the other root.

(A) $\frac{-10}{3}$ (B) $\frac{8}{3}$

(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

13. If the value of p in the equation $x^2 + 2(p + 1)x + 2p = 0$, is real, the roots of the equation are

(A) rational and unequal.
(B) irrational and unequal.
(C) real and unequal.
(D) real and equal.

14. Find the equation whose roots are twice the roots of the equation $3x^2 - 7x + 4 = 0$.

(A) $3x^2 - 14\,x + 8 = 0$ (B) $3x^2 + 14x + 16 = 0$
(C) $3x^2 + 14\,x - 16 = 0$ (D) $3x^2 - 14x + 16 = 0$

15. The length of a rectangle is 1 cm more than its breadth. If its diagonal is 29 cm, what is the measure of its breadth? (in cm)

(A) 18 (B) 20
(C) 17 (D) 21

16. A is any single-digit prime number and B is any natural number. How many equations of the form $x^2 - 4\sqrt{A}\,x + 3B = 0$ have both real roots?
(A) 15 (B) 18
(C) 21 (D) 24

17. In a class, eight students play basketball. The remaining students, who represent 7 times the square root of the strength of the class, play football. Find the strength of the class.
(A) 36 (B) 16
(C) 64 (D) 100

18. If the price of a book goes down by ₹20 per dozen, a person can purchase 50 dozen books more for ₹30,000. Find the original price of each book.
(A) ₹10 (B) ₹12
(C) ₹9 (D) ₹8

19. If $-9 \le p \le -5$ and $-17 \le q \le -12$ then which of the following can be concluded?
(A) $\frac{5}{12} \le \frac{p}{q} \le \frac{9}{17}$ (B) $\frac{17}{9} \le \frac{p}{q} \le \frac{12}{5}$
(C) $\frac{5}{17} \le \frac{p}{q} \le \frac{3}{4}$ (D) $\frac{12}{9} \le \frac{p}{q} \le \frac{17}{5}$

20. If $|3x-4| = |5x-12|$, then the sum of the possible values of x is _____.
(A) 4 (B) 6
(C) –4 (D) –6

21. If $|4x-9| = 7$, then the values of $4|x| - |-x^3|$ is _____.
(A) $48, \frac{15}{8}$ (B) $-48, \frac{-15}{8}$
(C) $48, \frac{-15}{8}$ (D) $-48, \frac{15}{8}$

22. Find the range of values of x that satisfy the relation $|2x - 1| - 1 < |x - 2| + 3$.
(A) $-4 < x < 4$ (B) $-6 < x < \frac{1}{2}$
(C) $-6 < x < 4$ (D) $-5 < x < 3$

23. If $E = |x + 4| + |x + 7| + |x - 1|$, then how many integral values of x satisfy the inequality $E \le 14$?
(A) 8 (B) 10
(C) 11 (D) More than 11

24. Which of the following inequalities gives a finite range of values for x?
(A) $6x^3 - x^2 - x < 0$
(B) $x^4 + x^3 - 3x^2 - x + 2 < 0$
(C) $x^3 - x^2 - 5x - 3 < 0$
(D) $x^4 + 3x^3 + 2x^2 > 0$

25. If $\frac{x}{x+1} - \frac{x+2}{x-1} < 0$, then find the range of x.
(A) $\left(-1, -\frac{1}{2}\right) \cup (1, \infty)$ (B) $(-2, -1) \cup (0, 1)$
(C) $(-\infty, -1) \cup \left(-\frac{1}{2}, 1\right)$ (D) $\left(-\frac{1}{2}, \infty\right)$

26. Find the range of x, for which $|x + 2| - 3|x - 1| + 4 \ge 0$.
(A) $-2 \le x \le 1$ (B) $-2 \le x \le \frac{9}{2}$
(C) $-\frac{3}{4} \le x \le 4$ (D) None of these

27. a, b, c and d are four positive real numbers whose sum is equal to 4. If $p = \frac{abcd}{(abc + bcd + acd + abd)}$, then find the maximum value of p.
(A) 16 (B) 4
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

28. If $2 < x < 5$ and $10 < y < 30$, then $\frac{y}{x}$ lies between
(A) 5 and 6 (B) 2 and 6
(C) 2 and 15 (D) 6 and 15

29. If $|x| > 6$ and $y > -4$, then which of the following is necessarily true?
(A) $/xy| > 24$ (B) $|xy| < 24$
(C) $|x|\,|y| > 0$ (D) None of these

30. Let $f(x) = \max(3x + 5, 7 - 2x)$, where x is any real number. Then the minimum possible value of $f(x)$ is
(A) $\frac{31}{5}$ (B) $\frac{27}{5}$
(C) $\frac{21}{5}$ (D) $\frac{29}{5}$

31. If $20 \le x \le 35$ and $3y - 2x = 5$, then the minimum value of $\frac{x}{x+y}$ is
(A) 1 (B) $\frac{1}{3}$
(C) $\frac{4}{9}$ (D) $\frac{4}{7}$

32. If a, b and c are positive real numbers. Find the minimum value of $\left(1 + a + \frac{1}{a}\right)\left(1 + b + \frac{1}{b}\right)\left(1 + c + \frac{1}{c}\right)$.
(A) 9 (B) 12
(C) 27 (D) 81

33. If $a \le 25$ and $a + b \ge 10$, then which of the following is always true?

(A) $a - b \ge 40$ (B) $b - a \ge -40$

(C) $a + b \ge 40$ (D) $ab \le 250$

34. If $1 \le x \le 3$, $4 \le y \le 10$ and $2 \le z \le 5$, what is the maximum possible value of $\frac{y}{x+y+z}$?

(A) 5 (B) $\frac{10}{3}$

(C) $\frac{10}{13}$ (D) $\frac{10}{7}$

35. Find the range of values of x for which $\left|\frac{18-2x}{4}\right| < 3$.

(A) $6 < x < 30$ (B) $-3 < x < 15$

(C) $-15 < x < 3$ (D) $3 < x < 15$

Answer Key

1. B	**2.** D	**3.** C	**4.** D	**5.** B	**6.** C	**7.** D	**8.** D	**9.** A	**10.** D
11. B	**12.** A	**13.** C	**14.** D	**15.** B	**16.** C	**17.** C	**18.** A	**19.** C	**20.** B
21. D	**22.** D	**23.** B	**24.** B	**25.** A	**26.** D	**27.** D	**28.** C	**29.** D	**30.** A
31. D	**32.** C	**33.** B	**34.** C	**35.** D					

Hints and Explanations

1. The quadratic equation which has α and β are the roots is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Quadratic equation taken by the first student is

$x^2 - (-15 + 16)x + (-15 \times 16) = 0$

$x^2 - x - 240 = 0$ ——————(1)

Quadratic equation taken by the second student is

$x^2 - (-10 + 21)x + (-10 \times 21) = 0$

$x^2 - 11x - 210 = 0$ ——————(2)

$\therefore$ Required correct quadratic equation is $x^2 - x - 210 = 0$

$\Rightarrow x^2 - 15x + 14x - 210 = 0$

$\Rightarrow x(x - 15) + 14(x - 15) = 0$

$\Rightarrow (x - 15)(x + 14) = 0$

$\therefore$ Required roots are -14 and 15. Choice (B)

2. Let $X = \sqrt{132 + \sqrt{132 + \sqrt{132 + ...}}}$

$\Rightarrow x = \sqrt{132 + x}$

Squaring on both sides

$\Rightarrow x^2 = 132 + x \Rightarrow x^2 - x - 132 = 0$

$\Rightarrow x^2 - 12x + 11x - 132 = 0$

$\Rightarrow (x - 12)(x + 11) = 0$

$\therefore x = 12 (\because x$ is always positive) Choice (D)

3. Given $6x^2 - kx + 9 - k^2 = 0$

The sum of the roots $= \frac{k}{6}$

The product of the roots $= \frac{9 - k^2}{6}$

$\frac{k}{6}$ and $\frac{9 - k^2}{6}$ are both negative

$\Rightarrow \frac{k}{6} < 0$ and $\frac{9 - k^2}{6} < 0$

$\Rightarrow k < 0$ and $9 < k^2$

$\Rightarrow k < 0$ and $k > 3$ or $k < -3$

$\therefore k < -3 \Rightarrow k \in (-\infty, -3)$. Choice (C)

4. It is given that $-x^2 + 4kx + 3k - 1 < 0$

$\Rightarrow -(x^2 - 4kx) + 3k - 1 < 0$

$\Rightarrow -[(x - 2k)^2 - 4k^2] + 3k - 1 < 0$

$\Rightarrow -[(x - 2k)^2] + 4k^2 + 3k - 1 < 0$

Now, for the above expression to be always negative $4k^2 + 3k - 1 < 0 \Rightarrow (4k - 1)(k + 1) < 0$

This is true when $-1 < k < \frac{1}{4}$. Choice (D)

5. The given equation is $2x^2 - 15x + k = 0$

The sum of the roots, $a + b = \frac{15}{2}$ and

the product $ab = \frac{k}{2}$

It is given that $a^2 - b^2 = 45 \Rightarrow a - b = 6$

$a + b = \frac{15}{2}$, $a - b = 6$, $\Rightarrow a = \frac{27}{4}$, $b = \frac{3}{4}$

$\therefore$ The product of the roots $ab = \left(\frac{27}{4}\right)\left(\frac{3}{4}\right) = \frac{81}{16}$

Now $\frac{k}{2} = \frac{81}{16} \Rightarrow k = \frac{81}{8} = 10\frac{1}{8}$ Choice (B)

6. Sum of the roots $= \frac{-(2k+1)}{3}$

Product of the roots $= \frac{-(k+5)}{3}$

$\frac{-(2k+1)}{3} = \frac{-(k+5)}{3}$

$\Rightarrow 2k + 1 = k + 5 \Rightarrow k = 4$

Choice (C)

7. Roots are to be diminished by one and then multiplied by two. i.e., if A, B are roots of given equation, then $2(A-1)=A_1$ and $2(B-1)=B_1$, where A_1 and B_1 are the roots of the new equation. i.e.,

$\Rightarrow \quad A = 1 + \frac{A_1}{2} = \frac{A_1+2}{2}$ and $B = \frac{B_1+2}{2}$

i.e., x of the given equation is to be replaced by $\frac{x+2}{2}$, to obtain the required equation. Given equation is: $x^2 - 7x - 12 = 0$.

Required equation is $\left(\frac{x+2}{2}\right)^2 - \frac{7(x+2)}{2} - 12 = 0$

$(x+2)^2 - 7(2)(x+2) - 4(12) = 0$.

$\Rightarrow \quad x^2 + 4x + 4 - 14x - 28 - 48 = 0$.

$\Rightarrow \quad x^2 - 10x - 72 = 0$. Choice (D)

8. When rewritten, the equation becomes:

$(k^2+2)x^2 - kx + 1 = 0$

Discriminant, $D = (k)^2 - 4(1)(k^2+2)$

$= -3k^2 - 8 = -(3k^2+8)$

$3k^2$ is positive for all real values of k, and hence $(3k^2+8)$ is positive; and so $-(3k^2+8)$ is negative.

As the discriminant is negative, roots are complex.

Choice (D)

9. Let the roots of the first quadratic equation be α and β and those of the second equation be γ and δ respectively.

Given $\alpha^2 + \beta^2 = \gamma^2 + \delta^2$

Also $\alpha + \beta = 10$ and $\gamma + \delta = 8$.

The possible values of $\alpha^2 + \beta^2$ are 50, 52, 58, 68 and 82 while the possible values $\gamma^2 + \delta^2$ as are 32, 34, 40 and 50. As only 50 is a common value, $\alpha = 5$, $\beta = 5$, $\gamma = 7$ and $\delta = 1$

$\therefore$ The greatest possible root is 7. Choice (A)

10. Dividing both sides of the given equation by $a + b$,

$x^2 + \frac{2abx}{a+b} + \frac{(a+b)^2}{16} = 0$

Discriminant

$= \left(\frac{2ab}{a+b}\right)^2 - \frac{4(a+b)^2}{16} = \left(\frac{2ab}{a+b}\right)^2 - \left(\frac{a+b}{2}\right)^2$

Shown below is the proof that this is always non-positive provided a and b are positive.

$(a-b)^2 \geq 0 \Rightarrow a^2 + b^2 + 2ab \geq 4ab$

dividing both sides by $2(a+b)$ $\frac{a+b}{2} \geq \frac{2ab}{a+b}$

As the expressions on both sides of the inequality are positive, $(\frac{a+b}{2})^2 \geq (\frac{2ab}{a+b})^2$

$\therefore \quad \Delta < 0$ or $\Delta = 0$

If $\Delta = 0$, the roots are real and equal.

If $\Delta < 0$, the roots are non-real and distinct.

Choice (D)

11. Let the required original number be x.

$(x+3)^2 = 23 + x$.

Hence $x^2 + 6x + 9 = 23 + x$

$\Rightarrow \quad x^2 + 5x - 14 = 0$.

$(x+7)(x-2) = 0$

$\Rightarrow \quad x = -7$ or $x = 2$.

Since the original number is positive, $x = 2$.

Choice (B)

12. If one of the roots is α, the other root is α^2.

Hence the product of the roots $= \alpha(\alpha^2)$.

$\alpha^3 = 64 \Rightarrow \alpha = \sqrt[3]{64} = 4$ and $\alpha^2 = 4^2 = 16$

The sum of roots $= -\left(\frac{6R}{1}\right) = -6R = 4 + 16 = 20$

$R = \left(\frac{20}{-6}\right) = -\frac{10}{3}$ Choice (A)

13. For the equation $x^2 + 2(p+1)x + 2p = 0$

$b^2 - 4ac = [2(p+1)]^2 - 4(2p)] = 4p^2 + 8p + 4 - 8p$

$= 4p^2 + 4$ which is always positive.

Hence the roots of the equation are always real and unequal. Choice (C)

14. For the equation, whose roots are twice the roots of the equation A : $3x^2 - 7x + 4 = 0$, the sum of the roots is twice the sum of the roots of A and the product of the roots is 4 times the product of the roots of A.The required equation is $x^2 - \left(2\left(\frac{7}{3}\right)\right)x + 4\left(\frac{4}{3}\right) = 0$

i.e., $3x^2 - 14x + 16 = 0$ Choice (D)

15. Let ℓ and b be the length and breadth in cm.

Given that $\ell = b + 1$

Also given that diagonal = 29 cm

$\Rightarrow \quad \sqrt{\ell^2 + b^2} = 29$

By squaring on both sides, $(b+1)^2 + b^2 = 29^2$

$\Rightarrow \quad 2b^2 + 2b - 840 = 0$

$\Rightarrow \quad b^2 + b - 420 = 0$

$\Rightarrow \quad (b+21)(b-20) = 0$

$\therefore \quad b = 20$ Choice (B)

16. $\left(4\sqrt{A}\right)^2 - 4(3B) \geq 0$

$\frac{4}{3} A \geq B$

As A is a single digit prime number, A can be 2, 3, 5 or if $A = 2$, B has 2 possibilities. If $A = 3$, B has 4 possibilities. If $A = 5$, B has 6 possibilities. If $A = 7$, B has 9 possibilities. A total of 21 equations are possible.

Choice (C)

17. Let the strength be x. The number of students who play basketball = 8

The number of students who play football

$= x - 8 = 7\sqrt{x}$

Substituting the choices in place of x in the equation above, only choice (C) satisfies it. Choice (C)

18. Let the initial number of books in dozens = b
Let initial price (in ₹) of books per dozen be p.
$pb = 30{,}000.$ →(I)
$(50 + b)(p - 20) = 30{,}000$
$50p - 1000 + pb - 20b = 30{,}000$
or, $50p - 20b = 1000$
$5p - 2b = 100.$ →(II)
From (I) and (II) $5p - \frac{60{,}000}{p} = 100$
$5p^2 - 100p - 60{,}000 = 0$
$5p^2 - 600p + 500p - 60{,}000 = 0$
$5p(p - 120) + 500(p - 120) = 0 \Rightarrow p = 120$
The price of each book = $\frac{120}{12} = 10$ Choice (A)

19. $\frac{p}{q} = \frac{-p}{-q}$
$5 \le -p \le 9$ and $12 \le -q \le 17$
$\frac{-p}{-q}$ is maximum when p is maximum and q is minimum
$\therefore \quad \text{Max}\left(\frac{-p}{-q}\right) = \frac{9}{12} = 3/4$
$\frac{-p}{-q}$ is minimum when p is minimum and q is maximum.
$\therefore \quad \text{Min}\left(\frac{-p}{-q}\right) = \frac{5}{17}$
$\frac{5}{17} \le \frac{p}{q} \le \frac{3}{4}$ Choice (C)

20. $|3x - 4| = |5x - 12|$
When ever $|p| = |q|$ it follows that $p = \pm q$
$3x - 4 = \pm(5x - 12)$
$\Rightarrow \quad 3x - 4 = 5x - 12$ or $3x - 4 = -5x + 12$
$\Rightarrow \quad 2x = 8$ or $8x = 16$
$\Rightarrow \quad x = 4$ or $x = 2$
$\therefore$ Required sum of the possible values of x is 6.
Choice (B)

21. $|a| = b \Rightarrow a = \pm b$
$|4x - 9| = 7$
$\Rightarrow \quad 4x - 9 = 7$ or $4x - 9 = -7$
$\Rightarrow \quad x = 4$ or $x = 1/2$
$4|x| - |-x|^3$
$= 4(4) - (4)^3$ or $4\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^3$
$= -48$ or $15/8$ Choice (D)

22. $|2x - 1| - 1 < |x - 2| + 3$
$\Rightarrow \quad |2x - 1| - |x - 2| < 4$
We need to consider 3 cases
(1) $x < \frac{1}{2}$ (2) $\frac{1}{2} \le x < 2$ and (3) $2 \le x$.
For $x < \frac{1}{2}$, we get
$-(2x - 1) + (x - 2) < 4$
$\Rightarrow \quad -x < 5 \Rightarrow x > -5$
$\therefore \quad -5 < x < \frac{1}{2}$
For $\frac{1}{2} \le x < 2$, $2x - 1 + x - 2 < 4$
$\Rightarrow \quad x < \frac{7}{3} \quad \therefore \frac{1}{2} < x < 2$
For $x \ge 2$, $(2x - 1) - (x - 2) < 4$
$\Rightarrow \quad x < 3 \quad \therefore 2 \le x < 3$
Thus the range of x such that the given relation is satisfied is $-5 < x < 3$ Choice (D)

23. For $x = -9$, $E = |-9 + 4| + |-9 + 7| + |-9 - 1| = 17$
For $x = -8$, $E = |-8 + 4| + |-8 + 7| + |-8 - 1| = 14$
For $x = 1$, $E = |1 + 4| + |1 + 7| + |1 - 1| = 13$
For $x = 2$, $E = |2 + 4| + |2 + 7| + |2 - 1| = 16$
Therefore the integral values of x for which the given inequality is satisfied are $-8, -7, -6, -5, -4, -3, -2, -1$, 0 and 1 i.e. a total of 10 values. Choice (B)

24. We need to considering each option separately,
Option A:
$6x^3 - x^2 - x < 0$
$x(2x - 1)(3x + 1) < 0$
The above inequality is satisfied for
$x < -\frac{1}{2}$ or $0 < x < \frac{1}{2}$
which does not give a finite range of values for x
Option B:
$x^4 + x^3 - 3x^2 - x + 2 < 0$
$(x - 1)^2(x + 1)(x + 2) < 0$ which gives the same solution set as $(x + 1)(x + 2) < 0$ $((x - 1)^2 \ge 0]$
The above inequality is satisfied for $-2 < x < -1$ this is a finite range of values for x.
Option C:
$x^3 - x^2 - 5x - 3 < 0$
$(x + 1)^2(x - 3) < 0$ which gives the same solution set as $x - 3 < 0 \Rightarrow x < 3$
It does not give a finite range of values for x.
Option D:
$x^4 + 3x^3 + 2x^2 > 0$
$x^2(x^2 + 3x + 2) > 0$
$x^2(x + 2)(x + 1) > 0$
The above inequality gives the same solution set as $(x + 2)(x + 1) > 0$ $[x^2 \ge 0]$

The inequality is satisfied for $x < -2$ or $x > -1$ which does not give a finite range of values for x.

Note: A polynomial of odd degree can take values from $-\infty$ to ∞ but a polynomial of even degree has a finite range of values for which it has values of a particular sign. If the coefficient of the leading term (say a) is positive, $f(x) < 0$ for a finite range if $a < 0$, $f(x) > 0$ for a finite range. Choice (B)

25. $\frac{x}{x+1} - \frac{x+2}{x-1} < 0$

$$\frac{x(x-1)-(x+2)(x+1)}{(x+1)(x-1)} < 0$$

$$\frac{x^2 - x - x^2 - 3x - 2}{(x+1)(x-1)} < 0$$

$$\frac{-4x-2}{(x+1)(x-1)} < 0$$

$$\frac{-2(2x+1)}{(x+1)(x-1)} < 0$$

$$\frac{(2x+1)}{(x+1)(x-1)} > 0$$

Multiplying both Nr & Dr by $(x + 1)(x - 1)$ we get

$$\frac{(2x+1)(x+1)(x-1)}{(x+1)^2(x-1)^2} > 0$$

The solution set for the above inequality is the same as that for $(2x + 1)(x + 1)(x - 1) > 0$

Therefore the inequality holds true for $-1 < x < -\frac{1}{2}$ or $x > 1$, i.e., $x \in \left(-1, -\frac{1}{2}\right) \cup (1, \infty)$ Choice (A)

26. $|x + 2| - 3|x - 1| + 4 \geq 0$

For $x < -2$, $|x + 2| - 3|x - 1| + 4 = -(x + 2) + 3(x - 1) + 4$

$\therefore \quad -x - 2 + 3x - 3 + 4 \geq 0 \Rightarrow 2x \geq 1 \Rightarrow x \geq \frac{1}{2}$

But we have taken $x < -2$, thus no solution exists in this range.

For $-2 \leq x < 1$,

$|x + 2| - 3|x - 1| + 4 = (x + 2) + 3(x - 1) + 4$

$\Rightarrow \quad x + 2 - 3x + 3 + 4 \geq 0$

$\Rightarrow \quad 9 - 2x \geq 0$

$\Rightarrow \quad 9 \geq 2x \qquad \Rightarrow \quad 2x \leq 9$

$\therefore \quad x + 2 + 3x - 3 + 4 \geq 0 \Rightarrow 4x \geq -3 \Rightarrow x \geq -\frac{3}{4}$

Therefore, the range of x satisfying the given condition is $-\frac{3}{4} \leq x < 1$

For $x \geq 1$,

$|x + 2| - 3|x - 1| + 4 = (x + 2) - 3(x - 1) + 4 - 2x \leq -9$

$\therefore \quad x + 2 - 3x + 3 + 4 \geq 0$

$\Rightarrow \quad 9 - 2x \geq 0$

$\Rightarrow \quad 2x \leq 9$

$\Rightarrow \quad x \leq \frac{9}{2}$

$\therefore \quad 1 \leq x \leq \frac{9}{2}$

Thus, the range of x satisfying the given inequality is $-\frac{3}{4} \leq x \leq \frac{9}{2}$ Choice (D)

27. $AM(a, b, c, d) \geq HM(a, b, c, d)$

$$\frac{a+b+c+d}{4} \geq \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d} \geq \frac{16}{a+b+c+d}$$

$$\frac{bcd+acd+abd+abc}{abcd} \geq \frac{16}{a+b+c+d}$$

$$\frac{abcd}{abc+bcd+acd+abd} \leq \frac{a+b+c+d}{16}$$

$(\because a + b + c + d = 4)$

$\therefore \quad p \leq \frac{1}{4}$. Choice (D)

28. Given $2 < x < 5$ and $10 < y < 30$.

The value of y/x is minimum, for the minimum value of y and the maximum value of x.

$\therefore \quad y/x > \frac{10}{5}$ or $y/x > 2$

The value of y/x is maximum, for the maximum value of y and the minimum value of x.

$\therefore \quad y/x < \frac{30}{2}$ or $y/x < 15$

$\Rightarrow \quad 2 < y/x < 15$ Choice (C)

29. Given $|x| > 6$, $y > -4$.

Consider $x = 7$ and $y = 2$; $xy = 14$

$\Rightarrow \quad |xy| = 14 |xy| > 24$ is not necessarily true.

Consider $x = 8$ and $y = 5$; $xy = 40$; $|xy| > 40 > 24$

$\therefore$ The second option is not necessarily true.

For $y = 0$; $|x|\,|y| = 0$, hence none of the given options is necessarily true. Choice (D)

30. Given $f(x) = \max(3x + 5, 7 - 2x)$

$f(x)$ has the minimum value when the two expressions are equal.

$\therefore \quad 3x + 5 = 7 - 2x$

$5x = 2 \Rightarrow x = \frac{2}{5}$

$\therefore$ The minimum value of $f(x)$ is

$$f\left(\frac{2}{5}\right) = \max\left(\frac{3(2)}{5} + 5, 7 - \frac{2(2)}{5}\right)$$

$$= \max\left(\frac{31}{5}, \frac{31}{5}\right) = \frac{31}{5}$$ Choice (A)

31. $20 \le x \le 35$

$$y = \frac{2x+5}{3}$$

$$\therefore \quad \frac{x}{x+y} = \frac{x}{x+\frac{2x+5}{3}}$$

$$= \frac{3x}{5x+5} = \frac{3}{5+\frac{5}{x}}$$

This expression is positive for the given range of values of x and it has its minimum value when $5/x$ has its maximum value, i.e. when $x = 20$.

The corresponding value is $\frac{3(4)}{21} = \frac{4}{7}$.

Choice (D)

32. If x is a positive number, the minimum value of $x + \frac{1}{x}$ is 2.

Hence, the minimum value of $1 + x + \frac{1}{x}$ is 3 and for the given expression, it is $3(3)(3) = 27$. Choice (C)

33. Given $a \le 25$ and $a + b \ge 10$

$\Rightarrow$ $a \le 25$ and $b \ge 10 - a$

$\Rightarrow$ $a \le 25$ and $b \ge 10 - 25$

$\Rightarrow$ $a \le 25$ and $-b \ge 15$

$\Rightarrow$ $a - b \le 40$

$\Rightarrow$ $b - a \ge -40$ Choice (B)

34. $\frac{y}{x+y+z} = \frac{1}{\frac{x}{y}+1+\frac{z}{y}}$; to maximize the given expression, $x + z$ should take minimum and y should take maximum possible value.

$\therefore$ Maximum value $= \frac{1}{\frac{1}{10}+1+\frac{2}{10}} = \frac{10}{13}$ Choice (C)

35. Given $\left|\frac{18-2x}{4}\right| < 3$

$\Rightarrow$ $|18 - 2x| < 12$

$\Rightarrow$ $|x - 9| < 6$

The expression $|x - a|$ denotes the distance of the point x from the point a on the number line.

$|x - 9| < 6$ Þ x lies within a distance 6 units from the point 9. i.e., $3 < x < 15$.

Choice (D)

UNIT II
LOGICAL REASONING TEST

Logical Ability Test 1 ..... **1.83**
Logical Ability Test 2 ..... **1.90**

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Logical Ability Test I

Number of Questions: 35 **Time: 35 min**

Directions for questions 1 to 5: Complete the following series.

1. 4, 27, 25, 343, 121, _____ .
(1) 169 (B) 2197
(C) 3197 (D) 2457

2. 11, 25, 77, 157, 473, _____.
(A) 978 (B) 1421
(C) 949 (D) 1431

3. 12, 30, 56, 132, 182, _____.
(A) 240 (B) 300
(C) 316 (D) 306

4. 53, 61, 71, 79, 89, _____.
(A) 91 (B) 93
(C) 101 (D) 95

5. 19, 58, 175, 526, _____.
(A) 1578 (B) 1238
(C) 1458 (D) 1579

Directions for questions 6 to 10: Find the missing term.

6. 24 : 576 : : 32 : _____.
(A) 961 (B) 1000
(C) 1225 (D) 1024

7. 4 : 27 : : 25 : _____.
(A) 64 (B) 216
(C) 125 (D) 36

8. BILK : DLPP : HMTO : _____.
(A) JOWQ (B) JRWS
(C) JPVS (D) JPXT

9. Cricket : Game : : Kangaroo : _____.
(A) Animal (B) Team
(C) Bird (D) Fish

10. Driver : Bus : : _____ : Horse
(A) Saddle (B) Jockey
(C) Horseman (D) Cowboy

Directions for questions 11 to 15: Find the odd man out.

11. (A) 11 (B) 21
(C) 31 (D) 41

12. (A) 3527 (B) 2357
(C) 5723 (D) 7532

13. (A) Brown (B) Green
(C) Yellow (D) Red

14. (A) June (B) May
(C) November (D) September

15. (A) Radish (B) Carrot
(C) Potato (D) Cabbage

Directions for questions 16 to 20: Select the correct alternative from the given choices.

16. If 'CENTURY' is coded as 'AGLVSTW', then what is the code for 'SACHIN'?
(A) QCAFKL (B) UCEFGL
(C) QCAJGP (D) UCAJGP

17. If the code for 'AMBITION' is 'GSHOZOUT', then which of the following is coded as 'VXOTZUAZ'?
(A) PRINTOUT (B) PRINTING
(C) PREDATOR (D) PROFOUND

18. If 'PRESIDENT' is coded as 'KIVHRWVMG', then 'MAHENDAR' is coded as _____.
(A) NZTVMWZI (B) NZTUMWZI
(C) NZSVMWZI (D) NZSUMWZI

19. In a code language, if pen is called pencil, pencil is called eraser, eraser is called paper, paper is called book, book is called table, table is called chair and chair is called desk, then on which of the following do we sit? (according to that language)
(A) Table (B) Paper
(C) Desk (D) Book

20. In a code language, if shirt means shoe, shoe means wallet, wallet means spectacle, spectacle means fan, fan means cabin and cabin means card, then which of the following do we use when we want some air? (according to that language)
(A) Wallets (B) Spectacles
(C) Fans (D) Cabins

Directions for questions 21 to 25: In a certain code language, the codes for sentences given in column I are given in column II. Each word has a unique code. Answer the questions based on these codes.

Column I	**Column** II
earth gets heat from sun	pep tep nep mep wep
moon gets light from sun	hep kep tep pep nep
sun gave energy to plants	bep pep dep zep lep
human gets food from plants	qep tep nep dep rep
heat and light gave life	hep fep sep wep bep
life needs food, food needs light	fep qep gep qep gep hep

21. What is the code for the word 'food'?
(A) gep (B) fep
(C) qep (D) pep

22. What is the code for the word 'sun'?
(A) pep (B) nep
(C) mep (D) wep

23. Which word is coded as 'lep'?
(A) gave (B) energy
(C) to (D) Cannot be determined

24. what is the code for 'earth sun and moon'?
(A) mep kep sep pep (B) mep tep nep sep
(C) kep qep sep mep (D) sep pep rep tep

25. What can be the meaning of 'fep gep zep sep hep'?
(A) life needs energy and light
(B) sun gave light and energy
(C) human needs sun and moon
(D) plants need sun and moon

Directions for questions 26 to 30: These questions are based on the following data.

A, *B*, *C*, *D*, *E*, *F* and *G* are the seven members in a family. Among them, there are two couples and each couple has exactly two children. *B*, who is married, has no siblings and he is not married to *E*, a female, who is also married. *D* is the father of *G*. *F*, the youngest in the family, has a paternal uncle. *A* is unmarried while *F* and *C* are of the same gender. *A* and *G* are of different gender.

26. How is *F* related to *A*?
(A) Daughter (B) Niece
(C) Nephew (D) Son

27. How is *C* related to *E*?
(A) Mother (B) Daughter
(C) Daughter-in-law (D) Mother-in-law

28. How is *A* related to *E*?
(A) Brother-in-law (B) Husband
(C) Brother (D) Father-in-law

29. Which among the following is the complete group of females in the family?
(A) *E*, *F* and *G* (B) *A*, *C*, *E* and *F*
(C) *C*, *E*, *B* and *F* (D) *C*, *E*, *F* and *G*

30. How is *C* related to *G*?
(A) Mother (B) Father
(C) Uncle (D) Grandmother

Directions for questions 31 to 35: These questions are based on the following data.

Five artists - a violinist, a pianist, a singer, a dancer and an actress-have to present their work one after the other, not necessarily in that order. The five artists are Anu, Gowri, Radhika, Sudha and Mythili. Also

(i) Mythili presents her work after the singer - not necessarily immediately.
(ii) the dancer presents her work immediately after Radhika.
(iii) Gowri, the violinist, plays second.
(iv) Radhika's item is not immediately next to Gowri's.
(v) Anu is not an actress.
(vi) Radhika is the singer.

31. Who is the dancer?
(A) Mythili (B) Sudha
(C) Anu (D) Radhika

32. If the actress plays first, who plays third?
(A) Anu (B) Mythili
(C) The dancer (D) Radhika

33. Who is the actress?
(A) Anu (B) Sudha
(C) Mythili (D) Radhika

34. The order in which the artists present their programmes is
(A) pianist, violinist, actress, singer, dancer.
(B) actress, violinist, pianist, dancer, singer.
(C) actress, violinist, dancer, pianist, singer.
(D) Cannot be determined.

35. If the actress plays third, when does Sudha play?
(A) Immediately after Radhika.
(B) Immediately before the singer.
(C) After the dancer.
(D) After Gowri but not immediately.

Directions for questions 36 to 39: These questions are based on the following data.

A man goes to work in his car on all days except Sundays. There are 4 different parking spaces near his office. Out of these, the cellar and the ground floor are the closest to his office while the garage and the parking lot are the farthest. Whenever he comes to office, he parks his car in one of the four parking spaces. It is known that

(i) the parking lot is open on all days of the week but he can afford it for only 2 days a week.
(ii) the Garage is open on Mondays, Tuesdays and Thursdays but he can use it only once a week.
(iii) he can use the cellar for 2 days of the week but he cannot use it on Tuesdays, Thursdays and Saturdays.
(iv) he can use the ground floor for one day of the week, but not on Mondays, Wednesdays, and Fridays.
(v) as he is always late on Mondays, he likes to park his car close to his office.

36. If he parks his car on the ground floor on Tuesday and in the parking lot on Wednesday, then where should he park it on Friday?
(A) Cellar (B) Parking lot
(C) Ground floor (D) Garage

37. If he uses the garage on Tuesday and the parking lot on Thursday, where does he park his car on Wednesday?
(A) Cellar (B) Parking lot
(C) Ground floor (D) Either (A) or (B)

38. If he uses the garage on Tuesday and parking lot on Saturday, then which is the place he uses on Thursday?
(A) Ground floor or Parking lot
(B) Cellar
(C) Ground floor
(D) Parking lot

39. If he uses the parking lot on Tuesday, then what must he use on Thursday?
(A) Parking lot
(B) Garage or Ground floor
(C) Ground floor
(D) Garage

Directions for questions 40 to 42: These questions are based on the following information.

Eight persons – Anand, Brijesh, Chandak, Dweepesh, Sayan, Jagat Rupak and Palak – are sitting around a square table such that two persons are sitting along each side. The following information is known about them.

(i) Jagat, who is sitting to the immediate right of Rupak, is sitting opposite Chandak who is sitting to the immediate right of Brijesh.

(ii) Sayan is sitting opposite Dweepesh, who sits along the same side as Brijesh.

(iii) Palak is not sitting along the same side as Sayan.

40. Who is sitting along the same side as Chandak?

(A) Anand (B) Palak
(C) Sayan (D) Rupak

41. Who is sitting opposite Rupak?

(A) Palak (B) Anand
(C) Brijesh (D) Data inadequate

42. Who is sitting to the immediate right of Sayan?

(A) Anand (B) Rupak
(C) Chandak (D) Data inadequate

Directions for questions 43 to 45: These questions are based on the following information.

Three people are to be selected from a group of six people – M, N, P, Q, R and S under the following constraints.

(i) If M is not selected, then N is selected.

(ii) If P is not selected, then Q is selected.

(iii) If R is not selected, then S is selected.

43. In how many ways can the team be selected?

(A) 2 (B) 4
(C) 8 (D) None of these

44. Who must be there in the team?

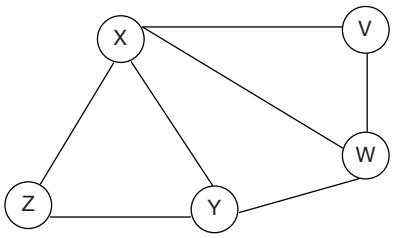
(A) N (B) Q
(C) S (D) None of these

45. Which of the following is a possible team?

(A) P, R, S
(B) P, Q, S
(C) M, Q, S
(D) More than one of the above

Directions for questions 46 and 47: Answer the questions on the basis of the following information.

Shown below is the layout of the major cities of a state and the rail tracks, connecting those cities.



Five trains – T_1, T_2, T_3, T_4 and T_5 run only on two days (Saturday and Sunday), along the following routes, between these cities.

T_1 : $Y - X - V$
T_2 : $Z - Y - X - V$
T_3 : $Z - Y - W - V$
T_4 : $Z - X - W - V$
T_5 : $Z - X - W$

Route $Y - W$ cannot be used on Sunday. On any day, no two trains are scheduled to run on the same track connecting two adjacent cities.

Each train should run exactly once in these two days.

46. T_4 can run

(A) only on Saturday
(B) only on Sunday
(C) on either day
(D) only if $W - Y$ route is used on Sunday.

47. Which of the following is NOT true?

(A) T_2 and T_4 can be scheduled to run on the same day.
(B) T_5 cannot be scheduled to run on Sunday.
(C) T_3 can be scheduled to run on Saturday.
(D) T_4 and T_1 can be scheduled to run on the same day.

Directions for questions 48 to 50: These questions are based on the data given below.

Six persons – A, B, C, D, E and F – stand in a row. A is to the left of B. C is to the right of D. E and F have two persons standing between them and neither of these two persons is C or A.

48. What is the total number of possible arrangements?

(A) 2 (B) 4
(C) 6 (D) 5

49. Who among the following stand at the extreme ends of the row?

(A) E and F (B) E and C
(C) A and C (D) F and A

50. If A sits to the immediate left of E, then who sits to the immediate right of B?

(A) D
(B) F
(C) C
(D) Cannot be determined

Answer Keys

1. B	**2.** C	**3.** D	**4.** C	**5.** D	**6.** D	**7.** B	**8.** D	**9.** A	**10.** B
11. B	**12.** D	**13.** A	**14.** B	**15.** D	**16.** C	**17.** A	**18.** C	**19.** C	**20.** B
21. C	**22.** A	**23.** D	**24.** A	**25.** A	**26.** B	**27.** D	**28.** A	**29.** D	**30.** D
31. A	**32.** A	**33.** B	**34.** D	**35.** B	**36.** A	**37.** D	**38.** C	**39.** D	**40.** B
41. A	**42.** B	**43.** C	**44.** D	**45.** C	**46.** B	**47.** D	**48.** B	**49.** C	**50.** D

Hints and Explanations

1. The given series can be expressed as follows.
$2^2, 3^3, 5^2, 7^3, 11^2$ where 2, 3, 5, 7, 11 are prime numbers.
The next in the series is $13^3 = 2197$ Choice (B)

2. The given series can be expressed as follows.
$(11 \times 2) + 3 = 25$; $(25\ 3) + 2 = 77$; $(77 \times 2) + 3 = 157$
$(157 \times 3) + 2$, $= 473$; $(473\ 2) + 3 = 949$ Choice (C)

3. The given series can be expressed as follows.
$3^2 + 3$, $5^2 + 5$, $7^2 + 7$, $11^2 + 11$, $13^2 + 13$ with 3, 5, 7, 11, 13 being prime numbers. The next number in the series is $17^2 + 17 = 289 + 17 = 306$ Choice (D)

4. The given series is the series of alternate prime numbers. The next in the series is 101. Choice (C)

5. The given series can be expressed as
$6 \times 3 + 1 = 19$; $19 \times 3 + 1 = 58$; $58 \times 3 + 1 = 175$
$175 \times 3 + 1 = 526$; $526 \times 3 + 1 = 1579$ Choice (D)

6. $24 : (24)^2 : : 32 : \underline{(32)^2}$
Square of the first number is the second number.
$(32)^2 = 1024$. Choice (D)

7. $4 : 27 : : 25 :$ _____
$(2)^2 : (3)^3 : : (5)^2 : \underline{(6)^3}$
This is of the form $(n)^2 : (n + 1)^3$.
$(6)^3 = 216$ is the next number. Choice (B)

8. BILK : DLPP : : HMTO : _____

B	I	*L*	*K*	
+2	+3	+4	+5	
D	*L*	*P*	*P*	
Similarly, H		*M*	*T*	*O*
+2	+3	+4	+5	
J	*P*	*X*	*T*	

Hence, JPXT is the next term. Choice (D)

9. Cricket is a type of game.
Similarly, kangaroo is type of animal. Choice (A)

10. Bus is driven by a driver.
Similarly, jockey rides a horse. Choice (B)

11. All the given numbers except
21 are prime numbers where as
21 is a composite number. Choice (B)

12. All the given numbers except
7532 are odd numbers whereas
7532 is an even number. Choice (D)

13. All the given colours except brown are the colours in (rainbow) VIBGYOR. Choice (A)

14. All the given months except May have 30 days where as in May there are 31 days. Choice (B)

15. All except Cabbage, grow under the soil. Choice (D)

16.

Word:	C	E	N	T	U	R	Y
Logic:	–2	+2	–2	+2	–2	+2	–2
Code:	A	G	L	V	S	T	W

Similarly,

Word:	S	A	C	H	I	N
Logic:	–2	+2	–2	+2	–2	+2
Code:	Q	C	A	J	G	P

∴ QCAJGP is the code for 'SACHIN'. Choice (C)

17.

Word:	A	M	B	I	T	I	O	N
Logic:	+6	+6	+6	+6	+6	+6	+6	+6
Code:	G	S	H	O	Z	O	U	T

Similarly,

Code:	V	X	O	T	Z	U	A	Z
Logic:	–6	–6	–6	–6	–6	–6	–6	–6
Word:	P	R	I	N	T	O	U	T

'PRINTOUT' is coded as 'VXOTZUAZ' Choice (A)

18. The code for the letter whose place value is '*n*' is the letter whose place value is $(27 - n)$.
∴ MAHENDAR is coded as NZSVMWZI.
Choice (C)

19. We sit on a chair and chair is called desk. Choice (C)

20. We use fans when we want air and spectacles means fan. Choice (B)

Solutions for questions 21 to 25:
The given statements and their codes are as follows.
(1) earth gets heat from sun – pep tep nep mep wep
(2) moon gets light from sun – hep kep tep pep nep
(3) sun gave energy to plants – bep pep dep zep lep

(4) human gets food from plants – qep tep nep dep rep
(5) heat and light gave life – hep fep sep wep bep
(6) life needs food, food needs light – fep qep gep qep gep hep

From (6), the words 'food' and 'needs' are repeated and the codes 'qep' and 'gep' are repeated. And now from (4) and (6) as only the word food is repeated the code for 'food' is 'qep' and hence the code for 'needs' is 'gep'.

From (2) and (6) the word 'light' and the code 'hep' are common.

Hence, the code for 'light' is 'hep'. The code for the remaining word in (6), i.e., 'life' is 'fep'.

The words 'gets' and 'from' are common for (1), (2) and (4).

Similarly the codes 'tep' and 'nep' are common. But the codes for 'gets' and 'from' cannot be individually obtained. Except the word 'and' and the code 'sep' in (5), all other words and codes are used in at least one of the other sentences. Hence, the code for 'and' is 'sep'.

By using comparison and elimination procedures we can find the codes for other words.

Word	earth	sun	heat	gets/ from	moon	light	gave	energy/ to	plants	human	and	life	needs	food
Code	mep	pep	wep	nep/tep	kep	hep	bep	zep/lep	dep	rep	sep	fep	gep	qep

21. The code for 'food' is 'qep'. Choice (C)

22. The code for 'sun' is 'pep'. Choice (A)

23. Either 'energy' or 'to' is coded as 'lep'. Choice (D)

24. The code for 'earth sun and moon' is 'mep kep sep pep'. Choice (A)

25. The meaning of 'fep gep zep sep hep' can be 'life needs energy and light' . Choice (A)

Solutions for questions 26 to 30:

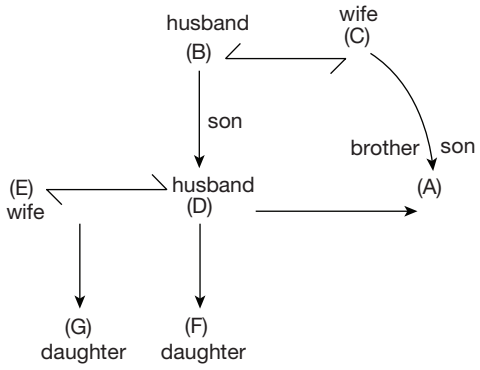
It is given that there are seven members in the family. *A*, *B*, *C*, *D*, *E*, *F* and *G*.

There are two couples in the family and each couple has exactly two children.

F, the youngest in the family has a paternal uncle which implies that *F*'s father has a brother. *B*, who is a male has no siblings and he is married. Hence, *B* will come in the first generation and he has two children and a spouse.

E is married and is a female. Hence, *E* is the mother of *F* and she is married to *D*. *G* is the child of *D* and *A* is the brother of *D*. Hence, *C* is the wife of *B*.

F and *C* are of same gender. Hence, *F* is female. *A* and *G* are of different gender. Since *A* is male, *G* is female. The given information can be represented in the diagram as follows:



26. *F* is *A*'s brother's daughter. Hence, *F* is the niece of *A*. Choice (B)

27. *C* is *E*'s husband's mother. Hence, *C* is the mother-in-law of *E*. Choice (D)

28. *A* is *E*'s husband's brother. Hence, *A* is the brother-in-law of *E*. Choice (A)

29. *C*, *E*, *F* and *G* are the females. Choice (D)

30. *C* is the grandmother of *G*. Choice (D)

Solutions for questions 31 to 35:

From the given information, we have

Gowri, the violinist, plays second. Radhika is a singer who does not come immediately after Gowri, Radhika cannot be 3rd since the dancer presents her work immediately after Radhika. Radhika cannot be 1st or 5th so, Mythili comes in the 5th place. Radhika has to be 4th and dancer 5th.

So, Anu is a Pianist and Sudha is an actress and they come in the first and the third places – not necessarily in that order. Thus we have

Order	Artist	Profession
1		
2	Gowri	Violinist
3		
4	Radhika	Singer
5	Mythili	Dancer

31. Mythili is the dancer. Choice (A)

32. If actress plays first, then Anu plays third. Choice (A)

33. Sudha is the actress. Choice (B)

34. It is not clear whether the actress or the pianist plays first. Hence, cannot be determined. Choice (D)

35. If the actress (Sudha) plays third, then Sudha plays immediately before the singer.(Radhika) Choice (B)

Solutions for questions 36 to 39:

Let us tabulate the days of the week when different slots are available.

	Cellar	Ground Floor	Garage	Parking lot
Monday		X	✓	
Tuesday	X		✓	
Wednesday		X		
Thursday	X		✓	
Friday		X		
Saturday	X			
Can use for	2 days of the week	Only one day	Only one day	2 days of the week

In addition, on Mondays he should park the car closest to his office, i.e., cellar or ground floor. But since ground floor cannot be used on Mondays, only the cellar can be used on Mondays. Hence, garage cannot be used on Mondays. Now, we can answer the questions.

36. If ground floor is used on Tuesday & parking lot on Wednesday; then we have

	Cellar	Ground Floor	Garage	Parking lot
Monday	✓	X	X	X
Tuesday	X	✓	X	
Wednesday	X	X	X	✓
Thursday	X	X	✓	
Friday		X	X	
Saturday	X	X	X	
Total	2	1	1	2

As can be seen, he has to park his car in the cellar on Friday. Otherwise he won't be able to use the cellar 2 times a week. Choice (A)

37. On Wednesday, he can park his car in the cellar or in the parking lot. Choice (D)

	Mon	Tue	Wed	Thur	Fri	Sat
Cellar	✓	X		X		X
Ground Floor	X	X	X	X	X	✓
Garage	X	✓	X	X	X	X
Parking Lot	X	X		✓		

38

	Mon	Tue	Wed	Thur	Fri	Sat	Total
Cellar	✓	X		X		X	2
Ground Floor	X	X	X	✓	X	X	1
Garage	X	✓	X	X	X	X	1
Parking Lot	X	X				✓	2

As can be seen, on Thursday only ground floor can be used (because, if he uses the parking lot on Thursday, then no day is available for the ground floor).

Choice (C)

39.

	Mon	Tue	Wed	Thur	Fri	Sat
Cellar	✓	X				
Ground Floor	X	X	X		X	
Garage	X	X	X		X	X
Parking Lot	X	✓				

Now, garage has only Thursday and no other day left. Hence, garage should be used on Thursday.

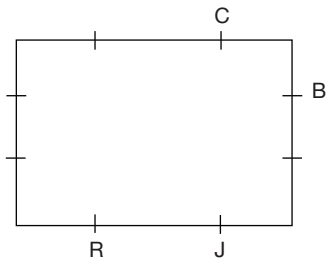
Choice (D)

Solutions for questions 40 to 42:

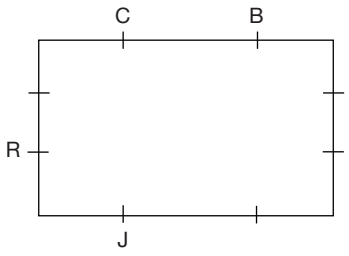
Let us represent the persons by the first letters of each name.

From (i), we get the following possibilities

Case (*a*),

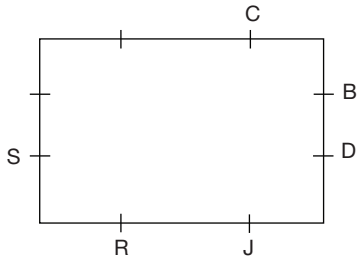


Case (*b*)

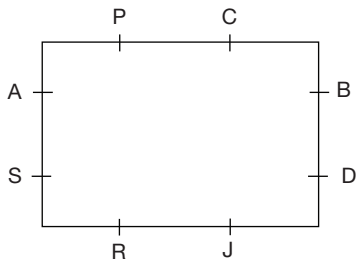


From (ii) as Brijesh and Dweepesh are sitting along the same side, case (*b*) is not possible.

From (ii) we get



From (iii) the possibility is as follows.



40. Palak is sitting along the same side as Chandak. Choice (B)

41. Palak is sitting opposite Rupak. Choice (A)

42. Rupak is sitting to the immediate right of Sayan. Choice (B)

Solutions for questions 43 to 45:

From (i), M and N can be selected as follows.

$\rightarrow$ Only M is selected.

$\rightarrow$ Only N is selected.

$\rightarrow$ Both M and N are selected.

This implies, at least one between M and N must be selected.

Similarly, from (ii) and (iii)

At least one among P and Q must be selected.

At least one among R and S must be selected.

As only 3 persons are to be selected, both M and N, both P and Q and both R and S cannot be selected.

$\therefore$ Exactly one among M and N, exactly one among P and Q and exactly one among R and S must be selected.

$\Rightarrow$ $M / N, P / Q, R / S$

43. The number of possible ways to select the team.

$= 2 \times 2 \times 2 = 8$ Choice (C)

44. There is no such person who must always be there in the team. Choice (D)

45. M, Q, S is a possible team. Choice (C)

Solutions for questions 46 and 47:

Given that the route $Y - W$ cannot be used on Sunday. Hence T_3 can be scheduled to run on Saturday. As it is given that, on any day, no two trains are scheduled to run on the same track connecting two adjacent cities, $Z - Y$ and $W - V$ should not run on Saturday ($\because$ T_3 is covering the route).

As T_2 is covering $Y - X - V$ on Sunday, T_1 has to be scheduled on Saturday. Similarly, T_5 is to scheduled on Saturday.

Finally, T_1, T_5, T_3 are to be scheduled on Saturdays and T_2 and T_4 are to be scheduled on Sundays.

46. T_4 should on run Sunday. Choice (B)

47. From the choices,

T_4 and T_1 can be scheduled on two different days. Choice (D)

Solutions for question 48 to 50:

The data is as given below:

(i) Six persons – A, B, C, D, E and F stand in a row.

Left of

(ii) A B

Right

(iii) D C

(iv) $\frac{E}{F} \text{ —— } \frac{F}{E}$

x C/A (neither C nor A)

Let us make all the possible arrangements as per the above data.

–	E/F	–	–	F/E	–
1	2	3	4	5	6

Positions 2 and 5 would be occupied by E or F. A cannot be at 3 and 4 (condition (iv)) and also A cannot be at 6 (condition (ii)). Hence, A must be at position 1. Similarly, C must be at position 6. Hence, we will get the following arrangement:

A	E/F	B/D	D/B	F/E	C

Therefore, the total number of arrangements are 4.

48. Total possible arrangements are four. Choice (B)

49. A and C stand at extreme ends. Choice (C)

50. The arrangement will be as shown below:

A	E	B/D	D/B	F	C

Hence, the person sitting to the immediate right of B cannot be determined. Choice (D)

Logical Ability Test 2

Number of Questions: 35 **Time: 35 min**

Directions for questions 1 to 4: These questions are based on the following data.

Out of a group of 315 students who went to Mumbai, 125 visited Essel World, 140 visited Lumbini Garden and 160 visited Film Nagar. Twenty Five of them visited all the three places while 200 visited exactly one of the three places. The number of students, who visited exactly 2 out of the three places, is five times as many as those who have not visited any of the places.

1. How many did not visit any of the three places?
(A) 75 (B) 25
(C) 125 (D) 15

2. How many students visited not more than one place?
(A) 200 (B) 180
(C) 250 (D) 215

3. If the number of students who visited at least one of the two places, Lumbini Gardens and Film Nagar is 255, then how many students visited only Essel World?
(A) 45 (B) 25
(C) 125 (D) 75

4. If the number of students who visited at least one of the two places, Lumbini Gardens and Film Nagar is 255, then how many students visited only one of the two places, Lumbini Gardens and Film Nagar (and not any of the other two places)?
(A) 215 (B) 125
(C) 155 (D) 175

Directions for questions 5 to 9: These questions are based on the following data.

In a colony, a survey was conducted regarding the ownership of three different types of vehicles – car, scooter and bicycle.

-- The number of residents owning all three vehicles is the same as those owning none.
-- The number of residents owning any two out of the three vehicles is the same as those owning any other two which in turn is the same as those owning none of the three.
-- The number of residents owning scooters alone is the same as those owning cars alone and each in turn is twice those owning bicycles alone.
-- Half the number of residents who own a bicycle also own at least one of the other two vehicles.

5. If the number of residents who own only bicycles is 150, then what is the total number of residents in the colony?
(A) 500 (B) 1000
(C) 750 (D) 1250

6. What percentage of the residents, who own a car, also own at least another vehicle?
(A) $66^{2}/_{3}\%$ (B) $13^{2}/_{3}\%$
(C) $28^{4}/_{7}\%$ (D) $33^{1}/_{3}\%$

7. If 15 residents do not own any of the three vehicles, then how many residents are there in the colony?
(A) 100 (B) 200
(C) 300 (D) 400

8. What percentage of the colony residents own exactly one type of vehicle?
(A) 15% (B) 25%
(C) 55% (D) 75%

9. What percentage of the residents own a scooter or a car but not a bicycle?
(A) 65% (B) 55%
(C) 75% (D) 45%

Directions for questions 10 to 12: These questions are based on the following data.

There are three trade unions – Viram, Vishram and Be-kam – and three thousand six hundred workers in a company. Becoming a member of a trade union is optional. A worker can be a member of more than one of the three trade unions also.

There are 500 workers who are members of at least two trade unions while Vishram has 1400 members. There are 100 workers who are members of only Viram and Be-kam, whereas 200 Vishram members also are Be-kam members; 550 workers are members of only Be-kam, whereas 20% of Viram members are members of exactly one more union. An eighth of all the workers in the company are members of exactly two unions.

10. How many workers are members of only Viram or only Be-kam?
(A) 3200 (B) 2700
(C) 1400 (D) 1700

11. If 10 workers give up their Be-kam membership and take up Vishram membership, then how many workers will now have membership of all the three unions?
(A) 40 (B) 50
(C) 60 (D) 45

12. How many workers are members of Vishram but not members of Be-kam?
(A) 400 (B) 800
(C) 1200 (D) 1600

Directions for questions 13 to 15: These questions are based on the following data.

In a school, 60% of the students passed in English, and 25% of the students who passed in English passed in the foreign language also, whereas $66^{2}/_{3}\%$ of the students who passed

in the foreign language failed in English. Twenty students failed in both English and the foreign language.

13. What percent of the students passed in exactly one of the two subjects – English and the foreign language?
(A) 15% (B) 65%
(C) 45% (D) 75%

14. The students who failed in exactly one subject are allowed to take a re-exam and it was found that the number of students who passed in both the subjects increased by 20%. What is the least value for the percentage of students in the school who pass only in English?
(A) 42% (B) 46%
(C) 34% (D) 28%

15. All the students, who failed in one or more subjects, are given grace marks and it was found that the number of students passing in exactly one subject went up by 4 and the number of students who failed in both the subjects dropped by 40%. What percent of the school now pass in both subjects?
(A) 40% (B) 15%
(C) 12% (D) 17%

Directions for questions 16 to 35: Select the correct alternative from the given choices.

16. The angle between the two hands of a clock at 9:00 a.m. is 90°. What will the angle between them be one minute later?
(A) 84.5° (B) 95.5°
(C) 101° (D) 79°

17. The angle between the two hands of a clock at 5:00 p.m. is 150°, what will the angle between them be one minute later?
(A) 144.5° (B) 155.5°
(C) 161° (D) 139°

18. What is the angle between the two hands of a clock at 7:28 p.m.?
(A) 56° (B) 58°
(C) 60° (D) 63°

19. At what time between 6 and 7 O' clock will the angle between the two hands of a clock be 50°?
(A) 6 hr.$41^{9}/_{11}$ min (B) 6 hr. $23^{7}/_{11}$ min
(C) 6 hr.$^{2}/_{11}$ min (D) Either (A) or (B)

20. At which of the following times between 8 and 9 O' clock, will the angle between the hands of the clock be 120°?
(A) 8 hr.$21^{9}/_{11}$ min (B) 8 hr. $16^{9}/_{11}$ min
(C) 8 hr. $34^{6}/_{11}$ min (D) 8 hr. $29^{3}/_{11}$ min

21. A watch, which gains uniformly was observed to be 10 minutes slow at 12 noon and 5 minutes fast at 6:00 p.m. on the same day. When did the watch show the correct time?
(A) 2:00 p.m. (B) 3:00 p.m.
(C) 4:00 p.m. (D) 5:00 p.m.

22. A watch, which loses uniformly was observed to be 5 minutes fast at 5:00 p.m. and 4 minutes slow at 8:00 p.m. on the same day. When did the watch show the correct time?
(A) 6:20 p.m. (B) 6:40 p.m.
(C) 7:00 p.m. (D) 6:30 p.m.

23. Two clocks show the correct time at 3:00 p.m. One clock gains 4 minutes in an hour, while the other loses 4 minutes. At 6:00 p.m. on the same day, by how much time will the two clocks differ?
(A) 8 minutes (B) 16 minutes
(C) 12 minutes (D) 24 minutes

24. The minutes hand of a clock overtakes the hours hand after every 60 minutes of correct time. How much time does the clock gain or lose in eleven hours of normal time?
(A) Gains 11 minutes (B) Loses 11 minutes
(C) Gains 60 minutes (D) Loses 60 minutes

25. If the time in a clock is 8 hours 20 minutes, then what time does its mirror image show?
(A) 3 hours 20 minutes (B) 3 hours 50 minutes
(C) 3 hours 40 minutes (D) 4 hours 40 minutes

26. If 22nd April, 1982 was a Thursday, then what day of the week was 3rd November, 1982?
(A) Monday (B) Wednesday
(C) Friday (D) Sunday

27. If 30th June, 1989 was a Friday, then what day of the week was 17th September, 1993?
(A) Monday (B) Wednesday
(C) Friday (D) Sunday

28. If 10th April, 1963 was a Wednesday, then what day of the week was 23rd August, 1959?
(A) Sunday (B) Monday
(C) Friday (D) Tuesday

29. If today is Sunday, then what day of the week will be the 426th day from today?
(A) Saturday (B) Friday
(C) Tuesday (D) Wednesday

30. If today is Wednesday, what day will it be, 1 year and 10 days from today?
(A) Sunday
(B) Friday
(C) Monday
(D) Cannot be determined

31. In a year, if two successive months start with the same day of the week, then the year is
(A) a century year.
(B) a leap year.
(C) a non-leap year.
(D) only a non-leap century year.

32. If a year has 53 Sundays, how many Saturdays will be there in that year?

(A) 52 (B) 53
(C) 54 (D) Cannot be determined

33. Three days ago I met my friend and asked him to lend me his maths book. He promised that he will lend it on the eighth day from that day. If today is Thursday, on which day will he lend me the book?
(A) Friday (B) Tuesday
(C) Monday (D) Sunday

34. There are three more days to go for my cousin to arrive. Eight days ago when I talked to him he said he would write his last exam three days later and that he would leave three days after his last exam. If his last exam is on a Saturday, on which day of the week will my cousin arrive?
(A) Saturday (B) Friday
(C) Tuesday (D) Sunday

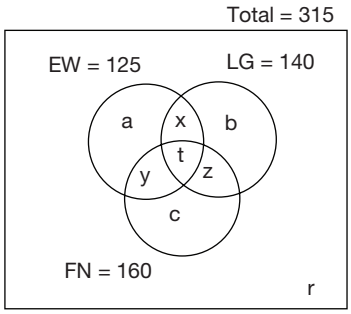
35. If the second Sunday of a month falls on the 8th, what is the date of the fourth day after the third Saturday of that month?
(A) 2 (B) 15
(C) 25 (D) 21

Answer Keys

1. D	**2.** D	**3.** A	**4.** C	**5.** B	**6.** D	**7.** C	**8.** D	**9.** A	**10.** D
11. B	**12.** C	**13.** D	**14.** A	**15.** D	**16.** B	**17.** A	**18.** A	**19.** D	**20.** A
21. C	**22.** B	**23.** D	**24.** C	**25.** C	**26.** B	**27.** C	**28.** A	**29.** A	**30.** D
31. C	**32.** D	**33.** B	**34.** D	**35.** C					

Hints and Explanations

Solutions for questions 1 to 4:



EW - Essel World
LG - Lumbini Gardens
FN - Film Nagar
25 visited all three $\Rightarrow t = 25$
125 visited $EW \Rightarrow a + x + y + t = 125$
$\Rightarrow$ $a + x + y = 100$ …. (1)
140 visited $LG \Rightarrow b + x + t + z = 140$
$\Rightarrow$ $b + x + z = 115$ …. (2)
160 visited $FN \Rightarrow c + y + z + t = 160$
$\Rightarrow$ $c + y + z = 135$ …. (3)
Number of students who visited exactly 2 places = 5 times those who did not visit any.
$\Rightarrow$ $x + y + z = 5r$
200 students visited exactly one place = $a + b + c = 200$.

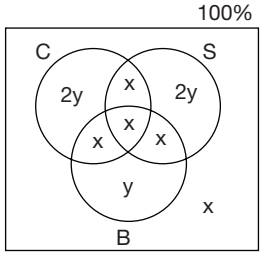
1. Total no of students = 315
$\Rightarrow$ $a + b + c + x + y + z + t + r = 315$
$\Rightarrow$ $200 + 5r + 25 + r = 315$
$6r = 90 \Rightarrow r = 15$
$\therefore$ 15 did not visit any of the three places. Choice (D)

2. Number of students who did not visit more than one place = Number of students who visited exactly one place + those who did not visit any = $a + b + c + r$
= 200 + 15 = 215. Choice (D)

3. Number of students who visited at least one of *LG* and *FN*
= 255 $\Rightarrow b + x + z + t + y + c = 255$ … (4)
We know $a + b + c + x + y + z + t$
$= 315 - r = 300$ …… (5)
(5) – (4) $\Rightarrow a = 45$ visited only *EW*. Choice (A)

4. Number of students who visited only one among *LG* and *FN* = $b + c = 200 - a = 200 - 45 = 155$.
Choice (C)

Solutions for questions 5 to 9:



The first three statements can be represented as shown in the diagram and hence, $5x + 5y = 100$
(because we have taken x & y as percentages)
or $x + y = 20$ …. (1)
From the fourth condition, we get $(y + 3x) = B$
i.e., $y = 3x$ …. (2)
From equations (1) and (2).
We get $x = 5\%$ and $y = 15\%$.

5. $y = 15\% = 150$.
Hence, total number % of residents
$= x = \frac{150 \times 100}{15} = 1000$. Choice (B)

6. $\frac{3x}{3x+2y} = \frac{15}{45} = \frac{1}{3} = 33^{1}/_{3}\%$. Choice (D)

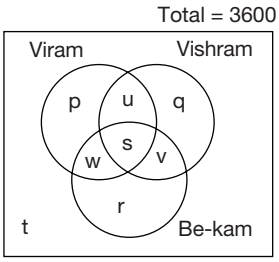
7. $x = 5\% = 15$. Hence total $= \frac{15}{0.05} = 300$. Choice (C)

8. $2y + 2y + y = 75\%$. Choice (D)

9. $2y + 2y + x = 65\%$. Choice (A)

Solutions for questions 10 to 12:

Represent in various segments as in the following diagram,



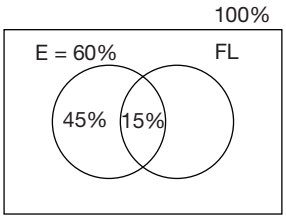
Members of at least two unions = $u + v + w + s = 500$ ….. (1)
Vishram members = $q + s + u + v = 1400$ …… (2)
Only Viram and Be-kam = $w = 100$ ……. (3)
Vishram and Be-kam = $s + v = 200$ ….. (4)
Only Be-kam = $r = 550$ …… (5)
Members of Viram who are members of only one more union = $w + u = 20\%$ of $(p + u + s + w)$.. (6)
$u + v + w = {}^{1}/_{8}$ (Total workers) = 450. ….. (7)
From (1), (3) and (4), $u = 200$.
From equations (2),
$q = 1400 - u - (s + v)$
$= 1400 - 200 - 200 = 1000$
From (7), $v = 450 - 200 - 100 = 150$
From (4), $s = 200 - 150 = 50$
From (6), $p = 1150$
$t = 3600 - (p + q + r + s + u + v + w)$
$= 3600 - (1150 + 1000 + 550 + 50 + 200 + 150 + 100)$
$= 3600 - (3200) = 400$.
Now, we have all the figures and the questions can be answered.

10. $p + r = 1150 + 550 = 1700$. Choice (D)

11. Since 10 workers have given up their Be-kam membership and taken Vishram membership, it means these 10 workers were initially Be-kam members but not Vishram members, i.e., they must be a part of r or w. When they give up Be-kam and take up Vishram, they will move to q or u respectively. So, s does not undergo any change at all. Hence, 50 is the answer. Choice (B)

12. $q + u = 1000 + 200 = 1200$. Choice (C)

Solutions for questions 13 to 15:



-- 25% of 60%, i.e., 15% of the school passed in both English and Foreign Language.
-- Since $66^{2}/_{3}\%$ of the students who passed in Foreign Language failed in English, $33^{1}/_{3}\%$ of students who passed in Foreign Language passed in English also, i.e., ${}^{1}/_{3}$ Foreign Language = 15%
$\Rightarrow$ Foreign Language = 45%
So, we have only passed in (English) = 60 – 15 = 45%,
Only Foreign Language pass = 45 – 15 = 30%
Passed both in English & Foreign Language = 15%.
A total of 90% passed in at least one of the subjects. So, 10% failed in both.
Number of students in the school $= \frac{20}{0.10} = 200$.

13. 45% + 30% = 75%. Choice (D)

14. 20% of 15% = 3% increase in pass in both subjects. Hence, least value of pass only in English will come when all the new people who pass in both subjects are from the group which passed only in English. i.e., 45 – 3 = 42%. Choice (A)

15. 40% of 20 students = 8 students. Out of this, 4 students passed in exactly one subject. Hence, the remaining 4 students (which is 2% of the school strength) pass in both subjects. So, pass in both the subjects = 15 + 2 = 17%. Choice (D)

Solutions for questions 16 to 35:

16. As the minute hand is 5.5°/minute faster than the hour hand and the minute hand is ahead of the hour hand, the angle after one minute, i.e., at 9:01 a.m., will be 90° + 5.5° = 95.5°. Choice (B)

17. The angle between the hands will change by 5.5°/min. In this case, since the minute hand is behind the hour hand the angle will decrease by 5.5°. Hence, the angle at 5:01 = 150 – 5.5 = 144.5°. Choice (A)

18. The angle $= \left|\frac{11}{2}(28) - 30(7)\right| = 56°$. Choice (A)

19. $\theta = \left|\frac{11}{2}(m) - 30(h)\right| \Rightarrow m = (30h \pm \theta)\left(\frac{2}{11}\right)$.
Here $h = 6$ and $\theta = 50°$
$\Rightarrow \quad m = (180 \pm 50°)\left(\frac{2}{11}\right) = \frac{460}{11}$ or $\frac{260}{11}$.
The two hands are 50° apart at $6\,\text{hr.}\ 41\frac{9}{11}$ min and at $6\ hr.\ 23\frac{7}{11}$ min. Choice (D)

20. $m = (30h \pm \theta)\left(\frac{2}{11}\right)$
Here $h = 8$ and $\theta = 120°$
$\Rightarrow \quad m = (240 \pm 120)\left(\frac{2}{11}\right) \Rightarrow 65\frac{5}{11}$ or $21\frac{9}{11}$

If $m = 65\frac{5}{11}$, then the time will be more than 9 O'clock.

∴ The time is 8 hr. $21\frac{9}{11}$ min Choice (A)

21. In 6 hours, the watch gained 15 minutes.
∴ In order to gain 10 minutes, (the initial difference) it takes 4 hours.
∴ At 4:00 p.m. it shows the correct time. Choice (C)

22. In 3 hours, the watch loses 9 minutes.
∴ In order to lose 5 minutes, (the initial difference) it takes 5 *x* 3/9 hours, i.e., 1 hour 40 minutes.
∴ At 6 : 40 p.m., it shows the correct time.
Choice (B)

23. In one hour, the clocks will differ by (4 + 4) minutes i.e., 8 minutes.
∴ In three hours they will differ by 24 minutes.
Choice (D)

24. As $60 < 65\frac{5}{11}$, the watch is gaining time.

The gain is $65\frac{5}{11} - 60 = 5\frac{5}{11}$ minutes per hour.

∴ In eleven hours, the watch will gain $11\left(\frac{60}{11}\right)$ = 60 minutes. Choice (C)

25. Actual time + mirror image time = 12 hours
∴ Mirror image time = 12 – 8 : 20 = 3 : 40
= 3 hours 40 minutes. Choice (C)

26. It is given that 22nd April, 1982 was a Thursday.
Number of days from 22nd April 1982 to 3rd November 1982. Month: April + May + June + July + August + September + October + November
In each month number of odd days :
1 + 3 + 2 + 3 + 3 + 2 + 3 + 3 = 6 odd days
6^{th} day after Thursday is Wednesday. Choice (B)

27. Number of odd days from 30th June, 1989 to 30th June, 1993 are five.
Number of days from 30th June, 1993 to 17th September, are
Month: July + August + September
Odd days : 3 + 3 + 3 = 9
Number of odd days = 9 + 5 =14 = 0 odd days
Hence, 17th September 1993 was a Friday. Choice (C)

28. Number of days from 10th April, 1963 to 23rd August, 1963
Month: April + May + June + July + August
Days: 20 + 31 + 30 + 31 + 23 = 135
Number of odd days in 135 days = 135/7 = 20 days
23rd August 1963 is 2 days to Wednesday i.e., Friday
Number of odd days form 23rd August 1963 to 23rd August 1959 are five odd days.
Hence, 23rd August 1959 is five days back to Friday is Sunday. Choice (A)

29. Number of odd days in 426 days
= 426/7 = 60 complete weeks + 6 odd days.
6th day after Sunday is a Saturday. Choice (A)

30. Whether the given year is a leap year or a non-leap year, is not given, hence the answer cannot be determined. Choice (D)

31. Two successive months can start with the same day of the week, only if the previous month does not have any odd days. This is possible only when February has 28 days. Hence, the year is a non-leap year. Choice (C)

32. A leap year has 52 weeks and two additional days, while a non-leap year has 52 weeks and one additional day. In any year each day of the week occurs at least 52 times.
A non-leap year starts and ends with the same day of the week. In a leap year the first two days of the year repeat. As it is not known whether the 53rd Sunday is the last day of the year or not, the number of Saturdays could be 52 or 53. Choice (D)

33. Today is a Thursday. Hence, the required day
= Thursday – 3 (ago) + 8 (from 3days ago)
= Thursday + 5 = Tuesday. Choice (B)

34. Eight days ago my cousin said that his last exam is three days later i.e. five days ago from today. He is going to arrive three days later from today. i.e. eight days after the last exam. As the last exam is on a Saturday, he is arriving on a Sunday. Choice (D)

35. Since, 8th is the second Sunday, 7th is the first Saturday. Hence, the third Saturday is on 21st and four days later it is 25th. Choice (C)

PART II
ENGINEERING MATHEMATICS

Engineering Mathematics Test 1 ..... 2.3
Engineering Mathematics Test 2 ..... 2.9
(Ordinary Differential Equations, Calculus (Vector Calculus)) 2.9
Engineering Mathematics Test 3 ..... 2.16
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(Probability And Statistics) 2.23
Engineering Mathematics Test 5 ..... 2.29
Numerical Methods 2.29

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ENGINEERING MATHEMATICS TEST I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. $\underset{x\to 0}{Lt}\dfrac{5^x-1}{\sqrt{x^2-x+1}-1}=$

(A) $\log_e 5$ (B) $2\log_e 5$

(C) $-2\log_e 5$ (D) 1

2. If $f(x)=x^2+5x-13$, if $x<1$,

$x-8$ if $x\geq 1$, then $\underset{x\to 0}{Lt} f(x)$ =

(A) –8 (B) –7

(C) 7 (D) does not exist

3. $f(x)=\dfrac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}$. for $x\neq 0$ is

$=0$ for $x=0$

(A) continuous at $x=0$.

(B) not continuous at $x=0$.

(C) continuous everywhere.

(D) none of these.

4. If $f(x)=a[x-5]+b[x+5]$ is continuous at $x=5$, then the value of $a+b$ ($[x]$ is the greatest integer less than or equal to x) is

(A) 0 (B) 1

(C) 5 (D) 2

5. If $f(x)=\begin{cases}\dfrac{\sqrt{x}-3}{x-9}; \text{for } x\neq 9\\ 3k; \text{ for } x=9\end{cases}$ is continuous everywhere,

then k is equal to ———.

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$

(C) $\dfrac{1}{9}$ (D) $\dfrac{1}{18}$

6. If $y=ax^2+3\log x+bx$ has its extreme value at $x=1$ and $x=-1$, then the value of $2a-b$ is

(A) 3 (B) 0

(C) –1 (D) –3

7. If $xy=6$, then find the minimum value of $2x+3y$, $x, y\in R^+$.

(A) 12 (B) 15

(C) 9 (D) 6

8. Which of the following functions have neither maximum nor minimum?

(A) $\dfrac{1}{2x}$ (B) $4x+7$

(C) e^{5x+1} (D) All the above

9. Find the maximum value of the function $f(x)=3x^4-2x^3-6x^2+6x+1$ in the interval $[-1, 2]$.

(A) $\dfrac{39}{16}$ (B) 2

(C) 21 (D) 18

10. Let $f(x)$ and $g(x)$ be two continuous functions in $[a, b]$ and differentiable in (a, b) and $g^1(x)\neq 0$ for any $x\in(a, b)$ then there exists at least one value $c\in(a, b)$, by Cauchy mean value theorem which of the following is true?

(A) $\dfrac{f^2(c)}{g^1(c)}=\dfrac{f(b)-g(a)}{g(b)+f(a)}$

(B) $\dfrac{g^1(c)}{f^1(c)}=\dfrac{f(b)+g(b)}{f(a)+f(b)}$

(C) $\dfrac{f^1(c)}{g^1(c)}=\dfrac{f^1(b)-f^1(a)}{g^1(b)-g^1(a)}$

(D) $\dfrac{f'(c)}{g'(c)}=\dfrac{f(b)-f(a)}{g(b)-g(a)}$

11. If the function $f(x)=(\sin 2x)\,e^{-2x}$ satisfies Rolle's theorem in the interval $\left(0,\dfrac{\pi}{2}\right)$, then the value of $c\in\left(0,\dfrac{\pi}{2}\right)$ such that $f'(c)=0$ is

(A) $\dfrac{\pi}{8}$ (B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{6}$ (D) $\dfrac{\pi}{3}$

12. By using Lagrange's mean value theorem for $f(x)=x(x+2)(x-1)$, find c for $c\in(-1, 0)$.

(A) $\dfrac{-1}{3}$ (B) $\dfrac{-3}{4}$

(C) $\dfrac{-2}{3}$ (D) $\dfrac{-4}{5}$

13. If $u=\tan^{-1}\left(\dfrac{x^3-y^3}{x+y}\right)$, then the value of $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$ is

(A) $2\sin u$ (B) $2\tan u$

(C) $\sin 2u$ (D) $2\cos 2u$

14. If $f(x, y)$ is a homogeneous function of degree n, then the value of

$y\dfrac{\partial^2 f}{\partial y^2}+x\dfrac{\partial^2 f}{\partial x\partial y}=$

(A) $n.\frac{\partial f}{\partial x}$ (B) $(n-1)\frac{\partial f}{\partial y}$

(C) $(n-1)\frac{\partial f}{\partial x}$ (D) $n.\frac{\partial f}{\partial y}$

15. If $u = \cos^{-1}\left(\frac{x^3+2y^3-4z^3}{x^7-y^7-z^7}\right)$, then the value of $\sum_{x,y,z} x\frac{\partial u}{\partial x}$ is

(A) $4\cot u$ (B) $-4\tan u$

(C) $-4\cos u$ (D) $-4\sin u$

16. If $z = \ln(x^2+y^3)$, then

(A) $3x\frac{\partial z}{\partial x} = 2y\frac{\partial z}{\partial y}$

(B) $3x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y} = 6$

(C) $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$

(D) $2y\frac{\partial z}{\partial x} + 3x\frac{\partial z}{\partial y} = 6$

17. The value of the definite integral $\int_0^1 (x^2-2x+3)e^{-x}dx$ is ———.

(A) $3+4e$ (B) $3-4e$

(C) $3+\frac{4}{e}$ (D) $3-\frac{4}{e}$

18. $\int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\operatorname{cosec} x}{\sec x + \operatorname{cosec} x}dx =$

(A) $\frac{7\pi}{4}$ (B) $\frac{7}{2}$

(C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{2}$

19. $\int_0^{\infty} \frac{x^2}{(1+x^2)^{\frac{5}{2}}}dx =$

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

20. Evaluate $\int_1^4 \frac{dx}{(x-2)(x-3)}$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) diverges

21. $\int_0^{\infty} \frac{dx}{9+x^2} =$

(A) $\frac{\pi}{2}$ (B) —

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

22. The area bounded by the cure $y^2 = x+2$ and $y = x-4$ (in sq units) is

(A) $20\frac{5}{6}$ (B) $15\frac{5}{6}$

(C) $12\frac{2}{3}$ (D) $18\frac{3}{4}$

23. The area bounded by the curves $y^2 = 9ax$ and $x^2 = 9ay$ is

(A) $54a^2$

(B) $48a^2$

(C) $36a^2$

(D) $27a^2$

24. The volume of the solid formed by the revolution of the area A about y – axis is

(A) $\iint_A 2\pi dxdy$ (B) $\iint_A 2\pi y dxdy$

(C) $\iint_A \pi x^2 dxdy$ (D) $\iint_A \pi y^2 dxdy$

25. The taylor's series expansion of $2e^x + 3\sin x$ about $x = 0$ is

(A) $2+5x+x^2+\frac{x^3}{6}+\frac{x^4}{12}+\frac{x^5}{24}+\ldots\ldots$

(B) $2+5x+x^2-\frac{x^3}{6}+\frac{x^4}{12}+\frac{x^5}{24}+\ldots\ldots\ldots$

(C) $2+5x+x^2+\frac{x^3}{6}-\frac{x^4}{12}+\frac{x^5}{24}+\ldots\ldots\ldots$

(D) $2+5x+x^2+\frac{x^3}{+}+\frac{x^4}{12}+\frac{x^5}{24}+\ldots\ldots\ldots\ldots$

Answer Keys

1. C	**2.** B	**3.** B	**4.** A	**5.** D	**6.** D	**7.** A	**8.** D	**9.** C	**10.** D
11. A	**12.** C	**13.** C	**14.** B	**15.** A	**16.** B	**17.** D	**18.** A	**19.** B	**20.** D
21. C	**22.** A	**23.** D	**24.** A	**25.** B					

Hints and Explanations

1. $\underset{x\to 0}{Lt}\dfrac{5^x-1}{\sqrt{x^2-x+1}-1}\times\dfrac{\sqrt{x^2-x+1}+1}{\sqrt{x^2-x+1}+1}$

$=\underset{x\to 0}{Lt}\dfrac{(5^x-1)\left(\sqrt{x^2-x+1}+1\right)}{x^2-x+1-1}$

$=\underset{x\to 0}{Lt}\dfrac{(5^x-1)\sqrt{x^2-x+1}+1}{x(x-1)}$

$=\underset{x\to 0}{Lt}\dfrac{5^x-1}{x}\underset{x\to 0}{Lt}\dfrac{\sqrt{x^2-x+1}+1}{x-1}.$

$=\log_e 5\times\dfrac{2}{-1}=2\log_e 5$ Choice (C)

2. we know that $\underset{x\to 1}{Lt}\ f(x)=\underset{x\to 1^-}{Lt}\ f(x)=\underset{x\to 1^+}{Lt}\ f(x)$

$\underset{x\to 1^-}{Lt}\ f(x)=\underset{x\to 1^-}{Lt}\ (x^2+5x-13)=-7$

$\underset{x\to 1^+}{Lt}\ f(x)=\underset{x\to 1^+}{Lt}\ (x-8)=-7$

$\therefore\quad \underset{x\to 1^-}{Lt}\ f(x)=\underset{x\to 1^+}{Lt}\ f(x)=-7$

$\Rightarrow\quad \underset{x\to 1}{Lt}\ f(x)=-7.$ Choice (B)

3. $\underset{x\to 0^-}{Lt}\dfrac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}=\dfrac{0}{1-0}=0$ ---------- (1)

$\underset{x\to 0^+}{Lt}\dfrac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}=\underset{x\to 0^+}{Lt}\dfrac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}\left(e^{\frac{-1}{x}}-1\right)}$

$=\underset{x\to 0^+}{Lt}\dfrac{1}{e^{\frac{-1}{x}}-1}=\dfrac{1}{0-1}=-1$ ---------- (2)

From (1) and (2) $\underset{x\to 0^-}{Lt}\ f(x)\neq\underset{x\to 0^+}{Lt}\ f(x)$

$\therefore$ $f(x)$ is not continuous at $x=0$. Choice (B)

4. $f(x)=a[x-5]+b[x+5]$

$\therefore\quad f(x)=a([x]-5)+b([x]+5)$

$\underset{x\to 5^-}{Lt}\ f(x)=\underset{x\to 5^{-1}}{Lt}\ \{a([x]-5)+b([x]+5)$

$=-a+9b$ ------ (1)

$\underset{x\to 5^+}{Lt}\ f(x)=\underset{x\to 5^+}{Lt}\ \{a([x]-5)+b([x]+5)\}$

$=10\,b$ ---- (2)

As $f(x)$ is continuous at $x=5$

$\Rightarrow\quad \underset{x\to 5^-}{Lt}\ f(x)=\underset{x\to 5^+}{Lt}\ f(x)$

$\Rightarrow\quad -a+9b=10b$

$\Rightarrow\quad a+b=0.$ Choice (A)

5. Given $f(x)=\begin{cases}\dfrac{\sqrt{x}-3}{x-9}, & \text{for } x\neq 9\\ 3k, & \text{for } x=9\end{cases}$

As $f(x)$ is continuous everywhere

$f(x)$ is continuous at $x=9$

$\therefore\quad \underset{x\to 9}{Lt}\ f(x)=f(9)$

$\Rightarrow\quad \underset{x\to 9}{Lt}\left(\dfrac{\sqrt{x}-3}{x-9}\right)=3k$

$\Rightarrow\quad \underset{x\to 9}{Lt}\left(\dfrac{\frac{1}{2\sqrt{x}}}{1}\right)=3k$ (By L'Hospital's Rule)

$\Rightarrow\quad \dfrac{1}{2\sqrt{9}}=3k$

$\Rightarrow\quad \dfrac{1}{6}=3k$

$\Rightarrow\quad k=\dfrac{1}{18}.$ Choice (D)

6. Let $f(x)=ax^2+3\log x+bx$

$f^1(x)=2ax+\dfrac{3}{x}+b$

Given at $x=-1$ and 1 $f(x)$ has extreme values

$\Rightarrow\quad f^1(-1)=0$ and $f^1(1)=0$

$f^1(-1)=2a(-1)+\dfrac{3}{-1}+b=0$

$\Rightarrow\quad -2a+b=3$

$\Rightarrow\quad 2a-b=-3.$ Choice (D)

7. Given $xy=6\Rightarrow y=\dfrac{6}{x}$

$2x+3y=2x+\dfrac{3\times 6}{x}$

Let $f(x)=2x+\dfrac{18}{x}$

$f^1(x)=2-\dfrac{18}{x^2}$

$\Rightarrow\quad f^{11}(x)=\dfrac{36}{x^3}$ which is always positive as x, y and R^+

For maximum or minimum $f^1(x)=0$

$=2-\dfrac{18}{x^2}=0$

$\Rightarrow\quad x=3$

$\therefore\quad y=\dfrac{6}{x}$

$\Rightarrow\quad y=2$

$\therefore$ At $x=3$ and $y=2$, $2x+3y$ has minimum value

The minimum value is $2(3)+3(2)=12$. Choice (A)

8. Choice A: Let $f(x) = \frac{1}{2x} \Rightarrow f^1(x) \neq 0$ for any real value of x

Choice B: $f_1(x) = 4x + 7 \Rightarrow f_1^1(x) \neq 0$ for any real value of x

Choice C: $f_2(x) = e^{5x+1} \Rightarrow f_2^1(x) \neq 0$ for any real value of x

$\therefore$ None of the options have either maximum or minimum values. Choice (D)

9. Let $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$

$f^1(x) = 12x^3 - 6x^2 - 12x + 6$

$f^1(x) = 0 \Rightarrow 2x^3 - x^2 - 2x + 1 = 0$

$(x - 1)(x + 1)(2x - 1) = 0$

$\Rightarrow x = \pm 1, \frac{1}{2}$

Max of $f(x)$ in $[-1, 2]$ = Max $\{f(-1), f\left(\frac{1}{2}\right), f(1), f(2)\}$

= Max $\left\{-6, \frac{39}{16}2, 21\right\} = 21$. Choice (C)

10. Standard result of Cauchy mean value theorem. Choice (D)

11. Given $f(x) = e^{-2x} \sin 2x$

Clearly $f(x)$ is continuous on $\left[0, \frac{\pi}{2}\right]$ and differentiable in $\left(0, \frac{\pi}{2}\right)$ and $f(0) = f\left(\frac{\pi}{2}\right)$ then there exists $c \in \left(0, \frac{\pi}{2}\right)$ such that $f^1(c) = 0$

$f^1(x) = \sin 2x\,(-2e^{-2x}) + 2\cos 2x e^{-2x}$

$f^1(c) = 2e^{-2c}(\cos 2c - \sin 2c) = 0$

$\Rightarrow \sin 2c = \cos 2c$ or $\tan 2c = 1$

$\Rightarrow c = \frac{\pi}{8} \in \left(0, \frac{\pi}{2}\right)$

The required value of $c = \frac{\pi}{8}$. Choice (A)

12. Given $f(x) = x(x + 2)(x - 1)$

Clearly $f(x)$ is continuous on $[-1, 0]$ and differentiable on $(-1, 0)$

$a = -1, b = 0$;

$f(b) = 0, f(a) = (-1)(1)(-2) = 2$

$f^1(x) = 3x^2 + 2x - 2$ and $f^1(c) = 3c^2 + 2c - 2$

By Lagrange's mean value theorem

$f^1(c) = \frac{f(b) - f(b)}{b - a}$

$\Rightarrow 3c^2 + 2c - 2 = \frac{0 - 2}{1}$

$\Rightarrow 3c^2 + 2c = 0$

$c(3c + 2) = 0 \Rightarrow c = 0$ or $c = \frac{-2}{3}$ As $c \neq 0$,

$c = \frac{-2}{3} \in (-1, 0)$. Choice (C)

13. $U = \tan^{-1}\left(\frac{x^3 - y^3}{x + y}\right) \Rightarrow \tan u = \frac{x^3 - y^3}{x + y}$

Let $x = kx$ and $y = ky$ then

$\tan u = \frac{k^3(x^3 - y^3)}{k(x + y)} = k^2\left(\frac{x^3 - y^3}{x + y}\right) = f(x)$ say

$\therefore$ f is a homogeneous function of degree 2

$\therefore$ By Euler's theorem $x\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = = n(f)$ (n is degree of f)

$= x.\frac{\partial(\tan u)}{\partial x} + y\frac{\partial(\tan u)}{\partial y} = 2\tan u$

$= x\sec^2 u\,.\,\frac{\partial u}{\partial x} + y\,\sec^2 u\,\frac{\partial u}{\partial y}\,\frac{\partial u}{\partial y} = 2\tan u$

$\therefore$ $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. Choice (C)

14. Given $f(x, y)$ is a homogeneous function of degree n then by Euler's theorem we know that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$

$= nf$ ------- (1)

Differentiating (1) w. r. to y partially we have

$x.\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y} + y.\frac{\partial^2 f}{\partial y^2} = n.\frac{\partial f}{\partial y}$

$\Rightarrow y\frac{\partial^2 f}{\partial y^2} + x\frac{\partial^2 f}{\partial x \partial y} = (n - 1)\frac{\partial f}{\partial y}$ Choice (B)

15. $U = \cos^{-1}\left(\frac{x^3 + 2y^3 - 4z^3}{x^7 - y^7 - z^7}\right)$

$\Rightarrow \cos u = \frac{x^3 + 2y^3 - 4z^3}{x^7 - y^7 - z^7} = f(\text{say})$

Clearly f is a homogeneous function of degree -4

$\therefore$ By Euler's theorem $x.\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = n.f$

$\therefore$ $x\frac{\partial(\cos u)}{\partial x} + y\frac{\partial(\cos u)}{\partial y} + z\frac{\partial(\cos u)}{\partial z}s = -4.\cos u$

$= -\left(x.\frac{\partial u}{\partial x}.(\sin u) + y.\frac{\partial u}{\partial y}(\sin u) + z\frac{\partial u}{\partial z}(\sin u)\right)$

$= -4\cos u$

$\therefore$ $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 4\cot u$. Choice (A)

16. Given $z = \text{in}\,(x^2 + y^3)$

$$\Rightarrow \quad \frac{\partial z}{\partial x} = \frac{2x}{x^2+y^3} \text{ and } \frac{\partial z}{\partial y} = \frac{3y^2}{x^2+y^3}$$

Consider $3x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y} = \frac{6x^2}{x^2+y^3} + \frac{6y^3}{x^2+y^3}$

$$= \frac{6\left(x^2+y^3\right)}{x^2+y^3} = 6. \qquad \text{Choice (B)}$$

17. We have $\int_0^1 (x^2 - 2x + 3)e^{-x}dx$

$$= (x^2-2x+3)(-e^{-x}) - (2x-2)(e^{-x}) + 2(-e^{-x})_0^1$$
$$= (-2e^{-1} - 0 - 2e^{-1}) - (-3 + 2 - 2)$$
$$= 3 - 4e^{-1} = 3 - \frac{4}{e} \qquad \text{Choice (D)}$$

18. We know that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Let $I = \int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\,\text{cosec}\,x}{\sec x + \text{c}\,\text{osec}\,x}$

$$= \int_0^{\frac{\pi}{2}} \frac{3\sec\left(\frac{\pi}{2}-x\right) + 4\,\text{c}\,\text{osec}\left(\frac{\pi}{2}-x\right)}{\sec\left(\frac{\pi}{2}-x\right) + \text{cosec}\left(\frac{\pi}{2}-x\right)}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{3\text{cosec}\,x + 4\sec x}{\text{cosec}\,x + \sec x}dx$$

$I + I$

$$= \int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\,\text{cosec}\,x}{\text{cosec}\,x + \text{cosec}\,x}dx + \int_0^{\frac{\pi}{2}} \frac{3\text{cosec}\,x + 4\sec x}{\sec x + \text{cosec}\,x}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{3(\text{cosec}x + \sec x) + 4(\sec x + \text{cosec}x)}{\sec x + \text{cosec}x}dx$$

$$= \int_0^{\frac{\pi}{2}} 7dx = 7(x)_0^{\frac{\pi}{2}} = 7\frac{\pi}{2}$$

$$\therefore \quad 2I = 7\frac{\pi}{2}$$

$$\Rightarrow \quad I = 7\frac{\pi}{4} \qquad \text{Choice (A)}$$

19. $\int_0^{\infty} \frac{x^2}{(1+x^2)^{\frac{5}{2}}}dx$ Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$ *L.L*

When $x = 0$ $\theta = 0$ and *U.L* when $x = \infty$, $\theta = \frac{\pi}{2}$

$$\therefore \quad \int_0^{\frac{\pi}{2}} \frac{\tan^2\theta.\sec^2\theta}{(1+\tan^2\theta)^{\frac{5}{2}}}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^2\theta.\sec^2\theta}{\cos^2 0.\sec^5\theta}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2\theta.\cos\theta d\theta = \frac{\sin^3\theta}{3}\Big]_0^{\frac{\pi}{2}} = \frac{1}{3} \qquad \text{Choice (B)}$$

20. $\int_1^4 \frac{dx}{(x-2)(x-3)}$

$$= \int_1^2 \frac{dx}{(x-2)(x-3)} + \int_2^3 \frac{dx}{(x-2)(x-3)} + \int_3^4 \frac{dx}{(x-2)(x-3)}$$

$I = I_1 + I_2 + I_3$ (say)

Clearly as I has discontinuity at $x = 2$, I_1 diverges

As I_1 diverges I also diverges

$$\therefore \quad \int_1^4 \frac{dx}{(x-2)(x-3)} \text{ diverges.} \qquad \text{Choice (D)}$$

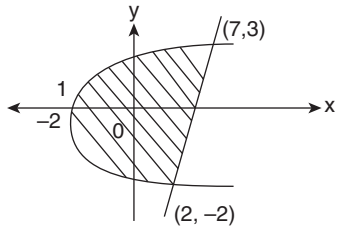
21. $\int_0^{\infty} \frac{dx}{9+x^2} = \underset{a\to\infty}{Lt} \int_0^a \frac{dx}{3^2+x^2} = \underset{a\to\infty}{Lt} \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^a$

$$\underset{a\to\infty}{Lt} \frac{1}{3}\tan^{-1}\left(\frac{a}{3}\right) = \frac{\pi}{6}$$

The given improper integral converges to $\frac{\pi}{6}$

Choice (C)

22.



$y^2 = x + 2$ and $y = x - 4$

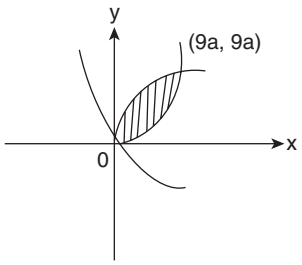
The above two intersect at (2, –2) and (7, 3).

The area bounded $= \int_{-2}^{+3} \left[(y+4) - (y^2-2)\right]dy$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{3} + 2y\right]_{-2}^{3}$$

$$= \frac{125}{6} = 20\frac{5}{6} \text{ sq units.} \qquad \text{Choice (A)}$$

23.



Given $y^2 = 9ax$ and $x^2 = 9ay$

The above two curves intersect at (0, 0) and $(9a, 9a)$.

The required area $= \int_0^{9a} \left(\sqrt{9ax} - \frac{x^2}{9a} \right) dx$

$$= 3\sqrt{a} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{9a} - \frac{1}{9a} \left(\frac{x^3}{3} \right)_0^{9a}$$

$$= 2\sqrt{a} \left((9a)^{\frac{3}{2}} \right) - \frac{1}{27a} (9a)^3$$

$$= 54a^2 - 27a^2 = 27a^2.$$ Choice (D)

24. Standard Result. Choice (A)

25. We have $2e^x + 3\sin x$

$$= 2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} ..... \right) + 3\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} ..... \right)$$

$$= \left(2 + 2x + 2\frac{x^2}{2!} + 2.\frac{x^3}{3!} + 2\frac{x^4}{4!} + 2\frac{x^5}{5!} + ... \right)$$

$$+ \left(3x - 3\frac{x^3}{3!} + 3.\frac{x^5}{5!} - 3\frac{x^7}{7!} + ... \right)$$

$$= 2 + 5x + x^2 - \frac{x^3}{3!} + \frac{x^4}{12} + \frac{x^5}{24} + ...$$ Choice (B)

ENGINEERING MATHEMATICS TEST 2

Number of Questions: 25 **Time: 60 min.**

(ORDINARY DIFFERENTIAL EQUATIONS, CALCULUS (VECTOR CALCULUS))

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The directional derivative of $f(x, y) = x^3y^2 + 3xy$ at (1, 2) in the direction of unit vector which makes an angle of $\frac{\pi}{4}$ with the x-axis is _____.

(A) $16\sqrt{2}$ (B) $8\sqrt{2}$

(C) $\frac{15}{\sqrt{2}}$ (D) $\frac{25}{\sqrt{2}}$

2. If the vector $e^{px-y-z}\left(\bar{i}+\bar{j}+\bar{k}\right)$ is solenoidal, then the value of p is _____.

(A) –3 (B) 3

(C) –2 (D) 2

3. Evaluate $\int_c \left(x^2+3yz\right)ds$ where c is the curve defined by $x = 6y$; $z = 4$ from $\left(3, \frac{1}{2}, 4\right)$ to (6, 1, 4).

(A) $15\sqrt{37}$ (B) $2\sqrt{37}$

(C) 199 (D) $37\sqrt{33}$

4. The unit normal vector to the surface $xy^3 + 3yz = 3$ at the point (3, –1, –2) is _____

(A) $\frac{-\bar{i}-3\bar{j}-3\bar{k}}{\sqrt{19}}$ (B) $\frac{-\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{19}}$

(C) $\frac{\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{19}}$ (D) $\frac{\bar{i}-3\bar{j}-3\bar{k}}{\sqrt{19}}$

5. Evaluate the integral $\int_c \left(yz+z+z^2\right)dx + (xz - 1)\,dy + (xy + x + 2xz)\,dz$ from (2, 3, 3) to (3, 4, 5).

(A) 146 (B) 107

(C) 39 (D) 185

6. Evaluate the integral $\oint_c \left(x+2y\right)dx + x^2y\,dy$, C is the triangle with vertices at (0, 0), (3, 0) and (3, 3) taken in that order.

(A) 18 (B) $\frac{45}{4}$

(C) $\frac{37}{3}$ (D) 15

7. Evaluate the surface integral $\iint_S F.\bar{n}\,dA$, where $\bar{F} = z^2\,\bar{i} + xy\,\bar{j} + y^2\,\bar{k}$ and S is the portion of the surface of the cylinder $x^2 + y^2 = 49$; $0 \le z \le 5$ included in first octant.

(A) 518 (B) 2590

(C) $\frac{2590}{3}$ (D) 624

8. If $\bar{V} = \left(2x-y\right) - yz^2\,j - y^2z\,\bar{k}$, then evaluate $\oint_c \bar{V}.d\bar{r}$ where C is the curve bounding the projection on the semi sphere $x^2 + y^2 + z^2 = 9$, $z > 0$; in the xy plane.

(A) 18 π (B) 6 π

(C) 8 π (D) 9 π

9. If $\bar{r} = 2x\bar{i} + 3y\,\bar{j} + z\bar{k}$ and V is the volume of the sphere $x^2 + y^2 + z^2 = 16$ then $\iint_S \bar{r}.\bar{n}\,dA =$ _____.

(A) 38 π (B) 512 π

(C) 438 π (D) 348 π

10. The work done by the force $\bar{F} = 2xy\bar{i} + y^2\,\bar{j} + z\bar{k}$ in moving a particle over the circular path $x^2 + y^2 = 9$; $z = 0$ from (3, 0, 0) to (0, 3, 0) is _____.

(A) 17 (B) –12

(C) –9 (D) 6

11. The solution of $3(xdy + ydx) = 2xydy$ when $x = 1$, $y = 1$ is

(A) $3 \log (xy) - 2y + 2 = 0$

(B) $\log(xy) + y - 2 = 0$

(C) $\log x + \log y + 2 = 0$

(D) $\log(xy) - 5y + 2 = 0$

12. Solve $dy = (9x + y - 1)^2\,dx$ when $x = 0$, $y = 1$.

(A) $3 \tan x = 3x + y - 1$

(B) $3 \tan (3x) = 9x + y - 1$

(C) $\tan x = 9x + y - 1$

(D) $3 \tan 3x = 9x - y + 1$

13. The solution of the differential equation $\frac{x\,dy - ydx}{x} = \cos^2\left(\frac{y}{x}\right)dx$ is

(A) $\tan\left(\frac{y}{x}\right) = \log (cx)$ (B) $\tan\left(\frac{y}{x}\right) = x + c$

(C) $\tan^{-1}\left(\frac{y}{x}\right) = \log(cx)$ (D) $\tan\left(\frac{x}{y}\right) = \log x + c$

14. The integrating factor of $\frac{dy}{dx} - y\tan x - \cos x = 0$ is

(A) $\cos x$ (B) $\sin x$
(C) $\sec x$ (D) $\operatorname{cosec} x$

15. Solve the differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ with the boundary conditions $x = 1, y = 1$.

(A) $x^2 + y^2 = 3$ (B) $x^2 - y^2 = 0$
(C) $x^2 - y^2 = 2$ (D) $x^2 + y^2 = 2$

16. The solution of the differential equation $ydx = (x + 3y^3)dy$ when $x = 1, y = 1$ is

(A) $x = 3y^2 - 1$ (B) $3x = y(2y^2 - 1)$
(C) $2x = y(3y^2 + 1)$ (D) $2x = y(3y^2 - 1)$

17. The particular integral solution of the differential equation

$$\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 3y = e^{4x}\sinh x \text{ is}$$

(A) $\frac{1}{8}\left[e^{5x} - 2e^{3x}\right]$

(B) $\frac{1}{16}\left[e^{3x} - xe^{-5x}\right]$

(C) $\frac{1}{64}\left[e^{5x} - 8xe^{3x}\right]$

(D) $\frac{1}{4}\left[xe^{5x} - 2e^{3x}\right]$

18. The solution of the differential equation $(D^3 + 5D^2)y = 4$ is

(A) $y = C_1 + C_2e^{-5x} + \frac{1}{5}x^2$

(B) $y = C_1 + C_2x + C_3e^{-5x} + \frac{2}{5}x^2$

(C) $y = (C_1 + C_2x)\,e^{5x} + \frac{2}{5}x^2$

(D) $y = (C_1 + C_2x)\,e^{x} + \frac{2}{5}x^2 + e^{5x}$

19. The particular integral of the differential equation given by $(D^2 - 2D + 4)y = x^2e^x$ is

(A) $\frac{1}{9}e^x\left(3x^2 - 2\right)$ (B) $\frac{1}{6}e^x\left(2x^2 - 3\right)$
(C) $\frac{1}{8}e^x\left(3x^2 - 1\right)$ (D) $\frac{1}{3}e^x\left(2x^2 - 3\right)$

20. The solution of the differential equation $(D^4 + D^2 + 36D + 52)y = 0$ is

(A) $y = (C_1 + C_2x)e^{-2x} + (C_3 + C_4x)e^{2x}$
(B) $y = (C_1 + C_2x + C_3\cos 3x + C_4\sin 3x)e^{2x}$
(C) $y = (C_1 + C_2x + (C_3\cos 3x + C_4\sin 3x)e^{-2x}$
(D) $y = (C_1 + C_2x)e^{-2x} + (C_3\cos 3x + C_4\sin 3x)e^{2x}$

21. The Laplace Transform of the function $f(t) = t^2\sin 3t$, $t > 0$ is _____.

(A) $\frac{\left(s^2 - 2s + 9\right)}{\left(s^2 + 9\right)^3}$ (B) $\frac{18\left(s^2 - 3\right)}{\left(s^2 + 9\right)^3}$
(C) $\frac{-\left(s^2 - 2s + 9\right)}{\left(s^2 + 9\right)^3}$ (D) $\frac{6\left(s^2 - 2s + 9\right)}{\left(s^2 + 9\right)^3}$

22. If $u(t - a)$ denotes the unit step function, then the Laplace Transform of $(t^2 + 3)\,u(t - 2)$ is _____.

(A) $\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{7}{s}\right)e^{-2s}$ (B) $\left(\frac{2}{s^3} + \frac{3}{s}\right)e^{-2s}$
(C) $\left(\frac{2}{s^2} + 3\right)e^{-2s}$ (D) $\left(\frac{1}{s^3} + \frac{4}{s^2} + \frac{7}{s}\right)e^{-2s}$

23. The Laplace Transform of solution of the initial value problem $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0$, $y(0) = 1$ and $y^1(0) = -2$ will be ____

(A) $\frac{1}{(s-4)(s+2)}$ (B) $\frac{1}{s-4}$
(C) $\frac{-1}{(s-4)(s+2)}$ (D) $\frac{1}{s+2}$

24. The inverse Laplace Transform of $\frac{3}{2}\left[\frac{1}{\sqrt{s^5}} - \frac{1}{\sqrt{s^3}}\right]$ is _____.

(A) $\sqrt{\frac{t}{\pi}}\,(2t - 3)$ (B) $\sqrt{\frac{t}{\pi}}\,(4t - 3)$
(C) $\sqrt{\frac{\pi}{t}}\,(2t + 3)$ (D) $\sqrt{\frac{\pi}{t}}\,(4t + 3)$

25. The inverses Laplace Tranform of $\frac{4(2s+3)}{\left(s^2 + 4s + 20\right)}$ is

(A) $e^{2t}[2\cos 4t - \sin 4t]$
(B) $e^{2t}\{2\sin 4t - \cos 4t]$
(C) $e^{-2t}[2\cos 4t - \sin 4t]$
(D) $e^{-2t}[2\sin 4t - \cos 4t]$

Answer Keys

1. D	**2.** D	**3.** A	**4.** B	**5.** B	**6.** B	**7.** C	**8.** D	**9.** B	**10.** C
11. A	**12.** B	**13.** A	**14.** A	**15.** B	**16.** D	**17.** C	**18.** B	**19.** A	**20.** D
21. B	**22.** A	**23.** D	**24.** A	**25.** C					

Hints and Explanations

1. Given $f(x, y, z) = x^3 y^2 + 3xy$

$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} \bar{j} = (3x^2 y^2 + 3y)\bar{i} + (2x^3 y + 3x)\bar{j}$

$(\nabla f)_{(1,2)} = (12 + 6)\bar{i} + (4 + 3)\bar{j} = 18\bar{i} + 7\bar{j}$

Given that unit vector makes an angle $\frac{\pi}{4}$ with x-axis

$\therefore$ unit vector must be $\bar{b} = \cos\frac{\pi}{4}\bar{i} + \sin\frac{\pi}{4}\bar{j} = \frac{\bar{i} + j}{\sqrt{2}}$

The directional derivative of f in the direction $\bar{b}$ is $\nabla f.\bar{b}$

$\left(18\bar{i} + 7j\right).\frac{\left(\bar{i} + \bar{j}\right)}{\sqrt{2}} = \frac{18 + 7}{\sqrt{2}} = \frac{25}{\sqrt{2}}$ Choice (D)

2. Given $\bar{V} = e^{px-y-z}\left(\bar{i} + \bar{j} + \bar{k}\right)$ is solenoidal

We know that if $\bar{V}$ is solenoidal, div $\bar{V} = 0$

$\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} = 0$

$= e^{px-y-z}.p + e^{px-y-z}.(-1) + e^{px-y-z}(-1) = 0$

$\therefore p - 1 - 1 = 0 \Rightarrow p = 2$ Choice (D)

3. Let $x = t$; $y = \frac{t}{6}$; $z = 4$ and $3 \le t \le 6$.

$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}.dt = \sqrt{1 + \frac{1}{36}}dt = \frac{\sqrt{37}}{6}dt$

$\int_C \left(x^2 + 3yz\right)ds = \int_3^6 \left(t^2 + 3\frac{t}{6}.4\right)\frac{\sqrt{37}}{6}dt = d\left(\tan\left(\frac{y}{x}\right)\right)$

$= \tan\left(\frac{y}{x}\right) = \tan\left(\frac{y}{x}\right)$ Choice (A)

4. Let $f(x, y, z) = xy^3 + 3yz - 3$

The normal vector to the surface $f(x, y, z)$ is

$\frac{dy}{dx} - y\tan x - \cos x$

$\nabla f = y^3\bar{i} + \left(3xy^2 + 3z\right)\bar{j} + 3y\bar{k}$

The normal vector at (3, –1, –2)

$(\nabla f)_{(3,-1,-2)} = -\bar{i} + (9 - 6)\bar{j} - 3\bar{k} = -\bar{i} + 3\bar{j} - 3\bar{k}$

$\therefore$ The unit normal vector to the surface f is $\frac{\nabla f}{|\nabla f|}$

$= \frac{-\bar{i} + 3\bar{j} - 3\bar{k}}{\sqrt{19}}$ Choice (B)

5. $\int \left(yz + z + z^2\right)dx + (xz - 1)dy + (xy + x + 2xz)dz$

Let $f(x, y, z) = yz + z + z^2$

$g(x, y, z) = xz - 1$

$h(x, y, z) = xy + x + 2xz$

$\frac{\partial f}{\partial y} = z = \frac{\partial g}{\partial x}$

$\frac{\partial f}{\partial z} = y + 1 + 2z = \frac{\partial h}{\partial x}$

$\frac{\partial g}{\partial z} = x = \frac{\partial h}{\partial y}$

The integral is independent of the path C.
The integral is exact differential
So there exists a function Φ

Such that $\frac{\partial \phi}{\partial x} = yz + z + z^2$ $\rightarrow (1)$

$\frac{\partial \phi}{\partial y} = xz - 1$ $\rightarrow (2)$

$\frac{x}{y} = \int 3y^2.\frac{1}{y}dy + c = xy + x + 2xz$ $\rightarrow (3)$

$\frac{\partial \phi}{\partial x} = yz + z + z^2$

Integrate wrt x.

$\phi = xyz + xz + xz^2 + Q(y, z)$ $\rightarrow (4)$

Diff wrt y

$\frac{\partial \phi}{\partial y} = xz + \phi^1(y, z)$ $\rightarrow (5)$

Comparing (2) and (5)

$\phi^1(y, z) = -1 \Rightarrow \phi(y, z) = -y + R(z)$

$\therefore \phi = xyz + xz + xz^2 - y + R(z)$ $\rightarrow (6)$

$\frac{ydx - xdy}{y^2} = 3y\,dy = xy + x + 2xz + R^1(z)$

Comparing (3) and (6)

$R^1(z) = 0$

$\Rightarrow R(z) = k$

$\therefore \phi = xyz + xz + xz^2 - y + k$

$\therefore \int_{(3,4,5)} \left(yz + z + z^2\right)dx + (xz - 1)dy + (xy + x + 2xz)dz$

$= \int_{(2,3,3)} d\left(xyz + xz + xz^2 - y\right)$

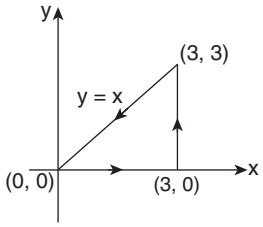
$= xyz + xz + xz^2 - y\Big]_{(2,3,3)}^{(3,4,5)}$

$= 146 - 39 = 107$ Choice (B)

6. $\oint_C (x + 2y)dx + x^2 y\,dy$

$f(x, y) = x + 2y$ $g(x, y) = x^2 y$

$\frac{\partial f}{\partial y} = 2$ and $\frac{\partial g}{\partial x} = 2xy$



By green's theorem

$$\oint_C (x+2y)dx + x^2 y dy = \iint_R (2xy-2)dxdy$$

$$= \int_0^3 \int_0^x (2xy-2)\,dydx$$

$$= \int_0^3 xy^2 - 2y\Big]_0^x dx$$

$$= \int_0^3 (x^3 - 2x)dx$$

$$= \frac{x^4}{4} - x^2\Big]_0^3 = \frac{81}{4} - 9 = \frac{45}{4}$$

Choice (B)

7. Let $f(x, y, z) = x^2 + y^2 - 49$

Surface then

$\text{grad}\, f = 2x\bar{i} + 2y\,j$

$$\hat{n} = \frac{\text{grad}\, f}{|\text{grad}\, f|} = \frac{2x\bar{i} + 2y\,j}{2\sqrt{x^2+y^2}} = \frac{1}{7}\left(x\bar{i} + y\bar{j}\right)$$

Consider the projection of S on the yz plane. It is a rectangle with sides 7 and 5.

$$dA = \frac{dy\,dz}{\bar{n}.\bar{i}} = \frac{dy\,dz}{\frac{x}{7}}$$

$$\bar{F}.\hat{n} = \left(z^2\bar{i} + xy\bar{j} + y^2\bar{k}\right).\left(\frac{x\bar{i} + y\bar{j}}{7}\right) = \frac{xz^2 + xy^2}{7}$$

$$\therefore \quad \iint_S \bar{F}.\hat{n}\,dA = \iint_s \frac{xz^2 + xy^2}{7}\frac{dy\,dz}{\frac{x}{7}}$$

$$= \int_{z=0}^{5}\int_{y=0}^{7} \left(z^2 + y^2\right)dy\,dz = \int_{z=0}^{5} z^2 y + \frac{y^3}{3}\Big]_0^7 dz$$

$$= \int_{z=0}^{5} (7z^2 + \frac{343}{3})dz$$

$$\frac{7z^3}{3} + \frac{343}{3}z\Big]_0^5 = \frac{875}{3} + \frac{1715}{3} = \frac{2590}{3}$$

Choice (C)

8. Given $\bar{V} = (2x - y)\bar{i} - yz^2\,\bar{j} - y^2 z\,\bar{k}$

$$\oint \bar{V}.d\bar{r} = \iint_S \nabla \times V.ndA$$

(By stoke's theorem)

$$\nabla \times V = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x - y & -yz^2 & -y^2 z \end{vmatrix}$$

$$= i(-2yz + 2yz) - j(0) + \bar{k}\,(0 - (-1)) = \bar{k}$$

$f(x, y, z) = x^2 + y^2 + z^2 - 9$ be the surface

$\text{grad}\, f = 2x\bar{i} + 2y\bar{j} + 2z\bar{k}$

$$\hat{n} = \frac{\text{grad}\, f}{|\text{grad}\, f|} = \frac{2x\bar{i} + 2y\bar{j} + 2z\bar{k}}{2\sqrt{x^2+y^2+z^2}}$$

$$\hat{n} = \frac{x\bar{i} + y\bar{j} + z\bar{k}}{3}$$

$$\therefore \quad \nabla \times \bar{V}.\hat{n} = \bar{k}.\frac{\left(x\bar{i} + y\bar{j} + z\bar{k}\right)}{3} = \frac{z}{3}$$

$$\iint_S \nabla \times V.\hat{n}\,dA = \iint_R \frac{z}{3}\frac{dx\,dy}{\bar{n}.\bar{k}}$$

$$= \iint_R \frac{z}{3}\frac{dx\,dy}{\frac{z}{3}} = \iint_R dx\,dy$$

Area of circular region in x-y plane = 9 π Choice (D)

9. By using divergence theorem

$$\iint_S \bar{r}.\bar{n}\,dA = \iiint_V div\,\bar{r}\,dV$$

$$\bar{r} = 2x\bar{i} + 3y\,\bar{j} + z\bar{k}$$

$$\text{Div}\ \bar{r} = 2 + 3 + 1 = 6 = \iiint_V div\,\bar{r}\,dv = \iiint_V 6dV = 6V$$

V is the volume of the sphere

$$= 6 \times \frac{4}{3} \times \pi(4)^3 = 512\,\pi$$

Choice (B)

10. Given $\bar{F} = 2xy\bar{i} + y^2\,\bar{j} + z\bar{k}$

Work done by the force is $\int_C F.d\bar{r}$

$$\bar{r} = x\bar{i} + y\,\bar{j} + z\bar{k} \Rightarrow d\bar{r} = dx\bar{i} + dy\,\bar{j} + dz\bar{k}$$

$$\bar{F}.d\bar{r} = \left(2xy\bar{i} + y^2\,\bar{j} + z\bar{k}\right).\left(dx\bar{i} + dy\,\bar{j} + dz\bar{k}\right)$$

$$\bar{F}.d\bar{r} = 2xy\,dx + y^2\,dy + z\,dz$$

$$\int_C F.d\bar{r} = \int (2xy\,dx + y^2\,dy + z\,dz$$

Convert x, y, z in parametric form
$x = 3\cos t, y = 3\sin t, z = 0$
$dx = -3\sin t\,dt, dy = 3\cos t\,dt$
The limit of t is 0 to $\pi/2$

$$\int_C F.d\bar{r} = \int_0^{\frac{\pi}{2}} 2.3\cos t.3\sin t.(-3\sin t)\,dt + 9\sin^2 t\,3\cos t\,dt$$

$$= -\int_0^{\frac{\pi}{2}} 27\sin^2 t\cos t\,dt$$

$$= -27\frac{\text{Sin}^3 t}{3}\Big]_0^{\frac{\pi}{2}} = \frac{-27}{3} = -9$$ Choice (C)

11. $3(x\,dy + y\,dx) = 2xy\,dy$
$x(3 - 2y)\,dy + 3y\,dx = 0$

$$\frac{(3-2y)dy}{y} + \frac{3dx}{x} = 0$$

$$\int\left(\frac{3}{y} - 2\right)dy + 3\int\frac{dx}{x} = C_1$$

$3\log y - 2y + 3\log x = \log C$
$3\log(xy) - 2y = \log C$
Given when $x = 1, y = 1$
$\Rightarrow \quad -2 = \log C$
$\therefore$ The required solution is $3\log(xy) - 2y + 2 = 0$.
Choice (A)

12. $dy = (9x + y - 1)^2 dx$ or $\frac{dy}{dx} = (9x + y - 1)^2$ ..... (1)

Let $9x + y - 1 = u$

$$9 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\Rightarrow \quad \frac{du}{dx} - 9 = u^2 \text{ (from 1)}$$

$$\frac{du}{dx} = u^2 + 9 \text{ or } \frac{du}{u^2+9} = dx$$

$$\int\frac{du}{u^2+9} = \int dx + c \Rightarrow \frac{1}{3}\text{Tan}^{-1}\left(\frac{u}{3}\right) = x + c$$

i.e., $\frac{1}{3}\tan^{-1}\left(\frac{9x+y-1}{3}\right) = x + c$ or

$3\tan(3x + 3c) = 9x + y - 1$, when $x = 0, y = 1$

$$\frac{1}{3}\tan^{-1}(0) = C \Rightarrow C = 0$$

$\therefore$ The required solution is $3\tan(3x) = 9x + y - 1$.
Choice (B)

13. $\frac{x\,dy - y\,dx}{x} = \cos^2\left(\frac{y}{x}\right)dx$

Dividing both sides by $x\cos^2\left(\frac{y}{x}\right)$, we have

$$\left(\frac{x\,dy - y\,dx}{x^2}\right)\sec^2\left(\frac{y}{x}\right) = \frac{dx}{x}$$

$$d\left(\tan\left(\frac{y}{x}\right)\right) = d(\log x)$$

Integrating on both sides, we have

$$\tan\left(\frac{y}{x}\right) = \log x + \log c$$

$$\tan\left(\frac{y}{x}\right) = \log(cx).$$ Choice (A)

14. $\frac{dy}{dx} - y\tan x - \cos x = 0$

$$\Rightarrow \quad \frac{dy}{dx} - y\tan x = \cos x$$

which is in the form of $\frac{dy}{dx} + Py = Q$

The integrating factor of above equation $e^{\int p dx} = e^{-\int \tan x\,dx}$
$= e^{\log\cos x} = \cos x$. Choice (A)

15. $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ --------- (1)

$(x^2 + y^2)dx - 2xy\,dy = 0$
which is in the form of $Mdx + Ndy = 0$
Here $M = x^2 + y^2, N = -2xy$

$$\frac{\partial M}{\partial y} = 2y, \frac{\partial N}{\partial x} = -2y$$

$$\Rightarrow \text{ Here } \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-1}{2xy}(2y + 2y) = \frac{-2}{x}$$

which is a function of x alone say $f(x)$ then the Integrating factor ($I.F$) is

$$e^{\int f(x)dx} = e^{\int\frac{-2}{x}dx} = e^{-2\log x} = e^{\log x^{-2}} = x^{-2}$$

Multiplying (1) by $I.F$, we have

$\left(1 + \frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0$ which is in the form of $M\,dx + N\,dy = 0$

$\therefore$ Solution is

$$\int m\,dx\,(\text{taking } y \text{ constant}) + \int(\text{terms of } N \text{ not containing } x)\,dy = C$$

$$\Rightarrow \quad \int\left(1 + \frac{y^2}{x^2}\right)dx + \int 0\,dy$$

$$x - \frac{y^2}{x} = C$$

Given when $x = 1, y = 1 \Rightarrow C = 0$

$\therefore$ The required solution is $x - \frac{y^2}{x} = 0$ or $x^2 - y^2 = 0$.

Choice (B)

16. $y\,dx = (x + 3y^3)\,dy$

or $y\frac{dx}{dy} = x + 3y^3$

or $\frac{dx}{dy} - \frac{x}{y} = 3y^2$, which is a linear equation in y of the form $\frac{dx}{dy} + px = Q$ here $P = -\frac{1}{y}$ and $Q = 3y^2$

$\therefore$ $I.F = e^{\int p dy} = e^{-\int \frac{1}{y} dy} = e^{\log y^{-1}} = \frac{1}{y}$

$\therefore$ The solution is $x \,.\, e^{I.F} = \int Q.e.^{I.F}\,.dy$

$\Rightarrow$ $\frac{x}{y} = \int 3y^2 . \frac{1}{y} dy + c$

$\frac{x}{y} = \frac{3y^2}{2} + c$

Given when $x = 1, y = 1$

$\Rightarrow$ $1 = \frac{3}{2} + c$ or $c = \frac{-1}{2}$

$\therefore$ The required solutions is $\frac{x}{y} = \frac{3y^2}{2} - \frac{1}{2}$

i.e., $2x = y(3y^2 - 1)$

Alternate solution:

Given

$ydx = (x + 3y^3)dy$

$ydx - xdy = 3y^3\,dy$

$\frac{ydx - xdy}{y^2} = 3y\,dy$

$d\left(\frac{x}{y}\right) = 3y\,dy$

Integrating on both sides

$\left(\frac{x}{y}\right) = \left(\frac{3y^2}{2}\right) + C$

$2x = y(3y^2 + C_1)$

Given $x = 1; y = 1$

$2 = 3 + C_1 \Rightarrow C_1 = -1$

$\therefore$ required solution is $2x = y\,(3y^2 - 1)$. Choice (D)

17. $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 3y = e^{4x} \sin hx$

i.e., $(D^3 - 5D^2 + 7D - 3)y$

$= e^{4x}\left[\frac{e^x - e^{-x}}{2}\right] = \frac{e^{5x} - e^{3x}}{2}$

i.e., $(D - 1)^2\,(D - 3)y = \frac{e^{5x} - e^{3x}}{2}$

Particular integral is $\frac{e^{5x} - e^{3x}}{(D-1)^2\,(D-3)} . \frac{1}{2}$

$\frac{1}{2}\frac{e^{5x}}{(D-1)^2\,(D-3)} - \frac{1}{2(D-1)^2\,(D-3)} . e^{3x}$

$= \frac{1}{2} . \frac{e^{5x}}{(5-1)^2\,(5-3)} - \frac{1}{2}\frac{1}{(3-1)^2} . \frac{e^{3x}}{(D-3)}$

$= \frac{e^{5x}}{64} - . \frac{1}{8} . x . e^{3x} = \frac{1}{64}\left[e^{5x} - 8x\,e^{3x}\right]$ Choice (C)

18. $(D^3 + 5D^2)\,y = 4$

Auxiliary equation $m^3 + 5m^2 = 0$

$m^2(m + 5) = 0$

$\Rightarrow$ $m = 0, 0, -5$

$\therefore$ $C.F$ is $(C_1 + C_2x)e^{0.x} + C_3e^{-5x} = (C_1 + C_2x) + C_3e^{-5x}$

$P.I = \frac{1}{D^2\,(D+5)} 4 . e^{0.x}$

$= \frac{1}{5} . \frac{1}{D^2} 4 = \frac{1}{5}\left[\frac{1}{D^2} . 4\right] = \frac{2x^2}{5}$.

Complementary Solution $y = C.F + P.I$

$y = C_1 + C_2x + C_3\,e^{-5x} + \frac{2x^2}{5}$ Choice (B)

19. $(D^2 - 2D + 4)\,y = x^2\,e^x$

$P.I = \frac{e^x\,x^2}{(D^2 - 2D + 4)} = e^x \frac{1}{(D+1)^2 - 2(D+1) + 4} . x^2$

$= e^x \frac{1}{D^2 + 3} x^2$

$\Rightarrow$ $e^x \frac{1}{3\left(1 + \frac{D^2}{3}\right)} x^2 = e^x \frac{1}{3}\left(1 + \frac{D^2}{3}\right)^{-1} x^2$

$= e^x \frac{1}{3}\left(1 - \frac{D^2}{3}\right) x^2$

$= e^x \frac{1}{3}\left(x^2 - \frac{2}{3}\right) = e^x \frac{1}{9}\left(3x^2 - 2\right)$ Choice (A)

20. $(D^4 + D^2 + 36D + 52)\,y = 0$

Auxiliary equation of the above is $m^4 + m^2 + 36m + 52 = 0$

By trail and error we notice m = –2, –2, are the roots of the above

$\therefore$ $(m + 2)^2\,(m^2 - 4m + 13) = 0$

The roots are $m = -2, -2$, and $2 \pm 3i$

$\therefore$ The solution is

$y = (C_1 + C_2x)e^{-2x} + e^{2x}\,(C_3 \cos 3x + C_4 \sin 3x)$

Choice (D)

21. Let $g(t) = \sin 3t \Rightarrow f(t) = t^2 g(t)$

Now $L[g(t)] = L:[\sin 3t] = \dfrac{3}{s^2+9}$

$$L[f(t)] = L[t^2 g(t)] = \frac{d^2}{ds^2}\ (L[g(t)])$$

$$= \frac{d}{ds}\left(\frac{d}{ds}\left(\frac{3}{s^2+9}\right)\right)$$

$$= \frac{d}{ds}\left(\frac{-6s}{(s^2+9)^2}\right)$$

$$= \frac{-6\left[(s^2+9)^2.1 - s.2(s^2+9).2s\right]}{(s^2+9)^4}$$

$$= \frac{-6\left[s^2+9-4s^2\right]}{(s^2+9)^3}$$

$$= \frac{18(s^2-3)}{(s^2+9)^3}$$ Choice (B)

22. Let $f(t) = (t^2+3)\ u(t-2)$

$L[f(t)] = L[(t^2+3)\ u(t-2)]$

$L[((t-2)+2)^2+3)\ u(t-2)]$

$\therefore \quad L[f(t)] = L[(t-2)^2 + 4(t-2)+7)\ u(t-2)]$ ----- (1)

Let $g(t) = t^2 + 4t + 7$

$\therefore \quad L[g(t)] = L[t^2+4t+7] = L[t^2] + 4L[t] + 7L[1]$

$\therefore \quad L[g(t)] = \dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{7}{s}$

Now from (1)

$L[f(t)] = L[(t-2)^2 + 4(t-2)+7)\ u(t-2)]$

$= L[g(t-2).\ U(t-2)$

$= L[g(t)]\ e^{-2s}$ (By second shifting theorem)

$$= \left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{7}{s}\right)e^{-2s}.$$ Choice (A)

23. Given initial value problem is

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0 \qquad \text{---------- (1)}$$

Where $y(0) = 1$ and $y^1(0) = -2$

Applying laplace transform on both sides of (1)

$$L\left[\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y\right] = L[0]$$

$$\Rightarrow \quad L\left[\frac{d^2y}{dt^2}\right] - 2L\left[\frac{dy}{dt}\right] - 8L[y] = 0$$

$$\Rightarrow \quad \left(s^2\bar{y} - sy(0) - y^1(0)\right) - 2\left(s\bar{y} - y(0)\right) - 8\bar{y} = 0$$

Where $\bar{y} = L[y]$

$$\Rightarrow \quad s^2\bar{y} - s \times 1 - (-2) - 2s\bar{y} + 2 \times 1 - 8\bar{y} = 0$$

$$\Rightarrow \quad \left(s^2 - 2s - 8\right)\bar{y} - s + 4 = 0$$

$$\Rightarrow \quad \left(s^2 - 2s - 8\right)\bar{y} = s - 4$$

$$\Rightarrow \quad \bar{y} = \frac{(s-4)}{\left(s^2-2s-8\right)} = \frac{(s-4)}{(s-4)(s+2)}$$

$$\bar{y} = \frac{1}{s+2}$$

The laplace transform of the solution of (1) is

$\bar{y} = L[y] = \dfrac{1}{s+2}$. Choice (D)

24. We have to find $L^{-1}\left[\dfrac{3}{2}\left(\dfrac{1}{\sqrt{s^5}} - \dfrac{1}{\sqrt{s^3}}\right)\right]$

$$= \frac{3}{2}\left(L^{-1}\left[\frac{1}{s^{\frac{5}{2}}}\right] - L^{-1}\left[\frac{1}{s^{\frac{3}{2}}}\right]\right)$$

$$= \frac{3}{2}\left[\frac{t^{\frac{3}{2}}}{\Gamma\left(\frac{5}{2}\right)} - \frac{t^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)}\right]$$

$$= \frac{3}{2}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}\times\frac{1}{2}\left(\sqrt{\pi}\right)} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}\left(\sqrt{\pi}\right)}\right]$$

$$= \frac{3}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{4}\sqrt{\pi}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}\sqrt{\pi}}\right) = -\times\frac{\sqrt{t}}{\sqrt{\pi}}\left[\frac{4t}{3} - 2\right]$$

$$= \sqrt{\frac{t}{\pi}} \times 3\left[\frac{2t}{3} - 1\right]$$

$$= \sqrt{\frac{t}{\pi}}\ (2t-3).$$ Choice (A)

25. We have $L^{-1}\left[\dfrac{4(2s+3)}{s^2+4s+20}\right]$

$$= 4\ L^{-1}\left[\frac{2s+3}{(s^2+4s+4)+16}\right]$$

$$= 4L^{-1}\left[\frac{2(s+2-2)+3}{(s+2)^2+4^2}\right] = 4L^{-1}\left[\frac{2(s+2)-1}{(s+2)^2+4^2}\right]$$

$$= 4L^{-1}\left[\frac{2(s+2)}{(s+2)^2+4^2}\right] - 4L^{-1}\left[\frac{1}{(s+2)^2+4^2}\right]$$

$$= 8 \times \frac{1}{4}e^{-2t}\cos 4t - 4 \times \frac{1}{4}e^{-2t}\sin 4t$$

$$= e^{-2t}[2\cos 4t - \sin 4t].$$ Choice (C)

ENGINEERING MATHEMATICS TEST 3

(LINEAR ALGEBRA, PARTIAL DIFFERENTIAL EQUATIONS)

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. If A is a square matrix of order 5 with $A^{-1} = A^T$ and non-negative determinant, then the determinant of A is _____.
(A) 0 (B) 1
(C) 2 (D) 5

2. For two matrices A and B, if $AB = A$ and $BA = B$, then which of the following statements is/are correct?
I. A is an idempotent matrix.
II. B is an idempotent matrix.
(A) I only
(B) II only
(C) Both I and II
(D) Neither I nor II

3. Consider the matrix $A = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$. Which of the following is NOT equal to the determinant of A?

(A) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ (B) $\begin{vmatrix} a+bc & a & 1+a \\ b+ca & b & 1+b \\ c+ab & c & 1+c \end{vmatrix}$

(C) $\begin{vmatrix} 0 & a-b & bc-ac \\ 0 & b-c & ca-ab \\ 1 & c & ab \end{vmatrix}$ (D) $\begin{vmatrix} 1 & a+1 & a^2+a \\ 1 & b+1 & b^2+b \\ 1 & c+1 & c^2+c \end{vmatrix}$

4. For a non-singular square matrix A, if $A^3 = A$, then A must be _____.
(A) a nilpotent matrix
(B) an idempotent matrix
(C) an involutory matrix
(D) None of these

5. If A is a matrix of order 6×9 with rank 5, then which of the following is true?
(A) All the rows of A are linearly independent.
(B) 5 columns of A are linearly independent.
(C) AA^T is invertible.
(D) A^TA is invertible.

6. The rank of the matrix $P = \begin{bmatrix} 1 & 2 & 4 & -3 \\ 2 & -3 & 5 & -4 \\ 4 & 1 & 13 & -10 \\ 3 & -8 & 6 & -5 \end{bmatrix}$ is _____.
(A) 1 (B) 2
(C) 3 (D) 4

7. The value of x_3 in the solution of the system of linear equations $x_1 + 2x_2 + 2x_3 = 4$, $2x_1 - 2x_2 - x_3 = -3$, $4x_1 + x_2 + 2x_3 = 3$ is —
(A) 1 (B) −1
(C) 2 (D) −2

8. For a homogeneous system of linear equations $AX = O$ with four equations in four unknowns, if the number of linearly independent solutions is one, then the rank of A is _____.
(A) 1 (B) 2
(C) 3 (D) 4

9. If $A = \begin{bmatrix} 1 & 10 & 16 & -20 \\ 0 & -1 & 159 & 237 \\ 0 & 0 & 1 & -431 \\ 0 & 0 & 0 & 1 \end{bmatrix}$; then the determinant of $A^9 - 7A^5 + 4A$ is _____.
(A) 4 (B) −4
(C) 16 (D) −16

10. If the characteristic equation of a 2×2 matrix A is $\lambda^2 - 4\lambda + 1 = 0$, then the trace and determinant of A respectively are _____.
(A) −1 and 4 (B) 1 and −4
(C) 4 and 1 (D) 4 and −1

11. If λ_1 and λ_2 are the eigenvalues of a 2×2 non-singular matrix A, then the eigenvalues of ad joint of A are _____.
(A) λ_1 and λ_2 (B) λ_1^2 and λ_2^2
(C) $\lambda_1 + \lambda_2$ and $\lambda_1 - \lambda_2$ (D) $\lambda_1 \times \lambda_2$ and $\frac{\lambda_1}{\lambda_2}$

12. If A is a 3×3 matrix with the characteristic equation $\lambda^3 - 5\lambda^2 + 2\lambda - 3 = 0$, then $3A^9 - 15A^8 + 6A^7 - 11A^6 + 10A^5 - 4A^4 + 10A^3 - 20A^2 + 8A - 9I$ is equal to _____.

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

13. Which of the following is NOT an eigenvector of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & -1 & 4 \end{bmatrix}$?

(A) $\begin{bmatrix} 3 \\ -3 \\ -4 \end{bmatrix}$ B) $\begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix}$

(C) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

14. If the system of linear equations
$2x + 3y + 4z = 1$
$5x - y + z = 4$
$3x + ay - 3z = 3$
has a unique solution, then the value of $a + 4$ _____
(A) must be equal to 0,
(B) should not be equal to 0
(C) can be any real number,
(D) can be any rational number

15. If a_1, b_1, c_1, d_1, a_2, b_2, c_2 and d_2 are any non zero real numbers, then which of the following types of solution is NOT possible for the system of linear equations.
$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
(A) Unique solution
(B) No solution
(C) Infinitely many solution
(D) None of these

16. The partial differential equation of
$z = f(x + at) - g(x - at)$ is _____.

(A) $\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$ (B) $\frac{\partial z}{\partial t} = a\frac{\partial z}{\partial x}$

(C) $\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}$ (D) $\frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial x^2}$

17. The first order partial differential equation by eliminating the arbitary function from $z = f(x^3 - y^3)$ is
(A) $p + q = 0$ (B) $yp + xq = 0$
(C) $y^2p + x^2q = 0$ (D) $2y^2p + 3x^2q = 0$

18. The general solution of the partial differential equation $x^3(y - z)p + y^3(z - x)q = z^3(x - y)$ is

(A) $\phi\left(xyz, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$

(B) $\phi\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$

(C) $\phi\left(\frac{1}{x} - \frac{1}{y} - \frac{1}{z}, \frac{1}{x^2} + \frac{1}{y^2} - \frac{1}{z^2}\right) = 0$

(D) $\phi\left(\frac{1}{x^2} - \frac{1}{y^2} - \frac{1}{z^2}, \frac{1}{x} - \frac{1}{y} + \frac{1}{z}\right) = 0$

19. The solution of the partial differential equation $xy^2z^2p + x^2yz^2q = x^2y^2z$ is
(A) $x^2 + y^2 = \phi(x^2 - y^2)$
(B) $x^2 + z^2 = \phi(x^2 - z^2)$
(C) $x^2 - y^2 = \phi(y^2 - z^2)$
(D) $y^2 - z^2 = \phi(x^2 - y^2 - z^2)$

20. The solution of $(p - q)(z - x\,p - y\,q) = 1$ is _____

(A) $z = ax - by + \frac{1}{a+b}$ (B) $z = ax - by$

(C) $z = ax + by$ (D) $z = ax + by + \frac{1}{a-b}$

21. If $u(x, y) = X(x).Y(y)$ be the solution of the partial differential equation $4\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = 0$, which is obtained by solving it by the method of separation of variables, then $X(x)$ (the function of x only in $u(x, y)$) is _____
[Note: Here c and k are arbitrary constants]

(A) $X(x) = ce^{(kx)}x^2$ (B) $X(x) = ce^{\left(\frac{4k}{x}\right)}$

(C) $X(x) = ce^{\left(\frac{k}{4}\right)x}$ (D) $X(x) = ce^{(-5k)}x^2$

22. Which of the following second order partial differential equations is an elliptic equation?

(A) $3\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} - 5\frac{\partial^2 u}{\partial y^2} + 7\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 7x^2$

(B) $3\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 6x^2y$

(C) $-3\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} + 4x\frac{\partial u}{\partial x} - 7y\frac{\partial u}{\partial y} = 0$

(D) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = 6xy^2$

23. The Fourier cosine series of the function $f(x) = \frac{1}{2}$, $0 \le x \le 1$ is
(A) 1 (B)
(C) 0 (D)

24. The Fourier series of $f(x) = e^{2x}$ in the $\frac{1}{4}\frac{a_0}{2}$ interval $(0, 2\pi)$ is

$f(x) = + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, then the value of $\frac{a_0}{2}$ is

(A) $\frac{e^{4\pi} - 1}{4\pi}$ (B) $\frac{e^{2\pi} - 1}{2\pi}$

(C) $\frac{e^{4\pi} - 1}{2\pi}$ (D) $\frac{e^{2\pi} - 1}{4\pi}$

25. The Fourier series of the function

$$f(x) = \begin{cases} -2 \text{ for } -\pi < x < \dfrac{-\pi}{2} \\ 0 \text{ for } \dfrac{-\pi}{2} < x < \dfrac{\pi}{2} \\ 2 \text{ for } \dfrac{\pi}{2} < x < \pi \end{cases}$$

is $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, then $b_n =$

(A) $\dfrac{1}{n\pi}\left[\sin\dfrac{n\pi}{2} - \cos n\pi\right]$

(B) $\dfrac{4}{n\pi}\left[\cos\dfrac{n\pi}{2} - \cos n\pi\right]$

(C) $\dfrac{1}{4n\pi}\left[\cos n\pi - \cos\dfrac{n\pi}{2}\right]$

(D) $\dfrac{4}{n\pi}\left[\sin\dfrac{n\pi}{2} - \sin n\pi\right]$

Answer Keys

1. B	**2.** C	**3.** B	**4.** C	**5.** B	**6.** B	**7.** A	**8.** C	**9.** D	**10.** C
11. A	**12.** C	**13.** D	**14.** B	**15.** A	**16.** A	**17.** C	**18.** B	**19.** C	**20.** D
21. C	**22.** B	**23.** B	**24.** A	**25.** B					

Hints and Explanations

1. Given $A^{-1} = A^T$ and det of A is non-negative

$\therefore$ $A.A^T = A^T.A = I_5$

$\Rightarrow$ Det of $(A.A^T)$ = Det of I_5

$\Rightarrow$ $|A.A^T| = |I_5|$

$\Rightarrow$ $|A|\,|A^T| = 1$

$\Rightarrow$ $|A| \,.\, |A| = 1$ ($\because$ $|A| = |A^T|$)

$\Rightarrow$ $|A|^2 = 1 \Rightarrow |A| = \pm 1$

$\therefore$ The determinant of $A = 1$.

($\because$ $|A|$ is non-negative) Choice (B)

2. Given $AB = A$ and $BA = B$

Consider $BA = B$

$\Rightarrow$ $A\,(BA) = AB$

$\Rightarrow$ $(AB)A = A$ ($\because$ $AB = A$)

$AA = A \Rightarrow A^2 = A$

$\Rightarrow$ A is an idempotent matrix $\rightarrow$ (1)

Consider $AB = A$

$\Rightarrow$ $B\,(AB) = BA$

$\Rightarrow$ $(BA)B = B$ ($\because$ $BA = B$)

$\Rightarrow$ $BB = B \Rightarrow B^2 = B$

$\Rightarrow$ B is an idempotent matrix $\rightarrow$ (2)

$\therefore$ From (1) and (2), both I and II are correct.

Choice (C)

3. Given $A = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$

Consider the determinant given in option (B)

$$\begin{vmatrix} a+bc & a & 1+a \\ b+ca & b & 1+b \\ c+ab & c & 1+c \end{vmatrix} = \begin{vmatrix} a & a & 1+a \\ b & b & 1+b \\ c & c & 1+c \end{vmatrix} + \begin{vmatrix} bc & a & 1+a \\ ca & b & 1+b \\ ab & c & 1+c \end{vmatrix}$$

$$= 0 + \begin{vmatrix} bc & a & 1 \\ ca & b & 1 \\ ab & c & 1 \end{vmatrix} + \begin{vmatrix} bc & a & a \\ ca & b & b \\ ab & c & c \end{vmatrix}$$

$$= \begin{vmatrix} bc & a & 1 \\ ca & b & 1 \\ ab & c & 1 \end{vmatrix} + 0$$

$$= -1 \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$= -$Det of $A \neq$ Det of A

$\therefore$ The determinant given in option B is NOT equal to det of A. Choice (B)

4. Given A is non-singular and $A^3 = A$

$\Rightarrow$ $AA^2 = A$

$\Rightarrow$ $A^{-1}\,(AA^2) = A^{-1}\,A$

$\Rightarrow$ $(A^{-1}A)A^2 = A^{-1}\,A$

$\Rightarrow$ $A^2 = I$

$\Rightarrow$ A must be an involutory matrix. Choice (C)

5. Given A is a matrix of order 6×9.

Rank of $A = 5$

$\therefore$ Maximum number rows/columns of A that are linearly independent = 5.

$\therefore$ Option (B) is TRUE. Choice (B)

6. Given matrix is $P = \begin{bmatrix} 1 & 2 & 4 & -3 \\ 2 & -3 & 5 & -4 \\ 4 & 1 & 13 & -10 \\ 3 & -8 & 6 & -5 \end{bmatrix}$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 - 4R_1$ and $R_4 \rightarrow R_4 - 3R_1$

$$\sim \begin{bmatrix} 1 & 2 & 4 & -3 \\ 0 & -7 & -3 & 2 \\ 0 & -7 & -3 & 2 \\ 0 & -14 & -6 & 4 \end{bmatrix}$$

$R_3 \to R_3 - R_2$ and $R_4 \to R_4 - 2R_2$

$$\sim \begin{bmatrix} 1 & 2 & 4 & -3 \\ 0 & -7 & -3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2 \to \frac{-1}{7} R_2$$

$$\therefore \quad P \sim \begin{bmatrix} 1 & 2 & 4 & -3 \\ 0 & 1 & 3/7 & -2/7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which is in Row Echelon form.

$\therefore$ The rank of P = the number of non-zero rows in its Row Echelon form = 2. Choice (B)

7. Given system of equations is

$x_1 + 2x_2 + 2x_3 = 4$

$2x_1 - 2x_2 - x_3 = -3 \to (1)$

$4x_1 + x_2 + 2x_3 = 3$

It can be written in matrix form as

$AX = B \to (2)$

Where $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & -2 & -1 \\ 4 & 1 & 2 \end{bmatrix}$; $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -3 \\ 3 \end{bmatrix}$

Consider the augmented matrix

$$[A/B] = \left|\begin{matrix} 1 & 2 & 2 & 4 \\ 2 & -2 & -1 & -3 \\ 4 & 1 & 2 & 3 \end{matrix}\right|$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 4R_1$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & -6 & -5 & -11 \\ 0 & -7 & -6 & -13 \end{bmatrix}$$

$R_3 \to 6R_3 - 7R_2$

$$\therefore [A/B] \sim \begin{bmatrix} 1 & 2 & 2 & 4 \\ 0 & -6 & -5 & -11 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

Hence the system of equations that has same solution as that of $AX = B$ is

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -6 & -5 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -11 \\ -1 \end{bmatrix}$$

$\Rightarrow$ $x_1 + 2x_2 + 2x_3 = 4$

$-6x_2 - 5x_3 = -11$

$-x_3 = -1 \Rightarrow x_3 = 1$. Choice (A)

8. Given the system of equations $AX = O$ has

Number of unknowns = $n = 4$

Number of equations = 4

$\therefore$ A is a 4×4 matrix

Also given the number of linearly independent solutions = 1. We know that the number of linearly independent solutions of a system of homogeneous linear equations.

$AX = O$ is $= n - r$

Where n = the number of unknowns and r = the rank of A.

$\therefore$ $n - r = 1$

$\Rightarrow$ $4 - r = 1$

$\Rightarrow$ $r = 4 - 1 = 3$

$\therefore$ The rank of $A = 3$ Choice (C)

9. Given A $= \begin{bmatrix} 1 & 10 & 16 & -20 \\ 0 & -1 & 159 & 237 \\ 0 & 0 & 1 & -431 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The eigenvalues of A are 1, –1, 1 and 1.

If $\lambda = -1$ is an eigenvalue of A, then $(-1)^9 - 7(-1)^5 + 4(-1) = 2$ is an eigenvalue of $A^9 - 7A^5 + 4A$.

Also if $\lambda = 1$ is an eigenvalue of A then

$(1)^9 - 7(1)^5 + 4(1) = -2$ is an eigenvalue of $A^9 - 7A + 4A$

$\therefore$ The eigenvalues of $A^9 - 7A^5 + 4A$ are –2, 2, –2 and –2.

$\therefore$ The determinant of $A^9 - 7A^5 + 4A$ = Product of the eigenvalues of $A^9 - 7A^5 + 4A = (-2)\ (2)\ (-2)\ (-2)$ $= -16$. Choice (D)

10. Given the characteristic equation of a 2×2 matrix A is $\lambda^2 - 4\lambda + 1 = 0$

Let λ_1 and λ_2 be the eigenvalues of A.

$\therefore$ Trace of A = sum of the eigenvalues of A $= \lambda_1 + \lambda_2 = (-(-4) = 4$

Determinant of A = Product of the eigenvalues of $A = \lambda_1 . \lambda_2 = 1$. Choice (C)

11. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix with λ_1 and λ_2 as its eigenvalues.

The characteristic equation of A is

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $(a - \lambda)\ (d - \lambda) - bc = 0$ ---- (1)

The adjoint of A is adj$(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\therefore$ The characteristic equation of adj(A) is

$$\begin{vmatrix} d-\lambda & -b \\ -c & a-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (d-\lambda)(a-\lambda)-bc=0$ ---- (2)

As (1) and (2) are one and the same and λ_1 and λ_2 being the roots of (1), λ_1 and λ_2 will be the roots of (2).

$\therefore$ The eigenvalues of adj(A) are λ_1 and λ_2.

Choice (A)

12. The characteristic equation of A is $\lambda^3-5\lambda^2+2\lambda-3=0$

$\therefore$ By Cayley Hamilton theorem, we have

$A^3-5A^2+2A-3I=0$ ----- (1)

Consider

$3A^9-15A^8+6A^7-11A^6+10A^5-4A^4+10A^3-20A^2+8A-9I$

$=3A^6(A^3-5A^2+2A-3I)-2A^6+10A^5-4A^4+10A^3-20A^2+8A-9I$

$=3A^6\times 0-2A^3(A^3-5A^2+2A-3I)+4A^3-20A^2+8A-9I$

(From (1)) $=0-2A^3\times 0+4(A^3-5A^2+2A-3I)+3I$

(From (1)) $=4\times 0+3I$ (From (1))

$$=3I=3\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$$ Choice (C)

13. Given $A=\begin{bmatrix}1&0&0\\2&3&0\\3&-1&4\end{bmatrix}$

The eigenvalues of A are 1, 3 and 4

If $x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ is an eigenvector of A, then x should satisfy any one of the three conditions. $AX=x$, $AX=3X$ and $AX=4X$

From the options given, it can be easily observed that the vectors given in options (A), (B) and (C), will satisfy one of these three conditions.

Consider the vector given in option (D),

$$AX=\begin{bmatrix}1&0&0\\2&3&0\\3&-1&4\end{bmatrix}\begin{bmatrix}0\\2\\0\end{bmatrix}=\begin{bmatrix}0\\6\\-2\end{bmatrix}\neq\lambda\begin{bmatrix}0\\2\\0\end{bmatrix}$$

For, $\lambda=1, 2$ or 3

$\therefore$ Its not an eigenvector of A Choice (D)

14. Given system of linear equations is

$2x+3y+4z=1$

$5x-y+z=4$

$3x+ay-3z=3$ --------- (1)

It can be written in matrix form as $AX=B$

Where $A=\begin{bmatrix}2&3&4\\5&-1&1\\3&a&-3\end{bmatrix}$;

$X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}1\\4\\3\end{bmatrix}$ consider the augmented matrix

$$[A|B]=\begin{bmatrix}2&3&4&1\\5&-1&1&4\\3&a&-3&3\end{bmatrix}$$

$R_2\to 2R_2-5R_1$, $R_3\to 2R_3-3R_1$

$$\sim\begin{bmatrix}2&3&4&1\\0&-17&-18&3\\0&2a-9&-18&3\end{bmatrix}$$

$R_3\to R_3-R_2$

$$[A|B]\sim\begin{bmatrix}2&3&4&1\\0&-17&-18&3\\0&2a+8&0&0\end{bmatrix}$$

The given system of equations has a unique solution, if $P(A)=p([A/B])=3$ (= The no. of unknowns)

This is possible only if $2a+8\neq 0$

$\Rightarrow \quad a+4\neq 0$. Choice (B)

15. Given system of equations is

$a_1x+b_1y+c_1z=d_1$ _____(1)

$a_2x+b_2y+c_2z=d_2$

It can be written in matrix form as

$AX=B$

Where $A=\begin{bmatrix}a_1&b_1&c_1\\a_2&b_2&c_2\end{bmatrix}$; $x=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}d_1\\d_2\end{bmatrix}$

Here two possibilities arise

(i) $P(A)\neq P(A/B])$

In this case, (1) has no solution

(ii) $P(A)=P([A/B])<3$(= The no. of unknowns)

In this case, (1) has infinitely many solutions.

So, the given system (1) do not have a unique solution. Choice (A)

16. $z=f(x+at)-g(x-at)$

$$\frac{\partial z}{\partial x}=f^1(x+at)-g^1(x-at)$$

$$\frac{\partial^2 z}{\partial x^2}=f^{11}(x+at)-g^{11}(x-at)$$

$$\frac{\partial z}{\partial t}=af^1(x+at)+ag^1(x-at)$$

$$\frac{\partial^2 z}{\partial t^2}=a^2f^{11}(x+at)-a^2g^{11}(x-at)$$

$$\frac{\partial^2 z}{\partial t^2}=a^2\frac{\partial^2 z}{\partial x^2}$$ Choice (A)

17. Given $z = f(x^3 - y^3)$

Let $x^3 - y^3 = u$

$z = f(u)$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}.\frac{\partial u}{\partial x} = f\ (u).3x^2$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}.\frac{\partial u}{\partial y} = -f\ (u)3y^2$$

$$y^2\frac{\partial z}{\partial x} + x^2\frac{\partial z}{\partial y} = f\ (u)3x^2y^2 - f^1(u)3x^2y^2 = 0$$

$\therefore$ The first order partial dE is $y^2p + x^2q = 0$

Choice (C)

18. $x^3(y-z)\,p + y^3\,(z-x)q = z^3(x-y)$

The subsidiary equation of the given differential equation is

$$\frac{dx}{x^3(y-z)} = \frac{dy}{y^3(z-x)} = \frac{dz}{z^3(x-y)} \quad \ldots\ldots\ldots\ldots (1)$$

using the multipliers $\frac{1}{x^2}, \frac{1}{y^2}$ and $\frac{1}{z^2}$ each fraction of (1) is equal to $\dfrac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{0}$

$$\Rightarrow \quad \frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz = 0$$

On integrating the above, we get

$$-\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = C \text{ or } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = C_1 \quad \ldots\ldots\ldots\ldots (2)$$

using the multipliers $\frac{1}{x^3}, \frac{1}{y^3}$ and $\frac{1}{z^3}$ each of the fraction (1) equal to $\dfrac{\frac{1}{x^3}dx + \frac{1}{y^3}dy + \frac{1}{z^3}dz}{0}$

On integrating both the sides, we get

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = C_2 \quad \ldots\ldots\ldots\ldots (3)$$

From (2) and (3) the general solutions is

$$\varphi\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) = 0.$$ Choice (B)

19. $xy^2z^2p + x^2yz^2q = x^2\,y^2\,z \quad \ldots\ldots\ldots\ldots (1)$

The subsidiary equations of (1) are

$$\frac{dx}{xy^2z^2} = \frac{dy}{x^2yz^2} = \frac{dz}{x^2y^2z} \quad \ldots\ldots\ldots\ldots (2)$$

Considering first two fractions of (2) we have $x\,dx = y\,dy$.

On integrating we get $x^2 = y^2$ or $x^2 - y^2 = C_1 \ldots\ldots\ldots (3)$

Considering the last two fractions of (2), we have $y\,dy = z\,dz$. On integrating, we get

$y^2 - z^2 = C_2 \quad \ldots\ldots\ldots\ldots (4)$

From (3) and (4) the general solution of (1) is

$x^2 - y^2 = \phi\,(y^2 - z^2)$. Choice (C)

20. Given $(p-q)\,(z - x\,p - yq) = 1$

$$z = x\,p + yq + \frac{1}{p-q}$$

This is a clairaut equation and its solution is

$$z = ax + by + \frac{1}{a-b}$$ Choice (D)

21. Given $u = x(x), y(y)$ ------ (1) is the solution of the PDE

$$4\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = 0 \text{ ---------- (2)}$$

Obtained by solving (2) by the method of separation of variables

$$\therefore \quad \frac{\partial u}{\partial x} = x^1y \text{ and } \frac{\partial u}{\partial y} = xy^1$$

where $x^1 = \dfrac{dx}{dx}$ and $y^1 = \dfrac{dY}{dy}$

$\therefore$ (2) becomes

$4x^1y + 5xy^1 = 0$

$$\Rightarrow \quad \frac{4x^1}{x} + 5\frac{y^1}{y} = 0$$

$$\Rightarrow \quad \frac{4x^1}{x} = -5\frac{y^1}{y} = k \text{ (say) where } k \text{ is a constant}$$

$$\Rightarrow \quad \frac{4x^1}{x} = k \text{ and } \frac{-5y^1}{y} = k$$

$$\Rightarrow \quad x^1 = \frac{kx}{4} \qquad y^1 = \frac{-ky}{5}$$

$$\Rightarrow \quad x^1 - \frac{kx}{4} = 0 \Rightarrow y^1 + \frac{ky}{5} = 0$$

$$\Rightarrow \quad \frac{dX}{dx} = \frac{kx}{4} \Rightarrow \frac{dx}{x} = \frac{k}{4}\,dx$$

$$\Rightarrow \quad \int\frac{dx}{x} = \frac{k}{4}\int dx \Rightarrow = \frac{k}{4}x + c^1$$

$$\Rightarrow \quad x = e^{\frac{kx}{4}} + c^1 = e^{\frac{kx}{4}}c^1$$

$$\Rightarrow \quad x = ce^{\left(\frac{kx}{4}\right)}; \text{ where } c = e^{c^1}$$

$$\therefore \quad x\,(x) = ce^{\left(\frac{kx}{4}\right)}.$$ Choice (C)

22. A PDE is of the form

$Au_{xx} + Bu_{xy} + Cu_{yy} + F(x, y, u, u_x, u_y) = 0$ ----- (1)

is elliptic, if $B^2 - 4AC < 0$

From the PDE in the options, consider the PDE in option (B)

Comparing it with (1), we have

$A = 3, B = -4$ and $C = 5$

$\therefore \quad B^2 - 4AC = (-4)^2 - 4 \times 3 \times 5 = -44 < 0$

$\Rightarrow \quad B^2 - 4AC < 0$

Hence the PDE given in option (B) is elliptic

Also, it can be easily observed that the PDE given in options (A), (C) and (D) do not satisfy the property, $B^2 - 4AC < 0$. Choice (B)

23. The coefficients of Fourier cosine series are given by

$$a_0 = \frac{2}{1}\int_0^1 f(x)dx \text{ and}$$

$$a_n = \frac{2}{1}\int_0^1 f(x)\cos\left(\frac{n\pi x}{1}\right)dx$$

$$\therefore \quad a_0 = \frac{2}{1}\int_0^1 \frac{1}{2}dx = x\Big]_0^1 = 1$$

$$a_n = \frac{2}{1}\int_0^1 \frac{1}{2}\cos(n\pi x)dx = \int_0^1 \cos(n\pi x)dx$$

$$= \frac{\sin(n\pi x)}{n\pi}\Big]_0^1 = 0$$

$\therefore$ The required series is $f(x) = \frac{1}{2}$. Choice (B)

24. In the Fourier series of e^{2x}; a_0 is given by $\frac{1}{\pi}\int_0^{2\pi} f(x)dx$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} e^{2x}dx = \frac{1}{2\pi}\left[e^{2x}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[e^{4\pi} - 1\right] \quad \frac{a_0}{2} = \frac{e^{4\pi}-1}{2\times 2\pi} = \frac{e^{4\pi}-1}{4\pi}.$$ Choice (A)

25. In the Fourier series of $f(x)$, b_n is given by $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nxdx$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\frac{-\pi}{2}} (-2)\sin nxdx + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} 0.\sin nxdx + \int_{\frac{\pi}{2}}^{\pi} 2\sin nxdx\right]$$

$$= \frac{2}{\pi}\left[\frac{\cos nx}{n}\right]_{-\pi}^{\frac{-\pi}{2}} + \frac{2}{\pi}\left[\frac{-\cos nx}{n}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\cos n\frac{\pi}{2}}{n} - \frac{\cos n\pi}{n} - \frac{\cos n\pi}{n} + \frac{\cos n\frac{\pi}{2}}{n}\right]$$

$$= \frac{4}{n\pi}\left[\cos\frac{n\pi}{2} - \cos n\pi\right].$$ Choice (B)

ENGINEERING MATHEMATICS TEST 4

Number of Questions: 25 **Time: 60 min.**

(PROBABILITY AND STATISTICS)

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. If A, B and C are mutually exclusive such that $5P(B) = 8\ P(A)$, $4P(C) = 3P(B)$ and $19P(A \cup B) = 13$ then $P(A \cup B \cup C)$ is _____.
(A) 0.75
(B) 1
(C) 0.625
(D) Cannot be determined

2. 240 passengers travelling in a plane from Hyderabad to Sharjah like one or more of the three meals among sandwich, burger and pizza as given below. 140 passengers like sandwich,110 passengers like burger and 90 passengers like pizza. 40 of them like both sandwich and burger, 50 of them like both Sandwich and Pizza, 40 of them like both burger and pizza where as 30 of them like all the three meals. What is the probability that a randomly selected passenger likes pizza only?
(A) 0.125 (B) 0.250
(C) 0.375 (D) 0.500

3. Let S be the set of all 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9. Probability that a randomly selected number of S has all digits distinct is _____.
(A) $\frac{5}{12}$ (B) $\frac{5}{18}$
(C) $\frac{5}{24}$ (D) $\frac{5}{36}$

4. An unbiased coin is tossed until it shows up the same face in two consecutive throws. What is the probability that the number of tosses is not more than 4?
(A) $\frac{3}{4}$ (B) $\frac{1}{8}$
(C) $\frac{7}{8}$ (D) $\frac{1}{4}$

5. What is the probability that a quadratic equation $ax^2 + bx + c = 0$ has equal roots if a, b and c are distinct and are taken from $\{1, 2, 3, 4, 6, 8, 9\}$?
(A) $\frac{1}{35}$ (B) $\frac{2}{35}$
(C) $\frac{1}{105}$ (D) $\frac{2}{105}$

6. A bag contains 4 five rupee coins, 3 two rupee coins and 3 one rupee coins. If 6 coins are drawn from the bag at random, what are the odds in favour of the draw yielding maximum amount?
(A) 1 : 70 (B) 1 : 69
(C) 69 : 70 (D) 70 : 1

7. Kids and Toys factory is transporting balls of 5 different colours – yellow, blue, red, green and white. Mr. Bholeram, a worker in the factory has to separate these balls as per their colours into different boxes and label them with the corresponding coloured labels.

Mr. Bholeram, after separating the balls, sealed the boxes and then labelled the boxes at random. What is the probability that all the boxes are incorrectly labelled?
(A) 1 (B) 0
(C) $\frac{11}{120}$ (D) $\frac{11}{30}$

8. While shuffling a pack of cards, 4 cards are accidentally dropped. The probability that all of them are numbered cards (2 to 10) of the same suit is
(A) $\frac{4 \times {}^{9}C_4}{{}^{52}C_4}$ (B) $\frac{\left({}^{9}C_4\right)^4}{{}^{52}C_4}$
(C) $\frac{4 \times {}^{9}C_1}{{}^{52}C_4}$ (D) $\frac{\left({}^{9}C_1\right)^4}{{}^{52}C_4}$

9. Arpit and Bipin pick up a ball at random from a bag containing 5 violet, 2 red and 3 orange balls one after the other, replacing it every time till one of them gets an orange ball and the one who first gets an orange ball is declared a winner. If Arpit begins the game, then the probability of Bipin winning the game is
(A) $\frac{10}{17}$ (B) $\frac{7}{17}$
(C) $\frac{7}{10}$ (D) $\frac{3}{10}$

10. An urn A contains 6 white balls and 7 black balls. And urn B contains 8 white balls and 6 black balls. A person draws a ball at random from one of the two urns. It turns out to be black. What is the probability that the ball was drawn from urn A?
(A) $\frac{7}{14}$ (B) $\frac{49}{88}$
(C) $\frac{39}{88}$ (D) None of the above

11. The bivariate probability distribution of X and Y is as follows.

X \ Y	0	1	2
0	$\frac{1}{40}$	$\frac{2}{40}$	$\frac{3}{40}$
1	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{1}{40}$
2	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{7}{40}$
3	$\frac{4}{40}$	$\frac{5}{40}$	$\frac{8}{40}$

Find $P(X \le 1, Y = 2)$.

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$

(C) $\frac{3}{10}$ (D) $\frac{1}{10}$

12. In a book of 500 pages, there are 50 typing errors. Assuming that the number of errors per page follows poission distribution, find the probability that randomly chosen 5 pages will contain no error.

(A) 0.6065 (B) 0.6078

(C) 0.6538 (D) 0.3935

13. The continuous random variable X is uniformly distributed with mean 2 and variance 12. Find $P(X > 0)$.

(A) $\frac{1}{3}$ (B) $\frac{4}{5}$

(C) $\frac{2}{3}$ (D) $\frac{1}{5}$

14. X and Y are two independent normal variates with means 3, 6 and variances, 1, 9 respectively. Find the value of k such that $P(X + Y \le k) = P(9X - Y \ge 2k)$.

(A) 9.3 (B) 9.6

(C) 8.6 (D) 10.3

15. Bag A contains 9 white balls and 5 green balls. Bag B contains 6 white balls and 7 green balls. One ball is drawn from bag A and is placed in bag B. Now one ball is drawn at random from bag B. It is found that the ball is green. Find the probability that white ball is transferred from bag A.

(A) $\frac{20}{103}$ (B) $\frac{63}{103}$

(C) $\frac{80}{103}$ (D) $\frac{75}{103}$

16. A dice is rolled twice the sum of the numbers appearing is 7, what is the probability that at least one dice shows 3?

(A) $\frac{3}{7}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{4}{7}$

17. A random variable X has the following probability distribution.

X = xi	0	1	2	3	4
P(x = xi)	K	2K	3K	5K	4K

Then find $P(X \ge 2)$.

(A) $\frac{4}{5}$ (B) $\frac{1}{5}$

(C) $\frac{2}{5}$ (D) $\frac{1}{15}$

18. The standard error is _____.

(A) accepting the null hypothesis when it is false.

(B) rejecting the null hypothesis when it is true.

(C) the standard deviation of the sampling distribution of a statistic.

(D) the probability that the test statistic does not lie in the critical region.

19. In large sampling, the sampling distribution of means follows _____

(A) Normal distribution

(B) t – distribution

(C) F – distribution

(D) x^2 – distribution

20. Which of the following distributions is used to test the equality of variances of two populations from which two small random samples are drawn?

(A) Normal distribution

(B) t – distribution

(C) F – distribution

(D) x^2 – distribution

21. If a statistic s follows t – distribution with $\nu = 10$ degrees of freedom, then s^2 follows F – distribution with degrees of freedom (ν_1, ν_2) = _____.

(A) (1,9) (B) (1,10)

(C) (1,11) (D) (9,1)

22. In testing of hypothesis, if the test statistic is outside the critical region, then we will

P : Accept the null hypothesis

Q : Reject the null hypothesis

R : Accept the alternative hypothesis

S : Reject the alternative hypothesis

Which of the following is true?

(A) P only

(B) R only

(C) P and S only

(D) Q and R only

23. Three letters are placed into three addressed envelopes randomly. A random variable X denotes the number of letters placed into corresponding envelopes. Find the variance of X.

(A) $\frac{5}{6}$ (B) 2

(C) 1 (D) 3

24. The variance of the data $x, x+3, x+5, x+7, x+10$ is

(A) 11.2 (B) 11.6

(C) $11.6 + x$ (D) $11.2 + x$

25. The median of the following data can be 3, 8, 12, 28, 16, 15, x

(A) 13 (B) 14

(C) 15 (D) Any of the above

Answer Keys

1. B	**2.** A	**3.** B	**4.** C	**5.** C	**6.** B	**7.** D	**8.** A	**9.** B	**10.** B
11. D	**12.** A	**13.** C	**14.** B	**15.** B	**16.** C	**17.** A	**18.** C	**19.** A	**20.** C
21. B	**22.** C	**23.** C	**24.** B	**25.** D					

Hints and Explanations

1. Given $5P(B) = 8P(A)$ and $4P(C) = 3P(B)$

$\Rightarrow P(A) = \frac{5}{8}P(B)$ and $P(C) = \frac{3}{4}P(B)$ ----- (1)

Now $19\,P(A \cup B) = 13 \Rightarrow P(A \cup B) = \frac{13}{19}$

$P(A) + P(B) = \frac{13}{19}$ ($\because$ A and B mutually exclusive)

$\Rightarrow \frac{5}{8}P(B) + P(B) = \frac{13}{19}$

$\Rightarrow \frac{13}{8}P(B) = \frac{13}{19}$

$\Rightarrow P(B) = \frac{8}{19}$

Now $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= \frac{5}{8}P(B) + P(B) + \frac{3}{4}P(B)$

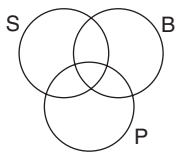
$= \left(\frac{5}{8} + 1 + \frac{3}{4}\right)P(B) = \frac{19}{8}P(B) = \frac{19}{8} \times \frac{8}{19} = 1.$

Choice (B)

2. The total number of passengers = 240

Let S, B and P denote the sets of passengers who like sandwich, burger and pizza respectively.

$\therefore$ $n(S) = 140$, $n(B) = 110$, $n(P) = 90$, $n(S \cap B) = 40$, $n(B \cap P) = 40$, $n(P \cap S) = 50$ and $n(S \cap B \cap P) = 30$



$\therefore$ Probability that a randomly selected passenger likes only pizza

$= \frac{(\text{The number of passengers who like only pizza})}{(\text{The total number of passengers})}$

$= \frac{n(P) - n(P \cap S) - n(B \cap P) + n(S \cap B \cap P)}{240}$

$= \frac{90 - 40 - 50 + 30}{240} = \frac{1}{8} = 0.125.$ Choice (A)

3. The number of 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9

= The number of elements of $S = 6^4$

The number of 4 digit numbers of S that have all digits distinct = The number of 4 digit numbers that can be formed using the digits 2, 3, 5, 7, 8 and 9 = ${}^6P_4$

$\therefore$ Probability that a randomly selected number of S has all digits distinct $= \frac{{}^6P_4}{6^4} = \frac{5}{18}$ Choice (B)

4. The number of tosses may be 2 or 3 or 4.

The possible cases and their corresponding probabilities:

Case 1 : HH OR TT $\rightarrow 2\left(\frac{1}{2}\right)^2$

Case 2 : HTT OR THH $\rightarrow 2\left(\frac{1}{2}\right)^3$

Case 3 : $HTHH$ OR $THTT$ $\rightarrow 2\left(\frac{1}{2}\right)^4$

Hence, the required probability is

$2\left[\frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right] = \frac{7}{8}$ Choice (C)

5. Considering different values of a, b and c from the set $\{1, 2, 3, 4, 6, 8, 9\}$, we get different quadratic equations. As a, b and c are distinct, ${}^7P_3 = 210$ different quadratic equations can be formed.

$\therefore$ Total ways are 210

For the quadratic equation $ax^2 + bx + c = 0$ to have equal roots, $b^2 = 4ac$.

The possible combinations of a, b and c respectively are 1, 6, 9 and 9, 6, 1.

Hence favourable cases are 2

$\therefore$ Required probability $= \frac{2}{210} = \frac{1}{105}$. Choice (C)

6. We have 4 five rupee coins, 3 two rupee coins and 3 one rupee coins.
For the draw to yield a maximum amount, of the 6 coins drawn 4 should be five rupee coins and 2 should be two rupee coins. The required probability is

$$\frac{{}^4C_4 \times {}^3C_2}{{}^{10}C_6} = \frac{3}{210} = \frac{1}{70}$$

Hence, odds in favour are favourable ways : unfavourable ways = 1 : 69. Choice (B)

7. There are 5 boxes and 5 labels. Hence the boxes can be labelled in 5! i.e. 120 different ways

$$P \text{ (all labelled incorrectly)} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}$$

$$= \frac{44}{120} = \frac{11}{30}$$ Choice (D)

8. There are 9 numbered cards in each suit.
P(all the 4 cards are numbered cards of same suit)

$$= \frac{{}^9C_4 + {}^9C_4 + {}^9C_4 + {}^9C_4}{{}^{52}C_4} = \frac{4 \times {}^9C_4}{{}^{52}C_4}$$ Choice (A)

9. The probability of picking up an orange ball is $\frac{3}{10}$ while not picking up an orange ball is $\frac{7}{10}$.

We compute the probability of Arpit (the beginner) winning the game.
Let A and B be the events of Arpit and Bipin picking up an orange ball respectively
The winning sequence of Arpit can be

$A, \overline{A}\,\overline{B}\,A, \quad \overline{A}\,\overline{B}\,\overline{A}\,\overline{B}\,A, \ldots\ldots$

As the above sequence indicates, Arpit may pick an orange ball right in the 1st trial with a probability of $\frac{3}{10}$ (or) in the third trial (as the 2nd trial is made by Bipin, and for Arpit to win, Bipin should not be getting an orange ball). The probability here being $\left(\frac{7}{10}\right)^2 \times \frac{3}{10}$ (or) in the fifth trial with a probability of $\left(\frac{7}{10}\right)^4 \times \frac{3}{10}$ and so on.

$$\therefore \quad P(A) = \frac{3}{10} + \left(\frac{7}{10}\right)^2 \times \frac{3}{10} + \left(\frac{7}{10}\right)^4 \times \frac{3}{10} + \ldots\ldots$$

$$= \frac{\frac{3}{10}}{1 - \left(\frac{7}{10}\right)^2} = \frac{30}{51} = \frac{10}{17}$$

Probability of Bipin winning is the same as probability of Arpit losing i.e.,

$$\therefore \quad P(B) = P(\overline{A}) = 1 - \frac{10}{17} = \frac{7}{17}$$ Choice (B)

Note: If 'p' is the probability of success (in this case picking up an orange ball), the probability that the beginner wins the game $= \frac{1}{2-p}$

10. Probability of selecting urn A is $P(A) = \frac{1}{2}$.

and that of selecting urn B is $P(B) = \frac{1}{2}$

Probability of drawing a black ball (event E) when urn A is selected $P\left(\frac{E}{A}\right) = \frac{{}^7C_1}{{}^{13}C_1}$ and probability of E when urn B is selected $P\left(\frac{E}{B}\right) = \frac{{}^6C_1}{{}^{14}C_1}$

Probability of selecting black ball

$$= P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right)$$

$$\frac{1}{2} \cdot \frac{{}^7C_1}{{}^{13}C_1} + \frac{1}{2} \cdot \frac{{}^6C_1}{{}^{14}C_1}$$

$$\text{Required Probability} = \frac{\frac{1}{2} \cdot \frac{{}^7C_1}{{}^{13}C_1}}{\frac{1}{2} \cdot \frac{{}^7C_1}{{}^{13}C_1} + \frac{1}{2} \cdot \frac{{}^6C_1}{{}^{14}C_1}}$$

$$= \frac{\frac{7}{13}}{\frac{7}{13} + \frac{6}{14}} = \frac{\frac{7}{13}}{\frac{98+75}{13 \times 14}} = \frac{7 \times 14}{176} = \frac{49}{88}$$ Choice (B)

11. The marginal distributions are given below.

X \ Y	0	1	2	$P_x(x)$
0	$\frac{1}{40}$	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{6}{40}$
1	$\frac{2}{40}$	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{6}{40}$
2	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{7}{40}$	$\frac{11}{40}$
3	$\frac{4}{40}$	$\frac{5}{40}$	$\frac{8}{40}$	$\frac{17}{40}$
$P_y(y)$	$\frac{10}{40}$	$\frac{11}{40}$	$\frac{19}{40}$	1

$P(X \le 1, Y = 2) = P(X = 0, Y = 2) + P(X = 1, Y = 2)$

$= \frac{3}{40} + \frac{1}{40} = \frac{4}{40} = \frac{1}{10}$ Choice (D)

12. Average number of errors per page $\lambda = \frac{50}{500} = \frac{1}{10}$

Average number of errors per 5 pages = 5 x $\frac{1}{10} = \frac{1}{2}$

Probability of k errors per page is $P(x = k) = \frac{\lambda^k}{k!} e^{-\lambda}$.

$\therefore$ here $k = 0$

$\therefore$ $P(k = 0) = \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\frac{1}{2}}$

$\therefore$ Probability that a random sample of 5 pages has no error = $e^{-0.5} = 0.6065$ Choice (A)

13. We know that X is uniform random variable in the interval $[a, b]$ then $p(x) = \frac{1}{b-a}$, $a < x < b$ and mean = $\frac{a+b}{2}$, variance = $\frac{(b-a)^2}{12}$

Given mean = 2 $\Rightarrow \frac{a+b}{2} = 2$

$\Rightarrow$ $a + b = 4$ ---- (1)

Variance = 12 $\Rightarrow \frac{(b-a)^2}{12} = 12$

$\Rightarrow$ $(b - a)^2 = 144 \Rightarrow b - a = 12$ ------ (2)

Solving (1) and (2) $a = -4$, $b = 8$

$\therefore$ $P(x) = \frac{1}{b-a} = \frac{1}{8-(-4)} = \frac{1}{12}$

$$P(x > 0) = \int_0^b p(x)dx = \int_0^8 \frac{1}{12} dx = \frac{1}{12} x \Big]_0^8 = \frac{8}{12} = \frac{2}{3}$$

Choice (C)

14. Given mean of $X = 3$
Variance of $X = 1$
Mean of $Y = 6$
Variance of $Y = 9$
$X = N(3, 1)$ $Y = N(6, 3)$ and X and Y are independent.
Let $u = x + y$; and $v = 9x - y$
Then u, v are also normal variates
$U = x + y = N(3 + 6, 1 + 9) = N(9, 10)$
$V = 9x - y = N(9(3) - 6, 81(1) + 9) = N(21, 90)$
By definition

$Z = \frac{u-9}{\sqrt{10}}$ and for $u = k \Rightarrow Z = \frac{k-9}{\sqrt{10}} = z$

Again $z = \frac{v-21}{\sqrt{90}}$ and for $v = 2k \Rightarrow Z = \frac{2k-21}{\sqrt{90}} = z_1$

Given $P(x + y \le k) = p(9x - y \ge 2k)$
$P(z \le z_1) = p(z \ge z_2)$
$P(z \le z_1) = p(z \le -2z)$

$$= \frac{k-9}{\sqrt{10}} = \frac{-(2k-21)}{\sqrt{90}} = k - 9$$

$$= \frac{-2k+21}{3}$$

$3k - 27 = -2k + 21$

$5k = 48 \Rightarrow k = \frac{48}{5} = 9.6$ Choice (B)

15. Let B_1: transfer of white ball to bag B.
B_2: transfer of green ball to bag B.

$P(B_1) = \frac{9}{14}; P(B_2) = \frac{5}{14}$

Let E be the event of drawing a green ball from bag B after transfer.

$P\left(\frac{E}{B_1}\right)$ = probability of drawing green ball if white ball is transferred to bag $B = \frac{7}{14}$

$P\left(\frac{E}{B_2}\right)$ = probability of drawing a green ball if green ball is transferred to bag $B = \frac{8}{14}$.

$\therefore$ $P(E) = P(B_1) \cdot P\left(\frac{E}{B_1}\right) + P(B_2) \cdot P\left(\frac{E}{B_2}\right)$

$$= \frac{9}{14} \cdot \frac{7}{14} + \frac{5}{14} \cdot \frac{8}{14} = \frac{63}{196} + \frac{40}{196} = \frac{103}{196}$$

$\therefore$ The required probability $P\left(\frac{B_1}{E}\right)$

$$= \frac{P(B_1).P\left(\frac{E}{B_1}\right)}{P(E)} = \frac{\frac{9}{14} \cdot \frac{7}{14}}{\frac{103}{196}} = \frac{63}{103}$$ Choice (B)

16. Let A be the event that the number 3 appears atleast once.
B be the event that sum of the numbers appearing is 7.
$A \cap B$ be the event that the sum is 7 and 3 appear atleast once $P(B) = \frac{6}{36} \Rightarrow P(A \cap B) = \frac{2}{36}$

A/B denotes atleast one number show 3 while the sum of the numbers is 7.

$$P\left(\frac{A}{B}\right) = \frac{P(B \cap A)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$$ Choice (C)

17. We know $\sum P(X = x_1) = 1$

$\therefore\ k + 2k + 3k + 4k + 5k = 1$

$15k = 1 \Rightarrow k = \dfrac{1}{15}$

$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$

$= 3k + 5k + 4k = 12k = \dfrac{12}{15} = \dfrac{4}{5}$ Choice (A)

18. By definition. Choice (C)

19. Standard Result. Choice (A)

20. Standard Result. Choice (C)

21. We know that, if a statistic 's' follows t-distribution with degrees of freedom = ν, then 's^2' follows F–distribution with degrees of freedom $(1, \nu)$

Here $\nu = 10$

$\therefore$'s^2' follows F-distribution with degrees of freedom $= (1, \nu) = (1,10)$. Choice (B)

22. When the test statistic is outside the critical region, it lies in the acceptance region. So, we will accept the null hypothesis and reject the alternative hypothesis.

$\therefore$ Both P and S are true. Choice (C)

23. Three letters are placed into 3 addressed envelopes randomly in 3! = 6 ways.

X denotes the number of letters placed into corresponding addressed envelopes. The probability distribution table is as follows.

$X = x_i$	0	1	2	3
$P(x = x_i)$	$\frac{2}{6}$	$\frac{3}{6}$	0	$\frac{1}{6}$

$\therefore$ mean $(M) = \sum X_i\, P\,(x = x_i)$

$= 0 \times \dfrac{2}{6} + 1 \times \dfrac{3}{6} + 2 \times \dfrac{0}{6} + 3 \times \dfrac{1}{6}$

$= \dfrac{0 + 3 + 0 + 3}{6} = \dfrac{6}{6} = 1$

Variance $= \sum x_i^2\, P(x = x_i) - \mu^2$

$= 0 \times \dfrac{2}{6} + 1 \times \dfrac{3}{6} + 4 \times 0 + 9 \times \dfrac{1}{6} - 1$

$= \dfrac{3 + 9}{6} - 1 = 2 - 1 = 1.$ Choice (C)

24. We know that variance $(x, x + 3, x + 5, x + 7, x + 10)$ = variance (0, 3, 5, 7, 10)

$AM(0, 3, 5, 7, 10) = \dfrac{0 + 3 + 5 + 7 + 10}{5} = \dfrac{25}{5} = 5$

Variance $(0, 3, 5, 7, 10) = \dfrac{\sum (x_1 - A)^2}{n}$

$= \dfrac{(0-5)^2 + (3-5)^2 + (5-5)^2 + (7-5)^2 + (10-5)^2}{5}$

$= \dfrac{25 + 4 + 0 + 4 + 25}{5} = \dfrac{58}{5}$

Variance $= \dfrac{58}{5} = 11.6$. Choice (B)

25. The ascending order of the given data except x is 3, 8, 12, 15, 16, 28

If $x < 12$, the fourth observation is 12 hence median is 12 if $x > 15$, the fourth observation is 15, hence median is 15. If $12 < x < 15$, the fourth observation is x, hence median is x.

Median is always lies between [12, 15]. Choice (D)

ENGINEERING MATHEMATICS TEST 5

Number of Questions: 25 **Time: 60 min.**

NUMERICAL METHODS

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. In the process of finding an approximate root of $f(x) = 0$ in $[a, b]$ (where $f(a)$ and $f(b)$ are of opposite signs) by Regula – Falsi method, we assume that the curve $f(x) = 0$ in between $x = a$ and $x = b$ can be approximated to _____.

(A) a parabola
(B) a straight line
(C) a hyperbola
(D) a rectangular hyperbola

2. The iterative formula to find a root of the equation $f(x) = x^3 - 5x + 7 = 0$ by Newton Raphson method is _____.

(A) $x_{k+1} = \dfrac{x_k^3 + 5x - 7}{3x_k^2 + 5}$ (B) $x_{k+1} = \dfrac{2x_k^3 + 5x}{3x_k^2 + 7}$

(C) $x_{k+1} = \dfrac{2x_k^3 - 7}{3x_k^2 - 5}$ (D) $x_{k+1} = \dfrac{x_k^3 - 5x}{3x_k^2 + 7}$

3. With $x_0 = 0.5$ as the initial approximation, the value of the root of $f(x) = x + \sin x - 1 = 0$, after first iteration by Newton Raphson method is _____.

(A) 0.7456 (B) 0.5110
(C) 0.4998 (D) 0.2644

4. Applying the secant method, the first approximation to the root of $f(x) = xe^x - 2 = 0$, starting with function value at $x = 0.5$ and $x = 1$ is _____.

(A) 1.1756 (B) 0.4035
(C) 0.8104 (D) 0.5473

5. The extreme (minimum or maximum) point of a function $f(x)$ is to be determined by solving $\dfrac{df(x)}{dx} = 0$ using the Newton Raphson method. Let $f(x) = x^3 - 4x^2 + 5$ and $x_0 = 3$ be the initial guess of x. The value of x after first iteration (x_1) is _____.

(A) 2.70 (B) 4.33
(C) 3.30 (D) 1.77

6. In the process of evaluating $\int_0^{\frac{\pi}{2}} \left(x^3 + \sin 2x + 5\right) dx$ using Simpson's Rule with $h = \dfrac{\pi}{8}$, the absolute value of the error does not exceed _____.

(A) 0.12351×10^{-4} (B) 1.03503×10^{-4}
(C) 3.01243×10^{-4} (D) 6.2475×10^{-4}

7. The following table gives the velocity ν of a particle at time t.

T(in seconds)	0	2	4	6	8	10	12
n(in m/sec)	6	10	16	20	22	30	40

The distance moved by the particle in 12 seconds, when calculated by the Trapeziodal rule with $h = 2$ is _____.

(A) 200 meters (B) 210 meters
(C) 242 meters (D) 262 meters

8. A curve is drawn to pass through the points given by the following table

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Y	1.0	1.7	2.5	3.4	4.1	3.7	2.9

The area bounded by the curve, the x – axis and the lines $x = 1$ and $x = 4$, when calculated by the Simpson's $\dfrac{3}{8}th$ Rule is _____ square units.

(A) 8.7562 (B) 5.7435
(C) 6.7134 (D) 8.4296

9. The absolute error (correct up to 4 decimal places) in calculating the value $\log_e 2$ by trapezoidal rule, with 4 intervals using the formulae $\log_e 2 = \int_1^2 \dfrac{dx}{x}$ is _____.

(A) 0.1314 (B) 0.0039
(C) 0.0000 (D) 0.0004

10. With reference to finding solution of a differential equation by numerical methods, which of the following methods is NOT a predictor correct method?

(A) Picard's method
(B) Modified Euler's method
(C) Adams – Bash forth method
(D) Milne's method

11. The differential equation $\dfrac{dy}{dx} - x^2 = y$; $y(0) = 1$ is to be solved by the modified Euler's method. With $h = 0.1$, the value of y_1 correct to four decimal places is _____.

(A) 1.2046 (B) 1.1058
(C) 0.9954 (D) 0.8764

12. Using Taylor's series method, the solution of the differential equation $\dfrac{dy}{dx} - xy = 1$ with $y(0) = 3$ at $x = 0.1$ with $h = 0.1$ is correct upto three decimal places is _____.

(A) 3.1153 (B) 2.9847
(C) 4.1572 (D) 3.7893

13. The solution of the differential equation $\frac{dy}{dx} = x + y$; $y(0) = 0$ at $x = 0.2$ by Runge Kutta method of fourth order with $h = 0.2$ is _____.
(A) 1.0034 (B) 0.0456
(C) 0.9984 (D) 0.0214

14. Consider an equation $f(x) = 0$ for which $x = 4.50$ is an exact root. In the process of finding a root of $f(x) = 0$ by a numerical method, the approximations obtained in four successive iterations are 4.45, 4.54, 4.47 and 4.52 respectively. Then these approximate values of the root of $f(x) = 0$ are _____.
(A) precise but not accurate
(B) not precise but accurate
(C) both precise and accurate
(D) neither precise nor accurate

15. For an equation $f(x) = 0$, if x_e is the exact root and x_a is the approximate root, then the percentage error is _____.
(A) $(x_e - x_a) \times 100$ (B) $|x_e - x_a| \times 100$
(C) $\frac{(x_e - x_a)}{x_e} \times 100$ (D) $\frac{|x_e - x_a|}{|x_e|} \times 100$

16. Consider the following two statements
P: Truncation error in numerical analysis arise when approximations are used to estimate some quantity.
Q: Round off error in numerical analysis occurs because of the computing devices inability to deal with certain number.
Then
(A) Both *P* and *Q* are true
(B) *P* is true but *Q* is false
(C) *P* is false but *Q* is true
(D) Both *P* and *Q* are false

17. In the process of fitting a quadratic equation of the form $y = a + bx + cx^2$ to a set of n points (x_1, y_1), (x_2, y_2), ….., (x_n, y_n) by the method of least squares, which of the following is not a normal equation?
(A) $\sum y_i = na + b\sum x_i + c\sum x_i^2$
(B) $\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3$
(C) $\sum x_i y_i^2 = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$
(D) $\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$

18. If $y = 3x + 7$ is the best fit for 6 pairs of values of x and y by the method of least squares and $\sum y = 150$, then $\sum x$ is _____.
(A) 144 (B) 102
(C) 46 (D) 36

19. In the process of fitting a curve $y = \frac{x^2}{ax + b}$ to a given set of *n* pairs of values of x and y by converting it into a linear form $Y = a + bX$, X and Y respectively stand for _____.
(A) $\frac{1}{x^2}$ and $\frac{x^2}{y}$ (B) $\frac{1}{x}$ and $\frac{x}{y}$
(C) x^2 and $\frac{y}{x^2}$ (D) x and $\frac{y}{x}$

20. In the process of fitting a curve $\exp(y) = ab^x$ to a given set of n pairs of values of x and y by converting it into a linear form $y = A + Bx$, A and B respectively stand for _____
(A) ln *a* and ln *b* (B) ln *a* and $\log_{10} b$
(C) $\log_{10} a$ and ln *b* (D) $\log_{10} a$ and $\log_{10} b$

21. If Δ denotes the forward difference operator then the value of $\Delta^{18}\left[\left(1+2x^3\right)\left(1-3x^4\right)\left(1+4x^5\right)\left(1-5x^6\right)\right]$ is _____.
(A) $5! \times 18!$
(B) $6! \times 18!$
(C) $5! \times 17!$
(D) $6! \times 17!$

22. The central difference operator δ is defined as
$y_r - y_{r-1} = \delta y_{r-\frac{1}{2}}$
Then which of the following is an identity? (Note that Δ and ∇ denote the forward and the backward difference operators respectively)
(A) $\Delta y_5 = \nabla y_4 = \delta y_{\frac{7}{2}}$
(B) $\Delta y_5 = \nabla y_4 = \delta y_4$
(C) $\Delta y_4 = \nabla y_5 = \delta y_{\frac{9}{2}}$
(D) $\Delta y_4 = \nabla y_5 = \delta y_4$

23. Match the following

	Group – I		Group – II
P.	To extrapolate the values of y to the left of y_0 when x values are equally spaced	1.	Newton's divided difference formula
Q.	To interpolate the values of y near the end value y_n when x values are equally spaced	2.	Lagrange's interpolation formula
R.	To split the given function into partial fractions	3.	Newton's forward interpolation formula
S.	To interpolate the values of y when x values are unequally spaced.	4.	Newton's backward interpolation formula

(A) $P-(1), Q-(2), R-(3), S-(4)$
(B) $P-(3), Q-(2), R-(4), S-(1)$
(C) $P-(3), Q-(4), R-(2), S-(1)$
(D) $P-(2), Q-(1), R-(4), S-(3)$

24. The 9th divided difference of a polynomial of degree 8 is _____.
(A) zero
(B) a non – zero constant
(C) a linear polynomial
(D) a quadratic polynomial

25. If $f(0) = -12$, $f(3) = 6$ and $f(4) = 12$, then the value of $f(6)$ obtained by the Lagrange's interpolation formula is _____.
(A) 18 (B) 24
(C) 20 (D) 26

Answer Keys

1. B	**2.** C	**3.** B	**4.** C	**5.** A	**6.** B	**7.** C	**8.** A	**9.** B	**10.** A
11. B	**12.** A	**13.** D	**14.** B	**15.** D	**16.** A	**17.** C	**18.** D	**19.** B	**20.** A
21. A	**22.** C	**23.** C	**24.** A	**25.** B					

Hints and Explanations

1. Standard Result. Choice (B)

2. Given, $f(x) = x^3 - 5x + 7 = 0$
$\Rightarrow f^1(x) = 3x^2 - 5$.
By Newton Raphson's method, the interative formulae to find a root is

$$x_{k+1} = x_k - \frac{f(x_k)}{f^1(x_k)} = x_k - \frac{(x_k^3 - 5x_k + 7)}{(3x_k^2 - 5)}$$

$$\therefore \quad x_{k+1} = \frac{2x_k^3 - 7}{3x_k^2 - 5}.$$ Choice (C)

3. Given, $f(x) = x + \sin x - 1 = 0$
$\Rightarrow f^1(x) = 1 + \cos x$ and $x_0 = 0.5$
By Newton Raphson's method

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$

$$= (0.5) - \frac{(0.5 + \sin(0.5) - 1)}{(1 + \cos(0.5))}$$

$$= 0.5110.$$ Choice (B)

4. Here, $f(x) = xe^x - 2 = 0$
By the secant method, the approximate root of $f(x) = 0$ after first iteration is given by

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$ ---------- (1)

Here, $x_0 = 0.5$ and $x_1 = 1$
$\therefore \quad f(x_0) = f(0.5) = -1.1756$ and $f(x_1) = f(1) = 0.7183$
Substituting these in (1) we have

$$x_2 = \frac{(0.5)(0.7183) - (1)(-1.1756)}{(0.7183) - (-1.1756)}$$

$\therefore \quad x_2 = 0.8104.$ Choice (C)

5. Given, $f(x) = x^3 - 4x^2 + 5$

$$\frac{df(x)}{dx} = 0 \Rightarrow 3x^2 - 8x = 0$$

Let, $g(x) = 3x^2 - 8x = 0$.

$\therefore$ We have to find the approximate root of $g(x) = 0$ after first interaction by the Newton Raphson method with $x_0 = 3$.
$\therefore \quad g^1(x) = 6x - 8$
By Newton Raphson method

$$x_1 = x_0 - \frac{g(x_0)}{g^1(x_0)}$$

$$= 3 - \frac{(3(3)^2 - 8(3))}{6(3) - 8} = 2.7.$$ Choice (A)

6. We have $\int_0^{\frac{\pi}{2}} (x^3 + \sin 2x + 5)dx$.

Let $y = f(x) = x^3 + \sin 2x + 5$.

The absolute value of the maximum error in Simpson's Rule is

$$|E|_{\max} = \frac{(b-a)h^4}{180} m$$ ---------- (1)

Where $m = \max - \{y_0^{(iv)}, y_2^{(iv)}, y_4^{(iv)}\}$

Here, $h = \frac{\pi}{8}$ and $y^{(iv)} = 16 \sin 2x$, $a = 0$, $b = \frac{\pi}{2}$

$\therefore \quad m = \max [16\sin 0, 16\sin\left(2 \times \frac{\pi}{4}\right),$

$16 \sin\left(2 \times \frac{\pi}{2}\right)] = 16$

From (1)

$$|E_{\max}| = \frac{\left(\frac{\pi}{2}\right)\left(\frac{\pi}{8}\right)}{180} \times 16 = 1.0350 \times 10^{-4}$$

The absolute value of the maximum error cannot exceed 1.03503×10^{-4}.

Choice (B)

7. Given velocity of the particle at various times is

T	0	2	4	6	8	10	12
n	6	10	16	20	22	30	40

Distance traveled in 12 seconds = $\int_0^{12} v\,dt$.

By trapezoidal rule

$$\int_0^{12} v\,dt = \frac{h}{2}[(v_0 + v_6) + 2(v_1 + v_2 + v_3 + v_4 + v_5)]$$

$$= \frac{2}{2}[(6 + 40) + 2(10 + 16 + 20 + 22 + 30)]$$

$$= 242 \text{ meters.}$$ Choice (C)

8. Let $y = f(x)$ be the curve, that pass through the points

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Y	1.0	1.7	2.5	3.4	4.1	3.7	2.9

$\therefore$ The area bounded by the curve $y = f(x)$, x – axis andthe lines $x = 1$ and $x = 4$ is $\int_1^4 f(x)dx$.

By Simpson's $\frac{3}{8}th$ Rule, we have

$$\int_1^4 f(x)dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3]$$

$$= \frac{3\times(0.5)}{8}[(1.0 + 2.9) + 3(1.7 + 2.5 + 4.1 + 3.7) + 2 \times 3.4] = 8.7562.$$

Choice (A)

9. Let $y = f(x) = \frac{1}{x}$

Here, $a = 1$, $b = 2$ and $n = 4$

$$\therefore \quad h = \frac{b-a}{n} = 0.25$$

X	1	1.25	1.5	1.75	2
F(x)	1	0.8	0.667	0.5714	0.5

We have $\log_e 2 = \int_1^2 \frac{dx}{x}$

By the trapezoidal rule, we have

$$\int_1^2 \frac{dx}{x} = \int_1^2 f(x)dx = \frac{h}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$$

$$\frac{0.25}{2}[(1+ 0.5) + 2(0.8 + 0.6667 + 0.5714)]$$

$$\therefore \quad \log_e 2 = \int_1^2 \frac{dx}{x} = 0.6970 \qquad \text{---------- (1)}$$

The exact value of $\log_e 2 = 0.6931$ -------- (2)

The absolute error in calculating $\log_e 2$ by the trapezoidal rule = 0.6970 – 0.6931 = 0.0039. Choice (B)

10. A predictor corrector method is one in which we predict the solution first and then we improve it for accuracy Picard's method is not a predictor corrector method and all other methods are predictor corrector methods. Choice (A)

11. Given different equation is $\frac{dy}{dx} - x^2 = y$ and $y(0) = 1$

$$\Rightarrow \quad \frac{dy}{dx} = x^2 + y$$

$\therefore \quad f(x, y) = x^2 + y$, $x_0 = 0$, $y_0 = y(x_0) = 1$ and $h = 0.1$

By Euler's method

$$y_1^{(0)} = y_0 + h\,(f(x_0, y_0) = y_0 + h\left(x_0^2 + y_0\right)$$

$$= 1 + (0.1)\,(0 + 1)$$

$$y_1^{(0)} = 1.1$$

By modified Euler's method

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f\left(x_0, y_0\right) f\left(x_1 y_1^{(0)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(0)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + (10.1)^2 + 1.1)] = 1.1055$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(1)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(1)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + ((0.1)^2 + 1.1055)]$$

$$= 1.1058$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(2)}\right)\right]$$

$$= y_0 + \frac{h}{2}\left[\left(x_0^2 + y_0\right) + \left(x_1^2 + y_1^{(2)}\right)\right]$$

$$= 1 + \frac{(0.1)}{2}[(0 + 1) + ((0.1)^2 + 1.1058)]$$

$$= 1.1058$$

The solution of a given different equation at $x_1 = 0.1$ is $y_1 = 1.1058$. Choice (B)

12. Given differential equation is $\frac{du}{dx} - xy = 1$ and $y(0) = 3$

$$\Rightarrow \quad \frac{dy}{dx} = 1 + xy$$

Here $f(x, y) = 1 + xy$, $x_0 = 0$, $y_0 = y(x_0) = 3$ and $h = 0.1$

By Taylar's series we have

$$y_1 = y(x_1) = y_0 + h_0 y_0^1 + \frac{h^2}{2!} y_0^{11} + \frac{h^3}{3!} y_0^{111} + \ldots\ldots \infty$$

----(1)

$$y_0^1 = \left(\frac{dy}{dx}\right)_{x=0} = f(x_0, y_0) = 1 + x_0 y_0 = 1 + (0)(3) = 1$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(1 + xy) = xy^1 + y$$

$$\therefore \quad y_0^{11} = \left(\frac{d^2y}{dx^2}\right)_{atx=x_0} = x_0\ y_0^1 + y_0 = 0 \times 1 + 3 = 3$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}(xy^1 + y) = xy^{11} + 2y^1$$

$$\therefore \quad y_0^{111} = \frac{d^3y}{dx^3}_{atx=x_0} = x_0\ y_0^{11} + 2\ y_0^1 = 0 \times 3 + 2 \times 1 = 2$$

$$\frac{d^4y}{dx^4} = \frac{d}{dx}\left(\frac{d^3y}{dx^3}\right) = \frac{d}{dx}(xy^{11} + 2y^1) = xy^{111} + 3y^{11}$$

$$y_0^{(iv)} = \frac{d^4y}{dx^4}_{atx=x_0} = x_0\ y_0^{111} + 3\ y_0^{11} = 0 \times 2 + 3 \times 3 = 9$$

Substituting these in (1), we have

$$y_1 = 3 + (0.1) \times 1 + \frac{(0.1)^2}{2!} \times 3 + \frac{(0.1)^3}{3!} \times 2 + \frac{(0.1)^4}{4!} \times$$

$9 + \ldots = 3.1153$. Choice (A)

13. Given differential equation is $\frac{dy}{dx} = x + y, y(0) = 0$

Here $f(x, y) = x + y$, $x_0 = 0$; $y_0 = 0$ and $h = 0.2$

$\therefore \quad x_1 = x_0 + h = 0.2$

By R – K method of fourth order we have

$$Y_{\text{at } x=0.1} = y_1 = y_0 + \Delta y \qquad \text{---------- (1)}$$

Where $\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$ ------- (2)

Here $k_1 = h\, f(x_0, y_0) = h(x_0 + y_0)$

$= (0.2)\ (0 + 0)$

$\therefore \quad k_1 = 0$

$$k_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= h\left(\left(x_0 + \frac{h}{2}\right) + \left(y_0 + \frac{k_1}{2}\right)\right)$$

$$= (0.2)\left[\left(0 + \frac{(0.2)}{2}\right) + \left(0 + \frac{0}{2}\right)\right]$$

$\therefore \quad k_2 = 0.02$

$K_3 = 0.022$ and $k_4 = h\, f(x_0 + h, y_0 + k_3)$

$= h\,[(x_0 + h) + (y_0 + k_3)]$

$= (0.2)\,[(0 + 0.2) + (0 + 0.022)]$

$\therefore \quad k_4 = 0.0444$

$\therefore$ From (2),

$$\Delta y = \frac{1}{6}\,[0 + 2 \times 0.02 + 2 \times 0.022 + 0.0444]$$

$\Delta y = 0.0214$

$\therefore$ From (1), $y_1 = y_0 + \Delta y = 0 + 0.0214 = 0.0214$. Choice (D)

14. The four approximations given are not relatively close to each other. So, they are not precise. All the four approximations are close to the exact root $x = 4.50$. So, they are accurate. Choice (B)

15. By definition. Choice (D)

16. By definitions of the round off and the truncation errors. Choice (A)

17. Given set of n points are $(x_1, y_1,)$, $(x_2, y_2), \ldots, (x_n, y_n)$

We have to fit the quadratic equation

$y = a + bx + cx^2$ → (1)

to the given set of n points

Here a, b and c are constants to be determined such that

$$S = \sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]^2 \qquad \rightarrow (2)$$

is minimum

S is minimum for those values of a, b and c at which

$$\frac{\partial S}{\partial a} = 0, \frac{\partial S}{\partial b} = 0 \text{ and } \frac{\partial S}{\partial c} = 0.$$

i.e., $-2\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$

$$-2x_i\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$$

and $-2x_i^2\left(\sum\left[y_i - \left(a + bx_i + cx_i^2\right)\right]\right) = 0$

$\therefore$ the normal equations are

$\Rightarrow \quad \sum y_i = na + b\sum x_i + c\sum x_i^2$

$$\sum x_i y_i = a\sum x_i + b\sum x_i^2 + c\sum x_i^3$$

And $\sum x_i^2 y_i = a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^4$

$\therefore$ the equation given in option (C) is not a normal equation. Choice (C)

18. Given that $y = 3x + 7$ is the best fit for 6 pairs of values of x and y also given $\sum y = 150$

$\therefore$ We know that

$\sum y = 3\sum x + n7$

(Here n = number of points = 6)

$\Rightarrow \quad 150 = 3\ \sum x + 6\ x\ 7$

$\Rightarrow \quad 3\ \sum x = 108$

$\Rightarrow \quad \sum x = 36$. Choice (D)

19. Given curve is $y = \frac{x^2}{ax + b}$

$$\Rightarrow \quad \frac{1}{y} = \frac{ax + b}{x^2}$$

$$\Rightarrow \quad \frac{}{y} = \frac{a}{x} + \frac{b}{x}$$

$$\Rightarrow \quad \frac{x}{y} = a + \frac{b}{x}$$

$$\Rightarrow \quad \frac{x}{y} = a + \frac{b}{x}$$

Which is of the linear from $Y = a + bX$

Where $X = \frac{1}{x}$ and $Y = \frac{x}{y}$ Choice (B)

20. Given curve is $\exp(y) = ab^x$

i.e., $e^y = ab^x$

Applying logarithm (ln) on both sides,

We have

$\ln(e^y) = \ln(ab^x)$

$\Rightarrow \quad y = \ln a + \ln b^x$

$\Rightarrow \quad y = \ln a + x \ln b$

Which is of the form

$y = A + Bx$

Where $A = \ln a$ and $B = \ln b$. Choice (A)

21. We have $\Delta^{18}\left[\left(1+2x^3\right)\left(1-3x^4\right)\left(1+4x^5\right)\left(1-5x^6\right)\right]$

$= \Delta^{18}\left[2x(-3)x4x(-5)x^{18} + k_1x^{17} + k_2x^{16} + ... + k_{15}x^3 + 1\right]$

$= 5!\ \Delta^{18}[x^{18}]$

$(\because \Delta^{18}[x^n] = 0$ for $n < 18)$

$= 5\ ! \times 18\ !.$ Choice (A)

22. We know that $\Delta y_{r-1} = y_r - y_{r-1}$ $\rightarrow (1)$

$\nabla y_r = y_r - y_{r-1}$ $\rightarrow (2)$

and given that $\delta y_{r-\frac{1}{2}} = y_r - y_{r-1}$ $\rightarrow (3)$

From (1), (2) and (3), we have

$\Delta y_{r-1} = \nabla y_r = \delta y_{r-\frac{1}{2}}$ $\rightarrow (4)$

Among the options given, we can get option (C) by taking $r = 5$ in (4),

$\therefore \quad \Delta y_4 = \nabla y_5 = \delta y_{\frac{9}{2}}$ Choice (C)

23. Standard result Choice (C)

24. We know that the n^{th} divided difference of any polynomial of degree less than n is always zero. Choice (A)

25. Given pairs of values of x and $f(x)$ are

X	0	3	4
f(x)	–12	6	12

By Lagrange's interpolation formula, we have

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{(x-3)(x-4)}{(0-3)(0-4)} \times (-12) + \frac{(x-0)(x-4)}{(3-0)(3-4)} \times 6 + \frac{(x-0)(x-3)}{(4-0)(4-3)} \times 12$$

By taking $x = 6$ on both sides,

We have

$$f(6) = \frac{(6-3)(6-4)}{3\times 4} \times (-12) + \frac{(6-0)(6-4)}{3\times(-1)} \times 6 + \frac{(6-0)(6-3)}{4\times 1} \times 12$$

$\therefore \quad f(6) = 24$ Choice (B)

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UNIT I
ENGINEERING MECHANICS

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Engineering Mechanics Test 1

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The value of coefficient of restitution is one for
(A) perfectly elastic collision
(B) perfectly inelastic collision
(C) neither plastic nor elastic collision
(D) None of these

2. The radius of gyrations for a sphere and cylinder of radius 'R' are respectively.
(A) 0.6324 R and 0.707 R
(B) 0.6234 R and 0.77 R
(C) 0.6432 R and 1.414 R
(D) 0.6324 R and 1.414 R

3. Which of the following relation represents motion under variable acceleration?
(A) $v = a\dfrac{dv}{ds}$ (B) $a = v\dfrac{dv}{ds}$
(C) $v = \dfrac{1}{a}\dfrac{dv}{ds}$ (D) None of these

4. If a projectile motion with usual notations is expressed is
$y = xP - \dfrac{gx^2}{2u^2Q^2}$ (α = Angle of projection), then 'P' and 'Q' are
(A) $\tan\alpha$ and $\cos^2\alpha$
(B) $\tan\alpha$ and $\cos\alpha$
(C) $\tan\alpha$ and $\sec\alpha$
(D) $\tan\alpha$ and $\sec^2\alpha$

5. A mechanism has 5 numbers of joints and 6 members. The number of additional members needed to make it a perfect frame will be
(A) 4 (B) 3
(C) 2 (D) 1

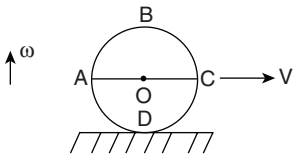
6. The rate of change of velocity and the rate of change of momentum of a moving body respectively are
(A) acceleration and impulse
(B) acceleration and force
(C) displacement and force
(D) force and displacement

7. In the equation of virtual work, which of the following force is neglected?
(A) reaction at any smooth surface with which the body is in contact
(B) reaction of rough surface of a body which rolls on it without shipping
(C) reaction at a point on an axis fixed in space, around which a body is constrained to turn.
(D) All of these

8. Two metallic balls having potential energy in the ratio 3 : 5 are made to slide down a frictionless inclined plane with zero position. What will be the ratio of their kinetic energy when they reach at bottom of inclined plane?
(A) 5 : 3 (B) 3 : 5
(C) 1 : 1 (D) 2 : 3

9. Two forces form a couple only when
(A) magnitude is same have parallel lines of action and same sense
(B) magnitude is different, have parallel lines of action but same sense
(C) magnitude is same have non parallel lines of action but same sense
(D) magnitude is same and have parallel lines of action and opposite sense

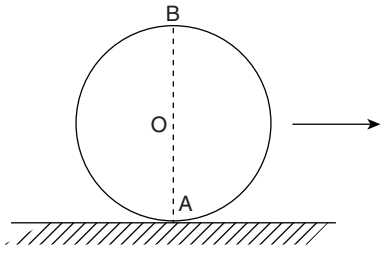
10. A wheel is rolling on a straight road as shown below. For this wheel the acceleration of the center 'O' and its instantaneous center are



(A) $\omega^2 r$ and O (B) $\omega^2 r$ and D
(C) V^2/r and D (D) zero and O

11. A particle moving from rest moves in a straight line. Its acceleration is given by the equation
$a = 10 - 0.006\,S^2$
Velocity of the particle when it has travelled 40 m is
(A) 19.16 m/s (B) 23.32 m/s
(C) 26.84 m/s (D) 30.14 m/s

12.



A wheel of radius 1 m rolls on a flat horizontal ground without slipping as shown in figure. Resultant velocity at point B is 1 m/s. Angular velocity of the wheel about its centre in rad/s is
(A) $\dfrac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
(C) 1 (D) $\dfrac{1}{2}$

13. Acceleration of a particle is given by
$a = t^3 - 3t^2 + 5$
Where t = time in seconds and a = acceleration in m/s^2. Velocity of particle when t = 2 sec is 8 m/s. Velocity of the particle when t = 4 sec is
(A) 22 m/s (B) 25 m/s
(C) 28 m/s (D) 32 m/s

Statement for Linked Data Questions 14 and 15:

14. A body of mass 5 kg falls from a height of 50 m and penetrates into the ground by 90 cm. Average resistance to penetration is
(A) 2668 N (B) 2774 N
(C) 2814 N (D) 2892 N

15. Time taken for penetration is
(A) 19.7 s (B) 12.7 s
(C) 17.4 s (D) 15.4 s

16. Angular displacement of a body is given by
$\theta = 6t^2 + 3t + 10$
Where t is in seconds. Angular velocity and angular acceleration of the body when
t = 10 seconds are
(A) 123 rad/s, 12 rad /s^2
(B) 135 rad /s, 14 rad /s^2
(C) 142 rad/s, 16 rad/s^2
(D) 153 rad/s, 18 rad/s^2

Statement for Linked Answer Questions 17 and 18:
A ball can be projected with a maximum velocity of 50 m/s. On an inclined plane, the maximum range obtained on projecting the ball is 190 m.

17. Inclination of the plane to the horizontal is
(A) 20° (B) 18°
(C) 16° (D) 14°

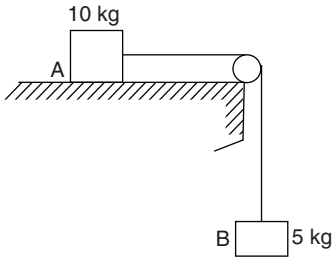
18. The projection angle from horizontal is
(A) 68° (B) 65°
(C) 60° (D) 55°

Statement for Linked Answer Questions 19 and 20:
A projectile is fixed at an angle of 30° in a horizontal level ground with a velocity of 50 m/s.

19. Time taken by the projectile to reach the ground after firing is
(A) 9.8 s (B) 8.6 s
(C) 7.2 s (D) 5.1 s

20. Horizontal range of the projectile is
(A) 220.7 m (B) 208.5 m
(C) 192.6 m (D) 186.1 m

Common Data for Questions 21 to 23:



Block A of mass 10 kg placed on a rough horizontal plane is connected to another block B of mass 5 kg by a string passing over a pulley as shown in figure. Coefficient of friction between block A and horizontal plane is 0.25. If the system is released from rest and block B is falling,

21. Tension on the string is
(A) 28.43 N (B) 33.41 N
(C) 37.62 N (D) 40.88 N

22. Acceleration of block B is
(A) 1.268 m/s^2 (B) 1.635 m/s^2
(C) 1.824 m/s^2 (D) 2.116 m/s^2

23. Velocity acquired by block B when it falls through a vertical distance of 1 m, is
(A) 1.24 m/s (B) 1.56 m/s
(C) 1.81 m/s (D) 2.35 m/s

Common Data for Questions 24 and 25:
Angular displacement of a particle, moving in a circular path of 150 m radius is given by
$\theta = 18t + 3t^2 - 2t^3$

24. Angular acceleration at 2 seconds from start is
(A) 15 rad/s^2 (B) 18 ard/s^2
(C) –15 ard/s^2 (D) –18 rad/s^2

25. Maximum angular velocity is
(A) 16.4 rad/s (B) 19.5 rad/s
(C) 22.3 rad/s (D) 25.4 rad/s

Answer Keys

1. A	**2.** A	**3.** B	**4.** B	**5.** D	**6.** B	**7.** D	**8.** B	**9.** D	**10.** D
11. B	**12.** D	**13.** A	**14.** B	**15.** C	**16.** A	**17.** A	**18.** D	**19.** D	**20.** A
21. D	**22.** B	**23.** C	**24.** D	**25.** B					

Hints and Explanations

2. Radius of gyration = $\sqrt{\frac{I_{min}}{m}}$

I = mass moment of inertia about the central axis
R = radius of sphere or cylinder.

For sphere: $I = \frac{2}{5} mR^2$ or $mK^2 = \frac{2}{5} mR^2$

k = radius of gyration

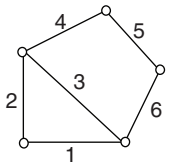
$\therefore \quad k = \sqrt{\frac{2}{5}} R = 0.6324R$

For cylinder:

$I = \frac{mR^2}{2}$ or $mk^2 = \frac{mR^2}{2}$

$\therefore \quad k = R = 0.707R.$ Choice (A)

5. Kutzbach equation
$F = 3(\ell - 1) - 2j - h$
F = degree of freedom
ℓ = number of links or member
J = number of lower pair
H = number of higher pair
When 5 joints and 6 members are there



$\ell = 6$
$J = 7$
$\therefore \quad F = 1$(kinematic chain)
For perfect frame, $F = 0$
$\therefore$ When 5 joints and 7 members then
$\ell = 7$
$J = 9$
$\therefore \quad F = 0$ (perfect frame)
Number of additional member = 1. Choice (D)

8. Since plane is frictionless, so KE at ground will be equal to $P.E$ at top.

$m_1 gh = \frac{1}{2} m_1 v_1^2$ and $m_2 gh = \frac{1}{2} m_2 v_2^2$

$\frac{KE_1}{KE_2} = \frac{m_1 gh}{m_2 gh} = \frac{PE_1}{PE_2} = \frac{3}{5}$ Choice (B)

11. $a = \frac{dv}{dt} = \frac{dv}{dS} \cdot \frac{dS}{dt} = v \cdot \frac{dv}{dS}$

$\therefore \quad v \frac{dv}{dS} = 10 - 0.006S^2$

$vdv = (10 - 0.006S^2)\, dS$

Integrating,

$\frac{v^2}{2} = 10S - \frac{0.006S^3}{3} + C$

When $S = 0$, $v = 0$
$\therefore \quad C = 0$
$\therefore \quad \frac{v^2}{2} = 10S \;\; 0.002S^3$

When $S = 40$ m

$\frac{v^2}{2} = 10 \times 40 - 0.002 \times (40)^3$

$v = 23.32$ m/s. Choice (B)

12. Resultant velocity at B
$R = 2\, v = 2\, \omega\, r$
$\therefore \quad 1 = 2\, \omega \times 1$
$\omega = \frac{1}{2}$ radian. Choice (D)

13. $a = \frac{dv}{dt} = t^3 - 3t^2 + 5$

$dv = (t^3 - 3t^2 + 5)\, dt$
Integrating,
$v = \int (t^3 - 3t^2 + 5)\, dt$

$= \frac{t^4}{4} - t^3 + 5t + C$

when $t = 2$, $v = 8$

$\therefore \quad 8 = \frac{2^4}{4} - 2^3 + 5 \times 2 + C$, $C = 2$

$\therefore \quad v = \frac{t^4}{4} - t^3 + 5t + 2$

When $t = 4$

$v = \frac{4^4}{4} - 4^3 + 5 \times 4 + 2 = 22$ m/s. Choice (A)

14. $m = 5$ kg, h = 50 m
$x = 90$ cm $= 0.9$ m
$v_1 = \sqrt{2gh} = 4.43 \sqrt{h} = 4.43 \sqrt{50} = 31.32$ m/s
$v_2 = 0$
Let R be the average resistance of penetration

$\frac{1}{2} m(v_2^2 - v_1^2) = (mg - R)\, x$

$\frac{1}{2} \times 5(0 - 31.32^2) = (5 \times 9.81 - R)\, 0.9 - 2724.84$

$= 49.05 - R$

$R = 2773.89$ N. Choice (B)

15. Applying impulse momentum equation,
$F \times t = m(v_2 - v_1)$
$F = mg - R = 49.05 - 2773.89 = -2724.84$ N

$\therefore \quad -2724.84 \times t = 5(0 - 31.32)$

$t = 17.4$ sec. Choice (C)

16. $\theta = 6t^2 + 3t + 10$

Angular velocity

$$\omega = \frac{d\theta}{dt} = 12t + 3$$

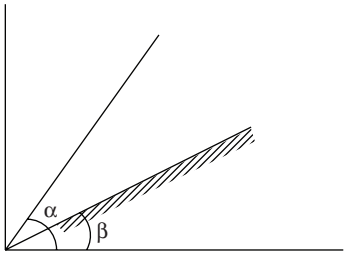
when $t = 10$

$\omega = 12 \times 10 + 3 = 123$ rad/s

Angular acceleration

$$\alpha = \frac{d\omega}{dt} = 12 \text{ rad/s}^2.$$ Choice (A)

17.



$$R_{max} = \frac{u^2}{g(1+\sin\beta)}$$

$$190 = \frac{50^2}{9.81(1+\sin\beta)}$$

$\sin\beta = 0.3413$

$\beta = 19.95°$ or $20°$. Choice (A)

18. $\beta = 19.95° = 0.3482$ radian

$$\frac{1}{2}\left(\frac{\pi}{2} - \beta\right) = \alpha - \beta$$

$$\frac{1}{2}\left(\frac{\pi}{2} - 0.3482\right) = \alpha - 0.3482$$

$\alpha = 0.9595$ radian $= 54.98$ or $55°$. Choice (D)

19. Time taken is given by

$$t = \frac{2u\sin\alpha}{g} = \frac{2 \times 50 \times \sin 30}{9.81}$$

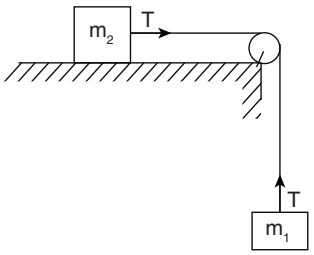
$= 5.097$ sec or 5.1 sec . Choice (D)

20. Horizontal range

= Horizontal velocity × time of flight = ($u \cos\alpha$) × t

= 50 cos 30 × 5.097 = 220. 7 m. Choice (A)

21. Let mass of block B be m_1 and block A be m_2



$m_1 g - T = m_1 a$

$T - \mu m_2 g = m_2 a$

$$\therefore \quad a = g\frac{(m_1 - \mu m_2)}{(m_1 + m_2)}$$

$$T = \frac{m_1 m_2 g(1+\mu)}{m_1 + m_2}$$

$$T = \frac{5 \times 10 \times 9.81(1+0.25)}{5+10} = 40.88 \text{ N.}$$ Choice (D)

22. $$a = \frac{g(m_1 - \mu m_2)}{m_1 + m_2}$$

$$a = \frac{9.81(5 - 0.25 \times 10)}{5+10} = 1.635 \text{ m/s}^2.$$ Choice (B)

23. $v^2 = u^2 + 2as = 0 + 2 \times 1.635 \times 1$

$v = 1.81$ m/s. Choice (C)

24. $\theta = 18\,t + 3t^2 - 2t^3$

Angular velocity

$$\omega = \frac{d\theta}{dt} = 18 + 6t - 6t^2$$

Angular acceleration

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = 6 - 12t$$

When $t = 2$

$\alpha = 6 - 12 \times 2 = -18$ rad/s^2. Choice (D)

25. Angular velocity is maximum when

$$\frac{d\omega}{dt} = 0$$

i.e., $6 - 12\,t = 0$ or $t = 0.5$ sec

Maximum angular velocity

$= 18 + 6 \times 0.5 - 6 \times (0.5)^2 = 19.5$ rad/s. Choice (B)

Engineering Mechanics Test 2

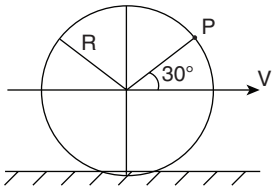
Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The velocity-time graph of a body is passing through the velocity axis with intercept of 4. If the slope of the graph is 3, the distance travelled by the body in 6 seconds would be

(A) 40 m (B) 60 m
(C) 78 m (D) 80 m

2. A circular disc of radius 'R' rolls without slipping at a velocity 'V'. The magnitude of the velocity at point P(see figure) is



(A) $\sqrt{3}V$ (B) $\frac{\sqrt{3}}{2}V$
(C) $\frac{V}{2}$ (D) $\frac{2}{\sqrt{3}}V$

3. A particle starts with velocity 2 m/s and accelerates at a rate of 3 m/s^2 for 15 seconds and then retards at 6 m/s^2 until it stops. The total distance covered is

(A) 184.08 m (B) 551.58 m
(C) 367.5 m (D) None of these

4. A stone is projected horizontally from a cliff at 10 m/s and lands on the ground below at 20 m from the base of the cliff. Find the height 'h' of the cliff. Use g = 10 m/s^2.

(A) 18 m (B) 20 m
(C) 22 m (D) 24 m

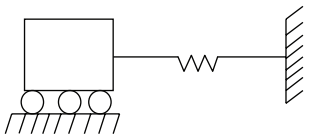
5. Two cars are going with constant speeds, round concentric circles of radii r_1 and r_2 and take the same time to complete their circular paths. Their speeds will correspond to the ratio

(A) 1 : 1 (B) $\frac{r_1}{r_2}$
(C) $\left(\frac{r_1}{r_2}\right)^2$ (D) $\frac{r_2}{r_1}$

6. A point 'P' moves along a straight line as per the law $x = 4t^2 + 12t + 1$, the velocity of the point after 3rd and 4th seconds are respectively.

(A) 36 m/s and 48 m/s
(B) 36 m/s and 44 m/s
(C) 34 m/s and 44 m/s
(D) 34 m/s and 46 m/s

7. A truck weighing 150 kN and traveling at 2 m/s impacts with a buffer spring, which compresses 1.25 cm per 10 kN. The maximum compression of the spring is



(A) 20 cm
(B) 22.85 cm
(C) 27.65 cm
(D) 30 cm

8. A particle moving in space with velocity $J = 3t^2i + 4tj - 7t^3k$. The acceleration of the particle at t = 1 will be

(A) $3i + 8j - 7k$
(B) $6i + 4j + 21k$
(C) $6i + 4j - 21k$
(D) zero

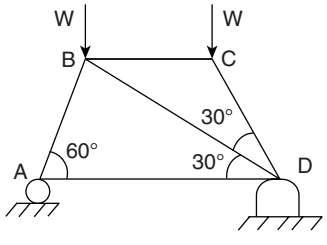
9. Match the following

	List – I		List – II
a.	Two parallel forces acting on a body moving with uniform velocity	1.	Collision
b.	A moving particle	2.	Forces in equilibrium
c.	Two coplanar forces equal in magnitude but opposite in direction	3.	Kinetic energy
d.	Co-efficient of restitution	4.	Couple

	a	b	c	d
(A)	4	3	2	1
(B)	1	2	3	4
(C)	2	3	4	1

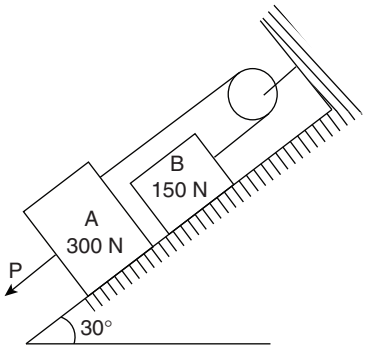
(D) None of these

10. For the truss shown in the figure, the force (N) in the member BC is



(A) 0 N(compressive)
(B) 0.577 W(tensile)
(C) 0.577 W(compressive)
(D) 0.866 W(compressive)

11.



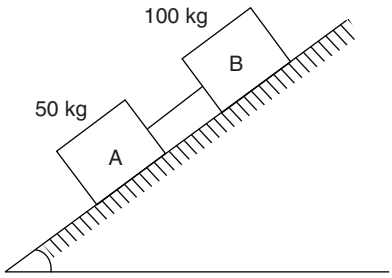
Two blocks *A* and *B* weighing 300 N and 150 N respectively are placed on a rough inclined plane of angle 30° and connected through a string over a pulley as shown in the figure. Coefficient of friction of the contact surfaces are 0.25. Force *P* required on block *A* for impending motion of the blocks is

(A) 22.43 N (B) 25.24 N
(C) 28.62 N (D) 30.14 N

12. A ball of mass 5 kg moving with a velocity of 6 m/s makes impact with another ball of mass 3 kg moving in the same direction with a velocity of 4 m/s. If coefficient of restitution is 0.5, velocities of the balls after impact are

(A) 4.875 m/s, 5.875 m/s
(B) 4.962 m/s, 6.125 m/s
(C) 5.125 m/s, 6.536 m/s
(D) 5.565 m/s, 6.926 m/s

Common Data for Questions 13 to 15:



Two blocks *A* and *B* of mass 50 kg and 100 kg are placed on an inclined plane, by connecting them by a string as shown in them figure. Coefficient of friction between block *A* and inclined plane is 0.15 and between block *B* and inclined plane is 0.4

13. Inclination of the plane when the two blocks just start to move is

(A) 10.15° (B) 17.57°
(C) 14.24° (D) 16.28°

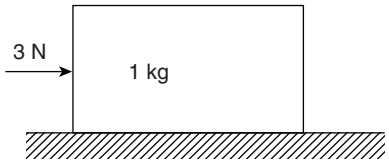
14. Tension on the string is

(A) 77.9 N (B) 31.96 N
(C) 74.3 N (D) 67.4 N

15. Angle of the inclined plane is increased to 20° and the connecting string is removed. If the coefficients of kinetic friction for blocks *A* and *B* are 0.1 and 0.35 respectively the frictional forces on *A* and *B* are

(A) 46.1 N, 369 N (B) 49.44 N, 398 N
(C) 52.14 N, 404 N (D) 56.48 N, 410 N

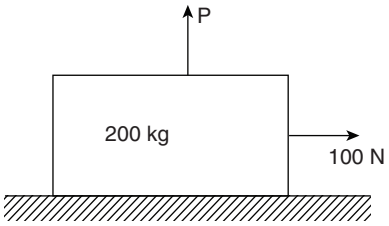
16.



A body of mass 1 kg is resting on a plane surface as shown in figure. A force of 3 N is gradually applied on one side as shown. Coefficient of static friction is 0.35 and coefficient of kinetic friction is 0.3. The friction force acting is

(A) 3.4335 N (B) 2.943 N
(C) 3 N (D) 0 N

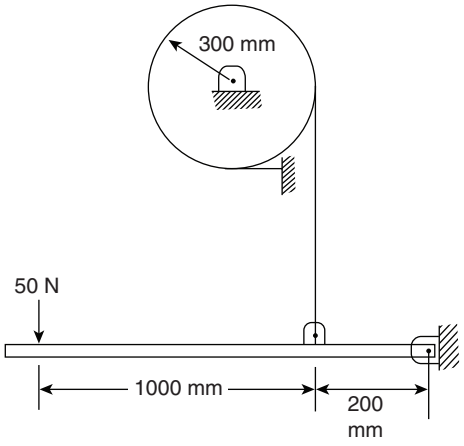
17.



A body of mass 200 kg rests on a horizontal surface as shown in the figure. Coefficient of friction between the body and surface is 0.2. If a horizontal pull of 100 N can be exerted on the body, the vertical force P required to move the body is

(A) 1462 N (B) 1418 N
(C) 1360 N (D) 1322 N

Common Data for Questions 18 and 19:

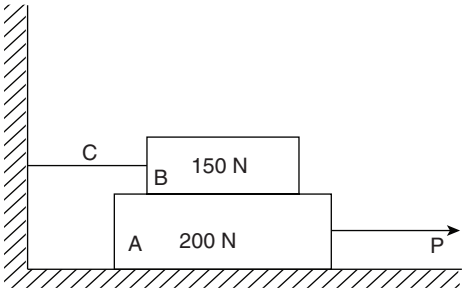


A band brake is used to control the speed of a flywheel as shown in figure. The coefficient of friction between the band and flywheel is 0.3. Radius of the flywheel is 300 mm. A force of 50 N is applied at the end of the lever as shown in the figure

18. Torque applied on the flywheel when it is rotating clockwise is
(A) 262 Nm (B) 280 Nm
(C) 315 Nm (D) 326 Nm

19. Torque applied on the flywheel when it is rotating counter clockwise is
(A) 94 Nm
(B) 82 Nm
(C) 76 Nm
(D) 68 Nm

Common Data for Questions 20 and 21:

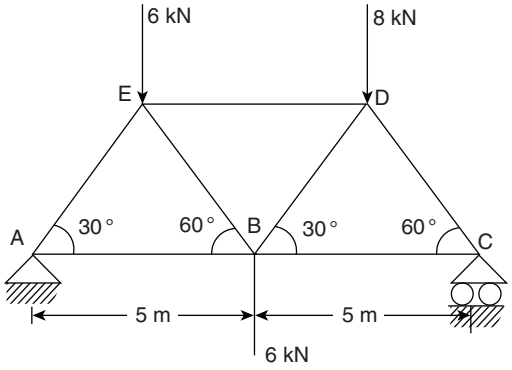


Block *A* weighing 200 N is placed on plane floor and block *B* weighing 150 N is placed over block *A*. Block *B* is constrained by a string *C* and a force *P* is applied on block *A* as shown in figure. For the contact surfaces, coefficient of static friction is 0.3 and coefficient of kinetic friction is 0.25.

20. The smallest force *P* required to start block *A* moving is
(A) 143 N
(B) 150 N
(C) 156 N
(D) 160 N

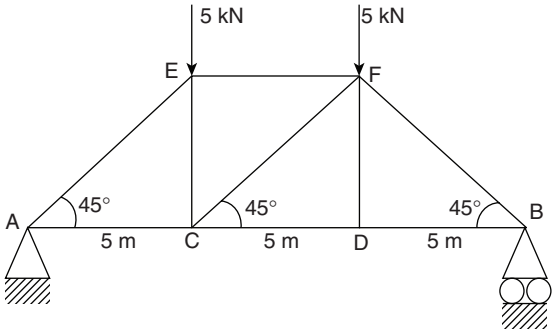
21. If a force *P* of 160 N is applied, the resultant friction forces exerted on block *A* is
(A) 110 N (B) 120 N
(C) 125 N (D) 150 N

22.



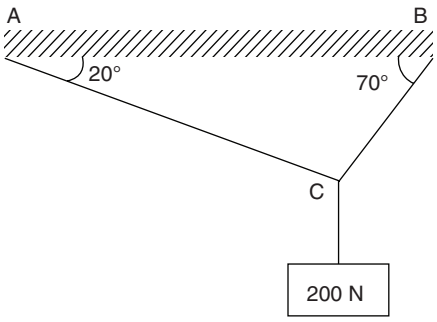
A simply supported structure is loaded as shown in the figure. Force in the member *AB* is
(A) 10.26 kN
(B) 13.42 kN
(C) 15.75 kN
(D) 17.83 kN

23.



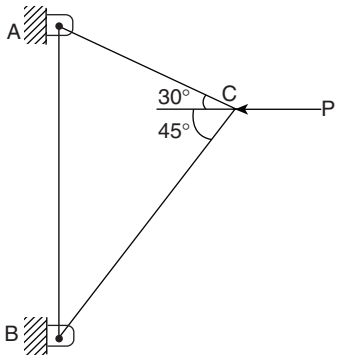
For the truss loaded as shown in figure, force in the member *CD* is
(A) 5 kN (B) 2.5 kN
(C) $\frac{5}{\sqrt{2}}$ kN (D) $5\sqrt{2}$ kN

24.



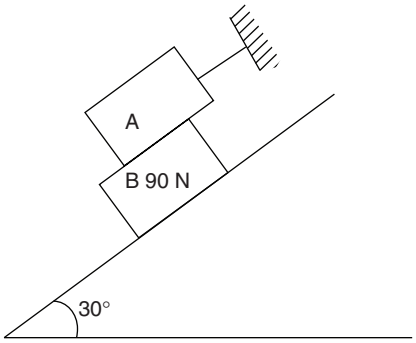
A weight of 200 N is hung using a cable as shown in the figure. Tensions in portions of cable *AC* and *BC* are respectively
(A) 59.6, 171.7 N
(B) 62.4, 176.8 N
(C) 62.5, 182.7 N
(D) 68.4, 187.9 N

25.



Two steel truss members *AC* and *BC* with cross section area 100 mm^2 is subjected to a horizontal force *P* kN as shown in figure. Maximum value of *P* such that axial stress in any of the members does not exceed 50 MPa is
(A) 10.15 kN (B) 9.22 kN
(C) 7.92 kN (D) 6.83 kN

26.



Refering to the figure given above, coefficient of friction for all surfaces of contact is 0.3. The minimum weight of block *A* required to keep block *B* in position is

(A) 35.6 N (B) 38.4 N
(C) 41.6 N (D) 44.5 N

27. A wheel at rest is accelerated uniformly from rest to 3000 rpm in 30 seconds. Its angular acceleration is

(A) 6.624 rad/s (B) 8.368 rad/s
(C) 10.472 rad/s (D) 14.376 rad/s

Common Data for Questions 28 and 29:
A pile of mass 500 kg is driven by a mass 350 kg falling on it vertically through a distance of 1 m. After impact, the falling mass and pile remain in contact and move together. The pile is moved 150 mm at each blow.

28. Energy lost in each blow is

(A) 1676 Nm (B) 1762 Nm
(C) 1915 Nm (D) 2020 Nm

29. Average resistance against the pile is

(A) 17.765 kN (B) 18.625 kN
(C) 20.516 kN (D) 22.835 kN

30. A body of mass 5 kg falls from a height of 50 m and penetrates into the ground by 90 cm. Average resistance to penetration is

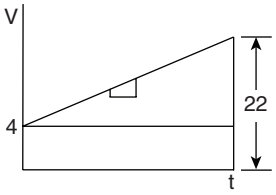
(A) 2668 N (B) 2774 N
(C) 2814 N (D) 2892 N

Answer Keys

1. C	**2.** A	**3.** B	**4.** B	**5.** B	**6.** B	**7.** C	**8.** C	**9.** C	**10.** C
11. A	**12.** A	**13.** B	**14.** A	**15.** A	**16.** C	**17.** A	**18.** B	**19.** D	**20.** B
21. C	**22.** B	**23.** A	**24.** D	**25.** D	**26.** C	**27.** C	**28.** D	**29.** A	**30.** B

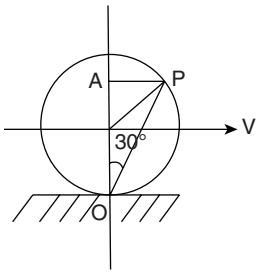
Hints and Explanations

1.



$a = 3$.
$v = u + at = 4 + 3 \times 6$
$= 22$ m/s
Distance travelled = Area under the curve
$= 6 \times 4 + \frac{1}{2} \times 18 \times 6 = 24 + 54 = 78$ m Choice (C)

2.



the instantaneous center of rotation of the disk is *O*, i.e, its point of contact with ground every other point on the disk is rotating with respect to this center with an angular velocity.

$\omega = \frac{V}{R}$

Velocity of $P = \omega \cdot OP$

$OP = \sqrt{R^2 + R^2 - 2R^2 \cos 120°}$

$= \sqrt{3R^2} = \sqrt{3}R$

$V_P = \frac{V}{R} \cdot \sqrt{3}R = \sqrt{3}V$ Choice (A)

3. $S = ut + \frac{1}{2}at^2$

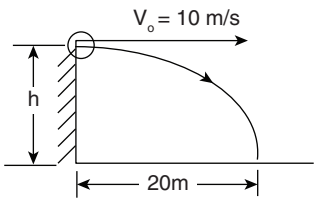
$u = 2$ m/s
$a = 3$ m/s^2

$S_1 = 2(15) + \frac{1}{2} \times 3(15)^2 = 367.5$ m

$v = u + at = 2 + 3(15) = 47$m/s
$v^2 - u^2 = 2as$
$0 - 47^2 = 2(-6)s_2$
$s_2 = \frac{47^2}{12} = 184.08$ m/s

Total distance covered is $= S_1 + S_2 = 367.5 + 184.08$
$= 551.58$ m/s Choice (B)

4.



time of flight = $\frac{20}{10}$ = 2 sec

Since horizontal component of velocity remains constant therefore

$h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 2^2 = 20$ m Choice (B)

5. We know $t_1 = \frac{2\pi r_1}{v_1}$

And $t_2 = \frac{2\pi r_2}{v_2}$

But $t_1 = t_2$

Therefore $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

Since $v = \omega r$, therefore

$$\frac{\omega_1 r_1}{r_1} = \frac{\omega_2 r_2}{r_2}$$

$\omega_1 = \omega_2$

$$\frac{v_1}{r_1} = \frac{v_2}{r_2}$$

$\therefore \quad \frac{v_1}{v_2} = \frac{r_1}{r_2}$ Choice (B)

6. $X = 4t^2 + 12t + 1$

Velocity, $v = \frac{dX}{dt} = 8t + 12$

$(v)\ t = 3\text{sec} = 8(3) + 12 = 36$ m/s

$(v)\ t = 4\text{sec} = 8(4) + 12 = 44$ m/s Choice (B)

7. $\frac{1}{2}mV^2 = \frac{1}{2}KX^2$

$m = \frac{w}{g} = \frac{150\times 10^3}{9.81} = 15290$ kg

$15290\ (2)^2 = \frac{10000}{1.25\times 10^{-2}}x^2$

$\Rightarrow \quad x = 27.65$ cm. Choice (C)

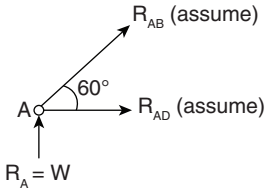
8. Velocity, $J = 3t^2 i + 4tj - 7t^3 k$

Acceleration, $\frac{dJ}{dt} = 6ti + 4j - 21t^2 k$

At $t = 1$

Acceleration, $\frac{dJ}{dt} = 6i + 4j - 21k$. Choice (C)

10. Considering pin A

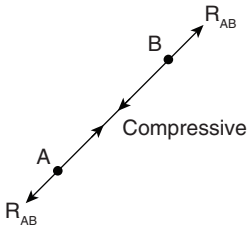


$\Sigma F_y = 0$

$\therefore \quad R_{AB}\sin 60° + R_A = 0$

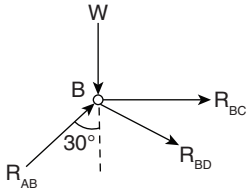
$\therefore \quad R_{AB} = -1.1547$W

Considering tress member *AB*



$R_{AB} = +1.1547$W

Considering pin *B*



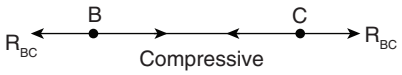
$\Sigma f_x = 0$

$\therefore \quad R_{AB}\sin 30° + R_{BC} = 0$

$\therefore \quad R_{Bc} = -1.1547\text{W}\ \sin 30°$

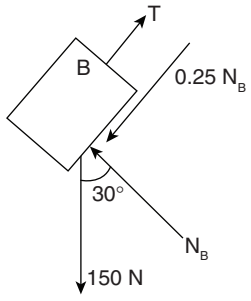
$R_{Bc} = -0.577$W

Considering tress member *BC*



$\therefore \quad R_{Bc} = 0.577$W(compressive) Choice (C)

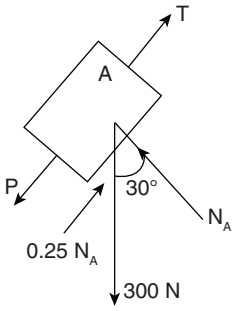
11. Consider free body diagram of block B. Let T be the tension on the string



$N_B = 150\cos 30 = 129.9$ N

$T = 150\sin 30 + 0.25\ N_B = 75 + 0.25\times 129.9 = 107.48$ N

Consider free body diagram of block A



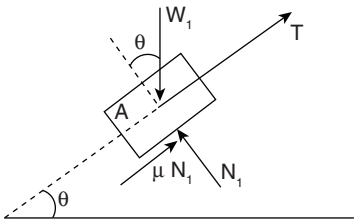
$N_A = 300 \cos 30 = 259.8$ N
$P + 300 \sin 30 = T + 0.25\ N_A$
$P = 107.48 + 0.25 \times 259.8 - 150 = 22.43$ N.
Choice (A)

12. $m_1 = 5$ kg, $u_1 = 6$ m/s
$m_2 = 3$ kg, $u_2 = 4$ m/s, $e = 0.5$
Applying momentum equation
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
i.e., $5 \times 6 + 3 \times 4 = 5 \times v_1 + 3 \times v_2$
$5v_1 + 3v_2 = 42$ ——————————(1)
From Newton's law of collision of elastic bodies,

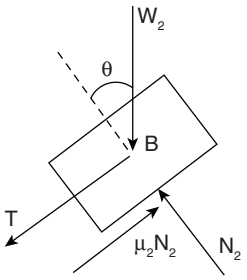
$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$v_2 - v_1 = e(u_1 - u_2) = 0.5\ (6 - 4)$
$v_2 - v_1 = 1$ ——————————(2)
From (1) and (2)
$v_1 = 4.875$ m/s
$v_2 = 5.875$ m/s. Choice (A)

13. Let T be the tension in the spring
Considering block A alone



$N_1 = W_1 \cos \theta$
$W_1 \sin \theta = T + \mu N_1 = T + 0.15\ W_1 \cos \theta$
$T = 50 \times 9.81 \sin \theta - 0.15 \times 50 \times 9.81 \cos \theta$
$= 490.5 \sin \theta - 73.575 \cos \theta$ ————————(1)
Considering block B



$N_2 = W_2 \cos \theta$
$T + W_2 \sin \theta = \mu_2 N_2 = 0.4 \times 100 \times 9.81 \cos \theta$
$T + 981 \sin \theta = 392.4 \cos \theta$ ——————————(2)
From (1) and (2)
$(490.5 \sin \theta - 73.575 \cos \theta) + 981 \sin \theta = 392.4 \cos \theta$
$1471.5 \sin \theta - 465.975 \cos \theta = 0$

$$\tan \theta = \frac{465.975}{1471.5} = 0.3167$$

$\theta = 17.57°$. Choice (B)

14. $T = 490.5 \sin 17.57 - 73.575 \cos 17.57 = 77.925$ N.
Choice (A)

15. Consider block A for static condition
$T = 490.5 \sin \theta - 73.575 \cos \theta$
As there is no connecting string
$490.5 \sin \theta - 73.525 \cos \theta = 0$
$\tan \theta = 0.15$
$\theta = 8.525°$
As θ is given as 20° the block moves down
Frictional force $= \mu_k \times N_1 = \mu_k \times W_1 \cos \theta$
$= 0.1 \times 50 \times 9.81 \cos 20 = 46.1$ N
For the static condition of block B
$981 \sin \theta = 392.4 \cos \theta$
$\tan \theta = 0.4$
$\theta = 21.8°$
$\therefore$ The block is stationary at an angle 20° and static friction is active
Frictional force $= \mu_s \times N_2 = \mu_s \times W_2 \cos \theta$
$= 0.4 \times 100 \times 9.81 \cos 20$
$= 368.735$ N. Choice (A)

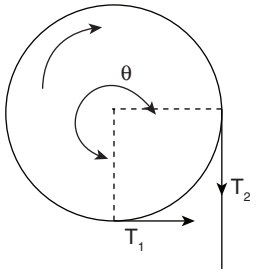
16. Weight of the body $W = mg = 1 \times 9.81 = 9.81$ N
Static friction force $= W \times \mu_s = 9.81 \times 0.35 = 3.4335$ N
Kinetic friction force $= W \times \mu_k = 9.81 \times 0.3 = 2.943$ N
The applied force is 3 N. The body will start moving only when applied force exceeds the static friction force. After this the kinetic friction force will come into action.
So the friction force acting is 3 N. Choice (C)

17. Weight of the body $= mg = 200 \times 9.81 = 1962$ N
Frictional force $= (mg - P)\mu = (1962 - P)0.2$
For moving the body
$100 \geq (1962 - P) \times 0.2$

$$1962 - P = \frac{100}{0.2}$$

$P = 1962 - 500 = 1462$ N. Choice (A)

18.



For clock wise rotation of the flywheel, band connected to the lever is slack side and corresponding tension is T_2

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

where θ = wrapping angle = 270° = $\frac{3\pi}{2}$ radian

$$\frac{T_1}{T_2} = e^{0.3\times\frac{3\pi}{2}} = 4.11$$

$T_2 \times 200 = 50 \times 1200$

$T_2 = 300$ N

$\therefore \quad T_1 = T_2 \times 4.11 = 300 \times 4.11 = 1233$ N

Torque on the fly wheel = $(T_1 - T_2)\, r$

$= (1233 - 300)\, 0.3 = 280$ Nm. Choice (B)

19. For anti clockwise rotation tight side of the band is connected to the lever i.e., T_1 acts on the lever

$\therefore \quad T_1 \times 200 = 50 \times 1200$

$T_1 = 300$ N

$$\frac{T_1}{T_2} = 4.11$$

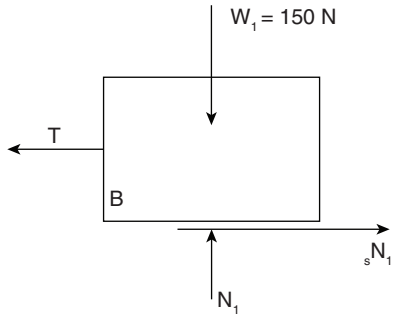
$$T_2 = \frac{T_1}{4.11} = \frac{300}{4.11} = 73 \text{ N}$$

Torque = $(T_1 - T_2)r$

$= (300 - 73)0.3 = 68$ Nm. Choice (D)

20. Consider block B,

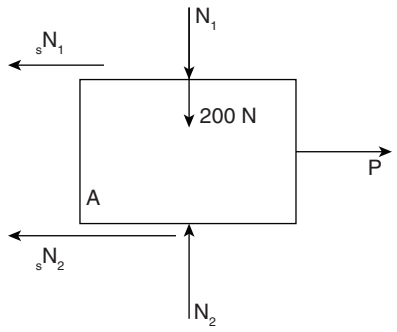
Let T be the tension in the string



$N_1 = W_1 = 150$ N

$T = \mu_s \times N_1 = 0.3 \times 150 = 45$ N

Consider Block A



$N_2 = N_1 + 200 = 150 + 200 = 350$ N

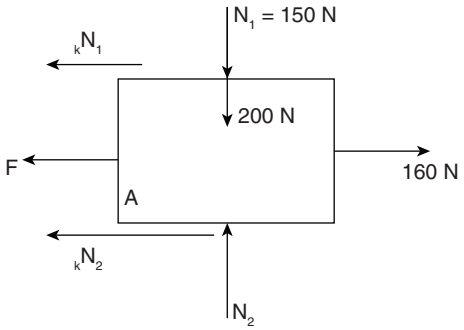
$P = \mu_s N_1 + \mu_s N_2 = 45 + 0.3 \times 350 = 150$ N. Choice (B)

21. Static friction forces to overcome for the movement of A = 150 N

For force P above 150 N

Kinetic friction is active

Considering block A

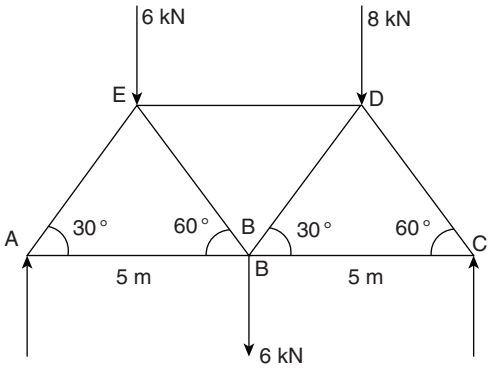


Resultant friction forces = $\mu_k N_1 + \mu_k N_2$

$= 0.25 \times 150 + 0.25\,(150 + 200)$

$= 37.5 + 87.5 = 125$ N. Choice (C)

22.



$V_A + V_C = 6 + 8 + 6 = 20$ kN

Taking moment about A

$$V_C \times 10 = 6 \times \frac{5\times3}{4} + 6\times5 + 8\left(5 + \frac{5\times3}{4}\right)$$

$V_C = 12.25$ kN

$\therefore \quad V_A = 20 - 12.25 = 7.75$ kN

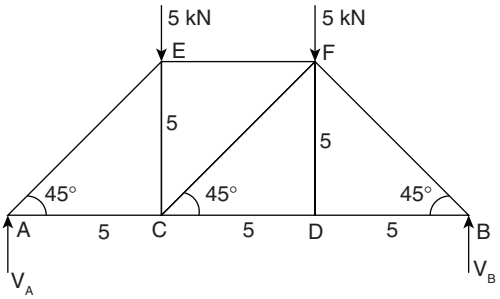
considering joint A $T_{AE} \sin 30 = V_A$

$$T_{AE} = \frac{7.75}{0.5} = 15.5 \text{ kN}$$

$T_{AB} = T_{AE} \cos 30 = 15.5 \times \cos 30 = 13.42$ kN.

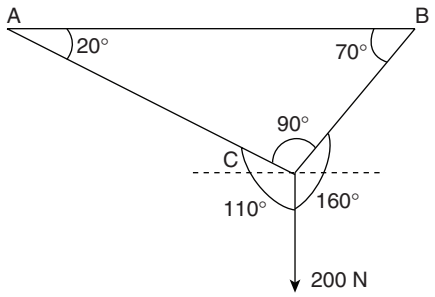
Choice (B)

23.



By symmetry
$V_A = V_B = 5$ kN
Considering joint B,
$T_{FB} \sin 45 = V_B = 5$
$T_{DB} = T_{FB} \cos 45 = 5$ kN as sin 45 = cos 45
considering joint D $T_{CD} = T_{DB} = 5$ kN. Choice (A)

24.

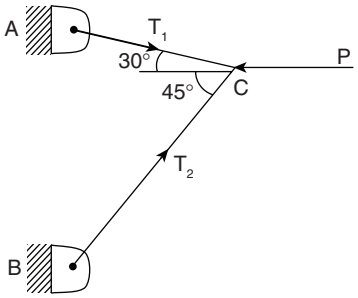


Applying Lami's theorem

$$\frac{T_{AC}}{\sin 160} = \frac{T_{BC}}{\sin 110} = \frac{200}{\sin 90} = 200$$

$T_{AC} = 200 \times \sin 160 = 68.4$ N
$T_{BC} = 200 \times \sin 110 = 187.94$ N Choice (D)

25.



Considering the joint P,
$T_1 \cos 30 + T_2 \cos 45 = P$
$T_1 \sin 30 = T_2 \sin 45$

$$\frac{T_1}{T_2} = \frac{\sin 45}{\sin 30} = 1.414$$

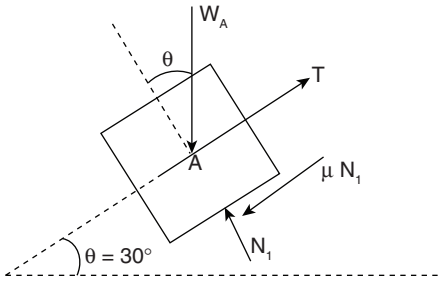
$T_1 = 1.414\ T_2$
$1.414\ T_2 \cos 30 + T_2 \cos 45 = P$
$T_2 = 0.5177\ P$
$T_1 = 0.732\ P$
Maximum force is in the member AC
$T_{AC} = T_1 = 0.732\ P$ kN

$$\text{Axial stress} = \frac{0.732 \times 10^3 P}{100} = 7.32\ P \text{ N/mm}^2$$

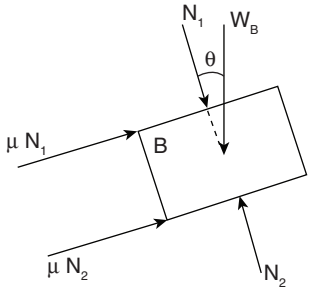
$$= 7.32\ P \text{ MPa}$$

$\therefore$ $7.32\ P \le 50$ MPa
$P \le 6.83$ kN. Choice (D)

26.



Consider the free body diagram of block A
$N_1 = W_A \cos 30$ ——————(1)
$\mu N_1 + W_A \sin 30 = T$



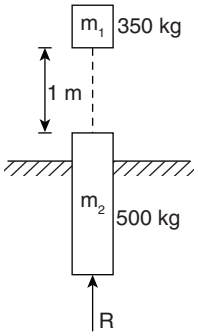
Consider free body diagram of B
$N_1 + W_B \cos \theta = N_2$
i.e., $N_1 - N_2 = -90 \times \cos 30$
$= -77.942$ ——————(2)
$\mu(N_1 + N_2) = W_B \sin 30$
$0.3(N_1 + N_2) = 90 \times 0.5$
$N_1 + N_2 = 150$ ——————(3)
From (2) and (3)
$2\ N_1 = 72.06$
$N_1 = 36$ N
Substituting in (1)
$36 = W_A \cos 30$
$\therefore$ $W_A = 41.6$ N. Choice (C)

27. $\omega_0 = 0$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60} = 100\ \pi \text{ rad/s}$$

$t = 30$ s
$\omega = \omega_0 + \alpha\ t$
$100\ \pi = 0 + \alpha \times 30$
$\alpha = 10.472$ rad/s. Choice (C)

28.



$m_1 = 350$ kg, $m_2 = 500$ kg
$S = 1$ m, $x = 150$ mm $= 0.15$ m
Let V_1 be the velocity of mass m_1 when hitting m_2

$$V_1 = \sqrt{2gS} = \sqrt{2 \times 9.81 \times 1} = 4.43 \text{ m/s}$$

V_2 = Velocity of pile before impact
V = Common velocity after impact

$$m_1V_1 + m_2V_2 = (m_1 + m_2)V$$
$$350 \times 4.43 + 0 = (350 + 500)V$$
$$\therefore \quad V = 1.824 \text{ m/s}$$

Kinetic energy before impact $= \frac{1}{2} m_1V_1$

$$= \frac{1}{2} \times 350 \times (4.43)^2 = 3434.36 \text{ Nm}$$

Kinetic energy after impact $= \frac{1}{2} (m_1 + m_2) V^2$

$$= \frac{1}{2} (350 + 500) \times (1.824)^2 = 1413.96 \text{ Nm}$$

Energy lost in blow = 3434.36 – 1413.96
= 2020.4 N/m. Choice (D)

29. Let R be the average resistance against the pile.
Net work done = kinetic energy after impact

$$(R - m_1g - m_2g)x = \frac{1}{2} (m_1 + m_2)V^2$$

$$(R - 350 \times 9.81 - 500 \times 9.81)\ 0.15 = 1413.96$$
$R = 17765$ N $= 17.765$ kN. Choice (A)

30. m = 5 kg, $h = 50$ m
$x = 90$ cm $= 0.9$ m

$$v_1 = \sqrt{2gh} = 4.43 \sqrt{h} = 4.43 \sqrt{50} = 31.32 \text{ m/s}$$

$v_2 = 0$
Let R be the average resistance of penetration

$$\frac{1}{2} \text{m}(v_2^2 - v_1^2) = (\text{mg} - R)\ x$$

$$\frac{1}{2} \times 5(0 - 31.32^2) = (5 \times 9.81 - R)\ 0.9$$

$$\Rightarrow \quad 49.05 - R = 2724.84$$
$R = 2773.89$ N Choice (B)

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UNIT II
SOLID MECHANICS

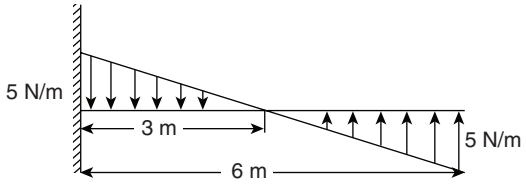
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SOLID MECHANICS TEST I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1.



A cantilever beam of 6 m span is subjected to a uniformly varying load as shown. The bending moment at the middle of the beam is

(A) 27.5 N–m (B) 15.0 N–m

(C) 22.0 N–m (D) 18.7 N–m

2.

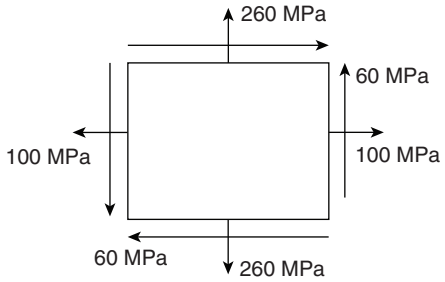
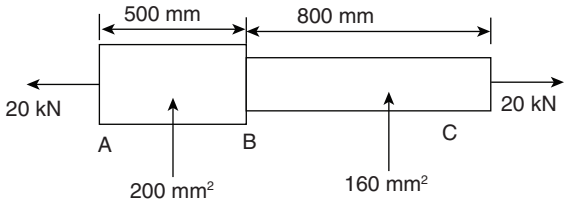


Figure shows state of stress at a point in a stressed body. Radius of Mohr's circle representing the state of stress is

(A) 60 (B) 80

(C) 120 (D) 100

3.

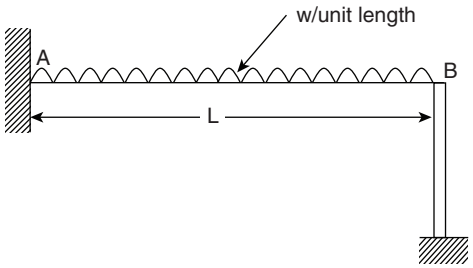


A bar ABC with cross sectional area 200 mm^2 at portion AB and 150 mm^2 at portion BC is subjected to an axial pull of 20 kN. If $E = 2 \times 10^5$ N/mm^2, strain energy stored in the bar is

(A) 15.5 Nm (B) 18.6 Nm

(C) 8.7 Nm (D) 10.5 Nm

4.

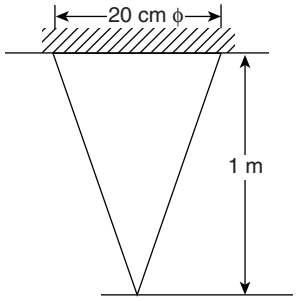


The free end of the cantilever AB is supported by a prop. The cantilever is loaded by a uniformly distributed load as shown in the figure. Assuming that there is no deflection at the free end, force on the prop is

(A) $\frac{4}{5}wL$ (B) $\frac{3}{8}wL$

(C) $\frac{3}{4}wL$ (D) $\frac{3}{5}wL$

5. A solid conical bar of uniformly varying cross section is hung vertically as shown



If specific weight is 80000 N/m^3 and modulus of elasticity is $E = 2 \times 10^5$ N/mm^2, extension of its length due to self weight is

(A) 6.67×10^{-5} m (B) 1.33×10^{-4} m

(C) 1×10^{-4} m (D) 4.45×10^{-5}m

6. The bulk modulus is K, modulus of elasticity E, and passion ratio is $\frac{1}{m}$ then which of the following is true?

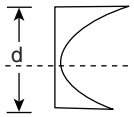
(A) $E = 3K\left(1+\frac{2}{m}\right)$ (B) $E = 3K\left(1-\frac{1}{m}\right)$

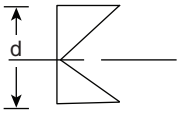
(C) $E = 3K\left(1-\frac{2}{m}\right)$ (D) $E = 3K\left(1+\frac{1}{m}\right)$

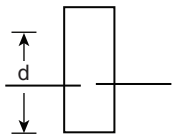
7. A solid circular shaft is subjected to bending & twist. The ratio of maximum shear to maximum bending stress at any point would be ($M = T$)

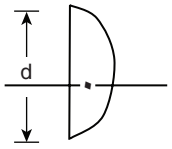
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 2 : 3

8. The shear stress distribution diagram of a beam of rectangular cross section, subjected, to transverse loading will be

(A) 

(B) 

(C) 

(D) 

Where '*d*' is the depth of the beam

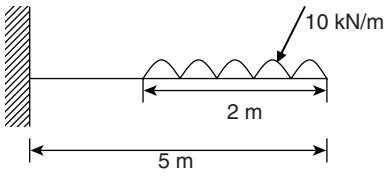
9. Proof resilience is the maximum energy stored at
(A) limit of proportionality
(B) elastic limit
(C) plastic limit
(D) None of these

10. Which of the following will give the value of deflection at any point?

(A) $EI\frac{dy}{dx} = M$ (B) $EIY = M$

(C) $EIY = \int M$ (D) $EIY = \int\int M$

11.



The displacement of the free end of the cantilever beam shown in figure is
[Take $E = 2 \times 10^5$ N/mm^2,
$I = 180 \times 10^6$ mm^4]
(A) 16.39 mm (B) 14.93 mm
(C) 12.72 mm (D) 10.68 mm

12. During an experiment on a steel column using Rankine's formula, the following results were available

Slenderness ratio	65	160
Average stress at failure	200 N/mm^2	70 N/mm^2

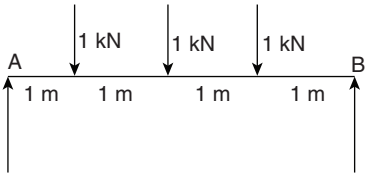
Rankine's constant for the material of the column is
(A) 1.865×10^{-4} (B) 2.194×10^{-4}
(C) 1.623×10^{-4} (D) 1.373×10^{-4}

Common Data for Questions 13 and 14:
At a cross section in a shaft of diameter 100 mm it is subjected to a bending moment of 2.5 kNm, and a twisting moment of 5 kNm.

13. Maximum principal stresses induced in the section in N/mm^2 are
(A) 37.5, 12.64 (B) 41.2, –15.74
(C) 52.8, –17.92 (D) 49.3, –16.78

14. The direct stress in N/mm^2 that produces same strain on that produced by the principal stresses is
(Poisson's ratio is 0.3)
(A) 36.78 (B) 52.76
(C) 45.92 (D) 39.62

Statement for Linked Answer Questions 15 and 16:



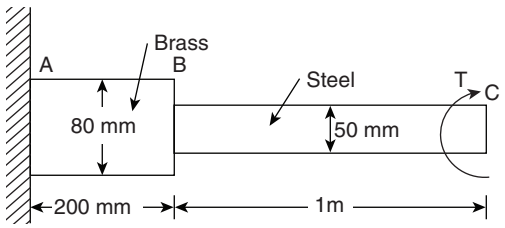
A simply supported beam *AB* is loaded as shown in the figure. The beam has a rectangular cross section of 100 mm width and 240 mm depth.

15. At a section 1.5 m from *A* maximum shearing stress is
(A) 0.0625 N/mm^2
(B) 0.0848 N/mm^2
(C) 0.0313 N/mm^2
(D) 0.0565 N/mm^2

16. Principal stresses at a point in neutral axis of the above section in N/mm^2 is
(A) +0.0313, 0
(B) +0.0313, –0.0313
(C) +0.0625, 0.0313
(D) +0.0625, –0.0625

17. A 2 m long wooden column bottom fixed and top end free, has a square cross section and has to take a load of 100 kN. Modulus of elasticity is 12 GPa. Size of the column, using Euler's formula and a factor of safety 3, is
(A) 148.5 mm (B) 135.8 mm
(C) 162.3 mm (D) 156.7 mm

Common Data for Questions 18 and 19:

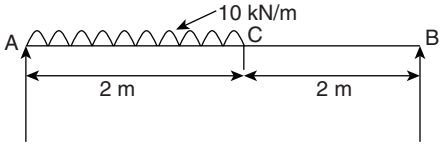


For the stepped shaft *ABC*, fixed at *A*, portion *AB* is made of brass and portion *BC* is made of steel, Allowable shear stress for brass is 80 N/mm^2 and for steel is 100 N/mm^2. Modulus of rigidity for brass is 40 kN/mm^2 and for steel is 80 kN/mm^2

18. Maximum value of torque that can be applied at the end of the shaft is
(A) 8042 Nm (B) 6053 Nm
(C) 2454 Nm (D) 3064 Nm

19. Total rotation at the free end in degrees is
(A) 4.18° (B) 3.04°
(C) 3.62° (D) 2.51°

Common Data for Questions 20 and 21:

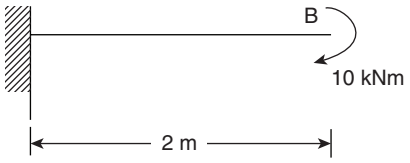


The simply supported beam loaded as shown above has a flexural rigidity 833.33 kN – m^2

20. Slope at end *A* is
(A) 12 rad (B) 15 rad
(C) 10 rad (D) 17 rad

21. Maximum deflection occurs between *A* and *C*. Distance from *A* is
(A) 2.00 m (B) 1.00 m
(C) 1.84 m (D) 1.95 m

22.

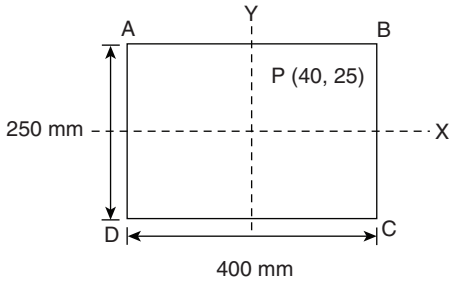


A cantilever *AB* of length 2 m and 100 mm breadth and 200 mm depth is fixed at end *A*. It is subjected to a moment of 10 kNm at the free end *B*. Flexural rigidity is 13,340 kN–m^2. Magnitude of maximum deflection is
(A) 1.8 mm (B) 2.2 mm
(C) 1.5 mm (D) 2.8 mm

23. A cylindrical tank of 750 mm internal diameter and 4 m length is made of 18 mm thick sheet. If it is subjected to an internal fluid pressure of 2 N/mm^2, maximum intensity of shear stress induced is
(Take $E = 2 \times 10^5$ N/mm^2 and $\mu = 0.3$)
(A) 11.78 N/mm^2 (B) 9.57 N/mm^2
(C) 8.62 N/mm^2 (D) 10.42 N/mm^2

24. Change in volume of the tank in cm^3 is
(A) 699 (B) 1119
(C) 1520 (D) 386

25.



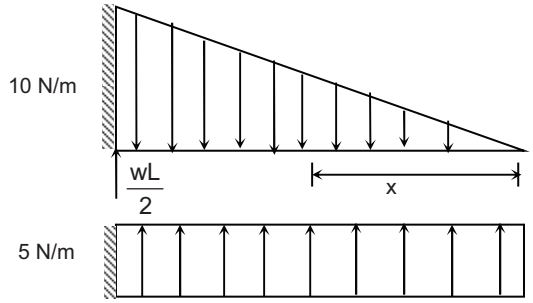
On a short masonry column of cross section as shown above, a concentrated load of 500 kN is applied at point *P*, + 40 mm from *y* axis and + 25 mm from *x* – axis. Moment of inertia about *x*– axis = 520.833×10^6 and moment of inertia about *y* – axis = 1333×10^6
Stress developed at point *D* is
(A) 2.10 N/mm^2 (B) 1.15 N/mm^2
(C) 1.00 N/mm^2 (D) 2.67 N/mm^2

Answer Keys

1. B	**2.** D	**3.** D	**4.** B	**5.** A	**6.** C	**7.** B	**8.** D	**9.** B	**10.** D
11. A	**12.** D	**13.** B	**14.** C	**15.** C	**16.** B	**17.** A	**18.** C	**19.** B	**20.** B
21. C	**22.** C	**23.** D	**24.** A	**25.** C					

Hints and Explanations

1. The load acting can be split in to two as shown in figure
(i) A uniformly varying load of 10 N/m at fixed end and zero at free end.
(ii) A uniformly distributed negative load of 5 N/m acting through out the beam.



Bending moment due to uniformly varying load at $x = 3$ m.
Bending moment due to the uniformly distributed load

$$= + \frac{wx^2}{2} = \frac{5 \times (3)^2}{2} = 22.5 \text{ Nm}$$

Total bending moment = 22.5 – 7.5 = 15 Nm
Choice (B)

2. Radius of Mohr's circle is $= \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$

$$= \sqrt{\left(\frac{260 - 100}{2}\right)^2 + 60^2}$$

$= \sqrt{80^2 + 60^2}$

$= \sqrt{6400 + 3600}$

$= \sqrt{1000} = 100.$ Choice (D)

3. Stress in portion AB $\sigma_1 = \dfrac{20000}{200} = 100$ N/mm^2

Stress in portion BC $\sigma_2 = \dfrac{20000}{100} = 200$ N/mm^2

Strain energy stored $= \sum\left(\dfrac{\sigma^2}{2E} \times \text{volume}\right)$

$= \dfrac{(100)^2}{2\times 2\times 10^5} \times (200\times 500) + \dfrac{200^2}{2\times 2\times 10^5} \times (100\times 800)$

$= 10{,}500$ N – mm $= 10.5$ Nm. Choice (D)

4. Deflection of beam at the free end if no prop is there $= \dfrac{wL^4}{8EI}$

Upward deflection of the end due to the force $F = \dfrac{FL^3}{3EI}$

When there is no resultant deflection

$$\frac{FL^3}{3EI} = \frac{wL^4}{8EI}$$

$\therefore \quad F = \dfrac{3}{8}wL$. Choice (B)

5. Extension of vertical conical bar due to self weight

$= \dfrac{wl^2}{6E} = \dfrac{80{,}000 \times 1^2}{6\times 2\times 10^5 \times 10^6}$ m

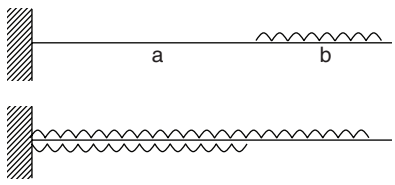
$= 6.67 \times 10^{-5}$ m Choice (A)

7. $\tau = \dfrac{16T}{\pi d^3}$

$\sigma = \dfrac{32M}{\pi d^3}$

$\dfrac{\tau}{\sigma} = \dfrac{16T}{\pi d^3} \times \dfrac{\pi d^3}{32M} = 1:2$ Choice (B)

11.



The given uniformly distributed loading of the cantilever can be treated as a combination as shown above. i.e., a full loading + a negative loading through length a

$\therefore$ Deflection at free end

$= \dfrac{w\ell^4}{8EI} - \left[\dfrac{wa^4}{8EI} + \dfrac{wa^3.b}{6EI}\right]$

$= \dfrac{10\times 5^4}{8EI} - \left[\dfrac{10\times 3^4}{8EI} + \dfrac{10\times 3^3\times 2}{6EI}\right]$

$= \dfrac{1}{EI}\left[781.25 - (101.25 + 90)\right] = \dfrac{590}{EI}$

$E = 2 \times 10^5$ N/mm^2 $= 2 \times 10^8$ kN/m^2

$I = 180 \times 10^6$ mm^4 $= 180 \times 10^{-6}$ m^4

$\therefore$ Deflection $= \dfrac{590}{2\times 10^8 \times 180 \times 10^{-6}}$

$= 0.01639$ m $= 16.39$ mm. Choice (A)

12. Rankine's formula is

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\ell/k\right)^2} \quad \text{Or} \quad \frac{P_{cr}}{A} = \frac{\sigma_c}{1 + a\left(\ell/k\right)^2}$$

When ℓ/k = slenderness ratio = 65

$$\frac{P_{cr}}{A} = 200 \text{ N/mm}^2$$

$$\therefore \quad 200 = \frac{\sigma_c}{1 + a(65)^2} \qquad \text{----------(1)}$$

When $\ell/k = 160$

$$\frac{P_{cr}}{A} = 70$$

$$\therefore \quad 70 = \frac{\sigma_c}{1 + a(160)^2} \qquad \text{----------(2)}$$

(1) ÷ (2) gives,

$$\frac{200}{70} = \frac{1 + a(160)^2}{1 + a(65)^2}$$

$200 + 845{,}000\,a = 70 + 1792{,}000\,a$

$947000\,a = 130$

$a = 1.373 \times 10^{-4}$. Choice (D)

13. $M = 2.5$ kNm $= 2.5 \times 10^6$ Nmm

$T = 5$ kNm $= 5 \times 10^6$ Nmm

Maximum principal stress

$$p_1 = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

$$= \frac{16}{\pi 100^3}\left[2.5 + \sqrt{2.5^2 + 5^2}\right] \times 10^6 = 41.2 \text{ N/mm}^2$$

$$p_2 = \frac{16}{\pi d^3}\left[M - \sqrt{M^2 + T^2}\right] \times 10^6$$

$$= \frac{16}{\pi 100^3}\left[2.5 - \sqrt{2.5^2 + 5^2}\right] \times 10^6$$

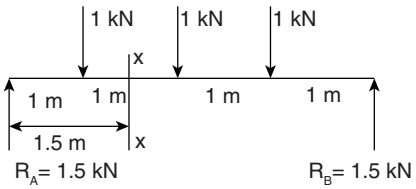
$= -15.74$ N/mm^2. Choice (B)

14. Let p be the direct stress

$$\frac{p}{E} = \frac{p_1}{E} - \frac{\mu p_2}{E}$$

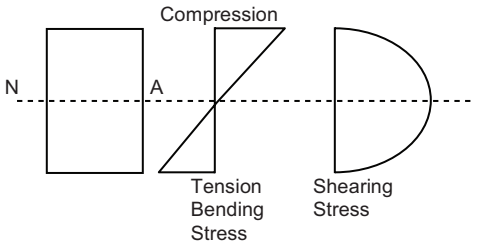
$P = 41.2 - 0.3\,(-15.74) = 45.92$ N/mm^2. Choice (C)

15.



Shear force at the section xx is $1.5 - 1.0 = 0.5$ kN

Average shearing stress $= \dfrac{0.5\times1000}{bd} = \dfrac{500}{100\times240}$



Shearing stress is maximum at neutral axis

Maximum shearing stress

$= 1.5 \times$ average shearing stress

$= 1.5\times\dfrac{500}{24000} = 0.03125$ N/mm^2. Choice (C)

16. At Neutral axis bending stress (Normal stress) is zero

$\therefore \quad p_x = 0,\ p_y = 0$ and $q = 0.03125$ N/m^2

Principal stress

$$= \frac{p_x + p_y}{2} + \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2} = 0 \pm \sqrt{0 + (0.03125)^2}$$

$= \pm\, 0.03125$ N/mm^2. Choice (B)

17. Critical load = working load × factor of safety

$= 100 \times 3 = 300$ kN

Applying Euler's formula

$$\text{Critical load} = \frac{\pi^2 EI}{L^2}$$

When L = Effective length = 2 × actual length

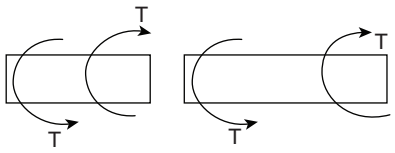
$= 2 \times 2 = 4$ m $= 4000$ mm

$$\therefore \quad 300 \times 10^3 = \frac{\pi^2 \times 12 \times 10^3 \times I}{(4000)^2}$$

$$I = 40{,}528{,}473 = \frac{a^4}{12}$$

$\therefore \quad a = 148.5$ mm. Choice (A)

18.



Both portions are subjected to same torque T

For brass portion maximum torque that can be applied

$$T_b = \frac{\pi}{16} d^3 \tau_b = \frac{\pi}{16}(80)^3 \times 80$$

$= 8042{,}477$ N–mm $= 8042.477$ Nm

For steel portion, $T_s = \dfrac{\pi}{16}(50)^3 \times 100$

$= 2454{,}369$ N–mm

$= 2454.369$ Nm

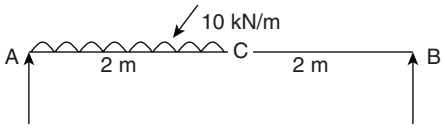
So maximum torque that can be applied at the end of the shaft is 2454 Nm = 2454 kN mm Choice (C)

19. Rotation of the free end $= \theta_B + \theta_s = \dfrac{TL_b}{G_b J_b} + \dfrac{TL_s}{G_s J_s}$

$$= 2454 \times \frac{32}{\pi}\left[\frac{200}{40\times80^4} + \frac{1000}{80\times50^4}\right]$$

$= 0.053$ rad $= 3.04°$. Choice (B)

20.



$V_A + V_B = 2 \times 10 = 20$ kN

$V_A \times 4 = 2 \times 10 \times 3$

$V_A = 15$ kN

$V_B = 5$ kN

Using Macaulay's method

$$Mx = 5\,x - \frac{10(x-2)^2}{2} = \text{EI}\frac{d^2y}{dx^2}$$

$$\text{EI}\frac{dy}{dx} = C_1 + \frac{5x^2}{2} - \frac{5(x-2)^3}{3}$$

$$\text{EI}y = C_2 + C_1 x + \frac{5}{2}\frac{x^3}{3} - \frac{5}{3}\frac{(x-2)^4}{4}$$

At $x = 0,\ y = 0$

$0 = C_2 + 0 + 0$

At $x = 4,\ y = 0$

$$0 = 0 + 4C_1 + \frac{5}{2}\times\frac{4^3}{3} - \frac{5}{3}\times\frac{2^4}{4}$$

$-4C_1 = 53.33 - 6.67 = 46.66$

$C_1 = -11.67$

$$\text{EI}\frac{dy}{dx} = -11.67 + \frac{5x^2}{2} - \frac{5(x-2)^3}{3}$$

At A, $x = 4$

$$\therefore \quad EI\left(\frac{dy}{dx}\right)_A = -11.67 + \frac{5\times 4^2}{2} - \frac{5\times 2^3}{3}$$

$$= -11.67 + 40 - \frac{40}{3} = 14.996 \text{ rad.}$$

Choice (B)

21. Maximum deflection occurs where slope is zero

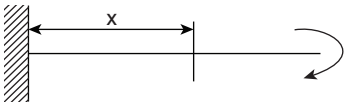
$$\therefore \quad -11.67 + \frac{5x^2}{2} - \frac{5}{3}(x-2)^3 = 0$$

$$2.5x^2 - \frac{5}{3}(x-2)^3 = 11.67$$

$\therefore$ $x = 2.16$ m (distance from C)

$\therefore$ Distance from $A = 4 - 2.16 = 1.84$ m Choice (C)

22.



At any section bending moment is 10 kNm

i.e., $M_x = 10$ kNm

$$EI\frac{d^2y}{dx^2} = 10 \text{ kNm}$$

$$EI\frac{dy}{dx} = 10x + C_1$$

At $x = 0$, $\frac{dy}{dx} = 0$

$\therefore$ $C_1 = 0$

$$EIy = \frac{10x^2}{2} + C_2$$

At $x = 0$, $y = 0$

$$\therefore \quad EIy = \frac{10x^2}{2}$$

$$EI\ y_{max} = \frac{10\times 2^2}{2}$$

Flexural rigidity EI = 13, 340 kN – m²

$$\therefore \quad y_{max} = \frac{20}{13340} \text{ m}$$

$$= 1.5 \times 10^{-3} \text{ m}$$

$$= 1.5 \text{ mm or}$$

$$y_{max} = y_{max} = \frac{ML^2}{2EI} = \frac{10\times 10^3 \times 2^2}{2\times 13340}$$

$$= 1.5 \text{ mm}$$ Choice (C)

23. $d = 750$ mm

$t = 18$ mm

$$\frac{d}{t} = \frac{750}{18} = 41.67 > 15$$

$\therefore$ Thin cylinder formula can be applied

Hoop stress $\sigma_1 = \frac{pd}{2t} = \frac{2\times 750}{2\times 18} = 41.67$ N/mm²

Longitudinal stress $\sigma_2 = \frac{pd}{4t} = \frac{41.67}{2} = 20.83$ N/mm²

$q_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{20.83}{2} = 10.42$ N/mm². Choice (D)

24. Diametral strain $\frac{\delta d}{d} = e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$

$$= \frac{41.67}{2\times 10^5} - 0.3\times\frac{20.83}{2\times 10^5} = 17.71 \times 10^{-5}$$

Longitudinal strain $\frac{\delta L}{L} = e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$

$$= \frac{20.83 - 0.3\times 41.67}{2\times 10^5} = 4.165 \times 10^{-5}$$

Volumetric strain $\frac{\delta V}{V} = 2e_1 + e_2$

$$= (2 \times 17.71 + 4.165) \times 10^{-5} = 39.58 \times 10^{-5}$$

$\delta V = 39.58 \times 10^{-5} \times V$

$$= \frac{39.58}{10^5} \times \frac{\pi\times(0.75)^2}{4} \times 4 \text{ m}^3$$

$$= 69.94 \times 10^{-5} \text{ m}^3 = 699.43 \text{ cm}^3.$$ Choice (A)

25. $\sigma = \frac{p}{A} + \frac{M_x}{I_x}y + \frac{M_y}{I_y}x$

$$= \frac{500\times 10^3}{250\times 400} + \frac{500\times 10^3 \times 25}{520.833\times 10^6}y + \frac{500\times 10^3 \times 40}{1333\times 10}x$$

$$f_D = 500\times 10^3\left[\frac{1}{250\times 400} - \frac{25\times 125}{520.833\times 10^6} - \frac{40\times 200}{1333\times 10^6}\right]$$

$= -1$ N/mm². Choice (C)

Solid Mechanics Test 2

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. If Poisson's ratio of a material is 0.5, modulus of elasticity of the material is

(A) $\frac{1}{3}$ times the shear modulus

(B) 3 times the shear modulus

(C) 4 times the shear modulus

(D) equal to the shear modulus

2. A circular shaft subjected to torsion undergoes a twist of 1° in a length of 1.6 m. If the maximum shear stress induced is 10,000 N/cm^2 and if modulus of rigidity is 8×10^6 N/cm^2 then radius of the shaft is

(A) $\frac{27}{\pi}$ cm (B) $\frac{36}{\pi}$ cm

(C) $\frac{\pi}{36}$ cm (D) $\frac{\pi}{27}$ cm

3. A 2 m long mild steel bar of 2000 mm^2 cross sectional area is subjected to an axial load of 40 kN. If Young's modulus for the shaft is 2×10^5 N/mm^2, extension of the shaft in mm is

(A) 0.5 mm (B) 1 mm

(C) 0.2 mm (D) 2 mm

4. A steel bar of 1 m length is heated from 30°C to 60°C. Coefficient of linear expansion is 12×10^{-6}/°C and Young's modulus is 2×10^5 MN/m^2. Stress developed in the bar is

(A) 18 N/mm^2 (B) zero

(C) 36 N/mm^2 (D) 72 N/mm^2

5. Slope of a beam under load is

(A) rate of change of deflection

(B) rate of change of bending moment

(C) rate of change of bending moment x flexural rigidity

(D) rate of change deflection x flexural rigidity

6. Relationship between modulus of elasticity E, modulus of rigidity G and bulk modulus K is

(A) $E = \frac{6KG}{3K+G}$ (B) $E = \frac{9KG}{3K+G}$

(C) $E = \frac{3K+G}{6KG}$ (D) $E = \frac{3K+G}{9KG}$

7. A column with 80mm diameter is fixed at both the ends. If the crippling load calculated by Rankine formula is 750 kN then what will be the crushing load of the column. The length of column is 8m. E = 180 GPa

(A) 949.4 kN (B) 994.4 kN

(C) 317.77 kN (D) 984.6 kN

8. A bar of 3m in length 30 mm breadth and 20 mm thickness is subjected to a compressive stress of 50 kN/m^2. What will be the final volume of the bar if the poisson's ratio is 0.30 and modulus of rigidity is 90 GN/m^2.

(A) will increase by 0.4615 mm^3

(B) will decrease by 0.5625 mm^3

(C) will decrease by 0.4615 mm^3

(D) will increase by 0.5625 mm^3

9. When 'C' is the modulus of rigidity and 'q' is the intensity of shear stress, the strain energy due to shear is given by

(A) $\frac{q}{2C} \times$ volume of block

(B) $\frac{q^2}{2C} \times$ volume of block

(C) $\sqrt{\frac{q^2}{2C}} \times$ volume of block

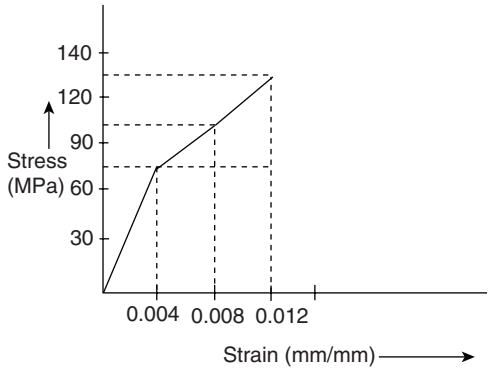
(D) $\frac{q}{2} \times$ Volume of block

10. According to maximum shear stress failure criterion, yielding in material occurs when maximum shear stress is

(A) $\frac{1}{2}$ yield stress (B) $\sqrt{2}$ yield stress

(C) $\frac{\sqrt{2}}{3}$ yield stress (D) 2 yield stress

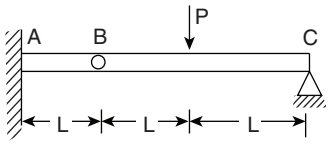
11. Stress-strain behaviour of a material is shown in the figure. Proof Resilience in Nm/m^3 is



(A) 10×10^4 (B) 15×10^4

(C) 76×10^4 (D) 130×10^4

12.

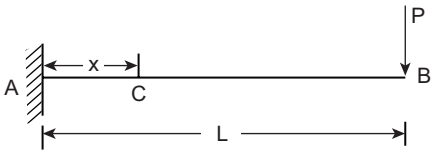


A beam is made of 2 bars *AB* and *BC* hinged at *B*, fixed at *A* and simply supported at *C*. If it is loaded at mid point of *BC* as shown in figure, bending moment at *A* is

(A) PL (B) $\frac{PL}{2}$

(C) $2PL$ (D) $\frac{2}{3}PL$

13. A cantilever beam is loaded as shown in the figure



If L is the length of the beam and EI, the flexural rigidity, slope at point C at a distance x from fixed end is

(A) $\frac{Px}{EI}(2L-x)$ (B) $-\frac{Px}{2EI}(2L-x)$

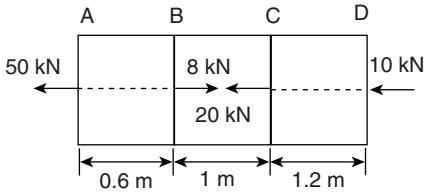
(C) $\frac{Px}{2EI}(L-x)$ (D) $-\frac{Px}{EI}(L-x)$

14. The extension of a circular bar tapering uniformly from diameter d_1 to d_2 is same as that of a uniform circular bar of same length, under same load. Diameter of the uniform bar is

(A) $\sqrt{d_1 d_2}$ (B) $\sqrt{d_1^2 - d_2^2}$

(C) $\frac{d_1 + d_2}{2}$ (D) $\frac{d_1 - d_2}{2}$

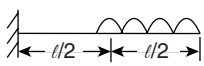
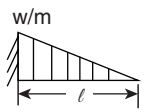
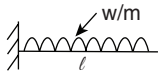
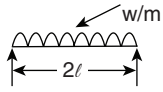
15. A brass bar having a cross sectional area of 1000 mm^2 is subjected to axial forces as shown in the figure. The total change in length of the bar is.
Take $E = 1.05 \times 10^5$ N/mm^2.



(A) – 0.15 mm (B) + 0.15 mm

(C) – 0.1143 mm (D) + 0.1143 mm

16. Match the following List – I (Loaded beam) and List – II (Maximum bending moment).

	List – I		List – II
a.		1.	$\frac{w\ell^2}{2}$
b.		2.	$\frac{w\ell^2}{6}$
c.		3.	$\frac{3}{8}$ w ℓ^2
d.		4.	$\frac{w\ell^2}{4}$

Codes:

	a	b	c	d		a	b	c	d
(A)	3	2	4	1	(B)	3	4	2	1
(C)	1	2	3	4	(D)	4	3	2	1

Directions for questions 17 and 18: A circular bar made of C.I is to resist on occasional torque of 2.2 kNm acting in transverse plane. The allowable stresses in compression, tension and shear are 100, 50, 35 MN/m^2 respectively.
Take $G = 40$ GN/m^2.

17. The diameter of the bar will be

(A) 64.8 mm (B) 68.4 mm

(C) 66.8 mm (D) 67.4 mm

18. The angle of twist under the applied torque per meter length of bar will be

(A) 1.86° (B) 1.26°

(C) 1.46° (D) 1.16°

19. If diameter of long column is reduced by 20% then percentage of reduction in Euler buckling is

(A) 4 (B) 36

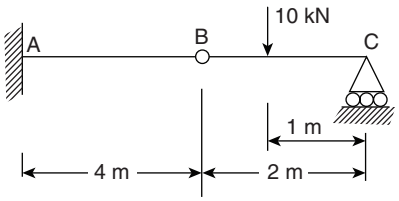
(C) 49 (D) 59

20. A shaft subjected to a maximum bending stress of 80 N/mm^2 and maximum shearing stress equal to 30 N/mm^2 at a particular section. If the yield point in the tension of the material is 280 N/mm^2, and maximum shear stress theory of failure is used, then the factor of safety obtained will be

(A) 2.5 (B) 2.8

(C) 3.0 (D) 3.5

21. A Cantilever beam *AB* is connected to another beam *BC* with a pin joint at *B* as shown in the figure. For the loading as shown in the figure, the magnitude of bending moment at *A* (in kN-m) is



(A) 5 (B) 10

(C) 20 (D) 50

22. A solid metal tube with modulus of elasticity E and Poisson's ratio μ is constrained on all faces. It is heated so that temperature rises uniformly. If coefficient of thermal expansion is α, the compressive stress developed in the cube due to the heating is

(A) $\frac{E\alpha\Delta T}{2(1-2\mu)}$ (B) $\frac{E\alpha\Delta T}{(1-2\mu)}$

(C) $\frac{2E\alpha\Delta T}{(1-2\mu)}$ (D) $\frac{E\alpha\Delta T}{3(1-2\mu)}$

23. A bar of length L, breadth b and thickness t is subjected to an axial pull of P. If e_x is the strain in the direction of pull, volumetric strain produced is (μ = Poisson's ratio)

(A) $e_x(1+2\mu)$
(B) $e_x(1-2\mu)$
(C) $e_x(1+\mu)$
(D) $e_x(1-\mu)$

24. A simply supported beam of length L has a cross section of depth d and width $\frac{d}{2}$. If it is loaded with a uniformly distributed load of w/unit length, maximum deflection is (Young's modulus = E)

(A) $\frac{5}{8}\frac{wL^4}{Ed^4}$ (B) $\frac{5}{16}\frac{wL^4}{Ed^4}$

(C) $\frac{5}{8}\frac{wL^3}{Ed^4}$ (D) $\frac{5}{16}\frac{wL^3}{ed^4}$

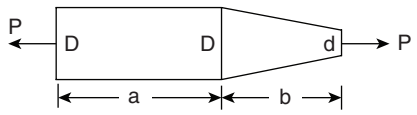
25. When a material is subjected to uniaxial tension, to avoid failure due to shear in 45° planes, the shear strength of the material should be atleast

(A) half the tensile strength
(B) $\frac{1}{\sqrt{2}}$ times tensile strength
(C) tensile strength
(D) $\frac{3}{4}$ times tensile strength

26. At a point in a strained material, direct stresses 120 N/mm^2 (tensile) and 100 N/mm^2 (compressive) are acting. If major principal stress is 150 N/mm^2, maximum shearing stress at the point is

(A) 87 N/mm^2
(B) 140 N/mm^2
(C) 130 N/mm^2
(D) 280 N/mm^2

27.



A bar is having uniform diameter D for a length a and tapering diameter from D to d for a length b as shown in figure. If the bar is subjected to an axial pull P, the extension produced is

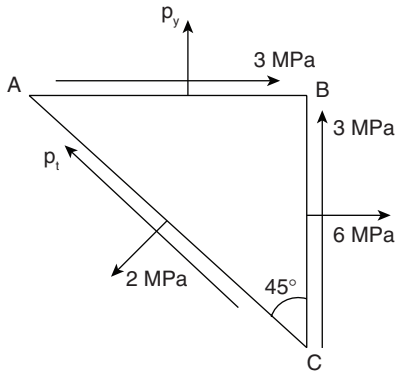
(A) $\frac{4P}{\pi DE}\left(\frac{a}{D}+\frac{b}{d}\right)$

(B) $\frac{4P}{\pi dE}\left(\frac{a}{D}+\frac{b}{d}\right)$

(C) $\frac{2P}{\pi DE}\left(\frac{a}{D}+\frac{b}{d}\right)$

(D) $\frac{2P}{\pi dE}\left(\frac{a}{D}+\frac{b}{d}\right)$

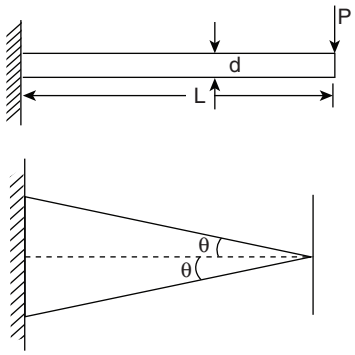
28.



At a point in a stressed body stresses acting are as shown in the figure. Value of P_y is

(A) –8 MPa (B) 8 MPa
(C) –4 MPa (D) 4 MPa

29.

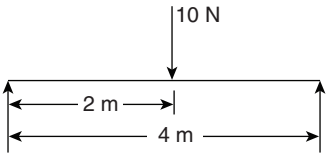


A cantilever beam of varying width and constant depth is loaded as shown in the figure. Maximum bending stress at the fixed end of the beam is

(A) $\frac{PL}{d^2\tan\theta}$ (B) $\frac{2P}{d^2\tan\theta}$

(C) $\frac{3P}{d^2\tan\theta}$ (D) $\frac{3PL}{d^2\tan\theta}$

30.



A beam with cross section 10 cm width and 20 cm depth is loaded as shown in the figure. Maximum shear stress at a section 1 m away from end is

(A) 0 (B) 0.375 MPa
(C) 3.75 MPa (D) 37.5 MPa

Answer Keys

1. B	**2.** B	**3.** C	**4.** B	**5.** A	**6.** B	**7.** C	**8.** C	**9.** B	**10.** A
11. B	**12.** B	**13.** B	**14.** A	**15.** C	**16.** A	**17.** B	**18.** C	**19.** D	**20.** B
21. C	**22.** B	**23.** B	**24.** B	**25.** A	**26.** B	**27.** A	**28.** A	**29.** C	**30.** B

Hints and Explanations

1. $E = 2G(1+\mu)$
$= 2G(1+0.5) \Rightarrow E = 3G$ Choice (B)

2. $L = 1.6$ m
$\theta = 1° = \dfrac{\pi}{180}$ radian
$G = 8 \times 10^6$ N/cm^2 $= 8 \times 10^{10}$ N/m^2
$\tau = 10,000$ N/cm^2 $= 1 \times 10^8$ N/m^2

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

$$\Rightarrow R = \frac{\tau L}{G\theta} = \frac{10^8 \times 1.6 \times 180}{8 \times 10^{10} \times \pi}\text{m} = \frac{0.36}{\pi}\text{m} = \frac{36}{\pi}\text{cm}.$$

Choice (B)

3. $L = 2$ m $= 200$ mm; $A = 2000$ mm^2;
$P = 40$ kN $= 40 \times 10^3$ N
$E = 2 \times 10^5$ N/mm^2

$$\text{Extension } \delta L = \frac{PL}{AE} = \frac{40 \times 10^3 \times 2000}{2000 \times 2 \times 10^5} = 0.2 \text{ mm}.$$

Choice (C)

4. As the expansion of the bar is not blocked, no stress is developed. Choice (B)

7. If both the ends fixed

$$L_e = \frac{L}{2} = \frac{8}{2} = 4\text{m}$$

$E = 180 \times 10^9$N/m^2

$$\text{Euler's load } P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 180 \times 10^9 \times I}{4^2}$$

$d = 80$mm $= 80 \times 10^{-3}$m

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\left(80 \times 10^{-3}\right)^4 = 2.01 \times 10^{-6}\,\text{m}^4$$

$$P_E = \frac{\pi^2 \times 180 \times 10^9 \times 2.01 \times 10^{-6}}{16 \times 1000} = 223.2 \text{ kN}$$

$$\frac{1}{P_R} = \frac{1}{P_E} + \frac{1}{P_C}$$

$$\frac{1}{750} = \frac{1}{223.2} + \frac{1}{P_C}$$

$$\frac{1}{P_C} = \frac{1}{750} - \frac{1}{223.2} = \frac{223.2 - 750}{750 \times 223.2}$$

$P_C = 317.77$ kN Choice (C)

8. $\ell = 3\text{m} = 3000$ mm
$b = 30$ mm $= 30 \times 10^{-3}$ m
$t = 20$ mm $= 20 \times 10^{-3}$m
$\sigma = 50$ kN/m^2
$\mu = 0.3$
$G = 90 \times 10^9$ N/m^2
$V = \ell b\, t = 3000 \times 30 \times 20$
$= 18 \times 10^5$ mm^3
$E = 2G(1+\mu)$
$E = 2 \times 90 \times 10^9 (1 + 0.3) = 234 \times 10^9$ N/m^2
$E = 3K(1 - 2\mu)$
$234 \times 10^9 = 3K(1 - 0.6)$

$$K = \frac{234 \times 10^9}{3 \times 0.4} = 195 \times 10^9$$

$$K = \frac{\text{Compressive stress}}{\text{volumetric strain}}$$

$$195 \times 10^9 = \frac{50 \times 10^3}{\left(\frac{dv}{v}\right)}$$

$$\frac{dv}{v} = \frac{50 \times 10^3}{195 \times 10^9} = 0.256 \times 10^{-6}$$

$dv = 0.4615$ mm^3
Will decrease by 0.4615 mm^3. Choice (C)

11. Proof resilience is the strain energy at elastic limit

$$= \frac{1}{2}(\text{stress} \times \text{strain} \times \text{volume})$$

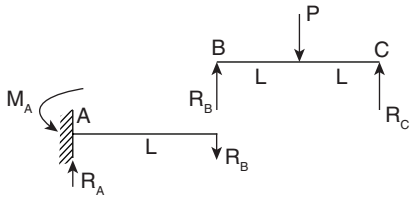
Proof resilience/ unit volume

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \text{ at elastic limit}$$

$$= \frac{1}{2} \times 75 \times 10^6 \times 0.004 = 15 \times 10^4 \text{ Nm/m}^3$$

Choice (B)

12.



Free body diagram is as shown above

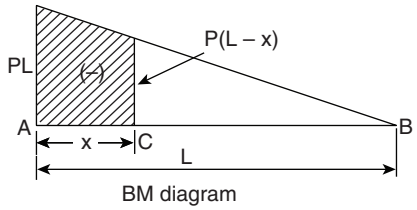
$$R_B = R_C = \frac{P}{2}$$

Bending moment at A

$$M_A = R_B \times L = \frac{PL}{2}.$$

Choice (B)

13.



BM diagram

Slope at C

$Q_C = Q_C - Q_A$ as $QA = 0$

$$= \frac{\text{Area of bending moment diagram between } A \text{ and } C}{EI}$$

$$= \frac{-\left[PL + P(L-x)\right]x}{2EI} = \frac{-(2PL - Px)}{2EI}x$$

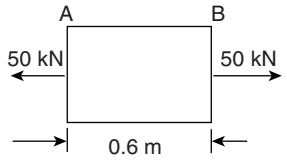
$$= \frac{-Px}{2EI}(2L - x)$$ Choice (B)

14. Extension $= \dfrac{PL}{AE}$

$$\therefore \quad \frac{PL}{\frac{\pi d^2}{4}E} = \frac{PL}{\frac{\pi d_1 d_2}{4}E}$$

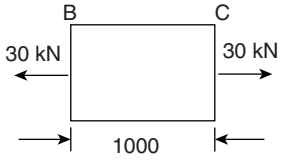
$$\Rightarrow \quad d = \sqrt{d_1 d_2}$$ Choice (A)

15.

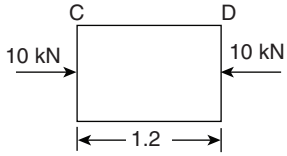


$$\delta l_1 = \frac{p\ell}{AE} = \frac{50\times10^3\times600}{1000\times1.05\times10^5}$$

$\delta l_1 = 0.2857$ mm



$$\delta l_2 = \frac{p\ell}{AE} = \frac{30\times10^3\times1000}{1000\times1.05\times10^5} = -0.2857\text{mm (–)}$$



$$\delta l_3 = \frac{p_l}{A_E} = \frac{10\times10^3\times1200}{1000\times1.05\times10^5} = -0.1143 \text{ mm}$$

$\delta l = \delta l_1 + \delta l_2 + \delta l_3 = -0.1143$ mm Choice (C)

17. $T = 2.2$ kNm

$\sigma_C = 100$ MN/m^2

$\sigma_t = 50$ MN/m^2

$\tau = 35$ MN/m^2

$G = 40$ GN/m^2

$$\frac{T}{J} = \frac{\tau_{max}}{R} = \frac{2.2\times10^3}{\frac{\pi}{32}d^4} = \frac{35\times10^6}{\frac{d}{2}}$$

$d = 68.4$ mm Choice (B)

18. $$\frac{\tau_{max}}{R} = \frac{G\theta}{L} = \frac{35\times10^6}{\left(\frac{68.4}{2000}\right)} = \frac{40\times10^9\times\theta}{1}$$

$\theta = 0.026$ radians $\times 180/\pi$

$\theta = 1.46°$ Choice (C)

19. $$F_{euler} = \frac{\pi^2 EI}{L^2}$$

$F \propto I$

$$I = \frac{\pi}{64}d^4$$

$F \propto d^4$

$$\frac{F_1 - F_2}{F_1} = \frac{d_1^4 - d_2^4}{d_1^4}$$

$$D_2 = 0.8d_1 = \frac{d_1^4 - (0.8d_1)^4}{d_1^4} = \left(\frac{1-0.8^4}{1}\right)\times100 = 59\%$$

Choice (D)

20. $\sigma_b = 80$ N/mm^2

$\tau = 30$ N/mm^2

$\sigma_y = 280$ N/mm^2

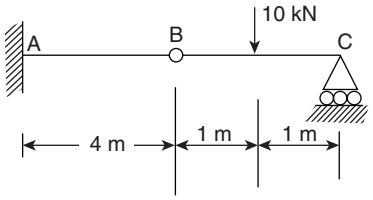
$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \frac{280}{FS\times2}$$

$$= \frac{1}{2}\sqrt{80^2 + 4(30)^2} = \frac{280}{FS\times2} = 50 = \frac{280}{FS}$$

F . S = factor of safety $= \dfrac{280}{50\times2} = 2.8$ Choice (B)

21. Bending moment at hinge B

$M_B = 0$



Considering the portion BC, reaction at C,

$R_c = 5$ kN

Let M_A be the bending moment at A

Taking moments about A

$10 \times 5 - 5 \times 6 = 20$ k Nm. Choice (C)

22. Let side length be L

Elongation due to heating $= \alpha\Delta TL$

As elongation is blocked compressive strain in the x-direction

$$e_x = \alpha \Delta T = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

As $\sigma_x = \sigma_y = \sigma_z = \sigma$

$$e_x = \frac{\sigma}{E}[1-2\mu]$$

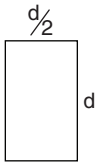
i.e., $\frac{\sigma}{E}[1-2\mu] = \alpha\Delta T \Rightarrow \sigma = \frac{E\alpha\Delta T}{(1-2\mu)}$ (compressive)

Choice (B)

23. Volumetric strain

$e_v = e_x + e_y + e_z = e_x + 2(-e_x \cdot \mu) = e_x(1-2\mu)$ Choice (B)

24.



Moment of inertia $= \frac{bd^3}{12} = \frac{\left(\frac{d}{2}\right)\cdot d^3}{12} = \frac{d^4}{24}$

Maximum deflection $= \frac{5wL^4}{384EI} = \frac{5}{384}\frac{wL^4}{E}\times\frac{24}{d^4}$

$= \frac{5}{16}\frac{wL^4}{Ed^4}$ Choice (B)

25. Maximum shear stress $= \sqrt{\left(\frac{p_x + p_y}{2}\right)^2 + q^2}$

$= \sqrt{\left(\frac{p_x}{2}\right)^2 + 0} = \frac{p_x}{2}$

Maximum shear stress $= \frac{1}{2}$ the uni axial tensile stress

Choice (A)

26. $p_x = 120$ N/mm^2, $p_y = -100$ N/mm^2

$p_1 = 150$ N/mm^2

$p_1 + p_2 = p_x + p_y = 120 - 100 = 20$ N/m^2

$\Rightarrow \quad 150 + p_2 = 20 \Rightarrow p_2 = -130$

Maximum shearing stress $= \frac{p_1 - p_2}{2}$

$= \frac{(150+130)}{2} = 140$ N/mm^2 Choice (B)

27. Mean area for tapering portion $= \frac{\pi Dd}{4}$

Total extension produced $= \frac{Pa}{\frac{\pi D^2}{4}E} + \frac{Pb}{\frac{\pi Dd}{4}E}$

$= \frac{4P}{\pi DE}\left(\frac{a}{D} + \frac{b}{d}\right)$ Choice (A)

28. Normal stress on plane AC

$$p_n = \frac{p_x + p_y}{2} + \frac{p_x - p_y}{2}\cos 2\theta + q\sin 2\theta$$

$p_n = 2$MPa; $p_x = 6$ MPa

$q = 3$ MPa; $\theta = 45°$

$\therefore \quad 2 = \frac{6 + p_y}{2} + 0 + 3\times 1 = 3 + \frac{p_y}{2} + 3$

$\Rightarrow \quad \frac{p_y}{2} = -4 \Rightarrow p_y = -8$ MPa

$= 8$ MPa (compressive) Choice (A)

29. Maximum bending stress $f = \frac{M}{Z}$

where M = bending moment

and Z = section modulus

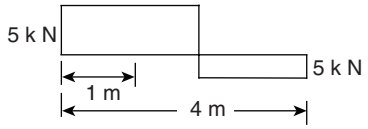
At fixed end,

$M = P \times L$

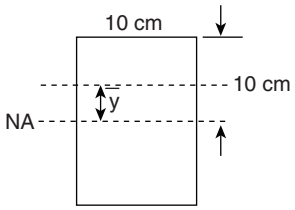
$Z = \frac{bd^2}{6} = \frac{2L\tan\theta d^2}{6}$

$\therefore \quad f = \frac{P\times L\times 6}{2Ld^2\tan\theta} = \frac{3P}{d^2\tan\theta}$ Choice (C)

30. Shear force diagram is given below



Shear force at a distance 1m from end $F = 5$kN



$\bar{y} = 50$ mm

Area above neutral axis

$a = (100)^2$ mm^2

breadth $b = 100$ mm

shear stress is maximum at neutral axis and is given by

$q = \frac{F}{bI}a\bar{y}$

$q_{max} = \frac{5\times 10^3 \times (100)^2 \times 50}{100 \times \frac{100\times(200)^3}{12}} = 0.375$ MPa

Alternately,

Average shear stress $q_{av} = \frac{F}{bd}$

Maximum shear stress

$q_{max} = q_{av} \times 1.5 = \frac{F}{bd}\times 1.5 = \frac{5\times 10^3 \times 1.5}{100\times 200} = 0.375$ MPa

Choice (B)

UNIT III
STRUCTURAL ANALYSIS

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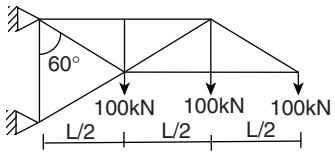
Structural Analysis Test I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following statements regarding statically determinate structures are correct?
(a) Can be analyzed by equilibrium equations
(b) Stresses are caused due to temperature changes
(c) *BM* at a section does not depend on material or sectional properties of structure.
(A) a, b, c are correct (B) a, b are correct
(C) a, c are correct (D) b, c are correct

2. The pin jointed frame shown in the figure is



(A) Perfect frame (B) Redundant frame
(C) deficit frame (D) None

3. Which of the following method comes under the category of displacement method to analyze statically indeterminate structure?
(A) Elastic center method
(B) Minimum strain energy theorem
(C) Moment distribution method
(D) Column analogy method

4. The cantilever beam *AB* of length '*L*' fixed at *A* and free at *B* is subjected to a concentrated load '*W*' at its free end. The strain energy (*U*) stored in a beam is [EI: Constant)

(A) $\frac{W^2 L^2}{4EI}$ (B) $\frac{WL^3}{6EI}$

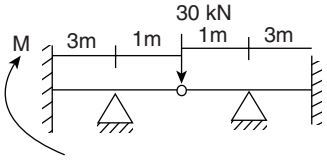
(C) $\frac{W^2 L^3}{6EI}$ (D) $\frac{WL}{EI}$

5. The Bending moment induced at fixed end of cantilever beam of span '*L*' if the free end undergoes a unit displacement without rotation is

(A) $\frac{3EI}{L^2}$ (B) $\frac{5EI}{L^2}$

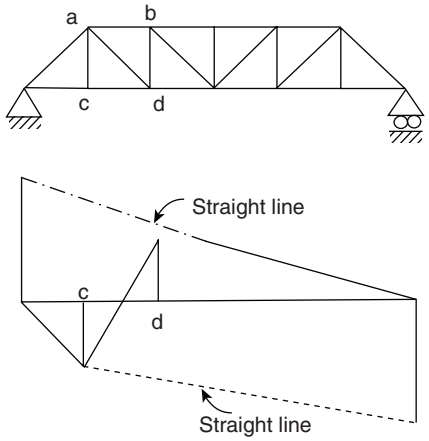
(C) $\frac{6EI}{L^2}$ (D) $\frac{4EI}{L^2}$

6. The value of support moment *M* for the beam shown below (in kN-m)



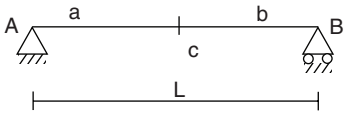
(A) 12.5 kN-m (B) 7.5 kN-m
(C) 15 kN-m (D) None

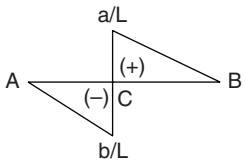
7. The Influence line diagram (I.L.D) shown is for the member ——

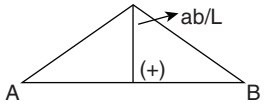


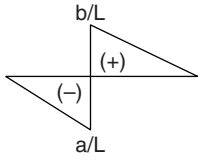
(A) *ab* (B) *ac*
(C) *cd* (D) *ad*

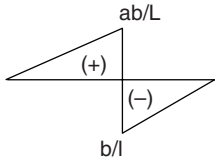
8. The ILD for shear force at a section '*c*' of simply supported beam of length '*L*', when unit load moves from one end to other is



(A)


(B)


(C)


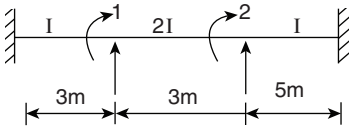
(D)


9. Which of the following statements below are correct?
(a) The stiffness coefficient k_{ji} indicates force at *j* due to a unit deformation at *i*.

(b) Stiffness matrix is a square symmetric matrix.
(c) Stiffness matrix is possible for both stable and unstable structures also.
(A) a, b, c are correct (B) a, b are correct
(C) a, c are correct (D) b, c are correct

10. Which of the following statements are correct?
(a) The internal forces at any c/s of an arch are *SF*, *BM* only
(b) The effect of arching a beam is to reduce *BM* in the span
(c) A two hinged arch is indeterminate by one degree
(d) The internal forces at any c/s of an arch are *SF*, *BM* and normal thrust also.
(A) a, b, c, d are correct
(B) a, b, c are correct
(C) b, c, d are correct
(D) a, c, d are correct

11. Determine the stiffness matrix for a beam for the given coordinates shown in the figure

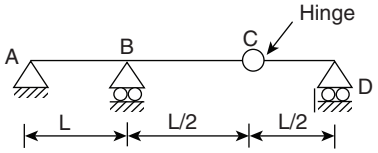


(A) $\begin{bmatrix} 4EI & \frac{4EI}{3} \\ \frac{4EI}{5} & 52EI \end{bmatrix}$ (B) $\begin{bmatrix} 4EI & \frac{4EI}{3} \\ \frac{4EI}{3} & \frac{52EI}{15} \end{bmatrix}$

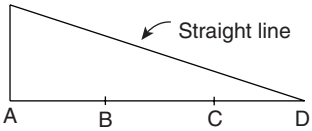
(C) $\begin{bmatrix} \frac{52EI}{15} & \frac{-4EI}{3} \\ \frac{-4EI}{3} & 4EI \end{bmatrix}$ (D) $\begin{bmatrix} 4EI & \frac{+4EI}{3} \\ \frac{4EI}{3} & 4EI \end{bmatrix}$

12. A cable carrying a load of 40 kN/m run of horizontal span, is stretched between supports 150m apart. The supports are at same level and the central dip is 15 m. The greatest tension and least tension in cable are
(A) 8100 kN, 7500 kN
(B) 10,000 kN, 7500 kN
(C) 9500 kN, 6000 kN
(D) None

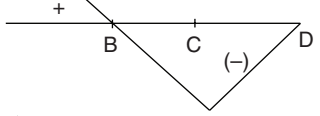
13. For the continuous beam shown below, the I.L.D for Reaction at *A* is



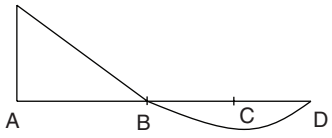
(A)



(B)



(C)

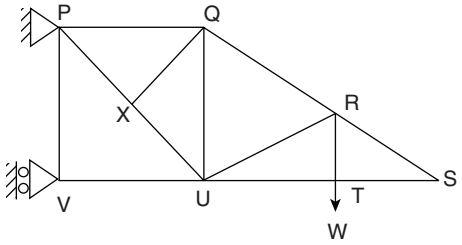


(D)



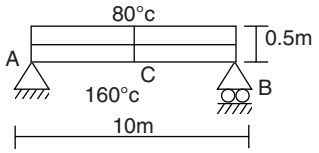
14. A beam *ABCD* is 15m long and is simply supported at *B* and *C* 8 m apart. Overhangs *BA* and *CD* are 3m and 4m respectively. A train of two point loads of 150 kN and 100 kN, 3 m apart, crosses the beam from left to right with 100 kN load leading. The maximum sagging *B.M.* under 150 kN load anywhere is
(A) 150 kN-m (B) 250 kN-m
(C) 360 kN-m (D) 400 kN-m

15. In the truss shown below, indicate how many number of members with zero forces



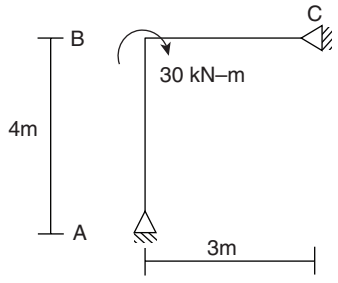
(A) 4 (B) 5
(C) 6 (D) None

16. A simply supported beam of length L = 10 m and depth = 0.5 m is subjected to a temperature differential of 80°C at top and 160°C at bottom. Determine the vertical deflection of beam at its mid point (c) due to temperature gradient take $\alpha = 10 \times 10^{-6}$ /°C.



(A) 25 mm (B) 30 mm
(C) 40 mm (D) None

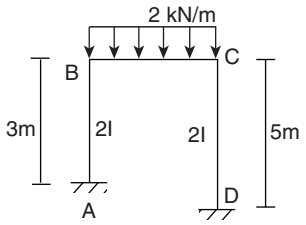
17. What is the rotation of the member at '*C*" for a frame as shown in figure below?



(A) $\frac{30}{3EI}$ (B) $\frac{60}{7EI}$

(C) $\frac{90}{EI}$ (D) $\frac{75}{EI}$

18. In the portal frame shown in the given figure, the ratio of sway moments in column *AB* and *CD* will be equal to



(A) 25/9 (B) 3/5

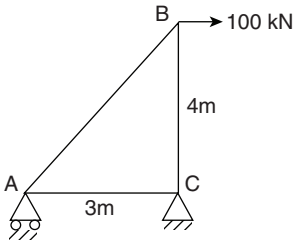
(C) 2/3 (D) 15/8

19. U_1 and U_2 are the strain energies stored in a prismatic bar due to axial tensile force w_1 and w_2 respectively. The strain energy 'U' stored in the same bar due to combined action of w_1 and w_2 be

(A) $U = U_1 U_2$ (B) $U > U_1 + U_2$

(C) $U < U_1 + U_2$ (D) $U = U_1 + U_2$

20. The right triangular truss is made of members having equal c/s area of 1000 mm^2 and youngs modulus of 2×10^5 MPa. The horizontal deflection at *B* is



(A) 15 mm (B) 20 mm

(C) 12 mm (D) None

21. A two spans continuous beam having equal spans each of length $L/2$ is subjected to a uniformly distributed load $2w$ per unit m length. The beam has constant flexural rigidity. The reaction at middle support is

(A) $\frac{3wl}{4}$ (B) $\frac{3wl}{8}$

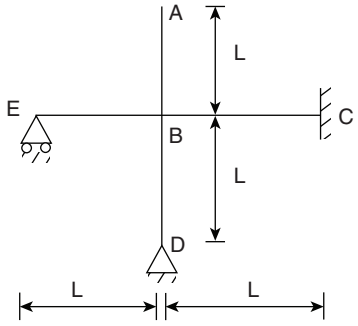
(C) $\frac{5wl}{4}$ (D) $\frac{5wl}{8}$

22. Using the data in Q No 21; find the Bending moment at the middle support

(A) $\frac{wl^2}{16}$ (B) $\frac{wl^2}{4}$

(C) $\frac{wl^2}{8}$ (D) $\frac{3wl^2}{16}$

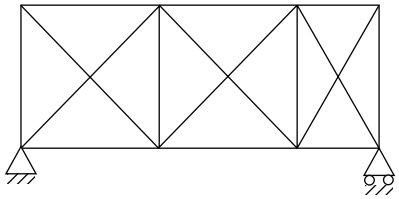
23. In the frame shown below; what are the distribution factors for members *BA*, *BC* and *BD* respectively



(A) 0, 0.4, 0.6 (B) 0, 0.3, 0.7

(C) 0, 0.4, 0.3 (D) None

24. Examine the given truss below



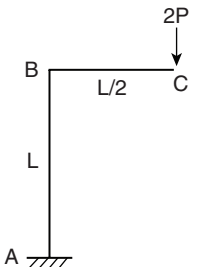
(A) statically determinate

(B) statically indeterminate but kinematically determinate

(C) statically indeterminate and kinematically indeterminate

(D) statically determinate and kinematically indeterminate

25. The horizontal deflection at *C* for the following frame shown below



(A) $\frac{PL^2}{2EI}$ (B) $\frac{PL^3}{3EI}$

(C) $\frac{PL^3}{2EI}$ (D) $\frac{2PL^2}{EI}$

Answer Keys

1. C	**2.** A	**3.** C	**4.** C	**5.** C	**6.** B	**7.** D	**8.** C	**9.** B	**10.** C
11. B	**12.** A	**13.** B	**14.** C	**15.** B	**16.** C	**17.** B	**18.** A	**19.** B	**20.** C
21. C	**22.** A	**23.** C	**24.** C	**25.** C					

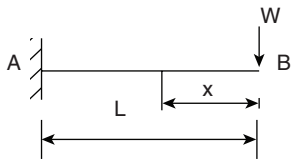
Hints and Explanations

1. Statements (a) and (c) are correct Choice (C)

2. $m = 2j - 3$
m = number of members = 11
j = number of joints = 7
$m = 2(7) - 3 = 14 - 3 = 11$
Hence, a perfect frame Choice (A)

3. Moment distribution method comes under the category of displacement method. Choice (C)

4.



Strain energy (U) is given by

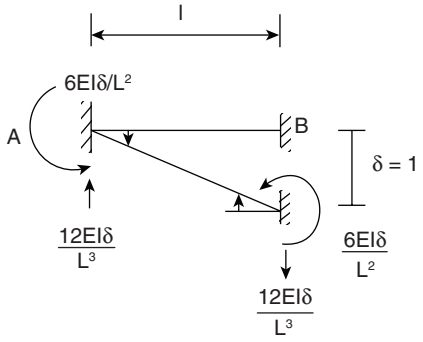
$$U_x = \int_0^L \frac{M_x^2\, dx}{2EI}$$

$$M_X = -wx$$

$$U_x = \int_0^L \frac{(WX)^2}{2EI} dx$$

$$= \frac{w^2}{2EI}\left[\frac{X^3}{3}\right]_o^L = \frac{W^2 L^3}{6EI}$$ Choice (C)

5. Given, free end undergoes unit displacement and without rotation, so the beam can be shown as



Moment at $A = \frac{6EI\delta}{L^2}$

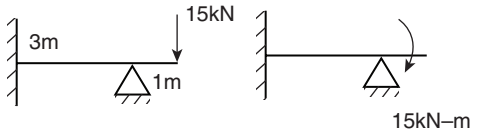
$\delta = 1$

$M_A = \frac{6EI}{L^2}$

[Due to sinking of support 'B'; Rotations develop at A and B; But fixed end does not allow rotations and hence fixed end moments of magnitude $\frac{6EI\delta}{L^2}$ develop]

Choice (C)

6. Because of symmetry,

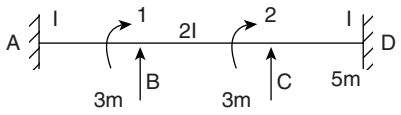


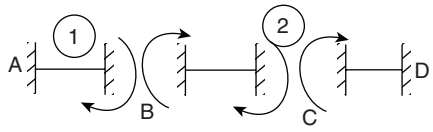
From concept of carry over moment half the moment is transferred to fixed end in some direction.

i.e., $\frac{15}{2}$ = 7.5 kN-m Clockwise

Choice (B)

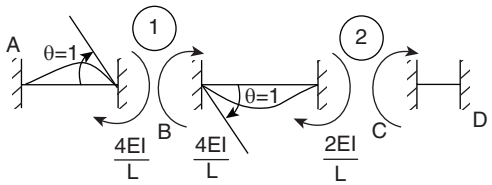
11.





$$K = \begin{bmatrix} K_1 & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

K_{11}: moment at (1) due to unit rotation at (1)
K_{21}: moment at (2) due to unit rotation at (1)
So at first coordinate given unit rotation in C.W direction

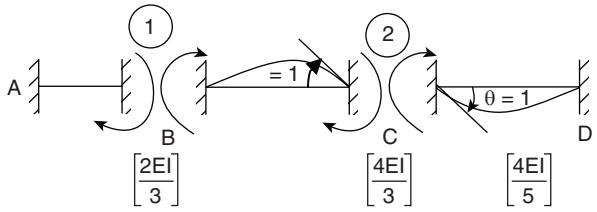


$$K_{11} = \left[\frac{4EI}{L}\right]_{BA} + \left[\frac{4EI}{L}\right]_{BC}$$

$$= \left[\frac{4EI}{3}\right] + \left[\frac{4E(2I)}{3}\right] = \frac{12EI}{3} = 4\text{EI}$$

$$K_{21}=\left[\frac{2EI}{L}\right]_{BC}=\frac{2E(2I)}{3}=\frac{4EI}{3}$$

K_{22}: moment developed at (2) due to unit rotation at (2)
K_{12}: moment developed at (1) due to unit rotation at (2)
So apply unit rotation at (2)



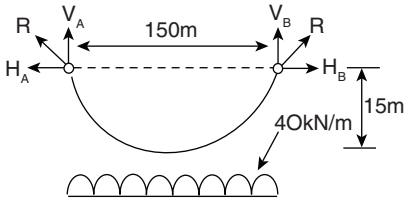
$$K_{22}=K_{22}=\left[\frac{4EI}{L}\right]_{CB}+\left[\frac{4EI}{L}\right]_{CD}=\left[\frac{4E(2I)}{3}\right]+\left[\frac{4EI}{5}\right]$$

$$=\frac{8EI}{3}+\frac{4EI}{5}=\frac{40EI+12EI}{15}=\frac{52EI}{15}$$

$$K_{12}=\left[\frac{2EI}{L}\right]_{CB}=\left[\frac{2E(2I)}{3}\right]=\frac{4EI}{3}$$

$$[K]=\begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}=\begin{bmatrix}4EI & \frac{4EI}{3}\\ \frac{4EI}{3} & \frac{52EI}{15}\end{bmatrix}$$ Choice (B)

12.



$$V_A=VB=\frac{wl}{2}=\frac{40\times150}{2}=3000\text{ kN}$$

$$H_A=H_B=\frac{Wl^2}{8h}=\frac{40\times150^2}{8\times15}$$

$$=\frac{5\times150\times150}{15}=7500\text{ kN}$$

Maximum tension $=R=\sqrt{V^2+H^2}$

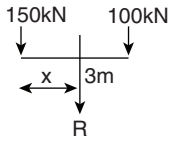
$$=\sqrt{(3000)^2+(7500)^2}=8077\text{ kN}$$

Minimum tension occurs at deepest point and is equal to Horizontal thrust

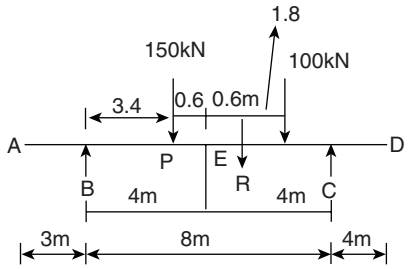
$T_{min}=7500$ kN Choice (A)

13. By Muller Breslau's principle, apply a unit vertical displacement at *A*. The resulting deflection profile is the I.L.D at *A*. Since *A*, *B* and *C* are one part and *CD* are the other part; *ABC* deforms as a single member and *CD* deforms due to downward movement of *C*. At '*B*', there won't be any displacement because of support of *B*. Since the beam is statically determinate the ILD is a straight line, not a curve. Choice (B)

14. To get maximum Bending moment under a load, the Resultant and load should be kept equidistant from center of span.

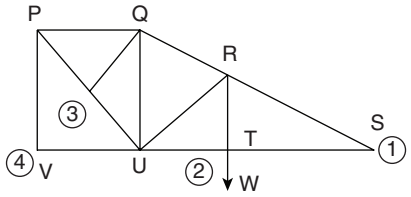


$$\bar{x}=\frac{100\times3}{150+100}=\frac{300}{250}=1.2\text{m}$$

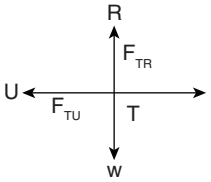


$V_b+V_C=150+100=250$ kN
$(V_C)(8)-100(6.4)-150(3.4)=0$
$V_C=143.75$ kN
$V_B=106.25$ kN
Maximum $B.M$ at $P=(V_B)(3.4)$
$=361.25$ kN-m Choice (C)

15.

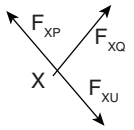


Go by Joint no's.
At joint (1); Since no load is acting at '*s*' force in these members to be zero
At Joint (2) i.e., at *T*



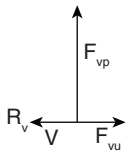
Three forces are acting at a joint; [i.e., W; F_{TR}; F_{TU}] and two of the forces are in same line i.e., W and F_{TR} the force in F_{TU} to be zero since for equilibrium of a joint

At Joint (3) i.e., at X;



The same statement as above; Hence the force in $F_{XQ} = 0$

At Joint (4) i.e., at V

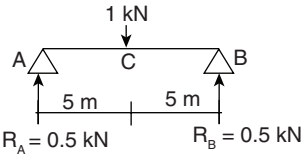


$F_{VP} = 0$

$\therefore$ no of members with zero forces = 5. Choice (B)

16. Since the vertical deflection at center of beam is to be determined

Apply a unit load at center of beam



From symmetry;

$R_A = R_B = 0.5$ kN

$$\delta_C = \int_0^L \frac{m\alpha\Delta T_m\, dX}{C}$$

m: Internal virtual moment in beam

$\alpha = 10 \times 10^{-6}/°c$

ΔT_M : temperature difference between mean temp and the temp at the top or bottom of beam

$$= \left[\frac{80+160}{2}\right] - 80 = 120 - 80 = 40°C$$

C: mid depth of beam $= \dfrac{0.5}{2} = 0.25$m

$$\delta_v = \int_0^{10m} \frac{(0.5x)\left(10\times10^{6}/°C\right)(40°C)}{0.25\text{m}}\, dx$$

$$= \left[\frac{0.5\times10\times10^{-6}\times40}{0.25}\frac{x^2}{2}\right]_0^{10}$$

$$= \frac{0.5\times10\times10^{-6}\times40\times10^2}{2\times0.25}$$

$\delta_V = 0.04$m

$\delta_V = 40$ mm Choice (C)

17. Rational stiffness at a joint B;

$$S = \frac{M}{\theta} = \frac{3EI}{L} + \frac{3EI}{L} = \frac{3EI}{3} + \frac{3EI}{4}$$

$$\frac{M}{\theta} = \frac{12EI + 9EI}{12}$$

$$\frac{M}{\theta_B} = \frac{21EI}{12}$$

$$\theta_B = \frac{12M}{21EI} \Rightarrow \theta_B = \frac{12\times30}{21EI}$$

$$\theta_B = \frac{120}{7EI}$$

From slope deflection method

$$M_{CB} = \frac{2EI}{L}\left[2\theta_C + \theta_B - \frac{3\delta}{L}\right] + M_{FCB}$$

$\delta = 0$, $M_{FCB} = 0$

$M_{CB} = 0$ [Since hinge support]

$$0 = \frac{2EI}{3}\left[2\theta_C + \theta_B\right]$$

$$\frac{2EI}{3}\left[2\theta_C + \frac{120}{7EI}\right] = 0$$

$$2\theta_c + \frac{120}{7EI} = 0$$

$$\theta_C = \frac{-120}{14EI}$$

$$\theta_C = \frac{+60}{7EI}$$ [Neglect sign] Choice (B)

18. $$M_{AB}\,\alpha\,\frac{6EI\delta}{L^2} = \frac{6E(2I)\delta}{(3)^2} = \frac{12EI\delta}{9}$$

$$M_{CD}\,\alpha\,\frac{6EI\delta}{L^2} = \frac{6E(2I)\delta}{5^2} = \frac{12EI\delta}{25}$$

$$\frac{M_{AB}}{M_{CD}} = \frac{12EI\delta(25)}{9(12\,E\,I\,\delta)} = 25/9$$ Choice (A)

19. U_1 = strain energy stored in bar due to W_1

$$U_1 = \frac{P^2\,L}{2AE} = \frac{W_1^2\,L}{2AE}$$

U_2 = strain energy stored in bar due to W_2

$$U_2 = \frac{W_2^2\,L}{2AE}$$

U = strain energy stored in bar due to combined $(W_1 + W_2)$

$$U = \frac{(W_1+W_2)^2\,L}{2AE}$$

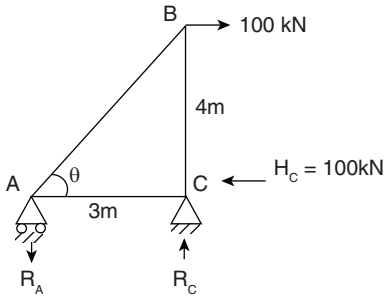
$\therefore$ $U > U_1 + U_2$ Choice (B)

20. $$\delta = \Sigma\frac{PkL}{AE}$$

P: Force in member due to applied loads

K: Force in member for a unit load in the direction in which deflection is desired

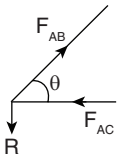
L: Length of members



$(R)(3) = 100 \times 4$

$$R_A = R_C = R = \frac{100 \times 4}{3} = 133.33 \text{ kN}$$

At Joint *A*:



$$\sin\theta = \frac{4}{5}$$

$\cos\theta = 3/5$

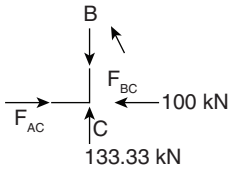
$F_{AB} \sin\theta = R$

$$F_{AB} = \frac{R}{\sin\theta} = \frac{133.33}{4/5} = 166.67 \text{ kN(Tension)}$$

$F_{AB} \cos\theta = F_{AC}$

$\Rightarrow \quad F_{AC} = 166.67 \times 3/5 = 100 \text{ kN(compression)}$

At joint *C*:



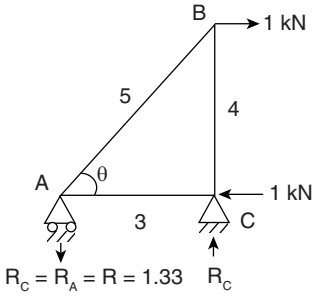
$\Sigma F_y = 0$

$F_{BC} = 133.33$ kN

$F_{BC} = 133.33$ kN (Compressive)

$F_{AC} = 100$ kN (Compressive)

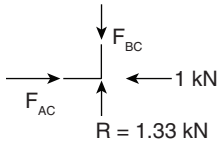
Due to unit load



$1 \times 4 = R \times 3$

$R = 4/3 = 1.33$ kN

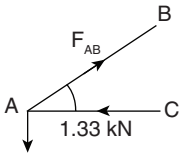
Joint C:



$F_{AC} = 1$ kN (Comp)

$F_{BC} = 1.33$ kN (Comp)

At Joint *A*:



$F_{AB} \sin\theta = 1.33$ kN

$$F_{Ab} = \frac{1.33}{\sin\theta} = \frac{1.33}{415}$$

$F_{AB} = 1.66$ kN (tension)

Take tension: + *ve*

compression: – *ve*

Member	Length (L)	P	K	PkL
AB	5	166.67	1.66	1383.36
BC	4	–133.33	–1.33	709.31
AC	3	–100	–1	300

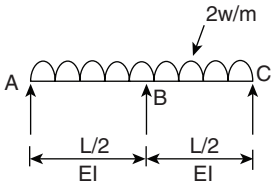
$\Sigma PKL = 2392.67$

$$\delta = \frac{\Sigma PkL}{AE}$$

$$= \frac{1}{\left(1000 \text{ mm}^2\right)\left(2 \times 10^5 \text{ N/mm}^2\right)} (2392.67)$$

$\delta = 11.96$ mm Choice (C)

21.

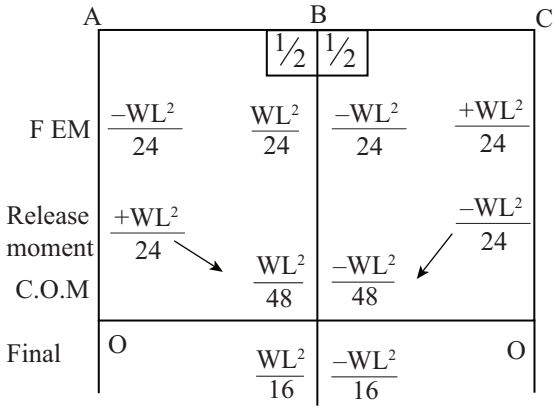


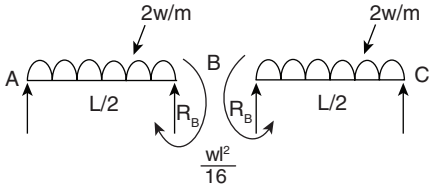
$$M_{FAB} = -\frac{Wl^2}{12} = \frac{-(2w)(L/2)^2}{12} = \frac{-(2w)(L^2/4)}{12} = \frac{-WL^2}{24}$$

$$M_{FBA} = \frac{+wl^2}{12} = \frac{(2w)(L/2)^2}{12} = \frac{+WL^2}{24}$$

$$M_{FBC} = \frac{-WL^2}{24}, M_{FCB} = \frac{+Wl^2}{24}$$

Joint	Member	Relative stiffness k – I/L	Σk	Distribution factor = k/ Σk
B	BA	$\frac{3}{4}\frac{I}{L/2}=\frac{6I}{4L}$	$\frac{12I}{4L}$	$\frac{6I\ 4L}{4L(12I)}=\frac{1}{2}$
	BC	$\frac{3}{4}\frac{I}{L/2}=\frac{6I}{4L}$		1/2





For SPAN *AB*:

$$(R_B)\frac{L}{2}=2w\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)=\frac{wL^2}{16}=0$$

$$R_B\frac{L}{2}=\frac{2\times2\times wL^2}{2\times8}+\frac{WL^2}{16}$$

$$R_B\frac{L}{2}=\frac{5wL^2}{16}w$$

$$R_B=R_B=\frac{5WL}{8}$$

For Span BC:

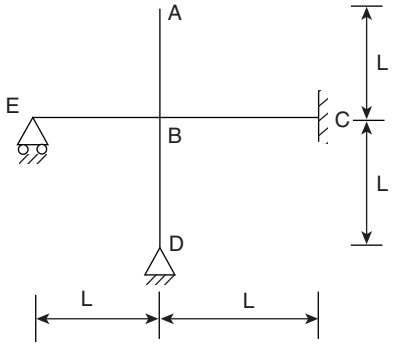
$$(R_B)\frac{L}{2}=\frac{WL^2}{16}=(2w)\frac{L}{2}\left(\frac{L}{4}\right)=0$$

$$R_B=\frac{5WL}{8}$$

$\therefore$ Prop Reaction at $B=\frac{5WL}{8}+\frac{5WL}{8}=\frac{5WL}{4}$

Choice (C)

23.



Joint	Members	K = I/L	ΣK	DF = K/ ΣK
B	BA	0	$\frac{5}{2}$I/L	0
	BC	I/L		$\frac{I2L}{L.5I}=0.4$
	BD	$\frac{3}{4}$I/L		$\frac{3I\ 2L}{4L\ 5I}=\frac{6}{20}=0.3$
	BE	$\frac{3}{4}$I/L		0.3

Choice (C)

24. Internal static indeterminacy, $D_{si}=m-(2j-3)$

$m=16, j=8$

$D_{si}=16-(2\times8-3)$

$D_{si}=3$

External indeterminacy $D_{se}=r-3=3-3=0$

$D_s=3+0=3$

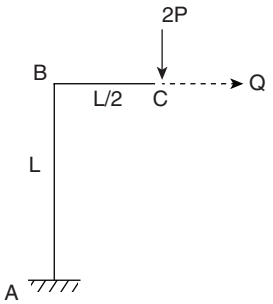
Kinematic indeterminacy, $D_k=Nj-C$

$D_k=2\times8-3$

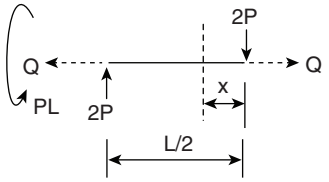
$D_k=13$

statically and kinematically indeterminate Choice (C)

25. Since no horizontal load acting at *c*, apply a fiction load '*Q*' at *'c'*.



Span *BC*:

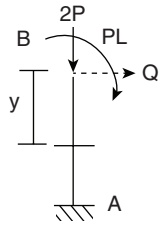


$$M_X = -2PX$$

$$\frac{\partial M_x}{\partial Q} = 0$$

$$(\delta_H)_{BC} = \int_0^{L/2} (M_x)\frac{\partial M_x}{\partial Q}\frac{dX}{EI} = 0$$

Span *AB*:



$$M_y = PL + Qy$$

$$\frac{\partial M_y}{\partial Q} = y \quad y = 0 \text{ to } L$$

$$(\delta H)_{AB} = \int_0^L (M_y)\left(\frac{\partial M_y}{\partial Q}\right)\frac{dy}{EI}$$

$$= \int_0^L (PL + Qy)(y)\frac{dy}{EI}$$

$$= \left[\frac{PLy^2}{2EI}\right]_0^L = \frac{PL^3}{2EI}$$

$$(\delta H)_c = (\delta H)_{Bc} + (\delta H)_{AB}$$

$$(\delta H)_c = \frac{PL^3}{2EI}$$

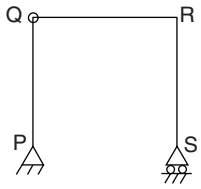
Choice (C)

Structural Analysis Test 2

Number of Questions: 30 **Time: 75 min.**

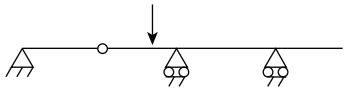
Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The frame shown below is



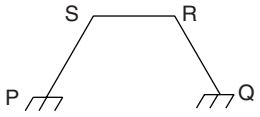
(A) statically indeterminate but unstable
(B) unstable
(C) determinate and stable
(D) none of the above

2. The static indeterminacy of a continuous beam with an internal hinge shown below is



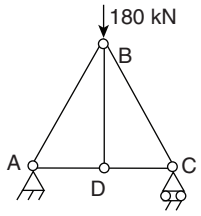
(A) zero (B) 1
(C) 2 (D) None

3. The degrees of freedom of the rigid frame with clamped ends at *P* and *Q* as shown in figure is



(A) 2 (B) 3
(C) 4 (D) zero

4. A truss, as shown in figure is carrying 180 kN at *B*. the force in member *BD* is



(A) 180 kN (Tensile)
(B) 180 kN (compressive)
(C) $180\sqrt{2}$ kN (Tensile)
(D) zero

5. Match List – I and List – II and select the correct answer using the code given below.

	List – I		List – II
a.	Column analogy method	1.	Stiffness
b.	Kanis rotation contribution method	2.	Force method
c.	Castiglianos theorem of minimum strain energy	3.	
d.	Moment distribution method	4.	

Codes:

	a	b	c	d		a	b	c	d
(A)	2	1	2	1	(B)	1	2	1	2
(C)	1	1	2	2	(D)	1	2	2	1

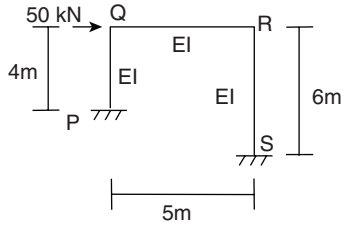
6. The cantilever beam *AB* of length 'ℓ' fixed at *A* and free at other end is subjected to a concentrated load *W* at its free end. The strain energy (*U*) stored in a beam is (*EI* const)

(A) $\frac{W^2\ell^2}{4EI}$ (B) $\frac{W\ell^3}{6EI}$
(C) $\frac{W^3\ell^3}{6EI}$ (D) $\frac{W\ell}{EI}$

7. The bending moment induced at fixed end of cantilever beam of span 'ℓ' if the free end undergoes a unit displacement without rotation is

(A) $\frac{3EI}{\ell^2}$ (B) $\frac{5EI}{\ell^2}$
(C) $\frac{6EI}{\ell^2}$ (D) $\frac{4EI}{\ell^2}$

8. The slope deflection equation at the end *Q* of member *QR* for the frame shown in the given figure below is



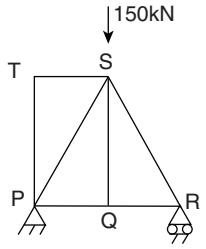
(A) $M_{QR} = \frac{2EI}{5}(2\theta_Q + \theta_R)$
(B) $M_{QR} = \frac{2EI}{5}(2\theta_Q - \theta_R)$
(C) $M_{QR} = \frac{2EI}{5}(2\theta_R - \theta_R)$
(D) $M_{QR} = \frac{EI}{5}(2\theta_Q + \theta_R)$

9. Which of the following cross sections has the highest shape factor?
(A) Rectangle (B) Diamond
(C) Triangle (D) Circle

10. The stiffness coefficient K_{ji} indicates
(A) force at *j* due to a unit deformation at *i*
(B) force at *I* due to a unit deformation at *j*

(C) deformation at j due to a unit force at i
(D) deformation at I due to a unit force at j

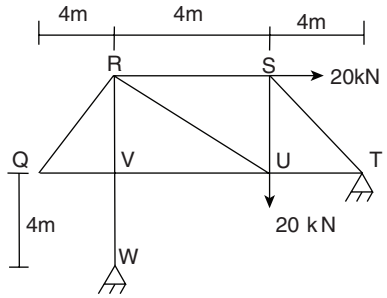
11. The force in the member QS of the truss shown in the figure is;



(A) 150 kN (B) $\frac{150}{\sqrt{2}}$ kN

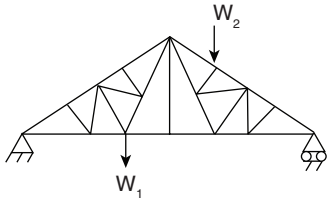
(C) zero (D) 35.5 kN

12. The pin jointed 2-D truss is loaded with a horizontal force of 20 kN at joint 'S' and another 20 kN vertical force at joint 'U' as shown. Find the force in the member RS (in kN). (taking tension as positive and compression as negative)



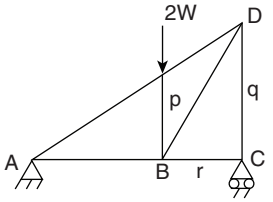
(A) zero (B) 20 kN
(C) 40 kN (D) −20 kN

13. For the plane truss shown in the figure, the number of zero force members for the given loading is



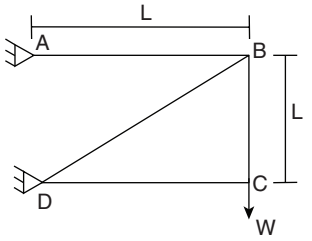
(A) 8 (B) 4
(C) 11 (D) 13

14. The force in members p, q, r in the truss shown



(A) $2W$, W, 0 (B) $2W$, W, W
(C) $\frac{W}{2}$, W, 0 (D) $\frac{W}{2}$, $\frac{W}{2}$, 0

15. For the truss shown in the figure, the force in member AB is

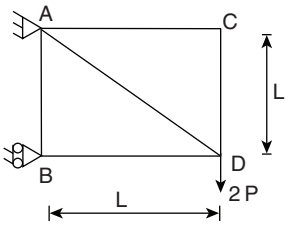


(A) W (B) $\frac{W}{2}$

(C) zero (D) None

16. Which of the following statements is true with regard to the flexibility method of analysis?
(A) the method is used to analyse determinate structures.
(B) the method is used only for manual analysis of indeterminate structures
(C) the method is used for analysis of flexible structures.
(D) the method is used for analysis of indeterminate structures with lesser degree of static indeterminacy.

17. The strain energy stored in the member AB of the pin jointed truss shown below. A and E is same for all members.



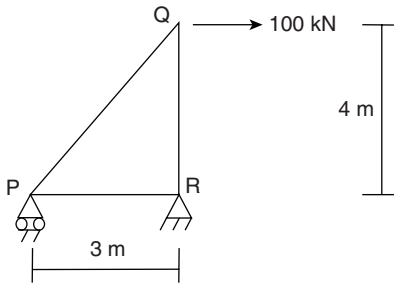
(A) $\frac{4P^2L}{2AE}$ (B) zero

(C) $\frac{P^2L}{AE}$ (D) $\frac{2P^2L}{3AE}$

18. The unit load method used in structural analysis is
(A) applicable only to indeterminate structures.
(B) derived from castigliano's, theorem
(C) another name for stiffness method
(D) none

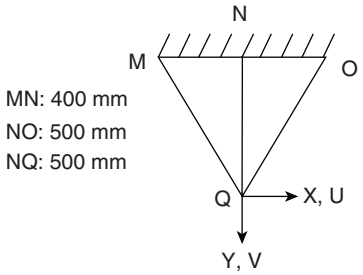
19. For linear elastic systems, the type of displacement function for strain energy is
(A) quadratic (B) linear
(C) cubic (D) quartic

20. The right triangular truss is made of members having equal cross sectional area of 1500 mm^2 and young's of $10^5 \times W^5$Mpa. The horizontal deflection at join Q is



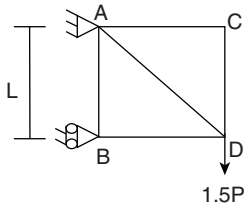
(A) 7.79 mm (B) 4.60 mm
(C) 8.20 mm (D) none

21. In a redundant joint model, three bar members are pin connected at Q as shown in figure. Under some load placed at Q, the elongation of the members MQ and OQ are found to be 40 mm and 25 mm. then the horizontal displacement 'U' and vertical displacement 'V' of the node Q_1 in mm will be respectively,



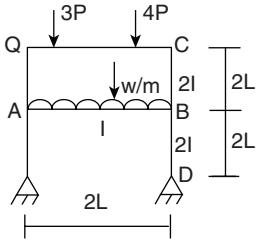
(A) 8.84 mm, 44.20 mm (B) 5 mm, 20 mm
(C) 0 mm, 44.20 mm (D) 7 mm, 35 mm

22. The strain energy started in the member AB of the pin jointed truss shown aside when E and A are same for all members is



(A) $\frac{1.5P^2L}{AE}$ (B) $\frac{P^2L}{AE}$
(C) zero (D) $\frac{2P^2L}{AE}$

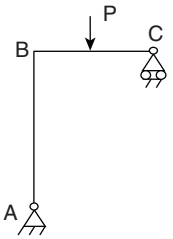
23. In the portal frame shown below, what are the distribution factors for member BA, BC BD respectively?

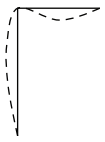


(A) 1/5, 2/5, 2/5 (B) 2/5, 1/5, 1/5
(C) 1/3, 1/3, 1/3 (D) none

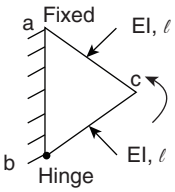
24. A propped cantilever beam PQ with fixed edge 'P' is propped at 'Q' and carries a UDL of w/m over the entire span. If the prop displaces upward by 2 mm, which one of the following is true? (if prop reaction = R_Q moment at $P = M_P$)
(A) both R_Q and M_P increase
(B) R_Q increases, and M_P decreases
(C) R_Q decreases and M_P increases
(D) both R_Q and M_P decreases

25. Identify the correct deflection diagram corresponding to the loading in the plane frame



(A) 
(B) 
(C) 
(D) None

26. Rotational stiffness coefficient, K_{11} at joint 'C' for the frame having two members of equal $\frac{EI}{\ell}$ is given by



(A) $\frac{7EI}{\ell}$ (B) $\frac{5EI}{\ell}$
(C) $\frac{4EI}{\ell}$ (D) $\frac{6EI}{\ell}$

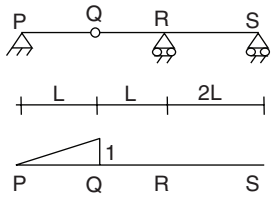
27. A propped cantilever beam of span 'L' is loaded with UDL of intensity w/unit length, all through the span. Bending moment at fixed end is _____

(A) $\frac{WL^2}{8}$ (B) $\frac{WL^2}{2}$
(C) $\frac{WL^2}{12}$ (D) $\frac{WL^2}{24}$

28. Influence lines for redundant structures can be obtained by
(A) Castigliano;s theorem
(B) Muller Berslau principle
(C) Unit load theorem
(D) Maxwell-Betti reciprocal theorem

29. A homogeneous, simply supported prismatic beam of Width B, depth D and span 'L' is subjected to a concentrated load of magnitude P. The load can be placed anywhere along the span of beam. The maximum flexural stress developed in the beam is
(A) $\frac{3}{4}\frac{PL}{BD^2}$ (B) $\frac{4}{3}\frac{PL}{BD^2}$
(C) $\frac{3}{2}\frac{PL}{BD^2}$ (D) $\frac{2}{3}\frac{PL}{BD^2}$

30. Consider the beam $PQRS$ and the influence line as shown below. The influence line pertains to



(A) Reaction of P, R_P
(B) Shear force Q, V_Q
(C) Shear force on left of Q, V_Q^-
(D) Shear force on right of Q, V_Q^+

Answer Keys

1. B	**2.** A	**3.** B	**4.** D	**5.** A	**6.** C	**7.** C	**8.** A	**9.** C	**10.** A
11. C	**12.** A	**13.** A	**14.** A	**15.** A	**16.** D	**17.** B	**18.** B	**19.** B	**20.** A
21. A	**22.** C	**23.** A	**24.** B	**25.** A	**26.** A	**27.** A	**28.** B	**29.** C	**30.** B

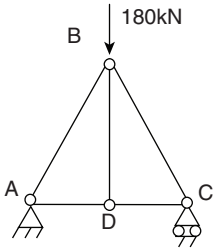
Hints and Explanations

1. Given rigid jointed plane frame with internal hinge at Q
$D_s = (3m + r) - 3J - C$
Where m number of members = 3
r : no of reactions = 2 + 1 = 3
J : number of joints = 4
C : number of releases = (2 – 1) = 1
$\therefore \quad D_s = [3(3) + 3] - 3(4) - 1$
$= 12 - 12 - 1 = -1$
$\Rightarrow \quad D_s < 1$
The given frame is unstable Choice (B)

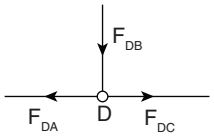
2. Degree of static indeterminacy = $D_s = D_{si} + D_{se} - C$
D_{se} : external indeterminacy
r : − 2 [since only vertical loading so '2' equilibrium equations]
r : number of reactions = 3
$D_{se} = 3 - 2 = 1$
D_{si} = Internal indeterminacy = 0
C : number of releases = 1
[Due to the presence of internal hinge, we get an additional equation since it does not allow moment from one part to other]
$\therefore \quad D_s = D_{se} + D_{si} - C$
$= 1 + 0 - 1$
$D_s = 0$ Choice (A)

3. Kinematic indeterminacy/Degrees of freedom $D_K = NJ - C$
Where N : number of degrees of freedom of each joint = 3 [since rigid plane frame]
J : number of joints = 4
C : number of restraints = 3 + 3 = 6
$\therefore \quad D_K = (3)(4) - 6 = 12 - 6$
$D_K = 6$ [with axial deformations]
$D_K = 6 - 3 = 3$ [neglecting axial deformation]
Choice (B)

4.



At joint D

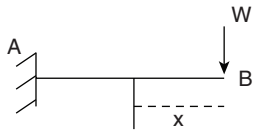


If two forces are in same line the force in third member is equal to zero when three forces are acting at a joint
$\therefore \quad F_{BD} = 0$ Choice (D)

5. These are basic method of structural analysis to analyse statically indeterminate structures.
Column analogy method and castiglianos theorem of minimum strain energy comes under force method/flexibility method/equilibrium method and kanis rotation

contribution method and moment distribution method comes under compatibility method/stiffness coefficient method/displacement method. Choice (A)

6.



Strain energy (U) is given by

$$U = \int_O^L \frac{M_X^2 d_X}{2EI}$$

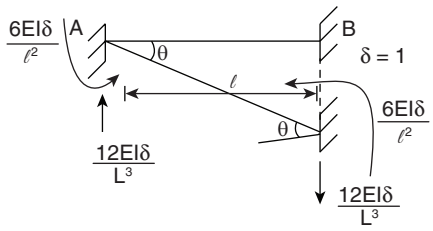
Where M_x : moment at a section (x)

$= -wx$ where x varies from o to L

$$U = \int_O^L \frac{(-W_X)^2 d_X}{2EI}$$

$$= \int_O^L \frac{W^2X^2}{2EI} = \left[\frac{W^2X^3}{6EI}\right]_O^L = \frac{W^2\ell^2}{6EI}$$ Choice (C)

7. Given free end undergoes unit displacement and without rotation so the beam can be shown as



∴ Moment at $A = \frac{6EI\delta}{\ell^2}$

Where $\delta = 1$

$M_A = \frac{6EI}{\ell^2}$

[Due to sinking of support B; rotations develop at A & B; but fixed end does not allow rotations and hence fixed end moments of magnitude $\frac{6EI\delta}{\ell^2}$ develops]

Choice (C)

8. $M_{QR} = \frac{2EI}{L}(2\theta_Q + \theta_R - \frac{3\delta}{L}) + M_{FQR}$

$\delta = 0$; $M_{FRQ} = 0$

∴ $M_{QR} = \frac{2EI}{L}(2\theta_Q + \theta_R)$ Here $L = 5$m Choice (A)

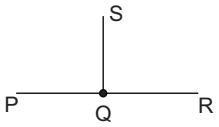
9.

C/S sections	Shape factors values
(a) Rectangle	1.5
(b) Diamond	2.0
(c) triangle	2.34
(d) circle	1.7

Choice (C)

10. Choice (A)

11. Consider joint Q



Three forces are acting at joint Q. two of them are in same line; the force in third member (QS) to be zero form equilibrium of joint. Choice (C)

12. $\Sigma Fy = 0$

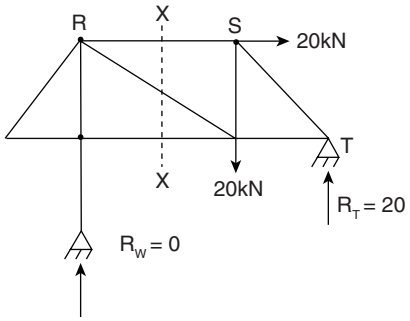
$R_W + R_J = 20$kN

$\Sigma M_T = 0$

$(R_W)8 + (20)(4) - (20(4) = 0$

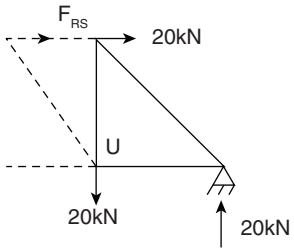
$R_W = 0$

$R_T = 20$kN



Draw a section (X) – (X) passing through (RS)

Taking equilibrium of right hand side

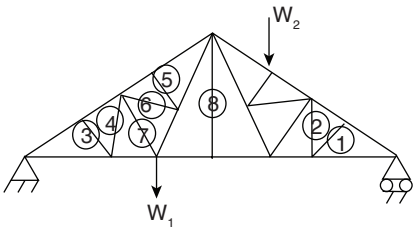


Taking moments about U

$(F_{RS})(4) + (20)4 - 20(4) = 0$

$(F_{RS}) = 0$ Choice (A)

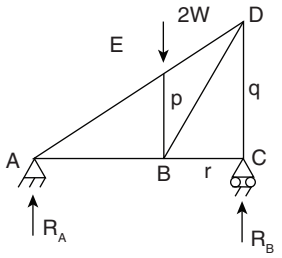
13.



Using the principle : If three forces act at a joint and two of them are in same line; the force in third member will be zero.

∴ Total number of zero forces = 8 Choice (A)

14.



$R_A + R_B = 2W$

Due to symmetry $R_A = R_B = \dfrac{2W}{2} = W$

R_B and q are in same line at joint C

$\therefore$ $R_B = q = W$

so the value of $r = 0$

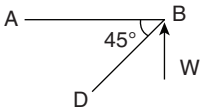
at joint E;

$2W$ and p are in same line

So, $p = 2W$ Choice (A)

15. At joint C; the force W and F_{BC} are in same line so the force in member DC will be zero

At joint B:



$\Sigma F_X = 0$

$F_{AB} = F_{BD} \cos 45°$

$\Sigma F_Y = 0$

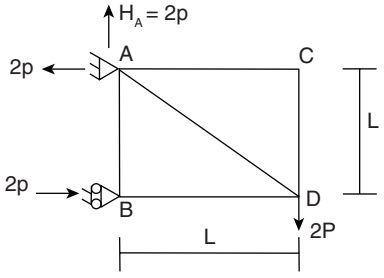
$F_{DB} \sin 45° = W$

$F_{DB} = W\sqrt{2}$

$F_{AB} = W\sqrt{2} \cos 45°$

$F_{AB} = W$ Choice (A)

17.



At joint B;

Reaction $2p$ and force in BD are in same line

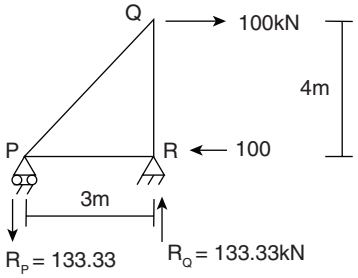
Therefore force in member $AB = 0$

Strain energy $= \dfrac{P^2 L}{2AE}$

$p = 0$;

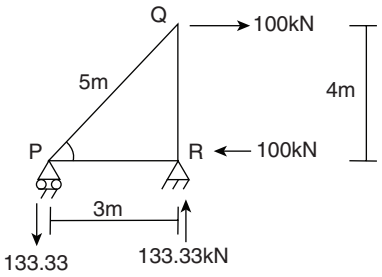
$U_{AB} = 0$ Choice (B)

20.

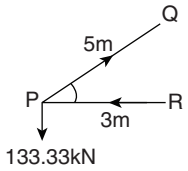


$R_P \times 3 = 100 \times 4$

$R_P = 100 \times 4/3 = 133.33$kN



At joint (P):



$\cos\theta = \dfrac{3}{5}$; $\sin\theta = \dfrac{4}{5}$

$F_{PQ} \sin\theta = 133.33$ KN

$F_{PQ} = \dfrac{133.33}{\sin\theta} = \dfrac{133.33}{(4)15}$

$F_{PQ} = 166.66$kN

$F_{PQ} \cos\theta = F_{PR}$

$F_{PR} = (166.66)(315)$

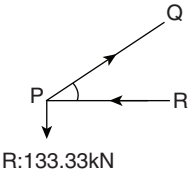
$F_{PR} = 99.99$kN

$F_{RQ} = 133.33$kN

$R \times 3 = 1 \times 4$

$R = 4/3 = 1.33$ kN

At joint P:



$F_{PQ} \sin\theta = 1.33$

$F_{PQ} = \dfrac{1.33}{\sin\theta} = \dfrac{1.33}{415}$

$F_{PQ} = 1.662$ kN

$F_{PQ}\cos\theta = F_{PR}$

$F_{PR} = 1.662 \times \frac{3}{5} = 0.9975$ kN

$F_{RQ} = 1.33$ kN

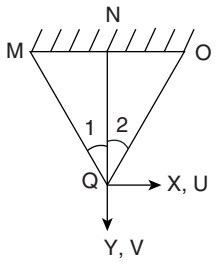
Member	P	K	ℓ
PQ	166.66	1.662	5
QR	133.33	1.33	4
PR	99.99	0.9975	3

$$\delta_H = \frac{\Sigma PKL}{AE}$$

$$= \frac{166.66 \times 1.662 \times 5 + (133.33)(1.33)(4) + (99.99)(0.9975)(3)}{1500 \times 200}$$

$\delta_H = 7.978$mm Choice (A)

21.



MN: 400mm

NO: 500mm

NQ: 500mm

$$\sin\theta_1 = \frac{400\text{mm}}{\sqrt{400^2 + 500^2}} = 0.624$$

$$\cos\theta_1 = \frac{500}{\sqrt{400^2 + 500^2}} = 0.780$$

$$\sin\theta_2 = \frac{500}{\sqrt{500^2 + 500^2}} = 0.707$$

$$\cos\theta_2 = \frac{500}{\sqrt{500^2 + 500^2}} = 0.707$$

By resolving the displacements along OQ;

$-U\sin\theta_2 + V\cos\theta_2 = 25$ - (1)

Along member MQ;

$U\sin\theta_1 + V\cos\theta_1 = 40$ - (2)

$U = 8.84$mm

$V = 44.20$mm Choice (A)

22. At joint B;

Reaction at B and force in member CD are in same line.

Therefore the force in member $AB = 0$

$F_{AB} = 0$

$U = \frac{P^2 L}{AE} = 0$ Choice (C)

23.

Joint	Members	Relative stiffness(k)	Σk	Distribution factor $\left(\frac{K}{\Sigma K}\right)$
B	BA	I/2L	5I/2L	1/5
	BC	2I/2L		2/5
	BD	2I/2L		2/5

Choice (A)

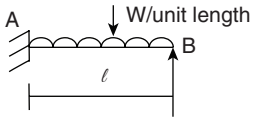
24. Choice (B)

25. Choice (A)

26. Rotational stiffness at joint = Sum of rotational stiffness of each member

$= \frac{4EI}{\ell} + \frac{3EI}{\ell} = \frac{7EI}{\ell}$ Choice (A)

27.



Initial FEM	$\frac{-WL^2}{12}$	$\frac{WL^2}{12}$
Balancing	$\frac{-WL^2}{12}$	$\frac{-WL^2}{12}$
Carry over	$\frac{-WL^2}{24}$	
Final FEM	$\frac{-WL^2}{8}$	0

Choice (A)

29. To get maximum BM place load at enter of beam,

Maximum bending stress $f = \frac{M_{max}}{Z}$

$= \frac{\frac{PL}{4}}{\frac{BD^2}{6}} = \frac{3}{2}\frac{PL}{BD^2}$ Choice (C)

30. ILD shown is for shear force at Q, V_Q Choice (B)

UNIT IV
CONSTRUCTION MATERIALS AND MANAGEMENT

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Construction Materials and Management Test 1

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The bearing strength of M25 grade concrete in limit state method of design as per IS 456:2000 is
(A) 25 MPa (B) 11.25 MPa
(C) 15 MPa (D) None

2. Modulus of Elasticity of M30 grade concrete is
(A) 25000 N/mm^2 (B) 27386 N/mm^2
(C) 30000 N/mm^2 (D) None

3. Flexural tensile strength of M25 grade concrete as per IS 456-2000 is
(A) 12.5 N/mm^2 (B) 25 N/mm^2
(C) 22.5 N/mm^2 (D) 3.5 N/mm^2

4. Minimum grade of concrete used for pretensioned and post tensioned prestressed concrete are
(A) M40 and M30 (B) M40 and M20
(C) M30 and M40 (D) None

5. The 7-days strength of M30 grade concrete should be at least
(A) 30 MPa (B) 20 MPa
(C) 25 MPa (D) none

6. The target mean strength (f_m) for concrete mix design obtained from the characteristic strength (f_{ck}) and standard deviation (σ) as defined in IS456-2000 is:
(A) $f_{ck} + 1.35\sigma$
(B) $f_{ck} + 1.65\sigma$
(C) $f_{ck} + 1.45\sigma$
(D) $f_{ck} + 1.55\sigma$

7. Minimum cement content to be used in Reinforced cement concrete for mild exposure is
(A) 300 kg/m^3 (B) 320 kg/m^3
(C) 340 kg/m^3 (D) 450 kg/m^3

8. Nominal cover to main reinforcement in case of slabs with mild exposure should be
(A) 30 mm (B) 25 mm
(C) 20 mm (D) 40 mm

9. The individual variation in compressive strength of three cubes in the sample should not exceed
(A) ±10% (B) ±15%
(C) ±20% (D) ±25%

10. The pozzolanas added to improve the properties of concrete are
(A) Fly ash (B) Silica fume
(C) Slag (D) All the above

11. Which of the following statements regarding the cube strength of concrete are correct?
(i) Strength increases with decrease in cube size
(ii) Strength decreases with increase in slenderness ratio
(iii) Strength increases with increase in slenderness ratio
(iv) Strength decreases with decreases in cube size
(A) (i) and (ii) are correct
(B) (i), (ii), (iii) are correct
(C) (i) and (iii) are correct
(D) All the above

12. Which of the following statements regarding properties of concrete are correct?
(p) Modulus of elasticity of M25 grade of concrete is 25000 MPa.
(q) Approximate value of shrinkage strain of concrete is 0.0003
(r) pH value of water used in concrete construction should not be less than 6.
(A) *p* and *q* are correct
(B) *p* and *r* are correct
(C) *q* and *r* are correct
(D) all the above are correct

13. The long term modulus of elasticity of M25 grade concrete with θ value at 7 days to be 2.2 is
(A) 25000 MPa (B) 7812.5 MPa
(C) 3500 MPa (D) None

14. The probability of failure of a structure as per IS456-2000 (according to the concept of limit state design) is ____
(A) 0.0975 (B) 0.95
(C) 0.975 (D) 0.20

15. Group – I contains some properties of concrete / cement and Group II contains list of some tests on concrete / cement.
Match the property with corresponding test.

	Group – I		Group – II
P.	Direct tensile strength of conc rete	1.	Cylinder splitting test
Q.	Workability of concrete	2.	Surface area test
R.	Bond between steel and concrete	3.	Vee – bee tests
S.	Fineness of cement	4.	Fineness modulus test
		5.	Pullout test

Codes:

	P	Q	R	S
(A)	1	3	5	4
(C)	2	3	1	4

	P	Q	R	S
(B)	5	2	1	3
(D)	2	1	5	3

16. Consider the following statements regarding the air entrained concrete?
(1) Increased resistance to freezing and thawing
(2) Improvement in workability.
(3) Increase in strength.

(4) Permits reduction in water content of these,
(A) 1, 2, 4 are correct
(B) 2, 3, 4 are correct
(C) 1, 3, 4 are correct
(D) All the above are correct

17. Which of the following statements regarding admixtures are correct?
(A) Retards the setting of cement
(B) Accelerates the setting of cement
(C) Improves the workability of concrete
(D) All the above

18. Consider the following statements:
I. The compressive strength of concrete decreases with increase in water cement ratio of the concrete mix.
II. Water is added to the concrete mix for hydration of cement and workability.
III. Creep and shrinkage of concrete are independent of the water cement ratio in the concrete mix.
The true statements are
(A) I and III (B) I, II, III
(C) II and III (D) I and II

19. Consider the following statements:
I. Modulus of elasticity of concrete increases with increase in compressive strength of concrete
II. Brittleness of concrete increases with decrease in compressive strength of concrete.
III. Shear strength of concrete increases with increase in compressive strength of concrete.
The true statements are
(A) I and III (B) I, II, III
(C) II and III (D) I and II

20. Consider the following statements:
(p) Nominal mix proportions for M20 grade concrete is 1 : 1.5 : 3
(q) Weight batching is preferred compared to nominal (volume) batching
(r) Maximum cement content as per IS456-2000 is 450 kg/m^3
(A) p, q are correct (B) p, r are correct
(C) q, r are correct (D) p, q and r are correct

21. Which of the following statements given below are correct.
(p) Nominal cover to reinforcement is based on serviceability or durability requirements
(q) Factors affecting the durability of concrete are w/c and maximum cement content
(r) Minimum cement content is not based on exposure conditions.
(A) p, q, r are correct (B) p and q are correct
(C) p and r are correct (D) only p is correct

22. Consider the following statements regarding the addition of pozzolanas to cement causes
(p) Increase in strength
(q) Less heat of hydration
(r) Decrease in workability
The true statements are
(A) p, q, r are correct (B) p and q are correct
(C) p and r are correct (D) q only is correct

23. The composition of air entrained concrete is given below:
Water : 180 kg/m^3
Ordinary Portland cement: 360 kg/m^3
Sand : 601 kg/m^3
Coarse aggregate: 1160 kg/m^3
Assume the specific gravity of OPC, sand and coarse aggregate to be 3.10, 2.65 and 2.74 respectively, the air content in liters/m^3 is _____
(A) 53 liters/m^3 (B) 50 liters/m^3
(C) 45 liters/m^3 (D) None

24. Consider the following statements
(p) Nominal maximum size of coarse aggregate to be used in *R.C.C* is 20 mm
(q) As per IS456-2000; fine sand to be used in *R.C.C* should confirm to zone II and medium sand.
(r) Minimum grade of concrete to be used in *R.C.C* is M30
The true statements are
(A) p and q are true (B) p and r are true
(C) p, q and r are true (D) q and r are true

25. Which of the following statements given below are correct?
(p) In mild environment, surface crack width should not exceed 0.3 mm as per IS456-2000.
(q) Crack width increases with increase in stress in reinforcement bar.
(r) Concrete and steel exhibit high strength after being subjected to high temperature.
(A) p and r are correct
(B) p, q and r are correct
(C) p and q are correct
(D) None

Answer Keys

1. B	**2.** B	**3.** D	**4.** A	**5.** B	**6.** B	**7.** A	**8.** C	**9.** B	**10.** D
11. A	**12.** D	**13.** B	**14.** A	**15.** A	**16.** A	**17.** D	**18.** D	**19.** B	**20.** D
21. B	**22.** D	**23.** A	**24.** A	**25.** C					

Hints and Explanations

1. Bearing strength of concrete = 0.45 *fck*
= 0.45 (25)
= 11.25 MPa Choice (B)

2. $E_c = 5000\sqrt{fck}$
$= 5000\sqrt{30}$
$= 27386 \text{N/mm}^2$ Choice (B)

3. Flexural tensile strength of concrete $= 0.7\sqrt{f_{ck}}$
$= 0.7\sqrt{25} = 3.5$ MPa Choice (D)

5. 7 days strength $= \frac{2}{3}$ cube strength
$= \frac{2}{3} \times 30 = 20 \text{ N/mm}^2$ Choice (B)

6. $f_{ck} = f_m - 1.65\,\sigma$
$\Rightarrow \quad f_m = f_{ck} + 1.65\,\sigma$ Choice (B)

7. Minimum cement content to be used in reinforced concrete for mild exposure is 300 kg/m^3 Choice (A)

8. Minimum cover to Reinforcement for slabs = 20 mm
Choice (C)

9. Individual variation is compressive strength of 3 cubes should not exceed ±15%. Choice (B)

10. Fly ash, silica fume and slag are the three types of pozzolanas used in concrete to improve the properties of concrete. Choice (D)

11. Statements (i) and (ii) are correct
Strength increases with decrease in cube size because of better homogeneity and strength decreases with increase in slenderness ratio. Choice (A)

12. All statements are correct.
Modulus of elasticity of concrete of M25 grade is
$E_c = 5000\sqrt{25} = 5000\sqrt{f_{ck}}$ 25000 MPa and approximate value of shrinkage strain of concrete is 0.0003. and pH value of water used in construction should not be less than 6. Choice (D)

13. $E_{ce} = \frac{E_c}{1+\theta} = \frac{5000 \times \sqrt{25}}{1+2.2}$
$= 7812.5 \text{ N/mm}^2$ Choice (B)

14. The structure may fail when
(i) The load exceeds the design load.
(ii) The strength is less than the characteristic strength
(iii) Both load exceeds design load and strength less than characteristic strength.
The probability of load exceeding design load $p_1 = 5\%$
The probability of strength less than characteristic strength $p_2 = 5\%$
The probability of failure $= 1 - q_1 q_2 = 1 - (0.95)^2$
$= 0.0975$ Choice (A)

15. Group I correctly matches with Group II for Choice (A).
Choice (A)

16. There is an increased resistance to freezing and thawing & improvement in workability and also permits reduction in water content in case of air entrained concrete.
Choice (A)

17. All the above statements regarding admixtures are correct. Choice (D)

18. I and II statements are correct. Choice (D)

19. I, II, III statements are correct. Choice (B)

20. Statements p, q and r are correct. Choice (D)

21. p and q statements are correct. Choice (B)

22. Statement 'q' only is correct. Choice (D)

23. $\frac{M_c}{\rho_c} + \frac{M_s}{\rho_s} + \frac{M_{cA}}{\rho_{cA}} + V_w + V_a = 1$

$$\frac{360}{3.10 \times 1000} + \frac{601}{2.65 \times 1000} + \frac{1160}{2.74 \times 1000} + \frac{180}{1000} + V_a = 1$$

$V_a = 0.053 \text{ m}^3/\text{m}^3$
$= 0.053 \times 1000 \text{ litres/m}^3$ [$\because$ 1 m^3 = 1000 litres]
$= 53 \text{ litres/m}^3$ Choice (A)

24. Statements (p) and (q) are true. Choice (A)

25. Statements (p) and (q) are correct. Choice (C)

Construction Materials and Management Test 2

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

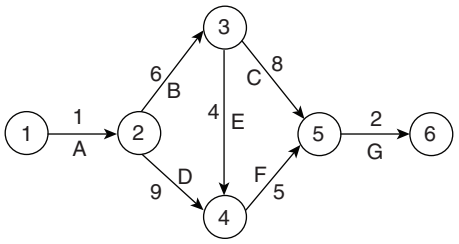
1. The probability distribution taken to represent the completion time in PERT analysis is _____
(A) Gamma distribution
(B) Normal distribution
(C) Beta distribution
(D) Log normal distribution

2. The probability of completion of any activity within its expected time is _____
(A) 50% (B) 84.1%
(C) 99.9% (D) 100%

3. The proportion of cement : *F.A* : *C.A* in a given concrete is 1 : 2 : 4; then the mix refers to
(A) M20 (B) M15
(C) M10 (D) M5

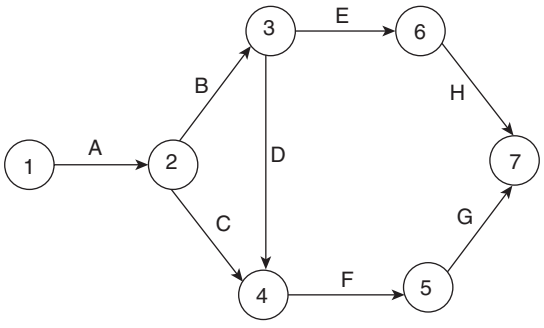
4. Negative slack occurs when
(A) Dummy activities are large in number
(B) Events stick to their schedule
(C) Dummy activities do not exist
(D) Deficiency of resources occurs

5.



Duration of the project shown in the network is ____
(Note : Durations given above are in days)
(A) 18 (B) 17
(C) 16 (D) 15

6. In the network shown in the figure, the activity '*F*' can be started only when



(A) Activity *B* is completed
(B) Activity *C* is completed
(C) Activity *D* is completed
(D) Activity *C* and *D* both are completed

7. Which of the following pairs are incorrect with reference to ordinary Portland cement?
(i) Initial setting time - 30 minutes
(ii) Final setting time – 10 hours
(iii) Normal consistency – 10%
(iv) All are correct
(A) iv (B) ii and iii
(C) iii only (D) i and iii

8. The most commonly used admixture to accelerate the initial setting time of concrete is _____.
(A) Gypsum
(B) Calcium Carbonate
(C) Calcium Chloride
(D) Calcium Ferrate

9. A sand is said to be unsuitable for construction if it has *F.M.* more than _____
(A) 2.9 (B) 3.2
(C) 3.4 (D) 3.9

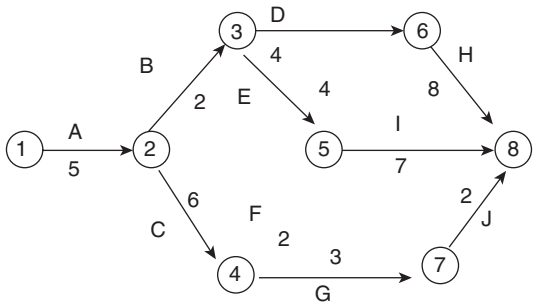
10. The standard size of specimen for conducting the modules of rupture of concrete is
(A) 15 × 15 × 60 cm
(B) 15 × 15 × 65 cm
(C) 15 × 15 × 70 cm
(D) 15 × 15 × 75 cm

11. To make one cubic meter of 1 : 2 : 4 by volume centrete, the volume of coarse aggregate required is ____
(A) 0.95 m^3 (B) 0.85 m^3
(C) 0.75 m^3 (D) 0.65 m^3

12. In drawing *AOA* network and making time computations, following processes are involved.
1. Activity listing
2. Work breakdown structure
3. Activity time allotment
4. Consideration of available resources for each activity
5. Activity dependencies
6. Float computations
7. Backward path computation
8. Project duration
9. Forward path computation.

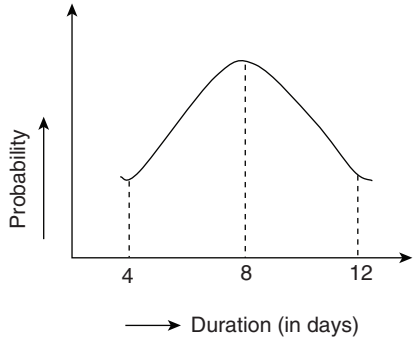
What is the correct sequence of the process given above?
(A) 1,2,4,3,9,7,5,6,8
(B) 4,1,3,2,9,5,7,8,6
(C) 2,1,4,3,5,9,8,7,6
(D) 1,3,2,4,5,9,7,8,6

13. A building project consist of 10 activities, represented by the network shown below. Find the critical path of the network?



(A) 1 - 2 - 4 - 7 - 8 (B) 1 - 2 - 3 - 5 - 8
(C) 1 - 2 - 4 - 5 - 8 (D) 1 - 2 - 3 - 6 - 8

14. For a given probability distribution curve,

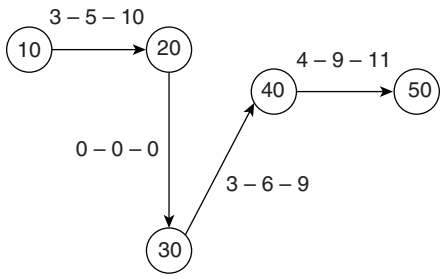


Match the following:

(i) Optimistic time (a) 12
(ii) Most likely time (b) 4
(iii) Expected time (c) 8
(iv) Pessimistic time

(A) i - b, ii - b, iii - c, iv - a
(B) i - a, ii - b, iii - b, iv - c
(C) i - a, ii - c, iii - c, iv - b
(D) i - b, ii - c, iii - c, iv - a

15. For the path of a certain network shown below, the expected time and standard deviation will be _____



(A) 15 and 1.75 (B) 20 and 1.93
(C) 15 and 1.5 (D) 20 and 1.75

16. Match List – I with List – II

	List – I		List – II
a.	Piece work contract	1.	Not practiced in government
b.	Lump sum contract	2.	Payment made by detailed measurement of different items
c.	Item rate contract	3.	Adopted for buildings, roads, bridges and electrical works.
d.	Labour contract	4.	Petty works and regular maintenance work.

Codes:

	a	b	c	d
(A)	1	2	3	4
(B)	2	3	4	5
(C)	5	4	3	2
(D)	4	3	2	1

17.

	List – I		List – II
1.	Setting time of cement	a.	Le- chatelier's apparatus
2.	Consistency of cement	b.	Air Permeability test
3.	Soundness of cement	c.	Vicat apparatus
4.	Fineness	d.	Pycnometer

Choose the correct one from the following

(A) 1 - c, 2 - c, 3 - b, 4 - a
(B) 1 - c, 2 - a, 3 - a, 4 - d
(C) 1 - c, 2 - c, 3 - a, 4 - b
(D) 1 - c, 2 - a, 3 - a, 4 - d

18.

	Type of work	Slump recommended
1.	Concrete for road works	20 - 28mm
2.	Ordinary RCC work	50 - 100mm
3.	Mass concrete	75 - 175mm
4.	Columns – retaining walls	12 - 25mm

Which of the following pairs are correctly matched

(A) 1,3 and 4 (B) 1 and 3
(C) 3 and 4 (D) 2 and 4

19. The fineness modulus of *C.A* and *F.A* are given as 7.6 and 2.78 respectively. The economical value of fineness modulues of combined aggregate is 6.4, then the proportion of fine aggregate is _____

(A) 25% (B) 33.33%
(C) 50% (D) 66.67%

20. Considering following strengths of concrete, choose the correct sequence in increasing order.

(1) Cube strength
(2) Cylinder strength
(3) Split tensile strength
(4) Modulus of rupture

(A) 3,4,2,1 (B) 3,4,1,2
(C) 4,3,2,1 (D) 4,3,1,2

21. Number of bricks required for one cubic meter of brick masonry is _____.

(A) 450 (B) 500
(C) 550 (D) 600

22. UPV method of non – destructive testing for concrete is used to determine _____

(1) Compressive strength
(2) Existence of voids
(3) Tensile Strength
(4) Static Modulus of concrete
(5) Dynamic modulus of concrete

(A) 1, 2, 3 and 4 (B) 1 and 3 only
(C) 2 and 5 only (D) 3 and 5 only

23. In shape test of aggregate which one of the following gives the correct slot for flakiness index for a material passing 50 mm sieve and retained on 40 mm sieve?

(A) 25 mm (B) 27 mm
(C) 81 mm (D) 30 mm

24. Identify the sequence of determination of components of a concrete mix as per IS method of mix design Select the correct answer using codes given below.

	List – I		List – II
a.	Cement content	1.	First step
b.	Aggregate content	2.	Second step
c.	Water content	3.	Third step
d.	Water cement ratio	4.	Fourth step

	a	b	c	d
(A)	1	2	3	4
(B)	3	2	4	1
(C)	1	4	2	3
(D)	3	4	2	1

25. The relation between the strength of brick masonry f_w, the strength of bricks f_b, and the strength of mortar f_m is given by (where k_w is coefficient based on layout of the bricks and the joints)

(A) $f_w = \sqrt{k_w \frac{f_b}{f_m}}$ (B) $f_W = K_W \frac{f_b}{f_m}$

(C) $f_W = \sqrt{k_w, f_b f_m}$ (D) $f_W = K_W \sqrt{f_b . f_m}$

26. Match List - I (Wood elements) With List - II (Description)

	List – I		List – II
a.	Pith	1.	Inner most portion of the tree
b.	Sap wood	2.	Inner annual rings Surrounding the pith
c.	Heart wood	3.	Outer most annual rings
d.	Cambium layer	4.	Thin layer of Sap between Sapwood and inner bark

Codes:

	a	b	c	d
(A)	1	3	2	4
(B)	2	4	1	3
(C)	1	4	2	3
(D)	2	3	1	4

27. For an activity $i - j$, the EST,EFT, LST and LFT are given as are 5,24,9, and 29, respectively. Activity duration is 6. Then, Match the List - I, with List - II

	List – I		List – II
a.	Free float	1.	5
b.	Total float	2.	9
c.	Interfering float	3.	13
d.	Independent float	4.	18

Codes:

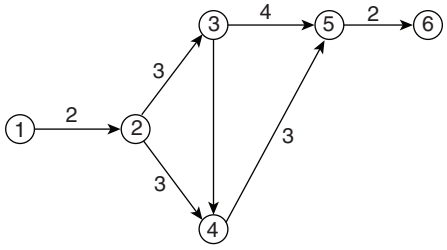
	a	b	c	d
(A)	4	3	1	2
(B)	3	4	1	2
(C)	3	4	2	1
(D)	4	3	2	1

28. Match the List - I with List - II

	List – I		List – II
a.	PERT	1.	Activity based
b.	Node	2.	Event oriented
c.	Dummy	3.	End of job
d.	CPM	4.	Imaginary activity

(A) a - 1, b - 4, c - 3, d - 2
(B) a - 2, b - 4, c - 3, d - 1
(C) a - 2, b - 3, c - 4, d - 1
(D) a - 1, b - 3, c - 4, d - 2

29.



In the network shown, total float for the activities 2 - 4 and 3 - 5 are respectively

(A) 0 and 0 (B) 2 and 2
(C) 2 and 1 (D) 1 and 1

30. Match List - I with List - II

	List – I (Cement mortar for different works)		List – II (Cement : Sand in mortar)
a.	For normal brick work	1.	1 : 4
b.	For plastering work	2.	1 : 3
c.	For grouting the cavernous rocks	3.	1 : 6
d.	For guniting	4.	1 : 1.5

Codes:

	a	b	c	d
(A)	3	2	4	1
(B)	4	1	3	2
(C)	4	2	3	1
(D)	3	1	4	2

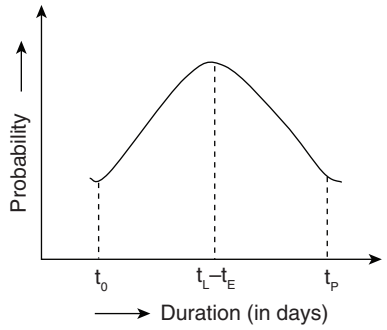
Answer Keys

1. C	**2.** A	**3.** B	**4.** D	**5.** A	**6.** D	**7.** C	**8.** C	**9.** B	**10.** C
11. B	**12.** C	**13.** C	**14.** D	**15.** B	**16.** D	**17.** C	**18.** B	**19.** B	**20.** A
21. B	**22.** C	**23.** B	**24.** D	**25.** D	**26.** A	**27.** B	**28.** C	**29.** C	**30.** D

Hints and Explanations

1. Beta distribution Choice (C)

2.

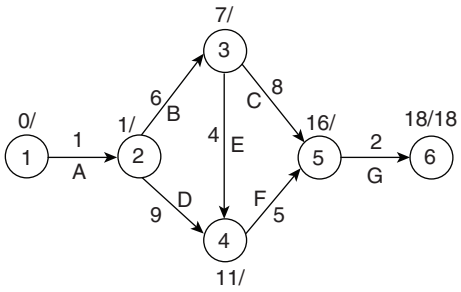


t_E = Expected time
So, 50% Choice (A)

3. M10 – 1:3:6
M15 – 1:2:4
M20 – 1:1.5:3 Choice (B)

4. Negative slacks are occur when there is a deficiency in available resources. Choice (D)

5.



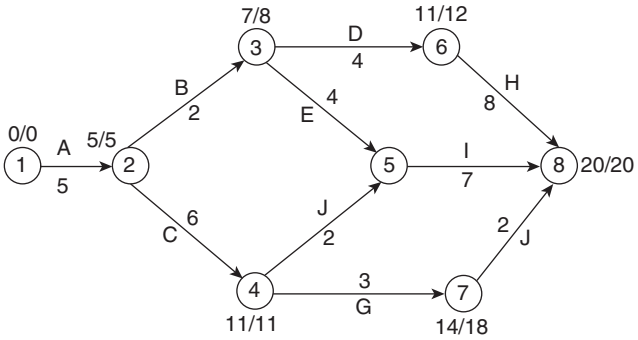
To find the duration of project, forward computation is enough. No need of backward computation of durations.
So, total duration of the project is 18 days. Choice (A)

6. To start one particular activity, the preceding activities must get completed. So to start an activity *F*, both *C* and *D* activities are need to get completed. Choice (D)

11. To make 1m^3 of wet concrete, 1.5 m^3 of dry concrete is required.
Volume of *C.A* = proportion of *C*. *A* × 1.5 m^3
$= \frac{4}{7} \times 1.5 = 0.857 m^3$ Choice (B)

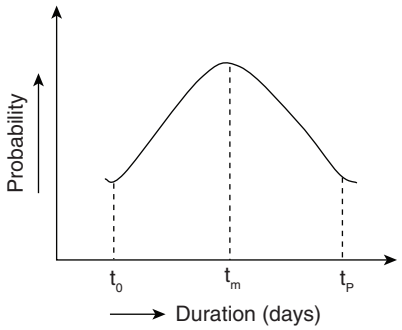
12. Correct sequence is:
2 -1 -4 -3 – 5 – 9 – 8 – 7 – 6 Choice(C)

13.



Critical path : 1 – 2 - 4 – 5 – 8 Choice (C)

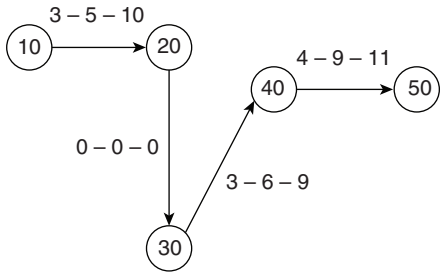
14.



$$t_e = \frac{t_o + 4t_m + t_p}{6} \frac{t_o + 4_{tm} + t_p}{6} = \frac{4 + 4(8) + 12}{6} = 8 \text{ days}$$

$\therefore$ Expected time = 8days Choice (D)

15.



$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$

Expected time $t_e = \left(\frac{3+20+10}{6}\right) + \left(\frac{3+24+9}{6}\right) + \left(\frac{4+36+11}{6}\right) = \frac{120}{6} = 20 \text{days}$

variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

$S.D = \sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{6}{6}\right)^2 + \left(\frac{7}{6}\right)^2} = 1.93$ Choice (B)

16. A – 4, B – 3, C – 2, D – 1 Choice (D)

17. 1 – c, 2 – c, 3 – a, 4 – b Choice (C)

18. Columns retaining walls - 75 -150mm
Mass concrete – 25 -50mm
Vibrated concrete – 12 -25mm Choice (B)

19. Proportion of *F.A* in

$= \frac{\text{F.M of C.A-Desired F.M}}{\text{Desired F.M-F.M of C.A}} \times 100$

Proportion of $F.A = \frac{7.6 - 6.4}{6.4 - 2.78} \times 100 = 33.33\%$

Choice (B)

20. Cube strength > Cylinder strength > Modulus of rupture > Split tensile Strength Choice (A)

21. Nominal size of brick is 19 × 9 × 9cm
With mortar in brick masonry, size = 20 × 10 × 10cm
∴ Volume of one brick in Masonry
$= 0.2 \times 0.1 \times 0.1\ m^3$
$= 2 \times 10^{-3}\ m^3$
For $1m^3$; Number of bricks required is

$= \frac{1}{2 \times 10^{-3}} = 500$ Choice (B)

23. Mean dimension $= \frac{50 + 40}{2} = 45mm$

Slot for flakiness index $= \frac{3}{5} \times 45 = 27mm$ Choice (B)

27. Given, $T_E^i = 5$; $T_E^j = 24$, $T_L^i = 9$, $T_L^i = 29$ and $t^{ij} = 6$

Total float = (LFT – EST) – Duration

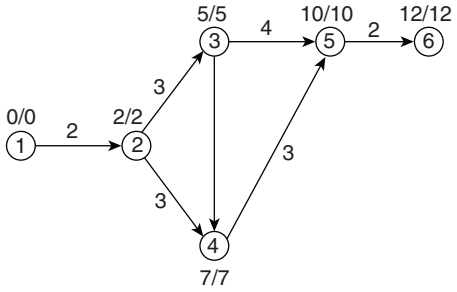
$= T_L^j - T_E^i - t^{ij} = (29 - 5) - 6 = 18$

Free float $= T_E^j - T_E^i - T^{ij}$

Free float = (24 – 5) – 6 = 13
Interfering float = Total float – Free float = 18 – 13 = 5
Independent float $= \left(T_E^i - T_L^i\right) - t^{ij}$

$= 24 - 9 - 6 = 9$ Choice (B)

29.



For an activity 2 – 4,
Total float = (7 – 2) – 3 = 2
For an activity 3 – 5,
Total float = (10 – 5) – 4 = 1 Choice (C)

UNIT V
CONCRETE STRUCTURES

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Concrete Structures Test 1

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The following two statements are made with reference to a simply supported Under Reinforced RCC beam.

I. Steel reaches ultimate stress prior to concrete reaching ultimate stress.

II. There is a shift in neutral axis upwards as the load is increased.

(A) Both the statements are false
(B) I is true but II is false
(C) I is false but II is true
(D) Both statements are true.

2. As per the provisions of IS 456-2000, the (short term) modulus of elasticity of M40 grade concrete (in N/mm^2) can be assumed to be
(A) 31600 (B) 28500
(C) 30000 (D) 36000

3. As per IS 456-2000, the effective length of column in a Reinforced concrete building frame is independent of
(A) height of column
(B) loads acting on frame
(C) frame type
(D) span of beam

4. Maximum strains in an extreme fiber in concrete and in the tension Reinforcement (Fe-250 grade and $Es = 200$ kN/mm^2) in a balanced section at limit state of flexure are respectively
(A) 0.0035 and 0.0041
(B) 0.002 and 0.0038
(C) 0.0035 and 0.0030
(D) 0.002 and 0.0018

5. An isolated foot bridge has a slab of 4 m width. The central supporting beam is of 8 m length, width of web-350 mm. The effective width of flange is,
(A) 1.25 m (B) 1.68 m
(C) 1.75 m (D) 2.0 m

6. The percentage of minimum shear Reinforcement as per IS456 using HYSD bars of Fe 415 grade is
(A) 0.11% (B) 0.15%
(C) 0.3% (D) 0.2%

7. Which of the following is the correct expression to estimate the development length of deformed reinforced bars used in compression as per IS456 in limit state of design?
(A) $\frac{\phi\sigma_s}{8\tau_{bd}}$ (B) $\frac{\phi\sigma_s}{4\tau_{bd}}$
(C) $\frac{\phi\sigma_s}{64\tau_{bd}}$ (D) $\frac{\phi\sigma_s}{5\tau_{bd}}$

8. What is the value of minimum percentage of reinforcement in case of Fe 250 steel in slabs?
(A) 0.1% (B) 0.12%
(C) 0.2% (D) 0.15%

9. Minimum grades of concrete to be used for pre tensioned and post tensioned structural elements are respectively [As per IS1343-1980]
(A) M30 and M40
(B) M40 and M30
(C) M20 and M30
(D) M30 and M30

10. Which of the following are subjected to primary torsion?
(A) Isolated L-beam
(B) Ringbeam of circular water tank
(C) A and B
(D) Grid system

11. A singly reinforced rectangular section of 300mm wide and 550 mm effective depth, reinforced with 3 bars of 16 mm diameter Fe415 steel bars. The concrete used is M20 grade. The ultimate moment of resistance of beam is _____ (in kN-m)
(A) 110 (B) 150
(C) 120 (D) 100

12. A reinforced beam of size 300 mm width and 700 mm overall depth is subjected to a service moment of 80 kN-m. If M25 and Fe 415 is used, it is be designed as (use effective cover = 50 mm)
(A) doubley reinforced section
(B) singly reinforced section
(C) over reinforced section
(D) None

13. The effective flange width of T-beams spaced at 4 m with web depth of 1.2 m, web width of 0.5 m spanning 10 m with a flange slab of 150 mm thickness is
(A) 4 m (B) 2 m
(C) 3 m (D) 5 m

14. An *RC* beam of 350 mm width and effective depth 550 mm is subjected to a factored shear force of 120 kN. M20 grade concrete is used for the beam. For shear reinforcement 6 mm diameter two legged mild steel stirrups are used. The spacing of shear reinforcement in beam is [take $\tau_{c,max} = 2.8$ MPa and $\tau_c = 0.60$ MPa]
(A) 1500 mm (B) 410 mm
(C) 90 mm (D) 300 mm

15. A bar of 12 mm diameter is embedded in concrete for a distance of 15 cm. Calculate the maximum load which the bar can take if bond stress is not to exceed 0.5 N/mm^2?

(A) 1.4 KN (B) 3.5 KN
(C) 2.0 KN (D) 2.8 KN

16. The minimum extension of steel bars of 12 mm diameter bar of Fe250 grade steel in M25 grade concrete with a design bond strength of 1.6 MPa and 135° standard bend at the end is
(A) 410 mm (B) 265 mm
(C) 150 mm (D) None

17. A reinforced concrete beam of 12 m effective span and 1.5 m effective depth is simply supported. If the total udl on the beam is 5 kN/m, the design shear force for the beam is
(A) 22.5 KN (B) 50 KN
(C) 30 KN (D) 40 KN

18. At the limit state of collapse, an RCC beam is subjected to total flexural moment of 300 kN-m, shear force of 30 kN, torque of 18 kN-m, the beam is of 350 mm wide and 450 mm gross depth with an effective cover of 30 mm. The equivalent nominal shear stress (τ_{ve}) as calculated using the code turns out to be lesser than the design shear strength (τ_c) of concrete. The equivalent shear force is
(A) 30 KN
(B) 115 KN
(C) 80 KN
(D) 200 KN

19. Based on the above data, the equivalent bending moment (Me_1) for designing the longitudinal tension steel is
(A) 400 KN-m
(B) 300 KN-m
(C) 325 KN-m
(D) 18 KN-m

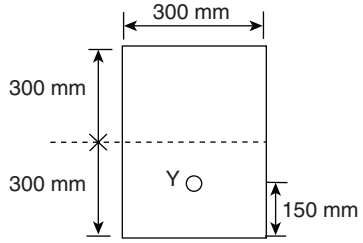
20. A rectangular column of 350 mm × 650 mm is reinforced with 0.8% reinforcement based on gross area. Fe500 steel and M25 grade concrete is used. The ultimate load carrying capacity of column is
(A) 4.8 MN
(B) 2.8 MN
(C) 3.2 MN
(D) 5.6 MN

21. The composition of an air entrained concrete is given below:
Water : 180 kg/m^3
Ordinary Portland cement: 360 kg/m^3
Sand : 600 kg/m^3
Coarse aggregate : 1200 kg/m^3
Assume the specific gravity of OPC, sand and coarse aggregate to be 3.0, 2.68 and 2.70 respectively. The air content in liters/m^3 is _____
(A) 30 (B) 40
(C) 50 (D) None

22. A column of size 300 × 550 mm has unsupported length of 4.0 m and is braced against side sway in both directions. According to IS456-2000, the minimum eccentricities (in mm) with respect to major and minor principle axes are
(A) 18 mm, 26.33 mm
(B) 26.33 mm, 18 mm
(C) 20 mm, 18 mm
(D) 26.33 mm, 20 mm

23. A simply supported reinforced concrete beam of length 12 m sags while undergoing shrinkage. Assuming a uniform curvature of 0.006 m^{-1} along the span, the maximum deflection at mid span is
(A) 0.20 m
(B) 0.11 m
(C) 0.30 m
(D) 0.25 m

24. In a pre stressed concrete beam section shown in figure. The net loss is 15% and the final prestressing force applied at *y* is 700 KN. The initial fiber stresses (in N/mm^2) at the top and bottom of beam were:



(A) + 49 and 58
(B) – 58 and – 49
(C) – 49 and 58
(D) – 58 and 49

25. A concrete column carries an axial load of 350 KN and a bending moment of 45 KN-m at its base. An isolated footing of 3 m × 5m with 5m side along the plane of bending moment, is provided under column. The C.G. of column and footing coincides, the net maximum and minimum pressures in KN/m^2 on the soil under the footing are respectively?
(A) 150, 98 (B) 100, 75
(C) 150, 120 (D) 134, 98

Answer Keys

1. D	**2.** A	**3.** B	**4.** C	**5.** B	**6.** A	**7.** A	**8.** D	**9.** B
10. C	**11.** A	**12.** B	**13.** C	**14.** C	**15.** D	**16.** B	**17.** A	**18.** B	**19.** B
20. B	**21.** A	**22.** D	**23.** B	**24.** C	**25.** D				

Hints and Explanations

2. $E_c = 5000\sqrt{fck} = 5000\sqrt{40}$

$= 31622$ N/mm^2 Choice (A)

4. Maximum strain in steel at extreme tension fiber

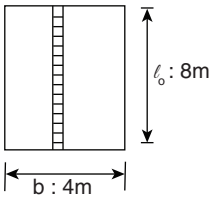
$\le 0.002 + \dfrac{0.87fy}{Es}$

$= 0.002 + \dfrac{0.87\times 250}{2\times 10^5}$

$= 0.0030.$

Miximum strain in concrete = 0.0035 Choice (C)

5. $b_f = \dfrac{\frac{\ell_o}{6}}{\frac{\ell_o}{b}+4} + bw \le b$



Isolated T-beam : so use the above formula

$b_f = \dfrac{8}{\frac{8}{4}+4} + 0.35 = \dfrac{8}{6} + 0.35$

$b_f = 1.68$ m Choice (B)

6. $\dfrac{A_{sv}}{b\times s_v} = \dfrac{0.4}{0.87fy}$

% minimum shear Reinforcement $= \dfrac{0.4}{0.87\times 415}\times 100$

$= 0.110\%$

Choice (A)

7. $L_d = \dfrac{\phi\sigma_s}{4\times\tau_{bd}}$

For HYSD bars increase τ_{bd} by 60% and for compression increase τ_{bd} by 25%

$L_d = \dfrac{\phi\sigma_s}{4\times 1.6\times 1.25\tau_{bd}} = \dfrac{\phi\sigma_s}{8\tau_{bd}}$ Choice (A)

11. Given

Singly Reinforced section

$b = 300$ mm

$d = 550$ mm

$A_{st} = 3\times\dfrac{\pi}{4}\times(16)^2$

$f_y = 415$ N/mm^2

$f_{ck} = 20$ N/mm^2

$M_u = ?$

$X_{umax} = (0.48)\,(d) = 0.48\times 550 = 264$ mm

Calculation of x_u:

$0.36\,f_{ck}bx_u = 0.87\,fyA_{st}$

$X_u = \dfrac{0.87\times 415\times 3\times\frac{\pi}{4}\times 16^2}{0.36\times 20\times 300}$

$X_u = 100.82$ mm

$X_u < x_{umax} \Rightarrow$ under reinforced section

$\therefore\quad M = T\times$ lever arm

$M_u = 0.87\,f_yA_{st}\,(d - 0.42\,x_u)$

$M_u = 0.87\times 415\times 3\times\dfrac{\pi}{4}\times 16^2\,(550 - 0.42\times 100.82)$

$M_u = 110.55$ KN-m Choice (A)

12. $b = 300$ mm;

$D = 700$ mm; $d = 650$ mm

Design moment, $M_u = 80\times 1.5 = 120$ KN-m

$f_{ck} = 25$ MPa

$fy = 415$ MPa

$d^1 = 50$ mm

M_u, limit $= 0.138\,fck\,bd^2 = 0.138\times 25\times 300\times 650^2$

$= 437.28$ KN-m

$M_u \le M_u$, lim $\Rightarrow$ singly Reinforced section. Choice (B)

13. $C = 4$ m

$b_w = 0.5$ m

$D_f = 150$ mm

$\ell_o = 10$ m

$b_f = \dfrac{\ell_o}{6} + b_w + 6D_f \le c = 4\text{ m} = \dfrac{10}{6} + 0.5 + 6\,(0.15)$

$b_f = 3.06$ m Choice (C)

14. $b = 350$ mm

$d = 550$ mm

$V_u = 120$ KN

$f_{ck} = 20$ MPa

$\tau_{cmax} = 2.8$ MPa

$\tau_c = 0.60$ MPa

$\tau_v = ?$

Nominal (average) shear stress due to external load is

$\tau_v = \dfrac{V_u}{bd} = \dfrac{120\times 10^3}{350\times 550}$

$\tau_v = 0.623$ N/mm^2

$\tau_v < \tau_{cmax} \Rightarrow$ No. diagonal compression failure.

$\tau_v > \tau_c \therefore$ not safe in shear

Shear reinforcement is required for

$V_{us} = v_u - \tau_c\,(bd)$

$= 120\times 10^3 - 0.60\,(350\times 550) = 4500$ N

$V_{us} = 4.5$ KN

$V_{us} = \dfrac{0.87\,fy\,A_{sv}d}{s_v}$

$$4500\ \text{N} = \frac{0.87\times250\times2\times\frac{\pi}{4}\times6^2\times550}{s_v}$$

$S_v = 1503$ mm

S_v based on minimum shear RFM

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87\,fy}$$

$$S_v = \frac{(A_{sv})(0.87\,fy)}{(0.4)(b)}$$

$$S_v = \frac{\left(2\times\frac{\pi}{4}\times6^2\right)(0.87\times250)}{0.4\times350}$$

$S_v = 87.85$ mm

$S_v = 0.75 \times 550 = 412.5$ mm

S_v should be minimum of

(1) s_v calculated based on vertical stirrups
(2) s_v based on minimum shear reinforcement
(3) $\le 0.75\,d$
(4) 300 mm

$\therefore$ $s_v = 87.85$ mm Choice (C)

15. $\tau_{bd} = 0.5$ N/mm^2
$L_d = 15$ cm
$\phi = 12$ mm
$P = ?$
$P = (\tau_{bd})\ (\prod\phi L_d)$
$= (0.5)\ (\pi \times 12 \times 150)$
$P = 2827.43$ N
$P = 2.82$ kN Choice (D)

16. $\phi = 12$ mm
$fy = 250$ MPa
$fck = 25$ MPa
$\tau_{bd} = 1.6$ MPa
135° bend

$(L_d)_{\text{required}} = (L_d)_{\text{straight bar}}$ – anchorage value

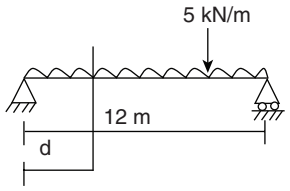
$$(L_d)_{\text{straight bar}} = \frac{\phi\sigma_s}{4\tau_{bd}} = \frac{12\times0.87\times250}{4\times1.6}$$

$= 407.81$ mm

Anchorage value for a 135° bend
$= 12\phi = 12 \times 12 = 144$ mm

$(L_d)_{\text{required}} = 407.81 - 144$
$= 263.81$ mm Choice (B)

17. $L = 12$ m
$d = 1.5$ m



Critical section for shear is at a distance 'd' from support

$\therefore$ Design shear force $= \frac{wl}{2} - wd = \frac{5\times12}{2} - 5(1.5)$

$F = 22.5$ kN Choice (A)

18. $M_u = 300$ kN-m
$V_u = 30$ kN
$T_u = 18$ kN-m
$b = 350$ mm
$D = 450$ mm
$d^1 = 30$ mm
$d = 420$ mm
$\tau_{ve} < \tau_c$
Equivalent shear force;

$$V_e = V_u + V_T = V_u + \frac{1.6T_u}{b} = 30 + \frac{1.6\times18}{0.35}$$

$V_u = 112.28$ KN Choice (B)

19. Longitudinal tension steel is based on M_u only.
[ignoring the effect of torsion] If $\tau_{ve} < \tau_c$
$\therefore$ $M_{e1} = M_u = 300$ KN-m Choice (B)

20. $b = 350$ mm
$D = 650$ mm
$A_{sc} = 0.8\%\ (A_g)$

$$= \frac{0.8}{100}(350\times650)$$

$A_{sc} = 1820$ mm^2
$A_c = A_g - A_{sc}$
$= (350 \times 650) - (1820)$
$A_c = 225680$ mm^2
$P_u = (0.4\,fckA_c) + (0.67\,fyA_{sc})$
$= (0.4 \times 25 \times 225680) + (0.67 \times 500 \times 1820)$
$P_u = 2866$ KN
$P_u = 2.8$ MN Choice (B)

21. $$\frac{Mc}{\rho_c} + \frac{M_s}{\rho_s} + \frac{M_a}{\rho_a} + v_w + v_a = 1$$

$$\frac{360}{3\times1000} + \frac{600}{2.68\times1000} + \frac{1200}{2.7\times1000} + \frac{180}{1000} + v_a = 1$$

$V_a = 0.0316$
$= 0.0316 \times 1000$
$= 31.67$ Choice (A)

22. $e_{\min} = \frac{L}{500} + \frac{b}{30}$ or 20 mm which ever is more

$$e_{xx} = \frac{4000}{500} + \frac{550}{30} = 26.33\ \text{mm}$$

$$e_{yy} = \frac{4000}{500} + \frac{300}{30} = 18.0\ \text{mm}$$

Subject to a minimum of 20 mm,so, take 20 mm.
Choice (D)

23. $EI\dfrac{d^2y}{dx^2} = M_x$

$$\frac{d^2y}{dx^2} = \frac{M_x}{EI}$$

$$\frac{d^2y}{dx^2} = \rho$$

$$\frac{dy}{dx} = \rho x$$

$$y = \frac{\rho x^2}{2}$$

$$y_{at\ x=\frac{L}{2}} = \frac{(0.006)\left(\frac{L}{2}\right)^2}{2}$$

$$= \frac{(0.006)(6)^2}{2} = 0.108 \text{ m}$$ Choice (B)

24. $P_i = 1.15 \times 700 = 805$ KN

$$\sigma_i = \frac{P}{A} \pm \frac{M}{I} \times y$$

$$= \frac{1.15\times 700\times 10^3}{300\times 600} \pm \frac{1.15\times 700\times 10^3\times 150}{300\times \frac{300^3}{12}} \times 300$$

$$= 4.47 \pm 53.66$$

$$= -49.18 \text{ and } 58.13 \text{ (in N/mm}^2\text{)}$$ Choice (C)

25. $P = 350$ KN

$M = 45$ KN-m

$$e = \frac{M}{P} = \frac{45}{350} = 0.128 \text{ m}$$

$$\left.\begin{matrix}\sigma_{max}\\ \sigma_{min}\end{matrix}\right\} = \frac{P}{A} \pm \frac{P.e}{z}$$

$$= \frac{P}{A} \pm \frac{P.e(6)}{bd^2}$$

$$= \frac{P}{bd}\left[1 \pm \frac{6.e}{d}\right]$$

$$= \frac{350}{3\times 5}\left[1 \pm \frac{6\times 0.128}{5}\right]$$

$\sigma_{max} = 134.58$ kN/m^2

$\sigma_{min} = 98.74$ kN/m^2 Choice (D)

Concrete Structures Test 2

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Minimum Grade of concrete for structural purpose
(A) M15 (B) M20
(C) M25 (D) M30

2. Balanced Neutral axis depth for a singly reinforced cross section depends on
(A) Grade of concrete
(B) Grade of steel
(C) Amount of steel reinforcement
(D) All the above

3. A rectangular concrete beam of width 230 mm and effective depth 300 mm is reinforced with 4 – 12 mm bars in tension zone. M20 Grade concrete and Fe 415 steel are used. Find the Moment of Resistance of the beam.
(A) 42.23 KNm (B) 35.26 KNm
(C) 64.13 KNm (D) 72.54 KNm

4. Minimum percentage of steel in both directions in a slab when HYSD bars are used is _____% of (*bD*)
(A) 0.1% (B) 0.15%
(C) 0.12% (D) 0.2%

5. In design for shear in Reinforced concrete structures, which of the following is considered explicitly.
(A) Dowel action
(B) Aggregate interlocking
(C) Concrete in compression zone
(D) All the above

6. As per IS: 456 – 2000, span/depth ratio of two way simply supported slabs is
(A) 40 (B) 30
(C) 20 (D) 35

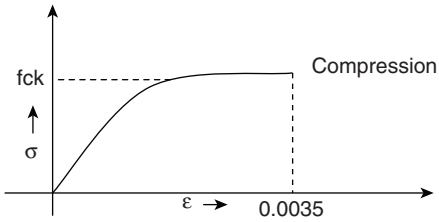
7. A hook of Fe 415 Grade is provided in Compression in M20 Grade Concrete with τ_{bd} = 1.2 MPa, Then the development length L_d = _____ ϕ
(A) 44.17 (B) 37.6
(C) 21.61 (D) Not allowed (hook)

8. In a plain concrete pedestal of M35 Grade, the maximum bearing pressure at the base is found to be 40 N/mm^2. Find the depth of footing if the projection beyond the column is 300 mm
(A) 3.1 m (B) 2.6 m
(C) 2.4 m (D) 1.9 m.

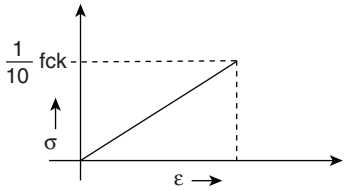
9. In the limit state design of serviceability the deflection after erection of partitions and erection of finishes is limited to
(A) span/250 (B) san/325
(C) span/350 (D) span/150

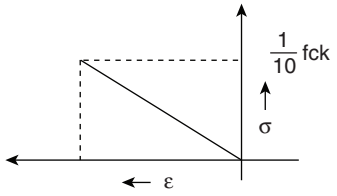
10. Calcium lignosulphate is an example of
(A) retarder (B) accelerator
(C) dispersal agent (D) hardness agent

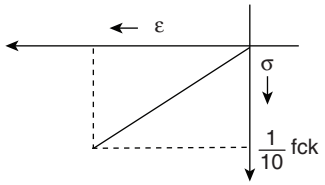
11. If the following figure represents the idealized σ – ε curve of concrete in compression.

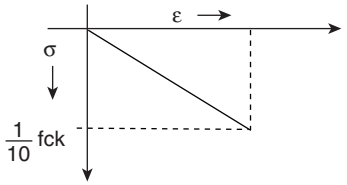


Then the σ – ε curve of concrete in Tension is

(A)


(B)


(C)


(D)


Common Data for Questions 12 and 13:
A reinforced concrete beam, size 250 mm wide and 400 mm deep effective is simply supported over a span of 6 m. It is subjected to a point load of *P* at centre of the beam. The point load is increased gradually. Beam is reinforced with 5 HYSD bars of Fe 415 grade of 12 mm diameter placed at an effective cover of 40 mm on bottom face and nominal shear reinforcement. The characteristic compressive strength and bending tensile strength of concrete are 20MPa and 2.2MPa respectively.

12. Ignoring the pressure on tension reinforcement, find the value of load P in KN when the first flexural crack will develop on the beam

(A) 26.4 KN (B) 11.83 KN
(C) 16.4 KN (D) 35.5 KN

13. The theoretical failure load of the beam for attainment of limit sate of collapse in flexure is

(A) 33 KN (B) 48 KN
(C) 52 KN (D) 64 KN

14. A reinforced beam of size 230 mm width and 350 mm overall depth is subjected to a working moment of 65 KNm. If M20 grade concrete and Fe415 grade steel are used, it is to be designed as (effective cover = 50 mm)

(A) Balanced section
(B) Singly reinforced section
(C) Doubly reinforced section
(D) Over reinforced section

15. A hall of 10 m × 24 m consists of a number of beams 4 m centre to centre parallel to the shorter span of the hall. Width of web = 300 mm. Thickness of slab 120 mm, the beams are cast monolithic with the columns at their ends. The effective width of flange of an intermediate beam is

(A) 1.52m (B) 0.94m
(C) 2.01m (D) 2.19m

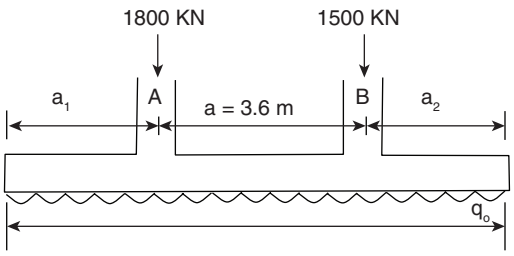
16. Match List – I with List – II and select the correct answer from the codes given below.

	List – I (Admixtures)		List – II (Example)
P.	Retarder	1.	Volcanic Tuff
Q.	Accelerator	2.	Natural wood resins
R.	Pozzolona	3.	Calcium sulphate
S.	Air entraining agent.	4.	Calcium chloride

Codes:

	P	Q	R	S
(A)	1	2	3	4
(B)	1	2	4	3
(C)	3	4	1	2
(D)	2	1	3	4

17. Two columns A and B carrying loads are shown in figure below with different notations. Soil bearing capacity of soil q_o = 200 KN/m^2. width of footing for both columns is 2.4 m. Find a_1 and a_2 shown in figure (assume footing weight as 10% of column load)



(A) a_1 = 1.82 m and a_2 = 2.14 m
(B) a_1 = 2.14 m and a_2 = 1.82 m
(C) a_1 = 1.24 m and a_2 = 2.72 m
(D) a_1 = 2.72 m and a_2 = 1.24 m

18. A reinforced concrete beam of 12m effective span and 1 m effective depth is simply supported. If the total *udl* of the beam is 10MN/m the design shear force for the beam is

(A) 20MN (B) 30MN
(C) 40MN (D) 50MN

19. A column is of 500 × 300 mm and unsupported length of 3 m. The design criteria of the column as per IS 456-2000 will be.

(A) short along long and short dimensions.
(B) long along short and short along long dimension
(C) long along long and short dimensions
(D) long along long and short along short dimensions

20. The factored load carrying capacity of a column of 300 mm × 500 mm size with minimum percentage of steel is (M20 and Fe415)

(A) 1234KN
(B) 1468KN
(C) 1524KN
(D) 1632KN

21. Match the following with reference to RCC.

	List – I		List – II
a.	Torsional Analysis	1.	Truss Analogy
b.	Shear stress Analysis	2.	Skew Bending Theory
c.	Limit state method	3.	Semi Probabilistic Approach
d.	Working stress method	4.	Deterministic

Codes:

	a	b	c	d
(A)	1	2	3	4
(B)	2	1	3	4
(C)	2	1	4	3
(D)	1	2	4	3

Common Data for Questions 22 and 23:

A post tensioned concrete beam 120mm wide and 300mm deep is prestressed by three cables each with a cross sectional area of 60 mm^2 and with an initial stress of 1100 MP$_a$. All the three cables are straight and located 100mm from the sofit of the beam. If the modular ratio is 6, the loss of stress due to elastic shortening in the beam

22. When simultaneous tensioning and anchoring of all the three cables is done will be.

(A) 24.51 MPa (B) 43.92 MPa
(C) 78.26 MPa (D) zero

23. When successive tensioning of the three cables are done (one at a time)

(A) 24.51MPa (B) 43.92MPa
(C) 78.26MPa (D) zero

24. Consider the following statements regarding the PORTLAND PUZZOLANA CEMENT
(1) It produces less heat of hydration
(2) Addition of Pozzolano does not contribute to the strength at early stages
(3) Strength of this cement at any time is always less than the strength of the Portland cement
(4) It is particularly useful in marine and hydraulic construction
(A) 1, 2, 3 are correct (B) 2, 3, 4 are correct
(C) 1, 3, 4 are correct (D) 1, 2, 4 are correct

25. A reinforced concrete beam of rectangular cross section of breadth 300 mm and effective depth 500 mm is subjected to maximum factored shear force of 400KN. The grades of concrete and steel are M25 and Fe415 respectively. Based on the area of main steel provided grade of concrete the design shear stress τ_c as per IS 456 : 2000 is 0.64N/mm^2. If 2 - 16ϕ bars are used as bent up bars $\alpha = 50°$) Design shear reinforcement spacing of (8 mm ϕ) 2 legged vertical stirrups ($\tau_{cmax} = 3.1$ N/mm^2)
(A) 90 mm (B) 110 mm
(C) 130 mm (D) 280 mm

26. A rectangular beam of 500 mm × 700 mm with effective cover of 40mm is subjected to a factored values of shear force 12KN, Bending Moment 150KNm and a torsional moment 15KNm. Find the design bending moment for the design in KN- m. Use share resistance of the cross section $\tau_c = 1.3$MPa.
(A) 100 (B) 114
(C) 150 (D) 171

27. A T – beam roof section has the following particulars. Width of flange = 600 mm, Thickness of slab = 120 mm. Width of web = 250 mm depth of web = 300mm Effective cover = 50mm. If 3 - 20ϕ bars reinforcement is provided and M20 Grade concrete and Fe415 steel are used. Find the moment of Resistance of the beam.
(A) 92KNm (B) 106KNm
(C) 114KNm (D) 138KNm

28. A concrete column carries an axial load of 500 KN and a bending moment of 50 KN–m at its base. An isolated footing of 2 m × 3m, with 3m side along the plane of bending moment is provided under column. The *CG* of column and footing coincides. The net maximum and minimum pressures in KN/m^2 on the soil under the footing are respectively
(A) 100 and 66.67 (B) 95 and 55.32
(C) 72 and 46.18 (D) 120 and 75

29. Unfactored maximum bending moments at a section of a reinforced concrete beam resulting from a frame analysis are 330, 420 and 150KN-m under dead, live and wind loads respectively. The design moment(KN – m) as per IS 456 : 2000 for the limit state of collapse is
(A) 720KN – m (B) 840KN – m
(C) 1125KN – m (D) 1530KN – m

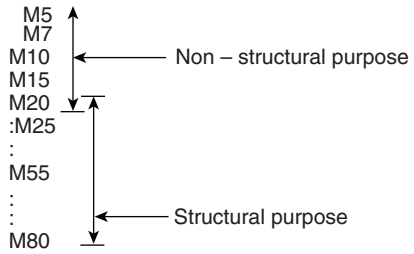
30. In under reinforced concrete beam, which of the following statements are correct?
1. Actual depth of neutral axis is less than the critical depth of neutral axis
2. Concrete reaches ultimate stress prior to steel reaching the ultimate stress.
3. Moment of resistance is less than that of balanced section
4. Lever arm of resisting couple is less than the balanced section.
(A) 1 and 2 only (B) 1 and 3 only
(C) 2, 3 and 4 (D) 1, 2 and 4

Answer Keys

1. B	**2.** B	**3.** A	**4.** C	**5.** A	**6.** D	**7.** D	**8.** A	**9.** C	**10.** A
11. C	**12.** B	**13.** B	**14.** C	**15.** D	**16.** C	**17.** B	**18.** D	**19.** A	**20.** C
21. B	**22.** D	**23.** B	**24.** D	**25.** A	**26.** C	**27.** C	**28.** A	**29.** C	**30.** B

Hints and Explanations

1.



M 20 can be used for both structural and non structural purpose. Minimum Grade for structural purpose starts from M 20. Choice (B)

2. Neutral axis can be found from either σ diagram or ε diagram.
By considering σ diagram:
Compressive force = tensile force

$$0.36\, fck\, bx = 0.87\, f_y\, \text{Ast}$$

$$X = \left[\frac{0.87 f_y Ast}{0.36 f_{ck}(bd)}.d\right] \qquad \text{--------- (1)}$$

By considering ε diagram

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{(d-x)}$$

$$\frac{0.0035}{x} = \frac{\frac{0.87fy}{E_s} + 0.002}{d - x}$$

On simplifying

$$X = \left[\frac{0.0035}{0.005 + \frac{0.87fy}{E_s}}\right] d \qquad \text{-------- (2)}$$

From (1) and (2) equations
We can say that
N.A depends on only
(i) Grade of steel
(ii) Depth of cross section. Choice (B)

3. $X = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \dfrac{0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 12^2}{0.36 \times 20 \times 230} = 98.63$ mm

Balanced Neutral axis for Fe415 Grade
$X_b = 0.48 \times d = 0.48 \times (300) = 144$ mm
$X < x_b \Rightarrow$ under reinforced.
$\therefore \quad M = T.\ a = (0.87\, fy\, Ast)\,(d - 0.42x)$

$$= 0.87 \times 415 \times \frac{\pi}{4} \times 4 \times 12^2 \times (300 - 0.42 \times 98.63)$$

$= 42.23 \times 10^6$ N mm
$= 42.23$ KNm. Choice (A)

4. Minimum % of steel in slab
$= 0.15\%\ b\ D$ (for Mild steel)
$= 0.12\%\ b\ D$ (for HYSD bars). Choice (C)

5. Aggregate interlocking and concrete in compression Zone effect are considered implicitly
But dowel action is taken explicitly like in inclined bars

$V_{us} = 0.87\, fy\, As_v \dfrac{d}{Sv} (\sin\beta + \cos\beta)$. Choice (A)

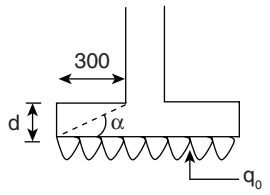
6. $\dfrac{\text{span}}{\text{depth}} = \begin{cases} 20(\text{beams}) \\ \text{same as beam for one ways labs} \\ 35(\text{two way simply supported slabs}) \\ 40(\text{2way continuous slabs}) \end{cases}$.

Choice (D)

7. In actual sense for every 45° bend, anchorage value $= 4\phi$
And for hook it is $180° = 4(45°)$
$\therefore \quad (L_d)_{req} = L_d - 4(4\phi)$
But maximum angle of bend in HYSD is only 135°
$\therefore$ Hook is not allowed with HYSD bars. Choice (D)

8. $\tan\alpha \geq 0.9 \sqrt{\dfrac{100 q_o}{f_{ck}} + 1}$

$q_0 = 40$ Mp$_a$
$f_{ck} = 35$ Mp$_a$



$$\tan\alpha \geq 0.9 \sqrt{\frac{100 \times 40}{30} + 1}$$

$$\tan\alpha = \frac{d}{300}$$

$$\frac{d}{300} \geq 0.9 \sqrt{\frac{100 \times 40}{30} + 1}$$

$$\frac{d}{300} \geq 10.43$$

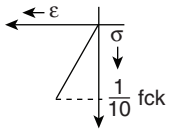
$d \geq 3129$ mm
≥ 3.13 m
$\therefore \quad d \simeq 3.1$ m. Choice (A)

9. The final deflection of horizontal members should not exceed span/250.
The deflection after construction of partitions or application of finishes should not exceed span/350 or 20 mm whichever is less. Choice (C)

10. Calcium lignosulphate is an example of retarder.
Choice (A)

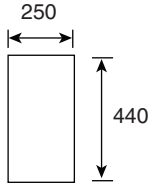
11. If compressive stress is considered positive, Tensile stress is considered negative
If strain in compression is positive, strain in tension is negative
Tensile stress in concrete $= \dfrac{1}{10}$(compressive stress in concrete) $= \dfrac{1}{10} fck$

In compression $\delta\ell$ is reduction, In tension $\delta\ell$ is increasing.
$\therefore$ if $\sigma - \varepsilon$ of compression is in I quadrant
$\sigma - \varepsilon$ of tension is in III quadrant.

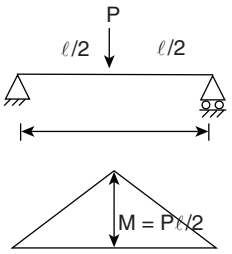


Choice (C)

12.



$d = 400$ mm
$D = 400 + 40 = 440$ mm
$b = 250$ mm
$\ell = 6$ m



From bending equation

$$\frac{M}{I} = \frac{f}{y}$$

$$f_{cr} = \frac{M}{Z}$$

$$2.2 = \frac{M}{\frac{250 \times 440^2}{6}}$$

$M = 17.75$ KN – m

Bending Moment is maximum at centre.

Therefore flexural crack develops at centre

$$\frac{P\ell}{4} = 17.75$$

$$\frac{P \times 6}{4} = 17.75 \Rightarrow P = 11.83 \text{ KN.}$$ Choice (B)

13. $$X = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 5 \times \frac{\pi}{4} \times 12^2}{0.36 \times 20 \times 250}$$

$X = 113.43$ mm

Balanced Neutral axis

$X_b = 0.48\ d = 0.48 \times (400) = 192$ mm

$X < x_b \Rightarrow$ under reinforced beam

$\therefore$ Moment of Resistance

M = $T.a$

$= 0.87\ fy\ Ast(d - 0.42x)$

$= 0.87 \times 415 \times 5 \times \frac{\pi}{4} \times 12^2\ (400 - 0.42 \times 113.43)$

$= 72$ KNm

For simply supported with point load at centre M = $\frac{P\ell}{4}$

$$72 = \frac{P \times 6}{4} \Rightarrow P = 48 \text{ KN.}$$ Choice (B)

14. $b = 230$ mm

$D = 350$ mm

Effective cover $d^1 = 50$ mm

Effective depth $d = 300$ mm

$X_{u\,max} = 0.48d = 144$ mm

$M_{u\,limit} = 0.36\ fck\ bx_{umax}\ (d - 0.42\ x_{umax})$

$= 0.36 \times 20 \times 230 \times 144\ (300 - 0.42 \times 144)$

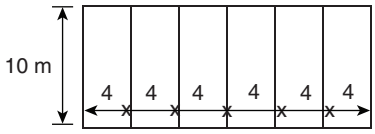
$= 57.12$ KNm

$M_u = 65\ k$ N m

$M_u > M_{u\,limit}$

$\therefore$ Design as doubly reinforced beam. Choice (C)

15.



Beam monolithic with columns means beam are fixed to columns

$L_0 = 0.7\ell = 0.7 \times 10 = 7$m

$$b_f = \frac{\ell_0}{6} + b_w + 6D_f$$

$$= \frac{7}{6} + 0.3 + (6 \times 0.12)$$

$b_f = 2.19$ m

$b_f \not> C = 4$m } use minimum

$\therefore$ $b_f = 2.19$ m. Choice (D)

16. Retarder – calcium sulphate

Accelerator – calcium chloride

Pozzolone – volcanic Tuff

Air entraining agent – Natural word resins. Choice (C)

17. $a = 3.6$ m

$B = 2.4$ m

$SBC = q_0 = 200$ KN/m^2

$$\frac{P_1 \times 1.1}{q_0} = 9.9 = A_1; \quad \frac{P_2 \times 1.1}{q_0} = 8.25 = A_2$$

$$L_1 = \frac{A_1}{B} = 4.125 \quad L_2 = \frac{A_2}{B} = 3.44$$

$$\frac{L_1 + L_2}{2} = 3.78 > 3.6 = a$$

$\therefore$ combined footting is provided

$$A = \frac{(P_1 + P_2) \times 1.1}{q_0} = 18.15 \text{ m}^2$$

$$L = \frac{A}{B} = 7.56 \text{ m}$$

Distance of centroid from column A

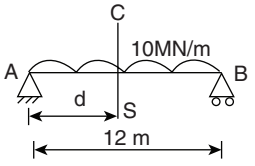
$$\bar{x} = \frac{P_2 a}{P_1 + P_2} = 1.64 \text{ m}$$

$$a_1 + \bar{x} = \frac{L}{2}$$

$a_1 = 2.14$ m

$a_2 = 1.82$ m Choice (B)

18.



Design shear force at critical section. i.e., at $d = 1$ m from one end

$V_u = R_A - 10d$

$= \dfrac{10\times12}{2} - 10 \times 1 = 50$ MN. Choice (D)

19. $\dfrac{\ell}{b} = \dfrac{3000}{300} = 10 < 12$

$\dfrac{\ell}{D} = \dfrac{3000}{500} = 6 < 12$

$\therefore$ short columns along both directions. Choice (A)

20. Minimum % of steel = 0.8% (Ag)

$A_{SC} = \dfrac{0.8}{100} \times (300 \times 500) = 1200$ mm^2

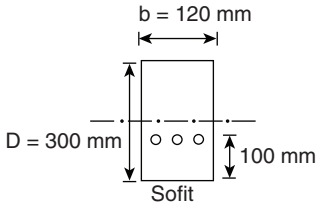
$A_c = Ag - A_{SC} = 148.8 \times 10^3$ mm^2

$P_u = 0.4\, fck\, A_c + 0.67\, fy\, A_{SC}$

$= 0.4 \times 20 \times (148.8 \times 10^3) + (0.67 \times 415 \times 1200)$

$= 1524 \times 10^3$N $= 1524$ KN. Choice (C)

22.



In simultaneous tensioning and anchoring, there will be no losses

$\therefore$ losses = 0. Choice (D)

23. prestressing force in each cable

$P = \sigma_s . A_S = 1100 \times 60 = 66$KN

$f_c = \dfrac{P}{A_C} + \dfrac{Pe}{I}(e)$

$= \dfrac{66\times10^3}{120\times300} + \dfrac{66\times10^3\times50}{\dfrac{120\times300^3}{12}} \times 50$

$f_c = 2.44$ MPa

Loss $= \dfrac{n(n-1)}{2}[mf_c]$

Given m $= 6 = \dfrac{3(3-1)}{2}[\,6 \times 2.44]$

$= 43.92$ MP$_a$. Choice (B)

24. Portland puzzolana cement produces less heat of hydration and so less cracks and used in hydraulic construction.

Less strength at early ages, but more strength at later ages. Choice (D)

25. $\tau_{cmin} = 0.64$ MP$_a$ $< \tau_{cmax} = 3.1$ MP$_a$

$V_c = (\tau_{cmin})\, bd$

$= (0.64) \times (300 \times 500) = 96$ KN

$v_{us} = V_u - V_c = 400 - 96 = 304$ KN

$304\text{KN} = V_{bentup} + V_{stirrups}$

$V_{bentup} = 0.87\, fy\, A_{sb} \times \sin\alpha$

$= 0.87 \times 415 \times 2 \times \dfrac{\pi}{4} \times 16^2 \times \sin(50°)$

$V_{bentup} = 111$KN

$V_{stirrrups} = 304 - 111 = 193$ KN

$V_{bentup} \not> 50\%\,(V_{us}) = \dfrac{304}{2} = 152$KN

Satisfied

$V_{stirrups} = \dfrac{0.87\, fyA_{sv}d}{S_v}$

$193 \times 10^3 = \dfrac{0.87\times415\times2\times\dfrac{\pi}{4}\times8^2\times500}{S_v}$

$S_v \not> 94$ mm

$\not> 0.75\, d = 375$ mm

$\not> 300$ mm

$\therefore$ 2 legged 8ϕ @ $S_v = 90$ mm. Choice (A)

26. $V_e = V_u + V_T = V_u + \dfrac{1.6T_u}{b}$

$= 12 + \dfrac{1.6\times15\times1000}{500} = 60$KN

$\tau_{vc} = \dfrac{V_e}{bd} = \dfrac{60\times10^3}{500\times(700-40)}$

$= 0.182$ MPa

$\tau_{ve} < \tau_c$

$\Rightarrow$ Ignore Torsion effect

$\therefore$ $M_u = 150$ KN – m. Choice (C)

27. $x_b = 0.48d = 0.48 \times (420 - 50) = 177.6$ mm

Let NA lies in flange

$0.36\, fck\, b_f\, x = 0.87\, fy\, Ast$

$0.36 \times 20 \times 600 \times x = 0.87 \times 415 \times 3 \times \dfrac{\pi}{4} \times 20^2$

$X = 78.77$ mm

$X < D_f = 120$ mm $\quad\therefore$ NA is with in flange

$X < x_b \Rightarrow$ under reinforced

M $= T.a$

$= 0.87 \times fy \times A_{st}\,(d - 0.42x)$

$= 0.87 \times 415 \times 3 \times \dfrac{\pi}{4} \times 20^2 \times (370 - 0.42 \times 78.77)$

$= 114.65$ KNm. Choice (C)

28. $\left.\begin{matrix}\tau_{max}\\ \tau_{min}\end{matrix}\right\} = \sigma \pm f = \dfrac{P}{A} \pm \dfrac{M}{Z} = \dfrac{500}{2\times3} \pm \dfrac{50}{\dfrac{2\times3^2}{6}}$

$\sigma_{max} = 100$ KN/m^2

$\sigma_{min} = 66.67$ KN/m^2. Choice (A)

29. Design load for collapse

(i) $1.5\,DL + 1.5\,LL$
$= 1.5\,(330 + 420)$
$= 1125$ KN – m

(ii) $1.5\,DL + 1.5\,WL$
$= 1.5\,(330 + 150)$
$= 720$ KN – m

(iii) $1.2\,DL + 1.2\,LL + 1.2\,WL$
$= 1.2\,(330 + 420 + 150)$
$= 1080$ KN – m

Design moment = maximum of above combinations
$= 1125$ KN – m.

Choice (C)

30. For under reinforced beam

(i) $x < x_b$(balanced)
X = depth of neutral axis

(ii) Steel yields prior to concrete

(iii) $M_{UR} = 0.87\,fy\,A_{st}\,(d - 0.42x)$
Which is also $= 0.36\,fck\,bx(d - 0.42x)$ -------- (1)
As x was found by
$0.36\,fck\,bx = 0.87\,fy\,A_{st}$
$M_B = 0.36\,fck\,bx_b\,(d - 0.42\,x_b)$ -------- (2)
From (1) and (2)
$\Rightarrow \quad M_{UR} < M_B\,(as\ x < x_b)$

(iv) Lever arm $= (d - 0.42x)$
$(d - 0.42x) > (d - 0.42\,x_b)$ [$\because\ x < xb$] Choice (B)

UNIT VI
STEEL STRUCTURES

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STEEL STRUCTURES TEST I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. How are the most commonly produced and used structural elements in frames, floor beams, with high moment of inertia about *X*-axis, are designated?
(A) ISWB – section
(B) ISLB – section
(C) ISMB – section
(D) ISHB – section

2. Match Group – A with Group – B and select the correct answers

	Group – A		Group – B
P.	Used when two plates are placed one below the other.	1.	Fillet weld
Q.	Pressure applied continuously	2.	Plug weld
R.	Member subjected to direct axial loads	3.	Slot weld
S.	Joining two surfaces in two different planes.	4.	Seam weld
		5.	Butt weld

Codes:

	P	Q	R	S
(A)	4	5	3	2
(B)	4	3	2	1
(C)	5	4	1	2
(D)	3	4	5	3

3. For economical spacing of roof truss, if t, p, r are the cost of truss, purlin and roof coverings respectively, then
(A) $t = p + r$
(B) $t = 2p + r$
(C) $t = p + 3r$
(D) $t = p + 2r$

4. Gantry girders are usually designed
(A) formultistorey buildings
(B) using channel sections only
(C) as laterally supported beams
(D) as laterally unsupported beams

5. Bearing stiffners in a plate girder is used wherever there is concentrated load to
(A) increase shear resistance
(B) prevent excessive deflection
(C) prevent buckling of web
(D) to transfer the load from compression flange to the tension flange

6. Which of the bolted connections have maximum efficiency?
(A) Zig – Zag (B) Diamond
(C) Chain (D) Both A and B

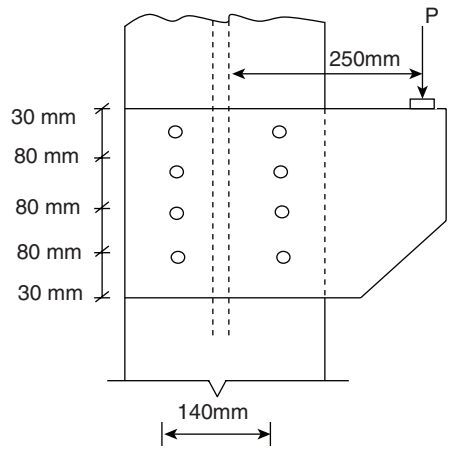
7. _____ beams are used for large spans and light loads
(A) ISLB
(B) Tubular beams
(C) Castellated beams
(D) ISWB

8. A steel c/s has less capacity to resist torsion, when
(A) Shear center is above center of gravity
(B) Shear center is below center of gravity
(C) Shear center coincides with Center of gravity
(D) Not related with their locations

9. The design normal strength of a fillet weld is
(A) f_u (B) $\frac{f_y}{\sqrt{3}}$
(C) f_y (D) $\frac{f_u}{\sqrt{3}\,\gamma_{mw}}$

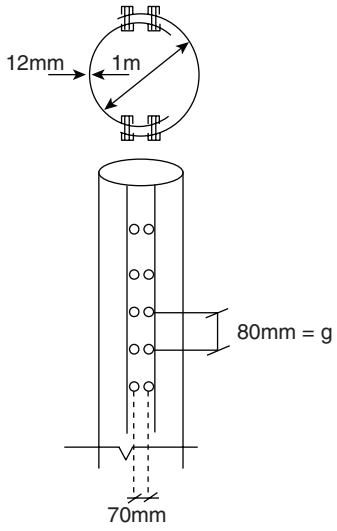
10. Maximum pitch for a tension member whose thickness is "t" is
(A) 12t or 200 mm
(B) 16t or 200 mm
(C) Least of a and b
(D) Only 200 mm

11. Determine the safe load P that can be carried by the joint shown in figure below. The bolts used are 20 mm diameter of grade 4.6. The thickness of flange of I-section is 9 mm and that of bracket plate is 10 mm



(A) 93.68 kN
(B) 89.49 kN
(C) 65.68 kN
(D) 72.92 kN

12. Boiler plates of t = 15 mm thickness are lap jointed with bolts 18 Φ of 4.6 Grade. If the diameter of boiler is 1m determine maximum pressure that can be allowed in boiler.



(A) 0.12 N/mm^2
(B) 0.26 N/mm^2
(C) 0.38 N/mm^2
(D) 0.45 N/mm^2

13. Match List – I of types of sections of beams with List – II the cases for which beams are designed.

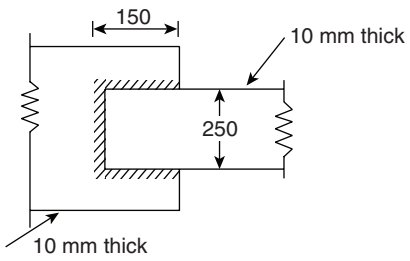
	List – I		List – II
P.	Plastic section	1.	Elastic design
Q.	Compact section	2.	Indeterminate frames
R.	Semi-compact section	3.	Plate Girdes
S.	Slender section	4.	Simply supported beams

Codes:

	P	Q	R	S
(A)	2	4	1	3
(B)	4	2	3	1
(C)	3	1	2	4
(D)	1	3	4	2

14. Statement I: In long columns the axial buckling stress remains below the proportional limit and they buckle elastically.
Statement II: The failure loads for such columns are proportional to the flexural rigidity (EI) of the column and independent of the strength of steel.
(A) Both I and II are true and II is correct explanation of I
(B) Both I and II are true and II is not correct explanation of I
(C) I is true and II is false
(D) Both I and II are false

15. Determine the block shear strength of the welded tension member shown in figure. Plates are of Fe 410 grade
P.S.F for yielding = 1.1
P.S.F for ultimate stress = 1.25

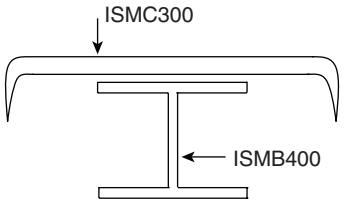


(A) 1543 kN (B) 1728 kN
(C) 1648 kN (D) 1483 kN

16. Determine the flenural design strength of the following welded members. The girders are simply supported and have continuous lateral support. Consider that only flanges resist BM.
Flanges: 250 × 12mm
Web: 1200 × 8mm
Span: 12m
(A) 1652.72 kN (B) 826.36 kN
(C) 727.19 kN (D) 1454.38 kN

17. A built up section is composed of an I section ISMB 400 and C section ISMC 300 connected on top of I section as shown in figure. The minimum radius of gyration of built up section in cm is ____

ISMB 400	ISMC 300
I_{zz} = 20458.4 cm^4	I_{zz} = 6362.6 cm^4
I_{yy} = 422.1 cm^4	I_{yy} = 310.8 cm^4
A = 78.46 cm^2	C_{yy} = 2.36 cm
	t_w = 7.6 cm
	A = 45.64 cm^2



(A) 10.6 cm (B) 13.5 cm
(C) 12.1 cm (D) 14.7 cm

18. Match List – I with List – II and select the correct answer using the codes given below the lists:

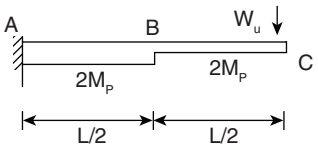
	List – I (Methods of Analysis)		List – II (Conditions satisfied)
a.	Exact Plastic Analysis	1.	Equilibrium, sufficient plastic hindges and non-violation of plastic moment capacity.
b.	Mechanism method of plastic analysis	2.	Equilibrium and non-violation of plastic moment capacity

c.	Equilibrium method of plastic analysis	3.	Equilibrium, continuity and non-violation of plastic moment capacity
		4.	Equilibrium and sufficient plastic hindges.

Codes:

	a	b	c
(A)	3	1	2
(B)	1	2	4
(C)	3	4	2
(D)	1	4	2

19. Find out the collapse load for the following cantilever beam in figure below



(A) 0.75 Mp/L (B) Mp/L
(C) 2 Mp/L (D) 1.5 Mp/L

20. Two framing angles ISA 150 × 150 × 10 mm are used to make beam to column connection. One angle is placed on either side of the web of the beam as in figure. 3 bolts of 16 mm diameter of 4.6 Grades are used to connect the angle legs to the beam web. Determine the reaction that can be transferred through the joint. Given pitch $P = 65$ mm and end distance $e = 40$ mm

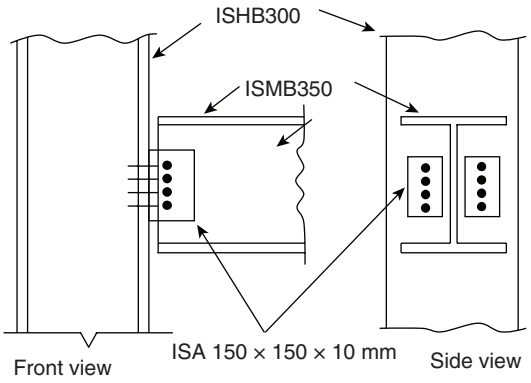
Column ISHB 300 @ 618.03 N/m

$t_f = 10.6$ mm

Beam section ISMB 350 @ 514.04 N/m

$t_w = 8.1$ mm

Grade of steel is Fe 410

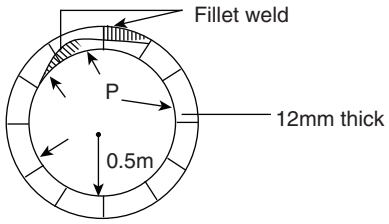


(A) 198.36 kN (B) 174.14 kN
(C) 200.68 kN (D) 183.42 kN

21. A circular plate, 200 mm in diameter is welded to another plate by means of 6 mm fillet weld. Calculate the ultimate twisting moment capacity that can be resisted by the weld use steel grade Fe 410 and shop welding

(A) 49.97 kNm (B) 36.31 kNm
(C) 57.68 kNm (D) 61.31 kNm

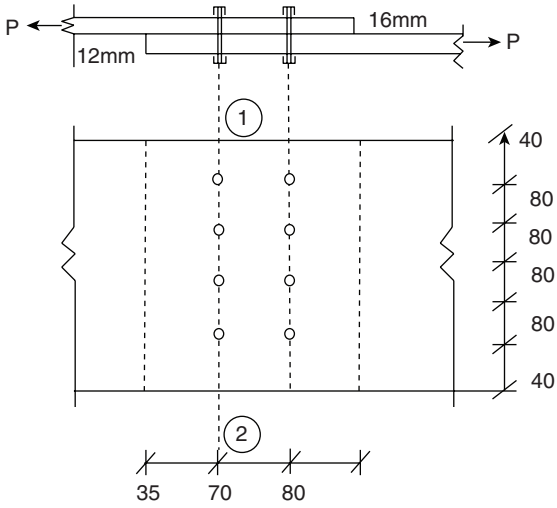
22. A circular penstock of mild steel of grade Fe 410, 1.0 m diameter is fabricated in works shop with 12 mm thick plates. The plates are secured by 8 mm size fillet weld provided on inside and outside of lapped ends as shown in figure. Determine safe internal pressure than can be allowed in the penstock. (in N/mm^2)



(A) 3.55 (B) 7.01
(C) 4.24 (D) 2.36

Common Data for Questions 23 and 24:

23. Find the design strength of Lap joint between 2 plates shown in figure Bolts 20 Φ, 4.6 Grade plates *E* 250 Fe(410) are used.



(A) 271 kN (B) 362 kN
(C) 308 kN (D) 420 kN

24. Find the efficiency for the bolted connection in problem no 23.

(A) 82% (B) 50%
(C) 32% (D) 41%

25. Match List – I (of different types of structural beams) List – II (functions of the beams)

	List–I		**List–II**
a.	Girder	1.	Provided in buildings to support roofs.
b.	Purlins	2.	These carry roof loads in trusses
c.	Joists	3.	Supports a number of joists
d.	Spandrels	4.	Carry part of floor that of the exterior wall.

Codes:

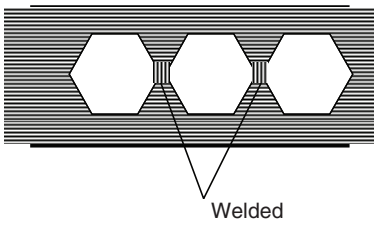
	a	b	c	d		a	b	c	d
(A)	1	2	3	4	(B)	2	1	4	3
(C)	3	2	1	4	(D)	2	3	4	1

Answer Keys

1. C	**2.** D	**3.** B	**4.** D	**5.** C	**6.** B	**7.** C	**8.** C	**9.** D	**10.** B
11. D	**12.** B	**13.** A	**14.** A	**15.** C	**16.** B	**17.** D	**18.** D	**19.** C	**20.** A
21. A	**22.** C	**23.** B	**24.** D	**25.** C					

Hints and Explanations

7. Castellated Beams



Choice (C)

11. For Fe 410 Grade of steel $f_u = 410$ MPa
For 4.6 Grade bolts $f_{ub} = 400$ MPa
Partial safety factor for the material of bolt $\gamma_{mb} = 1.25$

Net Area $A_{nb} = 0.78 \times \frac{\pi}{4} \times 20^2 = 245 \text{ mm}^2$

Diameter $d = 20$ mm $\Rightarrow d_o = 22$ mm
Pitch $P = 80$ mm
Edge distance $(e) = 30$ mm
Strength of bolt in single shear

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3}} \times \left[\frac{A_{nb}}{\gamma_{mb}}\right] = \frac{400}{\sqrt{3}} \times \frac{245}{1.25} \times 10^{-3}$$

$= 45.26$ kN

Strength of bolt in bearing $V_{dbb} = 2.5\, K_b dt \frac{f_u}{\gamma_{mb}}$

$$K_b = \frac{e}{3d_o} = \frac{30}{3 \times 22} = 0.454$$

$$= \frac{P}{3d_o} - 0.25 = \frac{80}{3 \times 22} - 0.25 = 0.96.$$

$$= \frac{f_{ub}}{f_u} = \frac{400}{410} = 0.975 = 1$$

$\therefore \quad K_b = 0.454$

$$V_{dpb} = 2.5 \times 0.454 \times 20 \times 9 \times \frac{410}{1.25} \times 10^{-3} = 67 \text{ kN}$$

$\therefore$ Strength of bolt $V_{sd} = 45.26$ kN
No. of bolts $n = 8$

Direct shear force in each bolt $F_1 = \frac{P_1}{h} = \frac{P_1}{8}$

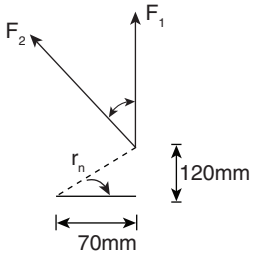
Force due to torque in bolt $F_2 = \frac{P.e_o\, r_n}{\Sigma r^2}$

$$r_n = \sqrt{(40+80)^2 + 70^2} = 138.92 \text{mm}$$

$\Sigma r^2 = 4\,[(40 + 80)^2 + 70^2 + 40^2 + 70^2] = 103200$ mm

$$F_2 = \frac{P_1 \times 250 \times 138.92}{103200} = 0.3365 P_1$$

$$\cos\theta = \frac{70}{\sqrt{120^2 + 70^2}} = 0.5038$$



Resulted force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

$$= \sqrt{\left(\frac{P_1}{8}\right)^2 + (0.3365P_1)^2 + \left(2 \times \frac{P_1}{8} \times 0.3365 P_1 \times 0.5038\right)}$$

$R = 0.4138\, P_1$
Resulted force $\le$ Strength of bolt
$0.4138 P_1 \le 45.26$
$P_1 \le 109.38$ kN
Service load

$$P = \frac{P_1}{\text{load factor}} = \frac{109.38}{1.5}$$

$= 72.92$ kN Choice (D)

12. $d = 18$mm
$d_o = 20$mm
$A_{sb} = 254\text{mm}^2$
$A_{nb} = 198\text{mm}^2$
Considering width of plate $g = 80$mm
Strength of Bolt:

Shear $V_{dsb} = \frac{1}{1.25}\left[\frac{400}{\sqrt{3}}(198)\right] = 36.58$ kN

2 × Bolt value = 73.16 kN

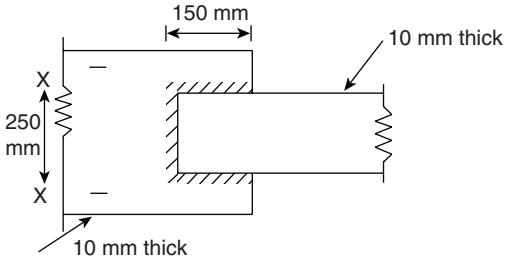
Stress in the plate/area $= \frac{73.16 \times 10^3}{70 \times 80} = 13.06 \text{ N/mm}^2$

Hoop stress $= \frac{Pd}{2t} = \frac{P \times 1.5 \times 10^3}{2 \times 15}$

$$13.06 = \frac{P \times 1.5 \times 10^3}{2 \times 15}$$

$P = 0.26 \text{ N/mm}^2$ Choice (B)

15.



$A_{vg} = A_{vn} = 2(150 \times 10) = 3000 \text{ mm}^2$

$A_{tg} = A_{tn} = 2(250 \times 10) = 5000 \text{ mm}^2$

$$T_{db1} = \frac{A_{vg} f_y}{\sqrt{3}\gamma_{mo}} + \frac{0.9 A_{tn} f_u}{\gamma_{ml}}$$

$$= \left[\frac{3000 \times 250}{\sqrt{3} \times 1.1} + \frac{0.9 \times 5000 \times 410}{1.25}\right] \times 10^{-3}$$

$= 1869.65$ kN

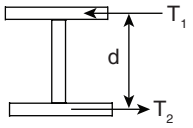
$$T_{db2} = \frac{0.9 A_{vn} f_u}{\sqrt{3}\gamma_{ml}} + \frac{A_{tg} f_y}{\gamma_{mo}}$$

$$= \left[\frac{0.9 \times 3000 \times 410}{\sqrt{3} \times 1.25} + \frac{5000 \times 250}{1.1}\right] \times 10^{-3}$$

$= 1647.66$ kN

$\therefore$ Block shear strength = 1648 kN Choice (C)

16. $M_d = A_f \times \dfrac{f_y}{\gamma_{mo}} \times d$



$$M_d = (250 \times 12) \times \frac{250}{1.1} \times (1212)$$

$= 826.36 \times 10^6$ Nmm

= 826.36 kNm Choice (B)

17. $I_{zz} = I_{zz(I)} + I_{zz(C)}$

$= 26821 \text{cm}^4$

$$\text{Centroid} = \frac{A_1 C_{yy} + A_2 (7.6 + 20)}{A_1 + A_2}$$

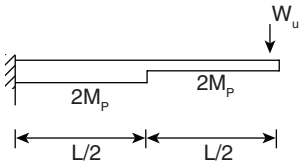
$$= \frac{(45.64 \times 2.36) + (78.46 \times (7.6 + 20))}{45.64 + 78.46}$$

$= 18.32$ cm

$I_{yy} = [422.1 + 78.46 \times (18.32 - 20)^2 + 310.8 + 45.64 \times (18.32 - 2.36)^2] = 12579 \text{ cm}^4$

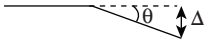
$$r_{\min} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{12579}{(45.64 + 78.46)}} = 14.7\text{cm}$$ Choice (D)

19.



There is a change of forming one plastic hindge

Case (i): Plastic hindge at the center where c/s changes



Internal work done

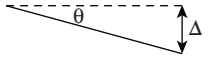
$$= W_u \times \Delta = W_u \times \frac{L}{2} \theta$$

External work done = $M_p \theta$

$$M_p \theta = W_u \times \frac{L}{2} \theta$$

Collapse Load $W_u = \dfrac{2M_p}{L}$

Case (ii): Plastic hindge at the fixed end



Internal work done = $W_u \Delta = W_u \times L \theta$

External work done = $2 M_p \theta$

$W_u L \theta = 2 M_p \theta$

$W_u = \dfrac{2M_p}{L}$ Choice (C)

20. For *Fe* 410 grade of steel $f_u = 410$ MPa

For 4.6 Grade bolts $f_{ub} = 400$ MPa

$d = 16$mm $d_o = 18$mm

$\gamma_{mb} = \gamma_{ml} = 1.25$

Bolts connecting both angle legs to web of beam are in double shear,

$$V_{dsb} = \frac{f_{ub}}{\gamma_{mb}} \left(\frac{n_n A_{nb} + n_s A_{sb}}{\sqrt{3}}\right)$$

$$A_{sb} = \frac{\Pi}{4} \times 16^2 = 201.06 \text{ mm}^2$$

$A_{nb} = 0.78 \times A_{sb} = 156.83 \text{ mm}^2$

$$V_{dsb} = \frac{400}{1.25 \times \sqrt{3}} (201.06 + 156.83) \times 10^{-3} = 66.12 \text{ kN}$$

Strength of bolt in bearing,

$$V_{dpb} = 2.5 K_b \frac{dt f_u}{\gamma_{mb}}$$

$$K_b = \frac{e}{3d_o} = \frac{40}{3 \times 18} = 0.74$$

$$= \frac{P}{3d_o} - 0.25 = \frac{65}{3 \times 18} - 0.25 = 0.95$$

$$= \frac{f_{ub}}{f_u} = \frac{400}{410} = 0.975 = 1$$

$\therefore \quad K_b = 0.74$

$$V_{dpb} = 2.5 \times 0.74 \times 16 \times 8.1 \times \frac{410}{1.25} \times 10^{-3}$$

$$= 78.64 \text{ kN}$$

$\therefore$ Strength of one bolt = 66.12 kN

As 3 bolts are provided, maximum end reaction that can be transferred

$= 3 \times 66.12 = 198.36$ kN Choice (A)

21. Diameter $D = 200$ mm

Fe 410 Grade $\therefore f_u = 410$ MPa

Shop weld: $\gamma_{mw} = 1.25$

Ultimate twisting moment capacity

$T_u = P_{dw} \times r$

$$= L_w \times t_t \times \frac{f_u}{\sqrt{3} \times \gamma_{mw}} \times \frac{200}{2}$$

$$= (\pi \times 200) \times (0.7 \times 6) \times \frac{410}{\sqrt{3} \times 1.25} \times \frac{200}{2}$$

$= 49.97 \times 10^6$ Nmm = 49.97 kNm Choice (A)

22. Fe 410 $f_u = 410$ MPa

Size of weld $S = 8$ mm

Throat thickness = $KS = 0.7 \times 8 = 5.6$ mm

$\gamma_{mw} = 1.25$

Let internal pressure = p

Hoop force/mm length $F_h = \frac{pd}{2}$

Design strength of fillet weld/mm length

$$P_w = t_t \times \frac{f_u}{\sqrt{3} \times r_{mw}} \times 2 = 5.6 \times \frac{410}{\sqrt{3} \times 1.25} \times 2$$

$P_w = 2120.95$ N

For safe conditions

Hoop force/mm length = Strength of weld/mm length

$$2120.95 = \frac{pd}{2} = \frac{p \times 1000}{2}$$

$p = 4.24$ N/mm^2 Choice (C)

23.

Bolt	Plate
d = 20 mm	$t_1 = 12$ mm
$d_o = 22$ mm	$t_2 = 16$ mm
$A_{sb} = \frac{\pi}{4} \times 20^2 = 314$ mm^2	$f_u = 410$ MPa
$A_{nb} = 0.78 A_{sb} = 245$ mm^2	$f_y = 250$ MPa
$f_{ub} = 400$ MPa	$\gamma_{mb} = 1.25$
$f_{yb} = 0.6 \times 400 = 240$MPa	

Single shear $n_n = 1$ $n_s = 0$

Shear strength

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3}} \frac{[\text{nn Anb} + \text{ns Asb}]}{\gamma_{mb}}$$

$$= \frac{400}{\sqrt{3} \times 1.25} [245 \times 1] = 45.27 \text{ KN}$$

Bearing strength

$$V_{dpb} = \frac{1}{\gamma_{mb}} [2.5 K_b dt f_u]$$

$$K_b = \frac{e}{3d_o} = \frac{35}{3 \times 22} = 0.53$$

$$= \frac{p}{3d_o} - 0.25 = \frac{70}{3 \times 22} - 0.25 = 0.81$$

$$= \frac{f_{ub}}{f_u} = \frac{400}{410} = 0.975 = 1$$

$K_b = 0.53$

$$V_{dpb} = \frac{1}{1.25} [2.5 \times 0.53 \times 20 \times 12 \times 410]$$

$= 104.3$ kN

Plate yield of gross c/s:

$$T_{dg} = \frac{A_g f_y}{\gamma_{mo}} = \frac{(320 \times 12) \times 250}{1.1}$$

$= 872.7$ kN

Tearing/Rupture:

$$T_{dh} = \frac{0.9 A_n f_u}{\gamma_{ml}}$$

$A_n = (320 - 4 \times 22) \times 12 = 2784$ mm^2

$$T_{dh} = \frac{0.9 \times 2784 \times 410}{1.25} = 821.8 \text{ kN}$$

Joint Strength:

Strength of bolt in shear = $45.2 \times 8 = 362$ kN

Strength of bolt in bearing = $104 \times 8 = 832$.kN

Yielding = 872 kN

Rupture = 821 kN

Strength of Joint = 362 kN Choice (B)

24. Efficiency $\eta = \frac{\text{strength of Joint}}{\text{yield strength}} \times 100$

$$= \frac{362}{872} \times 100 = 41\%$$ Choice (D)

Steel Structures Test 2

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. Minimum edge distance of bolted joint for hand flame cut edges is not less than
(A) 1.5 × diameter of bolt hole
(B) 1.5 × diameter of bolt
(C) 1.7 × diameter of bolt hole
(D) 1.7 × diameter of bolt

2. The type of weld used for joining two surfaces in two different planes is
(A) Fillet weld (B) single *V* butt weld
(C) double groove weld (D) None

3. If Bolts in any Bolt group is subjected to shear and tension; the interaction equation need to be satisfy as per IS 800 : 2007 is [V_b, V_{db} : Factored shear force and design shear strength and T_b, T_{db} : Factored tensile force and design tensile strength]
(A) $\left[\frac{V_B}{V_{db}}\right]+\left[\frac{T_b}{T_{db}}\right]\le 2.0$
(B) $\left[\frac{V_b}{V_{db}}\right]^2+\left[\frac{T_b}{T_{dc}}\right]^2\le 2.0$
(C) $\left[\frac{V_b}{V_{db}}\right]^2+\left[\frac{T_b}{T_{db}}\right]^2\le 1.0$
(D) None

4. The maximum length of a tension member with minimum radius of gyration of 20 mm carrying load reversals other than wind or earthquake forces as per IS 800 is _____?
(A) 5.0 m (B) 1.5 m
(C) 3.6 m (D) 6.0 m

5. Which of the following is the most efficient section for column for a given equal cross section area?
(A) solid circular section
(B) Angle section
(C) I-section
(D) tubular section

6. When the column is effectively held in position and restrained against rotation at both ends, the effective length of column is '*K*' times the unsupported length (*L*) of column where *K* is _____
(A) 1.2 (B) 0.8
(C) 0.65 (D) 1.0

7. A beam section is selected and provided on the basis of
(A) shear (B) deflection
(C) section modulus (D) All the above

8. While designing, for a steel column of Fe250 grade, a base plate resting on a concrete pedestal of M30 grade, the bearing strength of concrete (in N/mm^2) in limit state method of design as per IS 456 : 2000 is
(A) 3.84 (B) 11
(C) 13.5 (D) 15

9. In a roof truss if pitch is $\frac{\sqrt{3}}{2}$ and slope is $\sqrt{3}$, the angle of inclination with horizontal would be
(A) 60° (B) 30°
(C) 45° (D) 90°

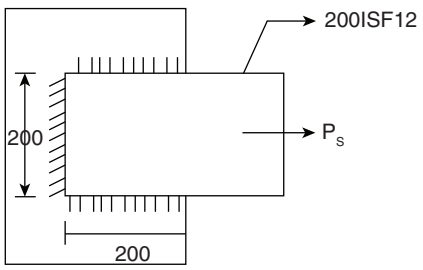
10. In an industrial building gantry girder of effective span 25.0 m carries a manually operated crane of 350 kN. The maximum deflection as per IS 800 – 2007 is
(A) 50 mm (B) 30 mm
(C) 25 mm (D) 75 mm

11. Two plates 12 mm and 20 mm thick are to be joined by a double cover butt joint with 8 mm thick packing plate. What will be the effect of packing on the design shear strength of bolt?
(A) decreases by 10% (B) increases by 10%
(C) decreases by 15% (D) increases by 15%

12. Calculate the design tensile capacity of M20 bolt of grade 4.6 is
(A) 50 kN (B) 68 kN
(C) 75 kN (D) 35 kN

13. Calculate the number of bolts required for a lap joint between two plates of 12 mm and 24 mm thick to transmit a factored load of 100 kN using M16 bolts of grade 4.6 and Fe410 plates. (assume minimum end distance = 27 mm; minimum pitch = 40 mm and thread intercept shear plane)
(A) 2 (B) 6
(C) 4 (D) 8

14. Determined the service load which can be applied to the fillet weld for the figure shown with a weld size of 5 mm. Use plates of grade 410 steel and workshop welding.

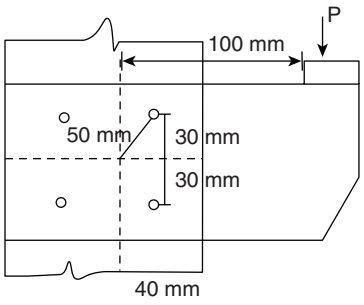


(A) 450 kN (B) 500 kN
(C) 350 kN (D) 265 kN

15. When length of side fillet weld is 400 times the effective throat thickness. Then the design shear capacity of fillet weld is
(A) decreased by 33%
(B) increased by 33%
(C) decreased by 20%
(D) decreased by 66%

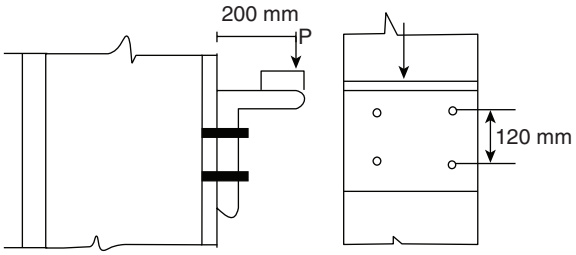
16. Two 12 mm thick plates are joined in the field by a single '*V*' bolt weld. The effective length of weld is 250 mm. Determine the design strength of welded joint. The yield and ultimate tensile strength of weld and steel are 250 Mpa and 410 Mpa respectively.
(A) 330.50 kN
(B) 437.50 kN
(C) 530.50 kN
(D) 357. 50 kN

17. Four the bolt systems shown in figure, the bolts are at a distance of 50 mm from Center of gravity of bolt group. The resultant force in the critical bolt is



(A) $P/4$ (B) $0.50P$
(C) $0.70P$ (D) $1.0P$

18. Each bolt shown in figure is capable of resisting design shear capacity of 25 kN and design tension capacity of 20 kN. The interaction equation between of forces as per limit state method of IS 800 : 2007?



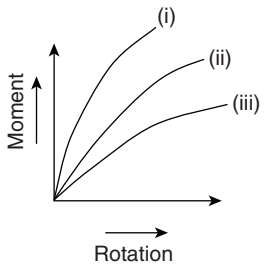
(A) $\left[\frac{P}{100}\right]+\left[\frac{P}{24}\right]\le 10$

(B) $\left[\frac{P}{50}\right]^2+\left[\frac{P}{24}\right]^2\le 1.0$

(C) $\left[\frac{P}{50}\right]^2+\left[\frac{P}{30}\right]^3\le 1.0$

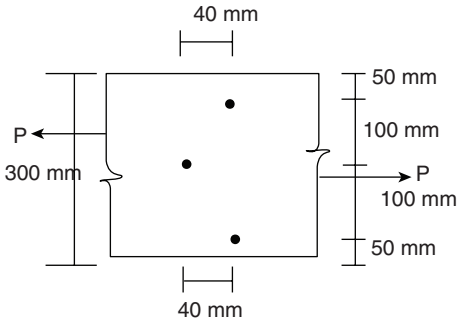
(D) $\left[\frac{P}{100}\right]^2+\left[\frac{p}{24}\right]^2\le 1.0$

19. The moment rotation curve shown in figure is that of a



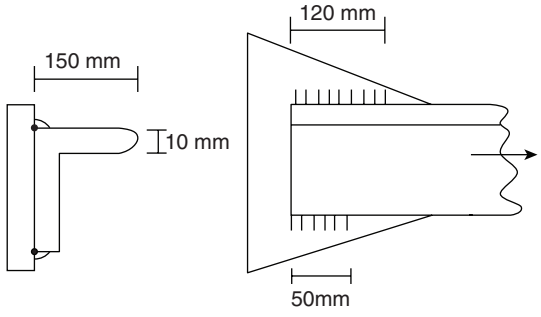
(A) (i) Semi Rigid Joint
(ii) Rigid.
(iii) Flexible
(B) (i) Rigid Joint
(ii) Semi Rigid.
(iii) Flexible
(C) (i) Flexible Joint
(ii) Semi Rigid.
(iii) Rigid
(D) None

20. What is the net effective sectional area of plate of thickness 12 mm as shown in the figure for carrying tension?
[take dh : 18 mm]



(A) 2500 mm^2 (B) 3048 mm^2
(C) 2750 mm^2 (D) 3500 mm^2

21. Compute the tensile strength of an angle section ISA 150 × 150 × 10 mm of Fe 410 grade of steel connected with guest plate as shown in figure based on gross sectional yielding.



(A) 450 kN (B) 350 kN
(C) 660 kN (D) none

22. The best tension member section will be a
(A) bolted single angle section
(B) welded single angle section
(C) channel section
(D) double angle section on opposite side of gusset plate

23. Determine service axial load on column section ISMB 350. Given that the height of column is 3.2 m and that is fixed on both ends. Assume f_y : 250 Mpa, f_u 410 Mpa and $E = 2 \times 10^5$ Mpa (properties of ISMB 350 are $A = 6670$ mm^2, $t_f = 14.2$ mm, $t_w = 8.1$ mm, $b = 140$ mm, $h = 350$ mm, $r_{zz} = 143$ mm and $r_{yy} = 28.4$ mm
(A) 720 KN (B) 850 KN
(C) 350 KN (D) None

24. A built up column consists of ISMC 300 channels placed back to back at a spacing of 250 mm and carries working load of 2000 kN, the double lacing provided with an angle of 50° with longitudinal axis. As per IS 800 – 2007 lacing member should be designed to resist design axial load of
(A) 19.5 kN (B) 24.5 kN
(C) 30.8 kN (D) 54.2 kN

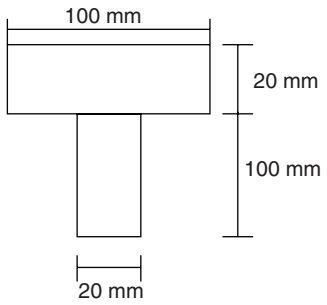
25. In laced columns, end tie plates are provided to
(A) check the buckling of column as a whole
(B) check the buckling of component column
(C) check the distortion of the column sections at ends because of unbalanced horizontal force from lacings
(D) keep the column components in position

26. A steel beam of circular c/s is clamped at both ends. Deformation is just observed when the U.D.L on the beam is 20 kN/m. At the instant of collapse, the load on the beam will be
(A) 15 kN/m (B) 30 kN/m
(C) 20 kN/m (D) 45 kN/m

27. For an rectangular beam the shape factor is 1.5. The factor of safety in bending is 1.5. If the allowable stress is increased by 15% for wind and earth quake loads, then the load factor is
(A) 1.95 (B) 1.40
(C) 1.65 (D) 1.80

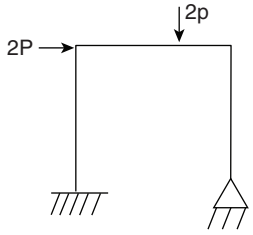
28. A propped cantilever beam AB of length 'L' fixed at 'A' and propped at B is subjected to a concentrated load 'w' at its center. By kinematic approach calculate the ultimate collapse load (w) in terms of Mp.
(A) $w = \frac{2Mp}{L}$ (B) $w = \frac{6Mp}{L}$
(C) $w = \frac{4Mp}{L}$ (D) $w = \frac{8Mp}{L}$

29. The distance of plastic neutral axis from top of T-section shown below:



(A) 15 mm (B) 60 mm
(C) 20 mm (D) 40 mm

30. The number of possible independent mechanisms for a portal frame shown in fig. is



(A) 1 (B) 3
(C) 4 (D) 2

Answer Keys

1. C	**2.** A	**3.** C	**4.** C	**5.** D	**6.** C	**7.** D	**8.** C	**9.** A	**10.** A
11. A	**12.** B	**13.** C	**14.** D	**15.** A	**16.** B	**17.** C	**18.** D	**19.** B	**20.** B
21. C	**22.** D	**23.** A	**24.** B	**25.** C	**26.** D	**27.** A	**28.** B	**29.** C	**30.** D

Hints and Explanations

1. For hand flame cut edges miniumum edge distance = 1.7 × dia. of bolt hole. Choice (C)

2. For joining two surfaces in different planes, Fillet weld is used. Choice (A)

3. Interaction equation need to be satisfy as per IS : 8OO is $\left[\frac{V_b}{v_{db}}\right]^2 + \left[\frac{T_b}{T_{db}}\right]^2 \le 1.0$ Choice (C)

4. $\frac{\ell}{r} \le 180$

$\ell = (180)\ (20)$

$\ell = 3600\text{mm}$

$\ell = 3.6\text{m}$ Choice (C)

5. For a given cross section area, tubular section is more efficient due to more (r) min. Choice (D)

6. Given end conditions are translations and rotations restrained at both ends. (i.e., fixed at both ends)
Effective length constant $K = 0.65$ Choice (C)

7. A beam section is selected based on all of the above. Choice (D)

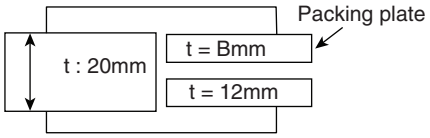
8. Maximum allowable bearing strength = 0.45×30
$= 13.5\ \text{N/mm}^2$ Choice (C)

9. Slope of roof truss;
$\tan\theta = \sqrt{3}$ where θ : Angle of inclination with horizontal
$\theta = \tan^{-1}(\sqrt{3}) = 60°$ Choice (A)

10. Limiting deflection $= \frac{\text{Span}}{500}$

$= \frac{25000}{500} = 50\text{mm}$ Choice (A)

11.



Reduction factor (β_{pkg}) : [$t_{pkg} > 6\text{mm}$]

$B_{pkg} = 1 - 0.0125\ t_{pkg}$

$= 1 - 0.0125\ (8) = 0.90$

$\therefore$ Design shear strength of bolt decrease by 10% Choice (A)

12. Design tensile capacity of bolt = T_{db}

$$T_{db} = \frac{0.9 f_{ub}}{\gamma_{mb}} \times A_{nb} \le \frac{f_{yb}}{\gamma_{mo}} A_{sb}$$

$$= \frac{0.9 \times 400}{1.25} \times 0.78 \times \frac{\pi}{4}(20)^2 \le \frac{240}{1.10} \times \frac{\pi}{4}(20)^2$$

$= 70.572\ \text{kN} \le 68.543$

T_{db} : 68.543 kN Choice (B)

13. Number of bolts $n = \frac{P}{V_{db}}$

V_{db} is least of V_{dsb}, V_{dpb}

$V_{dsb} = \frac{f_{ub}}{\sqrt{3}\gamma_{mb}}(n_n A_{nb} + n_s A_{sb})$ [$n_s = 0$ since thread intercept shearplane]

$$= \frac{400}{\sqrt{3} \times 1.25}[1 \times 0.78 \times \frac{\pi}{4} \times 16^2] = 28.974\ \text{kN}$$

$V_{dpb} = \frac{2.5dt\ K_b f_u}{\gamma_{mb}}$

K_b is least of $\frac{e}{3dh}; \frac{P}{3dh} - 0.25; \frac{f_{ub}}{f_u}; 1.0$

$\frac{e}{3dh} = \frac{27}{3 \times 18} = 0.50$

$\frac{P}{3dh} - 0.25 - \frac{40}{3 \times 18} - 0.25 = 0.490$

$\frac{f_{ub}}{f_u} = \frac{400}{410} = 0.97$

$K_b = 0.490$

Take least among these

$$V_{dpb} = \frac{2.5 \times 16 \times 12 \times 0.490 \times 400}{1.25} = 75.264\ \text{kN}$$

$\therefore\ V_{db} = 28.974\ \text{kN}$

$n = \frac{P}{V_{db}} = \frac{100}{28.97} = 3.45 \sim 4$ Choice (C)

14. Given $S = 5\text{mm}$
Design requirement:
Design action ≤ design strength
$\therefore\ P = P_{dw}$
Design shear strength of fillet weld

$$(P_{dw}) = (L_w \times t_t) \times \frac{f_u}{\sqrt{3}\gamma_{mw}}$$

$$= 600 \times 0.70 \times 5 \times \frac{410}{\sqrt{3} \times 1.25} = 397.678\ \text{kN}$$

$\therefore\ P = P_{dw} = 397.678\ \text{kN}$

$P_s = \frac{P}{1.5} = \frac{397.678}{1.5} = 265.11\ \text{kN}$ Choice (D)

15. $L_J = 400t_t$
If $L_J > 150t_t \Rightarrow$ a reduction factor need to be applied for design shear capacity of fillet weld.
Reduction factor (β_{LW}):

$$\beta_{LW} = 1.20 - \frac{0.20 L_J}{150 t_t} = 1.20 - \frac{0.20 \times 400t}{150 t_t}$$

$= 1.20 - 0.533 = 0.66$
So strength reduced by 33% Choice (A)

16. Design strength of single 'v' butt weld = $L_w \times \frac{5}{8} \times T \times \frac{f_y}{\gamma_{mw}}$

$$= 350 \times \frac{5}{8} \times 12 \times \frac{250}{1.50} = 437.500\ \text{kN}$$ Choice (B)

17. Load/moment is lying in plate of bolt group.
So bolt group is subjected to Direct concentric load (P) and Inplane moment/twisting moment ($M = Pe$)
Vertical shear force in any bolt due to direct concentric load is F_a

$$F_a = \frac{P}{n} = \frac{P}{4}$$

Shear force in any bolt due to twisting moment ($M = Pe$) is

$$F_m = \frac{p.e.r}{\Sigma r^2} = \frac{p.(100)(50)}{4(50)^2}$$

$$F_m = 0.5\,p \qquad \cos\theta = \frac{40}{50} = \frac{4}{5}$$

$(F_R)_{max}$: maximum Resultant force

$$= \sqrt{\left(\frac{P}{4}\right)^2 + \left(\frac{P}{2}\right)^2 + 2\left(\frac{P}{4}\right)\left(\frac{P}{2}\right)\frac{4}{5}}$$

$= 0.71\,P$ Choice (C)

18. Load/moment is not lying in plane of bolt group.
So bolt group is subjected to direct Concentric load (P) and bending moment (M)
Vertical shear force in any bolt due to P is

$$q_1 = \frac{P}{n} = \frac{P}{4} = V_b$$

Stress in any bolt due to M is q_2

$$q_2 = \frac{M^1 y_n}{\Sigma y^2}$$

assume CG of bolt coincides with bending axis

$$M_1 = \frac{M}{2} = \frac{p.e}{2}$$

$$T_b = q_2 = \frac{p(200)(60)}{2\left(2(60)^2\right)} \qquad T_b = \frac{5P}{6}$$

$\therefore$ Interaction equation is

$$\left[\frac{V_b}{V_{db}}\right]^2 + \left[\frac{T_b}{T_{db}}\right]^2 \le 1.0$$

$$\left[\frac{P}{4}\times\frac{1}{25}\right] + \left[\frac{5P}{6}\times\frac{1}{20}\right]^2 \le 1.0$$

$$\left[\frac{P}{100}\right]^2 + \left[\frac{P}{24}\right]^2 \le 1.0$$ Choice (D)

20. $A_{net} = (B - n.dh)t + \frac{P_1^2}{4g_1}t_1 + \frac{P_2^2}{4g_2}t_2$

$$= [300 - 3(18)]12 + \frac{40^2\times12}{4\times100} + \frac{40^2\times12}{4\times100}$$

$= 3048\text{mm}^2$ Choice (B)

21. $T_{dg} = \frac{f_y}{r_{mo}} \times A_g$

$A_g = 150 \times 10 + (150 - 10) \times 10 = 2900\ \text{mm}^2$

$$T_{dg} = \frac{250}{1.10} \times 2900 = 659.090\ \text{kN}$$ Choice (C)

23. For $\frac{h}{b_f} = \frac{350}{140} = 2.5 > 12$ and $t_f < 40\text{mm}$

Buckling class about z-z axis – a
Buckling class about y-y axis is – b
$\alpha = 0.21$ (Buckling class a)
$= 0.34$ (Buckling class b)
Effective slenderness ratio $\frac{KL}{r_{min}}$

$K = 0.65$ for ends fixed
Effective slenderness ratio

$$= \frac{0.65L}{r_{min}} = \frac{0.65\times3.2}{28.4} = 73.23\text{mm}$$

Non dimensional effective slenderness ratio = λ

$$\lambda = \sqrt{\frac{f_y\left(\frac{KL}{r}\right)^2}{\pi^2 E}} = \sqrt{\frac{250(73.25)^2}{\pi^2\times2\times10^2}}$$

$\lambda = 0.82$
$\phi = 0.5\,(1 + \alpha\,(\lambda - 0.2) + \lambda^2)$
$= 0.5\,(1 + 0.34\,(0.82 - 0.2) + 0.82^2)$
$= 0.9416$
Design strength of axial compression

$$f_{c_d} = \frac{\frac{f_y}{r_{mo}}}{\phi + \left(\phi^2 - \lambda^2\right)^{0.5}} \le \frac{f_y}{r_{mo}}$$

$$= \frac{\frac{250}{1.10}}{0.9416 + \left(0.9416^2 - 0.82^2\right)^{0.5}}$$

$= 161.82\ \text{Mpa} \le 22727\text{Mpa}$
Design axial load on column $P = f_{c_d} \times A_e$

$= 161.82 \times 6670$
$= 1079.3\ \text{kN}$

Service load $= \frac{1079.3}{1.5} = 719.53\ \text{kN}$ Choice (A)

24. Design load on column $P = 1.5 \times 2000 = 3000\ \text{kN}$

Transverse shear $(V) = \frac{25}{100} \times 2000 \times 1.5 = 75\ \text{kN}$

Force in lacing member $(F) = \frac{V}{4\sin\theta}$

$$= \frac{75}{4\times\sin50} = 24.47\ \text{kN}$$ Choice (B)

26. Given deformation just taken place

i.e., $M_y = \frac{wl^2}{12}$

At collapse $M_p = \frac{w_c l^2}{16}$

Shape factor $S = 1.70$

$$1.70 = \frac{w_c l^2 \times 12}{16 \times wl^2} = \frac{w_c \times 12}{16 \times 20}$$

$W_c = 45.33$ kN Choice (D)

27. Load factor = factor of safety × shape factor

$$= \frac{f_y}{f} \times S = \left[\frac{f_y}{1.15f}\right] \times 1.50$$

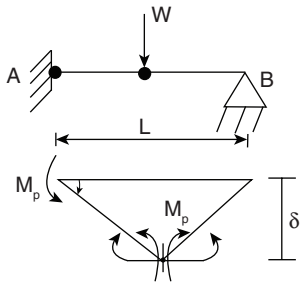
$$= \left[\frac{1}{1.15}\right] \times 1.5 \times 1.5 = 1.95$$ Choice (A)

28. Number of plastic hinges = $D_s + 1 = n$

$D_s = (2 + 1) - 2 = 1$

$n = 1 + 1 = 2$

one plastic hinge is formed at fixed end and other at point load.



$$\delta = \frac{L}{2}\theta$$

From principle of virtual work,

External work done by loads = Internal work done

$$w\frac{L}{2}\theta = M_p\,\theta + M_p\,(\theta + \theta)$$

$$w = \frac{6M_p}{L}$$ Choice (B)

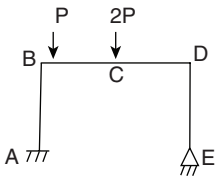
29. Area of flange = $100 \times 20 = 2 \times 10^3$ mm^2

Area of web = $100 \times 20 = 2 \times 10^3$ mm^2

Area of flange and area of web are equal so plastic *NA* lies at junction of flange and web

$\therefore$ distance of *N.A.* from top = 20mm Choice (C)

30.



Number of Independent mechanisms = $N - Ds$

Where N – number of possible location of plastic hinges

D_s – Degree of static Indeterminacy

$D_s = r - 3 = 5 - 3 = 2$

$N = A, B, C, D = 4$

Number of independent mechanisms = $4 - 2 = 2$

Choice (D)

UNIT VII
GEOTECHNICAL ENGINEERING

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Soil Mechanics Test 1

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following is a type of chemical weathering
(A) oxidation (B) wedging
(C) abrasion (D) temperature effect

2. Which of the following is the characteristic of a flocculated clay structure
(A) low shear strength
(B) low permeability
(C) low compressibility
(D) None of these

3. In oven drying method a temperature of 60°C to 80°C is preferred when
(A) high organic soils are present
(B) Gypsum is present
(C) inorganic particles are present
(D) high clay content is present

4. When $C_u > 4$ and C_c is lies between 1 and 3 the soil can be classified as
(A) uniformly graded soil
(B) well graded soil
(C) gap graded soil
(D) Coarse grained soil

5. The notations GP and SM represent
(A) silty gravel and silty sand
(B) clayey gravel and clayey sand
(C) well graded gravel and well graded sand
(D) poor graded gravel and silty sand

6. A soil is said to be highly permeable when
(A) $K > 10^{-1}$ cm/sec (B) $K > 10^{-3}$ cm/sec
(C) $K < 10^{-1}$ cm/sec (D) $K < 10^{-3}$ cm/sec

7. The process of softening of soil due to increase in water content caused by melting of ice formed in soil is
(A) frost heave (B) frost boil
(C) Thawing (D) Capillary fringe

8. Effective stress in soil increased if the flow is
(A) downwards (B) zig-zag
(C) upwards (D) uniform

9. Space between any two adjacent flow lines and adjacent equi potential lines is called
(A) flow net (B) flow line
(C) flow field (D) flow path

10. The chart used to find the vertical stress on westergaard's equation is known as
(A) influence chart
(B) isobar chart
(C) Fenske's chart
(D) None of the above

11. Match the following

	Source of Transportation		Type of Soil
1.	River	i.	colluvial soil
2.	Gravitation	ii.	Aeolian soil
3.	Wind	iii.	Alluvial soil
4.	Lakes	iv.	Lacustrine soil

(A) (1 – iii), (2 – i), (3 – ii), (4 – iv)
(B) (1 – ii), (2 – iii), (3 – iv), (4 – i)
(C) (1 – iv), (2 – iii), (3 – i), (4 – ii)
(D) (1 – i), (2 – iv), (3 – ii), (4 – iii)

12. A sample of soil deposit has a void ratio of 1. If the void is reduced to 0.3 by compaction, the % volume loss is
(A) 58% (B) 56%
(C) 54% (D) 34%

13. The following data is obtained from the liquid limit test conducted on soil sample

No. of blows	20	25	30	35	40
Water content	64.2	63.9	62.5	61.9	61.8

(A) 61.9% (B) 61.8%
(C) 63.9% (D) 64.2%

14. In falling head permeability test on a sample 13.4 cm high and 48.4 cm^2 in cross sectional area, the water level in a stand pipe of 5.25 mm internal diameter dropped from a height diameter dropped from a height of 65 cm to 25 cm in 20 min. The coefficient of permeability ($\times 10^{-4}$ cm/sec) is
(A) 0.58 cm/sec (B) 0.47 cm/sec
(C) 0.53 cm/sec (D) 0.54 cm/sec

15. A glass container with pervious bottom has a sand with void ratio = 0.6. If the specific gravity of sand particles = 2.65, area of c/s = 20 m^2, head of water required to cause quick sand condition is (take L = 10 m)
(A) 10.1 m (B) 11.3 m
(C) 10.8 m (D) 10.3 m

16. In a flow net there are 10 flow channel and 20 equipotential drops, the quantity of seepage if head loss is 4 m and $k = 3 \times 10^{-5}$ m/s is
(A) 24×10^{-5} m^3/s (B) 6×10^{-5} m^3/s
(C) 8×10^{-5} m^3/s (D) 22×10^{-5} m^3/s

Common Data for Questions 17 and 18:
A soil profile consists of a surface layer of sand 4 m thick (γ = 1.8 t/m^3), an intermediate layer of clay 3.8 m thick (γ = 2.3 t/m^3) and the bottom layer of gravel 5 m thick (γ = 1.98 t/m^3). The water table is at upper surface of clay layer (take $_w$ = 0.98 t/m^3)

17. Effective stress at 7.8 m from the surface is
(A) 8.58 t/m^3 (B) 8.64 t/m^3
(C) 12.21 t/m^3 (D) 8.58 t/m^3

18. Effective stress at 12.8 m from the surface is
(A) 14.9 t/m^3 (B) 17.21 t/m^3
(C) 14.8 t/m^3 (D) 15.3 t/m^3

A saturated clay has water content 39.3% and bulk sp-gravity 1.84

19. Specific gravity of soil is
(A) 2.73 (B) 2.78
(C) 2.74 (D) 2.79

20. Void ratio of soil is
(A) 1.05 (B) 1.2
(C) 1.07 (D) 1.8

21. (i) Soil with largest void ratio has less permeability
(ii) Permeability of partially saturated soils is considerably smaller than that of fully saturated soils
(A) (i) is true and (ii) is false
(B) (i) is false and (ii) is true
(C) (i) and (ii) are false
(D) (i) and (ii) are true

22. The Plastic limit and liquid limit of the soil are 33% and 45% respectively. The percentage of clay fraction 30%. The activity of clay is
(A) 0.3 (B) 0.4
(C) 2.5 (D) 2.8

23. The unit weight of sand back fill was found to be 1746 kg/m^3. The water content is 6.6% and unit weight of soil constituents is 2.6 g/cc. In laboratory the void ratio of loosest and densest states were found to be 0.842 and 0.622 respectively. The relatively density of soil is
(A) 1.23 (B) 1.86
(C) 1.18 (D) 1.15

24. A soil has the liquid limit of 50% and plastic limit of 30%. Then the classification of soil will be
(A) CL (B) CI
(C) CH (D) MH

25. Sedimentation method generally used in the field of soil mechanics is
(A) successive sedimentation
(B) observation of the amount of sediment per unit volume at a given point
(C) observation of total amount of soil in suspension above a given elevation
(D) observation of total sedimentation soil

Answer Keys

1. A	**2.** C	**3.** B	**4.** B	**5.** D	**6.** A	**7.** B	**8.** A	**9.** C	**10.** C
11. A	**12.** D	**13.** C	**14.** B	**15.** D	**16.** B	**17.** C	**18.** B	**19.** C	**20.** C
21. D	**22.** B	**23.** D	**24.** B	**25.** B					

Hints and Explanations

12. $\frac{V_1}{V_2} = \frac{1+e_1}{1+e_2}$ given: $e_1 = 1$

$e_2 = 0.3$

$\frac{V_1}{V_2} = \frac{1+1}{1+0.3} = \frac{2}{1.3}$

$V_1 = 1.53\ V_2$

$\%\ \text{loss in volume} = \frac{V_1 - V_2}{V_1} \times 100$

$= \frac{1.53V_2 - V_2}{V_1} \times 100$

$= \frac{0.53V_2}{V_1} \times 100 = 34\%$ Choice (D)

13. The liquid limit of soil is water content at which part of soil cut by a groove will flow together for a distance of 12 mm under an impact of 25 blows in casagrande's apparatus.

$w_L = 63.9\%$ Choice (C)

14. In falling head permeability test

$$K = \frac{a.\mathrm{L}}{A.t}\log_e\left(\frac{h_1}{h_2}\right)$$

Given

$$a = \frac{\pi d^2}{4} = \frac{\pi}{4}\left(52.5\times10^{-1}\right)^2 = 0.216\ \text{cm}^2$$

$L = 13.4$ cm
$A = 48.4$ cm^2
$h_1 = 55$ cm
$h_2 = 25$ cm
$t = 20$ min

$$k = \frac{0.216\times13.4}{48.4\times20}\times\log_e\left(\frac{65}{25}\right)$$

$= 28.61$ cm/min $= 0.47 \times 10^{-4}$ cm/sec Choice (B)

15. Head required to cause quick sand condition is

$$h = \frac{r^1 L + q}{r_w}$$

$r^1 = r_{sat} - r_w$

$$r_{sat} = \frac{r_w(G+e)}{1+e}$$

$$= \frac{9.8(2.65+0.6)}{1+0.6} = 19.9\text{kN/m}^3$$

$r^1 = 19.9 - 9.8 = 10.1\ \text{kN/m}^3$

In the given problem $q = 0$

$$\therefore \quad h = \frac{10.1 \times 10}{9.8} = 10.3\text{m}$$ Choice (D)

16. Seepage quantity

$$q = K.H.\left(\frac{N_f}{N_d}\right)$$

$N_f = 10$
$N_d = 20$
$K = 3 \times 10^{-5}$ m/s
$H = 4$m

$$q = 3 \times 10^{-5} \times 4 \times \left(\frac{10}{20}\right)$$

$= 6 \times 10^{-5}\ \text{m}^3/\text{s}$ Choice (B)

17. Effective stress = total stress – Pore water pressure total stress at 7.8 m from surface
$= (1.8 \times 4) + (3.8 \times 2.3)$
$= 15.94\ \text{t/m}^3$
Pore water pressure
$= 3.8 \times 0.98 = 3.724\ \text{t/m}^3$
$\therefore$ Effective stress = 15.94 – 3.72
$= 12.216\ \text{t/m}^3$ Choice (C)

18. Total stress at 12.8m from the surface is
$= (1.8 \times 4) + (3.8 \times 2.3) + (5 \times 1.98)$
$= 25.84\ \text{t/m}^3$
$\therefore$ Effective stress = $25.84 - (0.98 \times 8.8)$
$= 17.216\ \text{t/m}^3$ Choice (B)

19. $(G_m)_{sat} = 1.84$

$$\frac{\gamma_{sat}}{\gamma_w} = 1.84$$

$$e = \frac{wG}{s}[s=1]$$

$$\frac{(G+e)}{1+e} = 1.84$$

$$\frac{G + 0.393\,G}{1 + 0.393\,G} = 1.84$$

$G = 2.74$ Choice (C)

20. $e = \dfrac{wG}{s}$

$$e = 2.74 \times \frac{39.3}{100} = 1.07$$ Choice (C)

22. Activity of clay $= \dfrac{lp}{c}$

$$= \frac{W_L + W_R}{C} = \frac{45\ 33}{30} = 0.4$$ Choice (B)

23. Relative density $= \dfrac{e_{\max} - e_o}{e_{\max} - e_{\min}}$

$$\gamma_d = \frac{\gamma}{1+w} = \frac{r_W G}{1+e_O}$$

$$\frac{1.746}{1+\frac{6.6}{100}} = \frac{1 \times 2.6}{1+e_O}$$

$$1.637 = \frac{2.6}{1+e_O}$$

$e_o = 0.587$

$$\text{Relative density} = \frac{0.842\ \ 0.587}{0.842\ \ 0.622} = 1.15$$ Choice (D)

Soil Mechanics Test 2

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The intensity of radial shear stress at a point 8 m below vertically and 5 m horizontally below a point load of 3 tonnes is
(A) 1.438 t/m^2 (B) 1.583 t/m^2
(C) 1.875 t/m^2 (D) 2.013 t/m^2

2. The curves indicating the distribution of excess hydrostatic pressure in the soil are known as
(A) isobars (B) isochrones
(C) isotopes (D) isohyts

3. The soil which is compacted to the dry of optimum has
(A) low swelling
(B) low shear strength
(C) high swelling
(D) None of the above

4. The critical shear plane will have an angle of _____ with reference to major principle plane
(A) $45° + \frac{\Phi}{2}$ (B) $35° + \frac{\Phi}{2}$
(C) $90° + \Phi$ (D) $135° + \frac{\Phi}{2}$

5. Under active pressure condition the failure wedge moves
(A) towards right
(B) towards left
(C) towards upward
(D) towards downward

6. Westergaard's theory is applicable for which type of soils
(A) Sandy soils (B) Startified soils
(C) humus soils (D) gravel

7. During consolidation process
(A) effective pressure on soil decreases
(B) void ratio increases
(C) degree of saturation remains same
(D) excess hydrostatic pressure increases

8. The compactive effort in the modified proctor test is about _____ times than that of the standard proctor test
(A) 4.85 (B) 4.65
(C) 4.25 (D) 4.55

9. The sensitivity of soil indicates the
(A) moisture holding capacity of soil
(B) shear strength of the soil
(C) consolidation of soil
(D) weakening due to remolding of soil

10. The unsupported vertical cut of the embankment if $C = 40$ kN/m^2, $\gamma = 30$ kN/m^3 and $k_a = 1$ is
(A) 5.23 m (B) 5.33 m
(C) 5.43 m (D) 5.53 m

11. A retaining wall of height 10 m retains dry sand. The soil is loose and has a void ratio of 0.8, $\gamma d = 18.8$ kN/m^3 and $\Phi = 50°$. The backfill is compacted to a state of 0.5, $\gamma d = 20.8$ kN/m^3 and $\Phi = 65°$. The ratio of initial passive thrust to the final passive thrust according to Rankine's earth pressure theory is
(A) 0.35 (B) 2.9
(C) 0.33 (D) 2.7

12. A cohesive soil yield a maximum dry density of 1.4 g m/cc at OMC of 16% during a standard proctar test. If the value of $G = 2.65$ then degree of saturation of the soil is
(A) 1.23 (B) 1.86
(C) 1.43 (D) 1.69

13. A saturated clay of 6m thick takes 1.6 years for 50% primary consolidation when drained on both sides. Its coefficient of volume charge is 1.5×10^{-3} m^2/kN. The coefficient of permeability of soil will be
(A) 0.013 m/yr (B) 0.016 m/yr
(C) 1.58 m/yr (D) 2.54 m/yr

14. A footing of 3 m × 2 m exerts uniform pressure of 150 kN/m^2 on soil.
Assuming a load dispersion of 2 vertical to 1 horizontal, the average vertical stress (kN/m^2) at 1 m below the footing is
(A) 50 (B) 80
(C) 45 (D) 100

15. Match the following:

	Compaction equipments		**Usage**
1.	Tampers	i.	for cohesive and non cohesive soils
2.	pneumatic tyred rollers	ii.	for clays
3.	sheep foot roller	iii.	For confined trenches
4.	Vibratory compactors	iv.	For granular soils

(A) 1 – iii, 2 – i, 3 – ii, 4 – iv
(B) 1 – iv, 2 – iii, 3 – ii, 4 – i
(C) 1 – iii, 2 – ii, 3 – iv, 4 – i
(D) 1 – ii, 2 – i, 3 – iii, 4 – iv

16. (i) In rankine's theory the retaining wall is assumed to be smooth and vertical.
(ii) In coulomb's wedge theory the retaining wall is assume to be rough.
(A) (i) is true, (ii) is false
(B) (i) is false, (ii) is true

(C) (i) and (ii) are true
(D) (i) and (ii) are false

Common Data for Questions 17 and 18:
When an unconfined compression test is conducted on a cylinder of soil, it fails under axial stress of 4.3 kg/cm^2. The failure plane makes an angle of 60° with horizontal

17. The cohesion of soil is
(A) 0.58 kg/cm^2 (B) 0.63 kg/cm^2
(C) 0.68 kg/cm^2 (D) 1.24 kg/cm^2

18. The angle of internal friction of soil is
(A) 6.5° (B) 20°
(C) 30° (D) 10°

19. A 5 m thick clay has coefficient of consolidation 0.025 cm^2/min and final settlement 10 cm. The time required for 80% of settlement to occur is

U	Tv
80%	0.567
25%	0.078

(A) 5.6×10^6 minutes (B) 6.5×10^6 minutes
(C) 7.5×10^6 minutes (D) 5.8×10^6 minutes

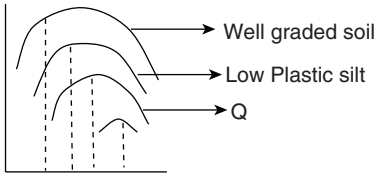
20. Time required for 2.5 cm settlement is
(A) 7.5×10^5 min (B) 7.8×10^5 min
(C) 7.6×10^5 min (D) 7.2×10^5 min

21. A 50 kN load acts on the surface of an infinite elastic medium the vertical pressure intensity in kN/m^2 at 10 m below and 4 m away from the load will be
(A) 0.048 kN/m^2 (B) 0.096 kN/m^2
(C) 0.049 kN/m^2 (D) 0.035 kN/m^2

22. With the increase of water content incompaction the maximum dry density will
(A) increase
(B) decrease
(C) first increase and then decrease
(D) first decrease and then increase

23. Which of the following property increases with increase in compaction
(A) permeability
(B) shear strength
(C) void ratio
(D) compressibility

24.



The type of soil present at Q is
(A) high plastic clay
(B) high plastic silt
(C) low plastic clay
(D) poorly graded soil

25. When the OCR = 1 then the type of clay is known as
(A) over consolidated clay
(B) Normal consolidated clay
(C) under consolidation clay
(D) None of these

Answer Keys

1. C	**2.** B	**3.** C	**4.** A	**5.** D	**6.** B	**7.** C	**8.** D	**9.** D	**10.** B
11. C	**12.** D	**13.** B	**14.** C	**15.** A	**16.** C	**17.** D	**18.** C	**19.** A	**20.** B
21. D	**22.** B	**23.** C	**24.** B	**25.** B					

Hints and Explanations

1. Radial shear stress $T_{rz} = \sigma_z \left(\frac{T}{z}\right)$

Given $\sigma_2 = 3$ tonnes $= 3 \times 10^3$ kg
$= 3 \times 10^4$ N

$r = 5$m
$z = 8$m

$$T_{rz} = 3 \times 10^3 \times 10 \times \left(\frac{5}{8}\right)$$

$= 18750$ N/m^2
$= 1.875$ t/m^2. Choice (C)

10. Unsupported vertical cut $H_C = \frac{4C}{\gamma\sqrt{k_a}}$

$$H_C = \frac{4 \times 40}{30 \times 1} = 5.33 \text{ m}$$ Choice (B)

11. Initial passive thrust
$P_P = K_P \times \sigma_v \qquad \sigma_v = \gamma \times z$
$\sigma_V = \gamma \times z = 18.8 \times 10 = 18.8$ kN/m

$$K_P = \frac{1+\sin(50)}{1-\sin(50)} = 7.54$$

$(P_p)_1 = 7.54 \times 188 = 1419.1$ kN/m

Final passive thrust

$\sigma_V = r \times z = 20.8 \times 10 = 208$ kN/m

$$k_p = \frac{1+\sin(65)}{1-\sin(65)} = 20.34$$

$(P_P)_2 = 208 \times 20.34 = 4232.0$ kN/m

$$\frac{(P_P)_1}{(P_P)_2} = \frac{1419.1}{4232.0} = 0.33$$ Choice (C)

12. $eG = Gw$

$$\gamma_d = \frac{\gamma_w G}{1+e}$$

$$\therefore \quad 0.25 \times S = 2.65 \times \frac{16}{100}$$

$$1.4 = \frac{1 \times 2.65}{1+e} \qquad S = 1.69$$

$e = 0.25$ Choice (D)

13. $$T_v = \frac{C_V \times t}{d^2}$$

$$d = \frac{H}{2} \rightarrow \text{double drainage}$$

$$\therefore \quad T_V = \frac{C_V \times t}{\left(H/2\right)^2} = \frac{C_v \times t \times 4}{H^2} = \frac{C_V - 3 \times 16 \times 4}{6^2}$$

T_V for 50% consolidation is $= \frac{\pi}{4} \times \left(\frac{50}{100}\right)^2$

$$\therefore \quad \frac{\pi}{4} \times \left(\frac{50}{100}\right)^2 = \frac{C_V \times 3 \times 1.6 \times 4}{6^2}$$

$$\therefore \quad C_V = 1.103$$

$$C_V = \frac{k}{m_v r_w} \quad \therefore K = C_V m_v r_w$$

$= 1.103 \times 1.5 \times 10^{-3} \times 9.8$

$= 0.016$ m/year. Choice (B)

14. The average vertical stress

$$= \frac{Q}{(L+z)(B+z)}$$

$Q = A \times q = (3 \times 2)(150) = 900$ kN/m

$$\therefore \quad \sigma = \frac{900}{(3+2)(2+2)} = 45 \text{ KN/m}^2$$ Choice (C)

16. Choice (C)

17. Unconfined compression test $\sigma = 2 \times C_u \times \tan(45 + \Phi/2)$

$4.3 = 2 \times C_u \times \tan 60°$

$\therefore \quad C_u = 1.24$ kg/cm^2. Choice (D)

18. $45 + \frac{\phi}{2} = \alpha = 60°$ Angle of internal friction

$= 7.5°.\phi = (60 - 45) \times 2 = 30°$ Choice (C)

19. $C_V = T_V d^2/t$

$0.025 = T_V \times (500)^2/t$

$t = 10 \times 10^6 T_V$

when U = 80%, $T_V = 0.567$

$\therefore \quad t = 10 \times 10^6 \times 0.567$

$= 5.67 \times 10^6$ minutes. Choice (A)

20. $$U = \frac{2.5}{10} \times 100 = 25\%$$

When $U = 25\%$, $T_V = 0.078$

$\therefore \quad t = 10 \times 10^6 T_V$

$= 10 \times 10^6 \times 0.078$

$= 7.8 \times 10^5$ minutes. Choice (B)

21. $$\sigma_z = \frac{Q}{Z^2} \times \frac{3}{2\pi} \left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{\frac{5}{2}}$$

$$r = \sqrt{10^2 + 4^2} = \sqrt{100+16} = 10.7\text{m}$$

$$= \frac{50}{100} \times \frac{3}{2\pi} \left[\frac{1}{1+\left(\frac{10.7}{10}\right)^2}\right]^{\frac{5}{2}}$$

$= 0.035$ kN/m^2 Choice (D)

GEOTECHNICAL ENGINEERING TEST 3

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. If w represents natural water content & w_L, w_p, w_s represents Liquid limit, Plastic limit, Shrinkage limit respectively; choose the incorrect pair from the following.

(A) Plasticity Index $(I_p) = w_L - w_P$

(B) Shrinkage Index $(I_s) = w_p - w_s$

(C) Liquidity Index $(I_L) = \dfrac{w - w_p}{w_L - w_p}$

(D) Consistency Index $(I_C) = \dfrac{w - w_L}{w_L - w_P}$

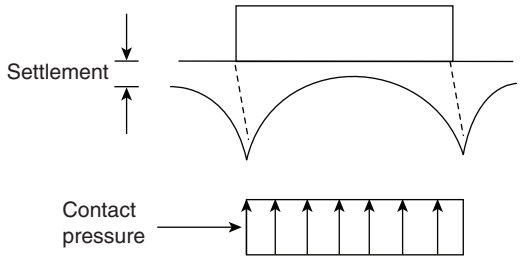
2. The symbol 'SM' indicates

(A) Sandy silt
(B) Medium silt
(C) Silty sand
(D) Medium sand

3. For a given flow net, if number of flow channels and number of potential drops are found as 12 and 8; then what would be the shape factor of the flow net?

(A) 4
(B) 1.67
(C) 1.5
(D) 1.71

4. The figure given below represents the contact pressure distribution underneath a

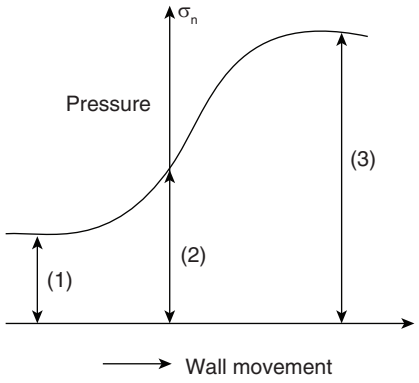


(A) Rigid footing on Sand
(B) Flexible footing on Clay
(C) Flexible footing on Sand
(D) Rigid footing on Clay

5. If uniform surcharge of 120 kN/m^2 is placed on the backfill with $\Phi = 30°$, the increase in pressure is (in kN /m^2)

6. The cohesion and density of a soil are 1.8 t/m^2 and 2 t/m^2. If Stability number is taken as maximum, for a factor of safety against 2.5 what will be the safe height of the slope in meters? _____

7.



Identify the correct one from the following. (p_o, p_a, p_p indicates at rest, active and passive earth pressures respectively)

(A) (1) – p_o, (2) – p_a, (3) – p_p
(B) (1) – p_a, (2) – p_o, (3) – p_p
(C) (1) – p_p, (2) – p_o, (3) – p_a
(D) (1) – p_o, (2) – p_p, (3) – p_a

8. In a falling head permeability test the initial head of 1.2 m dropped to 0.4 m in 4 hours, the diameter of the stand pipe being 5 mm. The soil specimen was 300 mm long and of 150 mm diameter. The coefficient of permeability of the soil is _____

(A) 2.54×10^{-5} cm/sec
(B) 2.54×10^{-6} cm/sec
(C) 2.54×10^{-4} cm/sec
(D) 2.54×10^{-7} cm/sec

9. Sand Bath method is used to determine ___

(A) Specific gravity
(B) Unit weight
(C) Moisture content
(D) Particle Size distribution

10. The number of blows observed in a Standard Penetration test for different penetration depths are given as follows.

Penetration of Sampler	Number of blows
0 – 100 mm	2
100 – 200 mm	4
200 – 350 mm	7
350 – 400 mm	10

The observed 'N' value is _____

11. For a Sand deposit having Specific gravity 2.65 and moisture content 25%; what will be the relative density in saturated condition if loose and Compacted void ratios are given as 0.92 and 0.41 respectively

(A) 49% (B) 51%
(C) 39% (D) 31%

12. For a particular soil sample, if D_{10}, D_{30} and D_{60} is given as 425 μ, 2.36 mm and 4.75 mm respectively. Then match the following

	Group – I		Group – II
1.	Coefficient of Curvature	a.	1.81
2.	Coefficient of Uniformity	b.	2.76
3.	Permeability	c.	11.2

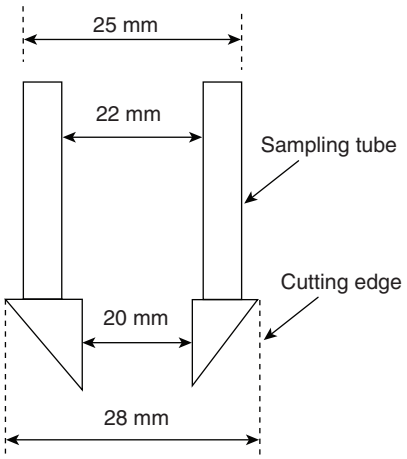
(A) 1 – a, 2 – b, 3 – c
(B) 1 – c, 2 – b, 3 – a
(C) 1 – a, 2 – c, 3 – b
(D) 1 – b, 2 – c, 3 – a

13. In a 8m thick stratum of fine sand having submerged density of 11 kN/m^3, quick sand condition occurred at a depth of 5.2 m of excavation. What is the depth of lowering of ground water table required for making an excavation 6 m deep?
(Take γ_w as 10 kN/m^3)
(A) 1.76 m (B) 1.68 m
(C) 0.88 m (D) 0.74 m

14. The vertical stress at some depth below the centre of 3 m × 4 m rectangular footing due to certain load intensity is 100 kN/m^2. What will be the vertical stress in kN /m^2 below the corner of 1.5 m × 2 m rectangular footing at the same depth and same load intensity?

15. An unsupported excavation is made to the maximum depth in a clay soil having $\gamma = 21$ kN/m^3, $C = 80$ kN/m^2 and $\Phi = 30°$. What will be the active earth at pressure the base level of excavation, according to Rankine's theory?
(A) 184.8 kN/m^2 (B) 92.4 kN/m^2
(C) 1462.4 kN/m^2 (D) 277.18 kN/m^2

16. For the sampler shown in the figure Area ratio, Inside clearance and outside clearance are respectively



(A) 96%, 12%, 10%
(B) 49%, 11%, 9%
(C) 49%, 9%, 11%
(D) 96%, 10%, 12%

17. Match List – I (Roller type) with List – II (Soil type)

	List – I		List – II
1.	Sheep foot roller	a.	Gravel in WBM Road
2.	Pneumatic roller	b.	Dry sand
3.	Smooth heavy roller	c.	Hearting of earthen dam
4.	Vibratory roller	d.	Casing of earthen dam

(A) 1 – b, 2 – c, 3 – a, 4 – d
(B) 1 – c, 2 – d, 3 – a, 4 – b
(C) 1 – d, 2 – c, 3 – a, 4 – b
(D) 1 – d, 2 – c, 3 – b, 4 – a

18. The water content of saturated soil and the specific gravity were found to be 30% and 2.65 respectively. Assuming the unit weight of water to be 10 kN/m^3, submerged unit weight (in kN/m^2) and porosity of the soil are _____
(A) 19,0.6 (B) 9,0.45
(C) 9,0.6 (D) 19,0.78

19. When an unconfined compression test is conducted on a cylinder of soil, it fails under axial stress of 1.5 kg/cm^2. If an angle of internal friction of the soil is 30°, what will be the cohesion of the soil? _____
(A) 0.43 kg/cm^2 (B) 0.67 kg/cm^2
(C) 0.75 kg/cm^2 (D) 0.35 kg/cm^2

20. A square footing of size 5 m × 5 m is resting on the surface of a deposit of saturated clay having an unconfined compressive strength of 54 kPa, What will be the net safe bearing capacity of the footing (in kPa) if factor of safety is given as 2.5? _____

21. A square group of 16 piles was driven into soft clay extending to a large depth. The diameter and length of the piles were 30 cm and 12 m respectively. If the cohesion of clay is given as 5 t/m^2, for the pile spacing 100 cm c/c what is the capacity of the pile group? (Take adhesion factor as 0.75 and FOS = 1.75)
(A) 733 *t* (B) 417 *t*
(C) 550 *t* (D) 623 *t*

22. In a constant head permeameter with cross section area of 10 cm^2, when the flow was taking place under a hydraulic gradient of 0.6 the amount of water collected in 60 seconds is 720 CC. The permeability of the soil is ____.
(A) 0.2 cm/sec (B) 0.02 cm/sec
(C) 0.002 cm/sec (D) 2 cm/ sec

23. What is the shear strength in terms of effective stress on a plane with in the saturated soil mass at a point where total normal stress is 245 kPa and pore water pressure is 80 kPa. The effective shear stress parameters are c^1 = 12 kPa, and $\phi' = 30°$.
(A) 105.3 kPa (B) 106.3 kPa
(C) 107.3 kPa (D) 108.3 kPa

24. Sieve analysis on a dry soil sample of mass 1000 g showed that 980 g and 270 g of soil pass through

4.75mm and 75 μ sieve, respectively. The liquid limit and plastic limits of the soil fraction passing through 425 μ sieves are 55% and 25% respectively. The soil may be classified as _____

(A) *GC* (B) *SM*
(C) *GM* (D) *SC*

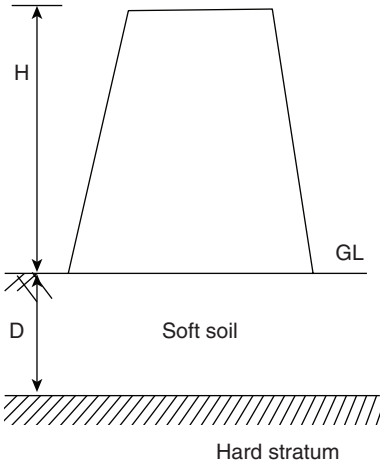
25. Identify the incorrect pair from the following
(A) Alluvial soils - Transported by running water
(B) Lacustrine soils - Deposited at the bottom of lakes
(C) Talus - soil transported by gravitational force
(D) Loess - soil transported by glaciers

26. In a plate load test with size of plate 30 × 30 cm; bearing capacity and settlement were noted as 15 kPa & 6 mm respectively in a sandy soil. Then find the bearing capacity and settlement under a footing of size 2.1 m × 2.1 m under the same pressure intensity?
(A) 15 kPa and 42 mm
(B) 15 kPa and 18.4 mm
(C) 105 kPa and 18.4 mm
(D) 105 kPa and 42 mm

27. A footing carries a load of 1200 tons and is of 3.2 m square. It rests in dense sand of 8 m thickness overlaying a clay layer of 2.8 m. The clay layer overlies hard rock. The depth of foundation is 1.5 m Liquid limit of clay is 48% and void is 0.95. The saturated unit weight of sand and clay are given as 1.86 t/m^3 and 1.76 t/m^3 respectively Take the load distribution as 2V to 1H. Assume that the site is flooded and determine the ultimate settlement due to consolidation of clay layer?
(A) 150 mm (B) 170 mm
(C) 190 mm (D) 210 mm

28. If a clay test specimen of 25 mm thick, under double drainage condition attained 50% of primary consolidation in 50 minutes. How long will it take for the same clay layer of 10 m thick to reach the same degree of consolidation under the condition that 'clay is drained on the top surface only'.
(A) 15.4 years (B) 61.7 years
(C) 85.6 years (D) None

29.



For the given slope, the failure expected to be ___
(A) Toe failure
(B) Base failure
(C) Face failure
(D) Can't say/ Data inadequate

30. Identify the false statements from the list given.
(i) Skempton's theory is suitable for clays only.
(ii) The discharge between any two adjacent flow lines is constant
(iii) If water table rises σ^1 and u increases but σ decreases.
(iv) The westergaard analysis is suitable for stratified soils
(A) (iii) and (iv) (B) (i) and (iii)
(C) (iii) only (D) (iv) only

Answer Keys

1. D	**2.** C	**3.** C	**4.** C	**5.** 40 kN/m^2		**6.** 1.37 to 1.40		**7.** B	**8.** B
9. C	**10.** 21	**11.** B	**12.** D	**13.** B	**14.** 25	**15.** B	**16.** D	**17.** B	**18.** B
19. A	**20.** 80	**21.** B	**22.** D	**23.** C	**24.** D	**25.** D	**26.** C	**27.** B	**28.** B
29. B	**30.** C								

Hints and Explanations

1. consistency Index, $Ic = \dfrac{w_L - w}{w_L - w_p}$ Choice (D)

3. Shape factor $= \dfrac{N_f}{N_d}$

$= \dfrac{12}{8} = 1.5$ Choice (C)

5. Increase in pressure $= Ka.\ q$

$Ka = \dfrac{1-\sin\phi}{1+\sin\phi} = \dfrac{1-\frac{1}{2}}{1+\frac{1}{2}} = \dfrac{1}{3} = \dfrac{1}{3}$

$Ka.\ q = \dfrac{1}{3} \times 120 = 40$ kN/m^2 Ans: 40 kN/m^2

6. $S_n = \dfrac{C}{Fc.\gamma H}$

$0.261 = \dfrac{1.8}{2.5 \times 2 \times H}$

$H = 1.38\text{m}$

Ans: 1.37 to 1.40

7. $p_a < p_o < pp$ Choice (B)

8. $K = 2.303\ \dfrac{aL}{At} \log_{10}\left(\dfrac{h_1}{h_2}\right)$

Given; $h_1 = 1.2\text{m}$ and $h_2 = 0.4\text{m}$

Area of stand pipe $(a) = \dfrac{\pi}{4} \times 5^2 = 19.635\text{mm}^2$

Area of soil specimen $(A) = \dfrac{\pi}{4} \times 150^2 = 17671.46\text{mm}^2$

$\therefore \quad k = 2.303 \times \dfrac{19.635 \times 300}{17671.46 \times 4 \times 60 \times 60} \log_{10}$

$K = 2.54 \times 10^{-5}$ mm/sec

$= 2.54 \times 10^{-6}$ cm/sec Choice (B)

9. Oven drying method, pycnometer method, calcium carbide method, Sand Bath method, are used to determine water content of the soil. Choice (C)

10. $N = 4 + 7 + 10 = 21$

N value is taken as the sum of blows for the last 300 mm penetration. Ans : 21

11. Given that soil is in Saturated condition, then void ratio,

$e = wG$

$e = 0.25 \times 2.65 = 0.66$

$$\text{Relative Density} = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

$$= \frac{0.92 - 0.66}{0.92 - 0.41} \times 100$$

$$= 50.98 \approx 51\%$$

Choice (B)

12. Given,

$D_{10} = 425\ \mu = 425 \times 10^{-3}\text{ mm} = 0.425\text{ mm}$

$D_{30} = 2.36\text{ mm}$

$D_{60} = 4.75\text{ mm}$

(i) $C_c = \dfrac{D_{30}^2}{D_{60} \times D_{10}} = \dfrac{2.36^2}{4.75 \times 0.425} = 2.76$

(ii) $C_u = \dfrac{D_{60}}{D_{10}} = \dfrac{4.75}{0.425}$

(iii) Permeability; $k = 100\ D_{10}^2$

It should be noted that 'D' is in cm's here

$\Rightarrow \quad K = 100\ (0.425 \times 10^{-1})^2 = 0.181$ cm/sec

$K = 1.81$ mm/sec Choice (D)

13. Quick sand conditions

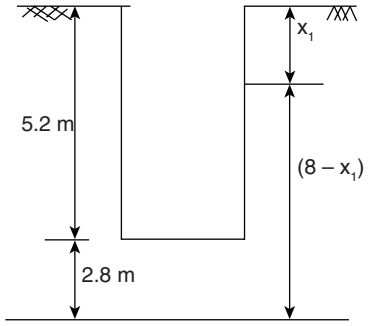
$\gamma_{sat}\, z - \gamma_w h = 0$

Let WT is at x_1, initially

$\sigma^1 = \gamma_{sat}\, Z - \gamma_w h = 0$

$21\ (2.8) - 10\ (8 - x_1) = 0$

$x_1 = 2.12\text{ m}$



To make an excavation of 6 m deep,

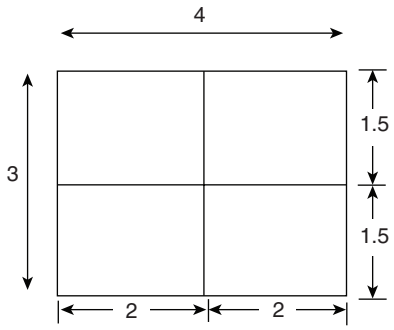
$21\ (2) - 10\ (8 - x_2) = 0$

$x_2 = 3.8\text{ m}$

$\therefore$ Lowering of $WT = 3.8 - 2.12 = 1.68$ m.

Choice (B)

14.



Since 3 m × 4 m rectangle consists of 4 no. of 1.5 m × 2 m;

Vertical stress at the corner of 1.5 m × 2 m rectangle = $\dfrac{1}{4}$ × (vertical stress at centre of 3 m× 4 m rectangle)

$= \dfrac{1}{4} \times 100 = 25\text{ kN/m}^2$ Ans: 25

15. Maximum depth of Unsupported excavation $= H_c$

$= \dfrac{4c}{\gamma\sqrt{K_a}}$

$K_a = \dfrac{1 - \sin\phi}{1 + \sin\phi} = \dfrac{1 - \frac{1}{2}}{1 + \frac{1}{2}}$

$\therefore \quad K_a = \dfrac{1}{3}$

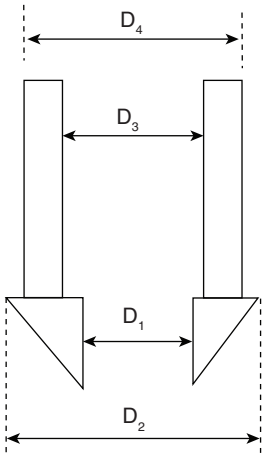
$\Rightarrow \quad H_c = \dfrac{4 \times 80}{21\sqrt{\frac{1}{3}}} = 26.40\text{m}$

$P_a = K_a.\ \gamma H - 2c\ \sqrt{K_a}$

$= \dfrac{1}{3} \times 21 \times 26.40 - 2 \times 80 \times \sqrt{\dfrac{1}{3}} = 92.42\text{ kN/m}^2$

Choice (B)

16.



$$\text{Area ratio} = \frac{D_2^2 - D_1^2}{D_1^2} \times 100 = \frac{28^2 - 20^2}{20^2} \times 100 = 96\%$$

$$\text{Inside clearance} = \frac{D_3 - D_1}{D_1} \times 100$$

$$= \frac{22 - 20}{20} \times 100 = 10\%$$

$$\text{Outside clearance} = \frac{D_2 - D_4}{D_4} \times 100 = \frac{28 - 25}{25} \times 100$$

$$= 12\%$$ Choice (D)

17. Sheep foot roller – Hearing of earthen down
Pneumatic roller – Casing of earthen down
Smooth heavy roller – Gravel in WBM road
Vibratory roller – Dry Sand Choice (B)

18. $G = 2.65$
$w = 0.30$
$\gamma_w = 10$ kN/m^3

$$e = \frac{wG}{s\gamma}$$

$$e = \frac{0.3 \times 2.65}{1} = 0.80$$

$$\gamma_{sat} = \frac{\gamma_w (G + e)}{1 + e} = \frac{10(2.65 + 0.80)}{1.8}$$

$$= 19.14 \text{ kN/m}^3 = 19 \text{ kN/m}^3$$

$$\gamma^1 = \gamma_{sat} - \gamma_w = 19 - 10 = 9 \text{ kN/m}^3$$

$$\text{Porosity, } n = \frac{e}{1 + e}$$

$$n = \frac{0.8}{1.8} = 0.45$$ Choice (B)

19. $\Phi = 30°$
Unconfined compression test, so

$$\sigma_1 = 2C \tan \alpha$$

$$\alpha = 45 + \Phi/2 = 60°$$

$$\Rightarrow \quad 1.5 = 2C \tan 60°$$

$$\therefore \quad C = 0.43 \text{ kg/cm}^2$$ Choice (A)

20. Given is a square footing.
So,

$$q_{ult} = 1.3\, CN_C + \gamma D N_q + 0.4\, \gamma\, BN\gamma$$

Unconfined compressive strength,

$$q_u = 2c$$

$$\Rightarrow \quad C = \frac{54}{2} = 27 \text{ kPa}$$

For clay; $(\Phi = o)$

$$N_c = 5.7,\ N_q = 1 \text{ and } N\gamma = 0$$

$$q_{netult} = (1.3CNc + \gamma\, DN_q) - \gamma D$$

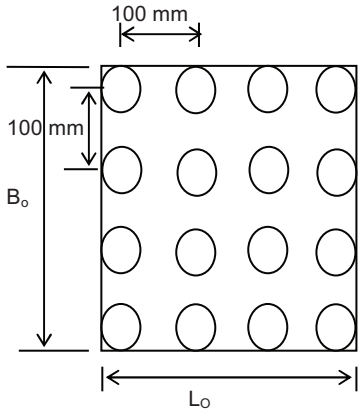
$$q_{netult} = 1.3\, CNc + \gamma D\, N_q - \gamma D$$

$$\therefore \quad q_{netult} = 1.3\, CN_c$$

$$q_{netsafe} = \frac{1.3CN_c}{F.S} = \frac{1.3 \times 27 \times 5.7}{2.5} = 80 \text{ kPa}$$

Ans: 80 kPa

21.



(i) Based on individual pile failure mode

$$Q_{gi} = n\,[A_b CN_c + A_s\, \alpha C]$$

$$= 16\left[\frac{\pi}{4} \times 0.3^2 \times 5 \times 9 + \pi \times 0.3 \times 12 \times 0.75 \times 5\right]$$

$$= 729.46 \text{ t}$$

(ii) Based on Block failure mode

$$Q_{gb} = A_B.\, CN_c + A_{s.} C$$

A_B = Area of block = $B_o.\, L_o$
A_s = Surface area of block = $2\,(B_o .+ L_o)\,.\,L$
$B_o = 3s + d = L_o = 330$cm

$$\therefore \quad A_B = 3.3 \times 3.3 = 10.90 \text{ m}^2$$

Similarly, $A_s = 2\,(3.3 + 3.3)\,12 = 158.4\text{m}^2$

$$Q_{gb} = 10.90(5)\,(9) + (158.4)\,(5) = 1282\ .5\text{t}$$

$$\therefore \quad Q_g = \text{smaller of } Q_{gi} \text{ and } Q_{gb} = 729.46\text{t}$$

$$Q_s = \frac{729.46}{1.75} = 416.8t = 417\text{t}$$ Choice (B)

22. $Q = \text{KiA}$

$$Q = \frac{720\text{cm}^3}{60\,\text{sec}} = 12 \text{ cm}^3/\text{sec}$$

$$\Rightarrow \quad 12 = K \times 0.6 \times 10$$

$$K = 2 \text{ cm/sec}$$ Choice (D)

23. $S = c^1 + \sigma^1 \tan \Phi^1$

$\sigma^1 = 245 - 80 = 165$

$\therefore \quad S = 12 + (165 \times \tan 30°) = 107.26$ kPa Choice (C)

24. As more than 50% of soil passes through 4.75mm sieve and retains on 75 μ sieve, it is Sand.

$I_p = w_L - w_p = 30$

Equation of A line $I_p = 0.73\ (w_L - 20)$

$= 0.73\ (35) = 25.55$

Falls above the A line so it clayey soil

$\therefore$ Given Soil is Clayey Sand. Choice (D)

25. Loess is the soil transported by wind. Choice (D)

26. Given is a Sandy soil,

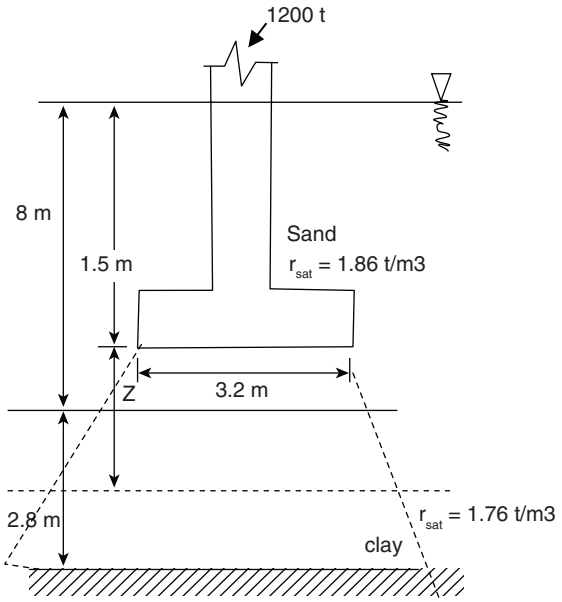
$$\frac{S_F}{S_P} = \left[\frac{B_F\left(B_P + 0.3\right)}{B_P\left(B_F + 0.3\right)}\right]^3$$

$S_P = 6$mm, $B_F = 2.1$m and $B_p = 0.3$ m

$$S_F = \left[\frac{2.1(0.6)}{0.3(2.4)}\right]^2 \times 6 \text{ mm} = 18.375 = 18.4 \text{ mm}$$

$$\frac{q_F}{q_p} = \frac{B_F}{B_p} q_F = \frac{2.1}{0.3} \times 15 = 105 \text{ kPa}$$ Choice (C)

27.



$$\frac{\Delta H}{H} = \frac{c_c}{1+e_0} \log_{10}\left(\frac{\sigma_f^1}{\sigma_0^1}\right)$$

$C_c = 0.009\ (w_L - 10) = 0.009\ (48 - 10) = 0.342$

$\sigma_0^1 = \gamma^1\ (8) + \gamma^1\ (2.8/2)$

$\therefore \quad \sigma_0^1 = 0.86\ (8) + 0.76\ (1.4) = 7.944$ t/m^3

$$\Delta\ \sigma^1 = \frac{1200}{(3.2+z)^2}$$

$Z = 8 - 1.5 + 1.4 = 7.9$m

$$\Delta\ \sigma^1 = \frac{1200}{(3.2+7.9)^2} = 9.74 \text{ t/m}^3$$

$\sigma_f^1 = \sigma_0^1 + \Delta\sigma^1$

$= 7.944 + 9.74 = 17.684$ t/m^3

$$\therefore \quad \Delta H = H\ \frac{C_c}{1+e_0} \log_{10}\left(\frac{\sigma_f^1}{\sigma_0^1}\right)$$

$$= 2.8 \times \frac{0.342}{1+0.95} \log_{10}\left(\frac{17.684}{7.944}\right) = 0.17\text{m}$$

$\therefore \quad \Delta H = 170$ mm Choice (B)

28. $T_v = \dfrac{C_v t}{d^2}$

T_v and C_v are same for the given soil as % degree consolidation is same

$t \propto d^2$

$$\frac{t_1}{t_2} \propto \left(\frac{d_1}{d_2}\right)^2$$

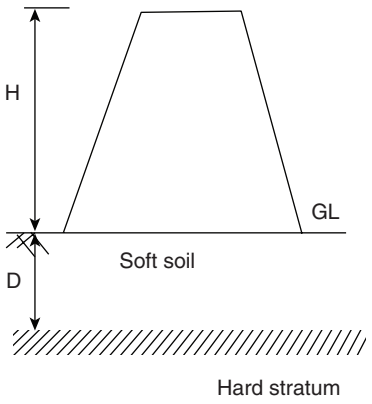
$$d_1 = \frac{25}{2} = 0.0125\text{m},\ d_2 = 10\text{m}$$

$t_1 = 50 \times 60$ sec $= 3000$ sec

$$\Rightarrow \quad \frac{3000}{t_2} = \left(\frac{0.0125}{10}\right)^2$$

$\therefore \quad t_2 = 61.728$ years Choice (B)

29.



Depth factor, $D_F = \dfrac{H+D}{H}$

For any value of D

$D_F > 1$

From the given figure; $D > O$ and $H > D$

$\Rightarrow$ So; irrespect of H and D values, $D_F > 1$

So; Base failure

$D_F = 1 \rightarrow$ Toe failure

$D_F < 1 \rightarrow$ Face failure

$D_F > 1 \rightarrow$ Base failure Choice (B)

30. If water table rises, σ and u increases but σ^1 decreases. Choice (C)

FOUNDATION ENGINEERING TEST 4

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. For toe failure the depth factor D_F is
(A) < 1 (B) > 1
(C) $= 1$ (D) None of these

2. The maximum value of stability number is
(A) 0.261 (B) 0.281
(C) 0.241 (D) 0.291

3. Which of the following is true for General shear failure?
(A) $I_D < 20\%$ (B) $e > 0.75$
(C) $N > 30$ (D) $N < 5$

4. The ultimate bearing capacity (q_f) and net ultimate bearing capacity (q_{nf}) are connected by relation
(A) $q_{nf} = q_f + \gamma D$ (B) $q_f = q_{nf} + \gamma D$
(C) $q_f = q_{nf}/F + \gamma D$ (D) $q_f = (q_{nf} + \gamma D)/F$

5. The allowable settlement for isolated foundations as per in sand and hard clay is
(A) 75 mm (B) 50 mm
(C) 100 mm (D) 80 mm

6. The piles that are provided at an inclination, to resist lateral forces (or) inclined forces is known as
(A) Tension piles (B) Anchor piles
(C) Fender piles (D) Batter piles

7. If the actual observed value of standard penetration resistance N is greater than 15 in a fine sand layer below water table, then equivalent penetration resistance will be
(A) $15 + \frac{(N+15)}{2}$ (B) $15 + \frac{(N+15)}{2}$
(C) $15 + \frac{(N-15)}{2}$ (D) $15 + \frac{(15-N)}{2}$

8. Cantilever sheet pile is generally suitable for depths
(A) $d \le 5$ (B) $5 \le d \le 10$
(C) $d \ge 10$ (D) $d \ge 12$

9. Geotextiles are used for
(A) compacting loose soils
(B) reducing settlements
(C) separation and drainage
(D) Improve bearing capacity

10. For determining the index properties of soil the sample should be
(A) disturbed (B) undisturbed
(C) intact (D) None of these

11. In standard penetration test conducted at site the recorded values of blow count for every 15 cm penetration at a depth of 45 cm are 5, 10, 15 respectively. The value of SPT blow count (N) that should be used is
(A) 15 (B) 17
(C) 25 (D) 19

12. Match the following

	Type of Boring		Usage
a.	Auger boring	1.	For drilling holes
b.	Rotary drilling	2.	Advancing holes in the ground
c.	Core drilling	3.	drilling holes in clay
d.	Percussion drilling	4.	Sampling for highways, railways etc.

a b c d
(A) 4 1 3 2
(C) 2 3 1 4

a b c d
(B) 4 3 1 2
(D) 1 2 3 4

13. (i) Cement stabilization is done by using mixture of soil + cement + water + compaction + curing
(ii) Chemical stabilization is done by using calcium chloride and sodium silicate
(A) (i) and (ii) are true
(B) (i) and (ii) are false
(C) (i) is true and (ii) is false
(D) (i) is false and (ii) is true

14. A plate load test is conducted on sand on a 500 mm diameter plate. If the plate settlement is 10 mm at a pressure of 200 kPa, the settlement of 0.5 m × 8 m footing will be
(A) 17 mm (B) 20.3 mm
(C) 23.7 mm (D) 18.6 mm

Common Data for Questions 15 and 16:
A group of piles of 20 m length and 0.25 m diameter is installed in a 10 m thick stiff clay layer underlain by rock. The pile soil adhesion factor is 0.3, average shear strength of soil on the sides is 200 kPa, undrained shear strength of soil at base is 200 kPa.

15. The base resistance of a single pile is
(A) 88 kN (B) 84.6 kN
(C) 86.2 kN (D) 88.3 kN

16. The side friction resistance of single pile is
(A) 92.3 kN (B) 94.2 kN
(C) 98 kN (D) 96.5 kN

17. A canal having side slopes 1 : 1 is proposed to be constructed in a cohesive soil to a depth of 10 m below ground surface. The soil properties are $\Phi_U = 20°$.

$C_u = 25$ kPa, $e = 1$, $G_S = 2.65$

If taylor's stability number, S_n is 0.08 and canal is full, the factor of safety with respect to cohesion against failure of canal bank slopes is
(A) 3.85 (B) 3.65
(C) 7.85 (D) 1.7

18. The incorrect statement among the following is
(A) The area ratio should be low
(B) The cutting edge should be thick
(C) The inside clearance should be small
(D) The outside clearance should be small

19. The seismic method of soil exploration cannot be used for
(A) sub surface investigation
(B) hard layers
(C) clays
(D) sandy soils

20. The term mobilized shear strength is referred to as
(A) shear strength
(B) Maximum shear stress
(C) applied shear stress
(D) None of the above

21. The efficiency of pile group for clays is
(A) >100% (B) <100%
(C) =100% (D) None of these

22. Negative skin frictions developed from
(A) A cohesive fill placed over cohesion less soil deposit
(B) lowering of ground water table with resulting ground subsidence
(C) pile driving operations
(D) All the above

23. When the water table rises to ground level in case of cohesion less soil, the bearing capacity is reduced by – %. If it is cohesive soil, the reduction will be____
(A) 20%, 50%
(B) 40%, negligible
(C) 50%, negligible
(D) negligible, 50%

24. Which of the following is false according to terzaghi's theory?
(A) zone I is elastic zone
(B) zone II is radial shear zone
(C) zone III is Rankine's passive zone
(D) zone IV is surcharge zone

25. The cohesion and density of a soil are 4 t/m^2 and 8 t/m^2 respectively. For a factor of safety of 2 and stability number 0.1 the safe height of slope is
(A) 5 m (B) 50 m
(C) 25 m (D) 2.5 m

Answer Keys

1. C	**2.** A	**3.** C	**4.** B	**5.** B	**6.** D	**7.** C	**8.** A	**9.** C	**10.** A
11. C	**12.** B	**13.** A	**14.** C	**15.** D	**16.** B	**17.** A	**18.** B	**19.** B	**20.** C
21. B	**22.** D	**23.** C	**24.** D	**25.** D					

Hints and Explanations

1. Choice (C)

2. Choice (A)

3. Choice (C)

4. Choice (B)

5. Choice (B)

6. Choice (D)

7. Choice (C)

8. Choice (A)

9. Choice (C)

10. Choice (A)

11. The SPT value have to be calculated by leaving the reading of first 15cm and adding the other penetration readings.
i.e., The SPT value = 10 + 15 = 25. Choice (C)

12. Choice (B)

13. Choice (A)

14. Dia of plate = 500 mm
Area of plate = 0.196 m^2
Area of footing = 40 m^2
Settlement of plate = 10 mm
Settlement of footing = ?
Width of plate = 500×10^{-3} m = 0.5 m
Width of footing = 8 m

$$\frac{S_f}{S_p} = \left[\frac{B_f\left(B_p + 0.3\right)}{B_p\left(B_f + 0.3\right)}\right]^2$$

$$\frac{S_f}{10\times10^{-3}} = \left[\frac{8\left(0.5+10.3\right)}{0.5\left(8+0.3\right)}\right]^2$$

S_f = 23.7 mm Choice (C)

15. Base resistance of single pile in clay = $A_b\, CN_C$

$$= \frac{\pi}{4} d^2 CN_c = \frac{\pi}{4} \times (0.25)^2 \times 200 \times 9$$

= 88.31 kN Choice (D)

16. Side friction resistance
$= A_S \propto C$
$= 2 \times 3.14 \times 0.25 \times 0.3 \times 200$
= 94.2 kN. Choice (B)

17. $S_n = \dfrac{c}{F \times H \times \gamma^1}$

$$\gamma^1 = \frac{\gamma_w (G+e)}{1+e}$$

$$= \frac{9.8 \times (2.65+1)}{1+1} = 8.08 \text{ kN/m}^2$$

$$0.08 = \frac{25}{F \times 10 \times 8.08}$$

$\Rightarrow \quad F = 3.85$ Choice (A)

18. Choice (B)

19. Choice (B)

20. Choice (C)

21. Choice (B)

22. Choice (D)

23. Choice (C)

24. Choice (D)

25. $S_n = \dfrac{C}{F_c \gamma H}$

$\Rightarrow \quad 0.1 = \dfrac{4}{2 \times 8 \times H}$

$$H = \frac{4}{2 \times 8 \times 0.1} = 2.5 \text{ m}$$ Choice (D)

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UNIT VIII
FLUID MECHANICS AND HYDRAULICS

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Fluid Mechanics Test I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. When pressure is increased, the bulk modulus of elasticity 'K'+
(A) decreases
(B) increases
(C) remains same
(D) decreases then increase

2. The viscosity of water and the viscosity of air with increase in temperature
(A) decrease and increases
(B) increases and decreases
(C) decreases and decreases
(D) decreases and remains same

3. Pascal's law for a fluid is not valid if
(A) fluid is at rest
(B) fluid is at constant rotational velocity in a container
(C) fluid is at constant linear acceleration
(D) None of the above

4. An inverted U tube manometer is more sensitive than an upright manometric because
(A) the height of levels is greater
(B) the manometric fluids are heavier than working fluids
(C) the manometric fluids are lighter than working fluids
(D) None of the above

5. A vertical wall is holding a liquid of specific weight 'w' and height 'h' on one side. The total pressure on the wall per unit length is
(A) wh (B) $wh/2$
(C) $\frac{wh^2}{2}$ (D) $\frac{2}{3}wh$

6. If the surface of liquid is concave, then cohesive pressure is
(A) increased
(B) decreased
(C) absent
(D) negligible or doesn't matter

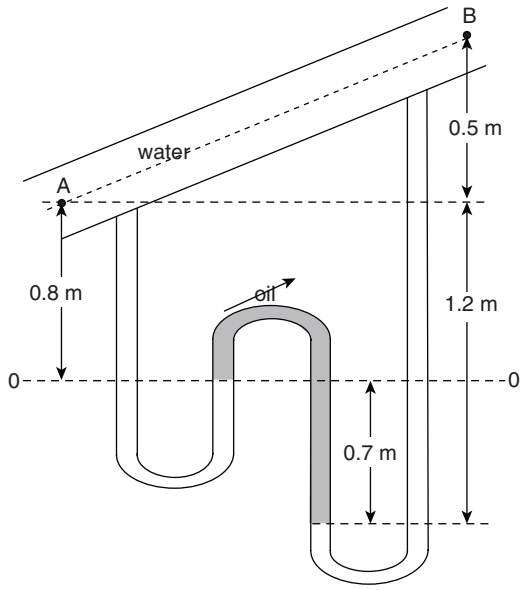
7. A body weight 4 kg in air was found to weigh 3.5 kg when submerged in water. Its specific gravity
(A) 1 (B) 3
(C) 6 (D) 8

8. If density of liquid $\rho = 1000$ kg/m^3 and area $A = 1$ m^2. Then flow rate Q at $t = 0$, (x, y) is (0, 1)
(A) 100 (B) 1000
(C) 0 (D) Can't be known

9. Surface Tension is
(A) also known as capillarity
(B) is a function of curvature of interface
(C) decreases with fall in temperature
(D) acts in a plane of interface normal to any line in the surface

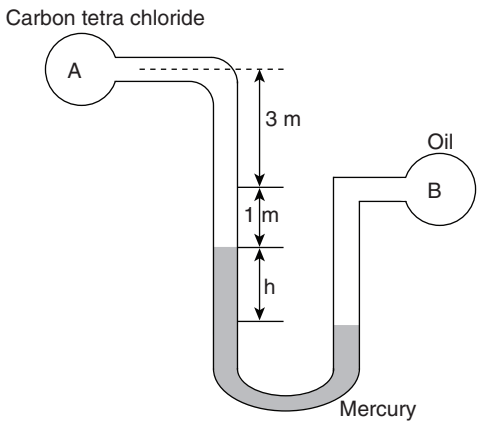
10. For stability of a floating body
(A) the meta centre 'M' should lie between the centre of gravity 'C' and centre of buoyancy 'B'
(B) M should lie above B and C
(C) M should coincide with B and C
(D) M should lie below B and C

11. An inverted U – tube differential manometer is used to measure pressure difference in an inclined water pipe as shown in figure. The manometer fluid is oil of specific gravity 0.75



Pressure difference between points A and B in N/m^2 is
(A) 1792 (B) 2882
(C) 3679 (D) 4216

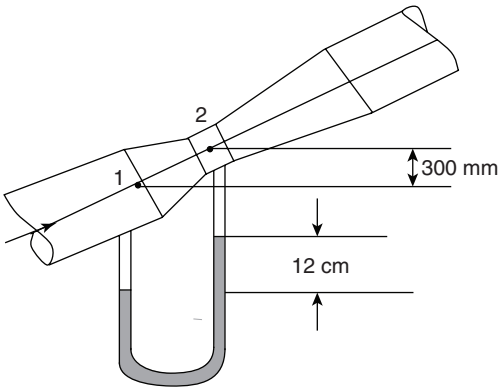
12.



Referring to the figure, pipe *A* contains carbon tetrachloride of sp gravity 1.59 under a pressure of 105 kN/m^2 and pipe *B* contains oil of sp gravity 0.8 under pressure 170 kN/m^2. Level difference h shown by the manometric fluid mercury is

(A) 72 mm (B) 83 mm
(C) 95 mm (D) 115 mm

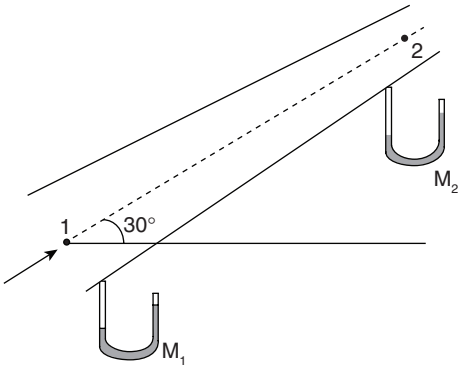
13. Water flows through an inclined venturimeter as shown in figure. Inlet and throat diameters are 100 mm and 50 mm respectively. Inlet and throat sections have a level difference of 300 mm. The differential mercury manometer connected across inlet and throat indicates 12 cm of mercury level difference at a given flow rate. Coefficient of discharge is 0.99.



The rate of flow in litres/s is

(A) 14.76 (B) 12.85
(C) 10.91 (D) 8.86

14.



Water flows through a tapering pipe inclined at 30° to the horizontal. At points 1 and 2 manometers are connected. Point 1 is at an elevation of 1 m from ground level and 2 is 3 m from ground level. Diameter at section 1 and 2 are 15 cm and 10 cm respectively. Velocity at 1 is 6 m/s. If manometer M_2 reads 10 cm of mercury, the reading shown by manometer M_1 in cm of mercury is

(A) 79.5 (B) 65.6
(C) 58.3 (D) 49.4

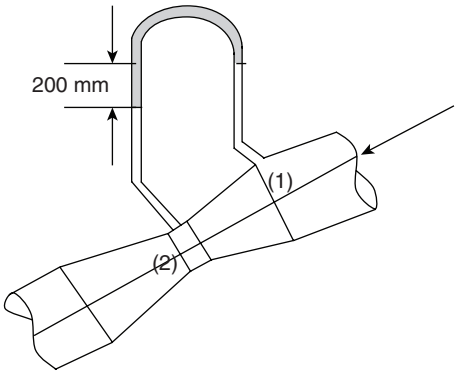
15. Mass flow rate of the oil in kg/s is

(A) 2.82 (B) 2.64
(C) 2.41 (D) 2.22

16. Power required to pump oil per 100 m length of pipe is

(A) 6.12 kw (B) 6.34 kw
(C) 6.63 kw (D) 6.82 kw

17. Water flows through an inclined pipe in which a venturimeter is installed for discharge measurement. The inlet and throat sections of the venturimeter have areas of cross sections 0.07 m^2 and 0.0177 m^2 respectively. An inverted U-tube manometer is used for measurement of differential pressure head. A liquid of specific gravity 0.7 is used in the manometer, which gives a reading of 250 mm. Inlet and throat sections have a level difference of 400 mm



Neglecting frictional losses the rate of flow through the pipe in m^3/s is

(A) 0.028 (B) 0.022
(C) 0.019 (D) 0.016

18. Vertical distance between the two orifices is

(A) 3.16 m (B) 2.82 m
(C) 2.25 m (D) 2.06 m

19. Vertical distance of point of intersection of the jets from the water level in the tank is

(A) 9.32 m (B) 10.25 m
(C) 10.78 m (D) 11.16 m

20. Velocity of kerosene at point *B* is

(A) 7.8 m/s (B) 6.7 m/s
(C) 5.6 m/s (D) 4.2 m/s

21. Pressure of kerosene at point *B* is

(A) –29.32 kPa (B) – 21.97 kPa
(C) –18.14 kPa (D) – 12.82 kPa

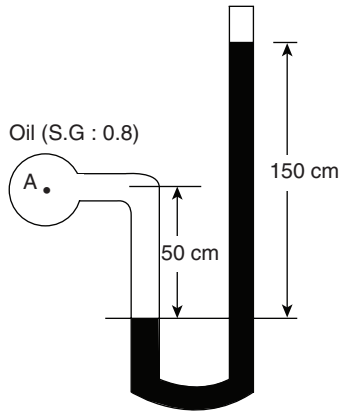
22. Match List – I with List – II and select the correct answer

	List – I (Forces)		**List – II (Dimensionless number)**
a.	Gravity force	1.	Weber number
b.	Pressure force	2.	Mach number
c.	Surface tension	3.	Froude's Number
d.	Elastic force	4.	Euler's number

Codes:

	a	b	c	d
(A)	1	2	4	3
(B)	4	3	2	1
(C)	3	4	1	2
(D)	2	1	3	4

23. A U-tube mercury manometer is used to measure pressure of oil flowing through a pipe at a point. Specific gravity of oil is 0.8 and the level of mercury is as shown in the figure. The pressure in kPa is



(A) 196.20 (B) 147.15
(C) 110.36 (D) 73.58

24. Match List – I with List – II and select the correct answer

	List – I (condition of floating bodies)		List – II (Result)
a.	M below G	1.	Floating body
b.	M above G	2.	Unstable equilibrium
c.	M and G coincides	3.	Stable equilibrium
d.	B below G	4.	Neutral equilibrium

[M - Meta centre
G - Centre of gravity
B - centre of buoyancy]

Codes:

	a	b	c	d
(A)	3	4	1	2
(B)	2	3	4	1
(C)	1	2	3	4
(D)	4	1	2	3

25. A glass tube of 3.7 mm diameter is dipped in water. If the contact angle at the meniscus is 0° and surface tension is 0.074 N/m determine the capillary effect in mm. (Take specific weight of water as 10000 N/m^2)

(A) 4 mm(dep) (B) 4 mm(rise)
(C) 8 mm(dep) (D) 8 mm (rise)

Answer Keys

1. A	**2.** A	**3.** D	**4.** C	**5.** C	**6.** B	**7.** D	**8.** B	**9.** C	**10.** B
11. C	**12.** B	**13.** C	**14.** A	**15.** A	**16.** C	**17.** B	**18.** C	**19.** B	**20.** D
21. B	**22.** C	**23.** A	**24.** B	**25.** D					

Hints and Explanations

7. $E_b g = 4\text{kg}\ (\rho_b - \rho_w)\ g = 3.5$ kg
$E_w g = 0.5$ kg

Specific gravity $= \dfrac{\rho_b}{\rho_w} = 8$ Choice (D)

8. $Q = \rho AV$
$V = (6xt + 3y^2)\ i + (3t + y^2)\ j = j$
$Q = 1000 \times 1 \times 1 = 1000$ kg/s. Choice (B)

11. Considering pressure balance on the horizontal plane 0 – 0

$p_A + 0.8 \times 9810 = p_B + (0.5 + 1.2) \times 9810 - 0.7 \times 0.75 \times 9810$

$p_A - p_B = 9810[(0.5 + 1.2) - 0.7 \times 0.75 - 0.8]$
$= 9810 \times 0.375 = 3679$ N/m^2. Choice (C)

12. Starting from pipe A the manometric equation is

$p_A + 4 \times 1.59 \times 9810 + h \times 13.6 \times 9810 - (h + 1) \times 0.8 \times 9810 = p_B$

$\dfrac{4fL}{D} \times \dfrac{v^2}{2g} = 6.36 + h \times 13.6 - 0.8h - 0.8$

$= 12.8\ h + 5.56$

$\dfrac{7}{40}\delta = 12.8h + 5.56$

$h = 0.083$ m $= 83$ mm. Choice (B)

13. For manometer,

$$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) - \left(\frac{s_m}{s_w} - 1\right) h_m = h$$

where h_m = the manometric level difference
Applying Bernoulli's equation,

$$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \frac{v_2^2 - v_1^2}{2g}$$

$$\frac{v_2^2 - v_1^2}{2g} = h$$

Also $A_1 v_1 = A_2 v_2$

The discharge equation for venturimeter

$Q = \dfrac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$ is derived from the above

$\therefore$ Irrespective of inclination h corresponding to manometric reading can be directly used in the above equation

$$h = h_m\left(\frac{s_m}{s_m}-1\right) = 0.12\left(\frac{13.6}{1}-1\right) = 1.512 \text{ m}$$

$C_d = 0.99$

$$A_1 = \frac{\pi}{4}(0.1)^2 = 7.85\times10^{-3}$$

$$A_2 = \frac{\pi}{4}(0.05)^2 = 1.96\times10^{-3}$$

$$Q = \frac{0.99\times7.85\times1.96\times10^{-6}}{10^{-3}\sqrt{7.58^2-1.96^2}}\times\sqrt{2\times9.31\times1.512}$$

$= 0.01091 \text{ m}^3/\text{s} = 10.91 \ \ell/\text{s}.$ Choice (C)

14. $d_1 = 15$ cm: $z_1 = 1$ m; $v_1 = 6$ m/s; $d_2 = 10$ cm; $z = 3$ m

$$v_2 = \frac{A_1}{A_2}v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{15}{10}\right)^2\times6 = 13.5 \text{ m/s}$$

Applying Bernoulli's theorem

$$\frac{p_1}{w}+\frac{v_1^2}{2g}+z_1 = \frac{p_2}{w}+\frac{v_2^2}{2g}+z_2$$

$$\frac{p_1}{w} = \frac{v_2^2-v_1^2}{2g}+(z_2-z_1)+\frac{p_2}{w}$$

$$= \frac{13.5^2-6^2}{2\times9.81}+(3-1)+(0.1\times13.6)$$

$$= 7.45 + 2 + 1.36 = 10.81 \text{ m of water} = \frac{10.81}{13.6}$$

$= 0.795$ m of mercury $= 79.5$ cm of mercury.

Choice (A)

15. $\mu = 1.2$ poise

$= 0.12 \text{ Ns/m}^2$

sp. gravity $= 0.85$

$\rho = 850 \text{ kg/m}^3$ $D = 0.03$ m

Pressure drop/m $= 20 \text{ kN/m}^2$

$$\Delta p = \frac{32\mu\bar{u}L}{D^2}$$

$$20\times10^3 = \frac{32\times0.12\times\bar{u}\times1}{(0.03)^2}$$

$\bar{u} = 4.6875$ m/s

$$\text{mass flow rate} = \rho A\ \bar{u} = (850)\times\frac{\pi}{4}\times(0.03)^2\times4.6825$$

$= 2.816$ kg/s. Choice (A)

16. Power required per metre

$p = w\,Q\,h_f = g(\rho Q)\,h_f$

where $\rho Q = \rho A\ \bar{u}$ = mass /s = 2.816

$$h_f = \frac{p_2-p_1}{\rho g} = \frac{20\times1000}{850\times9.81} = 2.4 \text{ m of oil}$$

$\therefore$ $P = 9.81\times2.816\times2.4 = 66.26$ W

power required per 100 m

$= 6626$ W $= 6.626$ kW. Choice (C)

17. Differential pressure head

$$h = h_1\left(1-\frac{s_l}{s_w}\right) = 0.25\left(1-\frac{0.7}{1.0}\right) = 0.075 \text{ m of water}$$

$$Q = \frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

$$= \frac{0.07\times0.0177\sqrt{2\times9.81\times0.075}}{\sqrt{0.07^2-0.0177^2}}$$

$= 0.022 \text{ m}^3/\text{s}.$ Choice (B)

18. horizontal distance $x = ut$

vertical distance $z = \frac{1}{2}gt^2$

$$u = \sqrt{2gH}$$

From the above,

$$z = \frac{x^2}{4H}$$

where H = height of water level above orifice

For the upper orifice,

$$z_1 = -\frac{x^2}{4\times4} = \frac{x^2}{16} = \frac{100}{16} = 6.25$$

For the lower orifice,

$$z_2 = \frac{100}{4(4+h)} = \frac{100}{4(4+h)} = \frac{25}{(4+h)}$$

$$z_1 - z_2 = h = 6.25 - \frac{25}{(4+h)} \text{ or}$$

$$h+\frac{25}{(4+h)} = 6.25$$

$(4 + h)\,h + 25 = 6.25\,(4 + h)$

$4h + h^2 + 25 = 25 + 6.25\,h$

$h^2 + 4h - 6.25\,h = 0$

$h - 2.25 = 0$

$h = 2.25$ m. Choice (C)

19. $$z_2 = \frac{25}{(4+h)} = \frac{25}{4+2.25} = 4$$

Vertical distance of water level from point of intersection of jets

$L = 4 + h + z_2 = 4 + 2.25 + 4 = 10.25$ m. Choice (B)

20 Velocity of kerosene at point B = velocity at C

Applying Bernoulli's theorem, between oil surface in the tank and pipe outlet

$$\frac{p_1}{w}+\frac{v_1^2}{2g}+z_1 = \frac{p_2}{w}+\frac{v_2^2}{2g}+z_2+h_{f(1-2)}$$

$$0+0+4=0+\frac{v_2^2}{2g}+1.5+(0.4+1.2)$$

$$\frac{v_2^2}{2g}=4-1.5-1.6\ =0.9\text{ m}$$

$$v_2^2=0.9\times 2\times 9.81$$

$v_2=4.2$ m/s $=v_B$. Choice (D)

21. Applying Bernoulli's equation at points 1 and B

$$0+0+4=\frac{p_B}{w}+\frac{(4.2)^2}{2g}+5.5+0.4$$

$$\frac{p_B}{w}=4-5.9-\frac{4.2^2}{2\times 9.81}\ =-2.8\text{ m}$$

$p_B=-2.8\times(9810\times 0.8)$

$=-21967$ N/m^2

$=-21.97$ kPa. Choice (B)

23. Equating pressure heads above the line passing through A

$$h_A+\frac{50}{100}\times 0.8=\frac{150}{100}\times 13.6$$

$\therefore\quad h_A+0.4=20.4$

$\Rightarrow\quad h_A=20$ m of water

Pressure $p_A=wh_A=9810\times 20$ N/m^2

$=9.81\times 20$ kN/m$^2=196.2$ kN/m^2

$=196.2$ kPa. Choice (A)

25. Capillary effect $h=\dfrac{4\sigma\cos\theta}{wd}$

where σ = surface tension, θ = contact angle

w = specific weight

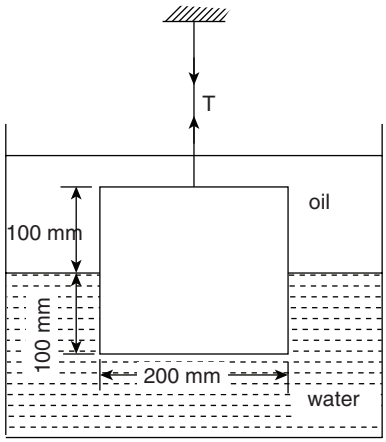
$$=\frac{4\times 0.074\times\cos 0^\circ}{10000\times 3.7\times 10^{-3}}=8\text{ mm(rise).}$$ Choice (D)

FLUID MECHANICS TEST 2

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. A metallic cube of side 200 mm and specific weight 26 kN/m^3 is suspended by a string in oil and water as shown in the figure. Half of the cube is submerged in water and the remaining half is submerged in oil. If specific gravity of the oil is 0.8, determine the tension in the string



(A) 98.2 N (B) 176.6 N
(C) 156.7 N (D) 192.4 N

2. In a 3-dimensional incompressible fluid flow, velocity components in x and y directions are
$u = x^2 + y^2z^3$
$v = -(xy + yz + zx)$
Velocity component in the z direction is

(A) $-xz + \frac{z^2}{2} + f(x, y)$ (B) $-xz + \frac{z^2}{2}$

(C) $xz - \frac{z^2}{2} + C$ (D) $-x + z$

3. Match List – I (measuring devices) with List – II (measuring parameter) and select the correct answer using the codes given below

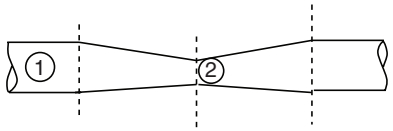
	List – I		List – II
a.	Pitot tube	1.	Rate of flow measurement
b.	Micro manometer	2.	Measurement of moderate pressure
c.	Venturimeter	3.	Velocity measurement
d.	Piezo meter	4.	Easier measurement of large pressures

Codes:

	a	b	c	d		a	b	c	d
(A)	1	3	2	4	(B)	4	2	3	1
(C)	2	1	4	3	(D)	3	4	1	2

4. A curved surface is submerged in a fluid. Consider the following statements relating to it
I. Vertical component of the hydrostatic force acting on the surface is equal to the weight of the fluid vertically above the surface
II. Horizontal component of the force acting on the curved surface is the hydrostatic force acting on the vertical projection of the curved surface
III. Horizontal component of the force acts through the centre of gravity of the vertical projection of the curved surface.
(A) I and II are correct
(B) I and III are correct
(C) II and III are correct
(D) I, II and III are correct

5. In a horizontal pipeline as shown in figure, point 2 is a contraction with reduced area of cross section. At point 1 the pressure head and velocity head are 60 cm and 4 cm respectively. If pressure head at point 2 is zero, the ratio of velocity at point 2 to that at point 1 is



(A) 2 (B) 3
(C) 4 (D) 6

6. For a flow, velocity components in the x and y directions are given by $u = y^2$, $v = -3x$
Component of rotation about the z-axis is
(A) $-(3 + 2y)$ (B) $(3 + 2y)$
(C) $\frac{-1}{2}(3+2y)$ (D) $\frac{1}{2}(3+2y)$

7. The velocity along the centre line of a nozzle of length 1.5 m is given by

$$v = 2t\left(1 - \frac{x}{2\ell}\right)^2 \text{ where } \ell = \text{length in m}$$

v = velocity in m/s,
t = time in seconds
from the commencement of flow and x = distance from inlet. The value of local acceleration at x = 1 m when t = 5 seconds is
(A) 0.67 m/s^2 (B) 0.89 m/s^2
(C) 1.33 m/s^2 (D) 1.67 m/s^2

8. Differential pressure head measured by a mercury oil differential manometer is 9.5 m of oil. If specific gravity of oil is 0.68, difference in level of mercury is
(A) 300 mm (B) 400 mm
(C) 500 mm (D) 600 mm

9. An orifice meter is calibrated with air in a geometrically similar model. Model to prototype scale ratio is $\frac{1}{4}$. The prototype has to carry water. Ratio of kinematic viscosity of air to water is 12.5. Dynamically similar flow will be obtained when the discharge ratio is

(A) 2.850 (B) 3.125
(C) 4.540 (D) 4.925

10. For a flow, the stream function is ψ. For the flow to be irrotational, the condition to be satisfied is

(A) $\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{dy} = 0$ (B) $\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 0$

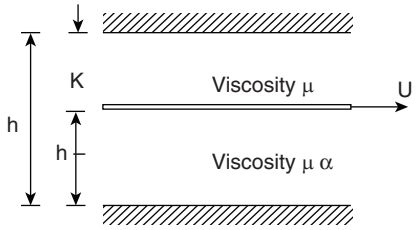
(C) $\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} = 0$ (D) $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

Common Data for Questions 11 and 12:
A liquid of viscosity 0.8 and sp. gravity 1.3 flows through a circular pipe of 100 mm diameter. Maximum shear stress at the pipe wall is 220 N/m^2

11. Pressure gradient of the flow in N/m^2 per m is
(A) – 6800 (B) – 8800
(C) 6800 (D) 8800

12. Average velocity of flow is
(A) 2.6 m/s (B) 2.9 m/s
(C) 3.2 m/s (D) 3.4 m/s

Statement for Linked Answer Questions 13 and 14:



A large thin plate is pulled at a constant velocity U through a narrow gap of height h. On one side of the plate is filled with oil of viscosity μ and the other side oil of viscosity αμ where α is a constant

13. Total drag force on the plate is

(A) $A\mu U\left(\frac{1}{k} + \frac{\alpha}{h-k}\right)$ (B) $A\mu U\left(k + \frac{h-k}{\alpha}\right)$

(C) $\frac{\mu U}{A}\left(\frac{1}{k} + \frac{\alpha}{h-k}\right)$ (D) $\frac{\mu U}{A}\left(k + \frac{h-k}{\alpha}\right)$

14. Value of k such that the drag force is minimum is

(A) $\frac{\sqrt{h}}{1+\alpha}$ (B) $\frac{\sqrt{h}}{1-\alpha}$

(C) $\frac{h}{1-\sqrt{\alpha}}$ (D) $\frac{h}{1+\sqrt{\alpha}}$

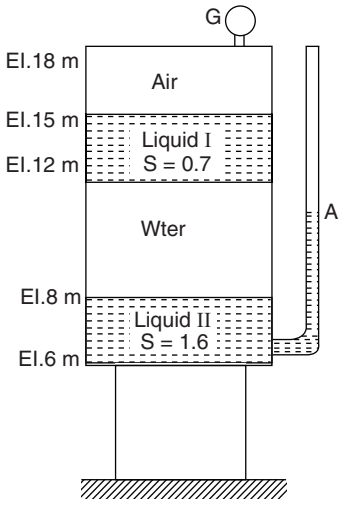
Common Data for Questions 15 to 17:
An 80 mm diameter composite solid cylinder consists of a 20 mm thick metallic plate and 650 mm long wooden cylinder of specific gravity 4 and 0.8 respectively. The cylinder floats in water its axis vertical

15. The position of centre of gravity from bottom is
(A) 0.3 m (B) 0.35 m
(C) 0.4 m (D) 0.45 m

16. Position of centre of buoyancy from bottom is
(A) 0.2 m (B) 0.25 m
(C) 0.3 m (D) 0.35 m

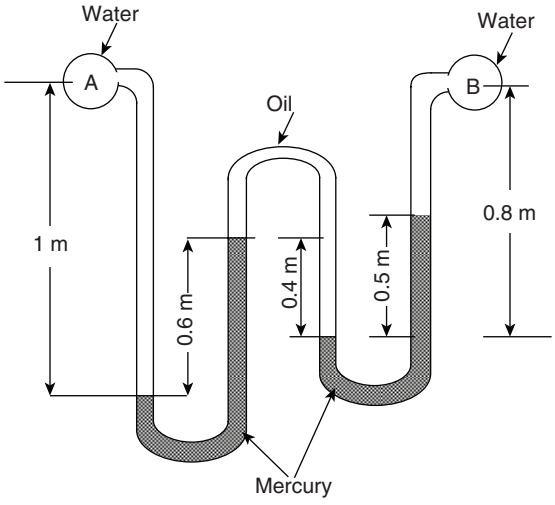
17. Metacentric height is
(A) 0.667 mm (B) 6.67 mm
(C) 3.33 mm (D) 8.63 mm

18.



A tank installed at an elevation of 6 m contains a liquid of specific gravity 1.6, water and another liquid of specific gravity 0.7 space over the liquids contain air. The gauge G show a pressure of –17 kN/m^2. The elevation of liquid level in the piezometer A is
(A) 4.73 m (B) 6.73 m
(C) 8.73 m (D) 10.73 m

19.

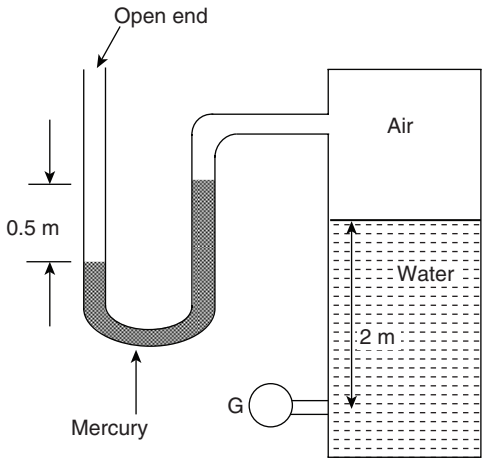


For the compound manometer shown in figure, the pressure difference between points A and B in kN/m^2 is ————————

Given that specific gravity of mercury = 13.6 and specific gravity of oil = 0.85

(A) 115 (B) 125
(C) 135 (D) 150

Statement for Linked Answer Question 20:



Refer to the figure given above. The tank is filled with water upto 2 m from the gauge G. The manometer shows a level difference of 0.5 m as shown. Local atmospheric pressure is 750 mm of mercury.

20. Absolute pressure of air above the water surface in the tank is

(A) 21.68 kN/m^2 (B) 33.35 kN/m^2
(C) 38.72 kN/m^2 (D) 42.83 kN/m^2

Common Data for Questions 21 and 22:

A propeller turbine is to develop 6250 kW under a head of 5m, having given that speed ratio 'k_u' based on outer diameter = 2.10, flow ratio $\psi = 0.65$, diameter of boss = 0.35 times external diameter of the runner and overall efficiency is 85%.

21. The diameter of the runner in 'm' is

(A) 5.81 (B) 4.91
(C) 5.21 (D) 6.35

22. The speed of the turbine in RPM is

(A) 78.27 (B) 68.37
(C) 58.35 (D) 48.22

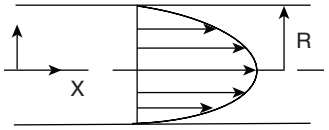
23. Draft tube is a pipe used in

(A) reaction turbine for discharge and it has gradually decreasing cross sectional area
(B) reaction turbine for discharge and it has gradually increasing cross sectional area
(C) Impulse turbine for discharge and it has gradually decreasing cross sectional area
(D) Impulse turbine for discharge and it has gradually increasing cross section area.

24. The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression

$$u(r) = \frac{-R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left(\frac{1-r^2}{R^2}\right)$$

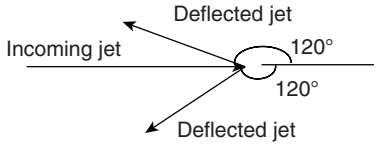
Where $\frac{\partial p}{\partial x}$ is a constant



The average velocity of fluid in the pipe is

(A) $\frac{-R^2}{8\mu}\left(\frac{dp}{dx}\right)$ (B) $\frac{-R^2}{4\mu}\left(\frac{dp}{dx}\right)$
(C) $\frac{-R^2}{2\mu}\left(\frac{dp}{dx}\right)$ (D) $\frac{-R^2}{\mu}\left(\frac{dp}{dx}\right)$

25. Water having a density of 1000 kg/m^3, issues from a nozzle with a velocity of 10 m/s and the jet strikes a bucket mounted on a petton wheel. The wheel rotates at 10 rad/s. The mean diameter of the wheel is 1m. The jet is split into two equal streams by the bucket, such that each stream is deflected by 120°, as shown in the figure. Friction in the bucket may be neglected. Magnitude of the torque exerted by the water on the wheel, per unit mass flow rate of the incoming jet is.



(A) 0 (Nm) / (Kg/s) (B) 1.25 (Nm) / (Kg/s)
(C) 2.5 (Nm) / (Kg/s) (D) 3.75 (Nm) / (Kg/s)

26. Match the following:

	List – I		List – II
P.	Compressible flow	1.	Nusselt number
Q.	Boundary layer flow	2.	Reynold's number
R.	Pipe flow	3.	Skin friction coefficient
S.	Heat convection	4.	Mach number

Codes:

	P	Q	R	S		P	Q	R	S
(A)	3	1	4	2	(B)	3	4	2	1
(C)	4	3	2	1	(D)	2	1	3	4

Common Data for Questions 27 and 28:

A pelton wheel has to be designed for the following data. Power to be developed = 6000 kW net head available = 300 m, speed = 550 RPM, ratio of jet diameter to wheel diameter = $\frac{1}{10}$ and overall efficiency = 85%

27. Find the no. of jets required
(A) 1 (B) 2
(C) 3 (D) 4

28. The diameter of jet is in mm
(A) 135.2
(B) 116.4
(C) 141.2
(D) 186.4

29. The maximum hydraulic efficiency for a pelton wheel is given by the expression (θ is the vane angle of outlet).
(A) $\frac{1-\cos\theta}{2}$ (B) $\frac{1+\cos\theta}{2}$
(C) $\frac{\cos\theta}{2}$ (D) None of these

30. Incase of Pelton turbine installed in a hydraulic power plant the gross head available is the vertical distance between
(A) forebay and tailrace
(B) reservoir level and turbine inlet
(C) forebay and turbine inlet
(D) reservior level and tail race

Answer Keys

1. B	**2.** A	**3.** D	**4.** A	**5.** C	**6.** C	**7.** B	**8.** C	**9.** B	**10.** D
11. B	**12.** D	**13.** A	**14.** D	**15.** A	**16.** C	**17.** A	**18.** D	**19.** C	**20.** B
21. A	**22.** B	**23.** B	**24.** A	**25.** D	**26.** C	**27.** C	**28.** B	**29.** B	**30.** B

Hints and Explanations

1. Tension in the string $T = W - F_B$
where W = weight of the cube
F_B = Buoyant forces
$\therefore \quad T = (0.2)^3 \times 26 \times 10^3 - [(0.2)^3 \times 0.5 \times 9810 \times 0.8]$
$= 26 \times 8 - 4 \times 9.81 \times 0.8 = 176.6$ N. Choice (B)

2. For a three dimensional, incompressible fluid flow the continuity equation must be satisfied

i.e., $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

$u = x^2 + y^2z^3$

$\frac{\partial u}{\partial x} = 2x$

$v = -(xy + yz + zx)$

$\frac{\partial v}{\partial y} = -(x+z)$

$\therefore \quad 2x - (x + z) + \frac{\partial w}{\partial z} = 0$

$\Rightarrow \quad \frac{\partial w}{\partial z} = -2x + x + z = -x + z$

Integrating with respect to z

$w = -xz + \frac{z^2}{2} + C$ where $C = f(x, y)$

or $w = -xz + \frac{z^2}{2} + f(x, y)$. Choice (A)

4. Statements I and II are correct
The horizontal component acts at a distance from top surface = $\bar{x} + \frac{I_G}{A\bar{x}}$ where $\bar{x}$ = the vertical distance of centre of gravity from surface Choice (A)

5. Applying Bernoullis theorem,
velocity head + pressure head = constant

i.e., $\frac{V_1^2}{2g}+\frac{p_1}{\rho g}=\frac{V_2^2}{2g}+\frac{p_2}{\rho g}$

$\therefore \quad 0.04 + 0.6 = \frac{V_2^2}{2g} + 0$

$\Rightarrow \quad \frac{V_2^2}{2g} = 0.64$

$\frac{\frac{V_2^2}{2g}}{\frac{V_1^2}{2g}} = \left(\frac{V_2}{V_1}\right)^2 = \frac{0.64}{0.04} = 16$

$\Rightarrow \quad \frac{V_2}{V_1} = 4$. Choice (C)

6. $u = y^2$, $v = -3x$
Rotation component about z-axis is

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) = \frac{1}{2}(-3-2y) = -\frac{1}{2}(3+2y)$

Choice (C)

7. $V = 2t\left(1-\frac{x}{2\ell}\right)^2$

Local acceleration $\frac{\partial V}{\partial t} = \left(1-\frac{x}{2\ell}\right)^2 \times 2$

$= \left(1-\frac{1}{2\times 1.5}\right)^2 \times 2 = \left(1-\frac{1}{3}\right)^2 \times 2$

$= \left(\frac{2}{3}\right)^2 \times 2 = \frac{8}{9} = 0.89$ m/s^2 Choice (B)

8. $h = y\left(\frac{S_m}{S_o} - 1\right)$

$9.5 = y\left(\frac{13.6}{0.68} - 1\right) = y \times 19$

$\Rightarrow \quad y = \frac{9.5}{19} = 0.5 \text{ m} = 500 \text{ mm}.$ Choice (C)

9. Scale ratio $\frac{L_m}{L_p} = \frac{1}{4}$

Ratio of kinematic viscosity

$\frac{\upsilon_{air}}{\upsilon_{water}} = \frac{\upsilon_m}{\upsilon_p} = 12.5$

For dynamically similar flow, Reynold number will be same

i.e., $(\text{Re})_m = (\text{Re})_p$

$\text{Re} = \frac{\rho VL}{\mu} = \frac{VL}{\upsilon}$

$\therefore \quad \frac{V_m L_m}{\vartheta_m} = \frac{V_p L_p}{\vartheta_p}$

$\Rightarrow \quad \frac{V_m}{V_p} = \frac{L_p}{L_m} \cdot \frac{\upsilon_m}{\upsilon_p} = 4 \times 12.5 = 50$

Discharge ratio $\frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)^2 \times \frac{V_m}{V_p}$

$= \left(\frac{1}{4}\right)^2 \times 50 = 3.125$ Choice (B)

10. If Ψ is stream function

$\frac{\partial \psi}{\partial x} = v$ and $\frac{\partial \psi}{\partial y} = -u$

For irrotational flow, $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

i.e., $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ Choice (D)

11. $\mu = 8$ poise $= 0.8$ Ns/m^2, sp. gravity $= 1.3$
$D = 100$ mm $= 0.1$ m, $\tau_o = 220$ N/m^2

$\tau_o = \frac{-\partial p}{\partial x} \times \frac{R}{2}$

$220 = \frac{-\partial p}{\partial x} \times \frac{0.1}{2 \times 2}$

$\therefore$ pressure gradient

$\frac{\partial p}{\partial x} = -8800$ N/m^2 per m. Choice (B)

12. Average velocity

$\bar{u} = \frac{1}{2} u_{\max} = \frac{1}{2}\left[\frac{-1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot R^2\right] = \frac{1}{8 \times 0.8} \times 8800 \times \left(\frac{0.1}{2}\right)^2$

$= 3.4375$ m/s. Choice (D)

13. $\tau_1 = \mu_1 \frac{du}{dy} = \mu \frac{U}{k}$

$\tau_2 = \mu_2 \frac{du}{dy} = \alpha\mu \frac{U}{h-k}$

Total drag force $F = A(\tau_1 + \tau_2) = A\ \mu\ U \left(\frac{1}{k} + \frac{\alpha}{h-k}\right)$.

Choice (A)

14. Drag force is minimum when $\frac{dF}{dk} = 0$

$\frac{-1}{k^2} + \frac{\alpha}{(h-k)^2} = 0$

$\frac{\alpha}{(h-k)^2} = \frac{1}{k^2}$

$\frac{\sqrt{\alpha}}{h-k} = \frac{1}{k}$

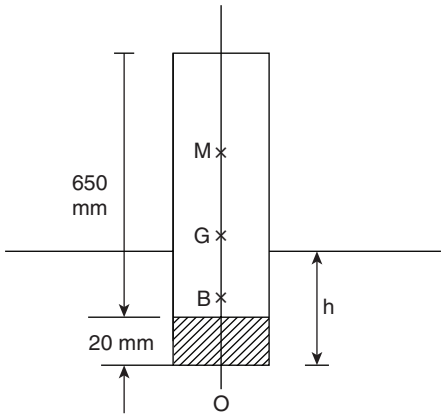
$k\sqrt{\alpha} = h - k$

$k + k\sqrt{\alpha} = h$

$k\left(1 + \sqrt{\alpha}\right) = h$

$k = h/(1 + \sqrt{\alpha})$. Choice (D)

15.



Position of centre of gravity

$OG = \frac{A\left[0.02 \times 4 \times 0.01 + 0.65 \times 0.8\left(0.02 + \frac{0.65}{2}\right)\right]}{A(0.02 \times 4 + 0.65 \times 0.8)}$

where A = area of cross section

$= \frac{8 \times 10^{-4} + 0.1794}{0.6} = 0.3$ m Choice (A)

16. Weight of cylinder = buoyancy force
$A\ (0.02 \times 4 + 0.65 \times 0.8) = A \times h \times 1$
$h = 0.6$ m

Position of centre of buoyancy $OB = \frac{h}{2} = 0.3$ m

Choice (C)

17. BM $= \frac{I}{V} = \frac{\pi}{64}\frac{(0.08)^4 \times 4}{\pi \times (0.08)^2 \times 0.6} = \frac{(0.08)^2}{16 \times 0.6} =$

6.67×10^{-4} m

$BG = OG - OB = 0.3 - 0.3 = 0$

$MG = BM - BG = 6.67 \times 10^{-4}$ m

$= 0.667$ mm Choice (A)

18. Air pressure $= -17$ kN/m^2

$= -17000$ N/m$^2 = \frac{-17000}{9810} = -1.733$ m of water

Pressure at the bottom of tank in metres of water

$= -1.733 + (15 - 12) \times 0.7 + (12-8) \times 1 + (8 - 6) \times 1.6$

$= 7.567$ m of water

In metres of liquid II, pressure $= \frac{7.567}{1.6} = 4.73$ m

Elevation of level in piezometer $= 4.73 + 6$

$= 10.73$ m. Choice (D)

19. Starting from point A and forming the manometric equation, in metres of water

$$\frac{p_A}{w} + 1 - 0.6\, S_m + 0.4\, S_o - 0.5\, S_m - (0.8 - 0.5) = \frac{p_B}{w}$$

Where w = sp. weight of water

S_m = sp. gravity of mercury

S_o = sp. gravity of oil

i.e., $\frac{p_A - p_B}{w} = (0.6 + 0.5)\, 13.6 - 0.4 \times 0.85 - 1 + 0.3$

$= 13.92$

$p_A - p_B = 13.92 \times 9810$ N/m$^2 = 136.56$ kN/m^2

Choice (C)

20. Atmospheric pressure

$p_{atm} = \frac{1}{2}\rho A U^2 \times 13.6 \times 9810 = 100062$ N/m^2

$p_{air} = 0 - 0.5 \times 13.6 \times 9810 = -66708$ N/m^2

Absolute pressure of air

p_{air} (abs) $= p_{atm} + p_{air} = 100062 - 66708$

$= 33354$ N/m$^2 = 33.354$ kN/m^2. Choice (B)

21. $\eta_0 = 0.85 = \frac{6250 \times 10^3}{9810 \times Q \times 5}$

$Q = 149.91$ m^3/sec

$0.65 = \frac{v_f}{\sqrt{2 \times 9.81 \times 5}}$

$V_f = 6.44$ m/s

$Q = 149.91 = \frac{\pi}{4}\left[D^2 - (0.35D)^2\right] \times 6.44$

$D = 5.81$ m Choice (A)

22. $K_u = 2.1 = \frac{u}{\sqrt{2 \times 9.81 \times 5}}$

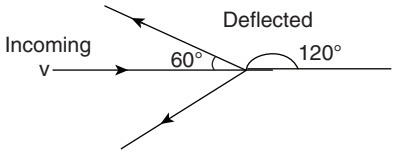
$u = 20.8$ m/s

$u = \frac{\pi DN}{60}$

$20.8 = \frac{\pi \times 5.81 \times N}{60}$

$N = 68.37$ RPM Choice (B)

25.



$\phi = 60°$, $K = 1$ (No friction)

$V = 10$ m/s

Wheel speed $= u = 10 \times \frac{1}{2} = 5$ m/s

Work done per sec by wheel $= \frac{w}{g}(v - u)(1 + k\cos\phi)\, u$

Power $= \frac{mg}{g}(v - u)(1 + k\cos\phi)\, u$

$P = T\omega$

$\frac{T}{m} = \frac{(v-u)(1+k\cos\phi)u}{\omega} = \frac{(10-5)(1+\cos 60°)5}{10}$

$= 3.75$ Nm / (kg/s) Choice (D)

27. $K_v = 0.98$, $K_u = 0.46$

$V = k_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.8 \times 300} = 75.19$ m/sec

$u = k_u\sqrt{2gh} = 0.46\sqrt{2 \times 9.8 \times 300} = 35.29$ m/s

$\eta_0 = \frac{p}{WQH}$

$0.85 = \frac{6000 \times 10^3}{9810 \times Q \times 300}$

$Q = 2.39$ m^3/s

$u = \frac{\pi DN}{60} = \frac{\pi \times D \times 550}{60}$

$D = 122.5$ m

$\frac{d}{D} = \frac{1}{10}$

$d = 0.1225$ m $= 122.5$ m

$A = \frac{\pi}{4}(0.1225)^2 = 0.0118$ m^2

Total jet area required $= \frac{Q}{V} = \frac{2.399}{75.19} = 0.0319$ m^2

No. of jets required $= \frac{0.0319}{0.0118} = 2.7 = 3$ Choice (C)

28. $d = \left[\frac{0.0319}{3\left(\frac{\pi}{4}\right)}\right]^{\frac{1}{2}} = 0.1164$ m $= 116.4$ mm Choice (B)

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HYDRAULICS TEST 3

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. In a fluid the velocity field is given by

$V = (3x + 2y)\, i + (2z + 3x^2)\, j + (2t - 3z)\, k$

The velocity at point (1, 1, 1) at time 2 second is

(A) 6.82 units (B) 7.14 units

(C) 7.93 units (D) 8.26 units

Statement for Linked Answer Questions 2 and 3:

The velocity along the centre line of a nozzleof length ℓ is given by,

$$V = 2\,t\left(1 - \frac{x}{2\ell}\right)^2$$

where V = velocity in m/s, t = time in seconds,

x = distance from inlet of the nozzle

2. Convective acceleration when x = 1 m and ℓ = 1.5 m and t = 5 seconds is

(A) -19.75 m/s^2 (B) 19.75 m/s^2

(C) 26.35 m/s^2 (D) -26.35 m/s^2

3. Total acceleration for the same conditions as above is

(A) 20.64 m/s^2 (B) -20.64 m/s^2

(C) -18.86 m/s^2 (D) 18.86 m/s^2

4. Velocity distribution in the boundary layer of fluid flow over a surface is given by

$$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}$$

When δ = the boundary layer thickness

U = maximum velocity of fluid

u = fluid velocity at y

The ratio of displacement thickness to boundary layer thickness is

(A) $\frac{5}{12}$ (B) $\frac{7}{12}$

(C) $\frac{5}{7}$ (D) $\frac{3}{7}$

5. A Kaplan turbine develops 20000 kW at an average head of 35 m. Assuming a speed ratio of 2, flow ratio of 0.6 and overall efficiency of 90% and taking boss to runner diameter ratio as 0.35, speed of the turbine is

(A) 365 rpm (B) 388 rpm

(C) 409 rpm (D) 418 rpm

6. A liquid is flowing between two parallel plates. One plate is moving relative to other with a velocity of 4 m/s in the negative direction. If pressure gradient $\frac{\partial p}{\partial x} = -100 \times 10^6$ N/m^3, viscosity μ = 0.4 poise and distance between the plates is 1 m, discharge per meter width is

(A) 182.5 L/s (B) 190.2 L/s

(C) 206.8 L/s (D) 242.4 L/s

7. A moving vane with velocity 20 m/s having an inlet angle zero degree and an outlet angle 25° receives water from a jet at a velocity of 48 m/s. Assuming mass flow rate of 1kg/s, the force acting on the vanes and its inclination is.

(A) 62.16 N, 15°

(B) 58.72N, 13.6°

(C) 54.68 N, 12.5°

(D) 50.25N, 10.4°

8. In the model study of a motor boat in a lake, 1 : 25 scale model is used. It is assumed that viscous resistance due to water and air is negligible compared to the wave resistance. If the speed of the model is 2 m/s, determine the speed of the prototype for dynamically similar conditions

(A) 8 m/s (B) 10 m/s

(C) 12 m/s (D) 15 m/s

9. A pipe line through which oil flows has a sudden expansion in it such that maximum pressure rise occurs. Energy loss in the sudden expansion in metres of oil is given by (V_1 = velocity before expansion)

(A) $\frac{V_1^2}{4g}$ (B) $\frac{V_1^2}{6g}$

(C) $\frac{V_1^2}{8g}$ (D) $\frac{V_1^2}{2\sqrt{2}g}$

10. Prandtl's velocity distribution for a boundary layer is given by

(A) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

(B) $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

(C) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$

(D) $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$

11. Specific speed of a hydraulic pump is the speed of geometrically similar pump working against unit head and

(A) consuming unit power

(B) having unit velocity of flow

(C) having unit radial velocity

(D) delivering unit quantity of water

12. Match List – I with List – II and select correct answer using the codes given

	List – I		List – II
a.	Pelton turbine (single jet)	1.	Mediumdischarge, low head
b.	Francis turbine	2.	High discharge, low head
c.	Kaplan turbine	3.	medium head
		4.	Low discharge high head

a b c
(A) 1 2 3
(B) 4 3 2
(C) 2 3 4
(D) 3 4 1

13. Efficiency of the turbine is least under part load condition in the case of *a*
(A) Pelton turbine
(B) Francis turbine
(C) Kaplan turbine
(D) Propeller turbine

14. Cavitation in a hydraulic turbine is most likely to occur at the turbine
(A) entry
(B) exit
(C) stator exit
(D) rotor exit

15. A hydraulic turbine develops 1000 kW power for a head of 40 m. If the turbine is to work under a head of 20 m the power developed in kW is
(A) 250
(B) 500
(C) $\frac{500}{\sqrt{8}}$
(D) $\frac{100}{\sqrt{8}}$

16. A water turbine delivering 16 MW power is to be tested with the help of a geometrically similar model of 1 : 4 scale. If the speed of the model is same as that of the prototype then determine, power developed by the model, assuming same efficiencies for model and prototype
(A) $\frac{800}{32}$ kW
(B) $\frac{1000}{128}$ kW
(C) $\frac{1000}{64}$ kW
(D) $\frac{500}{64}$ kW

17. A jet of water of cross sectional area '*a*' strikes on a series of flat plates mounted on a wheel. If the jet velocity is *v* and velocity of the plates is *u*, the force exerted on plate is
(A) $\rho a(v-u)^2$
(B) $\rho a v^2 (v-u)$
(C) $\rho a(v-u)^2 u$
(D) $\rho a v(v-u)$

18. A jet issuing from nozzle with diameter of 180 mm has a velocity of 90 m/s and strikes the pelton wheel and gives a shaft power of 7500 kW. If the coefficient of velocity of nozzle is 0.98, the overall efficiency of the turbine is
(A) 80%
(B) 77.65%
(C) 90%
(D) 78.83%

19. In two pelton wheels, the first wheel deflects the water through 160° and the other through 170°. The ratio of the maximum efficiencies of the first and second wheels is (all other operating parameters being same for both)
(A) 1.023
(B) 0.966
(C) 1.035
(D) 0.977

Common Data for Questions 20 and 21:
A pelton wheel develops 5520 kW under a head of 240 m at an overall efficiency of 80% when revolving at a speed of 200 rpm.

20. The unit discharge is
(A) 0.172
(B) 0.189
(C) 0.135
(D) 0.212

21. The unit power is
(A) 1.645
(B) 1.581
(C) 1.485
(D) 1.321

Common Data for Questions 22 and 22:
A 10 cm diameter water jet having a velocity of 45 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet.

22. What will be the normal force on the plate if the plate is stationary?
(A) 5.62 kN
(B) 11.25 kN
(C) 15.9 kN
(D) Data insufficient

23. What will be the normal force on the plate if the plate is moving towards the jet with a velocity of 15 m/s
(A) 5 kN
(B) 10 kN
(C) 20 kN
(D) 30 kN

Common Data for Questions 24 and 25:
A pipe line 1200 m long supplies water to 3 single jet pelton wheels. The head above the nozzle is 360 m. The velocity coefficient for the nozzle is 0.98 and coefficient of friction for the pipeline is 0.02. The turbine efficiency based on the head at the nozzle is 0.85. The specific speed of each turbine is 15.3 and the head loss due to friction in the pipeline is 12 m of water. If the operating speed of each turbine is 560 rpm.

24. The total power developed by the turbine in kW
(A) 6069
(B) 5059
(C) 7079
(D) 4079

25. The diameter of the nozzle in mm
(A) 45.7
(B) 95.6
(C) 76.2
(D) 81.9

Answer Keys

1. B	**2.** A	**3.** C	**4.** A	**5.** C	**6.** C	**7.** C	**8.** B	**9.** C	**10.** B
11. D	**12.** B	**13.** D	**14.** D	**15.** D	**16.** C	**17.** D	**18.** B	**19.** D	**20.** B
21. C	**22.** B	**23.** C	**24.** B	**25.** B					

Hints and Explanations

1. The velocity components are

$u = 3x + 2y$

$v = 2z + 3x^2$

$w = 2t - 3z$

Substituting,

$x = 1, y = 1, z = 1$ and $t = 2$

$u = 3 + 2 = 5$

$v = 2 + 3 = 5$

$w = 2 \times 2 - 3 = 1$

$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{5^2 + 5^2 + 1^2} = \sqrt{51}$

$= 7.14$ units. Choice (B)

2. Convective acceleration

$$= v\frac{\partial v}{\partial x} = 2t\left(1 - \frac{x}{2\ell}\right)^2 \times 2t \times 2\left(1 - \frac{x}{2\ell}\right)\left(-\frac{1}{2\ell}\right)$$

$$= -\frac{4t^2}{\ell}\left(1 - \frac{x}{2\ell}\right)^3$$

Given that $\ell = 1.5$ m

$x = 1$ and $t = 5$ seconds

$$\text{convective acceleration} = \frac{-4 \times (5)^2}{1.5}\left(1 - \frac{1}{2 \times 1.5}\right)^3$$

$= -19.75$ m/s^2. Choice (A)

3. Total acceleration

= Convective acceleration + local acceleration

$$\text{local acceleration} = \frac{\partial v}{\partial t} = \left(1 - \frac{x}{2\ell}\right)^2 \times 2$$

$$= \left(1 - \frac{1}{2 \times 1.5}\right)^2 \times 2$$

$= 0.889$ m/s^2

Total acceleration $= -19.75 + 0.889 = -18.86$ m/s^2. Choice (C)

4. $\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}$

Displacement thickness

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right)dy = \int_0^\delta 1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\frac{y^2}{\delta^2}$$

$$= \left[y - \frac{3}{2}\frac{y^2}{2\delta} + \frac{1}{2}\cdot\frac{y^3}{3\delta^2}\right]_0^\delta = \delta - \frac{3}{4}\delta + \frac{\delta}{6} = \frac{5}{12}\delta$$

$$\frac{\delta^*}{\delta} = \frac{5}{12}\frac{\delta}{\delta} = \frac{5}{12}.$$ Choice (A)

5. $u_1 = k_u\sqrt{2gH} = 2\sqrt{2 \times 9.81 \times 35} = 52.4$ m/s

$v_{f1} = k_f\sqrt{2gH} = 0.6\sqrt{2 \times 9.81 \times 35} = 15.7$ m/s

$$\frac{\text{shaft power}}{wQH} = \text{overall } \eta$$

i.e., $\frac{20000 \times 10^3}{9810 \times Q \times 35} = 0.9$

$Q = 64.72$ m^3/s

But $Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times v_{f1}$ or

$$64.72 = \frac{\pi}{4}D_o^2\left(1 - 0.35^2\right) \times 15.7$$

$D_o = 2.45$ m

$u_1 = \frac{\pi D_o N}{60}$ where N = speed of turbine

i.e., $52.4 = \frac{\pi \times 2.45 \times N}{60}$

$N = 409.2$ rpm. Choice (C)

6. $U = -3$ m/s; $\frac{dp}{dx} = -100 \times 10^6$ N/m^3

$\mu = 0.4$ poise $= 0.04$ Ns/m^2

$b = 1$ mm

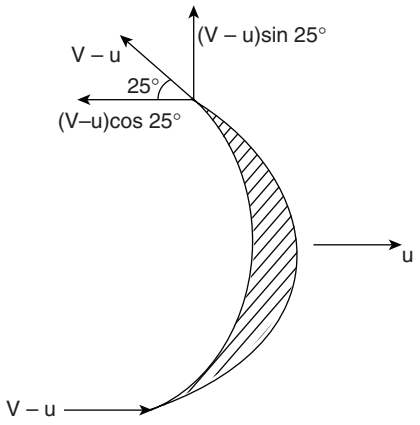
Discharge per unit width is given by,

$$q = U\frac{b}{2} - \frac{b^3}{12\mu}\cdot\frac{\partial p}{\partial x}$$

$$= (-3) \times \frac{0.001}{2} - \frac{(0.001)}{12 \times 0.04} \times (-100 \times 10^6)$$

$= 0.2068$ m^3/s $= 206.8$ L/s Choice (C)

7.



$u = 20$ m/s

$v_{1x} = (v - u) = 48 - 20 = 28$ m/s

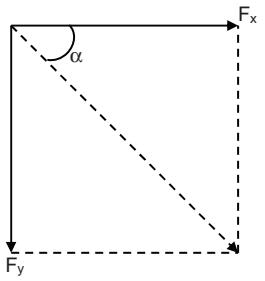
$v_{1y} = 0$

$v_{2x} = -(v - u)\cos 25 = -28\cos 25 = -25.38$ m/s

$v_{2y} = (v - u)\sin 25 = 28 \sin 25 = 11.83$ m/s

$F_x = 1\ x\ [v_{1x} - v_{2x}] = 28 - (-25.38) = 53.38$ m/s

$F_y = 1\ x\ [v_{1y} - v_{2y}] = 0 - 11.83 = -11.83$ m/s



Resultant force

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{53.38^2 + (-11.83)^2} = 54.68 \text{ N}$$

$$\tan\alpha = \frac{F_y}{F_x} = \frac{11.83}{53.38} = 0.2216$$

$\alpha = 12.5°$. Choice (C)

8. Since wave resistance is dominant, dynamic similarity will be attained when Froude number is same for model and prototype

$$\text{i.e., } \left(\frac{V}{\sqrt{Lg}}\right)_m = \left(\frac{V}{\sqrt{Lg}}\right)_p \Rightarrow \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{25} = 5$$

$\Rightarrow V_p = 2 \times 5 = 10$ m/s Choice (B)

9. Maximum pressure rise occurs when $\frac{D_1}{D_2} = \frac{1}{\sqrt{2}}$ or

$D_2 = \sqrt{2}\ D_1$

$\Rightarrow A_2 = 2A_1 \because A \propto D^2$

$\Rightarrow V_2 = \frac{V_1}{2}$

$\because A_1V_1 = A_2V_2 = Q$

Energy (head) loss in sudden expansion

$$= \frac{(V_1 - V_2)^2}{2g} = \frac{\left(V_1 - \frac{V_1}{2}\right)^2}{2g}$$

$$= \frac{V_1^2}{2g}\left(1 - \frac{1}{2}\right)^2 = \frac{V_1^2}{2g} \times \frac{1}{4} = \frac{V_1^2}{8g} \text{ m of oil}$$

Choice (C)

11. Specific speed of hydraulic pump $N_s = \frac{N\sqrt{Q}}{H^{\frac{3}{4}}}$

It is the speed when working against unit head and delivering unit quantity of water. Choice (D)

13. Blades of a propeller turbine are fixed and not adjustable under part load condition. So under part load, water enters with shock and eddies are formed which reduces its efficiency. Choice (D)

15. $P_1 = 1000$ kW, $H_1 = 40$ m

$H_2 = 20$ m

For both conditions, unit power is same

$$\text{i.e., } P_u = \frac{P_1}{(H_1)^{\frac{3}{2}}} = \frac{P_2}{(H_2)^{\frac{3}{2}}} \Rightarrow P_2 = P_1\left(\frac{H_2}{H_1}\right)^{\frac{3}{2}}$$

$$= 1000 \times \left(\frac{20}{40}\right)^{\frac{3}{2}} = 1000 \times \left(\frac{1}{2}\right)^{\frac{3}{2}}$$

$$= \frac{1000}{\sqrt{8}} \text{ kW}$$ Choice (D)

16. Given- $P_p = 10$ MW

$\frac{D_p}{D_m} = \frac{1}{4}$

$\frac{N_p}{N_m} = 1$

$\eta_p = \eta_m$

Power coefficient $\frac{P}{N^3D^5}$ is

same in both cases

$$\text{i.e., } \frac{P_p}{N_p{}^3 D_p{}^5} = \frac{P_m}{N_m{}^3\ D_m{}^5}$$

$$\therefore \quad P_m = P_p \times \left(\frac{N_m}{Np}\right)^3 \times \left(\frac{D_m}{D_p}{}^5\right)$$

$$= 16 \times 1 \times \left(\frac{1}{4}\right)^5 = \frac{1}{4^3} = \frac{1}{64} \text{MW}$$

$$= \frac{1000}{64} \text{ kW}$$ Choice (C)

17. Force exerted

= mass/second × change in velocity of the jet

$= \rho av(v - u)$ Choice (D)

18. $V = C_v\sqrt{2gH}$

$90 = 0.98\ \sqrt{2 \times 9.81 \times H}$

$H = 429.87$ m

$$\eta_0 = \frac{P}{wQH} = \frac{7500 \times 10^3}{9810 \times 2.29 \times 429.87} = 0.7766$$

$Q = \frac{\pi}{4}\ d^2 \times V$

$= \frac{\pi}{4}\ (0.18)^2 \times 90 = 2.29\ \text{m}^3/\text{s}$

$\eta_0 = 0.7766 \times 100 = 77.66\ \%$ Choice (B)

19. Maximum efficiency is expressed as $\eta_H = \dfrac{1+\cos\phi}{2}$

$\phi_1 = 180 - 160 = 20°$
$\phi_2 = 180 - 170 = 10°$

$$\eta_{H1} = \frac{1+\cos 20°}{2}$$

$$\eta_{H2} = \frac{1+\cos 10°}{2}$$

$$\frac{\eta_{H1}}{\eta_{H1}} = \frac{(1+\cos 20°)}{2} \times \frac{2}{1+\cos 10°}$$

$$= \frac{1.9397}{1.9848} = 0.977$$ Choice (D)

20. $\eta_0 = \dfrac{P}{wQH}$

$$0.8 = \frac{5520 \times 10^3}{9810 \times Q \times 240}$$

$Q = 2.931$ m³/sec
Unit Discharge

$$Q_u = \frac{Q}{\sqrt{H}} = \frac{2.931}{\sqrt{240}} = 0.189$$ Choice (B)

21. Unit power $Pu = \dfrac{P}{H^{\frac{3}{2}}} = \dfrac{5520}{(240)^{\frac{3}{2}}} = 1.485$ Choice (C)

22. $F_n = \rho A V^2 \sin\theta$
$V = 45$ m/s
$\theta = 45° = 1000 \times 7.8544 \times 10^{-3} \times (45)^2 \sin 45°$
$= 11246$ N

$$F_n = \frac{11246}{1000} = 11.25\text{kN}$$ Choice (B)

23. Relative velocity $= v + u = 45 + 15 = 60$ m/s
$F_n = \rho A (v+u)^2 \sin\theta$
$= 1000 \times 7.854 \times 10^{-3}(60)^2 \sin 45°$
$= 19993$ N

$$F_n = \frac{19993}{1000} = 20 \text{ kN}$$ Choice (C)

24. Net head available at the base of the nozzle
$= 360 - 12 = 348$m

$$N_s = \frac{N\sqrt{p}}{H^{\frac{5}{4}}}$$

$$15.33 = \frac{560\sqrt{p}}{348^{\frac{5}{4}}}$$

$P = 1686.37$ kW
Total power $= 1686.37 \times 3 = 5059.11$ kW Choice (B)

25. $\eta_0 = \dfrac{P}{wQH}$

$$0.85 = \frac{1686.37 \times 10^3}{9810 \times Q\ 348}$$

$Q = 0.581$ m³/sec
Total discharge $= 3 \times 0.581$
$Q = 1.743$ m³/s

$$V = k_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 348}$$

$= 80.98$ m/s
Thus, if the diameter of the nozzle is 'd' then

$$0.581 = \frac{\pi}{4} d^2 \times 80.98$$

$D = 0.0956$ m $= 95.6$ mm Choice (B)

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UNIT IX
WATER RESOURCES ENGINEERING

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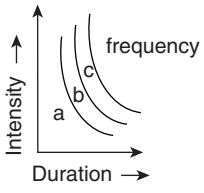
Hydrology Test 1

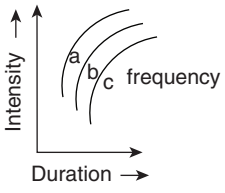
Number of Questions: 25 **Time: 60 min.**

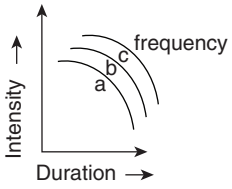
Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The region where air coming from the pole (cooler and denser) and the air of the middle cell (warmer and lighter) meet is called _____.
(A) Cold front
(B) Warm front
(C) Polar front
(D) Occluded front

2. The intensity – duration – frequency Curve from the following is
(Where $a < b < c$)

(A) 

(B) 

(C) 

(D) None

3. The rate of evaporation from a water body increases directly with increase in
1. Radiation
2. Wind upto a critical value
3. Atmospheric pressure
4. Quality of water
(A) Only 2, 3, 4 are correct
(B) Both 1 and 2 are correct
(C) Both 2 and 3 are correct
(D) Only 1, 2, 4 are correct

4. The infiltration capacity curves which are developed from infiltrometer tests or the hydrograph analysis methods are used to estimate _____ from a given storm.
(A) Infiltration
(B) Rainfall
(C) Runoff
(D) All the above

5. _____ hydrograph is independent of rainfall duration.
(A) Instantaneous unit hydrograph
(B) Synthetic unit hydrograph
(C) Direct runoff hydrograph
(D) Unit hydrograph

6. When the seepage takes place from the stream into the ground, it is called _____ stream.
(A) Perennial stream
(B) Influent stream
(C) Effluent stream
(D) Ephemeral stream

7. A structure with a useful life period of 100 years is designed for a 50-year flood. Then the risk in the design is given by _____.
(A) 0.68 (B) 0.71
(C) 0.87 (D) 0.99

8. The peak flow in outflow hydrographs in a channel routing occurs at _____.
(A) Intersection point of inflow and outflow hydrographs
(B) Before intersection
(C) After intersection
(D) Any of the above

9. For unconfined aquifers, the storage coefficient
(A) is essentially the same as the specific yield.
(B) does not exist
(C) is essentially the same as the specific retention.
(D) is essentially the same as the porosity.

10. In surface float method, the actual velocity of flow (V_a) is equal to _____ times of surface velocity (V_s).
(A) 0.2 (B) 0.4
(C) 0.8 (D) 0.85

11. In a river flow which has shallow depth velocity at different depths at a c/s $0.2d$, $0.4d$ and $0.8d$ from the bottom are 0.1, 0.2, 0.5 m/s respectively. Find the mean velocity at that cross-section.
(A) 0.3 m/s (B) 0.2 m/s
(C) 0.4 m/s (D) 0.35 m/s

12. A sample has a hydraulic conductivity of 12 m/day. What would be its intrinsic permeability? (indarcys)
(A) 12.68 (B) 13.12
(C) 14.35 (D) 16.89

13. During a recuperation test conducted on a open well in a region, the water level in the well was depressed by 4 m and it was observed to rise by 2 m in 90 minutes. What would be the yield from that well having a diameter of 6 m under a depression head of 3 m?
(A) 35.1 m^2/h (B) 39.19 m^3/h
(C) 48.32 m^3/h (D) 51.6 m^3/h

14. Calculate the peak of the outflow hydrograph in a river reach using Muskingham method given the following inflow hydrograph. Take $Co = 0.032$, $C_1 = 0.53$ and the starting value of outflow hydrograph as 10^3/s.

Time (hrs)	0	1	2	3	4	5	6	7
Inflow (m^3/s)	10	20	30	40	50	40	30	20

(A) 30.2 m^3/s (B) 42.4 m^3/s
(C) 51.6 m^3/s (D) 20.8 m^3/s

15. An urban area has a runoff coefficient of 0.35 and an area of 0.8 km^2. The maximum depth of rainfall with a 30 – year return period is as below:

Duration (min)	3	5	10	20	30	40
Depth of rainfall (mm)	10	15	20	25	30	35

If a culvert for drainage at the outlet of this area is to be designed for a time period of 30 years, estimate the peak flow rate. Take the time of concentration for the drainage area as 20 minutes.
(A) 5.83 m^3/s
(B) 15.55 m^3/s
(C) 2.78 m^3/s
(D) 10.13 m^3/s

Common Data for Questions 16 and 17:
The drainage area of water shed is 60 km^2. The Φ – index is 0.4 cm/hr. Base flow at outlet 12 m^3/s, 1 hour – UHG of water shed is triangular in shape with a time base of 10 hour. Peak ordinate occurs at 5 hours.

16. Peak ordinate of UHG in (m^3/s) is _____.
(A) 13.33 (B) 23.33
(C) 33.33 (D) 43.33

17. For a storm of depth of 6.4 cm and duration of 1 hr, the peak ordinate in m^3/s of hydrograph is _____.
(A) 153 (B) 181
(C) 212 (D) 240

18. A storm of 3-hour duration occurred over a basin of area 555.2 km^2. The resulting flow measurement is as follows:

Time (hr)	0	2	4	6	8	12	15
Q (m^3/s)	10	210	310	360	260	60	10

If Base flow = 10 m^3/s, find the depth of runoff over a catchment.
(A) 2.53 cm
(B) 0.81 cm
(C) 3.14 cm
(D) 1.43 cm

19. The total observed runoff volume during a 7-hour storm with a uniform intensity of 2 cm/h is $25 \times 10^6 m^3$. If the area of the basic is 300 km^2, find the average infiltration rate for the basin in (mm/hr).
(A) 2.18 (B) 3.63
(C) 5.72 (D) 8.14

20. The infiltration rate for excess rain on a small area was observed to be 5 cm/hr at the beginning of rain and decreased exponentially toward an equilibrium of 0.6 cm/hr. A total of 35 cm of water infiltrated during 10 hours interval. Determine *k* of the Horton's equation.
(A) 0.05/hr
(B) 0.11/hr
(C) 0.15/hr
(D) 0.2/hr

21. Match the following in Group – A with Group – B.

	Group – A		Group – B
P.	Transpiration	1.	Phytometer
Q.	Evapotranspiration	2.	Rainfall simultator
R.	Evaporation	3.	Lysimeter
S.	Infiltration	4.	Water balance method

P Q R S P Q R S
(A) 2 4 3 1 (B) 1 3 4 2
(C) 3 2 1 4 (D) 4 1 2 3

22. If 9.2 liters of water is added to an evaporation pan of 1.3m diameter to bring the water surface to the stipulated level and if a nearby rainguage measured a rainfall of 8.8 mm. what is the evaporation recorded for the day?
(A) 0.6 mm
(B) 1.87 mm
(C) 2.3 mm
(D) 3.2 mm

Common Data for Questions 23 and 24:
A one-day rainfall of 16 cm in Hyderabad is found to have a return period of 100 years.
Calculate the probability that one-day rainfall of this magnitude or larger magnitude

23. Will not occur in Hyderabad during the next 50 years.
(A) 0.01 (B) 0.99
(C) 0.605 (D) 0.53

24. Will occur in next year.
(A) 0.01 (B) 0.99
(C) 0.605 (D) 0.53

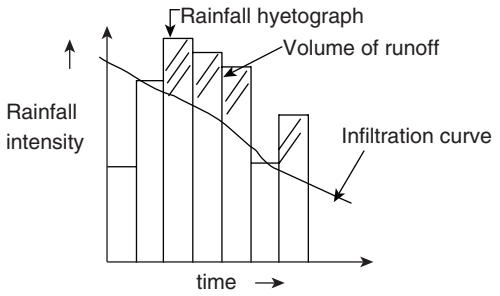
25. Find out the mean precipitation of a catchment which is in a triangular shape of side 10 km. Rainguages installed at each corner recorded 10 cm, 15 cm, 20 cm respectively.
(A) 15 cm (B) 12 cm
(C) 17 cm (D) 14 cm

Answer Keys

1. C	**2.** A	**3.** D	**4.** C	**5.** A	**6.** B	**7.** C	**8.** D	**9.** A	**10.** D
11. B	**12.** C	**13.** B	**14.** B	**15.** A	**16.** C	**17.** C	**18.** D	**19.** D	**20.** C
21. B	**22.** B	**23.** C	**24.** A	**25.** A					

Hints and Explanations

4.



Choice (C)

7. Risk = $1 - q^n$

$P = \frac{1}{T_r} = \frac{1}{50} = 0.02$

$q = 0.98$

$\therefore$ Risk = $1 - (0.98)^{100}$

$= 0.867 \cong 0.87$ Choice (C)

8. In reservoir routing, the peak of outflow hydrograph is at the intersection of inflow and outflow, but not for channel routing. Choice (D)

9. **Storage coefficient:**
The volume of water that an aquifer releases per unit surface area of the aquifer per unit change in component of head normal to that surface.
In an unconfined aquifer it is equal to specific yield. (water extracted by force of gravity). Choice (A)

10. $V_a = 0.85\ V_s$ Choice (D)

11. For shallow depth, one point method is used
$V_m = V_{0.4} = 0.2$ m/s. Choice (B)

12. Permeability $k = \frac{K\mu}{\rho g}$

$k = 12$ m/day
$\mu = 0.01$ gm–cm/s
$\rho = 1$ gm/cc
$g = 981$ cm/s^2

$k = \frac{12 \times 100 \times 0.01}{24 \times 60 \times 60 \times 1 \times 981}$

$= 1.416 \times 10^{-7}$cm^2
$= 1.416 \times 10^{-11}$m^2
1 Darcy = 0.987×10^{-12}m^2

$\therefore \quad k = \frac{1.416 \times 10^{-11}}{0.987 \times 10^{-12}} = 14.35$ Darcy's. Choice (C)

13. $h_1 = 4$ m
$h_2 = 4 - 2 = 2$m
$T = 90$ minutes $= 1.5$ h

$C = \frac{k}{A} = \frac{2.303}{T} \log\left(\frac{h_1}{h_2}\right)$

$= \frac{2.303}{1.5} \log\left(\frac{4}{2}\right)$

$= 0.462$ h^{-1}

$Q = CAH$

$= 0.462 \times \left(\frac{\pi}{4} \times 6^2\right) \times 3$

$= 39.19$ m^3/h. Choice (B)

14. $C_0 = 0.032$, $C_1 = 0.53$
$C_2 = 1 - C_0 - C_1 = 0.438$
Outflow $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$
$= (0.032 \times 20) + (0.53 \times 10) + (0.438 \times 10)$
$= 10.32$ m^3/s

Similarly

Inflow (m^3/s)	Outflow (m^3/s)
10	10
20	10.32
30	16.08
40	24.22
50	33.41
40	**42.41**
30	40.74
20	34.38

$\therefore$ Peak discharge = 42.41 m^3/s. Choice (B)

15. Area = 0.8 km^2 = 80 ha

$Q = \frac{AIR}{360}$

Duration to be taken = time of concentration = 20 minutes

Depth of rainfall = $\frac{25}{20} \times 60 = 75$ mm/h

$Q = \frac{80 \times 0.35 \times 75}{360}$

$= 5.83$ m^3/s. Choice (A)

16. Volume = $A \times$ depth
$= 60 \times 10^6 \times 0.01$
$= 6 \times 10^5$m^2

From UHG

$\frac{1}{2} \times 10 \times 60 \times 60 \times Q = 6 \times 10^5$

$Q = 33.33$ m^3/s. Choice (C)

17. $\Phi = 0.4$ cm/hr $= \frac{6.4 - R}{1}$

Runoff $R = 6$ cm
Peak $DRH = 33.33 \times 6 = 200$ m^3/s
Peak $SHG = DRH$ + Base flow
$= 200 + 12 = 212$ m^3/s. Choice (C)

18. **Base flow** = B

Let O_{s1}, O_{s2}, are storm hydrograph ordinates

Let OD_1, OD_2. OD_3 ….. are direct runoff hydrograph ordinates

$OD_1 = O_{s1} - B$

$OD_2 = O_{s2} - B$

OD	0	200	300	350	250	50	0

By Trapezoidal formula:-

Volume

$$= \Delta t\left[\frac{OD_1 + OD_7}{2} + \Sigma(\text{remaining ordinates})\right]$$

$$V = 2 \times 60 \times 60\left[\frac{0+0}{2} + (200+300+350+200+50)\right]$$

$V = 7.92 \times 10^6 \text{m}^3$

$$\text{Depth} = \frac{V}{A} = \frac{7.92\times10^6}{555.2\times10^6}$$

$= 0.01426$ m

$= 1.43$ cm. Choice (D)

19. Total rainfall = Intensity of rainfall × duration

$= 2 \times 7 = 14$ cm

Volume of runoff $= 25 \times 10^6 \text{m}^3$

Area of the basin $= 300 \text{ km}^2 = 300 \times 10^6 \text{m}^2$

$$\text{Depth of runoff} = \frac{\text{Volume of runoff}}{\text{Area of runoff}}$$

$$= \frac{25\times10^6}{300\times10^6} = 0.083 \text{ m}$$

$= 8.3$ cm

Total infiltration = rainfall – runoff = 14 – 8.3 = 5.7 cm

$$\text{Average infiltration rate} = \frac{5.7}{7} = 0.814 \text{ cm/h}$$

$= 8.14$ mm/h. Choice (D)

20. $$35 = \int_0^{10}\left(0.6 + (5-0.6)e^{-kt}\right)dt$$

$$35 = 0.6t - \frac{4.4}{k}e^{-kt}\Big]_0^{10}$$

$$35 = 6 - \frac{4.4}{k}e^{-k\times10} + \frac{4.4}{k}e^{o}$$

$$\frac{4.4}{k}[1 - e^{-10k}] = 29$$

$e^{-10k} \simeq 0$

$\therefore \quad k = 0.152$/hr Choice (C)

22. Evaporation recorded $= 8.8 - \dfrac{(9.2\times10^{-3})\times1000}{\frac{\pi}{4}\times1.3^2}$

$= 1.87$ mm ($1\text{m}^3 = 1000$ liters). Choice (B)

23. $p = \dfrac{1}{T} = \dfrac{1}{100} = 0.01$

$q = 0.99$

Probability for not occurring in next 50 years.

$P = {}^{50}C_0\, p^0\, q^{50}$

$= 1 \times 1 \times (0.99)^{50}$

$= 0.605$. Choice (C)

24. Will occur next year:

$\Rightarrow$ at least once in 1 year

$P = {}^{1}C_1\, p^1\, q^0 = 0.01$. Choice (A)

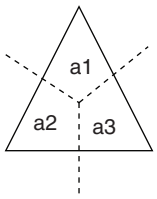
25. $a_1 = a_2 = a_3$ = areas represented by rain gauges = $\dfrac{A}{3}$

Mean precipitation

$$\overline{P} = \frac{P_1a_1 + P_2a_2 + P_3a_3}{A} \downarrow$$

$$= \frac{\left(10\times\frac{A}{3}\right) + \left(15\times\frac{A}{3}\right) + \left(20\times\frac{A}{3}\right)}{A}$$

$= 15$ cm.



Choice (A)

IRRIGATION TEST 2

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Consumptive use of water for a crop is equal to the depth of water
(A) evaporated by the crop.
(B) transpired by the crop.
(C) transpired and evaporated by the crop
(D) used by the crop in transpiration, evaporation and also the quantity of water evaporated from adjacent soil.

2. The cross-section of weir in order from upstream to downstream is _____.
(A) launching Apron → Block protection → sheet pile → impervious floor → sheet pile → filter → launching apron
(B) block protection → sheet pile → filter → sheet pile → impervious floor → launching apron
(C) impervious floor → launching apron → sheet pile → block protection → sheet pile → filter → launching apron
(D) filter → impervious floor → sheet pile → block protection → launching apron → sheet pile → launching apron

3. According to Lacey's method for design of alluvial channel, the velocity of flow
(A) increases with increase in design discharge
(B) increases with increase in diameter of silt particle
(C) increases with increase in silt factor
(D) All the above

4. Drainage gallery in a dam is used
(A) to provide drainage of the dam
(B) for post cooling of concrete
(C) Both a and b
(D) None of the above

5. Effective Precipitation for a crop may be defined as
(A) total precipitation from sowing of seeds to cutting of crop.
(B) total precipitation minus loss due to evaporation and infiltration.
(C) total precipitation minus runoff.
(D) available water stored in soil within root zone of the crop.

6. The best method of irrigation for mango trees is
(A) border strip method
(B) basin method
(C) checks or Leeves method
(D) furrow method

7. _____ are called safety valves of a dam.
(A) Diversion headwork's (B) Canal outlets
(C) Spill ways (D) Drainage Gallery

8. Non-modular outlet is the one in which discharge
(A) is independent of water levels in the distributing channel and water course.
(B) varies only with water level in the distributing channel.
(C) Varies only with water level in the water course.
(D) depends on difference in water levels in distributing channel and water course.

9. Main causes of water logging are
(A) surface runoff
(B) steep ground profile
(C) excessive irrigation
(D) All the above

10. If pH < 8.5, the soil is called
(A) acidic soil (B) saline soil
(C) basic soil (D) alkaline soil

11. Match the following.

	Group – A		Group – B
1.	Aqueduct	P.	Bed of drain well above the canal F.S.L
2.	Siphon Aqueduct	Q.	F.S.L of Canal higher than bed of drain
3.	Super passage	R.	High flood level HFL) of drain higher than canal bed
4.	Siphon	S.	Bed of canal is well above the HFL of drain

1 2 3 4 1 2 3 4
(A) *S*, *R*, *P*, *Q* (B) *P*, *Q*, *R*, *S*
(C) *Q*, *P*, *R*, *S* (D) *R*, *S*, *Q*, *P*

12. To irrigate a strip of area of size 100 m × m, the time taken is 50 minutes. Assuming average depth of water is 8 cm and average infiltration is 4 cm/hr. Find the discharge of stream flow in cumecs?
(A) 0.04 cumecs
(B) 0.0163 cumecs
(C) 0.023 cumecs
(D) 0.0368 cumecs

13. The slope of a channel in alluvium is $\frac{1}{3036}$, mean soil particle size is 0.5 mm, velocity = 0.618 m/s. Find the wetted perimeter of the regime channel in meters.
(A) 10.62 (B) 9.46
(C) 11.58 (D) 12.9

14. Find the Exchangeable Sodium Ratio (ESR) where the concentrations of all Sodium, Calcium, Magnesium and Potassium are 161, 200, 108, and 156 in ppm respectively.
(A) 83.33% (B) 43.63%
(C) 68.73% (D) 23.33%

15. An area of 1 hectare has a root zone depth of 1.2 m and available moisture holding capacity is 14 cm per meter depth. It is irrigated through a stream of 0.04 m^3/s. Water is applied to field when 60% of the available moisture is depleted. Irrigation period is 9 hours. If the water application efficiency is 70% determine the storage efficiency.
(A) 64.3% (B) 72%
(C) 58% (D) 49%

16. The moisture content of soil in the root zone of an agricultural crop at certain stage is found to be 0.06. The Field Capacity of the soil is 0.14. The root zone depth is 1.6 m. The consumption use of crop at this stage is 3mm/day and there is no precipitation during this period. Irrigation efficiency is 60%. It is intended to raise the moisture content to the field capacity in 9 days through irrigation. The necessary depth of irrigation in (mm) is _____.
(A) 250 (B) 257
(C) 300 (D) 310

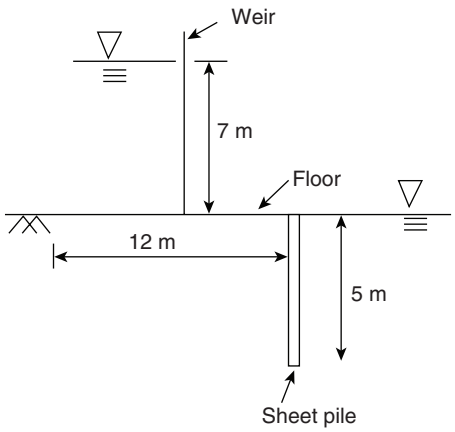
17. The transplantation of chilli crop takes 20 days and the total depth of irrigation water required by the crop is 50 cm on field. During this period, useful rainfall on field is 10 cm. Find the duty of irrigation water for this crop on the field during transplantation period. Assuming 30% losses of water in water course, find the duty at the head of water course in hectares/cumec.
(A) 208 (B) 250
(C) 302 (D) 350

18. In a gravity dam the friction coefficient is 0.8. Sum of vertical forces = 7000*t* and sum of Horizontal forces = 4000*t*. Base width is 80m and shear strength is 150 t/m^2. Find FOS against sliding and shear friction factor at base?
(A) 4.4 and 1.4 (B) 1.4 and 4.4
(C) 3.2 and 1.6 (D) 1.6 and 3.2

19. The limiting height of gravity dam with material of concrete having specific gravity of 2.5 is equal to 100m. Find crushing stress of concrete (in kg/cm^2).
(A) 20 (B) 25
(C) 30 (D) 35

20. Find the design head (H_d) of the Ogee spill way when a coordinate (– 10, 5) of the point on the D/s profile with origin at the crest of the spill way is _____.
(A) 0.56 m (B) 0.72 m
(C) 0.98 m (D) 1.1 m

21. A weir on the permeable foundation with downstream sheet pile is shown in figure below. The exit gradient as per Khosla's method is _____.



(A) 1 in 2 (B) 1 in 3
(C) 1 in 4 (D) 1 in 5

22. Match the following.

	Group – A		Group – B
1.	Bligh's theory	P.	UpperBariDoabCanal
2.	Khosla's theory	Q.	Regime Channel
3.	Lacey's theory	R.	Sheet pile is more useful at D/S end
4.	Kennedy's theory	S.	Sheet pile is more useful at U/S end

1 2 3 4 1 2 3 4
(A) Q S R P (B) R P S Q
(C) P R Q S (D) S R Q P

23. For a channel to be in regime, conditions to be established are:
(A) Channel should be flowing uniformly in unlimited coherent alluvium.
(B) Silt grade and silt charge should be constant.
(C) Both a and b
(D) None of the above

24. Calculate the bed width for an irrigation channel to carry a discharge of 6 cumecs and side slopes of the channel are $\frac{1}{2}$H : 1V. The critical velocity ratio is 0.9 and depth of flow is 0.8 m. Bed slope is 0.3 m/km.
(A) 13.75 m (B) 15.25 m
(C) 17.04 m (D) 19.21 m

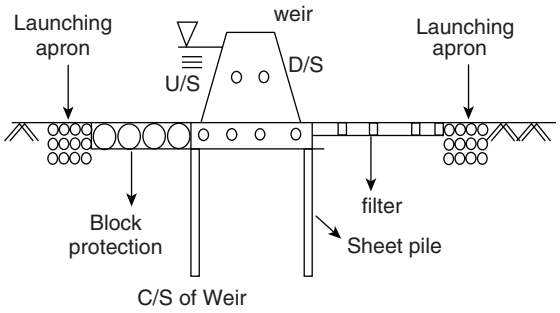
25. Find the spacing of drains in case of closed drains where the depth of impermeable layer from *GL* is 10 m and depth of drain below *GL* is 2 m. Minimum depth of drained *W.T* below *GL* is 1.5 m. Permeability of soil = 1 cm/s. Discharge through drain is 0.02 m^3/s.
(A) 14.5 m (B) 16.5 m
(C) 18.5 m (D) 20.5 m

Answer Keys

1. D	**2.** A	**3.** D	**4.** C	**5.** D	**6.** B	**7.** C	**8.** D	**9.** C	**10.** B
11. A	**12.** B	**13.** A	**14.** D	**15.** A	**16.** C	**17.** C	**18.** B	**19.** D	**20.** C
21. B	**22.** D	**23.** C	**24.** C	**25.** B					

Hints and Explanations

2.



Choice (A)

3. $V = \left(\frac{Qf^2}{140}\right)^{\frac{1}{6}} = \sqrt{\frac{2}{5} fR}$

But $f = 1.76\sqrt{d} \Rightarrow f \propto \sqrt{d}$

$\therefore \quad V \propto Q^{\frac{1}{6}}$

$\propto f^{\frac{1}{3}}$

$\propto d^{\frac{1}{6}}$. Choice (D)

4. Drainage gallery is used
1. To provide drainage of the dam.
2. For drilling and grouting of foundation
3. Post cooling of concrete
4. To lay mechanical equipment for operation of outlet gates and spill way gates. Choice (C)

5. Effective precipitation can mean rainfall and losses. But for a crop it means only available water within root zone that can be easily extracted by plants.
Choice (D)

6. A basin in created surrounding each tree and field channel supplies water to the basin used for irrigation of orchards (fruit trees) Choice (B)

9. Causes of water logging
→ Excessive rainfall irrigation
→ Seepage from canals and lands beside
→ Flat ground profile
→ Improper drainage of surface runoff Choice (C)

12. The time required to irrigate the land

$$t = \frac{y}{I}\log_e\left(\frac{Q}{Q - IA}\right)$$

$$50 \times 60 = \frac{0.8}{\frac{0.04}{60\times60}}\log_e\left(\frac{Q}{Q - \frac{0.04}{60\times60}\times 500}\right)$$

$$1.517 = \frac{Q}{Q - \frac{1}{180}}$$

$0.517Q = 8.43 \times 10^{-3}$

$Q = 0.0163$ m³/s. Choice (B)

13. Silt factor $f = 1.76\sqrt{d} = 1.76\sqrt{0.5} = 1.244$

Velocity $V = \left(\frac{Qf^2}{140}\right)^{\frac{1}{6}}$

$$0.618 = \left(\frac{Q\times 1.244^2}{140}\right)^{\frac{1}{6}}$$

$Q = 5$ m³/s
(OR)
Longitudinal slope

$$S = \frac{f^{\frac{5}{3}}}{3340\,Q^{\frac{1}{6}}}$$

$$\frac{1}{3036} = \frac{(1.244)^{\frac{5}{3}}}{3340Q^{\frac{1}{6}}}$$

$Q = 5$ m³/s
Perimeter $P = 4.75\sqrt{Q}$
$= 4.75\sqrt{5} = 10.62$ m Choice (A)

14. Exchangeable Sodium Ratio (ESR)

$$= \frac{Na^+}{Ca^{++} + Mg^{++} + Na^+ + K^+}$$

All concentrations are in meq/lt

$$Na\ (meq/lt) = \frac{161}{23} = 7$$

$$Ca\ (meq/lt) = \frac{\frac{200}{40}}{2} = 10$$

$$Mg\ (meq/lt) = \frac{\frac{108}{24}}{2} = 9$$

$$K\ (meq/lt) = \frac{156}{39} = 4$$

$$\therefore \quad ESR = \frac{7}{7+10+9+4}\times 100 = 23.33\%$$ Choice (D)

15. Depth of root zone = 1.2 m
Total available depth of water

$y = 14\frac{cm}{m} \times 1.2\ m = 16.8$ cm

Water consumed = (100 – 60%) y = 6.72 cm
Water needed to raise upto Field capacity = 16.8 – 6.72 = 10.28 cm
Volume of water supplied = 0.04 × 9 × 60 × 60
= (1296 cm^3)

Application efficiency $\eta_a = 60\% = \dfrac{W_{\text{root zone depth}}}{1296}$

Water in root zone = 778 m^3

Depth of water stored $= \dfrac{778}{1.2\times10^4} = 0.06483$ m

$= 6.483$ cm

$\eta_{\text{storage}} = \dfrac{\text{water stored in root zone}}{\text{water needed to raise the w/c to } F.C}$

$\therefore \quad \eta_{\text{storage}} = \dfrac{6.483}{10.08} = 0.643 = 64.3\%.$ Choice (A)

16. d = 1.6m, OM = 0.06
FC = 0.14, C_u = 3 mm/day
Trying with S = 1
Depth of water in root zone $d_w = d.s[F.C - OM]$
= 1.6 × 1[0.14 – 0.06]
= 0.128 m = 128 mm
Total depth of water to be kept in root zone
= 128 + Cu × time
= 128 + (3 × 9)
= 155 mm

Total depth to be released at head canal $= \dfrac{155}{\eta_a} = \dfrac{155}{0.6}$

= 258 mm

Answer must be > 258 mm (assumed S = 1)
$\therefore$ depth of irrigation = 300 mm (>258 mm)
Choice (C)

17. Irrigation depth of water
Δ = 50 – 10 = 40 cm = 0.4 m
Base period, B = 20 days

D_F, duty at field $= \dfrac{8.64\,B}{\Delta} = \dfrac{8.64\times20}{0.4}$

= 432 hectares/cumec
Duty at head of canal = $D_F \times \eta_C$
= 432 × [1 – 0.3] [$\because$ losses = 30%]
= 302.4 hectares/cumec. Choice (C)

18. Sliding $FOS = \dfrac{\mu\Sigma V}{\Sigma H} = \dfrac{0.8(7000)}{4000} = 1.4$

Shear Friction Factor (SFF) $= \dfrac{\mu\Sigma V + (b\times1\times q)}{\Sigma H}$

$= \dfrac{0.8(7000)+(80\times150)}{4000}$

= 4.4. Choice (B)

19. Critical Height of dam

$H_C = \dfrac{f_C}{w[S+1]}$

$100 = \dfrac{f_C}{1000[2.5+1]}$

$f_c = 3.5 \times 10^5$ kg/m^2
Crushing stress f_c = 35 kg/cm^2 Choice (D)

20. Equation of Ogee spill way

$\dfrac{y}{H_d} = -0.5\left(\dfrac{x}{H_d}\right)^{1.85}$

$(x, y) = (5, -10)$

$\dfrac{-10}{H_d} = -0.5\left(\dfrac{5}{H_d}\right)^{1.85}$

$\dfrac{H_d^{1.85}}{H_d} = 0.982$

$H_d^{0.85} = 0.982$

H_d = 0.978 m. Choice (C)

21. Exit Gradient $G_E = \dfrac{H}{d\pi\sqrt{\lambda}}$

$\propto = \dfrac{b}{d} = \dfrac{12}{5} = 2.4$

$\lambda = \dfrac{1+\sqrt{1+\propto^2}}{2} = \dfrac{1+\sqrt{1+2.4^2}}{2} = 1.8$

$G_E = \dfrac{7}{5\pi\sqrt{1.8}} = 0.332$

$\Rightarrow$ 1 in 3.01. Choice (B)

22. Bligh's Theory:

$t_{\min} = \dfrac{h}{(G_C - 1)}$

To minimize floor thickness sheetpile is more useful at U/S end
Khosla's theory:
Exit Gradient, $G_E = \dfrac{H}{d\pi\sqrt{\lambda}}$

H = head causing flow
To avoid piping Khosla's proposed sheet pile @ D/s end.
Kennedy made his observations on Upper Basi Drab Canal
Lacey's equations are applicable for regime channel.
Choice (D)

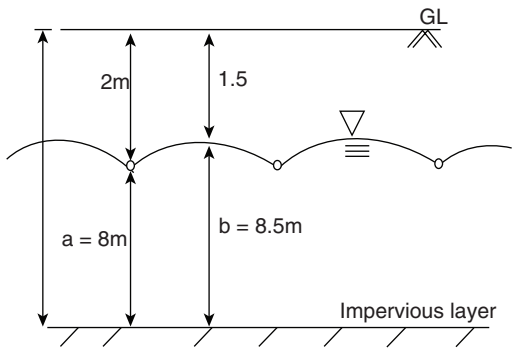
23. For a channel in regime conditions
1. Channel should be flowing uniformly.
2. Silt grade and silt charge should be constant
3. Discharge should be constant Choice (C)

24. $Q = 6$ cumecs
Critical velocity ratio. $M = 0.9$
$D = 0.8$ m
$V_K = 0.55 \text{ m } D^{0.64}$
$= 0.55\ (0.9)\ (0.8)^{0.64} = 0.43$
$A = (B + 0.5D)D = 0.8\ B + 0.32$
$Q = A.V_K$
$6 = (0.8\ B + 0.32)\ 0.43$
$B = 17.04$ m Choice (C)

25. Spacing of drains

$$L = \frac{4k\left(b^2 - a^2\right)}{Q_o}$$

$b = 10 - 1.5 = 8.5$
$a = 10 - 2 = 8$



$$L = \frac{4 \times 0.01\left(8.5^2 - 8^2\right)}{0.02} = 16.5 \text{ m}$$

$\therefore$ Spacing of drains = 16.5 m Choice (B)

Water Resource Engineering Test 3

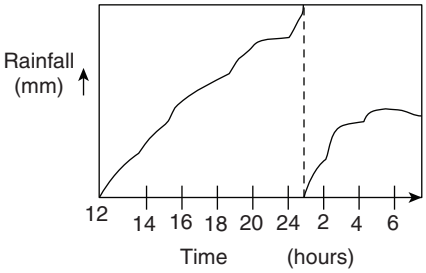
Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The slope of the rainfall mass curve is zero, when it is horizontal. This happens when the intensity for that period is
(A) constant (B) Increasing
(C) Decreasing (D) Zero

2. Which of the following is not a continuous random variable?
(A) Annual peak flood
(B) Annual runoff
(C) Number of cyclones in a year
(D) Wind velocity

3. The following rainfall chart is from which type of rain gauge.



(A) Weighing bucket type
(B) Float type Raingauge
(C) Tipping bucket type
(D) Both A and B

4. The average pan coefficient of ISI standard pan is
(A) 0.95 (B) 0.8
(C) 0.7 (D) 0.6

5. Storage in the channel is equal to
(A) Prism storage (B) Wedge storage
(C) Largest of A and B (D) (A + B)

6. Which of the following formations does not contain any ground water?
(A) aquifee (B) acquifuge
(C) aquitard (D) aquiclude

7. Moisture equivalent is equal to
(A) Field capacity
(B) Saturation capacity
(C) Available Moisture
(D) Ultimate wilting

8. Indentify the incorrect statement related to the adverse effects of soil salinity or alkalinity, from the following:
(A) Causing low yields of crops
(B) Limiting of the type of crops
(C) Rendering the quality of folder poor
(D) Causing high infiltration, resulting in damage to crops

9. As a result of the construction of a diversion structure across a river, there will be a rise in the flood level on the upstream side of the structure and it is called as
(A) Free board (B) uplift
(C) aggradation (D) afflux

10. Coefficient of discharge of Ogee spillway is
(A) 2 (B) 1.8
(C) 2.2 (D) 2.4

11. A steady groundwater flow is running through an unconfined aquifer of coarse sand under laid by a horizontal impervious formation as shown in figure. The depths of water table below the ground surface in the two observation wells, fixed at a spacing of 300 m along the direction of flow are recorded as 6 m and 6.5m. The sand layer is 30 m thick and its coefficient of permeability is 5×10^{-3} cm/sec. Determine the rate of flow in m^3/day/m width of the quifer.
(A) 0.17 (B) 0.23
(C) 0.38 (D) 0.42

12. Match the parameters in Group - A with Group - B.

	Group – A		Group – B
P.	Isonif	1.	Line joining points having equal rainfall.
Q.	Isohyte	2.	Line joining points having equal snowfall.
R.	Isopleth	3.	A line joining points having equal depths of rainfall of particular duration with particular return period.
S.	Isopluvial	4.	Line joining points having equal depth of evapotranspiration

	P	Q	R	S		P	Q	R	S
(A)	4	3	2	1	(B)	2	1	4	3
(C)	4	1	3	2	(D)	3	1	4	2

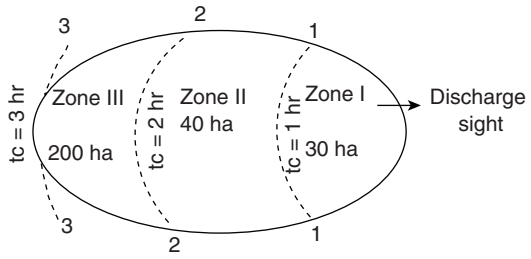
13. Sodium dichromate solution with a concentration of 30mg/cc is introduced into a stream at a rate of 1.7 litres/minute. The samples collected at a downstream section sufficiently far away indicated on equilibrium concentration of 0.001 ppm. Determine the discharge in the stream (Assume no initial concentration of sodium dichromate in the stream)
(A) 424.5 m^3/s (B) 380.6 m^3/s
(C) 298.3 m^3/s (D) 484.1 m^3/s

Common Data for Questions 14 and 15:
Ordinates of 1 hr UGH at 1 hr intervals are 6, 8, 11, 14, 12, 2, 1 m^3/s.

14. Calculate the equilibrium discharge of S-curve in m^3/s.
(A) 53 (B) 54
(C) 55 (D) 56

15. Calculate the maximum ordinate of 3 hr UHG in m^3/s.
(A) 9.33 (B) 8.33
(C) 12.33 (D) 13.33

Common Data for Questions 16 and 17:
A 4 hour rain of average intensity 1.6 cm/hr falls over the catchment as shown below:



The time of concentration from the lines 11, 22, 33 are 1hr, 2hr, and 3hr respectively to the discharge measuring site. The values of run-coefficients are 0.4, 0.5, and 0.6 for 1st, 2nd, 3rd hours of rainfall respectively and there after attains a constant value of 0.65.

16. The discharge at the end of 3 hours period, at the measuring site is
(A) 1.03 m^3/s (B) 1.51 m^3/s
(C) 1.77 m^3/s (D) 2.04 m^3/s

17. The discharge at the end of 5 hours period, at the measuring site, is
(A) 0.58 m^3/s (B) 0.93 m^3/s
(C) 1.21 m^3/s (D) 1.53 m^3/s

18. The isohyets drawn for a storm which occurred over a drainage basin of area 950 km^2 yielded the following information:

Isohyet interval in (mm)	95-85	85-75	75-65	65-55
Area between isohyets in km^2	126	198	224	175

Determine the average depth of rainfall over the basin.
(A) 60.11 mm (B) 68.21 mm
(C) 73.8 mm (D) 80.64 mm

19. Normal annual precipitation of 5 Rain Gauge stations *P*, *Q*, *R*, *S*, *T* are 125, 102, 76, 113 and 137 cm. During a particular storm the precipitation recorded by stations *P*, *Q*, *R*, S are 13.2, 9.2, 6.8 and 10.2 cm. Station *T* was not working. Estimate rainfall during this storm at *T*.
(A) 10.21 cm (B) 12.86 cm
(C) 13.43 cm (D) 7.89 cm

Common Data for Questions 20 and 21:
A 600 sq.km watershed received a 8 hr storm which produced hourly intensities of 4, 10, 16, 20, 11, 2, 13, 4 mm/hr. If the initial abstractions are estimated to be 14 mm and Φ – index is 5 mm/hr.

20. What would be the runoff volume produced by the storm (in hr.m)
(A) 2400 (B) 2000
(C) 3000 (D) 2800

21. What would be the area that can be irrigated, if the above runoff is utilized without wastage to irrigate a land where the depth of irrigation required is 20 cm.
(A) 12000 ha
(B) 10000 ha
(C) 15000 ha
(D) 14000 ha

22. The following data pertains to the healthy growth of a crop. Field capacity of soil = 40%. Permanent witting percentage = 11%. Density of soil = 1400 kg/m^3. Effective depth of root zone = 800 mm. Daily consumptive use of water for the given crop = 15 mm. For healthy growth moisture content must not fall below 25% of the water holding capacity between the field capacity and the permanent witting point. Determine the watering interval in days.
(A) 13 (B) 14
(C) 15 (D) 16

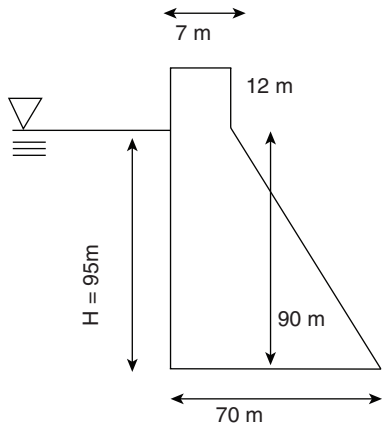
23. Match List - A with List - B

	List – A		**List – B**
P.	CIR	1.	$\frac{C_u + R_e + w}{\eta_a}$
Q.	NIR	2.	$\frac{C_u - R_e + w}{\eta_a . \eta_c}$
R.	FIR	3.	$C_u - R_e$
S.	GIR	4.	$C_u - R_e + w$

CIR – Consumptive Irrigation Requirement
NIR – Net Irrigation Requirement
FIR – Field Irrigation Requirement
GIR – Gross Irrigation Requirement
C_u – Consumptive Use
R_e – Effective Rainfall
w – Water lost in deep percolation
η_a – water application efficiency
η_c – water conveyance efficiency

	P	Q	R	S
(A)	4	3	1	2
(B)	3	4	1	2
(C)	3	4	2	1
(D)	4	3	2	1

24. The given figure gives the profile of a gravity dam with reservoir level as shown. If the coefficient of friction is 0.75, Find the *F.O.S* against sliding and if the dam is safe? (∫concrete = 2.4 tonnes/m^3) (Neglect uplift pressure)



(A) 1.1 and not safe (B) 1.42 and safe
(C) 2.1 and not safe (D) 1.8 and safe

25. A weir across an alluvial river has a horizontal floor of length 70m and retains 7m of water under full pond condition. If the downstream sheet pile is driven to a depth of 6m below the average bed level, calculate the exit gradient if porosity is 30% and the relative density of soil particles as 2.7. Estimate the vertical exit gradient.
(A) $G_E = 0.181$ and $i_C = 1.48$
(B) $G_E = 0.147$ and $i_C = 2.12$
(C) $G_E = 0.147$ and $i_C = 1.19$
(D) $G_E = 0.181$ and $i_C = 2.12$

26. Which of the incorrect statement among the drawbacks in Lacey's theory.
(A) Silt transportation is incorporated in a single factor.
(B) Equations are empirical
(C) Regime conditions are only theoretical
(D) Silt charge and silt grade have not been properly defined.

27. Match the terms of List - A with List - B

	List – A		List – B
P.	Nappe (sheet of water)	1.	Canal regulation work
Q.	Aqueduct	2.	Spillways
R.	Rigid module	3.	Cross drainage work
S.	Canal drop	4.	Canal outlet

	P	Q	R	S		P	Q	R	S
(A)	1	2	3	4	(B)	2	3	1	4
(C)	4	1	3	2	(D)	2	3	4	1

28. Find the discharge over an ogee weir with coefficient of discharge equal to 2.4 at a head of 3m. The length of spillway is 100m. Crest of weir is 10m above the bottom of the approach channel having the same width as that of the spillway
(A) 1247 m^3/s (B) 1276 m^3/s
(C) 1301 m^3/s (D) 1348 m^3/s

Common Data for Questions 29 and 30:
For border strip method of irrigation, discharge in an area from tube well was 0.01 cumecs. The infiltration capacity of the soil may be taken as 6 cm/hour and the average depth of the flow on the field as 10 cm.

29. Determine the time required to irrigate strip of land of 0.05 hectares.
(A) 0.48 hours (B) 48 minutes
(C) 2.98 hours (D) 298 minutes

30. Determine the maximum area that can be irrigated from this tube well.
(A) 0.001 ha (B) 0.02 ha
(C) 0.06 ha (D) 0.2 ha

Answer Keys

1. D	**2.** C	**3.** B	**4.** B	**5.** D	**6.** B	**7.** A	**8.** D	**9.** D	**10.** C
11. A	**12.** B	**13.** A	**14.** B	**15.** C	**16.** D	**17.** A	**18.** C	**19.** B	**20.** A
21. A	**22.** D	**23.** B	**24.** B	**25.** C	**26.** A	**27.** D	**28.** B	**29.** C	**30.** C

Hints and Explanations

11. $Q = k\,I\,A$
$= (5 \times 10^{-3} \times 10^{-2}) \times \left(\frac{6.5-6}{300}\right) \times 1 \times \frac{24+23.5}{2}$
$= 1.979 \times 10^{-6}$ $m^3/s/m$
$= 0.171$ $m^3/day/m$. Choice (A)

13. $C_o = 0$
$C_T = 30$ mg/cc $= 0.03$ µg/cc
$C = 0.002$ ppm
$= \frac{0.002}{10^6} = 2 \times 10^{-9}$ µg/cc

$Q_T = 1.7$ litres/minute $= \frac{1.7 \times 10^3}{60}$
$= 2.83 \times 10^{-5}$ m^3/s

$C_{\min} = \frac{Q_T C_T + Q_R C_R}{Q_T + Q_R}$

$2 \times 10^{-9} = \frac{(2.83 \times 10^{-5} \times 0.03) + 0}{2.83 \times 10^{-5} + Q_R}$

$Q_R = 424.5$ m^3/s. Choice (A)

14.

Time	1 hr UH	S Curve addition	S_A	S_B	$\frac{S_A - S_B}{3} \times 1$ 3 Hr UH
0	0	–	0	–	0
1	6	0	6	–	2
2	8	6	14	–	14/3
3	11	14	25	0	25/3
4	14	25	39	6	11
5	12	39	51	14	37/3
6	2	51	53	25	28/3
7	1	53	**54**	39	5
8	0	54	**54**	51	1
		54	**54**	53	1/3
				54	0

Equilibrium discharge $Q_C = 54$ m³/s. Choice (B)

15. Maximum ordinate = 37/3 = 12.33 m³/s. Choice (C)

16. $Q_{3hr} = \frac{A_1 I_3 R_1}{360} + \frac{A_2 I_2 R_2}{360} + \frac{A_3 I_1 R_3}{360}$

$\frac{1.6 \times 10}{360}$ [(30 × 0.6) + (40 × 0.5) + (20 × 0.4)]

= 2.04 m³/s. Choice (D)

17. $Q_{5hr} = \frac{A_3 I_3 R}{360} = \frac{20 \times 0.65 \times (1.6 \times 10)}{360}$

= 0.58 m³/s. Choice (A)

18. $d = \frac{\Sigma(\text{avg of isohyte}) \times \text{Area}}{\Sigma\text{Area}}$

$= \frac{(90 \times 126) + (80 \times 198) + (70 \times 224) + (60 \times 175)}{(126 + 198 + 224 + 175)}$

= 73.8 mm. Choice (C)

19. $N_x \pm 10\%\ N_x = 137 \pm (10\%\ 137) = 123.3$ to 150.7
Some normal precipations are out of these ranges.
Using Normal ratio method.

$P_x = \frac{N_x}{m}\left[\frac{P_1}{N_1} + \frac{P_2}{N_2} + \ldots \frac{P_m}{N_m}\right]$

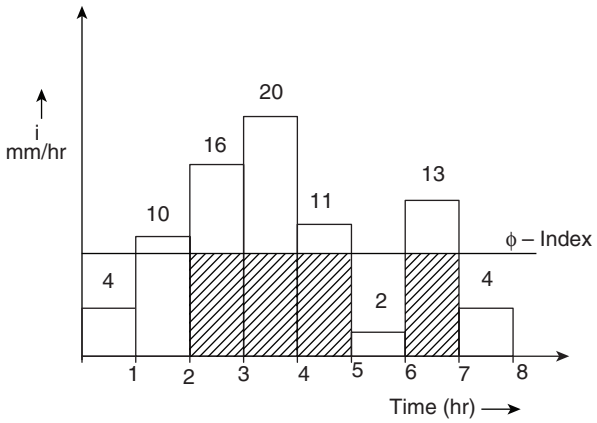
$= \frac{137}{4}\left[\frac{13.2}{125} + \frac{9.2}{102} + \frac{6.8}{76} + \frac{10.2}{113}\right]$

= 12.86 cm. Choice (B)

20. Initial abstractions = losses = 14 mm = 4 + 10
∴ (first 2 hour rainfall not considered)
Φ_{index} = 5 mm/hr

$= \frac{[(16 + 20 + 11 + 13) \times 1 - \text{Runoff}]}{4}$

Runoff = 40 mm = 4 cm



Runoff volume $= \frac{4}{100} \times 600 \times 10^6$

$= 24 \times 10^6$ m³ = 2400 ha.m. Choice (A)

21. Area = $\frac{\text{Volume}}{\text{depth}} = \frac{2400}{0.2}$ = 12000 ha. Choice (A)

22. Water holding capacity above wilting point = $F_C - w_c$ = 40 – 11 = 29%
Optimum moisture
$m_o = 40 - 0.75 \times 29 = 18.25\%$
$d_w = S.d[F_C - m_o]$

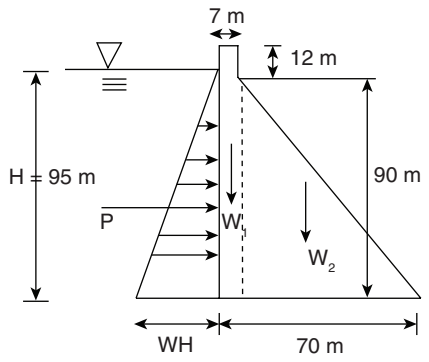
$= \frac{1400}{1000} \times 800[0.4 - 0.1825] = 243.6$ mm

Daily consumptive use C_u = 15 mm

∴ frequency of watering = $\frac{d_w}{C_u} = \frac{243.6}{15}$

= 16.24 ≅ 16 days. Choice (D)

24.



Horizontal water pressure P

$= \frac{wH^2}{2} = \frac{1000}{2} \times (95)^2 \times 10^{-3}$

= 4512.5 tonnes/m

Selfweight $W_1 = 7 \times 102 \times 2.4 = 1713.6$ tonnes/m

$W_2 = \frac{1}{2} \times (70 - 7) \times 90 \times 2.4$

= 6804 tonnes/m

Total self $wt\ W = W_1 + W_2 = 8517.6$ tonnes/m

$$F.O.S \text{ against sliding} = \frac{\mu W}{P} = 0.75 \times \frac{8517.6}{4512.5}$$

$$= 1.416 > 1$$

$\therefore$ safe against sliding. Choice (B)

25. $b = 70$m

$H_s = 7$m

$d = 6$m

$$\therefore \quad = \frac{b}{d} = \frac{70}{6} = \frac{35}{3}$$

$$\lambda = \frac{1\sqrt{1+\propto^2}}{2} = \frac{1+\sqrt{1+\left(\frac{35}{3}\right)^2}}{2} = 6.35$$

Exit hydraulic

Gradient $G_E = \frac{H_s}{d} \times \frac{1}{\pi\sqrt{\lambda}} = \frac{7}{6} \times \frac{1}{\pi\sqrt{6.35}} = 0.147$

Critical hydraulic Gradient

$$i_C = \frac{G-1}{1+e}$$

$$e = \frac{n}{1-n} = \frac{0.3}{1-0.3} = 0.429$$

$$i_C = \frac{2.7-1}{1+0.429} = 1.19.$$ Choice (C)

28. Neglecting approach velocity

$$Q = C_d L H^{\frac{3}{2}} = 2.4 \times 100 (3)^{\frac{3}{2}} = 1247.07 \text{ m}^3/\text{s}$$

Approach velocity

$$V_a = \frac{Q}{\text{Height} \times \text{Width of channel}}$$

$$= \frac{1247.07}{(10+3)\times 100}$$

$$= 0.959 \text{ m/s}$$

$$H_a = \frac{V_a^2}{2g} = \frac{(0.959)^2}{2 \times 9.81} = 0.047 \text{ m}$$

$$H_e = H + h_a = 3 + 0.047 = 3.047 \text{ m}$$

$$Q = 2.4 \times 100 \times (3.047)^{\frac{3}{2}}$$

$= 1276.49$ m^3/s. Choice (B)

29. $Q = 0.01$ cumecs $= 0.01 \times 3600$ m^3/hr

$= 0.0036$ hectare-m/hr

$y = 10$cm $= 0.1$m

$I = 6$cm/hour $= 0.06$ m/hr

$A = 0.05$ hectare

$$t = 2.303 \frac{y}{I} \log_{10}\left(\frac{Q}{Q - IA}\right)$$

$$= \frac{2.303 \times 0.1}{0.06} \log_{10}\left(\frac{0.0036}{0.0036 - 0.06 \times 0.05}\right)$$

$= 2.986$ hours. Choice (C)

30. Maximum area that can be irrigated $= \frac{Q}{I} = \frac{0.0036}{0.06}$

$= 0.06$ ha. Choice (C)

UNIT X
ENVIRONMENTAL ENGINEERING

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WATER SUPPLY ENGINEERING TEST I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Baylisturbidimeter is generally used to measure turbidities in the range of
(A) 0 – 10 mg/l (B) 0 – 5 mg/l
(C) 5 – 15 mg/l (D) 5 – 10 mg/l

2. Water supply projects, under normal circumstances, may be designed for a period of (from the completion of project)
(A) 10 years (B) 30 years
(C) 25 years (D) 25 years

3. The presence of silver, (Ag) in drinking water causes
(A) argyria (B) hypertension
(C) Anaemia (D) Blue baby disease

4. Ferric Chloride ($FeCl_3$) is used as a coagulant if the pH range is
(A) > 8.5 (B) < 6.5 and >8.5
(C) < 7 (D) None

5. Design a circular sedimentation (diameter (D) and depth (d) of tank) to capture flocculants particles after coagulant with a surface loading rate of 30 $m^3/day/m^2$. Detention time = 3 hrs to treat 15 MLD of water
(A) D = 20.3 m and d = 5.6 m
(B) D = 25.2 m and d = 5.6 m
(C) D = 20.3 m and d = 3.12 m
(D) D = 25.2 m and d = 3.12 m

6. Plain sedimentation follows
(A) Type – I settling
(B) Type – II settling
(C) Type – III settling
(D) Both B and C

7. Filtration helps in removing ___ from water.
(A) color and odour only
(B) turbidity
(C) Some pathogenic bacteria
(D) All the above

8. In water treatment, slow sand filters when compared to rapid sand filters produce
(A) lesser contaminated effluent
(B) More contaminated effluent
(C) Equally contaminated effluent
(D) Cannot be judged

9. An air bubble is present in water. If concentration of a gas (C_t) in water is more than saturated concentration (C_s), then
(A) absorption first, desorption next
(B) absorption takes place
(C) desorption takes place
(D) desorption first, absorption next

10. The valve which allows flow only in one direction is
(A) Reflux valve (B) Sluice valve
(C) Blow off valve (D) Air valve

11. Match Group – A with Group – B

	Group – A		Group – B
P.	Radial system	1.	More valves are used
Q.	Grid iron system	2.	Design calculations are simple
R.	Dead end system	3.	Distribution area divided into rectangular circular blocks.
S.	Ring system	4.	Suitable for direct pumping and gravity system

P Q R S P Q R S
(A) 3 4 1 2 (B) 2 3 4 1
(C) 1 2 3 4 (D) 4 1 2 3

12. Chlorine gas is used for disinfection combined with water to form hypochlorous acid (HOCl). The HOCl ionizes to form hypochlorite (OCl^-) in a reversible reaction: $HOCl \leftrightarrow H^+ + OCl^-$ ($k = 3 \times 10^{-8}$ at 20°C), the equilibrium of which is governed by pH. The sum of HOCl and OCl is known as free chlorine residual and HOCl is the more effective disinfectant. The 95% fraction of HOCl in the free chlorine residual is available at a pH value
(A) 5.38 (B) 6.24
(C) 7.82 (D) 8.38

Common Data for Questions 13 and 14:
In a rapid sand filter, the time for reaching particle breakthrough (T_B) is defined as the time elapsed from start of filter run to the time at which the turbidity of the effluent from the filter is greater than 2.5 *NTU*. The time for reaching terminal head loss (T_H) is defined as the time elapsed from the start of the filter run to the time when head loss across the filter is greater than 3 m.

13. The effect of increasing the porosity (while keeping all other conditions same) on T_B and T_H is
(A) T_B increases and T_H decreases
(B) T_B decreases and T_H increases
(C) Both T_B and T_H increase
(D) Both T_B and T_H decrease

14. The effect of increasing the concentration of impurities (while keeping all other conditions same) on T_B and T_H is
(A) T_B increases and T_H decreases
(B) T_B decreases and T_H increases
(C) Both T_B and T_H increase
(D) Both T_B and T_H decrease

15. A coagulation treatment plant with a flow of 0.7 m^3/s is dosing alum at 25 mg/L. No other chemicals are

added. The raw water suspended solid concentration is 40 mg/L. The effluent suspended solids concentration is measured as 15 mg/L. Specific gravity of sludge solids is 3.01. Find the volume of sludge solids produced in m^3/day?

(A) 0.53 (B) 0.41
(C) 0.62 (D) 0.86

16. In a continuous flow settling tank 3.5 m deep and 60 m long, what flow velocity of water would you recommend for effective removal of 0.026 mm particles at 25°C. The specific gravity of particles is 2.5 and kinematic viscosity for water is 0.01 cm^2/sec. (take free board as 0.5 m)

(A) 0.8 cm/s (B) 1.1 cm/s
(C) 2.0 cm/s (D) 1.6 cm/s

17. Match the type of settling Group – A with where it occurs Group – B

	Group – A		Group – B
P.	Type – I settling	1.	Secondary settling tank
Q.	Type – II settling	2.	Sedimentation with coagulation
R.	Type – III settling	3.	Waste water treatment with excessive solid concentration
S.	Type – IV settling	4.	Plain sedimentation

P Q R S P Q R S
(A) 4 2 1 3 (B) 2 4 3 1
(C) 1 3 2 4 (D) 3 1 4 2

18. Medium amount of coagulant dosage is used when turbidity (T) and Alkalinity (A) are

(A) T is high and A is low
(B) T is low and A is high
(C) Both T and A are high
(D) Both T and A are low

19. In order to test filtration process, clear water is made to pass through a bed of uniform sand at a filtering velocity of 4 m/hour. Sand bed has the following properties.
Depth of bed : 0.8 m
Sand grain size : 0.6 mm
Sand specific gravity : 2.65
Sand shape factor Φ : 0.9
Porosity of sand bed n : 0.5
Kinematic viscosity r : $1 \times 10^{-6} m^2/s$
Calculate the loss of head in filtration in (cm).

(A) 7.86 (B) 8.43
(C) 10.61 (D) 9.45

20. A water sample has pH of 10.3. The concentration of hydroxyl ions in the water sample is

(A) $10^{-10.3}$ moles/L
(B) 10^{-37} moles/L
(C) 3.39 mg/L
(D) 0.339 mg/L

Common Data for Questions 21 and 22:
Following chemical species were reported for water sample from a well:

Species	Concentration (milli equivalent/L)
Chloride (Cl^-)	10
Sulphate (SO_4^{2-})	20
Carbonate (CO_3^{2-})	7
Bicarbonate (HCO_3^-)	25
Calcium (Ca^{2+})	15
Magnesium (Mg^{2+})	20
pH	9

21. Total hardness in mg/L as $CaCO_3$ is

(A) 120.8 (B) 32.16
(C) 1600 (D) 1750

22. Carbonate hardness (mg/L × as $CaCO_3$) present in the above water sample is

(A) 32.16 (B) 1600
(C) 88.64 (D) 1750

23. Match the Characteristics of water in Group – A with corresponding test used for measuring in Group – B.

	Group – A		Group – B
P.	Color	1.	Nephelometer
Q.	Turbidity	2.	EDTA
R.	pH	3.	Tintometer
S.	Hardness	4.	Potentiometer

P Q R S P Q R S
(A) 3 1 4 2 (B) 4 3 1 2
(C) 1 4 3 2 (D) 2 1 3 4

24. Determine the future population of Mumbai town by Geometric increase method for the year 2021, given the following data.

Year	1951	1961	1971	1981	...	2021
Population in thousand	95	121	135	164	...	?

(A) 4,51,622 (B) 3,80,460
(C) 3,28,323 (D) 4,89,671

25. To treat 6 MLD of water 1 mg/L of bleaching power is added to water to have free chlorine residual of 0.1 mg/l. Find out Cl demand of water in mg/l and amount of bleaching powder/month in kgs if the available Cl in bleaching power is 40%.

(A) 0.9 mg/l and 180 kg/month
(B) 0.9 mg/l and 72 kg/month
(C) 0.3 mg/l and 72 kg/month
(D) 0.3 mg/l and 180 kg/month

Answer Keys

1. B	**2.** B	**3.** A	**4.** B	**5.** D	**6.** A	**7.** D	**8.** A	**9.** C	**10.** A
11. D	**12.** B	**13.** B	**14.** D	**15.** C	**16.** B	**17.** A	**18.** B	**19.** D	**20.** C
21. D	**22.** B	**23.** A	**24.** C	**25.** D					

Hints and Explanations

3. Argyria (blue-gray) discoloration of skin and also toxic to aquatic organisms. Choice (A)

5. $\text{Area} = \dfrac{Q}{v_o} = \dfrac{15\times10^6\times10^{-3}\ \text{m}^3/\text{day}}{30\ \text{m}^3/\text{day}/\text{m}^2} = 500\ \text{m}^2$

$\text{Volume} = Q \times DT$

$\dfrac{15\times10^6\times10^{-3}}{24}\times2.5$

$= 1562.5\ \text{m}^3$

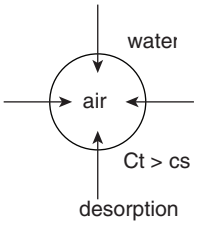
$\text{Depth} = \dfrac{\text{Volume}}{\text{Area}} = 3.125\ \text{m}$

$A = \dfrac{\pi}{4}d^2 = 500$

$d = 25.2\ \text{m}$ Choice (D)

6. Type I settling i.e., settling under gravity because specific gravity of impurities is greater than water. Choice (A)

9.



Gas tries to reach to saturated concentration. Choice (C)

12. $\dfrac{[\text{HOCl}]}{[\text{HOCl}]+[\text{OCl}^-]}\times100 = \dfrac{1}{1+\dfrac{k}{\text{H}^+}} \times 100 = \%\ \text{of HOCl}$

$0.95 = \dfrac{1}{1+\dfrac{(3\times10^{-8})}{\text{H}^+}}$

$[\text{H}^+] = 5.7\times10^{-7}$

$\text{pH} = -\log_{10}[\text{H}^+] = \log_{10}\left[\dfrac{1}{\text{H}^+}\right] = \log_{10}\left[\dfrac{1}{5.7\times10^{-7}}\right]$

$\text{pH} = 6.24$ Choice (B)

13. As porosity increases, more particles are allowed to pass through voids and turbidity reaches early

$\therefore$ T_B decreases

$h_f = \dfrac{fLV^2}{gd}\times\dfrac{(1-n)}{n^3\phi}$

$h_f \propto \dfrac{(1-n)}{n^3} \propto \dfrac{1}{n^2}$

n = porosity

$\dfrac{1}{T_H} \propto h_f \propto \dfrac{1}{n^2}$

$\therefore$ As porosity increases, head loss reduces and time for terminal head loss increases.

$\therefore$ T_H increases. Choice (B)

14. As concentration of impurities increase, as all voids get filled by impurities shearing off starts early.

$\therefore$ T_B (time for turbidity) reduces.

As concentration of impurities increase, more particles are clogged, more resistance to flow

$\Rightarrow$ more friction

$\therefore$ More head loss in less time

$\therefore$ T_H decrease Choice (D)

15. $\text{Al}_2(\text{SO}_4)_3.18\text{H}_2\text{O} + 3\ \text{Ca(OH)}_2$
(Alum)
$\rightarrow \text{CaSO}_4 + 2\ \text{Al(OH)}_3\downarrow + 18\ \text{H}_2\text{O}$

1 mole of Alum $\rightarrow$ 2 moles of Al $(\text{OH})_3$

666 gm of Alum $\rightarrow 2\times78$ gm of $\text{Al(OH}_3)$

1 gm of alum $= \dfrac{2\times78}{666} = 0.24$ gm of Al(OH)_3

$\therefore$ 25gm of alum will produce

$= 25\times0.24 = 6$ mg/l of solid sludge $\rightarrow$ (1)

Suspended solids (turbidity) removed

$= 40 - 15 = 25$ mg/l $\rightarrow$ (2)

Total dry sludge (solids) removed

$= (1) + (2) = 31$ mg/l

Discharge of plant $= 0.7\ \text{m}^3/\text{s} = 0.7\times24\times60\times60$

$Q = 60480\ \text{m}^3/\text{day}$

Total dry sludge solids produced per day

$= 31\ \text{mg/l}\times60480\ \text{m}^3/\text{day}$

$= 31\ \text{gm/m}^3\times60480\ \text{m}^3/\text{day}$

$= 1.87\times10^6\ \text{gm/day} = 1.87\ \text{t/day}$

Volume of solid sludge produced

$= \dfrac{1.87}{\text{Density of sludge solids}} = \dfrac{1.87\ \text{t/day}}{3.01\ \text{t/m}^3}$

$= 0.62\ \text{m}^3/\text{day}$ Choice (C)

16. Settling velocity for particles 0.026 mm (<0.1 mm) dia

$V_s = \dfrac{g}{18}(\text{S}-1)\dfrac{d^2}{\vartheta}$

$$V_s = \frac{981}{18}(2.5-1)\frac{(0.0026)^2}{0.01} = 0.055 \text{ cm/sec}$$

In settling tank,

$$\frac{V_S}{V_H} = \frac{H}{L}$$

$$\frac{V_S}{V_H} = \frac{30}{60}$$ [assuming freeboard = 0.5m,

$H = 3.5 - 0.5 = 3.0$m]

$$\frac{0.055}{V_H} = \frac{3}{60}$$

$V_H = 1.1$ cm/s

∴ Flow velocity = 1.1 cm/s Choice (B)

19. $$R_e = \frac{\rho_w V d}{\mu}\times\phi = \frac{Vd}{\gamma}\times\phi$$

$$= \left(\frac{4}{60\times60}\right)\times\left(\frac{0.6\times10^{-3}}{1\times10^{-6}}\right)\times0.9 = 0.6$$

Friction factor $f = \frac{150(1-n)}{R_e} + 1.75$

$$= \frac{150(1-0.5)}{0.6} + 1.75 = 126.75$$

$$h_f = \frac{fLV^2}{gd}\frac{(1-n)}{n^3\phi}$$

$$= \frac{126.75\times0.8\times}{9.81\times0.6\times10^{-3}}\left(\frac{4}{60\times60}\right)^2\times\left(\frac{1-0.5}{(0.5)^3\times0.9}\right)$$

= 0.0945 m = 9.45 cm. Choice (D)

20. $p[H] + p[OH] = 14$

$p[OH] = 14 - 10.3 = 3.7$

$-\log_{10}[OH] = 3.7$

Concentration of [OH]

$$= 10^{-3.7} \text{ moles/L} = 10^{-3.7}\times\frac{[17]}{1}$$

[∵ 17 is eqwt of OH]

$= 3.39 \times 10^{-3}$gm/L = 3.39 mg/l Choice (C)

21. TH in mg/l as $CaCO_3$

$$= Ca^{++} \text{ in mg/l} \times \left[\frac{\text{eq wt of } CaCO_3}{\text{eq wt of } Ca^{2+}}\right]$$

$$+Mg^{++} \text{ in mg/l} \times \left[\frac{\text{eq wt of } CaCO_3}{\text{eq wt of } Mg^{2+}}\right]$$

$[Ca^{2+}] = 15$ milli equivalent/L

$[Ca^{2+}] = 15 \times 10^{-3} \times 20$ gm/L

$= 0.3$ gm/L = 300 mg/l

$[Mg^{2+}] = 20$ mill equivalent/L

$$= 20 \times \left[\frac{24}{2}\right] \text{mg/l} = 240 \text{ mg/l}$$

$$[CO_3^{2-}] = 7 \times \left[\frac{60}{2}\right] = 210 \text{ mg/l}$$

$$[HCO_3^-] = 25 \times \left[\frac{61}{1}\right] = 1525 \text{ mg/l}$$

$$TH \text{ in mg/l as } CaCO_3 = Ca^{2+}\times\left[\frac{50}{20}\right] + Mg^{2+}\times\left[\frac{50}{12}\right]$$

$$= 300\times\left[\frac{50}{20}\right] + 240\times\left[\frac{50}{12}\right]$$

= 1750 mg/l as $CaCO_3$ Choice (D)

22. Alkalinity in mg/l as

$$CaCO_3 = CO_3^{2-}\times\left[\frac{50}{30}\right] + HCO_3^-\times\left[\frac{50}{61}\right]$$

$$= 210\times\left[\frac{50}{30}\right] + 1525\times\left[\frac{50}{61}\right]$$

= 1600 mg/l as $CaCO_3$

Alkalinity $< TH$

Carbonate hardness = least of both

= 1600 mg/l as $CaCO_3$ Choice (B)

24.

Year (1)	Population in Thousand (2)	Increase in Population in Thousands (3)	% increase in population = growth rate = Col(3)/Col(2) × 100 (4)
1951	**95**		
1961	121	26	$\frac{26}{95}\times100 = 27.37$
1971	135	14	11.57
1981	164	29	21.48

Constant growth rate assumed for future

r = geometric mean of past growth rates

$$= \sqrt[3]{27.37\times11.57\times21.48}$$

= 18.95% per decade

The population after n decades

$$P_n = P_o\left(1+\frac{r}{100}\right)^n$$

P_{2021} = Population 4 decades from 1981

$$= P_{1981}\left(1+\frac{18.95}{100}\right)^4 = 164000\times(1.1895)^4$$

= 3, 28,323 Choice (C)

25. Cl_2 dose = 1 mg/l × 40% Cl_2 = 0.4 mg/l

Cl demand = Cl dose added – residual

= 0.4 – 0.1 = 0.3 mg/l

Total bleaching powder added

= Q × dosage of bleach = 6 × 1 mg/l

= 6 kg/day = (6 × 30) kg/month

= 180 kg/month Choice (D)

WASTE WATER ENGINEERING TEST 2

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Maximum runoff will be obtained from the rain having a duration equal to time of concentration, and this is called the
(A) Point rainfall intensity
(B) Time of equilibrium
(C) One hour rainfall
(D) Critical rainfall duration

2. A well oxidized sewage will contain
(A) Nitrites and sulphur
(B) More Ammonia and H_2S but less nitrates and sulphates
(C) Less Ammonia and H_2S but more nitrates and sulphates
(D) H_2S, CO_2 and water

3. Particles whose size is greater than 1 μm are
(A) Colloidal solids (B) Suspended solids
(C) Settleable solids (D) Dissolved solids

4. In determination of Chemical Oxygen Demand, sewage is titrated with ferrous ammonium sulphate using potassium dichromate as oxidant in the presence of silver sulphate and mercuric sulphate as catalyst and inhibitor respectively. In this the excess amount of _____ left in sample after digestion is found.
(A) Potassium dichromate
(B) Ferrous Ammonium sulphate
(C) Mercuric Sulphate
(D) Oxygen

5. The degree and amount of treatment given to raw sewage before disposing off into river stream will depend upon
1. Quantity of raw sewage only
2. Self purification capacity of river stream
3. Intended use of its water
(A) 1, 3 (B) 2 only
(C) 1, 2, 3 (D) 1 only

6. Preliminary treatment reduce the BOD of the waste water by
(A) 5 – 10% (B) 10 – 20%
(C) 15 – 30% (D) 25 – 40%

7. The girt chambers of a sewage treatment plant normally need cleaning
(A) Every hour (B) Every day
(C) Every fortnight (D) Every year

8. Activated sludge treatment plants are normally preferred for
(A) Large sized cities
(B) Towns and smaller cities
(C) Medium sized cities
(D) All of them

9. Prechlorination of sewage before it enters the sedimentation tank may help in
1. Controlling odour
2. Prevent flies in trickling filter
3. Assist in removal of grease
4. Reduce bacterial count
(A) 1 and 4 (B) 1, 2, 3 only
(C) 2, 3, 4 only (D) All of the above

10. In the final stage of sludge digestion, more resistant materials like proteins and organic acids are attacked and broken up by anaerobic bacteria called
(A) Acid formers (B) Suspended culture
(C) Mesophilic organisms (D) Methane formers

11. A river with a flow of 0.25 m^3/s receives waste water at a rate of 0.1 m^3/s. River has 8 mg/l of DO with no BOD. The waste water with BOD 30 mg/l having no DO disposed into it. If deoxygenating and reoxygenation constant are 0.36 d^{-1} and 0.7 d^{-1} respectively. Find the critical deficit (D_C) and distance at which it occurs on D/s of disposal of waste water?
Assume $(DO)_{saturation}$ = 9 mg/l and velocity of flow in river = 0.4 m/s.
(A) D_C = 3.825 mg/l at 30.96 km
(B) D_C = 3.4 mg/l at 5 km
(C) D_C = 3.29 mg/l at 30 km
(D) D_C = 2.8 mg/l at 15 km

12. In a community of 1500 people water is supplied at 200 l/head/day. If BOD produced is 40 g/head/day and BOD loading rate for oxidation pond is 20 kg/ha/day. (Assume depth of pond, d = 2 m and efficiency of pond as 80%).
Find the BOD of the effluent and whether it can be used for irrigation?
(A) 100 mg/l and suitable for irrigation
(B) 150 mg/l and not suitable for irrigation
(C) 75 mg/l and not suitable for irrigation
(D) 50 mg/l and suitable for irrigation

13. Following data pertaining to Activated sludge process is given below:
Flow rate Q = 12,000 m^3/day
BOD of influent y_i = 280 mg/l
BOD of effluent y_e = 30 mg/l
X: MLSS concentration = 3000 mg/l
X_u: MLSS in underflow = 10,000 mg/l
X_e: MLSS in effluent = 0
If = $\frac{F}{M}$ 0.3 day^{-1} and θ_C = 12 days. Find mass of solids wasted in kg/day?

(A) 1000 (B) 1120
(C) 1500 (D) 800

14. Find out the Theoretical Oxygen Demand (in mg/l) for glucose of 300 mg/lpresent in waste waters if the following chemical reaction is given

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$$

(A) 320 (B) 420
(C) 120 (D) 220

15. A population of 20,000 is residing in a town having an area of 60 hectares. Water supply per capita is 120 lt/day. If the average runoff coefficient of the area is 0.5 and time of concentration of design rain is 30 minutes, Calculate the discharge for which the sewers of a proposed combined system will be designed for the town? [Use rational method for runoff. Assume 80% of water supply as wastage and maximum sewage discharge as 3 × (Average discharge)]

(A) 1.67 m^3/s (B) 2.1 m^3/s
(C) 1.74 m^3/s (D) 0.06 m^3/s

16. Match List – I with List – II and select the correct answer using the code given below.

	List – I (Process)		**List – II (Biological agent)**
a.	Oxidation ditch	1.	Facultative bacteria
b.	Waste water stabilization	2.	Anaerobic bacteria
c.	Imhoff tank	3.	Suspended Culture (Aerobic bacteria)
d.	Rotating Biological Contactors (RBC)	4.	Attached Culture (Aerobic bacteria)

a b c d
(A) 4 1 2 3
(C) 1 2 3 4

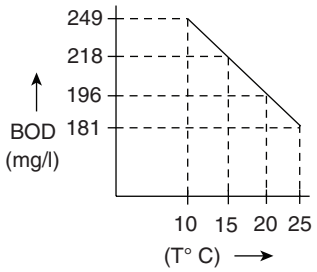
a b c d
(B) 3 1 2 4
(D) 3 4 1 2

17. The sewage is flowing at 5 MLD from a primary clarifer to a standard rate trickling filter. The $y_5^{20°C}$ = 180 mg/l. The value of the adopted organic loading is to be 150 $gm/m^3/day$ and surface loading 1500 $l/m^2/day$. Calculate the efficiency of this filter unit.

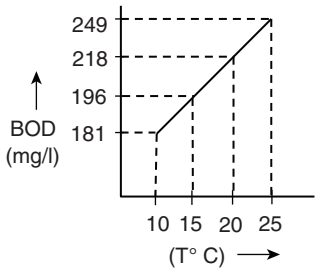
(A) 85.44% (B) 78.51%
(C) 80.03% (D) 69.91%

18. The 3 day 20°C BOD of a sample of sewage is 200 mg/l. Draw a graph of 5 day BOD as a function of temperature in the range, 10°C to 25°C in steps of 5°C. Assume K_D at 20°C = 0.1

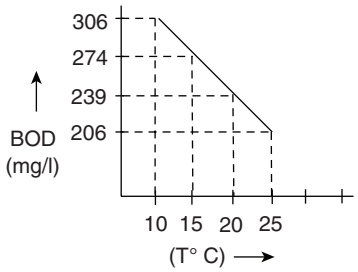
(A)



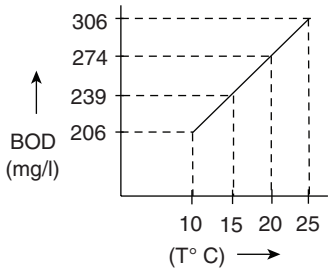
(B)



(C)



(D)



Common Data for Questions 19 and 20:
A completely mixed activated sludge process is used to treat a waste water flow of 2 MLD having a BOD at 5 days as 250 g/l. The biomass concentration of aeration tank is 2100 mg/l and the concentration of the net biomass leaving the system is 50 mg/l. The aeration tank has a volume of 250 m^3.

19. What is the hydraulic retention time of the waste water in aeration tank? (in hours)

(A) 1 (B) 3
(C) 9 (D) 12

20. What is the average time for which the biomass stays in the system?

(A) 3.18 hours (B) 4.61 days
(C) 5.25 days (D) 8.21 hours

21. Match Column – I with Column – II

	Column – I		**Column – II**
P.	Grit Chamber	1.	Zone settling
Q.	Secondary settling tank	2.	Stoke's law
R.	Activated sludge process	3.	Aerobic process
S.	Trickling filter	4.	Contact stabilization

P Q R S
(A) 1 2 3 4
(C) 1 2 4 3

P Q R S
(B) 2 1 3 4
(D) 2 1 4 3

22. Statement I: Nitrates are non-objectionable end products in aerobic treatment of sewage.
Statement II: Nitrates > 45 ppm cause nitrate poisoning in infants.
(A) I is true and II is false
(B) I is false and II is true
(C) Both I and II are true
(D) Both Iand II are false

23. A sewer has a diameter of 250 mm and slope of 1 in 500. While running full it has a mean velocity of 0.8 m/s. If both diameter and slope are doubled, what will be the changed mean velocity when running half full? (Use Manning's formula)
(A) 1.0 m/s (B) 1.2 m/s
(C) 1.6 m/s (D) 1.8 m/s

24. A combined sewer is serving 30,000 persons having BOD 80 gm per capita for day and 60,000 liters industrial effluent per day having BOD 500 mg/l. If average standard BOD of domestic sewage is 0.09 kg/day/person. Find the population equivalent of sewage
(A) 27,000 (B) 20,000
(C) 22,000 (D) 25,000

25. Match List – I with List – II

	List – I (Empirical Formula)		**List – II (Catchments)**
P.	Inglis formula	1.	North India
Q.	Ryve's formula	2.	South India
R.	Nawab Jung Bahadur formula	3.	Old Bombay State
S.	Burge's formula	4.	Hyderabad Deccan
		5.	Based on Indian records

P Q R S P Q R S
(A) 2 1 3 4 (B) 1 3 4 5
(C) 2 3 1 5 (D) 3 2 4 5

Answer Keys

1. D	**2.** C	**3.** B	**4.** A	**5.** C	**6.** C	**7.** C	**8.** A	**9.** B	**10.** D
11. A	**12.** D	**13.** B	**14.** A	**15.** C	**16.** B	**17.** A	**18.** D	**19.** B	**20.** C
21. B	**22.** C	**23.** D	**24.** A	**25.** D					

Hints and Explanations

2. Free ammonia, H_2s and nitrites indicate initial stages of decomposition. Nitrites and sulphates indicates fully oxidized sewage Choice (C)

3.
$$\begin{bmatrix} \text{Pariticlessize} & > 10\ \mu_m \Rightarrow \text{settleable solids.} \\ & > 1\ \mu_m \Rightarrow \text{suspended solids.} \\ & 1-10^{-3}\ \mu_m \Rightarrow \text{Colloidals solids.} \\ & < 10^{-3}\ \mu_m \Rightarrow \text{dissolved solids.} \end{bmatrix}$$
Choice (B)

7. Grit chambers of a sewage treatment plant are cleaned every fort night Choice (C)

8. Activated sludge treatment plants are prepared large sized cities as they are costly. Choice (A)

9. Prechlorination helps for first three whereas Post chlorination helps for reducing bacterial count and BOD. Choice (B)

10. In Acid Fermentation (1st stage) sewage is acted upon by acid formers. In Alkaline Fermentation (3rd stage) sewage is acted upon by methane formers. Choice (D)

11.

River	**Waste water**
Q_R: 0.25 m³/s	Q_W = 0.1 m³/s
$(DO)_R$: 8 mg/l	$(DO)_W$ = 0
y_R : 0	y_w : 30 mg/l

$K_1 = 0.36\ d^{-1}$
$K_2 = 0.7\ d^{-1}$

$$y_{mix} = \frac{(0+0.1\times 30)}{(0.25+0.1)} = 8.57 \text{ mg/l}$$

$$(DO)_{mix} = \frac{0.25\times 8}{0.25+0.1} = 5.71 \text{ mg/l}$$

$$D_O = (DO)_{sat} - (DO)_{mix}$$
$$= 9 - 5.71 = 3.29 \text{ mg/l}$$

$$y_{mix} = L_o\left[1-e^{-k_1 t}\right]$$

$$8.57 = L_o\left[1-e^{-0.36\times 5}\right]$$

$$\Rightarrow\ L_o = 10.27 \text{ mg/l}$$

Critical time

$$t_c = \frac{1}{K_2-K_1}\ln\left[\frac{K_2}{K_1}\left(1-D_o.\frac{K_2-K_1}{K_1 L_0}\right)\right]$$

$$= \frac{1}{0.7-0.36}\ln\left[\frac{0.7}{0.36}\left(1-3.29\times\frac{(0.7-0.36)}{0.36\times 10.27}\right)\right]$$

$$= 0.896 \text{ days}$$

Critical deficit: $D_C = \frac{K_1}{K_2} L_o e^{-k_1 t_c}$

$$= \frac{0.36}{0.7}\times 10.27\ e^{-0.36\times 0.896}$$

$$= 3.825 \text{ mg/l}$$

Velocity flow in river $V = 0.4$ m/s
Distance at which critical flow occurs
$= 0.4 \times 0.896 \times 24 \times 60 \times 60$
$= 30.96 \times 10^3$ m
$= 30.96$ km. Choice (A)

12. Population = 1500
Water supplied = 200 l/head/day
80% of water is converted to waste water
$Q_w = 1500 \times 200 \times 0.8 = 24 \times 10^4$ l/day
$= 240$ m^3/day $= 0.24$ MLD
Total BOD produced $= 1500 \times 40$
$= 60 \times 10^3$g/day
$= 60$ kg/day
Total BOD applied $= Qy_i$
60×10^6mg/day $= 0.24 \times 10^6$l/day $\times y_i$
$y_i = 250$ mg/l
$\eta = 80\%$

$$80 = \frac{250 - y_e}{250} \times 100$$

$y_e = 50$ mg/l ≤ 100 mg/l which is the limitation value for irrigation
$\therefore$ Effluent water is suitable for Irrigation. Choice (D)

13. We know,

$$\frac{F}{M} = \frac{Qy_i}{VX}$$

$$0.25 = \frac{12,000 \times 280}{V \times 3000}$$

$V = 4480$ m^3

$$\theta_c = \frac{\text{Mass of solids in aeration}}{\text{Mass of solids wasted/day}}$$

$$12 = \frac{VX}{Q_w X_u + Q_e X_e}$$

$$Q_w X_u = \frac{4480 \times 10^3 \times 3000}{12} \quad [\because 1\text{m}^3 = 1000 \text{ L and } X_e = 0]$$

$= 1.12 \times 10^9$ mg/day
$= 1120$ kg/day. Choice (B)

14. Mol. Wt of $C_6H_{12}O_6$ (Glucose)
$= (6 \times 12) + (12 \times 1) + (6 \times 16) = 180$
Mol. wt of $6O_2 = 6[2 \times 16] = 192$
180 parts of glucose demand: 192 parts of O_2
1 part glucose demand $= \frac{192}{180}$ parts of O_2
300 mg/l of glucose demand $= \frac{192}{180} \times 300$
$= 320$ mg/l of O_2
$\therefore$ TOD = 320 mg/l. Choice (A)

15. Quantity of sanitary sewage produced per day

$$= \left(\frac{80}{100}\right) \times 120 \times 20000 = 1.92 \times 10^6 \text{ltrs.}$$

Quantity of sewage produced per second

$$= \frac{1920}{24 \times 60 \times 60} = 0.022 \text{ cumecs}$$

Maximum sewage discharge $= 3 \times 0.022$
$Q_s = 0.066$ cumecs
Storm water discharge by Rational Method

$$Q_R = \frac{KP_C A}{36}$$

$$\left[\begin{array}{l} P_C = \dfrac{a}{t_c + b} \text{ cm/hr} \\ \text{as } t_c \ge 20 \text{ minutes} \\ a = 100; b = 20 \end{array}\right]$$

$$P_C = \frac{100}{30 + 20} = 2\text{cm/hr}$$

$$Q_R = \frac{1}{36} \times 0.5 \times 2 \times 60$$

$= 1.67$ cumecs
Total peak discharge $= Q_s + Q_R = 0.066 + 1.67$
$= 1.736$ cumecs. Choice (C)

17. Efficiency $\eta = \dfrac{100}{1 + 0.0044\sqrt{u}}$

u = organic loading in kg/ha–m/day
$u = 150$ gm/m^3/day
1 hectare $= 10^4$m^2
$u = 150 \times 10^4$gm/ha–m/day
$= 1500$ kg/ha–m/day

$$\eta = \frac{100}{1 + 0.0044\sqrt{1500}}$$

$= 85.44\%$. Choice (A)

18. $y_3^{20^\circ C} = L\left[1 - (10)^{-K_D \times t}\right]$

$200 = L[1 - (10)^{-0.1 \times 3}]$
$L = 400.95$ mg/l

$$K_{D(10^\circ C)} = K_{D(20^\circ C)} [1.047]^{T - 20^\circ C}$$

$$= 0.1[1.047]^{10-20}$$
$$= 0.063$$

$$K_{D(15^\circ C)} = 0.1[1.047]^{15-20} = 0.079$$

$$K_{D(25^\circ C)} = 0.1[1.047]^{25-20} = 0.1258$$

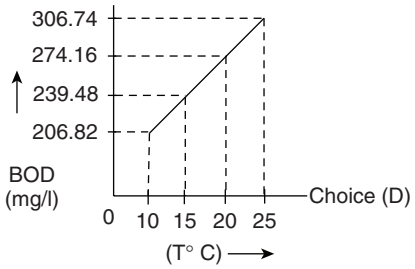
$$y_5^{10^\circ C} = 400.95[1 - 10^{-0.063 \times 5}]$$
$$= 206.82 \text{ mg/l}$$

$$y_5^{15^\circ C} = 400.95[1 - 10^{-0.079 \times 5}]$$
$$= 239.48 \text{ mg/l}$$

$$y_5^{20^\circ C} = 400.95[1 - 10^{-0.1 \times 5}]\text{s}$$
$$= 274.16 \text{ mg/l}$$

$y_5^{25^\circ C} = 400.95\ [1 - 10^{-0.1258 \times 5}]$

$= 306.74$ mg/l



Choice (D)

19. $Q = 2 \times 10^6$ lit/day

$y_i = 250$ mg/l

$X = 2100$ mg/l

$X_e = 50$ mg/l

$V = 250\ m^3 = 250 \times 10^3$ lit

$$HRT = \frac{V}{Q} = \frac{250 \times 10^3}{2 \times 10^6}$$

$= 0.125$ days $= 3$ hours. Choice (B)

20. $$\theta_C = \frac{VX}{Q_w X_w + Q_e X_e}$$

Neglecting X_w (being small and not given)

$$Q_C = \frac{250 \times 10^3 \times 2100}{2 \times 10^6 \times 50}$$

$= 5.25$ days. Choice (C)

22. Statement I and II seems to be contradicting.
In infants there exists lower acidity in intestines which permits growth of nitrate reducing bacteria, which convert nitrate to nitrites. Nitrates' having great affinity for haemoglobin than oxygen and cause suffocation and body turns blue called blue baby disease or mathaemoglobinemia. This is only in children <6 months.

Choice (C)

23. $$V = \frac{1}{N} R^{2/3} S^{1/2}$$

Hydraulic mean depth $R = \frac{D}{4}$

$$\frac{V_1}{V_2} = \frac{R_1^{2/3} S_1^{1/2}}{R_2^{2/3} S_2^{1/2}}$$

$$R_1 = \frac{D_1}{4}$$

$$R_2 = \frac{D_2}{4} = \frac{2D_1}{4}$$ [though running half/full $R = \frac{D}{4}$]

$$S_1 = \frac{1}{500}$$

$$S_2 = \frac{1}{250}$$

$$\frac{0.8}{V_2} = \frac{R_1^{2/3}}{(2R_1)^{2/3}} \times \frac{\left(\frac{1}{500}\right)^{1/2}}{\left(\frac{1}{250}\right)^{1/2}}$$

$V_2 = 1.795$ m/s

$\simeq 1.8$ m/s. Choice (D)

24. (i) BOD of sewage produced per day
$= 30000 \times 80 = 2400 \times 10^3$ gm/day
$= 2400$ kg/day

(ii) BOD of industrial effluent per day
$= 60000 \times 500$
$= 30 \times 10^6$ mg/day
$= 30$ kg/day

Total BOD entering the sewer
$=$ (i) + (ii)
$= 2430$ kg/day

Population equivalent

$$= \frac{\text{Total BOD}}{\text{Standard BOD of domestic sewage}}$$

$$= \frac{2430}{0.09} = 27000.$$ Choice (A)

Solid Waste Management and Pollution Test 3

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following factors is primarily responsible for causing air pollution in modern days is
(A) Dust storms
(B) Forest fires
(C) Industries and automobiles
(D) None

2. The most hazardous gaseous air pollutant for human health is:
(A) Nitrogen
(B) Carbondioxide
(C) Oxygen
(D) Sulphurdioxide

3. Among the given choices below; which of the following is not a primary air pollutant?
(A) PAN
(B) Volatile organic compounds like hydrocarbon
(C) Suspended particulate matter
(D) Oxides of nitrogen

4. During temperature inversion in atmosphere, air pollutants tend to.
(A) Accumulate above inversion layer
(B) Accumulate below inversion layer
(C) Disperse laterally
(D) Disperse vertically

5. For ambient air quality standard, the permissible SO_2 for residential and industrial areas in India is
(A) 40 $\mu g/m^3$ (B) 50 $\mu g/m^3$
(C) 65 $\mu g/m^3$ (D) 35 $\mu g/m^3$

6. The frequency of sound is measured in (units):
(A) Hertz (B) Doboson unit (DU)
(C) Decibel (dB) (D) None

7. As per IS:4954 – 1964; An acceptable noise level for residential and business urban areas is
(A) 40-50 dB (B) 30-40 dB
(C) 15-25 dB (D) 50-60 dB

8. Noise is measured in units of:
(A) Bacqueral (B) Doboson
(C) Hertz (D) Decibel

9. The method of refuse disposal, involving burial in trenches, is called
(A) Incineration (B) Composting
(C) Polverisation (D) None

10. The quantity of refuse produced in an average Indian city or a town is of the order of:
(A) $\frac{1}{4}-\frac{1}{5}$ t/c day (B) 4 – 6 t/c day
(D) 2 – 4 t/c day (D) 1 – 2 t/c day

11. The sound pressure level for a jet plane on the ground with sound pressure of 200 μ bar should be
(A) 40 dB (B) 80 dB
(C) 100 dB (D) 120 dB

12. Express 350 $\mu g/m^3$ of SO_2 in ppm at STP?
(A) 0.122 (B) 0.142
(C) 0.20 (D) 0.31

13. An air parcel having 50°C temperature moves from ground level to 800 m elevation in dry air following the adiabatic lapse rate. The resulting temperature of air parcel at 800 m elevation will be?
(A) 40°C (B) 35°C
(C) 42°C (D) 50°C

14. The maximum dispersion of pollutants in atmosphere occur when
(A) Environmental lapse rate is equal to adiabatic lapse rate
(B) Environmental lapse rate is less than adiabatic lapse rate
(C) Environmental lapse rate is greater than adiabatic lapse rate
(D) None

15. Elevation and temperature data for a place are tabulated below:

Elevation, m	Temperature °C
10 m	25.25°C
250 m	15.70°C

Based on the above data, lapse rate can be referred as:
(A) Sub-adiabatic (B) Super adiabatic
(C) Neutral (D) inversion

16. If energy content of solid waste as discarded is 14,700 kJ/kg. Find energy content on dry basis if moisture content of solid waste is 20%
(A) 18375 kJ/kg (B) 18000 kJ/kg
(C) 14,700 kJ/kg (D) 15,000 kJ/kg

17. The composition of certain MSW sample and specific weight of its various components are given below:

Component	Percent by weight	Specific weight (kg/m3)
Food waste	70	300
Dirt ash	20	500
Plastics	10	65

The specific weight (kg/m^3) of the MSW sample is
(A) 209 (B) 319
(C) 217 (D) 234

18. A waste water stream (flow = $3m^3/s$) with ultimate BOD 110 mg/ℓ is joining a small river (flow = $10m^3/s$),

ultimate BOD = 5 mg/ℓ. Both water streams get mixed up simultaneously at where cross sectional area of the river is 70 m^2.Assuming K = 0.25 / day, the BOD (in mg/ℓ) of the river water, 10 km downstream of the mixing point is

(A) 8.832 gm/m^3 (B) 24 gm/m^3
(C) 15 gm/m^3 (D) none

19. The reference pressure used in determination of sound pressure level is:

(A) 20 db (B) 10 μPa
(C) 20 μPa (D) 10 db

20. Which one of the following is the correct sound intensity expression with usual notations.

(A) dB = 10 $\log_{10}(I/I_0)^2$
(B) dB = 10 $\log_{10}(I/I_0)$
(C) dB = 10 $\log_{10}(I - I_0)^2$
(D) dB = 10 $\log_{10}(I - I_o)$

21. A source emitting 40 dB, 70 dB, 110 dB of different times in a day. What is average noise produces by source in a day?

(A) 220 dB (B) 80 dB
(C) 74 dB (D) 100 dB

22. Which among the following are two biodegradable components of municipal solid waste

(A) Leather and tin cans
(B) Plastics and wood
(C) Cardboard and glass
(D) Food waste and garden trimmings

23. Two electrostatic precipitators are in series. The fractional efficiencies of the upstream and downstream ESPS for size *dp* are 90% and 80% respectively. What is the overall efficiency of the system for same *dp*?

(A) 93% (B) 95%
(C) 98% (D) 90%

24. SO_2 and CO adversely effects

(A) Oxygen carrying capacity of blood and functioning of lungs respectively.
(B) Functioning of the respiratory system and brain respectively.
(C) Functioning of the respiratory system and oxygen carrying capacity of blood respectively.
(D) Functioning of air passage and chess respectively.

25. Two air pollution control devices that are usually used to remove very fine particles from the flue gas are:

(A) Cyclone and venture scrubber
(B) Cyclone and packed scrubber
(C) Electrostatic precipitator and fabric filter
(D) Settling chamber and tray scrubber

Answer Keys

1. C	**2.** D	**3.** A	**4.** B	**5.** B	**6.** A	**7.** A	**8.** D	**9.** B	**10.** A
11. D	**12.** A	**13.** C	**14.** C	**15.** B	**16.** A	**17.** D	**18.** A	**19.** C	**20.** A
21. D	**22.** D	**23.** C	**24.** C	**25.** C					

Hints and Explanations

1. Industries and automobile is primarily responsible for causing air pollution in modern days. Choice (C)

2. Sulphur dioxide is the most hazardous gaseous air pollutant. Choice (D)

3. PAN is not a primary air pollutant. Choice (A)

5. The permissible SO_2 for residential and Industrial area in India is 50 μg/m^3. Choice (B)

6. Frequency of sound is measured in Hertz. Choice (A)

7. An acceptable noise level of 40–50 dB in residential area is allowed. Choice (A)

8. Decibel is the unit to measure noise. Choice (D)

11. Sound pressure level in (dB)

$$= 20 \log_{10}\left[\frac{P_{rms}}{20}\right]$$

P_{rms} = 200 μ bar
= 200 × 10^5μPa [1 bar = 100kPa= 10^5Pa]
= 20 × 10^6μ.Pa

$$dB = 20 \log_{10}\frac{20 \times 10^6\ \mu.\ P_a}{20\ \mu.\ P_a}$$

= 20 × 6 = 120 dB Choice (D)

12. At STP

$$1PPM = \frac{M}{22.4} \times 10^3 \mu g l m^3$$

$$1\mu g/m^3 = \frac{22.4}{M \times 10^3}\ PPM$$

$$350\ \mu g/m^3 = \frac{22.4}{M \times 10^3} \times 350\ PPM$$

Where M = molecular weight of SO_2
= 32 + (2 × 16) = 64

$$= \frac{22.4 \times 350}{64 \times 10^3}$$

= 0.122 PPM. Choice (A)

13. For adiabatic lapse rate: There is a decrease in temperature of 1°C for every 100 m

i.e., $\frac{dT}{dZ} = -1°C/100m$

$100m \rightarrow -1°C$

$800m \rightarrow ?$

$$\frac{800(-1\,°C)}{100} = -8\,°C$$

$\therefore$ The resulting temperature

$= 50 - 8 = 42°C.$ Choice (C)

15. Lapse rate $= \frac{dT}{dZ} = \frac{25.25 - 15.70}{250 - 10}$

$$= \frac{9.55}{240} = 0.039$$

$$= \frac{-3.97\,°C}{100\ m}$$

Adiabatic lapse rate $= \frac{-1\,°C}{100\ m}$

$\therefore$ Lapse rate > ALR

$\Rightarrow$ Super adiabatic. Choice (B)

16. Energy content on dry basis

$= \text{Energy content as discarded} \times \left[\frac{100}{100 - mc}\right]$

where mc - moisture content.

$$= 14,1700 \times \frac{100}{100 - 20}$$

$= 18375$ KJ/kg. Choice (A)

17. $$\frac{100}{\text{specific weight of MSW}} = \frac{\%\text{Foodwaste}}{\text{specific weight of foodwaste}} + \frac{\%\text{dirt ash}}{\text{specific weight of dirt ash}} + \frac{\%\text{plastics}}{\text{specific weight of plastics}}$$

$$\frac{100}{\text{specific weight of MSW}} = \frac{70}{300} + \frac{20}{500} + \frac{10}{65}$$

Specific weight of MSW (kg/m^3) = 234. Choice (D)

18. BOD of mixture $= \frac{Q_1 \times (BOD)_1 + Q_2 \times (BOD)_2}{Q_1 + Q_2}$ s

$$= \frac{(3)(110) + (10 \times 5)}{3 + 10}$$

$= 29.23$ gm/m^3

(BOD) of mixture at 10 km downstream of mixing point

$y_{10km} = y_0(1 - 10^{-Kt})$

Time taken to reach 10 km,

$t = \frac{\text{distance}}{\text{velocity}}$;

velocity of flow $= \frac{Q_1 + Q_2}{A} = \frac{10 + 3}{70}$

$V = 0.185$ m/sec

$\therefore \quad t = \frac{10 \times 1000}{0.185} \times \frac{1}{24 \times 60 \times 60}$,

$t = 0.625$ day

$y_{10} = 29.23\ [1 - 10^{-0.25 \times 0.625}]$

$= 8.832\ \frac{gm}{m^3}$ Choice (A)

21. $\overline{L_P} = 20 \log_{10} \frac{1}{N} \Sigma (10)^{L_{N/20}}$

$$= 20 \log_{10} \frac{1}{3}\left[10^{\frac{40}{20}} + 10^{\frac{70}{20}} + 10^{\frac{110}{20}}\right]$$

$= 100.54$dB. Choice (D)

23. Particles removed by ESP1 = 90%

Only 10 % particles remains left and flows through ESP 2.

So, Particles removed by ESP2 = 80 % of 10% = 8%

$\therefore$ Overall efficiency = 90 + 8 = 98% Choice (C)

Environmental Engineering Test 4

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. SO_x in atmosphere is measured by
(A) Non-dispersive infrared analyzer
(B) West and Gack method
(C) Sodium arsenate method
(D) Gas Chromatography

2. Algae dies out, though fish life may survive, in a river zone, known as
(A) Zone of degradation
(B) Zone of active decomposition
(C) Zone of recovery
(D) None of these

3. Man hole covers are circular in shape to
(A) strengthen the cover
(B) make entry convenient
(C) for architectural reasons
(D) prevent falling of cover into manhole

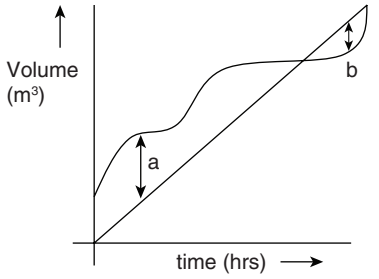
4. The factors which affect the sludge digestion are
1. Temperature
2. pH
3. Mixing and stirring of raw sludge
4. Seeding of sludge
(A) 1 and 2 (B) 2 and 3
(C) 2, 3, 4 only (D) All the above

5. Primary Clarifier used in waste water treatment follows _____ type of settling.
(A) Type I settling
(B) Type II settling
(C) Type III settling
(D) Type IV settling

6. If the Cl demand of water is 0.5 mg/l to have a residual Cl of 0.1 mg/l. What dose of bleaching powder is added to the water if bleaching powder contains 32% of available chlorine.
(A) 1.875 mg/l
(B) 1.25 mg/l
(C) 1.562 mg/l
(D) 2.188 mg/l

7. If the methyl orange alkalinity of water equals or exceeds total hardness, all of the hardness is
(A) Non-Carbonate hardness
(B) Carbonate hardness
(C) Pseudo hardness
(D) Negative non-carbonate hardness

8. The given figure shows roughly the daily mass curves of supply and demand from an elevated reservoir. The minimum required capacity of the reservoir is given by

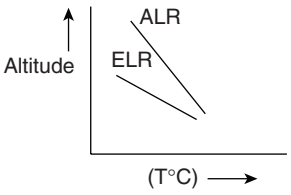
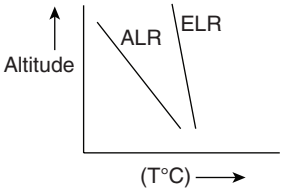
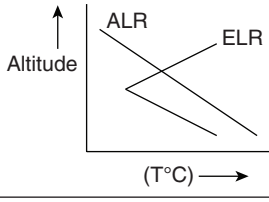


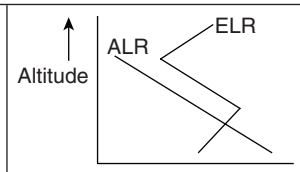
(A) $a - b$
(B) $a + b$
(C) $a \times b$
(D) larger of a or b

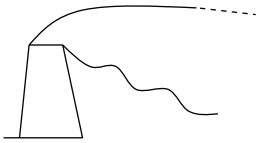
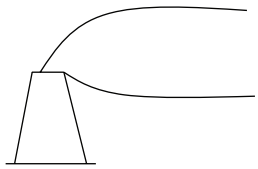
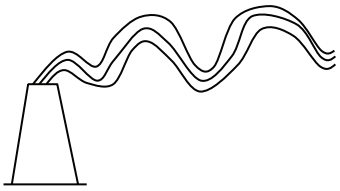
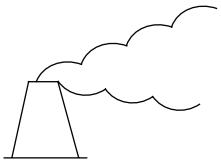
9. Which of the following are removed by rapid sand filter from water?
1. Dissolved solids
2. Suspended solids
3. Bacteria
4. Helminths
(A) 1 and 2
(B) 2 and 3
(C) 1 and 3
(D) 2, 3 and 4

10. The yield of a well depends upon
(A) permeability
(B) area of aquifer opening into the wells
(C) actual flow velocity
(D) All of the above

11. Match List - A (lapse rate) with List - B (different types of plumes) at stack level.

List – A (Lape rate)	
P.	
Q.	
R.	

S.	

	List – B
1.	 Fumigation plume
2.	 Trapping plume
3.	 Looping plume
4.	 Coning plume

	P	Q	R	S		P	Q	R	S
(A)	3	4	1	2	(B)	3	1	2	4
(C)	1	4	2	3	(D)	4	2	1	3

12. Find the equivalent noise level for fluctuating noise level of 95 minutes. The one with 80 dB lasted for 10 minutes, followed by 60 dB for 80 minutes followed by 100 dB for 5 minutes.

(A) 68 dB (B) 74 dB
(C) 87 dB (D) 96 dB

13. In Delhi it has been decided to provide 200 Litre per head per day in 2020. Estimate the domestic water requirements of this city in 2020 by projecting the population of the town by incremental increase method.

Year	Population
1970	2, 37, 98, 624
1980	4, 50, 78, 325
1990	5, 53, 86, 432
2000	6, 91, 87, 241

(A) 18641 MLD (B) 19094 MLD
(C) 20986 MLD (D) 22782 MLD

14. A standard multiple tube fermentation test was conducted on a sample of water from a stream. The results of the analysis of the confirmed test are given below.

Sample size (ml)	No. of positive results out of 5 tubes	No. of negative results out of 5 tubes
10	2	3
1	0	5
0.1	2	3

MPN Index and 95% confidence limits for combination of positive results when 5 tubes used per dilution (1ml, 0.1ml, 0.01ml)

Combination of positives	MPN Index per 100 ml	95% Confidence limit	
		Lower	Upper
1 – 3 – 5	280	120	360
2 – 0 – 2	330	160	380
3 – 5 – 3	140	100	210

Using the above *MPN* Index table, the Most Probable number (*MPN*) of the sample is

(A) 25 (B) 14
(C) 28 (D) 33

15. 12 mg of copperas is consumed with lime at a coagulation basin per litre water. Determine the quantity of copperas and the quick lime required to treat 20 MLD. [F_e = 56, S = 32, O = 16, Ca = 40, H = 1]

(A) 240 and 16 kg/day
(B) 240 and 48.4 kg/day
(C) 180 and 16 kg/day
(D) 180 and 48.4 kg/day

Common Data for Questions 16 and 17:
In a slow sand filter, water is supplied at the rate of 24 million litres per day and rate of filtration is 5 $m^3/hr/m^2$. If Backwashing is done for 15 minutes at the rate of 5 times of rate of filtration for every 24 hours (Let L : B = 2:1)

16. Find the volume of water filtered between backwashing in m^3.

(A) 23750 m^3 (B) 24000 m^3
(C) 24,860 m^3 (D) 25,390 m^3

17. Volume of water used in backwashing in m^3.

(A) 250 m^3 (B) 500 m^3
(C) 750 m^3 (D) 1250 m^3

18. Calculate the requirement of soda ash or softening 2 MLD of water formed to have the following chemical composition.

CO_2 = 39.6 mg/l
Ca^{2+} = 44 mg/l
Mg^{2+} = 18 mg/l
HCO_3 = 122 mg/l

(A) 168.4 kg/day (B) 180.2 kg/day
(C) 230.1 kg/day (D) 242.8 kg/day

19. Which among the following are dechlorinating agents.
1. Sulphur dioxide gas
2. Ammonia
3. Sodium sulphate
4. Sodium sulphite

(A) 1 and 2 (B) 2, 3 and 4
(C) 1, 2 and 4 (D) All the above

20. A town with a population of one lakh is to be supplied with water, daily at 120 l/head. The variation in demand is as follows.

6am – 10am – 50% of total demand
10am – 12pm – 10% of total demand
12pm – 6pm – 5% of total demand
6pm – 12am – 30% of total demand
12am – 6am – 5% of total demand

Determine the capacity of service reservoir assuming pumping to be at uniform rate and the period of pumping from 6am – 10am and6pm – 10pm (Neglect fire demand)

(A) 1.2 ML (B) 2.4 ML
(C) 3.6 ML (D) 4.8 ML

21. A 30 cm diameter sewer with an invert slope of 1 in 400 is running full. Calculate the rate of flow in the sewer. (use mannings equation and $N = 0.015$)

(A) 0.02 m^3/s (B) 0.03 m^3/s
(C) 0.04 m^3/s (D) 0.05 m^3/s

22. If the percapita contribution of suspended solids and B.O.D is 100 gm and 60 gm, find the population equivalent of 50,000 liters daily of industrial waste water containing 1800 mg/l of suspended solids.

(A) 800 (B) 900
(C) 1000 (D) 1100

23. The following data are given for a channel type grit chamber of length 8 m.
1. Flow through velocity = 0.4 m/s
2. Depth of waste water at peak flow in the channel = 1m
3. Specific Gravity of inorganic particles = 2.6
4. $g = 9.8\ m/s^2$, $\mu = 1.002 \times 10^{-3}$ N–s/m^2 at 20°C, $\rho_w = 1000\ kg/m^3$

Assuming that the stokes law is valid, the largest diameter particle that would be removed with 100%. efficiency is.

(A) 0.08 mm (B) 0.12 mm
(C) 0.18 mm (D) 0.24 mm

24. A sedimentation tank is treating 5 million litres of sewage per day containing 300 ppm of suspended solids. The tank removes 60% of suspended solids. Calculate the quantity of sludge produced per day is bulk and weight respectively if moisture content of sludge is 98% (assume specific gravity of wet sludge = 1.02)

(A) 78.43m^3 and 80 tonnes
(B) 66.67m^3 and 68 tonnes
(C) 50.98m^3 and 52 tonnes
(D) 44.12m^3 and 45 tonnes

25. Match List - I with List - II

	List – I (Treatment units)		**List – II (Types of Processes)**
a.	Trickling filter	1.	Symbiotic
b.	Activated sludge	2.	Extended aeration
c..	Oxidation ditch	3.	Suspended growth
d.	Oxidation pond	4.	Attached growth

	a	b	c	d
(A)	3	4	2	1
(B)	4	3	1	2
(C)	3	4	1	2
(D)	4	3	2	1

Common Data for Questions 26 and 27:
The sewage is flowing at 6 MLD from a primary clarifier to a standard rate tricking filter. The 5 day BOD of the influent is 150 mg/l. The adopted organic loading is to be 160 gm/m^3/day and surface loading 2000 l/m^2/day.

26. Determine the volume of the trickling filter.

(A) 4532 m^3 (B) 5138 m^3
(C) 5625 m^3 (D) 6100 m^3

27. Calculate the efficiency of the filter

(A) 85% (B) 74%
(C) 61% (D) 91%

28. The composition of a certain MSW sample and specific weight of its various components are given below.

Component	**% by weight**	**Specific weight (kg/m^3)**
Food waste	60	250
Dirt and Ash	20	400
Plastics	10	70
Wood and Yard waste	10	130

Specific weight (kg/m^3) of the MSW sample is

(A) 120 (B) 143
(C) 196 (D) 219

29. In a stream flowing at 6 m^3/s has no concentration of chemical. Industrial water is released into stream at 25 MLD with chemical concentration as 30 mg/l. If rate of dissipation of impurities in stream is 0.13 mg/l/hr. Calculate the distance at which the chemical is removed completely from stream (velocity of stream is 0.3 m/s)

(A) 9.72 km
(B) 11.47 km
(C) 8.61 km
(D) 12.43 km

30. A sewage containing 300 mg/l of suspended solids is passed through primary settling tank where 60% of suspended solids are removed. 70% of suspended solids are volatile. The solids from the primary settling tank are digested to recover the gas where volatile matter is reduced by 70%. Methane and CO_2 produced in the digestion of the sludge from 2000 m^3 of sewage are in 3:2 ratio. If the fuel value of methane is 36000 kJ/m^3. Find the fuel value of gas produced (Assume gas production is at the rate of 0.9 m^3/kg of volatile) in MKJ.

(A) 26.3 (B) 31.4
(C) 48.5 (D) 34.8

Answer Keys

1. B	**2.** A	**3.** B	**4.** D	**5.** C	**6.** A	**7.** B	**8.** B	**9.** D	**10.** A
11. A	**12.** C	**13.** D	**14.** D	**15.** B	**16.** A	**17.** D	**18.** B	**19.** C	**20.** B
21. C	**22.** B	**23.** D	**24.** D	**25.** D	**26.** C	**27.** A	**28.** C	**29.** B	**30.** D

Hints and Explanations

1. SO_x are measured by West and Gack method.
Choice (B)

2. In zone of degradation, degradation just begins and all aquatic organisms disappear except fish. Choice (A)

3. Circular man hole covers are convenient to easy entry at times of repair. Choice (B)

4. Factors affecting sludge digestion are
1. Temperature
2. pH
3. Seeding of sludge
4. Mixing and stirring of raw sludge. Choice (D)

5. Primary clarifies is aimed to remove settleable organic solids of large size in waste water. These organic solids are similar to flocculent particles and influence other particles surrounding them and all of them settle together. This is called hindered/Type-III settling.
Choice (C)

6. Cl_2 added = 0.5 + 0.1 = 0.6 mg/l

$$\text{Bleaching powder} = \frac{Cl_2 \text{ added}}{\%\text{ of } Cl_2}$$

$$= \frac{0.6}{0.32} = 1.875 \text{ mg/l.}$$ Choice (A)

7. $$\text{Carbonate hardness} = \text{least of} \begin{pmatrix} \text{Alkalinity} \\ (\text{or}) \\ \text{Total hardness} \end{pmatrix}$$

∴ When Alkalinity > Total hardness
Carbonate Hardness = Total Hardness

∴ "Methyl Orange Alkalinity" indicates use of methyl orange indicator is titration, adopted for determining Alkalinity which changes its color from orange at pH = 4.6 to pink at pH = 4.
Choice (B)

8. Minimum/Balancing storage capacity of reservoir = maximum surplus + maximum deficit = $b + a$.
Choice (B)

9. Rapid sand filters are less efficient in removing bacteria (80 to 90%) but efficient in removing color and suspended solids.
Helminths are worms, which naturally would get removed in rapid sand filters. These are larger than bacteria. Choice (D)

10. Discharge $Q = KIA$
I = Hydraulic gradient
K = permeability coefficient
A = c/s area of well. Choice (A)

12. $L_{eq} = 10 \log_{10} \Sigma 10^{\frac{Li}{10}} \times T_i$

$$T_i = \frac{t_i}{\Sigma t_i}$$

$$\therefore \quad T_2 = \frac{10}{10+80+5} = 0.105$$

$$T_2 = \frac{80}{95} = 0.842$$

$$T_3 = \frac{5}{95} = 0.053$$

$$L_{eq} = 10 \log_{10} \left[\left(10^{\frac{80}{10}} \times 0.10.5 \right) + \left(10^{\frac{60}{10}} \times 0.842 \right) + \left(10^{\frac{100}{10}} \times 0.053 \right) \right]$$

= 87.33 dB. Choice (C)

13.

Year	Population	Increase in Population	Incremental Increase
1970	2,37,98,624	2,12,79,701	(+) 1,09,71,594
1980	4,50,78,325	1,03,08,107	(+) 34,92,702
1990	5,53,86,432	1,38,00,809	
2000	6,91,87,241		
Total		4,53,88,617	1,44,64,296
Average per decade		$\bar{x}$ = 1,51,29,539	$\bar{y}$ = 48,21,432

Expected population at the end of year 2020 (after 2 decades from 2000)

$$P = P_o + 2\bar{x} + \frac{2\times(2+1)}{2}.\bar{y}$$

$$= 69187241 + 2(15129539) + \frac{2\times3}{2} \times (4821432)$$

$$= 11,39,10,615$$

∴ Water requirement in 2020 @ 200 l/head/day

$$= \frac{200\times11,39,10,615}{10^6}\text{MLD}$$

= 22,782 MLD. Choice (D)

14. Since the volume of samples is 10 times more than specified sample size, the *MPN* from table is divided by 10 to get actual *MPN* of the sample

$$MPN = \frac{330}{10} = 33.$$ Choice (D)

15. The equations involved are

$$Fe\,SO_4.7H_2O + \underset{\text{(hydrated lime)}}{Ca(OH)_2}$$

$$\rightarrow Ca\,SO_4 + F_e(OH)_2 + 7H_2O$$

$$Ca(OH)_2 \rightarrow Ca\,O + H_2O$$

Q = 20 MLD

Dosage of copperas = 12 mg/l

$Fe\,SO_4.7H_2O = 56 + 32 + 64 + 7(2 + 16) = 278$

$Ca\,O = 56$

Total quantity of ferrous sulphate consumed

= 20 × 12 = 240 kg/day

278 parts of copperas = 56 parts of Ca O

$$1 \text{ part of copperas} = \frac{56}{278}\ Ca\,O$$

$$12 \text{ mg/l of copperas} = \frac{56}{278} \times 12 \text{ as } Ca\,O$$

= 2.42 mg/l as Ca O

∴ Ca O required = 2.42 × 20

= 48.4 kg/day. Choice (B)

16. Volume of water filtered between backwashing

= *R.O.F* × time of filtration × area of filter

$$= 5 \times \left(24 - \frac{15}{60}\right) \times 200 = 23750\ m^3.$$ Choice (A)

17. Volume of water used in backwashing

= (Rate of backwashing × time of backwashing × area of filter)

$$= (5 \times 5) \times \frac{15}{60} \times 200 = 1250\ m^3.$$ Choice (D)

18. Concentration of soda ash required

$$\underset{\text{(m.eq/lt)}}{Na_2CO_3} = \underset{\text{(m.eq/lt)}}{Ca^{2+}} + Mg^{2+} - \text{Alkalinity}$$

$$Ca^{2+} = \frac{44}{40/2} = 2.2 \text{ m.eq/lt}$$

$$Mg^{2+} = \frac{18}{24/2} = 1.5 \text{ m.eq/lt}$$

$$HCO_3 = \frac{122}{61/1} = 2 \text{ m.eq/lt}$$

$Na_2CO_3 = 2.2 + 1.5 - 2 = 1.7$ m.eq/lt

Soda ash = soda ash in m.eq/lt × eq. wt of Na_2CO_3

$$= 1.7 \times \frac{2(23)+12+3(16)}{2} = 90.1 \text{ mg/l}$$

Total soda ash required

= 2 × 90.1 = 180.2 kg/day. Choice (B)

19. Sodium sulphate is not a dechlorinating agent whereas sodium thisulphate, sodium meta bisulphate, sodium sulphite, sodium bisulphate, ammonia, sulphur dioxide gas are dechlorinating agents. Choice (C)

20. Total demand = 1,00,000 × 120 = 12 MLD

Volume of water demand in a day = 12 ML

$$\text{Rate of supply} = \frac{12}{8} = \frac{3}{2}\text{ML/hr}$$

Time	Demand % of total	Demand (ML)	Supply (ML)	Cumulative demand (ML)	Cumulative supply (ML)	Deficit (ML)	Surplus (ML)
6am-10am	50	6	$\frac{3}{2} \times 4 = 6$	6	6	–	–
10am-12pm	10	1.2	–	7.2	6	1.2	–
12pm-6pm	5	0.6	–	7.8	6	1.8	–
6pm-12am	30	3.6	$\frac{3}{2} \times 4 = 6$	11.4	12	–	0.6
12am-6am	5	0.6	–	12	12	–	–

∴ storage capacity

= Maximum surplus + Maximum deficit

= 0.6 + 1.8 = 2.4 ML. Choice (B)

21. D = diameter of server = 30 cm = 0.3 m

Area of sewer $A = \frac{\Pi}{4}D^2 = \frac{\Pi}{4} \times (0.3)^2 = 0.07 m^2$

$$R = \frac{A}{P} = \frac{\frac{\pi}{4}.D}{\pi.D} = \frac{D}{4} = \frac{0.3}{4} = \frac{3}{40}$$

$$S = \frac{1}{400}$$

$N = 0.015$

Use Mannings equation

$$V = \frac{1}{N} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$= \frac{1}{0.015}\left(\frac{3}{40}\right)^{\frac{2}{3}}\left(\frac{1}{40}\right)^{\frac{1}{2}}$$

$= 0.59$ m/s

$Q = A \times V$

$= 0.07 \times 0.59$

$= 0.0413$ m^3/s. Choice (C)

22. 1 liter of industrial waste water contains 1800 mg of suspended solids 50,000 liters of industrial water produce $= \frac{1800 \times 50000}{10^3}$ gm of suspended solids = 90000 gm

Standard percapita solid contribution of suspended solids = 100 gm

$\therefore$ Population equivalent $= \frac{90000}{100} = 900.$

Choice (B)

23. $L = 8$ m

$V_H = 0.4$ m/s

$H = 1$ m

Kinematic viscosity $\vartheta = \frac{\mu}{\rho}$

$$= \frac{1.002 \times 10^{-3}}{1000} = 1.002 \times 10^{-6} \text{ m}^2/\text{sec}$$

$\eta = 100\%$

$$\therefore \quad \frac{L}{V_H} = \frac{H}{V_S}$$

$$\frac{8}{0.4} = \frac{1}{V_S}$$

$V_S = 0.05$ m/s

$$V_S = \frac{g}{18}(S-1)\frac{d^2}{\vartheta}$$

$$0.05 = \frac{9.80}{18}(2.6-1) \times \frac{d^2}{1.002 \times 10^{-6}}$$

$d = 2.398 \times 10^{-4}$ m

$= 0.24$ mm. Choice (D)

24. Volume of sewage treated

= 5 M-liters/day

Mass of suspended solids in sewage = 300 × 5

= 1500 kg/day

As 60% solids are removed in sedimentation tank.

Mass of solids removed in sedimentation tank

$$= 1500 \times \frac{60}{100}$$

= 900 kg/day

When moisture content is 98%

2kg of solids → 100kg of wet sludge

900 kg of solids → $\frac{100}{2} \times 900$

= 45000 kg

Wet sludge produced per day = 45000 kg = 45 tonnes

Sp.gravity of wet sludge

= 1.02 × 1 t/m^3

= 1.02 t/m^3

Volume of wet sludge per day

$= \frac{45}{1.02} = 44.12$ m^3. Choice (D)

26. $y_5 = 150$ mg/l

Total BOD = 150 × 6 = 900 kg/day

= 9,00,000 gm/day

Volume of filter media required

$$= \frac{\text{Total BOD}}{\text{Organic loading rate}}$$

$= \frac{900000}{160} = 5625$ m^3. Choice (C)

27. Organic loading $u = 160$ gm/m^3/day

$= \frac{160}{1000} \times 10^4$ kg/ha-m/day

= 1600 kg/ha-m/day

$$\eta = \frac{100}{1 + 0.0044\sqrt{1600}}$$

$= \frac{100}{1+0.176} = 85.03\%$. Choice (A)

28. $\frac{100}{S_D} = \frac{\%A}{S_A} + \frac{\%B}{S_B} + \frac{\%C}{S_C}$

$$\frac{100}{S_D} = \frac{60}{250} + \frac{20}{400} + \frac{10}{70} + \frac{10}{130}$$

$S_D = 196.16$ kg/m^3 Choice (C)

29. $C_{mix} = \frac{Q_R + C_R + Q_W C_W}{Q_R + Q_W}$

$$= \frac{0 + (25 \times 10^6 \times 30)}{[25 \times 10^6 + (6 \times 1000 \times 60^2 \times 24)]} = 1.38 \text{ mg/l}$$

Rate of dissipation = Rate of removal = 0.13 mg/l/hr

Time to remove chemical from water = $\frac{1.38}{0.13} = 10.62$ hr

Distance travelled by river in 10.62 hr = 0.3 × 10.62 × 60^2 = 11469.6 m = 11.47 km Choice (B)

30. Total suspended solids in sewage = 300 mg/l
Suspended solids removed = 60% (300) = 180 mg/l
Volatile solids removed as sludge = 70% (180)
= 128 mg/l
Volatile solids are reduced by 70% in digestion tank
= 70% (128) = 89.6 mg/l

Volatile matter reduced in 20000 m^3 of sewage

$= 89.6 \times \frac{(20000 \times 1000)}{10^6}$

= 1792 kg

1kg volatile matter = 0.9 m^3 *G* as 1792 kg volatile matter = 0.9 × 1792 = 1612.8 m^3

Methane produce = $\frac{3}{3+2} \times 1612.8 = 967.68$ m^3

Fuel value of gas = 36000 × 967.68 = 34.8 MKJ
Choice (D)

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UNIT XI
TRANSPORTATION ENGINEERING

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Transportation Infrastructure Test 1

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Indian Road Congress was established in the year ____.
(A) 1932 (B) 1930
(C) 1934 (D) 1936

2. The maximum grade compensation necessary to ease the gradients at horizontal curves is _____.
(A) $\frac{60}{R}$ (B) $\frac{30+R}{R}$
(C) $\frac{75}{R}$ (D) $\frac{40+R}{R}$

3. The length of the summit curve for a stopping sight distance of 100 m with an upward gradient of 1% and downward gradient of 2% is _____.
Take the height of the eye of the driver to be 1.2 m and the height of the object above the roadways to be 0.15 m.
(A) 62 m (B) 65 m
(C) 73 m (D) 69 m

4. The extra widening necessary on a two lane pavement for a curve of radius 80 m and wheel base 6 m with a design speed of 65 kmph is _____.
(A) 1.23 m (B) 1.43 m
(C) 1.17 m (D) 1.13 m

5. Width of roadway of two lane national highway in mountainous and steep terrain is
(A) 6.25 m (B) 8.8 m
(C) 4.75 m (D) 9 m

6. An aggregate is said to be exceptionally strong when the aggregate impact value is
(A) less than 10. (B) 10 to 20.
(C) 20 to 30. (D) above 35.

7. Bitumen of viscosity 80/100 penetration grade is used for
(A) spraying applications
(B) paving applications in cold regions
(C) paving applications in hot regions
(D) high stress areas like toll plaza, terminals etc.

8. Length of the transition curve for comfort condition is _____.
(A) $L_S = \frac{V^3}{CR}$ (B) $L_S = \frac{L}{2}$
(C) $L_S = \sqrt{\frac{NV^3}{C}}$ (D) All of the above

9. The minimum length of overtaking zone should be $(d_1 + d_2 + d_3 = OSD)$
(A) $2(d_1 + d_2 + d_3)$ (B) $5(d_1 + d_2 + d_3)$
(C) $3(d_1 + d_2 + d_3)$ (D) $4(d_1 + d_2 + d_3)$

10. The sight distance available to a driver travelling on the road at any instance depends on the following, factors.
I. height of drivers eyes above the road surface
II. height of object above the road surface
(A) Both I and II are true
(B) I is false and II is true
(C) Both I and II are false
(D) I is true and II is false

11. Which of the following relation is true
(A) Ruling gradient < limiting gradients < exceptional gradient
(B) limiting gradient > ruling gradient > exceptional gradient
(C) Exceptional gradient < limiting gradient < Ruling gradient
(D) Ruling gradient > limiting gradients > exceptional gradient

12. Calculate the safe stopping sight distance on a level road, stretch for design speed of 40 kmph for a two way traffic on a single lane road assuming coefficient of friction as 0.37 and reaction time of driver as 2.5 seconds.
(A) 44.79 m (B) 45.68 m
(C) 89.59 m (D) 88.63 m

Common Data for Questions 13 and 14:
The radius of horizontal circular curve is 400 m. The design speed is 80 kmph and coefficient of lateral friction is 0.1.

13. Calculate the super elevation required (if the lateral friction is assumed to develop).
(A) 0.58 (B) 0.025
(C) 0.007 (D) 0.48

14. Coefficient of friction needed if no super elevation is provided is _____.
(A) 0.15 (B) 0.19
(C) 0.125 (D) 0.087

Common Data for Questions 15 and 16:
A vertical summit curve is to be designed when two grades $+\frac{1}{20}$ and $-\frac{1}{50}$ meet on the highway. The stopping sight distance and over taking sight distance required are 120 m and 600 m respectively. The length of vertical the curve is restricted to a maximum value of 500 m.

15. The length of the summit curve needed to fulfill the requirements of stopping sight distance is _____.
(A) 228 m (B) 229 m
(C) 230 m (D) 222 m

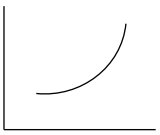
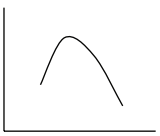
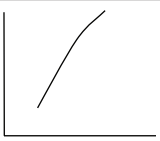
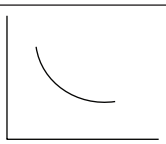
16. The length of summit curve needed to fulfill the requirements of overtaking sight distance is _____.
(A) 1052.7 m (B) 1048.9 m
(C) 1062.8 m (D) 1073.9 m

Common Data for Questions 17 and 18:
The wheel load of 5100 kg is applied on a pavement of thickness 18 cm, modulus of elasticity 3×10^5 kg/cm^2, radius of contact area 15 cm and Poisson's ratio = 0.15. modulus of subgrade reaction = 6kg/cm^3.

17. The stress at the corner of the slab is __.
(A) 24.27 kg/cm^2
(B) 22.56 kg/cm^2
(C) 25.96 kg/cm^2
(D) 26.5 kg/cm^2

18. The location where a crack develops due to corner load is _____.
(A) 85 cm (B) 84.5 cm
(C) 84 cm (D) 84.3 cm

19. According to the bituminous mix design by Marshall method, match the following correctly (Binder content % on x – axis)

	A		B
(i)	Marshall stability Vs Binder content%	1.	
(ii)	Flow value Vs Binder content%	2.	
(iii)	Voids% Vs Binder content%	3.	
(iv)	Voids filled with Bitumen Vs Binder content %	4.	

(A) (i – 2), (ii – 3), (iii – 4), (iv – 1)
(B) (i – 4), (ii – 2), (iii – 1), (iv – 3)
(C) (i – 3), (ii – 4), (iii – 1), (iv – 2)
(D) (i – 2), (ii – 1), (iii – 4), (iv – 3)

20. (i) Viscosity test on bitumen is done to assess the ability to be sprayed through jets.
(ii) Coating test on bitumen is done to assess the stability in presence of fines in aggregates.
(A) (i) is true and (ii) is false
(B) (i) is false and (ii) is true
(C) Both (i) and (ii) are true
(D) Both (i) and (ii) are false

21. The number of commercial vehicles in the year of completing overlay construction is estimated to be 2850 per day with an average growth rate of 6.5 percent per year, if the average *VDF* value is 5.4 and the traffic design period is 10 years. The design traffic volume if the *LDF* = 0.75 is _____.
(A) 110 msa (B) 98 msa
(C) 83 msa (D) 57 msa

22. The following data are related to a horizontal curved portion of a two lane highway: length of curve = 300 m, radius of curve = 600 m, width of pavement = 9.5 m. In order to provide an *SSD* of 80 m the set back distance of the inner lane of the pavement is
(A) 3.71 m
(B) 2.35 m
(C) 4.62 m
(D) 2.97 m

23. The speeds of overtaking and overtaken vehicles on the highways are 70 kmph and 65 kmph respectively. Assuming acceleration of overtaking vehicle as 2.5 kmph/sec and speed of vehicle in opposite direction as 65 kmph, the overtaking sight distance needed for two way traffic is (assume reaction time of driver = 2sec) _____.
(A) 451 m (B) 326 m
(C) 564 m (D) 275 m.

24. In a concrete pavement
(i) Temperature stress is tensile at bottom during day time
(ii) Load stress is compressive at the bottom.
Identify the correct choice from the following.
(A) Both the statement (A) and (B) are correct
(B) Statement (A) is correct and (B) is incorrect.
(C) Statement (B) is incorrect.and (A) is orrect
(D) Both statements (A) and (B) are incorrect

25. A valley curve is formed by a descending grade of 1 in 35 meeting an ascending grade of 1 in 30. Assuming allowable rate of change of centrifugal acceleration is 0.6 m/sec^3, design speed is 50 kmph, then the length of the valley curve to fulfill comfort condition is _____.
(A) 128 m (B) 33 m
(C) 73 m (D) 94 m

Answer Keys

1. C	**2.** C	**3.** D	**4.** C	**5.** B	**6.** A	**7.** A	**8.** D	**9.** C	**10.** A
11. A	**12.** C	**13.** B	**14.** C	**15.** B	**16.** C	**17.** A	**18.** C	**19.** D	**20.** A
21. D	**22.** A	**23.** A	**24.** A	**25.** B					

Hints and Explanations

3. $L = \dfrac{NS^2}{4.4} = \dfrac{(0.01+0.02)\times 100\times 100}{4.4} = 69$ m.

Choice (D)

4. $W_e = \dfrac{n\times \ell^2}{2\,R} + \dfrac{0.1\times V}{\sqrt{R}}$.

Where
V = design speed of road in kmph
n = no of lanes
R = Radius of the curve

$W_e = \dfrac{2\times(6)^2}{2\times 80} + \dfrac{0.1\times 65}{\sqrt{80}} = 1.17$ m Choice (C)

8. $L_S = \dfrac{V^3}{CR} = \dfrac{L}{2} = \sqrt{\dfrac{NV^3}{C}}$. Choice (D)

12. Stopping sight distance (SSD) $= vt + \dfrac{v^2}{2gf}$,

where $V = 40 \times \dfrac{5}{18} = 11.11$ m/s

$SSD = (11.11 \times 2.5) + \dfrac{(11.11)^2}{(2\times 9.8\times 0.37)}$

$\Rightarrow$ $SSD = 44.79$ m.
SSD when there is a single lane 2 way $= 2\ SSD$
$= 2 \times 44.79 = 89.59$ m. Choice (C)

13. $e + f = \dfrac{V^2}{gR} = \dfrac{V^2}{127R} \Rightarrow e + 0.1 = \dfrac{80^2}{127\times 400}$

$\Rightarrow 0.025$. Choice (B)

14. If no super elevation is provided $e = 0$

$\Rightarrow$ $e + f = \dfrac{V^2}{127R}$

$f = \dfrac{80^2}{27\times 400} = 0.125$. $[\because e = 0]$ Choice (C)

15. Let $L > SSD$,

Deviation angle, $N = \dfrac{+1}{20} - \left(\dfrac{-1}{50}\right) = \dfrac{7}{100}$

$L = \dfrac{NS^2}{4.4} = \dfrac{7\times(120)^2}{4.4\times 100} = 229$m. Choice (B)

16. Length of the curve restricted to 500 m,
$L < OSD$ $(500 < 600)$

$L = 2S - \dfrac{9.6}{N} = 2\times(600) - \dfrac{9.6\times 100}{7} = 1062.8$ m.

Choice (C)

17. Radius of relative stiffness

$\ell = \left[\dfrac{Eh^3}{12k(1-\mu^2)}\right]^{\frac{1}{4}} = \left[\dfrac{3\times 10^5\times 15^3}{12\times 6\times(1-0.15^2)}\right]^{\frac{1}{4}} = 70.6$ cm.

$S_C = \dfrac{3P}{h^2}\times\left[1-\left(\dfrac{a\sqrt{2}}{\ell}\right)^{0.6}\right] = \dfrac{3\times 5100}{18^2}\left[1-\left(\dfrac{15\sqrt{2}}{70.6}\right)^{0.6}\right]$

$= 24.27$ kg/cm^2. Choice (A)

18. $X = 2.58\sqrt{aL} = 2.58\sqrt{(15\times 70.6)}$

$= 83.96 \cong 84$ cm. Choice (C)

20. Coating test is assess coating of stone aggregates.

Choice (A)

21. $N = 2650$
$r = 6.5\%$
$VDF = 5.4$
$LDF = 0.75$
$n = 10$ years

Design traffic $= 365\left\{N\times VDF\times LDF\times\dfrac{[1+r)^n - 1]}{r}\right\}$

$= \dfrac{365\times\left[2850\times 5.4\times 0.75\times\left[1.065^{10} - 1\right]\right]}{0.065}$

$= 56.8\times 10^6 \cong 57$ msa. Choice (D)

22. Set back distance $= R - (R - d)\cos\left(\dfrac{\infty}{2}\right)$

$d = \dfrac{9.5}{4}$ $\infty = \dfrac{SSD\times 180}{\Pi\times(R-d)} = \dfrac{80\times 180}{\pi\times\left(600 - \dfrac{9.5}{4}\right)} = 7.64°$

$\therefore$ Set back distance $= 600 - \left(600 - \dfrac{9.5}{4}\right)\times\cos\left(\dfrac{7.66°}{2}\right)$

$\Rightarrow$ set back distance $= 3.71$ m. Choice (A)

23. $V_b = 65$ kmph
$a = 2.5$ kmph/sec $= 0.694$ m/sec^2

$S = 0.7\ V_b + 6 = 0.7\times\left(65\times\dfrac{5}{18}\right) + 6 = 18.64$ m

$T = \sqrt{\dfrac{4S}{a}} = \sqrt{\dfrac{4\times 18.6}{0.694}}$

$= 10.36$ sec (reaction time of driver $t = 2$ sec.)
OSD $= 0.28V_b + 0.28V_bT + 2\ s + 0.28\ VT$.
$= (0.28\times 65\times 2) + (0.28\times 65\times 10.36)$
$+ (2\times 18.64) + (0.28\times 65\times 10.36)$
$= 450.78$ m $\simeq 451$ m. Choice (A)

25. $L_s = 2\left[\dfrac{Nv^3}{C}\right]^{\frac{1}{2}}$ $v = 50\times\dfrac{5}{18} = 13.88$

$N = \dfrac{1}{-35} - \dfrac{1}{30} = \dfrac{-13}{210}$

$L_s = 2\left[\dfrac{13}{210}\times\dfrac{13.88^3}{0.6}\right]^{\frac{1}{2}} = 33.2$ m. Choice (B)

Traffic Engineering Test 2

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The vehicle speed affects the design of
(A) sight distance
(B) super elevation
(C) length of transition curve
(D) All of the above

2. The wheel base of the vehicle is 8.5 m. The off-tracking while negotiating curved path with mean radius of 32 m is
(A) 1.18 m (B) 1.13 m
(C) 1.12 m (D) 1.15 m

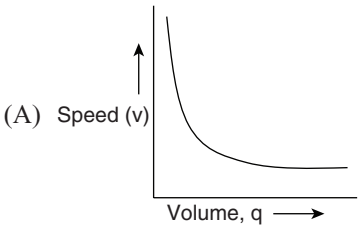
3. The speed at which (or) below which 85 percent of vehicles are passing the point on highway can be assessed is known as
(A) 85^{th} percentile speed
(B) 20^{th} percentile speed
(C) 15^{th} percentile speed
(D) 25^{th} percentile speed

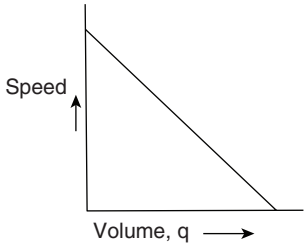
4. The method of origin and destination studies in which the car is struck with a pre-coded card as it enters the area under study is
(A) Road side interview method
(B) Home interview method
(C) License plate method
(D) Tag on car method

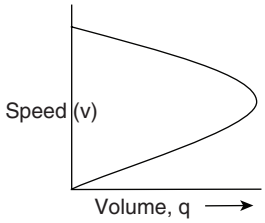
5. A vehicle *a* of weight 2 tonne skids through a distance equal to 50 m before colliding with other parked vehicle of weight 1 tonne. After collision both the vehicles together skid through a distance of 10 m before stopping. The initial speed of moving vehicle is (Assume coefficient of friction $f = 0.5$).
(A) 9.89 m/s (B) 8.93 m/s
(C) 7.96 m/s (D) 9.82 m/s

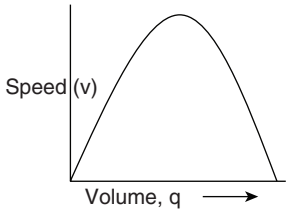
6. When a vehicle moves obliquely across the path of another vehicle moving in same direction at small angle of crossing is termed as
(A) Merging (B) Weaving manoeuvre
(C) Crossing manoeuvre (D) Diverging

7. Which of the following graph represents relation between speed and volume?

(A) 

(B) 

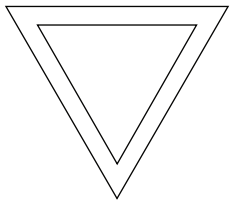
(C) 

(D) 

8. The free mean speed on a roadway is found to be 100 kmph under stopped condition the average spacing between vehicles is 8.9m. The Jam density of flow is
(A) 113 vehicles/km
(B) 118 vehicles/km
(C) 145 vehicles/km
(D) 148 vehicles/km

9. The maximum number of vehicles that can pass a given point on a lane or roadway during one hour under prevailing traffic conditions is known as
(A) Basic capacity
(B) Practical capacity
(C) Possible capacity
(D) Highway capacity

10.



The above sign is categorized under
(A) Warning signs
(B) Regulatory signs
(C) Informatory signs
(D) None of these

11. The theoretical capacity of traffic lane with one way traffic flow at a stream speed of 80 kmph. The average

space gap $S_g = 0.278\ Vt$ and average length of vehicles = 8m

(A) 3450 veh/hr/lane (B) 3395 veh/hr/lane
(C) 3530 veh/hr/lane (D) 3834 veh/hr/lane

12. The average normal flow of traffic on cross roads *A* and *B* during design period are 400 and 250 PCU per hour, the saturation flow values on these roads is estimated to be 1350 and 1200 PCU per hour respectively. The all-red time required for pedestrian crossing is 15 sec. The total cycle time using Webster's method is

(A) 62.4 sec (B) 63.3 sec
(C) 65.53 sec (D) 67.5 sec

13. (i) At intersection the area of conflict should be as small as possible
(ii) Sudden change of path should be avoided

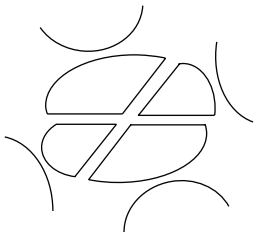
(A) (i) and (ii) are true (B) (ii) and (iii) are false
(C) (i) is true (ii) is false (D) (i) is false (ii) is true

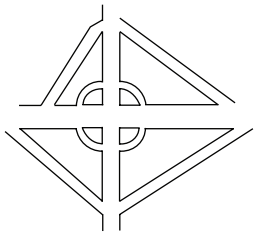
14. In a street light system
Street width = 20m
Mounting height = 10m
Lamp size = 5000 lumen
Luminaire type – II
Coefficient of utilization = 0.36
Spacing between lighting units if average lighting intensity is 6 Lux
(assume maintenance factor = 0.8)

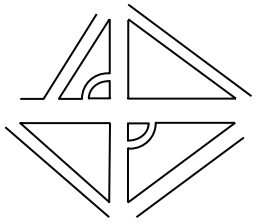
(A) 16 m (B) 18 m
(C) 15 m (D) 12 m

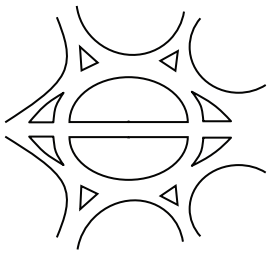
15. Match the following

1. Diamond a.
2. Rotary b.
3. Partial clover leaf c.
4. Full clover leaf d.









1 2 3 4
(A) a b c d
(C) d c b a

1 2 3 4
(B) a d c b
(D) c b d a

16. The no parking symbol is represented by

(A) triangle (B) circle
(C) octagonal (D) hexagonal

17. The practical capacity of a rotary is given by the formula

(A) $Q_P = \dfrac{280W(1+W)\left(1-\dfrac{p}{3}\right)}{\left(1+\dfrac{W}{L}\right)}$

(B) $Q_P = \dfrac{280w\left(1+\dfrac{w}{L}\right)\left(1-\dfrac{P}{3}\right)}{\left(1+\dfrac{e}{w}\right)}$

(C) $Q_P = \dfrac{280w\left(1+\dfrac{e}{w}\right)\left(1+\dfrac{w}{L}\right)}{\left(1-\dfrac{P}{3}\right)}$

(D) None of the above

18. The average width of entry e_1 is 150 m and average width of exit is 200 m. The width of the rotary roadway is

(A) 182 m (B) 178 m
(C) 176 m (D) 179 m

19. Match the following

	Type of Marking		Areas
1.	Markings at intersections	a.	slow & stop
2.	Carriage way marking	b.	speed change lanes and stop lines
3.	object markings	c.	no parking zones and traffic lanes
4.	word messages	d.	kerb markings and objects within the carriage way

Codes:

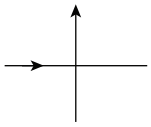
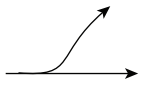
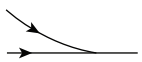
1 2 3 4
(A) c a b d
(B) d c b a
(C) b c d a
(D) a d c b

20. Desired lines are used in
(A) speed and delay studies
(B) origin and destination study
(C) spot speed study
(D) None of these

21. The spacing between the vehicles allowed by the driver of the following vehicle does not depends on
(A) speed of leading vehicle
(B) Average length and width of vehicle class
(C) tyre and characteristics of two vehicles
(D) driver characteristics of following vehicle

22. The speed of road is 100 kmph and average center to center spacing of vehicles (or) space headway is 10m. The capacity of single lane is
(A) 10^3 veh/hr/lane (B) 10^5 veh/hr/lane
(C) 10^4 veh/hr/lane (D) 10^2 veh/hr/lane

23. Match the following

Traffic Man oeuvres		Figures	
1.	diverging left	a.	
2.	diverging right	b.	
3.	Crossing right	c.	
4.	merging left	d.	

1 2 3 4 1 2 3 4
(A) b a c d (B) c a b d
(C) d c a b (D) c d b a

24. The charts showing the volume variations over a period of years are known as
(A) Trend charts
(B) Variation charts
(C) Traffic flow maps
(D) Volume flow diagram

25. A vehicle moving at 40 kmph speed was stopped by applying brakes and length of skid mark is 13.2 m. Average skid resistance of pavement is 0.5. The brake efficiency of test vehicle is
(A) 64.6% (B) 73.7%
(C) 86.9% (D) 95.4%

Answer Keys

1. D	**2.** C	**3.** A	**4.** D	**5.** A	**6.** B	**7.** C	**8.** A	**9.** C	**10.** B
11. B	**12.** D	**13.** A	**14.** D	**15.** B	**16.** B	**17.** A	**18.** D	**19.** C	**20.** B
21. B	**22.** C	**23.** B	**24.** A	**25.** D					

Hints and Explanations

1. Choice (D)

2. Off tracking $= \frac{L^2}{2R} = \frac{8.5^2}{2\times 32} = 1.12\text{m}$ Choice (C)

3. Choice (A)

4. Choice (D)

5. Loss of kinetic energy of both vehicles together = work done against frictional force

$$\left(\frac{w_a + w_b}{2g}\right)\left(V_2^2 - V_1^2\right) = (w_a + w_b) f S_2$$

$$= \frac{V_2^2 - V_1^2}{2g} = fS_2$$

But $V_4 = 0$

$$\frac{V_3^2}{2g} = 0.5 \times 10$$

$$V_3 = \sqrt{0.5\times 10\times 2\times 9.8} = 9.89 \text{ m/s}$$ Choice (A)

6. Choice (B)

7. Choice (C)

8. Spacing between vehicles $s = 8.9$ m

$$\text{Jam density} = \frac{1000}{s} = \frac{1000}{8.9} = 112.3$$

$KM_{ax} \simeq 113$ vehicles/km (per lane) Choice (A)

9. Choice (C)

10. It is a give way sign categorized as Regulatory sign Choice (B)

11. Reaction time of ideal driver $t = 0.7$ sec

$S = 0.278\ Vt + L$

$= (0.278 \times 80 \times 0.7) + 8$

$= 23.568\text{m}$

$\therefore$ Theoretical capacity $= \frac{1000\text{V}}{S} = \frac{1000\times 80}{23.568}$

$= 3394.4$

$\cong 3395$ veh/hr/lane

Choice (B)

12. $y_a = \frac{q_a}{S_a} = \frac{400}{1350} = 0.296$

$y_b = \frac{q_b}{S_b} = \frac{250}{1200} = 0.208$

$Y = y_a + y_b = 0.296 + 0.208 = 0.504$

Lost time $= 2n + R = (2 \times 2) + 15 = 19$ sec

Optimum cycle time $C_o = \frac{1.5L + 5}{1 - Y}$

$= \frac{(1.5 \times 19) + 5}{1 - 0.504} = 67.54$ sec

$G_a = \frac{y_a}{Y}(C_o - L)$

$= \frac{0.296}{0.504}(67.54 - 19) = 28.5$ sec

$G_b = \frac{y_b}{Y}(C_o - L) = \frac{0.208}{0.504}(67.54 - 19) = 20$ sec

Total cycle time $= 28.5 + 20 + (15 + 2 + 2)$

$= 67.5$ sec Choice (D)

13. Choice (A)

14. Spacing

$$= \frac{\text{lamp lumem} \times \text{coefficient of utilization} \times \text{Maintenance factor}}{\text{Average lux} \times \text{width of road}}$$

$= \frac{5000 \times 0.36 \times 0.8}{6 \times 20} = 12$m Choice (D)

15. Choice (B)

16. Choice (B)

17. Choice (A)

18. Average width of rotary is

$= \left[\frac{e_1 + e_2}{2}\right] + 3.5$

$= \frac{150 + 200}{2} + 3.5 = 178.5\text{m} \cong 179$ m. Choice (D)

19. Choice (C)

20. Choice (B)

21. Choice (B)

22. Capacity of single lane is $= \frac{1000\text{V}}{S}$

$= \frac{1000 \times 100}{10}$ (V in 'kmph' S in 'm')

$= 10^4$veh/hr/lane Choice (C)

23. Choice (B)

24. Choice (A)

25. Brake efficiency $= \frac{100 f^1}{f}$

Average skid resistance developed, $f^1 = \frac{v^2}{2gL}$

$V = \frac{v}{3.6} = \frac{40}{3.6} = 11.11$ m/s

$f^1 = \frac{11.11^2}{2 \times 9.81 \times 13.2} = 0.477.$

Brake efficiency $= \frac{100 \times 0.47}{0.5} = 95.4\%$ Choice (D)

Transportation Engineering Test 3

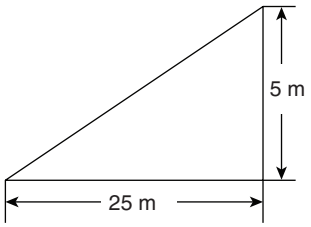
Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. The factors affecting the highway alignment are
(A) Traffic (B) Geometric design
(C) Economy (D) All the above.

2. A test car of mass 1400 N is travelling at a speed of 85 kmph, when it is suddenly braked the wheels. The average vehicles comes to a stop in a distance of 50 m. Skid resisting force is
(A) 7862 N (B) 7928 N
(C) 7804.7 N (D) 7642 N

3. The height and width of the pavement are as given below figure



If $f = 0.15$ and ruling design speed is 60 kmph. Find the absolute minimum radius on the curve in 'm' is _____.
(A) 8.94 m (B) 9.34 m
(C) 6.62 m (D) 7.34 m

4. The turning angle of the curve is 30° and tractive force on the vehicle is 300 N. Then the loss of tractive force due to turning of vehicle in horizontal curve is
(A) 38 N (B) 40 N
(C) 41 N (D) None of these

5. If width of the vehicle is 6 m and height of the vehicle is 10 m and coefficient of friction 0.15 then
(A) Vehicle overturns prior to skidding
(B) Vehicle skids prior to overtaking
(C) Overturning is avoided
(D) Skid is avoided.

6. The design speed of a road is 40 kmph and the radius of curve is 200 m. Then find the length of transition curve for the road of plain and rolling terrain.
(A) 21.6 m (B) 26.1 m
(C) 16.2 m (D) 24.2 m

7. A summit curve is to be designed with two gradients +2% and –6%. The rate of change of gradient is 1% per 100 m length. The minimum radius of curve is.
(A) 100 m (B) 1000 m
(C) 200 m (D) 300 m

8. The material obtained by the destructive distillation of wood is _____.
(A) Bitumen (B) Cutback
(C) Emulsion (D) Tar

9. The mix design of concrete pavement is based on
(A) Flexural strength
(B) Compressive strength
(C) Shear strength
(D) Bond strength

10. The speed at which greatest number of vehicles travel is called
(A) Medium speed (B) Model speed
(C) 15^{th} percentile speed (D) 98^{th} percentile speed

11. A circular curve of radius 300 m, coefficient of lateral friction of 0.15 and the design speed is 40 kmph. The super elevation at which equal pressure is distributed on inner and outer wheel would be
(A) 0.02 (B) 0.06
(C) 0.05 (D) 0.04

12. What will be the non passing sight distance on a highway for a design speed of 100 kmph when its ascending gradient is 2%. Assuming coefficient of friction as 0.7 and brake efficiency is 50%
(A) 176.2 m (B) 174.5 m
(C) 172.3 m (D) 175.05 m

13. A summit curve is formed at the intersection of 3% upgrade and 5% downgrade. What is the length of the summit curve in order to provide a stopping distance of 128 m is
(A) 223 m (B) 248 m
(C) 298 m (D) 300 m

14. Match List - I (traffic survey) with List - II and select the correct answer using the codes given below

	List – I		List – II
a.	Spot speed	1.	By video tape
b.	Traffic volume	2.	By road side interview
c.	O-D survey	3.	By Doppler radar
d.	Parking survey	4.	By pneumatic tube

	a	b	c	d		a	b	c	d
(A)	3	1	2	4	(B)	2	4	3	1
(C)	3	4	2	1	(D)	4	2	1	3

15. What will be the initial traffic after construction, in the Commercial Vehicles per day (CVpd) for following data? Rate of traffic growth per annum = 7%. The road is proposed to be completed in 3 years and present traffic existing is 400 CVpd
(A) 50 (B) 449
(C) 490 (D) 421

16. What is the deflection at the surface of a flexible pavement due to a wheel load of 40 kN and a tyre pressure of 0.5 MPa? The value of ϵ for pavement and sub grade is 20 MPa.

(A) 15 mm (B) 11 mm
(C) 9 mm (D) 6 mm

17. Which one of the following is the set of physical requirements of coarse aggregates for construction of WBM roads as per *IRC* recommendation?

	LAV(%)	AIV(%)	FI(%)
(A)	< 50	< 40	< 15
(B)	< 50	< 30	< 15
(C)	< 40	< 30	< 20
(D)	< 40	< 30	< 15

18. In 500 gm sample of course aggregate are 100 gm of flaky particles and 80 gm elongated particles. What are the flakiness and elongation particles as per Is.
(A) 40% (B) 3.6%
(C) 18% (D) 4%

19. The design speed of a traffic lane is 70 kmph. What is the theoretical capacity per hour taking the total reaction time to be 2 seconds and average length of vehicles as 8 m.
(A) 828 veh/m/day (B) 735 veh/m/day
(C) 628 veh/m/day (D) 428 veh/m/day

20. Consider following factors
1. Length of the vehicle
2. Width of the vehicle
3. Approach speed
4. Stopping time for approaching vehicle
5. Passing sight distance

Which of these factors are taken into consideration for determing yellow time of traffic signal at intersection?
(A) 1, 2 and 5 (B) 2, 3 and 4
(C) 1, 3 and 5 (D) 1, 3 and 4

21. On a road the free speed was 65 kmph and the space headway at jam density was 6.25 m. What is the maximum flow which could be expected on this road?
(A) 2600 vph (B) 1625 vph
(C) 1300 vph (D) 406 vph

22. In marshall method of mix design, the course aggregates, fine aggregates, filler material and bitumen, having respective specific gravities of 2.62, 2.72, 2.70 and 1.02 are mixed in the ratio of 55, 34.6, 4.8 and 5.6 percent respectively. The theoretical specific gravity of mix would be.
(A) 2.36 (B) 2.4
(C) 2.44 (D) 2.5

23. Compute the equalent radius of resisting section of 15 cm slab, if ratio of radius of wheel load distribution to thickness of slab is 0.5
(A) 7.82 cm (B) 7.93 cm
(C) 7.48 cm (D) 7.62 cm

24. The centrifugal ratio of a vehicle is 0.25, width of vehicle is 2.4 m, height of vehicle to its *C.G* is 4.2 m, lateral friction is 0.15, assuming no superlevation.
(A) Lateral skid occurs first
(B) Overturning occurs first
(C) Neither lateral skid nor overturning
(D) Both simultaneously

25. Find minimum sight distance to avoid head on collision of two cars approaching at 90 kmph and 60 kmph. Used reaction time of driver $t = 2.5$ sec, coefficient of longitude friction, $f = 0.7$ and brake efficiency of 50% in either case is.
(A) 235.8 m
(B) 243.2 m
(C) 256.8 m
(D) 292.3 m

26. The last time due to starting delay on a traffic signal approach is noted to be 3 seconds, the actual green time is 20 seconds and amber time is 3 seconds. How much is the effective green time.
(A) 19 sec (B) 22 sec
(C) 27 sec (D) 31 sec

27. The free mean speed on a road wing is found to be 60kmph under stopped condition the average spacing between vehicle is 6 m. The capacity of flow, assuming linear speed density relation is
(A) 2333 veh/hr
(B) 3333 veh/hr
(C) 2870 veh/hr
(D) 3838 veh/hr

28. A road 10 m wide is to deflect through and angle of 65° with the centreline radius 350 m. A transition curve is to be used at each end of a circular curve of such a length that the rate of gain of radial acceleration $0.4\ m^2/sec$, when speed is 60 kmph. Find the shift of the transition curve.
(A) 0.13 m (B) 3.12 m
(C) 0.18 m (D) 3.42 m

29. If the lamp lumen is 30 lux coefficient of utilization is 0.3, maintenance factor is 0.25, average lux on road is 15 and width of road is 7.5
(A) 2 m (B) 3 m
(C) 2.5 m (D) 3.5 m

30. Consider the following statements with reference to pavements
(1) Flexible pavement are more suitable an rigid pavements in regions. Where sub grade strength is uneven
(2) Load carrying capacity of rigid pavements depends more on properties of concrete than strength of sub grade
(3) Compared to flexible pavements, rigid pavements are more affected by temperature variations

Which of these statements are correct.
(A) 1 and 2 (B) 1 and 3
(C) 2 and 3 (D) 3 alone

Answer Keys

1. D	**2.** C	**3.** A	**4.** B	**5.** B	**6.** A	**7.** A	**8.** D	**9.** A	**10.** B
11. B	**12.** D	**13.** C	**14.** B	**15.** C	**16.** D	**17.** D	**18.** A	**19.** B	**20.** D
21. A	**22.** D	**23.** D	**24.** A	**25.** A	**26.** B	**27.** B	**28.** A	**29.** A	**30.** C

Hints and Explanations

2. $\frac{1}{2}mv^2 = (wf) \times d$

$\downarrow$

Skid resistance force

$\therefore$ skid resistance force $= \frac{1}{2}\frac{mv^2}{d}$

$$= \frac{1}{2} \times \frac{(1400) \times \left(85 \times \frac{5}{18}\right)^2}{50}$$

$= 7804.7$ kg Choice (C)

3. $e + f = \frac{V_{min}^2}{gR_{min}}$

$e = \frac{E}{B} = \frac{1}{5} = 0.04$

$$0.04 + 0.15 = \frac{\left(60 \times \frac{5}{18}\right)^2}{9.8 \times R_{min}}$$

$$R_{min} = \frac{\left(60 \times \frac{5}{18}\right)^2}{9.8(0.04 + 0.15)}$$

$= \frac{16.66}{9.8 \times 0.19} = 8.94$ m Choice (A)

4. Loss of tractive force $T(1 - \cos\theta)$

$= 300(1\text{-}\cos 30°) = 40.1$ N Choice (B)

5. $b = 6$ m

$h = 10$ m

$f = 0.15$

$\frac{b}{2h} = \frac{6}{2 \times 10} = 0.3$

$F = 0.15 < \frac{b}{2h}$

$\therefore$ Vehicle skids prior to overtaking Choice (B)

6. $V = 40$ kmph

$R = 200$ m

$L = \frac{2.7 \times 40^2}{200} = 21.6$ m Choice (A)

7. $R = L/N$

$N = g_1 - g_2 = 2 - (-6)$

10 %

$L = \frac{\text{Total change of grade}}{\text{rate of change of grade}} = \frac{10 \times 100}{1} = 1000$

$R = \frac{1000}{10} = 100$ m Choice (A)

11. $e + f = \frac{V^2}{gR}$

For equilibrium super elevation (equal pressure on two wheels) $f = 0$

$e = \frac{V^2}{gR}$

$V = 40$ kmph $= 40 \times \frac{5}{18} = 11.11$ m/s

$\therefore$ $e = \frac{(11.11)^2}{9.8 \times 200} = 0.06$ Choice (B)

12. Non passing sight distance $= vt + \frac{v^2}{2g(f + n)}$

$V = 100$ kmph $= 100 \times \frac{5}{18} = 27.7$ m/s

Non passing sight distance

$$= (27.7 \times 2.5) + \frac{(27.7)^2}{2 \times 9.8(0.7 \times 0.5 + 0.02)}$$

$= 69.25 + 105.8 = 175.05$ m Choice (D)

13. **Deviation angle**

$N = 3 - (-5) = 8\% = 0.8$

Assuming $L > S$

$L = \frac{NS^2}{4.4} = \frac{0.08 \times (128)^2}{4.4} = 298$ m Choice (C)

15. $A = P\left(1 + \frac{r}{100}\right)n$

$P = 400$ commercial veh/day

$r =$ rate of traffic growth $= 7\%$

$n = 3$ years completion time

$A = 400\left(1 + \frac{7}{100}\right)^3 = 490$ cvd Choice (C)

16. Deflection,

$\Delta = \frac{1.5pa}{E_S}$ (for flexible plate)

Where,

$p =$ contact pressure due to wheel load $= 0.5$MPa

a = radius of contact area

$$\text{Contact area} = \frac{\text{Wheel load}}{\text{tyre pressure}} = \frac{40\times10^3\ \text{N}}{0.5\ \text{N/mm}^2}$$

$$= 80 \times 10^3\ \text{mm}^2$$

And area

$$\pi a^2 = 80\times10^3$$

$$A = \sqrt{\frac{80\times10^3}{\pi}} = 159.615\ \text{mm}$$

$$E_s = 250\ \text{MPa}$$

$$\text{So, } \Delta = \frac{1.5\times0.5\times159.617}{20} = 6\ \text{mm}$$ Choice (D)

17. For *WBM* the prescribed value of *LAV*, *AIV* and *FI* for Base course are < 50%, < 40%, and < 15% respectively and Surface course are < 40%, < 30% and < 15% respectively

Where,

LAV is los-angles abrasion value

AIV is aggregate Impact value

FI is flakiness index. Choice (D)

18. $$\text{Flakiness index} = \frac{100}{500}\times100 = 20\%$$

$$\text{Elongated index} = \frac{80}{400}\times100 = 20\%$$

Total = 20 + 20 = 40% Choice (A)

19. Theoretical capacity

$$C = \frac{1000V}{S}$$

$$S = SSD = L = 0.278\ vt + \frac{(0.278v)^2}{2fg} + L$$

$$= (0.278\times 70\times 2) + \frac{(0.278\times70)^2}{2\times0.4\times9.81} + 8 = 95.1736\ \text{m}$$

$$\text{Capacity} = \frac{1000v}{S} = \frac{1000\times70}{95.136}$$

$$= 735.5\ \text{veh/m/day.}$$ Choice (B)

21. $$\text{Jam density} = \frac{1000}{\text{space head way (H)}}$$

$$= \frac{1000}{6.25} = 160\ \text{veh/ km}$$

$$\text{Maximum flow} = \frac{\text{jam density}\times\text{free speed}}{4}$$

$$= \frac{160\times65}{4} = 2600\ \text{vph}$$ Choice (A)

22. Theoretical specific gravity

$$G_t = \frac{100}{\frac{w_1}{G_1}+\frac{w_2}{G_2}+\frac{w_3}{G_3}+\frac{w_4}{G_4}} = \frac{100}{\frac{55}{2.02}+\frac{34.6}{2.72}+\frac{4.8}{2.7}+\frac{5.6}{1.02}}$$

$$= 2.49 \simeq 2.5$$ Choice (D)

23. h = 15 cm, a/h = 0.5

$a = 0.5 \times 15 = 7.5$ cm

$$b = \sqrt{(1.6a^2 + h^2)} - 0.675h$$

$$= \sqrt{(1.6(7.5)^2 + 15^2)} - 0.675(15)$$

$$= 7.62\ \text{cm}$$ Choice (D)

24. $P/w = 0.25$

$b = 2.4$ m

$h = 4.2$ m

$f = 0.15$

$$\frac{P}{W} = \frac{b}{2h} = \frac{v^2}{gR}$$

To avoid overturn $\frac{b}{2h} > \frac{P}{w}$

$$\Rightarrow \frac{2.4}{2\times(4.2)} > 0.25$$

$$0.285 > 0.25$$

To avoid lateral skid $f > \frac{P}{w}$

$$\Rightarrow 0.5 > 0.25$$

$$F < \frac{b}{2h} \Rightarrow 0.15 < 0.285$$

$\Rightarrow$ vehicle skids prior to over turning

$$\frac{b}{2h} < f \Rightarrow 0.285 < 0.15\ \text{C}$$

$\Rightarrow$ vehicle overturns prior to skidding Choice (A)

25. $v_1 = 90$ km/ hr

$v_2 = 60$ km/hr

$t = 2.5$sec

Brake efficiency = 50%

$f = 0.7$

*SSD*1 for first car is = 153.6 m

*SSD*2 for second car is = 82.2 m

SSD = *SSD*1 + *SSD*2 = 235.8 m Choice (A)

26. Effective green time

$$G_e = G + A - L$$

G = 25 sec

A = 3 sec

L = intinal lost time (L_i) + final lost time (L_f)

$$G_e = 25 + 3 - (3+3) = 22\ \text{sec}$$ Choice (B)

27. Jam density= $\frac{1000}{6}$ = 166.67Veh/km

Maximum flow (or) capacity

$= \left(\frac{80}{2}\right)\left(\frac{166.67}{2}\right) = 3333.33$ veh / hr Choice (B)

28. $R = 350$ m

$V = 60$ kmph = 16.67 m/s

$\propto = 0.4$ m^3/sec

$L_S = \frac{v^3}{cR} = \frac{16.67^3}{2} = 33.07$ m

$S = \frac{L^2}{24R} = \frac{(33.07)^2}{(24 \times 350)} = 0.13$ m Choice (A)

29. Spacing of lamps = $\frac{100 \times 0.3 \times 0.25}{15 \times 7.5} = 2$ m

Choice (A)

30. 2 and 3 Choice (C)

Highway Engineering Test 4

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The mean weight of aggregates in the cylinder is 100 and weight of water required to fill the cylinder is 60 g and specific gravity of aggregate is 2.6. Then the angularity number is _____.

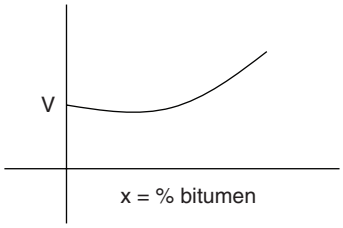
2. The grade of tar which is generally used for surface pointing and renewal coats of road is
(A) *RT* – 1 (B) *RT* – 2
(C) *RT* – 3 (D) *RT* – 4

3. The lowest point temperature at which a material gets ignited and burns under specified conditions of test
(A) Fire point
(B) Flash point
(C) Triple point
(D) None of these

4. The elastic modulus of granular layer is 5.5 MPa and thickness of granular layer is 3mm. The composite elastic modulus of granular sub base and base in MPa is.
(A) 1.4 (B) 1.8
(C) 2.4 (D) 1.3

5. If the wheel loads stress at edge is 40kg /cm^2, warping stresses at edge is 9kg/cm^2 and frictional stress is 5kg/cm^2 and then the critical stress in edge region for summer mid day is.
(A) 48 kg/cm^2
(B) 42 kg/cm^2
(C) 44 kg/cm^2
(D) 40 kg/cm^2

6.



In the above graph v = ?
(A) marshal stability
(B) unit weight
(C) flow value
(D) percentage voids of total mix

7. In a dual wheel assembly if (p) is equal to each wheel load, 's' is centre to centre spacing of dual wheels and 'd' is the clear distance between wheels, then the equivalent single wheel load for depth between d/2 and 2s is
(A) p (B) 2p
(C) between P and 2p (D) None

8. Match the following

	Types of Test		Property
a.	Impact	1.	Resistance to weather
b.	Soundness	2.	Hardness
c.	Crushing	3.	Toughness
d.	Abrasion	4.	Strength

Codes:
a b c d a b c d
(A) 4 3 2 1 (B) 2 1 4 3
(C) 3 4 1 2 (D) 3 1 4 2

9. The design traffic of a flexible pavement is based on 7 day 24 hours traffic count as per
(A) *IRC* : 39.2001 (B) *IRC* : 58 – 2002
(C) *IRC* : 9 (D) *IRC* : 7

10. The parameters required to get the value of radius of relative stiffness of *CC* pavement is (conventions are as per *IRC*)
(A) E, k, r, u (B) E, h, k, u
(C) h, k, u, r (D) P, h, k, u

11. A bitumen concrete mix has average specific gravity of 2.325 and theoretical specific gravity as 2.41. The density of bitumen is 1.03 g/cc with 4.5% bitumen content by weight in mix. The *VFB* of mix is
(A) 32.8% (B) 30.8%
(C) 20.73% (D) 10.15%

12. A subgrade soil sample was tested using standard *CBR* apparatus and the observations are given below:

Load, kg	Penetration, mm
40.8	2.5
60.5	5.0

The CBR value of the sample is
(A) 3.9% (B) 2.9%
(C) 4.8% (D) 12.2%

13. The width of expansion joint is 30 mm in a cement concrete pavement. The laying temperature is 10°C and the maximum slab temperature in summer is 50°C. The coefficient of termal expansion of concrete is 10×10^{-6}/°c and the joint filler compresses up to 50% of the thickness. The spacing between expansion joints should be ____ (in m).

14. What is the deflection at the surface of flexible pavement due a wheel load of 30 kN and a tyre pressure of 0.6 Mpa. Take E = 20 MPa
(A) 5.8mm (B) 5.6mm
(C) 5.4mm (D) 5.2mm

15. A plain cement concrete pavement is constructed at temperature of 20°c. The peak summer temperature is

40°c. The coefficient of thermal expansion of concrete is 8×10^{-6} per °c, the gap at expansion joint is 3.5 cm. The spacing of expansion joint is _____ (in m).

16. In marshall method of mix design, the coarse aggregates, fine aggregates, filler material and bitumen, having respective specific gravities of 2.82, 2.42, 2.68, and 1.02 are mixed in ratio of 52, 34.8, 4.2 and 5.4 percent respectively. The theoretical specific gravity of the mix would be ___

17. In case of governing equations for calculating wheel loads stress using westergaard's approach, the following statements are made.
(I) load stress are inversely proportional to wheel load.
(II) Modulus of sub grade reaction is useful for load stress calculation.
(A) Both statements are true
(B) I is true and II is false
(C) Both statements are false
(D) I is false and II is true

18. Match the following
Type of test property

	Types of cut backs		Materials
1.	Rapid curing (RC)	a.	Bitumen fluxed with kerosene
2.	Medium curing (MC)	b.	Bitumen blended with high boiling point gas oil
3.	Slow curing (BC)	c.	Bitumen fluxed with naphtha (or) gasoline

Codes:

	1 2 3		1 2 3
(A)	*c a b*	(B)	*b c a*
(C)	*a b c*	(D)	*c a b*

19. Which of the following are the purposes for use of steel bar reinforcement in cement concrete pavements. The correct answer using the code is
(1) To increase the flexural strength of concrete
(2) To prevent the onset of cracks to allow wider spacing of joints.
(3) To allow wider spacing of joints
(A) 1 and 2 only (B) 2 and 3 only
(C) 1 and 3 only (D) 1, 2 and 3

20. Variation of tyre pressure is
(A) Equal to 1.0 for an average tyre pressure of 7 kg/cm^2
(B) < 1.0 for tyre pressure greater than 7 kg /cm^2
(C) > 1.0 for tyre pressure less than 7 kg /cm^2
(D) all the above are correct

21. A combined value of flakiness and elongation index are to be determined for a sample of aggregates. The sequence in which two tests are conducted is
(A) elongation index test followed by flakiness index test
(B) flakiness index test followed by elongation index test
(C) flakiness index test followed by elongation index on non flaxy aggregates
(D) elongation index test followed by flakiness index test on non elongated aggregates

22. Compute the equivalent radius of resisting section of 20 cm slab, if ratio of radius of wheel load distribution to thickness of slab is 0.6.
(A) 14.22 cm (B) 14.02 cm
(C) 14.8 cm (D) 14.7 cm

23. Match the following

	Type of test		Purpose
a.	Penetration test	1.	Design of bitumen concrete mix
b.	Marshal test	2.	Overlay design
c.	Ring and ball test	3.	Gradation of Asphalts
d.	Benkelman beam test	4.	Determination of softening point

Codes:

	a b c d		a b c d
(A)	3 2 4 1	(B)	3 1 4 2
(C)	2 3 4 1	(D)	4 2 3 1

24. If 6.5 cm bituminous concrete surface with $E_C = 1000$ kg/cm^2 is equivalent to thickness t_b of base course, then equivalent thickness t_b of base course having $t_b = 400$ kg/cm^2 will be.
(A) 12.42 cm (B) 9.36 cm
(C) 10.48 cm (D) 8.82 cm

25. In the revised *CBR* design method recommended by *IRC* for design of flexible pavement total thickness depends upon
(A) *CBR* value of soil only
(B) *CBR* value of soil and magnitude of wheel load
(C) *CBR* value of soil and number of commercial vehicle per day
(D) *CBR* value of soil and cumulative standard axle loads

Answer Keys

1. 2.85 – 2.95	**2.** C	**3.** A	**4.** 1.8	**5.** C	**6.** C	**7.** C	**8.** D	**9.** C
10. B	**11.** A	**12.** B	**13.** 37.5 m	**14.** B	**15.** 21.2 m		**16.** 2.50 – 2.52	
17. B	**18.** A	**19.** D	**20.** D	**21.** B	**22.** A	**23.** B	**24.** D	**25.** D

Hints and Explanations

1. Angularity number = 67 -% solid volume

$$= 67 - \left(\frac{100}{60 \times 2.6} \times 100\right)$$

$= 2.89 = 3$ Ans: 2.85 – 2.95

2. $RT - 3$ Choice (C)

3. Fire point Choice (A)

4. $E_1 = E_2 \times 0.2 \times h^{0.45}$

$= 5.5 \times 0.2 \times (3)^{0.45}$

$= 5.5 \times 0.2 \times (3)^{0.45}$

$= 1.8\ MPa.$ Ans: 1.8

5. $\sigma_{\text{critical}}\ \sigma_e + \sigma_{w(e)} - \sigma_f$

$= 40 + 9 - 5 = 44\ \text{kg/cm}^2$ Choice (C)

6. Flow value Choice (C)

7. In between P and $2P$ Choice (C)

8. Choice (D)

9. $IRC - 9$ Choice (C)

10. E, h, k, u Choice (B)

11. $VFB = \dfrac{v_b \times 100}{VMA}$

$$VMA = \frac{V_a + V_b}{V_t} \times 100$$

$$V_b = \frac{G_m\, x W_b}{G_b} = \frac{2.325 \times 4.5}{1.03} = 10.15$$

$$V_a = \frac{G_t - G_m}{G_t} \times 100$$

$$= \frac{2.41 - 2.325}{241} \times 100 = 20.73$$

$$VMA = \frac{10.15 + 20.73}{1} \times 100 = 30.88$$

$$VFB = \frac{10.15 \times 100}{30.88} = 32.8\%$$ Choice (A)

12. $CBR = \dfrac{\text{load at 2.5mm}}{\text{standard load}} \times 100$

$$\text{At 2.5 mm} = \frac{40.8}{1370} \times 100 = 2.97\%$$ Choice (B)

13. $L_e = \dfrac{\delta^1}{\alpha(t_2 - t_1)} = \dfrac{\delta}{2(\alpha)(t_2 - t_1)}$

$$= \frac{30}{2x10x10^{-6}\,x(40)} = 37500\ \text{mm}$$

$= 37.5$ m Ans: 37.5 m

14. $\Delta = \dfrac{1.5pa}{E_s}$

$$\text{Contact area} = \frac{\text{wheel load}}{\text{Tyre Pressure}} = \frac{30 \times 10^3}{0.6\ \text{n/mm}}$$

$\pi \times a^2 = 50 \times 10^3$

$$a = \sqrt{\frac{50 \times 10^3}{\pi}}$$

$a = 126.1\text{mm}$

$$\Delta = \frac{1.5 \times 0.6 \times 126.1}{20} = 5.6\ \text{mm}$$ Choice (B)

15. $L_e = \dfrac{\delta}{\alpha(t_2 - t_1)}$

$\delta = 3.5$ cm

$\alpha = 8 \times 10^{-6}\text{per}^\circ c$

$t_2 = 40^\circ c$

$t_1 = 20^\circ c$

$$L_e = \frac{3.5}{8 \times 10^{-6}(40 - 20)} = 21.2\ \text{m}$$ Ans: 21.2 m

16. $G_t = \dfrac{100}{\frac{W_1}{G_1} + \frac{W_2}{G_2} + \frac{W_3}{G_3} + \frac{W_4}{G_4}}$

$W_1 = 52$ $G_1 = 2.82$

$W_2 = 34.8$ $G_2 = 2.42$

$W_3 = 4.2$ $G_3 = 2.62$

$W_4 = 5.4$ $G_4 = 1.02$

$$G_t = \frac{100}{\frac{52}{2.82} + \frac{34.8}{2.42} + \frac{4.2}{2.62} + \frac{5.4}{1.02}}$$

$= 2.517$ Ans: 2.50 – 2.52

17. Choice (B)

18. Choice (A)

19. Choice (D)

20. Choice (D)

21. Choice (B)

22. $h = 20$ cm, $a/h = 0.6$

$a = 0.6 \times 20 = 12$ cm

$$b = \sqrt{\left(1.6a^2 + h^2\right)} - 0.675h$$

$$= \sqrt{1.6(12)^2 + (20)^2} - (0.675 \times 20)$$

$= 14.22$ cm Choice (A)

23. Choice (B)

24. $$\frac{t_b}{t_c} = \left(\frac{E_c}{E_b}\right)^{\frac{1}{3}}$$

$$\frac{t_b}{6.5} = \left(\frac{1000}{400}\right)^{\frac{1}{3}}$$

$$t_b = 6.5\left(\frac{1000}{400}\right)^{\frac{1}{3}} = 8.82 \text{ cm}$$ Choice (D)

25. Choice (D)

UNIT XII
GEOMATICS ENGINEERING

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GEOMATICS ENGINEERING TEST 1

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. What is the principle of surveying?
(A) Working from part to whole
(B) Working from whole to part
(C) Working from higher to lower
(D) Working from lower to higher

2. An offset is measured with an accuracy of 1 in 40. If the scale of plotting is 1 cm = 30 m, the limiting length of offset, so that the displacement of point on paper from both sources of error may not exceed 0.025 mm, is _____.
(A) 21.3 m (B) 21.4 m
(C) 21.2 m (D) 21.8 m

3. The whole circle bearing of $S\ 31°\ 36^1E$ is _____
(A) $138°\ 24^1$ (B) $158°\ 24^1$
(C) $128°\ 24^1$ (D) $148°\ 24^1$

4. If the image formed by the objective lens is not in the same plane with cross hairs, then it is known as
(A) focusing of eye piece
(B) focusing of objective
(C) parallax
(D) aberration

5. The automatic check for leveling in case of height of instrument method is
(A) ΣB.S – ΣF.S = Last RL – First R.L.
(B) ΣB.S – ΣF.S = ΣRise – Σfall.
(C) ΣRise – Σfall = last 1 – First RL.
(D) None of the above.

6. A line lying throughout the surface of the ground and preserving a constant inclination to the horizontal is
(A) contour gradient.
(B) horizontal equivalent.
(C) contour interval.
(D) vertical control.

7. Surveyor's chain consists of 100 links along a length of
(A) 100 ft long (B) 33 ft long
(C) 66 ft long (D) 50 ft long

8. Correction for temperature in a chain when the temperature at field is more than the standard temperature is _____
(A) additive
(B) negative
(C) constant
(D) None of the these

9. In surveyor's compass the graduation are in
(A) whole circle bearing
(B) quadrantal bearing
(C) both whole circle bearing and quadrantal bearing
(D) None of the above

10. Small scale representation of map is known as
(A) scale (B) plan
(C) grid (D) survey map

11. The sag correction for a 30 m steel tape under a pull of 200 N in four equal spans of 8 m each. The weight of one cubic cm of steel = 0.078 N and area of c/s of tape = 0.08 sqm.
(A) 3.9N (B) 4.2N
(C) 4.99N (D) 4.6N

12.

Point	Co-ordinates N	E
A	O	O
B	3014	256
C	1764	1398
D	–	–

A straight tunnel is to be run between two points *A* and *B* whose co-ordinates are as given above.
The length and bearing of *CD* if *D* is the midpoint of *AB*, are _____.
(A) $74°\ 38^1$, 1380.2 m
(B) $75°\ 39^1$, 1278.4 m
(C) $78°\ 34^1$, 1295.7 m
(D) $76°\ 32^1$, 1287.6 m

13. In leveling between two points *A* and *B* on opposite banks of a river, the staff readings at *A* and *B* were 1.295 and 2.960 m respectively. The level was then removed and set up near *B* and the readings on *A* and *B* were 0.56 and 2.42. The true difference of levels between *A* and *B* is __.
(A) 3.486 m (B) 3.525 m
(C) 3.538 m (D) 3.624 m

14. The perpendicular offsets at 20 m intervals from survey line to an irregular boundary lines are 3.25, 5.6, 4.2, 6.65, 8.75, 6.2, 3.25, 4.2, 5.65. The area enclosed between survey line by the application of trapezoidal rule is _____.
(A) 820 m^2 (B) 833 m^2
(C) 860 m^2 (D) 866 m^2

15. An observation with a theodilite given staff readings of 1.052 and 2.502 for angles of elevation gave 8% and 6% respectively. The vertical angle was 5.25%. The horizontal distance of staff station if the instrument elevation is 942.5 m is _____
(A) 48 m (B) 52 m
(C) 56 m (D) 50 m

16. The following are bearings in closed traverse.

Line	FB	BB
AB	80° 10¹	259° 0¹
BC	120° 20¹	301° 50¹
CD	170° 50¹	350° 50¹
DE	230° 10¹	49° 30¹
EA	310° 20¹	130° 15¹

The interior angles between the bearings are _____.
(A) 50°, 65.3°, 230.5°, 100°, 123°
(B) 50.5°, 138.40°, 131°, 120.40°, 99.10°
(C) 53°, 48.4°, 123.2°, 121°, 100.10°
(D) 58°, 62.5°, 73o, 148°, 133°

17. In order to find the difference in elevation between two points *P* and *Q*, a level was set upon line *PQ*; 40 meters from *P* and 1.280 m from *Q*. The readings obtained on shaft kept at *P* and *Q* were 0.525 m and 4.92 m. respectively. The true difference in elevation between *P* and *Q* is _____.
(A) 3.87 m (B) 4.82 m
(C) 4.39 m (D) 4.31 m

18. Match the following.

1.	Vertical axis	i.	Axis about which telescope rotates in vertical plane
2.	Turnion axis	ii.	Line passing through intersection of horizontal and vertical cross hairs and optical center
3.	Line collination	iii.	Axis about which instrument rotates in horizontal plane
4.	Axis of level tube	iv.	Line tangential to longitudinal curve of level tube at its center

(A) (1 – ii), (2 – iii), (3 – i), (4 – iv)
(B) (1 – iv), (2 – i), (3 – iii), (4 – ii)
(C) (1 – iii), (2 – i), (3 – ii), (4 – iv)
(D) (1 – iii), (2 – iv), (3 – ii), (4 – i)

19. A surveyor measured the distance between two points on plan drawn to a scale of 1 cm = 30 m and the result was 500 m. Later it was discovered that 1 cm = 10 m scale was used. The true distance between the points would be _____.
(A) 168.3 m (B) 165.6 m
(C) 162.3 m (D) 166.6 m

20. (i) The durinal variation is the departure of the destination from its mean value a during a period of 24 hrs.
(ii) The variation which has a yearly period is known as annual variation.
(A) (i) is true.
(B) (ii) is true.
(C) Both (i) and (ii) are true.
(D) Both (i) and (ii) are false.

21. Match the following

	Purpose of survey		Scale
1.	Building site	i.	1 cm = 50 mts 200 m
2.	Location survey	ii.	1 cm = 10 m or less
3.	Town planning	iii.	1 cm = 50 m to 100 m

(A) (1 – ii), (2 – iii), (3 – i)
(B) (1 – iii), (2 – ii), (3 – i)
(C) (1 – ii), (2 – iii), (3 – i)
(D) (1 – i), (2 – ii), (3 – iii)

22. Two straight lines intersect at an angle of 50°. The radius of curve joining the two straight lines is 400 m. The length of long chord and mid ordinates in meters is. _____.
(A) 337.8 m, 36.8 m (B) 340.6 m, 38.3 m
(C) 338.09 m, 37.47 m (D) 360.7 m, 35.6 m

23. During the leveling work along a falling gradient using a dumpy level and staff of 3 m length, the following successive readings are taken:
1.623, 2.789, 0.260, 1.520.
What will be the correct order of booking these four readings in level book?
(A) *BS*, *FS*, *BS*, *FS* (B) *BS*, *IS*, *FS*, *FS*
(C) *BS*, *IS*, *IS*, *FS* (D) *BS*, *IS*, *BS*, *FS*

24. The bench mark with reduced level (*RL*) = 156.305 m has been established at the floor of a room. It is required to find the *R.L* of the underside of root. Back slight (BS) is 1.8 m whereas the Foresight (*FS*) is 0.675, The *R.L* will be _____.
(A) 152.3 m (B) 159.8 m
(C) 158.7 m (D) 153.3 m

25. *A*: Radiation method of plane table survey is employed for locating the details.
R: Radiation method is suitable when distances are small.
(A) *A* and *R* are true and *R* is the correct explanation of *A*.
(B) *A* and *R* are true *R* is not the correct explanation of *A*.
(C) *A* is true and *R* is false.
(D) *A* is false and *R* is true.

Answer Keys

1. B	**2.** C	**3.** D	**4.** C	**5.** A	**6.** A	**7.** C	**8.** A	**9.** B	**10.** B
11. C	**12.** C	**13.** B	**14.** D	**15.** D	**16.** B	**17.** A	**18.** C	**19.** D	**20.** C
21. A	**22.** C	**23.** A	**24.** C	**25.** B					

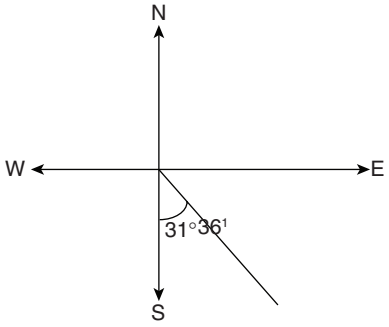
Hints and Explanations

2. Total displacement of paper $= \frac{\sqrt{2}L}{rs}$ cm

$$\frac{\sqrt{2}L}{rs} = 0.025$$

$$L = \frac{0.025}{\sqrt{2}} \times 40 \times 30 = 21.21 \text{ m.}$$ Choice (C)

3.



WCB $= 180° - R.B$
$= 180° - 31° 36^1$
$= 148° 24^1$ Choice (D)

11. Volume of tape/m $= 0.08 \times 200 = 16 \text{ m}^3$.
Weight of tape/m $= 16 \times 0.078 = 0.624$N.
Total weight of tape between two supports $= 0.624 \times 8$
$= 4.992$N. Choice (C)

12. Since D is midway between A and B, its co-ordinates are 1507 and 128
Latitude of $AD = 1507$.
Departure of $AD = 128$.
Latitude of $AC = 1764$.
Departure of $AC = 1398$.
Latitude of $DC = 1764 - 1507 = 257$.
Departure of $DC = 1398 - 128 = 1270$.
$\therefore$ Latitude of $CD(_L) = -257$.
Departure of $CD(_D) = -1270$.

Bearing of $CD = \tan\theta = \frac{D}{L} = \frac{1270}{257}$

$\Rightarrow \theta = 78° 34^{1.}$

Length of $CD = \sqrt{D^2 + L^2}$

$= \sqrt{1270^2 + 257^2}$

$= 1295.7$ m. Choice (C)

13. When instrument is at A.
$2.96 - 1.295 = 1.665$ m.
When instrument is at B.
$2.42 - 0.56 = 1.86$ m.

True difference in elevation $= \frac{1.665 + 1.86}{2}$

$= 3.525$ m. Choice (B)

14. According to trapezoidal rule

$$\Delta = \left(\frac{O_o + O_n}{2} + O_1 + O_2 + O_3 + ... O_{n-1}\right) \times d$$

$$\Rightarrow \left(\frac{3.25 + 5.65}{2} + 5.6 + 4.2 + 6.65 + 8.75 + 6.2 + 3.25 + 4.2\right)$$

$\times 20 = 866 \text{ m}^2$. Choice (D)

15. $\tan\propto_1 = 0.06$, $\tan\propto_2 = 0.08$.
Horizontal distance

$$D = \frac{S}{\tan\propto_1 - \tan\propto_2} = \frac{2.502 - 1.502}{0.08 - 0.06}$$

$= 50$ m. Choice (D)

16. $\angle A$ = Bearing of AE – Bearing of AB
$= 130° 15^1 - 80° 10^1 = 50° 5^1$.
$\angle B$ = Bearing of BA – Bearing of BC
$= 259 - 120° 20^1 = 138° 40^1$
$\angle C$ = Bearing of CB – Bearing of CD
$= 301° 50^1 - 170° 50^1 = 131° 0^1$
$\angle D$ = Bearing of CD – Bearing of DE
$= 350° 50^1 - 230° 10^1 = 120° 40^1$
$\angle E$ = Bearing of ED – Bearing of EA
$= 49° 30^1 - 310° 20^1 + 360° = 99° 10^1$ Choice (B)

17. Combined correction for Q
$= 0.067828 (1.280)^2 = 0.110$m (subtractive)
Correct staff reading at Q
$= 4.92 - 0.525 = 4.395$ m
Difference in elevation between P and Q
$= 4.395 - 0.525 = 3.87$ m. Choice (A)

19. Measured length = 500 m.

$R.F$ of wrong scale $= \frac{1}{30 \times 100} = \frac{1}{3000}$.

RF of correct scale $= \frac{1}{10 \times 1000} = \frac{1}{1000}$.

$$\therefore \text{ Correct length} = \left(\frac{\frac{1}{3000}}{\frac{1}{1000}}\right) \times 500$$

$= 166.6$ m. Choice (D)

22. $\Delta = 50$, $R = 400$ m,
Length of long chord $= 2RS \text{ in } \frac{\Delta}{2}$

$= 2 \times 400 \times \sin\frac{50}{2} = 338.09$ m.

Length of midordinate $= R\left(1 - \cos\frac{\Delta}{2}\right)$

$= 400\left(1 - \cos\frac{50}{2}\right) = 37.47$ m. Choice (C)

24. $RL = 156.305 + 0.675 + 1.8 = 158.7$ m. Choice (C)

GEOMATICS ENGINEERING TEST 2

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. A closed contour line with one or more higher contour lines inside represents
(A) cliff (B) hill
(C) valley (D) cave

2. Any convenient direction towards a permanent and prominent mark is called
(A) True meridian (B) Magnetic meridian
(C) Arbitrary meridian (D) None of the above

3. Theodilite is a measuring device which is included under category of
(A) First order measurements
(B) Second order measurements
(C) Third order measurements
(D) Fourth order measurements

4. A tape is standardized at 100N pull. If the load applied is 120N the sag correction
(A) $L_1\ W_1^2\ (1.12 \times 10^{-6})$
(B) $L_1\ W_1^2\ (1.273 \times 10^{-6})$
(C) $L_1\ w_1^2\ (1.73 \times 10^{-6})$
(D) $L_1\ W_1^2\ (1.53 \times 10^{-6})$

5. The long and short sides of a rectangle measure 9.32 m and 4.82 m, with errors ± 5 mm. Express the area of correct number of significant figures
(A) 44.98 m^2 (B) 44.96 m^2
(C) 44.92 m^2 (D) 44.85 m^2

6. An offset is laid out 3° from its true direction of the field. If the scale of plotting is 10 m to 1 cm, find the maximum length of offset so that the displacement of the point on paper may not exceed 0.25mm.
(A) 4.43 (B) 4.58
(C) 4.23 (D) 4.77

7. The combined correction of curvature and refraction for distance of 1.29 km is
(A) 0.234 m (B) 0.121 m
(C) 0.112 m (D) 0.187 m

8. The observation ray between two triangulation stations *A* and *B* just grazes the sea. If the heights of *A* and *B* are 6000 m and 2000 m respectively, the distance *AB* is (Let radius of earth *R* = 6440 km)
(A) 432.4 km (B) 438.3 km
(C) 450.2 km (D) 442.4 km

9. The constant for an instrument is 750, the value of *C* = 0.3 m and intercept *S* = 2 m. The distance from instrument to the staff when the micrometer readings are 4.326 and 4.283 and the line of sight is at +8° 361 when the staff was held vertical is
(A) 172.4 m (B) 173.2 m
(C) 174.8 m (D) 170.8 m

10. The survey which is done for fixing the property lines is known as
(A) Topographical survey
(B) Cadastral survey
(C) City survey
(D) Astronomical survey

11. The volume of 130 m long road of formation width 10 m, side slopes 1 : 1, average depth of cutting along center of line is 5m and slopes of ground in cross-section is 10 to 1 is
(A) 9880 m^3 (B) 9723 m^3
(C) 9624 m^3 (D) 9892 m^3

12. In a region with magnetic declination of 4°E, the magnetic fore bearing (*FB*) of a line *AB* was measured as N 82° 50^1 E. There was a local attraction at *A*. To determine the correct magnetic bearing of the line, a point *O* was selected at which there was no local attraction. The magnetic *FB* of line *AO* and *OA* were observed to be S 42° 40^1 E and N 48° 20^1 W respectively. What is the true *FB* of line *AB*?
(A) N 81° 50’ (B) N 82° 10’ E
(C) N 79° 30’ E (D) N 84° 10’ E

13. The length of the tape is 30 m and the sag is 30.35 cm at the mid span under a tension of 100 N, the weight of the tape is
(A) 0.248 N/m (B) 0.269 N/m
(C) 0.326 N/m (D) 0.459 N/m

14. Two distances of 30 and 100 meters were accurately measured out and the intercepts on the staff between the outer stadia webs were 0.176 m at the former distance and 0.892 m at the latter. The tacheometric constant *K* is
(A) 100 (B) 97.7
(C) 95 (D) 96.2

15. A flag staff of 2 m height was erected on topic of hill (*Q*) and the observations were made from two stations *P* and *R*, 50 meters apart. The horizontal angle measured at *P* between *R* and top of flag staff was 50° 30’ and that measured at *R* between the top of the flag-staff *P* was 50° 18’. Angle of elevation to top of staff was measured to be 10° 12^1 at *P*. The angle of elevation to top of flag staff and was measured to be 10° 48’ at *R*. Staff readings on *B.M* when the instrument was at *P* = 1.826 m and that with the instrument at *R* = 2.285 m. The elevation of the top of hill *Q* if the *BM* was 485.065 m is
(A) 485.36 m (B) 488.32 m
(C) 494.22 m (D) 498.32 m

16. Match the following

	List – I		List – I
i.	Correction for standard length	a.	$C_a = \frac{L.c}{l}$
ii.	Correction for tension	b.	$C_p = \frac{(P - P_o)}{AE}L$
iii.	Sag correction	c.	$C_t = \alpha(T_m - T_o)L$
iv.	Correction for Temperature	d.	$C_{sa} = \frac{W^2L}{24P^2}$

	i	ii	iii	iv		i	ii	iii	iv
(A)	d	a	b	c	(B)	a	b	d	c
(C)	d	c	b	a	(D)	c	a	b	d

17. (i) Optical square is better than prism square.
(ii) In both optical and prism squares, the principle of operation is same.
(A) only (i) is correct
(B) only (ii) is correct
(C) Both (i) and (ii) are correct
(D) None of the above

18. Match the following

	List – I		List – II
i.	Vertical cliff	a.	contour lines of different elevations unite to form one line
ii.	Steep slope	b.	contour lines of different elevations cross one another
iii.	Hill	c.	Contour lines are closely spaced
iv.	Overhanging cliff	d.	closed contour lines with higher values inside them

	i	ii	iii	iv
(A)	d	c	a	b
(B)	a	c	d	b
(C)	a	b	d	c
(D)	d	b	a	c

19. A Circular curve has 300 m radius and 55° deflection angle, then the apex distance is
(A) 38.23m (B) 38.21m
(C) 39.23m (D) 40.24m

20. The length of mid ordinate in the above question is
(A) 37.23m (B) 38.62m
(C) 33.89m (D) 32.43m

21. Find the area between line *AB* and the stream taken at a regular interval of 30 m along line *AB*, using simpson's rule.

Distance	0	30	40	90	120	150	180	210	240
Offset length	23	40	42	30	32	60	10	14	22

(A) 7980 m^2 (B) 6352 m^2
(C) 5652 m^2 (D) 4734 m^2

Common Data for Questions for 22 and 23:
In reciprocal levelling, the following readings are taken.

Instrument station	Staff reading		
	A	B	
A	1.286	2.768	Distance AB = 1150 m
B	1.292	2.432	RL of A = 100 m Collimation error = $\frac{0.003}{150m}$

22. The correction for collimation is
(A) 0.021 (B) 0.048
(C) 0.023 (D) 0.032

23. Correction for refraction is
(A) 1.33m (B) 1.34m
(C) 1.38m (D) 1.29m

24. The following observations were taken during testing of a dumpy level *s*

Instrument at	Staff readings at	
	A	B
A	1.342	2.125
B	1.485	1.683

If *A* and *B* are 100 m apart the angle of inclination of line of collimation is
(A) 14' 15.50' (B) 12' 23.30'
(C) 10' 2.29' (D) 13^1 3.40'

25. The distance between two points *A* and *B* by tachometer fitted with anallactic lens which made a vertical angle of +10° 46' and staff intercept of 1.763 m. Later on the constants of instrument were changed to 100 and 0.5. The percentage error is computed horizontal distance is
(A) 0.287% (B) 0.321%
(C) 0.262% (D) 0.213%

26. In a quadrilateral ABCD, the coordinates of points are as follows

Point	East	North
A	0	0
B	0	–842.8
C	600.1	742.8
D	1023.4	659.3

The area of the figure is
(A) 4.68 hectares (B) 7.06 hectares
(C) 9.12 hectares (D) 12.51 hectares

27. The incorrect statement among the following is
(A) The direction of magnetic meridian is variable
(B) The direction of true meridian is invariable
(C) The magnetic bearing of line varies with time
(D) Magnetic meridian through various stations are not parallel but converge at poles

28. After surveying an area with plane table at a station it was detected that the centering of the instrument was not done accurately. The displacement of the plotted point at right angles to the ray was 20 cm. The scale used was 1 cm = 20 m. Determine the error in the plotted position of the point (in cm)

(A) 0.03 (B) 0.04
(C) 0.01 (D) 0.02

29. To continue a survey line *AB* past an obstacle, a 400 m long line *BC* was set out perpendicular to *AB* and from *C* angles *BCD* and *BCE* were set out 60° and 45° respectively. Determine the lengths which must be chained off along *CD* and *CE* in order that *ED* may be in line with *AB* produced.

(A) 747 m, 620.4 m (B) 827 m, 580.3 m
(C) 800 m, 565.6 m (D) 852 m, 648.3 m

30. Match the following related to Electromagnetic distance measurement (EDM)

i.	short range	a.	less than 3km
ii.	Medium range	b.	less than 100km
iii.	long range	c.	less than 25km

	i	ii	iii
(A)	b	c	a
(B)	a	c	b
(C)	c	a	b
(D)	a	b	c

Answer Keys

1. B	**2.** C	**3.** B	**4.** B	**5.** C	**6.** D	**7.** C	**8.** B	**9.** D	**10.** B
11. A	**12.** C	**13.** B	**14.** B	**15.** C	**16.** B	**17.** C	**18.** B	**19.** B	**20.** C
21. D	**22.** C	**23.** A	**24.** C	**25.** A	**26.** B	**27.** D	**28.** C	**29.** C	**30.** B

Hints and Explanations

1. A closed contour line with one or more higher ones inside represents a hill. Choice (B)

2. Any convenient direction towards a permanent and prominent mark is called Arbitrary meridian. Choice (C)

3. The odilite is included under second order measurements. Choice (B)

4. Total sag correction = Sag correction for 100N pull – Sag correction for 120 N pull

$$= \frac{L_1 W_1^2}{24(100)^2} - \frac{L_1 W_1^2}{24(120)^2}$$

$$= \frac{L_1 W_1^2}{24}\left[\frac{1}{100^2} - \frac{1}{120^2}\right]$$

$= L_1 W_1^2\ (1.273 \times 10^{-6})$. Choice (B)

5. $A = 9.32 \times 4.82 = 44.92\ m^2$

Maximum error in individual measurements = 0.005 m

$$\therefore \quad \text{error ratios are} = \frac{\Delta L}{L} = \frac{0.005}{9.32} = \frac{1}{1864} \text{ and}$$

$$\frac{\Delta b}{b} = \frac{0.005}{4.82} = \frac{1}{964}$$

$$\delta A = 44.92\left[\frac{1}{1864} + \frac{1}{964}\right] = \pm 0.07\ m^2$$

Hence the area limits can be $44.92 + 0.07 = 44.99\ m^2$

$44.92 - 0.07 = 44.85\ m^2$

The compatible area is $44.92\ m^2$. Choice (C)

6. Displacement of point on the paper

$$= \frac{L \sin \propto}{S} = \frac{L \sin 3^o}{10} \text{cm}$$

This should not exceed 0.025 cm

$$\frac{L \sin 3^o}{10} = 0.025$$

$L \sin 3° = 0.025 \times 10$

$L = 4.77$ m. Choice (D)

7. Combined correction = $0.06728\ (1.29)^2 = 0.1196$ m. Choice (C)

8. $C_1 = A^1 A = 6$ km

Radius of earth (R) = 6440 km

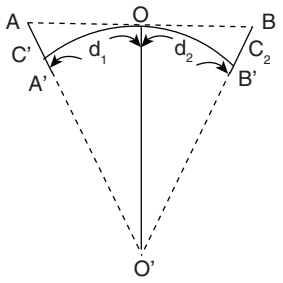
$C_2 = B^1 B = 2$ km

Correction of curvature

$$C_1 = \frac{d_1^2}{2R}$$

Distance $d_1 = \sqrt{2 \times R \times C_1}$

$$d_1 = \sqrt{2 \times 6440 \times 6} = 277.9 \text{ km}$$



Distance $d_2 = \sqrt{2 \times R \times C_2} = \sqrt{2 \times 6440 \times 2}$

$= 160.49$ km $= OA + OB = OA' + OB'$

Distance $AB = d_1 + d_2 = 438.3$ km. Choice (B)

9. The sum of micrometer reading = 4.326 + 4.283 = 8.609

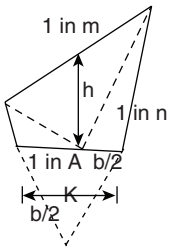
$$D = \frac{K.S}{n}\cos^2\theta + \cos\theta \not> 1 \text{ (one)}$$

$$= \left[\frac{750 \times 2}{8.609} \times \cos^2\left(8^o\,36^1\right)\right] + [0.3 \times \cos(8°\,36^1)]$$

$= 170.6$ m. Choice (D)

10. Cadastral survey is the survey in which fixing of property lines is done. Choice (B)

11.



$$A = \frac{n\left(\frac{b}{2}\right)^2 + m^2\left(bh + nh^2\right)}{m^2 - n^2}$$

Where $n = 1$, $m = 10$, $h = 5$, $b = 10$

$$\therefore \quad A = \frac{1\left(\frac{10}{2}\right)^2 + 10^2\left(10 \times 5 + 1 \times 5^2\right)}{10^2 - 1^2} = 76 \text{ m}^2$$

$V = A \times L = 76 \times 130 = 9880$ m^3. Choice (A)

12. Declination (D) = 2° E

Magnetic $F.B.$ of AB = N82° 50′ E = 82° 50′

Correct $F.B$ of OA = N48° 20′ W = 311° 40′

$\therefore$ Correct $B.B$ of OA = 131° 40′ = True variation (as O is free from local attraction) In general, $F.B.$ of $AO = B.B.$ of OA.

But $F.B.$ of $AO = S42°40'$ E = 137°20′. ≠ 131°40′.

As A has local attraction,

$\therefore$ Magnetic variation = 137°20′ ($F.B.$ of AO)

$\therefore$ Error = Magnetic variation – True Variation

= 137°20′ – 131°40′ = 540°.

Therefore Correction = –5°40′

True $F.B.$ of line AB = 2°50′+2°–5°40′

= 79°10′ = N79°10′E. Choice (C)

13. For sag $h = \frac{wL_1d_1}{8P}$

Here $L_1 = d_1$ (Considering approximately) as not given

$h = 30.35$ cm $= 0.3035$ m

$\therefore \quad 30.35 = \frac{wL_1^2}{8P}$

$$\Rightarrow \quad w = \frac{0.3035 \times 8 \times 100}{30 \times 30}$$

$= 0.269$ N/m. Choice (B)

14. In the first observation

$30 = K \times S + C$

$\therefore \quad 30 = K \times (0.176) + C$ ——— (1)

In the second observation

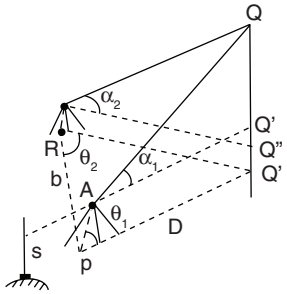
$100 = K \times (0.892) + C$ ——— (2)

Subtracting equations (2) – (1)

$K(0.892 - 0.176) = 100 - 30$

$K = 97.7$ Choice (B)

15.



$b = 50$ m, $\theta_1 = 58°\,18'$

$\theta_2 = 50°30'$

$\alpha_1 = 10°12'$

$\alpha_2 = 10°48'$

S = staff readings.

$$PQ_1 = D = \frac{b\sin\theta_2}{\sin(\theta_1 + \theta_2)}$$

$$= \frac{50 \times \sin(50^o30')}{\sin(50^o30' + 58^o18')} = 40.75 \text{ m}$$

$h_1 = D\tan\alpha_1 = (40.75)\tan(10°\,12^1) = 7.33$m

RL of Q = ($R.L$ of instrument axis at P) + h_1

= ($R.L$ of $BM + s$) + h_1

= 485.065 + 1.826 + 7.33

= 494.22 m. Choice (C)

19. The apex distance $= R\left(\sec\frac{\Delta}{2} - 1\right)$

$$= 300\left(\sec\frac{55}{2} - 1\right) = 38.21 \text{ m.}$$ Choice (B)

20. Length of mid ordinate $= R\left(1 - \cos\frac{\Delta}{2}\right)$

$$= 300\left(1 - \cos\frac{55}{2}\right)$$

$= 33.89$ m. Choice (C)

21. $A = \frac{d}{3}\begin{bmatrix}(0_1+0_n)+4(0_2+0_4+....+0_{n-1})\\+2(0_3+0_5+....0_{n-2})\end{bmatrix}$

$= \frac{30}{5}[(23+22)+4(40+30+60+14)+2(42+32+10)]$

$= 4734\ m^2$. Choice (D)

22. Collimation error $= \frac{0.003}{150} \times 1150 = 0.023$ m

Correction for collimation $= -0.023$ m. Choice (C)

23. Correction for refraction

Error due to curvature $(C_c) = 0.0785d^2 = 0.0785(1.15)^2$

$C_c = 0.104$ m

Correction for curvature $C_c = -0.104$ m

Combined correction for curvature and refraction

$= -0.104 + C_R$

Total correction $= -0.023 - 0.104 + C_R$

Corrected staff reading at B

$= 2.768 - 0.023 - 0.104 + C_R$

$= 2.641 + C_R$

Incorrect level difference between A and B at A

$= 2.768 - 1.286 = 1.482$ M

Incorrect level difference between A and B at B

$= 2.432 - 1.292 = 1.14$ m

True difference of level between A and B

$= \frac{1.482+1.14}{2}$ 1.311 m

$2.641 + C_R = 1.311$

$C_R = -1.33$ m. Choice (A)

24. If instrument is at A

Apparent difference of level $= 2.125 - 1.342 = (0.783)$

If instrument is at B

Apparent difference of level $= 1.683 - 1.485 = (0.198)$

True level difference $= \frac{0.783+0.198}{2} = 0.490$

Collimation error when instrument is at B

Correct reading on $B = 1.683$

Correct reading on $A = 1.683 - 0.490 = 1.193$m

The amount of inclination $= 1.485 - 1.193 = 0.292$m

Inclination of line of collimation

$\tan\theta = \frac{0.292}{100} = 2.93\times10^3$

$\theta = 10'\ 2.29''$. Choice (C)

25. Case 1:– $K = 100$, $C = 0$

$D = KS\cos^2\theta + C\cos\theta$

$= [100 \times 1.763 \times \cos^2(10° 46^1)] + [0 \times \cos(10° 46^1]$

$= 170.14$ m

Case 2:– $K = 100$, $C = 0.5$

$D = \text{KScos}^2\theta + C\cos\theta$

$= [100 \times 1.763 \times \cos^2(10° 46')]$

$+ [0.5 \times \cos(10° 46')] = 170.63$ m

Percentage error $= \frac{170.63-170.14}{170.14} \times 100$

$= 0.287\%$. Choice (A)

26. Area of quadrilateral $ABCD$

Let $y \rightarrow$ North, $x \rightarrow$ East Area

Area $A = \frac{1}{2}\begin{bmatrix}y_1(x_2-x_4)+y_2(x_3-x_1)\\+y_3(x_4-x_2)+y_4(x_1-x_3)\end{bmatrix}$

$\frac{1}{2}\begin{bmatrix}0(0-10234)-8428\times(600.1-0)\\+7428(10234-1)+659.3(0.600.1)\end{bmatrix}$

$= |-70614.345\ m^2|$

$= 7.0614 \times 10^4 m^2 = 7.06$ hectars Choice (B)

27. Magnetic meridian through various stations are not parallel, so donot converge at poles. Choice (D)

28. Scale is 1 cm = 20 cm

Representative fraction $K = \frac{1}{2000}$

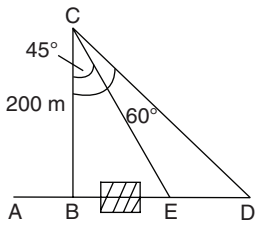
Displacement of the plotted position

$e = 20 \times \frac{1}{2000} = 0.01$ cm < 0.025 cm

As the permissible error in plane table survey is 0.025 cm. The above error is not much significant.

Choice (C)

29.



From ΔBCD

$CD = BC \sec 60° = 400 \sec 60° = 800$ m

From ΔBCE

$CE = BC \sec 45° = 400 \sec 45° = 565.6$ m Choice (C)

PART IV MOCK TESTS

Mock Test 1 ..... 4.3
Section – I General Aptitude 4.3
Section – II Civil Engineering 4.4
Mock Test 2 ..... 4.16
Section – I: General Aptitude 4.16
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Mock Test 3 ..... 4.29
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Mock Test 4 ..... 4.42
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Mock Test 1

Number of Questions: 65 **Total Marks: 100**

Section – I: General Aptitude

Questions 1 to 5 carry One Mark each.

Directions for questions 1 and 2: Select the correct alternative from the given choices.

1. $P = \{4, 6, 8, 10\}$
$Q = \{8, 10, 12, 14, 16\}$
Two integers were randomly selected such that one was selected from set P and the other was selected from set Q. What is the probability of the sum of these integers not being equal to 18?
(A) 0.67 (B) 0.70
(C) 0.75 (D) 0.80

2. In a certain code language, if the word SPRING is coded as TSWPWR, then how is the word ELASTIC coded in that language?
(A) FMBTUJD
(B) FOFZEVT
(C) FOFZTCZ
(D) FOFZCTP

Directions for question 3 and 4: Choose the word from the options given below, that is most nearly **similar** in meaning to the given word.

3. REIMBURSE
(A) Offset (B) Outshine
(C) Emanate (D) Indemnify

4. RAUCOUS
(A) Hoarse (B) Risky
(C) Phonetic (D) Bitter

Directions for question 5: Fill in the blank with the correct idiom/phrase.

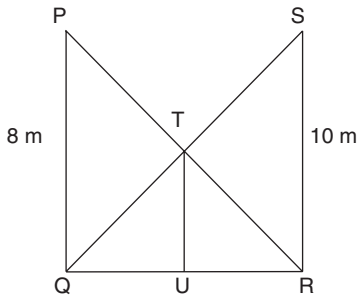
5. His association with bad companions got him _____.
(A) a piece of cake
(B) into hot water
(C) back to the drawing board
(D) beating around the bush

Questions 6 to 10 carry Two Marks each.

Directions for questions 6 to 8: Select the correct alternative from the given choices.

6. A right angled triangle GHI is to be formed in a rectangular coordinate system such that the triangle is right angled at H and HI is parallel to the y-axis. It is also intended for the x-and the y-coordinates of G, H and I to satisfy the inequalities $-8 \le x \le 8$ and $7 \le y \le 15$. How many triangles can be formed such that the above conditions are satisfied?
(A) 78 (B) 1224
(C) 19584 (D) 20566

7.



In the figure above, angle PQR = angle QRS = angle $TUR = 90°$, $PQ = 8$ m and $SR = 10$ m. Find TU (in m).
(A) $\frac{29}{9}$ (B) $\frac{31}{9}$
(C) $\frac{40}{9}$ (D) $\frac{34}{9}$

8. There are few pens in one box, few pencils in another box and a mixture of pens and pencils in a third box. But each of these boxes is wrongly labeled and the labels on the boxes are 'Pens' 'Pencils' and 'Mixture'. You are allowed to select only one box and only one item from that box. Which labeled box would you select to determine what each box contains?
(A) Pens
(B) Pencils
(C) Mixture
(D) Any one of the above

Directions for question 9: Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence.

9. The child had a _____ birth defect in the form of a cleft palate.
(A) hereditary (B) compulsive
(C) congenial (D) congenital

10. Mr. Dutta, a politician, said that he had visited England and he wanted to float the first green party in India. He said his greens were going to replace red in Bengal.
Which of the following can be inferred from the sentence above?
I. There is a green party in England.
II. Green party is not the name of a party.
III. The green party is going to be floated in Bengal.
(A) All follow
(B) Only I and III follow
(C) Only III follows
(D) Only II and III follow

SECTION – II: CIVIL ENGINEERING

Direction for questions 11 to 65: Select the correct alternative form the given choices.

Questions 11 to 35 carry One Mark each.

11. The length of the curve $x = y\sqrt{y}$ from (0, 0) to (8, 4) is _____
(A) 8.06 (B) 9.07
(C) 10.07 (D) 7.08

12. If 2, – 4 and 7 are the eigenvalues of a 3 × 3 square matrix A, then the rank of A is _____
(A) 3 (B) 2
(C) 1 (D) None of these

13. If the inverse Laplace transform of a function $F(s)$ is $L^{-1}[F(s)] = t^2 + 3t - 2\sin t$, then $L^{-1}[s\,F(s)]$ is _____.
(A) $t + 3 - \frac{2}{t}\sin t$
(B) $\frac{t^3}{3} + \frac{3}{2}t^2 + 2\cos t$
(C) $t^3 + 3t^2 - 2t\sin t$
(D) $2t + 3 - 2\cos t$

14. For larger values of the degrees of freedom υ, x^2 – distribution can be approximated to _____
(A) Poisson distribution
(B) Normal distribution
(C) F – distribution
(D) Exponential distribution

15. The order and degree of the partial differential equation $\frac{\partial^2 u}{\partial t^2} - 3\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + 4\frac{\partial^3 u}{\partial x \partial t^2} - 6\frac{\partial u}{\partial x} + 4u^2 = 0$ respectively are _____
(A) 2 and 1 (B) 2 and 2
(C) 3 and 1 (D) 3 and 2

16. The shear test that is more suitable in the field is _____.
(A) Unconfined compression test.
(B) Tri axial shear test.
(C) Box shear test.
(D) Vane shear test.

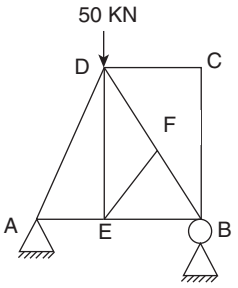
17. The camber provided on a sloping road is 1 in 28. Which of the following is ruling gradient?
(A) 1 in 15 (B) 1 in 14
(C) 1 in 41 (D) 1 in 28

18. EMD in contracts refer to
(A) Earlier Money Draft
(B) Earnest Money Draft
(C) Emergency Money Deposit
(D) Earnest Money Deposit

19. The return period for annual maximum flood of a given magnitude is 10 years. The probability that this flood will be exceeded once during next 5 years is
(A) 0.402 (B) 0.483
(C) 0.409 (D) 0.408

20. MPN index is a measure of _____.
(A) Hardness. (B) Turbidity.
(C) Coliform bacteria. (D) Sludge.

21.



The number of zero force members in the given truss is _____.
(A) 3 (B) 4
(C) 5 (D) 6

22. Shear force in a beam under load is
(A) Rate of change of loading
(B) Rate of change of bending moment
(C) Rate of change of slope
(D) Rate of change of deflection

23. In a two dimensional incompressible flow, x-component of velocity $u = y^2 + 4xy$. If y component of velocity $v = 0$ at $y = 0$ expression for v is
(A) $2xy$ (B) $-2y^2$
(C) $2y^2$ (D) $4y$

24. The target mean strength of concrete mix is give by (follow usual notations)
(A) $fck + s$ (B) $k.fck + s$
(C) $fck + ks$ (D) $k(fck + s)$

25. Match the List-I with List-II

	List–I (Type of transition curves)		List–II (Characteristics)
a.	Glover's spiral	1.	An autogenous curve of automobile
b.	Cubic spiral	2.	Radius of curve at any point varies inversely as the distance from the beginning of curve.
c.	Froude's transition curve	3.	$\sin\phi = \phi$
d.	Bernoulli's lemniscates	4.	X = L, $\cos\phi$=1

Codes:

	a	b	c	d		a	b	c	d
(A)	3	4	2	1	(B)	2	3	4	1
(C)	4	3	2	1	(D)	1	2	3	4

26. A column is fixed at the bottom and free at the top end. If L is the length of the column, EI is the flexural rigidity, Euler's critical load for the column is

(A) $\frac{\pi^2 EI}{4L^2}$ (B) $\frac{\pi^2 EI}{L^2}$

(C) $\frac{2\pi^2 E}{L^2}$ (D) $\frac{4\pi^2 EI}{L^2}$

27. The minimum area of tension reinforcement in a beam shall be greater than which of the following for the grade of Fe415 steel

(A) 0.002 bd (B) 0.0002 bd

(C) 0.003 bd (D) 0.0003 bd

28. The expression for specific speeds of a hydraulic turbine and a pump respectively are

(A) $\frac{N\sqrt{Q}}{H^{\frac{5}{4}}}, \frac{N\sqrt{P}}{H^{\frac{5}{4}}}$ (B) $\frac{N\sqrt{P}}{H^{\frac{3}{4}}}, \frac{N\sqrt{Q}}{H^{\frac{5}{4}}}$

(C) $\frac{N\sqrt{P}}{H^{\frac{5}{4}}}, \frac{N\sqrt{Q}}{H^{\frac{3}{4}}}$ (D) $\frac{N\sqrt{Q}}{H^{\frac{3}{4}}}, \frac{N\sqrt{P}}{H^{\frac{5}{4}}}$

29. The instrument used to measure contour interval is called

(A) Roughometer

(B) Gradienmeter

(C) Auto level

(D) Thacheometer

30. The phenomenon of salts coming in solution and forming a thin crust on surface of soil after evaporating of water is called _____.

(A) white alkali

(B) saline soil

(C) black alkali

(D) red alkali

31. The ultimate BOD of the waste water, whose 5 – day BOD and rate constant (base e) are respectively 180 mg/l and 0.23/day, is

(A) 164 mg/l (B) 204 mg/l

(C) 224 mg/l (D) 264 mg/l

32. In case of passive earth pressure, what would be the angle (in degrees) of failure plan with respect to horizontal plane (Take ϕ of the soil as 30°) _____.

(A) 30° (B) 45°

(C) 60° (D) 90°

33. For the soil having specific gravity 2.65 and void ratio as 0.4; the critical gradient would be _____.

(A) 1.89 (B) 1.18

(C) 1.03 (D) 1.65

34. Generally, fatigue life of welded steel structure to fatigue life of riveted steel structure ratio is

(A) smaller than 1

(B) equal to 1

(C) greater than 1

(D) greater than 2.1

35. The length of the runway under standard conditions is 1480. The airport site has on elevation of 260 m. Its reference temperature is 32°C. Assuming all over conditions to be under standard level then correction for elevation is _____.

(A) 1400 m (B) 1470 m

(C) 1500 m (D) 1570 m

Questions 36 to 65 carry Two Marks each.

36. The value of $\oint_C \left(xydy - \frac{3}{2} y^2 dx \right)$, C being the boundary of the region enclosed by the circle $x^2 + y^2 = 9$ and the coordinate axes in the first quadrant is _____

(A) 9 (B) 18

(C) 27 (D) 36

37. If the characteristic equation of a 3 × 3 matrix A is $\lambda^3 - 3\lambda^2 + \lambda - 3 = 0$, then the determinant of the matrix $B = 2A^7 - 6A^6 + 2A^5 - 6A^4 - 3A^3 + 7A^2 - 3A + 9I_3$ is _____

(A) 72 (B) –72

(C) 36 (D) –36

38. The Taylor's series expansion of $f(x) = x^3 - 5x^2 + 6x - 2$ about $x = -1$ is _____

(A) $(x-1)^3 - 8(x-1)^2 + 19(x-1) - 14$

(B) $(x+1)^3 + 16(x+1)^2 - 19(x+1) + 14$

(C) $(x+1)^3 - 8(x+1)^2 + 19(x+1) - 14$

(D) $(x-1)^3 + 16(x-1)^2 - 19(x-1) + 14$

39. In a binomial experiment, the chances of getting success and failure are in the ratio 1:2, then the probability of getting the 5th success in the 8th trial is _____

(A) $\frac{280}{2187}$ (B) $\frac{140}{6561}$

(C) $\frac{280}{6561}$ (D) $\frac{140}{2187}$

40. Consider the following table of values of x and $f(x)$

x	–1	0	3	6	7
f(x)	3	–6	39	822	1611

The 4th divided difference value is _____

(A) 1 (B) 13

(C) 132 (D) 789

41. A simply supported pre stressed concrete beam of 6m long, 300mm wide and 600mm of depth is pre stressed by horizontal cable tensioned at eccentricity of 100mm. If the pre stressing tensile force in the cable tendon is given as 1000 KN, what will be the maximum and minimum normal stress developed in beam at transfer?

(A) 11.11Mpa, 5.55Mpa

(B) 11.11Mpa, 0

(C) 0, 5.5Mpa

(D) 11.11Mpa, 11.11Mpa.

42. The following are the rates of rainfall of successive 20min period of a 150min storm 5.0, 4.0, 11.0, 8.5, 1.45, 1.85, 7.0 cm/hr. Taking the value of ϕ - index as 4.5 cm/hr, find the net runoff in cm.

(A) 2.3 cm (B) 6.4 cm
(C) 4.82 cm (D) 4.51 cm

43. Chose the correct pairs from the following

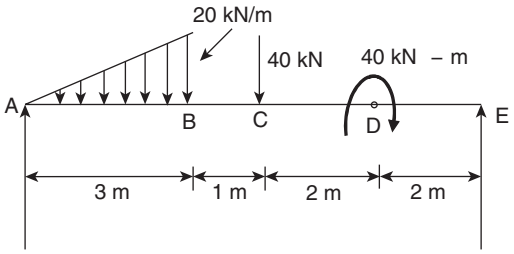
	Group–I		Group–II
1.	HC	a.	Primary air pollutant
2.	O_3	b.	Secondary pollutant
3.	PAN		
4.	NO_x		

(A) 1 – b, 2 – b, 3 – a, 4 – a
(B) 1 – b, 2 – a, 3 – a, 4 – b
(C) 1 – a, 2 – b, 3 – b, 4 – a
(D) 1 – a, 2 – a, 3 – b, 4 – b

44. A soil has a bulk density of 2.05 g/cc and water content of 21%. If G = 2.64, the degree of saturation of the soil is _____.

(A) 50% (B) 75%
(C) 100% (D) 0%

45.



A simply supported beam is loaded as shown above. Maximum bending at point *D* is

(A) 25 kNm
(B) 35 kNm
(C) 65 kNm
(D) 45 kNm

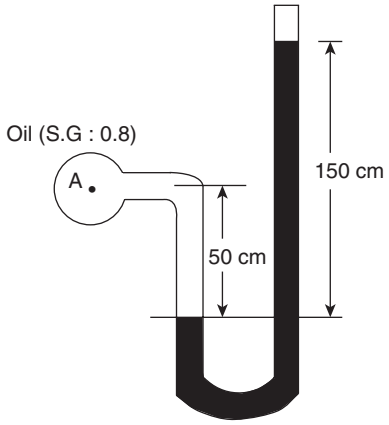
46. Match List-I with List-II and select correct answer using the codes given

	List–I		List–II
a.	Pelton turbine (single jet)	1.	Medium discharge, low head
b.	Francis turbine	2.	High discharge, low head
c.	Kaplan turbine	3.	Medium discharge, medium head
		4.	Low discharge, high head

Codes:

	a	b	c
(A)	1	2	3
(B)	4	3	2
(C)	2	3	4
(D)	3	4	1

47. A U-tube mercury manometer is used to measure pressure of oil flowing through a pipe at a point. Specific gravity of oil is 0.8 and the level of mercury is as shown in the figure. The pressure in kPa is



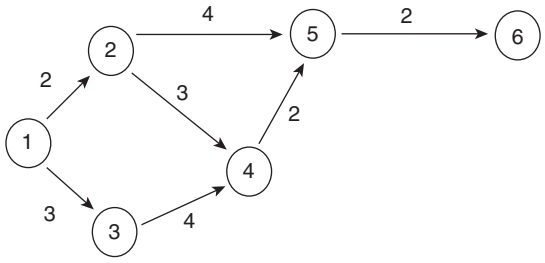
(A) 196.20
(B) 147.15
(C) 110.36
(D) 73.58

48. The development length of a deformed reinforced bar under compression can be expressed as $K.\left(\frac{\phi.\sigma_s}{4\tau_{bd}}\right)$.

From the IS 456:2000, the value of *K* can be calculated as _____.

(A) 2 (B) 0.75
(C) 0.5 (D) 0.125

49.



Critical path of the give network is _____.

(A) 1 - 2 - 4 – 5- 6
(B) 1 - 2 - 5 - 6
(C) 1 - 3 - 4 - 5 - 6
(D) None of the above

50. During a leveling work along a falling gradient using a dumpy level and a staff of 3m length, following successive readings were taken:-

1.745, 2.765, 0.243, 1.432. What will be the correct order of booking these four readings in level book (*BS*: Back Sight, *IS*: Intermediate Sight, *FS*: Fore Sight).

(A) *BS*, *FS*, *BS*, *FS*
(B) *BS*, *IS*, *FS*, *FS*
(C) *BS*, *IS*, *IS*, *FS*
(D) *BS*, *IS*, *BS*, *FS*.

51.

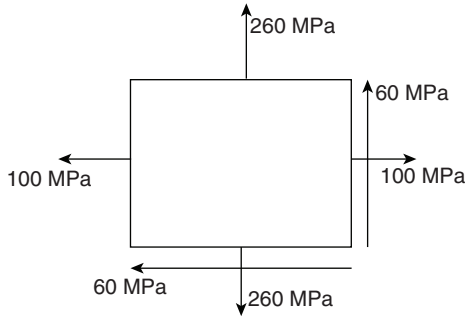


Figure shows state of stress at a point in a stressed body. Radius of Mohr's circle representing the state of stress is

(A) 60 (B) 80
(C) 120 (D) 100

52. A single lane unidirectional highway has design speed of 55 kmph. The perception brake reaction time of driver's is 2.5 seconds and average length of vehicles is 6m. The capacity of this road in terms of kmph is.

(A) 604 veh/hr/lane (B) 743 veh/hr/lane
(C) 402 veh/hr/lane (D) 516 veh/hr/lane

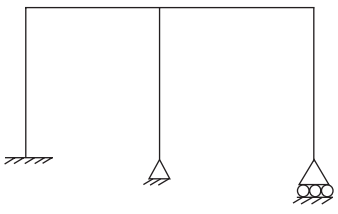
53. If the actual observed value of standard penetration resistance, N in a fine sand layer below water table is 25. What will be the equivalent penetration resistance after correction _____.

(A) 10 (B) 15
(C) 20 (D) 25

54. A water treatment plant of capacity 5 m^3/s has filter boxes of dimensions 8m × 15m. Loading rate to the filter is 240 $m^3/day/ m^2$. When three of the filters are out of service for back washing, the loading rate (in $m^3/day/ m^2$) is _____.

(A) 200 (B) 300
(C) 400 (D) 500

55. The static and kinematic indeterminacy of the frame shown below is _____.



(A) 3 and 3 (B) 6 and 9
(C) 6 and 7 (D) 3 and 7

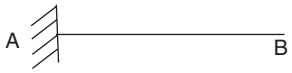
56. If the wheel base of a train moving on broad gauge track is 5.5m, the diameter of wheel base is 1.4m and depth of flanges below the top of rail is 3.2cm. The extra width of provided gauge, if radius of curve is 100m is _____.

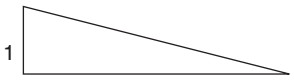
(A) 4.6 cm (B) 5.6 cm
(C) 6.6 cm (D) 7.2 cm

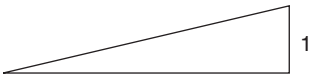
57. In a certain situation, waste water is discharged into a river mixes with the river water instantaneously and completely. Following is the data available.
Waste water DO = 2mg/l
Discharge rate = 1.5 m^3/sec
River water DO = 8.4 mg/l
Flow rate = 8.8 m^3/sec
Temperature = 20°C
What will be the initial amount of DO in the mixture of waste and river water?

(A) 7.39 mg/l
(B) 7.47 mg/l
(C) 5.29 mg/s
(D) 7.29 mg/l

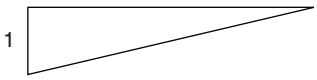
58. For a given beam AB, ILD for vertical reaction at fixed end A is _____.



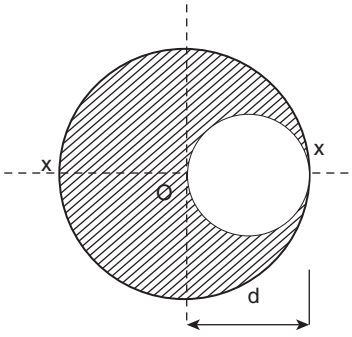
(A) 

(B) 

(C) 

(D) 

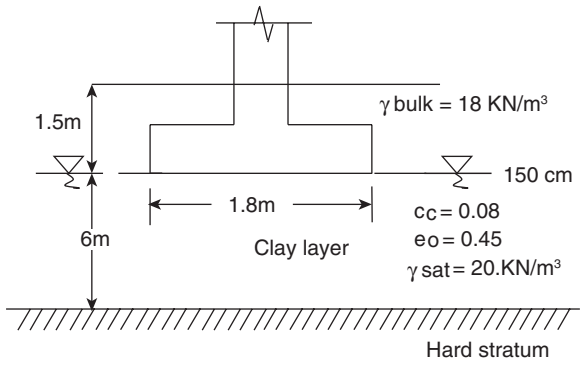
59.



Centroid of the shaded area shown in figure, is at a distance of _____ from O

(A) $\frac{d}{8}$ (B) $\frac{d}{6}$
(C) $\frac{d}{3}$ (D) $\frac{d}{2}$

60. Figure shows the geometry of a strip footing supporting the load bearing walls of a multi storied building and the properties of clay layer.
(Take γ_w = 10 KN/m^3)



If the pressure acting on the footing is 40 Kpa, the consolidation settlement of the footing will be _____

(A) 7.64 mm
(B) 76.4 mm
(C) 54.38 mm
(D) 5.43 mm

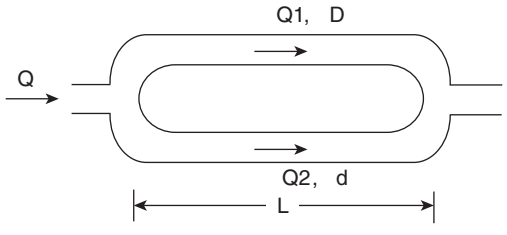
61. The gross commanded area for a distributary is 8000 hectares, 70% of which culturable irrigable. The intensity of irrigation for karif season is 35% and that for rabi season is 60%. If average duty of head of distributary is 800 hectares/cumec for karif season and 1500 hectares/cumec for rabi season, find the discharge required for design at head of distributary.

(A) 2.24 m^3/sec
(B) 2.45 m^3/sec
(C) 2.52 m^3/sec
(D) 2.25 m^3/sec

62. A non-homogeneous soil deposit consists of a clay layer sandwiched between a fine sand layer at top and a silt layer below. Permeability of the silt layer is given as five times the permeability of clay layer and $\frac{1}{10}$th of the sand layer. Thickness of sand layer is 2 times the thickness of the silt layer and half of the thickness of clay layer. Then, what will be the ratio of equivalent horizontal and vertical permeability of the deposit ____ (Round up to nearest integer)

(A) 5 (B) 10
(C) 15 (D) 20

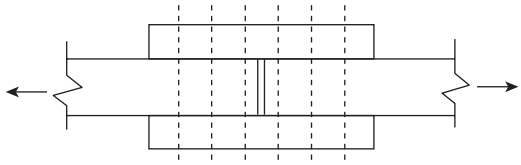
63.



Two pipes of equal length L are connected in parallel. Rate of flow through pipe of diameter D is Q_1 and pipe of diameter d is Q_2. If $d = 0.5\ D$, the ratio of $\frac{Q_1}{Q_2}$ is equal to

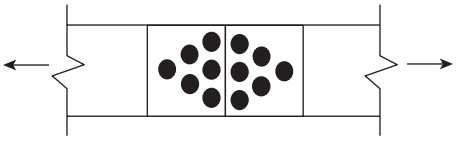
(A) 3.462 (B) 4.683
(C) 5.657 (D) 6.448

64. Identify the most effective but joint (with double cover plates) for a plate in tension from the patterns (plan view) shown below, each comprising 6 identical bolts with the same pitch and gauge:

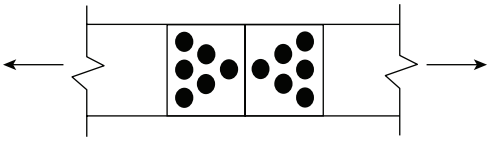


Common Elevation
(all plates have same thickness)

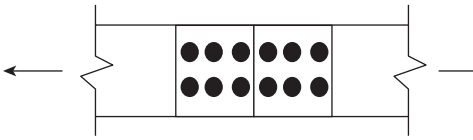
(A)



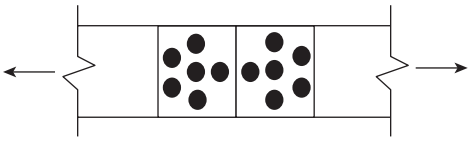
(B)



(C)



(D)



65. The following data pertains to number of commercial vehicles per day for the design flexible pavement for national highway as per *IRC*: 37-1984.

Type of commercial no. of vehicles per day	Vehicles considering no. of lanes	Vehicles damaging factor
Two axle trucks	3000	7
Tanden axle trucks	400	8

Assuming a traffic growth factor of 7.5% per annum for both the types of vehicles, the cumulative number of standard axle load repetitions for a design life of ten years is

(A) 13.42 msa
(B) 14.26 msa
(C) 12.49 msa
(D) 15.3 msa

Answer Keys

1. D	**2.** D	**3.** D	**4.** A	**5.** B	**6.** C	**7.** C	**8.** C	**9.** D	**10.** A
11. B	**12.** A	**13.** D	**14.** B	**15.** C	**16.** D	**17.** B	**18.** D	**19.** C	**20.** C
21. B	**22.** B	**23.** B	**24.** C	**25.** B	**26.** A	**27.** A	**28.** C	**29.** B	**30.** A
31. D	**32.** A	**33.** B	**34.** A	**35.** D	**36.** D	**37.** B	**38.** C	**39.** C	**40.** A
41. B	**42.** D	**43.** C	**44.** C	**45.** C	**46.** B	**47.** A	**48.** C	**49.** C	**50.** C
51. D	**52.** B	**53.** C	**54.** B	**55.** D	**56.** A	**57.** B	**58.** C	**59.** B	**60.** B
61. B	**62.** B	**63.** C	**64.** A	**65.** C					

Hints and Explanations

1. Probability (The sum not being equal to 18) = 1 – Probability (The sum being equal to 18)
The number of ways of selecting two integers one from each set is $4 \times 5 = 20$.
The sum can be 18 when the integers selected are 4 and 14, 6 and 12, 8 and 10, 10 and 8.

$\therefore$ Probability that the sum being $18 = \frac{4}{20}$

$\therefore$ Required probability $= 1 - \frac{4}{4 \times 5} = 0.8$

Choice (D)

2. Word: *S P R I N G*
Logic: +1 +3 +5 +7 +9 + 11
Code: *T S W P W R*
Similarly,
Word: *E L A S T I C*
Logic: +1 +3 +5 +7 +9 +11 +13
Code: *F O F Z C T P* Choice (D)

3. To 'reimburse' is to pay back the money spent. To 'indemnify' is also to pay back money (for some loss or damage). 'Offset' is an amount that diminishes or balances the effect of an opposite one. It does not mean a guarantee, as does 'indemnity'. Choice (D)

4. 'Raucous' is harsh or 'hoarse'. Other synonyms are, grating, discordant, jarring or strident. Choice (A)

5. When we get into bad company we get into hot water or get into trouble. Other idioms do not work in the context. 'A piece of cake' is a job/task that is very easy while 'back to the drawing board' means a failed attempt that has to be started again. To 'beat around the bush' is to avoid speaking openly/directly about an issue. Choice (B)

6. *HI* is parallel to the *y*-axis. Any line parallel to the *y*-axis must have its equation of the form $x = a$ constant.
$\therefore$ *HI* can be any of the lines $x = -8$ or $x = -7$ or …. $x = -1$ or $x = 0$ or $x = 1$ or ….. $x = 8$. ($\because -8 \le x \le 8$, $7 \le y \le 15$).
Suppose *HI* is the line $x = 8$. Then *H* can be any of (8, 7), (8, 8), …… (8, 15). $\therefore$ *H* has 9 possible positions. For each of these possible positions *I* can have any of the remaining 8 possible positions.
$\therefore$ *HI* has $9 \times 8 = 72$ possibilities.
As explained above, it similarly follows that when *HI* is the line $x = 7$ or $x = 6$ or ….. $x = -8$, *HI* has 72 possibilities in each case.
Total number of possibilities for *HI* = (72) (17) = 1224
The triangle is right angled at *H*.
$\therefore$ The *y*-coordinate of *G* must be the same as that of *H* and its *x*-coordinate can be any possible value other than that of *H*.
$\therefore$ The *x*-coordinate of *G* has 17 – 1 = 16 possibilities.
$\therefore$ *G* has 16 possible positions.
From (1) and (2)
$\therefore$ The triangle *GHI* has 1224 × 16 = 19584 possible arrangements i.e., 19584 triangles can be formed satisfying the given conditions.
Alternate Solution:
Given $-8 \le x \le 8$ and $7 \le y \le 15$ i.e. there are 17 vertical lines and 9 horizontal lines. The number of rectangles formed with these lines is ${}^{17}C_2 \times {}^{9}C_2$.
We know that one rectangle gives 4 right angled triangles.
$\therefore$ Total number of right angled triangles formed is $4 \times {}^{17}C_2 \times {}^{9}C_2 = 19584$. Choice (C)

7. In ΔTUR and ΔPQR,
$\angle R$ is common.
$\angle TUR = \angle PQR = 90°$
The above conclusions mean the third pair of angles of both Δ's must be equal. The third angle of each Δ = 180° − (sum of the other two of its angles).
$\therefore$ $\Delta TUR \sim \Delta PQR$.

$\therefore$ $\frac{TU}{PQ} = \frac{UR}{QR}$ ---------- (1)

Similarly $\Delta TUQ \equiv \Delta SRQ$.

$\therefore$ $\frac{TU}{SR} = \frac{QU}{QR}$ --------- (2)

$\frac{(1)}{(2)} \Rightarrow \frac{SR}{PQ} = \frac{UR}{QU}$

$\therefore$ $\frac{QU}{UR} = \frac{8}{10} = \frac{4}{5}$

$$\Rightarrow \quad 1+\frac{QU}{UR}=1+\frac{4}{5}$$

$$\Rightarrow \quad \frac{UR+QU}{UR}=\frac{9}{5}$$

$$\Rightarrow \quad \frac{QR}{UR}=\frac{9}{5}$$

$$\frac{UR}{QR}=\frac{5}{9}$$

From (1) $\Rightarrow \frac{TU}{8}=\frac{5}{9}$

$$TU=\frac{40}{9}\text{m}$$ Choice (C)

8. We have to select the box that is labeled as mixture. Now if we get a pen, as the box cannot have a mixture, it has pens. Now the box which is labeled as pens cannot have mixture. [$\because$If it happens then the box with label pencil must contain pencils]
$\Rightarrow$ The box with label pens has pencils and that with pencils has a mixture of them. Choice (C)

9. Birth defects are congenital (present from birth) and not hereditary, compulsive, or congenial (affable; friendly). Choice (D)

10. Mr. Dutta has decided to float the first green party in India after visiting England. It is clear that there is a green party in England and Mr. Dutta is impressed with the party. Assume the case where Mr. Dutta learnt about the green party while in England but there is no green party in England; in such a case, there would not be any relevance to the visit to England. Hence I follows.
According to the statement, Mr. Dutta wants to float the first green party. It implies that there can be several green parties. There cannot be more than one party with the same name. Hence, II follows.
Mr. Dutta says his green would replace the reds in Bengal. From this, it can be concluded that the green party in going to be floated in Bengal as well. Hence, III too follows.
Therefore all follow. Choice (A)

11. Given curve is $x = y\sqrt{y}=y^{\frac{3}{2}}$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{3}{2}y^{\frac{1}{2}}$$

$\therefore$ The length of the curve $x = y\sqrt{y}$ from (0, 0) to

$$(8,4)=\int_{y=0}^{4}\sqrt{\left(1+\left(\frac{dx}{dy}\right)^2\right)}dy$$

$$=\int_{y=0}^{4}\sqrt{\left(1+\left(\frac{3}{2}y^{\frac{1}{2}}\right)^2\right)}dy$$

$$=\int_{0}^{4}\sqrt{\left(1+\frac{9}{4}y\right)}dy$$

$$=\frac{2}{3}\left(1+\frac{9}{4}y\right)^{\frac{3}{2}}\times\frac{4}{9}\Big]_0^4$$

$$=\frac{8}{27}\left(1+\frac{9}{4}\times4\right)^{\frac{3}{2}}-\frac{8}{27}\left(1+\frac{9}{4}\times0\right)^{\frac{3}{2}}$$

$= 9.0734.$ Choice (B)

12. Since none of the eigenvalues of A are zero, A is a non-singular matrix.
$\therefore$ The rank of A = The order of $A = 3$. Choice (A)

13. Given $L^{-1}[F(s)] = t^2 + 3t - 2\sin t = f(t)$, (say)
$\therefore \quad f(0) = 0^2 + 3 \times 0 - 2 \sin 0 = 0$
We know that, if $L^{-1}[F(s)] = f(t)$ and
$f(0) = 0$, then $L^{-1}[s\,F(s)] = f^1(t)$

$\therefore \quad L^{-1}[s\,F(s)] = f^1(t) = \frac{d}{dt}\,[t^2 + 3t - 2\sin t] = 2t + 3 - 2\cos t.$ Choice (D)

14. Standard Result. Choice (B)

15. For the partial differential equation

$$\frac{\partial^2 u}{\partial t^2}-3\left(\frac{\partial^2 u}{\partial x^2}\right)^2+4\frac{\partial^3 u}{\partial x\,\partial t^2}-6\frac{\partial u}{\partial x}+4u^2=0$$

Order = order of the highest ordered partial derivative = 3
Degree = Degree (power) of the highest ordered partial derivative = 1. Choice (C)

17. $G = 2C = 2\times\frac{1}{28}=\frac{1}{14}$ Choice (B)

19. $p^1 = 1 - q^n$

$$p=\frac{1}{T}=\frac{1}{10}=0.1$$

$q = 1 - 0.1 = 0.9$

$p=1-(0.9)^5=0.409$ Choice (C)

21. Zero force members are:
DC, BC, EF, DE Choice (B)

23. Continuity equation must be satisfied for the flow

i.e., $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

$u = y^2 + 4xy$

$$\frac{\partial u}{\partial x}=4y$$

To satisfy continuity equation

$$\frac{\partial v}{\partial y}=-4y$$

On integration, $v=-\frac{4y^2}{2}+C=-2y^2+C$

It is given that $v = 0$ at $y = 0$

$\therefore \quad C = 0$

$\therefore$ The expression for v is $v = -2y^2$ Choice (B)

26. Euler's critical load $P_{cr} = \dfrac{\pi^2 EI}{L_e^2}$

where L_e = effective length

For the given case, $L_e = 2L$

$\therefore \quad P_{Cr} = \dfrac{\pi^2 EI}{4L^2}$ Choice (A)

27. $\dfrac{As}{bd} = \dfrac{0.85}{fy}$

$\therefore \quad \text{As} = \dfrac{0.85}{415} \times bd = 0.002\ bd$ Choice (A)

31. $y_t = y_0(1 - e^{-kt})$

$y_5 = y_0\left(1 - e^{-(0.23\times 5)}\right)$

$180 = y_0(0.683)$

$\therefore \quad y_0\ 263.4$ mg/l

$y_0 = 263.4$ mg/l Choice (D)

32. Passive earth pressure case,

45 + ϕ/2 w.r.t. major principal plane (vertical)

45 – ϕ/2 w.r.t. minor principal plane (horizontal)

Choice (A)

33. $G = 2.65$ and $e = 0.4$

$i_c = \dfrac{G-1}{1+e} = \dfrac{1.65}{1.4} = 1.18$ Choice (B)

35. Correction for elevation

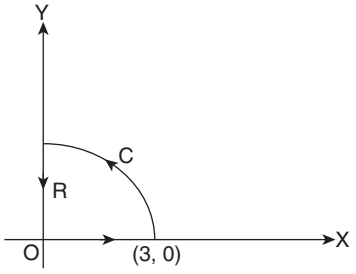
$= \dfrac{7}{100} \times 1480 \times \dfrac{260}{300} = 89.78\text{m}$

Corrected length = 1480 + 89.78m = 1569.7m

Choice (D)

36. We have $\oint_C \left(xy\,dy - \dfrac{3}{2}y^2 dx \right)$

$= \oint_C \left[\dfrac{-3}{2} y^2 dx + xy dy \right]$



By Green's theorem, we know that

$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy \quad \rightarrow (1)$$

Here $M = \dfrac{-3}{2} y^2$ and $N = xy$

$\therefore \quad \dfrac{\partial M}{\partial y} = -3y$ and $\dfrac{\partial N}{\partial x} = y$

In the region R,

y varies from $y = 0$ to $y = \sqrt{9 - x^2}$ and x varies from $x = 0$ to $x = 3$

$\therefore \quad \oint_C xy dy - \dfrac{3}{2} y^2 dx = \oint_C M\,dx + N\,dy$

$= \iint_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y} \right) dx\,dy$ [From (1)]

$= \iint_R \left(y - (-3y) \right) dx\,dy$

$= \int_{x=0}^{3} \left(\int_{y=0}^{\sqrt{9-x^2}} 4y\,dy \right) dx$

$= \int_{x=0}^{3} 2y^2 \Big]_{y=0}^{\sqrt{9-x^2}}\ dx$

$= \int_0^3 2\left(9 - x^2\right) dx$

$= 18x - \dfrac{2}{3}x^3 \Big]_0^3 = 36.$ Choice (D)

37. Given the characteristic equation of a 3×3 matrix A is $\lambda^3 - 3\lambda^2 + \lambda - 3 = 0$

$\therefore$ By Cayley – Hamilton theorem, we have $A^3 - 3A^2 + A - 3I_3 = O \rightarrow (1)$

Consider $B = 2A^7 - 6A^6 + 2A^5 - 6A^4 - 3A^3 + 7A^2 - 3A + 9I_3 = 2A^4(A^3 - 3A^2 + A - 3I_3) - 3(A^3 - 3A^2 + A - 3I_3) - 2A^2 = 2A^4 \times 0 - 3 \times 0 - 2A^2.$ (From (1))

$\therefore \quad B = -2A^2$

Now Det(B) = Det$(-2A^2)$

$= (-2)^3 |A^2|$ ($\because$ $|ka| = k^n |A|$, where n = order of A)

$= -8|A|^2$

($\because$ $|A^m| = |A|^m$ for any positive integer m)

$\therefore \quad |B| = -8|A|^2 \quad \rightarrow (2)$

We know that

$|A|$ = Product of the eigenvalues of $A = (-1)^n \times$ constant term in the characteristic equation of A (where n = order of A)

$= (-1)^3 \times (-3)$

$\therefore \quad |A| = 3$

Hence from (2), $|B| = -8 \times 3^2 = -72.$ Choice (B)

38. We have $f(x) = x^3 - 5x^2 + 6x - 2$

We know that the Taylor's series expansion of $f(x)$ about $x = a$ is

$$f(x) = f(a) + (x - a) f^1(a) + \frac{(x-a)^2}{2!} f^{11}(a) + \frac{(x-a)^3}{3!} f^{111}(a) + \ldots\ldots \infty \rightarrow (1)$$

Here $a = -1$

$f(x) = x^3 - 5x^2 + 6x - 2 \Rightarrow f(-1) = -14$

$f^1(x) = 3x^2 - 10x + 6 \Rightarrow f^1(-1) = 19$

$f^{11}(x) = 6x - 10 \Rightarrow f^{11}(-1) = -16$

$f^{111}(x) = 6 \Rightarrow f^{111}(-1) = 6$

And $f^{(IV)}(x) = f^{(V)}(x) = \ldots. = 0$

$\therefore$ From (1), the Taylor's series expansion of $f(x)$ about $x = -1$ is

$$f(x) = f(-1) + (x - (-1)) f^1(-1) + \frac{(x-(-1))^2}{2!} f^{11}(-1) + \frac{(x-(-1))^3}{3!} f^{111}(-1) + \ldots\ldots \infty$$

$$= -14 + (x + 1) \times 19 + \frac{(x+1)^2}{2!} \times (-16) + \frac{(x+1)^3}{3!} \times 6 + 0 + 0 + \ldots\ldots..$$

$\therefore$ $f(x) = (x + 1)^3 - 8(x + 1)^2 + 19(x + 1) - 14$

Alternative solution:

We know that the Taylor's series expansion of $f(x)$ about $x = -1$ is same as that of the Taylor's series expansion of $f(x)$ in powers of $x + 1$.

$\therefore$ $f(x) = x^3 - 5x^2 + 6x - 2$

$= [(x + 1) - 1]^3 - 5[(x + 1) - 1]^2 + 6[(x + 1) - 1] - 2$

$= [(x + 1)^3 - 3(x + 1)^2 + 3(x + 1) - 1] - 5[(x + 1)^2 - 2(x + 1) + 1] + 6(x + 1) - 6 - 2$

$= (x + 1)^3 - 8(x + 1)^2 + 19(x + 1) - 14$. Choice (C)

39. Given the ratio of success and failure in a binomial experiment is 1 : 2

$\therefore$ The probability of success $= p = \frac{1}{1+2} = \frac{1}{3}$

The probability of failure $= q = 1 - p = \frac{2}{3}$

Probability of getting the 5th success in the 8th trial = Probability of getting 4 successes in first 7 trails and a success in the 8th trial.

$$= \left({}^7C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^3 \right) \times \frac{1}{3}$$

$$= \frac{280}{3^8} = \frac{280}{6561}.$$ Choice (C)

40. Given pairs of values of x and $f(x)$ are

x	-1	0	3	6	7
f(x)	3	-6	39	822	1611

The divided difference table for this data is as shown below.

x	f(x)	1st divided differences	2nd divided differences	3rd divided differences	4th divided differences
-1	3	$\frac{-6-3}{0-(-1)} = -9$			
0	-6		$\frac{15-(-9)}{3-(-1)} = 6$		
		$\frac{39-(-6)}{3-0} = 15$		$\frac{41-6}{6-(-1)} = 5$	
3	39		$\frac{261-15}{6-0} = 41$		$\frac{13-5}{7-(-1)} = 1$
		$\frac{822-39}{6-3} = 261$		$\frac{132-41}{7-0} = 13$	
6	822		$\frac{789-261}{7-3} = 132$		
		$\frac{1611-822}{7-6} = 789$			
7	1,611				

$\therefore$ The 4th divided difference value for the given data is 1. Choice (A)

41. $\sigma = \frac{P}{A} \pm \frac{P.e}{z}$

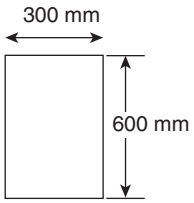
$z = \frac{bd^2}{6}$

$= \frac{300 \times 600^2}{6}$

$= 18 \times 10^6 \text{ mm}^3$

$\sigma = \frac{1000 \times 10^3}{300 \times 600} \pm \frac{1000 \times 10^3 \times 100}{18 \times 10^6} \text{Mpa}$

$= 5.55 \pm 5.55$



$\therefore \quad \sigma \max = 11.11$ Mpa and

$\sigma_{\min} = 0$ Mpa Choice (B)

42. The total rainfall is

$\left[5+4+11+8.5+1.45+1.85+7\right] \times \frac{20}{60} = 12.93 \text{cm}$

$\phi = 4.5 \text{cm/hr}$

$P_e = \left[5+11+8.5+7\right] \times \frac{20}{60} = 10.5 \text{cm}$

$\phi \text{ index} = \frac{P_e - R}{t_e}$

$t_e = \frac{4 \times 20}{60} = 1.33$

$4.5 = \frac{10.5 - R}{1.33}$

$R = 4.515$ cm Choice (D)

43. Primary pollutants: NO_x, HC
Secondary pollutants: O_3, PAN Choice (C)

44. $\gamma = 2.05$ g/cc
$w = 21\%$
$G = 2.64$

$\gamma = \frac{\gamma_w \left(G + e.s_\gamma\right)}{1+e}$

$2.05 = \frac{1\left(2.64 + e.s_\gamma\right)}{1+e}$

$e = \frac{wG}{s_\gamma}$

$\Rightarrow 2.05 = \frac{1\left(2.64 + wG\right)}{1 + \frac{wG}{s_\gamma}}$

$wG = 0.21 \times 2.64 = 0.55$

$S\gamma = 100\%$

(or)

$\gamma d = \frac{\gamma}{1+w} = \frac{2.05}{1+0.21} = 1.70$

$\gamma_d = \frac{\gamma_{wG}}{1+e}$

$1.70 = \frac{1 \times 2.64}{1+e}$

$e = 0.55$

$e = \frac{wG}{s_\gamma} \Rightarrow s_\gamma = \frac{0.21 \times 2.64}{0.55}$

$\therefore \quad S\gamma = 100\%$ Choice (C)

45. $R_A + R_E = \frac{20 \times 3}{2} + 40 = 70$ kN

Taking moment about E

$R_A \times 8 - \frac{20 \times 3}{2} \times 6 - 40 \times 4 + 40 = 0$

$R_A \times 8 = 180 + 160 - 40 = 300$
$R_A = 37.5$ kN
$R_E = 70 - 37.5 = 32.5$kN
Minimum bending moment at D
$= R_E \times 2 - 40 = 32.5 \times 2 - 40 = 25$ kNm
Maximum bending moment at D
$= R_E \times 2 = 32.5 \times 2 = 65$ kNm. Choice (C)

47. Equating pressure heads above the line passing through A

$h_A + \frac{50}{100} \times 0.8 = \frac{150}{100} \times 13.6$

$\therefore \quad h_A + 0.4 = 20.4$

$\Rightarrow \quad h_A = 20$ m of water

Pressure $p_A = wh_A = 9810 \times 20$ N/m^2
$= 9.81 \times 20$ kN/m$^2 = 196.2$ kN/m^2
$= 196.2$ kPa Choice (A)

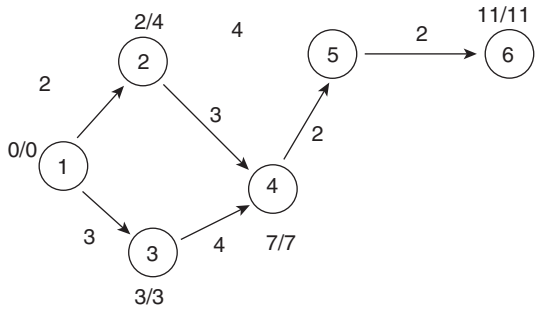
48. $Ld = \frac{\phi.\sigma_s}{4\tau_{bd}}$

For the deformed bars τ_{bd} can be increased by 60% and for bars under compression, τ_{bd} can further increased by 25% as per IS 456 : 2000.
So; τ_{bd} becomes $(1.6 \times 1.25)\tau_{bd}$

$\therefore \quad Ld = \frac{\phi}{4} \frac{\sigma_s}{2\tau_{bd}}$

$\Rightarrow \quad k = 0.5$ Choice (C)

49.



$\therefore$ Critical path = 1 – 3 – 4 – 5 – 6 Choice (C)

50. $BS = 1.745$
$IS = 2.765$
$IS = 0.243$
$FS = 1.432$ Choice (C)

51. Radius of Mohr's circle is $= \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$

$= \sqrt{\left(\frac{260-100}{2}\right)^2 + 60^2}$

$= \sqrt{80^2 + 60^2}$

$= \sqrt{6400 + 3600}$

$= \sqrt{1000} = 100.$ Choice (D)

52. $V = \frac{CS}{1000}$

$S = vt + \frac{v^2}{2gf}$

$= (0.278 \times 55 \times 2.5) + \frac{(0.278 \times 55)^2}{2 \times 9.8 \times 0.4}$

$= 68.04\text{m} = 68.04 + 6 = 74.04$ m

$C = \frac{1000 \times 55}{(68.01+6)} = 743$ veh/hr/lane Choice (B)

53. When $N_R > 15$ and fine sand present below the water table; diletancy correction should be applied.

Equivalent $N = 15 + \left(\frac{N_R - 15}{2}\right)$

$\therefore \quad N_{eq} = 15 + \left(\frac{25-15}{2}\right) = 20$ Choice (C)

54. Plant capacity = 5 m³/s = 5 × 24 × 60 × 60 m³/day/
= 43200 m³/day

Area of filter $= 8 \times 15 = 120$ m²

Surface loading rate = 240/m³/day/m²

$\Rightarrow$ Area required $= \frac{432000\,\text{m}^3/\text{day}}{240\ \text{m}^3/\text{day}/\text{m}^2} = 1800\text{m}^2$

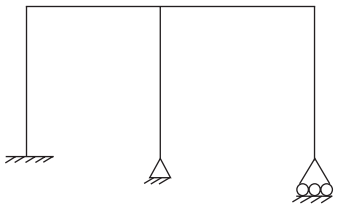
No. of filters $= \frac{1800}{120} = 15$

But, Given that 3 are out of service

$\therefore$ Surface loading rate $= \frac{43200\,\text{m}^3/\text{day}}{12 \times 120\ \text{m}^2}$

$= 300\text{m}^3/\text{day}/\text{m}^2$ Choice (B)

55.



$D_s = (3 + 2 + 1) - 3 = 3$
$D_k = (0 + 1 + 2 + 3 + 3 + 3) - \text{m}$
$= 12 - 5 = 7$ Choice (D)

56. $h = 3.2$cm
$D = 140$cm
$B = 5.5$m
$R = 100$m

$L = \sqrt{h^2 + D.h} = 0.02\sqrt{(3.2)^2 + (140 \times 3.2)} = 0.428$m

Extra width of gauge (d)

$d = \frac{13(B+L)^2}{R} = \frac{13(5.5+0.428)^2}{100}$

$= 4.568$cm Choice (A)

57. $\text{DO}_{\text{mix}} = \frac{Q_W.DO_W + Q_R.DO_R}{Q_W + Q_R}$

$= \frac{1.5(2) + 8.8(8.4)}{1.5 + 8.8} = 7.47$ mg/l Choice (B)

58. According to Muller-Breslau principle to find reaction at A, remove the constraint at A & apply unit force in the direction of reaction. But, here the other end B is not hinged, it is a free end. So, end B also lifted up along with end A when we apply unit reaction at A. Choice (C)

59. $\bar{x} = \frac{A_1 x_1 - A_2 x_2}{(A_1 - A_2)}$

$= \frac{\left[\frac{\pi(2d)^2}{4} \times 0\right] - \frac{\pi d^2}{4} \times \frac{d}{2}}{\frac{\pi(2d)^2}{4} - \frac{\pi d^2}{4}}$

$= \frac{-\frac{\pi d^3}{8}}{\frac{3\pi d^2}{4}} = -\frac{d}{6} = \frac{d}{6}$ to left of O. Choice (B)

60. $\Delta H = H.\dfrac{c_c}{1+e_0}\log\left(\dfrac{\sigma_f}{\sigma_0}\right)$

At the middle of the clay layer,

$\sigma_0 = (1.5\times 18)+(3\times(20-10))$

$= 27 + 30 = 57$ Kpa

$\sigma_f = 57 + 40 = 97$ Kpa

$\Delta H = \dfrac{6000\times 0.08}{1+0.45}\log_{10}\left(\dfrac{97}{57}\right)$

$\therefore \quad \Delta H = 76.43$mm Choice (B)

61. $GCA = 8000$ Ha

$CCA = 8000\times\dfrac{70}{100} = 5600$ha

Area under kharif $= \dfrac{35}{100}\times 5600 = 1960$ha

Area under rabi $= \dfrac{60}{100}\times 5600 = 3360$ha

Discharge required for kharif $Q = \dfrac{A}{D}$

$= \dfrac{1960}{800} = 2.45$ m^3/sec

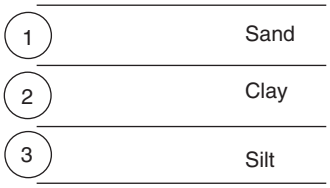
Discharge required rabi $Q = \dfrac{A}{D}$

$= \dfrac{3360}{1500} = 2.24$ m^3/sec

Design discharge = higher of both the discharge

$= 2.45$m^3/sec Choice (B)

62.



$K_3 = 5K_2 = \dfrac{1}{10}K_1$

$\Rightarrow \quad K_1 = 10K_3,\ K_2 = \dfrac{K_3}{5}$

$H_1 = 2H_3 = \dfrac{H_2}{2}$

$\Rightarrow \quad H_2 = 2H_1, H_3 = \dfrac{H_1}{2}$

$k_H = \dfrac{K_1H_1 + K_2H_2 + K_3H_3}{H_1 + H_2 + H_3}$

$= \dfrac{(10K_3)H_{1_{+\frac{K_3}{5}(2H_1)+K_3}}\left(\dfrac{H_1}{2}\right)}{H_1 + 2H_1 + \dfrac{H_1}{2}}$

$= 3.114K_3$

$K_V = \dfrac{H_1 + 2H_1 + \dfrac{H_1}{2}}{\dfrac{H_1}{10K_3} + \dfrac{2H_1}{\left(\dfrac{K_3}{5}\right)} + \dfrac{H_1}{K_3}}$

$= 0.315\ K_3$

$K_H : K_V = 9.87 \cong 10$ Choice (B)

63. When pipes are connected in parallel, head lost is same in both pipes

$\therefore \quad h = \dfrac{4fL}{D} = \dfrac{V_1^2}{2g} = \dfrac{4fl}{d}\dfrac{V_2^2}{2g}$

$\Rightarrow \quad \dfrac{flQ_1^2}{3D^5} = \dfrac{fLQ_2^2}{3d^5} \Rightarrow \dfrac{Q_1^2}{D^5} = \dfrac{Q_2^2}{d^5}$

$\Rightarrow \quad \left(\dfrac{Q_1}{Q_2}\right)^2 = \left(\dfrac{D}{d}\right)^5 = (2)^5 = 32$

$\Rightarrow \quad \dfrac{Q_1}{Q_2} = \sqrt{32} = 5.657$ Choice (C)

64. The most common type of rivet patterns are chain riveting and diamond riveting. Staggered pattern in option (a) yields more net area of the section and because of this reason this pattern is most suitable for tension members. Staggered and diamond pattern better as compared to the chain pattern. Choice (A)

65. $A_1 = 3000 \quad F_1 = 7 \quad r = 0.075$

$A_2 = 400 \quad F_2 = 8 \quad n = 10$yrs

$N = \dfrac{365\left[(1+0.075)^{10}-1\right]\left[(3000\times 7)+(400\times 8)\right]}{0.075}$

$= 12.49$msa Choice (C)

Mock Test 2

Number of Questions: 65 **Total Marks: 100**

Section – I: General Aptitude

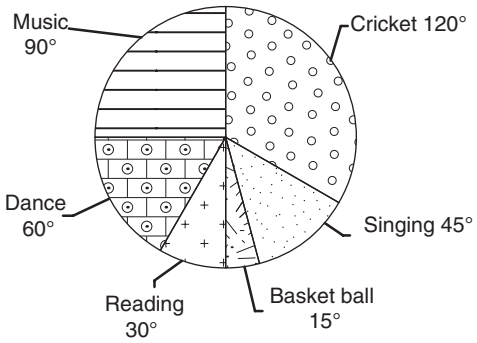
Questions 1 to 5 carry One Mark each.

Directions for question 1: Select the correct alternative from the given choices.

1. If $\left(x^2+\frac{1}{x^2}\right) = 79$, then find $x^3 + \frac{1}{x^3}$.

(A) ±692 (B) ±702
(C) ±712 (D) ±756

2. The following pie-chart gives the split up of number of students having different hobbies in a college of 1800 students.



Every student has exactly one of the above mentioned hobbies. The difference in the number of students with cricket and music as their hobbies is _____

Directions for question 3: The question given below gives a word followed by four choices. From the choices, select the most suitable **synonym** (word which means the same) for the question word and mark its number as the answer.

3. ADJOURN
(A) prevent
(B) withhold
(C) postpone
(D) abrogate

Directions for question 4: The question given below has a sentence with two blanks, followed by four pairs of words as choices. From the choices, select the pair of words that can best complete the given sentence.

4. Reason accepts the _____ of spontaneous intuition but asks that we check our intuition and _____ whether they be correct.
(A) problem . . . evaluate
(B) tranquility . . . promulgate
(C) challenge . . . question
(D) existence . . . ascertain

Directions for question 5: In the question below, determine the relationship between the pair of capitalised words and then select the pair of words which has a **similar** relationship to the capitalized words and mark the number of that pair as your answer.

5. ACRID : BITTER
(A) Mythical : Shallow
(B) Suave : Urbane
(C) Clairvoyant : Disgraceful
(D) Diminutive : Gargantuan

Questions 6 to 10 carry Two Marks each.

Directions for questions 6 to 8: Select the correct alternative from the given choices.

6. Pointing at a photograph Ms. Sudha told Mr. Bhujbal, "the person in the photograph is your father's wife's mother-in-law's only son and my mother's brother's only brother-in-law". How is Bujbal related to Sudha?
(A) Brother (B) Cousin
(C) Uncle (D) Brother-in-law

7. log cot1° + log cot2° + log cot3° + . . . + log cot 89° = _____.
(A) 1 (B) – 1
(C) 0 (D) None of these

8. It is high time that we address the problem of obesity with more seriousness as the ill effects of obesity is more diverse and more harmful than it was thought earlier.
Which of the following can be inferred from the above statement?
(A) Obesity was not considered a disease earlier.
(B) Obesity is the reason for all health related problems.
(C) An obese person cannot be free from a disease.
(D) Measures to address the problem of obesity has been initiated in the past.

Directions for question 9: In the question below, four different ways of writing a sentence are indicated. Choose the best way of writing the sentence.

9. A. The report discusses growth stories in sectors like IT and pharma where Indian companies have made a mark and describes how domestic industry has profitably restructured through the past ten years as economic reforms saw tariff wall fall and the license permit raj dismantled.
B. The report discusses growth stories in sectors like IT and pharma where Indian companies have made a mark and describes how domestic industry has profitably restructured over the past ten years into economic reforms seeing tariff walls fall and the license permit raj dismantled.

C. The report discusses growth stories in sectors like IT and pharma where Indian companies have made a mark and describes how domestic industry has profitably restructured over the past ten years as economic reforms saw tariff walls fall and the license permit raj dismantled.

D. The report discusses growth stories in sectors like IT and pharma where Indian companies have made a mark and describes how domestic industry has restructured profitably through the past ten years when economic reforms have seen tariff walls fall and the license permit raj dismantling.

Directions for question 10: Choose the best answer for the following question:

10. Media watchers (and readers) have found that there has been a sudden increase of 25% in crimes reported after laws were recently passed to reduce the punishment for crimes by lessening prison terms. Clearly the new laws have been solely responsible for the sudden spurt in crimes reported.

Which of the following can be logically inferred from argument above?

(A) As a part of the recent measures undertaken by the state, almost 20% of the entire police force has been laid off.

(B) There has been no appreciable increase in crimes reported in other countries where similar laws were passed.

(C) The crime rate is still 10% lower than that in any other country of comparable size.

(D) The state has recently increased the strength of judges in the courts from 10 to 20%.

SECTION – II: CIVIL ENGINEERING

Directions for questions 1 to 55: Select the correct alternative form the given choices.

Questions 11 to 35 carry One Mark each.

11. Which of the following statements is ALWAYS true with reference to the LU – decomposition $A = LU$ of a square matrix A with L and U being the lower triangular and upper triangular matrices respectively?

(A) All the principal diagonal elements will be equal to 1 for only L.

(B) All the principal diagonal elements will be equal to 1 for any U.

(C) All the principal diagonal elements will be equal to 1 for both L and U.

(D) All the principal diagonal elements will be equal to 1 for any one of L and U.

12. If $u(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$, then the ratio of the first partial derivatives of $u(x, y)$ with respect to x and y is same as the ratio of _____

(A) x and y,

(B) $-y$ and x

(C) x^2 and y^2

(D) $-y^2$ and x^2

13. In the process of finding the reciprocal value of 23 by the Newton – Raphson method with 0.08 as the initial guess, the value of the reciprocal of 23, after the first iteration will be _____

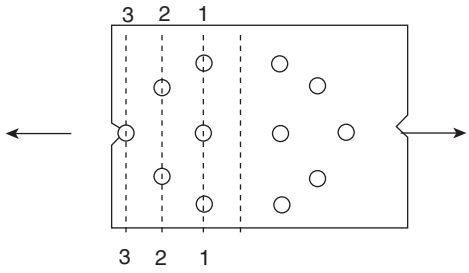
14. If $f(x) = \begin{cases} \pi + x; & -\pi < x < 0 \\ 0; & 0 \le x \le \pi \end{cases}$ and $f(x + 2\pi) = f(x)$, then the Fourier series expansion of $f(x)$ at $x = 0$ converges to _____.

15. The value of $\underset{x\to 0}{Lim} \dfrac{\sqrt{9+x^7} - \sqrt{9-x^7}}{x^7}$ is _____

(A) $\sqrt{7}$ (B) 3

(C) $\dfrac{1}{\sqrt{7}}$ (D) $\dfrac{1}{3}$

16. Two steel plates each of 10 mm thickness, are connected by a double cover butt joint by bolts as shown in figure. If bolt diameter is 20 mm and steel is of grade 410, which of the following section is the most critical section for a main plate?



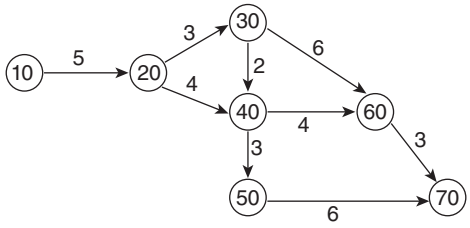
(A) section 1 – 1

(B) section 2 – 2

(C) section 3 – 3

(D) section 1 – 1 and 3 – 3

17.



For the network shown in figure, find the critical path?

(A) 10 – 20 – 30 – 60 - 70
(B) 10 – 20 – 30 – 40 – 50 – 70
(C) 10 – 20 – 40 – 60 – 70
(D) 10 – 20 – 30 – 40 – 60 – 70

18. The following constituent imparts plasticity to brick earth ___
(A) Alumina (B) Silica
(C) Lime (D) Magnesia

19. The contribution compounds of cement in decreasing order of rate of hydration are _____
(A) C_3S, C_3A, and C_2S
(B) C_2S, C_3S, and C_3A
(C) C_3A, C_2S, C_3S
(D) C_3A, C_3S, C_2S

20. The critical section considered for the shear design, if the end of beam is subjected to tension due to support reaction?
(A) section at a distance of effective depth (d) from the support
(B) section at a face of the support
(C) section at a distance $\frac{d}{2}$ from the support
(D) section at a distance $2d$ from the support

21. The equation of A line is ____
(A) $I_p = 0.83\ (W_L - 20)$ (B) $I_p = 0.63\ (W_L - 20)$
(C) $I_p = 0.53\ (W_L - 20)$ (D) $I_p = 0.73\ (W_L - 20)$

22. For a dense packing of regular spheres at the maximum density the void ratio is _____
(A) 0.91 (B) 0.81
(C) 0.65 (D) 0.35

23. In case of soil mechanics the following head is neglected
(A) Pressure head (B) Velocity head
(C) Elevation head (D) Total head

24. Consolidation of soil is due to load which is
(A) Static and short term
(B) Dynamic and short term
(C) Dynamic and long term
(D) Static and long term

25. A fully compensated raft foundation for a building is ____
(A) Designed as a completely flexible footing
(B) Designed as a completely Rigid footing
(C) Designed such that weight of excavated soil is equal to the load due to building
(D) None of these

26. The missing data (X_1 and X_2 respectively) from the table given below is

Station	BS	FS	HI	RL
BM	1.175		X_1	100
A		X_2		98.975

(A) 100.1 m, 1.9 m (B) 100.1 m, 2.2 m
(C) 101.173 m, 2.2 m (D) 101.175 m, 1.9 m

27. If the magnetic bearing = 40°30^1 and magnetic declination is 6° 30^1 E then true bearing is
(A) 47°0^1 (B) 34°
(C) 42° (D) 50°

28. If 10 ml of water sample is diluted with 190 ml distilled water at which odour is just detectable, then TON of sample is
(A) 10 (B) 20
(C) 40 (D) 30

29. A source emitting 40 dB, 20 dB and 90 dB at different times in a day. What is average noise produced by source in a day
(A) 96.2 dB (B) 96.4 dB
(C) 94 dB (D) None of the above

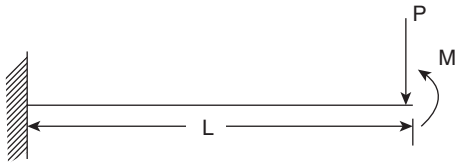
30. The concentration of dissolved oxygen (DO) may fall down to zero, causing anaerobic conditions in river zone known as
(A) zone of degradation
(B) zone of active decomposition
(C) zone of recovery
(D) zone of clear water

31. Determine the amount of lime required for treatment of one million liters of water per day if raw water contains 200 mg/L of total hardness is
(A) 110 kg/day (B) 140 kg/day
(C) 120 kg/day (D) 112 kg/day

32. Find the width of grit chamber to handle a waste water flow of 20 MLD with $V_H = 0.3$ m/sec, detention time = 1 min, depth = 0.8 m is
(A) 0.8 m (B) 1 m
(C) 0.6 m (D) 0.4 m

33. A wire of radius r is stretched by a force P. If another wire of radius $2r$ of same material is stretched by the same load, its modulus of elasticity will
(A) be doubled (B) be halved
(C) become 4 times (D) not be changed

34.



A cantilever beam of span L is subjected to a concentrated load P and a moment M as shown in the figure. Deflection at the free end is given by

(A) $\frac{PL^3}{3EI} + \frac{ML^2}{2EI}$ (B) $\frac{PL^2}{2EI} + \frac{ML^3}{3EI}$
(C) $\frac{PL^3}{3EI} - \frac{ML^2}{2EI}$ (D) $\frac{PL^3}{2EI} - \frac{ML^2}{3EI}$

35. Diameter of an air bubble in water is 0.01 mm. If the surface tension at air water interface is 0.07 N/m, the pressure difference between inside and outside of the air bubble is

(A) 14 kPa (B) 28 kPa
(C) 56 kPa (D) 62 kPa

Questions 36 to 65 carry Two Mark each.

36. The least possible value of the determinant of a 3 × 3 matrix A with all entries being either 0 or 1 is _____.

(A) 0 (B) −1
(C) −2 (D) −3

37. The value of the line integral $\int_c \bar{F} \cdot d\bar{r}$ where $\bar{F} = xy\,\bar{i} + 2yz\,\bar{j} - z^2\,\bar{k}$ and the curve c is given by $x = t$, $y = t^2$, $z = 3t$, $0 \le t \le 2$ is _____.

38. Rajeev goes to office either by car, scooter, bus or train with various possibilities as shown in the table below.

Mode of transport	Percentage chance of using	Chance of reaching the office lately
Car	20%	1 out of 25
Scooter	40%	2 out of 25
Bus	30%	1 out of 10
Train	10%	1 out of 5

If Rajeev reached the office lately, then the probability that he travelled by scooter is _____.

(A) $\frac{1}{3}$ (B) $\frac{8}{9}$
(C) $\frac{14}{27}$ (D) $\frac{16}{45}$

39. The value of the definite integral $\int_0^4 \frac{x}{1+3x^2}dx$ obtained by the Simpson's rule with step size $h = 1$ is _____.

(A) 0.6059 (B) 0.6486
(C) 0.6687 (D) 0.6984

40. The value of y at $x = 3$, if y satisfies the differential equation $x\frac{dy}{dx} - 2y = x^3$ and $y = 0$ at $x = 1$ is _____

(A) 9 (B) 18
(C) 27 (D) 36

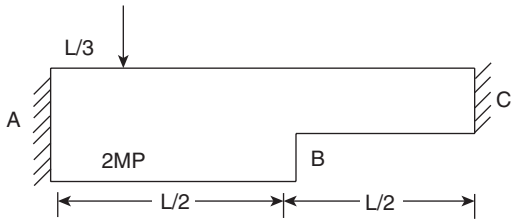
41. Velocity potential function for a flow is given by $\phi = x^2 - y^2$. The stream function for the flow is

(A) $2xy + f(y)$ (B) $2xy + f(x)$
(C) $2xy$ + constant (D) $2x - 2y$ + constant

42. In a 3 dimensional incompressible fluid flow, velocity components in x and y directions are
$u = x^2 + y^2z^3$
$v = -(xy + yz + zx)$
Velocity component in the z direction is

(A) $-xz + \frac{z^2}{2} + f(x, y)$ (B) $-xz + \frac{z^2}{2}$
(C) $xz - \frac{z^2}{2} + C$ (D) $-x + z$

43. A fixed beam subjected to load w at $\frac{1}{3}$ rd span as shown in figure. Estimate the collapse load?



(A) $12\frac{M_p}{L}$ (B) $9\frac{M_p}{L}$
(C) $15\frac{M_p}{L}$ (D) $6\frac{M_p}{L}$

44. For a particular activity of a project, time estimates received from engineers x, y and z are as follows

	Optimistic time	Most likely time	Pessimistic time
Engineer x	4	6	8
Engineer y	3	5	8
Engineer z	5	7	10

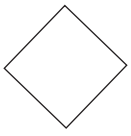
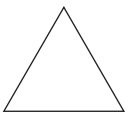
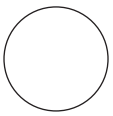
Who is more certain about time of completion of job?

(A) Engineer x (B) Engineer y
(C) Engineer z (D) Can't say

45. A concrete beam of 200 mm × 400 mm is pre-stressed with a force of 600 kN at an eccentricity of 120 mm. The maximum and minimum stresses developed are ____

(A) 21 MPa and 6 Mpa (B) 15 MPa and 9 MPa
(C) 6 MPa and 1.5 MPa (D) 12 MPa and 21 MPa

46. Identify the correct relation with respect to "shape factor" for different shapes in increasing order

(i) 
(ii) 
(iii) 
(iv) 

(A) iii, iv, ii, i (B) ii, iii, iv, i
(C) iv, iii, i, ii (D) iv, i, ii, iii

47. A saturated clay stratum draining both at the top and bottom undergoes 50 percent consolidation in 24 years under an applied load. If an additional drainage layer

were present at the middle of the clay stratum, 50 percent consolidation would occur in _____ years.

48. An unconfined compression test yielded a strength of 0.2 N/mm^2. If the failure plane is inclined at 60° to the horizontal, what are the values of shear strength parameters?
(A) 0.057 N/mm^2; 30°
(B) 0.35 N/mm^2; 60°
(C) 0.15 N/mm^2; 30°
(D) 0.4 N/mm^2; 60°

49. A 6 m high retaining wall is supporting a saturated sand (saturated due to capillary action) of bulk density 21 KN/m^3 and angle of shearing resistance 30°. The change in magnitude of earth pressure at the base due to rise in ground water table from the base of footing to the ground surface shall (γ_w = 10 kN/m^3)
(A) increases by 20 kN/m^2
(B) increases by 40 kN/m^2
(C) decreases by 20 kN/m^2
(D) decrease by 40 kN/m^2

50. A bed of sand consists of three horizontal layers of equal thickness. The value of Darcy's K for the upper and lowest layers is 1×10^{-1} cm/sec and that for the middle layer is 1×10^{-2} cm/sec. The ratio of permeability of the bed in the horizontal direction to that in vertical direction is _____.

51. If a square footing of size 4m × 4m is resting on the surface of a deposit of soft clay having cohesion of 13.5 kPa, the ultimate bearing capacity of the footing (as per Terzahi's equation) is _____.

52. Find the capacity of digester from the following data
Q = 20 ML, SS in waste water flow 200 mg/L
% of SS removed from clarifier= 70
Moisture content in fresh sludge P_1 = 98%
Moisture content of digested sludge P_2 = 90%
Specific gravity of fresh sludge = 1.06
Specific gravity of digested sludge = 1.02
Digestion period = 60 days

53. The specific gravities and weight proportions are given as under for the preparation of marshall mould. The volume and weight of marshall mould are 475 cc and 1100 g respectively

Specimen	Weight (g)	Specific gravity
A1	825	2.63
A 2	1200	2.51
A 3	325	2.46
A 4	150	2.43
Bitumen	100	1.05

The % voids in mineral aggregates is
(A) 15.25 (B) 16.85
(C) 12.49 (D) 13.42

54. Find the extra widening necessary for horizontal curves having R = 100 m, wheel base 6.1 m, pavement width = 7 m, Design speed, V = 70km/hr
(A) 1.112 m (B) 1.14 m
(C) 1.11 m (D) 1.32 m

55. A design speed of traffic lane is 80 kmph. Average length of vehicle is 6m. The capacity of the road when reaction time of driver is 2 sec and coefficient of longitudinal friction is 0.35 is
(A) 523veh/hr
(B) 656veh/hr
(C) 832veh/hr
(D) 932veh/hr

56. The rate of super elevation on a highway in plain terrain with design speed of 100 kmph with radius of curve 500 m under mixed traffic condition is
(A) 7% (B) 9.2%
(C) 8.8% (D) 15.72%

57. Consider
1. Creation of central road fund
2. National highway act
3. Formation of Indian Road congress
4. Creation of highway research board
The correct chronological order of the events is
(A) 4, 3, 2, 1 (B) 2, 1, 3, 4
(C) 1, 3, 2, 4 (D) 2, 3, 1, 4

58. Following test results were obtained by a CBR test on a subgrade soil?

Penetration in "mm"	0	2.5	5.0
Load (kg)	0	55	78

Area of plunger is 19.6 cm^2
The aggregates pressures at 2.5 mm and 5mm are 70 kg/cm^2 and 105 kg/cm^2 respectively. The CBR of the soil is?
(A) 3.8% (B) 3.5%
(C) 4% (D) 4.4%

59. From the analysis of rainfall data at a particular station it was found that a rainfall of 400mm had a return period of 20 years. What is the probability of rainfall equal to or greater than 400 mm occurring at least once in 10 successive years is
(A) 50.43 (B) 40.12
(C) 20.32 (D) 26.23

60. Route the following flood hydrograph through a river reach for which Muskingum coefficient k = 8 and x = 0.25

Time (h)	0
Inflow cm^3/s	8

If the initial out flow discharge from the reach is 8 m^3/s. Find the final outflow

(A) 10 m^3/s (B) 8 m^3/s
(C) 6 m^3/s (D) 4 m^3/s

61. In an irrigated plot the net irrigation requirement of crop is found to be 14.9 cm, the application efficiency is 80 % and the water conveyance efficiency is 70%. What is the gross irrigation requirement?
(A) 29.8 cm (B) 25.2 cm
(C) 31.3 cm (D) 26.60 cm

62. The area of the irrigation channel in alluvial soil according to lacey's sit theory with the following data, discharge is 50m^3/sec lacey's silt factor = 1, side slope = $0.5H : 1V$
(A) 56.4 m^2 (B) 62.3 m^2
(C) 64.3 m^2 (D) 59.38m^2

63.

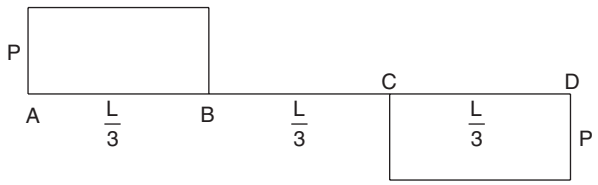
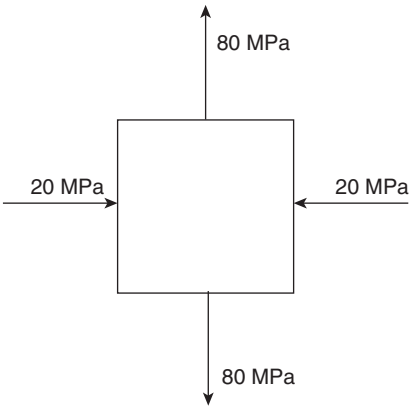


Figure shows shear force diagram of a beam *ABCD* of length *L*. Bending moment at the centre of the beam is

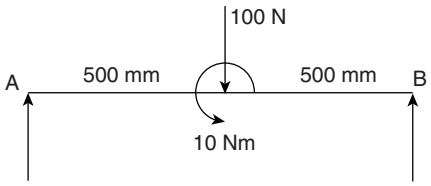
(A) 0 (B) $\frac{PL}{3}$
(C) $\frac{PL}{2}$ (D) $\frac{2PL}{3}$

64.



For the state of stress at a point in a stressed body, shown in figure radius of the Mohr's circle is
(A) 80 MPa (B) 60 MPa
(C) 50 MPa (D) 40 MPa

65.



A simply supported beam of 1 m length is loaded at the mid span as shown in the figure. Maximum bending moment is
(A) 30 Nm (B) 35 Nm
(C) 40 Nm (D) 45 Nm

Answer Keys

1. B	**2.** 150	**3.** C	**4.** D	**5.** B	**6.** A	**7.** C	**8.** D	**9.** C	**10.** A
11. D	**12.** B	**13.** 0.011 to 0.014		**14.** 1.55 to 1.59		**15.** D	**16.** C	**17.** B	**18.** A
19. D	**20.** B	**21.** D	**22.** D	**23.** B	**24.** D	**25.** C	**26.** C	**27.** A	**28.** B
29. D	**30.** B	**31.** D	**32.** B	**33.** D	**34.** C	**35.** B	**36.** C	**37.** 8.7 to 8.9	
38. D	**39.** A	**40.** B	**41.** C	**42.** A	**43.** C	**44.** A	**45.** A	**46.** C	
47. 6 years		**48.** A	**49.** B	**50.** 2.8	**51.** 100 – 102 Kpa		**52.** 478.35 m^3		**53.** C
54. C	**55.** B	**56.** A	**57.** C	**58.** C	**59.** B	**60.** B	**61.** D	**62.** D	**63.** B
64. C	**65.** A								

Hints and Explanations

1. Given $x^2 + \frac{1}{x^2} = 79$

$$x^2 + \frac{1}{x^2} + 2 = 81$$

$$\left(x + \frac{1}{x}\right)^2 = 81$$

$$\Rightarrow \quad x + \frac{1}{x} = \pm 9$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x.\frac{1}{x}\left(x + \frac{1}{x}\right)$$

$$= (\pm 9)^3 - 3(\pm 9)$$

$$= \pm[729 - 27] = \pm 702 .$$ Choice (B)

2. Required difference = $\frac{(120-90)}{360} \times 1800 = 150.$

Ans: 150

3. 'Adjourn' means 'put off' as in "The meeting was adjourned until December 4". So 'postpone' is apt. 'Abrogate' means override / revoke or cancel.

Choice (C)

4. The second blank needs a word synonymous with the word "check" since the sentence talks about how one needs to check whether one's intuition is correct. Hence, it could be "ascertain". Though evaluate also suits, the former suits better in this context. The first blank can take "existence". Tranquility means peace.

Choice (D)

5. 'Suave' means sophisticated, polished, 'urbane'. The question words 'acrid' and 'bitter' are synonyms too.

Choice (B)

6. Bhujbal's father's wife's mother-in-law is Bhujbal's father's mother, whose only child is Bhujbal's father. Hence, the person in the photograph is Bhujbal's father. Bhujbal's father is Sudha's mother's brother's only brother-in-law, i.e. Sudha is Bhujbal's father's daughter.

$\therefore$ Bhujbal is Sudha's brother. Choice (A)

7. log cot1° + log cot 2° + . . . log cot 89°
= log (cot1° . cot2° . . . cot 89°)
Now cot 89° = cot (90° – 1) = tan 1°
cot 88° = cot (90° – 2) = tan 2°
$\therefore$ log (cot 1° cot 2° . . . .cot 88° cot 89°)
= log (cot 1° cot 2° . . . .tan 2° tan 1°)
= log 1 = 0 ($\because$ cot 1° tan 1° = 1
cot 2° tan 2° = 1) Choice (C)

8. From the statement it is clear that certain action is being taken to address obesity. Hence, (D) can be inferred from the statement. Choice (A) is in contradiction to the statement. From the statement we know that obesity leads to certain problems. But the information does not allow us to infer that every health problem is caused by obesity. Hence, (C) cannot be inferred. Hence, (D) can be inferred. Choice (D)

9. In (A) and (D) the usage of 'through the past ten years' is incorrect. Further the usage of 'wall' in (A) is incorrect. Also in (D) 'dismantling' is awkward. In (B) the usage of the 'ing' form of the verb 'see' is incorrect. Choice (C) is correct in all ways. Choice (C)

10. Only (A) can be logically inferred from the sentences. If the laws have been changed to reduce punishment and prison terms, obviously it follows that some of the police staff will not be needed as there will be fewer prisoners. The other statements (B) and (C) are outside the scope of the text as what happens in other countries is beyond our scope of understanding. Choice (D) only weakens / goes against the arguments in the text.

Choice (A)

11. From the procedure of LU – decomposition

Choice (D)

12. Given $u(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\left(\frac{1}{y}\right) = s\frac{y}{x^2+y^2}$$

$$\text{and } \frac{\partial u}{\partial y} = \frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\left(\frac{-x}{y^2}\right) = \frac{-x}{x^2+y^2}$$

$$\therefore \frac{\partial u}{\partial x} : \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2} : \frac{-x}{x^2+y^2} = -y : x$$

Choice (B)

13. We have to find x such that

$$x = \frac{1}{23} \Rightarrow \frac{1}{x} = 23$$

$$\text{Let } f(x) = \frac{1}{x} - 23 = 0$$

$$\Rightarrow f'(x) = \frac{-1}{x^2}$$

Given the initial guess $= x_0 = 0.08$

$$\therefore f(x_0) = \frac{1}{0.08} - 23 = -10.5$$

$$\text{and } f'(x_0) = \frac{-1}{(0.08)^2} = -156.25$$

By Newton – Raphson method, the reciprocal value of 23 after first iteration is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.08 - \frac{(-10.5)}{(-156.25)}$$

$$\therefore x_1 = 0.0128.$$

Ans: 0.011 to 0.014

14. Given $f(x) = \begin{cases} \pi + x; & -\pi < x < 0 \\ 0; & 0 \le x < \pi \end{cases}$

and $f(x + 2\pi) = f(x)$
Clearly, $f(x)$ is discontinuous at $x = 0$,

$\therefore$ Fourier series of $f(x)$ at $x = 0$ converges to $\frac{1}{2}\left[f(0+) + f(0-)\right]$

where $f(0+) = \underset{x\to 0+}{Lt} f(x) = \underset{x\to 0+}{Lt} 0 = 0$

and $f(0-) = \underset{x\to 0-}{Lt} f(x) = \underset{x\to 0-}{Lt} (\pi + x) = \pi$

Hence the Fourier series of $f(x)$ at $x = 0$ converges to

$$\frac{1}{2}[0 + \pi] = \frac{\pi}{2} = 1.57$$

Ans: 1.55 to 1.59

15. We have $\underset{x\to 0}{Lt}\dfrac{\sqrt{9+x^7}-\sqrt{9-x^7}}{x^7}$

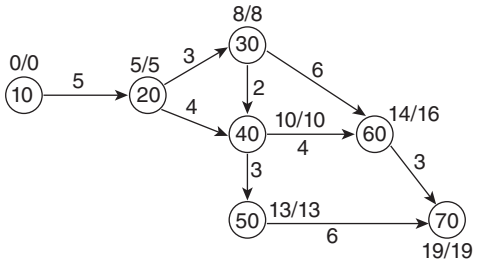
$= \underset{x\to 0}{Lim}\left[\dfrac{\sqrt{9+x^7}-\sqrt{9-x^7}}{x^7}\right]\times\left[\dfrac{\sqrt{9+x^7}+\sqrt{9-x^7}}{\sqrt{9+x^7}+\sqrt{9-x^7}}\right]$

$= \underset{x\to 0}{Lt}\dfrac{(9+x^7)-(9-x^7)}{\sqrt{9+x^7}+\sqrt{9-x^7}}$

$= \underset{x\to 0}{Lt}\dfrac{2}{\sqrt{9+x^7}+\sqrt{9-x^7}}=\dfrac{1}{3}$ Choice (D)

16. For cover plate, 1 – 1 is critical and for main plate 3 – 3 is critical section. Choice (C)

17.



Critical path: 10 – 20 – 30 – 40 – 50 – 70 (or) Critical path is the path along which duration is maximum.
From options, the path 10 – 20 – 30 – 40 – 50 – 70 has maximum duration. Choice (B)

18. Alumina imparts plasticity to brick earth. Choice (A)

19. $C_3A > C_3S > C_2S$. Choice (D)

20. If end of beam is subjected to tension; critical section is at face of the support. If end of beam is subjected to compression; critical section is at a distance of d, effective depth from the support. Choice (B)

21. $I_p = 0.73\ (W_L - 20)$. Choice (D)

22. Void ratio for diagonal packing (for densest condition) is 0.35 where as for loose packing (cubical array) void ratio is 0.91. Choice (D)

23. Velocity head is neglected in soil mechanics. Choice (B)

24. Consolidation is due to static and long term loading. Choice (D)

26. $X_1 = RL + BS = 100 + 1.175 = 101.175$m
$HI - X_2 = 98.975$
$X_2 = HI - 98.975 = X_1 - 98.975$
$= 101.175 - 98.975 = 2.2$m. Choice (C)

27. Forbearing $= 40°30^1 + 6°30^1 = 47°01$. Choice (A)

28. TON $= \dfrac{A+B}{A}=\dfrac{10+190}{10} = 20$ Choice (B)

29. $L = 20\log_{10}\dfrac{1}{3}\left[10^{\frac{40}{20}}+10^{\frac{20}{20}}+10^{\frac{90}{20}}\right]$

$= 96.69$dB. Choice (D)

31. Molecular weight of lime (CaO) = 56 mol.wt
100mg of TH requires 56mg of quick lime
200mg of TH requires 56mg of quick lime

$\Rightarrow \dfrac{200\times 56}{100} = 112$mg/lit

$\Rightarrow$ Total lime dosage required = $Q \times$ dosage
$= 1 \times 112 = 112$kg/day. Choice (D)

32. Volume of grit chamber = $Q_1 \times D_T$

$= \dfrac{20\times 10^6}{(10^3\times 24\times 60)}\times 1 = 13.86\text{m}^3$

Length of grit chamber = $V_H \times D_T$
$= 0.3 \times 1 \times 60 = 18$ m

$\dfrac{C}{S} = \dfrac{\text{volume}}{\text{length}} = \dfrac{13.86}{18} = 0.771$m

$\therefore$ Width $= \dfrac{0.771}{0.8} = 0.96\text{m} \backsimeq 1$ m. Choice (B)

35. $\sigma = 0.07$ N/m, d = 0.01 mm

$\Delta P = \dfrac{4\sigma}{d} = \dfrac{4\times 0.07}{0.01\times 10^{-3}}$

$= 28 \times 10^3$ N/m^2 $= 28$ kN/m^2 $= 28$ kPa. Choice (B)

36. Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ be a 3 × 3 matrix with all entries being either 0 or 1.

$\therefore$ Det $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$

$\therefore$ Det $A = a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_1b_3c_2 - a_2b_1c_3 - a_3b_2c_1$ $\rightarrow (1)$

As the entries of A being 0 or 1 only and from (1), we can observe that the least possible value for $|A|$ cannot be less than −3.
Also, from (1), we can observe that for no possible set of values of a_i, b_j and c_k ($i = 1$ to 3, $j = 1$ to 3 and $k = 1$ to 3), determinant of A can be −3.

And let $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$,

Det $A = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -2$

$\therefore$ The least possible value of determinant of $A = -2$ Choice (C)

37. Given $\overline{F} = xy\,\overline{i} + 2yz\,\overline{j} - z^2\,\overline{k}$

and C is $x = t$, $y = t^2$, $z = 3t$; $0 \le t \le 2$

$\Rightarrow$ $dx = dt$, $dy = 2tdt$ and $dz = 3dt$

$\therefore$ $\int_c \overline{F}.d\overline{r} = \int_c (xy\overline{i} + 2yz\overline{j} - z^2\overline{k}).(dx\overline{i} + dy\overline{j} + dz\overline{k})$

$= \int_{t=0}^{2} (t^3\overline{i} + 6t^3\overline{j} -).(dt\overline{i} + 2tdt\overline{j} + 3dt\overline{k})$

$= \int_{t=0}^{2} (12t^4 + t^3 - 27t^2)\,dt$

$= 12\frac{t^5}{5} + \frac{t^4}{4} - 27\frac{t^3}{3}\Big]_0^2$

$= 12 \times \frac{32}{5} + \frac{16}{4} - 9 \times 8$

$= \frac{384}{5} + 4 - 72 = \frac{44}{5} = 8.8.$ Ans: 8.7 to 8.9

38. Let B_1, B_2, B_3 and B_4 denote the events of Rajeev going to office by car, scooter, bus and train respectively.
Let A be the event of Rajeev reaching the office lately.

$\therefore$ $P(B_1) = 0.2$, $P(B_2) = 0.4$, $P(B_3) = 0.3$, $P(B_4) = 0.1$

$P\left(\frac{A}{B_1}\right) = \frac{1}{25} = 0.04$; $P\left(\frac{A}{B_2}\right) = \frac{2}{25} = 0.08$

$P\left(\frac{A}{B_3}\right) = \frac{1}{10} = 0.1$; $P\left(\frac{A}{B_4}\right) = \frac{1}{5} = 0.2$

$\therefore$ Probability that Rajeev reached the office lately given that he travelled by scooter.

$= P\left(\frac{B_2}{A}\right) = \frac{P(B_2)P\left(\frac{A}{B_2}\right)}{\sum_{i=1}^{4} P(B_i)P\left(\frac{A}{B_i}\right)}$

(By Bayes' theorem)

$= \frac{0.4 \times 0.08}{0.2 \times 0.04 + 0.4 \times 0.08 + 0.3 \times 0.1 + 0.1 \times 0.2}$

$= \frac{0.032}{0.09} = \frac{32}{90} = \frac{16}{45}$ Choice (D)

39. We have to evaluate $\int_0^4 \frac{x}{1+3x^2}\,dx$ by Simpson's rule.

Here $y = f(x) = \frac{x}{1+3x^2}$; $h = 1$; $a = 0$ and $b = 4$

x	0	1	2	3	4
y = f(x)	0	0.25	0.1538	0.1071	0.0816

$\therefore$ By Simpson's rule, we have

$\int_0^4 \frac{x}{1+3x^2}\,dx = \int_a^b y\,dx = \frac{h}{3}$

$[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2]$

$= \frac{1}{3}[(0 + 0.0816) + 4(0.25 + 0.1071) + 2(0.1538)]$

$= 0.6059$ Choice (A)

40. Given differential equation is

$x\frac{dy}{dx} - 2y = x^3$ $\rightarrow$ (1)

$\Rightarrow$ $\frac{dy}{dx} - \frac{2y}{x} = x^2$ $\rightarrow$ (2)

Clearly, (2) is a linear equation of first order in y of the form $\frac{dy}{dx} + P(x)y = Q(x)$

where $P(x) = \frac{-2}{x}$ and $Q(x) = x^2$

$\therefore$ Integrating factor = I.F. $= e^{\int \frac{-2}{x}dx} = e^{\ln x^{-2}}$

IF $= \frac{1}{x^2}$

$\therefore$ The general solution of (1) is

$y \times (I.F) = \int [Q(x) \times \text{I.F}]\,dx + c$

$\Rightarrow$ $y \times \frac{1}{x^2} = \int \left[x^2 \times \frac{1}{x^2}\right]dx + c$

$\Rightarrow$ $\frac{y}{x^2} = \int dx + c$

$\Rightarrow$ $\frac{y}{x^2} = x + c$ $\rightarrow$ (3)

Given $y = 0$ at $x = 1$

$\therefore$ From (3),

$\frac{0}{1^2} = 1 + c$

$\Rightarrow$ $c = -1$

Substituting the value of c in (3), we have

$\frac{y}{x^2} = x - 1 \Rightarrow y = x^3 - x^2$ $\rightarrow$ (4)

From (4), at $x = 3$, we have

$y = 3^3 - 3^2 = 18$ Choice (B)

41. Potential function $\phi = x^2 - y^2$

$\frac{\partial \phi}{\partial x} = u$ and $\frac{\partial \phi}{\partial y} = v$

$\therefore$ $2x = u$ or $u = 2x$
and $-2y = v$ or $v = -2y$

For stream function ψ

$\frac{\partial \psi}{\partial x} = -v$ -----(1)

$\frac{\partial \psi}{\partial y} = u$ ----(2)

From (1)

$\partial \psi = -v\,dx = (2y)dx$

$$\psi = \int 2y dx = 2xy + f(y)$$

$$\frac{\partial \psi}{\partial y} = 2x + f'(y)$$

Equating with (2)

$u = 2x + f'(y)$

or $2x = 2x + f'(y)$

$\Rightarrow \quad f'y = 0$

Or $f(y) = C$(constant)

$\therefore \quad \psi = 2xy + C$ Choice (C)

42. For a three dimensional, incompressible fluid flow the continuity equation must be satisfied

i.e., $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$u = x^2 + y^2z^3$

$$\frac{\partial u}{\partial x} = 2x$$

$v = -(xy + yz + zx)$

$$\frac{\partial v}{\partial y} = -(x + z)$$

$$\therefore \quad 2x - (x + z) + \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \quad \frac{\partial w}{\partial z} = -2x + x + z = -x + z$$

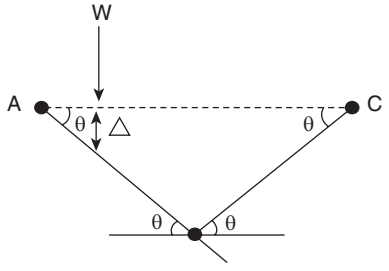
Integrating with respect to z

$w = -xz + \frac{z^2}{2} + C$ where $C = f(x, y)$

or $w = -xz + \frac{z^2}{2} + f(x, y)$. Choice (A)

43. No of plastic hinges required = $D_S + 1 = 2 + 1 = 3$

Case I: Two plastic hinges at supports, and one at point where cross section changes



Internal work done = $2M_p\theta + M_p(\theta + \theta) + M_p\theta$

External work done = $W.\Delta$

$\Rightarrow \quad 5M_p\theta = W.\Delta$

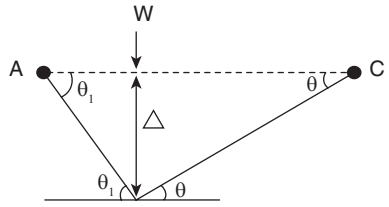
$$\tan\theta = \frac{\Delta}{\frac{L}{3}}$$

$$\Delta = \theta.\frac{L}{3}$$

$$5M_p\theta = W.\theta.\frac{L}{3}$$

$$\therefore \quad W = \frac{15M_p}{L}$$

Case II: Two plastic hinges at supports and one under concentrated load



$$\theta_1 = \frac{\Delta}{\frac{L}{3}} \text{ and } \theta = \frac{\Delta}{\frac{2L}{3}}$$

$$\Delta = \theta_1 \times \frac{L}{3} = \theta \times \frac{2L}{3}$$

$\Rightarrow \quad \theta_1 = 2\theta$

Internal work done = $2M_p\theta_1 + 2M_p(\theta + \theta_1) + M_P.\theta$

$= 4M_p\theta + 6M_p\theta + M_p\theta = 11\ M_p\theta$

$$M_p\theta = W.\frac{2L}{3}\theta$$

$$\therefore \quad W = \frac{16.5M_p}{L}$$

The collapse load will be minimum of above two values

So; collapse load = $\frac{15M_p}{L}$. Choice (C)

44. The degree of uncertainty is indicated by the variance of time estimates

$$\text{Variance} = \left(\frac{t_p - t_o}{6}\right)^2$$

Variance of time estimates given by

$$x: \ \sigma_x^2 = \left(\frac{8-4}{6}\right)^2 = 0.4356$$

Similarly: $\sigma_y^2 = \left(\frac{10-5}{6}\right)^2 = 0.69$ and $\sigma_z^2 = 0.69$

Greater than variance; greater the uncertainty. So, Engineer X's time estimates have less variance, hence more certain. Choice (A)

45. $\sigma = \frac{P}{A} \pm \frac{P.e}{z}$

Given: $P = 600$ KN

$A = 200 \times 400 = 80 \times 10^3 mm^2$

$E = 120$mm

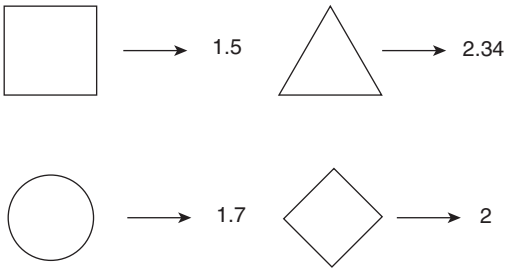
$$Z = \frac{bd^2}{6} = \frac{200 \times 400^2}{6} = 5.33 \times 10^6 mm^3$$

$$\therefore \quad \sigma = \frac{600\times10^3}{80\times10^3} \pm \frac{600\times10^3\times120}{5.33\times10^6} = 7.5 \pm 13.5$$

$\sigma_{max} = 21$ Mpa

$\sigma_{min} = -6$ Mpa. Choice (A)

46. Shape factor different shapes are



Choice (C)

47. For same degree of consolidation

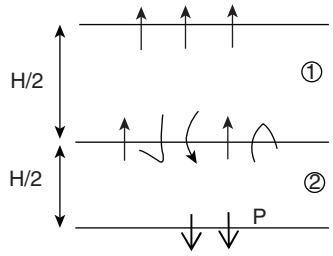
T_v is same

$$\frac{C_v t}{d^2} = \text{constant}$$

$\therefore \quad t \;\&\; d^2$

In, 1st case $d = \frac{H}{2}$

In 2nd case; an additional drainage layer is provided



So, d should be taken as higher of (1) and (2)

$$\therefore \quad d_2 = \frac{H}{4}$$

$$\therefore \quad \frac{t_1}{t_2} = \left(\frac{\frac{H}{2}}{\frac{H}{4}}\right)^2$$

$$\frac{24}{t_2} = 2^2$$

$T_2 = 6$yrs Ans: 6 years

48. $q_u = 0.2$ N/mm^2

$q_u = 2c\tan\alpha$

$$C = \frac{q_u}{2\tan\alpha} = \frac{0.2}{2\times\tan 60}$$

$C = 0.057$ N/mm^2

$$45 + \frac{\phi}{2} = 60$$

$\theta = 30°$. Choice (A)

49. $K_a = \frac{1-\sin 30}{1+\sin 30} = \frac{1}{3}$

Case I: $W.T$ at base

$$(P_o)_{\text{at base}} = K_a\,\gamma H = \frac{1}{3}\times 21 \times 6$$

$(P_a)_{\text{at base}} = 42$ kN/m^2

Case 2: $W.\ T$ at ground surface

$(P_a)_{\text{at base}} = ka\gamma^1 H + \gamma_w H$

$$= \frac{1}{3}\times(21-10)\times 6 + 10\times 6 = 22 + 60$$

$(P_a)_{\text{atbase}} = 82$KN/m^2

$\therefore$ From case (1) and case (2), P_a increased by 40 KN/m^2 Choice (B)

50. $$K_H = \frac{K_1Z_1 + K_2Z_2 + K_3Z_3}{Z_1+Z_2+Z_3}$$

$$= \frac{\left(1\times10^{-1}\right)1 + \left(1\times10^{-2}\right)1 + \left(1\times10^{-1}\right)1}{1+1+1}$$

$K_H = 0.07$ cm/sec

$$K_V = \frac{Z_1+Z_2+Z_3}{\frac{Z_1}{K_1}+\frac{Z_2}{K_2}+\frac{Z_3}{K_3}}$$

$$= \frac{1+1+1}{\frac{1}{1\times10^{-1}}+\frac{1}{1\times10^{-2}}+\frac{1}{1\times10^{-1}}}$$

$K_v = 0.025$

$$\frac{K_H}{K_V} = \frac{0.07}{0.025} = 2.8$$ Ans: 2.8

51. $Q_u = 1.3\,CN_c + \gamma DN_q + 0.4\gamma BN_r$

For pure clay; ($\phi = 0$)

$N_c = 5.7$; $N_q = 1$ and $N_r = 0$

$\therefore \quad q_u = 1.3 \times 13.5 \times 5.7 + \gamma\,(0)\,(1) + 0$

$q_u = 100.$kPa Ans: 100 – 102 Kpa

52. $M = 20 \times 200 \times \frac{70}{100} = 280$ kg/day

$e_f = S_f \times e_w = 1.06 \times 1000 = 1060$ kg/m^3

$ed = S_d \times e_w = 1.02 \times 1000 = 1020$ kg/m^3

$$V_f = \frac{100}{100-p1} \times \frac{M}{e_f}$$

$$= \frac{100}{100-98} \times \frac{280}{1060}$$

$= 13.2$ m^2/day

$$V_d = \frac{100}{100-P_2} \times \frac{M}{e_d}$$

$$= \frac{100}{100-90} \times \frac{280}{1020} = 2.745\text{m}^3/\text{day}$$

Capacity of digester $= \left[\frac{V_f + V_d}{2}\right] \times t$

$= \left[\frac{13.2 + 2.745}{2}\right] \times 60 = 478.35 m^3$

Ans : $478.35 m^3$

53. $VMA = \frac{v_a + v_b}{v_t} \times 100$

$V_a = \frac{G_t - G_m}{G_t} \times 100$

$G_t = \frac{100}{\frac{W_1}{G_1} + \frac{W_2}{G_2} + \frac{W_3}{G_3} + \frac{W_4}{G_4}}$

$W = w_1 + w_2 + w_3 + w_4 + w_5 = 825 + 1200 + 325 + 150 + 100 = 2600$

$W_1 = \frac{825}{2600} \times 100 = 31.7\%$

$W_2 = \frac{1200}{2600} \times 100 = 46.1\%$

$W_3 = \frac{325}{2600} \times 100 = 12.5\%$

$W_4 = \frac{150}{260} \times 100 = 5.76\%$

$W_5 = \frac{100}{2600} \times 100 = 3.84\%$

$G_t = \frac{100}{\frac{31.7}{2.63} + \frac{46.1}{2.51} + \frac{12.5}{2.46} + \frac{5.76}{2.43} + \frac{3.84}{1.05}} = 2.405,$

$G_m = \frac{1100}{475} = 2.315$

$V_a = \frac{2.405 - 2.315}{2.405} \times 100 = 3.74\ \%,$

$V_b = \frac{G_m \times W_b}{G_b} = \frac{2.315 \times 3.84}{1.05} = 8.479\%$

$VMA = 3.74 + 8.749 = 12.491.$ Choice (C)

54. Total extra widening on the road

7m wide road has two lanes as per IRC

$W_e = \frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}}$

$= \frac{2 \times (6.1)^2}{2 \times 100} + \frac{70}{9.5\sqrt{100}} = 1.11m.$ Choice (C)

55. $SSD = vt + \frac{v^2}{2gf}$

$= 22.2\ (2) + \frac{22.22^2}{2 \times 9.81 \times 0.35} = 116m$

$S = SSD + L = 116 + 6 = 122m$

$C = \frac{1000\ V}{S} = \frac{1000 \times 80}{122} = 656 veh/hr.$ Choice (B)

56. $e = \frac{(0.75v)^2}{gR}$

$= \frac{(0.75 \times 27.77)^2}{9.81 \times 500} = 0.08 = 8\%$ is greater than 7% as per IRC in plain terrain. Choice (A)

58. $CBR = \frac{\text{load at 2.5 mm penetration}}{\text{standard load at 2.5 mm penetration}}$

$= \frac{55}{(19.6 \times 70)} = 4.0\%$

$CBR\ (\%) = \frac{\text{load at 5.0 mm penetration}}{\text{standard load}}$

$= \frac{78 \times 100}{19.6 \times 105} = 3.79\%$

CBR (%) = 4.0% Choice (C)

59. $T = 20$ years

$P = \frac{1}{T} = \frac{1}{20} = 0.05$

$N = 10$ years

$Q = 1 - p = 1 - 0.05 = 0.95$

Probability of occuring at least once $= 1 - q^n$

$= 1 - (0.95)^{10}$

$= 0.4012 = 40.12\%.$ Choice (B)

60. $C_0 = \frac{-kx + 0.5\Delta t}{k - kx + 0.5\Delta t} = \frac{(-8 \times 0.25) + (0.5 \times 4)}{8 - (8 \times 0.25) + (0.5 \times 4)} = 0$

$C_1 = \frac{kx + 0.5\Delta t}{k - kx + 0.5\Delta t} = \frac{(8 \times 0.25) + (0.5 \times 4)}{8 - (8 \times 0.25) + (0.5 \times 4)} = 0.5$

$C_2 = 1 - C_0 - C_1 = 1 - 0 - 0.5 = 0.5$

$Q_2 = C_O I_2 + C_1 I_1 + C_2 Q_1$

$= 0 + (0.5 \times 8) + (0.5 \times 8)$

$= 8 m^3/sec.$ Choice (B)

61. $\eta_a = 0.8$, $\eta_c = 0.7$

Net irrigation requirement, NIR = 14.9

$FIR = \frac{NIR}{n_a} = \frac{14.9}{0.8} = 18.65cm$

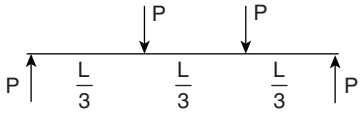
$GIR = \frac{FIR}{n_c} = \frac{18.625}{0.7} = 26.607cm.$ Choice (D)

62. $Q = 50 m^3/sec.$ $f = 1$, $x = 0.5$

$V = \left(\frac{Qf^2}{140}\right)^{\frac{1}{6}} = \left(\frac{50 \times 1^2}{140}\right)^{\frac{1}{6}} = 0.842 m/s$

$A = \frac{50}{0.842} = 59.38 m^2.$ Choice (D)

63.



Space diagram is as shown in the figure.
Bending moment at the centre

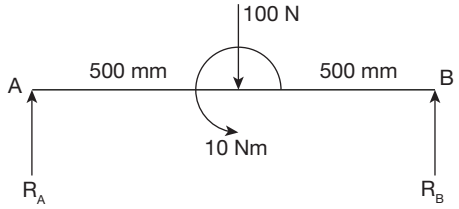
$$= P \times \frac{L}{2} - P \times \frac{L}{6} = PL\left(\frac{1}{2} - \frac{1}{6}\right) = \frac{PL}{3}$$ Choice (B)

64. As there is no shear stress the stresses shown may be taken as principal stresses
$p_1 = 80$ MPa; $p_2 = -20$ MPa

Radius of Mohr's circle $= \frac{p_1 - p_2}{2} = \frac{80 + 20}{2}$

$= 50$ MPa Choice (C)

65.



$R_A + R_B = 100$ N
Taking moment about A,

$R_B \times 1 + 10 - 100 \times 0.5 = 0$

$\Rightarrow R_B = 50 - 10 = 40$ N

$\Rightarrow R_A = 100 - 40 = 60$ N

Maximum bending moment $= 60 \times 0.5 = 30$ Nm

Choice (A)

Mock Test 3

Number of Questions: 65 **Total Marks: 100**

Section – I: General Aptitude

Questions 1 to 5 carry One Mark each.

Directions for question 1: Select the correct alternative from the given choices.

1. In a UG and PG College with a total strength of 1000, 26% of the students are girls. There are 520 PGs and 38 more male UGs than female UGs. What percent of the males are UGs?
(A) 45%
(B) 65%
(C) 26%
(D) 35%

Directions for question 2: In the question below, there is a main statement followed by four statements *a*, *b*, *c* and *d*. From the choices, choose the ordered pair where the first statement implies the second statement and the two are logically consistent with the main statement.

2. Unless the Indian government seals the borders illegal migration in India will not stop.
a. Indian government sealed the border.
b. Illegal migration in India stopped.
c. Indian government had not sealed the borders.
d. Illegal migration in India will not stop.
(A) *cd* (B) *ab*
(C) *cb* (D) *ad*

Directions for question 3: In the question below, determine the relationship between the pair of capitalised words and then select the pair of words which has a **similar** relationship to the capitalized words and mark the number of that pair as your answer.

3. BEAVER : INDUSTRIOUS
(A) Dog : Watchful
(B) Hyena : Frightening
(C) Vixen : Waiting
(D) Fox : Cunning

Directions for question 4: In the given sentence select the correct alternative that best explains the meaning of the **idiom** from the given choices.

4. The leader of the opposition party had an axe to grind against the *CM* ever since he lost the elections.
Here 'axe to grind' means
(A) a damaging condemnation.
(B) an authoritative air.
(C) having a selfish motive for doing something.
(D) an official disgrace.

Directions for question 5: Identify the underlined part that has an error (of grammar usage, word choice or idiom) and write the number of that underlined part as your answer.

5. In this special meeting, many individuals
(A) (B)
as well as some groups has participated.
(C) (D)

Questions 6 to 10 carry Two Marks each.

Directions for questions 6 to 8: Select the correct alternative from the given choices.

6. The terrorist attacks in Mumbai has raised security concerns. These attacks have raised serious doubts about the safety of the residents of Mumbai.
Which of the following can be an **inference** drawn from the above statements? (An inference is something which is not directly stated but can be inferred from the given facts)
(A) Terrorists attacked common citizens.
(B) Terrorist attacked a province of high importance.
(C) The residents of Mumbai are shifting to other cities.
(D) None of these

7. If ℓ, m and n are distinct and $\ell^a = m^b = n^c = \ell mn$, then $ab + bc + ca =$
(A) $(a+b+c)^2$ (B) $a(b+c)$
(C) abc (D) $abc(a+b+c)$

8. A can complete a job $2\frac{1}{2}$ times as big as job X in 25 hours. B can complete a job $3\frac{1}{2}$ times as big as X in 42 hours. C can complete a job $4\frac{1}{2}$ times as big as X in 49 $\frac{1}{2}$ hours. D can complete the job $5\frac{1}{2}$ times as big as X in $54\frac{9}{20}$ hours. Who is the most efficient among the four?
(A) D (B) C
(C) B (D) A

Directions for question 9: In the following question statements 1 and 6 are respectively the first and the last sentences of a paragraph and statements (2), (3), (4) and (5) come in between them. Rearrange (2), (3), (4) and (5) in such a way that they make a coherent paragraph together with statements 1 and 6. Select the correct order from the given choices and mark its number as your answer.

9. (1) strology is considered to be one of the six limbs of the Vedas.

(2) Astrology's inclusion as a subject in universities is not a retrograde step and certainly doesn't deserve to be derided.
(3) It will create a sensation in the West like yoga – ridiculed by pseudo secularists here but assimilated by the Europeans and Americans – did.
(4) Astrology attempts to interpret the influence of heavenly bodies on human affairs.
(5) It is, in fact, a pragmatic measure considering that our society is misguided by some unscrupulous elements in the garb of astrologers who have no basic understanding of the subject.
(6) Incidentally, when a dead subject like Marxism can continue to be taught, why not astrology in which a vast majority of Indians have faith?

(A) 2453
(B) 4253
(C) 2345
(D) 4352

Directions for question 10: Select the correct alternative from the given choices.

10. Cases of food poisoning have been reported from village '*X*'. After a dinner party arranged for 100 people, 68 have been admitted to the hospital, 36 cases are reported to be out of danger. The food, which was cooked and stored in an open space for almost 12 hours earlier was served after reheating it. Investigation is going on. A news report.
Which of the following can be hypothesized from the above information?
(A) Late night dinner parties for large number of people result into food poisoning.
(B) Stale food is likely to be the cause of food poisoning.
(C) Cases of food poisoning need to be handled carefully.
(D) Food poisoning is a matter of chance and no preventive measure can be suggested.

SECTION – II: CIVIL ENGINEERING

Directions for questions 11 to 65: Select the correct alternative form the given choices.

Questions 11 to 35 carry One Mark each.

11. By changing the order of integration, the double integral $\int_{-1}^{1}\int_{x-1}^{1-x} f(x, y)dxdy$ changes to _____.

(A) $\int_{-1}^{1}\int_{1+y}^{1-y} f(x, y)dxdy$

(B) $\int_{-1}^{0}\int_{1+y}^{1-y} f(x, y)dxdy$

(C) $\int_{0}^{1}\int_{1+y}^{1-y} f(x, y)dxdy$

(D) $\int_{-1}^{0}\int_{0}^{1+y} f(x, y)dxdy + \int_{0}^{1}\int_{0}^{1-y} f(x, y)dxdy$

12. The solution of the differential equation $\frac{dy}{dx}+\frac{2x}{y} = 0$; $y(0) = 2$ at $x = 0.1$ obtained by the Runge-Kutta method of second order with step size 0.1 is _____.

(A) 1.995 (B) 2.005
(C) 1.905 (D) 2.01

13. The value of $\int_{-\infty}^{-2} \frac{20}{x^6} dx$ is _____.

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$
(C) $\frac{1}{16}$ (D) $\frac{7}{8}$

14. If the trace of a 3×3 matrix A with all of its eigenvalues being one and the same is $\frac{15}{2}$, then the determinant of A is _____.

(A) $\frac{125}{4}$ (B) $\frac{25}{4}$
(C) $\frac{125}{8}$ (D) $\frac{25}{8}$

15. The partial differential equation of $z = e^y f(x^2 - y^2)$ is _____.

(A) $zx = py + qx$ (B) $zy = px + qy$
(C) $zx = py - qx$ (D) $zy = px - qy$

16. A rapid test to indicate the intensity of pollution in river water is_____

(A) *BOD* (B) *DO*
(C) *MPN* (D) *TDS*

17. The unit in which both sedimentation and digestion processes of sludge takes place simultaneously is ____

(A) Skimming tank (B) Imhoff tank
(C) Deritus tank (D) Digestion tank

18. Modulus of elasticity of concrete is _____

1. Tangent Modulus
2. Secant Modulus
3. Proportional to $\sqrt{fck}$
4. Proportional $\frac{1}{\sqrt{fck}}$

Which of these statements are correct?

(A) 1 and 3 only (B) 1 and 4 only
(C) 2 and 3 only (D) 2 and 4 only

19. The timber preservative "Creosote" belongs to group of
(A) Water soluble salts (B) Inorganic solvent
(C) Tar oil (D) Organic solvent

20. Minimum clear cover (in mm) to the main steel in slab, beam, column are respectively _____
(A) 15,25 and 40 (B) 20, 25 and 30
(C) 20,35 and 50 (D) 10,15 and 25

21. If optimistic, pessimistic and expected times of an activity are given as 8, 16 and 12 days respectively, find the most probable time of the given activity?
(A) 16 days (B) 12 days
(C) 8 days (D) 10 days

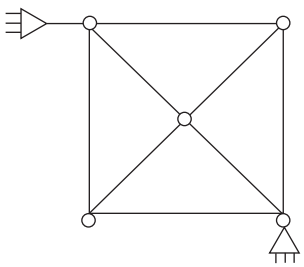
22. The maximum longitudinal pitch in bolted joints, subjected to tensile forces, (Where t – thickness of the plate and D – diameter of the bolt)
(A) 32D (B) 16D
(C) 32t (D) 16t

23. If the gross bearing capacity of a strip footing of 1.2 m wide located at a depth of 2 m in a clay is 340 kN/m^2, its net bearing capacity for $\gamma = 20$ kN/m^3 is_____ (in kN/m^2)
(A) 250 (B) 300
(C) 350 (D) 400

24. Which of the following statements below are correct?
(i) Vibratory rollers are best suited for compacting sandy soil.
(ii) Relative compaction is the same as relative density.
(iii) Zero air void line and 100% saturation line are identical.
(A) (i), (ii), (iii) are correct
(B) (i) and (iii) are correct
(C) (i) and (ii) are correct
(D) (ii) and (iii) are correct

25. A concentrated load of 500 kN acts vertically at a point on the soil surface. According to Boussinesq's equation the ratio of the vertical stresses at a depth of 2 m and 4 m is_____
(A) 0.5 (B) 0.25
(C) 2 (D) 4

26. The static indeterminacy of the truss shown below is___



(A) 0 (B) 1
(C) 2 (D) 3

27. Consumptive use refers to loss of water as a result of
(A) Evaporation and transpiration.
(B) Crop water requirement.
(C) Evaporation and Infiltration.
(D) Evaporation and transpiration from cropped area.

28. Garret's diagrams are used to
(A) Separate base flow from total run off.
(B) Correct inconsistency in rainfall data.
(C) Determine reservoir capacity.
(D) Design channels.

29. "Isogonic lines" passes through points of
(A) Zero declination (B) Equal declination
(C) Equal dip (D) None of above

30. Which of the following closely represents the shape of the earth.
(A) Spheroid (B) Ellipsoid
(C) Oblate spheroid (D) Oblate ellipsoid

31. The consistency and flow resistance of bitumen can be determined from the following
(A) Ductility test (B) Penetration test
(C) Softening point test (D) Viscosity test

32. A velocity potential function is given by $\phi = 3xy$. x and y components of velocity at point (1, 2) is
(A) 3, 6 (B) 6, 3
(C) –3, –6 (D) –6, –3

33. In a pipe of 300 mm diameter and 700 m length an oil of specific gravity 0.8 is flowing at the rate of 0.45 m^3/s. If coefficient of friction is 0.00498, head loss due to friction (in m) of oil is
(A) 96 (B) 115
(C) 124 (D) 136

34. For viscous flow through pipe, Darcy-Weisbach friction factor in terms of Reynold's number R_e is given by
(A) $\frac{16}{R_e}$ (B) $\frac{32}{R_e}$
(C) $\frac{48}{R_e}$ (D) $\frac{64}{R_e}$

35. State of stress at a point in a stressed body are, $\sigma_x = 900$ MPa, $\sigma_y = 300$ MPa, $\tau_{xy} = 400$ MPa. Radius of the circle representing this state of stress is
(A) 300 MPa (B) 400 MPa
(C) 500 MPa (D) 600 MPa

Questions 36 to 65 carry Two Marks each.

36. The directional derivative of $f(x, y, z) = x\ell nz - y^2 + 40$ at (–1, 2, 1) in the direction of the normal to the surface $4x^2y + z^3 = 4$ at (1, –1, 2) is _____.
(A) $\sqrt{\frac{7}{2}}$ (B) $\frac{7}{\sqrt{2}}$
(C) $-\sqrt{\frac{7}{2}}$ (D) $\frac{-7}{\sqrt{2}}$

37. If A and B are two events of the sample space S such that $P(B) = \frac{3}{5}$ and $P(A' \cap B') = \frac{1}{6}$ (where E' denote the complement of an event E), then $P\left(\frac{A}{B'}\right)$ is _____.

(A) $\frac{7}{12}$ (B) $\frac{2}{3}$

(C) $\frac{3}{4}$ (D) $\frac{5}{12}$

38. Which of the following functions is/are continuous at $x = 2$? (Here $[x]$ denotes the greatest integer less than or equal to x).

I. $f(x) = \begin{cases} 3x - [x]; & \text{if } x < 2 \\ 5 \quad\quad ; & \text{if } x = 2 \\ x + 3 \; ; & \text{if } x > 2 \end{cases}$

II. $g(x) = \begin{cases} \frac{x^2 + 2x - 8}{x^2 + 3x - 10}; & \text{if } x \neq 2 \\ \frac{5}{7} \quad\quad ; & \text{if } x = 2 \end{cases}$

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

39. If A is a real matrix of order 3×3 with trace -1 and $-2 - 5i$ as one of the eigenvalues, then the determinant of A is _____.

(A) 63 (B) −63

(C) 87 (D) −87

40. The Laplace transform of the solution of the initial value problem

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} - 14x = 0$; $x(0) = 1$ and $\left(\frac{dx}{dt}\right)_{at\ t=0} = 2$

is _____.

(A) $\frac{1}{s-2}$ (B) $\frac{1}{s+7}$

(C) $\frac{s+2}{s^2+5s-14}$ (D) $\frac{s-7}{s^2+5s-14}$

41. A slender column is fixed at one end and the other end is hinged. Euler's buckling load and effective length respectively are.

(A) $\frac{\pi^2 EI}{\ell^2}, \ell$ (B) $\frac{2\pi^2 EI}{\ell^2}, \frac{\ell}{\sqrt{2}}$

(C) $\frac{4\pi^2 EI}{\ell^2}, \frac{\ell}{2}$ (D) $\frac{\pi^2 EI}{4\ell^2}, 2\ell$

42. A mild steel flat subjected to tensile force of 84 tonnes is connected to a gusset plate using rivets. If the force required to sheer a single rivet, to crush the rivet and to tear the plate per pitch length are 5000 Kg, 8000 kg and 6000 kg respectively; then the number of rivets required is _____

(A) 12 (B) 14

(C) 16 (D) 17

43. For a fixed beam with a concentrated load W at $\frac{1}{4}$ th of span from an end; the collapse load is _____

(A) $\frac{16\ Mp}{3L}$ (B) $\frac{4\ Mp}{L}$

(C) $\frac{32\ Mp}{3L}$ (D) $\frac{6\ Mp}{L}$

44. Consider the singly reinforced beam section with following parameters. Calculate the moment of resistance by Limit state method?

Grade of concrete used: M30

Reinforcement: 5 No's of Fe 415 having dia of 16 mm

Cross section: 200 mm × 350 mm

Effective cover: 25 mm

(A) 73 KN-m (B) 87 KN-m

(C) 93 Kw-m (D) 112 KN-m

45. Results of a water sample analysis are as follows.

Cation	Concentration (mg/l)	Equivalent weight
Na^+	40	23
Mg^{+2}	10	12.2
Ca^{+2}	55	20
K^+	12	39

Hardness of the water sample in mg/L as $CaCO_3$ is ___

(A) 44.8 (B) 89.5

(C) 179 (D) 358

46. A landfill is to be designed to serve a population of 2,00,000 for a period of 20 years. The solid waste (SW) generated is 2.5 Kg/person/day. The density of un-compacted SW is 100 Kg/m^3 and a compaction ratio of 4 is recommended. The ratio of compacted fill (i.e; SW + cover) to compacted SW is 1.5. The landfill volume (in million m^3) required is _____

(A) 10.6 (B) 11.6

(C) 12.7 (D) 13.7

47. Two Electro static precipitators (ESPS) are in series. The fractional efficiencies of the up stream and down stream ESPS for a size dp are 80% and 65%, respectively. What is the overall efficiency of the system for the same *dp*?

(A) 100% (B) 93%

(C) 80% (D) 65%

48. A sample of dry soil weighs 50 gm. Find the volume of voids (in ml) if the total volume of the sample is 50 ml and the specific gravity of solids is 2.65.

(A) 31.5 (B) 25.2

(C) 15.4 (D) 42.1

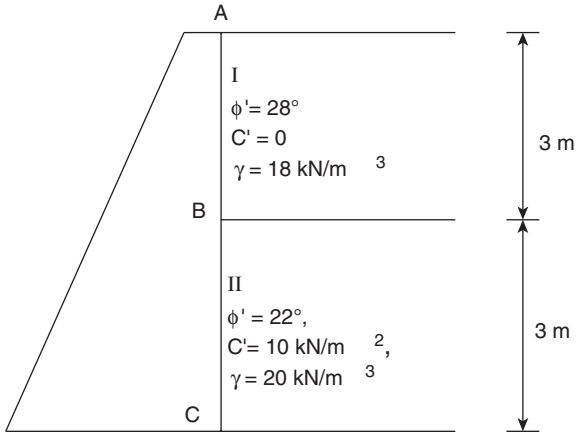
49. The consistency limits of a soil sample are;
Liquid limit = 40%
Plastic limit = 25%
Shrinkage limit = 5%.
If the specimen of this soil shrinks from a volume of 7.5 cm^3 at liquid limit to 4.5 cm^3 at the shrinkage limit, the specific gravity of soil solids is _____
(A) 2.25 (B) 2.75
(C) 2.10 (D) 2.02

50. In a consolidation test on a soil, the void ratio of the sample decreased from 1.25 to 1.10 when the pressure is increased from 250 kN/m^2 to 450 kN/m^2. The coefficient of consolidation of soil if the coefficient of permeability is 8×10^{-8} cm/sec is ____ (m^2/year).
(A) 6.7 (B) 7.7
(C) 8.7 (D) 9.7

51. A shear vane, 7.5 cm dia and 11.25 cm long, was pressed into soft clay at the bottom of a bore hole. The shear strength of clay if the torque required for failure was 50 N-m is____(in kN/m^2)
(A) 25 (B) 40
(C) 50 (D) 65

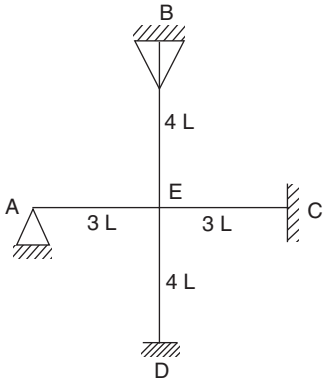
52. A long natural slope in an over consolidated clay (C' = 12 kN/m^2, ϕ' = 30°, γ_{sat} = 21 kN/m^3) is inclined at 10° to the horizontal. The water table is at the surface and the seepage is parallel to the slope. The factor of safety of slope at a depth of 5 m below the slope is_____ (take γ_w = 10 kN/m^3).
(A) 2.12 (B) 2.38
(C) 3.72 (D) 4.24

53. The retaining wall with backfill conditions are shown in below figure. The total passive force at bottom of the wall is____ (Take water table at level of B and γ_w = 10 kN/m^3).



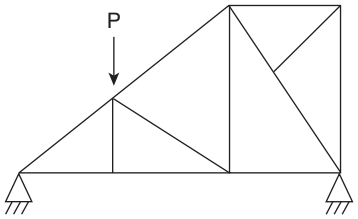
(A) 815 kN
(B) 755 kN
(C) 635 kN
(D) 952 kN

54. Distribution factor for DE in given figure is



(A) $\frac{12}{49}$ (B) $\frac{3}{13}$
(C) 1 (D) $\frac{49}{48}$

55. The number of Zero force members in the truss shown below is____



(A) 3 (B) 4
(C) 5 (D) 6

56. The following rainfall data refers to station A and B which are equidistant from station X

Long term	A	x	B
Normal Annual rainfall in mm	200	250	300
Annul rainfall in mm for the year 1940	140	P	270

The value of P will be.
(A) 250 (B) 220
(C) 205 (D) 200

57. A direct runoff hydrograph due to isolated storm was triangular in shape with a base of 80 h and peak of 200 m^3/s. If the catchment area is 1440 km^2, the effective rainfall of the storm is
(A) 20 cm (B) 10 cm
(C) 5cm (D) 2 cm

58. What is the maximum length of an offset so that the displacement of a point on the paper should not exceed 0.25 mm, gives that maximum error expected is 2.5° from its true direction and the scale is 1 : 2000
(A) 11.46 m (B) 1.48 m
(C) 14.32 m (D) 9.82 m

59. It is required to set up points on a sloping down ground of line 50 m at every 20 m contour interval. If the staff

reading over first point is 0.55 m. The staff reading over next point *B* is

(A) 0.95 m (B) 0.15 m
(C) 1.05 m (D) 0.25 m

60. Sliding considerations for stopped vehicle on super elevation. Horizontal curve provide the following bound on amount of super elevation.

(A) $e >$ coefficient of rolling friction
(B) $e >$ coefficient of side friction.
(C) $e >$ coefficient of rolling friction.
(D) $e >$ coefficient of side friction.

61. A vertical summit curve is formed at the intersection two gradients +3% and –6%. Design sped of two lane two way is 80 kmph. Driver reaction time is 2.5 sec. Over the curve to stop the vehicle frictional coefficient between road and tyres is 0.35. Then the length of the summit curve based on *SSD* is

(A) 296 m (B) 1182m
(C) 73.6 m (D) 1472 m

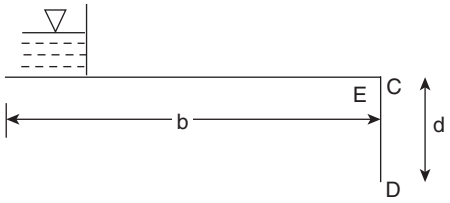
62. Consider the following factors:

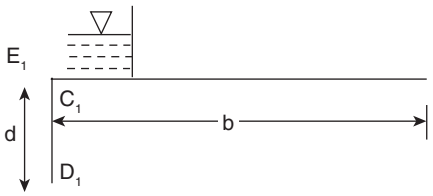
1. Reaction time
2. Speed
3. Coefficient of longitudinal fraction
4. Gradient

Which of these factors are taken into account for computing braking distance?

(A) 1 and 3 (B) 1, 2 and 4
(C) 2, 3 and 4 (D) 2 and 3

63. The uplift pressure at points *E* and *D* of a straight horizontal floor of negligible thickness with a sheet pile at down stream end are 30% and 40% respectively. If the sheet pile is at upstream end of floor the uplift pressure at point D_1 and C_1



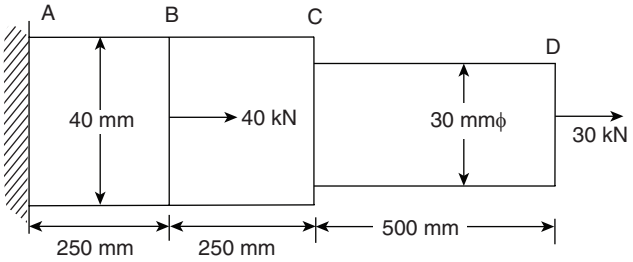


(A) 68% and 60% respectively
(B) 80% and 72% respectively
(C) 70% and 60% respectively
(D) 40% and 80% respectively

64. In a smooth inclined pipe of 250 mm diameter carrying water, point 2 is at a higher elevation of 2 m from point 1. Velocity through the pipe is 1.25 m/s. Pressures at points 1 and 2 are 50 kPa and 20 kPa respectively. Rate of flow (in m^3/s) through the pipe and direction of flow are

(A) 0.0614, upwards
(B) 0.0614, downwards
(C) 0.0307, upwards
(D) 0.0307, downwards

65.



In the composite bar as shown above portion *AC* is made of steel and portion *CD* is made of copper. The two materials are rigidly joined at *C*. Modulus of elasticities of steel and copper are 2×10^5 N/mm^2 and 1.1×10^5 N/mm^2 respectively. If the bar is loaded as shown in the figure, total extension produced in mm is _____.

(A) 0.2 mm
(B) 0.3 mm
(C) 0.4 mm
(D) 0.5 mm

Answer Keys

1. D	**2.** A	**3.** D	**4.** C	**5.** D	**6.** D	**7.** C	**8.** A	**9.** B	**10.** B
11. D	**12.** A	**13.** B	**14.** C	**15.** A	**16.** D	**17.** B	**18.** C	**19.** C	**20.** A
21. B	**22.** D	**23.** B	**24.** B	**25.** D	**26.** B	**27.** D	**28.** D	**29.** B	**30.** C
31. D	**32.** D	**33.** A	**34.** D	**35.** C	**36.** C	**37.** A	**38.** A	**39.** C	**40.** A
41. B	**42.** D	**43.** C	**44.** B	**45.** C	**46.** D	**47.** B	**48.** A	**49.** C	**50.** B
51. B	**52.** B	**53.** A	**54.** A	**55.** B	**56.** D	**57.** D	**58.** A	**59.** A	**60.** B
61. B	**62.** C	**63.** C	**64.** A	**65.** B					

Hints and Explanations

1. The data is tabulated below

	Girls	Boys	Total
PG	39	481	520
UG	221	259	480
Total	260	740	1000

Percentage of males who are UG's is

$\frac{259}{740}\times 100 = 35\%$. Choice (D)

2. p

Unless [The Indian government seals the borders,]

q

[illegal migration will not stop.]

The implications of the above statement will

(i) $\sim p \Rightarrow q$ and

(ii) $\sim q \Rightarrow p$.

It can be CD or BA. Choice (A)

3. A 'beaver's' nature is to be 'industrious' (hard working). Among all the options, D is the correct one since 'cunning' is the nature of a fox. A dog is 'faithful' and being watchful is its secondary trait. A Hyena is not necessarily frightening. 'Vixen' (a female fox) and 'waiting' are unconnected. Choice (D)

4. When someone has 'an axe to grind' against someone or something, it means he/she has a 'false grievance' or a selfish motive for doing something. Choice (C)

5. The reference is the 'some groups' (in the plural). Hence, 'has' is erroneous and should be replaced with 'have' as the subject is a plural one. Choice (D)

6. No information is given whom the terrorists targeted. Hence, neither (A) nor (B) can be inferred. It cannot be inferred that the residents are moving out of Mumbai. Hence (C) cannot be inferred. Hence, none can be inferred. Choice (D)

7. Let $\ell^a = m^b = n^c = lmn = \lambda$

$\ell = \lambda^{\frac{1}{a}}$, $m = \lambda^{\frac{1}{b}}$ and $n = \lambda^{\frac{1}{c}}$

$\ell mn = (\lambda)^{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\lambda^1 = \lambda^{\frac{ab+bc+ca}{abc}}$

$\Rightarrow \quad \frac{ab+bc+ca}{abc} = 1 \Rightarrow ab + bc + ca = abc.$

Choice (C)

8. Times in which A, B, C and D can complete 'X' (in hours) are $\frac{25}{2\frac{1}{2}}, \frac{42}{3\frac{1}{2}}, \frac{49\frac{1}{2}}{4\frac{1}{2}}$ and $\frac{54\frac{9}{20}}{5\frac{1}{2}}$ respectively.

$\frac{25}{2\frac{1}{2}} = \frac{25}{2.5} = 10$; $\frac{42}{3\frac{1}{2}} = \frac{42}{3.5}$ which is > 10.

Similarly it follows that $\frac{49\frac{1}{2}}{4\frac{1}{2}} > 10$ and $\frac{54\frac{9}{20}}{5\frac{1}{2}} < 10$

$\therefore$ D would take the least time to complete the job.

$\therefore$ D is the most efficient. Choice (A)

9. It would be more appropriate for statement (4) to follow statement 1 because it explains the function of "astrology" which has been introduced in the previous sentence. (4) cannot be followed by (3) because of the future tense in 'it will'. Choice (B)

10. The report says that 68 out of 100 people were admitted to the hospital because of the food poisoning. The most appropriate cause, which is stated in the statements, is stale food. Choice (B)

11. Given double integral is <<Eqn3767.eps>>= I, (say)

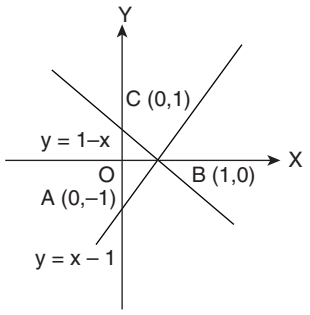
Here y varies from $y = x - 1$ to $y = 1 - x$

And x varies from $x = 0$ to $x = 1$

$\therefore$ The region of integration is the triangle ABC as shown in the figure.

By changing the order of integration, the given integral becomes,

$$I = \iint_{OAB} f(x, y)dxdy + \iint_{OBC} f(x, y)dxdy \quad \rightarrow (I)$$



Where in triangle OAB, x varies from $x = 0$ to $x = 1 + y$ and y varies from $y = -1$ to $y = 0$

And in triangle OBC,

x varies from $x = 0$ to $x = 1 - y$

and y varies from $y = 0$ to $y = 1$

$\therefore$ By changing the order of integration, we have

$$I = \int_0^1 \int_{x-1}^{1-x} f(x, y)dxdy = \int_{-1}^0 \int_0^{1+y} f(x, y)dxdy$$

$$+ \int_0^1 \int_0^{1-y} f(x, y)dxdy$$ Choice (D)

12. Given differential equation is $\frac{dy}{dx}+\frac{2x}{y}=0; y(0)=2$

$\Rightarrow \quad \frac{dy}{dx}=\frac{-2x}{y}$

$\therefore \quad f(x,y)=\frac{-2x}{y}; h=0.1;$

$x_0=0; y_0=y(0)=2$

By Runge-Kutta method of second order, we have

$$y_1=y_0+\frac{1}{2}[k_1+k_2] \qquad \rightarrow (1)$$

where $k_1=hf(x_0,y_0)$

$$=h\left(\frac{-2x_0}{y_0}\right)=(0.1)\left(\frac{-2\times 0}{2}\right)$$

$\therefore \quad k_1=0$

and $k_2=hf(x_0+h, y_0+k_1)$

$$=h\left(\frac{-2(x_0+h)}{y_0+k_1}\right)=(0.1)\left(\frac{-2(0+0.1)}{2+0}\right)$$

$k_2=-0.01$

$\therefore$ Substituting these in (1), we get

$$y_1=y(0.1)=2+\frac{1}{2}[0+(-0.01)]$$

$$=1.995$$ Choice (A)

13. We have $\int_{-\infty}^{-2}\frac{20}{x^6}dx=\underset{A\to\infty}{Lim}\int_{-A}^{-2}\frac{20}{x^6}dx$

$$=\underset{A\to\infty}{Lim}\frac{20}{-5x^5}\Big]_{-A}^{-2}$$

$$=\underset{A\to\infty}{Lim}\left[\frac{20}{-5(-2)^5}-\frac{20}{-5(-A)^5}\right]$$

$$=\underset{A\to\infty}{Lim}\left[\frac{1}{8}-\frac{4}{A^5}\right]=\frac{1}{8}-0=\frac{1}{8}$$ Choice (B)

14. Let λ be an eigenvalue of A.

$\therefore$ Trace of $A=\frac{15}{2}$

$\Rightarrow \quad \lambda+\lambda+\lambda=\frac{15}{2}$ ($\because$ Trace of A = Sum of the eigenvalues of A)

$\Rightarrow \quad \lambda=\frac{5}{2}$

$\therefore$ The determinant of A = Product of the eigenvalues of $A=\lambda\times\lambda\times\lambda=\lambda^3$

$$=\left(\frac{5}{2}\right)^3=\frac{125}{8}$$ Choice (C)

15. Given $z=e^y f(x^2-y^2)$ $\rightarrow (1)$

$\Rightarrow \quad f(x^2-y^2)=\frac{z}{e^y}$ $\rightarrow (2)$

Differentiating (2) *w.r.t* x partially,

$$f'(x^2-y^2).2x=\frac{1}{e^y}\frac{\partial z}{\partial x}$$

$\Rightarrow \quad 2x.f'(x^2-y^2)=\frac{p}{e^y}$ $\rightarrow (3)$

Differentiating (2) w.r.t y partially

$$f'(x^2-y^2)(-2y)=\frac{e^y\frac{\partial z}{\partial y}-ze^y}{(e^y)^2}$$

$\Rightarrow \quad -2yf'(x^2-y^2)=\frac{q-z}{e^y}$ $\rightarrow (4)$

$$\frac{(3)}{(4)}\Rightarrow\frac{2xf\left(x^2-y^2\right)}{-2yf\left(x^2-y^2\right)}=\frac{\frac{p}{e^y}}{\frac{(q-z)}{e^y}}$$

$\Rightarrow \quad \frac{-x}{y}=\frac{p}{q-z}$

$\Rightarrow \quad -qx+zx=py$

$\Rightarrow \quad zx=py+qx$

$\therefore$ The partial differential equation of (1) is

$zx=py+qx$ Choice (A)

16. Total dissolved solids, TDS indicates the intensity of the pollution in a rapid manner. Choice (D)

17. Imhoff tank Choice (B)

18. $E_c=5000\sqrt{fck}$ and it is secant modulus

Choice (C)

19. Creosote belongs to group of " Tar oil" Choice (C)

20. Min clear cover for slabs, beams, columns, and footings are 15mm, 25mm, 40mm and 50mm respectively.

Choice (A)

21. $t_O=8$ days

$t_p=16$ days

$t_e=12$ days

$$t_e=\frac{t_0+4t_m+t_p}{6}=\frac{8+4(t_m)+16}{6}=12$$

$\therefore \quad t_m=12$ days. Choice (B)

22. The maximum longitudinal pitch in tension $16t$ or 200mm, whichever is less and in compression is $12t$ or 200mm which ever is less. Choice (D)

23. $q_n=q_u-\gamma.D=340-20\times 2=300\ \text{kN/m}^2$

Choice (B)

24. Statement (i) and (iii) are correct. Choice (B)

25. $\frac{(\sigma_z)_1}{(\sigma_z)_2}=\frac{(Z_2)^2}{(Z_1)^2}=\frac{4^2}{2^2}=\frac{4\times 4}{4}=4.$ Choice (D)

26. $D_s=(m+r)-2j=(9+4)-2\times 6=13-12$

$D_s=1$ Choice (B)

32. For potential function

$$u = \frac{-\partial\phi}{\partial x} \text{ and}$$

$$v = \frac{-\partial\phi}{\partial y}$$

$$\therefore \quad u = \frac{-\partial(3xy)}{\partial x} = -3y$$

$$\text{and } v = \frac{-\partial(3xy)}{\partial y} = -3x$$

At point (1, 2)

$u = -3 \times 2 = -6$

$v = -3 \times 1 = -3$

i.e., x component $u = -6$

y component $v = -3$ Choice (D)

33. $D = 300$ mm

$= 0.3$ m

$L = 7$ m

$Q = 0.45\ m^3/s$

$f = 0.00498$

$$h_f = \frac{4fL}{d}\frac{v^2}{2g} = \frac{fLQ^2}{3D^5}$$

$$= \frac{0.00498 \times 700 \times (0.45)^2}{3 \times (0.3)^5}$$

$= 96.83$ m Choice (A)

34. Friction coefficient $f = \dfrac{16}{R_e}$

Friction factor $= 4f = \dfrac{64}{R_e}$ Choice (D)

35. Radius of the Mohr's circle

= Maximum shear stress

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{900 - 300}{2}\right)^2 + 400^2}$$

$= 500$ MPa Choice (C)

36. Given $f(x, y, z) = x\ell nz - y^2 + 40$

$$\therefore \nabla f = \frac{\partial f}{\partial x}\bar{i} + \frac{\partial f}{\partial y}\bar{j} + \frac{\partial f}{\partial z}\bar{k} = \ell nz\,\bar{i} - 2y\,\bar{j} + \frac{x}{z}\,\bar{k}$$

$$\therefore \quad \nabla f_{at(-1, 2, 1)} = -4\,\bar{j} - \bar{k}$$

Let $\phi(x, y, z) = 4x^2y + z^3 - 4 = 0$

$\therefore$ Normal to the surface $\phi(x, y, z) = 0$ is

$$\nabla\phi = \frac{\partial\phi}{\partial x}\bar{i} + \frac{\partial\phi}{\partial y}\bar{j} + \frac{\partial\phi}{\partial x}\bar{k}$$

$$= 8xy\,\bar{i} + 4x^2\,\bar{j} + 3z^2\,\bar{k}$$

$\therefore$ Normal to $\phi(x, y, z) = 0$ at $(1, -1, 2)$ is

$$\nabla\phi_{at(1, -1, 2)} = -8\,\bar{i} + 4\,\bar{j} + 12\,\bar{k}$$

$$\therefore \ \hat{N} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{-8\bar{i} + 4\bar{j} + 12\bar{k}}{\sqrt{(-8)^2 + 4^2 + 12^2}}$$

$$= \frac{4(-2\bar{i} + 4\,\bar{j} + 3\bar{k})}{4\sqrt{4 + 1 + 9}}$$

$$\hat{N} = \frac{-2}{\sqrt{14}}\,\bar{i} + \frac{1}{\sqrt{14}}\,\bar{j} + \frac{3}{\sqrt{14}}\bar{k}$$

$\therefore$ The directional derivative of $f(x, y, z)$ in the direction of the normal to the surface $\phi(x, y, z) = 0$ is $\nabla f.\ \hat{N}$

$$= (-4\,\bar{j} - \bar{k}).\left(\frac{-2}{\sqrt{14}}i + \frac{1}{\sqrt{14}}\,\bar{j} + \frac{3}{\sqrt{14}}\bar{k}\right)$$

$$= \frac{-4}{\sqrt{14}} - \frac{3}{\sqrt{14}} = -\sqrt{\frac{7}{2}}$$ Choice (C)

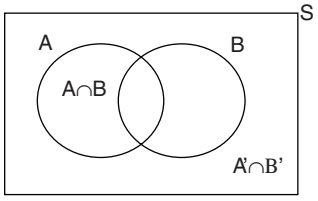
37. Given $P(B) = \dfrac{3}{5}$ and $P(A' \cap B') = \dfrac{1}{6}$

$$P\left(\frac{A}{B'}\right) = \frac{P(A \cap B')}{P(B')}$$

$$= \frac{P(A \cup B) - P(B)}{P(B')}$$

$$= \frac{1 - P(A' \cap B') - P(B)}{1 - P(B)}$$

$$= \frac{1 - \frac{1}{6} - \frac{3}{5}}{1 - \frac{3}{5}} = \frac{\frac{7}{30}}{\frac{2}{5}} = \frac{7}{12}$$



Choice (A)

38. (I) Given $f(x) = \begin{cases} 3x - [x]; & if\ x < 2 \\ 5 \quad\quad ; & if\ x = 2 \\ x + 3 \ ; & if\ x > 2 \end{cases}$

Left Limit $= \underset{x \to 2-}{Lt}\ f(x) = \underset{x \to 2-}{Lt}\ (3x - [x]) = 6 - 1 = 5$

Right Limit $= \underset{x \to 2+}{Lt}\ f(x) = \underset{x \to 2+}{Lt}\ (x + 3) = 2 + 3 = 5$ and

$f(2) = 5$

$$\therefore \quad \underset{x \to 2-}{Lt}\ f(x) = \underset{x \to 2+}{Lt}\ f(x) = f(2)$$

Hence $f(x)$ is continuous at $x = 2$

$\therefore$ (I) is true $\rightarrow$ (1)

(II) Given $g(x) = \begin{cases} \dfrac{x^2+2x-8}{x^2+3x-10}; & if\ x \neq 2 \\ \dfrac{5}{7} \quad\quad ; & if\ x = 2 \end{cases}$

$$\underset{\rightarrow}{Lt}\ g(x) = \underset{\rightarrow}{Lt}\ \frac{x^2+2x-8}{x^2+3x-10}$$

$$= \underset{x\to 2}{Lt}\ \frac{(x+4)(x-2)}{(x+5)(x-2)}$$

$$= \underset{x\to 2}{Lt}\ \frac{x+4}{x+5} = \frac{6}{7}$$

And $g(2) = \frac{6}{7}$

$\therefore$ $\underset{x\to 2}{Lt}\ g(x) \neq g(2)$

Hence $g(x)$ is not continuous at $x = 2$

$\therefore$ (II) is not true $\rightarrow$ (2).

Choice (A)

39. Given $-2 - 5i$ is an eigenvalue of a 3×3 real matrix A.

$\therefore$ $-2 + 5i$ is also an eigenvalue of A.

Let λ be the third eigenvalue of A

Given the trace of $A = -1$

i.e., $(-2 - 5i) + (-2 + 5i) + \lambda = -1$

$\Rightarrow$ $-4 + \lambda = -1 \Rightarrow \lambda = 3$

$\therefore$ The determinant of A = The product of the eigenvalues of A.

$= ((-2 - 5i)(-2 + 5i)\ 3$

$= 87$

Choice (C)

40. Given initial value problem is $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} - 14x = 0$ $\rightarrow$ (1)

where $x(0) = 1$ and $\left(\frac{dx}{dt}\right)_{at\ t=0} = 2$ $\rightarrow$ (2)

Applying Laplace transform on both sides of (1),

we have $L\left[\frac{d^2x}{dt^2}\right] + 5L\left[\frac{dx}{dt}\right]_{t=0} - 14L[x] = L[0]$

$\Rightarrow$ $s^2\bar{x} - sx(0) - x'(0) + 5(s\bar{x} - x(0)) - 14\bar{x} = 0$

where $\bar{x} = L[x]$

$\Rightarrow$ $(s^2 + 5s - 14)\bar{x} - s \times 1 - 2 - 5 \times 1 = 0$

$\Rightarrow$ $(s^2 + 5s - 14)\bar{x} = s + 7$

$\Rightarrow$ $\bar{x} = \frac{(s+7)}{(s+7)(s-2)}$

$\Rightarrow$ $\bar{x} = \frac{1}{s-2}$

$\therefore$ The Laplace transform of the solution is

$\bar{x} = \frac{1}{s-2}$

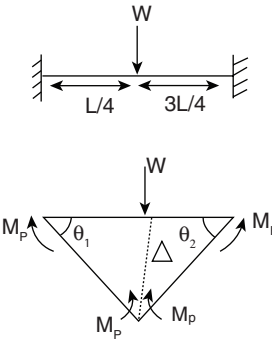
Choice (A)

42. Rivet value = Minimum of {5000, 8000, 6000} kg = 5000 kg.

Number of rivets = $\frac{84 \times 1000}{5000} = 16.8 \cong 17$ No.

Choice (D)

43.



$W.\Delta = 2\ M_p\theta_1 + 2M_p\theta_2$

$\theta_1 = \frac{\Delta}{\frac{L}{4}}$ and $\theta_2 = \frac{\Delta}{\frac{3L}{4}}$

$\Delta = \theta_1 . \frac{L}{4} = \theta_2 . \frac{3L}{4}$

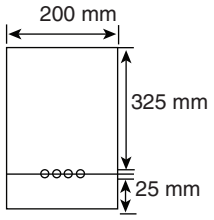
$\therefore$ $\theta_1 = 3\ \theta_2$

$W\Delta = 2M_p(3\ \theta_2 + \theta_2)$

$W . \frac{3L}{4} . \theta_2 = 2M_p . 4\ \theta_2$

$\therefore$ $W_c = \frac{32\ M_p}{3L}$. Choice (C)

44.



$b = 200$ mm and $d = 325$ mm

$Ast = 5 \times \frac{\pi}{4} \times 16^2 = 1005.3\ mm^2$

Depth of neutral Axis,

$$z_u = \frac{0.87 f_y\ Ast}{0.36\ fck.b} = \frac{0.87 \times 415 \times 1005.3}{0.36 \times 30 \times 200}$$

$= 167.988$ mm

$x_u, \max = 0.48\ d = 0.48 \times 325 = 156$ mm

$x_u > x_u \max \Rightarrow$ over reinforced section

So; $M_{u,\lim} = 0.36\ fck\ b\ x_{u,\max}(d - 0.42\ x_{u,\max})$

$= 0.36 \times 30 \times 200 \times 156(325 - 0.42(156))$

$= 87.43 \times 10^6$ N-mm $= 87.43$ kN-m. Choice (B)

45. Total hardness $= \left(Mg^{+2} \times \frac{50}{12.2}\right) + \left(Ca^{+2} \times \frac{50}{20}\right)$

$= \left(10 \times \frac{50}{12.2}\right) + \left(55 \times \frac{50}{20}\right)$

$= 179$ mg/l Choice (C)

46. Solid waste generated per day $= (2 \times 10^5) \times 2.5$ kg/day

$= 5,00,000$ kg/day

Total solid waste generated in 20 years $= 5 \times 10^5 \times 20 \times 365 = 365 \times 10^7$ kg

Compaction ratio $= 4 = \frac{\text{Volume before compaction}}{\text{Volume after compaction}}$

$\Rightarrow$ Volume before compaction,

$V = \frac{365 \times 10^7}{100} = 365 \times 10^5 \text{ m}^3$

$\frac{SW + \text{Cover}}{SW} = 1.5 \Rightarrow 1 + \frac{\text{Cover}}{SW} = 1.5$

Cover volume $= 0.5 \times 91.25 \times 10^5 = 45.625 \times 10^5$ m^3

$\therefore$ Total volume $= SW +$ Cover

$= (91.25 \times 10^5) + (45.625 \times 10^5)$

$= 13.7 \times 10^6$ m$^3 = 13.7$ million m^3 Choice (D)

47. Given that;

$\eta_1 = 80\%$ and $\eta_2 = 65\%$

Since, efficiency of up stream ESP is 80%; only 20% of particulates are not removed.

The remaining 20% particulates will face down stream ESP, which has an efficiency of 65%

So; particulates removed by down stream ESP = 65% of 20%

$= 20 \times \frac{65}{100} = 13\%$

Since they are connected in series, overall efficiency

$= 80\% + 13\% = 93\%$ Choice (B)

48. $\gamma_d = \frac{W_d}{V} = \frac{50}{50} = 1$ gm/cc

$G = \frac{\gamma_s}{\gamma_w} = \frac{w_s}{v_s \gamma_w}$

$\Rightarrow \quad v_s = \frac{w_s}{G\gamma_w} = \frac{50}{2.65 \times 1}$

$V_s = 18.868$ cm^3

$V = V_v + V_s$

$V_v = V - V_s = 50 - 18.868$

$V_v = 31.132$ ml Choice (A)

49. $SR = \left[\frac{\left(\frac{V_L - V_S}{V_S}\right)}{W_L - W_S}\right] \times 100 = \left[\frac{\left(\frac{7.5 - 4.5}{4.5}\right)}{40 - 5}\right] \times 100$

$SR = 1.904\%$

SR is nothing but $(G_m)_{\text{dry}}$

$\therefore \quad Ws = \left[\frac{1}{(G_m)_{dry}} - \frac{1}{G}\right] \times 100$

$5 = \left[\frac{1}{1.904} - \frac{1}{G}\right] \times 100$

$G = 2.10$ Choice (C)

50. $C_v = \frac{K}{m_v \gamma_w}$

$m_v = \frac{a_v}{1 + e_\circ} = \frac{\Delta e}{(1 + e_\circ)\Delta\sigma^1}$

$= \frac{(1.25 - 1.10)}{(1 + 1.25)(200)}$

$m_v = 3.33 \times 10^{-4}$ m^2/kN

$C_v = \frac{8 \times 10^{-8} \times 10^{-2} \text{ m/sec}}{3.33 \times 10^{-4} \text{ m}^2 \text{/KN} \times 9.81 \text{ kN/m}}$

$= 2.446 \times 10^{-5}$ m^2/sec

$C_v = 7.72$ m^2/year Choice (B)

51. $T = \pi d^2 C_u \left(\frac{H}{2} + \frac{d}{6}\right)$

$C_u = \frac{T}{\pi d^2 \left(\frac{H}{2} + \frac{d}{6}\right)} = \frac{50 \times 100 \text{ N} - \text{cm}}{\pi \times (7.5)^2 \times \left(\frac{11.25}{2} + \frac{7.5}{6}\right)}$

$C_u = 4.11 \frac{\text{N}}{\text{cm}^2}$

$= 4.11 \times 10^4$ N/m^2

$C_u = 41.1$kN/m^2 Choice (B)

52. Seepage parallel to the slope,

$F = \frac{C^1 + \gamma^1 z \cos^2 i \tan\phi^1}{\gamma_{sat} Z \cos i \sin i}$

$= \frac{12 + (21 - 10) \times 5 \cos^2 10^0 \tan 30^0}{21 \times 5 \times \cos 10^\circ \times \sin 10^\circ} = 2.38.$

Choice (B)

53. $(k_p)_1 = \frac{1 + \sin\phi^1}{1 - \sin\phi^1} = \frac{1 + \sin 28}{1 - \sin 28} = 2.77$

$(k_p)_2 = \frac{1 + \sin 22^\circ}{1 - \sin 22^\circ} = 2.197$

At top

$Z = 0$;

$\sigma_v = \gamma_z = 0$

$\left(P_p\right)_A = \left(Kp_1\right)\sigma_v + 2C_1\sqrt{\left(Kp\right)_1}$

$= 0 + 2\times 0\times\sqrt{2.77} = 0$

At *B*

Just above:

$Kp_1 = 2.7$

$\left(\sigma_v\right) = \left(\gamma\right)_z = 18\times 3 = 54\ \text{kN/m}^2$

$\left[\left(P_p\right)_B\right]_{at\ top} = 2.77\times 54 + 2\times 0\times\sqrt{2.77}$

$\left(P_p\right)_B = 149.58\ \text{kN/m}^2$

Just below

$\left(Kp\right)_2 = 2.197\ ;\ C_2 = 10\ \text{kN/m}^2$

$\left(\sigma_v\right) = \left(\gamma\right)\times z = 18\times 3 = 54\ \text{kN/m}^2$

$\left[\left(P_p\right)_B\right]_{below} = \left(2.197\right)\times 54 + 2\times 10\times\sqrt{2.197}$

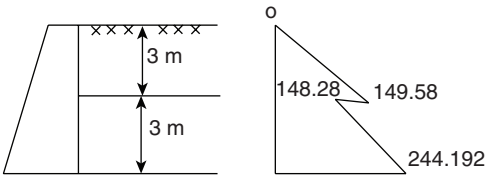
$= 148.28\ \text{kN/m}^2$

At C(i.e; at Bottom):

$\left(\sigma_v\right) = 18\times 3 + \left(20-10\right)\times 3 = 84\ \text{kN/m}^2$

$\left[\left(P_p\right)_c\right] = \left(2.197\right)\times 84 + 2\times 10\sqrt{2.197} + 3\times 10$

$= 244.192$



Pressure diagram

Total Passive force

$= \frac{1}{2}\times 149.58\times 3 + \left[\frac{148.28 + 244.192}{2}\right]\times 3$

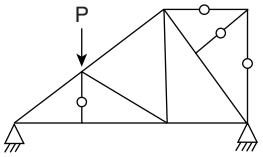
= 813.078 kN. Choice (A)

54.

Joint	Members	Relative stiffness, K	∑K	D.F = $\frac{K}{\Sigma K}$
E	EB	$\frac{3\,I}{4(4L)} = \frac{3\,I}{16L}$	$\frac{49I}{48L}$	$\frac{I\times 48L}{4L\times 49\,I} = \frac{12}{49}$
	EC	$\frac{I}{3L}$		
	ED	$\frac{I}{4\,L}$		
	EA	$\frac{3\,I}{4(3L)} = \frac{3\,I}{12\,L}$		

Choice (A)

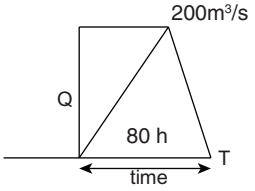
55. The Zero force members are marked below:



Choice (B)

56. $P = \frac{250}{2}\left[\frac{140}{200} + \frac{270}{300}\right] = 200\ \text{mm}$ Choice (D)

57.



Total runoff = Area of triangle

$= \frac{1}{2}\times 80\times 3600\times 200$

$= 28.8\times 10^6\ \text{m}^3$

Area of catchment = 1440 cm²

∴ Effective rainfall

$= \frac{\text{Total runoff}}{\text{Area of Catchment}} = \frac{28.8\times 10^6}{1440\times 10^6} = 0.02\ \text{m}$

= 2 cm Choice (D)

58. $S = 2000$

Displacement of field = $L\sin\alpha$

∴ Displacement on the paper = $\frac{L\sin\alpha}{S}$

$0.25 = \frac{L\sin(2.5)}{2000} \Rightarrow L = \frac{0.25\times 2000}{\sin 25} = 11462\ \text{mm}$

$= 11.462\ \text{m}$ Choice (A)

59. Difference between two consecutive points is

$\frac{1}{50}\times 20 = 0.4\ \text{m}$

Staff reading at *A* = 0.55 m staff reading at *B* should be 0.4 m lower than *A* (staff reading increases by 0.4 m)

i.e; = 0.55 + 0.4 = 0.95 m. Choice (A)

61. Design speed *V* = 80 kmph =22.22m/s

$\text{SSD} = vt + \frac{v^2}{2gf} = \left(22.22\times 2.5\right) + \frac{(22.22)^2}{2\times 9.81\times 0.35}$

$= 127.45\ \text{m}$

Safe SSD = 2 × SSD = 2 × 127.5 = 255 m

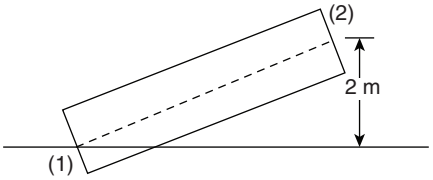
deviation angle *N*= 3 – (-5) = 8 %

length of summit curve

$L = \frac{NS^2}{4.4} = \frac{\left(\frac{8}{100}\right)(255)^2}{4.4} = 1182\ \text{m}$ Choice (B)

63. At Point E: $h_E = 100 - 30 = 70\%$
At Point D: $h_D = 100 - 40 = 60\%$. Choice (C)

64.



$D = 250$ mm $= 0.25$ m
$Z_2 - Z_1 = 2$ m
$V_1 = V_2 = 1.25$ m/s
$p_1 = 50$ k Pa
$p_2 = 20$ k Pa

$$A_1 = A_2 = \frac{\pi D^2}{4} = \frac{\pi \times (0.25)^2}{4} = 0.0491 \text{ m}^2$$

Rate of flow $= A_1 V_1$
$= 0.0491 \times 1.25$
$= 0.0614$ m^3/s

Total energy difference

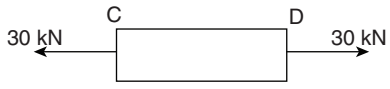
$$E_1 - E_2 = \frac{P_1 - P_2}{w} + \frac{V_1^2 - V_2^2}{2g} + (Z_1 - Z_2)$$

$$= \frac{(50-20)10^3}{9810} + 0 - 2$$

$= 1.0581$ m

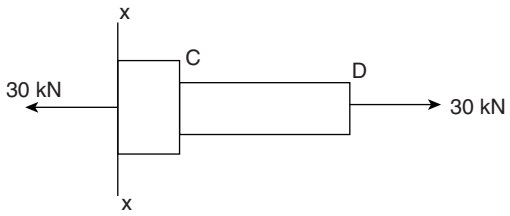
As $E_1 > E_2$, the flow is upwards Choice (A)

65. Portion CD



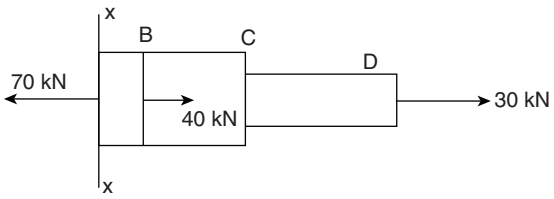
Force acting = 30 kN (Tensile).

Portion BC (consider section xx)



Force acting = 30 k N (Tensile).

Portion AB (consider section xx)



Force acting = 70 kN (Tensile)

Extension produced $= \sum \frac{PL}{AE}$

$$= \frac{30 \times 10^3 \times 500}{\frac{\pi}{4}(30)^2 \times 1.1 \times 10^5} + \frac{30 \times 10^3 \times 250}{\frac{\pi}{4}(40)^2 \times 2 \times 10^5} + \frac{70 \times 10^3 \times 250}{\frac{\pi}{4}(40)^2 \times 2 \times 10^5}$$

$$= \frac{4 \times 250}{\pi \times 10^2}\left[\frac{30 \times 2}{1.1 \times (30)^2} + \frac{30 \times 1}{2 \times (40)^2} + \frac{70 \times 1}{2 \times (40)^2}\right]$$

$= 0.2924$ mm Choice (B)

Mock Test 4

Number of Questions: 65 **Total Marks: 100**

Section – I: General Aptitude

Questions 1 to 5 carry One Mark each.

Directions for questions 1 to 3: Select the correct alternative from the given choices.

1. A bag contains a total of 12 kg of wheat, which is a mixture of three varieties of wheat x, y, z. If 4 kg of wheat comprising x and y in the ratio 3 : 1 and 12 kg of wheat comprising y and z in the ratio 1 : 3 are poured into the bag, the resulting wheat mixture in the bag will have x, y and z in the ratio 3 : 4 : 9. Find the ratio of x, y and z in the wheat mixture presently in the bag.

(A) 4 : 6 : 11
(B) 3 : 5 : 13
(C) 3 : 4 : 9
(D) Cannot be determined

2. Mohan and Sohan can complete a job in 12 days and 15 days respectively. With the help of Rohan, they completed the job in 3 days. The wage paid to Rohan for completing his part of the job was ₹ 320 more than the total wage paid to the other two for completing their parts of the job. Find Rohan's wage.

(A) ₹ 2340 (B) ₹ 1760
(C) ₹ 2520 (D) ₹ 1540

3. Six wrestlers – A, B, C, D, E, and F participated in a competition called "who is stronger" and at end of the competition they were ranked based on their strength, C and B are less stronger than D and they have atleast one wrestlers who is weaker than them. F is ranked second and E is ranked third in strength. How many wrestlers are stronger than A?

(A) 2 (B) 3
(C) 1 (D) 5

Directions for question 4: The question has a word followed by four choices. From the choices, identify the one which is **opposite** in meaning (antonym) to the question word and mark its number as your answer.

4. RISIBLE

(A) Grave (B) Funny
(C) Jovial (D) Insecure

Directions for question 5: Fill in the blank with the most suitable options from the choices given.

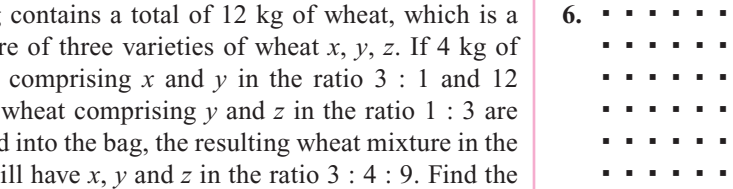
5. In places, the trunks of the fallen trees have been _____ white and ghostly by the strong sunlight.

(A) bleached (B) stained
(C) considered (D) furnished

Questions 6 to 10 carry Two Marks each.

Directions for questions 6 to 7: Select the correct alternative from the given choices.

6.



The number of triangles can be formed, whose vertices lie on the dots shown in the above pattern is _____

7. Most scientists now believe that the near extinction of the cobra in the North Indian forests was due to the extensive hunting down of the poor creature by the tribals of the forest. During the 40s and the 50s cobra was in great demand for its medicinal use which the tribals exploited greatly, the scientists say.
Which of the following, weakens the above argument?

(A) During the 40s and the 50s claws, antlers, horns, tusks, and other animal parts were in great demand for their medicinal uses which prompted most tribals to kill animals.
(B) During 40s and the 50s, the government built a dam in the vicinity of North Indian forests.
(C) The extensive government sponsored logging operations of the 40s and the 50s in the North Indian forests led to the immediate extinction of rodents, a major prey of snakes.
(D) The government undertook afforestation to revive the forests to prevent extinction of snakes in the North India but failed miserably.

Directions for question 8: In the following question, a part of a sentence is in bold. Indicate which among the answer choices given below the sentence can best replace the bold part to make it grammatically and logically correct. If the sentence is correct as it is, mark (*D*), i.e., 'No correction required' as the answer.

8. The next batch in which I am placed, will start 9 o' clock in the mornings.

(A) would start at 9 o' clock in the mornings
(B) starts at 9 o'clock in the morning
(C) will have started at 9 o'clock in the morning
(D) No correction required

Directions for question 9: In the following question given below, statements 1 and 6 are respectively the first and the last sentences of a paragraph and statements (2), (3), (4) and (5) come in between them. Rearrange (1), (2), (3) and (4) in such a way that they make a coherent paragraph together with statements 1 and 6. Select the correct order from the given choices and mark its number as your answer.

9. (1) With the decline of feudalism in ancient society, some important developments like merchant capital, emergence of wage labour, putting out system and enclosure movement set the stage for the industrial revolution.
(2) A new cyclical pattern of growth took place.
(3) Increased agricultural production and new technology further contributed to its growth.
(4) Though industrial development was marked by periods of depression, gradually this problem was overcome.
(5) With industrial development, the composition of capital also underwent changes.
(6) Soon the increased industrial production gave rise to capitalism and new social classes.
(A) 2345 (B) 4532
(C) 3524 (D) 5234

Directions for question 10: Select the correct alternative from the given choices.

10. The killings of Dalits in Uttar Pradesh proves that things have not improved over the years and the plight of Dalits is the same everywhere. The tragedy in UP is that despite the state having a Dalit Chief Minister who is seen by many as a future Prime Minister, the atrocities against Dalits continue.
Which of the following can be inferred from the above statement?
(A) A Dalit Chief Minister understands the agony of Dalits better.
(B) The chances of a Dalit Chief Minister becoming a Prime Minister will not be paralysed due to these events.
(C) Even a Dalit Chief Minister cannot prevent Dalits from being derogated.
(D) None of the above.

SECTION – II: CIVIL ENGINEERING

Directions for questions 11 to 65: Select the correct alternative form the given choices.

Questions 11 to 35 carry One Mark each.

11. Which among the following types of matrices is always non – singular?
(A) Symmetric matrix (B) Idempotent matrix
(C) Involutory matrix (D) Nilpotent matrix

12. If the Laplace transform of $f(t)$ is $F(s)$, then which of the following is necessary for $L^{-1}[sF(s)] = \frac{d}{dt}(f(t))$?
(A) $F(0) = 0$ (B) $f(0) = 0$
(C) $\left(\frac{dF}{ds}\right)_{at\ s=0}$ (D) $\left(\frac{df}{dt}\right)_{at\ t=0}$

13. The greatest value of C such that the partial differential equation

$$3\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} - 7x\frac{\partial u}{\partial x} + 5x^2 y\frac{\partial u}{\partial y} + 3u = 0$$

is not hyperbolic is _____.

14. Which of the following is a sufficient condition for the cross product of two vector point functions $\overline{F}$ and $\overline{G}$ to be solenoidal?
(A) Both $\overline{F}$ and $\overline{G}$ have to be solenoidal.
(B) $\overline{F}$ has to be solenoidal and $\overline{G}$ has to be irrotational.
(C) Both $\overline{F}$ and $\overline{G}$ have to be irrotational.
(D) $\overline{F}$ has to be irrotational and $\overline{G}$ has to be solenoidal.

15. The life time of bulbs manufactured by a company is normally distributed with mean 450 hours and standard deviation 30 hours. If a random sample of 36 bulbs has an average life time of 440 hours, then the test statistic in the process of testing the average life of bulbs is 450 hours or not is _____.

16. The contact pressure for a rigid footing resting on clay, at the centre and the edges are respectively _____.
(A) Maximum and zero
(B) Maximum and Minimum
(C) Zero and Maximum
(D) Minimum and Maximum

17. The width and depth of a footing are 2 and 1.5 m respectively. The water table at the site is at a depth of 2.5 m below the ground level. The water table correction factor for calculation of bearing capacity will be _____
(A) 1.0 (B) 0.75
(C) 0.50 (D) 0.25

18. The creep is caused due to _____.
(A) Initial Consolidation
(B) Primary Consolidation
(C) Secondary Consolidation
(D) Tertiary Consolidation

19. When water table rises _____
(A) Total stress increases, effective and pore pressures decreases.
(B) Effective stress decreases, total and pore pressures increases
(C) Pore pressure decreases, total and effective pressure increases
(D) Total stress decreases, effective and pore pressure increases.

20. If a R.C. beam fails in bond, its bond strength can be increased by _____
(A) Increasing the depth of beam
(B) Using thinner bars but more in numbers
(C) Using thicker bars but less in numbers
(D) Providing vertical stirrups

21. A 2 – way RCC slab is simply supported on all its edges with corners not free to lift. The area of steel in shorter span is 812 mm^2. The torsional reinforcement required at corner is _____.

(A) 0 mm^2 (B) 609 mm^2
(C) 406 mm^2 (D) 203 mm^2

22. The moisture content in a well seasoned timber is _____.
(A) 4 to 6% (B) 10 to 12%
(C) 15 to 20% (D) 50%

23. The most appropriate method to specify the concrete mix is by _____.
(A) Nominal mix ratio (B) Design mix ratio
(C) Degree of control (D) Grade of concrete

24. A combined sewer is the one which transports
(A) domestic sewage and storm water
(B) domestic sewage and industrial wastes
(C) domestic sewage and over head flow
(D) domestic sewage, industrial wastes and storm water.

25. The main constituents of gas generated during anaerobic digestion of sewage sludge are
(A) carbon dioxide and methane
(B) methane and ethane
(C) carbon dioxide and carbon monoxide
(D) carbon monoxide and nitrogen

26. The type of surveying in which the curvature of the earth is taken into account is called.
(A) Geodetic surveying
(B) Plane surveying
(C) Preliminary surveying
(D) Topographical surveying

27. A lysimeter is used to measure
(A) Infiltration (B) Evaporation
(C) Evapotranspiration (D) Radiation

28. The standard project flood is
(A) same as the probable maximum flood
(B) same as the design flood
(C) smaller than the probable maximum flood
(D) larger than the probable maximum flood by a factor implying safety factor

29. Stopping slight distance and frictional coefficients are
(A) directly proportional to each other
(B) inversely proportional to each other
(C) unrelated
(D) either directly (or) inversely proportional to each other depending on nature of pavement.

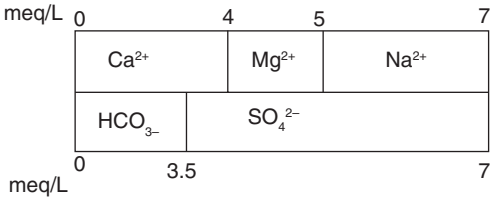
30. The traffic survey data plotted by means of desire lines is
(A) Accident (B) Classified volume
(C) Origin and destination (D) Speed and delay

31. The shape of stop sign as per IRC : 67 – 2001 is
(A) Circular (B) Triangular
(C) Octagonal (D) Rectangular

32. A thin walled spherical shell of diameter 500 mm and thickness 8 mm is subjected to an internal pressure of 6 MPa. The average hoop stress (in N/mm^2) is _____.

33. A fine grained soil has 40% (by weight) silt content. The Liquid limit, plastic limit and shrinkage limit of soil are given as 45%, 30% and 18% respectively. Using this data available, identify the correct statement from the following.
(A) The given fine grained soil is inactive soil.
(B) The given fine grained soil is normal active soil.
(C) The given fine grained soil is active soil.
(D) Data insufficient.

34. For a sample of water with ionic composition, the carbonate and non – carbonate hardness concentrations (in mg/L as $CaCO_3$) respectively are :



(A) 200 and 50 (B) 175 and 75
(C) 75 and 175 (D) 50 and 200

35. A wastewater sample has an initial BOD of 222 mg/l. The first order BOD decay coefficient is 0.4/day. The BOD consumed (in mgu) in 5 days is
(A) 150 (B) 192
(C) 30 (D) 50

Questions 36 to 65 carry Two Marks each.

36. If $g(x) = (x + 5)(x + 2)(x - 1)(x - 6)$ and $f(x) = \int g(x)dx$, then the number of local maxima for $f(x)$ is _____.

37. If the system of linear equations

$$a_1 x_1 + b_1 x_2 + c_1 x_3 = 0$$
$$a_2 x_1 + b_2 x_2 + c_2 x_3 = 0$$
$$a_3 x_1 + b_3 x_2 + c_3 x_3 = 0$$

(where a_i, b_i and c_i, $i = 1, 2, 3$ are all constants) has only a trivial solution, then the number of solutions of the system of linear equations

$$a_1 x_1 + b_1 x_2 + c_1 x_3 = 5$$
$$a_2 x_1 + b_2 x_2 + c_2 x_3 = 3$$
$$a_3 x_1 + b_3 x_2 + c_3 x_3 = 2 \text{ is } _____.$$

(A) 0 (B) 1
(C) infinitely many (D) Cannot be determined

38. If X_1 and X_2 are two independent normal variates with means 100 and 60 respectively and variances 16 and 9 respectively, then which of the following statements is true?
(A) $X_1 - X_2$ is a normal variate with mean 40 and standard deviation 7.
(B) $X_1 - X_2$ is a normal variate with mean 40 and standard deviation 5.
(C) $X_1 - X_2$ is a normal variate with mean 20 and standard deviation 1.
(D) $X_1 - X_2$ need not be a normal variate.

39. The area bounded by the curve $y = x^3$ and the line $y = x$ in square units is _____
(A) 0.25 (B) 0.5
(C) 0.75 (D) 1

40. Which of the following statements are true with reference to the evaluation of a definite integral

$I = \int_a^b f(x)dx$ by numerical integration methods?

I. If $[a, b]$ is divided into 5 subintervals, then we can use any of the Trapezoidal rule or Simpson's rule to evaluate the definite integral I.

II. If $y = f(x)$ represents a parabola, then the value of I obtained by the Simpson's $\frac{1}{3}$rd rule and by direct integration are same.

III. While evaluating by the Trapezoidal rule, the accuracy can be increased by dividing [a, b] into large number of sub intervals.

(A) All of I, II and III (B) Only I and II
(C) Only II and III (D) Only I and III

41. Match List - I with List - II

	List – I		List – II
a.	Coriolis effect	1.	Rotation of earth
b.	Fumigation	2.	Lapse rate and vertical temp. profile
c.	Ozone layer	3.	Inversion
d.	Max. mixing depth (mixing height)	4.	Dobson

	a	b	c	d		a	b	c	d
(A)	2	1	4	3	(B)	2	1	3	4
(C)	1	3	2	4	(D)	1	3	4	2

42. The tests on soil determines, following properties of a the particular sample.
Degree of saturation = 85%
Water content = 12%
Specific gravity = 2.65
If the factor of safety (FOS) against the quick sand condition is adopted as 2.5; the hydraulic gradient provided will be _____.

(A) 3 (B) 2.52
(C) 0.48 (D) 3.48

43. Two straight lines intersect at an angle of 60^0. The radius of a curve joining the two straight lines is 600 m. The length of long chord and mid – ordinates in meters of the curve are

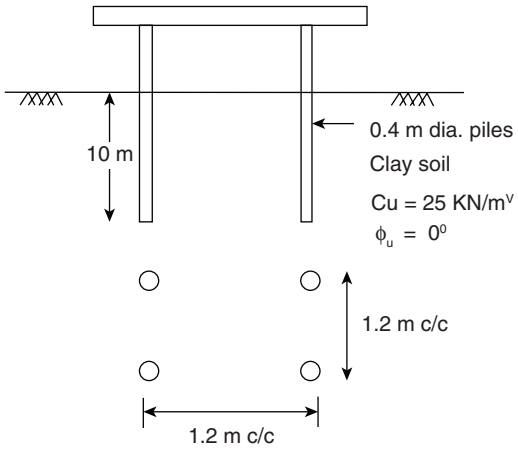
(A) 80.4, 600 (B) 600, 80.4
(C) 600, 39.89 (D) 49.89, 300

44. Match the following:

	Group – I		Group – II
P.	Alidade	1.	Chain surveying
Q.	Arrow	2.	Leveling
R.	Bubble tube	3.	Plain table surveying
S.	Stadia hair	4.	Teodilite surveying

	P	Q	R	S		P	Q	R	S
(A)	3	2	1	4	(B)	2	4	3	1
(C)	1	2	4	3	(D)	3	1	2	4

45. What is ultimate capacity of the pile group shown in figure assuming the group to fail as a single block



(A) 2861 KN (B) 2681 KN
(C) 2176 KN (D) 1874 KN

46. During sub surface investigation of the soil, the following record of number of blows were given, which was obtained from the standard penetration test conducted at certain depth.

Penetration depth (cm)	Number of blows
0 – 7.5	3
7.5 – 15	3
15 – 22.5	6
22.5 – 30	6
30.0 – 37.5	8
37.5 – 45	7

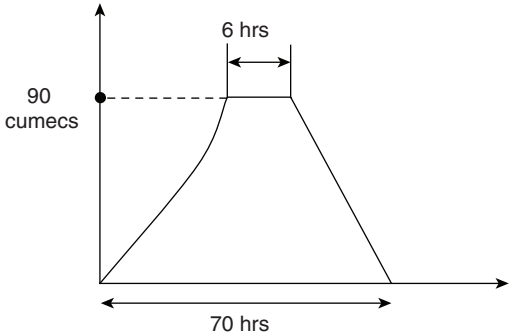
Assuming water table at ground level, soil as fine sand and correction for overburden pressure as 0.926; the corrected value of '*N*' for the soil would be _____.

(A) 27 (B) 21
(C) 20 (D) 15

47. A 1 hr rainfall of 10 cm has teturn period of 50 yr. The 1 hour of rainfall 10 cm (or) more will occur in each of two successive year is

(A) 0.04 (B) 0.2
(C) 0.02 (D) 0.0004

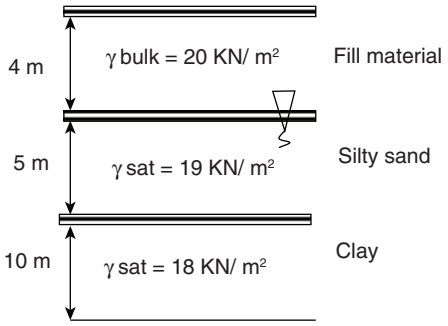
48. A direct runoff hydrograph due to an isolate storm with an effective rainfall of 2 cm was trapezoidal in shape as shown in figure. The hydrograph corresponds to a catchment area (in sq. km) of



(A) 790.2 km^2 (B) 615.6 km^2
(C) 599.4 km^2 (D) 435.3 km^2

49. An outlet irrigates an area of 40 ha. The discharge (L/S) required at this outlet to met the evaportransiration requirement of 40 mm occurring uniformly in 40 days neglecting other field losses is _____.

50. At a reclamation site having soil strata as shown in the figure, 4 m thick layer of fill material is to be laid instantaneously on the top surface. If the coefficient of volume compressibility, m_v for clay is 2.2×10^{-4} m^2/KN, the consolidation settlement of clay layer due to placing fill material will be _____.



(A) 211 mm (B) 189 mm
(C) 176 mm (D) 142 mm

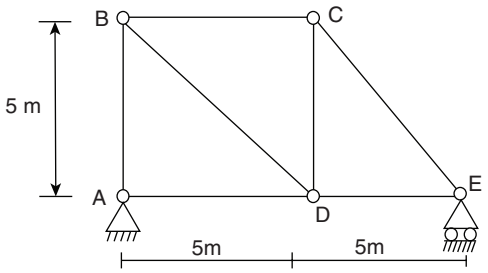
51. A homogeneous dam is 21.5 m high and has a free board of 1.5 m. A flow net was constructed and the following results were observed.
No. of potential drops = 12
No. of flow channels = 3
The discharge/m length of the dam if the coefficient of permeability of the dam material is 2×10^{-6} m/sec is _____ $\times 10^{-5}$ cumecs/m

52. An isolated T – beam has effective span of 10 m. Actual width of flange is 2 m, width of web is 250 mm; then effective flange width will be _____.
(A) 1.36 m (B) 1.57 m
(C) 1.82 m (D) 2 m

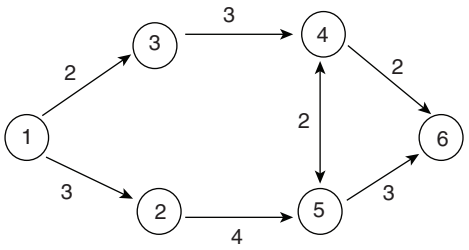
53. During CBR test, the load sustained by a remolded soil specimen at 5 mm penetration is 50 kg. The CBR value of the soil will be
(A) 10% (B) 5%
(C) 3.6% (D) 2.4%

54. For a road with camber of 3% an design speed of 80 kmph, the min radius of a curve beyond which no super elevation is needed is
(A) 1680 m (B) 948 m
(C) 406 m (D) 280 m

55. Determine the vertical displacement of joint D (in mm) if member BC of the pin – jointed frame as shown below is long by an amount 4×10^{-3} from the original length 5 m. All the members of frame have same AE value [$AE = 10 \times 10^0$ kN]

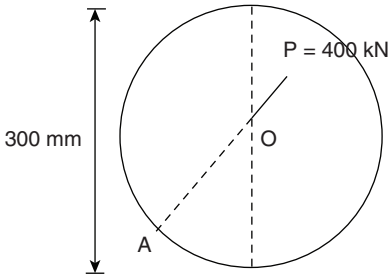


56. From the given network, the Inference float and independent float for the activity 5 – 4 will be _____



(A) Negative and Positive (B) Both positive
(C) Both negative (D) Both equals to zero

57.



On a short masonry column of circular cross section of diameter 300 mm a load of 400 kN acts axially at a radial distance of 100 mm from centre as shown in the figure. Strain developed at point A in N/mm^2 is
(A) 9.43(compressive) (B) 9.43(Tensile)
(C) 20.75(compressive) (D) 20.75 (Tensile)

58. A rod of length 1.2 m tapered from 100 mm. diameter to 50 mm diameter. If a torque of 3 kN m is applied, the angular rotation of free end (in radians) is.
(Take modulus of rigidity = 80 kN/mm^2)

59. For a Pelton wheel, head at the base of the nozzle is 80 m, discharge through nozzle is 0.3 m^3/s at a jet velocity of 38.2 m/s. Loss of power in the nozzle in kW) is
(A) 12.76 (B) 16.55
(C) 18.49 (D) 20.36

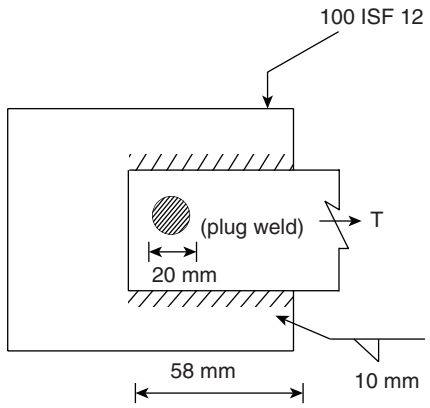
60. For a surface of area A immersed in a liquid of specific weight w, if $\bar{x}$ is the depth of centre of area and $\bar{h}$ is the depth of centre of pressure from the liquid surface, the total pressure acting on the surface is given by
(A) $wA\bar{x}$ (B) $wA\bar{h}$
(C) $wA\bar{x}^2$ (D) $wA\dfrac{(\bar{x}+\bar{h})}{2}$

61. If velocity potential satisfies, the Laplace equation, it is a case of
(A) unsteady, compressible and rotational flow
(B) steady, incompressible and irrotational flow.
(C) unsteady, incompressible and rotational flow.
(D) steady, incompressible and rotational flow.

62. A piping system consists of 3 pipes of lengths 1000 m, 800 m and 600 m arranged in series. Diameters of the pipes are 750 mm, 600 mm and 450 mm respectively. If the system is to be replaced by an equivalent pipe of diameter 600 mm, length of the equivalent pipe (in m) will be _____

63. A close coiled helical spring has to carrying a load of 800 N. Mean diameter of the spring is to be 75 mm. If the allowable shear stress is 100 N/mm^2 wire diameter required for the spring (in mm) is.
(A) 9.64 (B) 11.52
(C) 13.47 (D) 15.26

64. The service load permitted on the connection shown for the fig. below in kN is _____. Assume field welding and Fe 410 steel.



(A) 105 (B) 212
(C) 441 (D) 145

65. The stiffness matrix of a beam is given as $k\begin{bmatrix}4 & 3\\ 3 & 5\end{bmatrix}$. The flexibility matrix is _____.

(A) $\frac{k}{11}\begin{bmatrix}5 & -3\\ -3 & 4\end{bmatrix}$ (B) $\frac{11}{k}\begin{bmatrix}5 & -3\\ -3 & 4\end{bmatrix}$

(C) $\frac{1}{11\times k}\begin{bmatrix}5 & -3\\ -3 & 4\end{bmatrix}$ (D) $11k\begin{bmatrix}5 & -3\\ -3 & 4\end{bmatrix}$

Answer Keys

1. C	**2.** B	**3.** D	**4.** A	**5.** A	**6.** 6800	**7.** C	**8.** B	**9.** B	**10.** D
11. C	**12.** B	**13.** 3	**14.** C	**15.** –2	**16.** D	**17.** B	**18.** C	**19.** B	**20.** B
21. B	**22.** B	**23.** D	**24.** D	**25.** A	**26.** A	**27.** C	**28.** C	**29.** B	**30.** C
31. C	**32.** 93.25 to 94.25		**33.** A	**34.** B	**35.** C	**36.** 2	**37.** B	**38.** B	**39.** B
40. C	**41.** D	**42.** C	**43.** B	**44.** A	**45.** C	**46.** C	**47.** D	**48.** B	
49. 0.925 lit/sec		**50.** C	**51.** 1	**52.** A	**53.** D	**54.** B	**55.** 2 mm	**56.** D	**57.** B
58. 0.02135 to 0.02145			**59.** B	**60.** A	**61.** B	**62.** 3655.5 to 3656.5		**63.** B	**64.** A
65. C									

Hints and Explanations

1. In the conditional case (as opposed to the actual case) the 4 kg mixture will have 3 kg of x and 1 kg of y. Also the 12 kg mixture will have 3 kg of y and 9 kg of z. ∴ Total quantities of x, y and z added are 3 kg, 4 kg and 9 kg respectively, i.e., in the quantity added, (if it is added), the ratio of x, y, z is 3 : 4: 9. In the final mixture also, it is 3 : 4 : 9. Therefore, the ratio in the present mixture also, it has to be 3 : 4 : 9. Choice (C)

2. Part of the job completed by

(i) Mohan $= 3\left(\frac{1}{12}\right) = \frac{1}{4}$

(ii) Sohan $= 3\left(\frac{1}{15}\right) = \frac{1}{5}$

(iii) Rohan $= 1 - \left(\frac{1}{4}+\frac{1}{5}\right) = \frac{11}{50}$

Let the total wage paid to the three be ₹T.

$\frac{11}{20}T = \frac{T}{4}+\frac{T}{5} + 320$

$\Rightarrow T = 3200$

$\frac{11}{20}T = 1760$

∴ Wage of Rohan = ₹ 1760. Choice (B)

3. E and F are ranked 2nd and 3rd. There is at least one wrestler weaker than B, C and D. This indicates that A is the weakest among all. Hence, five wrestlers are stronger than A. Choice (D)

4. Something that is hilarious, funny or ludicrous is said to be 'risible'. The antonym of 'risible' is 'grave' or serious. Choice (A)

5. The blank is best filled by 'bleached', meaning, 'make the appearance white'. Other options fall short of representing the blank properly. Choice (A)

6. Number of combinations of three dots which can be taken is ${}^{36}C_3$. But for some combinations, all the 3 dots may be collinear, and we cannot form a triangle. In the given arrangement, by joining the dots we will have the following:
(1) 6 horizontal lines having 6 points each
(2) 6 vertical lines having 6 points each
(3) 2 diagonal lines having 6 points each
(4) 4 diagonal lines having 5 points each
(5) 4 diagonal lines having 4 points each and
(6) 4 diagonal lines having 3 points each.
Number of triangles which can be formed
$= {}^{36}C_3 - [6({}^6C_3 + {}^6C_3) + 2({}^6C_3) + 4\,({}^5C_3 + {}^4C_3 + {}^3C_3)]$
$= 7140 - [14(20) + 4(15)] = 6800.$ Ans: 6800

7. The correct answer is C. To weaken an argument, we should show the assumption in the argument to be false. The assumption in the given argument is that there is no other cause for the near extinction of the cobra. If we prove this wrong, it weakens the argument. Choice (C), by showing an alternative cause, does this. Others are either irrelevant or out of scope. Choice (C)

8. We use the verb in the simple present tense for a future event that is part of a fixed time table. Hence, choice (B) is apt. The reference is to a specific morning and hence 'mornings' is incorrect. Choice (B)

9. Statement 1 ends with '… set the stage for industrial revolution'. So it is followed by (4) which talks of 'industrial revolution.' (2) and (3) do not talk of industries. Though (5) talks of 'industrial development', the word 'also' indicates it follows something else. The sentences 4532 in that order tell us chronologically the steps that followed the industrial revolution and the growth of capitalism. Choice (B)

10. Choice (A) is an assumption of the people / the writer. Choice (B) is false. It is evident that the Chief Minister failed in his duties so it will hamper his chances of becoming the Prime Minister. 'Will not' in (B) rules it out. (C) is not an inference as it is openly stated that even a Dalit CM could not stop the killing of Dalits. The last word of the paragraph 'continue' suggests that in the past too Dalits were killed. Among the given sentences none of them qualify as inferences. Choice (D)

11. We know that, a matrix A is an involutory matrix, if $A^2 = I$ (*OR*) $A^{-1} = A$
$\therefore$ Every involutory matrix is non – singular
Counter Example for (A):
Let $A = \begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$
Clearly, $A^T = A$ but $|A| = 0$
$\therefore$ A is symmetric but not non – singular
Counter Example for (B):
Let $B = \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$
Clearly, $B^2 = B$ but $|B| = 0$
$\therefore$ B is idempotent but not non – singular
Also, we know that every nilpotent matrix is a singular matrix. Choice (C)

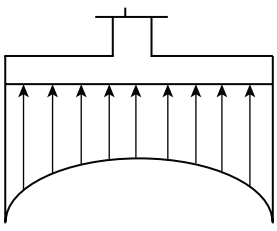
12. We know that if $L[f(t)] = F(s)$, then
$L^{-1}[sF(s)] = \frac{d}{dt}(f(t))$ only if $f(0) = 0$. Choice (B)

13. Given PDE is
$$3\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} - 7x\frac{\partial u}{\partial x} + 5x^2 y\frac{\partial u}{\partial y} + 3u = 0$$
$\rightarrow$ (1)
Comparing (1) with the general second order linear partial differential equation, we have
$A = 3, B = -6, C = C$
For (1) not to be hyperbolic, we have
$B^2 - 4AC \not> 0$
$\Rightarrow$ $B^2 - 4AC \leq 0$
$\Rightarrow$ $(-6)^2 - 4 \times 3 \times C \leq 0$
$\Rightarrow$ $36 - 12C \leq 0$
$\Rightarrow$ $C - 3 \leq 0$
$\Rightarrow$ $C \leq 3$
$\therefore$ The greatest value of C such that (1) is not hyperbolic is 3 Ans: 3

14. $\overline{F} \times \overline{G}$ is solenoidal
$\Rightarrow$ Div $(\overline{F} \times \overline{G}) = 0$
$\Rightarrow$ $\overline{G}$. curl $\overline{F}$ – $\overline{F}$. Curl $\overline{G} = 0$
So, $\overline{F} \times \overline{G}$ is solenoidal when curl $\overline{F} = \overline{0}$ and curl $\overline{G} = \overline{0}$
i.e., Both $\overline{F}$ and $\overline{G}$ have to be irrotational. Choice (C)

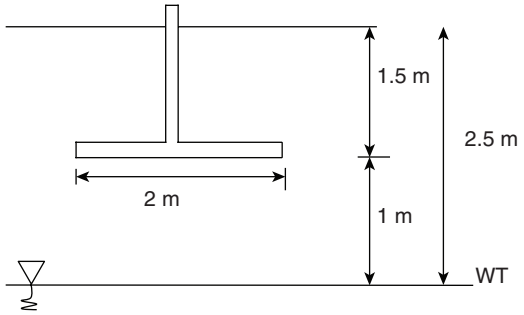
15. Population mean = μ = 450 hours
Population standard deviation = σ = 30 hours
Sample mean = $\bar{x}$ = 440 hours
Sample size = n = 36
$\therefore$ Test statistic $= Z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$
$= \dfrac{440 - 450}{\left(\dfrac{30}{\sqrt{36}}\right)} = \dfrac{-2}{6} \times \sqrt{36}$
$\therefore$ $Z = -2$ Ans: –2

16.



Minimum at centre, Maximum at edges. Choice (D)

17.



Correction factor = $0.5 + 0.5 \times \frac{1}{2} = 0.75$ Choice (B)

19. If water table rises; Total pressure and pure water pressure increases but effective pressure decreases.
Choice (B)

21. Torsional reinforcement = $\frac{3}{4} \times$ steel in shorter span
$= \frac{3}{4} \times 812 = 609 \text{ mm}^2$ Choice (B)

32. For a thin spherical shell, hoopstress (circumferential stress) = $\frac{pd}{4t}$
where p = internal pressure.
d = diameter
t = thickness.
$p = 6$ MPa $= 6$ N/mm^2
$d = 500$ mm
$t = 8$ mm
$\therefore$ Hoop stress = $\frac{6 \times 500}{4 \times 8}$
$= 93.75$ N/mm^2 Ans: 93.25 to 94.25

33. % clay content in the soil = 100 – 40 = 60%
Plasticity index = Liquid limit – Plastic limit
= 45 – 30 = 15
$\Rightarrow$ Activity number = $\frac{\text{Plasticity index, } I_P}{\text{\% clay fraction}}$
$= \frac{15}{60} = 0.25$
The activity number is less than 0.75. So, the given soil is said to be inactive soil. Choice (A)

34. Carbonate harness = 3.5×10^{-3} g eq [if NCH is present sodium alkalikrity will be absent i.e., $NaHCO_3$ absent]
$= 3.5 \times 10^{-3} \times 50$ g/L as $CaCO_3$
= 175 mg/L as $CaCO_3$
Non – Carbonate hardness = Total hardness – Carbonate hardness
Total hardness = 5×50 mg/L as $CaCO_3$ = 250 mg/L as $CaCO_3$
NCH = 250 – 175 = 75 mg/L as $CaCO_3$. Choice (B)

35. $K_D = 0.4 \times 0.434 = 0.1736$
$BOD_5 = L[1 - (10)^{-K_D t}]$
$= 222[1 - (10)^{-5 \times 0.1736}]$
= 192 mg/L
BOD remaining = 192 mg/L
Hence, BOD consumed = 222 – 192 = 30 mg/L
Choice (C)

36. Given $g(x) = (x + 5)(x + 2)(x - 1)(x - 6)$ and $f(x) = \int g(x)dx$
$\Rightarrow$ $g(x) = f^1(x) = (x + 5)(x + 2)(x - 1)(x - 6)$
$f^1(x) = 0 \Rightarrow (x + 5)(x + 2)(x - 1)(x - 6) = 0$
$\therefore$ The critical values of $f(x)$ are –5, –2, 1 and 6
Also, $f^1(x) > 0$ for $x < -5$
$f^1(x) < 0$ for $-5 < x < -2$
$f^1(x) > 0$ for $-2 < x < 1$
$f^1(x) < 0$ for $1 < x < 6$
and $f^1(x) > 0$ for $x > 6$
As $f^1(x) > 0$ for $x < -5$, $f^1(x) < 0$ for $-5 < x < -2$ and $f^1(x) = 0$ for $x = -5$, $f(x)$ has a local maximum at $x = -5$
Similarly, as $f^1(x) > 0$ for $-2 < x < 1$,
$f^1(x) < 0$ for $1 < x < 6$ and $f^1(x) = 0$ for $x = 1$
$f(x)$ has a local maximum at $x = 1$
$\therefore$ The number of local maxima is 2. Ans: 2

37. Given that the system of linear equations
$a_1 x_1 + b_1 x_2 + c_1 x_3 = 0$
$a_2 x_1 + b_2 x_2 + c_2 x_3 = 0$ $\rightarrow(1)$
$a_3 x_1 + b_3 x_2 + c_3 x_3 = 0$
has only a trivial solution.
$\therefore$ Its coefficient matrix $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is non – singular.
Consider the system of linear equations
$a_1 x_1 + b_1 x_2 + c_1 x_3 = 5$
$a_2 x_1 + b_2 x_2 + c_2 x_3 = 3$ $\rightarrow(2)$
$a_3 x_1 + b_3 x_2 + c_3 x_3 = 2$
The augmented matrix of (2) is
$$[A/B] = \begin{bmatrix} a_1 & b_1 & c_1 & : & 5 \\ a_2 & b_2 & c_2 & : & 3 \\ a_3 & b_3 & c_3 & : & 2 \end{bmatrix}.$$
As A is non – singular, we have
$\rho(A) = \rho([A/B]) = 3$(= The number of unknowns)
$\therefore$ The system (2) has a unique solution. Choice (B)

38. We know that, if X_1 and X_2 are two independent normal variates, then $X_1 - X_2$ is also a normal variate.
Mean of $X_1 = \mu_{X_1} = 100$
Mean of $X_2 = \mu_{X_2} = 60$
Variance of $X_1 = \sigma_{X_1}^2 = 16$
Variance of $X_2 = \sigma_{X_2}^2 = 9$

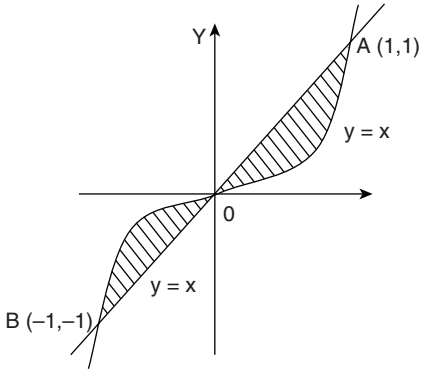
$\therefore$ Mean of $X_1 - X_2 = \mu_{X_1 - X_2} = \mu_{X_1} - \mu_{X_2}$

$= 100 - 60 = 40$

Standard deviation of $X_1 - X_2$

$= \sigma_{X_1 - X_2} = \sqrt{\sigma_{X_1}^2 + \sigma_{X_2}^2} = \sqrt{16+9} = 5$ Choice (B)

39. The area bounded by the curve $y = x^3$ and the line $y = x$ is the shaded region shown in the figure. Also, the required area = 2 × The shaded region in the first quadrant.



$\therefore$ The required area $= 2\int_{x=0}^{1} \left(x - x^3\right) dx$

$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

$= 2\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2}$ square units Choice (B)

40. We know that $[a, b]$ must be divided into even number of subintervals in order to apply the Simpson's Rule.
So, when $[a, b]$ is divided into 5 subintervals, we can't use the Simpson's rule.

$\therefore$ I is not true

II and III are properties of the respective numerical integration methods and are always true. Choice (C)

42. $G = 2.65$, $S_r = 85\%$ and $w = 12\%$

$e = \frac{WG}{S_r}$

$e = \frac{0.12 \times 2.65}{0.85} = 0.374$

critical hydraulic gradient, $i_c = \frac{(G-1)}{1+e} = \frac{1.6}{1.374} = 1.2$

$FOS = \frac{i_c}{i}$

$\therefore$ The hydraulic gradient provided $= \frac{1.2}{FOS} = \frac{1.2}{2.5}$

$\Rightarrow i = 0.48$ Choice (C)

43. Length of long chart $T_1 T_2 = 2R\sin(D/2)$

$= 2 \times 600 \times \sin(60/2) = 600$ m

Length of mid – ordinate $= R[1 - \cos(D/2)]$

$= 600[1 - \cos(60/2)]$

$= 600 \times 0.134 = 80.4$ m Choice (B)

45. $Q_{ug} = C_u N_c \,.\, A_b + P_b L \,.C_u$

P_b = Perimeter of block = 4(1.2 + 0.4) = 6.4 m

A_b = area of block = $(1.2 + 0.4)^2 = 2.56$ m^2

$\Rightarrow Q_{ug} = (25 \times 9 \times 2.56) + (6.4 \times 10 \times 25)$

$= 2176$ KN Choice (C)

46. The number of blows for first 15 cm is not taken into consideration, only blows required for last 30 cm of penetration is taken.

$\therefore$ Number of blows = 6 + 6 + 8 + 7 = 27

(i) Correction of over burden pressure

corrected value, $N^1 = 0.926 \times 27 = 25$

(ii) Correction for dilatancy

$N = 15 + \left(\frac{25-15}{2}\right) = 20$

$\therefore$ The correct value for N is 20. Choice (C)

47. Return period of rainfall $T = 50$ yrs

$\therefore$ Probability of occurrence once in 50 yrs.

$P = \frac{1}{50} = 0.02$

Probability of occurrence in each of 2 successive year $= P^2 = (0.02)^2 = 0.0004$ Choice (D)

48. Area of given DRH = Volume of direct runoff

$= \left[\frac{1}{2}(70+6) \times 60 \times 60 \times 90\right]$

$= 12312000$ m^3

Effective rainfall = 2 cm

$\therefore$ Catchment are $= \frac{\text{Volume of } DRH}{\text{Effective rainfall}}$

$= \left(\frac{12312000}{2 \times 10^{-2}}\right)$

$= 615.6$ km^2 Choice (B)

49. Volume of water required for evapotranspiration

$= 40 \times 10^4 \times 40 \times 10^{-3} \times 10^3$

$= 16 \times 10^5$ lit

$\therefore$ Discharge required at outlet

$= \frac{16 \times 10^5}{20 \times 24 \times 60 \times 60} = 0.925$ lit/sec

Ans: 0.925 lit/ sec

50. $m_v = 2.2 \times 10^{-4}$ m^2/KN

$\sigma_i^1 = 4(20) + 5(19 - 10) + 10(18 - 10) = 205$ KN/m

$\Delta\sigma^1 = 4 \times 20 = 80$ KN/m

$\sigma_f = \sigma_i + \Delta\sigma = 285$ KN/m

$\Delta H = m_v \,.\, H_0 \,.\, \Delta\sigma^1 = 2.2 \times 10^{-4} \times 10 \times 80$

$\therefore$ $\Delta H = 176$ mm Choice (C)

51. $q(\text{m}^3/\text{sec/m}) = K.H.\dfrac{N_f}{N_d}$

$= 2 \times 10^{-6} \times 20 \times \dfrac{3}{12}$

$= 1 \times 10^{-5}$ cumecs/m Ans: 1

52. For isolated T – beam;

Effective flange width, $b_f = \dfrac{l_0}{\frac{l_0}{b}+4} + b_w$

$l_0 = 10$ m, $b = 2$m and $b_w = 250$ mm

$\Rightarrow \quad b_f = \dfrac{10000}{\frac{10000}{2000}+4} + 250 = 1.36$ m Choice (A)

53. CBR(%) =

$$\frac{\text{load sustained by specimen at 5mm penetration}}{\text{load sustained by std aggregates at 5mm penetration}} \times 100$$

$= \dfrac{50}{2055} \times 100 = 2.4\%$ Choice (D)

54. Taking camber as super elevation

$e = \dfrac{3}{100} = 0.33 = \dfrac{v^2}{225R}$

$0.33 = \dfrac{(80)^2}{225R}$

$R = 948.14$ m Choice (B)

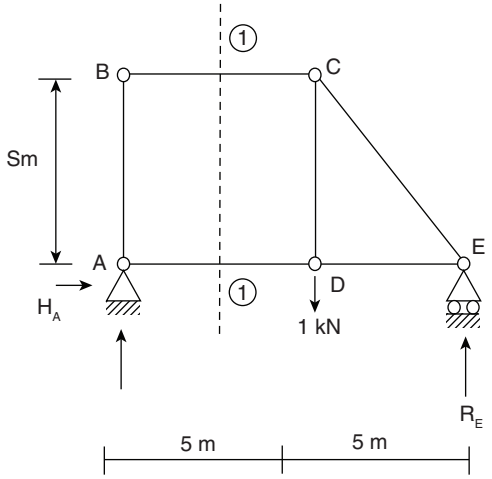
55. $S_D = \Sigma k\delta^1$

Where

k = force in members due to unit load applied where deflection is desired.

δ^1 = deformation of a members due to lack of fit.

Since only in member BC, there is lack of fit. So find force in member due to unit load in member BC only. Forces in other members are not required.



Reactions

Due to symmetry;

$R_A = R_B = \dfrac{1}{2}$kN

$\Sigma F_X = 0$

$H_A = 0$

Force in member BC:

Pass a section (1) – (1) through member BC.

Considering equilibrium of RHS

Taking moment at point 'D'

$(F_{BC}) \times 5 - R_E \times 5 = 0 \Rightarrow F_{BC} = +R_E = \dfrac{1}{2}$kN

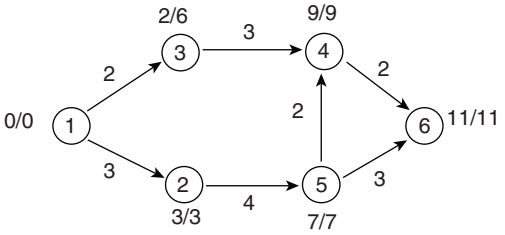
Table

Member	K	δ^1	k. δ^1
BC		4 mm	2.0 mm
AB	–	0	0
BD	–	0	0
CD	–	0	0
DE	–	0	0
AD	–	0	0

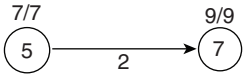
$\Sigma k\delta^1 = 2.0$ mm

$\therefore \quad S=D = 2.0$ mm Ans : 2 mm

56.



Activity:



Inference float = Total float – Free float

Total float = (9 – 7) – 2 = 0

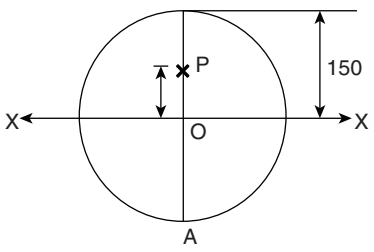
Free float = (9 – 7) – 2 = 0

$\therefore$ Inference float = 0

Independent float = (9 – 7) – 2 = 0

$\therefore$ Both inference float and independent floats are zero. Choice (D)

57.



The stress at a point is the sum of the compressive stress $\frac{P}{A}$ and stress due to moment $P \times 100$

Area of cross section $A = \pi R^2$
$= \pi \times (150)^2$
$= 22500\ \pi\ mm^2$

Direct stress $\frac{P}{A} = \frac{400 \times 10^3}{22500\pi} = 5.6588\ N/mm^2$

Stress at A due to moment (compressive)

$$= \frac{PeR}{I} = \frac{400 \times 10^3 \times 100 \times 150}{\left(\frac{\pi}{64}\right)(300)^4}$$

$= 15.0902$ (Tensile)

Resultant stress at A
$= -15.0902 + 5.6588$
$= -9.4314$
$= 9.4314$ (Tensile) Choice (B)

58. For a tapered rod angular rotation (angle of twist)

$$\theta = \frac{2TL}{3\pi G} \frac{r_1^2 + r_1 r_2 + r_2^2}{r_1^3 r_2^3}$$

where T = torque applied
L = length
r_1, r_2 = radii.

$$\therefore \quad \theta = \frac{2 \times 3 \times 10^6 \times 1200}{3\pi \times 80 \times 10^3} \times \frac{50^2 + 50 \times 25 + 25^2}{50^3 \times 25^3}$$

$= 0.02139$ radian Ans: 0.02135 to 0.02145

59. Discharge $Q = 0.3\ m^3/s$
Jet velocity $v = 38.2$ m/s
Power at the base of nozzle
$= wQH$
$= 9.81 \times 0.3 \times 80$
$= 235.44$ kN

Kinetic energy of jet $= \frac{1}{2}\left(\frac{wQ}{g}\right)v^2$

$$= \frac{1}{2} \times \frac{9.81 \times 0.3}{9.81} \times (38.2)^2$$

$= 218.89$ kW

Loss of power in nozzle
$=$ Power at base $-$ Kinetic energy in nozzle
$= 235.44 - 218.89$
$= 16.55$ kW Choice (B)

62. Diameter of the equivalent pipe $D = 600$ mm $= 0.6$ m
Using Dupit's equation.

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

$$\therefore \quad \frac{L}{(0.6)^5} = \frac{1000}{(0.75)^5} + \frac{800}{(0.6)^5} + \frac{600}{(0.45)^5}$$

$= 4213.99 + 10288.07 + 32{,}515.37$
$= 47{,}017.43$

$\Rightarrow \quad L = 3656.08$ m Ans: 3655.5 to 3656.5

63. $W = 800$ N
$\tau = 100\ N/mm^2$
Mean radius $= \frac{75}{2} = 37.5$ mm

$\tau = \frac{16WR}{\pi d^3}$ where d = wire diameter

$$\therefore \quad 100 = \frac{16 \times 800 \times 37.5}{\pi d^3}$$

$\Rightarrow \quad d = 11.52$ mm Choice (B)

64. Design strength of side fillet weld $= \frac{L_w (KS) f_u}{\sqrt{3} r_{mw}}$

L_w = Effective length of fillet weld
$= 50 + 50 = 100$ mm
t_t = Effective throat thickness
$= k \times s = 0.7 \times 10 = 7$
$fu = 410\ N/mm^2$
r_{mw} = (for field welding) $= 1.50$

$$\therefore \quad (P_{dw})_{\substack{side \\ fillet}} = \frac{100 \times 0.7 \times 10 \times 410}{\sqrt{3} \times 1.50} = 110.46\ kN$$

Design strength of plug weld $= (P_{dw})_{plug}$

$$(P_{dw})_{plug} = \frac{\frac{\pi}{4} \times (20)^2 \times 410}{\sqrt{3} \times 1.5}$$

$(P_{dw})_{plug} = 49.577$ kN

$\therefore$ Design strength of fillet weld
$= P_{dw} = (P_{dw})_{sidefillet} + (P_{dw})_{plug} = 110.46 + 49.577$

$P_{dw} = 160.037$ kN

$\therefore$ Service load carrying capacity $= \frac{160.037}{1.5}$

$= 106.69$ KN Choice (A)

65. $[F]$ = inverse of stiffness matrix

$$= \frac{1}{(ad - bc)} k \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix}$$

$$[F] = \frac{1}{11 \times k} \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix}$$ Choice (C)

Mock Test 5

Number of Questions: 65 **Total Marks: 100**

Section – I: General Aptitude

Questions 1 to 5 carry One Mark each.

Directions for questions 1 and 2: Select the correct alternative from the given choices.

1. A, B, C, D are four points on a plane. $AB = 5$, $AC = 7$ and $AD = 10$. Which of the following is not a possible value of $BC + CD$?

(A) 6.5 (B) 10.5
(C) 24 (D) 30

2. Seven persons – A, B, C, D, E, F and G – are sitting in a row. A, B and C are women, D and E are children, and F and G are men. No two children or no two men or no two women sit next to each other. B and A have two persons sitting between them. F is two places away from A. D has two persons sitting to his right. G has two persons sitting to his left. If the maximum number of people sit between the two children, then who sits to the right of F?

(A) D (B) C
(C) E (D) B

Directions for question 3: In the following question, determine the relationship between the pair of capitalised words and then select the pair of words which has a **similar** relationship to the capitalized words. Mark the number of that pair as your answer.

3. SICKLE : FARMER : :

(A) Shears : Gardener
(B) Painter : Brush
(C) Computer : Whizkid
(D) Workshop : Lathe

Directions for question 4: The following statement has a part missing. Choose the best option from those given below the statement to make up the missing part.

4. Not only was the tone and tenor of the interaction especially positive, the two principals and their delegations _____.

(A) managed to generate specific outcomes also on any number of issues
(B) also manage to generate specific outcomes on a number of issues
(C) also managed to generate specific outcomes on a number of issues
(D) will also manage to generate specific outcomes of any number of issues

Directions for question 5: In the following question, four alternatives are given for the idiom/phrase printed in bold in the sentence. Choose the alternative which best expresses the meaning of the idiom/phrase.

5. Although Jack volunteered to help me cook the dinner, he threw a spanner in the works when it came to the actual cooking.

(A) started laughing
(B) left the place
(C) caused hindrance
(D) wholeheartedly assisted

Questions 6 to 10 carry Two Marks each.

Directions for questions 6 and 8: Select the correct alternative from the given choices.

6. If a, b, c, p, q, r are non-zero integers, $a^p = b$, $b^q = c$, $c^r = a$ and $pqr = 15$, the possible values of $a + b + c$ are _____.

(A) 1, –1 (B) 3, –3
(C) 1 (D) 3

7. A dealer in travel goods sells four types of suitcases – Standard, Deluxe, Super-deluxe and Premium. The number of each type of suitcase sold during the four quarters of 2015 are tabulated below. The costs of four types are ₹ 700, ₹ 900, ₹ 1000 and ₹ 1200 per suitcase respectively.

Type/ Quarter	Standard	Deluxe	Super-deluxe	Premium
Q_1	18,500	17,500	15,010	13,000
Q_2	16,060	16,800	15,590	13,400
Q_3	19,800	18,900	16,040	12,500
Q_4	20,740	19,500	16,090	12,800

Which suitcase accounts for the greatest part of the revenue for the dealer?

(A) Standard (B) Deluxe
(C) Super - deluxe (D) Premium

8. Vikram, his brother, his daughter and his son are playing doubles tennis match. Vikram's son is diagonally across the net from the tallest player's sibling. Vikram's brother is directly across the net from Vikram's daughter. The shortest and the tallest players are on the same side of the net. Who is the shortest player?

(A) Vikram (B) Vikram's brother
(C) Vikram's son (D) Vikram's daughter

Directions for question 9: In each of the following questions, statements 1 and 6 are respectively the first and the last sentences of a paragraph and statements 2, 3, 4 and 5 come in between them. Rearrange 2, 3, 4 and 5 in such a way that they make a coherent paragraph together with statements 1 and 6. Select the correct order from the given choices and mark its number as your answer.

9. 1. A lot of us do not realize that to carry out our day-to-day functions we need a minimum amount of endurance strength and flexibility in our bodies.
2. Adding to this, 21st century seems to have many an unforeseen emergency in store for all of us; whether it be the 9/11 attack, outbreak of epidemics, or strikes of terrorism – one never knows where and when what might happen.
3. Our current ways of life in general, are no longer providing our bodies with sufficient exercise to maintain adequate levels of fitness.
4. The realization of this dawns only when the necessity occurs, and that could be too late.
5. When we give in to the comforts of modern day living and inadvertently let our bodies deteriorate, our abilities to carry out physical activities are so adversely affected that one may not derive the full pleasure of living.
6. In such circumstances proving the quality and most likely the longevity of our lives is a matter of personal choice, and depends greatly on how well we are able to pursue sound physical fitness programmes that help maintain a strong body and mind.

(A) 5432 (B) 3254
(C) 2345 (D) 5243

Directions for question 10: Select the correct alternative from the given choices.

10. Over the years, there is a change in the attitude of disciples. While some are symbols of dedication and grace, others want to become superstars overnight, and in the process, defocus from their path to the extent of questioning the guru.
Which of the following can be inferred from the above?
(A) Those disciples who question the guru cannot become superstars.
(B) Earlier all the disciples were graceful and dedicated.
(C) Gurus are always right.
(D) For the disciples who are graceful and dedicated, becoming super stars overnight is not the priority.

Section – II: Civil Engineering

Directions for questions 11 to 65: Select the correct alternative form the given choices.

Questions 11 to 35 carry One Mark each.

11. The order and degree of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}+5\left(\frac{\partial^2 z}{\partial x\partial y}\right)^3-4\left(\frac{\partial z}{\partial x}\right)^5+3\frac{\partial z}{\partial y}=4z$$ respectively

are _____.
(A) 2 and 1 (B) 2 and 3
(C) 2 and 5 (D) 1 and 5

12. If f and g are any two scalar point functions that are continuous and have continuous first partial derivatives and if $\overline{G}$ = grad g, then curl($f\,\overline{G}$) is equal to _____.
(A) f(curl $\overline{G}$) (B) grad $f\times\overline{G}$
(C) g(grad f) (D) None

13. 8 books are placed at random in a shelf. The probability that three books B_1, B_2 and B_3 are together is _____.

14. If A is a 3 × 4 matrix with rank 3, then for any system of linear equations $AX = B$ has
(A) a unique solution,
(B) no solution
(C) infinitely many solutions
(D) exactly three solutions

15. If $f(x)+f(6-x)=0$ and $f(x)$ is continuous in [0, 6], then the value of $\int_0^6 f(x)dx$ is _____.

16. Lateral ties in *RCC* columns are provide to resist _____.
(A) Bending moment
(B) Shear
(C) Buckling of longitudinal bars
(D) Bothe bending moment and shear

17. Anchorage value of hook in RCC is _____ (ϕ = diameter of bar)
(A) 16ϕ (B) 12ϕ
(C) 8ϕ (D) 4ϕ

18. The most active clay mineral in shrinkage and swelling is _____.
(A) Kaolinite (B) Illite
(C) Hollysite (D) Montmorillonite

19. Following the usual notations, the flow value $N\phi$ is given as _____.
(A) $\tan^2(45+\phi/2)$ (B) $\tan^2(45-\phi/2)$
(C) $\tan(45+\phi/2)$ (D) $\tan(45-\phi/2)$

20. For a soil, the void ratio is given as 0.68. If the specific gravity of soil is 2.75, the shrinkage limit of the soil is _____
(A) 24.7% (B) 2.7%
(C) 40.4% (D) 12.3%

21. The soil "loam" means,
(A) Silt with little sand
(B) Sandy silt with little clay
(C) Clayey silt exhibiting slight cohesion
(D) Mixture of sand, silt and clay sized particles in approximately equal proportion.

22. The organism, with exhibits very nearly the characteristics of an ideal pathogenic indicator is
(A) Entamoeba histolytica
(B) Escherichia coli
(C) Salmonella typhi
(D) Vibrio conma

23. Pathogens are usually removed by
(A) Chemical precipitation
(B) Sedimentation
(C) Activated sludge process
(D) Chlorination

24. The reoxygenation coefficient K of stream is 0.5 at 20^0C. Its K value at 32^0C is likely to be _____.

25. The load transfer to the lower layers of flexible pavements is by
(A) Bending action of layers
(B) Shear deformation
(C) Grain to grain contact
(D) Consolidation of sub grade

26. Camber on highway pavement is provided to take care of
(A) Centrifugal force
(B) Drainage
(C) Sight distance
(D) Off tracking

27. The type of surveying in which curvature of the earth is taken into account is called
(A) Geodetic surveying
(B) Plane surveying
(C) Preliminary surveying
(D) Topographical surveying

28. The ratio of actual evapotraspiration to potential evapotranspiration is in the range of
(A) 0 to 0.4
(B) 0.6 to 0.9
(C) 0 to 1.0
(D) 1.0 to 2.0

29. At a point in a strained body, normal stresses are zero and shear stresses are 100 MPa. Value of principal stress are
(A) 50 MPa
(B) 100 MPa
(C) 150 MPa
(D) 20 MPa

30. In laminar flow through a pipe, the pressure drop per unit length of pipe is given by
(A) $\frac{32\mu\bar{u}}{D}$ (B) $\frac{16\mu\bar{u}}{D^2}$
(C) $\frac{128\mu Q}{\pi D^4}$ (D) $\frac{128\mu Q}{\pi D^2}$

31. A closed cylindrical vessel of radius R completely filled with a liquid of density ρ is rotated at an angular velocity ω about its vertical axis. The total fluid pressure force acting on the top is
(A) $\frac{\rho}{4}\omega^2\pi R^4$ (B) $\frac{\rho}{4}\omega^2\pi R^3$
(C) $\frac{\rho}{4}\omega^2\pi R^2$ (D) $\frac{\rho}{4}\omega^2\pi R$

32. A three hinged parabolic arch of span 4 m and rise 8 m carries a uniformly distributed load of 8 KN per unit over the whole span. The horizontal thrust at each support (in KN) is _____.

33. As per 1S : 800, the maximum bending moment for design of purlins can be taken as
(A) $\frac{WL}{8}$ (B) $\frac{WL}{4}$
(C) $\frac{WL}{10}$ (D) $\frac{WL}{12}$

34. When length of side fillet weld is 400 times the effective throat thickness, the design shear capacity of fillet weld is
(A) Decreased by 33% (B) increased by 33%
(C) Increased by 66% (D) Decreased by 66%

35. State of stress at a point is as follows. $\sigma_x = 900$ MPa, $\sigma_y = 300$ MPa. Maximum shear stress = 500 MPa. Value of maximum principal stress (in MPa) is
(A) 900 (B) 1000
(C) 1100 (D) 1200

Questions 36 to 65 carry Two Marks each.

36. The system of linear equations
$2x + 3y - 4z = 0$
$4x + 5y + z = 0$
$2x + 4y - 13z = 0$
has
(A) only a trivial solution
(B) exactly one linearly independent solution.
(C) exactly two linearly independent solutions.
(D) exactly three linearly independent solutions.

37. The coefficient of x^8 in the Taylor's series expansion of $f(x) = 1 + \cos^2(x^2)$ about $x = 0$ is _____
(A) $\frac{2}{3!}$ (B) $\frac{3}{4!}$
(C) $\frac{5}{6!}$ (D) $\frac{7}{8!}$

38. Which of the following is a value of the function $f(x, y) = \frac{x^2y^2 + 8(x+y)}{xy}$ at one of its local extreme points?
(A) 12 (B) 23
(C) 17 (D) 10

39. If $y(x)$ satisfies the differential equation $y^{11} + 8y^1 + 7y = 49$, then the value of $\lim_{x\to\infty} y(x)$ is _____.

40. For a set of 5 pairs of values of x and y,
$\sum x = 20$, $\sum y = 325$, $\sum xy = 366$ and $\sum x^2 = 120$
If a straight line of the form $y = a + bx$ is fitted to this data using the method of least squares, then the value of y at $x = 10$ is _____
(A) –25 (B) –75.10
(C) 41.32 (D) 30.50

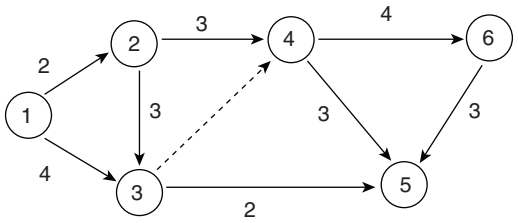
41. An *RC* short column 400 mm × 600 mm is made of M30 concrete and has 6 no's of 16 mm Fe500 steel bars. The ultimate load carrying capacity of the column is _____,

(A) 1478 KN (B) 3270 KN
(C) 3837 KN (D) 4904 KN

42. A pre stressing concrete beam 150 mm × 300 mm supports a load of 5 KN/m over a simply supported span of 8 m. The beam has parabolic cable of zero eccentricity at ends and 50 mm towards soffit at the central span. The pre – stressing force required to balance external load is _____.

(A) 800 KN (B) 850 KN
(C) 875 KN (D) 900 KN

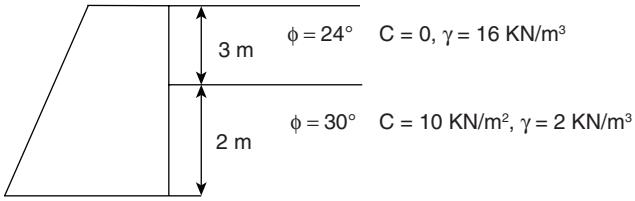
43.



Identify the critical path, for the given network.

(A) 1 – 2 – 3 – 4 – 5 – 6
(B) 1 – 3 – 4 – 5 – 6
(C) 1 – 2 – 3 – 5 – 6
(D) All the above

44. A retaining wall is shown below.
The active earth pressure at 5 m. depth is _____.



(A) 17.8 kpa (B) 29.3 kpa
(C) 40.8 kpa (D) 229.4 kpa

45. Two soil samples *A* and *B*, each weighing 1 kg and having water contents of 50% and 75% are mixed together. The resulting water content of the mixture is _____.

(A) 63.8% (B) 62.5%
(C) 61.7% (D) 60.8%

46. A clay sample originally 30 mm thick at a void ratio of 1.10 was subjected to a compressive load. After some period of time the clay sample was measured as 25 mm. The void ratio is _____.

(A) 0.385 (B) 1.45
(C) 0.68 (D) 0.75

47. A continuous footing of width 2.5 m rests at 1.5 m below the ground surface in clay. The unconfined compressive strength of the clay is 150 kpa and unit weight of soild is 16 KN/m³. For using factor of safety of 3, the safe bearing capacity of the footing as per terezaghi's theory is _____.

(A) 150.3 kpa
(B) 451 kpa
(C) 142.5 kpa
(D) 166.5 kpa

48. The composition of a certain *MSW* sample and specific weight of its various components are as given below:

Component	Percentage by weight	Specific weight (kg/m³)
Food	50	300
Dirt and Ash	30	500
Plastic	210	65
Wood and yard waste	20	125

(A) 319 (B) 139
(C) 217 (D) 199

49. A setting tank is designed for a surface overflow rate of 40 m²/day m². Assuming specific gravity of sediment particles = 2.65. Density of water γ_w = 1000 kg/m³, dynamic velocity of water = 0.00 NS/m² and stoke's law's valid. The approximate minimum size of particles which can be completely removed is _____.

50. Match

	List – I		List – II
a.	Evapora transpiration	1.	Penman method
b.	Infiltration	2.	Snyder's method
c.	Synthetic unit hydrograph	3.	Museingham method
d.	Channel routing	4.	Horton's method

	a	b	c	d
(A)	1	3	4	2
(B)	1	4	2	3
(C)	3	4	1	2
(D)	4	2	1	3

51. A road is being designed for a speed of 110 km/hr on a horizontal curve with a super elevation of 8%. If the coefficient of side friction is 0.1, the minimum radius of curve 9 in m) required for safe vehicular movement is _____.

52. Match the following:

P.	Resistance to impact	1.	Hardness
Q.	Resistance of wear	2.	Strength
R.	Resistance to weathering action	3.	Toughness
S.	Resistance to crushing	4.	Soundness

(A) $P-1, Q-3, R-4, S-2$
(B) $P-3, Q-1, R-4, S-2$
(C) $P-4, Q-1, R-3, S-2$
(D) $P-3, Q-4, R-2, S-1$

53. In a cylindrical bituminous mix VMA = 15% and V_v = 4.5%. The magnitude of VFB is
(A) 24 (B) 54
(C) 40 (D) 70

54. The speed density ($v - k$) relationship on a single lane road with uni directional flow is $v = 70 - 0.7\,k$, where v is km/hr and k is in veh/km. The capacity of road (in veh/hr) is _____.

55. The width of expansion joint gap is 2.5 cm in a cement concrete pavement. The spacing between expansion joint for a maximum rise in temperature of 25^0C is (Assuming coefficient of thermal expansion of concrete as 10×10^{-6} per degree C)
(A) 5 m
(B) 50 m
(C) 100 m
(D) 25 m

56. In a leveling work, sum of back sight (B S) and fore-sight (F S) have been found to be 3.985 m and 5.725 m respectively. If R.L of the starting station is 100.00 m, the RL (in m) of the last station is _____

57. The chainage of the intersection point of two straight is 1585.6 m and angle of intersection is 150^0. If the radius of a circular curve is 600.0 m, the tangent distance and length of the curve respectively are
(A) 418.88 and 1466.08
(B) 218.38 and 1648.49
(C) 314 m and 160.79
(D) 418.88 and 218.38

58. A hydraulic turbine is to develop 1010 kW while running at 120 rpm under a head of 12 m. Overall efficiency of the turbine at the best operating point is 90%. For the prediction of performance a 1 : 10 scale model is tested under a head of 7 m. Power output (in kW) of the model turbine for similar conditions of prototype is.

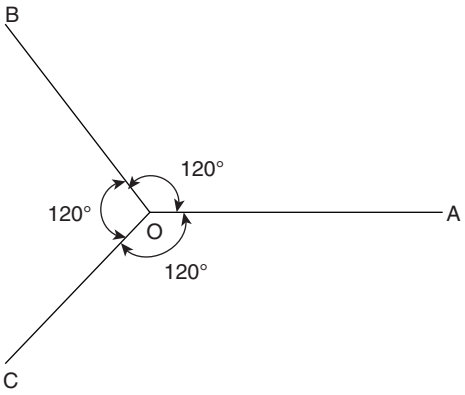
59. A Kaplan turbine works under the following conditions
Head = 20 m
Speed = 150 rpm.
Hydraulic efficiency = 95%
Outer diameter of runner = 4.5 m.
Hub diameter = 2 m
Velocity of flow = 10.2 m/s
Runner inlet vane angle (in degree) at outer periphery of the runner is
(A) 160.7
(B) 155.2
(C) 150.4
(D) 144.3

60. An inward flow reaction turbine has an external diameter of 1 m. Speed of the runner is 210 rpm and guide blades make an angle of 10° to the wheel tangent. If velocity of flow at inlet is 2.2 m/s, the runner vane a _____.

61. A cylindrical vessel 3 m long and 1 m diameter is subjected to an internal pressure of 1.5 N/mm^2. The hoop stress developed was 50 N/mm^2. If Young's modulus and Poisson's ratio are 2×10^5 N/mm^2 and 0.3 respectively change in volume (in cm^3) is _____.

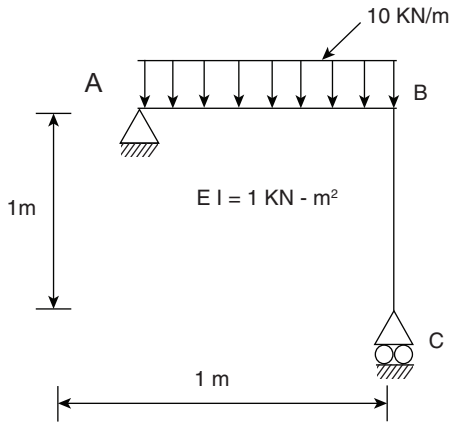
62. A cylindrical vessel of 1.5 m internal diameter and 4 m length has 10 mm shell thickness. If it is subjected to a fluid pressure of 2 MPa, maximum shear stress induced (in N/mm^2) is
(A) 25
(B) 37.5
(C) 50
(D) 62.5

63.



At a stressed point in a two dimensional stress system, strain were measured to 3 directions OA, OB and OC, 120° to each other. The observed strains are
$e_A = 300 \times 10^{-6}$, $e_B = -225 \times 10^{-6}$ and
$e_C = 100 \times 10^{-6}$.
Value of strain at an angle 90° clockwise to OA (in micron) is _____.

64. The rigid jointed plane frame ABC is as shown below. The deflection at center of AB is _____ (in mm)



65. The design bending strength of laterally supported beam of ISLB 350 @ 486 N/m, when the design shear force (V) is less than the design shear strength is _____. (in KN – m). (Assume plastic section, Z_{PZ} = 850 10^3 mm^3 Z_{ez} = 750 10^3 mm^3 and steel of grade Fe410]

Answer Keys

1. D	**2.** D	**3.** A	**4.** C	**5.** C	**6.** B	**7.** B	**8.** B	**9.** A	**10.** D
11. B	**12.** B	**13.** 0.106 to 0.108		**14.** C	**15.** 0	**16.** C	**17.** A	**18.** D	**19.** A
20. A	**21.** B	**22.** B	**23.** D	**24.** 0.604	**25.** C	**26.** B	**27.** A	**28.** C	**29.** B
30. C	**31.** A	**32.** 2	**33.** D	**34.** A	**35.** C	**36.** B	**37.** A	**38.** A	**39.** 7
40. B	**41.** B	**42.** A	**43.** A	**44.** A	**45.** C	**46.** B	**47.** D	**48.** B	
49. 5.14×10^{-9} m		**50.** B	**51.** 0.071	**52.** B	**53.** D	**54.** 1750 veh/hr		**55.** B	
56. 98.26		**57.** C	**58.** 4.49 to 5.55		**59.** A	**60.** 56.0 to 56.1			
61. 1119.15 to 1119.25			**62.** B	**63.** 183 to 184		**64.** 13.00 to 13.05			
65. 190 – 195 KN – m									

Hints and Explanations

1. $AB = 5, AC = 7 \therefore 2 \le BC \le 12$
$BC = 7, AD = 10 \therefore 3 \le CD \le 17$
$\therefore \quad 5 \le BC + CD \le 29$
Among the choices, 30 is not a possible value.
Choice (D)

2. Let us take down the data as given below:
(i) Seven persons – A, B, C, D, E, F and G - sit in a row.
(ii) 3 women : A, B and C
2 children : D and E
2 men : F and G
(iii) No two persons of the same gender (or no two children) sit next to each other.
(iv) $\frac{B}{A} - - \frac{A}{B}$
(v) $\frac{F}{A} - \frac{A}{F}$
(vi) D _ _ (2 persons sitting to the right of D)
(vii) _ _ G (G has two persons sitting on his left).
The positions of G and D are located as shown:

$G \quad D$
_ _ _ _ _ _ _
$(m) \quad (c)$

Here, $m \rightarrow$ Man, $w \rightarrow$ Woman and $c \rightarrow$ Child.
For the maximum number of people to sit between the two children, the other child has to be seated at the extreme left end. Also, other conditions are used to get the following arrangement:

E	C	G	A	D	F	B
---	---	---	---	---	---	---
(1)	(w)	(m)	(w)	(c)	(m)	(w)
$x(f)$		$x(f)$				

Hence, B sits to the right of F. Choice (D)

3. A 'sickle' is used by the farmer to cut the weeds i.e. for weeding. In the same way 'shears' are used by the 'gardener' to cut the bushes. (Shears are large pair of scissors). Hence option A is the correct answer. Option B can be ruled out as the order is reversed, it should be (Brush : Painter). A 'whizzkid' is a person who is successful at a very young age, not necessarily in computers. A 'lathe' is a machine that shapes pieces of wood or metal by holding and turning them against a fixed cutting tool and a 'workshop' is a place where things are repaired or made. Choice (A)

4. Since we have 'was' in the beginning of the sentence it should be 'managed' (past tense) in the second half. So we look at choices A and C. 'A number of' is better than 'any number of' since the latter suggests a very high number. Choice (C)

5. The idiom 'throw a spanner in the works' means 'to cause hindrance'. It means to do something that prevents a plan or activity from succeeding. Choice (C)

6. $a^p = b, b^q = c, c^r = a$ .... (1)
and $pqr = 15$ .... (2)
$a^p = b \Rightarrow c^{rp} = b \Rightarrow b^{qrp} = b$
$\therefore$ Either $b = 1, 0, -1$ or $pqr = 1$. As $pqr \neq 1$, $b = 1$ or -1
($\because$ a, b, c are nonzero integers). If $b = 1$, then $c = 1$ and $a = 1$
If $b = -1$, (as p, q, r are odd integers), it follows that $c = -1$ and $a = -1$
$\therefore \quad a + b + c = 3$ or -3. Choice (B)

7. The total sales (number), the cost of each type of suitcase and the total revenue from the four types of suitcases are tabulated below.

Type	Number	Cost	Revenue
S	75,100	700	5,25,70,000
D	72,700	900	6,54,30,000
SD	62,730	1000	6,27,30,000
P	51,700	1200	6,20,40,000

The greatest revenue is from Deluxe. Choice (B)

8. Let Vikram, his brother, his son, and his daughter be V, B, S and D.
If S is diagonally opposite to D, then S is the taller player. The arrangement is as follows:

S	B
V	D

And now, B is the shortest player ($\because$ shortest and tallest players are on same side)

If S is diagonally opposite to V, then D and B cannot be directly opposite.

If S is diagonally opposite to B, then V is the taller player. The arrangement is as follows:

S	D
V	B

And now, B is the shortest player. Choice (B)

9. Statement 1 says to carry out our day-to-day activities a minimum amount of endurance and flexibility in our bodies is needed. If we look at the statements, '5' is the most appropriate one to follow 1. '5' emphasizes on the fact that our physical activities are adversely affected, which is a continuation of the idea expressed in '1'. The words "the realization of this ...." refers to the reason for our abilities to be adversely affected. Hence '4' is a better statement to follow '5'. Further in '3' we find emphasis on the current ways of life. Thus '543' is the logical order which is further followed by '2'.

Choice (A)

10. According to the passage a disciple becomes defocused to the extent of questioning the guru once they want to become superstars overnight. But the statement does not explicitly indicate whether they would be successful or not. Hence, (A) cannot be inferred. The passage describes the attitude of present disciples but not the past ones. Hence, (B) cannot be inferred. The passage did not discuss about the gurus, hence, (C) cannot be inferred. The passage demarks the disciples as those who are dedicated and graceful and the other as those who want to become super stars overnight. From this it can be understood that one group is different from the other. Hence, (D) can be inferred. Choice (D)

11. Order = 2 and degree = 3. Choice (B)

12. Given $\overline{G} = G\text{rad } g$

Now $\text{curl}(f\overline{G}) = \text{Grad } f \times \overline{G} + f(\text{curl } \overline{G})$

$= \text{Grad } f \times \overline{G} + f(\text{curl (grad } g))$

$= \text{Grad } f \times . \overline{G} . + f(\overline{0})$

($\because$ curl (Grad g) = $\overline{0}$)

$= \text{Grad } f \times \overline{G}$. Choice (B)

13. Total number of ways of placing 8 books in a shelf = 8!

As three books B_1, B_2 and B_3 has to be together, take the three books as a single entity.

The number of ways of arranging the three books among themselves = 3!

The number of ways of arranging the remaining 5 books and this entity (The three books together) = 6!

$\therefore$ The number of ways of arranging the 8 books such that the three books are together = $3! \times 6!$

$\therefore$ Required probability $= \dfrac{3! \times 6!}{8!} = \dfrac{3}{28} = 0.1071$

Ans: 0.106 to 0.108

14. Given that A is a 3×4 matrix with rank 3.

$\therefore$ Any system of linear equations $AX = B$ will consist of 4 unknowns

Also, as $\rho(A) = 3$, we have

$\rho([A/B]) = 3$

$\therefore$ $\rho(A) = \rho([A/B]) = 3 < 4$ (The number of unknowns), the system of equations $AX = B$ has infinitely many solutions. Choice (C)

15. We know that, if $f(x) = -f(2a - x)$, then

$$\int_0^{2a} f(x)dx = 0$$

Now $f(x) + f(6 - x) = 0 \Rightarrow f(x) = -f(6 - x)$

$\therefore$ $a = 3$

Hence $\displaystyle\int_0^6 f(x)dx = 0$ Ans: 0

16. Lateral ties in columns are provided, to avoid buckling of longitudinal (main) steel. Choice (C)

17. 180^0 bend is called Hook

Anchorage value for 180^0 bend is 16ϕ Choice (A)

20. $e = \dfrac{WG}{S_\gamma}$

At shrinkage limit, $S\gamma = 1$

$0.68 = \dfrac{W \times 2.75}{1}$

$\therefore$ $W = 24.7\%$ Choice (A)

24. $K_{R(T^0C)} = K_{R(20^0C)}[1.016]^{T-20C}$

$= 0.5\,[1.016]^{32-20}$

$= 0.604$ Ans: 0.604

29. $p_x = 0$, $p_y = 0$, $\tau = 100$ MPa

$$p_{1,2} = \frac{p_x + p_y}{2} \pm \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + \tau^2}$$

$= 0 \pm \sqrt{0 + \tau^2} = \tau$

$= 100$ MPa Choice (B)

30. $\dfrac{\Delta P}{L} = \dfrac{32\mu\overline{u}}{D^2}$

Substituting $\overline{u} = \dfrac{Q}{\left(\dfrac{\pi d^2}{4}\right)}$

$\dfrac{\Delta P}{L} = \dfrac{128\mu Q}{\pi D^4}$ Choice (C)

32. Horizontal thrust at each support is $\frac{wl^2}{8h}$

$$\therefore \quad H = \frac{8\text{KN/m} \times (4)^2\,\text{m}^2}{8 \times 8\text{m}} = \frac{8 \times 16}{8 \times 8}$$

$$\boxed{H = 2\ \text{KN}}$$

Ans: 2

33. As per 1*S* : purlins are assumed as continuous beams in design. So maximum bending moment to be taken as $\frac{WL}{12}$. Choice (D)

34. $L_J = 400\ t_t$

If $L_J > 150\ t_t \Rightarrow$ a Reduction factor need to be applied for design shear capacity of fillet weld.

Reduction factor (β_{LW})

$$(\beta_{LW}) = 1.20 - \frac{0.20 L_J}{150 t_t}$$

$$= 1.20 - \frac{0.20 \times 400 t_t}{150 t_t}$$

$$(\beta_{LW}) = 0.66$$

So strength reduced by 33% Choice (A)

35. Maximum principal stress

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + (\tau_{\max})$$

$$= \left(\frac{900 + 300}{2}\right) + 500$$

= 1100 MPa Choice (C)

36. Given system of linear equations is

$$\left.\begin{aligned} 2x + 3y - 4z &= 0 \\ 4x + 5y + z &= 0 \\ 2x + 4y - 13z &= 0 \end{aligned}\right\} \quad \rightarrow (1)$$

It can be written in matrix form as

$AX = 0$

Where $A = \begin{bmatrix} 2 & 3 & -4 \\ 4 & 5 & 1 \\ 2 & 4 & -13 \end{bmatrix}$

$R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - R_1$

$$\sim \begin{bmatrix} 2 & 3 & -4 \\ 0 & -1 & 9 \\ 0 & 1 & -9 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$\therefore \quad A \begin{bmatrix} 2 & 3 & -4 \\ 0 & -1 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \quad \rho(A) = r = 2$

The number of variables = $n = 3$

So, the number of linearly independent solutions is $n - r = 3 - 2 = 1$ Choice (B)

37. We have $f(x) = 1 + \cos^2(x^2)$

$$= 1 + \left(\frac{1 + \cos 2x^2}{2}\right)$$

$$= \frac{3}{2} + \frac{1}{2}\cos 2x^2$$

$$= \frac{3}{2} + \frac{1}{2}\left[1 - \frac{(2x^2)^2}{2!} + \frac{(2x^2)^4}{4!} - \frac{(2x^2)^6}{6!} + \ldots\infty\right]$$

$$= 2 - 2\frac{x^4}{2!} + 2^3\frac{x^8}{4!} - 2^5\frac{x^{12}}{6!} + \ldots\ldots\infty$$

$\therefore$ The coefficient of $x^8 = \frac{2^3}{4!} = \frac{2}{3!}$ Choice (A)

38. Given $f(x,y) = \frac{x^2y^2 + 8(x+y)}{xy} = xy + \frac{8}{x} + \frac{8}{y}$

$$\therefore \quad f_x = y - \frac{8}{x^2} \text{ and } f_y = x - \frac{8}{y^2}$$

$$f_x = 0 \Rightarrow y - \frac{8}{x^2} = 0$$

$$\Rightarrow x^2y = 8 \quad \rightarrow (1)$$

and $f_y = 0 \Rightarrow x - \frac{8}{y^2} = 0$

$$\Rightarrow xy^2 = 8 \quad \rightarrow (2)$$

From (1) and (2), we have

$x^2y = xy^2 \Rightarrow x = y$

$\therefore$ From (1), $x^2 \times x = 8 \Rightarrow x = 2$

$\Rightarrow y = 2$

$\therefore$ The stationary point of $f(x, y)$ is (2, 2)

Now $r = f_{xx} = \frac{16}{x^3}$; $s = f_{xy} = 1$ and $t = f_{yy} = \frac{16}{y^3}$

At (2, 2), $rt - s^2 = 3 > 0$ and $r = 2 > 0$

$\therefore$ $f(x, y)$ has a local minimum at (2, 2) and the value of $f(x, y)$ at (2, 2) is 12. Choice (A)

39. Given differential equation is

$y^{11} + 8y^1 + 7y = 49 \quad \rightarrow (1)$

Its general solution is $y = y_c + y_p \quad \rightarrow (2)$

To find y_c:

The homogenous differential equation corresponding to (1) is

$y^{11} + 8y^1 + 7y = 0$

Its auxiliary equation is

$D^2 + 8D + 7 = 0$

$\Rightarrow (D + 1)(D + 7) = 0$

$\Rightarrow D = -1; D = -7$

$\therefore$ The complementary function of (1) is

$y_c = c_1e^{-x} + c_2e^{-7x} \quad \rightarrow (3)$

To find y_p:

$$y_p = P.I = \frac{1}{f(D)}X = \frac{1}{(D^2+8D+7)}49 = \frac{1}{7}\times 49$$

$y_p = 7$ → (4)

Substituting the values of y_c and y_p in (2)

We get the general solution of (1) as

$y(x) = c_1e^{-x} + c_2e^{-7x} + 7$

Now $\underset{x\to\infty}{Lt}\, y(x) = \underset{x\to\infty}{Lt}\, [c_1e^{-x} + c_2e^{-7x} + 7] = 7$ Ans: 7

40. Given $n = 5$, $\Sigma x = 20$, $\Sigma y = 325$, $\Sigma xy = 366$ and $\Sigma x^2 = 120$

The normal equations in the process of fitting a straight line $y = a + bx$ to the above data are

$\Sigma y = na + b\Sigma x$

and $\Sigma xy = a\Sigma x + b\Sigma x^2$

i.e., $325 = 5a + 20b$

$366 = 20a + 120b$

Solving these equations for a and b, we have

$a = 158.4$ and $b = -23.35$

$\therefore$ The straight line that fits to the given data is $y = 158.4 - 23.35\,x$

$\therefore$ y at $x = 10$ is

$y(10) = 158.4 - 23.35 \times 10 = -75.1$ Choice (B)

41. $A_{SC} = 6 \times \frac{\pi}{4} \times 16^2 = 1206\ m^2$

$A_C = A_g - A_{SC}$

$= (400 \times 600) - 1206$

$= 238794\ mm^2$

$P_u = 0.4\,fck\,A_C + 0.67\,fy\,A_{SC}$

$= 0.4(30)(1206) + 0.67(500)(1206)$

$= 3269838$ N

$\therefore$ $P_u = 3270$ KN Choice (B)

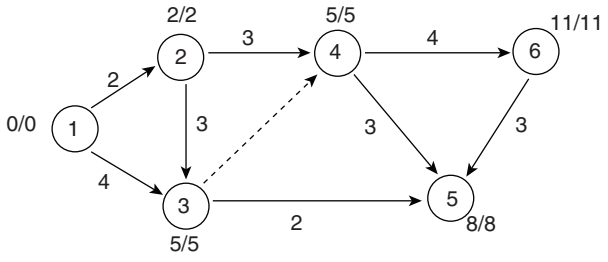
42. $P.e = \frac{wl^2}{8}$

$P(50) = \frac{5\times(8000)^2}{8}$

$P = 8{,}00{,}000$N

$\therefore$ $P = 800$ KN Choice (A)

43.



Total flat for 1 – 3 and 3 – 5 is not zero. Choice (A)

44. At 5m;

$\sigma_V = 3 \times 16 + 2 \times 20 = 88$ KPa

$P_a = K_{a2}\sigma_V - 2C_2\sqrt{Ka_2}$

$K_{a2} = = 0.33$

$\therefore$ $P_a = 0.33 \times 88 - 2(10)\sqrt{0.33}$

$= 17.80$ KPa Choice (A)

45. For sample A;

$W = \frac{W_w}{W_s}$

$W_w = 0.5\ W_s$

$W_w + W_s = 1$kg

$0.5\ W_s + W_s = 1$

$W_s = 0.667$ kg and $W_w = 0.333$ kg

Similarly, for sample B;

$W_s = 0.571$ kg and $W_w = 0.43$ kg

Total water weight; $W_w = 0.333 + 0.43 = 0.763$ kg

Total solids weight $W_g = 0.667 + 0.571 = 1.237$ kg

$\therefore$ Water content $(wg) = \frac{0.763}{1.237}\times 100$

$= 61.68\%$ Choice (C)

46. $\frac{\Delta e}{1+e_0} = \frac{\Delta H}{H_0}$

$\frac{\Delta e}{1+1.1} = \frac{30-25}{30}$

$\Delta e = 0.35$

$e_f = e + \Delta e$

$= 1.1 + 0.35$

$\therefore$ $e_f = 1.45$ Choice (B)

47. $C_u = \frac{150}{2} = 75$ kPa

$Q_s = \frac{1}{F}(CN_c) + r.D$

$= \frac{1}{3}(75 \times 5.7) + 16(1.5) = 166.5$ kPa Choice (D)

48. $\frac{100}{x} = \frac{50}{300} + \frac{30}{500} + \frac{20}{65} + \frac{20}{125}$ (Harmonic mean)

$\frac{100}{x} = 0.72$

$x = \frac{100}{0.72} = 138.8\ kg/m^3 \backsimeq 139\ kg/m^3$. Choice (B)

49. $V_o = V_s$

$V_o = \frac{(G_S - 1)r_w d^2}{18\mu}$

Now $V_o = \frac{40\text{m}^3}{\text{day.m}^2} = \frac{40}{24\times60\times60}$

$= \frac{(2.65-1)\times9.81\times10^3\times d^2}{18\times0.01}$

Solving we get

$d = 5.14\times10^{-9}$ m Ans: 5.14×10^{-9} m

51. $e+f = \frac{V^2}{gR}$ (or) $e+f = \frac{V^2}{127R}$

$e+f = \frac{(60)^2}{127\times128}$

$e = \frac{(60)^2}{127\times128} - 0.15$

$= 0.071$ Ans: 0.071

53. $VFB = \frac{V_B}{V_V} = \left(\frac{15-4.5}{15}\right)\times100 = 70\%$ Choice (D)

54. $C = VK$

$= 70k - 0.7\,kz$

$\frac{dc}{dk} = 70 - 1.4k = 0$

$K = 50$

$C = 70\times50 - 0.7(50)^2$

$= 1750$ veh/hr Ans: 1750 veh/hr

55. $\sigma_1 = \frac{2.5}{2} = 1.25$ cm $= 1.25\times10^{-2}$ m

$t_2 - t_1 = 25^0$C

coefficient of thermal expansion $\alpha = 10\times10^{-6}$/C

$\sigma_1 = L\alpha\,(t_2 - t_1)$

$\therefore$ spacing of expansion joint $L = \frac{\sigma_1}{\alpha(t_2-t_1)}$

$= \frac{1.25\times10^{-2}}{10\times10^{-6}\times25} = 50$ metres Choice (B)

56. Using rise and fall method

$\sum FS > \sum BS$

Fall = $\sum F.S - \sum B.S = R.L$ of first station $- R.L$ of last station

$= 5.725 - 3.985$

$= 1.74$ m

$R.L$ (last station = $R.L$ (first station) – fall

$= 100 - 1.74 = 98.26$ Ans: 98.26

57. Length of the curve $= \frac{\pi R}{180^0}\Delta$

$= \frac{\pi\times600}{180}\times30^0\ (\Delta = 180 - 150 = 30^0)$

$= 314$ m

Tangent distance $= R\tan\frac{\Delta}{2} = 600\times\tan(15^0)$

$= 160.79$m Choice (C)

58. Prototype –

$P_p = 1010$ kW

$N_p = 120$ rpm

$H_p = 12$ m

Model –

$H_m = 7$ m

Scale ratio = 1 : 10

Equating the head coefficient

$\left(\frac{H}{N^2D^2}\right)_p = \left(\frac{H}{N^2D^2}\right)_m$

$\Rightarrow \left(\frac{N_m}{N_p}\right) = \frac{H_m}{H_p}\times\left(\frac{D_p}{D_m}\right)^2$

$\Rightarrow \left(\frac{N_m}{120}\right)^2 = \frac{7}{12}\times10^2$

$\Rightarrow N_m = 916.5$ rpm

Equating power coefficient

$\left(\frac{P}{N^3D^5}\right)_p = \left(\frac{P}{N^3D^5}\right)_m$

$\Rightarrow \left(\frac{P_m}{P_p}\right) = \left(\frac{N_m}{N_p}\right)^3\times\left(\frac{D_m}{D_p}\right)^5$

$\Rightarrow \frac{P_m}{1010} = \left(\frac{916.5}{120}\right)^3\times\left(\frac{1}{10}\right)^5$

$\Rightarrow P_m = 4.4996$ kW Ans: 4.49 to 5.55

59. Head $H = 20$ m

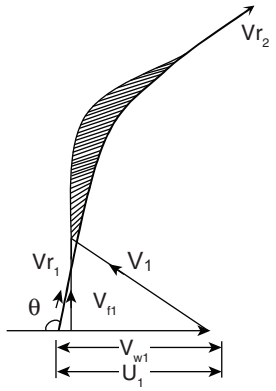
Speed $N = 150$ rpm

Hydraulic efficiency $\eta_h = 95\%$

Outer diameter $D = 4.4$ m

Hub diameter $d = 2$ m

Hydraulic efficiency = 95 %



At the outer periphery of the runner,

Velocity of flow,

$V_{f_1} = 10.2$ m/s

Blade velocity at inlet

$$u_1 = \frac{\pi DN}{60} = \frac{\pi \times 4.4 \times 150}{60} = 34.5575 \text{ m/s}$$

Hydraulic efficiency

$$\eta_h = \frac{V_{w_1} u_1}{gH} \text{ where } V_{w_1} = \text{velocity of whirl}$$

$$\therefore \quad 0.95 = \frac{V_{w_1} \times 34.5575}{9.81 \times 20}$$

$$\Rightarrow \quad V_{w_1} = 5.3936 \text{ m/s}$$

From inlet velocity triangle,

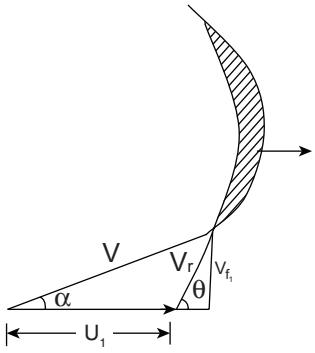
$$\tan(180 - \theta) = \frac{V_{f_1}}{u_1 - V_{w_1}}$$

$$= \frac{10.2}{34.5575 - 5.3936} = 0.3497$$

$$\Rightarrow \quad 180 - \theta = 19.277°$$

$$\Rightarrow \quad \theta = 160.7°$$ Choice (A)

60.



$D_1 = 1.2$ m.
$N = 210$ rpm.

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1 \times 210}{60}$$

$= 10.9956$ m/s

$V_{f_1} = 2.2$ m/s

Guide blade angle $\alpha = 10°$
Let θ be the runner vane angle at inlet.
From velocity triangle,

$$\tan\theta = \frac{V_{f_1}}{V_1 \cos\alpha - u_1}$$

But $\frac{V_{f_1}}{V_1 \cos\alpha} = \tan\alpha = \tan 10$

$$\Rightarrow \quad V_1 \cos\alpha = \frac{V_{f_1}}{\tan\alpha} = \frac{2.2}{\tan 10} = 12.4768$$

$$\therefore \quad \tan\theta = \frac{2.2}{12.4768 - 10.9956} = 1.4853$$

$$\Rightarrow \quad \theta = 56.04870$$ Ans: 56.0 to 56.1

61. $L = 3$ m $= 3000$ mm
$d = 1$ m $= 1000$ mm
$p = 1.5$ N/mm^2
$E = 2 \times 10^5$ N/mm^2
$\mu = 0.3$
Hoop stress $f_1 = 50$ N/mm^2
Longitudinal stress

$$f_2 = \frac{f_1}{2} = 25 \text{ N/mm}^2$$

Diametral strain

$$e_1 = \frac{\delta d}{d} = \frac{f_1}{E} - \mu \frac{f_2}{E}$$

$$= \frac{1}{E}(f_1 - \mu f_2)$$

$$= (50 - 0.3 \times 25) = \frac{42.5}{E}$$

Longitudinal strain

$$e_2 = \frac{\delta L}{L} = \frac{f_2}{E} - \mu \frac{f_1}{E}$$

$$= \frac{1}{E}(f_2 - \mu f_1)$$

$$= \frac{1}{E}(25 - 0.3 \times 50) = \frac{10}{E}$$

Volumetric strain

$$\frac{\delta V}{V} = 2e_1 + e_2 = \frac{2 \times 42.5}{E} + \frac{10}{E} = \frac{95}{E}$$

Change in volume

$$\delta V = \frac{V}{E} \times 95 = \frac{\pi d^2 L}{4E} \times 95$$

$$= \frac{\pi \times (1000)^2 \times 3000 \times 95}{4 \times 2 \times 10^5}$$

$= 1119{,}192.4$ mm^3
$= 1119.19$ cm^3. Ans: 1119.15 to 1119.25

62. $p = 2$ MPa $= 2$ N/mm^2
$d = 1.5$ m $= 1500$ mm
$L = 4$ m $= 4000$ m
$t = 10$ mm

Hoop stress $f_1 = \frac{pd}{2t} = \frac{2 \times 1500}{2 \times 10}$

$= 150$ N/mm^2

Longitudinal stress $f_2 = \frac{pd}{4t} = \frac{f_1}{2} = \frac{150}{2}$

$= 75$ N/mm^2

Maximum shear stress $\frac{f_1 - f_2}{2} = \frac{150 - 75}{2}$

$= 37.5$ N/mm^2 Choice (B)

63. Treating OA as x-axis, 90° clockwise to OA is y-axis

$e_A = e_x = 300$ micron.

$e_B = -225$ micron.

$e_C = 100$ micron.

$$e_B = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2}\cos 240 + \frac{1}{2}\gamma_{xy}\sin 240$$

$$\Rightarrow \quad -225 = \frac{300 + e_y}{2} + \frac{300 - e_y}{2} \times \left(\frac{-15.5}{2}\right)$$

$$+\frac{1}{2}\gamma_{xy}(-0.866) \qquad \rightarrow (1)$$

$$e_C = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2}\cos 480 + \frac{1}{2}\gamma_{xy}\sin 480$$

$$\Rightarrow \quad 100 = \frac{300 + e_y}{2} + \frac{300 - e_y}{2}(-0.5)$$

$$\frac{1}{2}\gamma_{xy}(0.866) \qquad \rightarrow (2)$$

Adding (1) and (2)

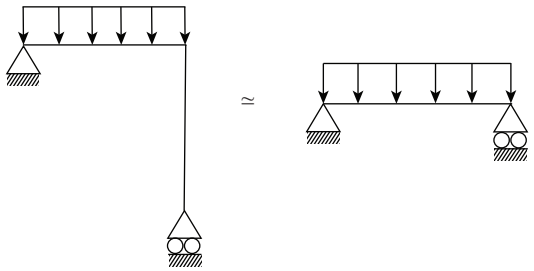
$$-225 + 100 = (300 + e_y) - \frac{(300 - e_y)}{2}$$

$$-125 = \frac{300}{2} + \frac{3}{2}e_y$$

$$\Rightarrow \quad \frac{3}{2}e_y = -125 - 150$$

$$\Rightarrow \quad e_y = 183.33 \text{ micron} \qquad \text{Ans: 183 to 184}$$

64. The above frame can be simplified as



Since there is only vertical load in column BC.

$\therefore$ Deflection at center of $AB = \dfrac{5WL^4}{384 \in \mathrm{I}}$

$$= \frac{5 \times (10)(1)^4}{384 \times 10} = 13.02 \text{ mm} \qquad \text{Ans: 13.00 to 13.05}$$

65. For Laterally supported beam;

Design bending capacity;

$$M_d = \beta_p Z_{pz} \frac{f_y}{r_{mo}} \text{ [For plastic section } \beta_p = 1.0]$$

$$= 1.0 \times 850 \times 10^0 \times \frac{250}{1.1}$$

$$= 193.18 \times 10^6 \text{ N} - \text{mm}$$

$M_d = 193.18$ KN – m

$$M_d < 1.2\, Z_e \frac{f_y}{r_{mo}} = 1.2 \times \frac{750 \times 10^3 \times 250}{1.0}$$

$$= 204.54 \text{ KN} - \text{m}$$

$\therefore$ Hence OK

$M_d = 193$ KN – m Ans: 190 – 195 KN – m