

Handbook *of* **Civil** **Engineering**

Useful For
**GATE, ESE, PSUs &
Other Competitive Exams**

■ **Praveen Dwivedi** ■ **Prachi Bajpai**

Hand Book of **Civil Engineering**



by
Praveen Dwivedi &
Prachi Bajpai



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About The Author



Praveen Dwivedi is currently employed with Reserve Bank of India as Technical Manager. He has formerly worked with Central Public Works Department. He obtained his B.Tech degree from Motilal Nehru National Institute of Technology, Allahabad in 2010. His main areas of interest include teaching and motivating students. All this while, he has been

extensively involved in teaching at some of the best coaching institutes for Engineers.

Praveen has qualified several written examinations of national repute including UPSC, RBI Grade-B (Dr) (Twice), DSSSB-Manager AIR-1, BEL- AIR 1, NBCC -AIR 2, WAPCOS- AIR 3, MPPSC- AIR 4, DSSSB Assistant Manager AIR-7, SIDBI- AIR 8, Syndicate Bank- AIR 12, DFCCIL- Top 20, SSC – AIR 21, UP Nirman Nigam Ltd- AIR 79, NPCC and GATE (several times). The experience from all of these exams has been fully utilised in the making of this book.

His love towards technology, reading and writing has also driven him to work as content developer for some of the best coaching institutes and publication houses in India. In terms of practical exposure, he has also been actively involved in 'Mahayojana Project' of Allahabad city, Sasan Ultra Mega Power Project and several Residential and Official projects throughout the country.

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I am thankful to all the teachers who taught me during the concept building session of life, especially the Super Masters of my Alma Mater (NIT Allahabad), Kanchan Sir, Ankit Sir, my Mentor- Arun Pratap Singh and Master Abhinav Trivedi.

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I particularly want to thank my wonderful and talented students whom I have taught over the years, who in turn taught me how to be a good teacher.

Special thanks to Prachi Bajpai and Team GKP for their diligent efforts to prepare a book which is thoroughly checked, so as to eliminate any possibility of error.

Despite our sincere efforts to keep the book void of errors, there is a possibility that some errors might have been left unnoticed while printing. I would sincerely welcome constructive criticism for improving the book for its subsequent edition. The feedback can be shared at Praveenmnit2k6@gmail.com.

Praveen Dwivedi

Preface

If you've got technical examinations such as GATE, ESE & PSUs around the corner and having to quickly go through a wide range of topics in a limited time, this is the book would be a complete solution for you. Technical exams being difficult in nature, has to be given enough priority and time during exam preparation. But it is also quite easy to score in these exams as you just need to be thorough with formulas, key points and techniques to quickly solve the questions.

GKP's Handbook Series is a collection of handbooks for Mechanical Engineering, Civil Engineering, Electrical Engineering, Computer Science Engineering and Electronics & Communication Engineering. This series serves as a quick reference guide for students preparing for exams such as GATE, ESE, PSUs recruitment and any other technical exam. The handbooks include last minute preparation points, formulae with conceptual clarity and definitions and equations with explanatory figures.

We, at GK Publications have specially designed this book in line with the varying needs of each aspirant. This book is not only to prepare you for the technical exams but is also a good asset for your semester exam preparations.

Hard work in the right direction will surely fulfill your desires. Have a lot of self-belief; load it with a lot of practice, top it off with a little smart work and you are good to go.

We hope this little effort of ours will be helpful in achieving your dreams. If you have any suggestions on improvement of this book, you can write to us at gkp@gkpublications.com.

All the Best!

Team GKP

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Civil Engineering

1

Strength of Materials

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Properties of Metals, Stress and Strain

1

Rigid and Deformable Material:

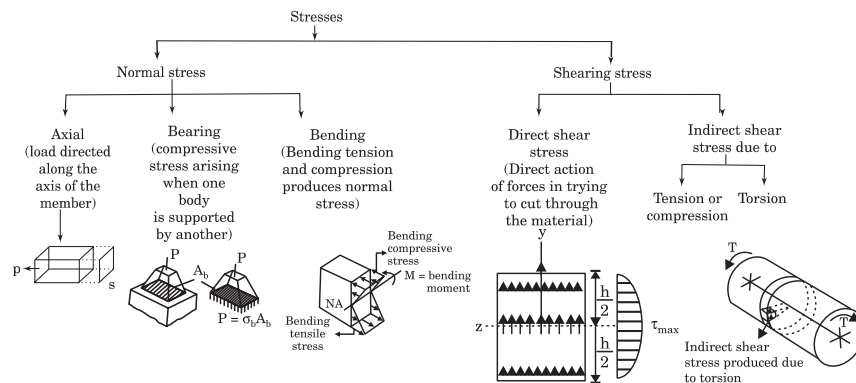
Rigid material is one which does not undergo any change in its geometry, size or shape. On the other hand, a deformable material is the one in which change in size, shape or both will occur when it is subjected to force/moment.

Stresses and strain:

Stresses (Force/Area) are generated as a resistance to the applied external forces or as a result of restrained deformations.

$$\text{Nominal stress (Engineering stress)} = \frac{\text{Load}}{\text{Original Area}}$$

$$\text{Actual/True stress} = \frac{\text{Load}}{\text{Original (Actual) Area}}$$

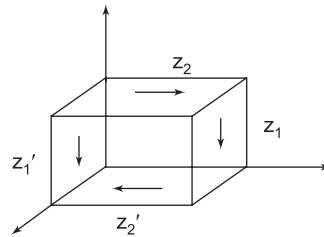


$$\text{Normal stress} = \frac{\partial P}{\partial A} = \sigma \Rightarrow P = \int \sigma dA$$

Equality of shear stress on perpendicular planes

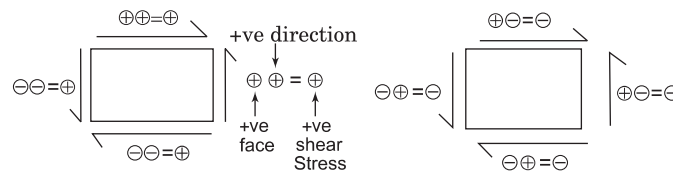
- (1) Shear stress on opposite faces of an element are equal in magnitude and opposite in direction.

- (2) Shear stress on adjacent and perpendicular faces of an element are equal in magnitude and have directions such that both stresses point towards or both point away from the line of intersection of the faces. These are called Complimentary shear stresses.



(Shear stress on opposite face are equal and opposite)

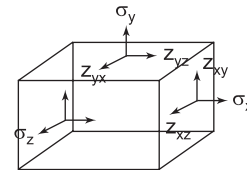
Sign convention for shear stress



Stresses under general loading conditions

1. Stress is NOT a Vector
2. **Stress** is a 2nd order Tensor.

$$3. \sigma \text{ (Stress tensor)} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$



4. Magnitude has only one dimension Hence it a 3rd = zero order tensor
5. Direction has three dimension. Hence it is 3rd = 1st order tensor
6. Stress has 9-dimension (3² = 2nd order tensor)
7. At any point in 3D condition 9 stress elements are there.
 - 3 Normal stress components ($\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$)
 - 6 shear stress components ($\tau_{xy}, \tau_{yx}, \tau_{xz}, \tau_{zx}, \tau_{yz}, \tau_{zy}$)

ONLY **6-stress** components are required to define conditions of stress at a point.

8. In 2-D condition, 4 stress elements exist ($\sigma_x, \sigma_y, \sigma_{xy}, \sigma_{zy}$) but ONLY 3-stress components are required to define conditions of stress at a point.

Design of members:

$\text{Allowable stress} = \frac{\text{yield stress}}{\text{F.O.S}}$ $\text{Margin of safety} = \text{FOS}-1$

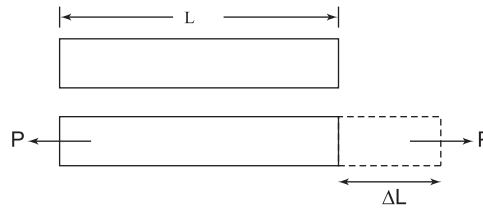
For **Ductile material**: FOS is applied on **yield stress**

For **Brittle material**: FOS is applied on **Ultimate stress**.

Normal Strain:

1. Deformation per unit length

2. Strain = $\frac{\Delta L}{L}$ or $\frac{\delta L}{\delta L}$



3. Measured by **EXTENSOMETER** It is a **dimensionless** quantity

Mathematical definition of strain

$$\epsilon_x = \frac{\partial u}{\partial x} \text{ Normal strain in } x\text{-direction}$$

$$\epsilon_y = \frac{\partial v}{\partial y} \text{ Normal strain in } y\text{-direction}$$

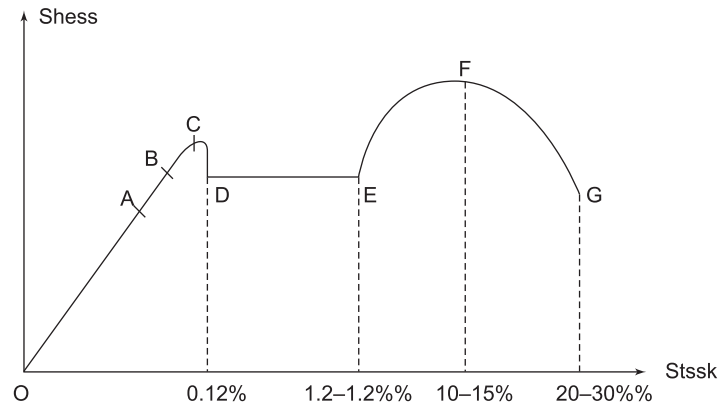
$$\epsilon_z = \frac{\partial w}{\partial z} \text{ Normal strain in } z\text{-direction}$$

$$\gamma_{xy} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \text{Shearing strain in } xy \text{ plane}$$

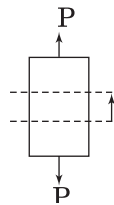
$$\gamma_{xz} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \text{Shearing strain in } xz \text{ plane}$$

$$\gamma_{yz} = \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \text{Shearing strain in } yz \text{ plane}$$

Stress-strain Curve of mild steel (Low carbon steel-Ductile Steel) in Tension



- OA = Linear curve
- A = Proportional limit
- B = Elastic limit
- C = upper yield point
- D = lower yield point
- DE = plastic region
- EF = strain hardening region
- FG = Neeking region
- F = ultimate stress point
- G = Fracture point.



L_0 = Gauge length
= Initiai length

$$\text{stress} = \frac{P}{A_0} \quad \text{strain} = \frac{P}{A_0}$$

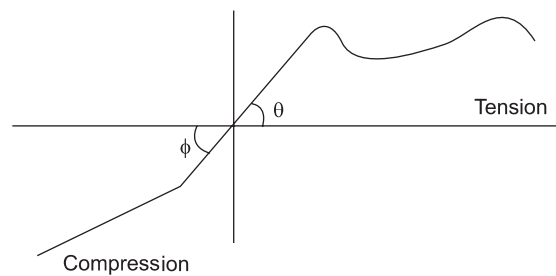
Salient points:

- (1) Volume of specimen increases from O to D
- (2) Lower yield point should be used to determine the yield strength of material
- (3) From D to E, large deformations but volume of specimen does not changes.
- (4) From E to F, its strain hardening, i.e material undergoes changes in its crystalline structure.

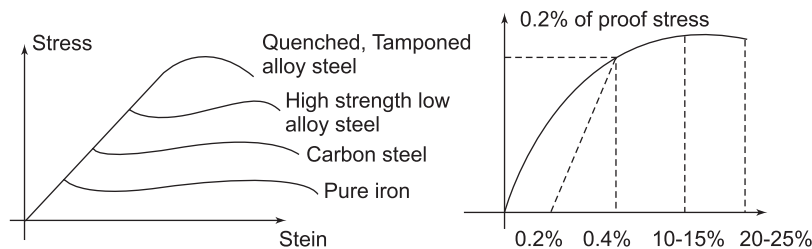
- (5) F to G, diameter of portion decreases due to instability called Necking.
- (6) Cup cone failure occurs at 45° with the load in ductile material.

Mild steel in compression

- (1) The stress strain curve will eventually be same through its initial straight line portion and through the beginning of the portion corresponding to yield and strain hardening



- (2) Modular of Elasticity in Tension= Modular of Elasticity in compression $\theta = \phi$



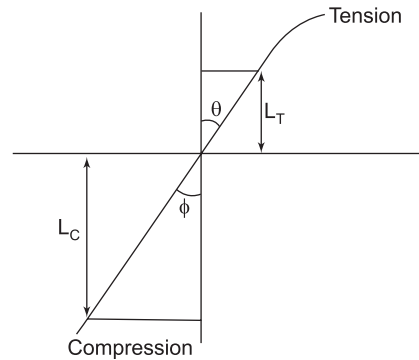
Stress-strain curves for other materials

- (1) All of them possess some Modulus of Elasticity.
- (2) As yield strength increases, Ductility falls.
- (3) For ductile materials like Aluminium and Copper, do not have defined yield point. Yield strength is defined by offset method.
- (4) $E_{AL} = \frac{1}{3} E_{st}$

Stress-strain diagram for Brittle material

- (1) $\phi = \theta$
- (2) Linear Elastic range in compression is more than Tension
- (3) Rupture stress = Ultimate stress

(4) No Necking occurs.



TRICK: to Remember failure surface:- Remember any one of the 4 given below and change at least two columns every time keeping the one constant.

(1) Ductile	Tension	45°
(2) Ductile	Torsion	90°
(3) Brittle	Tension	90°
(4) Brittle	Torsion	45°

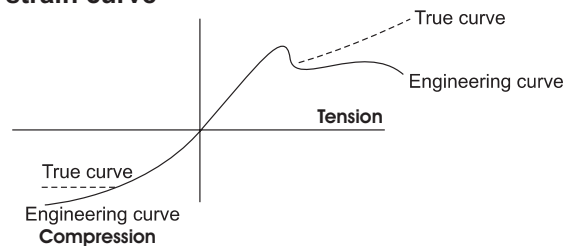
Eg. Remember

Ductile	Tension	45°
Same	Change	
Ductile	Torsion	90°

Brittle fracture:

- (1) Ductile material at normal temp. may become brittle at very low temp.
- (2) A Brittle material at low temp. may become ductile at very high temp.

True stress strain curve



- (1) True stress curve is below Engineering stress in compression because resisting area in compression increases

$$(2) \text{ Engineering stress} = \frac{P}{A_0} \quad \text{True stress} = \frac{P}{A}$$

$$\text{Engineering stress} = \frac{\delta}{L_0} \quad \text{True stress} = \frac{\Delta L}{L}$$

$A_0, L_0 \rightarrow$ Original Area & length

Relation between True stress and Engineering stress

In Tension:
$$A = \frac{A_0}{1 + \xi}$$

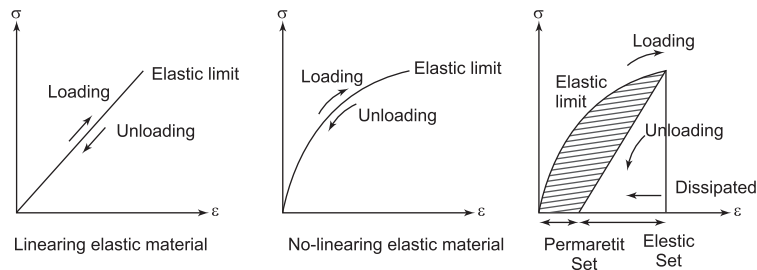
$$\sigma = \sigma_0(1 + \xi)$$

In compression
$$A = \frac{A_0}{1 - \xi}$$

$$\sigma = \sigma_0(1 - \xi)$$

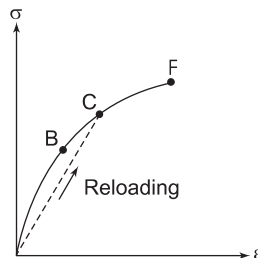
Properties of Materials

Elasticity:- Property by virtue of which material deformed under the load is enabled to **return** to its original dimension when the load is removed.



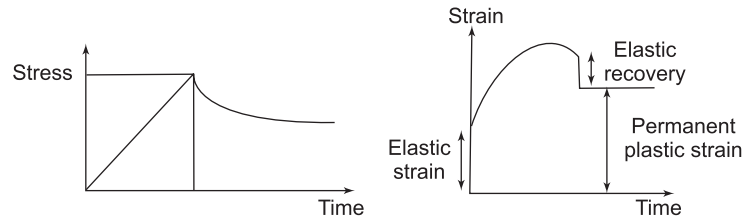
Plasticity:- The characteristics of material by which it undergoes **inelastic strain** beyond those at the **elastic limit**.

Reloading:- Proportional limit increases from B to C but ductility decreases from 'B to F' to 'C to F'



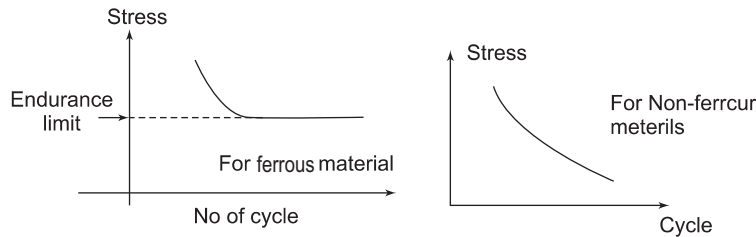
Creep: Property by virtue of which a material undergoes **additional deformation** (over and above due to applied load) with passage of time under sustained loading with in **elastic limit**

Relaxation:- The decrease in stress in steel as a result of creep with in steel under prolonged strain



Fatigue:- Deterioration of a material under repeated cycles of stress or strain resulting in progressive cracking that eventually produces fracture.

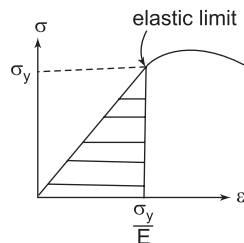
Endurance limit:- Stress level below which even large number of stress cycle cannot produce fatigue failure.



For structural steel, Endurance limit = $\frac{1}{2} \times$ ultimate strength

Resilience:- Property of material to absorb energy when it is deformed elastically and then upon unloading to have **this energy recovered**.

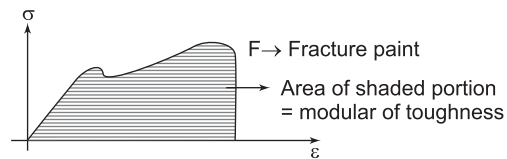
Modular of Resilience: Elastic strain energy stored **per unit volume**



$$= \frac{1}{2} \times \sigma_y \times \frac{\sigma_y}{E}$$

$$= \frac{\sigma_y^2}{2E}$$

Toughness:- Ability to absorb mechanical energy upto **failure**.



Toughness → Resists fracture

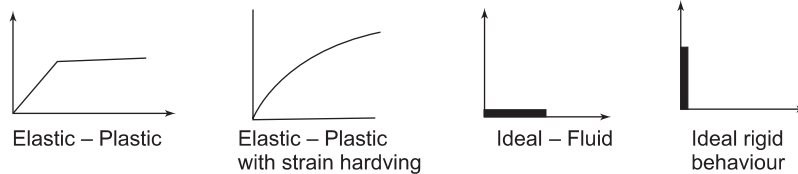
Hardness → Resists scratch or abrasion

Tenacity:- Property of material to resist fracture under the action of tensile load

Visco-Elastic material

Materials having both Viscous and Elastic properties and exhibit time dependent strain.

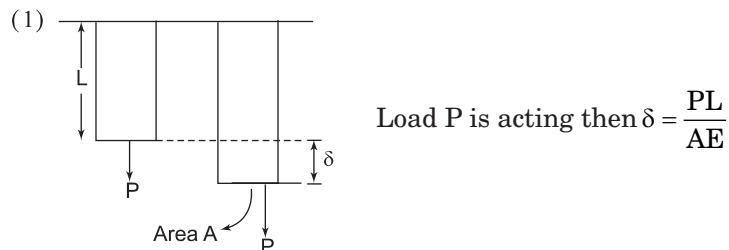
Approximate stress-strain curves



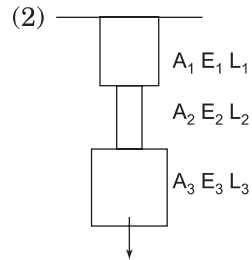
Hooke's law:-

- (a) Homogenous $\sigma = E \cdot \xi$
- (b) Isotropic
- (c) Linearly elastic materials

Deformation of member under axial load



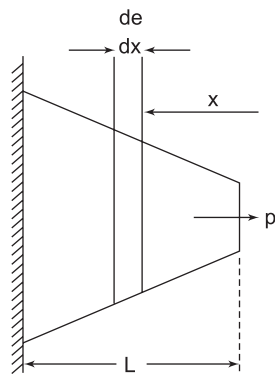
1.12 CIVIL ENGINEERING



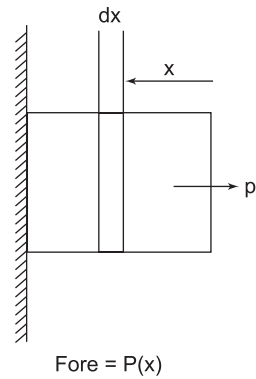
Load P is acting then

$$\delta = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3}$$

(3) $\delta = \int_0^L \frac{P(x) dx}{A(x) E}$ then



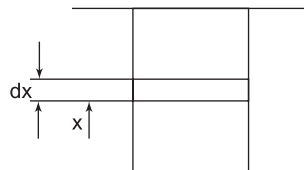
or



$$\delta = \int_0^L \frac{P dx}{A(x) E}$$

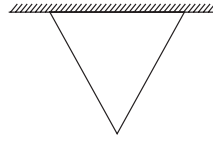
$$\delta = \int_0^L \frac{P(x) dx}{AE}$$

(4) (a) In prismatic bar due to self weight



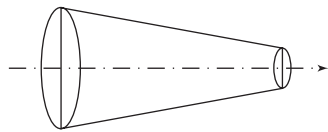
$$\delta = \frac{\gamma L^2}{2E} \text{ or } \frac{\left(\frac{W}{2}\right)L}{AE}$$

(b) Conical bar due to self weight



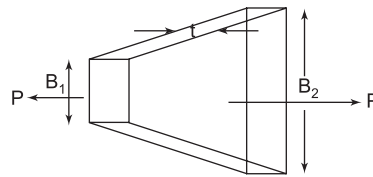
$$\delta = \frac{\gamma L^2}{6E} = \frac{1}{3} \text{ (deflection of prismatic bar of same length and same density)}$$

5.



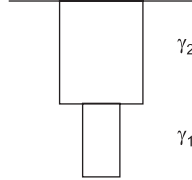
$$\Delta = \frac{4PL}{\pi D_1 D_2 E}$$

6.



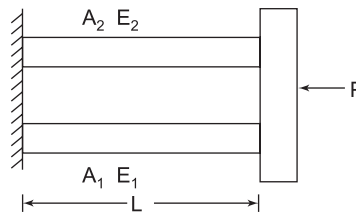
$$\Delta = \frac{PL \log_e \left(\frac{B_2}{B_1} \right)}{Et(B_2 - B_1)}$$

7.



$$\Delta = \frac{\gamma_1 L_1^2}{2E} + \frac{\gamma_2 L_2^2}{2E} + \frac{\gamma_1 \phi_1^2 L_1 L_2}{\phi_2^2 E}$$

Composite Bars



$$P = P_1 = P_2$$

$$\delta_1 = \delta_2 = \frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$$

$$P_1 = \frac{A_1 E_1 P}{A_1 E_1 + A_2 E_2} \quad P_2 = \frac{A_2 E_2 P}{A_1 E_1 + A_2 E_2}$$

- (1) Principle of superposition is applicable only when stress is within proportional limit
- (2) If temperature is increased and member is restrained, then force produced is compressive. If temperature is decreased the force produced is tensile.
- (3) Temp $\uparrow \rightarrow$ more value of $\alpha \rightarrow$ compression
Temp $\downarrow \rightarrow$ more value of $\alpha \rightarrow$ Tension

Nut and Bolt problem:

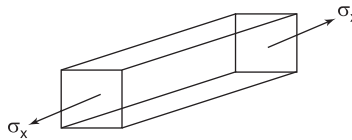
Extension of Bolt + Contraction of Tube = Movement of nut

$$\frac{\sigma_b L}{E_b} + \frac{\sigma_T L}{E_c} = np$$

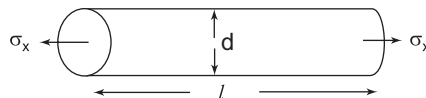
n = no. of rotations of bolt

p = pitch of thread.

Poisson's Ratio:- For **Homogenous** and **isotropic** material, Elongation (or contraction) produced by any Axial Force in the direction of force is accompanied by contraction (or elongation) in all transverse directions and all such contractions (or elongations) are same.



$$\mu = - \left(\frac{\text{Lateral Strain}}{\text{Axial Strain}} \right) = \frac{-\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$



$$\mu = \frac{-\Delta d / d}{\Delta l / l} \quad \begin{array}{l} \mu = 0 \text{ for cork} \\ \mu = 0.1 - 0.2 \text{ for concrete} \\ \mu = 0.5 \text{ Perfectly elastic rubber} \end{array}$$

Volume of rod remains unchanged as a result of combined effect of elongation and transverse condition.

Dilation, Bulk modulus:-

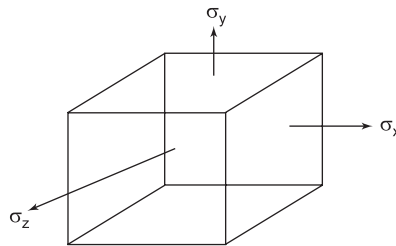
$$\varepsilon_v = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}(1 - 2\mu)$$

If

 $\sigma_x = \sigma_y = \sigma_z = p$ than

$$\varepsilon_v = \frac{3p}{E}(1 - 2\mu)$$

$$K = \frac{p}{\varepsilon_v} = \frac{E}{3(1 - 2\mu)}$$



$$\text{Hydrostatic Pressure} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

- (1) Stretching of material in one direction i.e due to σ_x will lead to increase in volume
- (2) During plastic deformation, volume of specimen remains constant.

Shearing Strain:-

- (1) Hooke's law for shearing stress and strain

$$\tau_{xy} = G\gamma_{xy}$$

- (2) Modulus of rigidity or shear modulus G , $G = \frac{E}{2(1 + \mu)}$

$$\text{as } 0 < \mu < 0.5 \quad \text{then} \quad \frac{E}{3} < G < \frac{E}{2}$$

- (3) If only shearing stresses are acting then volume of the specimen does not change.

Relationship between Elastic Constants

$$G = \frac{E}{2(1 + \mu)}$$

$$K = \frac{E}{3(1 - 2\mu)}$$

$$E = \frac{9KG}{3K + G}$$

$$\mu = \frac{3K - 2G}{6K + 2G}$$

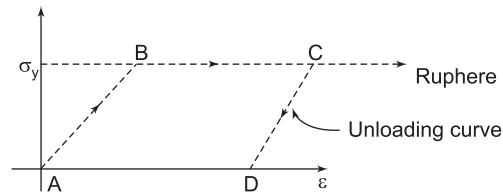
No. of Independent Elastic Constants

- (1) Homogenous and Isotropic $\rightarrow 2$
- (2) Orthotropic (wood) $\rightarrow 9$
- (3) Anisotropic $\rightarrow 21$

Saint-Venant Principle: Except in the immediate vicinity of application of loads, the stress distribution may be assumed independent of the actual mode of application of loads.

Plastic deformation:-

When yield stress of material is exceeded, plastic flow occurs.



Idealised curve for elasto plastic material

Residual stress:- When some part of an indeterminate structure undergoes plastic deformation, or different part undergoes different plastic deformation the stress in various parts of the structure will not return to zero after the load has been removed. These stresses are called Residual stresses

Thermal Stress and Strain:-

$$\sigma = E\alpha\Delta T$$

$$\Delta = L\alpha\Delta t$$

$$\text{Strain} = \frac{L\alpha\Delta t}{L} = \alpha\Delta t$$

$$\alpha_{\text{Aluminum}} > \alpha_{\text{Brass}} > \alpha_{\text{Copper}} > \alpha_{\text{Steel}}$$

$$\boxed{\text{TRICK } A > B > C > S}$$

When bar is not restrained, then there will be no induced temperature stresses due to change in temperature.

Shear Force and Bending Moment

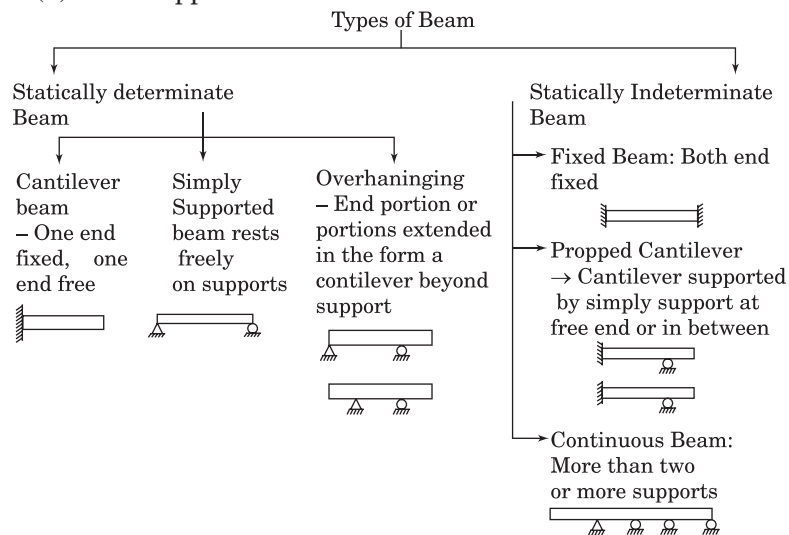
2

Span of a beam:-

- (i) The clear horizontal distance between the supports is called clear span of the beam.
- (ii) The horizontal distance between the centres of the end bearings is called the effective span of the beam.

Types of Support:-

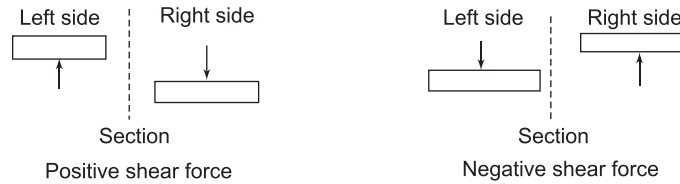
- (i) A Simple or free support/Roller Support/Rocker support
- (ii) Hinged or pinned support.
- (iii) A built in or fixed or encastre support
- (iv) Slider support
- (v) Link Support



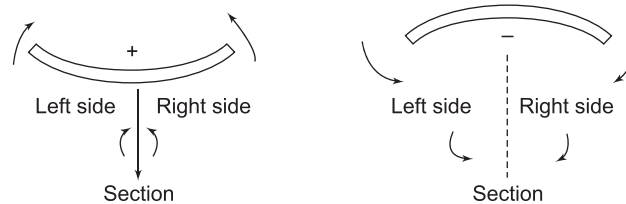
Note:- A continuous beam may or may not be an overhanging beam.

Shear force:- It is the resultant of all transverse forces to the right or left of the section.

S.F at a Section is +ve if the resultant of all transverse forces to the right of the section is downward or resultant of transverse forces to the left of section is upwards.



Bending moment:- It is the resultant moment at a section due to all the transverse forces either to the left or right of the section.

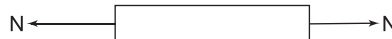


Positive Bending moment = Sagging

Negative Bending moment = Hogging

Note:- Bending moment is the algebraic sum of moments at that section while moment at a point is the summation of moment due to all loading on the beam produced at that point.

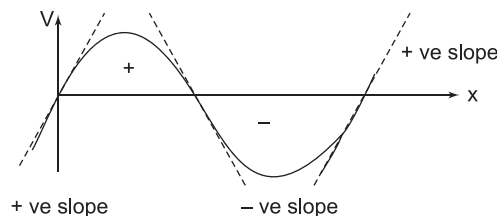
Axial Thrust → Force acting along the longitudinal axis of the members. Axial thrust is +ve if it tries to elongate the members.



Relationship between Bending moment, Shear force and Loading

(i) Slope of the shear force diagram = Intensity of distributed load

$$\frac{dV}{dx} = W_x$$



If the slope of SFD is positive, this implies the load intensity at that point is +ve i.e upwards and if the slope of SFD is negative, this implies the load intensity at that point is -ve i.e downwards.

(ii) Slope of Bending moment diagram = Shear force at that section.

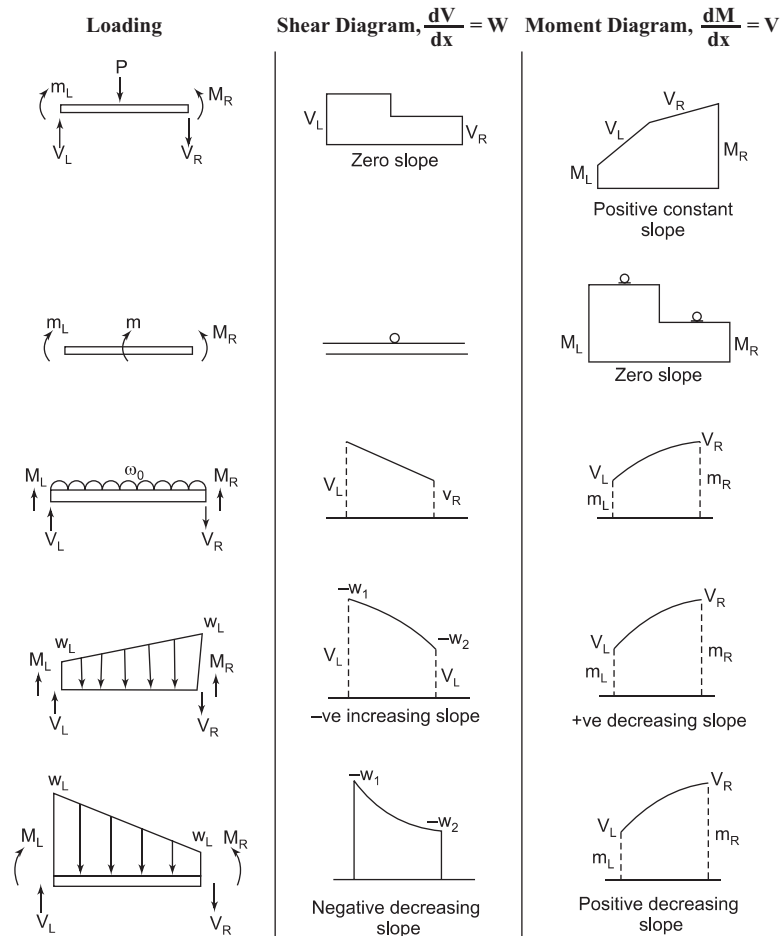
$$\frac{dM}{dx} = V$$

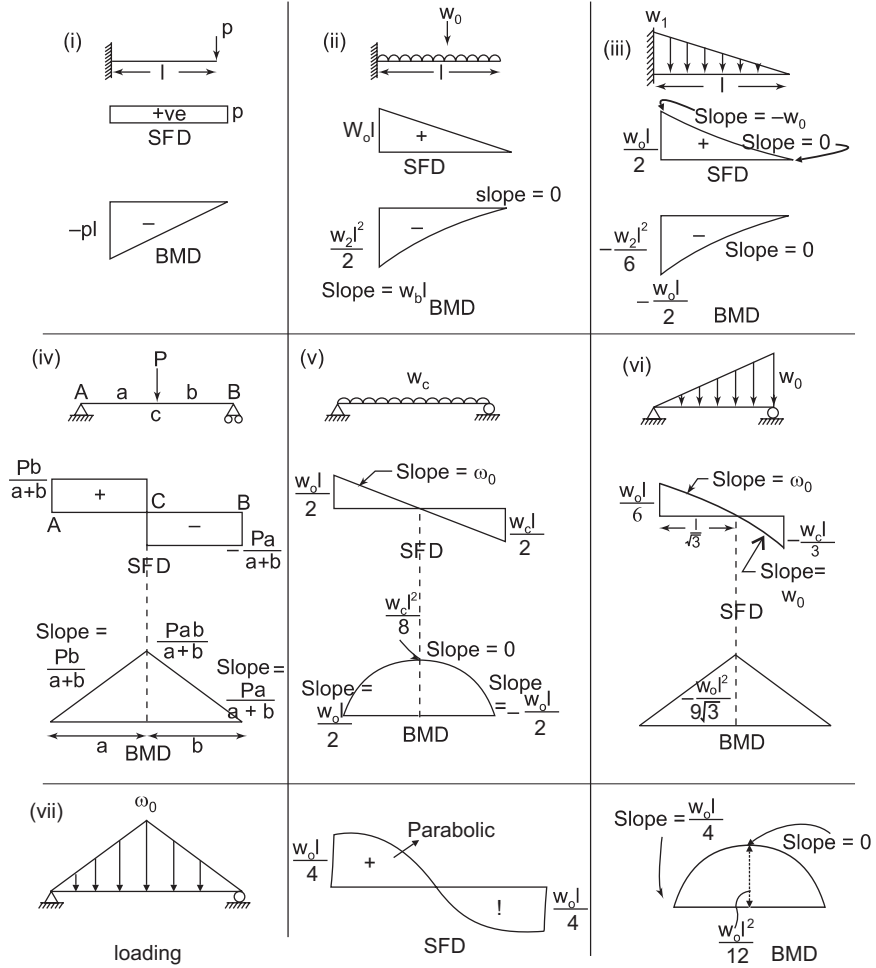
NOTE:-

$$\Delta V = \int W_x dx$$

$$\Delta M = \int V dx$$

$M_{\text{final}} - M_{\text{initial}} = \text{Area under the Shear force diagram between those two sections.}$





Principal Stress and Principal Strain

3

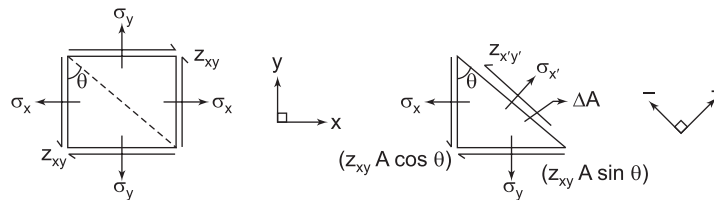
Whenever any structural component is under equilibrium due to external forces then each and every point inside the volume of the structural component must be in equilibrium and must have stress less than the permissible stress.

Plane stress:- When two faces of cubic elements are free of any stress, the stress condition is termed as plane stress condition

$$\sigma_z = \tau_{zx} = \tau_{zy} = 0$$

So plane stress components are σ_x , σ_y and τ_{xy}

Transformation of Plane Stress



$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + Z_{xy} \sin 2\theta$$

$$Z_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + Z_{xy} \cos 2\theta$$

$$\sigma_x + \sigma_y = \sigma'_x + \sigma'_y$$

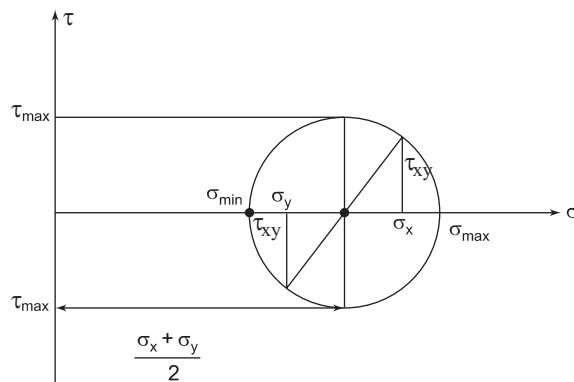
NOTE:- The sum of normal stresses exerted on a cubic element of a material is independent of the orientation of element.

Principal Stress and maximum shear stress:- It is the maximum or minimum normal stress which may be developed

on a loaded body. The plane of principal stress does not carry any shear stress.

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



Mohr's Circle for plane stress → It is the locus of points representing the magnitude of normal and shear stress at various plane in a given stress element.

1. $\sigma_{\max/\min}$ = Principal stresses, end points of diameter on σ -axis
2. $\tau_{\max}$ = max shear stress, whose magnitude is equal to radius of mohr's circle
3. $\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}$ $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

So $\tan 2\theta_s \times \tan 2\theta_p = -1$
 $\Rightarrow 2\theta_s$ and $2\theta_p$ are 90° apart.

Hence, plane of Max. Shear stress are 45° to the principal planes (i.e. θ_s and θ_p are 45° apart)

4. Normal stress on a plane of maximum shear stress is represented by co-ordinates of centre of Mohr's circle.
5. In hydrostatic loading → Mohr circle reduces to a point.
6. In Pure Shear case → centre of mohr circle will fall at origion.

Strain Energy per unit Volume:-

1. Plane Stress Condition:-

$$U = \frac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy})$$

When σ_1 and σ_2 principal stresses then

$$V = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2)$$

2. Under Triaxial Stress Condition:

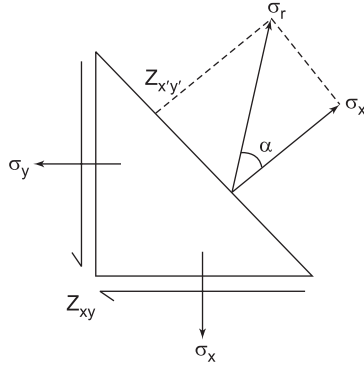
$$V = \frac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz})$$

$$V = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\mu(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x))$$

Angle of obliquity:- Angle that line of action of resultant stress on a plane makes with the normal to the plane is called angle of obliquity.

α = Angle of obliquity

$$\sigma_r = \sqrt{\sigma_x^2 + \tau_{x'y'}^2}$$



Plane Strain:- If the deformations are those in x-y plane only then only 3-strain components exist $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

NOTE:- Strain energy only leads to distortion of element. It does not lead to change in volume. Normal stresses on the other hand leads to change in volume.

Transformation of Plane Strain

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\varepsilon_{y1} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\varepsilon_{x1} + \varepsilon_{y1} = \varepsilon_x + \varepsilon_y$$

$$\frac{\gamma_{x1y1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$

Comparison of Plane Stress and Plane Strain

	Plane Stress	Plain Strain
Stress	$\sigma_z = 0$ $\tau_{xz} = 0$ $\tau_{yz} = 0$ σ_x, σ_y and $\tau_{xy} \rightarrow$ non zero	$\tau_{xz} = 0, \tau_{yz} = 0$ $\sigma_x, \sigma_y, \sigma_z, \tau_{xy} \rightarrow$ Non-zero
Strain	$\gamma_{xz} = 0$ $\gamma_{yz} = 0$ $\varepsilon_x, \varepsilon_y, \varepsilon_z, \varepsilon_{xy} \varepsilon$ Non zero	$\varepsilon_z = 0$ $\gamma_{xz} = 0$ $\gamma_{yz} = 0$ $\varepsilon_x, \varepsilon_y, \varepsilon_{xy} \varepsilon$ Non-zero

Mohr circle for Plane Strain:-

1. Principal Strains

$$\varepsilon_{\max/\min} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

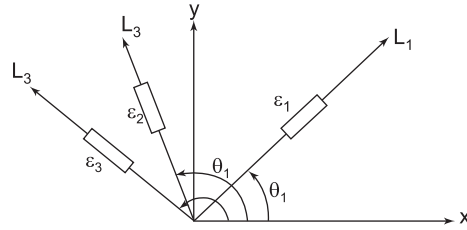
2. Maximum In plane shearing strain = Radius of Mohr's circle (R)

$$R = \left(\frac{\gamma_{\max}}{2}\right)_{\text{in plane}} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

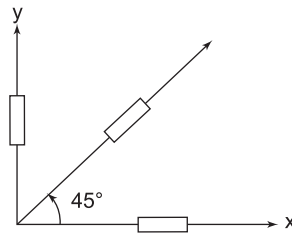
Strain Rosette → A group of three gauges arranged in a particular pattern such that it can measure **normal strain** in three different directions on the surface of a structural element.

$$\varepsilon_1 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta_1 + \frac{\gamma_{xy}}{2} \sin 2\theta_1$$

$$\varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta_2 + \frac{\gamma_{xy}}{2} \sin 2\theta_2$$



Special Case:- when $\epsilon_1 \rightarrow$ along x-axis i.e. ϵ_x
 $\theta_2 \rightarrow 45^\circ$
 $\epsilon_3 \rightarrow$ along y-axis i.e. ϵ_y



then,

$$\epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\gamma_{xy}}{2}$$

or

$$\gamma_{xy} = \epsilon_2 - (\epsilon_x + \epsilon_y)$$

Deflection of Beams

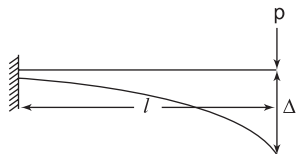
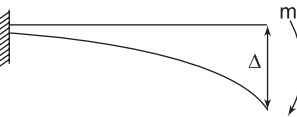
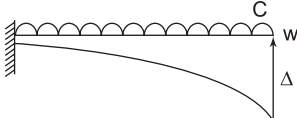
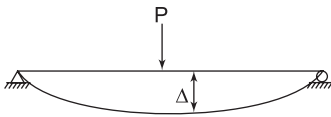
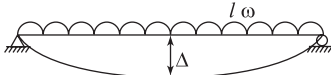
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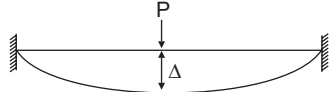
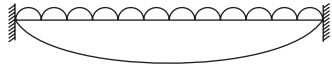
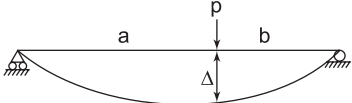
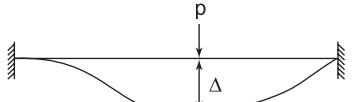
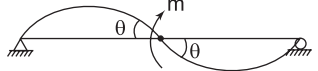
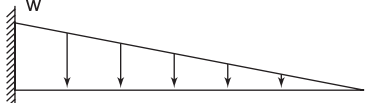
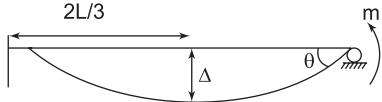
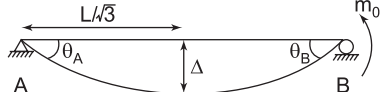
Deflection of structure is caused by its internal loadings such as Normal force, Shear force, Bending Moment, Torsion.

For **Beams** and **Frames**, major deflection is due to **Bending**

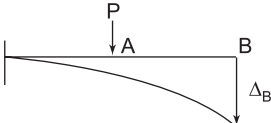
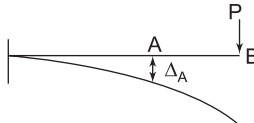
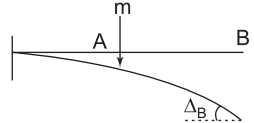
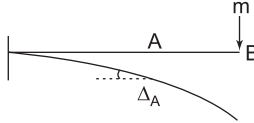
For **Trusses**, deflection is caused by internal Axial Forces

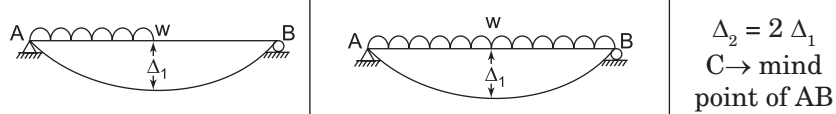
Some standard results of deflection and slopes.

Loading	Deflection	Slopes
	$\Delta = \frac{Pl^3}{3EI}$	$\theta = \frac{Pl^2}{2EI}$
	$\Delta = \frac{ML^2}{2EI}$	$\theta = \frac{ML}{EI}$
	$\Delta = \frac{wL^4}{8EI}$	$\theta = \frac{wL^3}{6EI}$
	$\Delta = \frac{PL^3}{48EI}$	$\theta = \frac{PL^2}{16EI}$
	$\Delta = \frac{5}{384} \frac{wL^4}{EI}$	$\theta = \frac{wL^3}{24EI}$

	$\Delta = \frac{1}{4} \left(\frac{PL^3}{48EI} \right)$	
	$\Delta = \frac{1}{5} \left(\frac{5}{384} \frac{wL^4}{EI} \right)$	
	$\Delta = \frac{Pa^2b^2}{3EIL}$	
	$\Delta = \frac{Pa^3b^3}{3EIL^3}$	
		$\theta = \frac{ML}{24EI}$
	$\Delta = \frac{wL^4}{30EI}$	$\theta = \frac{wL^3}{24EI}$
	$\Delta = \frac{ML^2}{27EI}$	$\theta = \frac{ML}{4EI}$
	$\Delta = \frac{ML^2}{9\sqrt{3}EI}$	$\theta_A = \frac{ML}{6EI} \quad \theta_B = \frac{ML}{3EI}$

Maxwell's Reciprocal Theorem:- In any beam, frame or truss, the deflection at any point due to load P at any point A is equal to deflection at any point A due to load P at any point B.

		$\Delta_A = \Delta_B$
		$\theta_A = \theta_B$



Methods of determining slope and deflection at a point

- (1) **Double Integration Method** $\Rightarrow$ Gives deflection only due to bending

$$\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}} \Rightarrow \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{M}{EI} \Rightarrow \boxed{\frac{d\theta}{dx} = \frac{M}{EI}}$$

where,

EI = Flexural rigidity

Flexural Stiffness = $\frac{\text{Flexural rigidity}}{\text{Length}}$

$$\frac{d^3y}{dx^3} = \frac{dM/dx}{EI} \Rightarrow \boxed{\frac{d^3y}{dx^3} = \frac{V}{EI}} \Rightarrow \frac{d^4y}{dx^4} = \frac{dV/dx}{EI} \Rightarrow \boxed{\frac{d^4y}{dx^4} = \frac{W}{EI}}$$

- (2) **Macauley's Method** $\Rightarrow$ Modification is done in loading pattern so that udl or uvl becomes continuous up to last segment.

$$EI \frac{d^2y}{dx^2} = M = \frac{-P}{2}x + \frac{3P}{2}(x - 2a)$$

Integrate and put boundary conditions

$$(a) \ x = 0, y = 0$$

$$(b) \ x = 2a, y = 0$$

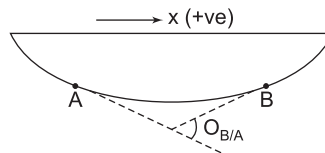
$$\text{Then, Deflection at } x = 3a, y = \frac{-Pa^3}{EI}$$

(3) Moment Area Method – Mohr's Method

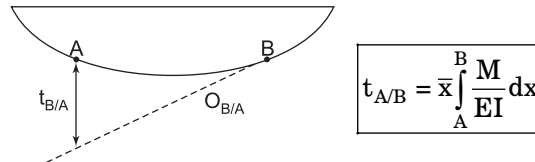
Theorem 1:- The change in slope between two points on elastic curve equals the area of $\frac{M}{EI}$ diagram between these two points.

$\theta_{B/A}$ = Slope of B with respect to tangent drawn on elastic curve at A

A = Area of $\frac{M}{EI}$ diagram between A and B.



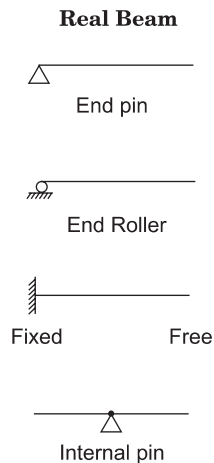
Theorem 2:→ Deflection of any point A on elastic curve with respect to tangent drawn at another point B (t_{AB}) equals the moment of area under diagram between A and B about point A.



- (4) **Moment diagram by parts:-** The resultant BM at any section is the algebraic sum of bending moments at that section caused by each loading separately (either from left or right of that section). So the effect of individual load can be considered instead of taking effects of all the loads together for drawing BMD.
- (5) **Conjugate beam method:-**

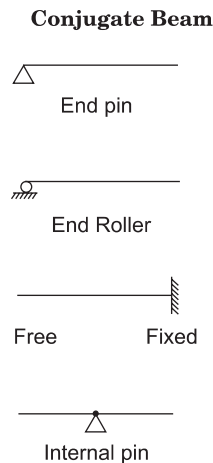
$$\begin{array}{c} V = \int w dx \\ \downarrow \quad \uparrow \\ \theta = \int \frac{M}{EI} dx \end{array}$$

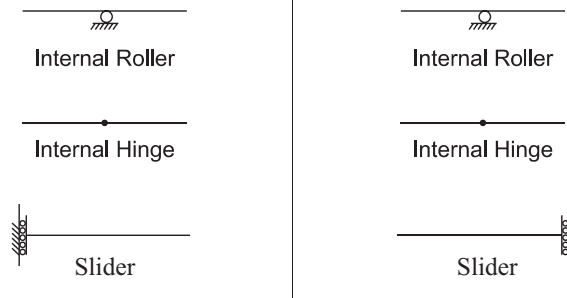
Slope at any point in real beam = Shear at that point in conjugate beam



$$\begin{array}{c} M = \int \left(\int w dx \right) dx \\ \downarrow \quad \uparrow \\ y = \int \left(\int \frac{M}{EI} dx \right) dx \end{array}$$

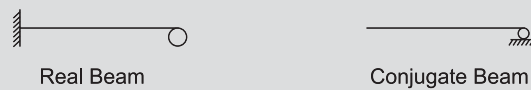
Deflection at any point in real beam = BM at that point in conjugate beam





NOTE:- (1) Area moment theorem requires understanding of geometry of deflected shape and applicable only when deflected shape is continuous, while in conjugate beam method, principle of statics is used. Hence, this method can also be used when deflected shape is not continuous i.e., Internal Hinge case.

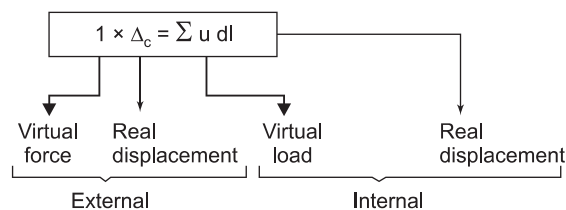
(2)



A **statically intermediate** real beam can have **unstable** conjugate beam.

(6) Method of virtual work (Unit load method)

External virtual work = Internal virtual work



So,

$$1 \times \Delta = \int_0^L \frac{m}{EI} M \, dx$$

$$1 \times \theta = \int_0^L \frac{m_\theta}{EI} M \, dx$$

Note:- Unit load method can be applied to plastic range of stress-strain also, but $d\theta$ will not be equal to $\frac{M}{EI} dx$

(7) **Castigliano's theorem (Method of least work – its 2nd theorem)**

$$\Delta = \frac{\partial U}{\partial P}$$

$$\theta = \frac{\partial U}{\partial M}$$

For beam's and frames $U = \frac{M^2 dx}{2EI}$

So,

$$\Delta = \int_0^L \frac{M \frac{\partial M}{\partial P} dx}{EI}$$

$$\theta = \int_0^L \frac{M \frac{\partial M}{\partial m} dx}{EI}$$

Note:- This theorem is applicable only when there is constant temperature, unyielding support and linear elastic material response.

Theories of Failure

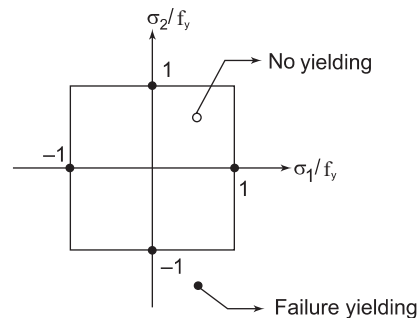
5

Under uniaxial tension or compression practically, yielding begins at the yield strength at which plastic deformation is significant. But when several components exist, the yielding depends on some combination of these components. The theory of failure is used to establish, the behaviour of material subjected to **simple tension** or **compression**, the point at which **failure** will occur under any type of combined loading. These theories are applicable to **static loading** only.

- (1) **Maximum principal stress theory (Rankine's theory, Lamé's theory are max stress theory)**

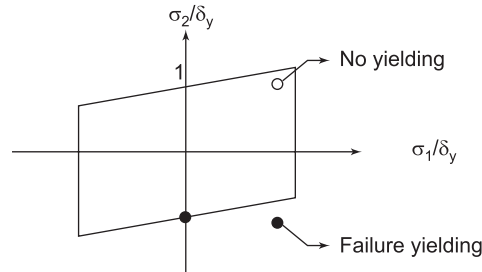
$$\sigma \leq \frac{f_y}{FOS}$$

Applicable for brittle material



- (2) **Max principal strain theory (St. venant theory)**

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \leq \frac{f_y}{FOS}$$

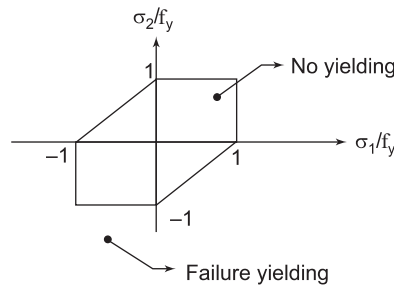


Satisfactorily applicable to brittle material, but over estimates the strength of ductile material. Even not suitable for pure shear case.

(3) **Max shear stress theory (Tresca, Guest, coulomb theory)**

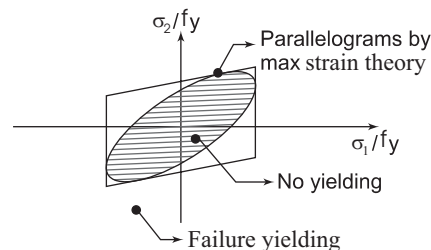
$$\text{Max of } \left[\left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|, \left| \frac{\sigma_{\max}}{2} \right|, \left| \frac{\sigma_{\min}}{2} \right| \right] \leq \frac{f_y}{2(\text{FOS})}$$

Applicable for ductile material and gives the most conservative design out of various other theories of failure



(4) **Maximum strain energy theory (Beltrami - Haigh theory)**

$$\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right] \leq \left(\frac{f_y}{\text{FOS}} \right)^2$$

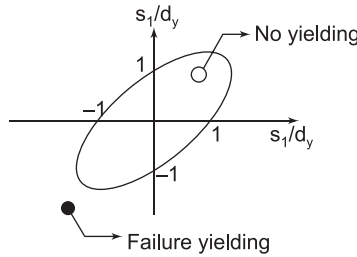


Applicable for ductile material and not suitable for brittle material or pure shear case.

(5) **Max shear strain energy theory (Distortion energy theory)**
– Huber-Hencky von mises theory

$$\left[\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \leq \left(\frac{f_y}{FOS} \right)^2 \right]$$

Applicable in pure shear case



(6) **Octahedral shear stress theory**

$$\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 \leq \left(\frac{f_y}{FOS} \right)^2 \right]$$

Applicable to ductile material in pure shear case

NOTE:

Total strain energy = volumetric strain energy + Distortion energy.

Volumetric strain energy = $\frac{1}{2} \times$ volumetric stress $\times$ volumetric strain

Volumetric strain energy = $\frac{1}{2} \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \right) \left(\frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E} \right)$

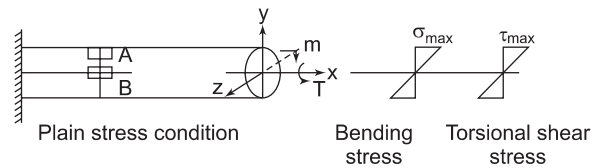
Total strain energy = $\frac{1}{2E} \left[\frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}{1} \right] \dots (1)$

Hence, (2)-(1) =

$$\left[\text{Distortion Energy} = \frac{1}{12G} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \right]$$

Combined Stress

Combined Bending and Torsion



For point A:-

$$\tau_{\max} = \frac{16T}{\pi D^3} \quad \sigma_{\max} = \frac{32M}{\pi D^3}$$

For point B:-

$$\tau_{xy} = \frac{-16T}{\pi D^3}$$

Principal Stresses at A:-

$$\sigma_{\max/\min} = \frac{16T}{\pi D^3} \left[M \pm \sqrt{M^2 + T^2} \right]$$

$$\tau_{\max} = \frac{16T}{\pi D^3} (\sqrt{M^2 + T^2})$$

Principal Stresses at B:- $\sigma_{\max/\min} = \pm \frac{16T}{\pi D^3}$

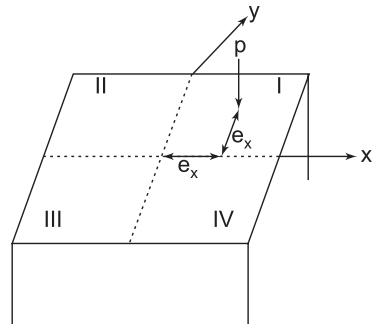
Equivalent Moment $M_e = \frac{1}{2} \left(M \pm \sqrt{M^2 + T^2} \right)$

Equivalent Torque $T_e = \sqrt{M^2 + T^2}$

Combined Bending and Axial Force

$$\sigma = \frac{-P}{A} - \frac{(Pe_x)x}{I_y} - \frac{(Pe_y)y}{I_x}$$

(-ve means compressive)

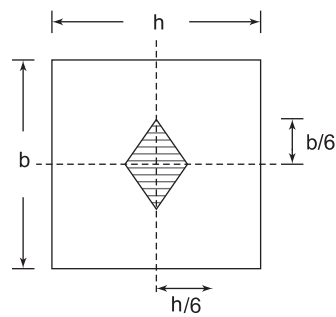


Equation of Neutral axis (put $\sigma = 0$)

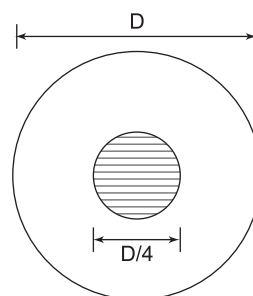
$$\left(\frac{e_x}{r_y^2} \right) x + \left(\frac{e_y}{r_x^2} \right) y = 1$$

Kern:- It is the area of the x-section on which if compressive loading occurs then there will be no tension anywhere on the entire x-section (when bending occurs due to axial force)

Kern for Rectangular Section Kern for Circular Section

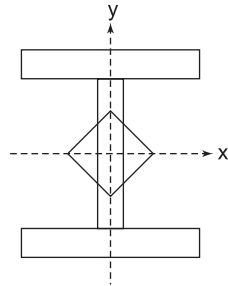


Rhombus shape



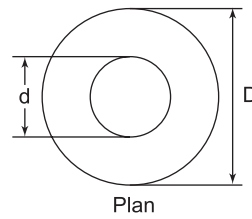
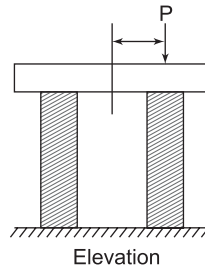
Circular shape of diameter= $D/4$

Kern for I-section



Rhombus shape

Kern for hallow circular section



$$\text{Dia of Kern} = \frac{D^2 + d^2}{4D}$$

Bending Stress in Beam

Symmetric Bending:- When member possess a **Plane of symmetry** and **loading acts in the plane of symmetry** then bending is called symmetric sending.

Unsymmetric Bending:- When bending couple does not acts in the plane of symmetry of member either because they act in different plane or because the member does not possesses a plane of symmetry

Pure Bending:- Bending of beam under constant Bending moment.

Non-uniform Bending:- Bending in presence of **Shear force**.

Flexure Formula: $\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$

where I = MOI about C.G axis about which bending occurs

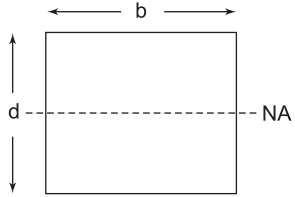
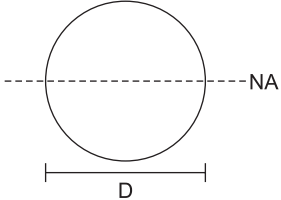
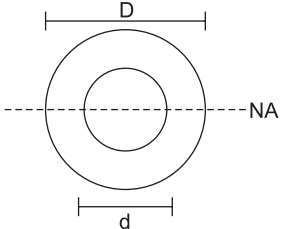
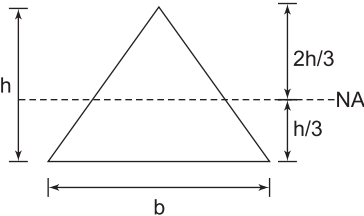
Here, $\boxed{\sigma_{\max} = \frac{My}{I}}$

Where, $\boxed{\frac{I}{y} = Z}$ = Section modulus about bending axis.

Moment of resistant (MOR) = $\sigma_{\max} \left(\frac{I}{y} \right)$

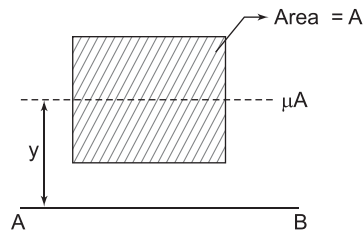
$$\boxed{\text{MOR} = \sigma_{\max} \times Z}$$

Section modulus (Z)

	I about N.A	y _{max}	Z=I/y _{max}
Rectangular section 	$\frac{bd^3}{12}$	$\frac{d}{2}$	$\frac{bd^2}{6}$
Solid circular section 	$\frac{\pi D^4}{64}$	$\frac{D}{2}$	$\frac{\pi D^3}{32}$
Hollow circular section 	$\frac{\pi(D^4 - d^4)}{64}$	$\frac{D}{2}$	$\frac{\pi(D^4 - d^4)}{32D}$
Triangular section 	$\frac{bh^3}{36}$	$\frac{2h}{3}$	$\frac{bh^2}{24}$

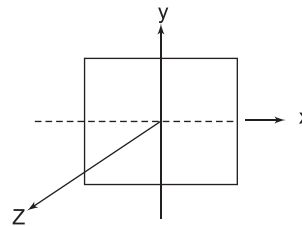
Parallel axis theorem

$$\Rightarrow I_{AB} = I_{NA} + AY^2$$



Perpendicular axis theorem

$$I_{ZZ} = I_{XX} + I_{YY}$$



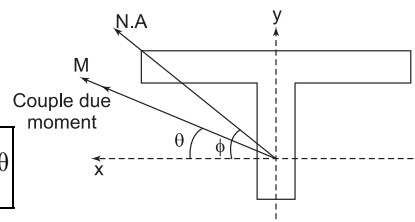
Beam of constant strength or fully stressed beam:- It is the beam in which max. stress at every X-Section of the beam is equal to the maximum allowable bending stress in the beam.

Rectangular beam loading	Max. bending stress	b= constant	d= constant
	$\frac{6Px}{b_x d_x^2}$	$d_x \propto \sqrt{x}$	$b_x \propto x$
	$\frac{3wx^2}{b_x d_x^2}$	$d_x \propto x$	$b_x \propto x$
	$\frac{3Wx}{b_x d_x^2}$	$d_x \propto \sqrt{x}$	$d_x \propto x$

Equation of Neutral axis in Unsymmetrical bending (No Twisting case)

$$y = \left(\frac{I_x}{I_y} \tan \theta \right) x$$

Slope of N.A = $\tan \phi = \frac{I_x}{I_y} \tan \theta$



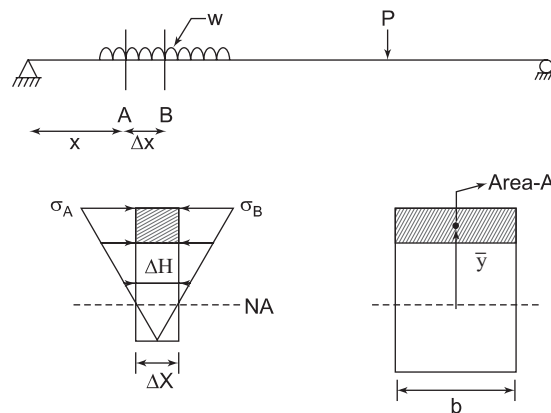
NOTE:- Neutral axis is always located between the couple vector and the minor principal axis.

Shear Stress in Beams

Normal stress is produced by bending and shear stress is produced by shear force. The Vertical shearing stress is accompanied by a horizontal shearing stress of equal magnitude known as **complimentary shear stress**.

$$\Delta H = \int_A (\sigma_A - \sigma_B) dA$$

$$\Delta H = \frac{VA}{I} \bar{y} \Delta x$$



Where,

ΔH = Shear force in length Δx of beam

V = S.F at the section where shear stress is to be found.

A = Moment of area of section above the level at which shear stress is to be found out.

I = Moment of inertia of complete section about N.A.

$$\text{Shear force per unit length} = \frac{\Delta H}{\Delta x} = \frac{VA\bar{y}}{I}$$

$$\text{Shear Stress at the level } y \text{ from N.A} = q = \frac{VA\bar{y}}{Ib}$$

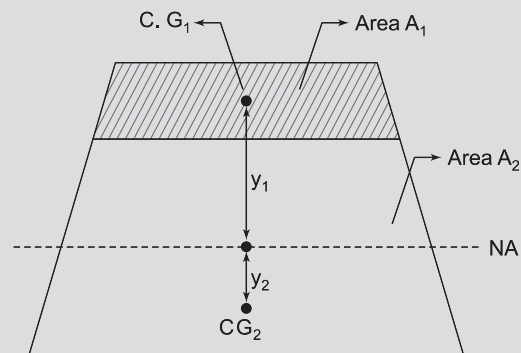
b = width of section at the level where shear stress is to be found.

Note:-

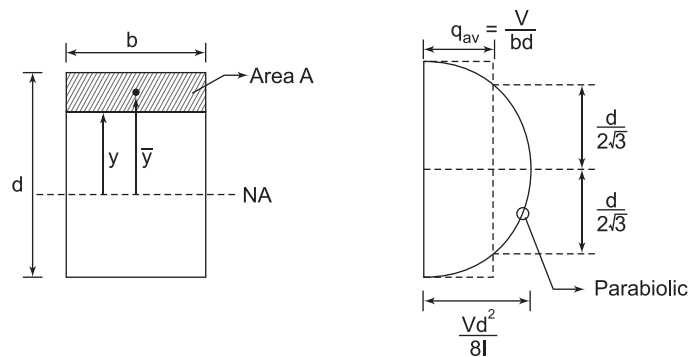
$$A_1 y_1 = A_2 y_2$$

A_1 = Shaded area

A_2 = unshaded area



Shear stress in rectangular section

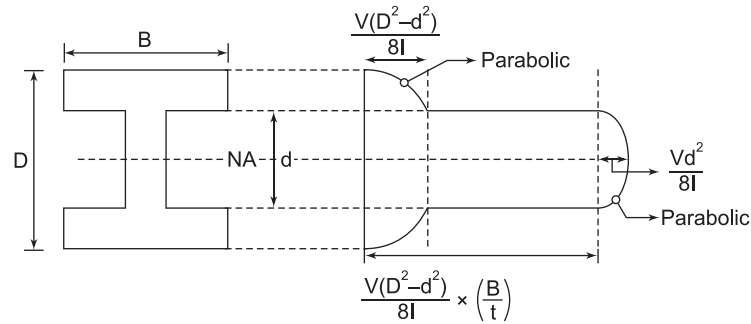


$$\text{Shear stress} = \frac{v}{2I} \left(\frac{d^2}{4} - y^2 \right)$$

$$\tau_{\max} = \frac{3}{2} \tau_{\text{avg}}$$

$$\tau_{\text{avg}} = \frac{V}{bd}$$

Shear stress in I - section

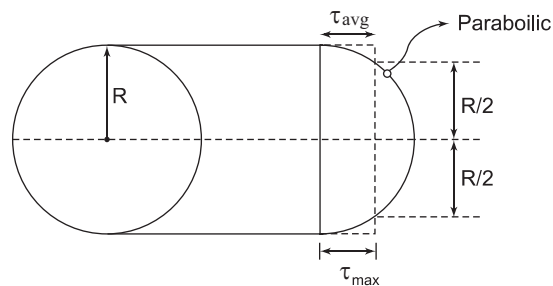


$$\text{Shear stress in flange} \Rightarrow \frac{V}{2I} \left(\frac{D^2}{4} - y^2 \right)$$

$$\text{Shear stress in flange} \Rightarrow \frac{V(D^2 - d^2)}{8I} \times \frac{B}{t} + \frac{V}{2I} \left(\frac{d^2}{4} - y^2 \right)$$

Note:- In I-section's nearly 80-85% shear is resisted by web.

Shear stress in circular section



$$\text{Shear stress} = \tau = \frac{4}{3} \left(\frac{V}{A} \right) \left(1 - \frac{y^2}{R^2} \right)$$

$$\tau_{\max} = \frac{4}{3} \tau_{\text{avg}}$$

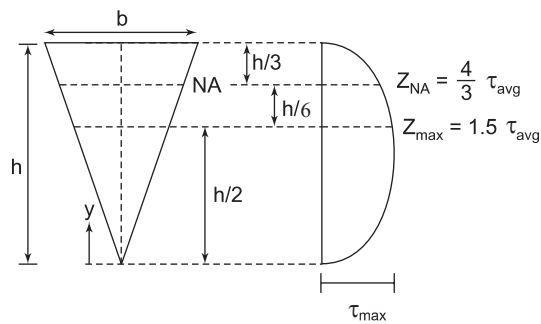
$$\tau_{\text{avg}} = \frac{V}{A}$$

Shear stress in triangular section

$$\text{Shear stress} = \frac{Vy(h-y)}{3I}$$

$$\tau_{\max} = \frac{3}{2} \tau_{\text{avg}}$$

$$\tau_{\text{NA}} = \frac{4}{3} \tau_{\text{avg}}$$

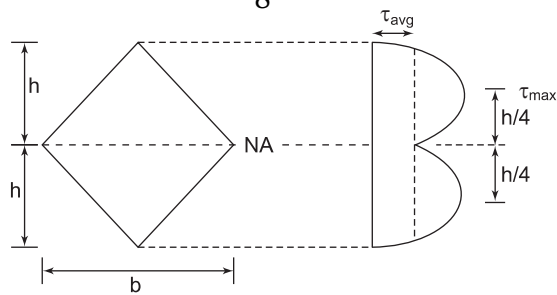


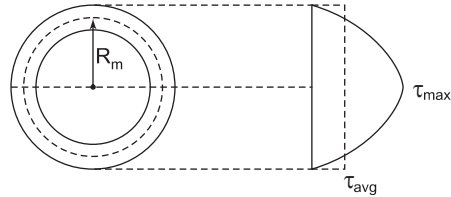
$$\text{Distance b/w N.A and } \tau_{\max} \text{ location} = \frac{h}{6}$$

Shear stress in Quadrilateral section about diagonal

$$\text{Shear stress} = \tau = \frac{V}{bh^3} (h-y)(2y+h)$$

$$\tau_{\max} = \frac{9}{8} \tau_{\text{avg}}$$

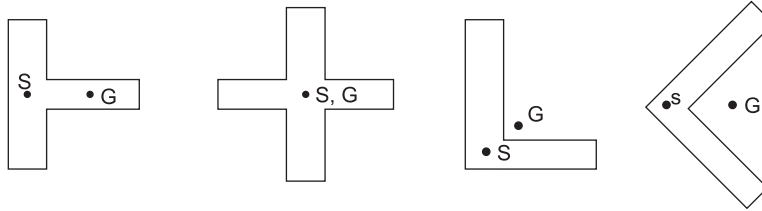


Shear stress of thin walled section

$$\tau_{max} = 2 \tau_{avg}$$

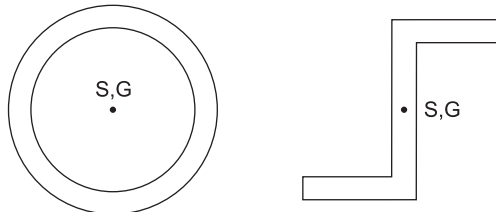
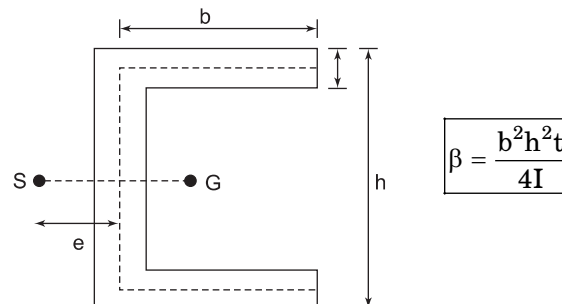
$$\tau_{avg} = \frac{V}{A}$$

Shear Centre:- It is the point through which if transverse bending load passes, the beam will have no twisting. It is the point through which resultant of shearing force on the section passes. Shear centre always lies on the axis of symmetry (if exists).

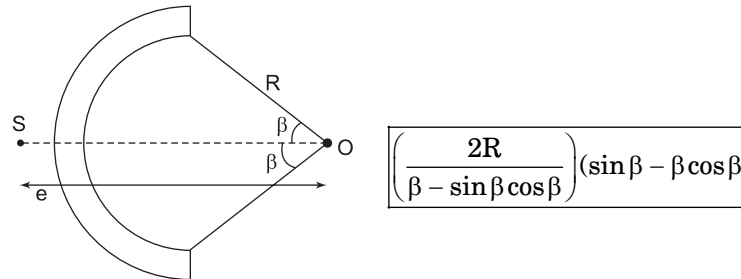


S = Shear Centre

G = Centre of gravity

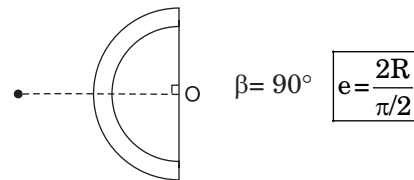
**Distance of Shear Centre of Important sections:-****1. Channel Section**

2. Circular arc

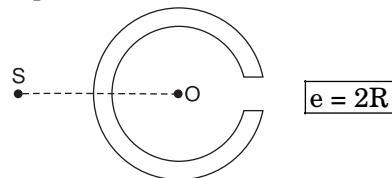


Special cases

(a) Semi-circular :

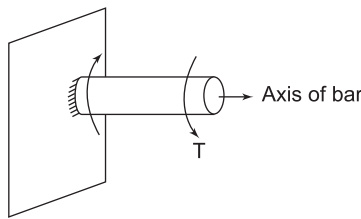


(b) Open slit case



Torsion of Circular Shaft

Torsion means twisting of a member when it is loaded by torques that tend to produce rotation about the longitudinal axis of the bar



Pure torsion:- When its cross-sections are subjected to only torsional moments and not accompanied by axial forces or bending moment or transverse shear. For pure torsion, bar should be prismatic.

Assumptions in Torsion formula

1. Circular section remains circular
2. Plane section remains plane and do not warp. (warping occurs in Non-circular sections)
3. Stress do not exceed proportional limits.
4. Shaft is loaded by twisting couples in planes that are perpendicular to the axis of shaft.

Torsion formula

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

T = Torque

J = Polar moment of Inertia

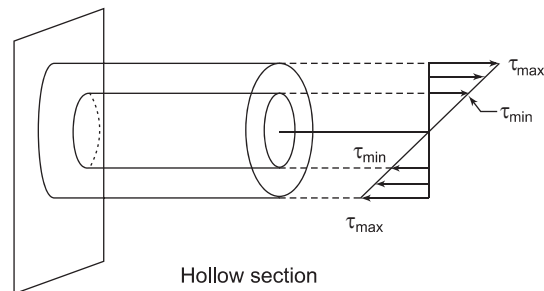
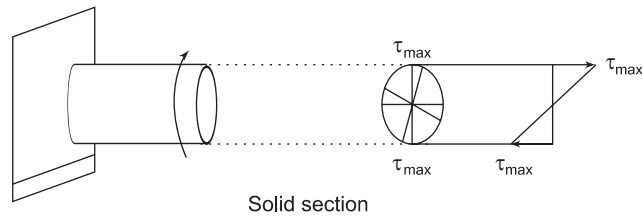
τ = Shear stress

r = distance from centre of shaft

G = shear modulus

θ = Angle of twist

L = Length of shaft

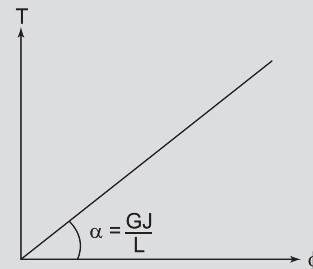


Note:-

$T - \phi$ plot is used to determine

The value of G using Torsion test

$$T = \left(\frac{GJ}{L} \right) \phi$$



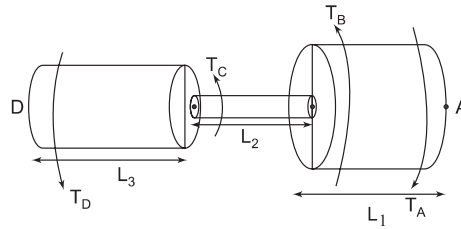
Get α from the graph and then by knowing J and L we can easily get the value of G .

Compound shaft

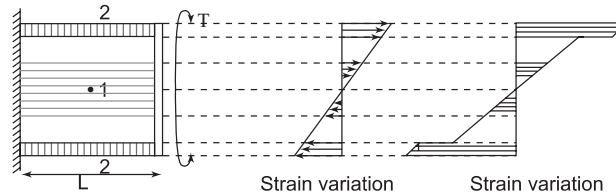
1. Series connection

$$\begin{aligned}\phi_{AD} &\Rightarrow \phi_A - \phi_D \\ &\Rightarrow (\phi_A - \phi_B) + (\phi_B - \phi_C) + (\phi_C - \phi_D)\end{aligned}$$

$$\boxed{\phi_{AD} \Rightarrow \phi_{AB} + \phi_{BC} + \phi_{CD}}$$



2. Parallel Connection



$$\boxed{\theta_1 = \theta_2} \Rightarrow \frac{T_1}{G_1} \frac{L}{J_1} = \frac{T_2}{G_2} \frac{L}{J_2}$$

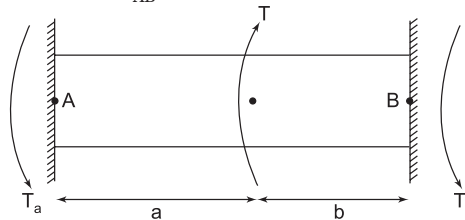
$$\boxed{\begin{aligned}\phi &= \frac{TL}{G_1 J_1 + G_2 J_2} \\ T_1 &= \frac{G_1 J_1}{G_1 J_1 + G_2 J_2} T \quad T_2 = \frac{G_2 J_2}{G_1 J_1 + G_2 J_2} T\end{aligned}}$$

Torsion in fixed beam (statically Indeterminate)

$$\boxed{T_a = \frac{Tb}{a+b}}$$

$$\boxed{T_b = \frac{Ta}{a+b}}$$

$$\Phi_{AB} = 0$$



Torsional strain energy

$$U = \frac{1}{2} T \cdot \theta = \frac{1}{2} T \cdot \frac{TL}{GJ} = \frac{\tau_{\max}^2}{4G} \times \text{Volume of shaft}$$

$$\text{Strain energy density} = \frac{\text{Strain energy}}{\text{Volume of shaft}} = \frac{1}{2} \times \tau \times \gamma$$

Power transmitted by shaft

$$P = T \times \omega$$

T = Torque

ω = Speed of rotation

N = Rotation per minute

Where $\omega = 2\pi f$

$$\text{or} \quad \omega = \frac{2\pi N}{60}$$

Thin walled Hollow shaft

$$\tau \times t = \frac{T}{2A_m}$$

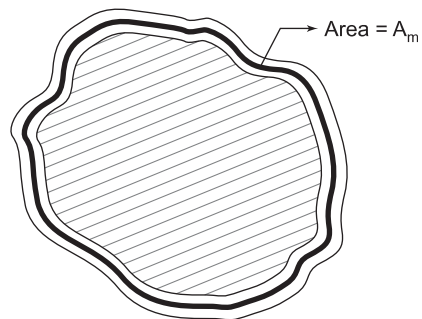
Where,

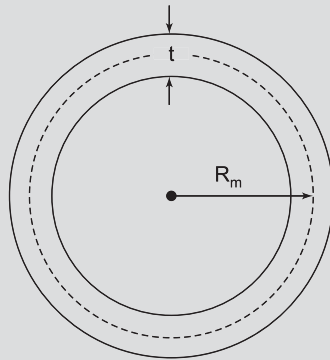
τ = Shear stress

t = thickness of the section

T = Torque applied

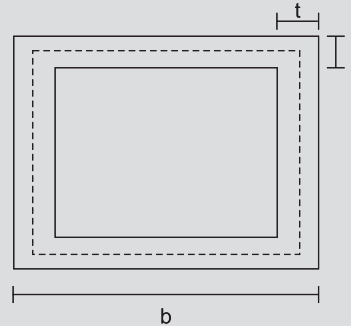
A_m = Area under mean circle.



Note:-

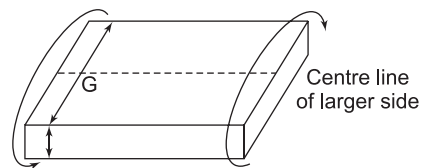
$$A_m = \pi R_m^2$$

Hollow Thin circular shaft



$$A_m = b^2$$

Hollow thin circular shaft

Torsion of Non-circular members

$$\tau_{\max} = \frac{T}{C_1 ab^2} \quad \text{Where } b < a$$

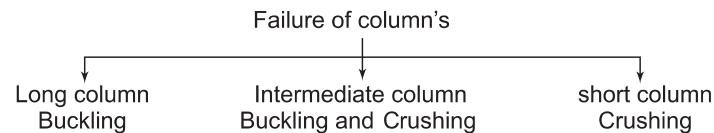
$$\phi = \frac{TL}{C_2 ab^3 G}$$

Shear stress at corners will be zero, Where as it will be maximum along the centre line of larger side.

Columns

10

Vertical members carrying vertical loading and moments is called column.



Assumptions of Euler's theory

1. Applicable to long columns only.(i.e Buckling failure only)
2. Material is isotropic, homogenous and linear elastic.
3. Purely axial loading.
4. Perfectly straight axis of column after unloading.

Euler's Formula

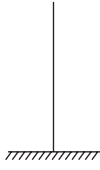
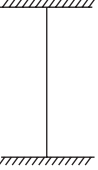
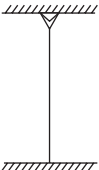
$$P_e = \frac{\pi^2 EI_{\min}}{L_{\text{eff}}^2}$$

P_e = Buckling load

$I_{\min}$ = Minimum MOI about centroidal axis.

L_{eff} = Effective length of column.

Effective length of column based on end conditions

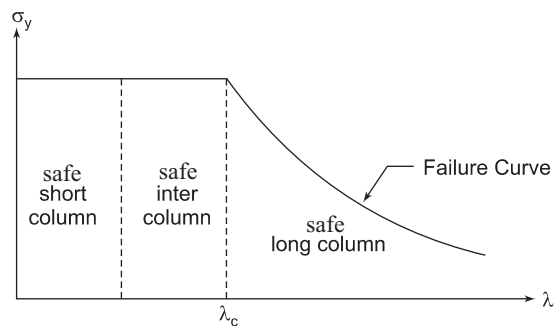
End condition	One end fixed one end free	Both end Hinged	Both end Fixed	One end fixed one end Hinged
L_{eff} (Theoretical)	$2L$ 	L 	$L/2$ 	$\frac{L}{\sqrt{2}}$ 
L_{eff} (As per IS code.)	$2L$	L	$0.65L$	$0.8L$

Validating of Euler's theory

$$P_e = \frac{\pi^2 E A r_{\min}^2}{L_{\text{eff}}^2}$$

$$P_{\text{critical}} = \frac{P_e}{A} = \frac{\pi^2 E}{\left(\frac{L_{\text{eff}}}{r_{\min}} \right)^2}$$

$$P_{\text{cr}} = \frac{\pi^2 E}{\lambda^2}$$



Where, λ = Slenderness ratio, ie ratio of effective length to least radius of gyration.

For mild steel $P_{\text{cr}} = 250$ $E = 2 \times 10^5 \text{ N/mm}^2$

Hence $\lambda_c = 88.89$

Rankine's Formula:- For Both **Short** and **Long** column

$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e}$$

P_R = Rankine load

P_c = Crushing load = $\sigma_c \times A$

P_e = Euler's load

$$P_R = \frac{P_c}{1 + \frac{P_c}{P_e}} = \frac{\sigma_c A}{1 + \frac{\sigma_c A}{\left(\frac{\pi^2 E}{\lambda^2}\right)}} = \boxed{\frac{\sigma_c A}{1 + \alpha \lambda^2}} = P_R$$

Where,

A = Area of column

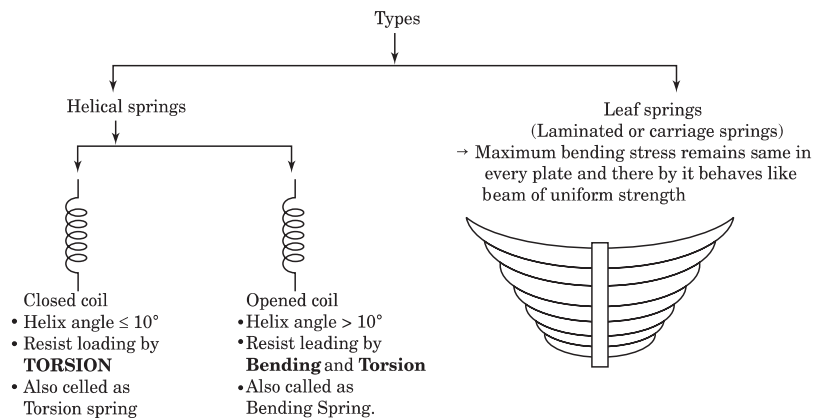
α = Rankines constant = $\frac{\sigma_c}{\pi^2 E}$

Note: For **long column** in which **eccentric loading** is applied, **secant formula** is used.

Springs

11

When load is applied on the spring it either gets deflected or distorted. and it recovers its original shape when load is released. During deflection or distortion it **absorbs** energy and **releases** the same as unloaded.



Proof load:- Greatest load that the spring can carry without getting permanently distorted.

Proof Stress:- Max stress in the spring when proof load is applied.

Proof Resilience:- Strain energy stored when proof load is applied.

Spring Constant:- It's the stiffness of the spring measured in load per unit deflection.

Closed coil helical spring

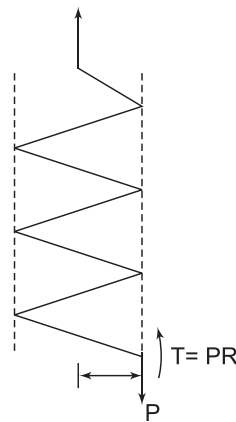
$$\text{Strain energy } U = \frac{1}{2} \frac{T^2 L}{GJ}$$

$$T = PR \quad L = 2\delta Rn \quad J = \frac{\pi d^4}{32}$$

$$\text{Axial deflection } (\delta) : \quad \delta = \frac{\partial U}{\partial P} = \frac{64PR^3n}{Gd^4}$$

$$\text{Stiffness of Spring (K)} \quad K = \frac{P}{\delta} = \frac{Gd^4}{64R^3n}$$

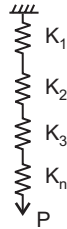
$$\text{Proof load} \quad P_{\max} = \frac{\pi d^3}{16R} \times \sigma_{\max}$$



Note:- If the spring is cut into two halves, then the stiffness of each half will be doubled to that of original spring.

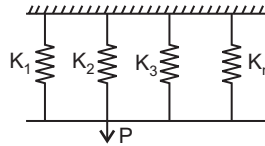
Equivalent Spring Constant

1. Series connection



$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \dots + \frac{1}{K_n}$$

2. Parallel connection

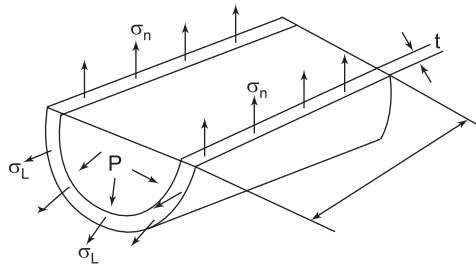


$$K_{eqa} = K_1 + K_2 + K_3 + \dots + K_n$$

Pressure Vessels

Thin shells are the shells in which thickness of the wall is less than $\frac{1}{10}$ th or $\frac{1}{15}$ th of its internal diameter. In thin shells normal stress is uniformly distributed throughout the thickness of the wall. While if thickness of wall is greater than $\frac{1}{10}$ th or $\frac{1}{15}$ th of internal diameter it's called Thick Shell.

Thin Cylinder subjected to internal pressure



1. Hoop Stress = $\sigma_h = \frac{pd}{2t}$
2. Longitudinal Stress = $\sigma_L = \frac{pd}{4t}$
3. Radial pressure = Inside = p
outside = 0
4. Hoop Strain $\epsilon_h = \frac{pd}{4tE}(2 - \mu)$
5. Longitudinal Strain $\epsilon_L = \frac{pd}{4tE}(1 - 2\mu)$

6. Volumetric Strain $\varepsilon_h = \varepsilon_1 + 2\varepsilon_h$

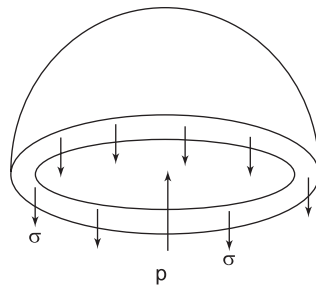
$$\varepsilon_v = \frac{pd}{4tE}(5 - 4\mu)$$

7. Maximum shear stress in plane of σ_h and σ_1

$$\tau_{\max (\text{in plane})} = \frac{pd}{8t}$$

8. Absolute maximum shear stress $\tau_{\text{abs max}} = \frac{pd}{4t} + \frac{p}{2}$

Thin sphere subjected to internal pressure



1. Hoop stress = Longitudinal stress

$$\sigma_h = \sigma_1 \frac{pd}{4t}$$

2. Hoop strain = Longitudinal strain

$$\sigma_h = \sigma_1 \frac{pd}{4t E} (1 - \mu)$$

3. Volumetric strain $\varepsilon_v = 3\varepsilon_h = 3\varepsilon_1$

$$\varepsilon_v = \frac{3pd}{4t E} (1 - \mu)$$

4. Maximum shear stress in plane = 0

5. Absolute maximum shear stress $\tau_{\text{abs max}} = \frac{pd}{8t}$

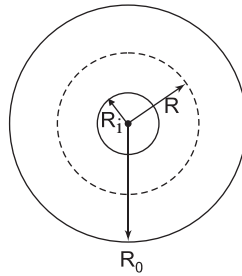
Lame's theorem:- Analysis of thick shells

Assumptions:-

1. Material is homogenous, isotropic and linear elastic.
2. Plane section of cylinder, perpendicular to longitudinal axis remains plane under pressure.

Lame's equation's for thick cylinder

1. Hoop stress $\rightarrow$ Tensile $\sigma_h = \frac{B}{R^2} + A$

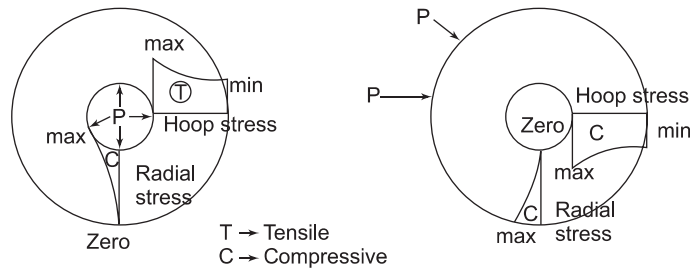


2. Longitudinal stress $\sigma_l = \frac{pR_i^2}{R_o^2 - R_i^2}$

3. Radial stress $\rightarrow$ compressive $\sigma_r = \frac{B}{R^2} - A$

Variation of stress in thick cylinder

- Note:-** 1. Radial and Hoop compression vary hyperbolically.
2. Longitudinal stress remains constant (Tensile)



Due to internal pressure

Due to external pressure

Lame's equation for thick sphere

1. Hoop stress = Longitudinal stress

$$\sigma_n = \sigma_l = \frac{B}{R^3} + A \text{ (Tensile)}$$

2. Radial stress

$$\sigma_r = \frac{2B}{R^3} - A \text{ (Compressive)}$$

Note:- Both lame's constant A and B are positive for internal pressure and both are negative for external pressure.

Civil Engineering

2

Structural Analysis

CONTENTS

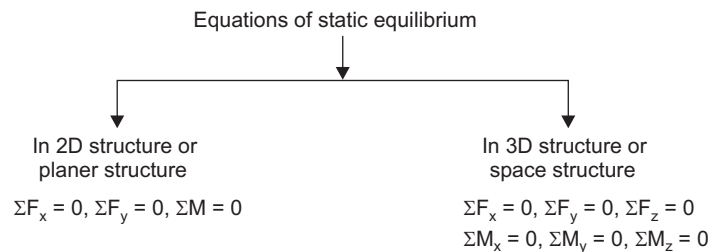
1. Determinacy-Indeterminacy
2. Force Method of Analysis
3. Slope-Deflection Method
4. Moment-Distribution Method
5. Arches
6. Trusses
7. Influence-Line Diagram
8. Matrix-Method of Analysis

Determinacy-Indeterminacy

1

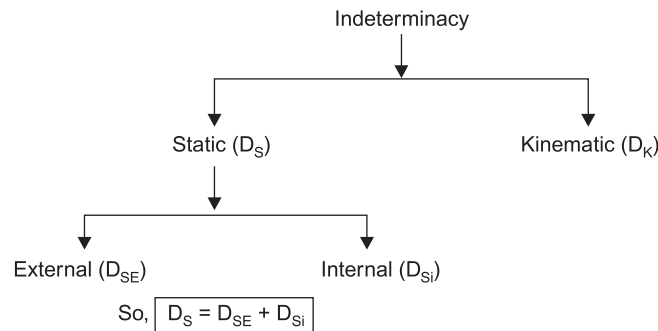
Statically Determine Structures: If the forces (or moments) in the members of the structure can be found out by use of equations of static equilibrium, then the structure is said to be statically determinate.

- Bending moment and shear force is independent of the cross-sectional area of the component and flexural rigidity of the members.
- No stresses are caused due to temperature change, lack of fit or differential settlement.



Statically Indeterminate Structures: If member forces cannot be found out by using equations of static equilibrium alone then the structure is statically indeterminate.

- Additional equations needed are obtained by relating the applied loads and reactions to the displacements or slopes known at different points on the structure. These equations are called compatibility equations.



2.4 CIVIL ENGINEERING

- Bending moment and shear force now depends upon cross-sectional area and flexural rigidity of the member.
- Stresses are caused due to temperature change, lack of fit or differential settlement.

Static Indeterminacy: (D_s)

$$D_s = \left[\begin{array}{c} \text{No. of unknown forces in} \\ \text{members or at support reactions} \end{array} \right] - \left[\begin{array}{c} \text{Equations of static} \\ \text{equilibrium available} \end{array} \right]$$

External Static Indeterminacy (D_{SE})

$$D_{SE} = \left[\begin{array}{c} \text{Total no. of Support} \\ \text{reactions in the structure} \end{array} \right] - \left[\begin{array}{c} \text{No. of equations of static} \\ \text{equilibrium available} \end{array} \right]$$

For 2D | plane Structures $D_{SE} = r - 3$, r = total external reactions

3D | space structures $D_{SE} = r - 6$

Internal Static Indeterminacy (D_{Si})

$$D_{Si} = D_s - D_{SE}$$

For 2D | plane Structure $D_{Si} = 3C - \gamma$ $\gamma = \Sigma (m_j' - 1)$

3D | plane structure $D_{Si} = 6C - \gamma$ $\gamma = 3\Sigma (m_j' - 1)$

where, C = No. of cuts to make structure determinate

γ' = No. of restraints applied to make all joints rigid

m_j' = No. of members connecting with j' number of joints

j' = No. of hybrid joint

$$D_s = (\alpha m + r) - (\beta j + \gamma)$$

		α	β
Truss	2D	1	2
	3D	1	3
Frame	2D	3	3
	3D	6	6

α = No. of unknown forces in each member

m = No. of members

r = No. of support reaction

β = No. of known equations at each joint

j = No. of joints

γ = Extra equations available due to hinge

$\Sigma (M_j - 1)$ for 2D

$3\Sigma (M_j' - 1)$ for 3D

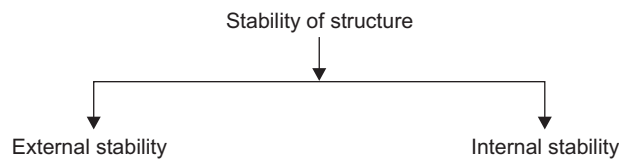
Kinematic Indeterminacy: If the number of unknown displacement components are greater than the number of compatibility equations, for these structures additional equations based on equilibrium must be written in order to obtain sufficient number of equations for the determination of all the unknown displacement components. The number of these additional equations necessary is known as degree of kinematic indeterminacy or degree of freedom of structure.

D_K = Total possible degree's of freedom – No. of available support reactions which generate as a result of restrained joint displacements.

Type of joints		Total possible degrees of freedom	D_K
Rigid jointed plane frame	3	$\Delta_x, \Delta_y, \theta$	$3J-r$
Rigid jointed space frame	6	$\Delta_x, \Delta_y, \Delta_z, \theta_x, \theta_y, \theta_z$	$6J-r$
Pin jointed plane structure	2	Δ_x, Δ_y	$2J-r$
Pin jointed space structure	3	$\Delta_x, \Delta_y, \Delta_z$	$3J-r$

$$D_K = (\beta j + \gamma) - (r + m)$$

m is used only when members are axially rigid or inextensible otherwise $m = 0$



External Stability: If a body is sufficiently constrained by external reaction such that rigid body movement of structure does not occur, then the structure is stable externally.

Necessary Conditions for External Stability

- (i) There should be three reactions that are neither concurrent nor parallel (in plane structure)
- (ii) Reactions should be non-parallel, non-concurrent and non-coplanar for space structure.

Internal stability: When part of the structure moves appreciably with respect to the other part, the structure is said to be unstable internally.

Force Method of Analysis

2

Statically indeterminate structures are always chosen over statically determinate structures because for a given loading, maximum stress and deflection is less as well as indeterminate structures have a tendency to redistribute its load to its redundant support in case of overloads.

Internal stresses are induced in indeterminate structures due to

- (i) Differential settlement of support
- (ii) Temperature variation
- (iii) Change in length due to fabrication error

In statically indeterminate structures, following conditions are to be satisfied:

- (i) Equilibrium equation
- (ii) Compatibility equation
- (iii) Force displacement relationship

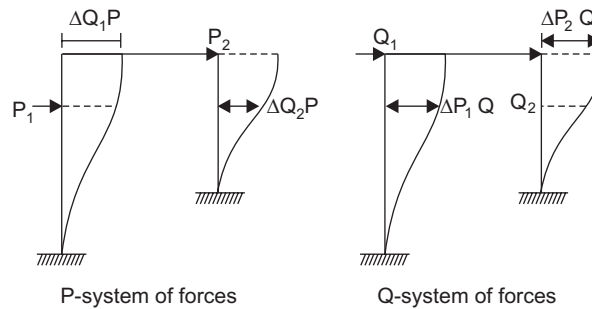
Force method	Displacement method
(i) Also known as compatibility method or method of consistent deformation or flexibility method	Also known as Equilibrium method or stiffness method
(ii) Forces (BM, SF) are taken as unknown.	Displacements (Δ , θ) are taken as unknown
(iii) Compatibility equations are written to find redundants	Equilibrium equations are written to find redundants
(iv) BM, SF are found using equilibrium equations	Δ , θ are found using load displacement equation
(v) Used when $D_s < D_k$	Used when $D_k < D_s$
(vi) Other force methods are: Castigliano's theorem, Strain energy method, Virtual work method, Clapeyron's three moment theorem (continuous beam's) Column analogy method (rigid frame with fixed support), flexibility matrix method.	Other methods are: Slope deflection, moment distribution, Kani's method, stiffness matrix method.

Principal of Superposition: The total displacement or internal loading at a point in a structure subjected to several external loading can be determined by adding together the displacements or internal loading caused by each of the external loads acting separately for application of superposition principal.

- (i) Hooke's law should be valid.
- (ii) Geometry of the structure must not undergo significant change when loads are applied.

Various Force Methods

- (i) **Method of Consistent Deformation:** Beam is assumed to be composed of:
 - (a) Primary structure which is obtained by removing the redundants and loading the resulting beam with external loading only.
 - (b) Secondary structure which contains loading of redundant reaction only.
 - (c) Redundant's are found out then by writing the compatibility conditions.
- (ii) **Betti's Theorem:** The virtual work done by a **P-force system** in going through deformation of **Q-Force system** is equal to the virtual work done by the **Q-force system** in going through the deformation of **P-force system**

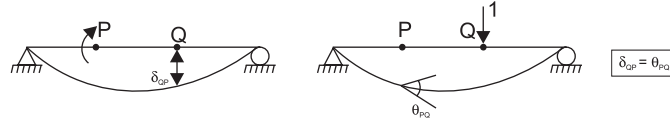


$$P_1 \Delta P_1 Q + P_2 \Delta P_2 Q = Q_1 \Delta Q_1 P + Q_2 \Delta Q_2 P$$

- (iii) **Maxwell's reciprocal theorem:** Special case of Bettis law. If only two force P and Q are acting and magnitude of P and Q are unity then $\Delta_{PQ} = \Delta_{QP}$ where

Δ_{PQ} = Deflection at P due to unit load at Q

Δ_{QP} = Deflection at Q due to unit load at P



- (iv) **Theorem of least work:** For any statically indeterminate structure, the redundant should be such as to make the total internal energy within the structure a minimum. This theorem is a special case of castigliano's 2nd theorem.
- (v) **Castigliano's Theorem:**

1st theorem $\frac{\partial U}{\partial \Delta_n} = P_n, \frac{\partial U}{\partial \theta_n} = M_n$	2nd theorem $\frac{\partial U}{\partial p_n} = \Delta_n, \frac{\partial U}{\partial M_n} = \theta_n$

It is also applied on self-straining systems. Self-straining is caused due to settlement of support of a redundant structure (λ) or by any initial misfit of a member by an amount λ too short or too long.

$$\frac{\partial U}{\partial R} = \lambda$$

λ = Small displacement in the direction of Redundant force R .

Slope-Deflection Method

3

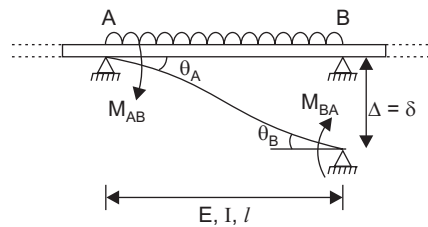
Slopes and deflections are combinedly called displacements.

In this method we establish a relationship between degrees of freedom (θ , Δ) and member end moments. This relationship is called slope deflection relationship. Method of super position is used to find out slope deflection relationship. It's also known as G.A. maney method.

Slope deflection equation:

(i) Continuous Beam

$$\begin{aligned} M_{AB} &= M_{FAB} + \frac{2EI}{l} \left(2\theta_A + \theta_B - \frac{3\delta}{l} \right) \\ M_{BA} &= M_{FBA} + \frac{2EI}{l} \left(2\theta_B + \theta_A - \frac{3\delta}{l} \right) \end{aligned}$$



M_{AB} = Internal member end moment at 'A'

M_{FAB} = Member end moment due to external load when all joints are considered fixed

θ_A = Rotation at A

θ_B = Rotation at B

δ = deflection of joint B with respect to A

Note: Axial and shear deformations are neglected.

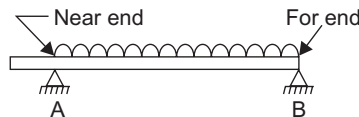
Sign Convention:

- (a) For member end moment, clockwise moment is taken as +ve,
- (b) Clockwise rotation is taken as +ve
- (c) If Δ gives clockwise rotation, it is taken as +ve

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(ii) When one end is pin supported

$$\begin{aligned} M_{AB} &= M_{FAB} + \frac{3EI}{L} \left(\theta_A - \frac{\delta}{L} \right) \\ M_{BA} &= 0 \end{aligned}$$



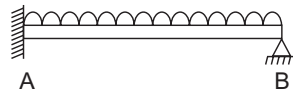
M_{FAB} = member end moment due to external load considering the continuous end (near end) to be fixed.

θ_A = Rotation at A.

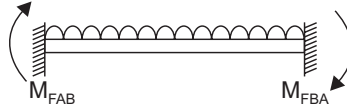
δ = Settlement of B with respect to A

Calculation of Fixed End Moments in case of one end being hinged, from the fixed end moments corresponding to the case of both end fixed.

$$\text{Fixed End Moment at A} = M_{FAB} - \frac{M_{FBA}}{2}$$



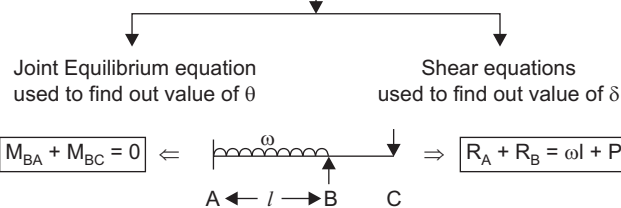
where, M_{FAB} and M_{FBA} are fixed end moments at A and B respectively.



Equilibrium Equation:

No. of equilibrium equations required = Degree of freedom of structure

Types of equilibrium equation



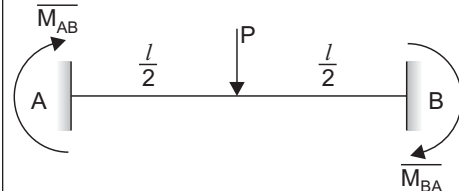
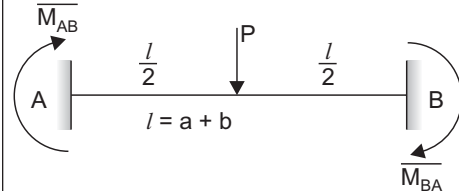
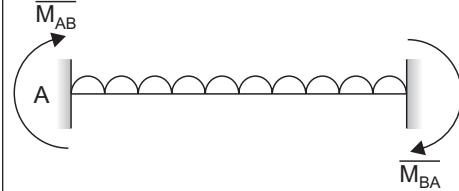
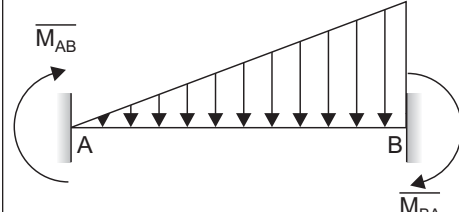
If δ is horizontal $\Sigma F_H = 0$

If δ is vertical $\Sigma F_V = 0$

Steps for Analysis in Slope Deflection Method:

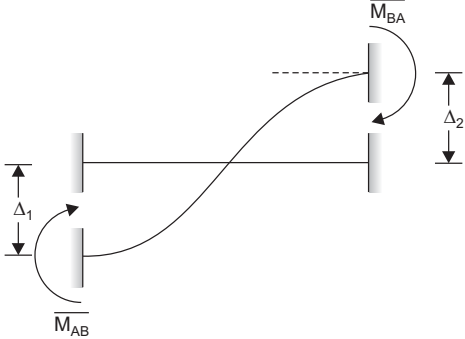
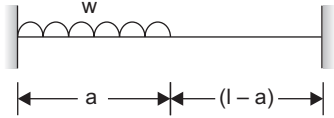
- (i) Computation of fixed end moments
- (ii) Relate member end moments to joint displacement
- (iii) Formulate equilibrium equations
- (iv) Solve the equations
- (v) Back-Substitution of known displacements in equations of (ii)
- (vi) Draw sketch of shear force diagram and bending moment diagram.

Fixed End Moments

Loading Diagram	M_{AB}	M_{BA}
	$-\frac{Pl}{8}$	$\frac{Pl}{8}$
	$-\frac{Pab^2}{l^2}$	$-\frac{Pa^2b}{l^2}$
	$-\frac{Wl^2}{12}$	$\frac{Wl^2}{12}$
	$-\frac{Wl^2}{30}$	$\frac{Wl^2}{20}$

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	$\frac{-11}{192} w l^2$	$\frac{5}{192} w l^2$
	$\frac{-5}{96} w l^2$	$\frac{5}{96} w l^2$
	$\frac{M_0 b(3a - l)}{L^2}$ $\alpha = \frac{l}{3} \quad 0$	$\frac{M_0 \alpha(3b - l)}{L^2}$ $M_{0/3}$
	$\frac{M_0}{4}$	$\frac{M_0}{4}$
	$\frac{-6EI\Delta}{l^2}$	$\frac{-6EI\Delta}{l^2}$
	0	$\frac{-3EI\Delta}{l^2}$

	$\frac{6EI(\Delta_1 + \Delta_2)}{l^2}$	$\frac{6EI(\Delta_1 + \Delta_2)}{l^2}$
	$\frac{wa^2}{12l^2}$ $(6l^2 - 8 / 6.a + 3a^2)$	$\frac{wa^2}{12l^2}$ $(4l - 3a)$

Moment-Distribution Method

4

Presented by **Prof. Hardy cross in 1929**. Also known as Relaxation method.

It's an iterative procedure of indirectly solving the equations of equilibrium as formulated in slope deflection method.

Initially assuming each joint in the analysis to be fixed. Then by unlocking and locking each joint in succession, the internal moments at the joints are distributed and balanced until the joints have rotated to then final or nearly final position.

Note: Last Cycle must end in balancing of hinged or continuous joints with carry over to fixed joint.

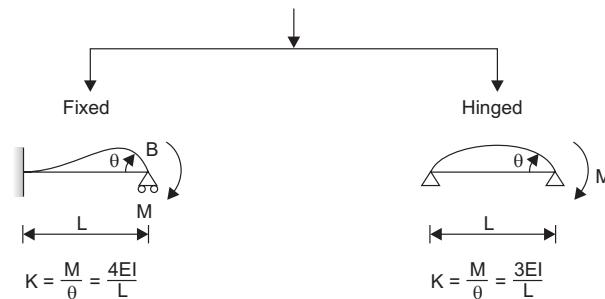
Stiffness: It is the force or moment required to be applied at a joint so as to produce unit deflection or rotation at that joint

$$\begin{matrix} F = K\Delta \\ M = K\theta \end{matrix}, K = \text{stiffness},$$

F = Force required for deflection Δ ,

M = Moment required for rotation θ

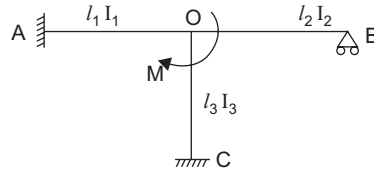
Member Stiffness: Stiffness of a member AB when farther end is



Joint Stiffness Factor: Assuming joint 'O' to be rigidly connected. Applying moment 'M' at O

$$M = M_1 + M_2 + M_3; M = K_1\theta + K_2\theta + K_3\theta$$

$$K = \frac{M}{\theta} = K_1 + K_2 + K_3 = \frac{4EI_1}{l_1} + \frac{3EI_2}{l_2} + \frac{4EI_3}{l_3}$$


Distribution Factor (D.F.):

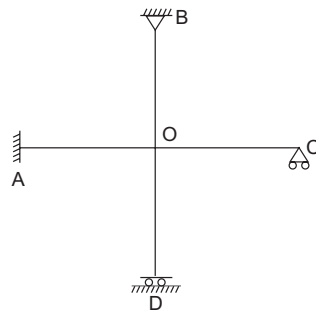
$$DF = \frac{\text{Stiffness of a member}}{\text{Sum of stiffness of all members at that joint}}$$

$$DF = \frac{\text{Relative stiffness of a member}}{\text{Sum of Relative stiffness of all members at that joint}}$$

Relative stiffness when far end is fixed = I/l

Relative stiffness when far end is hinged = $\frac{3I}{4l}$

Note: Sum of DF for all members at a joint is always one



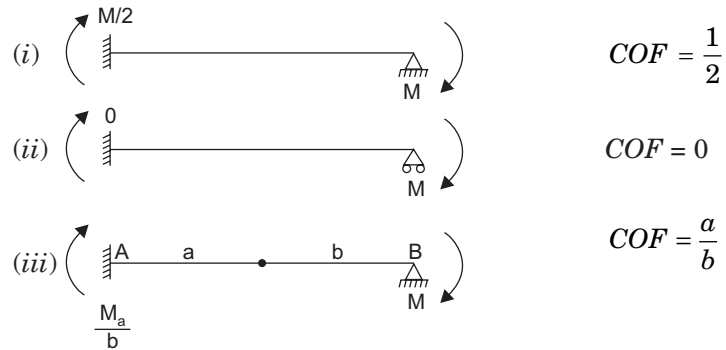
Length of $OA = OB = OC = OD = l$

Member	Stiffness
OA	$\frac{4EI}{l}$
OB	$\frac{3EI}{l}$
OC	$\frac{3EI}{l}$
OD	$\frac{EI}{l}$

Carry Over Factor = (COF)

$$COF = \frac{\text{Carry over moment}}{\text{Applied moment}}$$

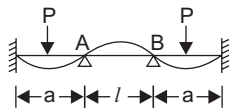
Standard Cases



$$\text{Stiffness at } B - \frac{M}{\theta_B} = \frac{3EIb^2}{a^3 + b^3}$$

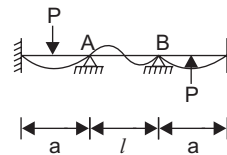
Modified stiffness factor for member AB

Symmetrical beam with symmetrical loading



$$K = \frac{2EI}{l}$$

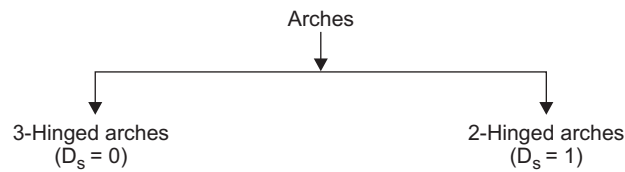
Symmetrical beam with symmetrical loading



$$K = \frac{6EI}{l}$$

Arches

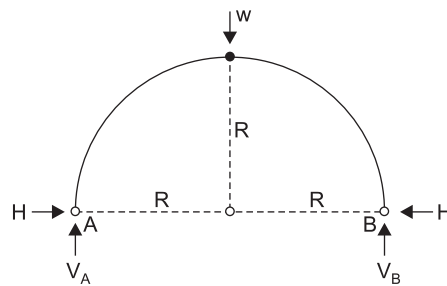
5



(i) Three-hinged arches

(a) Semi-circular with concentrated load at crown

$$H = V_A = V_B = \frac{W}{2}$$

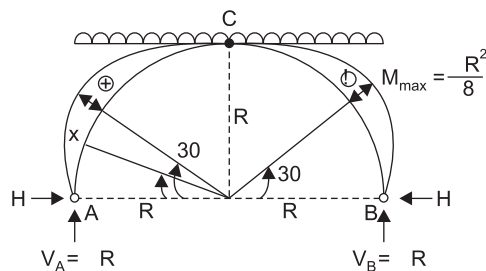


(b) Semi-circular with *udl* over the whole span

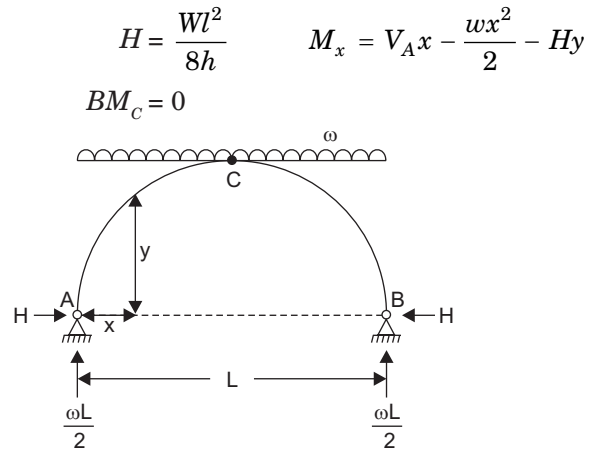
$$H = \frac{wR}{2} \quad M_x = \frac{wR^2}{2} (\sin \theta - \sin^2 \theta)$$

$$M_{Max} = \frac{-wR^2}{8} \quad BM_C = 0$$

Point of contraflexure = 0

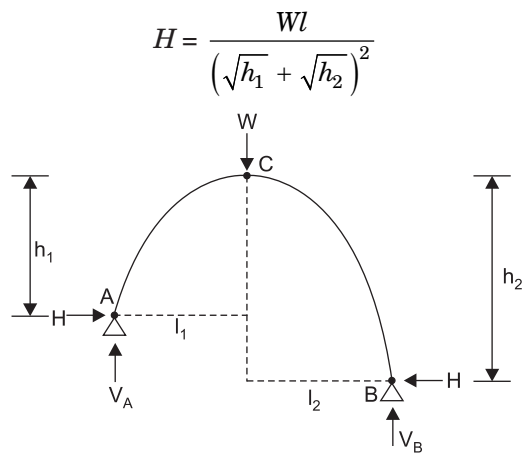


(c) Parabolic arch with udl over the whole span



(d) Parabolic arch with abutments at different levels

1. Concentrated load at crown

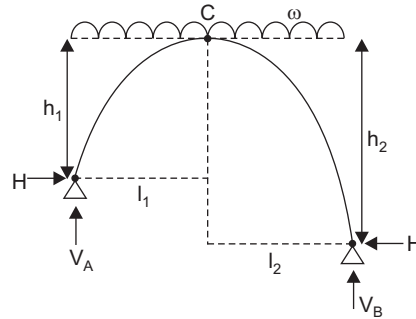


2. udl over the whole span

$$l_1 = \frac{\sqrt{h_1} l}{\sqrt{h_1} + \sqrt{h_2}}, \quad l_2 = \frac{\sqrt{h_2} l}{\sqrt{h_1} + \sqrt{h_2}}$$

$$H_A = H_B = \frac{wl^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}$$

$$BM_C = 0$$

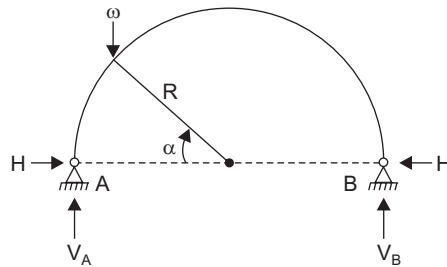


(ii) Two hinged Arches

(a) Semi circular arch with concentrated load

$$H = \frac{w}{\pi} \sin^2 \alpha$$

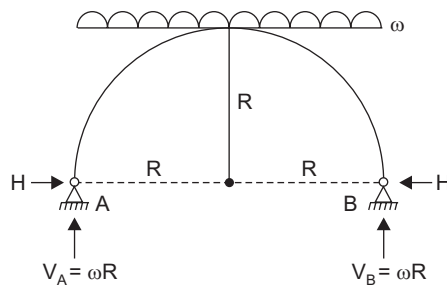
At Crown $\alpha = 90^\circ$, $H = \frac{w}{\pi}$



Note: H of 3-Hinged semi circular arch > H of 2 Hinged semi circular arch

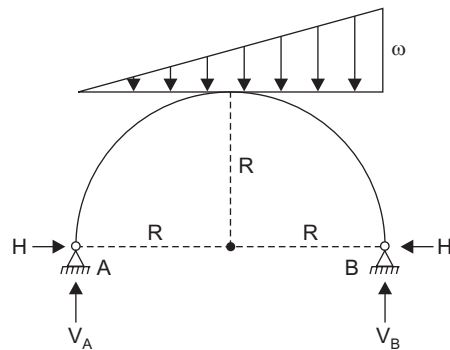
(b) Semi circular arch with *udl* over the whole span

$$H = \frac{4}{3} \frac{wR}{\pi}$$



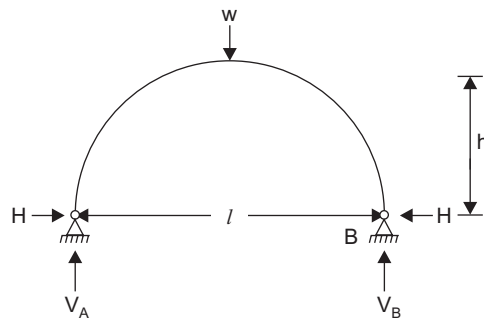
- (c) Semi-circular arch with uvl over the whole span

$$H = \frac{2}{3} \frac{wR}{\pi}$$



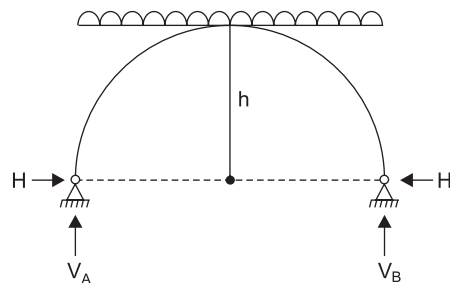
- (d) Parabolic arch with concentrated load at crown

$$H = \frac{25}{128} \frac{wl}{h}$$



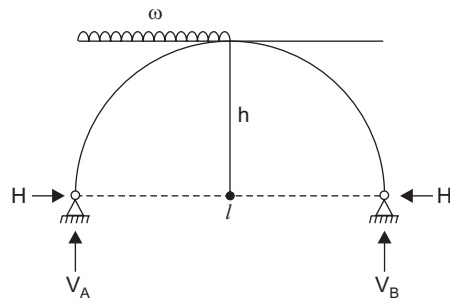
- (e) Parabolic arch with udl over the whole span

$$H = \frac{wl^2}{8h}$$



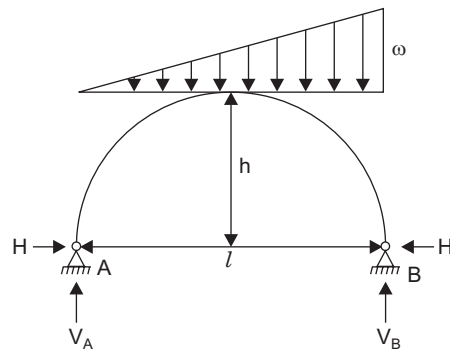
(f) Parabolic arch with udl over half span

$$H = \frac{wl^2}{16h}$$



(g) Parabolic arch with uvl over the whole span

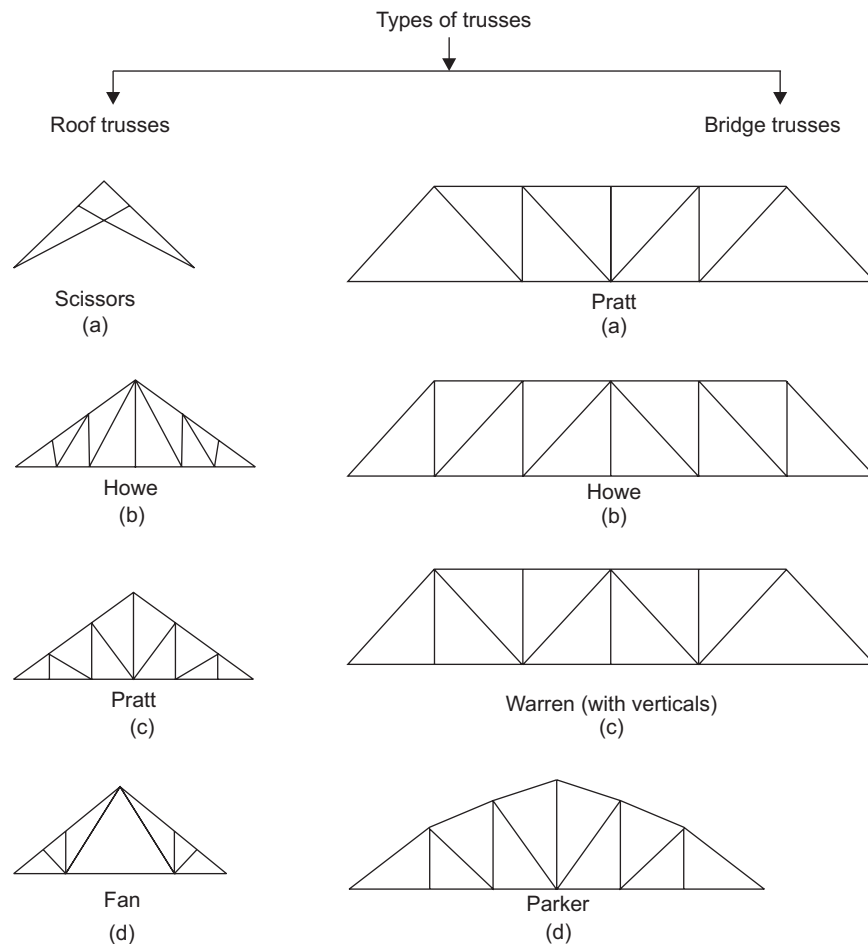
$$H = \frac{wl^2}{16h}$$

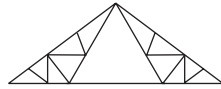


Trusses

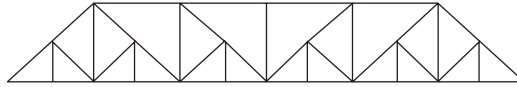
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A truss is a structure composed of slender members joined together at end points by bolting/riveting or welding. Ends of the members are joined to a common plate called gusset plate.

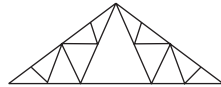




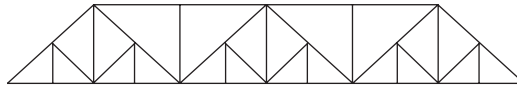
Fink
(e)



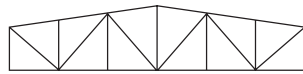
Baltimore
(e)



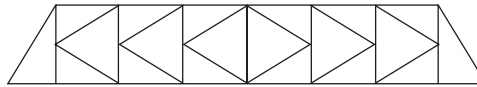
Combined Fink
(f)



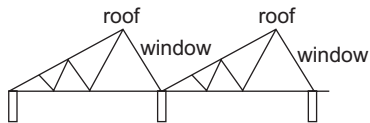
Subdivided warren
(f)



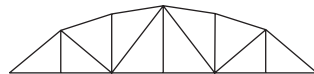
Warren
(g)



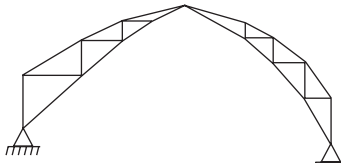
K-truss
(g)



Sawtooth
(h)



Bowstring
(i)



Three-hinged arch
(j)

Note: Pratt truss is better than Howe truss because the diagonal member in Pratt truss carries tension while in Howe truss, diagonal member carries compression. Thus if longer member carries compression, there is likely chance of buckling of truss member.

Assumptions for design of truss members and connection

- (i) The members are joined together by smooth pins and centre lines of joining members are concurrent.
- (ii) All loadings are applied to joints.
- (iii) Self weight of the members is neglected.

Degree of static indeterminacy (D_s):

$$D_s = m + r - 2j$$

* where, m = Number of members

r = Total external reactions

j = Total number of joints

**Zero-force Members:**

- (i) If only two non-collinear members exist at a truss joint and no external force or support reaction is applied to the joint, the members must be zero force member.
- (ii) If three members join at a point and out of them, two are collinear and also no external load acts at joint, the third member is a zero force member.

Methods of Analysis of Statically determinate truss**Deflection of Truss joint****(i) Castigliano's Method**

$$\Delta = \frac{\partial U}{\partial F}$$

Δ = deflection at the point of application of F and in the direction of F

U = Strain energy in the system

$$U = \sum_{i=1}^n \frac{p_i^2 l_i}{2A_i E_i} \quad \text{so} \quad \Delta = \frac{\partial}{\partial F} \left(\sum_{i=1}^n \frac{p_i^2 l_i}{2A_i E_i} \right)$$

p_i = force in the i^{th} member of truss due to combined action of external load and applied load (F) at the point at which deflection so to be found.

(ii) Maxwell's unit load method

$$\boxed{\text{External virtual work} = \text{Internal virtual work}}$$

$$\begin{array}{ccccc} 1 & \times & \Delta & = & \Sigma u_i & \times & dl_i \\ \text{Virtual} & & \text{Real} & & \text{Virtual} & & \text{Real} \\ \text{load} & & \text{displacement} & & \text{load} & & \text{displacement} \end{array}$$

$$1 \times \Delta = \Sigma u_i \left(\frac{p_i l_i}{A_i E_i} + l_i \alpha_i \Delta t_i + \lambda_i \right)$$

where, u_i = internal member forces in the i^{th} member due to unit load

p_i = force in the i^{th} member due to combined external and applied load

α_i = coefficient of thermal expansion for i^{th} member

Δt_i = Change in temperature of i^{th} member

L_i = length of i^{th} member

λ_i = Lack of fit or fabrication error

Sign Convention:

- (i) Tension +ve, compression –ve
- (ii) If temperature increases Δt +ve otherwise negative
- (iii) λ +ve if larger than normally required (i.e. too long case) otherwise –ve in too short case.

Note: Angle weights methods, joint displacement equation method and Williot-Mohr (Graphical) method are also used in calculating truss deflection.

Method's of Analysis of Statically Indeterminate Truss

When no. of external reactions is exactly three or more than three, then only member forces can be taken as redundant and redundants should be selected such that its removal does not make the truss unstable.

(i) Castigliano's Method:

- (a) Remove redundant member such that the truss becomes determinate and remains stable.
- (b) Apply equal and opposite force F at joints connecting the removed members.
- (c) Now calculate the member forces P_i due to F and external loading.

- (d) Find the strain energy of the system U

$$U = \sum_{i=1}^n \frac{P_i^2 l_i}{2A_i E_i} \quad n = \text{no. of members in the truss}$$

- (e) Put $\frac{\partial U}{\partial F} = 0$ (minimum strain energy condition) and calculate F

- (f) Once F is known then other member forces can be found out.

(ii) Maxwell's unit load method

- (a) Choose redundant such that its removal does not makes structure unstable.

- (b) Remove redundant and find out member forces due to external loading i.e. P_i .

- (c) Remove external loading and apply the unit force (equal and opposite) at joints connecting the removed member. Calculate the member force u_i .

- (d) Now net member force $F_i = P_i + u_i R$

- (e) Minimise the strain energy by $\frac{\partial U}{\partial R} = 0$ where $U = \sum \frac{F_i^2 l_i}{2A_i E_i}$

$$(f) \text{ Hence } R = \frac{-\sum_{i=1}^{n-1} u_i \frac{P_i l_i}{A_i E_i}}{\sum_{i=1}^n \frac{u_i^2 l_i}{A_i E_i}}$$

Note: If all factor's are taken into account then

$$R = \frac{-\sum_{i=1}^{n-1} u_i \left(\frac{P_i l_i}{A_i l_i} + l \alpha \Delta t + \lambda_i \right)}{\sum_{i=1}^n \frac{u_i^2 l_i}{A_i E_i}}$$

Development of force due to change in temperature or lack of fit

- (i) Externally as well as internally determinate truss: No force is developed in members.
- (ii) Externally determinate but internally indeterminate: Rigid body motion will occur.
- (iii) Externally Indeterminate but internally determinate: No force developed in all members except members joining support.

Influence-Line Diagram

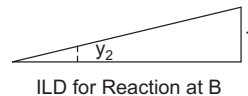
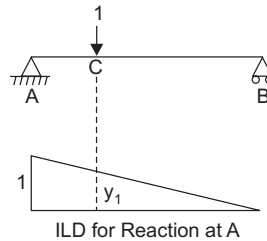
7

Influence line diagram represents the variation in the values of stress functions such as shear force, bending moment, Axial force, slope or deflections etc. at a specified point in a member as the concentrated unit load moves over the member.

Note: Influence line for stress functions for statically determinate structure consists of straight line segment. However, that for statically indeterminate structure it consists of curved line segment. *ILD* for deflection may even be curved for determinate structure.

(i) Simply supported beam

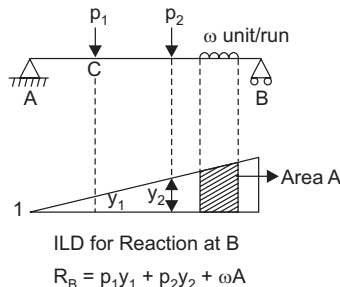
(a) End reactions due to unit load at C



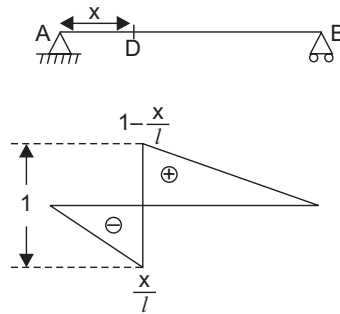
y_1 = Magnitude of reaction at A when unit load at C

y_2 = Magnitude of reaction at B when unit load at C

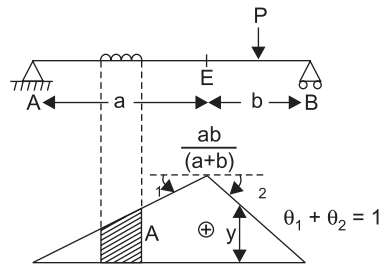
(b) End reaction at B due to a load system



(c) Shear force at D

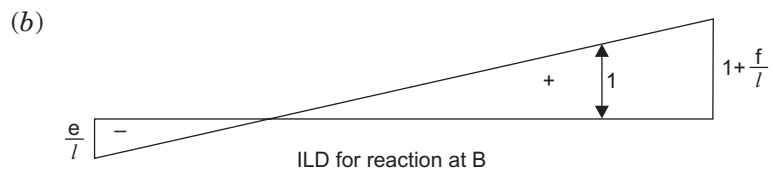
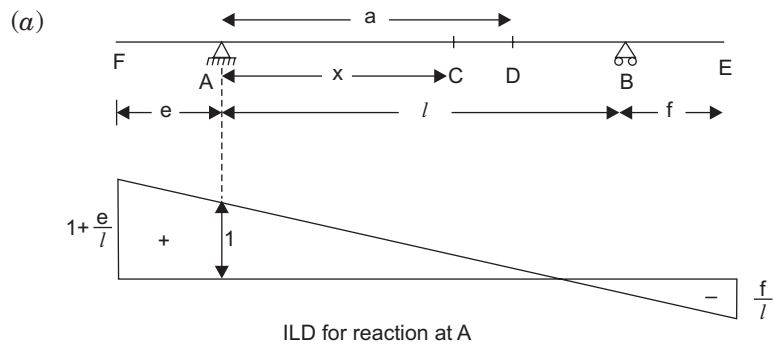


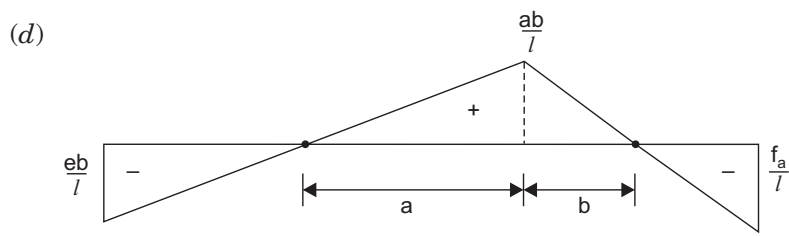
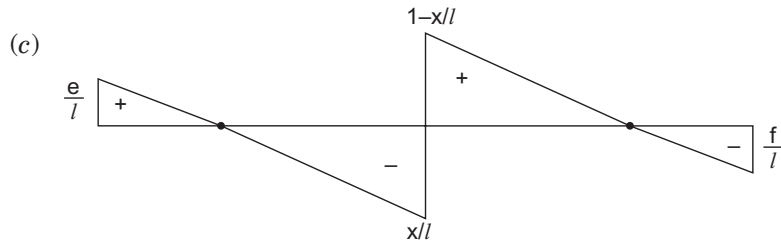
(d) Bending moment at E



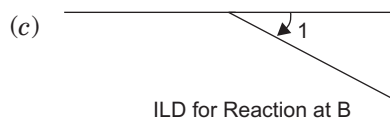
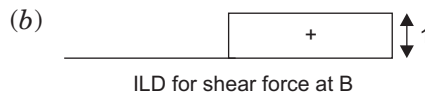
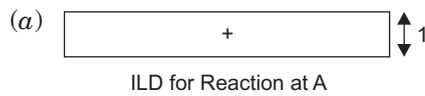
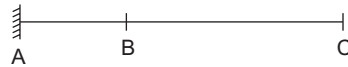
$$BM \text{ at } E = wA + Py$$

(ii) Simply supported beam with overhang

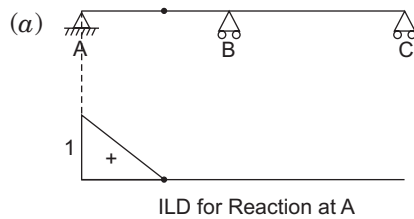


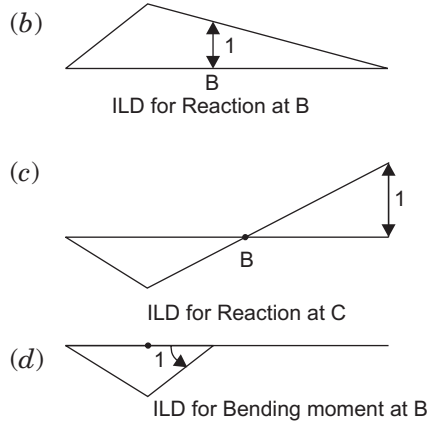


(iii) Cantilever beam



(iv) Simply supported beam with internal hinge

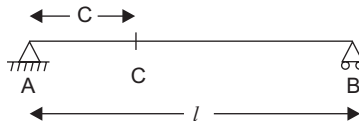




Note: The ordinate of ILD for reaction and shear force is dimensionless where as ordinate of BM in ILD has dimensions of length.

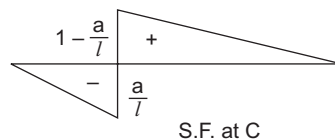
Muller-Breslau Principle: Influence line for any stress function may be obtained by removing the restraint offered by the function and introducing a directly related unit displacement at the location and in the direction of the function.

Effect of rolling Load

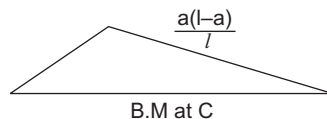


(i) Single point Rolling load

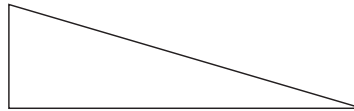
- (a) For max. SF at Section C, the point load should be just to the left or just to the right of the section where ordinate of ILD for SF is maximum.



- (b) For maximum Bending moment at section C, the point load should be on the section C itself.



- (c) For maximum +ve shear force, anywhere in the span the point load should be on the left hand support.



S.F at A

- (d) For maximum -ve shear force, anywhere in the span the point load should be on the right hand support.



SF at B

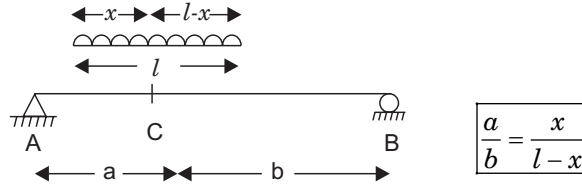
- (e) For absolute maximum BM, point load should be at mid span.

(ii) udl longer than the span

- (a) For maximum -ve SF at Section C, span should be loaded such that head of the udl reaches at C.
- (b) For maximum +ve SF at section C, the tail of the *udl* should reach section C.
- (c) For maximum Bending moment at Section C, complete span should be loaded.
- (d) For absolute maximum +ve or -ve shear force complete span should be loaded.
- (e) For absolute maximum BM complete span should be loaded and absolute max. BM will occur at mid span.

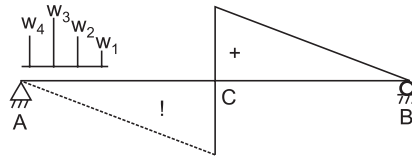
(iii) udl shorter than the span

- (a) For maximum -ve SF at section C, span should be loaded such that head of the udl reaches at C.
- (b) For maximum +ve SF at section C, the tail of the *udl* should reach section C.
- (c) For maximum bending moment at section C, the load should be placed such that the section divides the load in the same ratio as it divides the span.



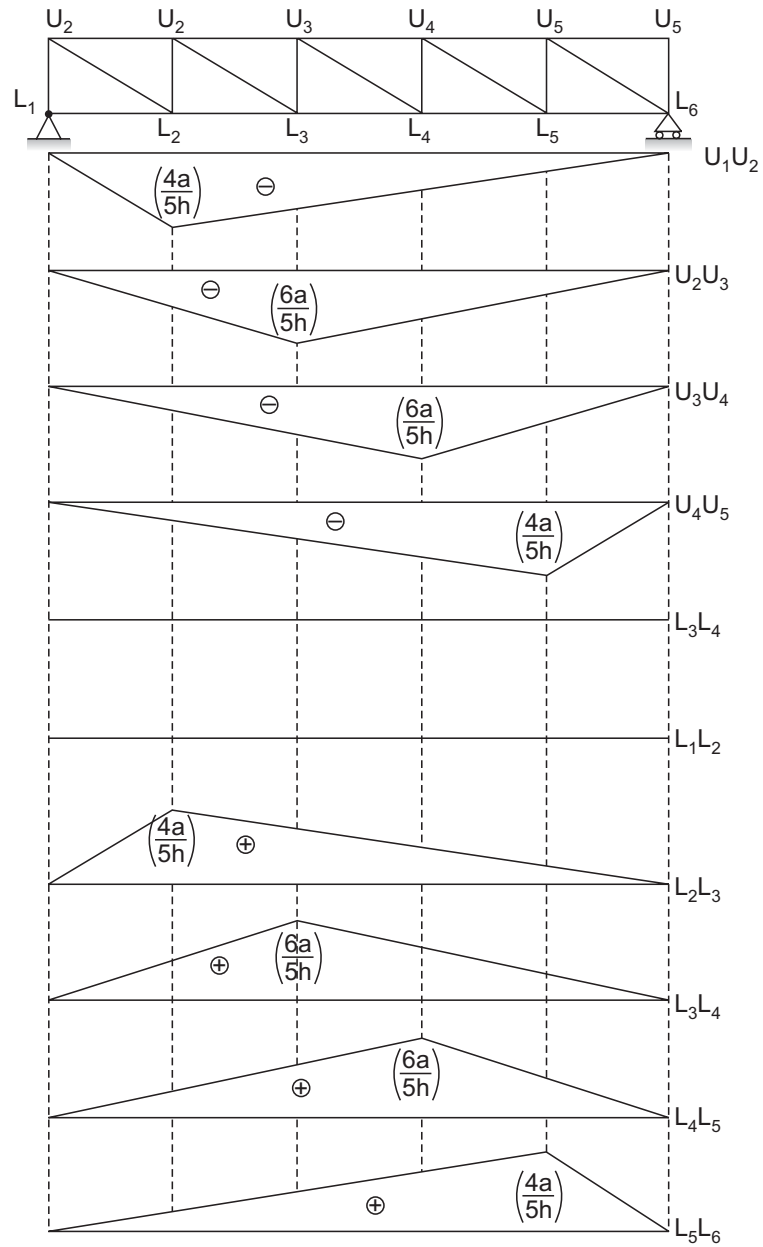
- (d) For absolute maximum +ve SF, the tail of the load should be at left support.
- (e) For absolute maximum -ve SF, the head of the load should be at right support.
- (f) For absolute maximum BM, the CG of the load should be at the mid span.

(iv) Train of concentrated loads

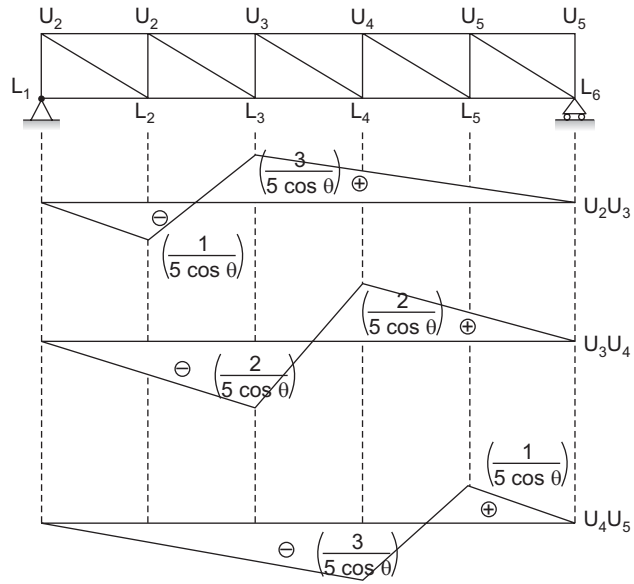


- (a) For maximum +ve/-ve SF at section C, various loads will be placed on the section and values of SF at the section will be calculated. Maximum of these +ve/-ve values will be the max. SF at the Section C.
- (b) Maximum B.M. at the section C, will develop **when one of the load is itself of the section**, and load on the section should be such that when it rolls over the section and comes to the other side average loading (load/length of one section) on the left side changes from heavier loading to lighter loading (or vice versa).
- (c) For absolute maximum BM, loads are placed in such a way that resultant and the heavier loads are equidistant from the midspan. Under this condition bending moment will be calculated below the heavier load only.

ILD influence line diagram for truss members.

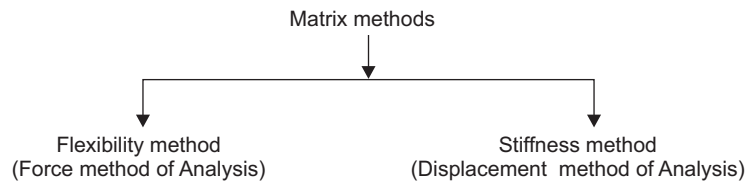


Influence line diagrams for inclined member of truss



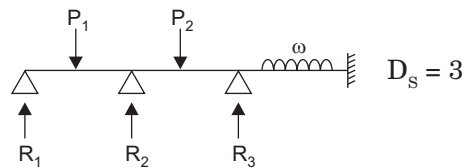
Matrix-Method of Analysis

8

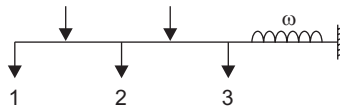


Flexibility Matrix-Method: Displacement caused by unit force.

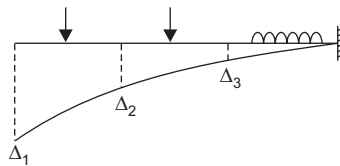
- (i) Identify the static indeterminacy of the structure and choose redundants.



- (ii) Assign coordinates to each redundant direction



- (iii) Release the redundants to obtain primary structure and find out deflections in the co-ordinate direction at the location of co-ordinate.



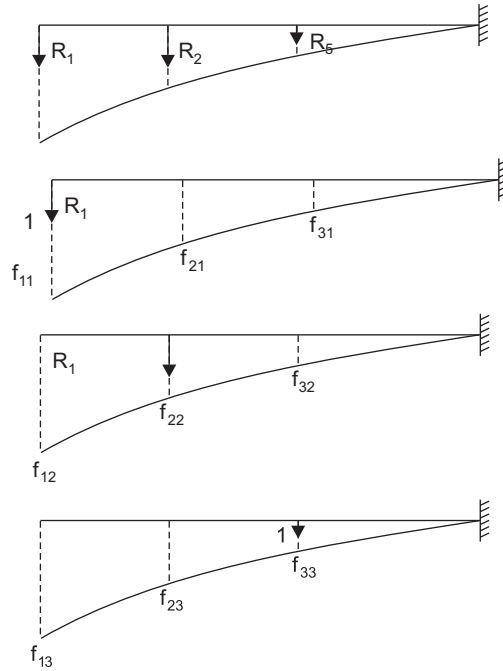
- (iv) Apply redundants (after removing external loading) and find out deflections in the co-ordinate directions.

$$\delta_1 = f_{11}R_1 + f_{12}R_2 + f_{13}R_3$$

$$\delta_2 = f_{21}R_1 + f_{22}R_2 + f_{23}R_3$$

$$\delta_3 = f_{31}R_1 + f_{32}R_2 + f_{33}R_3$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$



When D_1, D_2, D_3 are final deflections at co-ordinate locations 1, 2, 3 then $D_1 = \Delta_1 + \delta_1, D_2 = \Delta_2 + \delta_2, D_3 = \Delta_3 + \delta_3$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} + \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

$$[D] = [\Delta] + [f] [R]$$

$$[R] = [F]^{-1} ([D] - [\Delta])$$

Note: f_{21} = element of 2nd row, 1st column.
= deflection in direction 2 due to unit force in direction 1

Properties of flexibility matrix

- (i) Order of flexibility matrix = No. of co-ordinates chosen in the problem i.e., static indeterminacy.
- (ii) Flexibility matrix is a square symmetrical matrix.
- (iii) All elements of flexibility matrix are displacements (δ or θ)
- (iv) Principle diagonal elements of flexibility matrix are always +ve.
- (v) Symmetry of flexibility matrix is owing to maxwell's reciprocal theorem.
- (vi) Flexibility matrix is calculated only for stable structures. There is no rigid body displacement.
- (vii) Dimensions of all elements may not be same i.e.

$$f = \frac{\theta}{M} = \frac{1}{kNm} \quad \text{or} \quad f = \frac{\delta}{P} = \frac{1}{kN}$$

Stiffness Matrix-method: Force acquired to produce unit displacement.

- (i) Identify the kinematic indeterminacy of the structure and choose unknown displacements.
- (ii) Assign co-ordinates to each of the unknown displacements (θ 's)
- (iii) The joints are restrained and forces developed due to applied load in the co-ordinate directions are found out. Let that be P'_1, P'_2, P'_3 .
- (iv) Now the joints are released and deflections are permitted in the co-ordinate directions and forces required in the co-ordinate directions for these displacements are found out.

$$P_{1\Delta} = K_{11}\Delta_1 + K_{12}\Delta_2 + K_{13}\Delta_3$$

$$P_{2\Delta} = K_{21}\Delta_1 + K_{22}\Delta_2 + K_{23}\Delta_3$$

$$P_{3\Delta} = K_{31}\Delta_1 + K_{32}\Delta_2 + K_{33}\Delta_3$$

$$\begin{bmatrix} P_{1\Delta} \\ P_{2\Delta} \\ P_{3\Delta} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix}$$

When P_1, P_2, P_3 are final forces developed at various co-ordinates axis

$$P_1 = P'_1 + P_{1\Delta}, P_2 = P'_2 + P_{2\Delta}, P_3 = P'_3 + P_{3\Delta}$$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix}$$

$$[P] = [P'] + [K] [\Delta]$$

$$[\Delta] = [K]^{-1} ([P] - [P'])$$

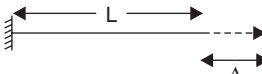
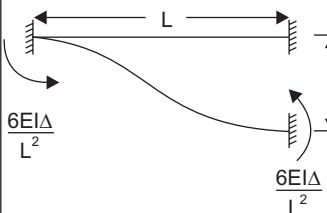
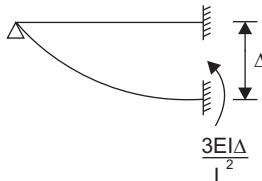
Note: K_{21} = element of 2nd row, 1st column
= force in the direction of 2 due to unit displacement in direction 1

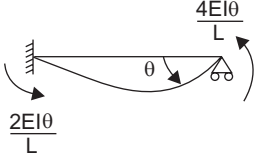
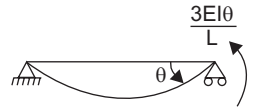
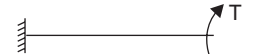
Properties of stiffness matrix

All the properties of flexibility matrix defined will also be the properties of stiffness matrix except that the following:

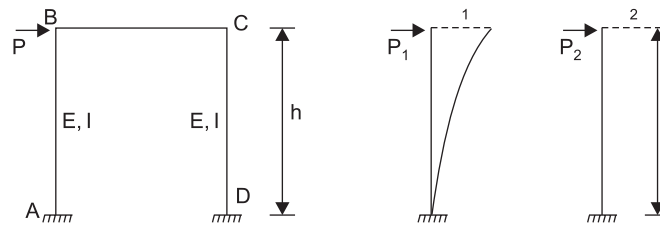
- (i) order of stiffness matrix will be degree of kinematic indeterminacy
- (ii) elements of stiffness matrix will be forces.

Simple Results

Type of displacement	Diagram	Flexi-bility	Stiff-ness
(i) Axial displacement		$\frac{L}{AE}$	$\frac{AE}{L}$
(ii) Transverse displacement (a) With fixed far end		$\frac{L^3}{12EI}$	$\frac{12EI}{L^3}$
(b) With far end hinged		$\frac{L^3}{3EI}$	$\frac{3EI}{L^3}$

(iii) Flexural displacement			
(a) With far end fixed		$\frac{L}{4EI}$	$\frac{4EI}{L}$
(b) With far end hinged		$\frac{L}{3EI}$	$\frac{3EI}{L}$
(iv) Torsional displacement		$\frac{L}{GI_P}$	$\frac{GI_P}{L}$

Special Case:

 (a) When $EI = 0$ for beam BC then


Deflection diagram

$$\delta_1 = \frac{P_1 h^3}{3EI}$$

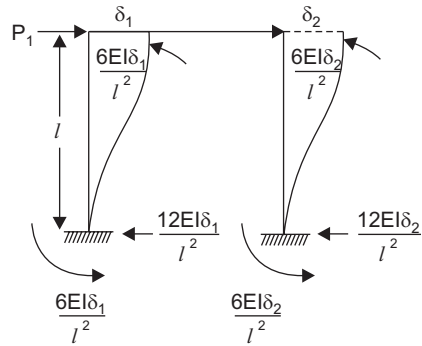
$$\delta_2 = \frac{P_2 h^3}{3EI}$$

$$\delta_1 = \delta_2 \Rightarrow P_1 = P_2 = P/2$$

So

$$\delta_1 = \delta_2 \Rightarrow \delta = \frac{Ph^3}{6EI}$$

(b) When $EI = \infty$ for beam BC then



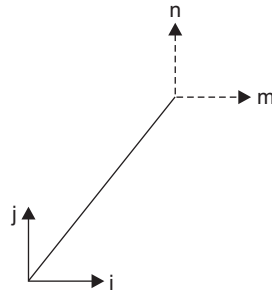
$$P = \frac{12EI\delta_1}{l^3} + \frac{12EI\delta_2}{l^3} \quad (\delta_1 = \delta_2 = \delta)$$

$$P = \frac{24EI\delta}{l^3} \quad \boxed{\delta = \frac{Pl^3}{24EI}}$$

Stiffness matrix method for Truss

There are four co-ordinate direction for 1 member

(i) Displacement in i -direction without displacement in any direction



$$K_{ij} = \frac{\Sigma AE}{L} \cos^2 \theta$$

$$K_{ji} = \frac{\Sigma AE \cos \theta \sin \theta}{L}$$

$$K_{mi} = \frac{\Sigma - AE}{L} \cos^2 \theta$$

$$K_{ni} = \frac{\Sigma - AE \cos \theta \sin \theta}{L}$$

(ii) Displacement in j -direction without displacement in any direction

$$K_{ij} = \Sigma \frac{AE}{L} \sin \theta \cos \theta$$

$$K_{mj} = \Sigma - \frac{AE}{L} \sin \theta \cos \theta$$

$$K_{jj} = \Sigma \frac{AE}{L} \sin^2 \theta$$

$$K_{nj} = \Sigma - \frac{AE}{L} \sin^2 \theta$$

Civil Engineering

3

Steel Structures

CONTENTS

1. Connections by Riveting
2. Limit State Method of Design
3. Bolts
4. Simple Welded Connections
5. Tension Member
6. Compression–Members
7. Column Bases and Caps
8. Beams(WSM)
9. Beams (LSM)
10. Plate-Girders
11. Industrial Building
12. Plastic-Analysis

Connections by Riveting

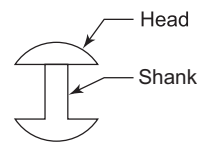
1

Connections are made by either Riveting, Bolting or welding.

Riveting:-

Note:- Strength of Cold rivet > Hot rivet

Quality control of Hot rivet > Cold rivet.



Strength calculation of rivet is done on the basis of gross diameter.

Nominal diameter (d):- It is shank diameter in cold condition. Various diameters available are 8, 10, 12, 16, 18, 20, 22, 25, 30 mm.

Gross diameter (d'):- Diameter of the hole, assuming that rivet fills the hole completely.

For $d \leq 25 \text{ mm}$ $d' = d + 1.5 \text{ mm}$

For $d > 25 \text{ mm}$ $d' = d + 2 \text{ mm}$

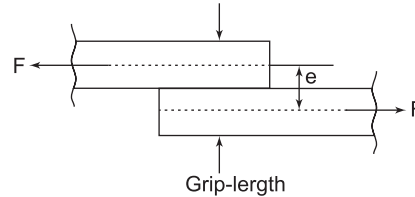
As per **Unwin's** formula $d_{mm} = 6.05 \sqrt{t_{mm}}$

where t_{mm} = minimum thickness of the plates being connected.

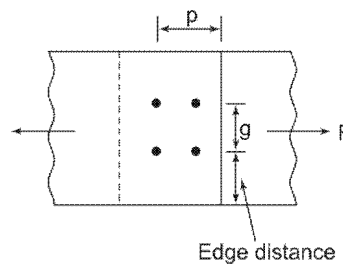
Assumptions in riveted Connection:-

- (i) Friction between plates is neglected
- (ii) Shear Stress is uniform over cross-section of rivet.
- (iii) Distribution of direct stress on the portion of plates between rivet hole is uniform.
- (iv) Rivets in group subjected to direct load share the load equally (Infact Stress is equally Shared)
- (v) Bending Stress in rivet is neglected
- (vi) Rivets fill the hole completely

Note:- When the grip of the rivet exceeds 6 times the diameter of hole, No. of rivets required by nominal calculation should be increased by not less than 1% for each additional 1.6mm of grip. Grip however should not exceed 8 times the diameter of holes.



Pitch:- It's the distance between two consecutive rivets in the direction of force.



Gauge:- It's the distance between two consecutive rivets perpendicular to the direction of force.

Note:- Diagonal pitch is the distance between centres of two adjacent rivets in the diagonal direction.

Net area:- It's the area at the root of the thread.

Edge distance:- It's the distance between the edge of a member or Cover plate and the centre of the nearest rivet hole.

Proof load:- Initial tension in HSFG bolts.

Slip factor:- It's coefficient of friction in friction type joint

Rivet Value (Rv):- minimum strength of rivet in shearing or

bearing
$$R_v = \min(P_s, P_b)$$

P_s = Strength of rivet in shear

$$P_s = n \frac{\pi}{4} d'^2 \sigma_s$$

$n = 1$, single shear case

$n = 2$, double shear case

d' = Gross diameter

σ_s = allowable shear stress

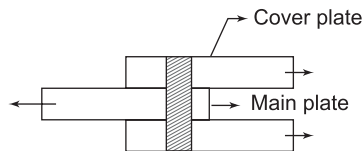
P_b = Strength of rivet in bearing

$$P_b = d' t \sigma_{br}$$

d' = Gross diameter

σ_{br} = allowable bearing resistance

t = min thickness of (sum of two cover plates, main plate)

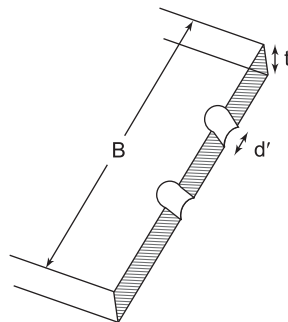


Strength of Plate:- Minimum strength of plate in shearing and tearing shearing failure can be prevented by sufficient edge distance

Tearing strength $P_t = (B - nd') t \sigma_{at}$

n = no. of holes

σ_{at} = allowable tensile strength of plate.



Strength of plate in tearing without any deduction (P)

$$P = B \cdot t \cdot \sigma_{at}$$

Efficiency of joint

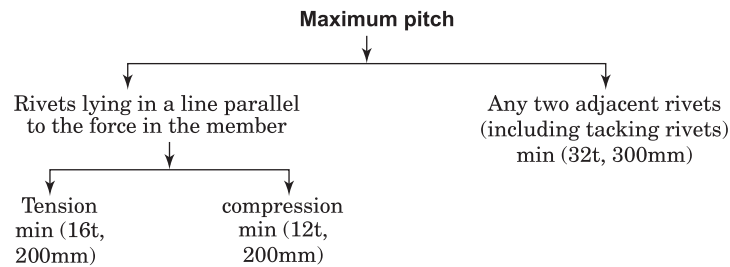
$$\eta = \frac{\text{minimum } (R_v, \text{Strength of plate})}{P}$$

Number of rivets = Force/ R_v

Maximum permissible stress in rivets and bolts (IS 800:1984)

Type of fastener	Axial tension σ_{at} (MPa)	Shear, τ_{vf} (MPa)	Bearing, σ_{pf} (MPa)
(i) Power driven			
(a) Shoprivets	100	100	300
(b) Fieldrivets	90	90	270
(ii) Hand driven rivets	80	80	250
(iii) Close tolerance and turned bolts	120	100	300
(iv) Bolts in clearance holes	120	80	250

Minimum pitch:- 2.5 times the nominal diameter i.e 2.5d



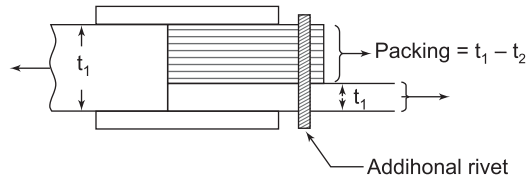
t = thickness of thinner outside plate

Permissible Stresses

Cases	Permissible stress
Axial tension and compression	$0.60 f_y$
In bending	$0.66 f_y$
In bearing (ex-at support)	$0.75 f_y$
In shear	max. permissible avg. = $0.40 f_y$ max. permissible = $0.45 f_y$

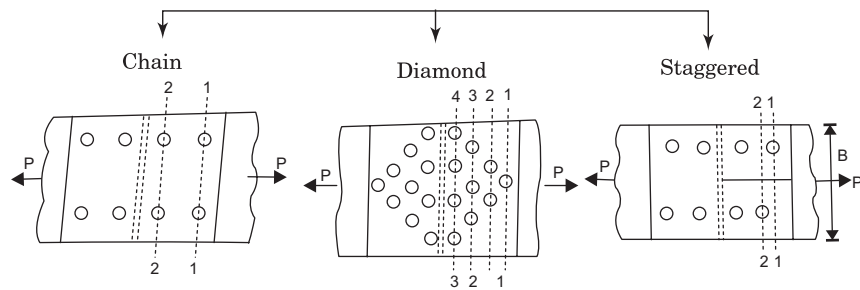
Note:- When wind and earthquake loads are considered, permissible stresses in steel structures are increased by 33.33% and in rivets and welds its increased by 25%.

Packing:- If the difference between two plates to be joined (i.e $t_1 - t_2 > 6\text{mm}$ then additional rivet (over and above that required from normal calculations) shall be provided on **Packing extension**.



So, No. of additional rivets = 2.5% of actual no. of rivets obtained from normal calculation per 2mm thickness of packing.

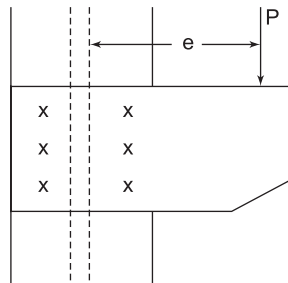
Pattern of riveted Joints:-



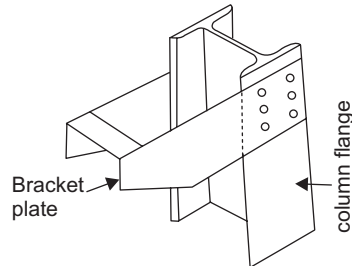
Eccentric Connection

$$F_{Di} = \frac{PA_i}{\Sigma A_i} \quad F_{Ti} = \frac{(Pe)r_i A_i}{\Sigma A_i r_i^2}$$

$$F_{ri} = \sqrt{(F_{Di})^2 + (F_{Ti})^2 + 2F_D F_T \cos \theta} \leq R_v$$



Direct load + Twisting



F_{Di} = Direct force in i th rivet

F_{Ti} = Force in i th rivet due to torsion

r_i = distance of i th rivet from C.G

$$A_i = \frac{\pi}{4}(d_i)^2$$

Note:-

F_{Di} = Acts in direction of applied load

F_{Ti} = Acts perpendicular to the line joining C.G of rivet group and the rivet under consideration.

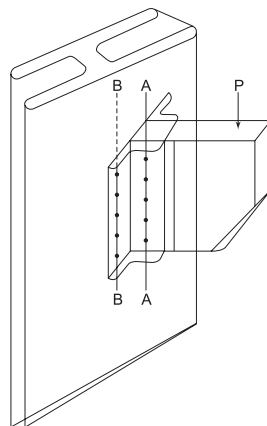
F_{ri} = Resultant force in i th rivet.

Most critical rivet has maximum value of r and minimum value of θ

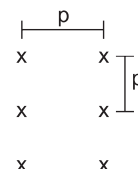
Rivets on Section A = Direct loading + Torsion

Rivets on section B = Direct loading + Bending

Minimum No. of rivets in Direct load + Twisting case



$$n = \sqrt{\frac{6M}{mpR_v}}$$



$$M = Pe$$

m = no. of lines of rivets = 2

n = no. of rivets in a line = 3

R_v = Rivet Value

p = Pitch (Same in both direction)

Note:- For rivets subjected to combined shear and Torsion

$$\frac{f_s \text{ calculated}}{\sigma_s} + \frac{f_t \text{ calculated}}{\sigma_t} \nless 1.4$$

This is as per maximum distortion energy theory

f_s = Calculated shear stress

f_t = Calculated tensile force

σ_s & σ_t = Permissible shear and tensile force.

Limit State Method of Design

2

Limitations of working stress method(WSM)

- (i) The main assumption of linear elastic behaviour (stresses can be kept within permissible limits) is not realistic.
- (ii) WSM fails to discriminate between various types of loads that act simultaneously but have different degree of uncertainty.
- (iii) In WSM there is no scientific basis for assumption of factor of safety. while in limit state method FOS is calculated from reliability analysis.

Note:-The partial safety factor takes into account, possible overloads and possible understrength.

Partial safety factor for load (γ_f)

$$\text{Factored load} = \text{Characteristic load} \times \gamma_f$$

Combination	DL	Limit State of Strength LL		WL/EL	AL	DL	Limit State of Serviceability		WL/EL
		Lead-ing	Accompanying (CL, SL etc.)				Lead-ing	Accompanying (CL etc.)	
DL+LL+ CL	1.5	1.5	1.05	—	—	1.0	1.0	1.0	—
DL+LL+ CL+	1.2	1.2	1.05	0.6	—	—	—	—	—
WL/EL	1.2	1.2	0.53	1.2	—	1.0	0.8	0.8	0.8
WL+WL/ EL	1.5	—	—	1.5	—	1.0	—	—	1.0
(0.9)*	—	—	—	—	—	—	—	—	—
DL+ER	1.2	1.2	—	—	—	—	—	—	—
(0.9)	—	—	—	—	—	—	—	—	—
DL+LL+ AL	1.0	0.35	0.35	—	1.0	—	—	—	—

*This value is to be considered when stability against overturning or stress reversal is critical

Abbreviations: DL = Dead Load, LL = Imposed Load (Live Loads), WL = Wind Load,

SL = Snow Load, CL = Crane Load (Vertical/horizontal), AL = Accidental Load,

ER = Erection Load, EL = Earthquake Load.

γ_f account for

- (i) Possibility of load exceeding characteristic load
- (ii) Possibility of inaccurate load assessment
- (iii) Uncertainties in the assessment of effect of load
- (iv) Uncertainties in the assessment of limit state being considered.

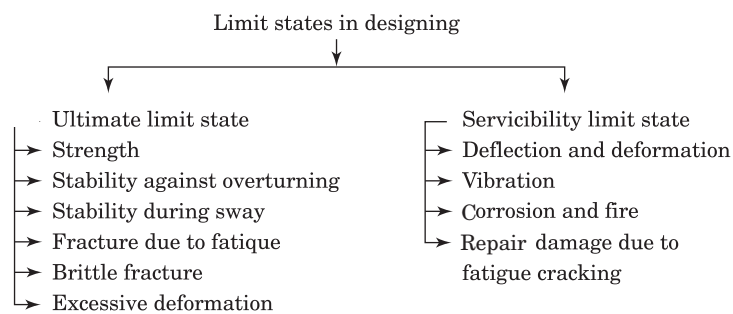
Partial safety factor for material strength (γ_m)

$$\text{Factored resistance} = \frac{\text{Characteristic Strength}}{\gamma_m}$$

S.No.	Definition	Partial Safety Factor	
1.	Resistance, governed by yielding γ_{m0}	1.10	
2.	Resistance of member to buckling γ_{m0}	1.10	
3.	Resistance, governed by ultimate stress γ_{m1}	1.25	
4.	Resistance of connection :	Shop Fabrications	Field Fabrication
	(i) Bolts–Friction type, γ_{mf}	1.25	1.25
	(ii) Bolts–Bearing Type, γ_{mb}	1.25	1.25
	(iii) Rivets, γ_{mr}	1.25	1.25
	(iv) Welds, γ_{mw}	1.25	1.50

γ_f accounts for

- (i) Possibilities of strength falling below characteristic strength.
- (ii) Reduction in member size.
- (iii) Reduction in strength due to fabrication and tolerance.
- (iv) uncertainties in theoretical assumptions.

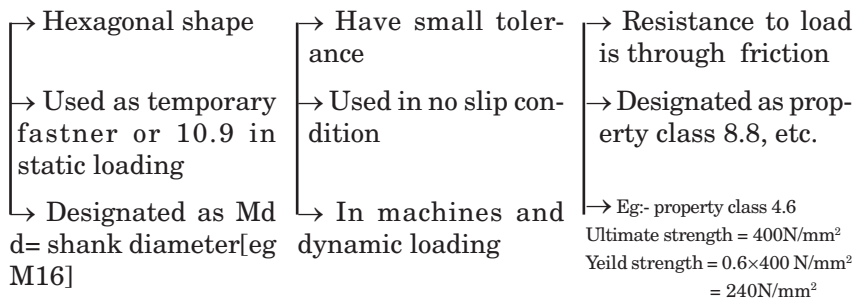
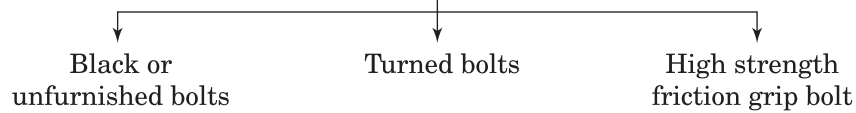


Bolts

3

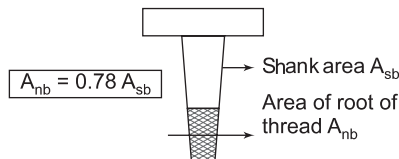
Bolts are used in place of rivets for structures not subjected to vibrations

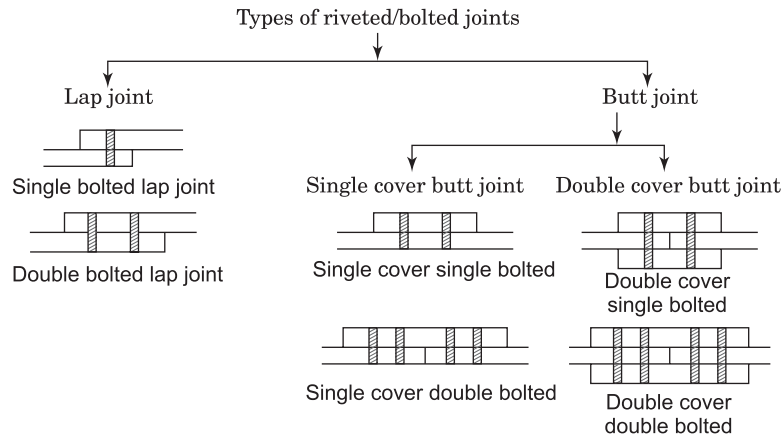
Types of bolts



Diameter of bolt holes:-

Nominal diameter (mm)	12	14	16	20	24	30	36
Diameter of hole	13	15	18	22	26	33	39





Specifications in bolting

Minimum pitch = $2.5d$

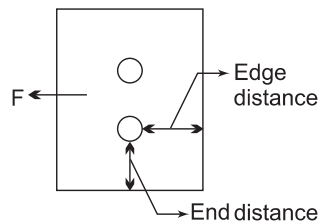
Maximum pitch

- (a) Tension = min (16t, 200mm)
- (b) Compression = min (12t, 200mm)
- (c) Tacking bolts
 - (i) Exposed to weather = min (16t, 200mm)
 - (ii) Not exposed to weather = min (32t, 300mm)

Note:- For 2 members placed back to back, tacking rivet should not exceed 1000mm in tension member, 600mm in compression member.

Minimum edge and end distance

- (a) Sheared or hand plane cut edges (Rough) = $1.7 \times \text{hole dia}$
- (b) machine cut = $1.5 \times \text{hole dia}$

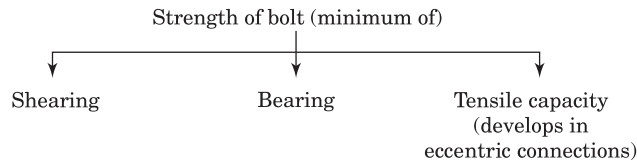


Maximum edge distance = $12t \xi$

where,

$$\xi = \sqrt{\frac{250}{f_y}} \quad t = \text{thickness of thinner plate}$$

Note:- when members are exposed to corrosive environment then maximum edge distance $\geq 40\text{mm} + 4t$ ($t \rightarrow$ thickness of thinner plate, mm)



(i) Nominal shear strength of bolt (V_{nsb})

$$V_{nsb} = \frac{f_{ub}}{\sqrt{3}} (n_n A_{nb} + n_s A_{sb}) \beta_{li} \beta_{lg} \beta_{pg}$$

Factored shear strength

$$(V_{sb}) = \frac{V_{nsb}}{\gamma_{m1}} \quad V_{sb} \leq \frac{V_{nsb}}{1.25}$$

where,

f_{ub} = ultimate tensile strength of bolt

n_n = no. of shear planes **with threads** intercepting the shear plane

n_s = no. of shear planes **without threads** intercepting the shear plane

A_{sb} = nominal shank area

A_{nb} = net tensile stress area

(a) $\beta_{lj} \rightarrow$ Reduction factor for long joints, applied when length of joint $> 15d$

$$\beta_{lj} = 1.075 - \frac{l_j}{200d}$$

$0.75 \leq \beta_{lj} \leq 1.0$ l_j = length of joint

(b) $\beta_{lg} \rightarrow$ Reduction factor for large grip lengths, applied when grip length between $5d$ and $8d$

$$\beta_{lg} = \frac{8d}{3d + l_g}$$

Note: $\beta_{lg} \neq \beta_{li}$

- (c) $\beta_{pkg} \rightarrow$ Reduction factor for packing plates, applied when thickness of packing plates $> 6\text{mm}$

$$\beta_{pkg} = 1 - 0.0125t_{pkg}$$

t_{pkg} = thickness of thicker plate in mm

(ii) Nominal Bearing strength of bolt (V_{npb})

$$V_{npb} = 2.5K_b d t f_u$$

$$V_{pb} \leq \frac{V_{npb}}{\gamma_{mb}}$$

d = nominal diameter of the bolt

t = aggregate thickness of connected plate experiencing bearing stress in same direction

f_u = ultimate tensile stress of plate in Mpa

$$K_b = \text{minimum} \left(\frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1 \right)$$

e = end distance p = pitch

d_0 = hole diameter

f_{ub} = ultimate tensile stress of bolt

f_u = ultimate tensile stress of plate

(iii) Nominal Tensile Strength of bolt (T_{nb})

$$T_{nb} = 0.9f_{ub}A_{nb} < f_{yb}A_{sb} \frac{\gamma_{mb}}{\gamma_{mo}} \quad T_{db} \leq \frac{T_{nb}}{\gamma_{mb}}$$

f_{ub} = ultimate tensile stress of bolt

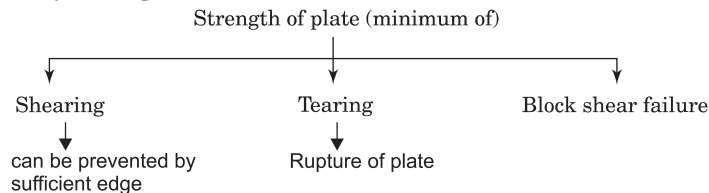
f_{yb} = yield stress of bolt

A_{nb} = Net tensile area of bolt

A_{sb} = Shank area of bolt

γ_{mb} = Partial safety factor for material of bolt = 1.25

γ_{mo} = Partial safety factor for material resistance governed by yielding = 1.10



(i) **Tensile strength of plate:-**

$$T_{nd} = 0.9 A_n \frac{f_u}{\gamma_{m1}}$$

f_u = ultimate strength of material

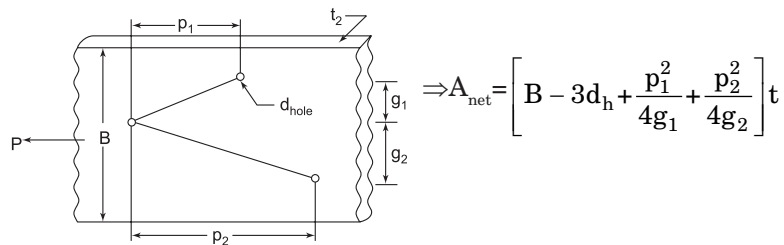
A_n = net effective area in mm^2

γ_{m1} = partial safety factor = 1.25

Net effective area $A_n = (B - nd_h)t$ (For chain bolting)

$$A_n = \left[B - nd_h + \sum_{i=1}^m \frac{p_{si}^2}{4g_i} \right] t$$

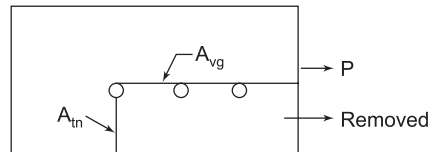
(For Staggered bolting)



(ii) **Block Shear failure:-** It's minimum of strength due to

(a) Shear yield + Tension rupture

$$\frac{\frac{f_y}{\sqrt{3}} \times A_{vg}}{1.1} + \frac{0.9 f_u A_{tn}}{1.25}$$

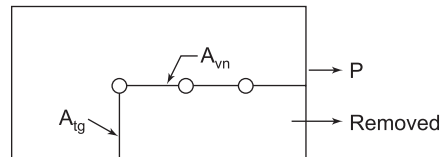


A_{vg} = Gross section area in shearing

A_{tn} = Net section area in tension

(b) Shear Rupture + Tension yielding

$$\frac{0.9 \frac{f_u}{\sqrt{3}} \times A_{vn}}{1.25} + \frac{f_y A_{tg}}{1.1}$$



A_{vn} = Net section area in shearing

A_{tg} = Gross section area in tension

Note:- Rupture always occurs on net area while yielding is assumed to occur on gross area.

1.1 and 1.25 are FOS with respect yielding and rupture respectively.

Strength of plate in tearing with out any deductions (P)

$$P = \min \left(\frac{f_y}{1.1} \times A_g, \frac{0.9f_u A_{tn}}{1.25} \right)$$

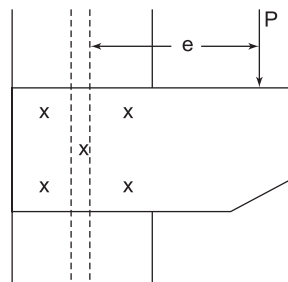
So $P = \frac{f_y A_g}{1.1}$ As gross-section yielding will always be Critical than

net section rupture

Efficiency of joint

$$\eta = \frac{\text{minimum of (Strength of bolt, Strength of plate)}}{\text{Strength of plate in tearing without deductions}}$$

Eccentric connection using limit state method:-



$$F_{D_i} = \frac{\gamma_f P}{n} \quad F_{T_i} = \frac{(\gamma_f P_e) r_i}{\sum A_i r_i^2} A_i$$

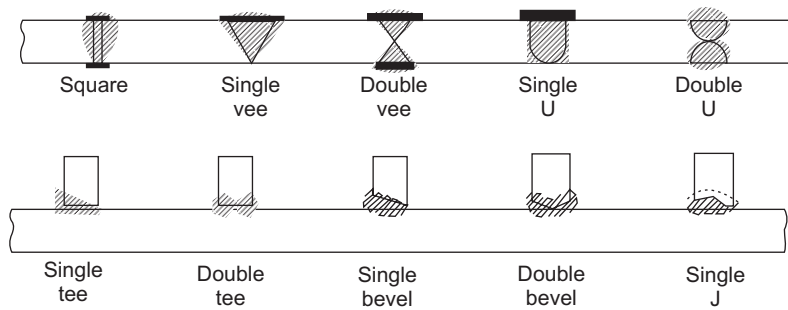
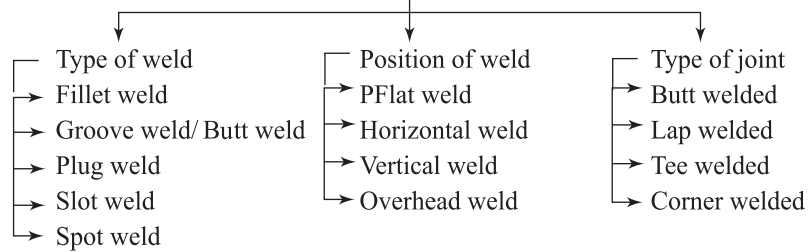
$$F_{t_1} = \sqrt{F_D^2 + F_T^2 + 2F_D F_T \cos \theta}$$

Simple Welded Connections

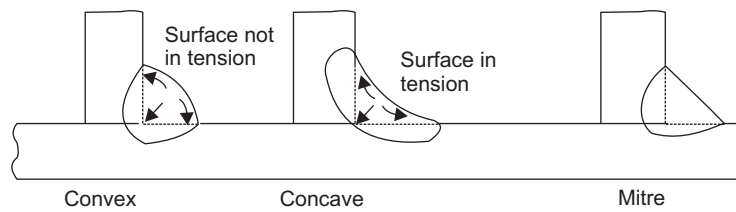
4

Welding is the best method for achieving a rigid connection

Types of welded joints

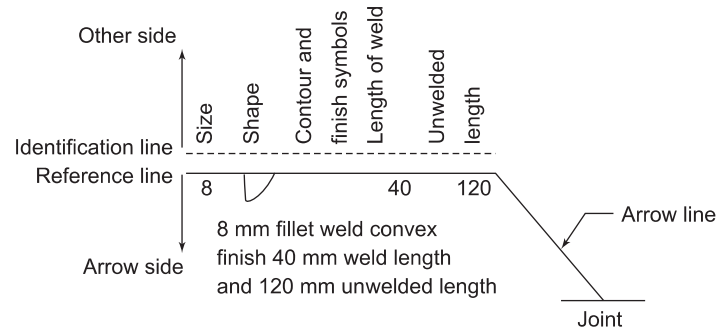


Types of groove welds



Types of fillet welds

Method of Representation



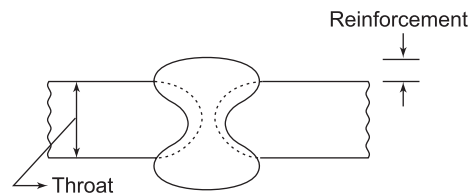
Weld defects:- Incomplete fusion, Porosity, Slag inclusions, Cracks and under cutting.

Inspection of welds by:-

- (i) Magnetic particle method
- (ii) Dye penetration method
- (iii) UltraSonic method
- (iv) Radiography

Specifications for Butt weld

(i) Reinforcement:- It is the extra weld metal which makes the throat dimension atleast 10% greater than the thickness of the welded material



Reinforcement is limited to 0.75mm to 3mm, due to concentration of stress.

(ii) Effective throat thickness:-

- (a) **Complete penetration:-** It's thickness of thinner member joined.
- (b) **Incomplete penetration:-** It's $\frac{5}{8}$ times the minimum thickness of plates being joined.

Design Strength of butt weld (V_{dw})

$$V_{dw} = \frac{f_{yw1} L_w t_e}{\gamma_{mw}}$$

f_{yw1} = Smaller of Shear Stress of weld $\left(\frac{f_{yw}}{\sqrt{3}} \right)$ and the parent metal $\left(\frac{f_y}{\sqrt{3}} \right)$

γ_{mw} = Partial safety factor

Shop welding = 1.25

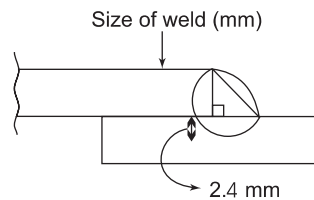
Site welding = 1.50

f_{yw} = yield stress of weld

t_e = effective throat thickness in mm

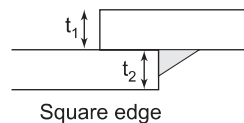
Specifications for fillet weld

(i) Size:- It's the minimum weld leg size in the largest right angled triangle that can be inscribed in the weld.

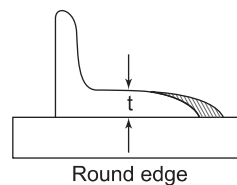


Note:- In deep penetration weld where depth of penetration beyond the root run is minimum 2.4mm, the size of weld is taken as 2.4mm+min leg size.

(a) Maximum size of weld:- depends on thinner member thickness



$$S_{max} = \min(t_1, t_2) - 1.5\text{mm}$$



$$S_{max} = \frac{3}{4} (\text{thickness of round edge})$$

(b) Minimum size of weld:- depends on thickness of thicker member

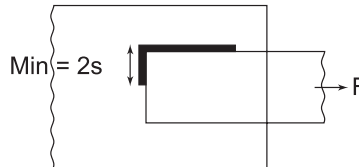
Thickness of thicker member	Minimum size
0 – 10	3
10 – 20	5
20 – 32	6
32 – 50	8 first run 10 second run

(ii) Effective throat thickness = $K \times \text{size of weld}$

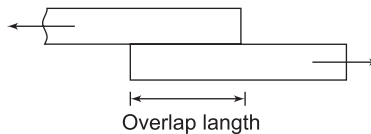
Angle b/w fusion faces	60°–90°	91°–100°	101°–106°	107°–113°	114°–120°
K	0.70	0.65	0.60	0.55	0.50

Note:- Fillet weld is not recommended if the angle of fusion faces is less than 60° or more than 120°.

(iii) Minimum end return = $2 \times \text{size of weld}$



(iv) Overlap length → max (4 (thickness of thinner plate), 40mm)



Design strength of fillet weld: (P_{dw})

$$P_{dw} = L_w (KS) \frac{f_u}{\sqrt{3} \gamma_{m\omega}}$$

L_w = effective length of the weld

3.22 CIVIL ENGINEERING

If length of joint in the direction of load transfer is $> 150t$ then length is reduced by a factor β

$$\beta = 1.2 - \frac{0.2 l_j}{150 t_t} \leq 0.1$$

t_t = KS = throat size of weld

l_j = length of joint

Combination of stresses

- (a) Fillet welds subjected to normal and shear stresses

$$f_e = \sqrt{f_a^2 + 3q^2} \leq \frac{fu}{3\gamma_{mo}}$$

f_a = normal stress, compression or tension

q = shear stress

- (b) Combined bearing, bending and shear

$$f_e = \sqrt{f_b^2 + f_{br}^2 + f_b f_{br} + 3q^2}$$

f_b = calculated stress due to bending

f_{br} = calculated stress due to bearing

q = shear stress

Tension Member

5

- (1) Net area is effective in tension member
- (2) There is no stability problem (as in compression members) but slenderness ratio is limited to safeguard against buckling during transportation & erection.
- (3) Permissible stress is fixed (0.6 fy) and design is straight forward

Maximum slenderness ratio

Types of member	Max slenderness ratio
1. A tension member in which reversal of direct stress due to loads other than wind or earthquake forces	180
2. A member normally acting as a tie in a roof truss or bracing system. But subjected to possible reversal of stress resulting from the action of wind or earthquake forces	350
3. A tension member permanently in tension except pretensioned member	400

Net sectional area (A_{net}):-

$$A_{\text{net required}} = \frac{f}{\sigma_{\text{at}}}$$

σ_{at} = Permissible axial tensile stress

$A_{\text{net Provided}} \geq A_{\text{net required}}$

(a) Net area for plate section

$$A_{\text{net}} = \left[B - nd' + \sum \frac{p^2}{4g} \right] t$$

3.24 CIVIL ENGINEERING

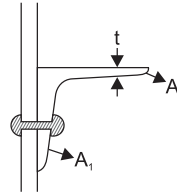
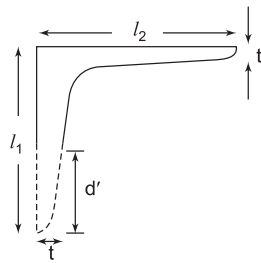
(b) Net area of angle section

(i) For single angle section $A_{\text{net}} = A_1 + KA_2$

where $K = \frac{3A_1}{3A_1 + A_2}$

A_1 = Net area of connected leg

$$A_1 = \left(l_1 - \frac{t}{2} - d' \right) t$$



A_2 = Net area of outstanding leg

$$A_2 = \left(l_2 - \frac{t}{2} \right) t$$

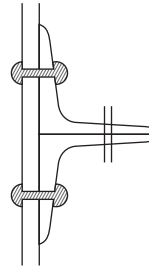
(ii) For double angle section

- on same side of gusset plate and tack-riveted

$$A_{\text{net}} = A_1 + KA_2$$

where

$$K = \frac{5A_1}{5A_1 + A_2}$$



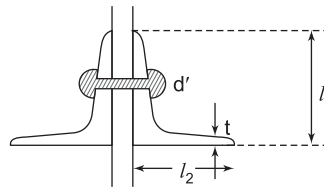
...(1)

- on same side of gusset plate and not tack riveted = Twice the net area obtained from case (i) above

- on opposite side of gusset plate and tack riveted

$$A_{\text{net}} = A_{\text{gross}} - \text{Area of hole}$$

$$= 2(l_1 + l_2 - t)t - 2d't$$



Permissible stress in design

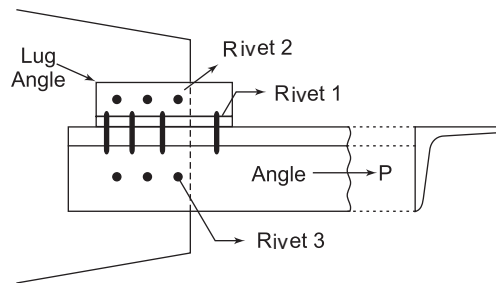
The direct stress in axial tension on the effective net area should not exceed σ_{at}

where $\sigma_{at} = 0.6 f_y$, f_y = minimum yield stress of steel in MPa

Allowable stress σ_{at} in Axial tension for steel

Form	Thickness/diameter	(mpa)
Plates, angles, tees and I-beams, channels and flats	upto and including 20mm	150
	20mm-40mm	144
	over 40mm	138
Bars(round, Square and hexagonal)	upto and including 20mm	150
	over 20mm	144

Lug Angle:- The lug angle is a short length of an angle between section used at a joint to connect the outstanding leg of a member, thereby reducing the length of the joint.

(a) When angle member is main member:-

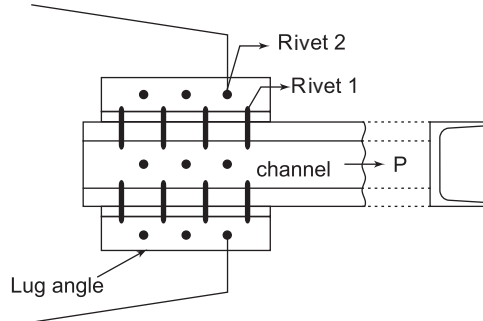
Rivet 1 → designed for 1.4 times force of outstanding leg.

Rivet 2 and lug angle → designed for 1.2 times force of outstanding leg.

(b) When Channel member is main member

Rivet 1 → designed for 1.2 times force of outstanding leg.

Rivet 2 and lug angle → designed for 1.2 times force of outstanding leg.



Note:- Rivet 1 in both the cases should start in advance of all other rivets.

Strength of tension member:- It's minimum of the following

(i) Net section rupture

(a) For plates

$$T_{dn} = \frac{0.9A_n f_u}{\gamma_{m1}}$$

T_{dn} = design rupture strength

A_n = Net effective area

f_u = Ultimate strength of material

γ_m = Partial safety factor in rupture = 1.25

(b) For Angles

$$T_{dn} = \frac{0.9A_{nc} f_u}{\gamma_{m1}} + \beta \frac{A_{g0} f_y}{\gamma_{m0}}$$

$$\text{where, } \beta = 1.4 - 0.076 \left(\frac{w}{t} \right) \left(\frac{b_s}{L_u} \right) \left(\frac{f_y}{f_u} \right)$$

$$\leq \left(\frac{f_u}{\gamma_{m1}} \right) \times \left(\frac{\gamma_{m0}}{f_y} \right)$$

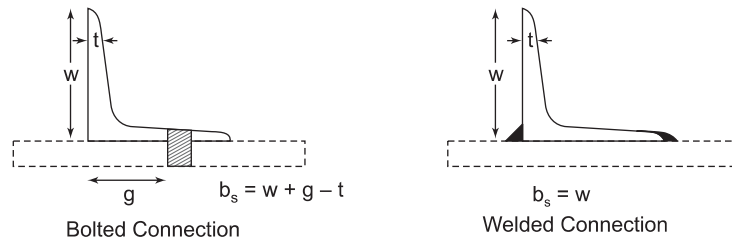
$$\geq 0.7$$

Note:- connected leg undergoes rupture (Net area is used) while outstanding leg undergoes yielding (gross area is used)

A_{nc} = Net area of connected leg

A_{go} = Gross area of outstanding leg

T = thickness of angle leg



For preliminary sizing of a tension member, the tearing strength of net section may be expressed as

$$T_{dn} = \alpha \frac{A_n f_u}{\gamma_{m1}}$$

$\alpha = 0.6$ for number of bolts ≤ 1 or 2

$= 0.7$ for 3 bolts

$= 0.8$ for number of bolts ≥ 4

or equivalent weld length

(ii) Gross – Section yielding

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}}$$

$\gamma_{m0} = 1.1$

A_g = Gross-sectional area

(iii) Block Shear failure

(a) For plates

(1) Shear yielding + Tension rupture.

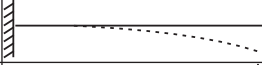
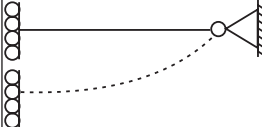
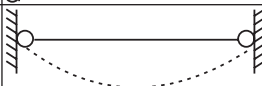
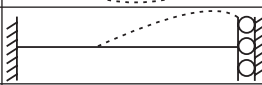
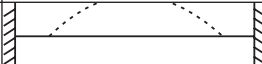
(2) Tension yielding + Shear rupture.

(b) **For Angles:-** Appropriate areas in shear and tension should be considered for failure. Also in case of welded angle tension members, it's the gross area only that is involved (No concept of net area).

Compression–Members

6

Effective length:- It's the distance between point of contra flexure's.

Boundary Conditions				Schematic representation	Effective Length (KL)
At One End		At the Other End			
Translation	Rotation	Translation	Rotation		
Restrained	Restrained	Free	Free		2.0L
Free	Restrained	Restrained	Free		2.0L
Restrained	Free	Restrained	Free		1.0L
Restrained	Restrained	Free	Re- strained		1.2L
Restrained	Restrained	Restrained	Free		0.8L
Restrained	Restrained	Restrained	Re- strained		0.65L

Effective length of Prismatic Compression members

S. No.	Type	Sections	Effective length	
			In the Plane of the Gusset	Perpendicular to the Gusset
1.	Continuous	Single or double angle	0.7 L to 1.00 L	1.00 L
2.	Discontinuous	*Single-angle connected with one bolt	1.00 L	1.00 L
3.	Discontinuous	*Single-angle connected with more than one bolt or welded	0.85 L	1.00 L

4.	Discontinuous	Double-angles placed back to back on opposite sides of gusset plate	0.70 L to 0.85 L	1.00 L
5.	Discontinuous	Double-angles placed back to back on same side of gusset plate	0.70 L to 0.85 L	1.00 L

Effective length of struts

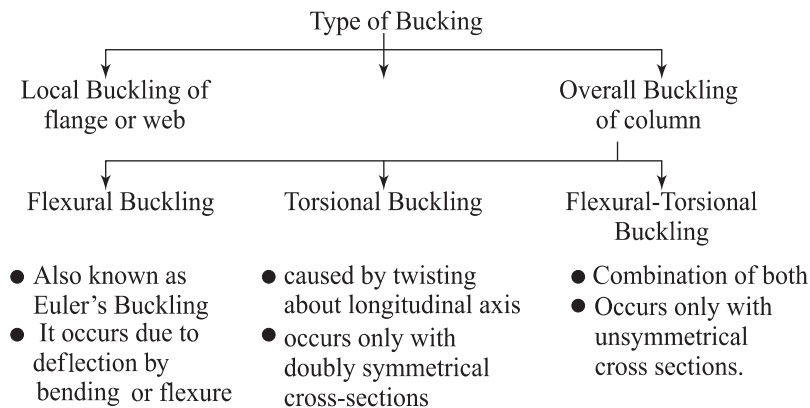
Slenderness ratio (λ): It's the ratio of effective length to appropriate radius of gyration.

$$\lambda_{\max} = \frac{\text{left}}{r_{\min}} \quad r_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

Maximum Slenderness Ratio (λ) for compression Members

S. No.	Type of Member	λ
1.	A tension member in which a reversal of direct stress occurs due to loads other than wind or seismic forces.	180
2.	A member carrying compressive loads resulting from dead loads and imposed loads.	180
3.	A member subjected to compressive forces resulting only from combination with wind/earthquake actions, provided the deformation of such members does not adversely effect the stress in any part of the structure.	250
4.	Compression flange of a beam restrained against lateral torsional buckling.	300
5.	A member normally acting as a tie in a roof truss or a bracing system is not considered effective when subjected to possible reversal of stresses resulting from the action of wind or earthquake forces.*	350

*Tension members such as bracings, pretensioned to avoid sag, need not satisfy the maximum slenderness ratio limit.



For prevention of local buckling of flange or web:-

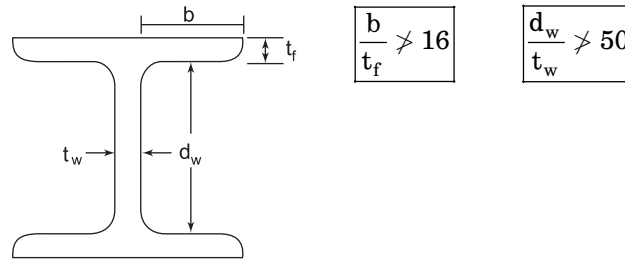
$$f_{cr} = \frac{K\pi^2 E}{12(1 - \mu^2) \left(\frac{b}{t}\right)^2}$$

$K \rightarrow$ depends on support conditions

f_{cr} = critical stress

Note:- The critical stress at which the plate buckles is inversely proportional to $(b/t)^2$

To prevent the buckling of flange plate and web plate



Effective sectional area

- (i) For slender cross-section other than circular hollow section

$$A_{eff} = \sum B_{eff} t$$

B_{eff} = Effective width of slender element $B_{eff} = \lambda_t T \varepsilon \leq b$

λ_t = limiting width to thickness ratio of semi-compact section

ε = modification factor for yield strength of material $\varepsilon = \sqrt{\frac{250}{f_y}}$

- (ii) For circular hollow sections

If $\frac{D}{t\varepsilon^2} > 88$, then its a slender section and its $A_{eff} = A_g \sqrt{\frac{88}{D/(t\varepsilon^2)}}$

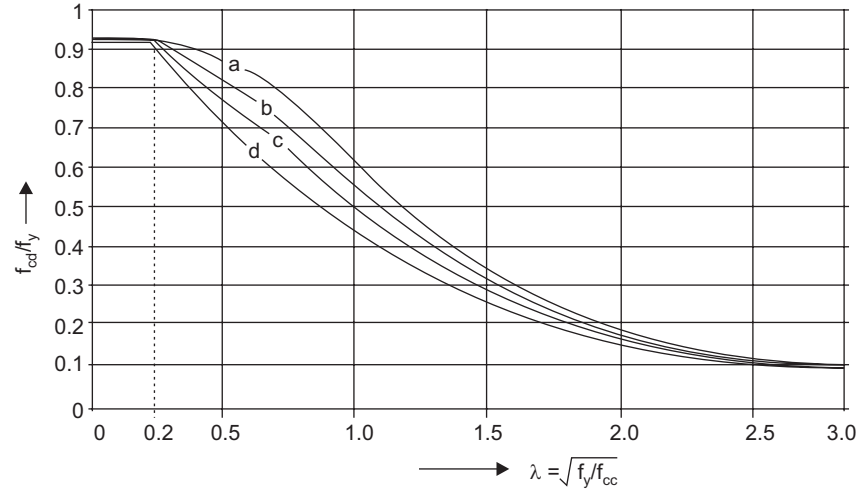
Merchant Rankine formula (for permissible compressive stress)

$$\sigma_{ac} = 0.6 \frac{f_{cc} f_y}{[(f_{cc})^n + (f_y)^n]^{1/n}}$$

$f_{cc} = \frac{\pi^2 E}{\lambda^2}$ Elastic critical stress in compression

n = factor assumed as 1.4 (ranges b/w 1-3)

Perry-Robertson approach



Column buckling Curves

$$f_{cd} = \frac{f_y / \gamma_{mo}}{\phi + (\phi^2 - \lambda^2)^{0.5}} = \chi \frac{f_y}{\gamma_{mo}} \leq \frac{f_y}{\gamma_{mo}}$$

$$\phi = 0.5(1 + \alpha(\lambda - 0.2) + \lambda^2)$$

λ = Non-dimensional effective slenderness ratio

$$\lambda = \sqrt{\frac{f_y}{f_{cc}}} \quad \text{where} \quad f_{cc} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

α = imperfection factor

Buckling curve	a	b	c	d
α	0.21	0.34	0.49	0.76

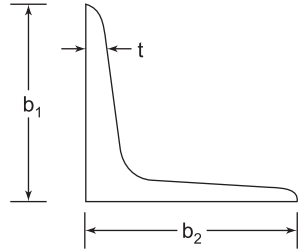
χ = Stress reduction factor to account for residual stress

γ_{mo} = partial safety factor for material strength

For single angle section loaded eccentrically

$$\lambda_e = \sqrt{K_1 + K_2 \lambda_{vv}^2 + K_3 \lambda_\phi^2}$$

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$$\lambda_{vv} = \frac{(l/r_{vv})}{\varepsilon \sqrt{\frac{\pi^2 E}{250}}}$$

$$\lambda_{\phi} = \frac{(b_1 + b_2)/2t}{\varepsilon \sqrt{\frac{\pi^2 E}{250}}}$$

Constants K_1, K_2, K_3 depend upon end conditions

No. of Bolts at Each End Connection	Gusset/ Connecting Member Fixity	k_1	k_2	k_3
≥ 2	Fixed	0.20	0.35	20
	Hinged	0.70	0.60	5
1	Fixed	0.75	0.35	20
	Hinged	1.25	0.50	60

l = Centre to centre length of supporting member

r_{vv} = radius of gyration about the minor axis

$$\varepsilon = \text{yield strength ratio} = \sqrt{\frac{250}{f_y}}$$

Design recommendations for tack riveting

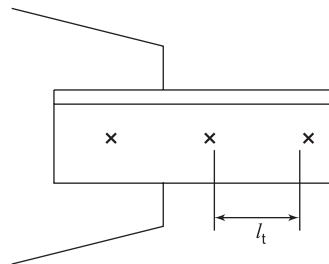
- (i) Slenderness ratio (λ) of each component between tack riveting should be such that

$$\lambda \nless 40, \lambda \nless 0.6(\lambda_{\text{whole section}})$$

$$\lambda = \frac{l_t}{r_{\min}}$$

$r_{\min} = r_{vv}$ for angle sector

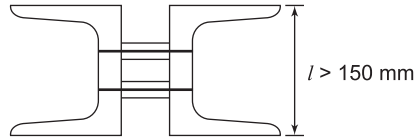
r_{xx} one for channel sector



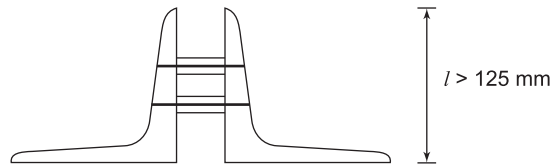
- (ii) The diameter of tack rivet should not be less than the minimum value given below

Thickness of member	Min. dia
Upto 10mm	16mm
10 – 16 mm	20mm
> 16mm	22mm

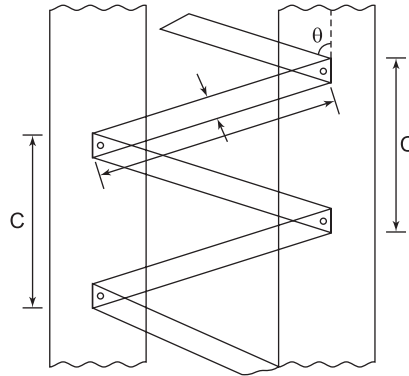
- (iii) Two rows of tack rivets are provided if
 (a) web of channel > 150mm



- (b) length of leg in angle > 125mm



Design recommendations for lacing



- (i) θ should be between $40^\circ - 70^\circ$

$$(ii) \frac{C}{r_{\min \text{ one}}} \not\geq 50$$

$$\not\geq 0.7(\lambda_{\text{whole section}})$$

- (iii) $l_{\text{eff}} = l$ (for single lacing)
 $= 0.7l$ (for double lacing/welded sections)

(iv) For lacing bar, $r_{\min} = \frac{t}{\sqrt{12}} \not\geq 14.5$

- (v) Minimum width of lacing bar

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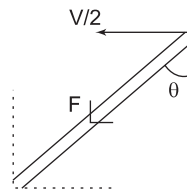
Diameter of rivet	Min width
16	50
18	55
20	60
22	65

- (vi) Thickness of lacing bar = $l/40$ in single lacing
 $= l/60$ in double lacing

Design of lacing

- (i) It is designed to resist a total transverse shear, equal to 2.5% the axial force in the column. (i.e 2.5% of $P = V$)
(ii) If No. of parallel sides is N (generally 2) then force in lacing

$$\text{bar} = \frac{V}{N \sin \theta}$$



$$\frac{V}{2} = F \sin \theta \Rightarrow F = \frac{V}{2 \sin \theta}$$

(iii) Tension check $(b - d')t\sigma_{ac} \geq \frac{V}{N \sin \theta}$

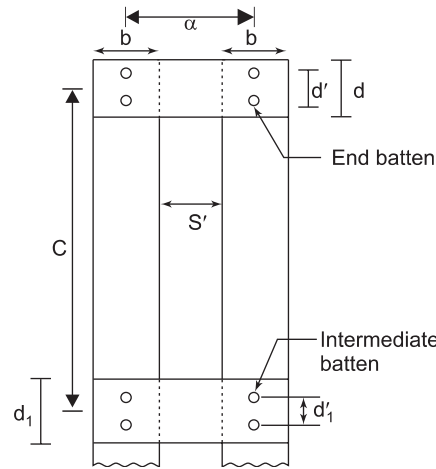
(iv) Compression check $\frac{l_{eff}}{t/\sqrt{12}} = \sigma_{ac} \Rightarrow \sigma_{ac} \times \text{Area} \leq F$

- (v) **Connection:-** No. of Bolts/Rivet

$$= \frac{V \cot \theta}{R_v \text{ in single shear}}, \quad \theta < 60^\circ$$

$$= \frac{V}{2 \sin \theta R_v} \quad \theta > 60^\circ$$

Design recommendations for batten



(i) Effective length of battened column is taken 10% more than the laced column.

(ii) Minimum No. of intermediate battens = 2

$$(iii) \frac{C}{(r_{min})_{one}} \not\geq 50$$

$$\not\geq 0.7(\lambda \text{ whole Section})$$

(iv) **Depth:-**

$$d_1 \not\geq \frac{3}{4}\alpha \quad d \not\geq \alpha$$

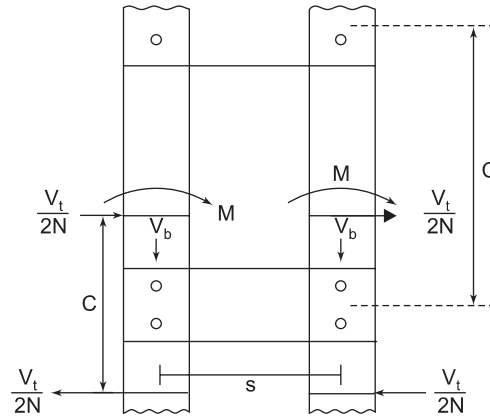
$$\not\geq 2b \quad \not\geq 2b$$

α = distance between C.G of component members.

(v) Thickness t of battens should not be less than $1/50$ of the distance between the innermost connecting lines of rivets/bolts or welds perpendicular to main member.

Note:- Battens are designed only for axial loading while laced column's are designed for both axial loading and moment.

Designing of battens:-



- (i) Designed to carry Bending moment and shear force arising from transverse shear force V_t which is 2.5% of total axial force on whole compression member.

$$(ii) V_b = \frac{V_t C}{NS} \quad M = \frac{V_t}{2N}$$

V_b = longitudinal Shear force

N = No. of parallel planes dividing the transverse shear force.

$$(iii) \frac{V_b}{A_1} < \frac{f_y}{\sqrt{3} \gamma_{m_0}}$$

where, $A_1 = td$

t = thickness of batten

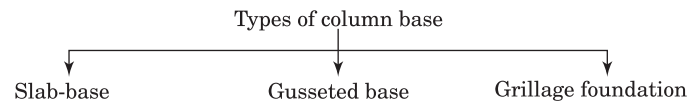
d = Overall depth of batten.

$$(iv) \sigma_{bc,cal} = \frac{6M}{td^2} < \frac{f_y}{\gamma_{m_0}}$$

Column Bases and Caps

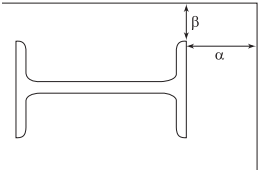
7

Sufficient fastenings are provided to retain the column firmly on the base plate and resist all moments and forces (except direct compression in the column) arising during transit, unloading and erection.



(i) When only axial load is acting

(a)



$$t = \sqrt{\frac{3\omega}{\sigma_{bs}} \left(a^2 - \frac{b^2}{4} \right)} \text{ (WSM)}$$

$$t = \sqrt{\frac{2.5\omega (a^2 - 0.3b^2)}{(f_y / \gamma_{m_0})}} \text{ (LSM)}$$

t = slab thickness in mm

ω = pressure or loading on the underside of the base

$$\omega = \frac{P}{B^2(\text{adopted}B)} \text{ in WSM, } \omega = \frac{1.5P}{\text{Area}} \text{ in LSM}$$

where, $B^2 > p/\sigma_c$ and $(\sigma_c = f_{ck}/4)$

$a = \max(\alpha, \beta)$ and $b = \min(\alpha, \beta)$

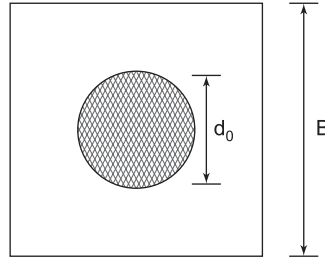
σ_{bs} = permissible bending stress in slab bases

= 185 Mpa for solid beams

= 165 Mpa for flanged beams.

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(b) Square slab base under solid round column



$$t = 10 \sqrt{\frac{90W}{16\sigma_{bs}} \left(\frac{B}{B - d_0} \right)}$$

$$B \not\prec 1.5(d_0 + 75) \text{ mm}$$

W = Total axial load (KN)

B = Length of the side of the cap or base.

d_0 = Diameter of the reduced end.

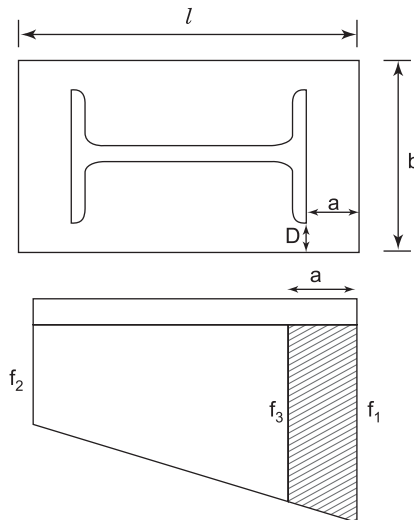
(ii) **When column is subjected to Bending moment also**

(a) When we have freedom to choose length (l)

$$e = \frac{M}{P}$$

1. Get $l > 6e$

$$2. \frac{p}{lb} \left(1 + \frac{6e}{l} \right) \geq \left(\frac{f_{ck}}{4} \right) \text{ Get } b$$



$$3. \quad t \geq \sqrt{\frac{6(M_{11} - \mu M_{22})}{\sigma_{bs}}}$$

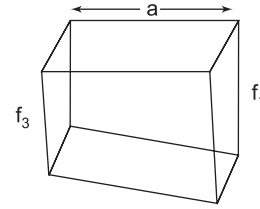
$$M_{11} = \left(\frac{f_1 + f_3}{2} \times a \times 1 \right) \left(\frac{2f_1 + f_3}{f_1 + f_3} \times \frac{a}{3} \right)$$

$$M_{22} = \frac{f_3 b^2}{2}$$

$$f_1 = \frac{p}{lb} \left(1 + \frac{6e}{l} \right)$$

$$f_2 = \frac{p}{lb} \left(1 - \frac{6e}{l} \right)$$

$$f_3 = f_2 + \left(\frac{f_1 - f_2}{l} \right) (l - a)$$



(b) When length is restricted and $e > l/6$

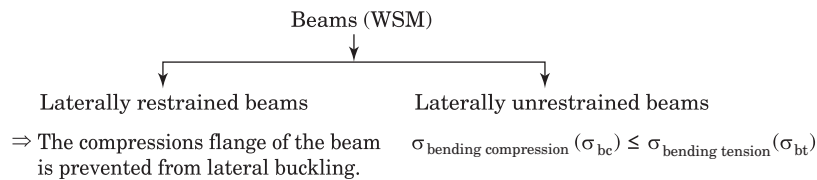
$$1. \quad l' = 3 \left(\frac{l}{2} - e \right) \quad f_1 = \frac{2p}{3B \left(\frac{l}{2} - e \right)} \quad f_3 = \frac{f_1}{l'} (l' - a)$$

$$2. \quad \text{For width } \frac{2P}{3B \left(\frac{l}{2} - e \right)} \leq \sigma_c$$

$$3. \quad t \geq \sqrt{\frac{6(M_{11} - \mu M_{22})}{\sigma_{bs}}}$$

Beams(WSM)

8



Primary design criteria for design of beams

(i) Safety against bending

Maximum permissible compressive or tensile bending stress

$$\sigma_{bc} = \sigma_{bt} = 0.66f_y$$

Permissible bending stresses (IS 226 – 1975)

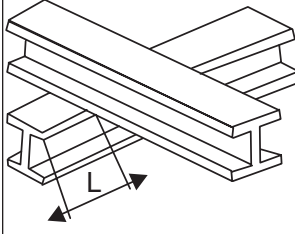
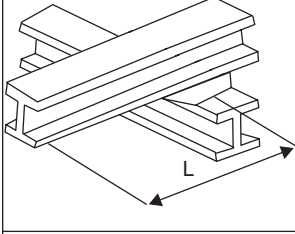
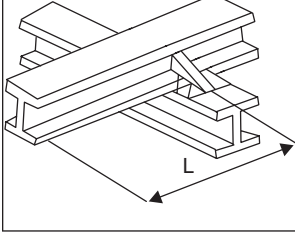
Nominal Thickness of plate	Yield Stress(f_y – Mpa)	$\sigma_{bc} = \sigma_{bt}$
Angle, Tee, I, Channel and flat sections upto and including 20mm	250	165
Over 20mm upto and including 40mm	240	158.4
Over 40mm	230	151.8

Effective length for simply supported beams

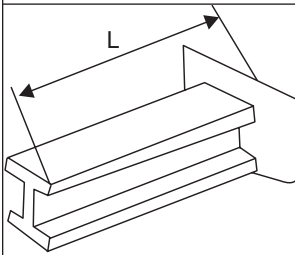
Sl. No.	Conditions of Restraints at supports		Loading condition	
	Torsional Restraint	Warping Restraint	Normal	Destabilizing
(i)	Fully restrained	Both flanges fully	0.70 L	0.85 L
(ii)	Fully restrained	Compression flange fully restrained	0.75 L	0.90 L
(iii)	Fully restrained	Both flanges partially restrained	0.80 L	0.95 L
(iv)	Fully restrained	Compression flange partially restrained	0.85 L	1.00 L

(v)	Fully restrained	Warping not restrained in both flanges	1.00 L	1.20 L
(vi)	Partially restrained by bottom flange support connection	Warping not restrained in both flanges	1.0 L + 2D	1.2 L + 2 D
(vii)	Partially restrained by bottom flange bearing support	Warping not restrained in both flanges	1.2 L + 2D	1.4 L + 2 D

Effective length, L_{LT} for cantilever beams of length L

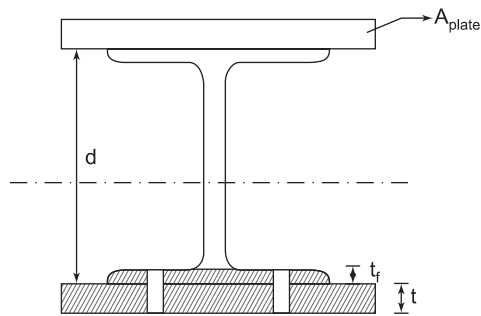
<i>Restraint Condition</i>		<i>Loading Condition</i>	
<i>At Support</i>	<i>At Top</i>	<i>Normal</i>	<i>Destabilizing</i>
	(i) Free	3.0 L	7.5 L
	(ii) Lateral restraint to top flange	2.7 L	7.5 L
	(iii) Torsional restraint	2.4 L	4.5 L
	(iv) Lateral and torsional restraint	2.1 L	3.6 L
	(i) Free	2.0 L	5.0 L
	(ii) Lateral restraint to top flange	1.8 L	5.0 L
	(iii) Torsional restraint	1.6 L	3.0 L
	(iv) Lateral and torsional restraint	1.4 L	2.4 L
	(i) Free	1.0 L	2.5 L
	(ii) Lateral restraint to top flange	0.9 L	2.5 L
	(iii) Torsional restraint	0.8 L	1.5 L
	(iv) Lateral and torsional restraint	0.7 L	1.2 L

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(d) Restrained laterally, torsionally and against rotation on plan	(i) Free	0.8 L	1.4 L
	(ii) Lateral restraint to top flange	0.7 L	1.4 L
	(iii) Torsional restraint	0.6 L	0.6 L
	(iv) Lateral and torsional restraint	0.5 L	0.5 L

$$Z_{\text{req}} = \frac{m_{\text{max}}}{\max(\sigma_{bc}, \sigma_{bt})} \text{ then provide } Z \geq Z_{\text{required}}$$

For designing of rolled Sections for bending moment



$$\text{Area due to holes} = 2l(t + t_f)$$

$$A_{p\text{required}} = \frac{Z_{\text{required}} - Z_{\text{rolled}}}{d}$$

$$\frac{My_t}{I_{\text{gross}}} \times \left\{ \frac{\text{Gross area of tension flange}}{\text{Net area of tension flange}} \right\} \leq \sigma_{bt}$$

Net area of tension slange

$$= \text{Gross area for tension} - \text{Area due to holes}$$

(ii) **Safety against Shear**

$$\text{Max. permissible shear stress } \tau_{vm} = 0.45 f_y$$

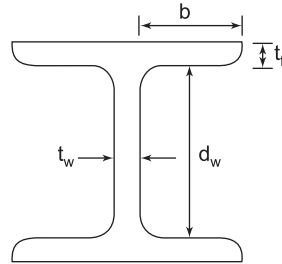
$$\text{Max. permissible average shear } \tau_{va} = 0.40 f_y$$

(iii) **Safety against deflection:-** Maximum deflection should not exceed $\frac{\text{span}}{325}$ where deflection and span are in mm.

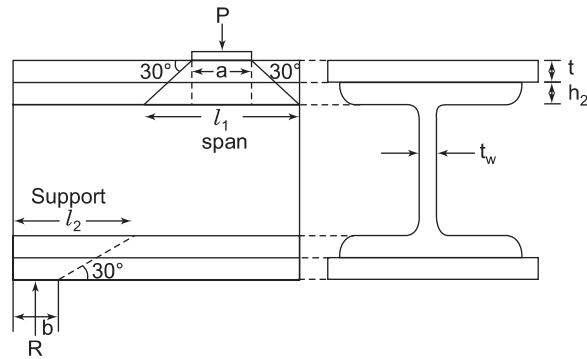
Secondary design criteria for design of beams

(i) Safety against local buckling

$$\frac{b}{t_f} \not\geq 16 \quad \frac{d_w}{t_w} \not\geq 50$$



(ii) Safety against web crippling



Area at **root of fillet**

(a) At span = $l_1 t_w$

(b) At support = $l_2 t_w$

$$l_1 = a + 2\sqrt{3}(h_2 + t)$$

$$l_2 = b + \sqrt{3}(h_2 + t)$$

For safety

$$\frac{P}{[a + 2\sqrt{3}(h_2 + t)] t_w} \leq 0.75f_y \quad \text{For span}$$

$$\frac{R}{[b + \sqrt{3}(h_2 + t)] t_w} \leq 0.75f_y \quad \text{For support}$$

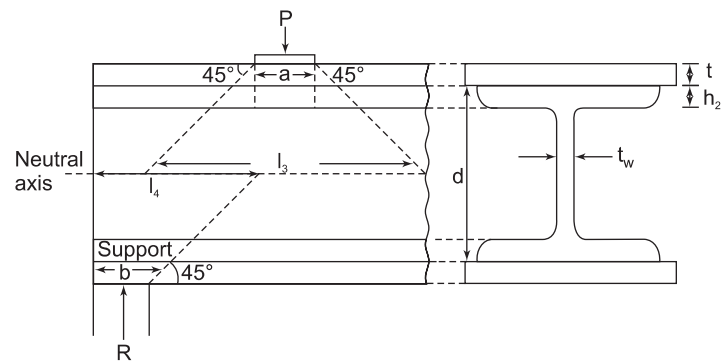
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(iii) Safety against web buckling

$$l_{\text{eff}} = \frac{d_w}{2} \quad r_{\text{min}} = \frac{t_w}{\sqrt{12}}$$

$$\lambda = \frac{\sqrt{3}d_w}{t_w}$$

where $d_w = d - 2h_2$ from λ get σ_{ac}



Area at Neutral axis

(a) At span = $l_3 t_w$

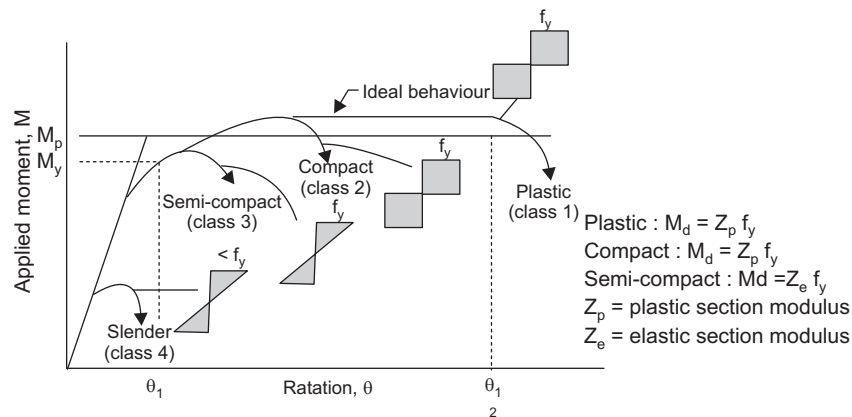
(b) At support = $l_4 t_w$

$$l_3 = a + 2\left(t + \frac{d}{2}\right)$$

$$l_4 = b + \left(t + \frac{d}{2}\right)$$

Now $\sigma_{\text{ac}} \times \text{Area} > P \text{ or } R$

Classification of cross-sections



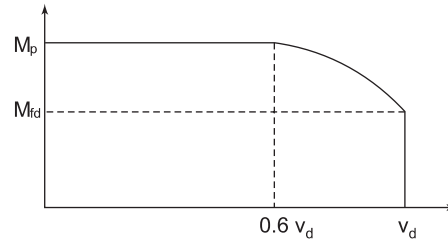
Moment-rotation behaviour of the four classes of cross sections

⇒ It is based on the least width-thickness ratio of the flanges and of the web

- (i) **Plastic Sections**, do not fail/buckle even after formation of plastic hinge. Used in indeterminate frames.
- (ii) **Compact Sections**, fail only after formation of one plastic hinge. Hence, suitable for simply supported beams.
- (iii) **Semi Compact Sections** can be used for elastic design, where section fails after reaching the first yield moment M_y , at the extreme fibres of beam.
- (iv) **Slender Sections** are those which fail even before reaching yield point. These are used for plate girders and cold form sections only.

Design of laterally supported beams

- (i) Low shear case ($V \leq 0.6V_d$) web will be fully effective and entire cross-section of the beam will be effective in resisting moment (M_d)



Interaction diagram between moment and shear

- (ii) High shear case ($V > 0.6V_d$) only flanges will resist moment. Because of high shear case, moment capacity of the beam will be reduced to M_{dv}

(i) **Low Shear Case**

$$M_d = \beta_b Z_p \frac{f_y}{\gamma_{mo}}$$

$$\leq 1.2 Z_e \frac{f_y}{\gamma_{mo}} \quad (\text{for simply supported beam})$$

$$\leq 1.5 Z_e \frac{f_y}{\gamma_{mo}} \quad (\text{for Cantilever beam})$$

Where, $\beta_b = 1$ (for plastic and compact sections)

$$= \frac{Z_e}{Z_p} \quad (\text{for Semi-compact sections})$$

$\gamma_{mo} = 1.1$ (partial safety factor)

- (a) **Effect of bolt holes:-** In tension flange it needs to be considered when

$$0.9 f_u \frac{A_{nf}}{\gamma_{m1}} \geq f_y \frac{A_{gf}}{\gamma_{m0}}$$

$\frac{A_{nf}}{A_{ng}}$ = Ratio of net area to gross area of the tension flange

$\frac{\gamma_{m1}}{\gamma_{m0}}$ = Ratio of partial safety factor against ultimate stress to yield stress

- (b) **Effect of shear lag effect:-** Shear lag effect in flanges may be disregarded when

$$b_0 \leq \frac{L_0}{20} \text{ (for outstand elements)}$$

$$b_1 \leq \frac{L_0}{10} \text{ (for internal elements)}$$

L_0 = length between point of zero moment (inflection) in the span

b_0 = width of flange outside

b_1 = width of an internal flange

(ii) High Shear Case

- (a) Plastic or Compact section

$$\begin{aligned} M_{dv} &= M_d - \beta(M_d - M_{fd}) \\ &\leq 1.2 Z_e \frac{f_y}{\gamma_{mo}} \end{aligned}$$

M_d = plastic design moment of the whole section disregarding high shear force effect but considering web buckling effect.

M_{fd} = plastic design strength of the area of the cross-section excluding the shear force, considering partial safety factor γ_{mo}

$$\beta = \left(\frac{2V}{V_d} - 1 \right)^2$$

V_d = design shear strength as governed by web yielding or web buckling

V = factored applied shear force

Z_e = elastic section modulus of whole section.

- (b) Semi – compact Section

$$M_d = Z_e \frac{f_y}{\gamma_{mo}}$$

Shear check:- Factored shear force should be less than designed shear force (V_d)

$$V_d = A_v \frac{f_y}{\sqrt{3}} \times \frac{1}{\gamma_{mo}}$$

A_v = Shear area f_y of material of web

Deflection check:- Maximum deflection should be less than $\frac{\text{span}}{300}$.

Web buckling and web crippling check

Not Needed in

(a) Low shear case $\frac{d}{t_w} > 67\xi$.

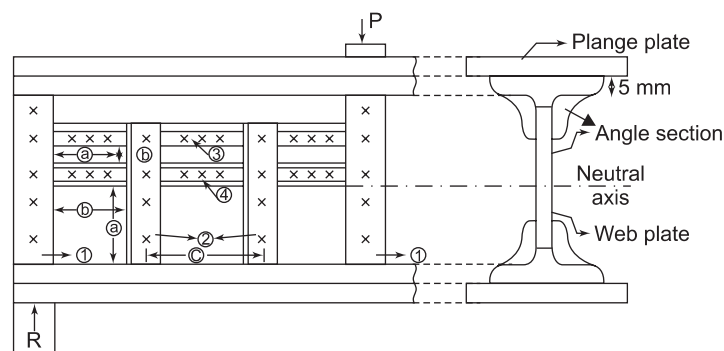
Needed in

(a) High shear case

(b) Low shear case $\frac{d}{t_w} > 67\xi$.

Plate-Girders

10



- (1) → Load bearing stiffener
- (2) → vertical stiffeners
- (3) → 1st horizontal stiffener
- (4) → 2nd horizontal stiffener

- (a) → larger unsupported length
- (b) → smaller unsupported length
- (c) → spacing between vertical stiffeners

$$a \nless 270t_w$$

$$b \nless 180t_w$$

For stiffened web avg. permissible shear depends upon d/t and c while for unstiffened web avg. permissible shear is $0.4f_y$.

Design of web:-

Average shear stress in the web $Z_{va,cal} = \frac{V}{d_w \times t_w}$ ◆ permissible average shear stress, Z_{va} .

Web-stiffeners

- (i) $\frac{d_1}{t_w} \leq \text{lesser of } \left(\frac{816}{\sqrt{Z_{va,cal}}}, \frac{1344}{\sqrt{f_y}}, 85 \right)$ No stiffener will be required
- (ii) $\frac{d_2}{t_w} \leq \text{lesser of } \left(\frac{3200}{\sqrt{f_y}}, 200 \right)$. Vertical stiffeners are provided.
- (iii) $\frac{d_2}{t_w} \leq \text{lesser of } \left(\frac{4000}{\sqrt{f_y}}, 250 \right)$

Note : Vertical stiffener and first horizontal stiffener are provided at a distance from the compression flange equal to two-fifth of the distance from the compression flange to the neutral axis.

- (iv) $\frac{d_2}{t_w} \leq \text{lesser of } \left(\frac{6400}{\sqrt{f_y}}, 400 \right)$

$d_2 = 2 \times \text{clear distance from compression flange angles or plate to the neutral axis.}$

(a) **Design of vertical stiffeners:-**

1. Spacing between vertical stiffeners should be such that average shear is greater than $\frac{V_{max}}{d.t}$
2. The maximum and minimum distance between vertical stiffeners is 1.5d and 0.33d respectively.
3. Vertical stiffeners can be provided at angles on 2 sides of web plate.
4. If it is to be provided on one side only, it is to be staggered.
5. Vertical stiffener should have size such that $I_{aa} = \frac{1.5d^3 t_w^3}{c^2}$

For design of connection's for vertical stiffener.

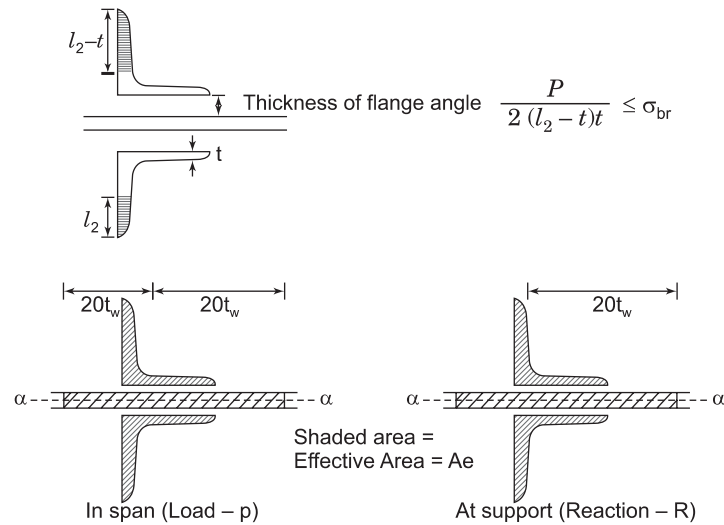
$$125 \frac{t_w^2}{h} \times p \leq R_v$$

1. Assume diameter of rivet and get Rivet value R_v
2. $h = \text{outstand of stiffener from the web, } h \nless 16t \text{ for angles, } h \nless 12t \text{ for flats.}$
3. Get pitch p .

(b) Design of horizontal stiffener:-

1. For first horizontal stiffener i.e., 3 (in diagram) $I_{\alpha\alpha} \nless 4ct_{\omega}^3$
 2. For second horizontal stiffener i.e., 4 (in diagram) $I_{\alpha\alpha} \nless dt_{\omega}^3$
- Connection of horizontal stiffener with web is similar to that of vertical stiffener with web.

(c) Design of load bearing stiffener



$$1. \quad r_{\min} = \sqrt{\frac{I_{\alpha\alpha}}{\text{Shaded Area } A_e}}$$

$$2. \quad \lambda = \frac{l_{\text{eff}}}{r_{\min}} \text{ where } l_{\text{eff}} = 0.7l$$

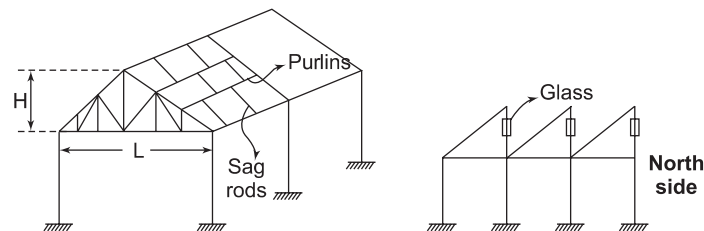
$$3. \quad \text{Get } \sigma_{ac}$$

$$4. \quad \sigma_{ac} \cdot A_e \geq P \text{ or } R \text{ as the case may be}$$

(d) Web splices are designed for shear while flange splices are designed for moment.

Industrial Building

11



We adopt windows on north side so that diffused light comes inside the building which creates no shadow and hence dark pockets are avoided

$$\text{Pitch of the truss} = \frac{H}{L}$$

$$\text{Slope of the truss} = \tan \theta = \frac{H}{L/2}$$

$$\boxed{\text{Slope} = 2 \times \text{Pitch}}$$

$$\text{For smaller Spans } \frac{H}{L} = \frac{1}{6}, \text{ for larger spans } \frac{H}{L} = \frac{1}{10}$$

Note:- For Galvanised Iron sheet pitch is 1/6 while for asbestos cement sheet, roof covering, flatter pitch of 1/12 is preferred.

$$\text{Spacing of truss (d) – for spans upto 15m } d = \frac{\text{span}}{4}$$

$$\text{for spans } > 15\text{m } d = \frac{\text{span}}{5}$$

Economy of the roof

$$\text{Cost of truss / unit area} = \frac{\text{Cost of purlin}}{\text{Plan area}} \times 2 + \frac{\text{Cost of roof covering}}{\text{Plan area}}$$

Sag rods are designed as tension member

Purlins:-

- (a) It's a biaxial bending member
- (b) Angle, Channel, I and Z Sections are used for purlins and girts to support the cladding
- (c) Maximum spacing between purlin's $\nless 1.4\text{m}$
- (d) If roof slope $< 30^\circ$ and minimum live load of 750 N/m^2 then following requirements are to be fulfilled (then uniaxial bending design)
 - (i) Width of angle leg in the plane perpendicular to the roof covering $\geq \frac{L}{45}$
 - (ii) Width of angle leg in the plane parallel to the roof covering $\geq \frac{L}{60}$
- (iii) Maximum bending moment in the purlin $M = \frac{\omega L^2}{10}$
 - ω = uniformly distributed load per unit length on purlin (including wind load)
 - L = Span of purlin
- (iv) $Z_x \text{ required} = \frac{M}{\sigma_{bc}} = \frac{\omega L^2}{10 \times 165} \text{ mm}^3$
- (e) Deflection of purlins = $\frac{\text{span}}{180}$

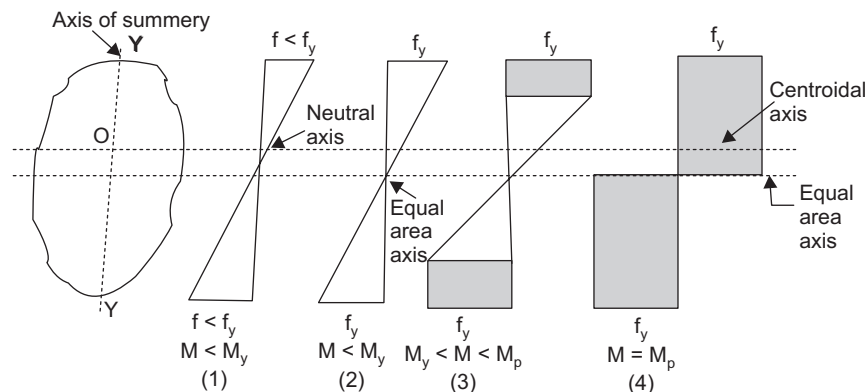
Plastic-Analysis

12

Assumptions in Plastic Analysis:-

- (i) stress-strain curve is elasto-plastic curve (strain hardening is neglected)
- (ii) Steel possess ductility so that it could be deformed into plastic state
- (iii) Strain variation is linear from Neutral axis.
- (iv) Relationship between tensile stress and tensile strain, compressive stress and compressive strain are same.
- (v) Joint should be sufficiently strong to transfer the moments.

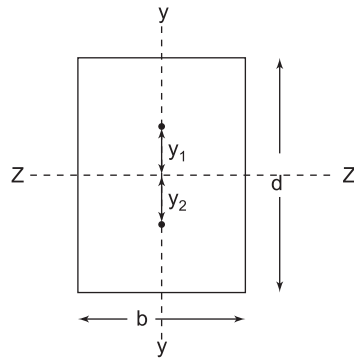
Plastic behaviour of beam



Development (in stages) of full plasticity of a beam section

Plastic moment
$$M_p = f_y \frac{A}{2} (\bar{y}_1 + \bar{y}_2) = f_y Z_p$$

 $\bar{y}_1, \bar{y}_2$ = distance of centre of gravity of the area above and below the neutral axis respectively



$$Z_{p(\text{about major } zz \text{ axis})} = \frac{d^2 b}{4}$$

$$Z_{p(\text{about minor } yy \text{ axis})} = \frac{db^2}{4}$$

Plastic Hinge:- It's a yielded zone in flexure in a structure in which infinite rotation can take place at a constant resisting moment M_p of the section.

- (i) For analysis plastic hinge is assumed at one section where stress at every point is f_y .
- (ii) Moment will be maximum, **if radius of curvature is minimum or curvature ($1/R$) is maximum.**
- (iii) In a beam, if plastic hinge is formed at various locations then they will form alternatively due to sagging and hogging moment.
- (iv) Length of plastic hinge depends on loading and shape of cross-section.
- (v) Due to formation of plastic hinge one after another redistribution of moments takes place.
- (vi) Sufficient no of plastic hinges have to be developed to render the structure to a collapse or unstable state.
- (vii) No of plastic hinges for complete collapse = $D_s + 1$
 D_s = degree of static indeterminacy.
- (viii) Plastic hinges are expected to form at
 - (a) At fixed ends
 - (b) At location of point loads
 - (c) At a section of sudden change in geometry.

- (d) At a point of zero shear in the span loaded with variable loads like udl, uvl etc.
- (ix) When two sections meet at a point, plastic hinge forms in a section having smaller M_p .

$$\text{Shape factor } (\alpha) = \frac{M_p}{M_y} = \frac{\text{Plastic moment capacity}}{\text{Yield moment capacity}} = \frac{Z_p}{Z_e}$$

Note: It represents the reserve strength of a section beyond the point of first yield.

$$\text{Load factor } (\lambda) = \frac{\text{Collapse load } W_c}{\text{Working load } W_o}$$

When all the hinges required for collapse are formed simultaneously, then

$$\text{Load factor } (\lambda) = \text{Factor of safety} \times \text{shape factor } (\alpha)$$

$$\text{Reserve strength } (\psi) = \frac{\text{Ultimate load } (W_u)}{\text{Load at first yield of structure } (W_y)}$$

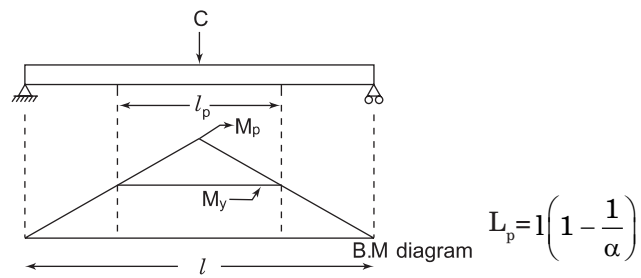
Shape factors for different shapes

Section	Shape Factor (α)
1. Rectangular section	1.5
2. (a) Triangular section (vertex upward) (b) Triangular section (vertex horizontal)	2.37 2.00
3. Solid circular section	1.7
4. Hollow circular section (K = Ratio of inner diameter to outer diameter)	$1.7 \times \frac{(1 - K^3)}{(1 - K^4)}$
5. Thin circular ring solid	1.25
6. (a) Diamond section (Rhombus) (b) Thin hollow rhombus	2.00 1.50

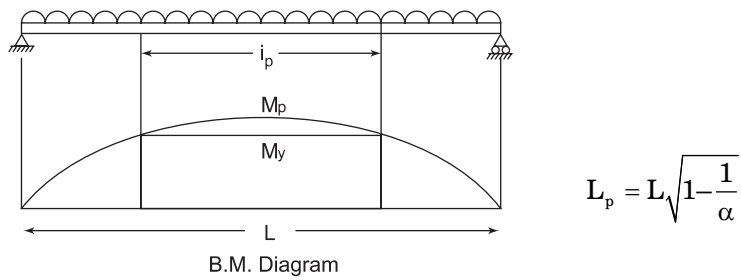
7. I-section	≈ 1.12
(a) About strong axis	≈ 1.55
(b) About weak axis	
8. T-section	≈ 1.90 to 1.95

Length of plastic hinge (L_p): Length of beam over which the moment is greater than (L_p) yield moment. (M_y)

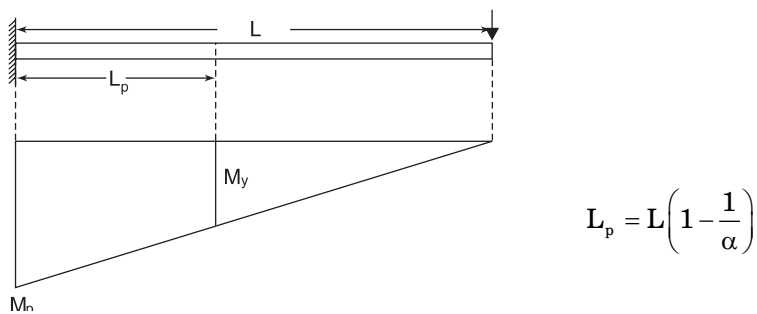
(i)



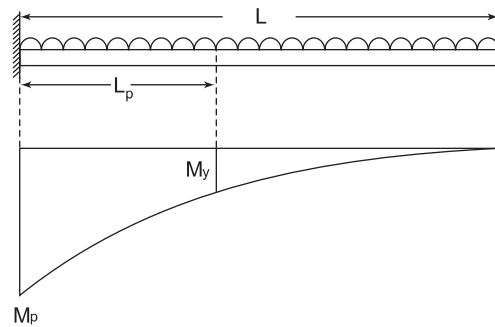
(ii)



(iii)



(iv)



$$L_p = L \left(1 - \frac{1}{\sqrt{\alpha}} \right)$$

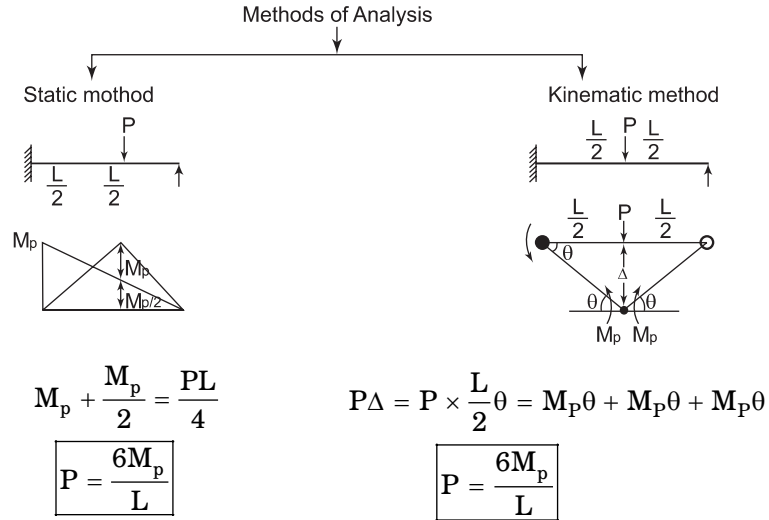
Conditions to be satisfied for complete collapse

- (i) **Equilibrium condition:-** $\sum F = 0$, $\sum M = 0$ both should be satisfied
- (ii) **Mechanism condition:-** At collapse sufficient no of plastic hinges must be developed so as to transform a part or whole of structure into mechanism leading to collapse.
- (iii) **Yield condition:-** At collapse, bending moment at any section must not exceed the fully plastic moment capacity of the section.

Note:- If all three conditions are satisfied we get a unique lowest value of collapse load.

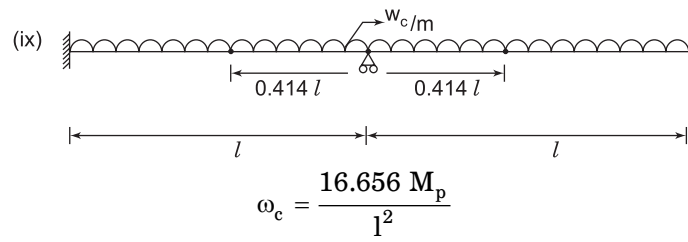
Theorem's of plastic analysis:-

- (i) **Upper bound theorem:-** It satisfies **equilibrium** and **mechanism** condition. Load determined by assuming a mechanism will always be greater than or equal to the actual collapse load. $P \geq P_u$
- (ii) **Lower bound theorem:-** It satisfies **equilibrium** and **yield** condition. Load determined by assuming a collapsed BMD will always be less than or equal to the actual collapse load. $P \leq P_u$



Collapse loads (W_c)

(i) $\omega_c = \frac{4M_p}{l}$	(ii) $\omega_c = \frac{16M_p}{l}$	(iii) $\omega_c = \frac{6M_p}{l}$
(iv) $\omega_c = \frac{411.656M_p}{l}$ $x = 0.414l$	(v) $\omega_c = \frac{8M_p}{l}$	(vi) $\omega_c = \frac{2 l M_p}{ab}$
(vii) $\omega_c = \frac{6M_p}{l}$	(viii) $\omega_c = \frac{18\sqrt{3} M_p}{l^2}$	



Civil Engineering

4

RCC

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2. Limit State Method
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5. Bond and Anchorage
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9. Footing
10. Pre-stressed Concrete
11. Analysis of Pre-stress and Bending Stresses
12. Losses in Pre-stress
13. Deflection of Pre-stressed Beam
14. IS Code Recommendations for Pre-stressed Concrete

Introduction

1

Plain concrete: It is a mixture of sand, gravel, cement and water resulting into solid mass usually used in mass concreting (*i.e.*, dams).

Note: Tensile strength is one-tenth of compressive strength.

Reinforced concrete: Concrete with reinforcement embedded in it. Bond between steel and surrounding concrete ensures **strain compatibility**. Reinforcing steel imparts ductility to concrete.

Grade of concrete: Compressive strength measured by standard test on concrete cube (150 mm size) tested 28 days after casting and continuous curing. Strength of cube is the average of three specimens of a sample, where **individual variation** should not be more than $\pm 15\%$ of average. If variation is more, test result of the **sample** is declared invalid.

Acceptance criteria of compressive strength

Specified Grade (1)	Mean of the group of 4 Non overlapping Consecutive test results in N/mm ² (2)	Individual test Result in N/mm ² (3)
M 15	$\geq f_{ck} + 0.825 \times \text{established standard deviation}$ (rounded off to nearest 0.5 N/mm ²) or $f_{ck} + 3 \text{ N/mm}^2$, whichever is greater	$f_{ck}^{-3} \text{ N/mm}^2$
M 20 or above	$> f_{ck} + 0.825 \times \text{established standard deviation}$ (rounded of to nearest 0.5 N/mm ²) or $f_{ck} + 4 \text{ N/mm}^2$, whichever is greater	$f_{ck}^{-4} \text{ N/mm}^2$

Flexural strength: Both the below condition should be satisfied.

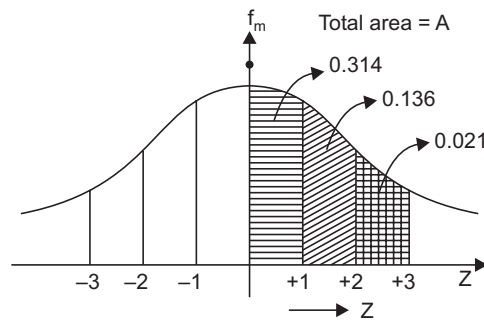
- (i) The mean strength determined from any group of four consecutive test results exceeds the specified characteristic strength by atleast 0.3 N/mm^2 .
- (ii) The strength determined from any test result is not less than the specified characteristic strength less 0.3 N/mm^2 .

Variation in strength: Strength of cube varies as **Normal probability distribution curve**.

(i) Mean strength
$$f_m = \frac{\Sigma(\text{Strength of sample i.e., } f)}{\text{No. of samples i.e., } m}$$

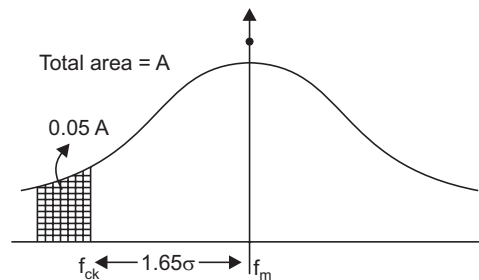
(ii) Standard deviation
$$\sigma = \sqrt{\frac{\Sigma(f - f_m)^2}{m}}$$

(iii) Frequency density
$$y = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, z = \frac{f - f_m}{\sigma}$$



Characteristic strength (f_{CK}): Strength below which not more than 5% of test results are expected to fall.

$$f_{CK} = f_m - 1.65\sigma$$



- Minimum curing period for OPC is 7 days.
- Grade's of concrete
 - (i) $M_{10}, M_{15}, M_{20} \rightarrow$ ordinary
 - (ii) $M_{25} - M_{55} \rightarrow$ standard
 - (iii) $M_{60} - M_{90} \rightarrow$ High strength
where 'M' stands for 'mix'
- Minimum grade for RCC $\rightarrow M_{20}$

Concrete Mix Design: It's economical selection of relative proportions of various ingredients of concrete such that it remains workable in fresh state and impermeable and durable in hardened state.



Nominal mix
(upto M_{20} grade only)

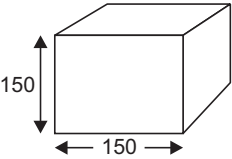
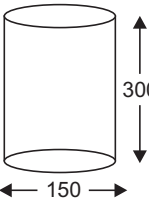
- Specified in terms of **total mass** of aggregate, and volume of water to be used per 50 kg of cement.
- 1 bag of cement = 34.5 litres

	cement : FA : CA			
M 5	1	:	5	: 10
M 7.5	1	:	4	: 8
M 10	1 +	:	3 +	: 6
M 15	1	:	2	: 4
M 20	1	:	1.5	: 3

Design mix
(IS 10262 – 1982)

- Steps involved are
- Target mean strength.
 - Water cement ratio from charts.
 - Water content based on workability and ratio of fine and coarse aggregate, by mass based on type and grading of aggregate.
 - Find cement content from (ii) and (iii) above
 - Mass of FA and CA from absolute volume principle.
 - Weight of ingredient per batch based on capacity mixer.

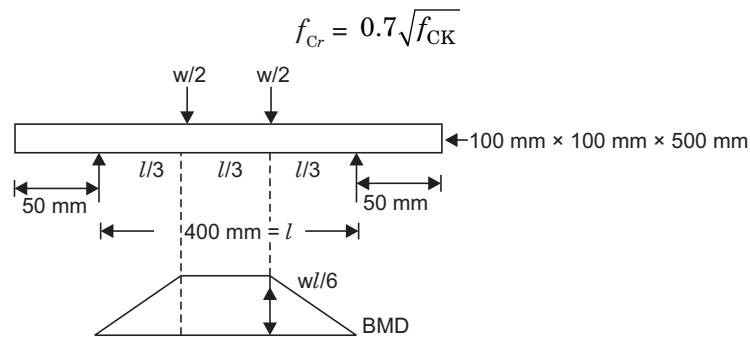
Compressive strength of concrete in structures:

 Compressive strength = f_{CK}	 $0.8 f_{CK}$	For size beyond 450 mm there is no decrease in compressive strength Hence taken as $0.67 f_{CK}$

4.6 CIVIL ENGINEERING

Modulus of Rupture of concrete

It is used to determine the onset of cracking or the load at which cracking starts.

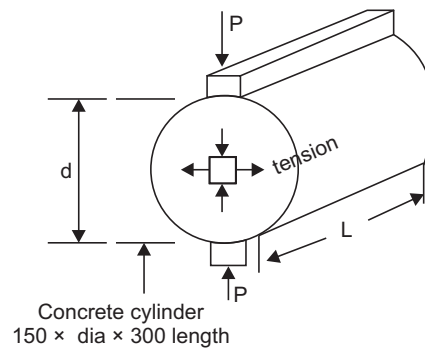


Tensile strength of concrete

Splitting tensile strength

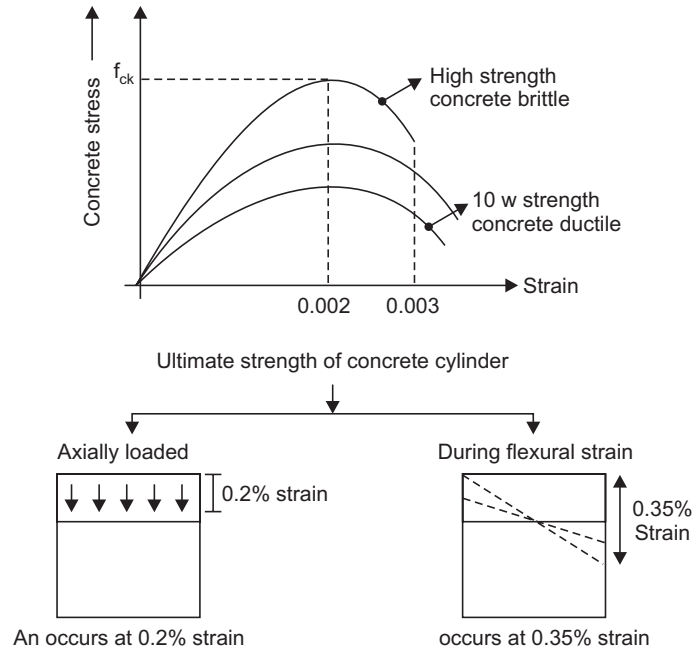
$$f_{Ct} = \frac{2P}{\pi dL} = 0.66 f_{Cr}$$

Direct tensile strength = \$(0.5 - 0.625) f_{cr}\$



Stress-strain curve of concrete

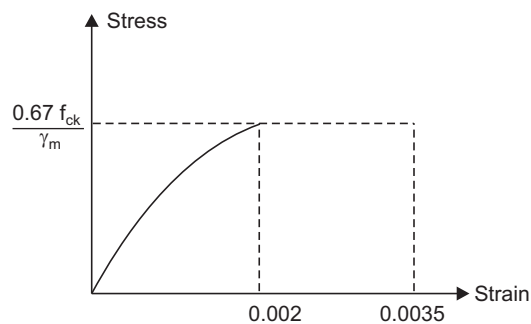
- Descending part of high strength concrete is steeper.
- Point where curve ends is crushing strain.
- Curves are linear upto 0.6 times the peak stress.
- Secant modulus of elasticity of concrete is taken at a stress of around $0.33 f_{CK}$.



Design stress-Strain curve for strength of concrete

Partial safety factor for material strength (γ_m)

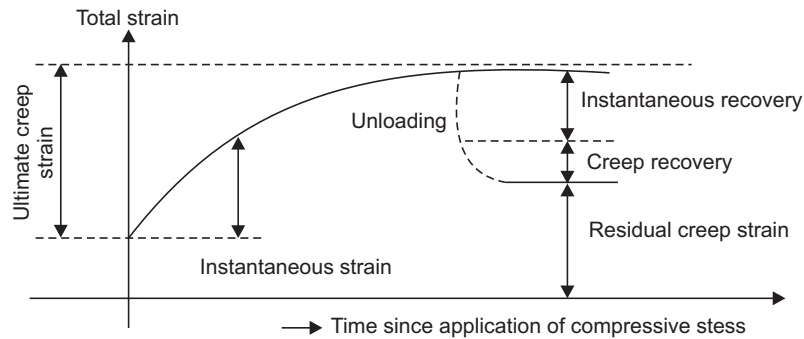
- (i) Collapse = 1.5
- (ii) Serviceability = 1.0



Creep in Concrete

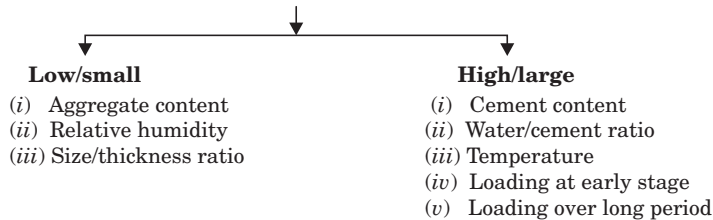
Time dependent component of total strain is creep.

- Occurs due to internal movement of adsorbed water, loss of moisture, growth of microcracks, sliding between gel particles.



- Advantages of creep
 - (a) Reduction in cracking stress developed due to restrained shrinkage.
 - (b) Reduction in stress due to differential settlement in indeterminate structures.
- Disadvantages of creep
 - (a) Increased deflection of beams, slabs and columns (which can even buckle).
 - (b) Gradual transfer of load from concrete to reinforcing steel in compression members.
 - (c) Loss of prestress.

Creep increases when following are



Note:

High strength concrete, Adding reinforcement, delaying application of partition wall and finishes, steam curing under pressure reduce the effect of creep.

Shrinkage: Shortening in length of a member or contraction of concrete per unit length due to drying when concrete sets.

Plastic Shrinkage

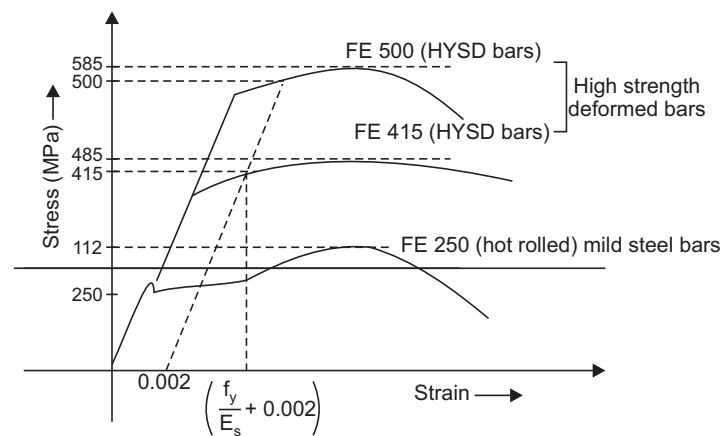
- Shrinkage when concrete is in its plastic stage itself due to loss of water by evaporation or by absorption by aggregate.
- Presented by using **expanding cement** or **Aluminium Powder**.

Drying Shrinkage

- Everlasting process by loss of water hold in gel pores when concrete is kept in drying condition.
- Presented by using harder and bigger size of aggregate, Bigger size of member and increasing humidity.

Reinforcement

In Fe 250, Fe 415, Fe 500 :- 250 MPa, 415 MPa and 500 MPa is the guaranteed minimum strength and can be treated as characteristic strength of steel respectively.



Note:

By cold working (stretching and twisting) yield strength increases but ductility falls (i.e., HYSD bars have high yield strength than mild steel).

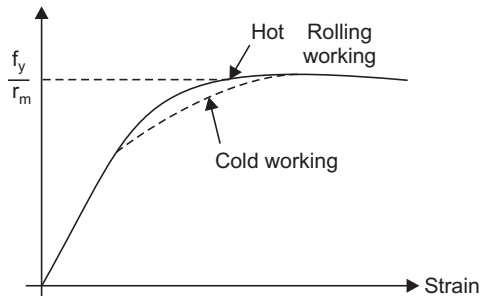
- HYSD bars fails at less elongation as compared to mild steel bars.

Design stress strain curve for strength of steel

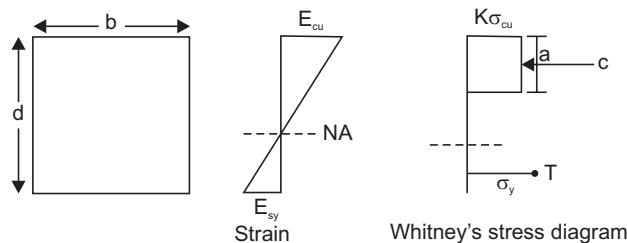
Partial Factor of safety for material strength (γ_m)

(i) Collapse = 1.5

(ii) Serviceability = 1.0


Design methods

1. Working stress method.
2. Unit state method.
3. Ultimate load method.

Ultimate load Method: (Whitney's theory)


a = Depth of rectangular stress block

= $0.537 d$ in accordance to whitney.

σ_{cu} = Ultimate compressive strength of concrete cubes (28 days)

$K\sigma_{cu}$ = average stress

= $0.85 \sigma_{cu}$ in accordance with whitney.

This theory is based on the assumption that ultimate strain in concrete is 0.3% and compressive stress at the extreme fiber of the section corresponds to this strain. Whitney replaced the actual parabolic stress diagram by a rectangular stress diagram.

Limit State Method

2

The acceptable limit for the safety and serviceability requirement of a structure or structural element before occurring of failure

Limit state of serviceability

- Satisfactory performance under service load
- Deflections, cracking, vibration, leakage, loss of durability etc.

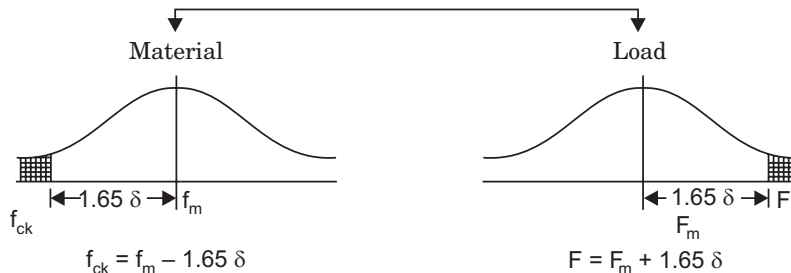
Limit state of collapse

- Adequate margin of safety for normal over loads.
- Flexure, compression, shear, torsion, over turning, sliding, buckling, fatigue.

Note:

Structure will return to its original state it has reached only till its limit state of serviceability. However, after reaching limit state of collapse it will not regain its shape.

Characteristic Shape



- To account for uncertainties **reliability based analysis** was performed and **partial factor of safety** were established for material as well as load.
- $(f_m - 1.65\delta)$ and $(F_m + 1.65\delta)$ are limits within which “probability of lying test result” is maximum and it is called **confidence limit**.

4.12 CIVIL ENGINEERING

(a) Partial factor of safety in material property

$$f_d = \frac{f_{ck}}{\gamma_{ms}}$$

Concrete

 $f_d = \frac{0.67 f_{ck}}{1.5} = 0.45 f_{ck}$

Steel

 $f_{ck} = \frac{f_y}{1.15} = 0.87 f_y$

(b) Partial factor of safety under various load combinations

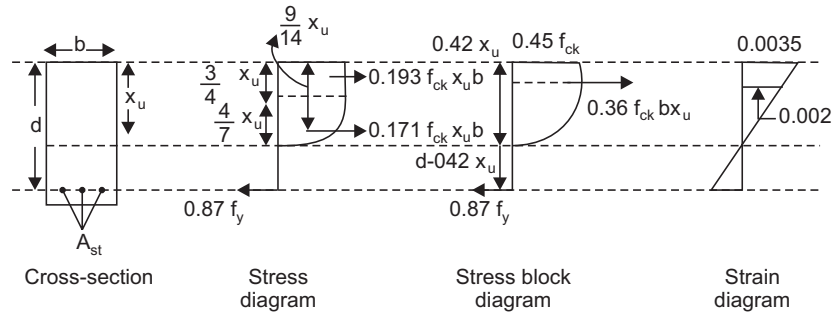
$$F_d = F \gamma_f$$

S. No.	Description	Collapse	Servicability
1.	D.L + L.L	1.5	1
2.	D.L + (W.L) or (E.L) combination		
	(i) for normal case		
	D.L + W.L (or E.L) D.L + W.L (or E.L)	1.5	1
	(ii) For checking stability against overturning/stress reversal		
	D.L + W.L (or E.L)	0.9	1
3.	(D.L.) + L.L + W.L (or E.L) combination		
	D.L	1.2	0.8
	W.L (or E.L)	1.2	0.8

Assumptions of limit state of collapse: Flexure

1. Plane section before bending remains plane after bending (**Strain compatibility**)
2. Maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending (Regardless of grade of concrete by **Max. Principal strain theory**)
3. Relationship between compressive stress distribution in concrete and strain in concrete may be assumed to be rectangular, Trapezoidal, parabolic or any other shape which results in prediction of strength in substantial agreement with the result of test.

4. Tensile strength of concrete is ignored.



5. For design purpose partial factor of safety for steel $\gamma_{ms} = 1.15$ and the stress in steel is derived from stress-strain curve.
6. **Maximum** strain in tension reinforcement in the section at failure shall not be less than

$$\epsilon_{st} = \frac{f_y}{1.15 E_s} + 0.002 = \frac{0.87 f_y}{E_s} + 0.002$$

This assumption restricts depth of neutral axis.

Limiting Depth of Neutral Axis

	$x_u \text{ lim}/d$	$R_u = \frac{M_u \text{ lim}}{bd^2}$	$p_t \text{ lim}$
Fe 250	0.53	$0.148 f_{ck}$	$0.088 f_{ck}$
Fe 415	0.48	$0.138 f_{ck}$	$0.0479 f_{ck}$
Fe 500	0.46	$0.133 f_{ck}$	$0.038 f_{ck}$

$$\left(\frac{x_u}{d} \right)_{\text{lim}} = \frac{0.0035}{0.0055 + \frac{0.87 f_y}{E_s}}$$

Note:

Failure always occurs due to crushing of concrete on compression face in both the cases.

Beams

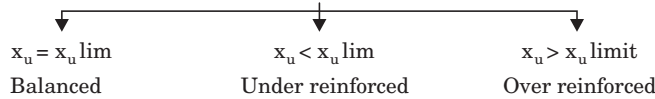
3

Singly Reinforced Beam

(i) To determine M.O.R (M_u) when beam cross-section is given

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{CK} b}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$



(a) **Balanced section** $M_u = M_{u \text{ lim}}$

$$M_{u \text{ lim}} = R_u b d^2 \quad \text{or} \quad M_{u \text{ lim}} = 0.87 f_y A_{st} (d - 0.42 x_{u \text{ lim}})$$

(b) **Under reinforced section**

$$M_u = 0.36 f_{ck} x_u (d - 0.42 x_u) \quad \text{or} \quad M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

(c) **Over reinforced section:** x_u is limited to $x_{u \text{ lim}}$ and M_u is calculated as in balanced section.

(ii) To determine area of steel, when concrete cross-section and applied moments are known.

$$M_u \text{ applied} = 0.87 f_y A_{st} \left(d - \frac{A_{st} f_y}{f_{CK} b} \right)$$

(iii) To determine the x-section for given bending moment (M_u)

(a) $M_u = R_u b d^2$

- Take $d = 2b$ and get 'b' then d
- Round off 'd' to the nearest **upper** 50 like 500, 550, 600, 650, etc.
- $D = d + 50$

$$(b) M_u = 0.87 f_y A_{st} \left(d - 0.42 \times \frac{0.87 f_y A_{st}}{0.36 f_{CK} b} \right)$$

Get A_{st}

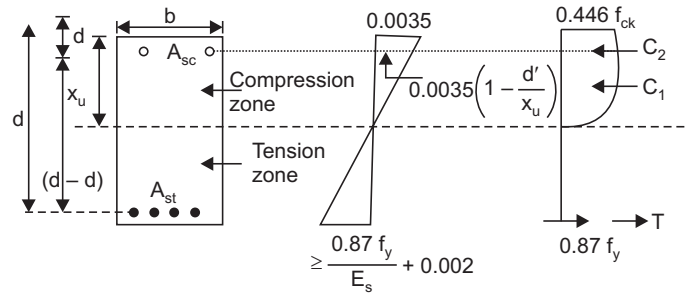
(c) Apply checks

$$\bullet \frac{A_{st \text{ min}}}{bd} > \frac{0.85}{f_y}, A_{st \text{ max}} = 0.04 bd, \quad x_u < x_{u \text{ lim}}$$

Doubly Reinforced Beam: If $M_u > M_{u \text{ lim}}$, then either section dimensions need to be modified or higher grade of steel/concrete to be used. If section dimensions are restrained then compression reinforcement is provided such that neutral axis does not shift downward by providing tension steel greater than $A_{st \text{ lim}}$.

Advantage

- (a) Prevents Beam in Reversal of moments.
- (b) Reduction in long term deflection due to Shrinkage and creep.
- (i) To determine Area of steel for given bending moment ($M_{u \text{ applied}} > M_{u \text{ lim}}$) and restrained dimension's (b and d).

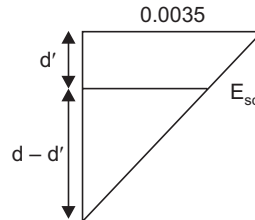


- (a) $M_{u \text{ lim}} = R_u b d^2$
- (b) $M_{u \text{ lim}} = 0.87 f_y A_{st1} (d - 0.42 x_{u \text{ lim}})$, Get A_{st1}
- (c) $M_{u \text{ applied}} - M_{u \text{ lim}} = 0.87 f_y A_{st2} (d - d')$, Get A_{st2}
where d' is selected such that

$$0.05 < \frac{d'}{d} < 0.2$$

- (d) Get E_{sc} from strain diagram

$$\frac{0.0035}{E_{sc}} = \frac{d}{d - d'}$$



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(e) Get f_{SC} from E_{SC} or from d/d' values. (given in question)

Stress (f_{SC})	Strain (ϵ_{SC})
$0.8 f_{yd}$	0.00144
$0.85 f_{yd}$	0.00163
$0.90 f_{yd}$	0.00192
$0.95 f_{yd}$	0.00241
$0.975 f_{yd}$	0.00276
$1.00 f_{yd}$	0.00380

For Fe 415 d'/d values

d'/d	0.05	0.10	0.15	0.20
$f_{st}(\text{N/mm}^2)$	355	353	342	329

(f) Get A_{SC} from $0.87 f_y A_{st2} = A_{SC} \cdot f_{SC}$.

(ii) To determine M.O.R (M_u) when beam cross-section is given

(a) Get $x_{u \text{ lim}}$ from d (i.e., $0.53 d$, $0.48 d$ or $0.46 d$)

(b) Get x_u from

$$0.36 f_{CK} b x_u + (f_{SC} - f_{CC}) A_{SC} = 0.87 f_y A_{st}$$

$$\text{where } f_{CC} = 0.45 f_{CK}$$

	f_{SC}
Fe 250	217
Fe 415	350
Fe 500	400

$$\text{if } 0.05 < \frac{d'}{d} < 0.2$$

otherwise trial and error.

(c) As $x_u < x_{u \text{ lim}}$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

Flanged beam: In monolithic construction, slab and beams are cast together. If slab in such cases is in compression zone they become effective in adding significantly to the area of concrete in compression in beam.

Effective flange width for T and L beams

Monolithic L-Beam

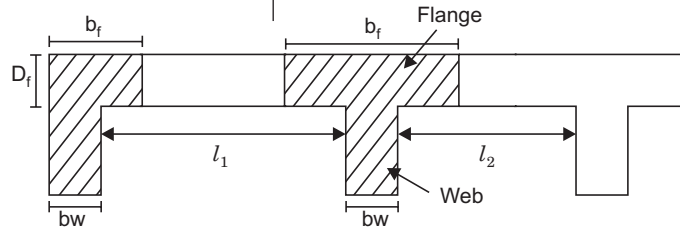
$$(i) \quad b_f = b_w + 3D_f + \frac{l_0}{12}$$

$$b_f \leq b_w + \frac{l_1}{2}$$

Monolithic T-Beam

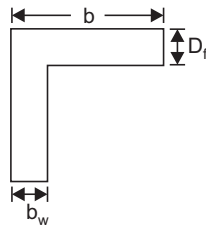
$$(ii) \quad b_f = b_w + 6D_f + \frac{l_0}{6}$$

$$b_f \leq b_w + \frac{l_1 + l_2}{2}$$



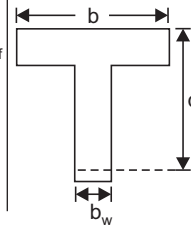
Isolated L-Beam

$$(i) \quad b_f = b_w + \frac{0.5 l_0}{\frac{l_0}{b} + 4} \leq b$$



Isolated T-Beam

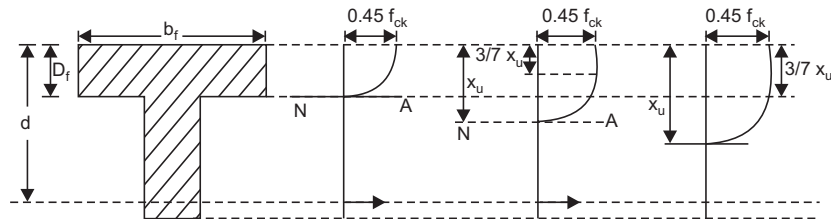
$$(ii) \quad b_f = b_w + \frac{l_0}{\frac{l_0}{b} + 4} \leq b$$



b_f = effective flange width (the width of flange with constant compressive stress equal to the peak actual flexural compressive stress which leads to the same longitudinal compressive force as due the original stress distribution).

l_0 = distance between point of zero moment in the beam.

(i) To determine M.O.R (M_u) when beam cross-section is given



(a) When Neutral axis lies in the flange ($x_u < D_f$)

$$x_u = \frac{0.87 f_y A_y}{0.36 f_{CK} b_f} \quad M_u = 0.36 f_{CK} b_f x_u (d - 0.42 x_u)$$

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

(b) When Neutral axis lies in the web $\left(x_u < \frac{7}{3} D_f\right)$

$$y_f = 0.65 D_f + 0.15 x_u$$

Get x_u from 0.36

$$f_{CK} b_w x_u + 0.45 f_{CK} (b_f - b_w) y_f = 0.87 f_y (A_{st1} + A_{st2})$$

If $x_u < x_{u \text{ lim}}$

$$M_u = 0.36 f_{CK} b_w x_u (d - 0.42 x_u) + 0.45 f_{CK} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$$

OR If $x_u > x_{u \text{ lim}}$, $M_{u \text{ lim}} = 0.36 f_{CK} b_w x_{u \text{ lim}} (d - 0.42 x_{u \text{ lim}})$
 $+ 0.45 f_{CK} (b_f - b_w)$

$$(0.65 D_f + 0.15 x_{u \text{ lim}}) \left(d - \frac{0.65 D_f + 0.15 x_{u \text{ lim}}}{2}\right)$$

(c) When Neutral axis lies in the web $\left(x_u > \frac{7}{3} D_f\right)$

Get x_u from $0.45 f_{CK} (b_f - b_w) D_f + 0.36 f_{CK} b_w x_u = 0.87 f_y A_{st}$

If $x_u < x_{u \text{ lim}}$

$$M_u = 0.45 f_{CK} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right) + 0.36 f_{CK} b_w x_u (d - 0.42 x_u)$$

OR If $x_u > x_{u \text{ lim}}$

$$M_u = 0.36 f_{CK} b_w x_{u \text{ lim}} (d - 0.42 x_{u \text{ lim}})$$

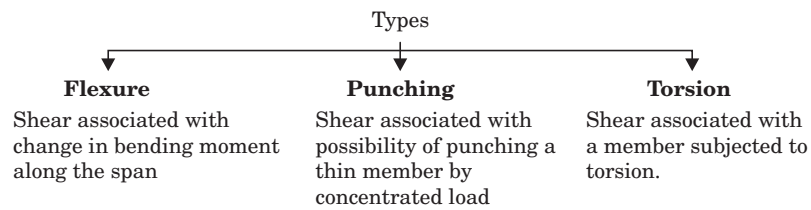
$$+ 0.45 f_{CK} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

Note:

Inverted beams are designed as rectangular beams, because slab in tension zone does to resist any compression. These beams are recommended for architectural requirement only.

Shear

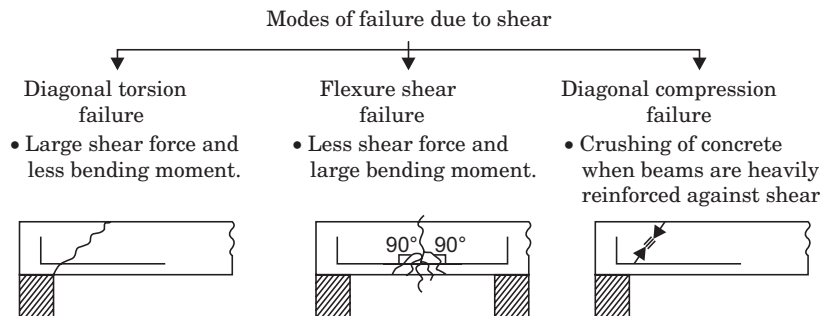
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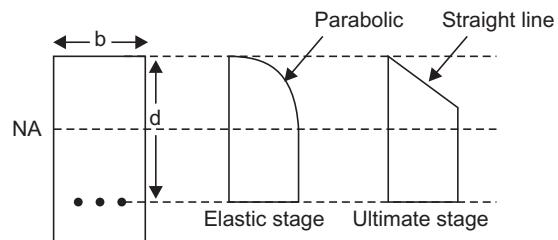
Note:

Flat slab is a **beam less** floor slab supported directly by columns.

Modes of Failure Due to Shear

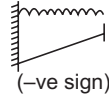


Shear stress distribution



$$\text{Nominal shear strength} = \tau_u = \frac{V_u}{b_d}$$

For beam of varying depth

$$\tau_u = \frac{Vu \pm \frac{M_u \tan \beta}{d}}{bd}$$


Design Shear Strength of Concrete in Beams

(i) Without shear reinforcement

$$\tau_c = \frac{0.85\sqrt{0.8f_{ck}}(\sqrt{1+5\beta}-1)}{6\beta}$$

where $\beta = \frac{0.8 f_{ck}}{6.89 p_t} > 1$

$$p_t = \frac{A_{st}}{bd} \times 100$$

A_{st} = Area of longitudinal reinforcement

which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing is as per code.

P	WSM		LSM	
	M20	M25	M20	M25
0 ≤ 0.15	0.18	0.19	0.28	0.29
0.25	0.22	0.23	0.36	0.36
0.50	0.30	0.31	0.48	0.49
0.75	0.35	0.36	0.56	0.57
1.00	0.39	0.40	0.62	0.64

Note:

τ_c depends on both f_c and A_{st} .

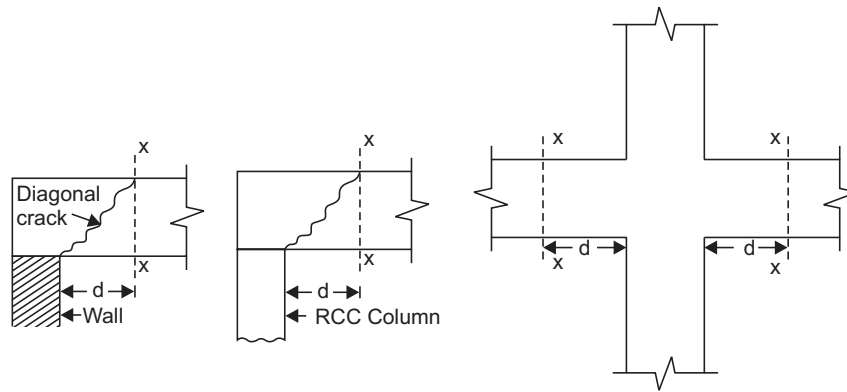
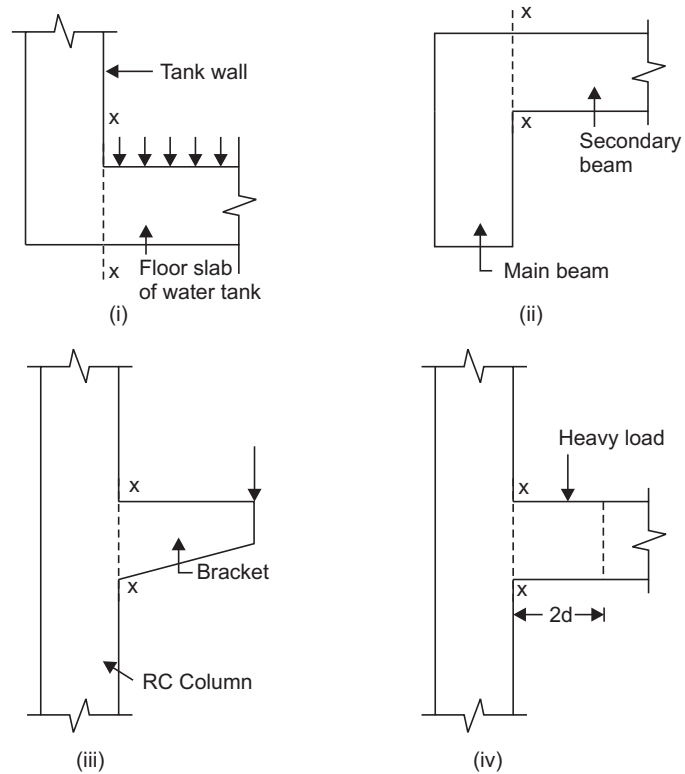
Maximum shear stress with shear reinforcement ($\tau_{c \max}$)

M	15	20	25	30	35	40 and above
LSM	2.5	2.8	3.1	3.5	3.7	4.0
WSM	1.6	1.8	1.9	2.2	2.3	2.5

for LSM $\tau_{c \max} \approx 0.63\sqrt{f_{ck}}, \tau_u \nless \tau_{c \max}$

Note: $\tau_{c \max}$ depends only on concrete (i.e., f_{ck})

Critical Section for Shear

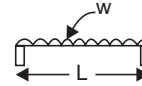
(a) Critical section X-X at d from the face of the support

(b) Critical section X-X at the face of the support

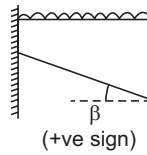
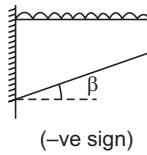
Design for shear

- (i) Get factored shear force V_u

$$V_u = 1.5 V \quad \text{or} \quad V_u = 1.5 \left(\frac{wL}{2} \right)$$



$$(ii) \tau_u = \frac{V_u}{bd} \quad \text{or} \quad \tau_u = \frac{V_u \pm \frac{M_u \tan \beta}{d}}{bd} \quad \text{if}$$



- (iii) Get $\tau_{c \max}$ from code or $\tau_{c \max} \approx 0.63 \sqrt{f_{ck}}$

- (iv) If $\tau_{c \max} < \tau_u$ Redesign the section by changing b or d .

- (v) If $\tau_u < \tau_{c \max}$ then get τ_c from I.S. code (which depends on f_{ck} and A_{st})

- (vi) Now if $\tau_u - \tau_c < 0.4 \text{ N/mm}^2$, provide reinforcement for minimum shear

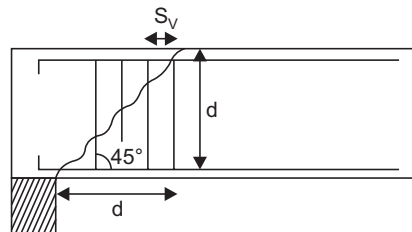
$$\frac{d}{S_v} (0.87 f_y) A_{sv} = 0.4 bd$$

Get S_v

$\frac{d}{S_v}$ = No. of stirrups till critical section for shear *i.e.* till distance d from face of support (as S_v = spacing between stirrups)

A_{sv} = Total cross-sectional area of stirrups leg effective in shear.

f_y = yield strength of stirrup's subjected to maximum of 415 N/mm^2



(vii) If $\tau_u - \tau_c > 0.4\text{N/mm}^2$ then get spacing S_v by

$$\frac{d}{S_v} (0.875 f_y) A_{sv} = (\tau_u - \tau_c)bd$$

(viii) Check for maximum spacing of stirrups as per code

Max. S_v

$$= \text{Minimum of } \begin{cases} 0.75 d (\text{Vertical stirrups}), d (\text{Inclined stirrups}) \\ S_v \text{ (as obtained from (vi) or (vii))} \\ 300 \text{ mm} \end{cases}$$

Note:

For inclined stirrups or a series of bent up bars

$$\left(\frac{d}{S_v} \right) (0.87 f_y) (\sin \alpha + \cos \alpha) A_{sv} = (\tau_u - \tau_c)bd$$

α = Angle of inclination of stirrups with the horizontal

Minimum shear reinforcement: It is provided

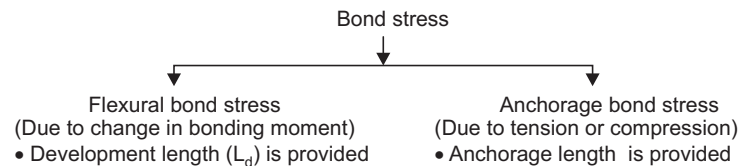
- (i) to prevent bursting of concrete cover.
- (ii) to prevent tension failure due to shrinkage, thermal stresses and internal cracks.
- (iii) to avoid brittle shear failure.
- (iv) to hold reinforcement in place while concreting.
- (v) to make the section effective with the tie effect.

Bond and Anchorage

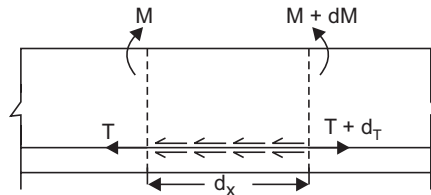
5

Bond is the means by which relative movement between concrete and steel is prevented and the intensity of adhesive force is called stress. Bond transfers the axial force by providing '**strain compatibility**' and '**composite action**', of concrete and steel.

Bond is due to combined effect of adhesive resistance, frictional resistance, and mechanical resistance (for deformed bars)



Flexural Bond Stress



Note: Bond stress can be reduced by providing an increased number of bars of small diameter rather than small number of large diameter bars.

$$\tau_{bd} = \frac{V}{jd(\sum O)}$$

$$V = \text{Shear force at any section} = \frac{dm}{dx}$$

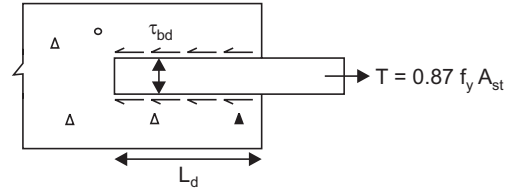
d = effective depth of the section

$\sum O$ = Summation of all perimeter of reinforcement

$$\sum O = n(\pi \phi)$$

n = number of bars

ϕ = diameter of reinforcement

Anchorage Bond Stress: (Due to tension)

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} \quad A_{st} = \frac{\pi \phi^2}{4}$$

L_d = development length

τ_{bd} = Average bond stress

ϕ = Nominal diameter

Permissible Bond Stress in Tension

Grade of concrete	M20	M25	M30	M35	M40 and above
τ_{bd} (N/mm ²)	1.2	1.4	1.5	1.7	1.9

For deformed bars above value is increased by **60%**.

For bars in compression above value is increased by **25%**.

For ready Reference

	M20		M25	
L_d	T	C	T	C
Fe 250	460 ϕ	37 ϕ	39 ϕ	32 ϕ
Fe 415	47 ϕ	38 ϕ	41 ϕ	33 ϕ

T = Tension

C = Compression

Development Length Due to Flexure

$$L_d \leq \frac{M_1}{V} + L_o$$

$$L_o = \max(d, 12\phi)$$

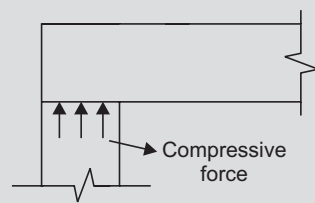
M_1 = MOR of the section to be assuming all reinforcement at the section to be stressed to $0.87 f_y$

V = Shear force at the section due to design load.

Note:

When the ends of the reinforcement are confined by compressive stresses then M_1 is increased by 30%

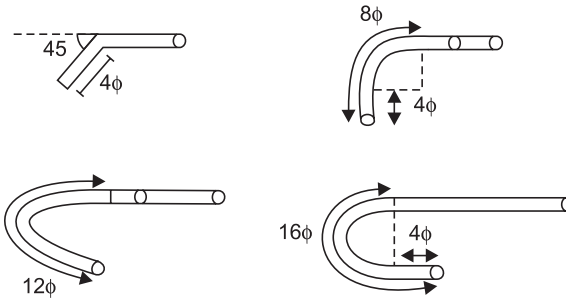
i.e.,
$$L_d \leq \frac{1.3M_1}{V} + L_o$$



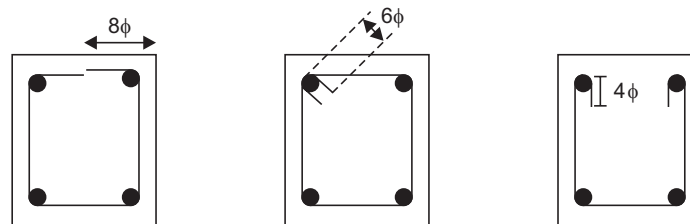
Bundled bars: The development length of each bundled bars shall be increased by 10% when two bars are bundled, by 20% when three bars are bundled and 33% when four bars are bundled.

Bends and Hooks: (IS : 2502)

The anchorage value of bend shall be taken as 4 times the diameter of bar for each 45° bend subject to a maximum of 16 times the diameter of bars.



For stirrups



Torsion

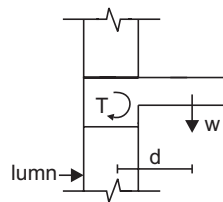
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Equilibrium Torsion/Primary Torsion

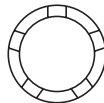
- Torsion which can be determined only by using static equilibrium conditions.
- Torsion induced in beams curved in plan and subjected to gravity loads.
- Also occurs in beams where the transverse loads are eccentric with respect to the shear centre of the x -section.

Compatibility Torsion or Secondary Torsion

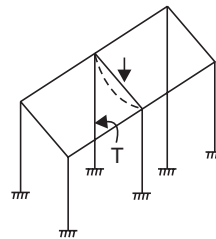
- It is induced in a structural member by rotation applied at one or more point along the length of member through inter connected members, instead of by directly applied load.



Cantilever slab with 'L' beam



Circular beam



Effect of Torsional moment

- Beam fails in diagonal tension.
- Longitudinal reinforcement is provided in the form of bars placed **closed to the periphery** where as transverse reinforcement is in the form of closed rectangular stirrups placed perpendicular to the beam axis.
- Longitudinal reinforcement helps in reducing the crack width through **dowel action**.
- Stirrups resist shear due to vertical loads and torsion.

Note:

As per IS code, clause 41, For design of torsion section located at a distance less than 'd' from the face of the support may be designed for the same torsion as computed at 'd' where 'd' is the effective depth.

Design for Torsion:

Given V_u = Ultimate Vertical Shear at the Section
 T_u = Ultimate Torsional moment
 M_u = Factored Bending moment at the cross-section
 b, d, D, f_{CK} and f_y

$$(i) \quad V_e = V_u + 1.6 \frac{T_u}{b}$$

V_e = Equivalent shear at the section

$$\tau_e = \frac{V_e}{bd}$$

$$\tau_{c \max} \approx 0.63 \sqrt{f_{CK}}$$

Get τ_c for minimum % of tensile reinforcement. If $\tau_c < \tau_{c \max}$ then Both Longitudinal and transverse reinforcement is required.

(ii) Longitudinal reinforcement

$$M_e = M_u + M_t$$

M_e = Equivalent Bonding moment at the section

$$M_t = \frac{T_u}{1.7} \left(1 + \frac{D}{b} \right)$$

Case (a): If $M_t < M_u$ = No compression reinforcement is required. Still provided two no's of hanger bar's (in compression side) of 10mm ϕ

Case (b): If $M_t > M_u$ = Compression reinforcement required (i.e., A_{SC})

$$M_t - M_u = 0.87 f_y A_{SC} (d - d'), \text{ get } A_{SC}$$

$$\text{For } A_{st}, \quad M_e = 0.87 f_y A_{st} \left(d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{CK} b} \right), \text{ Get } A_{st}$$

(iii) Transverse reinforcement

$$\left(\frac{d_1}{S_V} \right) (0.87 f_y) A_{SV} = \frac{T_u}{b_1} + \frac{V_u}{2.5}$$

Get S_V

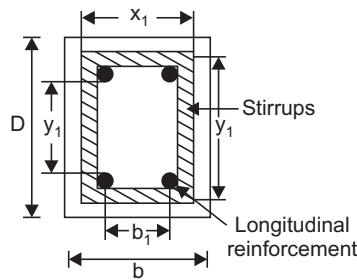
where,

$b_1 = c/c$ distance between corner bars in the direction of the width.

$d_1 = c/c$ distance between corner bars in the direction of the depth.

$b_1 = b$ -clear cover – 2 diameter of stirrups

$$- 2 \left(\frac{\text{diameter of longitudinal bar}}{2} \right)$$



A_{SV} = Area of cross-section of two legged closed loops enclosing the

corner longitudinal bar (For e.g. $2 \times \frac{\pi}{4} (8)^2$ if dia is 8 mm)

Now get x_1 from b_1 and y_1 from d_1

x_1 = Short dimension of stirrup

$$= b_1 + 2 \left(\frac{\text{diameter of longitudinal bar}}{2} \right) + 2 \left(\frac{\text{diameter of stirrups}}{2} \right)$$

y_1 = Long dimension of stirrup

$$= d_1 + 2 \left(\frac{\text{diameter of longitudinal bar}}{2} \right) + 2 \left(\frac{\text{diameter of stirups}}{2} \right)$$

Note:

Better to calculate b_1 , d_1 , x_1 and y_1 from diagram only.

$$(iv) S_v \text{ will be minimum of } \begin{cases} S_v \text{ obtained in (iii)} \\ x_1 \\ \frac{x_1 + y_1}{4} \\ 300 \text{ mm} \end{cases}$$

- (v) Additional Longitudinal reinforcement shall be provided along the two faces when the cross-sectional dimensions either b or D of the member exceeds 450 mm.

$$\text{Hence, side face reinforcement to be provided} = \frac{0.1}{100} bD$$

$$\text{on one side} = \frac{1}{2} \left(\frac{0.1bD}{100} \right)$$

Note:

Generally 10 mm ϕ bars provided, so number of bars

$$= \frac{\frac{1}{2} \left(\frac{0.1bd}{100} \right)}{\frac{\pi}{4} (10)^2} \approx 2$$

So provided 2 no's of 10 mm ϕ bar's with spacing < 300 mm.

$$(vi) \frac{A_{st \min}}{bd} = \frac{0.85}{f_y} \text{ and } A_{st \max} = 0.04 bD$$

- (vii) Minimum spacing between longitudinal tension bars

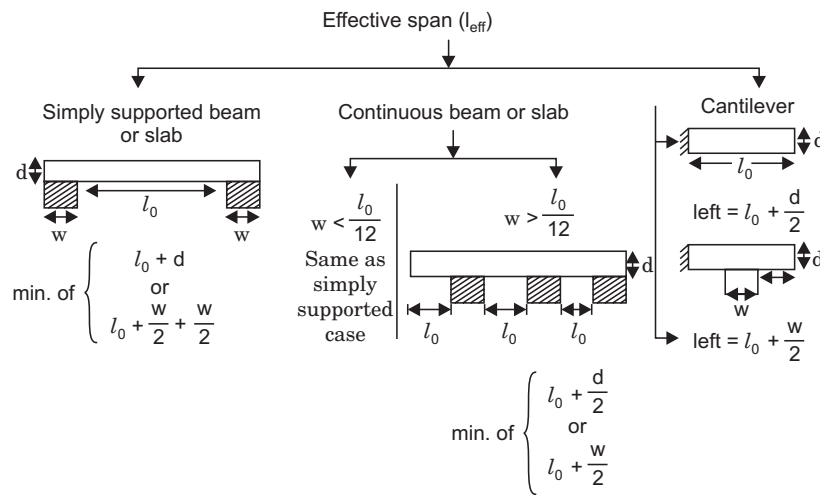
$$(a) \text{ In horizontal direction, min } \begin{cases} \text{max bar dia} \\ \text{coarse aggregate} + 5 \text{ mm} \end{cases}$$

$$(a) \text{ In vertical direction, min } \begin{cases} \text{max bar dia} \\ \frac{2}{3} \text{ coarse aggregate} \\ 15 \text{ mm} \end{cases}$$

Codal Provisions for Beam and Slab

7

EFFECTIVE SPAN (L_{eff})



Note:

For frames effective span is its centre to centre distance between members.

- In case of spans with roller or rocket bearings, the effective span shall always be the distance between the centres of bearings.

Check for Deflection

- Final deflection due to all loads including the effect of temperature, creep and shrinkage should not exceed $\frac{\text{span}}{250}$.
- Deflection including effect of creep, temperature and shrinkage occurring after creation of partition and application of finishes should not exceed $\frac{\text{span}}{350}$ or 20 m which ever is less.

Control of Deflection

$$\frac{\text{span}}{\text{depth}} < \left(\frac{l}{d} \right)_{\text{basic}} \times \frac{10}{\text{span}} \times k_t \times k_c \times k_f$$

(i) If span ≤ 10 m then $\left(\frac{l}{d} \right)_{\text{basic}}$ values are

Cantilever beam	7
Simply supported beam	20
Continuous beam	26

Type of slab	Type of reinforcement	
	Mild steel	Fe 415
Simply supported	35	$28 = 35 \times 0.8$
Continuous	40	$32 = 40 \times 0.8$

Where as $\left(\frac{10}{\text{span}} \right)$ factor is neglected.

Note:

For slab's its short span to overall depth ratio.

- For Fe 415 reinforcement values in case of slab, multiply mild steel value's by **0.8**.

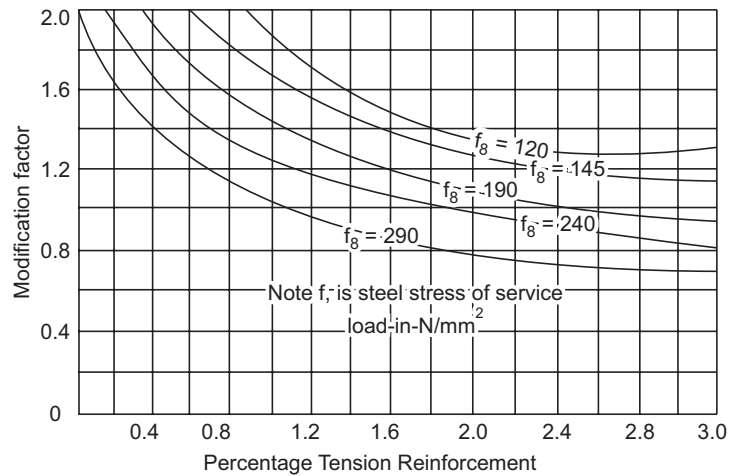
(ii) If span > 10 m, then $\left(\frac{10}{\text{span}} \right)$ factor is also multiplied (But if

the case is of cantilever beam > 10 m then actual deflection calculations should be made)

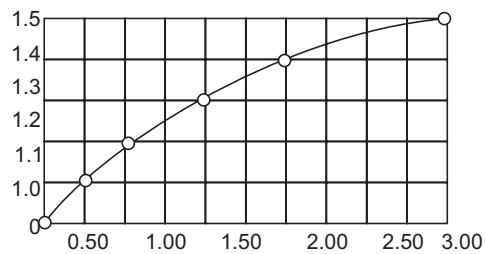
(iii) K_t = modification factor for tension reinforcement depending upon area and stress of steel for tension reinforcement.

Note:

This factor allows the designer to make shallow members by increasing area of tension reinforcement.



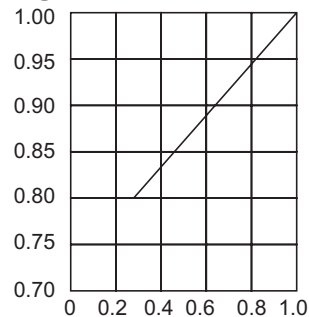
- (iv) K_c = modification factor for compression reinforcement depending on area of compression reinforcement



Note:

It's always greater than 1, as compression reinforcement reduces shrinkage and increases the stiffness of the beam.

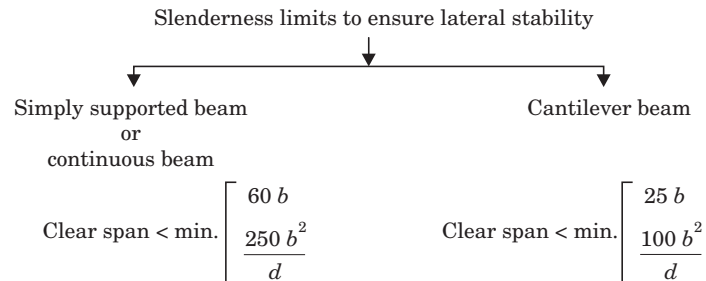
- (v) K_f = modification factor for flanged beams depends on ratio of web width to flange width.



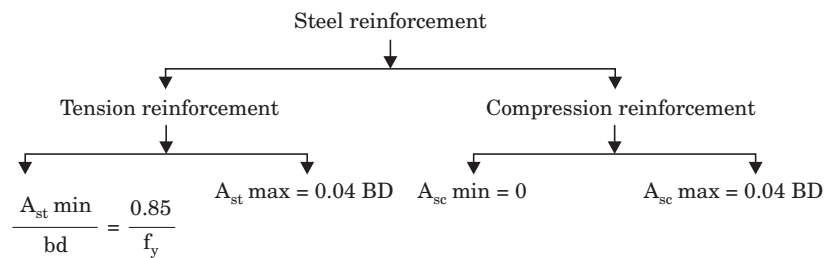
Note:

Here K_t and K_c should be calculated based on area $b_f d$.

Slenderness Limits to Ensure Lateral Stability



Steel Reinforcement

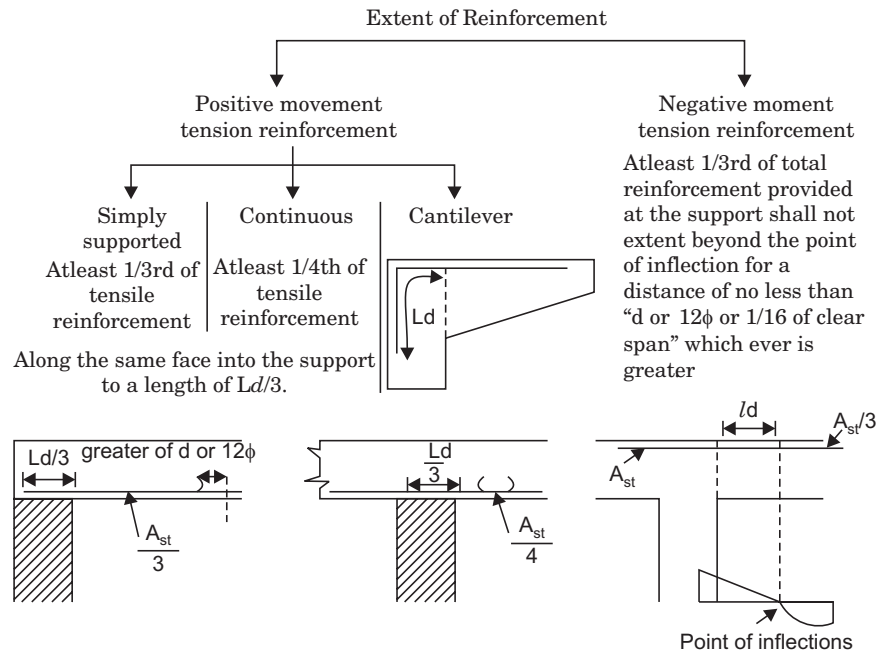


Side face reinforcement: Provided when **D > 750 mm**.

The total area of such reinforcement shall not be less than 0.1 percent of web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less.

Cover: To protect steel against corrosion

Environmental condition	Minimum grade of conc.	Nominal cover
Mild	M20	20 mm
Moderate	M25	30 mm
Severe	M30	45 mm
Very severe	M35	50 mm
Extreme severe	M40	75 mm



Arrangement of loads in Continuous beams

- (i) **For maximum moment:** Dead load on all spans while live loads on **two adjacent spans**.
- (ii) **For maximum span moment:** Dead load on all spans while live loads on **alternate spans**.

Types of load	Span moments		Support moments	
	Near middle of end span	At middle of interior span	At support next to the end support	At other interior supports
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	$+\frac{1}{12}$	$+\frac{1}{16}$	$-\frac{1}{10}$	$-\frac{1}{12}$
Imposed load (not fixed)	$+\frac{1}{10}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{9}$

Moment Coefficients

Types of load	At end support	At support next to the end support		At all other interior supports
		Outer side	Inner side	
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	0.4	0.6	0.55	0.5
Imposed load (not fixed)	0.45	0.6	0.6	0.6

Shear Coefficients

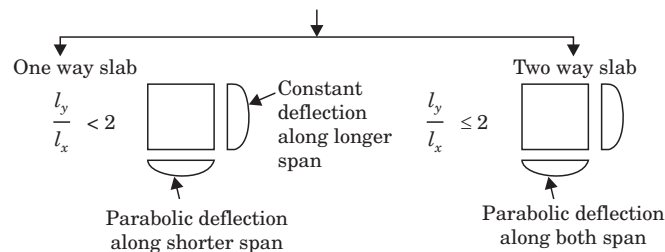
Note:

If the span of two sides of support are different or the loading are different then support moment will be calculated from both sides and average will be taken.

- Dia of reinforcement bars generally available are 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 36, 40 mm.
- Mild steel (Fe 250) is more ductile hence preferred for earthquake zones or where there are possibilities of vibration, impact, blast etc.

Slabs: Plate elements having depth much smaller than its other two dimensions.

They carry distributed load primarily by bending



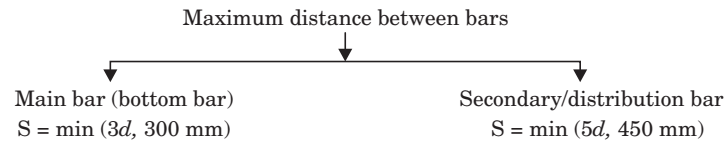
Note:

Even if $\frac{l_y}{l_x} \leq 2$ but the slab is supported only on two parallel edges then it will be treated as one way slab.

Minimum reinforcement in slabs (in either direction)

- (i) Fe 250 → 0.15% of total cross-sectional area.
- (ii) Fe 415/Fe 500 → 0.12% of total cross-sectional area.

Maximum diameter of reinforcing bar = $\frac{1}{8} \times \text{Depth of the slab}(D)$

Maximum Distance between Bars

Designs of One Way Slab

Given dimensions of room ($l_y \times l_x$), superimposed/dead load, f_y, f_{ck}

- (i) Get 'd' from deflection criteria (Let it be d_{provided}).
- (ii) Assume clear cover (20 mm) and dia of main bar (10 mm), get D.
- (iii) $L_{\text{eff } x}$ and $L_{\text{eff } y}$ from end conditions.
- (iv) Check for one way or two way slab. If one way slab then

(v) Get factored Bending moment, $M_u = 1.5 \frac{W l_{\text{eff}}^2}{8}$

where w = (dead load + super imposed load) per metre of **width**.

(vi) Check for d_{required}

$$M_u \leq 0.36 f_{ck} x_{u \text{ lim}} b (d - 0.42 x_{u \text{ lim}}) \text{ where } b = 1000 \text{ mm}$$

If $d_{\text{required}} < d_{\text{provided}}$ Adjust values of d_{provided} such that slab remains

under reinforced

(vii) Get A_{st} from, $M_u = 0.87 f_y A_{st} (d_{\text{provided}} - 0.42 x_u)$

$$\text{where } x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} (1000)}$$

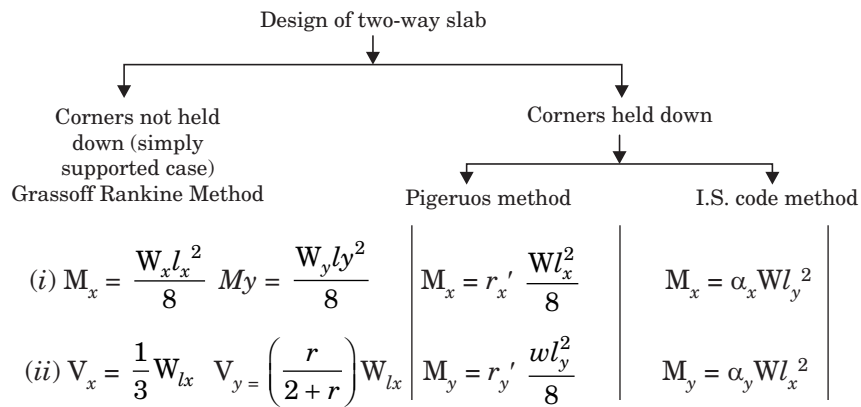
(viii) Check for $A_{st} \text{ min i.e. } A_{st} > A_{st} \text{ min}$

(ix) Spacing between bars = $\frac{1000}{\left(\frac{A_{st}}{\frac{\pi}{4} \phi^2} \right)}$

(x) Get number of bar's needed

- (xi) Provide distribution bar similarly and check its spacing as per criteria.
- (xii) Design for shear.
- (xiii) Check for development length.
- (xiv) Final diagram showing cross-section of slab and detail of bars and spacing in slab.

Design of Two way Slab



Column

8

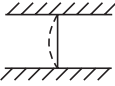
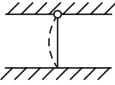
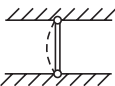
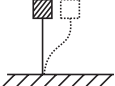
It is a compression member whose slenderness ratio is greater than 3. If slenderness ratio is less than 3 it is termed as pedestal.

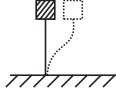
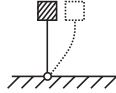
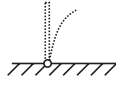
$$\text{Slenderness } (\lambda) \text{ ratio} = \frac{\text{Effective length}}{\text{Least lateral dimension}}$$

$\lambda < 12$ Short column, fails under ultimate loads by crushing.

$\lambda > 12$ Long column, fails due to large lateral deflection under relatively low compressive loads.

Effective length of column: Length between the points of contra-flexure of a buckled column.

Degree of end restraint of compression member (1)	Symbol (2)	Theoretical value of effective length (3)	Recommended value of effective length (4)
Effectively held in position and restrained		$0.50 l$	$0.65 l$
Effectively held in position at both ends, rotation at one end		$0.70 l$	$0.80 l$
Effectively held in position at both ends, but not restrained against rotation.		$1.00 l$	$1.00 l$
Effectively held in position and restrained against rotation at one end, and at other restrained against rotation but not held in position.		—	$1.2 l$

Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		$1.5 l$	$1.5 l$
Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position.		$2.00 l$	$2.00 l$
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end.			

Minimum Eccentricity

Rectangular section $e_{x, \min} = \max \left\{ \frac{l_{ex}}{500} + \frac{D_x}{30}, 20 \text{ mm} \right\}$ $e_{y, \min} = \max \left\{ \frac{l_{ey}}{500} + \frac{D_y}{30}, 20 \text{ mm} \right\}$	Non rectangular and non circular section (generally square) $e_{\min} = \max \left\{ \frac{l_e}{300}, 20 \text{ mm} \right\}$

Minimum longitudinal reinforcement

- (i) **Column:** 0.8% of gross cross sectional area
- (ii) **Pedestal with plain concrete columns:** 0.15% of gross

sectional area

(iii) **Concrete walls:**

- (a) In general, 0.15% of gross cross sectional area
- (b) If welded wire fabric or deformed bars (Fe 415/Fe 500) then 0.12% of gross cross-sectional area ($\phi < 16$ mm)
- (c) If wall thickness > 200 mm, two layers of vertical reinforcement needed.
- (d) Spacing of bars = min ($3d$, 450 mm)

Maximum Longitudinal Reinforcement

- Its 6% of gross cross-sectional area of the column.
- Can even be reduced to 4% at lapped splice locations for better placement and compaction.

Other specifications

- (i) Minimum diameter of longitudinal bar = 12 mm (but diameter should not exceed 12 mm for small sized column's *i.e.*, $D \leq 200$ mm)
- (ii) Max centre to centre spacing of reinforcement = 300 mm
- (iii) Number of bar's
 - (a) For rectangular columns = 4
 - (b) For circular columns = 6
- (iv) Longitudinal bars should be placed close to periphery for better flexural resistance.
- (v) Cover to reinforcement
 - (a) Minimum = 40 mm
 - (b) Can be reduced to 25 mm for small sized column
 - (c) Even in aggressive environment maximum cover is limited to 75 mm.

Lateral ties: Diameter of lateral ties is governed by criteria of stiffness not by strength. Hence, it is independent of grade of steel.

$$\text{Tie diameter } \phi_t \geq \begin{cases} \frac{\phi \text{ longitudinal max}}{4} \\ 6 \text{ mm} \end{cases}$$

$$\text{Tie spacing } S_t \leq \begin{cases} D \\ 16 \phi \text{ longitudinal min} \\ 300 \text{ mm} \end{cases}$$

Design of short column

- (i) Check for short column $\lambda < 12$
- (ii) If $e_{\min} \leq 0.05 D$ then its short axially loaded column
- (iii) For A_{SC}

(a) For short axially loaded column

$$P_u = 0.4 f_{CK} (A_g - A_{SC}) + 0.67 f_y A_{SC}$$

(b) For truly axially loaded column $e = 0$

$$P_u = 0.45 f_{CK} (A_g - A_{SC}) + 0.75 f_y A_{SC}$$

Note:

It can also be used when member is subjected to combined axial load and biaxial bending and also used when $e > 0.05 D$

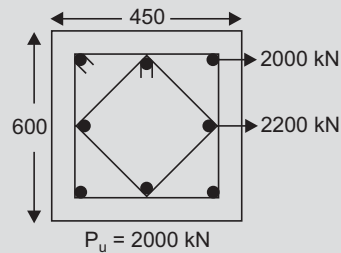
- (iv) Provide A_{SC} , check for maximum and minimum reinforcement criteria, check for minimum diameter of longitudinal bar.
- (v) Provide lateral ties check for diameter and spacing.

Note:

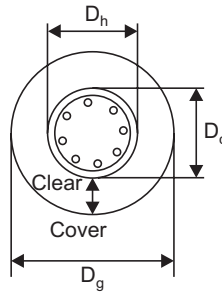
For general Idea

P_u	2000 kN	2200 kN
$B \times D$	450 × 600 mm	500 × 600 mm
Bars	4 # 25 + 4 # 20 ϕ	8 # 25 ϕ

Lateral ties are placed in BARFI cut


Short axially loaded column with helical reinforcement

$$(i) P_u = 1.05 (0.4 f_{CK} (A_g - A_{SC}) + 0.67 f_y A_{SC})$$



(ii) Diameter of helical reinforcement should be selected such that

$$0.36 \frac{f_{CK}}{f_y} \left(\frac{A_g}{A_C} - 1 \right) \leq \frac{V_h}{V_C}$$

$$A_g = \text{gross cross-sectional area} = \frac{\pi}{4} (D_g)^2$$

where,

$$D_c = D_g - 2d_c$$

d_c = clear cover to tie distance

$$V_n = \frac{1000}{p} \times \pi D_n \times \frac{\pi}{4} \phi_h^2$$

D_n = diameter of helix

ϕ_h = diameter of helical reinforcement

p = pitch of helix

V_C = Volume of core in unit length of column

$$V_C = 1000 \times A_C$$

$$(iii) \text{ pitch} < \begin{cases} \frac{\text{core diameter}}{6} \\ 75 \text{ mm} \end{cases} \quad \text{pitch} > \begin{cases} 3(\text{diameter of tie}) \\ 25 \text{ mm} \end{cases}$$

Other Specifications on Column's Slenderness Limit

(a) Unsupported length between end restrains $\nless 60$ times least lateral dimension

(b) If in any given plane one end of column is unrestrained then its unsupported length $\nless \frac{100B^2}{D}$

Design of column by WSM

(i) For short column ($\lambda \leq 12$)

$$P_u = \sigma_{SC} A_{SC} + \sigma_{CC} A_C$$

σ_{SC} = Stress in compression steel

$$A_C = A_g - A_{SC}$$

σ_{CC} = Stress in concrete

(ii) For long column

$$P_u = C_r (\sigma_{SC} A_{SC} + \sigma_{CC} A_C)$$

C_r = reduction factor

$$\lambda > 12 \quad C_r = 1.25 - \frac{l_{eff}}{48b}$$

l_{eff} = effective length

b = least lateral dimension

$$\lambda > 50 \quad C_r = 1.25 - \frac{l_{\text{eff}}}{160i_{\text{min}}}$$

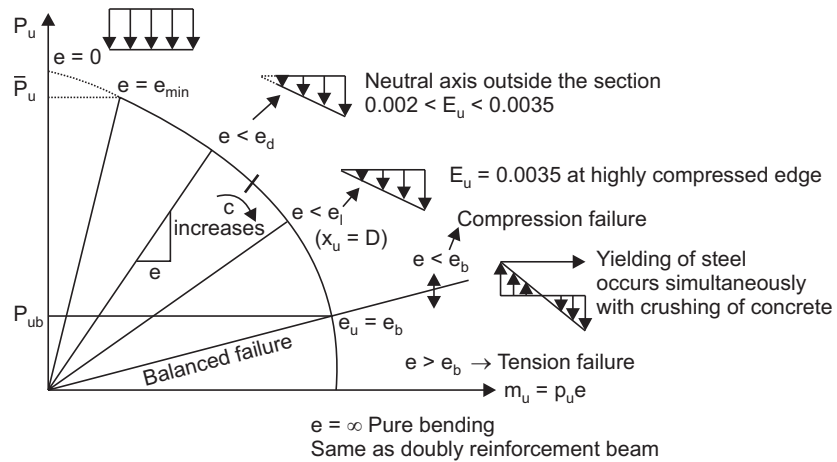
i_{min} = least radius of gyration.

$$i_{\text{min}} = \sqrt{\frac{I}{A}}$$

I = moment of inertia

A = cross-sectional area

Column Subjected to Axial Compression and Uniaxial Bending



Footing

9

Requirements of the Design of Foundation

- (i) Foundation should sustain without exceeding safe bearing capacity of the soil.
- (ii) Avoid differential settlement.

Depth of Foundation: Minimum 50 cm (IS 1080 – 1962)

As per Rankine's Formula

$$d = \frac{q_c}{\gamma} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2$$

d = minimum depth of foundation

q_c = gross bearing capacity of soil

γ = density of soil

ϕ = Angle of response of soil

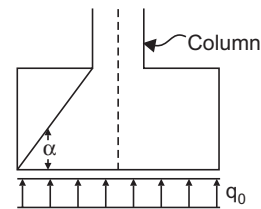
Design Considerations

- (i) Minimum Nominal Cover = 50 mm
- (ii) Minimum Thickness at the edge of Footing
 - (a) If footing rests on soil = 150 mm
 - (b) If footing rests on top of piles = 300 mm
 - (c) For plain Concrete pedestal footing without any longitudinal tension steel.

$$\tan \alpha \leq 0.9 \sqrt{\frac{100 q_o}{f_{ck}} + 1}$$

q_o = Calculated maximum bearing pressure at the base of pedestal.

f_{ck} = Characteristic strength of concrete at 28 days in N/mm²



- (iii) Critical section of maximum bending moment.

- (a) At the face of the column, pedestal or wall for footing supporting a concrete column, pedestal or reinforced concrete wall.

(b) Halfway between the centre-line and edge of the wall for footing under masonry wall.

(iv) Critical section for one way shear

Footing slab rests on	distance from face of column, pedestal or wall
Soil	d
Piles	$d/2$

(v) Critical section for two way shear → at a distance $d/2$ around the column on a perimeter.

Permissible shear stress when reinforcement is not provided should be less than $K_s \tau_c$ where

$$K_s = (0.5 + \beta_c) < 1,$$

$$\tau_c = 0.25 \sqrt{f_{CK}}$$

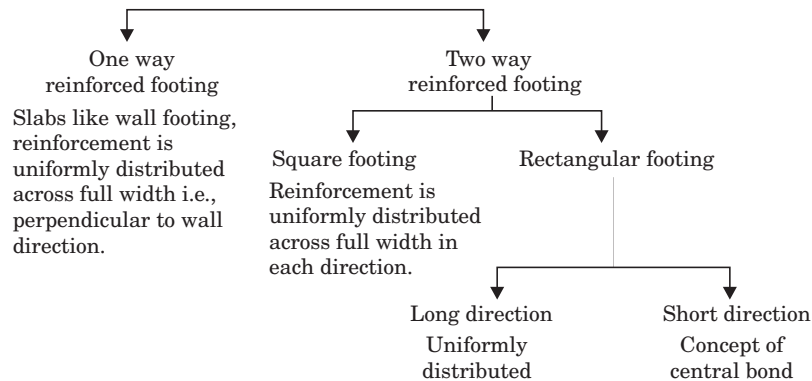
$$\beta_c = \frac{\text{Short side of column}}{\text{Long side of column}}$$

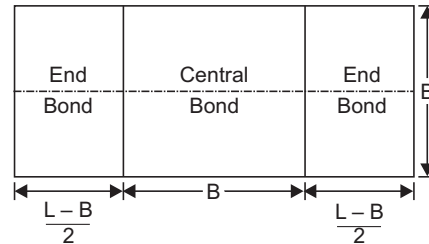
Note:

Generally, thickness of slab is governed by shear, for WSM its $K_s \times 0.16 \sqrt{f_{CK}}$

(vi) Critical section for checking development length → in a footing slab be the same planes as those of bending moments.

(vii) Tensile reinforcement

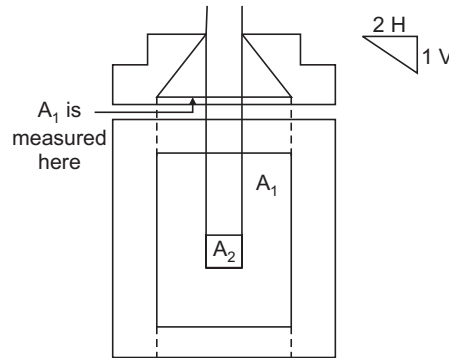


Concept of Central band

Reinforcement in central band = $\left(\frac{2}{\beta + 1} \right)$ (Total reinforcement in short direction)

where $\beta = \frac{\text{Longer dimension of footing}}{\text{Shorter dimension of footing}}$

(viii) **Transfer of load at the base of column:** Compression forces are transferred through direct bearing while tension forces are transferred through developed reinforcement.



Permissible bearing stresses on full area of concrete

(a) WSM $\sigma_{br} = 0.25 f_{CK}$

(b) LSM $\sigma_{br} = 0.45 f_{CK}$

$$\sigma_{br} = 0.45 f_{CK} \sqrt{\frac{A_1}{A_2}} \quad \text{where} \quad \sqrt{\frac{A_1}{A_2}} \leq 2$$

A_1 = maximum supporting area of footing for bearing which is geometrically similar to and concentric with loaded area A_2

A_2 = loaded area at the base of the column.

(xi) Dowel bars

- (a) Minimum number needed = 4
- (b) diameter of dowel < diameter of column bar + 3 mm
- (c) minimum area of dowel = 0.5% of gross cross-sectional area of supported column.
- (d) column bars of diameter larger than 36 mm, in compression only can be doweled at the footing with bars of smaller size of the necessary area.

Design of Footing

Given: load (p_u), Bearing capacity of soil (q_u), column size, f_{ck} , f_y

(i) Size of foundation

Column load = P , Foundation load (P_F) = $0.1 P$

Total load $P_T = P + 0.1 P = 1.1 P$ (even for LSM)

$$\text{Area required} = \text{Area} = \frac{P_T}{q_u}$$

Choose L and B such that $A_{\text{provided}} > A_{\text{required}}$

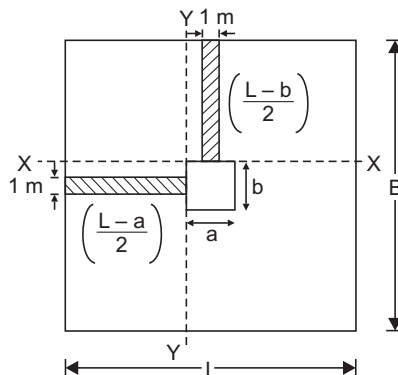
$$\text{Net soil pressure, } w = \frac{P}{A} = \frac{\text{Column load}}{\text{Area provided}} \quad (\text{WSM})$$

$$w = \frac{1.5P}{A} \quad (\text{LSM})$$

(ii) Bending moment

$$M_x = \frac{w(B-b)^2}{8}$$

$$M_y = \frac{w(L-a)^2}{8}$$



Note:

For LSM its 1.5 w

(iii) Get required depth 'd' (put $b = 1000$ mm)

$$M_{\max} = Q b d^2 \quad (\text{WSM})$$

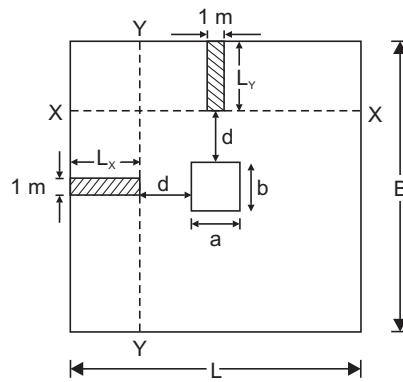
$$M_{u \max} = Q b d^2 \quad (\text{WSM})$$

(iv) One way shear check

$$V_x = W.1 \times L_y = W \left[\left(\frac{B-b}{2} \right) - d \right]$$

$$V_y = W.1 \times L_x = W \left[\left(\frac{L-a}{2} \right) - d \right]$$

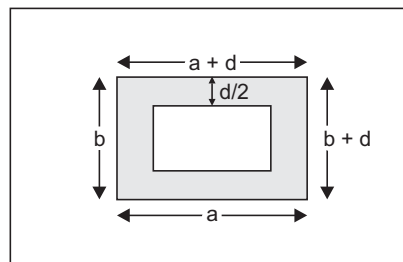
$$V_{\max} = \max(V_x, V_y)$$



Nominal shear stress $\tau_v = \frac{V_{\max}}{Bd}$ τ_c is taken from IS 456

(v) Two way shear check

$$\text{Punching stress developed} = \frac{\text{Net punching force}}{\text{Cross-sectional resisting area}}$$



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Net punching force, $P_{\text{net}} = P - W(a + d)(b + d)$

Cross-sectional resisting area = $2((a + d) + (b + d)) \times d$

Punching stress developed < Permissible punching stress

(vi) Area of steel for longer span

$$M_y = A_{st} (\sigma_{st} j d) \text{ WSM}$$

$$M_{uy} = 0.87 f_y A_{st} (d - 0.42 x_u) \text{ LSM}$$

This reinforcement is equally distributed over entire width B.

Here area of steel is calculated for 1 m width. Calculate for width B, then distribute uniformly

(vii) Area of steel for shorter span

$$M_x = \sigma_{st} A_{st} j d, \text{ WSM}$$

$$M_{ux} = 0.87 f_y A_{st} (d - 0.42 x_u), \text{ LSM}$$

This A_{st} is provided in the central band width B.

Then find total reinforcement in short direction **By central band concept formulae.**

Reinforcement in each end bands

$$= \frac{\text{Total reinforcement} - A_{st} \text{ in central band}}{2}$$

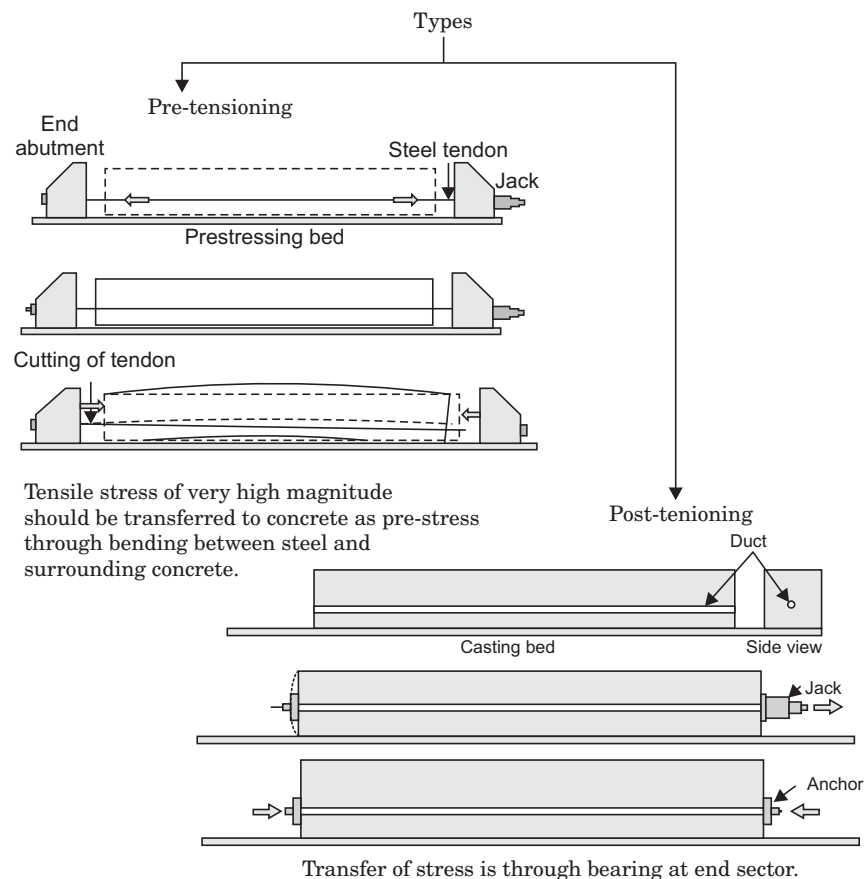
Pre-Stressed Concrete

10

A concrete in which internal stress of suitable magnitude and distribution are introduced so that stresses resulting from external load are counteracted to a desired degree.

Note:

In pipes or liquid storage tanks the hoop tensile stresses can be effectively counteracted by circular prestressing.



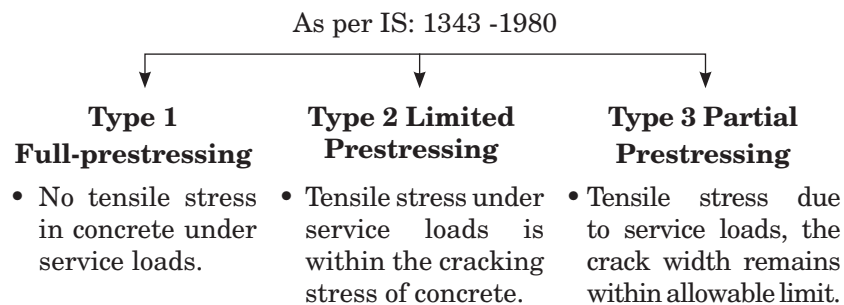
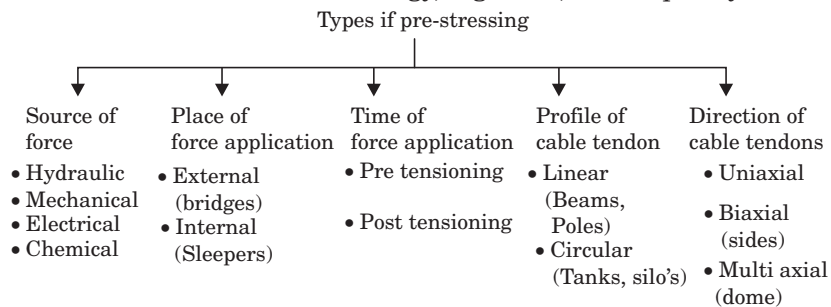
- By using high strength concrete, loss of prestress can be reduced.
- High strength concrete results in reduction of cross-sectional dimensions Hence, ultimately reduced dead weight.

Note:

Normally, due to creep and shrinkage loss in strain is approximately 0.0008 Hence, stress loss is $0.0008 \times 2 \times 10^5 = 160 \text{ N/mm}^2$. Hence, High strength steel is used

- **Advantages:** Full section is utilised as section remains uncracked at service load, shear resistance capacity is increased, high span/depth ratio is possible, increases speed of construction.

Limitations: Skilled technology, high cost, better quality control



	Pre-tensioning	Post-tensioning
Stages	Anchoring of tendons, placing jacks, applying tension, casting concrete then cutting of tendons.	Casting of concrete, placing tendons, placement of anchorage block and jack, applying tension to tendons, seating wedges then cutting of tendons

Devices Used	Prestressing bed, End Abutments, Shuttering/mould, Jack, Anchoring device, Harping device	Casting bed, mould/ Shuttering, Ducts, Anchoring devices, Jacks.
Advantages	• Suitable for Bulk production • Large Anchorage device not needed	• Heavy cast in place members can be easily post-tensioned • Less waiting period in casting bed. • Transfer of prestress is independent of length.
Disadvantages	• Prestressing bed required • Good bond necessary between transmission length.	• Requirement of anchorage device and grouting equipment.

Post-tensioning system

System (Country)	Type of tendon	Range of force	Arrangement of tendons in duct	Type of Anchorage
Freyssinet (France)	Wires and stands	Medium Large	Annular, spaced by helical wire core.	Concrete wedge
Gifford-up all-ccl (Great Britain)	Wires	Small medium	Evenly spaced by perforated spacers	Split Conical wedge
Lee-mc-call (Great Britain)	Bar threaded at ends	Small medium large	Single bars	High strength nut
Magnel-Blaton (Belgium)	wires	Small medium large	Horizontal rows of four wires spaced by metal griller	Flat steel wedge in sandwich plates

Note :- Great Britain has 'all'.

Analysis of Pre-stress and Bending Stresses

11

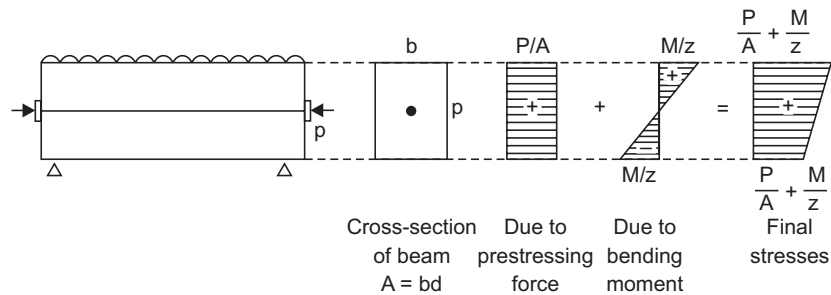
Assumptions:

1. Concrete is homogenous elastic material.
2. Withing working stresses, Hooke's law is valid.
3. Plane section before bending remains plane after bending.
4. Stress variation in steel due to external load is negligible.
5. Stress in reinforcement does not change along the length of member.

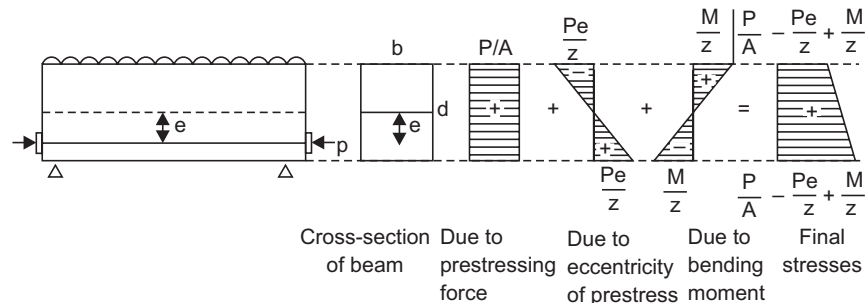
Analysis of Members Under Flexure

1. Stress concept

(a) Concentric tendon



(b) Eccentric tendon



Note:

The bending moment at which visible cracks develop in prestressed concrete members is called cracking moment. The tensile stresses developed when crack become visible at the soffit of beams depend upon the type and distribution of steel and quality of concrete in the beam.

Load factor against cracking

$$= \frac{\text{Live load required for cracking}}{\text{Live load actually acting}}$$

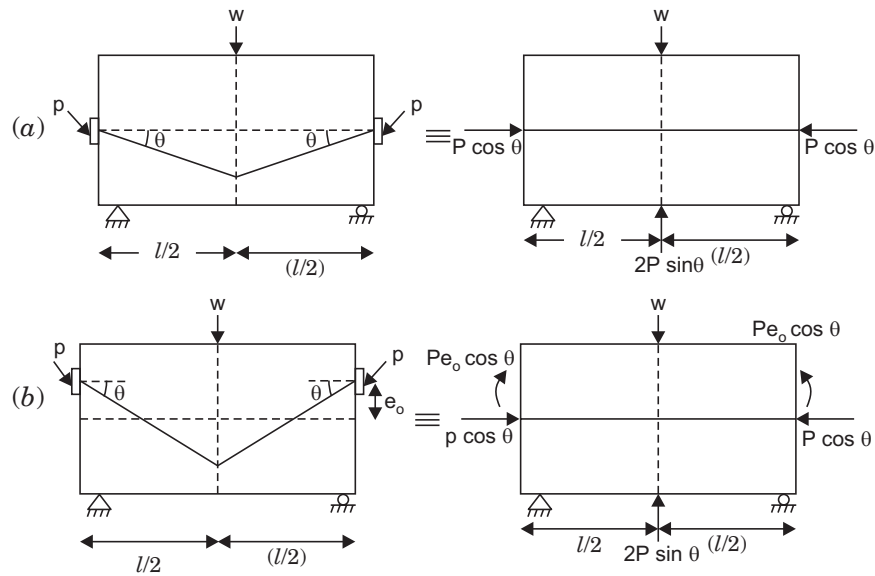
$$= \frac{\text{Live load moment required for cracking}}{\text{Actual live load moment}}$$

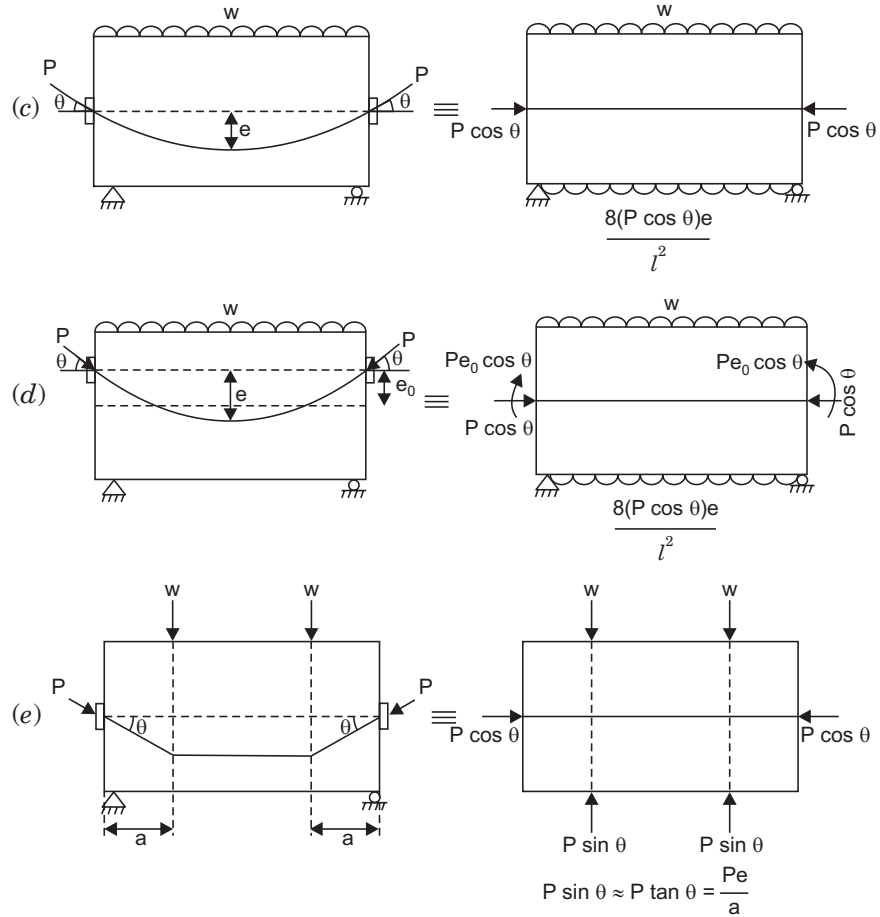
Load factor against cracking with respect to total load

$$= \frac{\text{Total load causing cracking}}{\text{Total load actually acting}}$$

$$= \frac{\text{Total Bending moment required to cause cracking}}{\text{Total Bending moment actually acting}}$$

2. Load Balancing Concept: Bent tendon exerts an upward pressure on the concrete beam and will therefore counteract a part or whole of the external downward loading.



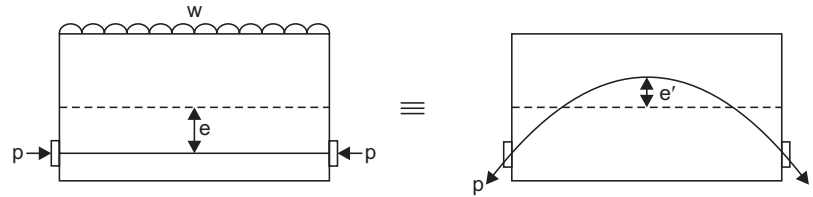


Note:

Equation of parabolic profile is $y = \frac{4ex(l-x)}{l^2}$

Type of load	Bending moment diagram	Cable profile
Point load	Triangular	Triangular
udl	Parabolic	Parabolic
Two point load	Trapezoidal	Trapezoidal

- 3. Thrust line or pressure line concept:** The combined effect of prestressing force ($P/A + Pe/Z$) and externally applied load (M/Z) will result in a distribution of concrete stresses that can be revolved into a single force and the locus of point of application of resultant force is called pressure or thrust line.



$$\sigma_{b/t} = \frac{P}{A} \pm \frac{Pe}{Z} \mp \frac{M}{Z}$$

↓

$$\sigma_{b/t} = \frac{P}{A} \pm \frac{Pe}{z} \mp \frac{P}{Z} \left(\frac{M}{P} \right) \quad \rightarrow$$

$$\sigma_{b/t} = \frac{P}{A} \mp \frac{Pe'}{Z}$$

↑

$$\sigma_{b/t} = \frac{P}{A} \mp \frac{P}{Z} \left(-e + \frac{M}{P} \right)$$

$$\text{So, } e' = -e + \frac{M}{P} \Rightarrow e + e' = \frac{M}{P}$$

Note:

As P is a constant force, hence if

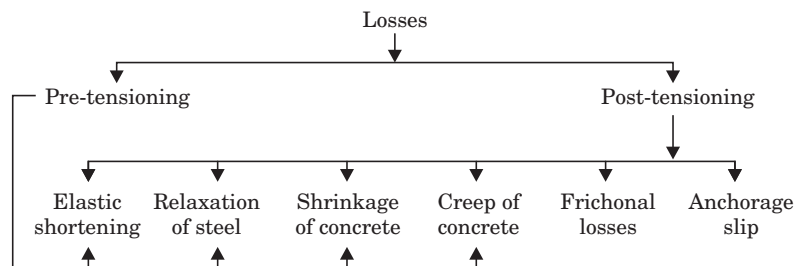
e	M	then e'
Constant	Parabolic	Parabolic
Constant	Linear	Linear
Parabolic	Parabolic	Parabolic if $\frac{M}{P} \neq e$ zero if $\frac{M}{P} = e$

- In prestressed section load carrying mechanism consist of constant force (F) and changing lever arm while in reinforced sections it consists of changing force with a constant lever arm.

Losses in Pre-stress

12

Various reductions of prestressing force are termed as losses in prestress.



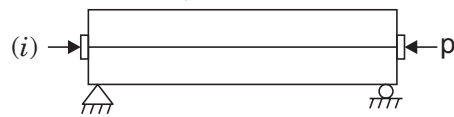
- Elastic shortening, frictional losses, Anchorage slip are short term losses where as other losses are time dependent.
- In Post-tensioning there is no loss of elastic shortening when all bars are simultaneously tensioned.
- There may be losses due to sudden change in temperature, especially in steam curing of pretensioned units.
- Total loss in Pretensioning is more than post tensioning.

1. Elastic Shortening Loss

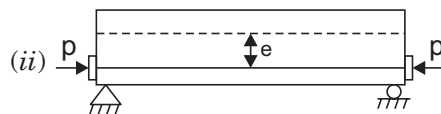
(a) Pre-tensioned members, Pre-stressing loss = $m \times f_{c \text{ average}}$

where m = modular ratio

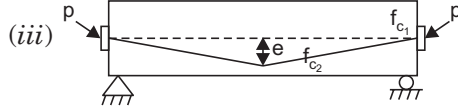
$f_{c \text{ average}}$ = stress in concrete at the level of tendon



$$\Delta f_s = m \left(\frac{P}{A} \right) \text{ as } e = 0$$



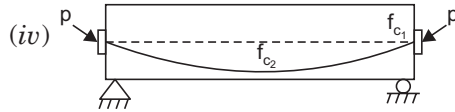
$$\Delta f_s = m \left[\frac{P}{A} + \frac{Pe.e}{I} - \frac{M_d e}{I} \right]$$



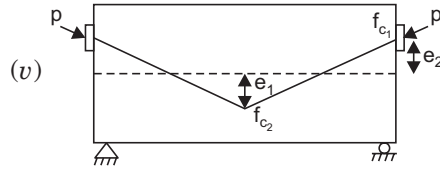
$$\Delta f_s = m \left(\frac{f_{c1} + f_{c2}}{2} \right)$$

$$f_{c1} = \frac{P}{A}$$

$$f_{c2} = \frac{P}{A} + \frac{Pe \cdot e}{I} - \frac{M_d e}{I}$$



$$\Delta f_s = m \left[f_{c1} + \frac{2}{3}(f_{c2} - f_{c1}) \right]$$



$$\Delta f_s = m \left[\frac{f_{c1} + f_{c2}}{2} \right]$$

$$f_{c1} = \frac{P}{A} - \frac{P_{e_o} \cdot e_o}{I}$$

$$f_{c2} = \frac{P}{A} + \frac{P_{e1} \cdot e1}{I} - \frac{M_d e1}{I}$$

(b) **Post-tensioned members:** In case of single tendon, there is no loss as the applied prestress is recorded after the elastic shortening of the member. However, when bars are successively tensioned and anchored then losses will occur as

- (i) when first bar tensioned, no loss
- (ii) when second bar tensioned, no loss in 2nd but loss in 1st bar
- (iii) when third bar tensioned, no loss in 3rd but loss in 1st and 2nd bar,

The same concept of averaging will be used when bar's are not stretched simultaneously.

2. **Relaxation of steel:** Decrease in stress with time under **constant strain**. It depends on type of steel, temperature and initial prestress, normally, it is taken as **2 to 5%** of initial stress in steel.

3. Shrinkage of concrete

$$\text{Loss of stress} = \varepsilon_{ss} \times E_s$$

$$\varepsilon_{ss} = \text{Total residual shrinkage strain}$$

$$= 3 \times 10^{-4} \text{ for pretensioning,}$$

$$= \frac{2 \times 10^{-4}}{\log_{10}(t + 2)} \text{ for post tensioning,}$$

$$t = \text{age of concrete at transfer in days.}$$

$$E_s = \text{Modulus of elasticity of steel.}$$

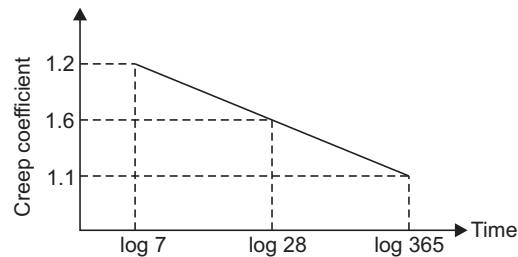
4. Creep of concrete:

$$\text{Loss of stress} = m \phi f_c$$

$$\phi = \text{creep-coefficient} = \frac{\text{ultimate creep strain}}{\text{elastic strain}}$$

$$m = \frac{E_s}{E_c}$$

Creep losses are generally 2–3% of initial prestressing force.

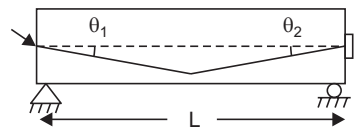
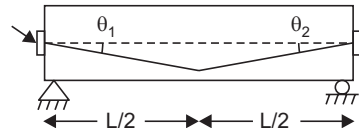


5. Frictional losses:

$$\text{Loss of stress} = \text{Initial stress} (\mu \alpha + Kx)$$

μ = Coefficient of friction in curve (Generally between 0.25 to 0.55)

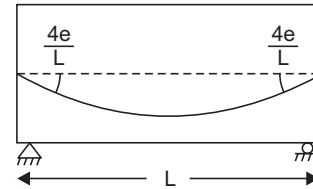
K = Wobble correction factor (0.0015 to 0.0050 per metre length of tendon)

		x	α
Jacking from one end		L	$\theta_1 + \theta_2$
Jacking from both ends		$\frac{L}{2}$	$\max(\theta_1, \theta_2)$

For parabolic profile,

Jacking at one end, $\alpha = 2\theta = \frac{8e}{L}$

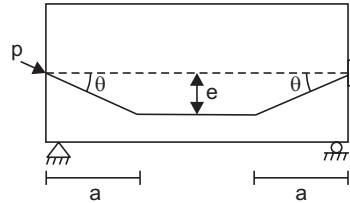
Jacking from both ends $\alpha = \theta = \frac{4e}{L}$



- For trapezoidal profile

Jacking at one end $\alpha = 2\theta = \frac{2e}{a}$

Jacking from both ends $\alpha = \theta = \frac{e}{a}$



Note:

In Pre tensioned members, as there is no concrete during stretching of tendons hence this loss doesn't occur.

In post tensioned members, frictional loss is generated due to **curvature of tendon** and **vertical component of prestressing force**.

6. Anchorage slip:

$$\text{Loss of stress} = \left(\frac{\Delta}{L} \right) E_s$$

E_s = young modulus of steel in N/mm²

Δ = Anchorage slip in mm

L = Length of cable in mm

Note:

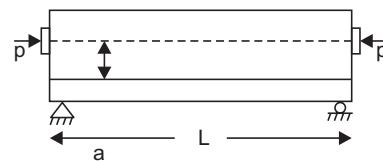
The percentage loss due to anchorage slip is higher for shorter members as compared to longer members.

Deflection of Pre-stressed Beam

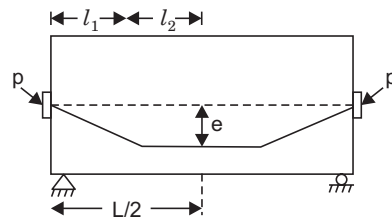
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Short term deflection under **uncracked condition** can be computed using elastic theory by using **area moment method** (Mohr's method). Concrete beam deflects **upwards** on the application or transfer of prestress.

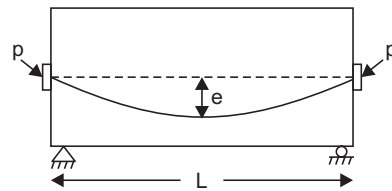
Bending moment at any section is the product of prestressing force and eccentricity at that section.



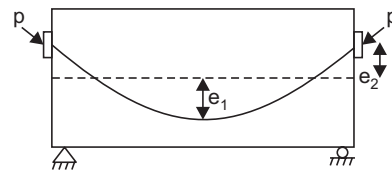
$$\Delta = \frac{PeL^2}{8EI}$$



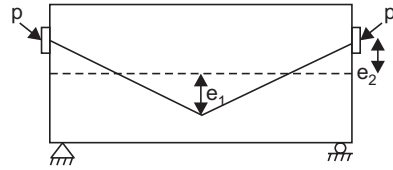
$$\Delta = \frac{-Pe}{6EI} (2l_1^2 + 6l_1l_2 + 3l_2^2)$$



$$\Delta = \frac{-5}{48} \frac{PeL^2}{EI}$$



$$\Delta = \left[\frac{-5}{48} \frac{PL^2}{EI} (e_1 + e_2) \right] + \left[\frac{Pe_2L^2}{8EI} \right]$$



$$\Delta = \left[\frac{-PL^2}{12EI} (e_1 + e_2) \right] + \left[\frac{Pe_2L^2}{8EI} \right]$$

Note:

Downward deflection due to self wt or imposed load is $\frac{5w}{384} \frac{L^4}{EI}$

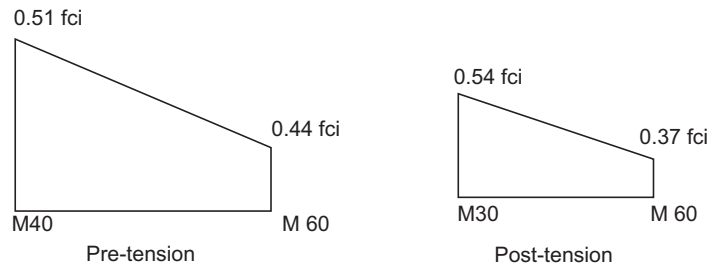
IS Code Recommendations for Pre-stressed Concrete

14

- (i) Grade of concrete
M 40 for pre tensioned members
M 30 for post tensioned members
- (ii) **Design mix:** Only 'design mix concrete' can be used with cement content preferably less than 530 kg/m³
- (iii) Flexure tensile strength $F_{cr} = 0.7\sqrt{f_{ck}}$
- (iv) Short term modulus of elasticity $E_c = 5000\sqrt{f_{ck}}$
- (v) Modulus of elasticity of steel

Type of steel	E_s (kN/mm ²)
Plain cold drawn wires	210
High tension steel bars rolled or heat treated	200
Strands	195

- (vi) Allowable stresses in concrete
 - (a) Allowable compressive stresses under flexure



where fci = cube strength at transfer

- (b) **Allowable compressive stresses under direct compression:** 80 % of the compressive stress under flexure

(c) Allowable tensile stresses under flexure

Type 1	Number tensile Stress
Type 2	3N/mm ² can be increased to 4.5 N/mm ² for temporary loads
Type 3	Table 8 of IS 1373 : 1980 provides hypothetical values

(d) **Maximum Initial Prestress:** Should be such that, maximum tensile stress immediately behind the anchorage should not exceed 80% of the ultimate tensile strength of wire

(vii) **Minimum cover:**

Pre tensioned work	200 mm
Post tensioned work	max (30 mm, size of cable)

Note:

For pre tensioned work in aggressive environment cover shall be increased by 10 mm.

(viii) Spacing of tendons

(a) For single wires = $\max \left(3d, \frac{4}{3} (\text{max aggregate size}) \right)$
where d = dia of wire

(b) For cable or large bars = $\max (40 \text{ mm, max size of cable, } 5 \text{ mm} + \text{max aggregate size})$

(c) For grouped cables

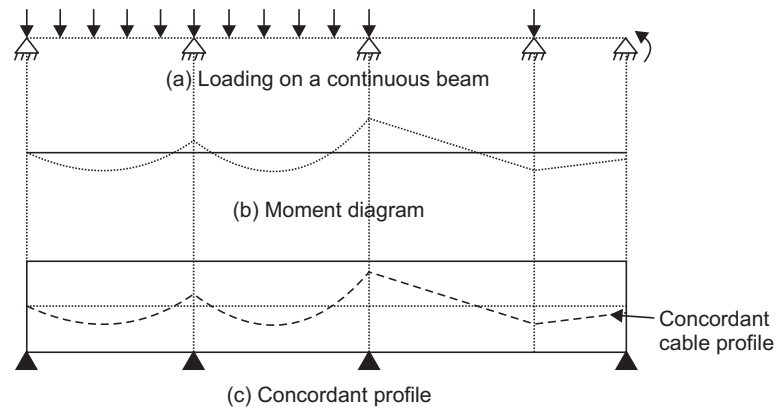
(i) Minimum horizontal spacing = $\max (40 \text{ mm, } 5 \text{ mm} + \text{max aggregate size})$

(ii) Minimum vertical spacing = 50 mm.

(ix) maximum deflection allowable

(a) Final deflection due to loads including effect of temp, shrinkage, creep	$\frac{\text{span}}{250}$
(b) Deflection including effect of temp. shrinkage, creep accruing after partition erection and application of finishes	$\max \left(20 \text{ mm, } \frac{\text{span}}{250} \right)$

- (x) **Concordant cable profile:** For continuous beams prestressing generates reactions at the support. These reactions cause additional moments along the length of beam (secondary moments). If the profile of cable is properly selected such that it does not produce reactions at the support or secondary moments in the span then it is called concordant cable profile.



Civil Engineering

5

CPM

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3. Programme Evaluation Review Technique
4. Critical Path Method
5. Crashing
6. Updating and Resource Allocation
7. Precedence Diagram
8. Engineering Economy
9. Fundamental of Equipments
10. Earthwork Equipment
11. Hoisting Equipment's
12. Compacting Equipment

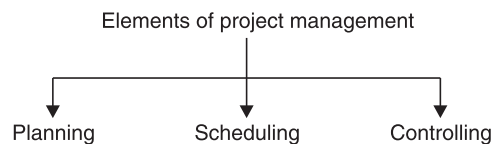
Project Management

1

Project management is the art of organizing the components of a project, whether the project is development of a new product, launch of new service or a wedding

Objectives of Project

- (a) To complete project in minimum time period
- (b) Use of available manpower and local resources.
- (c) Completion of project with minimum cost and without delay.



1. Project planning:

	Tools for planning
Setting goals	Time and cost ultimates
Defining the project	Budgets
Tying needs into timed project	Cash flow charts
Activities	Material availability details
Organising the term	Engineering diagrams

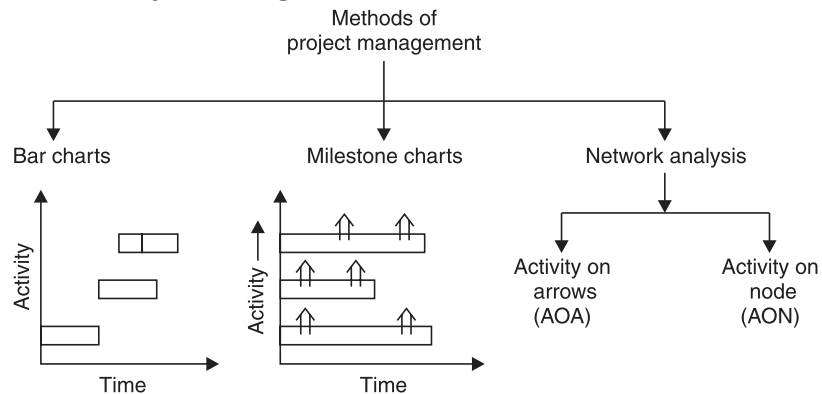
2. Project Scheduling:

	Tools for scheduling
Assigning resources	CPM and PERT
Relating activities to each other	Milestone charts
Updating and revising	Cash flow schedules
	Gantt chart

3. Project controlling

	Tools for controlling
Monitoring resources	PERT chart
Revising and changing plans	Reports of delayed activities, Slack activities and Quality of work completed
Shifting resources to meet time, Cost and quality demands	

Methods of Project Management



- 1. Bar charts:** Also known as **Gantt chart**. Introduced by **Henery Gantt** in 1900 AD

Advantages:

- Simple to draw
- No special skill required
- Progress expressed as percentage
- Resource requirement can be depicted
- Visual representation of the project

Limitations

- Lack of degree of details
- Review of project progress
- Activity inter-relationship not shown
- Uncertainty in time determination
- Critical activities not shown
- Optimum cost cannot be found out

- 2. Milestone charts:** Milestone can be considered to be specific event along the main activity. By milestone charts, the relationship b/w two specific milestones on an activity is revealed where as interdependency among milestones between different activities is not indicated.

Note:

Bar chart is activity oriented where as milestones chart is event oriented.

- 3. Network analysis:** It is a graphical and logical model or plan which lists out the sequence of various activities (with interdependencies) which are required for completion.

Note:

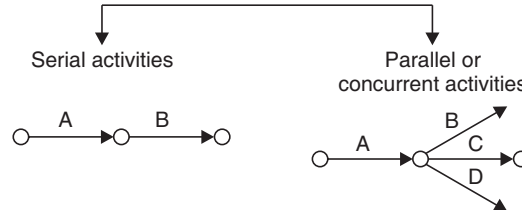
AON is widely used for repetitive task where as AOA are considered better for complex projects where there is not a high degree of repetitive work.

Fundamentals of Network

2

Network is a flow diagram consisting of activities and events connected **logically** and **sequentially**.

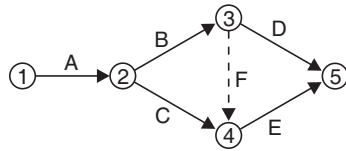
Activity: A project can be broken into various jobs in the form of operation's and each operation **consuming time or resource** is called an activity. It is denoted by an arrow, where length of arrow has no significance. Arrows are usually drawn from left to right.



Event: It's an **instant of time** at which some specific milestone has been achieved. An event **does not consumes any time or resource**. It denotes the start or completion of an activity, and is represented as nodes of different shapes like circular, rectangular, square or oval.

Tail event	Head event	Double role event
1 is tail event for activity <i>A</i> 2 is tail event for activity <i>B</i> i.e., Beginning of an activity	2 is the head event for <i>A</i> 3 is the head event for <i>B</i> i.e., end of an activity	Acts as tail event for some activities (<i>D</i> , <i>E</i>) and head event for some activities (<i>A</i> , <i>B</i> , <i>C</i>)

Dummy Activity: It is an **artificial** activity represented by dashed arrows which **neither consumes any time for resources**.



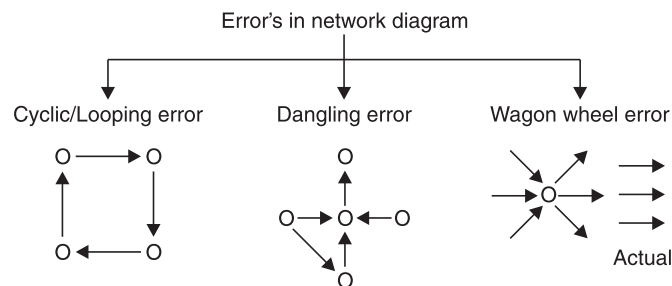
Activity *F* is a dummy activity.
Dummies maintain the logic of network diagram and keeps the numbering system of the network unique.

Rules for providing dummies:

1. If dummy is the only activity coming out from a node and terminating into some other node. The dummy can be removed and the two nodes at the two ends of dummy can be merged into one.
2. If dummy is the only activity terminating into a node, then dummy can be removed and the nodes at the two ends of the dummy can be merged into one.
3. If two or more activities, emanating from different nodes and having same set of preceding activities then the two activities should emanate from a single node.
4. If two or more activities having same set of successors and succeeding activities are having other predecessors as well, the two activities should terminate into one single node.
5. Dummy activities which shows predecessor relations already implied by other activities are called as Redundant dummies and can be removed.

Note:

System of numbering of events in a network diagram in order to maintain the logical sequence is given by FULKERSON'S Rule.

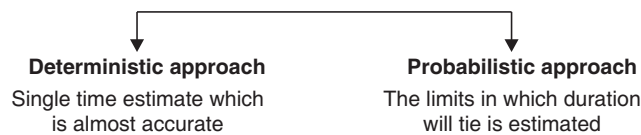


Programme Evaluation Review Technique

3

PERT was developed by **US Navy** in late 1950's to accelerate the development of Polaris Fleet Bullastic missile.

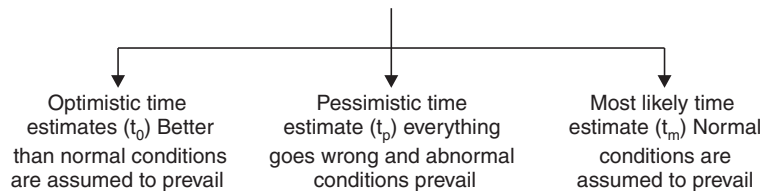
Total project duration depends on time taken by each activity and estimation of these time estimates can be done in two ways



Note:

PERT follows the probalistic approach and absorbs uncertainties into the time estimates for activity and project duration.

Three time estimates for activities in PERT



Mean time ($\bar{x}$):

$$\bar{x} = \frac{x_1 + x_2 + x_3 \dots + xn}{n} = \frac{\Sigma x}{n}$$

Deviation (δ):

$$\delta = x - \bar{x}$$

δ = deviation of any value x

x = value under consideration.

Variance (σ^2):

$$\sigma^2 = \frac{\Sigma \delta^2}{n} = \frac{\Sigma (x - \bar{x})^2}{n}$$

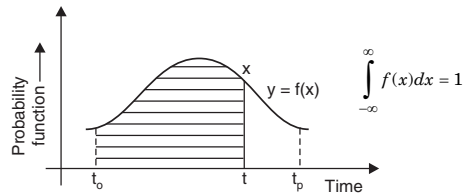
Standard deviation (σ):

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$$

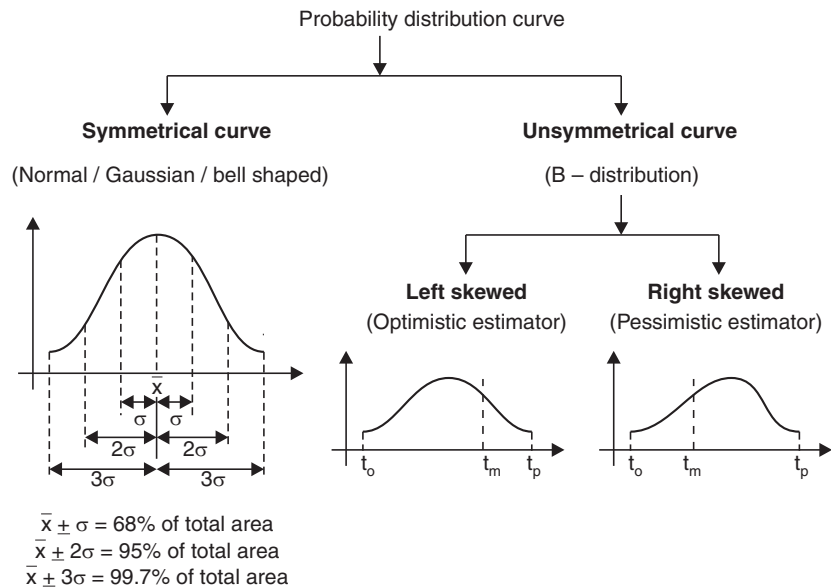
Note:

Variance is the measure of uncertainty. Greater the variance, greater will be uncertainty.

Probability distribution: It is the plot of probability density function $f(x)$ with the height of curve standardized such that area under the curve is equal to unity.



Probability at any point x is equal to the shaded area only as the total area is equal to unity.



Expected time (t_e) of an activity: The average of mean time taken for the completion of an activity is called expected time. It is calculated by weighted average method. and the probability of completion of work on expected time is 0.5.

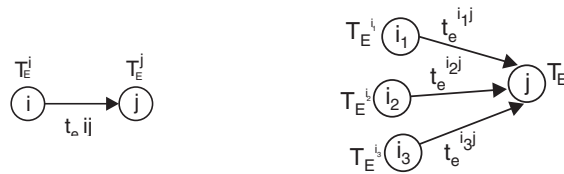
$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Note:

According to central limit theorem, in a network if number of activities are in series (as in critical path), the expected time for the path along the activities is the sum of the expected time of activities.

Event Time

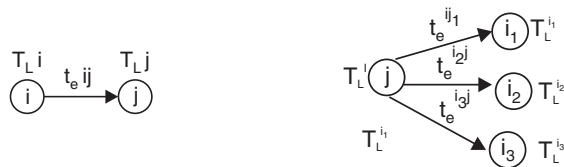
- 1. Earliest expected event occurrence time (T_E):** Minimum time in which an event can occur. It is determined by forward pass method.



$$T_E^j = T_E^i + t_{eij}$$

$$T_E^j = \text{maximum} \begin{cases} T_E^{i_1} + t_{e^{i_1j}} \\ T_E^{i_2} + t_{e^{i_2j}} \\ T_E^{i_3} + t_{e^{i_3j}} \end{cases}$$

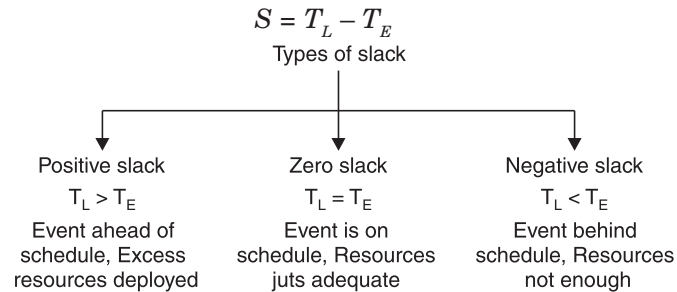
- 2. Latest allowable occurrence time (T_L):** Maximum time of an event by which it may be allowed to occur without affecting the completion time of project. It is determined by backward pass method.



$$T_L^i = T_L^j - t_{eij}$$

$$T_L^i = \text{Minimum of} \begin{cases} T_L^{j_1} - t_{e^{ij_1}} \\ T_L^{j_2} - t_{e^{ij_2}} \\ T_L^{j_3} - t_{e^{ij_3}} \end{cases}$$

Slack: It is associated with an **event**. It is the difference between the latest allowable occurrence time and earliest expected occurrence time of the event



Critical Path: Longest path in terms of time or most time consuming path from beginning to end of the network OR It is the shortest path in terms of time in which the project can be completed as early as possible. Events on critical path will have zero or minimum slack.

Note:

Most critical path in a network diagram having multiple critical path will be the critical path which has larger variance (or larger standard deviations) along the path.

Probability of Completion of Project (Steps)

1. Determine σ along critical path

$$\sigma = \sqrt{\sum \left(\frac{t_p - t_o}{\sigma} \right)^2}$$

Where t_p and t_o belong to activities lying on critical path only

2. Determine probability factor or normal deviate (Z)

$$Z = \frac{T_S - T_E}{\sigma}$$

T_S = Scheduled completion time,

T_E = Expected completion time

where

$$T_E = te_A + te_B + te_C + Te_D + \dots + te_Z$$

A, B, C ... Z are activities on critical path.

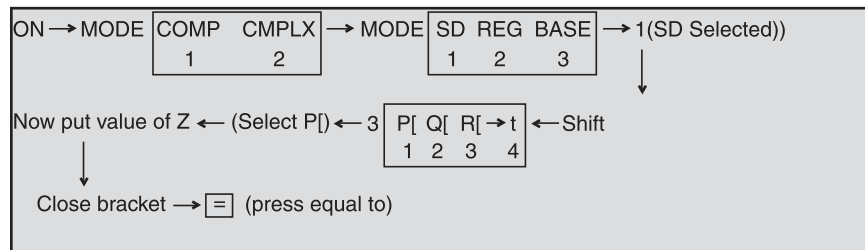
3. Find % Probability with respect to normal deviate from table or calculator.

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Note:

Calculation of probability from CASIO Fx-991 MS (where SD = std. deviation and P = probability)

ON → MODE → MODE → 1 → Shift → 3 → 1 → Now put value of Z → Close bracket



Critical Path Method

4

M.R. Walker and **J.E. Kelley** of Ramination Rand developed CPM for the constriction of chemical plants in united states.

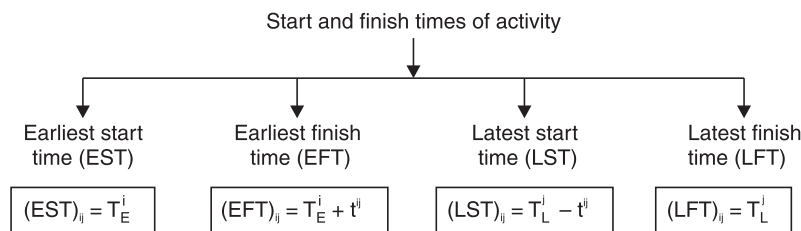
It is used in construction projects when there is prior experience in handling similar projects from which relationships between resources and job time are available.

Comparison between PERT and CPM

PERT	CPM
1. Probabilistic approach	1. Deterministic approach
2. Event oriented	2. Activity oriented.
3. Three time estimate	3. Single time estimate
4. β -distribution	4. Normal distribution
5. Cost is proportional to time	5. Cost model is developed
6. Concept of slack	6. Concept of float
7. Critical path by joining critical events	7. Critical path by joining critical activities.

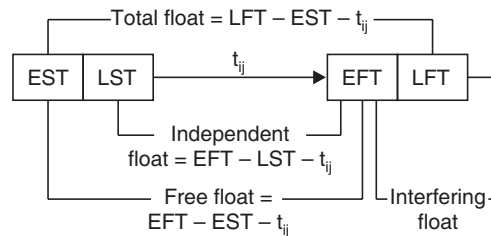
Note:

Time estimate of an event has greater range of variation hence greater uncertainty and therefore probabilistic approach is used. While the time estimate of an activity has narrow range of variation hence the single timed deterministic approach is used.

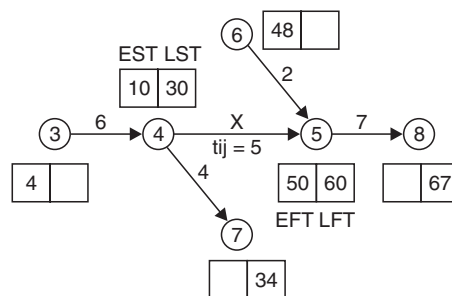


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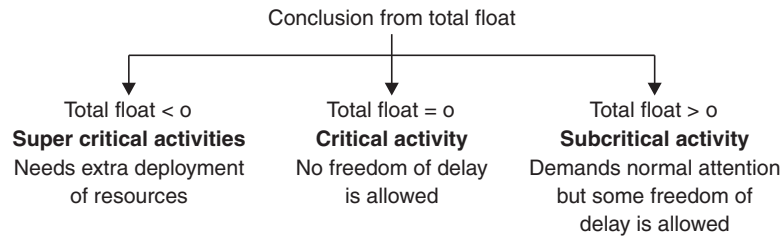
Float: It indicates the time by which, starting or finishing of an activity can be delayed without affecting the project completion time.



- 1. Total Float:** It is the difference between **maximum time available** and actual time required for the completion of the activity. It is the measure of particular activity but depends on the succeeding and preceding activities.
- 2. Free Float:** It is the amount of time by which an activity can be delayed **without affecting the EST of the succeeding activity**. Free float is the amount of time an activity can be delayed without affecting the commencement of a subsequent activity at its EST but may affect the float of a previous activity.
- 3. Independent Float:** It's the excess of minimum available time over the required activity duration.
- 4. Interfering Float :** It is the difference of total float and free float of an activity. It is also equal to the slack of head event of head activity.



For activity 'X' of 5 day duration.
 Total float = $60 - 10 - 5 = 45$
 Free float = $50 - 10 - 5 = 35$
 Independent float = $50 - 30 - 5 = 15$
 Interfering float = $60 - 50 = 10$
 or $TF - FF = 45 - 35 = 10$

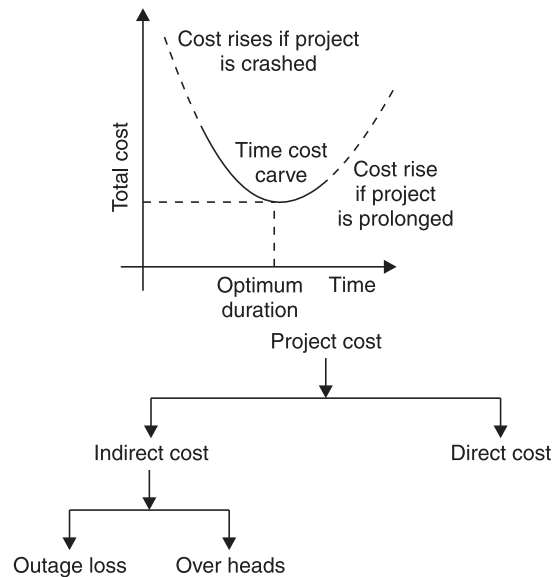
**Note:**

For critical path, all the four floats are generally zero because critical path passes through those events where slack is zero and this is only a necessary condition but it is not sufficient condition.

Crashing

5

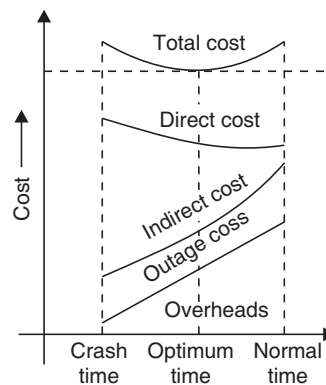
Cost Mode Analysis: It shows relationship of the cost versus the completion time. Project time corresponding to minimum value of the cost function gives the most optimum duration of the project



Indirect cost of the project are those expenses which cannot be associated or assigned to any individual activity of the project

Outage loss: Loss in profit, liquidity damage, or penalty for delay in project completion.

Overhead charges: Establishment charges, Insurance charges, expenditure for maintenance of service.



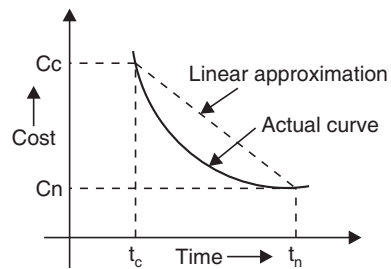
Note:

Indirect cost always increases with time. Hence in crashing when we reduce time, our indirect cost is saved although it increases non linearly with time but for ease in calculation we assume that indirect cost varies linearly with time.

Direct cost: Direct cost of the project are those expenses which are directly chargeable and can be identified by **activities** e.g., cost of material, machine, labour etc. The project has highest cost corresponding to crash duration and normal cost corresponding to normal duration.

Cost Slope of Direct cost Curve

1. **Normal time (t_n):** It is the standard time that a estimator will assign to an activity
2. **Crash time (t_c):** It is the minimum possible time in which an activity can be completed by assigning extra resources. OR it is that time before which an activity cannot be completed even by application of infinite number of resources.
3. **Normal Cost (C_n):** It is the direct cost of an activity associated with the normal time.
4. **Crash cost (C_c):** It is the direct cost of an activity corresponding to crash time



$$\text{Cost slope} = \frac{C_c - C_n}{t_n - t_c}$$

Crashing potential of an activity = Normal time – Crash time

Cost slope indicates the increase in direct cost when the activity is reduced by one day.

Note:

Optimum project duration is the time corresponding to that point on the total cost VS time curve where the tangent drawn to the curve is horizontal.

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Crashing : The process of reduction of the total project duration along the **longest path** (time wise) of the network i.e., along the critical path to obtain the optimum project cost and optimum duration.

Important Points in Crashing

1. Crashing is always done of the activities lying on critical path.
2. Activity which has least cost slope is to be crashed first.
3. Lost of the project increases by direct cost invested and reduced by indirect cost saved.

Updating and Resource Allocation

6

The process of reviewing the progress of project, execution and redrafting the network according to latest requirements is called **updating**.

When to Update:

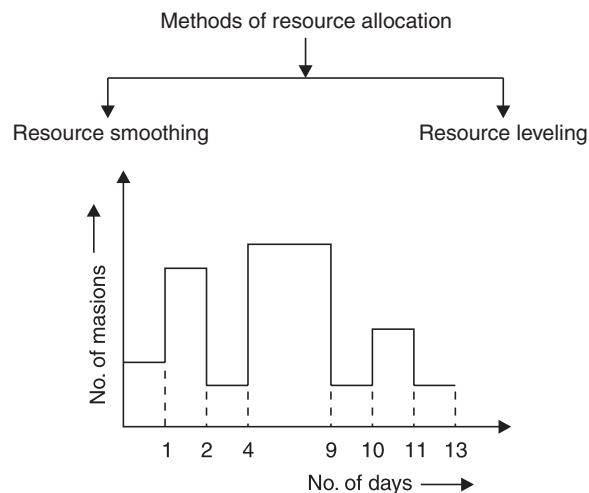
1. It is done whenever there is a major change in the duration of any of the activity.
2. For smaller project, frequent updating is necessary because any slippage will have large impact on project duration.
3. For large project, the process of updating must be increased as the project progresses towards the completion.

Note:

Updating is mandatory for critical activities.

- During updating, activities are neither added or deleted
- Critical path as well as critical activities may get changed.
- On any major change in duration, updating becomes compulsory.

Resource: It is the physical variable required for completion of activities. It can be material, man power, machinery, space, money or time. Resource allocation is deciding the resources to each activity.



Note:

Fluctuations in demand of easily available resource like unskilled labour or some material will not cause much effect of the project. But if it is a rare available resource like skilled professional or some special equipment then they have to be hired or employed on permanent or semi-permanent basis.

Resource Smoothing : In this the total project duration is not changed but the activities having float are rescheduled such that a uniform demand for the resources is achieved. In this the resources are considered to be unlimited and the critical path of the project remains unchanged.

Resource Levelling : In resource levelling, the activities are so rescheduled such that maximum or peak demand of the resource does not exceed the availability of resources. In levelling, the resources are limited and the critical path may get changed (so project duration will also get changed).

Note:

smoothing → Floats are re-scheduled.

levelling → Resources are limited.

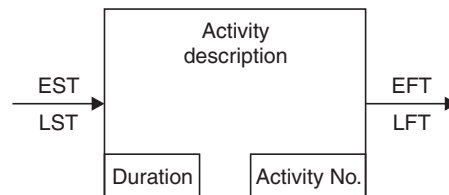
Precedence Diagram

7

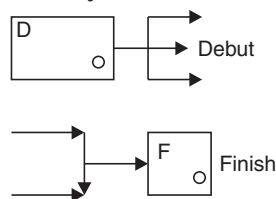
It is activity on nodes network system (AON) where nodes represent the **activities** and the arrow represent their interdependence or precedence relation ship. It represents the logical and sequential inter-relationship between various components or activities.

Representation of Nodes:

Usually square or rectangular shapes are to be used but circles and other geometrical shapes can also be used.



When a number of activities start simultaneously, an activity called **DEBUT** (D) is provided as the first activity, though such an activity may not exist similarly when a number of activities finish together an activity is called **FINISH** activity.



Note:

Both Debut and Finish activities do not consume time.

Advantages of AON over AOA

1. It Eliminates use of dummies
2. Easy revision and modification.

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3. Pre and post operation activities are clearly depicted.
4. It is self sufficient and self explanatory as all activity times are shown on network itself
5. Overlapping and separation of activities is easily shown

Note:

As Events have no place in AON network, hence it cannot be used in PERT analysis.

Time Value of Money: It means that two equal amount at different points of time do not have equal value if the interest rate is greater than zero. Therefore the value of money is not constant. The rate of interest is used to determine the difference between what is borrowed and what is to be repaid.

Interest: It is the rent paid for use of resource. It is expressed as a percentage of principal amount of an asset or a resource for its use over a period of time.

Simple Interest: When the total interest earned or charged is directly proportional to the principal involved, the interest rate and the no of interest periods for which the principal is committed, the interest is called simple Interest.

$$SI = P \times n \times i$$

SI = Simple Interest

P = Principal

n = no of years

i = rate of interest per year.

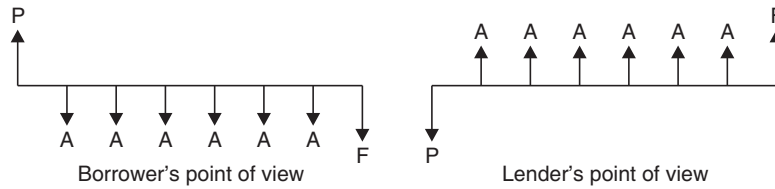
Compound Interest: Whenever the interest charged for any interest period is based on the remaining principal amount plus any accumulated interest charges upto the beginning of that period, the interest is said to be compound interest.

Note:

During simple interest calculation, principal amount remains same as the 1st basic principal amount. But during compound interest principal amount varies as the interest period varies.

Cash Flow Diagram: The graphical presentation of each value plotted at appropriate time is called cash flow diagram. In cash flow diagram, usually time is drawn on horizontal axis on an appropriate scale here as the y-axis represents the amount involved in the transaction, with the receipts (benefit) on the positive side and disbursement (cost) being drawn on the negative side of the y-axis.

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Notations:

i = Annual interest rate.

n = Number of annual interest periods

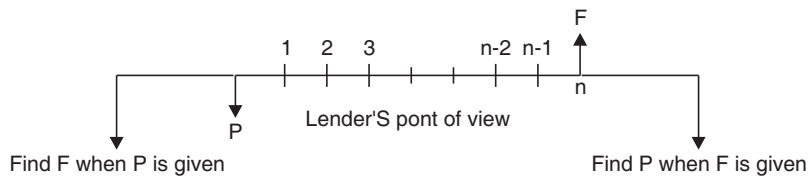
p = Present sum of money.

F = Future sum of money.

A = A Single payment, in a series of n equal payments.

G = Uniform period by period increase or decrease in amount
(the arithmetic gradient)

Interest Formula for Single Payment Series

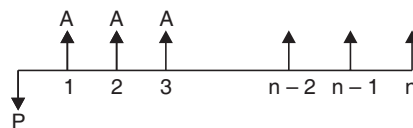


$$\frac{F}{P} = (1 + i)^n = \text{SPCAF}$$

$$\frac{P}{F} = \frac{1}{(1 + i)^n} = \text{SPPWF}$$

SPCAC = Single payment compound amount factor SPPWF = Single payment present worth factor

Interest formula for equal payment series.



1. Find P when A is given

$$\frac{P}{A} = \left[\frac{(1 + i)^n - 1}{i(1 + i)^n} \right] = \text{USPWF}$$

USPWF = Uniform series present worth factor.

2. Find A when P is given

$$\frac{A}{P} = \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] = \text{CRF}$$

CRF = Capital recovery factor

3. Find F when A is given

$$\frac{F}{A} = \left(\frac{(1+i)^n - 1}{i} \right) = \text{USCAF}$$

USCAF = Uniform series compound amount factor

Uniform series also known as equal series

4. Find A when F is given

$$\frac{A}{F} = \left[\frac{i}{(1+i)^n - 1} \right] = \text{SFF}$$

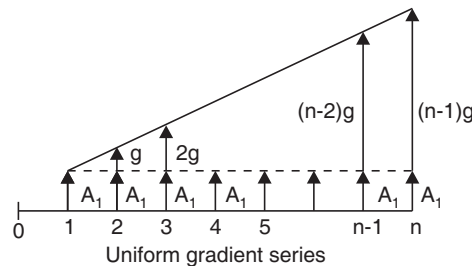
SFF = Sinking fund factor

Compound Interest Factor					
Single payment	To find	Given	Factor by which to multiply with given	Factor name	Functional rotation
	P	P	$(1+i)^n$	Single payment compound amount factor	$\left(\frac{F}{P}, i, n \right)$
Uniform series payment	P	F	$\frac{1}{(1+i)^n}$	Single payment present worth factor	$\left(\frac{P}{F}, i, n \right)$
	P	A	$\frac{(1+i)^n - 1}{i(1+i)^n}$	Uniform series present worth factor	$\left(\frac{P}{A}, i, n \right)$
	A	P	$\frac{i(1+i)^n}{(1+i)^n - 1}$	Capital recovery factor	$\left(\frac{A}{P}, i, n \right)$
	F	A	$\frac{(1+i)^n - 1}{i}$	Uniform series compound amount factor	$\left(\frac{F}{A}, i, n \right)$
	A	F	$\frac{i}{(1+i)^n - 1}$	Sinking fund factor	$\left(\frac{A}{F}, i, n \right)$

Interest Formula for Uniform Gradient Series

Certain economic analysis problems involve receipt or disbursement that are projected to increase or decrease by a uniform amount each period, hence forming an arithmetic series for *e.g.*: Maintenance and repair expenses on specific equipment may increase by a relatively constant amount each period, increase in rent each year,

- If, g = Annual change in gradient
 n = number of years
 A_1 = Payment at the end of first year
 A_2 = Equivalent annual payment of gradient series
 $(0, g, 2g \dots (n-1)g)$ at the end of successive years.

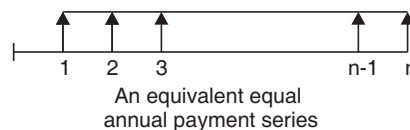


A = Equivalent Annual payment of the series.

$$A = A_1 + A_2$$

where
$$A_2 = g \left(\frac{1}{i} + \frac{n}{i} \left(\frac{P}{(1+i)^n - 1} \right) \right)$$

$\frac{A_2}{g}$ = Gradient factor for annual compounding interest.



Nominal and Effective Interest Rate

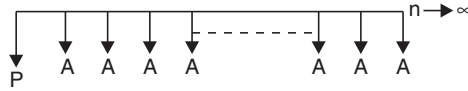
If the interest is compounded twice or thrice or more than once in a year then effective interest rate becomes greater than nominal interest rate

$$\text{Effective Interest rate} = \left(1 + \frac{i}{c} \right)^c - 1$$

i = nominal annual interest rate

c = number of times interest calculated in a year

Capitalised Cost: It is the present worth of a uniform series of amount paid till infinite time (or perpetual service)



$$P = A \lim_{n \rightarrow \infty} \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$P = A \lim_{n \rightarrow \infty} \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

$$\Rightarrow P = \frac{A}{i}$$

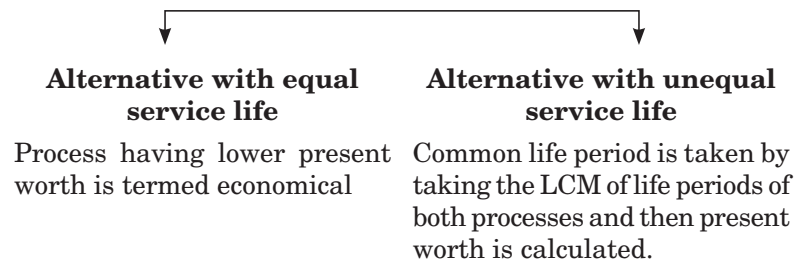
P is called as capitalised value of A

Comparison of Alternatives

Generally the comparison is done between processes which involve high initial cost and low operation and maintenance cost, and the one which involves low initial cost and high operation and maintenance cost.

Methods of comparison are as follows :

1. **Present worth Amount:** Present worth of the cash flow in the form of equivalent single sum is calculated using a discounted interest rate. This method is used when cash flow and interest rate is known



2. **Annual Equivalent Amount: (Most Widely Used)** cash flow is converted into a series of equal amount by at first calculating the present worth amount and then multiplying it with capital Recovery factor.
3. **The Capitalized Amount: (Used for long term projects).** The annual equivalent amount obtained is assumed to extend for infinitely long period and then capitalised amount is obtained as $\frac{A}{i}$.

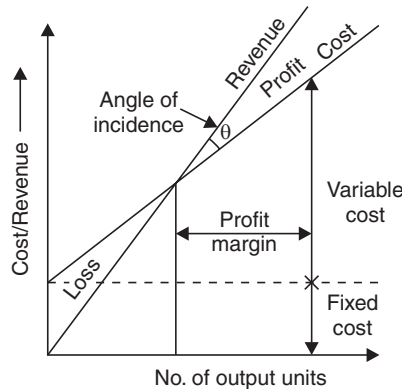
4. **Rate of return method:** Rate of return is the interest earned on the unrecovered balance of an investment such that unrecovered balance is zero at the end of investment life. It represents the interest rate which reduces the present worth amount of a series of receipts and disbursements to zero for each alternative.

Break Even Analysis

It is done to find out the point at which total revenue equals total cost and profit potential under varying conditions of output and cost

Break even point: is the point of no profit no loss.

Also called as **cost-volume-profit** Analysis



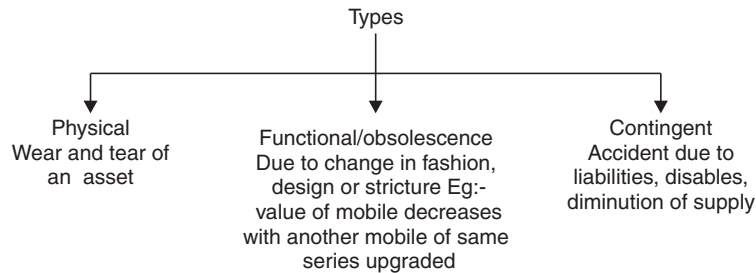
Functions:

1. Helps in selection of favourable business option
2. Possibility of profit is determined at any rate of production
3. Greater the angle of incidence more will be the profit margin.

Limitations:

1. Used generally for small business
2. Does not accounts for different varieties of product produced by the company.
3. Cost/Revenue related to No of output units only, no relation with time.
4. Market conditions do not remain static. Hence, not much usefull in that case.

Depreciation: It's the loss in value of asset with passage of time. It is used to calculate, the recovery that has been involved in the possession of the physical property. Or it can be said that, it signifies that an equal amount of fund that should be created out parallely out of profit, so that at the end of service life of the equipment we can buy another one.



Salvage Value (Resale Value): Value of asset at the end of utility period. It implies that asset has further utility but due to some reason it is selling *E.g*: Selling of household items due to relocation.

Scrap Value: The value of asset when its utility is considered to be as junk or scrap for *E.g*: Selling of vehicles after severe accidents, now vehicle cannot be used as whole while some parts can be used.

Book Value: It is the value of asset shown in account books in a particular year. At the end of utility period book value = Scrap value

$$\text{Book value} = \text{Initial cost} - \text{Depreciation cost}$$

Note:

Not all the assets do not depreciate with time, example land and sometimes even gold, platinum etc.

Notations:

C_i = Original cost or Initial cost (cost of asset + Transportation charges + Installation charge + Any other specific charges)

C_s = Salvage/scrap value at the end of utility period

n = Life of the asset

B_m = Book value at the end of period ' m '

D_m = Depreciation charge during period ' m '

Methods of calculating depreciation:

- 1. Straight time method:** Asset loses its value by a fixed amount every year

$$D_m = \frac{C_i - C_s}{n}$$

$$D_m = D_1 = D_2 \dots = D_n$$

$$B_m = C_i - m \left(\frac{C_i - C_s}{n} \right)$$

- 2. Decline Balance method/constant percentage method:** Assets lose their value at constant percentage of its book value.

Fixed declining balance

$$\text{FDB} = 1 - \left(\frac{C_s}{C_i} \right)^{1/n}$$

Note:

This method cannot be used when salvage value is zero.

- 3. Double decline balance method:** Assets lose its value by a fixed factor of the book value

Fixed factor for double decline balance method = FDDB

$$\text{FDDB} = \frac{2}{n}$$

$$B_n = C_i (1 - \text{FDDB})^n$$

- 4. Sum of year digit method:** Value of asset decreases at an decreasing rate. The digits corresponding to the number of each year of life are listed in reverse order and then sum of these digits is calculated.

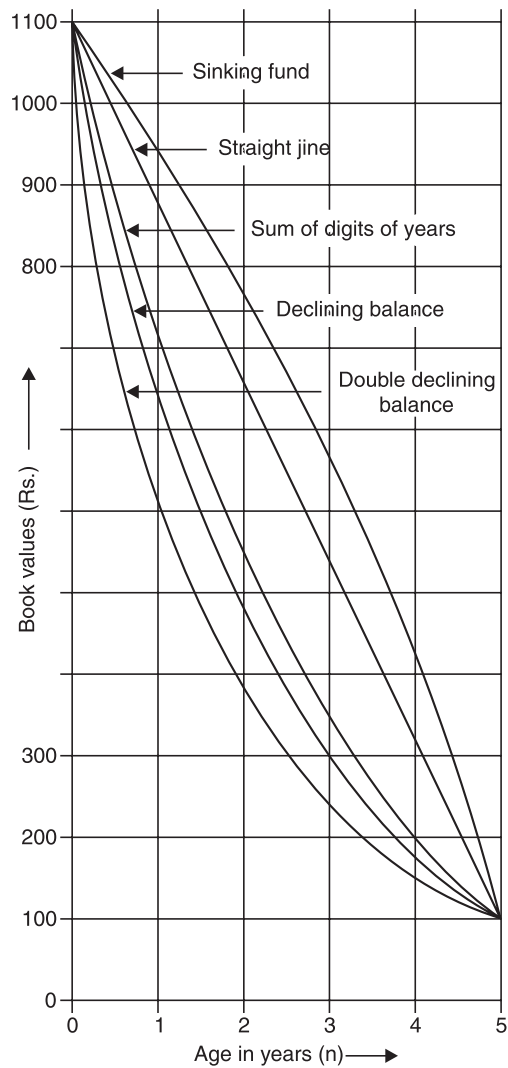
$$D_m = (C_i - C_s) \left(\frac{i}{(1+i)^n - 1} \right)$$

and

$$D_m = D(1+i)^{m-1}$$

$$D_1 = D$$

$$D_n = D(1+i)^{n-1}$$

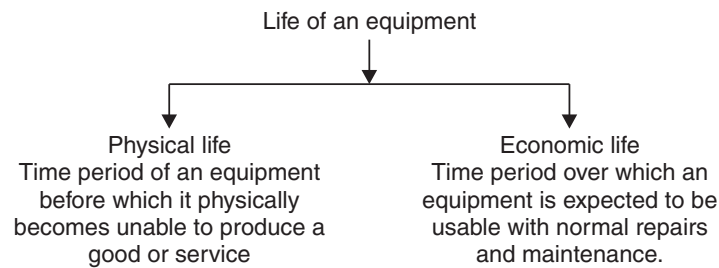


Depletion:

$$\text{Depletion for a year} = \frac{\text{cost of property} \times \text{units sold during that year}}{\text{No. of units in the property}}$$

Fundamental of Equipments

9



Costs related to an instrument:

- 1. Investment Cost:** It is the fixed cost which is incurred at the time of purchasing equipment, interest on the money invested in buying the equipment, taxes pertaining to the ownership of the equipment, insurance and storage.

(a) When there is no salvage value

$$P_{av} = \frac{p + \frac{p}{n}}{2}$$

(b) When there is salvage value

$$P_{av} = \frac{P + \frac{P - S}{n} + S}{2}$$

where,

P = Total initial cost

n = Life in years

P_{av} = Average value

S = Salvage value.

Here, book value is based on straight line depreciation

2. Depreciation and Replacement costs:

$$\text{Annual depreciation} = \frac{\text{Initial value} - \text{salvage value}}{\text{Useful life of equipment}}$$

Replacement cost of the equipment is to be increased by certain percentage every year to balance the increase in cost of equipment every year

3. Maintenance and repair cost

4. Downtime Cost: Downtime the time when machine is not working because it is undergoing repairs or-adjustments.

5. Obsolescence Cost: It by installation of new machine the production cost is reduced by some amount as compared to the production cost of existing machine, then existing machine is said to suffer a loss of that amount and its called obsolescence cost.

Fundamental of Equipment:

1. Rolling Resistance: It is a measure of force that must be overcome to pull a vehicle over the surface. It is analogous to dynamic coefficient μ .

$$R = \frac{P}{W}$$

R = Rolling resistance (kg/tonne)

P = Required tractive force (kg)

W = Gross weight of vehicle (tonnes)

2. Grade Resistance: It is the measure of force that is required to overcome or assist the vehicle up or downhill respectively. Grade resistance is taken as 10 kg/ tonne of total weight of unit for each 1% of grade.

Note:

Grade resistance is overcome with the help of **drawbar pull (crawler)** or **Rimpull (Rubber tyred)**. It acts against the total weight of both tractor and loading unit.

Total Resistance = Rolling Resistance $\pm$ Grade Resistance

Usable Tractive effort = Available tractive effort – Total resistance.

3. Coefficient of traction: Maximum frictional force produced between the surface without pulling. (Same as limiting friction)

Usable Traction = Coefficient of traction $\times$ weight

4. Drowbar Pull: It is the pulling force in kg which a crawler tractor can exert on a trailing load (to be pulled). It varies inversely with the speed of each gear. It being highest in the lowest gear and lowest in the top gear.

5. Rimpull: It is the force (in kg) that a rubber tyred tractor can exert between the rubber tyres of driving wheels and the surface on which they travel. It is expressed in kg.

(a) When slippage is eliminated

$$\text{Maximum Rimpull} = \frac{375 \times \text{HP} \times \text{Efficiency}}{\text{Speed in mph}}$$

$$\eta = 0.80 - 0.85$$

(b) When slippage starts before its rated capacity

$$\text{Maximum effective rimpull} = \text{Total pressure b/w tyres and surface} \times \text{coefficient of friction}$$

6. Gradability: It is the maximum slope (%) on which a crawler or wheeled tractor may move at uniform speed.

Combined effect of pressure and temp on I.C. Engine

The rated horse power of an I.C Engine is the power tested under standard conditions of temperature (15.5°C or 60°F) and pressure.

$$H_c = H_o \frac{P_s}{P_o} \sqrt{\frac{T_o}{T_s}}$$

H_c = Corrected horse power for standard condition.

H_o = Observed horse power as determined for test.

P_s = Standard barometric pressure (760 mm of Hg)

P_o = Observed barometric pressure in mm of kg

T_o = Absolute observed temp. (Temp in °C + 273)

T_s = Absolute temp for std. Condition (273 + 15.5)

Swell and Shrinkage

Bank Volume is the volume of earth measured in situ. It is the volume on which payment is usually made.

Loose Measure Volume is the volume of earth after it has been removed from its natural position and deposited in trucks, scrapers or spoil banks.

Compacted Volume or fill Volume, is the volume of earth it has been placed in a fill and compacted.

$$\% \text{ Swell} = \left(\frac{W_u}{W_l} - 1 \right) \times 100$$

$$\% \text{ Shrinkage} = \left(1 - \frac{W_u}{W_c} \right) \times 100$$

$$\text{Swell factor} = \frac{W_l}{W_u} = \frac{100}{100 - \% \text{ of swell}}$$

$$\text{Shrinkage factor} = \frac{W_c}{W_u} = \frac{100}{100 - \% \text{ of shrinkage}}$$

W_u = density of undisturbed earth.

W_l = density of loose earth.

W_c = density of compacted earth.

Capacity of Earth Moving Equipment

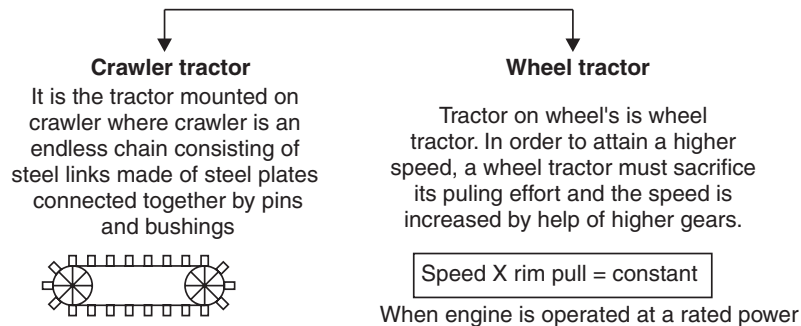
Struck capacity: Volume in the body at level measure or water level capacity

Heaped Capacity: Total volume of material as it is heaped up over the sides with 1 : 1 inclusive of struck volume.

Earthwork Equipment

10

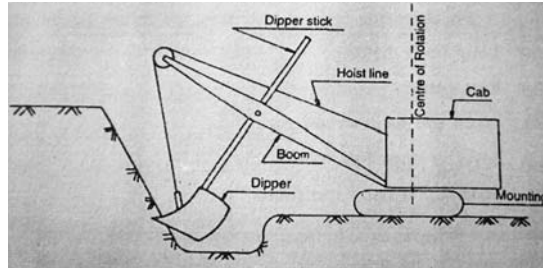
Tractor: The primary purpose of a tractor is to pull or push loads, and it may be also used as mount for many types of equipment such as bull dozer, shovel, dragline, hoe, trenchers etc.



Comparison of Crawler and Wheel Tractor.

1. Wheel tractors travel faster.
2. Crawler units are costly, compact, powerful and can handle heavier loads.
3. Wheeled units may slip over smooth surface.
4. Crawler units are controlled by stick hence need greater skill in operation.
5. Crawler units when transported over long distances, they are loaded over trailers as their speed is slow and they are likely to damage pavement.
6. Operational cost of crawler's is much higher as compared to wheeled units.

Power shovels: Shovel is an instrument used for digging, lifting and moving bulk materials. When a shovel is mounted on a power vehicle, it's called a power shovel.



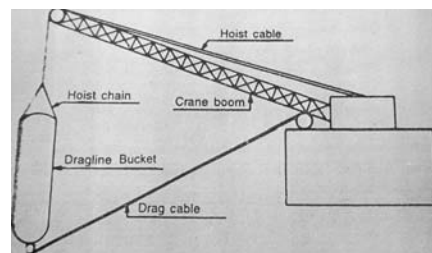
These are mainly used to excavate earth **except solid rock** without prior loosening and load it into trucks.

Note: For excavating hard and tough soil, the dipper of large shovel can exert greater downward pressure will be more suitable

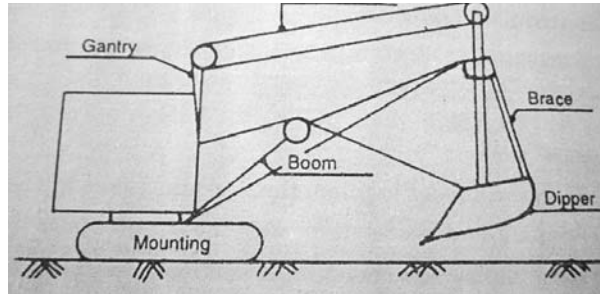
Output of shovel

$$= \frac{\text{Loose volume of dipper}}{1 + \text{swell fraction}} \times \frac{\text{Actual time in sec's per hr}}{\text{cycle time in sec's}} \times \text{Efficiency}$$

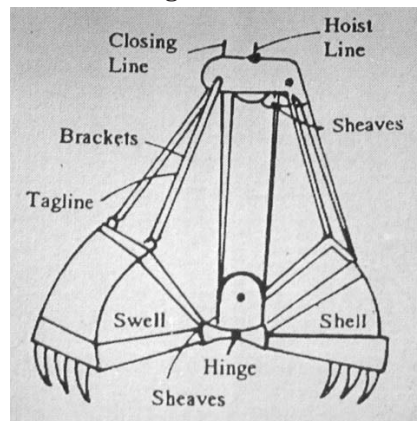
Drag lines: Drag lines are used to excavate earth and load it into haul units. Such as trucks or to deposit it on spoil banks and embankments near the place from where it is excavated. It has long boom to dispose of the earth without any hauling unit. It can excavate **below its level and under water**, and can also excavate trenches without shoring.



Hoes: Hoe is a long-handle hand equipment having a light blade to dig, scrap, till or weed. Similar machine of a power shovel group is also known as back hoe, back shovel and pull shovel. It is primarily used to **excavate below ground level**, or below machine track level.



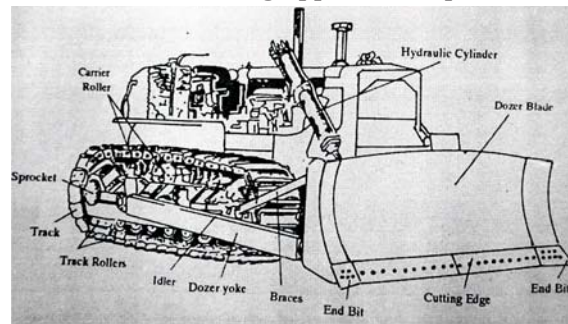
Clamshells: It's a machine having characteristics of crane and dragline. Digging is done like dragline and when bucket is full it works like crane. Especially suited to vertical lifting of materials from one location to another.



Sl. No.	Items of comparison	Power Shovel	Back Hoe	Drag line	Clam shell
1.	Excavation in hard soil or rock	Good	Good	Not Good	Poor
2.	Excavation in wet soil or mud	Poor	Poor	Moderately good long	Moderately good long
3.	Distance between footing and digging	Small	Small		

4.	Loading Efficiency	Very good	Good	Moderately good	Precise but slow
5.	Footing required	Close to work	Close to pit	Fairly away from pit	Fairly away from pit
6.	Digging level	Digs at or above footing	Digs below footing level	Digs below footing level	Digs at or below footing level
7.	Cycle time	Short	Slightly more than power shovel	More than the power shovel	More than the other equipment

Bulldozers: A heavy steel blade mounted on the front of a tractor. This blade pushes the material from one place to another. These are efficient excavating tools for short hauling application upto 100 m.



Output of a bull dozer in bank measure volume/hour

$$= \frac{\text{Loose volume of dipper}}{1 + \text{swell fraction}} \times \frac{\text{Actual time in sec's per hr}}{\text{cycle time in sec's}}$$

$$\times \frac{\text{Actual operating time in minutes perhour}}{\text{Time required per trip in minutes}}$$

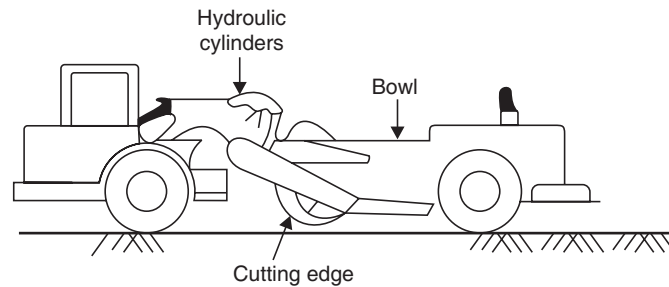
$$\text{Time required} = \frac{\text{Haul distance (in } m)}{(\text{forward} + \text{Backward}) \text{ speed}} + \text{Gear shift time}$$

Scrapers: Self operating earth moving equipment designed to dig, load, haul and discharge material over the required area without depending on other equipment there fore its also called as carry all. Equipment. But it is not used for Hard rock, sand which will not pile up into a scraper,

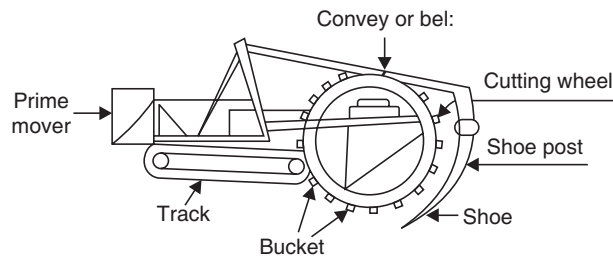
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wet and muddy material which make discharging of a scraper difficult.

$$\text{Output} = \text{Optimum loose volume per trip} \\ \times \text{Swell factor} \times 60 \times \text{Efficiency cycle time per trip (in minute)}$$



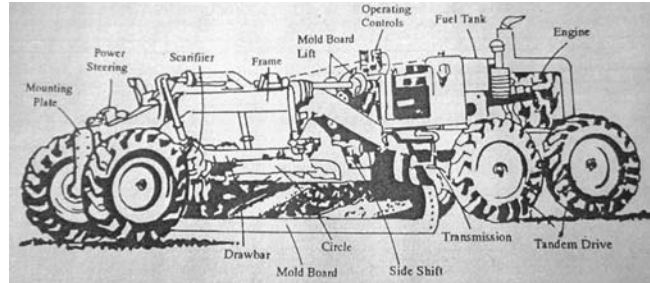
Trenchers: These provide fast digging with control of depth and width. These can dig any type of material **except rock**. Generally used to dig utility trenches for water, gas and oil pipelines, telephone cables, drainage ditches and sewers. These can also not to be used where there is large quantity of ground water as unstable soil will prevent the trench wall in remaining in its place.



Note:

In cohesive soil. It will require more power so better use power shovel or Backhoe.

Motor Graders: Generally used in making and maintaining road projects and land reclamation for earth construction. It is mainly used for levelling or finishing earthwork but sometimes also used for mixing gravel and trimming slopes.

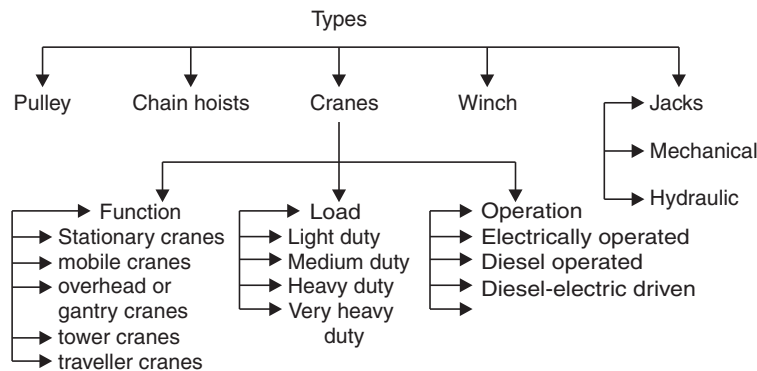
**Note:**

It's output is usually expressed in units of area covered by machine per hour.

Hoisting Equipments

11

It is the operation of lifting a weight from one location and moving it to another location which is at reasonable distance and then dumping it.

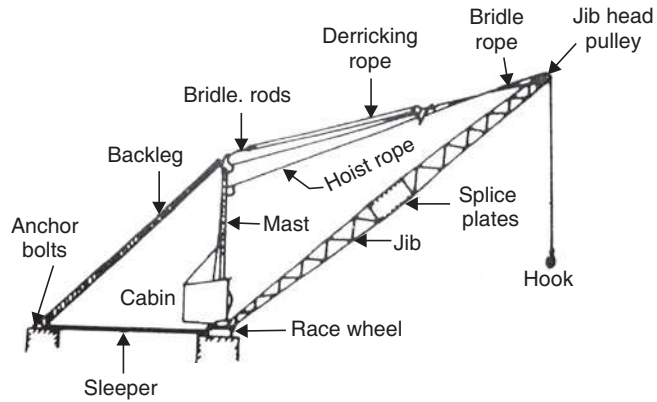


- Pulley and sheaves are used for lifting rough surfaced and heavy objects.
- Chain hoists are used for lifting loads upto 50 tonnes.
- Winch is combination of gears, clutches and brakes. Commonly used in lifting railway gates and its operation is controlled through series of levers.
- Jacks is the short name of screw jack and generally its capacity varies from 5 to 100 tonnes
- Cranes are the only hosting equipment which is capable of providing three dimensional movement of the weight. Lifting capacities varies from 0.5 to 500 tonnes.

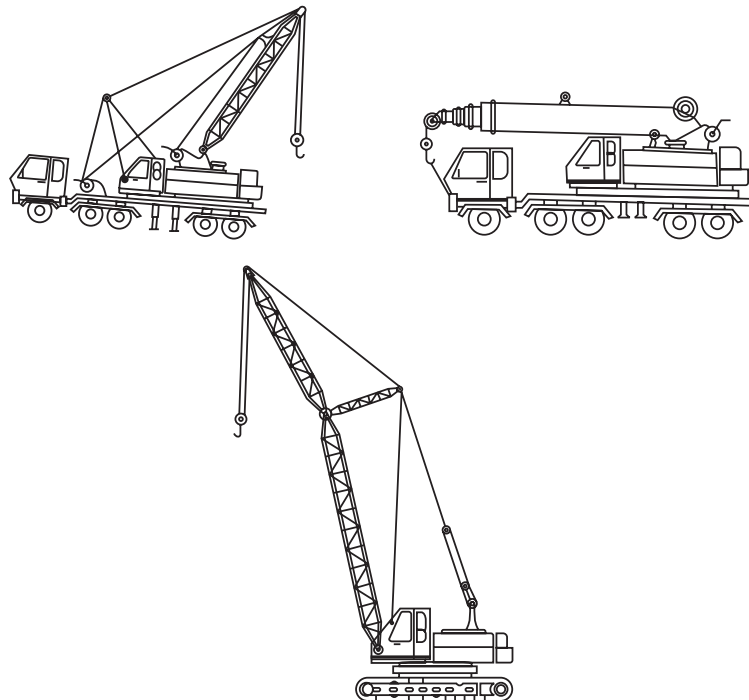
(a) Derrick cranes or stationary cranes:

Guy derrick: Used for heavy loads upto 200 tons with 360° rotation

Stiff leg derrick: Used for load from 7 to 50 tons with 270–290° rotation.

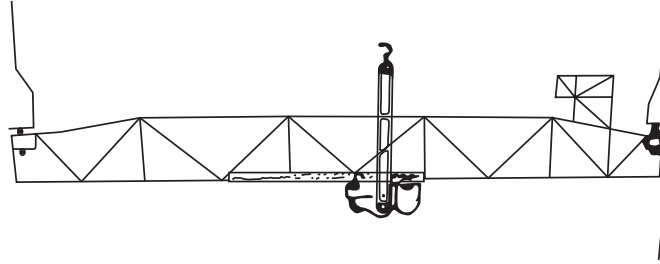


- (b) **Mobile cranes:** These cranes are mounted on mobile Units like rubber tyred truck or a crawler. Crawler cranes are economical where ground conditions are poor and operation is to be done with in small area.

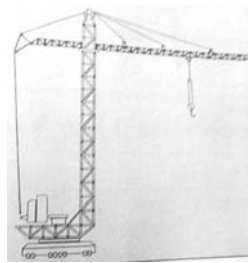


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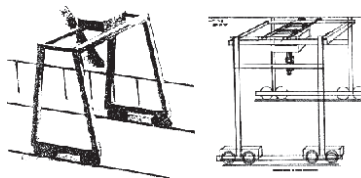
- (c) **Overhead or Gantry Cranes:** These type of cranes have freedom from floor obstruction three way motion and can cover large service area hence mostly used in **factories, steel plants, storage yards, welding yards**, etc.



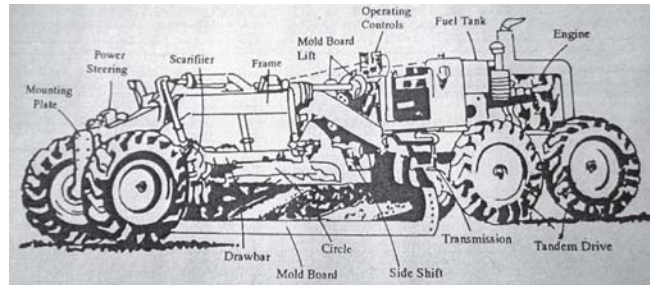
- (d) **Tower cranes:** Usually employed in erection of high industrial and residential buildings. Also used in assembling high industrial plants with element of steel structure.



- (e) **Traveller cranes:** A crane mounted on wheels especially an overhead crane consisting of a crab or other hoisting apparatus travelling on rails or beams fixed overhead.



They have their crabs moving on girders which are supported on legs instead of an overhead gantry track as in gantry girder.

**Note:**

It's output is usually expressed in units of area covered by machine per hour.

- (b) **Manually Operated Vibratory Tamping Compactors:** They have an engine drive reciprocating mechanism which acts on a spring system through which vertical oscillation with amplitude of about 10–80 mm are set up in the base plate. The most commonly used machines have a mass in the range of 50-150 kg and usually operate at a frequency of about 10 Hz.
- (c) **Manually Operated Rammer Compactors:** These are self propelled in which each blow moves them ahead slightly to contact new soil. These units range in impact from 40 to 120 per sec at an impact rate upto 850 per min.

Estimated Production of Rollers

$$Q = \frac{Shlnpt}{N}$$

$$\text{Overlap} = \frac{\text{width of overlap}}{\text{No. of drums} \times \text{width of each drum}}$$

Q = Quantity of compacted earth m^3

S = Speed of roller, m/min

h = Time of rolling, min

l = Length of strum, m

n = No. of drums

p = Percentage fraction of overlap

t = Layer thickness

N = No. of passes required

Civil Engineering

6

Building Material

CONTENTS

1. Important Indian Standard Codes
2. Cement
3. Concrete
4. Aggregates
5. Bricks
6. Brick Masonry
7. Mortar
8. Timber

Important Indian Standard Codes

1

- 455 Portland slag cement
- 456 Plain and reinforced concrete
- 457 Plain and reinforced concrete for dams and massive structures

- 269 Specification of OPC 33 grade
- 8112 Specification of OPC 43 grade
- 12269 Specification of OPC 53 grade

- 383 Coarse and fine aggregates from natural sources
- 516 Strength of concrete tests
- 650 Specification for standard sand for testing
- 1199 Sampling and analysis of concrete

- 2386 Aggregate (1 – 8)
- 2430 Sampling of aggregate for concrete
- 2502 Bending and fixing of bar

- 3370 Code of practice for concrete structures for storage of liquids (1 – 4)

- 4031 Testing of hydraulic cement (Part 1 – 15)
- 4032 Chemical analysis of hydraulic cement
- 4925 Concrete batching and mixing plant
- 4926 Ready mix concrete

Specification of Apparatus used in cement testing

5512 Flow table	5515 Compaction Factor Apparatus
5513 Vicat Apparatus	5516 Blaine Type-Air prematurity
5514 Le-Chatellier	5517 Lea & Nurse Apparatus
5816 Splitting tensile strength of concrete	
6461 Glossary of terms related to cement concrete (Part 1 – 12)	
6491 Method of sampling of fly ash	
7320 Specification of concrete slump test apparatus	

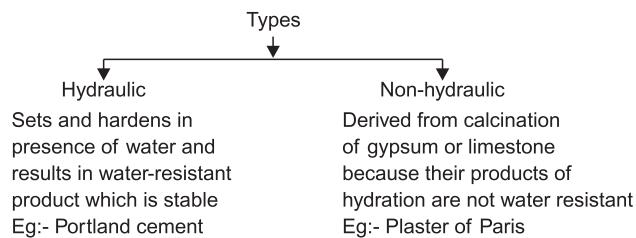
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- 7325 Apparatus for determining constituents of fresh concrete
- 7861 Extreme weather concreting, Part 1 for Hot weather, Part 2 for Cold weather
- 8041 Rapid hardening Portland cement
- 8042 White Portland cement
- 8043 Hydrophobic Portland cement
- 9012 Practise for short creating
- 9013 Accelerated cured concrete test specimen
- 9103 Admixture of concrete
- 10262 Guideline for concrete mixed design
- 13311 Part – 1 Ultrasonic pulse velocity test
Part – 2 Rebound hammer
- 875 Design loads (other than earthquakes) for building and structures
 - Part 1 : Dead load, Part 2 : Live load
 - Part 3 : Wind load, Part 4 : Snow load
 - Part 5 : Special loads and load combinations
- 1893 Earthquake resistant design for structures

Cement

2

Cement is a material which has **cohesive** and **adhesive** properties in the presence of water



Chemical Composition of Raw Materials

Oxide	Composition(%)	Function
Lime, CaO	60 – 65	Deficiency reduces strength and setting time
Silica, SiO ₂	17 – 25	Excess of it causes slow setting
Alumina, Al ₂ O ₃	3 – 8	Responsible for quick setting, excess of it lowers strength
Iron oxide Fe ₂ O ₃	0.5 – 6	Gives colour and helps in fusion of different ingredients
Magnesia, MgO	0.5 – 4	Colour and hardness
Soda and Potash Na ₂ O and K ₂ O	0.5 – 1	If in excess causes efflorescence and cracking
Sulphur trioxide SO ₃	1 – 2	Makes cement sound

Note: Rate of setting of cement paste is controlled by regulating the ratio of $\frac{\text{SiO}_2}{\text{Al}_2\text{O}_3 + \text{Fe}_2\text{O}_3}$

Bogue's Compound

Principal Mineral Compound	Formula	Sym-bol	Function
Tri calcium Silicate (Alite)	$3\text{CaO} \cdot \text{SiO}_2$	C_3S	7-day strength and Hardness
Dicalcium silicate (Belite)	$2\text{CaO} \cdot \text{SiO}_2$	C_2S	Ultimate strength
Tricalcium Aluminate (Celite)	$3\text{CaO} \cdot \text{Al}_2\text{O}_3$	C_3S	Flash-set
Tetra calcium Alumina Finite (Felite)	$4\text{CaO} \cdot \text{Al}_2\text{O}_3 \cdot \text{Fe}_2\text{O}_3$	C_4AF	Poorest cementing value

Hydration of Cement

Cement when reacts with water produces $\text{CaO} \cdot \text{SiO}_2 \cdot \text{H}_2\text{O}$ (Tobermorite Gel) and $\text{Ca}(\text{OH})_2$. $\text{Ca}(\text{OH})_2$ then reacts with sulphate present in soil or water to form CaSO_4 which further reacts with C_3A and causes deterioration of concrete which is known as **sulphate attack**.

Note: Product of hydration of C_2S are better than those of C_3S .

Only advantage of $\text{Ca}(\text{OH})_2$ is that, it makes pH alkaline (pH = 13) which resist the corrosion of reinforcement.

Rate of hydration $\text{C}_4\text{AF} > \text{C}_3\text{A} > \text{C}_3\text{S} > \text{C}_2\text{S}$

Rate of hydration $\text{C}_3\text{A} > \text{C}_3\text{S} > \text{C}_4\text{AF} > \text{C}_2\text{S}$

Water Requirement for hydration

Bound water = 23% by weight of cement.

Gel water = 15% by weight of cement.

Total minimum = 38%

Raw materials in cement manufacture

Argillaceous	Calcareous
Blast Furnace slag	Lime stone
Cement Rock	Chalk
Marl	Marine Shells
Shale and Clay	

- Ratio of Argillaceous to calcareous is 1 : 3.
- Cement is manufactured by dry as well as wet process.
- Ferric oxide is used as flux
- 2 – 3% of Gypsum is added to clinkers
- Fusion temperature = 1400 – 1500°C

Types of Cement

- 1. Portland cement:** Classified on the basis of manufacturing as 33 grade, 43 grade and 53 grade.
 - On the basis of compressive strength, As per IS 10262 classified from grade A to F.
- 2. Rapid hardening cement:** More C_3S and less C_2S as compared to OPC
 - Not-used in mass concrete
 - Large Shrinkage
- 3. Extra Rapid hardening cement:** Rapid hardening cement + 2% $CaCl_2$
 - Especially used in cold weather
 - Excessive Shrinkage
 - Not used in Pre-Stressed concrete
- 4. High Alumina cement:** Bauxite (40%), Lime stone (40%) Iron-oxide (15%)
 - Particularly suitable to sea and under-water work
 - Widely used in Pre-Casting, Expansion ≤ 5 mm
- 5. Sulphate Resisting Portland cement:** Very low C_3A and fine grounded OPC
 - Used in sea water, coastal areas and marshy lands
 - Not to be used in temp $< 40^\circ C$ where chloride is present

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6. Portland Slag cement: Portland cement + Granulated blast furnace slag + Gypsum

- High Sulphate resistance
- Used in mass concreting

7. Super Sulphated Portland cement: 80 – 85% Granulated slag + 10 – 15% calcium sulphate + 5% Portland cement clinker.

- Note to be used in freezing and thawing conditions
- Highly resistant to sea water

8. Low heat Portland cement: Low C_3S and C_3A and more contents of C_2S

- Rate of development of strength is low but ultimate strength is same

9. Portland Pozzolana cement: OPC + 10 – 30% of flyash by mass of PPC

- Free lime is removed, hence, resistant to chemical attack increases

Note: Pozzolana has no cementing property in itself but when it combines with lime it produces a stable lime pozzolana compound which has cementitious property.

10. Quick setting cement: Fine grounded OPC with reduced Gypsum content.

- Initial setting time = 5 minutes
final setting time = 30 minutes
- Used in under water concreting.

11. Masonary cement: Not to be used in concrete works

- Initial setting time 1.5 hrs,
Final max setting time 24 hrs
- 28 days strength = 5 MPa

12. White and Colured Portland cement: From Pure white chalk and China clay

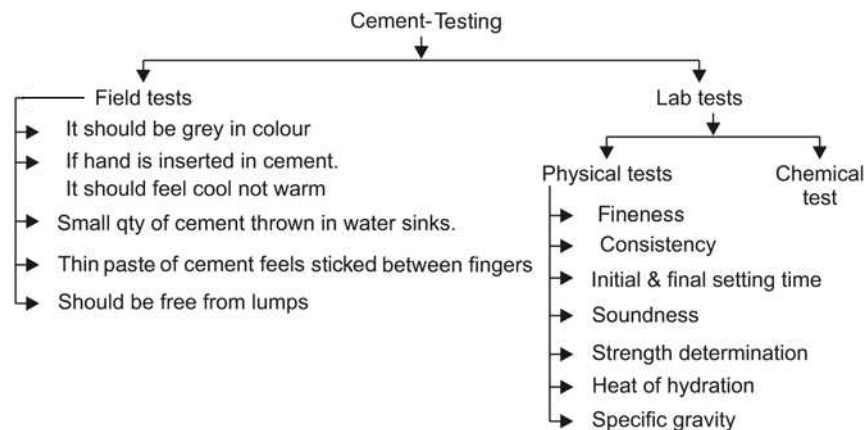
- Nil Iron Oxide
- 5 – 10% Colouring pigment before grinding

13. Air Entraining cement: OPC + Vinsol resin or vegetable fats or oils or fatty acids.

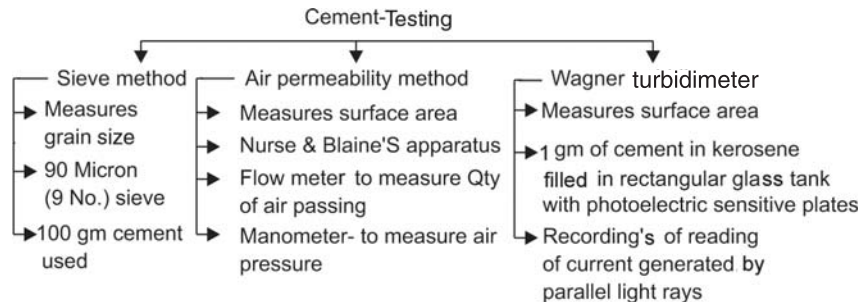
- Small amount of (0.1%) by weight of an air entraining agent.

- 14. Water Repellent or Hydrophobic cement:** OPC + fractions of oleic acid, stearic acid or pentachlorophenol.
- Suitable for basement and making water tight structures.
- 15. Water Proff cement:** OPC + Non sponifiable oil + metal Stearate like Ca, Al.
- 16. Expanding cement:** OPC + Expanding material like sulpho-aluminate + stabilising agent.
- 17. Oil Well cement:** Used as cement slurry in cementing oil-wells, under extreme atmospheric conditions of temperature and pressure.
- 18. Natural cement:** Clay (20 – 40%) + Carbonate of lime and Magnesia.
- 19. Acid resistant cement:** Acid containing aggregates + Aqueous solution of sodium silicate + Na_2SiF_6

Testing of cement: It should be taken from **12 points** from heap or from **12 different bags** or packages. Weight of the sample collected should be atleast **5 kg**.



- 1. Fineness Tests:** It's the measure of mean size of grain in it.



Note: Fineness increases shrinkage as well as cracking but reduces bleeding.

- Early development of strength increases with fineness, keeping the ultimate strength unchanged.
 - Fineness decreases the setting time
2. **Consistency Test:** Amount of water used to make paste of normal consistency. It is the percentage of water required for the cement paste, the viscosity of which will be such that Vicat's plunger penetrates upto 5 to 7 mm from bottom of the Vicat's mould.

Attachment		Use
1.	Plunger, 10 mm dia, 50 mm long	Consistency test
2.	1 mm × 1 mm square needle	Initial setting time
3.	5 mm dia Annular collar	Final setting time

3. Initial and Final Setting Time

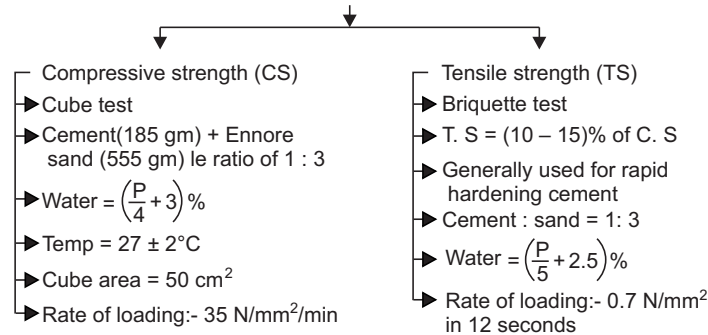
Initial Setting Time	Final Setting Time
• It is possible to remix cement paste during this period • 300 gm cement + 0.85 P of water • 1 mm² square needle penetrates by 35 – 37 from top. • For OPC Its 30 mins	• Annular collar replaces square needle • Needle makes an impression but collar fails to do so. • For OPC Final setting time $\nless 10$ hrs.

4. Soundness Test: To detect change in volume after setting.

Le-chatelier's method	Auto clave test
• Unsoundness due to free lime only • 100 gm of cement + 0.78 P • Result is given in "mm"	• Sensitive to both lime and magnesia • Internal mould dimension = 25x25x282 mm • Result is given in %

Note: If Gypsum more than 2 – 3% it will also cause Unsoundness.

5. Strength determination



6. Heat of hydration: Measured by **Calorimeter**.

7. Specific Gravity Test: Measured by Le-chatelier's flash.

Specific gravity of OPC = **3.15**

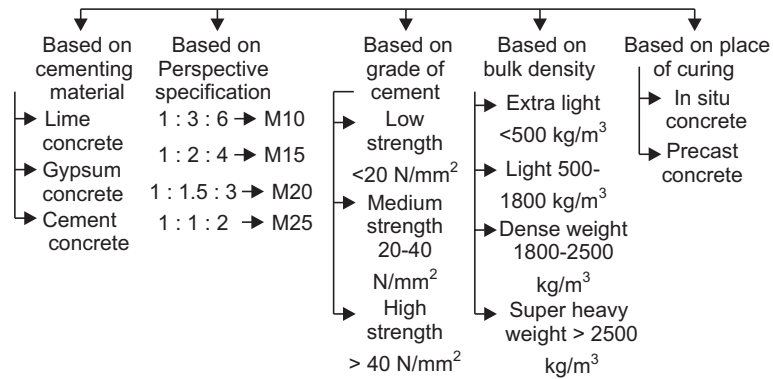
Chemical Test

1. Lime saturation factor (LSF) =
$$\frac{\text{CaO} - 0.7 \text{ SO}_3}{2.8 \text{ SiO}_2 + 1.2 \text{ Al}_2\text{O}_3 + 0.65 \text{ Fe}_2\text{O}_3}$$
 should be between **0.66** and **1.02**.
2. $\frac{\text{Al}_2\text{O}_3}{\text{Fe}_2\text{O}_3} \nless 0.66$
3. Weight of Insoluble residue < 4%
4. Weight of Magnesia < 4%
5. Total loss on ignition < 5%
6. Total sulphur content as sulphuric Anhydride
 - (a) < 2.5% when $\text{C}_3\text{A} < 5\%$
 - (b) < 3% when $\text{C}_3\text{A} > 5\%$

Concrete

3

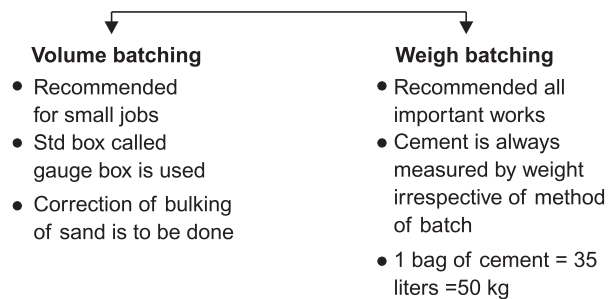
Concrete is a mixture of binding material such as lime or cement, well graded coarse and fine aggregate, water and some admixture.



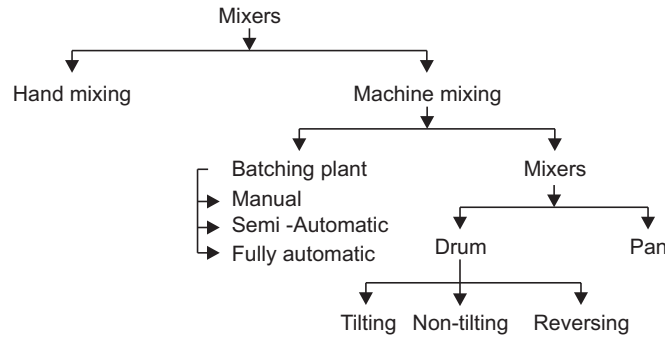
Stages of manufacturing of concrete

1. Batching
2. Mixing
3. Transporting
4. Placing
5. Compacting
6. Curing
7. Finishing.

1. Batching: Aggregate cement and water measured with accuracy of $\pm 3\%$ of batch quantity.



2. Mixing: Approximate mixing time as per IS – 456 is 2 minutes.



- Concrete mixers are specified by **volume of mixed concrete discharged** after mixing of each batch expressed in m^3 .
- Sometimes mixers are specified by two quantities, the total volume of ingredients added and the volume of concrete produced for eg. 285/200.
- Tilting mixers are used for large construction works where as Non-tilting mixers are used for small works.
- Non-tilting mixers cannot be used when aggregate size is more than 75 mm.
- Reversing mixers are forced action types mixers with two sets of blades.
- Pan type concrete mixers is an open pan equipped with a rotating star of blades moving in opposite direction relative to each other at a very low speed. These mixers are efficient with stiff mixes.

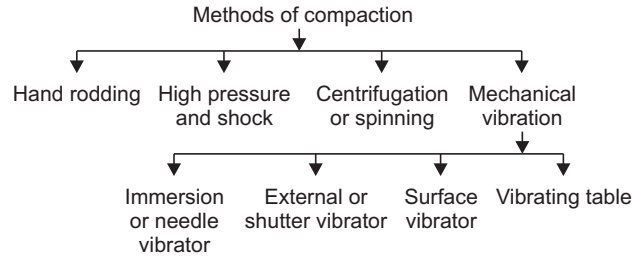
3. Transporting: It is done by

(a) Pan's	(b) Wheel Barrows	(c) Power buggies
(d) Tower bucket	(e) Crane bucket	(f) Dump truck
(g) Chutes	(h) Concrete pumps	(i) Transit mixers
(j) Agitating	(k) Belt-conveyer	

4. Placing: Delayed placing of concrete results in a gain in ultimate compressive strength provided the concrete is adequately compacted.

- Placing of concrete underwater is done by **tremie pipe**. (Slump – 150 – 200 mm)

5. Compaction: Process of removal of entrapped air and of uniform placement of concrete to form a homogenous dense mass is called compaction.



6. Curing: As per IS456, curing is the process of preventing the loss of moisture from the concrete while maintaining a satisfactory temperature regime.

Combined water: Is chemically combined with hydration products, it is non evaporable.

Gel water: Held physically or adsorbed on the surface area of the cement.

Capillary water: It partially occupies the capillary pores that constitute the space in the cement paste remaining after accounting for the volumes of cement gel and unhydrated cement. It is easily evaporated.

Methods of Curing

- (a) Shading
- (b) Covering surface with wet hessian cloth or gunny bags
- (c) Sprinkling
- (d) Ponding
- (e) Steam curing (For precast members)
- (f) Applying curing compounds

7. Finishing: Levelling or smoothing of top surface of freshly placed concrete. It is done by **Screeding, Floating** and **Trowelling**.

$\text{Maturity of Concrete} = \text{Time} \times \text{Temperature} = ^\circ\text{C hours or } ^\circ\text{C days}$

Yield of Concrete

If proportion of concrete is **a : b : c** then resulting concrete will have volume of $\frac{2}{3} (a + b + c)$ whose *a* is volume of cement, *b* is volume of sand

and *c* is the volume of coarse aggregate.

If *w*, *a*, *b*, *c*, are absolute volumes of water, cement, sand and coarse aggregate respectively then **$w + a + b + c = 1$** .

Methods of Proportioning concrete mixes

1. Fuller's maximum density method:

$$p = 100\sqrt{\frac{d}{D}}$$

D = maximum size of aggregate

p = % by weight of matter finer than diameter d

2. Minimum Voids method: Voids in coarse aggregate should be filled by sand and voids in sand should be filled by cement. Hence, in actual practice quantity of sand used in the mix is kept 10% more than the voids in the coarse aggregate and quantity of cement is taken as 15% more than the Voids in sand.

3. Abram's water-cement law: For any given conditions of the test, the strength of the workable concrete mix is dependent only on water-cement ratio.

According to the law, the strength of the mix increases with decrease in water content.

$$P_7 = \frac{984}{7^x} \frac{\text{kg}}{\text{cm}^2}$$

$$P_{28} = \frac{984}{4^x} \frac{\text{kg}}{\text{cm}^2}$$

P_7 = Cylinder crushing strength after 7 days

P_{28} = Cylinder crushing strength after 28 days

x = Water cement ratio by volume

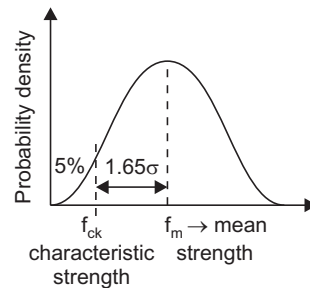
Strength of concrete increases with age

Months	Age Factor
1	1.00
3	1.10
6	1.15
12	1.20

Water-Cement Ratio: It is the **mass** of 'free water' (excluding that adsorbed by the aggregate) to that of cement in the mix. It is mainly responsible for porosity of the hardened cement paste. The ratio generally lies between 0.35 to 0.65.

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Strength of Concrete: It is designated in terms of numbers preceded by the letter 'M' where M refer to the '**Mix**' and number represents the characteristic strength.



Characteristic strength is the strength of material below which not more than 5% of the test results are expected to fall.

$$f_m = f_{ck} + 1.65 \sigma$$

Laboratory-test's for
strength of concrete

Compressive strength test Flexural tensile strength test Split tensile strength test

1. Compressive strength test:

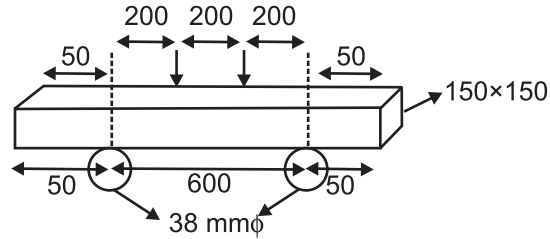
- Size of coarse aggregate upto 38 mm
- Size of cube $\rightarrow 150 \times 150 \times 150$ mm
- Size of cylinder $\Rightarrow 150$ mm dia, 300 mm height
- Cube mould filled in 3 layers, tampered 35 times per layer with tampering rod of 16 mm dia and 600 mm length.
- Stored at temp of $27 \pm 3^\circ\text{C}$ at 90% humidity for $24 \pm 1/2$ hour.
- Then immersed in water for 7 days or 28 days.
- Rate of loading in compression testing machine = $14 \text{ N/mm}^2/\text{minute}$.

Note: If aggregate size less than 19 mm then mould of size $100 \text{ mm} \times 100 \text{ mm} \times 100 \text{ mm}$ can also be used as per IS code.

$$\text{Cube strength} = 1.25 \times \text{Cylinder strength}$$

2. Flexural tensile Strength Test:

- Size of mould $\rightarrow 150 \times 150 \times 700$ mm
- Tamping bar $\rightarrow$ weight 2 kg, length 400 mm



$$\begin{aligned} \text{Modulus of rupture} &= \frac{pl}{bd^2} \quad (a > 200 \text{ mm}) \\ &= \frac{3pa}{bd^2} \quad (280 \text{ mm} < a < 170 \text{ mm}) \end{aligned}$$

a = distance b/w line of fracture and nearest support.

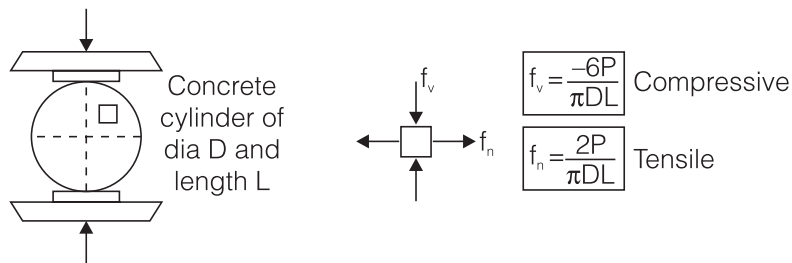
$b, d \Rightarrow$ width and depth of specimen.

$l \Rightarrow$ length of the span on which the specimen is supported.

$p \Rightarrow$ maximum load applied to the specimen.

Note: If aggregate size is less than 19 mm, then size of specimen can be 100 mm $\times$ 100 mm $\times$ 500 mm

3. Split tensile strength



Factors affecting Strength of Concrete

1. **Size of specimen:** Size decreases then strength increases
2. **Stress situation:** Concrete under triaxial state offers more resistance than uniaxial state.
3. **Moisture in specimen:** Reduces the strength
Strength of dry sample = (1.1 to 1.2) $\times$ strength of saturated sample
4. **Air Voids:** Formed by evaporation of water used in making concrete or by entrapped air. They decrease the strength considerably.

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5. **Rate of loading:** Strength increases with increase in rate of loading.
6. **Age of cement:** Strength reduces as it absorbs moisture from atmosphere.
7. **Cement aggregate ratio:** If cement is increased then ultimate strength will increase.
8. **Type of aggregate:** Rounded or cubical aggregate give higher compressive strength (but poor bonding).
9. **Size of aggregate:** Larger aggregate size reduces strength.
10. **Degree of compaction:** 5% air Void decreases strength by 30% where as 10% air Void decreases strength by 60%.

Workability: Property of concrete which determines the amount of useful internal work necessary to produce full compaction.

Or

It is the ease with which concrete can be compacted 100% with regard to mode of compaction and place of deposition.

Note: Consistency indicates fluidity or mobility. Concrete with high consistency may not be workable for a particular job and concrete having same consistency may vary in workability.

Factors affecting workability

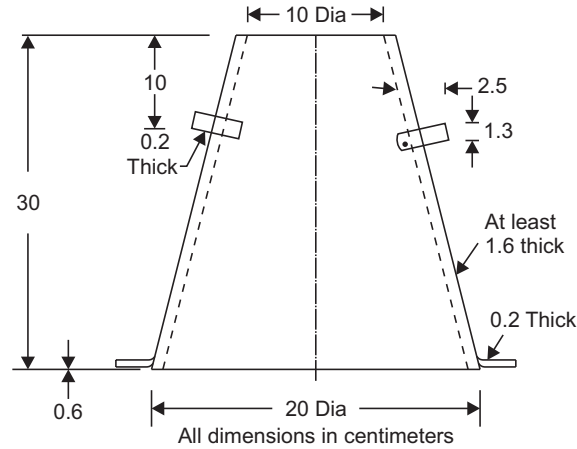
1. **Water content:** Increase in water content increases workability.
2. **Mix proportions:** Lower aggregate cement ratio more it will be workable.
3. **Size of Aggregate:** Bigger size, more workable.
4. **Shape of aggregate:** Round and cubical shapes are more workable.
5. **Surface texture:** Smooth surface aggregate more workable than rough.
6. **Grading of Aggregate:** Well graded aggregates more workable than poorly graded.
7. **Admixtures:** Air entrained admixtures enhance workability.

Measurement of workability

1. Slump test:

- Dimensions of mould = Top dia = 100 mm
Height = 300 mm
Bottom dia = 200 mm
- Four layers, 25 strokes/layer

- Tamping rod: dia 16 mm, length 600 mm
- Type: True slump, shear slump, collapse slump



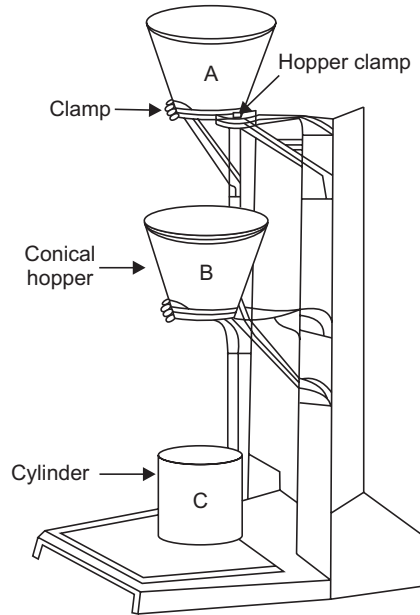
Note: The test is performed for maximum size of aggregate as 38 mm only, if aggregate size > 38 mm then wet sieving is done through 38 mm sieve.

Recommended slumps of concrete

No.	Type of concrete	Slump
1.	Concrete for road construction	20 to 40 mm
2.	Beams and slabs	50 to 100 mm
3.	Normal RCC work	80 to 150 mm
4.	Mass concrete	25 to 50 mm
5.	Concrete to be vibrated	10 to 25 mm
6.	Impermeable work	75 to 120 mm

- 2. Compacting factor test:** Degree of workability is measured in terms of **internal energy** required to compact the concrete thoroughly.

Compacting factor is the ratio of density actually achieved in the test to density of same concrete fully compacted.



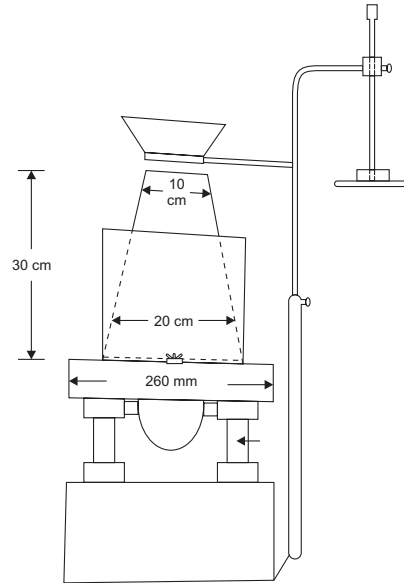
Values of workability for different placing conditions

Degree of Workability	Consistency	Slump (mm)	Compacting factor	Vee-Bee degree	Characteristics	Uses
Extremely	Moist Earth	0	0.65–0.7	>20	Particles of coarse aggregate in the concrete are adhesive, but concrete does not clot, risk of segregation	Precast paving slabs
Very Low	Very Dry	0–25	0.7–0.8	12–20	Concrete has the consistency of very stiff porridge, forms a stiff mound when dumped, and barely tends to shake or roll itself to form an almost horizontal surface when conveyed for a long time in, say, a wheel-barrow.	Roads (power vibrator)

Low	Dry	25–50	0.8-0.85	6-12	Concrete has the consistency of stiff porridge, forms a mound when dumped, and shakes or rolls itself to form a horizontal surface when conveyed for a long time in, say, a wheel-barrow stiff po	Mass concreting, light reinforced section,
Medium	Dry	50-100	0.85-0.95	3-6	Concrete can be shaped into a ball between the palms of the hands, and adheres to the skin.	Flat slabs, heavily reinforced section, RCC Sections (Manual Vibrator)
High	Semi-fluid	100-175	0.95	0-3	Concrete cannot be rolled into a ball between the palms of the hands, but spreads out even though slowly and without affecting the cohesion of the constituents so that segregation does not occur	RCC with congested reinforcement (can not be vibrated)

3. Vee Bee consistometer: Laboratory method to indirectly measure the workability of concrete.

This test determines the **time** required for transforming, by vibration, a concrete specimen in the shape of a conical frustum into a cylinder.



Consistency of the concrete is expressed in VB degree which is equal to the recorded time in seconds.

4. **Flow test:** It is the laboratory test which gives an indication of the quality of concrete with respect to consistency, cohesiveness and the proneness to segregation.

$$\text{Flow \%} = \frac{\text{Spread dia s in cm} - 25}{25} \times 100$$

Range = 0 to 150

Note: Compacting factor test measures the inherent characteristics of the concrete which relates very close to workability requirements of concrete.

Mix- Design: The task of deciding the proportions of the constituents of concrete is accompanied by use of certain established relationships, the concrete thus produced is termed as **Design Mix Concrete**. While when the proportions of cement, aggregate and water are adopted based on arbitrary standards the concrete produced is termed as **Nominal Mix Concrete**.

Steps in mix-design

1. Find target mean strength (f_m)

$$f_m = f_{ck} + 1.65 \sigma$$

2. Get **water cement ratio** based on 28-day strength of cement

f_{ck} = Characteristic mean strength

σ = Standard deviation

Grade of concrete	Assumed σ
M ₁₀ – M ₁₅	3.5
M ₂₀ – M ₂₅	4.0
M ₃₀ – M ₅₀	5.0

3. Get **water content** based on workability requirement.
4. Get **cement** required from (2) and (3)
5. Calculate mass of fine M_f and coarse aggregate M_c based on absolute volume principle.

$$\frac{M_c}{f_c} + \frac{M_{fa}}{f_{fa}} + \frac{M_{ca}}{f_{ca}} + V_w + V_v = 1$$

6. Determine weight of ingredients per batch, based on capacity of concrete mixer.

Exposure conditions and requirements for RC works with normal aggregates of 20 mm nominal size.

Exposure	Description	Min grade	Min cement (kg/m ³)	Max free w/c ratio
Mild	Normal	M20	300	0.55
Moderate	Alternate wetting and thawing	M25	300	0.50
Severe	Immersed in sea water	M30	320	0.45
Very Severe	Exposed to sea spray	M35	340	0.45
Extreme	Tidal zone	M40	360	0.40

Durability: A durable concrete is the one that performs satisfactorily under anticipated exposure conditions for stipulated life of the structure.

Factors affecting durability

1. **Permeability:** More permeable concrete will be obviously less durable.
2. **Frost action:** At temp below 0°C, expansion occurs due to freezing of adsorbed water.
3. **Sulphate attack:** Expansion due to chemical reaction b/w products of hydration of cement and solution containing sulphate of calcium, magnesium and sodium.
4. **Mineral oils:** Petroleum and its products do not directly affect hardened concrete.
5. **Organic acids:** Acetic acid, lactic and, Butyric acid severally attack concrete. Formic acid causes corrosion.
6. **Sugar:** Retarding agent, causes corrosion.
7. **Sewage:** H_2S gas evolved from septic sewage promotes the formation of H_2SO_4 affecting the concrete.

Defects in concrete

1. **Cracks:** Acceptable limits for the surface width of cracks is 0.1 to 0.3 mm
2. **Crazing:** It's the network of fine random cracks on the surface of concrete or mortar caused by shrinkage of surface layer.
3. **Efflorescence:** Fluffy white patches on the surface of concrete members.
4. **Segregation:** Separation of coarse aggregate from fine aggregate. It is reduced by increasing small size coarse aggregate, air entrainment using dispersing agents and puzzolona.
5. **Bleeding:** If excess water in the mix comes up at the surface causing small pores through the mass of concrete. Its called bleeding.

Physical properties of concrete

1. **Elastic modulus of concrete:** $E_c = 5000 \sqrt{f_{ck}}$

It varies from $14 \times 10^3 \text{ N/mm}^2$ to $40 \times 10^3 \text{ N/mm}^2$ and increases with density and to some extent with age.

2. Poisson's ratio: $\mu_{\text{concrete}} = 0.1 \text{ to } 0.3$

For dynamic tests its value is slightly high i.e., 0.24.

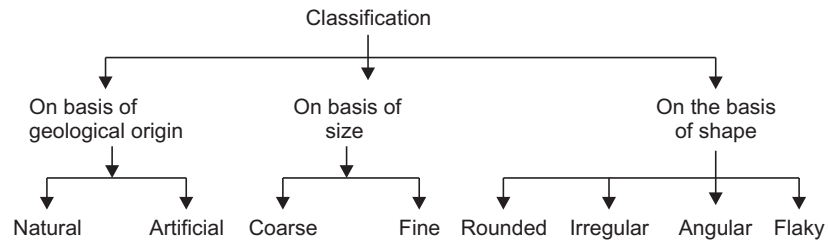
3. Creep: Continued deformation with time under a **constant load**. It is also known as plastic or yield flow. In general creep increases when Aggregate content and relative humidity is low or when cement content, water cement ratio, Air entrainment and temperature are high.

Rheology: It is the science of flow and deformation of matter and describes one interrelation between force, deformation and time. Rheology of fresh concrete like workability includes the parameter of stability, mobility and compatibility. It also includes the principles and technique applied in deformation of hardened concrete, handling and placing of freshly mixed concrete and behaviour of its constituent parts.

Aggregates

4

Aggregates are the materials basically used as fillers with binding material in the production of mortar and concrete. They give body to the concrete and occupy **70 to 80 percent of volume** of concrete.



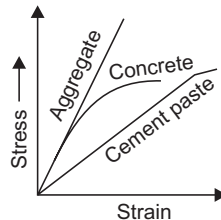
- **Natural aggregates** are obtained by crushing from quarries of igneous, sedimentary or metamorphic rocks. Most widely used are that from igneous rocks.
- Broken bricks, blast furnace slag and synthetic aggregates are **artificial aggregates**.
- Blast furnace slag aggregate has good fire resisting properties but are responsible for corrosion of reinforcement due to sulphur content of slag.
- Aggregates ranging from 80 mm – 4.75 mm are called **coarse aggregates** where as those passing from 4.75 mm sieve are **fine aggregates**.
- Max size of coarse aggregate to be used depends on the thickness of the member, reinforcement spacing, clear cover, mixing, handling and placing method.
- **Rounded aggregates** have minimum ratio of surface area to volume cement paste required is minimum and produce minimum voids (i.e., 32%).
- **Irregular aggregates** develop good bond and are suitable for making ordinary concrete (Voids produced is 36%).

- **Angular aggregates** provide maximum void (about 40%) but they provide very good bond and are suitable for high strength concrete.
- **Flaky aggregates** are those whose lateral dimension is less than 0.6 times the mean dimension while **Elongated aggregates** are those whose length is 1.8 times its mean dimension. Both these aggregates influence the concrete properties adversely.

S.No.	Classification	Examples
1.	Rounded	River or seashore gravels, desert and wind blown flints
2.	Irregular or partly rounded	Pit sands and gravel, land or dug flints
3.	Angular	Crushed rocks
4.	Flaky	Laminated rocks

Characteristics of Aggregate

1. **Strength of aggregate:** Test's for assessment of strength are
 - (a) Aggregate crushing value
 - (b) 10 percent fineness value
 - (c) Aggregate impact value
 - (d) Aggregate abrasion value
2. **Stiffness:** Modulus of elasticity of concrete is approximately equal to the weighted average of the moduli of the cement paste and the aggregate, therefore modulus of coarse aggregate has an important influence on the stiffness.



3. **Bond strength:** Difference between the coefficients of thermal expansion of paste and aggregates and due to the shrinkage of cement paste during hardening results in a state of internal stress even though no external forces are present. There is **no standard test** for bond but it is known that the rougher the surface texture of the particles, the better the bond.

Note: The greater specific surface of angular particles should enable greater adhesive force to be developed, but the angular shape probably causes more severe concentrations of internal stress.

4. **Shape and texture:** Rounded and irregular aggregates are highly workable but yield low strength concrete. Flaky aggregates require more cement paste, produce maximum voids hence not used.

Note: Crushed and uncrushed aggregates generally give same strength for the same cement content.

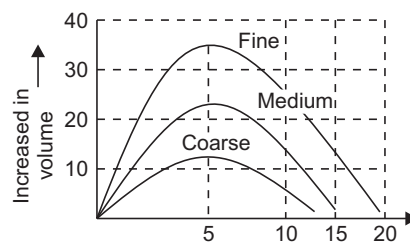
5. **Specific gravity:** Specific gravity of most of the natural aggregates lies between **2.6 to 2.7**. Low specific gravity indicate high porosity and therefore poor durability and low strength.
6. **Bulk density:** Shows how densely the aggregate is packed when filled in a standard manner. Higher the bulk density, lower is the void content to be filled by the sand and cement.

7. **Voids:**
$$\text{Void ratio} = 1 - \frac{\text{Bulk density}}{\text{Apparent specific gravity}}$$

8. **Moisture content:** Moisture as the percentage of the weight of the **saturated surface dry aggregate**.

$$\text{Total water content} = \text{Moisture content} + \text{Adsorbed water}$$

Bulking of sand: It is the **increase in the volume** of given mass of sand caused by the films of water pushing the sand particles apart is called bulking of sand. Extent of bulking depends on the percentage of moisture present in the sand and its fineness.



- If sand is measured by volume and no allowance is made for bulking then concrete will be richer.

$$\% \text{ Bulking} = \frac{\text{original volume} - \text{increased volume}}{\text{Increased volume}} \times 100$$

Fineness Modulus: It is an index number which is roughly proportional to the average size of the particles in the aggregate.

It is the sum of **cumulative percentage** retained on the sieves of the standard sieves: 150 μm , 300 μm , 600 μm , 1.18 mm, 2.36 mm, 4.75 mm, 10 mm, 12.5 mm, 20 mm, 63 m and 80 mm.

Aggregate	F.M
Coarse agg.	6 to 8.5
Fine agg.	2 to 3.5
Mixed agg.	4.7 to 7
Coarse sand	2.9 to 3.2
Medium sand	2.6 to 2.9
Fine sand	2.2 to 2.6

Higher Fineness modulus aggregate results in harsh concrete mixes and lower Fineness modulus results in uneconomical concrete mixes.

Soundness: Ability of aggregate to resist change in volume as a result in changes of physical conditions. The physical conditions responsible for the unsoundness are:

- (a) Freezing and Thawing
- (b) Thermal changes at temperature above freezing
- (c) Alternate wetting and drying.

Alkali-Aggregate Reaction: Silica present in the aggregates reacts with alkalies present in the cement forming alkali-silicate gel which imbibes water and increases in volume. As this gel is confined by surrounding hydrated cement paste, it results in internal pressure and eventually leads to expansion, cracking and disruption of hydrated cement paste.

Factors affecting alkali-aggregate reaction

1. Reactive type of aggregates
2. High alkali cement content
3. Availability of moisture content
4. Temperature condition (10 to 38°C)

Control of Alkali-Aggregate reaction

1. By selecting non-reactive aggregate
2. By using low alkali cement
3. By controlling moisture content
4. By using puzzolonas
5. By air entraining agents

Note: Coefficient of thermal expansion of concrete increases with that of coarse, aggregate. If the difference of coefficients *b/w* coarse aggregate and cement paste is appreciable then it may break the bond between the two.

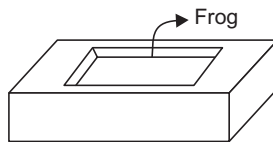
Bricks

5

Dimensions of Brick

Standard size = $19 \text{ cm} \times 9 \text{ cm} \times 9 \text{ cm}$

Nominal size (with mortar) – $20 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm}$



Traditional bricks = $9'' \times 4.5'' \times 3''$ ($23 \text{ cm} \times 11.4 \text{ cm} \times 7.6 \text{ cm}$)

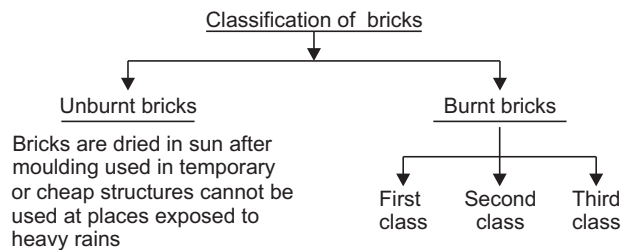
From (indent in the brick) = $10 \text{ cm} \times 4 \text{ cm} \times 1 \text{ cm}$

Weight of 1 m^3 of brick = 1800 kg

Avg. weight of the brick = 3 kg to 3.5 kg

Comparison between Bricks and Stone

1. Brick resist more fire than stones and also resist other atmospheric effects in a better way.
2. In brick work mortar joint are thin, hence more durable.
3. Brick work is cheaper, requires less skilled labour and no complicated lifting device.
4. Brick work is less water tight and absorbs more moisture.
5. Thickness of brick wall is 100 – 200 mm while stone wall has minimum thickness of 300 mm.
6. Stone work is stronger, gives solid appearance, attractive in texture.
7. Stone work has higher compressive strength (adopted in piers, docks, dams) and doesn't requires any finishing.



1. First class bricks:

- These are **table** mounted and burnt in **kiln's**.
- Smooth surface, rectangular in shape with parallel, sharp and straight edges with square corners.
- Water absorption = **12 – 15%** of its **dry weight** when immersed in cold water for 24 hrs.
- Crushing strength $\geq$ **10 N/mm²**
- Recommended for pointing, Exposed face work in masonry structures, flooring and reinforced brick work.

2. Second class bricks:

- These are **ground** moulded and burnt in **kiln's**.
- Small cracks and distortions permitted.
- Water absorption = **16 – 20%** of its **dry weight** in 24 hrs
- Crushing strength $\geq$ **7 N/mm²**
- Recommended for hidden masonry works and centering of reinforced brick and reinforced cement concrete structures.

3. Third class bricks:

- There are **ground** moulded and burnt in **clamps**.
- Soft and reddish yellow in colour.
- Water absorption = **25%** of its **dry weight**.
- Recommended for unimportant and temporary works where rainfall is not heavy.

4. Fourth class bricks:

- Over burnt, badly distorted.
- Used as ballast in foundations and floors in lime concrete.

Note: Heavy duty bricks have compressive strength more than 40 N/mm², used in structures such as bridges, foundation of industrial buildings etc. water absorption is limited to 5%.

Properties of good brick

1. Bricks should have uniform size, parallel sides and sharp edges.
2. Uniform deep red or cherry colour as indicative of uniformity.
3. Should not be too smooth to cause slipping of mortar.
4. On scratching a brick by finger nail no impression should be made.
5. Metallic sound is produced when bricks are struck together.

6. Water absorption $\leq 20\%$ of its dry weight.
7. Crushing strength $\geq 10 \text{ N/mm}^2$
8. Brick earth should be free from stones, grits, organic matter etc.

Composition of Good Brick earth

1. **Silica (50 – 60%):** Prevents **cracking** and shrinking of raw bricks.
 - Imports durability and uniform shape to bricks.
 - Excess of silica makes bricks brittle.
2. **Alumina (20 – 30%):** Absorbs water and imparts **plasticity** to the earth.
 - Clays with high alumina are found to be very **refractory**.
 - Excess of Alumina causes cracks on drying and become too hard when burnt.
3. **Lime (< 10%):** Prevents **shrinkage** on drying.
 - Helps in binding by melting silica on burning.
 - Excess of lime causes melting of bricks and hence, it loses shape.
4. **Magnesia (< 1%):** Imparts yellow tint to the bricks and decreases shrinkage.
 - Excess of magnesia leads to the decay of bricks.
5. **Iron oxide (< 7%):** Improves impermeability and durability.
 - Lowers fusion point of the clays especially if present as ferrous oxide.
 - Gives strength and hardness.
 - Provides red colour.

Harmful ingredients in brick earth

1. **Lime:** If in excess, colour of brick becomes **yellow** from red. Lumps of lime absorb moisture, swell and cause **disintegration** of the bricks.
2. **Iron pyrites:** Dis colourises the bricks. If present in brick earth, it gets oxidised and decompose during burning which may even **split** the brick.
3. **Pebbles, Gravels and Grits:** Result in **non-uniform mixing** of clay.
4. **Alkalies:** Brick may melt and lose shape. Results in **Efflorescence**.
5. **Organic matter:** It gets burnt completely during burning leaving behind pores and making brick.

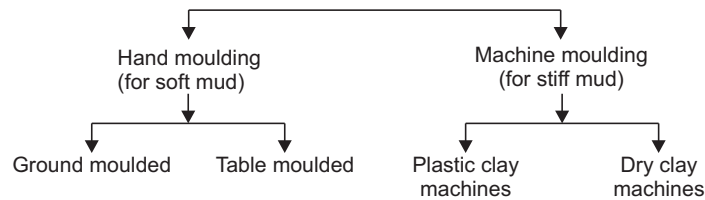
6. **Water:** Free water causes **shrinkage** on drying where as combined water causes shrinkage on burning.
7. **Sulphur:** If incomplete oxidation then it will cause **spongy** and **swollen** structure in the brick.

Manufacturing of bricks

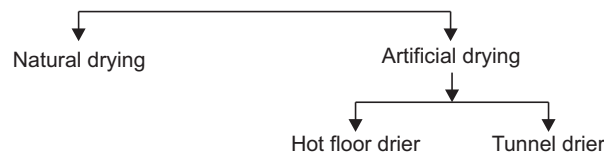
1. Preparation of brick earth

- (i) Unsoiling (ii) Digging
(iii) Cleaning (iv) Weathering
(v) Blending (vi) Tempering (Done in Pug mills)

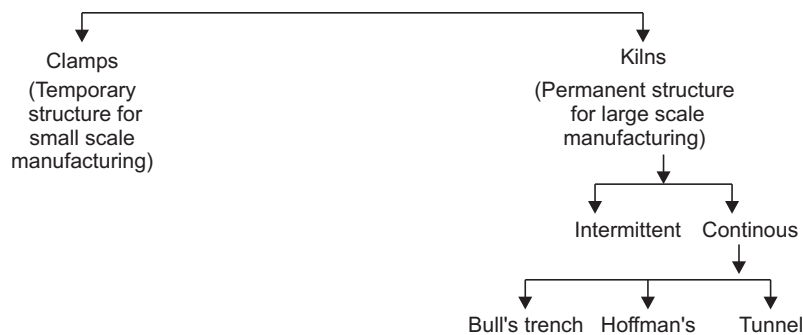
2. Moulding: Process of giving required shape to the brick earth.



3. Drying: Green bricks if burnt can get cracked and distorted.



4. Burning: Over burnt bricks become brittle while under burnt bricks will be soft and hence cannot carry loads.



Stages of burning are (a) Dehydration (400-650°C) (b) Oxidation (650-900°C) (c) Vitrification (900-1250°C)

Comparison between clamp Burning and Kiln Burning

S.No.	Item	Clamp burning	Kiln burning
1	Capacity	About 20000 to 100000 bricks can be prepared at time	Average 25000 bricks can be prepared per day.
2.	Cost	Low, as grass, cow dung litter, etc. may be used.	Generally high as coal dust is to be used.
3.	Initial cost	Very low as no structures are to be built	More as permanent structures are to be constructed.
4.	Quality of bricks	The percentage of good quality bricks is small about 60% or so.	The fire is under control throughout the process of burning.
5.	Regulation of fire	It is not possible to control or regulate fire during the process of burning	The fire is under control throughout the process of burning.
6.	Skilled supervision	Not necessary through-out the process of burning.	The continuous skilled supervision is necessary
7.	Structure	Temporary structure.	Permanent structure
8.	Suitability	Suitable when bricks are to be manufactured on a small scale and when the demand of brick is not continuous	Suitable when bricks are to be manufactured on a large scale and when there is continuous demand of bricks
9.	Time of burning and cooling	It requires about 2 to 6 months for burning and cooling of bricks.	Actual time for burning of one hamer is about 24 hours and only about 12 days are required for cooling of bricks.
10.	Wastage of heat	There is considerable wastage of heat from top and sides and hot fuel gas is not properly utilized.	The hot fuel gas is used to dry and pre heat raw bricks. Hence the wastage of heat is the least.

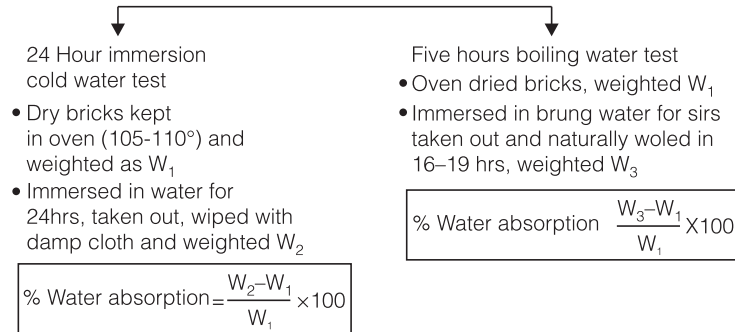
Comparison between Bull's trench Kiln and Hoffman's Kiln

S. No.	Item	Bull's Trench Kiln	Hoffman's Klin
1.	Burning capacity	About 3 lakhs in 12 days.	About 40 lakhs in one season.
2.	Cost of fuel	High as consumption of fuel is more.	Low as consumption of fuel is less.
3.	Cost of fuel	High as consumption of fuel is more	Low as consumption of fuel is less.
4.	Drying space	It requires more space for drying of bricks	It requires less space for drying of bricks.
5.	Initial cost	Low	High
6.	Nature	It is semicontinuous in loose sense.	It is perfectly Continuous.
7.	Popularity	More popular because of less initial cost.	Less popular because of high initial cost.
8.	Quality of bricks	Percentage of good quality bricks is small	Percentage of good quality bricks is more.
9.	Suitability	Suitable when demand of bricks in monsoon is not substantial	Suitable when demand of bricks is throughout the year.

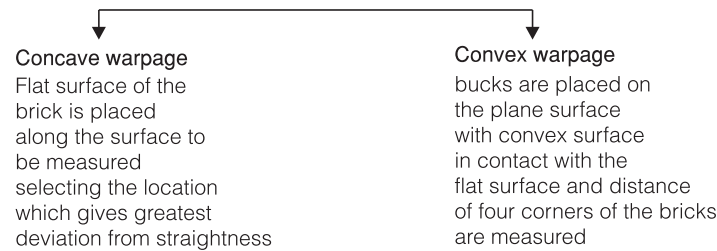
Testing of bricks

1. **Dimension Test (IS 1077):** 20 bricks are randomly selected of standard size ($19 \times 9 \times 9$ cm).

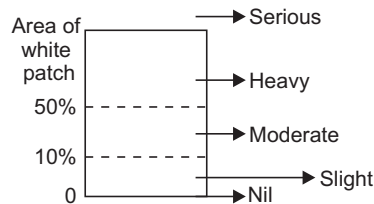
	Sub Class-A		Sub Class-B	
	Dimension (cm)	Tolerance (mm)	Dimension (cm)	Tolerance (mm)
Length	380	± 12	380	± 30
Width	180	± 6	180	± 15
Height				
(i) 9 cm	180	± 6	180	± 15
(ii) 4 cm	80	± 3	80	± 6

2. Water Absorption test: (IS 3495 – Part III)

3. Compressive Strength test (IS : 3495 – Part I): Bricks are immersed in water for 24 hours, Taken out, Frogs filled with 1 : 3 mortar, covered with Gunny bags for 24 hours, Again immersed in water for 3 days. Now taken out and placed in compression testing machine. (Rate of loading 14 N/mm²).

4. Warpage Test (IS : 3495 – Part IV):

5. Efflorescence test (IS 3495 – Part III): Brick is kept in a glass disc (dia 150 mm) with water upto 25 mm till entire water is absorbed or evaporated. Again water is filled upto 25 mm, and allowed to be absorbed or evaporated. Area of white patch is measured.

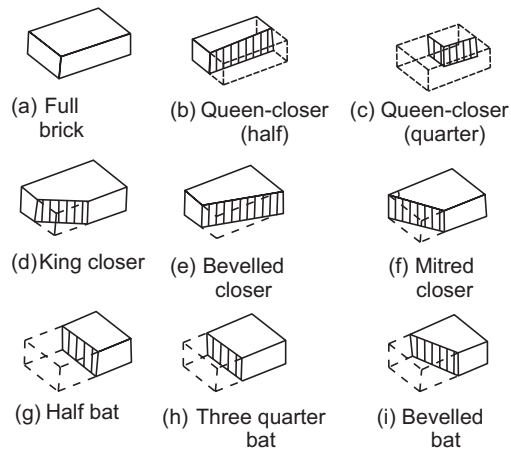


Defects of bricks

1. **Over burning:** Bricks loose their shape.
2. **Under burning:** Higher water absorption and less compressive strength.
3. **Bloating:** Spongy swollen mass over bricks surface due to excess of carbonaceous and sulphur matter.
4. **Black core:** Due to in complete oxidation of bituminous matter or carbon during burning.
5. **Efflorescence:** Due to alkalies.
6. **Chuffs:** Deformation of shape of the bricks caused by the rain water falling on hot bricks.
7. **Checks or cracks:** Because of lumps of lime or excess water.
8. **Spots:** Dark surface spots due to presence of sulphide.
9. **Blisters:** Formed to due to air imprisoned during their moulding.
10. **Laminations:** It is due to entrapped air in the Voids of day.

Brick Masonry

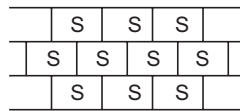
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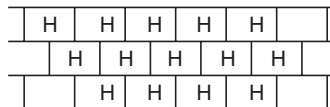
Bonds in brick work: It is method of arranging the bricks in courses such that individual units are tied together and vertical joint of the successive courses do not lie in same vertical line.

Types of bonds

- 1. Stretcher bond:** All the bricks are laid as Stretchers on the faces of the wall.



- 2. Header bond:** All the bricks are laid as headers on the faces of the wall.



3. **English bond:** Alternate courses of headers and stretchers.

H	H	H	H	H	H
S	S	S	S	S	S
H	H	H	H	H	H

4. **Flemish bond:** Each course has alternate header and stretcher.

H	S	H	S	H
S	H	S	H	S

5. **Facing bond:** Bricks of different thickness are to be used in the facing or backing of the wall.

Note: English bond is stronger than Flemish bond. But Flemish bond requires greater skill in construction and gives better appearance than English bond.

Defects in brick masonry

1. **Sulphate attack:** Sulphate salts in brick react with hydraulic lime (in case of lime mortar) or with alumina of cement (in case of cement mortar) resulting in increase in volume and finally spalling of bricks.
2. **Crystallization of salts from bricks:** Occurs due to excessive soluble salts in the bricks. Can be improved by brushing and washing on the affected surface from time to time.
3. **Corrosion of embedded fixtures:** Generally occurs when iron or steel fixtures are embedded in lime mortar.
4. **Drying shrinkage:** On evaporation of moisture from the brick, the brick shrinks resulting in development of cracks in masonry joint.

Slenderness ratio of brick masonry: Effective height of the wall divided by effective thickness or effective length, **whichever is less.**

Maximum slenderness ratio for load bearing walls

No. of storeys	Using Portland cement or pozzolana cement	Using lime mortar
Not exceeding 2	27	20
exceeding 2	27	13

Shape factor: It takes into account the effect of shape of the brick i.e., ratio of height to thickness.

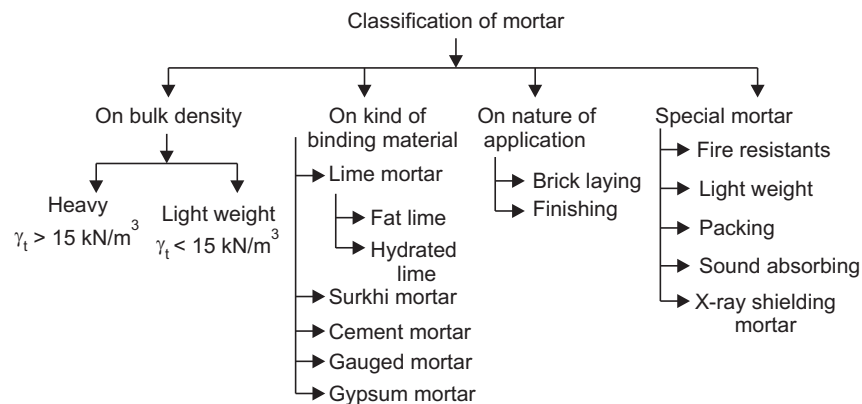
Ratio of height to thickness	Shape factor
0.75	1
1.0	1.2
1.5	1.6
2 to 3	2.0

Note: Permissible tensile stress of brick masonry is 0.1 N/mm^2 where as. Permissible shear stress of brick masonry is 0.15 N/mm^2 .

Mortar

7

It is a mixture containing a binding agent (cement), water and aggregate (sand) and acquiring stone like properties as a result of hardening. It is used for filling joints as a binder in stone and brick masonry.



- Fat lime is unsuitable for water logged areas or in damp situations where or hydrated lime is used in damp situations and it possesses more strength also.
- Surkhi is a pozzolanic material. Surkhi mortar cannot be used for plastering or pointing as Surkhi is likely to disintegrate after some time.
- Gauged mortar is a lime cement mortar. In lime mortar, cement is sometimes added to achieve early strength and it's called **gauging**.
- Gypsum mortar is prepared from gypsum binding materials.
- Fire resistant mortar is prepared by adding aluminous cement to the crushed powder of fire bricks. Used in lining furnaces, fire places, ovens etc.
- Light weight mortar is prepared by adding materials such as saw dust, wood powder etc. to lime or cement mortar. Used in sound proof and heat proof construction.
- Packing mortars are used to pack oil wells possess high water resistant property.

- Sound absorbing mortars have light weight porous material as aggregates.
- X-ray shielding mortars have bulk density over 22 kN/m³.

Selection of mortar

S.No.	Nature of work	Type of mortar	Proportions
1.	Water logged areas	Hydraulic lime	1 : 3
2.	Damp proof course and cement concrete roads	Cement mortar	1 : 2
3.	General RCC works	Cement mortar	1 : 3
4.	Internal walls	Lime cinder mortar	1 : 3
5.	For laying fire bricks	Fire resistant mortar	1 part aluminous
6.	Partition wall or Parapet wall	Cement mortar	1 : 3
7.	Plaster work	Cement mortar Lime mortar	1 : 3 to 1 : 4 1 : 2
8.	Pointing work	Cement mortar	1 : 1 to 1 : 2
9.	Stone masonry	Hydraulic lime	1 : 2
10.	Thin joints in brick work	Lime mortar	1 : 3

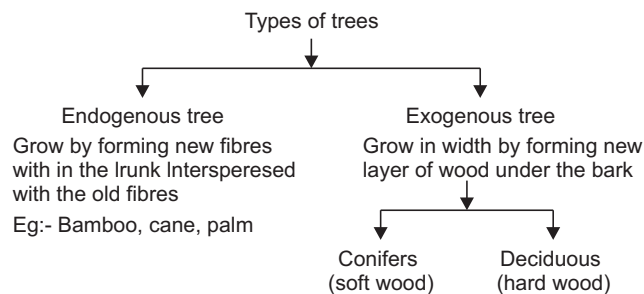
Grout: Cement mortar of **fluid consistency** used to fill the voids and joints in masonry and to repair the cracks water-cement ratio should be kept as low as possible to increase strength and reduce shrinkage.

Guniting: Application of mortar or concrete under pneumatic pressure through **cement gun**. It is done in layers of 40 – 50 mm. A thin layer of grout builds up and acts like a cushion reducing the percentage rebound in the successive layers. Average unit weight of gunite is 2300 kg/m³.

Timber

8

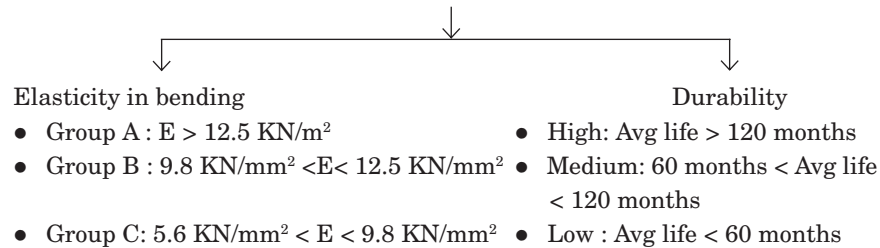
Timber applies to the trees measuring not less than **600 mm** in **circumference** of the trunk



Comparison of softwood and Hardwood

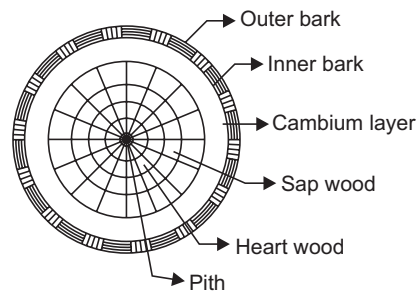
Property	Softwood	Hard-wood
Colour	Lighter	Darker
Growth	Faster	Slower
Weight	Lighter	Heavier
Density	Low	High
Annual rings	Distinct	Indistinct
Heart-wood	Cannot be distinguished	Can be distinguished
Strength	Strong along grains	Strong along and across grains
Conversion	Easy	Difficult
Resinous Material	Exist in pores	Does not exist
Examples	Chir, fir deodar, Kali Pine,	Teak, Sal, Shishum, Poplar and Maple

Other Classifications Based on



Structure of Timber

1. **Pith:** Innermost central portion consists of cellular tissues and nourishes plant at young age.



2. **Heart wood:** Dead portion of the tree, Dark in colour, Provides rigidity.
3. **Sap wood:** Light in colour, shows recent growth, sap moves in upward direction through it.
4. **Cambium layer:** Sap which has yet not converted into sap wood.
5. **Inner bark:** Protection of cambium from injury.
6. **Outer bark:** Consists of cells of wood fibre also known as cortex.
7. **Medullary ray:** Hold the annual rings of heart wood and sap wood.

Note: Wood added during early part is different from the wood added during the late part. Because of this difference between the early wood and late wood, **Growth rings** (Annual rings) becomes apparent.

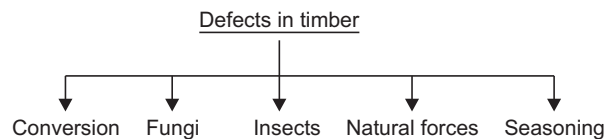
Properties of Timber:

1. Specific gravity of wood = **1.54**
2. Weight of timber should be noted at **12%** moisture content
3. Timber is **orthotropic**, shrinks least along length and maximum along circumference

4. Fibre saturation point is % moisture content at which cell walls are saturated and cell cavity contains no water.
5. Heat conductivity is low, but conductivity along the fibres is 1.8 times greater than across the fibres.
6. Timber has high sound conductivity, nearly 2 – 17 times greater than air.
7. Very high compressive force resistant property along the longitudinal axis Hence used in column's, posts, props etc.
8. Tensile strength along the grains is 2-4 times stronger than compressive strength.
9. Static bending strength measured by one point loading test.
10. Resistance of wood to cutting across the fibers (shear strength) is 3 to 4 times greater than along the fibers
11. Modulus of elasticity is generally $0.5 - 1 \times 10^4 \text{ N/mm}^2$ and its ratio of longitudinal to transverse direction is 1 – 2 times.

Advantages of Timber: High specific strength, Better insulating properties, light in weight, easily convertible to any size and shape, Easy connections in timber, even high resale and salvage value.

Disadvantages of Timber: Decay if not seasoned properly, high maintenance cost, not suitable for cyclonic weather, high risk of fire if not locally available it may prove to be very costly.



1. Conversion defects

- (i) **Chip mark:** Marks or signs placed by chips on finished surface of timber.
- (ii) **Diagonal grain:** Improper sawing, Indicated by diagonal mark on straight grained surface of timber.
- (iii) **Torn grain:** Small depression on the finished surface due to falling of tool
- (iv) **Wane:** Presence of original rounded surface on the manufactured timber.

2. Defects due to fungi:

- (i) **Blue stain:** Sap of wood is stained to bluish colour
- (ii) **Brown rot:** Fungi of certain type remove cellulose compounds from wood and it attains brown colour.

- (iii) **Dry rot:** occurs due to lack of ventilation for eg in basements, rooms or in damp situations like kitchen.
- (iv) **Heart rot:** Heart wood is exposed to atmosphere and attacked by atmospheric agents.
- (v) **Sap Stain:** Fungi feeds on cell contents of sap wood, then the sap wood loses its colour
- (vi) **Wet rot:** Chemical decompositions of wood due to alternate dry and wet conditions
- (vii) **White rot:** Fungi attacks lignin of wood and wood assumes the appearance of white mass consisting of cellulose compounds

3. Defects due to insects:

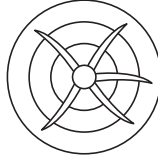
- (a) **Termites:** Also known as white ants, make tunnels inside timber in different directions at the centre leaving the outer shell intact.
- (b) **Beetles:** Small insects forming 2 mm diameter pin holes.
- (c) **Marine Borers:** Generally found in salty water. They do not feed on wood but make bore for taking shelter.

4. Defects due to Natural forces:

- (a) **Burls:** Also known as Excrescences, upset growth or irregular projection in a tree due to shock or injury received in its young age.
- (b) **Callus:** Soft tissue or skin which covers the wound of a tree.
- (c) **Chemical stain:** Wood is discoloured by the chemical action
- (d) **Coarse grain:** Widening of annual rings due to rapid growth
- (e) **Dead wood:** Timber from dead standing trees
- (f) **Druxiness:** White decayed spots concealed by healthy woods
- (g) **Foxiness:** Red or yellow tinge or reddish brown stains around the pith due to lack of ventilation or over maturity of tree.
- (h) **Knots:** Bases of branches which are broken or cut-off from the tree
- (i) **Rind galls:** Abnormal growth or curved swellings on the body of tree.
- (j) **Shakes:** Longitudinal separations in the wood between the annual rings.

(i) Heart Shake:

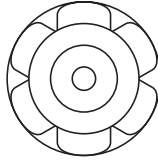
- Occurs in overmatured trees



Wide at centre and diminish outwards

(ii) Star shake

- Due frost and fierce heat of sun
- Wood dries below fibre saturation point
- Radial splits, wide cracks at circumference



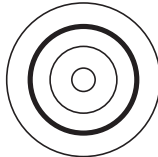
(iii) Cup shake

- Excessive frost action on the sap of young trees



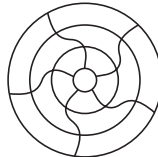
(iv) Ring shakes

- Cup shakes cover the entire ring



(v) Radial shakes

- When tree is exposed to sun for seasoning after being fell down



- (k) **Twisted fibres:** Twisting of young trees by fast blowing winds in one direction.
- (l) **Upsets:** Also known as ruptures due to improper felling of a tree
- (m) **Water Stain:** Wood is discoloured when it comes into contact with water
- (n) **Wind cracks:** When wood is exposed to atmospheric agencies, its exterior surface cracks.

5. Defects due to seasoning:

- (a) Bow (b) cup (c) Twist (d) Spring
 (e) Case hardening (f) Split (g) Honey Combing
 (h) Shakes (i) Collapse (j) check
 (k) warp.

Preservation of Timber (IS: 401)

It is carried out to

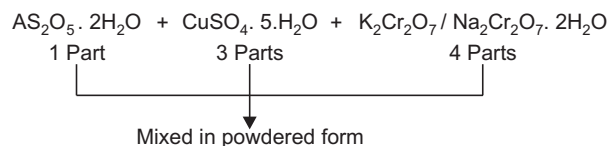
1. Increase the life of timber
2. Make the timber durable.
3. Protect the timber structures from the attack of destroying agencies.

Requirements of Good preservative

1. It should penetrate at least for a depth of 6 mm to 25 mm.
2. Cheap, easily available and free from unpleasant smell
3. Capable of covering large area with small quantity
4. Should allow decorative treatment on timber after being applied over timber surface.

Types of Preservatives:

- (a) **Ascu Treatment:** Developed by forest Research Institute, Dehradun

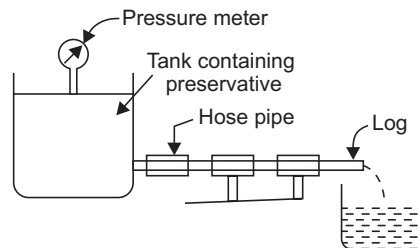


Solution is made by 6 parts of powder in 100 parts of water by weight. Preservative gives protection against white ants.

- (b) **Chemical Salts:** Water borne preservatives, mostly soluble in water like copper sulphate, mercury chloride, Sodium fluoride and zinc-chloride. Surface treated by these preservatives can be later painted or varnished after drying.
- (c) **Coal tar:** Surface is coated with hot coal tar with the help of brush. It is fire resistant.
- (d) **Creosote oil:** Obtained by distillations of tar. It is one of the best antiseptic. It is black or brown liquid weakly affected by water, neither volatile nor hygroscopic. Should not be used for interior surface of dwelling.
- (e) **Oil points:** Timber is coated with 2 to 3 coats of oil points. They preserve timber from moisture and make it durable.
- (f) **Solignum paints:** Highly toxic in nature and preserve timber from white ants. Can be mixed with coloured pigments and applied in hot state with a brush

Methods of Treatment (IS: 401)

1. **Surface Application:** Simplest method and generally adopted for seasoned timber. It does not give surface absorption and is limited to treatment at site or for retreatment of cut surface.
2. **Soaking:** Timber is debarked thoroughly and submerged in the preservative solution for a sufficiently long period till the required absorption is obtained.
3. **Hot and cold process:** Timber is submerged in a tank containing solution of preservative which is heated for few hours at temp of 85°C – 95°C . Tank is then allowed to cool down gradually while the timber is still submerged in the tank. This method is effective in giving protection to the sap wood.
4. **Boucherie process:** Treatment is carried out by attaching the butt end of a pole a hose pipe connected to a reservoir of the preservative at a higher level.



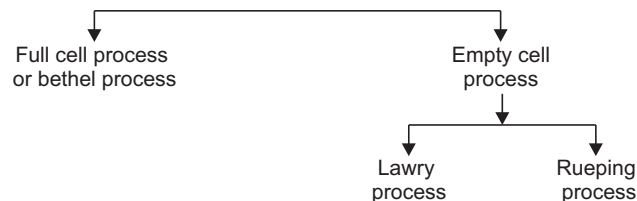
5. Diffusion process: It can be done as

- (a) Momentary dipping of timber in concentrated solution
- (b) Prolonged immersion in dilute solution
- (c) Application of a preservative paste all over the surface of the timber.

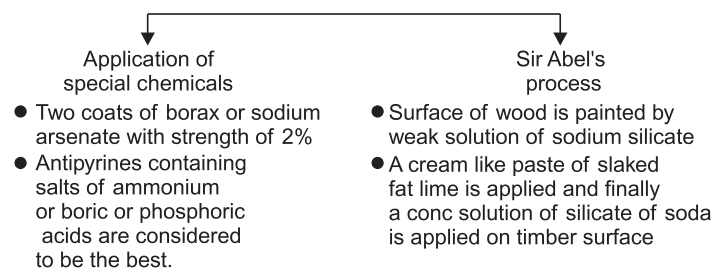
In this method, the inorganic toxic elements diffuse into timber from the place of application at high concentration to other zones through the medium of water contained in timber.

6. Charring : The surface to be charred is kept wet for about half an hour and then burnt to a depth of 15 mm over wood fire. Due to burning a layer of coal is formed on the surface. This layer is neither effected by moisture nor by fungi. Major disadvantages of charring is that, there is loss of strength of timber and it cannot be used for exterior surface.

7. Pressure or pneumatic process: It's a commercial process for treating large quantity of timber. Large metallic drums are employed into which the charge of timber stacked on trolleys is taken in, drum is then closed and the process of treatment is started

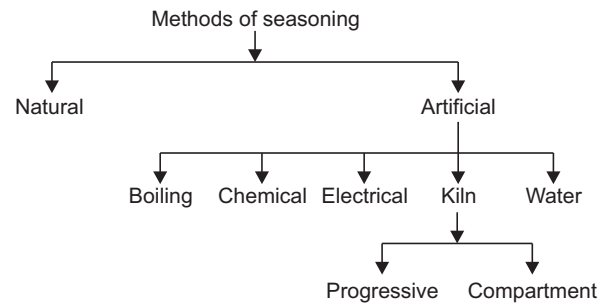


Fire Resistance of timber: Fire resistant characteristics of timber are better than steel which buckle and collapse much earlier than wood work. Timber cannot be made fire proof, can be only made fire resistant by



Seasoning of Timber: It is the process of **reducing the moisture content** of timber in order to prevent the timber from possible fermentation. During seasoning remaining moisture which cannot be extracted is uniformly distributed throughout the mass it drying is regular, the shrinkage of timber will also be irregular and it will set up internal stresses between the fibres. It is done to

- (a) Reduce its weight
- (b) Make timber burn readily as a fuel
- (c) Reduce shrinkage and warping
- (d) Increase strength and durability.
- (e) Make it suitable for painting.
- (f) Reduce its tendency to split and decay



Natural Seasoning: It's carried out by natural air hence even called air seasoning

Advantage

- Moisture content can be brought down to 10–20%
- Does not require skilled supervision.
- It's cheap and simple.
- Uncommercial if section is taken thicker than 100 mm

Disadvantages

- Difficult to control as process is dependent on natural air
- Drying may not be even or uniform
- Chances of end splitting.

Artificial Seasoning: Adopted for faster drying. It is possible to provide different drying conditions based on the species of timber.

- (a) **Boiling:** Timber is immersed in water and it is boiled. It effects the elasticity and strength of wood.

- (b) **Chemical (Salt seasoning):** Timber is immersed in solution of suitable salt. It is taken out and seasoned in the ordinary way.
- (c) **Electrical Seasoning:** High frequency alternating current is used. It's most rapid but uneconomical method.
- (d) **Kitn Seasoning:** Drying is carried out in air tight chamber or oven
- (e) **Water Seasoning:** Log's of wood immersed in running water with their larger ends pointing upstream Therefore sap, sugar and gum are leached out and are replaced by water.

Civil Engineering

7

Fluid Mechanics

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1. Properties of Fluid
2. Pressure and its measurement
3. Hydrostatic–Forces
4. Buoyancy and Flotation
5. Liquid in Relative Equilibrium
6. Fluid Kinematics
7. Fluid Dynamics
8. Momentum Equation and Application
9. Weir and Notches
10. Laminar Flow
11. Turbulent Flow
12. Boundary Layer Thickness
13. Dimensional Analysis and Model Studies
14. Pipe Flow

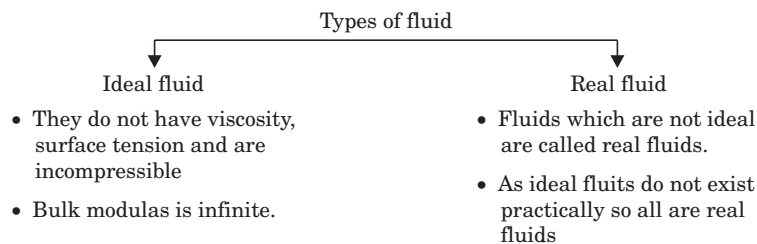
Properties of Fluid

1

Substance in liquid or gaseous phase is called fluid. They are capable of deforming continuously under the action of shear stress.

Note:

In solids stress is proportional to strain but in fluid stress is proportional to strain rate.



Some specific fluid properties

1. Density = $\frac{\text{mass}}{\text{volume}}$ measured in kg/m^3 .
2. Liquid density is constant while that of gas is directly proportional to pressure and inversely to temperature
3. Specific gravity/relative density = $\frac{\text{Density of substance}}{\text{Density of water at } 4^\circ\text{C}}$
4. If $R.D < 1$ then fluid is lighter than water.
5. Specific weight = $\frac{\text{Weight of substance}}{\text{Volume of substance}}$, denoted by $\gamma = \rho g$ in N/m^3
6. $\gamma_{\text{water}} = 9810 \frac{\text{N}}{\text{m}^3} = 9.81 \frac{\text{KN}}{\text{m}^3}$
7. $\gamma_{\text{mercury}} = 13.6 \gamma_w$
8. Specific volume = $\frac{1}{\text{Density}}$

Vapour Pressure and Cavitation

When liquid molecules are heated then the molecules on the surface of the liquid start converting into gaseous form called vapour. These vapours molecules exert partial pressure in the space called **vapour pressure**.

When the **absolute pressure** above a liquid surface becomes less than or equal to the vapour pressure then boiling starts. It leads to formation of cavity inside the liquid surface, and if this occurs in flowing liquid then this cavity is washed away in region of **higher pressure** where water from the cavity surrounding's rushes in, to fill the cavity and this bubble burst's. This phenomenon of formation and collapsing is called **cavitation**.

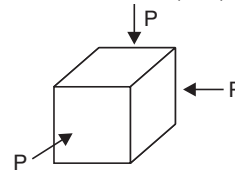
Note:

1. Higher temperature, more chances of cavitation.
2. Mercury has very low vapour pressure, hence used in pressure measuring equipment.

At 100°C, vapour pressure of water = Atmospheric pressure.

Bulk modulus: Bulk modulus of elasticity (k) = $\frac{-dp}{\left(\frac{dv}{v}\right)} = \frac{-dp}{\left(\frac{d\delta}{\delta}\right)}$

$$\text{Compressibility} = \frac{1}{K} = \frac{d\delta}{\delta dp}$$



Note:

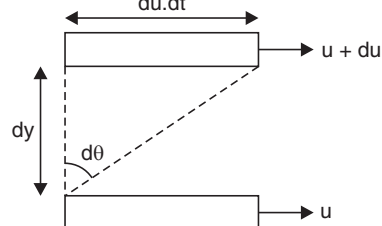
If density does not change with pressure *i.e.*, $\frac{\partial \delta}{dp} \neq 0$, then fluid is incompressible

- Isothermal bulk modulus $K_T = \text{Pressure}$
- Adiabatic bulk modulus $K_A = \gamma \times \text{pressure}$,
 γ – Adiabatic. Index

$$\gamma = \frac{C_P}{C_V} = \frac{\text{Specific heat at const. Pressure}}{\text{Specific heat at const. Volume}}$$

Viscosity: It's the measure of resistance of fluid to deformation.

It is due to the internal frictional forces that develop between different layers of fluid when they are forced to **move relative to each other**.



$$\text{Shear strain} = d\theta = \frac{du \cdot dt}{dy}$$

$$\text{Rate of change of shear strain} \frac{d\theta}{dt} = \frac{du}{dy} \text{ velocity gradient}$$

In Newtonian fluids,

$\tau \propto$ Rate of change of shear strain

$$\tau \propto \frac{d\theta}{dt} \propto \frac{du}{dy} \Rightarrow \tau = \mu \frac{du}{dy}$$

μ = absolute viscosity, dynamic viscosity or coefficient of viscosity

$$\text{Unit of } \mu = \frac{\text{NS}}{\text{m}^2} \text{ or } \frac{\text{kg}}{\text{m.s}} \text{ or pascal sec (SI) or poise}$$

$$(\text{CGS unit}) \text{ or } \frac{\text{Dyne} - \text{sec}}{\text{cm}^2}$$

$$1 \frac{\text{NS}}{\text{m}^2} (\text{SI}) = 10 \text{ poise}$$

Note: $\mu_{\text{water}} \approx 50 \mu_{\text{air}}$

$$\text{Kinematic viscosity} = \frac{\text{dynamic viscosity}}{\text{density}} = \frac{\mu}{\rho} \Rightarrow \nu = \frac{\mu}{\rho}$$

$$\text{SI unit} = \frac{\text{m}^2}{\text{sec}} \quad \text{CGS unit} = \frac{\text{cm}^2}{\text{sec}} \text{ or stoke}$$

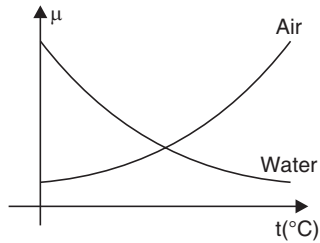
$$1 \text{ m}^2/\text{s} (\text{SI}) = 10^4 \text{ stoke}$$

Note: $\nu_{\text{air}} \approx 15.2 \nu_{\text{water}}$ at 20°C
 $\mu_{\text{water}} = 1 \text{ centipoise}$ at 1°C

Viscosity of liquids is due to **cohesion** but for gases it is due to **molecular momentum transfer**.

Effect of temperature and pressure on fluid's viscosity

7.6 CIVIL ENGINEERING



$$\mu_{\text{liq}} = \frac{\mu_0}{1 + \alpha t + \beta t^2}$$

$$\mu_{\text{gas}} = \mu_0 + \alpha t - \beta t^2$$

α, β are constants t = temp in $^{\circ}\text{C}$

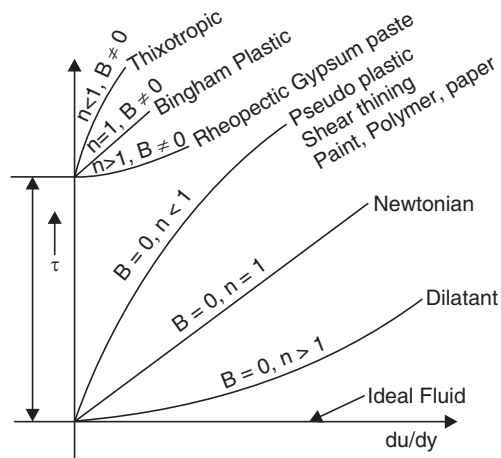
For liquids, μ does not depend on pressure except at high pressure.

For gases, μ_{gas} also doesn't depend on pressure but as δ is proportional to pressure

So,
$$v_{\text{gas}} \propto \frac{1}{\text{Pressure}}$$

Newtonian and Non-Newtonian fluids

- If $\tau = \mu \frac{du}{dy}$ then Newtonian fluids otherwise not



- $\tau = A \left(\frac{du}{dy} \right)^n + B$ General shear equation
- Slope of the curve gives **apparent viscosity**.
- Study of Now-Newtonian fluid is called **Rheology**.
- Pseduo plastic are **shear thinning** while Dilatants are **shear thickening** fluids.
- Eg
 - (a) Thixotropic Ink, ketchup, Enamels
 - (b) Bingham plastic Toothpaste, sewage, sludge, Drilling mud, gel (i.e., Related to Bath room).
 - (c) Rheopectic Gypsum in water and Bentonite slurry.
 - (d) Pseudo Plastic Paint, Polymer, Paper, Pulp, Blood, syrup
 - (e) Dilatant Quick sand, Butter, sugar in water

Note:

Tujhe	Bachk	Rchna	Padega	Nahitoh	Dil tot jaeyga
↓	↓	↓	↓	↓	↓
Thixotropic	Bingham	Rheopectic	Pseudo	Newtonian	Dilatant

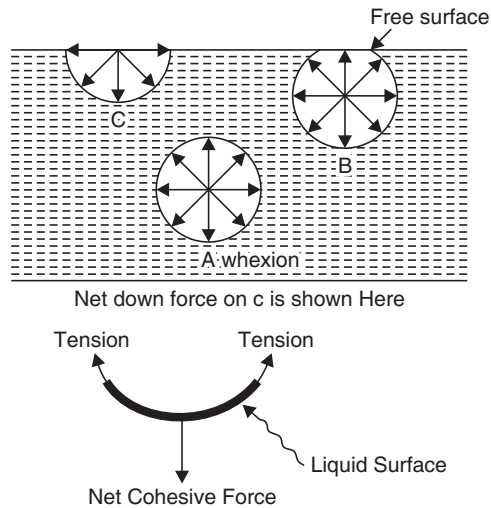
No-slip condition of viscous fluid: When ever a viscous fluid flows over a solid surface, the relative velocity between the solid surface and the adjacent fluid particle is zero. This condition is known as No-slip condition.

Note:

1. Wetting property is due to surface tension.
2. No slip condition is due to fluid viscosity.
3. Ideal fluids → No-viscosity → No “No slip” condition

Surface tension and capillary effect: It occurs at the **liquid-gas interface** or at the interface of **two immiscible liquids** where a thin film is apparently formed due to attraction of liquid in the surface which is similar to tension in stretched membrane known as surface tension

measured as $\frac{\text{force}}{\text{length}}$ Unit $\frac{\text{N}}{\text{m}}$



Note:

Surface tension occurs due to cohesion only.

Surface tension = $\frac{\text{Work done}}{\text{Change in area to work done}}$
 $\sigma_{\text{water/air}} = 0.0786 \text{ N/m}$, At critical point it becomes zero

P = Gauge pressure

σ = Surface tension

d = diameter

Then

(a) Pressure inside jet $P = \frac{2\sigma}{d}$

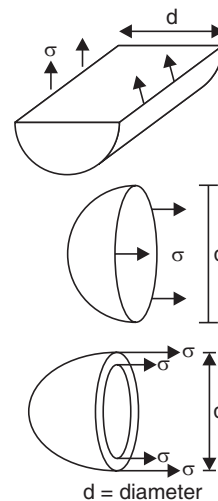
P = Gauge pressure

(b) Pressure in side $P = \frac{4\sigma}{d}$

σ = Surface tension

(c) Pressure inside bubble $P = \frac{8\sigma}{d}$

d = diameter.

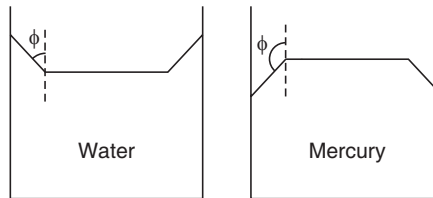


Rise or fall in the surface of liquid when a **small** diameter tube is inserted into the liquid is called capillary rise or capillary depression respectively.

Note:

Capillary effect is due to **Adhesion** and **surface tension** both

Force of attraction between molecules of different types is called adhesion while in molecules of same type it is called cohesion (for eg clay is a cohesive soil)



Note:

$\phi < 90^\circ$	Cohesion < Adhesion	Wetting of surface	Concave top surface	Rise in capillary tube
$\phi > 90^\circ$	Adhesion < cohesion	Does not wets the surface	Convex top surface	Drop in capillary tube

$$h = \frac{4\sigma \cos \theta}{(G_1 - G_2)\gamma_w d}$$

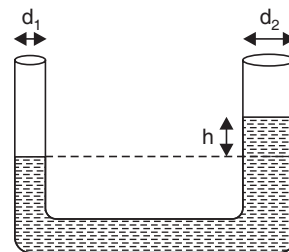
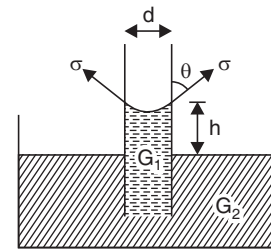
θ for water glass = 0° , mercury glass = 130° , Kerosene glass = 26°

Special case :-

$$h = \frac{4\sigma \cos \theta (d_2 - d_1)}{G\gamma_w d_2^2}$$

- $d_2 > d_1$, capillary rise is positive, that is water liquid rises in capillary tube of larger diameter (if $\phi < 90^\circ$ otherwise vice versa)
- $d_1 = d_2$ No capillary rise as $h = 0$
- If $d_2 \gg d_1$, then neglect d_1 wrt d_2

$$h = \frac{4\sigma \cos \theta}{G\gamma_w d_2}$$



Pressure and its Measurement

2

Normal force exerted by a fluid per unit area is called pressure. It is a **scalar Quantity** *i.e.*, it has magnitude but no direction.

Atmospheric Pressure: Pressure exerted by atmosphere. It is measured by **Barometer** At mean sea level it is equal to 10.3 m head of water or 76 cm head of mercury (specific gravity 13.6)

Note:

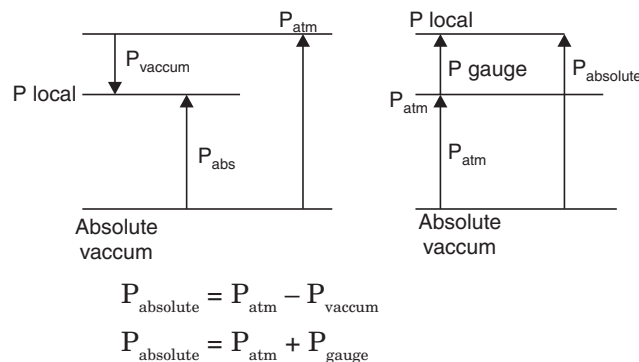
It head of water is ' h ' m then equivalent pressure is $\gamma_w h$ and if head of mercury is ' h ' m then equivalent pressure will be $\gamma_{Hg} h$

Absolute Pressure: Pressure with respect to absolute zero or complete Vacuum is called absolute pressure. It's the actual pressure and measured by **Aneroid Barometer**.

Gauge Pressure: Pressure with respect to atmospheric pressure as datum. It is measured using **Manometer** or **Bourdon gauge**

Steps to draw :-

1. Draw P_{atm} always above $P_{absolute}$ V_{acc} .
2. Then draw P_{gauge}/P_{vaccum} wrt to P_{atm} depending upon gauge reading



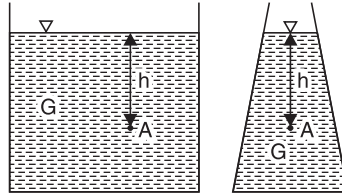
Note:

' h ' m of water vaccum means pressure of $-h \gamma_w$

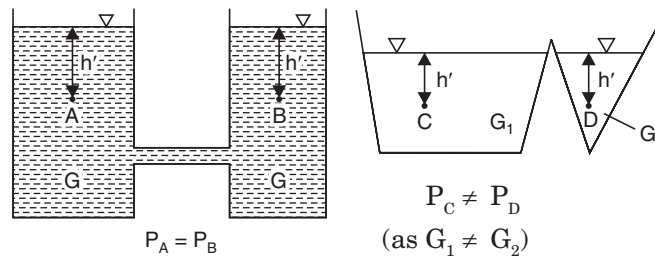
- A moving fluid exerts a tangential force on the surface apart from normal forces. However in absence of motion, fluid will exert normal force only.

Pressure Variation in Vertical direction for fluid at rest

$$P_{\text{gauge}} = G \gamma_{\text{o}} h \quad (G \text{ is Specific gravity of liquid})$$



Pressure at a point in a fluid at rest is independent of shape and cross-section of container in which it is kept. It varies in vertical direction and remains constant in horizontal direction.



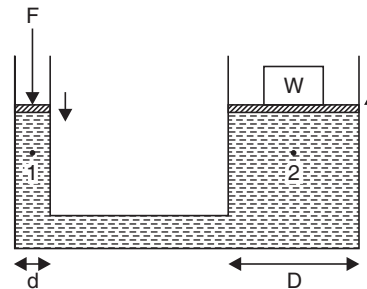
Note:

As per **Pascal's law**, pressure applied to a **confined** fluid increases the pressure throughout by the same amount.

$$P_1 = P_2$$

$$\frac{F}{\frac{\pi}{4}d^2} = \frac{W}{\frac{\pi}{4}D^2}$$

Hence $W = F \left(\frac{D}{d} \right)^2$



Principle of conservation of energy is still valid as if higher weight will be lifted up by small distance when smaller force comes down by large distance.

Toricceli-Barometer

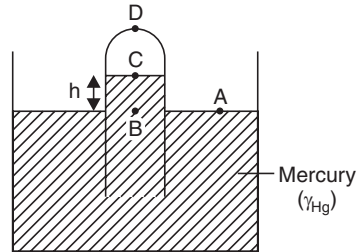
- Used to measure Atmospheric pressure
- A mercury filled tube is inverted to a mercury container that is open to atmosphere

$$P_A = P_B = P_{atm}$$

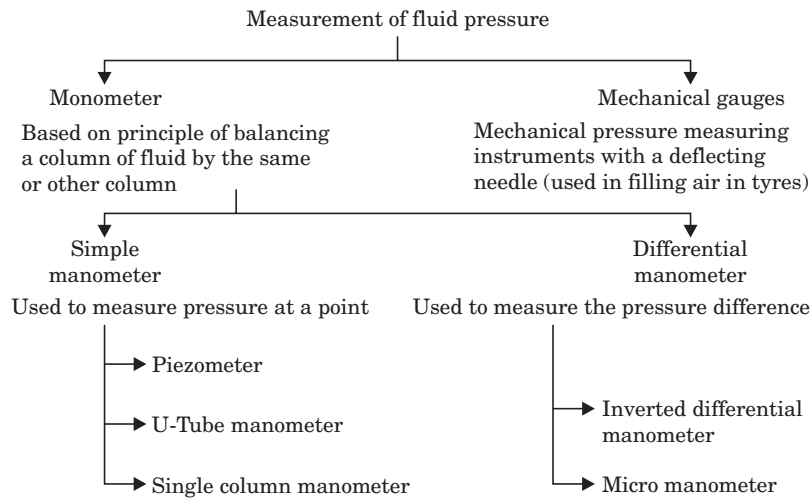
$$P_C = \text{Absolute zero}$$

$$P_B = P_C + \gamma_{Hg} h$$

Hence $P_{atm} = \gamma_{Hg} h$

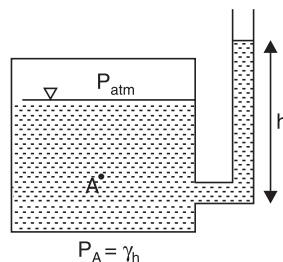


- If a hole is made at D, level of C will come down to B.



1. Piezometer

- -ve pressure cannot be measured
- Very long column of piezometer is required if pressure is large.
- Cannot measure the gas pressure.

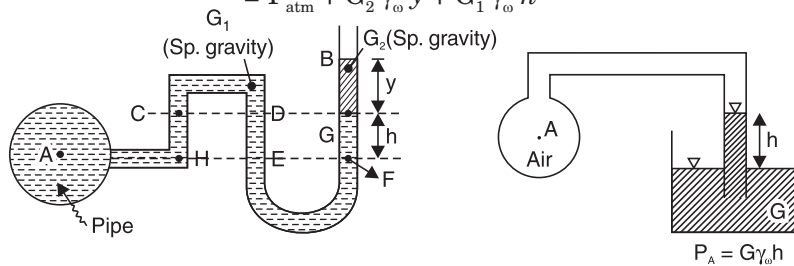


2. U-Tube manometer

- for large pressure measurement
- for measuring gas pressure
- for measurement of -ve pressure

Pressure at A = Pressure at H

$$= P_{\text{atm}} + G_2 \gamma_w y + G_1 \gamma_w h$$



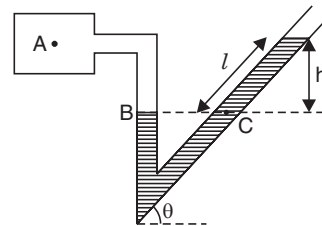
Note:

Liquid in U-tube manometer, should have specific gravity more than the liquid whose pressure is to be measured.

- Liquid should also have small thermal coefficient and small vapour pressure
- Liquid should be completely immiscible with the liquid whose pressure is to be measured.

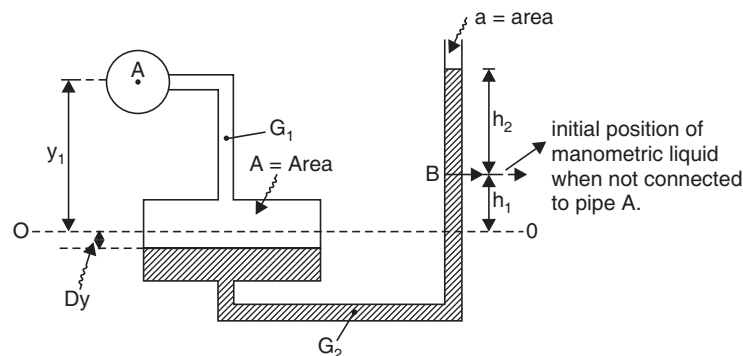
Special Case: To increase the **sensitivity** one leg is inclined.

$P_A = P_B = P_C = G \gamma_w h = G \gamma_w (l \sin \theta)$
measured reading of tube = 'l'



3. Single Column Manometer:

In this case **only one reading** is required as against the two readings on two limbs in case of U-tube manometer.



$$\% \text{ error} = \frac{G_2 \gamma_\omega h_2}{G_2 h_2 \gamma_\omega + (G_2 - G_1) \frac{a}{A} h_2 \gamma_\omega} \times 100 = 1-2\%$$

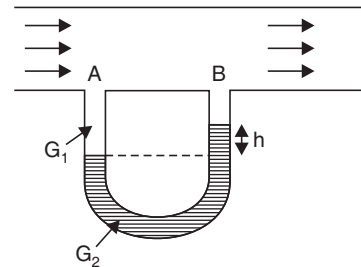
$$P_A = G_2 h_2 \gamma_\omega + (G_2 - G_1) \frac{a}{A} h_2 \gamma_\omega$$

Concept of differential manometer

$$P_A - P_B = (G_2 - G_1) \gamma_\omega h$$

$$P_A - P_B = (\text{head loss}) G_1 \gamma_\omega$$

$$\frac{P_A - P_B}{G_1 \gamma_\omega} = \left(\frac{G_2}{G_1} - 1 \right) h$$

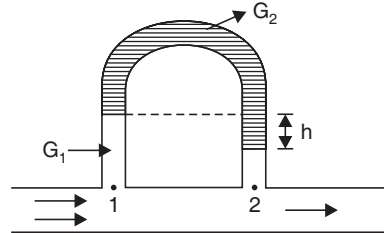


Inverted differential manometer (for measuring small pressure difference)

$$P_1 - P_2 = (G_1 - G_2) \gamma_\omega h$$

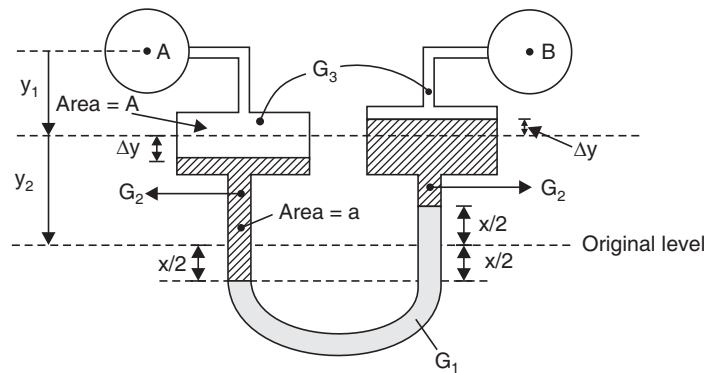
Here, $G_1 > G_2$ otherwise fluid G_2 will drop in pipe as soon as flow stops.

Here also sensitivity can be increased by inclining the gauge tube.



2. Micromanometer: measures very small pressure difference or for measuring the pressure difference with high precision.

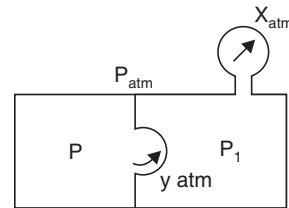
$$P_A - P_B = (G_1 - G_2) x \gamma_\omega$$



Note:

Multi fluid manometer is used in measuring pressure in a pressurized water tank.

$$\begin{aligned}
 P_1 &= P_{\text{atm}} + x_{\text{atm}} \\
 P &= P_1 + y_{\text{atm}} \\
 P &= P_{\text{atm}} + x_{\text{atm}} + y_{\text{atm}} \\
 P_{\text{atm}} &= 1 \text{ atm}
 \end{aligned}$$

**Factual facts about pressure**

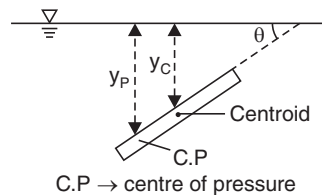
1. Longer runway's needed at higher altitude due to reduced drag and lift.
2. Aeroplane cruise at higher altitude's because of less drag, which increases fuel efficiency.
3. Nose bleeding starts at higher altitude because of difference in body's blood pressure and atmosphere pressure.
4. Motor capacity reduces at higher altitude.
5. Cooking takes longer time at higher altitudes.

Hydrostatic–Forces

3

Forces at every point on the **plane** surface can be added algebraically to obtain the magnitude of resultant force on the plane surface.

If the surface is **Curved**, then at every point, the direction of force due to stationary fluid is **Normal** to the surface



$$y_P = y_C + \frac{I_G \sin^2 \theta}{A y_C}$$

I_G = MOI about the centroidal axis

y_P = Centre of pressure from liquid surface

y_C = Centroid from the liquid surface

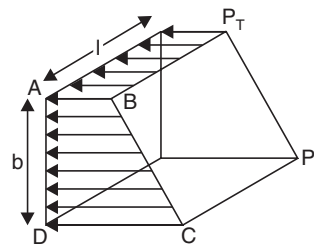
Note:

Magnitude of the resultant force acting on a plane surface of a **completely submerged plate** in a **homogenous** (constant density) fluid is equal to the product of pressure at centroid of surface and Area 'A' of the surface

$F = P_C A$ and this force acts at y_P

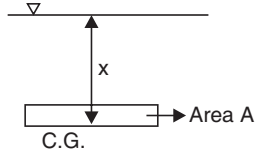
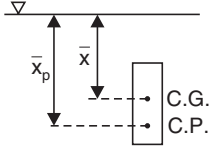
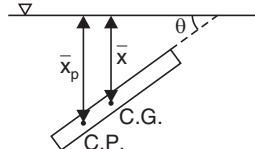
As we go deeper, difference of y_P and y_C will reduce.

Concept of Pressure Prism: Net force acting on a plane area is the product of average pressure acting on that area multiplied by the magnitude of area or it can be said as **Net force** acting on a plane area is equal to the **volume of the pressure prism** formed.

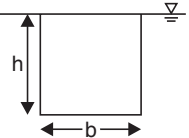
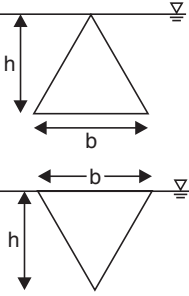
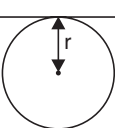


Area of trapezoid (ABCD) = $\frac{1}{2} (P_T + P_B) b$ (Force per unit length)

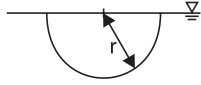
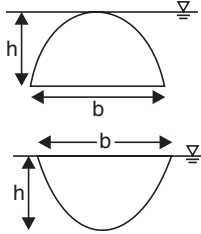
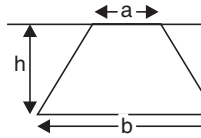
Total Force = $\frac{1}{2} (P_T + P_B) bl$ = Volume of prismoid

Horizontal Plane Surface	Vertical Plane Surface	Inclined Plane Surface
 $F = \gamma A \bar{x}$	 $F = \gamma A \bar{x}$ $x_p = \bar{x} + \frac{I_g}{A \bar{x}}$	 $F = \gamma A \bar{x}$ $x_p = \bar{x} + \frac{I_g \sin^2 \theta}{A \bar{x}}$

$\bar{x}$ and $\bar{x}_p$ for some vertical plane surface from liquid surface

Surface	C.G. ($\bar{x}$)	C.P. ($\bar{x}_p$)
1. 	$\frac{h}{2}$	$\frac{2h}{3}$
2. 	$\frac{2h}{3}$ $\frac{h}{3}$	$\frac{3h}{4}$ $\frac{h}{2}$
3. 	r	$\frac{5}{4} r$

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4. 	$\frac{4r}{3\pi}$	$\frac{3\pi r}{16}$
5. 	$\frac{3h}{5}$	$\frac{5h}{7}$
6. 	$\left(\frac{a+2b}{a+b} \right) \left(\frac{h}{3} \right)$	$\left(\frac{a+3b}{a+2b} \right) \frac{h}{2}$

Hydrostatic forces on the curved surface:

Horizontal Force (F_H): It is the resultant hydrostatic force ' f_x ' of curved surface may be computed by projecting the surface upon a vertical plane and multiplying the projected area by the pressure at its own centre of area.

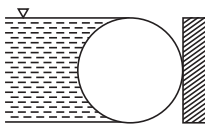
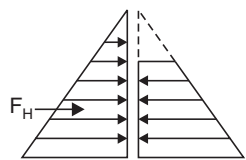
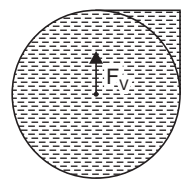
Vertical Force (F_V): It is the weight of the liquid contained in the zone bounded by two verticals drawn from the two ends of the curved surface, the curved surface and the free surface (which is applying pressure on the curved surface)

$$\text{Resultant force (F): } F = \sqrt{F_V^2 + F_H^2}$$

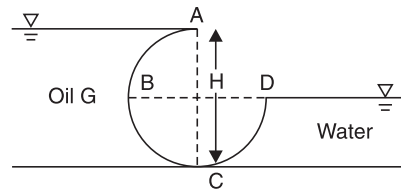
$$\tan \theta = F_y / F_x$$

$\theta \rightarrow$ Angle b/w line of action and Horizontal axis

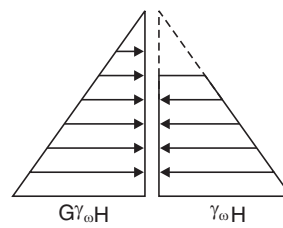
Special case (1)

	 Horizontal force (F_H) / length $\Rightarrow$ Resultant at these two pressure triangles.	 Vertical force (F_V)/length = weight of the liquid contained in the above diagram

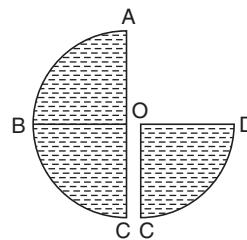
Special Case (2)



Horizontal force/length = Resultant of these two pressure triangles.



Vertical force/length = weight of oil filled in ABCO + weight of water filled in OCD



Buoyancy and Flotation

4

Archimedes Principle: When a body is wholly or partially submerged in a liquid then the vertical upward force acting on the body (called Buoyant force) is equal to the weight of the liquid displaced by the immersed part of the body.

Buoyant force = Net upward force = weight of liquid displaced

Note:

Point of application of this force is the C.G of the displaced liquid and it is called centre of buoyancy.

Flotation: A body will float in a liquid, if weight of body = weight of liquid displaced by its immersed part.

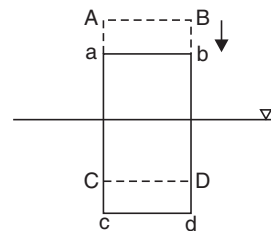
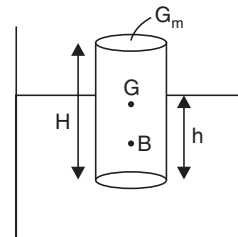
B = Centre of buoyancy at a distance of $h/2$ from base of cylinder.

$$h = G_m H$$

G = Centre of gravity, at a distance of $H/2$ from base of cylinder.

Where, G_m is specific gravity of material wrt liquid, which should be < 1

Linear Stability: When a small linear displacement sets up a restoring force, the body is in linear stability



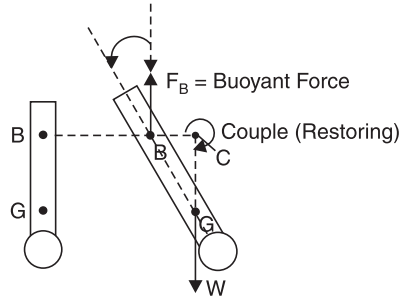
Submerged body

It remains in neutral equilibrium against linear displacement

Floating body

Remains in stable equilibrium against vertical displacement and in neutral equilibrium against horizontal displacement

Rotational Stability: When a small angular displacement sets up a restoring couple, then stability is known as rotational stability.



	Submerged body	Floating body
Stable equilibrium	G below B	M above G $BM > BG$ $GM = MB - BG = +Ve$
Unstable equilibrium	G above B	M below G $BM < MG$ $GM = BM - BG = -Ve$
Neutral equilibrium	G and B coincide	M at G $GM = 0$

Note:

Metacentre (M) is the point of intersection of lines of action of buoyant force before and after rotation.

GM = metacentre height

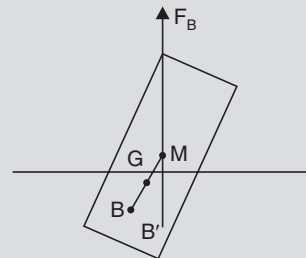
$$GM = BM - BG$$

Where $BM = \frac{I}{V}$,

V is the volume of liquid displaced

I = MOI of top view of the immersed part of the body about longitudinal axis.

Larger the metacentric height, greater the stability.



Special Case: For cylinder of specific gravity 'S' wrt liquid to be in

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neutral equilibrium.

$$\frac{L}{D} = \frac{1}{\sqrt{8S(1-S)}}$$

Time period of oscillation: If a floating body oscillates then its time period of transverse oscillation wrt metacentre is given by

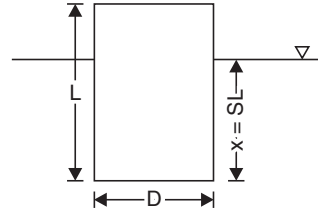
$I \rightarrow$ MOI about axis of rotation.

$$T = 2\pi \sqrt{\frac{I}{W(\overline{GM})}}$$

Larger the time period, more will be the comfort of passenger.

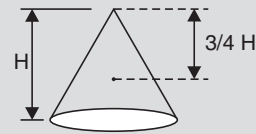
For passenger ship, $\overline{GM}$ is less so more comfortable.

For cargo ships $\overline{GM}$ is more so more stability.



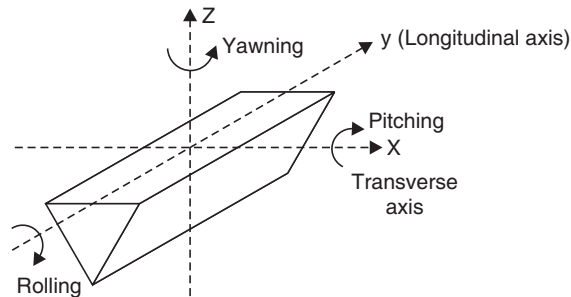
Note:

C.G of cone lies at $\frac{3}{4} H$ from the pointed end



Movements of a ship:

If a ship is safe in rolling, it will also be safe in pitching.



Liquid in Relative Equilibrium

5

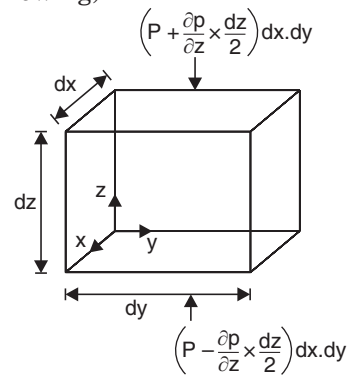
When a liquid is contained in a moving container then it behaves as a rigid body (*i.e.*, liquid is moving but not flowing)

By Newton's law of motion

$$\frac{-\partial P}{\partial x} = \gamma \frac{a_x}{g}$$

$$\frac{-\partial P}{\partial y} = \gamma \frac{a_y}{g}$$

$$\frac{-\partial P}{\partial z} = \gamma \frac{(a_z + g)}{g} \Rightarrow \text{Euler's equation}$$



Adding all three,

$$-\left(\frac{\partial P}{\partial x} \hat{i} + \frac{\partial P}{\partial y} \hat{j} + \frac{\partial P}{\partial z} \hat{k}\right) = \frac{\gamma}{g} (a_x \hat{i} + a_y \hat{j} + (a_z + g) \hat{k})$$

1. When fluid at rest $a_x = a_y = a_z = 0$

$$\text{then } \frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = 0 \quad \frac{\partial P}{\partial z} = -\delta g \Rightarrow P = -\delta g z$$

2. When fluid moves in upward direction with constant acceleration ($-a_z$) then

$$a_x = a_y = 0 \Rightarrow \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\rho(g - a_z)$$

$$\boxed{P = -\rho(g - a_z)z}$$

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3. When fluid moves in downward direction with constant acceleration (a_z) then

$$a_x = a_y = 0 \Rightarrow \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\delta(g + a_z)$$

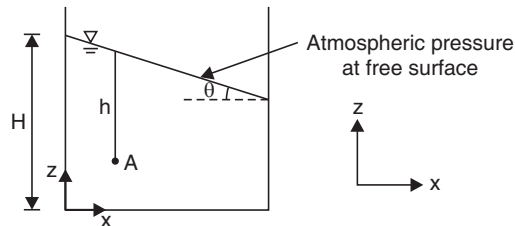
$$P = -\rho(g + a_z)z$$

4. With constant acceleration a_x an x -direction

$$P_A = \delta gh$$

$$\tan \theta = \frac{a_x}{g_{\text{eff}}}$$

$$Z = H - \frac{a_x x}{g_{\text{eff}}} \Rightarrow \text{Equation of free surface}$$



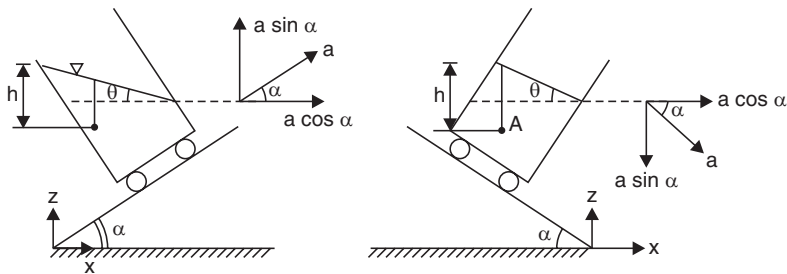
5. Constant acceleration on inclined slope

$$\tan \theta = \frac{a \cos \alpha}{g \pm a \sin \alpha}$$

$$P_A = \delta g_{\text{eff}} h$$

$$g_{\text{eff}} = g \pm a \sin \alpha$$

$+$ $\Rightarrow$ upward $- \Rightarrow$ downwards



Note:

θ is measured from the horizontal axis.

Rotation in Cylindrical Container.

$$\frac{\partial P}{\partial r} = \frac{\delta V^2}{r} \quad \dots(i)$$

$$\frac{\partial P}{\partial z} = -\delta(a_z + g) \quad \dots(ii)$$

Combining (i) and (ii)

$$dP = \frac{\delta V^2}{r} dr - \delta(a_z + g) dz$$

- **In free vortex motion**, angular momentum remains conserved as external torque is zero so $mvr = \text{constant} = C$

So $V \propto \frac{C}{r}$

Hence from (i) as radius increases pressure decreases. Eg: whirling mass of liquid in wash basin.

- **In forced vortex motion**, fluid is rotated about a vertical axis at constant speed such that every particle has the same angular velocity.

$$V = r \omega$$

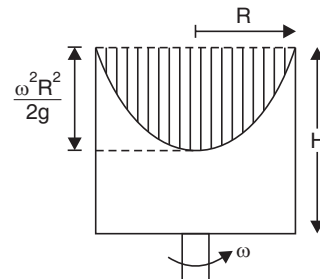
Hence from (i) as radius increases pressure increases. Eg: Flow inside centrifugal pump.

Amount of water spilled out = original volume – Remaining volume

Remaining volume = Volume of cylinder – volume of shaded paraboloid.

$$\text{Volume of cylinder} = \pi R^2 H$$

$$\text{Volume of paraboloid} = \frac{1}{2}(\pi R^2) \left(\frac{\omega^2 R^2}{2g} \right)$$

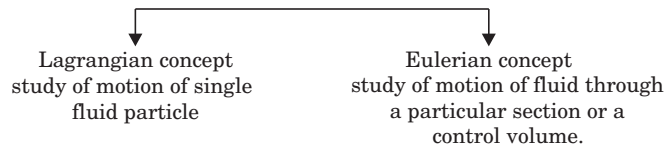

Note:

For No spilling case, Rise above original water level = Fall below original water level.

Fluid Kinematics

6

Fluid Kinematics: It deals with the motion of the fluids without necessarily considering the forces and moments that cause the motion.



Note:

We generally follow Eulerian concept, as its difficult to keep the track of a single fluid particle.

Types of fluid show:

1. **Steady and Unsteady Flow:** At any given location, the flow and fluid properties do not change **with time** then its steady flow otherwise unsteady.

$$\frac{\partial v}{\partial t} = 0, \frac{\partial p}{\partial t} = 0, \frac{\partial f}{\partial t} = 0 \Rightarrow \text{Steady flow}$$

2. **Uniform and Non-Uniform Flow:** At particular instant of time, the flow properties do not change with location then its uniform flow otherwise non-uniform flow $\left. \frac{\partial v}{\partial s} \right|_{t=0} = 0$ uniform flow

3. **One, two or three Dimensional Flow:** If flow parameters varies in one dimension wrt space only then its one dimensional otherwise its two or three dimension respectively.

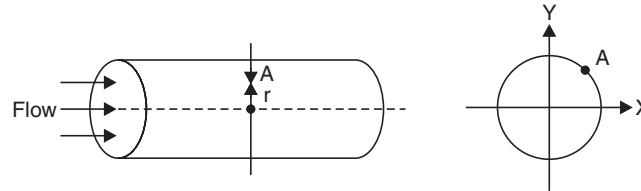
$$V = V(x, t) \rightarrow \text{one dimensional}$$

$$V = V(x, y, t) \rightarrow \text{two dimensional}$$

$$V = V(x, y, z, t) \rightarrow \text{three dimensional}$$

In cartesian co-ordinate system point A is represented as A(x, y)

While in polar co-ordinate system properties will be same all around the circumference of circle hence will depend only on r.



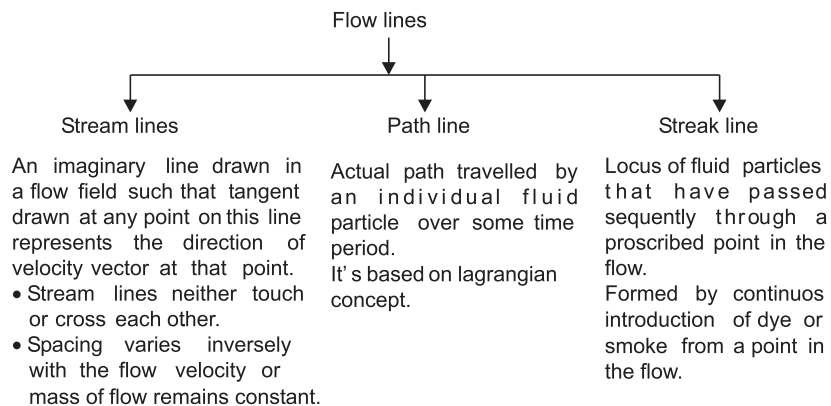
4. Laminar and Turbulent Flow: In Laminar flow, the particles move in layers sliding smoothly over the adjacent layers while in turbulent flow particles have the random and erratic movement, intermixing in the adjacent layers. Which causes continuous momentum transfer.

5. Rotational and Irrotational Flow: When fluid particles rotate about their mass centre during movement. Flow is said to be rotational otherwise irrotational.

Rotational Flow → Forced Vortex, Flow inside boundary layer.

Irrotational Flow → Free Vortex, Flow outside boundary layer.

6. Compressible and Incompressible Flow: In compressible flow density of fluid changes from time to time where in Incompressible flow it remains constant.



$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad \text{Equation of stream line}$$

Continuity Equation: Fluid mass can neither be created or can be destroyed so mass of fluid entering a fixed region should be equal to mass of fluid leaving that fixed region in a particular time. It is based on **principle of conservation of mass**.

Various forms of continuity Equation:

(a) Cartesian co-ordinate System:

$$(i) \text{ 3-D} \quad \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$(ii) \text{ 1-D} \quad \frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho A v)}{\partial s} = 0$$

$$(iii) \text{ Steady Flow in 1-D} \quad \rho AV = \text{Constant}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$(iv) \text{ Steady Incompressible in 1-D} \quad A_1 V_1 = A_2 V_2$$

$$(v) \text{ Steady flow in 3D} \quad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

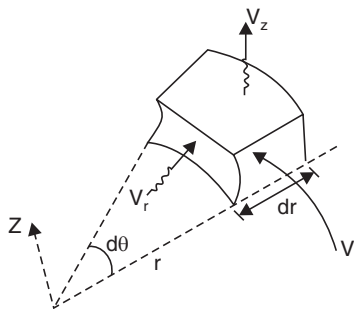
$$(vi) \text{ Steady Incompressible in 3D} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\text{Divergence of Velocity} = 0 \text{ i.e., } \vec{\nabla} \cdot \vec{v} = 0$$

(b) Cylindrical Polar coordinate System

$$(i) \text{ 3-D} \quad \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho r V_r)}{\partial r} + \frac{\partial(r V_\theta)}{r d\theta} + \frac{\partial(\rho V_z)}{\partial z} = 0$$

$$(ii) \text{ Steady Incompressible in 2D} \quad \frac{\partial(r V_r)}{\partial r} + \frac{\partial V_\theta}{\partial \theta} = 0$$



Acceleration of fluid:

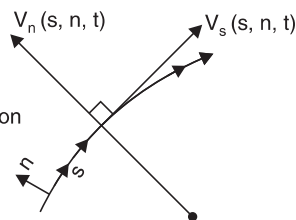
$$\vec{V} = u(x, y, z, t) \hat{i} + v(x, y, z, t) \hat{j} + w(x, y, z, t) \hat{k}$$

$$\begin{aligned}
 a_x &= u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \\
 a_y &= u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \\
 a_z &= u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}
 \end{aligned}$$

Local/Temporal acceleration i.e., acceleration component wrt time
 For steady Uniform flow, Total acceleration = 0
 Convective/Advective acceleration i.e., acceleration wrt place

Acceleration on a stream line

Tangential Acceleration
It is due to change in magnitude of velocity. If spacing b/w stream line changes tangential acceleration exists



Normal Acceleration
It is due to the change in the direction of fluid moving on a curved path

$$\begin{aligned}
 a_s &= V_s \frac{\partial v_s}{\partial s} + \frac{\partial v_s}{\partial t} & a_n &= V_s \frac{\partial v_n}{\partial s} + \frac{\partial v_n}{\partial t} \\
 \text{convective} & & \text{convective} & \\
 \text{tangential} & & \text{normal} & \\
 \text{acceleration} & & \text{acceleration} & \\
 & \text{local tangential} & & \text{local normal} \\
 & \text{acceleration} & & \text{acceleration}
 \end{aligned}$$

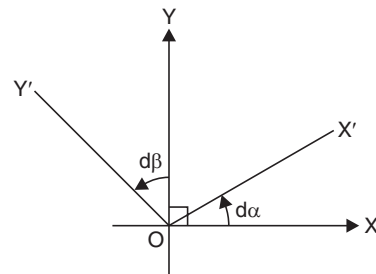
Angular Velocity: It is the average of rotation rate of two initially perpendicular lines that intersect at that point.

$$\frac{d\theta}{dt} = \frac{\left(\frac{d\alpha}{dt} + \frac{d\beta}{dt} \right)}{2}$$

$$\Omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$



$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\Omega = \frac{1}{2} \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix}$$

Note:

It angular velocity is zero, flow will be irrotational.

Vorticity (ξ) = Twice of Angular Velocity

Circulation (Γ) = line integral of tangential component of velocity vector along a closed curve.

Circulation = Vorticity $\times$ Area

Velocity Potential or Potential Function (ϕ): It is the scalar function of space and time such that its negative derivative wrt any direction gives velocity of flow in that direction.

In Cartesian co-ordinate System

$$\phi = f(x, y, z, t)$$

$$\frac{-\partial\phi}{\partial x} = u, \frac{-\partial\phi}{\partial y} = v, \frac{-\partial\phi}{\partial z} = w$$

In Cylindrical Polar co-ordinate System

$$\phi = f(r, \theta, z, t)$$

$$V_r = \frac{-\partial\phi}{\partial r}, V_\theta = \frac{-1}{r} \frac{d\phi}{d\theta}, V_z = \frac{-\partial\phi}{\partial z}$$

Note:

1. Velocity potential exists only for ideal and irrotational flow.
2. Velocity of flow is in direction of decreasing potential function.
3. Equipotential line is the line joining points having same potential function.

Stream Function (ψ): It is a scalar function of space and time such that its partial derivative wrt. any direction gives the velocity component at right angles (in anti clock wise direction) to this direction.

Cartesian co-ordinate system

$$\frac{\partial \Psi}{\partial x} = v, \quad \frac{\partial \Psi}{\partial y} = -u$$

Polar co-ordinate system

$$\frac{\partial \psi}{\partial r} = V_{\theta}, \quad \frac{\partial \psi}{r d\theta} = -V_r$$

Note:

If Stream function (ψ) satisfies the Laplace equation, then flow is irrotational otherwise rotational *i.e.*,

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0$$

It two points lie on same straight line then ψ will be constant.

Cauchy-Riemann Equation: For incompressible irrotational flow

$$u = \frac{-\partial \phi}{\partial x} = \frac{-\partial \Psi}{\partial y} \quad v = \frac{-\partial \phi}{\partial y} = \frac{\partial \Psi}{\partial x}$$

Fluid Dynamics

7

It is the study of motion of fluid along with the forces causing the motion.

(i) Newton's equation of motion

$$\vec{F}_g + \vec{F}_P + \vec{F}_V + \vec{F}_t + \vec{F}_c + \vec{F}_\sigma = m\vec{a}$$

(ii) Reynold's equation of motion

$$\vec{F}_g + \vec{F}_P + \vec{F}_V + \vec{F}_t = m\vec{a}$$

(iii) Navier-stock equation of motion

$$\vec{F}_g + \vec{F}_P + \vec{F}_V = m\vec{a}$$

(iv) Euler's equation of motion

$$\vec{F}_g + \vec{F}_P = m\vec{a}$$

where,

F_g = Gravity force

F_P = Pressure force

F_V = Viscous force

F_t = Turbulence force

F_c = Compressibility force

F_σ = Surface tension force

Note:

Euler equation based on momentum conservation while Bernoulli is based on energy conservation.

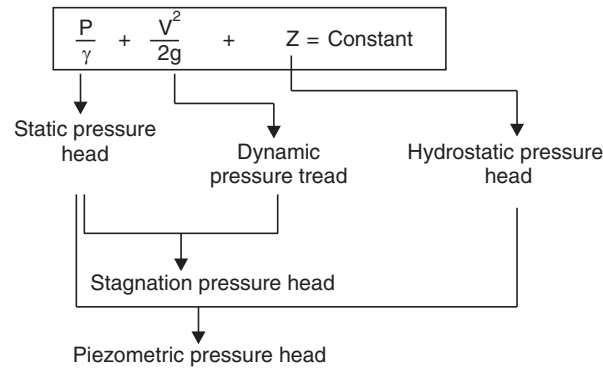
Bernoulli's Equation: It is the integration of Euler's equation of motion along a stream line under steady incompressible flow conditions.

Assumptions:

(i) Along Stream line

(ii) Ideal flow

(iii) Steady and Incompressible



Note:

When normal acceleration is zero *i.e.*, when particles move on a straight line then the piezometric head is a constant.

Kinetic Energy Correction Factor (α)

$$\alpha = \frac{\text{Actual K.E.}}{\text{K.E. Calculated from Average Velocity}}$$

$$\alpha = \frac{\int_A v^3 dA}{A V_{\text{avg}}^3} \quad V_{\text{avg}} = \frac{1}{A} \int_A v dA$$

Momentum Correction Factor (β)

$$\beta = \frac{\text{Actual linear momentum/sec}}{\text{Linear momentum calculated from Average Velocity}}$$

$$\beta = \frac{\int_A v^2 dA}{A V_{\text{avg}}^2} \quad V_{\text{avg}} = \frac{1}{A} \int_A v dA$$

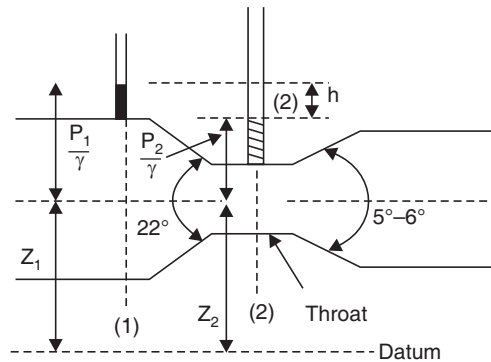
Note:

	α	β
Laminar flow between plates	2	4/3
Laminar flow between pipes	1.543	1.2
Turbulent flow in pipes	4/3	1.015

Applications of Energy Equation :-

1. Venturimeter

- To find discharge from a large diameter pipe
- Reduction in Area leads to increase in Velocity and decrease in pressure, this pressure decrease is noted and used in Bernoulli to calculate discharge.



$$\text{Piezometric head difference } h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\bullet Q_{\text{actual}} = c_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\Rightarrow a_1, a_2$ cross-sectional areas at section 1 and 2

$\Rightarrow c_d \rightarrow$ discharge coefficient

$\Rightarrow \frac{a_1}{a_2} =$ area ratio

$\Rightarrow \frac{a_1 a_2 \sqrt{2g}}{\sqrt{a_1^2 - a_2^2}}$, as this depends only on dimensions of venturimeter

it is called venturi-constant.

• $d \approx \left(\frac{1}{3} \text{ to } \frac{3}{4} \right) D$ where d = dia of throat

D = dia of pipe

$$c_d = \sqrt{\frac{h - h_L}{h}} \approx 0.98$$

- If inclined Venturimeter with differential manometer is used then piezometer head difference (h)

$$\left[h = \left(\frac{G_2}{G_1} - 1 \right) x \right] \text{ or } \left[h = \left(1 - \frac{G_2}{G_1} \right) x \right]$$

$$\left[\begin{array}{c} G_2 > G_1 \\ G_2 < G_1 \end{array} \right]$$

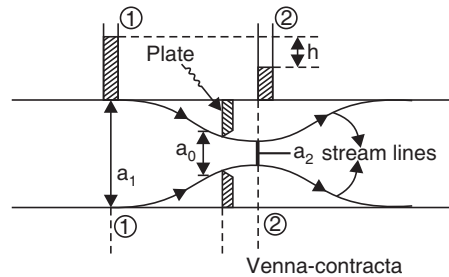
$G_2 \rightarrow$ always of liquid filled in manometer

$G_1 \rightarrow$ of flowing liquid.

$x \rightarrow$ reading on manometer

2. Orificemeter

- Circular plate with concentric sharp edged hole is installed in a pipe such that the plate is perpendicular to the axis of pipe.



- Cheaper instrument, measures discharge but has more losses hence $c_d = 0.64 - 0.76$
- Region of minimum flow area is called Vena contracta, here stream lines are assumed to be nearly parallel.

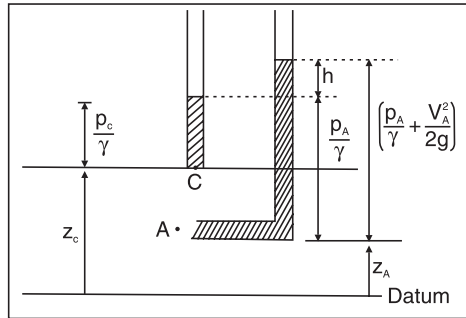
$$c_c = \frac{a_2}{a_0} = \frac{\text{Area of Vena contracta}}{\text{Area of opening}}$$

$$Q_{\text{actual}} = c_d \frac{a_1 a_0}{\sqrt{a_1^2 - a_0^2}} \sqrt{2gh}$$

Note:

If the discharge is changed, then the position of Vena contracta will also change and then stream lines will not be parallel at sec (2)-(2).

3. Pitot tube:



- Measures the velocity of fluid

- $\frac{V_A^2}{2g} = h = \text{measured}$

$$V_a = \sqrt{2gh}$$

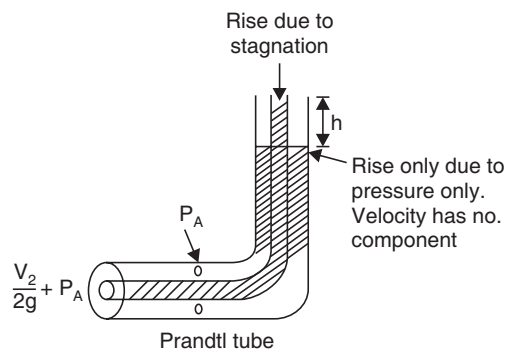
- $V_{a \text{ actual}} = C_v \sqrt{2gh}$

$$C_v = 0.98 \text{ (coefficient of velocity)}$$

- **Anemometer** measures gas and air velocity.

4. Pitot Static tube (Prandtl tube)

- Measures the velocity of fluid.
- It measures the piezometric head at the same point where velocity is to be measured.

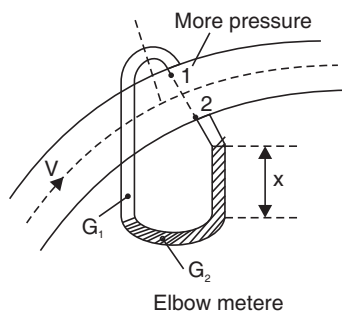


$$V_A = C_v \sqrt{2gh}$$

- Velocity head is found out from difference of total head and piezometric head.
- It can also used on rough boundaries.

5. Elbow meter or Bend meter

- Measures discharge
- When liquid moves along a bend pipe in free vortex $\left(V = \frac{c}{r}\right)$ then its pressure increases with radius.



- More r , less v , more $P \rightarrow$ at outer surface 1 or compared to 2
- $Q = c_d A \sqrt{2g \left(\frac{G_2}{G_1} - 1 \right) x}$

Note:

If in venturimeter, the pipe is not contracted such that $c_c = 1$ then it is termed as Nozzle meter and it is also used for calculating discharge.

- **Rotameter** is used to measure discharge while current meter is used to measure velocity in open channel.

Momentum Equation and Application

8

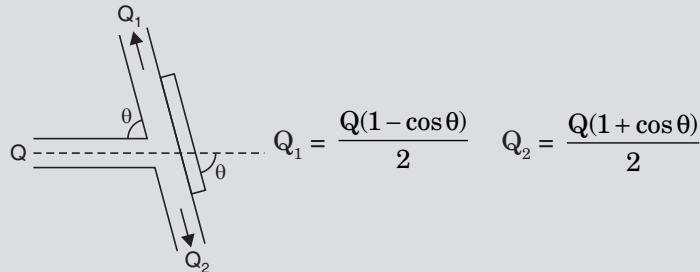
Rate of change of linear momentum in any direction of a body with respect to a fixed frame of reference is equal to external forces acting on the body in that direction.

$$\begin{aligned} \left[\begin{array}{c} \Sigma F_{\text{external}} \\ \text{on control} \\ \text{volume} \end{array} \right] &= \left[\begin{array}{c} \text{Net rate of flow} \\ \text{of linear} \\ \text{momentum out} \\ \text{of control volume} \\ \text{in x-direction} \end{array} \right] \\ &= \left[\begin{array}{c} \text{Rate of flow} \\ \text{of momentum} \\ \text{out of control} \\ \text{volume in} \\ \text{x-direction} \end{array} \right] - \left[\begin{array}{c} \text{Rate of flow} \\ \text{of momentum} \\ \text{into the control} \\ \text{volume in} \\ \text{x-direction} \end{array} \right] \end{aligned}$$

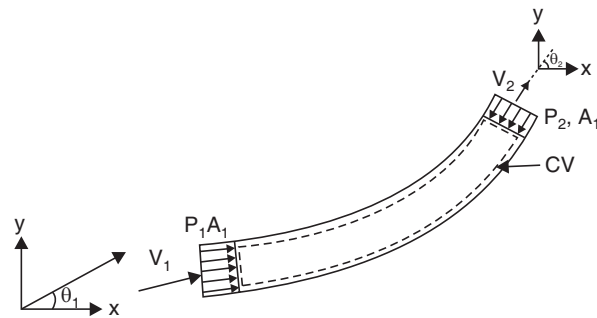
Rate of change of Angular momentum in any direction of a body with respect to a fixed frame of reference is equal to torque applied on the body in that direction.

$$\begin{aligned} \left[\begin{array}{c} \Sigma M_{\text{z external}} \\ \text{on control} \\ \text{volume} \end{array} \right] &= \left[\begin{array}{c} \text{Net torque on} \\ \text{control volume} \end{array} \right] \\ &= \left[\begin{array}{c} \text{Rate of change} \\ \text{of angular momentum} \\ \text{of control volume wrt} \\ \text{fixed frame of reference} \\ \text{in z-direction} \end{array} \right] + \left[\begin{array}{c} \text{Net rate of flow of} \\ \text{angular momentum} \\ \text{out of control volume} \\ \text{wrt fixed frame of} \\ \text{reference in z-direction} \end{array} \right] \end{aligned}$$

Note:



Force acting on a bend pipe:



In x -direction:

$$\Sigma F_x = \rho Q (v_2 \cos \theta_2 - v_1 \cos \theta_1)$$

$$P_1 A_1 \cos \theta_1 - P_2 A_2 \cos \theta_2 + R_x = \rho Q (v_2 \cos \theta_2 - v_1 \cos \theta_1)$$

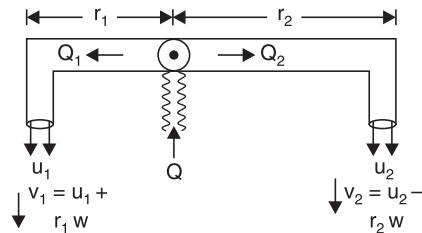
In y -direction: $\Sigma F_y = \rho Q (v_2 \sin \theta_2 - v_1 \sin \theta_1)$

$$P_1 A_1 \sin \theta_1 - P_2 A_2 \sin \theta_2 + R_y = \rho Q (v_2 \sin \theta_2 - v_1 \sin \theta_1)$$

$$\text{Resultant force } R = \sqrt{R_x^2 + R_y^2}$$

Sprinkler:

Torque on jet in z -direction = $\Sigma mvr = 0$



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$$\rho Q_2 v_2 r_2 - \rho Q_1 v_1 r_1 = 0$$

$$\text{Torque} = \rho(A_2 u_2) v_2 r_2 - \rho(A_1 u_1) v_1 r_1 = 0$$

Note:

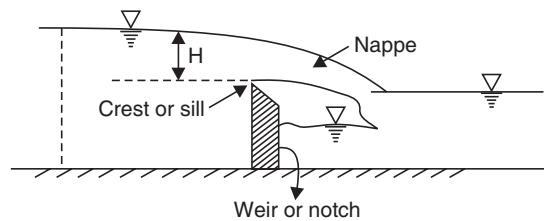
- Discharge Q is measured wrt relative velocity $Q_1 = A_1 u_1$, $Q_2 = A_2 u_2$
- v_1 and v_2 are absolute velocities of jet wrt ground

If net torque is zero then $\rho Q_1 v_1 r_1 = \rho Q_2 v_2 r_2$

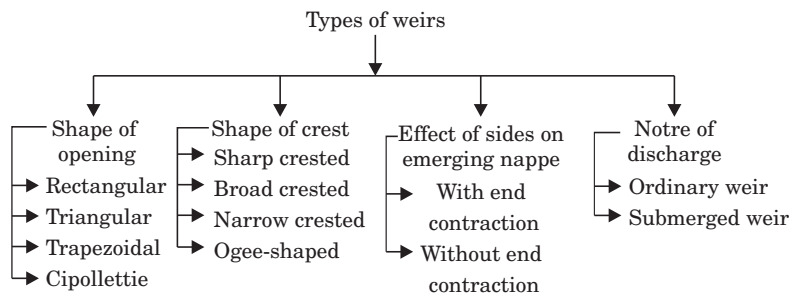
Weir and Notches

9

Weir: It is a concrete or masonry structure, constructed in an open channel to measure its discharge. It is generally in the form of vertical wall, with sharp edge at the top, running all the way across the open channel.



Notch: It is a device (generally metallic plate) used for measuring the discharge through a small channel or a tank.

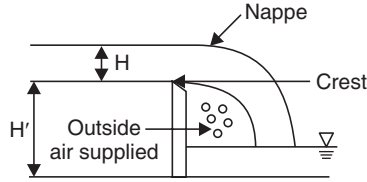


1. Rectangular sharp-crested Suppressed weir:

- Suppressed $\Rightarrow$ without end contraction.

$$Q_{\text{actual}} = \frac{2}{3} c_d L \sqrt{2g} H^{3/2} \quad c_d = 0.62$$

$H \rightarrow$ depth of water above crest level



- If velocity of approach (V_a) is also considered then

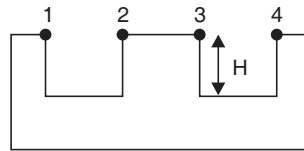
$$V_a = \frac{Q}{(H + H')L}, \quad h_a = \frac{V_a^2}{2g}, \quad Q = \frac{2}{3} c_d \sqrt{2g} L [(H + h_a)^{3/2} - h_a^{3/2}]$$

- Effect of end contraction *i.e.*, if not suppressed L is replaced by L_{eff}

$$L_{eff} = L - 0.1 n H$$

n = Number of end contractions (Here its 4)

$$Q = \frac{2}{3} c_d \sqrt{2g} L_{eff} H^{3/2}$$



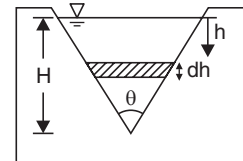
- When there is no ventilation of Nappe (*i.e.*, air is not supplied from outside) then the discharge will increase or the Nappe will be pulled down due to negative pressure created in the zone below Nappe.

2. Flow over V-Notch or triangular weir:

$$Q = \frac{8}{15} c_d \sqrt{2g} \tan \frac{\theta}{2} H^{5/2} \quad c_d = 0.52$$

- Considering velocity of approach then

$$Q = \frac{8}{15} c_d \sqrt{2g} \tan \frac{\theta}{2} [(H + h_a)^{5/2} - h_a^{5/2}]$$



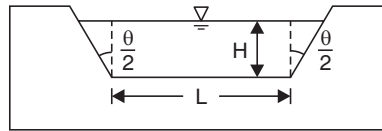
- End contraction is not taken into account in this case.

Advantages

- Only one dimension is to be measured hence more accurate
- c_d nearly constant with depth.
- Even for small discharge, high head is obtained. Hence no effect of viscosity and surface tension.

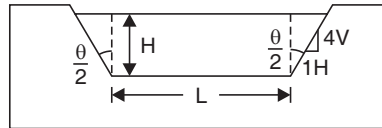
3. Trapezoidal Notch or weir:

$$Q = \frac{2}{3} c_{d_1} \sqrt{2g} L H^{3/2} + \frac{8}{15} c_{d_2} \sqrt{2g} \tan \frac{\theta}{2} H^{5/2}$$

**4. Cipolletti-Weir:**

It is a trapezoidal weir whose slopes are adjusted such that:

Decrease in discharge due to end contraction in rectangular weir = Increase in discharge due to triangular portion.



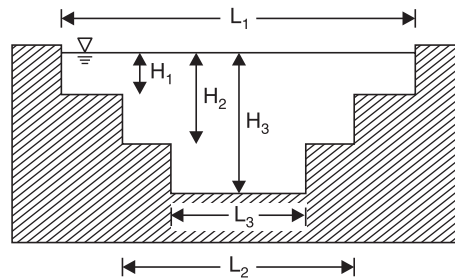
$$\tan \frac{\theta}{2} = \frac{1}{4}, \theta = 28^\circ$$

$$Q = \frac{2}{3} c_d \sqrt{2g} L H^{3/2}$$

$$c_d = 0.63$$

5. Stepped Notch

$$Q = \frac{2}{3} c_d \sqrt{2g} \left[L_1 H_1^{3/2} + L_2 H_2^{3/2} + L_3 H_3^{3/2} \right]$$



Note:

H_1, H_2, H_3 are measured from the top.

6. Broad crested weir

- Supports a Nappe such that stream lines become straight and pressure variation become hydrostatic over the weir.

- $Q = c_d L h \sqrt{2g(H - h)}$

- In this, flow adjusts itself to give max. discharge at available head H.
- For max, discharge

$$h = \frac{2}{3}H \quad Q = 1.7 c_d L H^{3/2}$$

$$c_d = 0.85 - 1,$$

h = critical depth as discharge is maximum

- If Velocity of approach is also considered

$$Q = 1.7 c_d L [(H + h_a)^{3/2} - h_a^{3/2}]$$

7. Proportional weir:

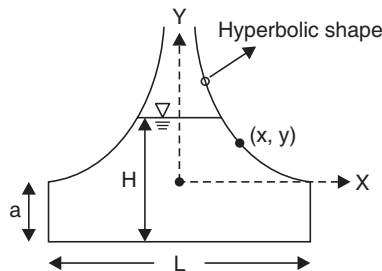
- $Q \propto H$

$$Q = K \left(H - \frac{a}{3} \right) \quad K = c_d L \sqrt{2ga}$$

$$c_d = 0.6 - 0.65$$

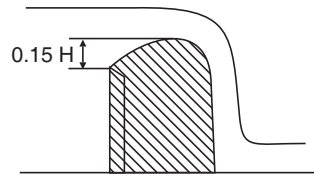
- Equation of curve $\frac{2x}{L} = 1 - \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{y}{a}} \right)$

- If there are fluctuations in discharge, then there will be less fluctuations in 'H' as compared to rectangular and triangular weirs.



8. Ogee spillway

- Profile of the crest is made such that it matches with the shape of water profile over sharp crested weir so as to avoid development of negative pressure below nappe (or Adhering Nappe).



$$Q = \frac{2}{3} c_d L \sqrt{2g} H^{3/2} \quad c_o = 0.62$$

9. Flow through orifice

- Orifice is small opening in tank
- $Q = c_d a \sqrt{2gh}$

$$c_c = \frac{a_c}{a} \quad c_d = c_c \times c_v$$

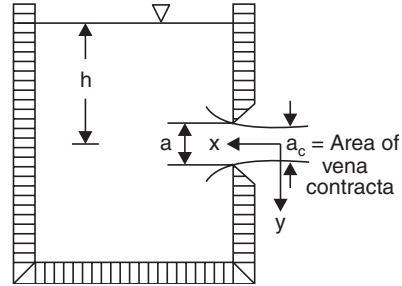
For equation of flow, eliminate 't' from below equations

$$x = vt \quad y = \frac{1}{2}gt^2 \quad \text{and get } V = \sqrt{\frac{gx^2}{2y}}$$

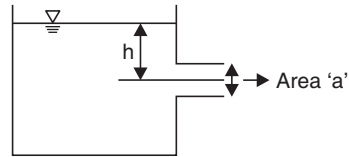
- For c_v , put $V = c_v \sqrt{2gh}$ and get $c_v = \sqrt{\frac{x^2}{4yh}}$

- Coefficient of resistance

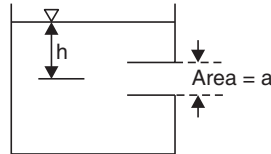
$$c_r = \frac{\text{loss of KE through orifice}}{\text{Actual KE}} = \left(\frac{1}{c_v^2} - 1 \right)$$

**10. Flow through mouth piece**

- Mouth piece is short length of tube with length $< (2 - 3)$ diameter
- $Q = c_d a \sqrt{2gh}$ $c_d = 0.82$

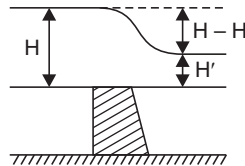
**11. Borda's weir**

$$Q = c_d a \sqrt{2gh}$$

**12. Submerged weir**

- When downstream water level is above the crest of the weir then it is said to be submerged

$$Q = \frac{2}{3} c_{d1} L \sqrt{2g} (H - H')^{3/2} + c_{d2} LH' \sqrt{2g(H - H')}$$



- Sharp crested weir is more susceptible to submergence than a broad crested weir

- Modular limit, is the limiting value of submergence ratio upto which submerged weir behaves as free weir.

$$\text{Submergence ratio} = \left(\frac{H'}{H + \frac{v_a^2}{2g}} \right) \text{ generally } = 0.83 \text{ to } 0.85$$

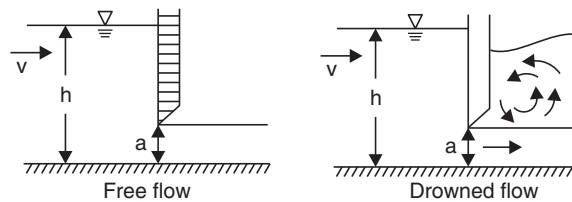
13. Discharge through sluice gate

$$v = \sqrt{2gh}$$

$$Q = c_d a L \sqrt{2gh}$$

L = Inside length

h = Water depth from ground



Effect on discharge due to error in head measurement

(i) For infinitesimal error's in head measurement

$$Q = KH^n$$

$$dQ = Kn H^{n-1} dH$$

$$\frac{dQ}{Q} \times 100 = n \left(\frac{dH}{H} \times 100 \right)$$

% error in discharge = $n \times$ % error in head measurement

$n = 1$ proportional weir,

$n = 1.5$ rectangular weir,

$n = 2.5$ triangular weir.

(ii) For Large error's in head measurement

$$\bullet \frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{KH_2^n - KH_1^n}{KH_1^n} \times 100$$

$$\bullet \text{ For V-notch } \frac{dQ}{Q} \times 100 = \left(\frac{\theta}{\sin \theta} \right) \left(\frac{d\theta}{\theta} \times 100 \right)$$

Laminar Flow

10

In Laminar flow fluid particles move along the straight parallel paths in layers. It occurs at a very low velocity, and here **Viscous force predominates the inertial forces**.

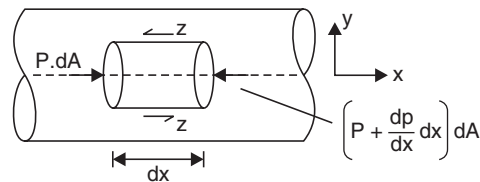
Nature of flow according to Reynold's number (R_e)

	Laminar	Transition	Turbulent
Flow in pipe	$R_e < 2000$	$2000 < R_e < 4000$	$R_e > 4000$
Flow between parallel plate	$R_e < 1000$	$1000 < R_e < 2000$	$R_e > 2000$
Flow in open channel	$R_e < 500$	$500 < R_e < 2000$	$R_e > 2000$
Flow through soil	$R_e < 1$	$1 < R_e < 2$	$R_e > 2$

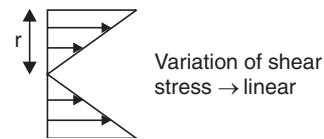
Flow through circular pipe (steady uniform flow)

$$1. \frac{dp}{dx} = \frac{dz}{dy}$$

x is the direction of flow
 y is perpendicular to x

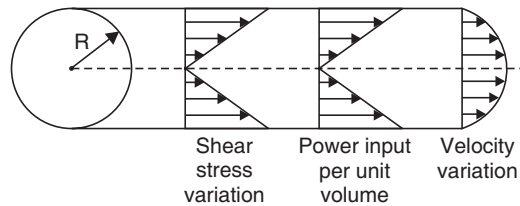


$$2. \tau = z = \frac{-dp}{dx} \left(\frac{r}{2} \right)$$



$$3. V = V_{\max} \left(1 - \frac{r^2}{R^2} \right)$$

$$4. V_{\max} = \frac{1}{4\mu} \left(\frac{-dp}{dx} \right) R^2$$



$$5. \tau_{\max} = \frac{2\mu V_{\max}}{R} = \left(\frac{P_1 - P_2}{L} \right) \frac{R}{2}$$

$$6. Q = \frac{\pi}{128\mu} \left(\frac{-dp}{dx} \right) D^4 \quad \text{Hagen Poiseuille Formula}$$

$$7. V_{\text{avg}} = \frac{Q}{\pi R^2} = \frac{1}{32\mu} \left(\frac{-dp}{dx} \right) D^2$$

$$8. V_{\max} = 2 V_{\text{avg}}$$

$$9. V = V_{\text{avg}} \text{ at } r = \frac{R}{\sqrt{2}} = 0.707 R$$

$$10. h_L = \frac{32\mu V^2 L}{\gamma D^2} = \frac{128\mu Q L}{\pi \gamma D^4}$$

$$11. h_L = \frac{f V^2}{2gD} = \frac{(4f') L V^2}{2gD}$$

$$f = \text{friction factor} = \frac{64}{R_e}$$

$$f' = \text{coefficient of friction}$$

Flow between two fixed parallel plates

$$1. u = \frac{1}{2\mu} \left(\frac{-dp}{dx} \right) (By - y^2)$$

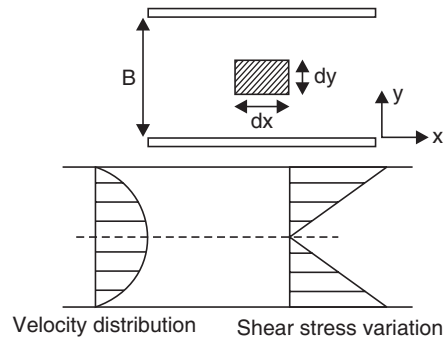
$$2. \tau = \mu \frac{du}{dy} = \frac{1}{2} \left(\frac{-dp}{dx} \right) (B - 2y)$$

$$3. Q = \frac{1}{12\mu} \left(\frac{-dp}{dx} \right) B^3$$

$$4. V_{\text{avg}} = \frac{Q}{A} = \frac{1}{12\mu} \left(\frac{-dp}{dx} \right) B^2$$

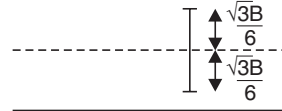
$$5. V_{\max} = \frac{1}{8\mu} \left(\frac{-dp}{dx} \right) B^2$$

$$6. V_{\max} = \frac{3}{2} V_{\text{avg}}$$



$$7. V = V_{avg} \text{ at } y = \frac{B}{2} \pm \frac{\sqrt{3}B}{6}$$

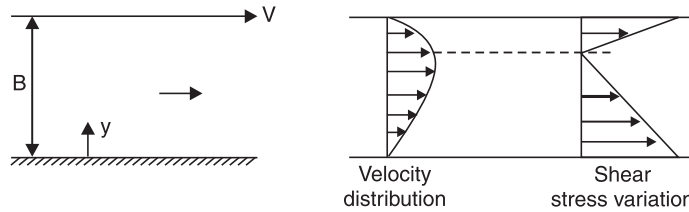
$$8. h_L = h_L = \frac{12\mu VL}{\gamma B^2}$$



For couette flow (one plate moving other at rest)

$$u = \frac{V_y}{B} + \frac{1}{2\mu} \left(\frac{-dp}{dx} \right) (By - y^2)$$

$$\tau = \frac{\mu V}{B} + \left(\frac{-dp}{dx} \right) \left(\frac{B}{2} - y \right)$$



Entrance length: The length of pipe from its entrance upto the point where flow attains fully developed velocity profile and which remains unaltered beyond that the known as entrance length.

For Laminar Flow $L_e = 0.07 R_e D$

For Turbulent Flow $L_e = 50 D$

NOTE:

For stability of laminar flow, a dimensionless parameter χ (chi) has been defined where $\chi(\text{Chi}) = \frac{y^2 \delta}{\mu} \left(\frac{du}{dy} \right)$ and if $\chi > 500$, then flow becomes unstable

- At the wall ($y = 0$) and at centre of pipe $\left(\frac{du}{dy} = 0 \right)$, x is zero.

Turbulent Flow

11

Turbulent flow results from the instability of laminar flow and due to continuous mixing between different layers. Momentum transfer occurs which gives rise to additional shear called Turbulent shear.

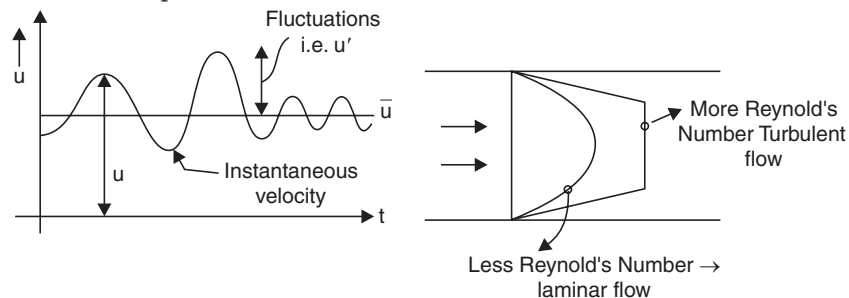
In turbulent flow every flow parameter is a combination of average value and fluctuating value

$$i.e. \quad u = \bar{u} + u'$$

Note :-

- Time average of fluctuating component is considered zero.

• As the Reynold's number of the flow increases, velocity profile becomes more flatter i.e. turbulent flow velocity profile is flatter than laminar flow profile



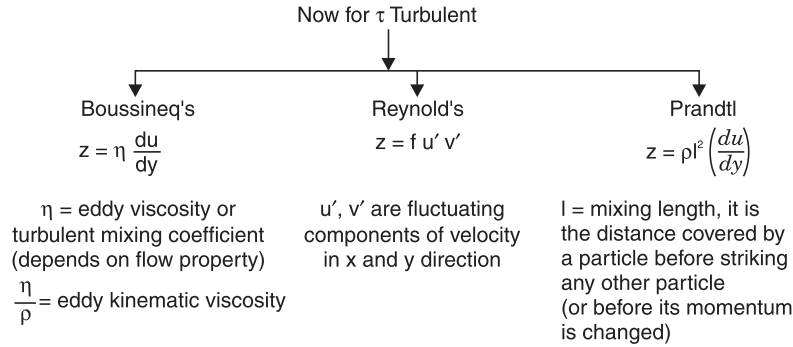
• **Shear stress at boundary** (τ_w) is **much less** in **Laminar flow** as compared to turbulent flow as velocity gradient near the boundary is large in turbulent flow.

Shear stress in turbulent flow

$$\tau_{\text{Total}} = \tau_{\text{Laminar}} + \tau_{\text{Turbulent}}$$

$$\tau_{\text{laminar}} = \mu \left(\frac{du}{dy} \right)$$

$\mu \rightarrow$ Dynamic Viscosity (depends on **fluid** characteristic)



Hydrodynamically smooth and Rough boundary

In hydrodynamically smooth boundaries, average height of roughness (K) is much less than the laminar sub layer (δ') while if average height of roughness (K) is more than the laminar sub layer its hydrodynamically rough boundary.

$$\delta' = 11.6 \frac{v}{u_*}$$

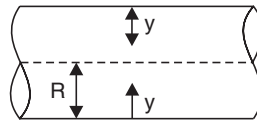
$v \rightarrow$ Kinematic Viscosity

$$u_* = \text{Shear velocity} = \sqrt{\frac{\tau_w}{\rho}} ; \quad \tau_w = \text{Boundary shear stress}$$

	Smooth	Transition	Rough
As per Nikuradse Experiment	$\frac{K}{\delta'} < 0.25$	$0.25 < \frac{K}{\delta'} < 6$	$\frac{K}{\delta'} > 6$
In terms of Roughness Reynold No. $\left(\text{put } \delta' = \frac{11.6v}{u_*} \right)$	$\frac{u_* K}{v} < 3$	$3 < \frac{u_* K}{v} < 70$	$\frac{u_* K}{v} > 70$

Velocity distribution for turbulent flow in smooth as well as Rough pipe

$$1. \quad \frac{u - u_{avg}}{u_*} = 5.75 \log_{10} \left(\frac{y}{R} \right) + 3.75$$



Here, y is measured from the boundary surface not from centre.

$$2. U = U_{\max} \text{ at } y = R \text{ so } \frac{U_{\max} - U_{\text{avg}}}{u_*} = 3.75 ; \log_{10} 1 = 0$$

$$3. \text{ As } u_* = \sqrt{\frac{f}{8}} U_{\text{avg}} \text{ then in 2, } \frac{U_{\max}}{U_{\text{avg}}} = 1 + 1.33 \sqrt{f}$$

$$4. \text{ In pipe flow, } z_w = \frac{dp}{dx} \left(\frac{R}{2} \right) \Rightarrow \frac{R}{2} \left(\frac{\Delta P}{L} \right) \Rightarrow \frac{R}{2L} (\gamma h_L)$$

$$\Rightarrow \frac{z_w}{\rho} = \frac{R}{2L} (g h_L) = \frac{R}{2L} g \frac{f V^2}{2gD} = \frac{D}{4L} g \frac{f V^2}{2gD}$$

$$\Rightarrow \frac{z_w}{\rho} = \frac{f}{8} V^2 \Rightarrow \sqrt{\frac{z_w}{\rho}} = u_* = \sqrt{\frac{f}{8}} V_{\text{avg}}$$

$$5. \frac{1}{7} \text{ th power law of velocity distribution for smooth pipes}$$

$$\frac{u}{u_{\max}} = \left(\frac{y}{R} \right)^{\frac{1}{7}} \text{ (As per Nikuradse)}$$

Friction factor ' f ' for Turbulent flow (Artificial Roughnes)

1. For smooth pipes

$$(a) f = \frac{0.316}{(R_e)^{1/4}}, \quad 4000 < R_e < 10^5$$

Note:

$$\text{For laminar flow, } f = \frac{64}{R_e} \text{ circular pipe}$$

$$(b) f = 0.0032 + \frac{0.221}{(R_e)^{0.237}}, \quad 10^5 < R_e < 4 \times 10^7$$

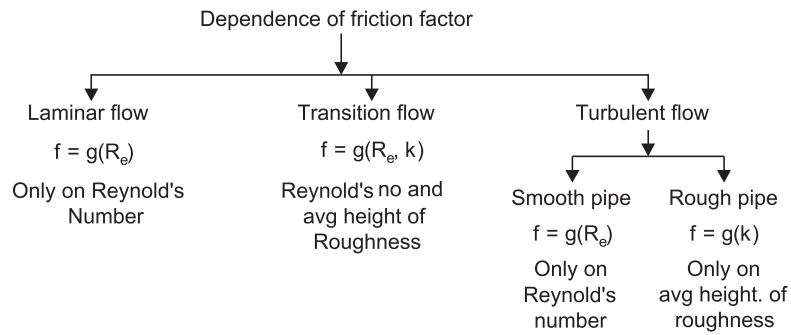
$$(c) \frac{1}{\sqrt{f}} = 2 \log_{10} (Re \sqrt{f}) - 0.8, \quad 5 \times 10^4 < Re < 4 \times 10^7 \text{ (Nikuradse)}$$

2. For Rough pipes

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{R}{K} \right) + 1.74, \quad \begin{array}{l} R \rightarrow \text{Radius of pipe} \\ \left(\frac{R}{K} \right) \rightarrow \text{Relative Smoothness} \end{array}$$

3. For Smooth Commercial pipes

$$\frac{1}{\sqrt{f}} = 2 \log_{10} (R_e \sqrt{f}) - 0.8 \quad 5 \times 10^4 < R_e < 4 \times 10^7 \quad (\text{Prandtl})$$

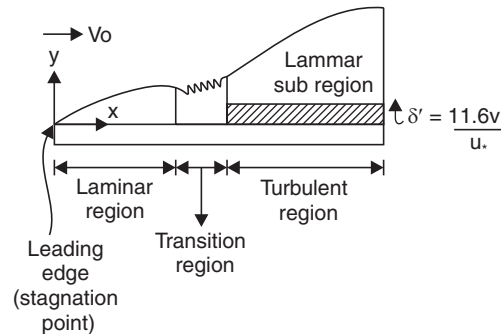


Boundary Layer Thickness

12

It is the region in the immediate Vicinity of the boundary surface in which the velocity of flowing fluid increases gradually from zero at the boundary surface to the velocity of the main stream.

- Developed by Prandtl in 1904 (even before Partition of Bengal)
- Valid for infinitely large medium of real fluid and Not for ideal fluid.



Essential boundary conditions:

1. $x = 0, \delta = 0$
2. $y = 0, u = 0$
3. $y = \delta, u = V_0$
4. $y = \delta, \frac{du}{dy} = 0$

Desirable boundary conditions: At $y = \delta, \frac{du}{dy} = 0, \frac{d^2u}{dy^2} = 0$

Salient points regarding boundary layer:

1. As the roughness of plate increases, length of laminar region decreases
2. Positive pressure gradient increases boundary layer thickness as well as reduces the length of laminar region.
3. With increase in velocity, boundary layer thickness decreases but with increase in viscosity boundary layer thickness increases.

4. On a smooth plate, in turbulent layer region, there is very thin layer adjacent to the boundary where the flow remains laminar. This region is called laminar sub layer.
5. $R_{ex} = 5 \times 10^5$ is called critical Reynold's number.
 If $R_e < R_{ex}$ then laminar boundary layer region in **flat plates**
 $R_e > R_{ex}$ then Turbulent boundary layer region, in **flat plates**

Velocity profiles

↓

Laminar region

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^2$$

$$\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

↓

Laminar sublayer

$$\frac{u}{u_*} = \frac{u_* \times y}{\nu}$$

↓

Turbulent Region

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

Boundary layer Thickness (δ): It is the distance from the boundary surface in which velocity reaches 99% of the free stream velocity.

At $y = \delta$, $V = 0.99 V_0$

Displacement Thickness (δ^*): Distance by which boundary should be shifted in order to compensate for the reduction in mass flow rate on account of boundary layer formation.

$$\delta^* = \int_0^\delta \left(1 - \frac{V}{V_0}\right) dy$$

For $\frac{V}{V_0} = \left(\frac{y}{\delta}\right)^{1/m}$

$\Rightarrow \delta^* = \frac{\delta}{m+1}$

V = Velocity at any distance y from the boundary

V_0 = Free stream velocity

Reduction in mass flow rate per unit width = $\rho \delta^* V_0$

Momentum Thickness (θ): Distance by which boundary should be shifted in order to compensate for the loss of momentum due to formation of Boundary layer.

$$\theta = \int_0^{\delta} \frac{V}{V_0} \left(1 - \frac{V}{V_0} \right) dy$$

$$\text{For } \frac{V}{V_0} = \left(\frac{y}{\delta} \right)^{1/m}$$

$$\theta = \frac{\delta m}{(m+1)(m+2)}$$

Loss of momentum per unit width of boundary layer = $\rho \theta V_0^2$

Energy thickness δ_E : Distance by which boundary should be shifted in order to compensate loss of energy due to boundary layer formation.

$$\delta_E = \int_0^{\delta} \frac{V}{V_0} \left(1 - \frac{V^2}{V_0^2} \right) dy$$

$$\text{Loss of energy due to boundary layer formation} = \frac{\delta V_0^3}{2} E$$

Note:

$$\bullet \delta^* > \delta_E > \theta$$

$$\bullet \text{ Shape factor} = \frac{\delta^*}{\theta}, \quad \text{For } \frac{V}{V_0} = \left(\frac{y}{\delta} \right)^{1/m}, \quad \text{Shape factor} = \frac{m+2}{m}$$

Equation's in boundary layer

$$1. \text{ Continuously equation, } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$2. \text{ Constant pressure gradient across boundary layer, } \frac{-1}{\rho} \frac{\partial p}{\partial y} = 0$$

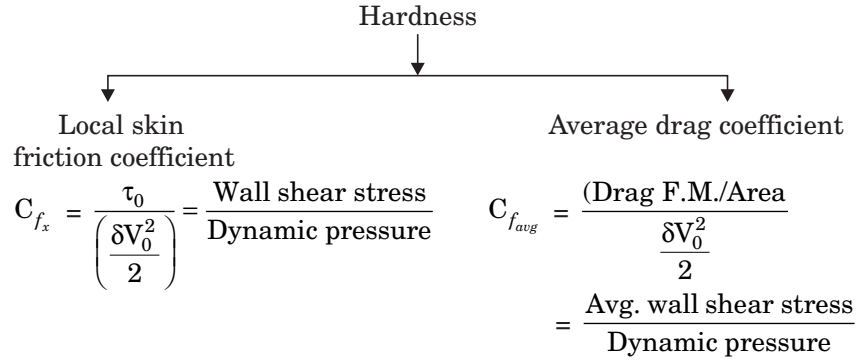
$$3. \text{ For steady 2-D Laminar flow, } u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{-1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial z}{\partial y}$$

$$4. \text{ Von-Karman momentum integral equation, } \frac{\tau_0}{\rho V_0^2} = \frac{d\theta}{dx}$$

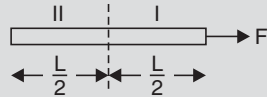
τ_0 = Boundary shear stress

V_0 = Free stream velocity

θ = Momentum thickness


Note:

It Boundary layer is laminar through out



$$\frac{\text{Drag force on I half}}{\text{Drag force on II half}} = 1 + \sqrt{2} \text{ i.e., } > 1$$

Blassius solution in absence of velocity profile on smooth plate

	Laminar	Turbulent	
	$R_e < 5 \times 10^5$	$5 \times 10^5 < R_e < 10^7$	$10^7 < R_e < 10^9$
δ	$\frac{5x}{\sqrt{R_{ex}}}$	$\frac{0.376 x}{(R_{ex})^{1/5}}$	$\frac{0.22 x}{(R_{ex})^{1/6}}$
C_{f_x}	$\frac{0.664}{(R_{ex})^{1/5}}$	$\frac{0.059}{(R_{ex})^{1/5}}$	$\frac{0.37}{(\log_{10} R_{ex})^{2.58}}$
$C_{f_{avg}}$	$\frac{1.328}{\sqrt{R_{ex}}}$	$\frac{0.074}{(R_{eL})}$	$\frac{0.455}{(\log_{10} R_{eL})^{2.58}}$
Applicable only if boundary layer is Turbulent through out			

Note:

- In Laminar region $\delta \propto \sqrt{x}$, while in turbulent region $\delta \propto x^{4/5}$, Hence δ increases more rapidly in turbulent region than in laminar region
- In Laminar region $C_{f_x} \propto \frac{1}{\sqrt{x}}$, while in turbulent region $C_{f_x} \propto \frac{1}{x^{1/5}}$ Hence, τ_0 decreases more rapidly in laminar region than in turbulent region.

Drag force on a plate which has both Laminar and Turbulent regions

$$\text{Drag Force} = C_{f_{avg}} \times \frac{\rho V_0^2}{2} \times \text{Area}$$

$$(a) \quad 5 \times 10^5 < R_e < 10^7, C_{f_{avg}} = \frac{0.074}{(R_{e_L})^{1/5}} - \frac{1700}{R_{e_L}}$$

$$(b) \quad 10^7 < R_e < 10^9, C_{f_{avg}} = \frac{0.455}{(\log_{10} R_{e_L})^{2.58}} - \frac{1700}{R_{e_L}}$$

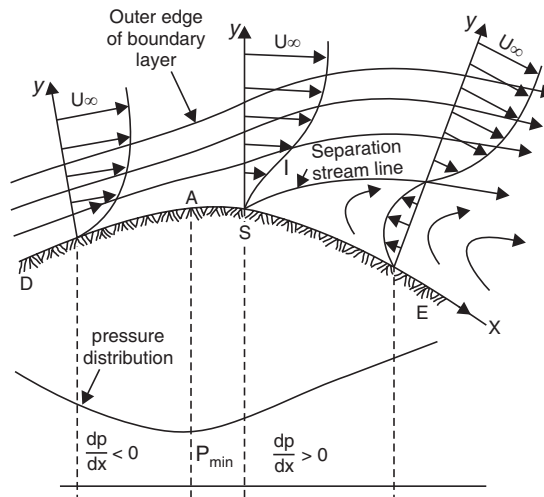
Separation of Boundary layer

Boundary layer thickness increases along the length of the body. So the fluid close to the body surface has to do work against skin friction at the expense of kinetic energy. This leads to decrease in velocity and increase in pressure in the forward direction. And at the point where

$$\left(\frac{du}{dy} \right)_{y=0} = 0 \text{ is called separation point}$$

$$\left(\frac{du}{dy} \right)_{y=0} > 0, \text{ Attached flow}$$

$$\left(\frac{du}{dy} \right)_{y=0} < 0, \text{ Already separated flow}$$



Separation of flow occurs in Pumps, Turbines, Aerofoils, open channel transitions etc.

Consequences of boundary layer separation

- (a) There is increase in pressure drag if there is boundary layer separation in case of external flow E.g. cars
- (b) Separation of boundary layer increases flow losses in case of internal flow like pipes.

Methods to control separation

1. Rotating boundary in flow direction
2. Supplying additional energy from blower
3. Streamlining of body shapes
4. Suction of flow moving fluid by suction slot
5. Injecting fluid into boundary layer
6. Providing guide blades on bends.

Dimensional Analysis and Model Studies

13

It helps in determining a systematic arrangement of the variations in the physical relationship, combining dimensional variable to form non-dimensional parameters. The various physical quantities can be expressed in terms of fundamental quantities of Mass (M), Length (L), Time (T) and Temperature (θ).

Dimensional homogeneity: States that every term in an equation when reduced to its primary (fundamental) dimensions must contain identical powers of each dimension.

Dimensions of Few Physical Quantities

(a) Kinematic Quantities:

1. Angular velocity	T^{-1}
2. Angular acceleration	T^{-2}
3. Vorticity	T^{-1}
4. Kinematic viscosity	L^2T^{-1}
5. Stream function	L^2T^{-1}
6. Circulation	L^2T^{-1}

(b) Dynamic Quantities:

1. Specific weight	$ML^{-2}T^{-2}$
2. Dynamic viscosity	$ML^{-1}T^{-1}$
3. Surface tension	MT^{-2}
4. Modulus of elasticity	$ML^{-1}T^{-2}$
5. Bulk modulus	$ML^{-1}T^{-2}$
6. Angular momentum	ML^2T^{-1}

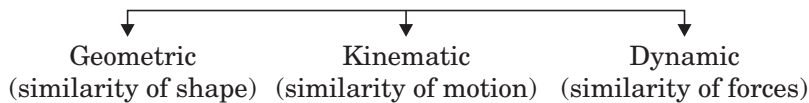
Methods of Dimensional Analysis

(a) **Rayleigh's Method:** It is used for determining the expression for a variable which depends upon maximum of three or four variables. It does not provide any information regarding the number of dimensionless groups to be obtained as a result of dimensional analysis.

(b) **Buckingham's π -theorem:** If there are n no. of variables in a dimensionally homogenous equation and these variables contain m fundamental dimensions then the no. of dimensionless groups which can be formed shall be $n-m$. These dimensionless groups are called π -terms.

Selection of 3 repeating variables should be from **geometry of flow** (size and shape of fluid passage, or diameter or length of moving body), **fluid property** (like density, surface tension, viscosity, elasticity, vapour pressure) and **fluid motion** (like velocity, acceleration, discharge, pressure, power, force)

Similitude: To achieve similarity between the flow in the model and its prototype, every dimensionless parameter referring to the conditions in the model must have the same numerical value as the corresponding parameter referring to the prototype



Note:

For kinetic similarity, Geometric similarity must exist and for dynamic similarity both Geometric and kinematic similarity must exist. These are necessary conditions but not sufficient conditions i.e. if kinematic similarity exists then geometry similarity will be definitely there but if geometric similarity exists then kinematic similarity may or may not exist. (same for dynamic similarity)

Forces acting on Fluid mass

1. Inertia Force (F_i) = $\delta L^2 V^2$
2. Viscous Force (F_v) = μVL
3. Gravity Force (F_g) = $\delta L^3 g$
4. Pressure Force (F_p) = PL^2
5. Surface tension Force (F_σ) = σL
6. Elasticity Force (F_e) = KL^2

Dimension less-Parameters

1. Reynold's Number

$$R_e = \frac{\text{Inertial Force}}{\text{Viscous Force}}$$

$$R_e = \frac{VL}{\nu}$$

2. Froude's Number

$$F_e = \sqrt{\frac{\text{Inertial Force}}{\text{Gravity Force}}}$$

$$F_e = \frac{V}{\sqrt{gL}}$$

3. Euler's Number

$$E_u = \sqrt{\frac{\text{Inertial Force}}{\text{Pressure Force}}}$$

$$E_u = \frac{V}{\sqrt{p / \delta}}$$

4. Weber Number

$$W_e = \sqrt{\frac{\text{Inertial Force}}{\text{Surface tension Force}}}$$

$$W_e = \frac{V}{\sqrt{(\sigma / \delta L)}}$$

5. Mach Number

$$M = \sqrt{\frac{\text{Inertial Force}}{\text{Elastic Force}}}$$

$$M = \frac{V}{\sqrt{K / \delta}} = \frac{V}{C}$$

(1) **Reynold's model law** $\frac{\delta_r V_r L_r}{\mu_r} = 1$

⇒ Pipe flow, Submarines, Aeroplanes, drag on parachutes

(2) **Froude's law** $\frac{V_r}{\sqrt{L_r g_r}} = 1$

⇒ when a free surface is present eg wiers, spillway, channels etc
where **gravity** is predominant.

Note: L = Hydraulic depth = $\frac{A}{P}$ (in case of open channel flow)

	Reynold's law	Froude's law
Velocity Ratio (V_r)	$\frac{\mu_r}{L_r \delta_r}$	$\sqrt{L_r}$
Time Ratio (T_r)	$\frac{\delta_r L_r^2}{\mu_r}$	$\sqrt{L_r}$
Acceleration Ratio (a_r)	$\frac{\mu_r^2}{\delta_r^2 L_r^2}$	1
Force Ratio (F_r)	$\frac{\mu_r^2}{\delta_r}$	$\delta_r L_r^3$
Power Ratio (P_r)	$\frac{\mu_r^3}{\delta_r^2 L_r}$	$\delta_r L_r^{3.5}$
Discharge Ratio (Q_r)	$\frac{\mu_r L_r}{\delta_r}$	$\delta_r L_r^{2.5}$

3. **Euler's law** $V_r = \sqrt{P_r / \delta_r}$

$\Rightarrow$ In case of cavitation, pressure due the sudden closure of valve, high pressure flow in pipes.

4. **Weber model law** $V_r = \sqrt{\frac{\sigma_r}{\delta_r L_r}}$

$\Rightarrow$ Flow of blood in arteries and veins, seepage through soil capillary rise, rising bubble, flow over weir for small head.

5. **Mach law** $V = \sqrt{k_r / \delta_r}$

Compressibility force are predominant when mach no ≥ 0.3

$M < 1 \rightarrow$ Subsonic

$M = 1 \rightarrow$ Sonic

$M > 1 \rightarrow$ Supersonic

$M \gg 1 \rightarrow$ Hyper sonic

River model law (Distorted model law)

$$(a) \text{ Horizontal scale ratio } L_{rH} = \frac{L_p}{L_m} = \frac{B_p}{B_m}$$

$$(b) \text{ Vertical scale ratio } L_{rv} = \frac{H_p}{H_m}$$

$$(c) \text{ Velocity ratio } V_r = \sqrt{L_{rv}}$$

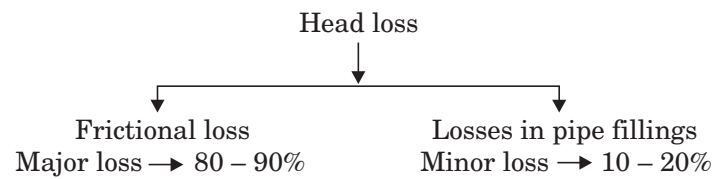
$$(d) \text{ Area ratio } A_r = L_{rv} \times L_{rH}$$

$$(e) \text{ Discharge ratio } Q_r = L_{rH} \cdot L_{rv}^{3/2}$$

Model Laws for

(a) Head ratio	$H_r = N_r^2 D_r^2$	$D_r = \text{diameter ratio}$
(b) Discharge ratio	$Q_r = N_r D_r^3$	$N_r = \text{rotational speed ratio}$
(c) Power ratio	$P_r = \delta_r N_r^3 D_r^5$	

Practically, all the flow in the pipes is **turbulent** in nature.



Major Losses

(a) Darcy's weisbach equation

$$h_f = \frac{f l Q^2}{12.1 D^5}$$

(b) Chezy's formula

$$V = C \sqrt{RS} \quad R = \frac{A}{P} = \frac{\pi D^2}{4 \pi D} \Rightarrow R = \frac{D}{4}$$

$$\Rightarrow S = \text{Slope} = \frac{h_L}{L}$$

Note:

Equating both the above equations we get $C = \sqrt{\frac{8g}{f}}$

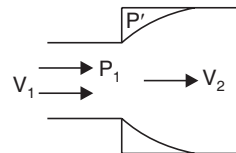
Minor losses

(a) Due to sudden expansion

$$h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g} \left(1 - \frac{A_1}{A_2} \right)^2$$

A_1 = Area of smaller dia pipe

A_2 = Area of bigger dia pipe



7.66 CIVIL ENGINEERING

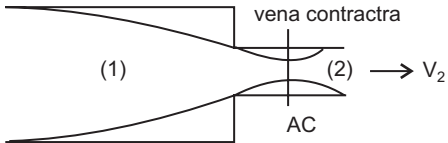
V_1 = Velocity of smaller dia pipe

$$h_L = K \frac{V_1^2}{2g} \quad \text{where } K = \left(1 - \frac{A_1}{A_2}\right)^2$$

Note:

- It is assumed that $P_1 = P'$ during derivation of this head loss
- Momentum equation and Bernoulli's equation are used in derivation of losses
- Losses are always expressed in terms of velocity of smaller diameter pipe.

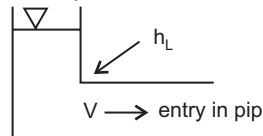
(b) Losses due to sudden contraction

$$h_L = \frac{(V_C - V_2)^2}{2g} = \frac{KV_2^2}{2g}$$


where $K = \left(\frac{1}{C_c} - 1\right)^2$, $C_c = \frac{A_c}{A_2}$

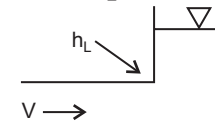
Note: Loss in expansion is much higher than loss in contraction.

(c) Entry loss



$$h_L = \frac{0.5 V^2}{2g}$$

(d) Exist loss (due to impact)



$$h_L = \frac{KV^2}{2g}$$

Note: In exist loss due to impact, K is the kinetic energy correction factor. For Turbulent its $K = 1$ and for Laminar its $K = 2$.

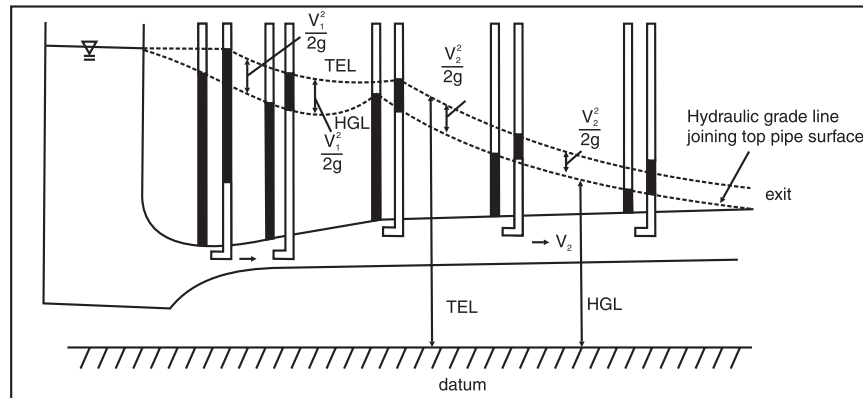
(e) Loss due to pipe fittings and bends

$$h_L = \frac{KV^2}{2g}$$

Type of fitting	K
Standard Tee	1.8
Standard Elbow	0.9
45° Elbow	0.4
90° Bend (Sharp)	1.2
Foot valve of pump	1.5
Gate valve half open	5.6
Angle valve	5.0

Hydraulic gradient line and Total energy line

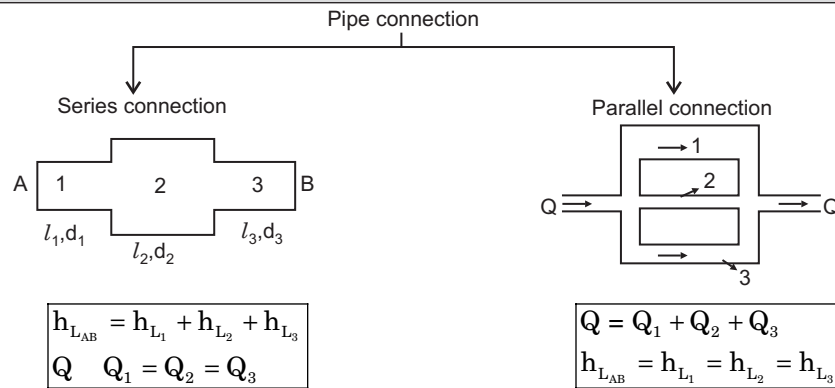
Line joining the points of **piezometric head** $\left(\frac{P}{\gamma} + z \right)$ at various points in a flow is called **hydraulic gradient line**.



Line joining the points of total energy $\left(\frac{P}{\gamma} + \frac{V^2}{2g} + z \right)$ at various points in a flow is called **total energy line**.

Note:

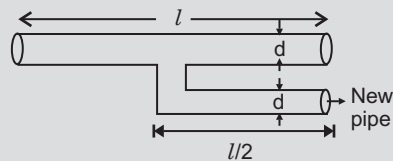
- Total energy line always fall down. But if there is a pump or turbine placed in the flow there will be sudden rise or fall respectively.
- Hydraulic grade line may rise or fall in the flow direction, depending upon the velocity head (which varies with the area of cross section)
- Total energy line is horizontal in case of idealised Bernoulli's flow because losses are zero.

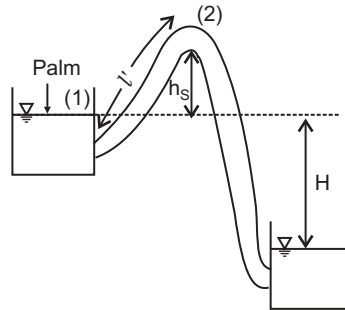


Equivalent pipe: A pipe which can replace existing compound pipe while carrying same discharge under same losses. For **series connection** equivalent pipe of length 'L' and diameter 'D' will be

$$\frac{L}{D^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}$$

Note: Increase in discharge by adding a pipe of same diameter in mid way of a pipe but keeping the head constant is 26.53%.



Flow through siphon:

Note: For No. vaporisation $P_s > P_{\text{vaporization}}$, otherwise vapours will form and flow will be stopped.

1. Get velocity of flow in pipe from head difference

$$H = \frac{f l V^2}{2gD} + 0.5 \frac{V^2}{2g} + \frac{V^2}{2g}$$

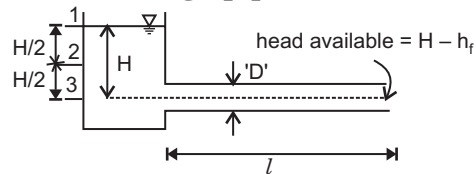
[Friction loss] [Entry loss] [Exist loss]

l = Total length of pipe

2. Apply Bernoulli's between 1 and summit (s)

$$\frac{P_{atm1}}{\gamma_w} = \frac{P_s}{\gamma_w} + \frac{V^2}{2g} + h_s + \frac{0.5V^2}{2g} + \frac{f l' V^2}{2gD}$$

l' = length of pipe b/w 1 and (s)

Power transmitted through pipe

$$\text{Efficiency } (\eta) = \frac{H - h_f}{H}$$

$$\text{Power } (P) = \gamma Q (H - h_f)$$

$$\text{For max power } \frac{dP}{dQ} = 0 \text{ i.e. } h_f = \frac{H}{3}$$

$$\eta_{\max} = 66.67\%, \text{ max power lost} = 33.33\%$$

Note: If Nozzle of area 'a' is attached at exit then for maximum efficiency $\frac{fLa^2}{DA^2} = \frac{1}{2}$ where A corresponds to area of diameter D.

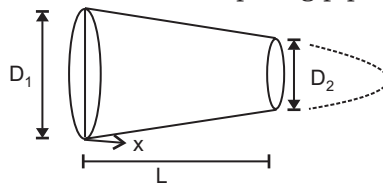
- Time required to empty the reservoir $\int_0^H \frac{dh}{\sqrt{h}} = \int_0^T \sqrt{\frac{2g}{K}} \times \frac{a}{A} dt$ where

K is the head loss constant.

- Time required to empty the top half of tank from 1 to 2 be t_1 and for bottom half from 2 to 3 be t_2 then $t_1 = 0.414 t_2$

Special cases of head loss

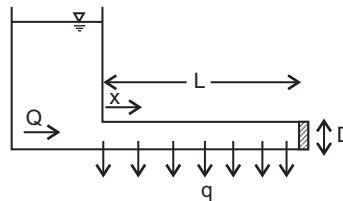
(a) Loss of head due to friction in tapering pipe



$$h_f = \int_0^L \frac{f Q^2 dx}{12.1(D_1 - kx)^5}$$

$$K = \left(\frac{D_1 - D_2}{L} \right)$$

(b) Head loss due to uniform discharge at regular interval from a closed end pipe

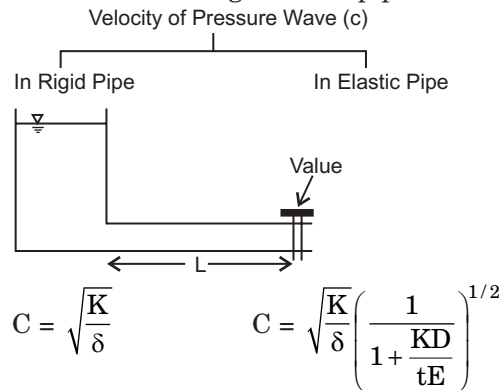


$$q' = \frac{Q}{L}$$

$$h_f = \frac{1}{3} \left(\frac{f l Q^2}{12.1 D^5} \right)$$

Head loss is one-third the head loss of constant discharge.

Water-hammer Pressure: Sudden closure of valve in a pipe carrying flowing liquid destroys the momentum of flowing liquid and sets up a high pressure wave. This pressure wave travels with the speed of sound and causes hammering action in pipe called Knocking or water hammer.



K = Bulk density of liquid

δ = mass density of liquid

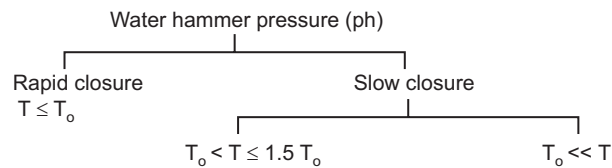
E = modulus of elasticity of material

t = thickness of pipe

D = diameter of pipe

- Time period for complete cycle of water hammer pressure = $\frac{4L}{C}$

Critical time $T_0 = \frac{2L}{C}$



$$P_{hr} = \delta C V \frac{P_{hs}}{P_{hr}} = \frac{T_0}{T} \quad p = \delta \left(\frac{L}{T} \right) V$$

V = Initial velocity of flow

Note: Equations used in solving branching of pipes connecting reservoirs at different levels are (a) Continuity equation (b) Bernoulli's equation (c) Darcy weis bach equation.

Hardy-Cross method of solving closed loop pipe networks.

1. Flow into any junction must be equal to flow out of each junction.
2. Loss of head due to flow in clockwise direction must be equal to loss of head due to flow in anti clockwise direction.

$$\text{Modification in discharge } \Delta Q = \frac{-\sum r Q^n}{\sum n Q^{n-1}}$$

where $h_f = rQ^n$ and ΔQ is algebraically added.

Civil Engineering

8

Open Channel Flow

CONTENTS

1. Introduction
2. Uniform– Flow
3. Energy-Depth Relationship
4. Gradually Varied Flow
5. Rapidly varied flow: Hydraulic Jump

Introduction

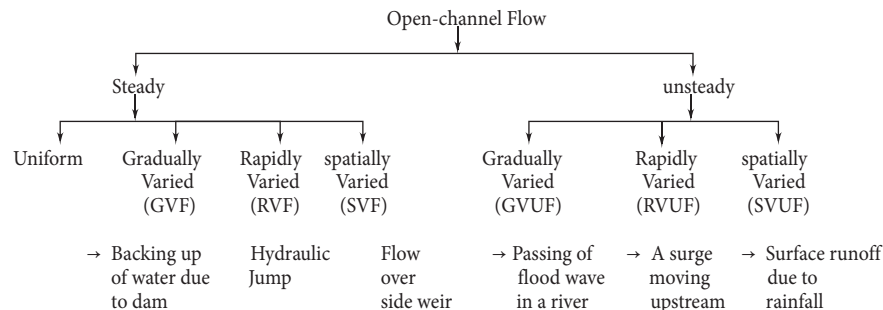
1

It's a natural or manmade structure in which liquid flows with a free surface at atmospheric pressure.

Prismatic channels are those in which X-sectional shape, size and bed slope is constant. While all natural channels having varying X-section are consequently known as Non-Prismatic channels.

Rigid channels are those in which the boundary is not deformable. Here shape and roughness factors is not a function of flow parameter while mobile channels are those in which boundaries undergo deformation due to continuous process of erosion and deposition due to flow.

Note: Rigid channels have only 1 degree of freedom (i.e. depth of flow) while mobile channels have 4 degree's of freedom.



Uniform flow: If the **flow** properties like **depth of flow** and **discharge** in an open channel remains constant along the length of the channel. Then its Uniform flow. The constant depth of flow is called the **normal depth**.

Note: → In Uniform flow, Bed Slope = Energy line Slope = Water Surface Slope

→ In GVF, loss of energy is mainly due to boundry friction.

→ In GVF, pressure distribution in Vertical direction is taken as hydrostatic.

Critical, Sub-Critical and Super-critical Flow

$$\text{Froude Number } (f_r) = \frac{v}{\sqrt{gL_e}} \quad L_e = \frac{\text{Area of flow}}{\text{Top width of flow}}$$

where L_c = characteristic length

for rectangular section $L_c = y$ (depth of flow)

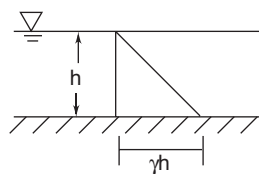
Note: $\sqrt{gy_e}$ represents the speed at which the disturbance wave travels in still water condition, denoted by celerity c_o

Type of flow	Depth of flow	Velocity of flow	Froude No	Comments
Subcritical	$y > y_c$	$v < v_c$	$F_r < 1$	Also known as streaming or tranquil flow
Critical	$y = y_c$	$v = v_c$	$F_r = 1$	Also known as shooting flow, rapid flow, torrential flow
Super critical	$y < y_c$	$v > v_c$	$F_r > 1$	

Control Section: A Section in which fixed relationship exists between **depth and discharge**. For sub-critical flow, downstream section is a control section. (i.e computation of water surface profile must start from down stream location and proceed upstream location). For super-critical flow control section is at upstream section.

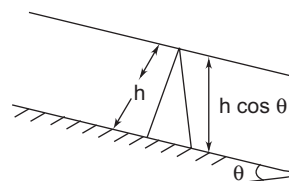
Note: In case of critical condition, disturbance wave will not travel at all.

Pressure distribution



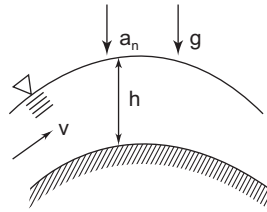
Small slope

$$P = \gamma h$$



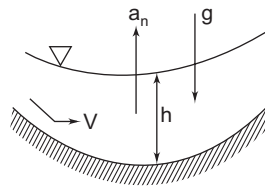
Large slope

$$P = \gamma h \cos \theta$$



Upward convex surface

$$P = \delta(g - a_n)h$$



Upward concave surface

$$P = \delta(g - a_n)h$$

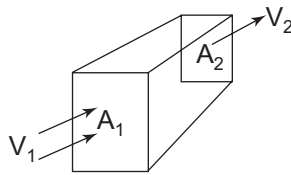
Continuity equation in open channel flow

Continuity equation in open channel flow

Steady State

Flow

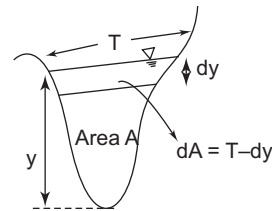
$$Q = AV = A_1V_1 = A_2V_2$$



Unsteady State

Flow

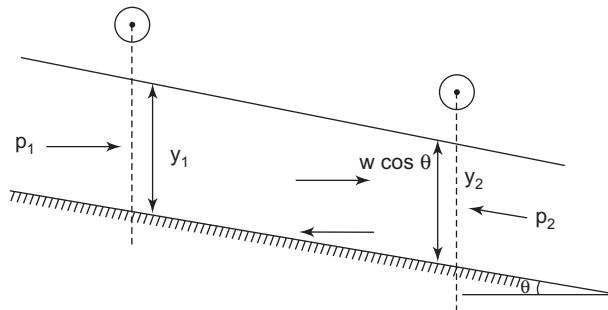
$$\frac{\partial Q}{\partial x} = -T \frac{\partial y}{\partial t}$$



Momentum equation in open channel flow:

$$M_2 - M_1 = P_1 - P_2 + w \sin \theta - F_t$$

$$-\rho A_1 V_1^2 + \rho A_1 V_1^2 = \gamma A_1 \bar{y}_1 - \gamma A_1 \bar{y}_2 + w \sin \theta - F_f$$



8.6 CIVIL ENGINEERING

M_1, M_2 = Momentum per unit time

P_1, P_2 = Pressure force

F_f = Friction force along the surface of contact b/w water and channel

W = Weight of water enclosed between sections.

θ = Bed slope.

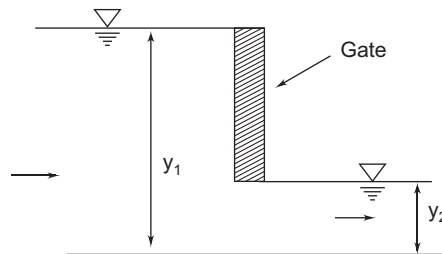
Note: Momentum equation is generally used where the loss of internal energy is high, because it deals with external forces only. For eg: Hydraulic Jump.

Specific force: Sum of pressure force and momentum flux per unit “unit weight” of the fluid at a section.

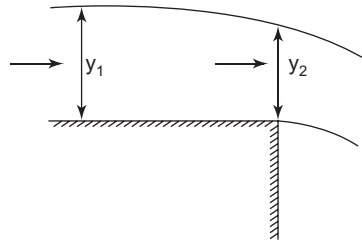
$$\text{Specific force} = \frac{P + M}{r} \quad \text{It is assumed to be constant if the channel}$$

is horizontal and frictionless.

Note:



$$\frac{F_{\text{on gate}}}{\text{Width}} = \frac{\gamma (y_1 - y_2)^3}{2 (y_1 + y_2)}$$



$$\frac{y_2}{y_1} = \frac{2F_1^2}{2F_1^2 + 1}$$

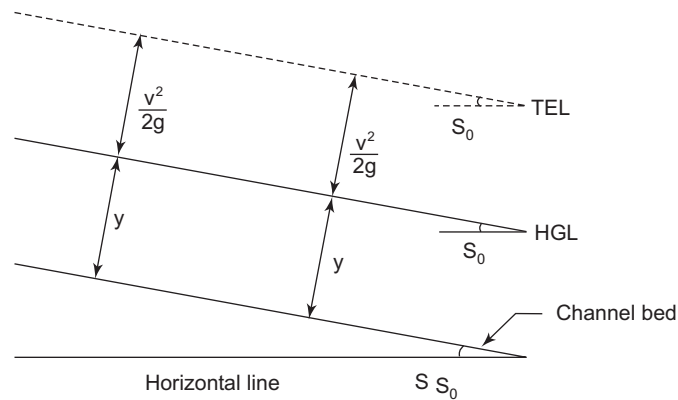
F_1 = Froude no at section 1

Uniform– Flow

2

In Uniform flow, frictional resistance between the fluid and channel boundary are balanced by the gravity forces.

Depth of flow corresponding to the slope of bed will be Normal depth.



Measurement of Velocity:

(1) Chezy's equation

$$\tau_0 = \gamma R S_0 \quad \tau_0 = K \left(\frac{\rho V^2}{2} \right)$$

Note: τ_0 is independent of Viscosity.

where τ_0 = Average shear stress on the wetted area under uniform flow condition.

R = Hydraulic radius (A/P) of the channel section.

S_0 = Slope of channel bed = Slope of HGL = Slope of TEL

K = Constant depending on roughness

ρ = Density of water.

Equating both equations, $v = C\sqrt{RS}$ → Chezy's equation

$$C = \sqrt{\frac{2r}{\delta K}} \rightarrow \text{Chezy's coefficient}$$

(2) Manning's equation

$$V = \frac{1}{n} R^{2/3} S_0^{1/2}$$

- By Robert Manning in 1889

n = Roughness coefficient, commonly known as Manning's " n ".

- Strikler's Formula $n = \frac{d_{50}^{1/6}}{21.10}$, $d_{50} \rightarrow$ in meters

Meyer's Formula $n = \frac{d_{90}^{1/6}}{26}$, $d_{90} \rightarrow$ in meters

Note: Relationship between c , n , f

$$c = \sqrt{\frac{8g}{f}} \quad f = \left(\frac{n^2}{R^{1/3}} \right) 8g \quad f = \text{Darcy's friction factor}$$

Economical channel section: A section whose construction cost is minimum for a given discharge. While the efficient section is the one whose discharge carrying capacity is maximum for a given cross-sectional area.

Note: Semi-Circular shape has a least amount of perimeter for a given area.

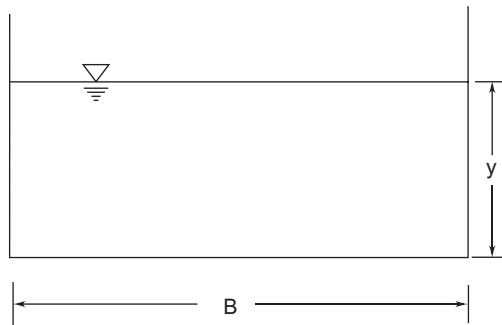
(1) Rectangular section

$$A = By$$

$$P = B + 2y$$

Keeping A constant, $P = \frac{A}{y} + 2y$ put $\frac{dP}{dy} = 0$

$$\text{Then, } y = \frac{B}{2} \quad R = \frac{y}{2}$$



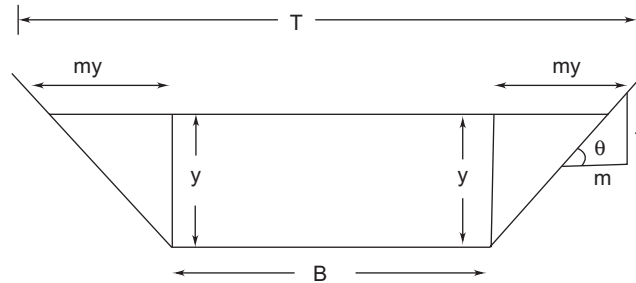
(2) Trapezoidal section

(a) Side slope is fixed i.e. $\tan \theta = \frac{1}{m}$

$$A = (B + my)y \quad P = B + 2y\sqrt{m^2 + 1}$$

$$P = \frac{A}{y} - my + 2y\sqrt{m^2 + 1}. \text{ Put } \frac{dP}{dy} = 0$$

$$\text{Then, } \boxed{\text{Length of side slope} = \frac{T}{2} = y\sqrt{m^2 + 1}}$$

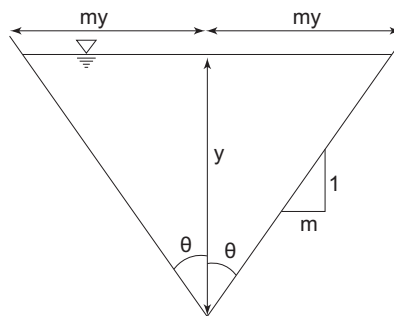


$$\boxed{R = \frac{y}{2}} \quad \boxed{\text{Circle of radius } y \text{ can be inscribed in trapezoidal section}}$$

(b) When side slope is variable.

$$P = 4y\sqrt{m^2 + 1} - 2my, \text{ Put } \frac{dp}{dm} = 0 \quad \boxed{m = \frac{1}{\sqrt{3}}} \text{ i.e. } \boxed{\theta = 60^\circ}$$

Most economical trapezoidal section should be half of regular hexagon.

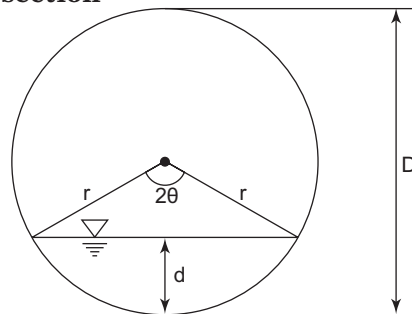
(3) Triangular section

$$A = my^2 \quad P = 2y\sqrt{m^2 + 1}$$

$$P = 2\sqrt{A}\sqrt{m + \frac{1}{m}}, \text{ Put } \frac{dp}{dm} = 0$$

$$m = 1 \Rightarrow \boxed{\theta = 45^\circ} \quad \boxed{r = \frac{y}{2\sqrt{2}}}$$

(4) Circular section



$$A = \frac{r^2}{2} (2\theta - \sin 2\theta) \quad P = 2r\theta$$

$$\frac{dA}{d\theta} = r^2(1 - \cos 2\theta) \frac{dp}{d\theta} = 2r$$

(a) For maximum discharge

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}, \text{ Put } \frac{dQ}{d\theta} = 0$$

$$\boxed{2\theta = 302^\circ 22'} \quad \boxed{d = 0.938 D}$$

(b) For maximum velocity

$$v = \frac{1}{n} \frac{A^{2/3}}{P^{2/3}} S^{1/2}, \text{ put } \frac{dv}{d\theta} = 0$$

$$\tan 2\theta = 2\theta$$

$$\boxed{2\theta = 257^\circ 27' 56''} \quad \boxed{d = 0.81 D}$$

Energy-Depth Relationship

3

Specific energy: Total energy at a section with respect to the channel bed as datum.

For small slopes

$$E = y + \alpha \frac{V^2}{2g}$$

For large slope

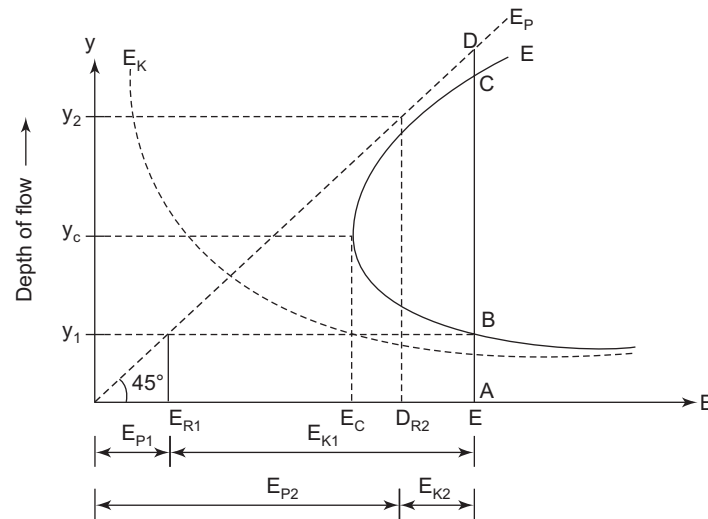
$$E = y \cos \theta + \frac{V^2}{2g}$$

α = K.E correction Factor

θ = Bed slope of channel Bottom.

Generally taken as 1

$$E = E_p + E_K$$



Energy-Depth Relationship

E_p = Potential energy = y

$$E_K = \text{Kinetic energy} = \frac{V^2}{2g}$$

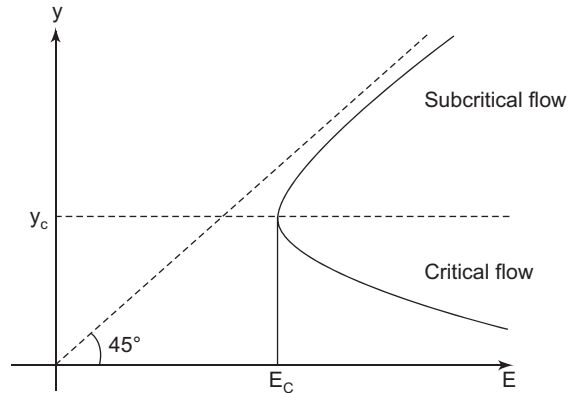
For a given Kinetic energy (E),

there are two possible depth of flow y_1 and y_2 . These are called

Alternate depth.

8.12 CIVIL ENGINEERING

$$\begin{array}{ll} \text{For } y_1 & E_{p1} = AB \\ \text{For } y_2 & E_{p2} = AC \end{array} \quad \begin{array}{ll} E_{K1} = BD \\ E_{K2} = CD \end{array}$$



Energy-Depth relationship

At the critical state of flow, two alternate depths become one, called as **Critical depth** (y_c). For a particular discharge, **specific energy is minimum** at the critical depth.

Note: At critical depth, water surface becomes wavy and unstable as the curve is nearly vertical. A slight change in energy, will give two alternate depths with greater difference

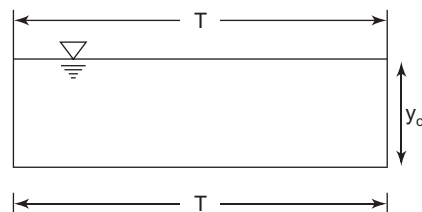
Note: At Critical Flow condition,

$$\boxed{\frac{Q^2 T}{g A^3} = 1} \quad \begin{array}{l} T = \text{Top width} \\ A = \text{Area of } x\text{-section} \end{array}$$

- (a) For a given discharge, specific energy is minimum
- (b) For a given specific energy, discharge will be maximum.
- (c) Specific Force is also minimum for a given discharge.

Calculation of critical depth

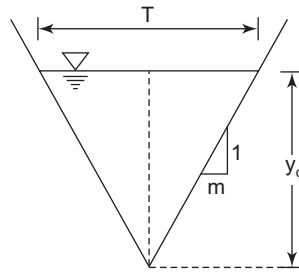
(1) Rectangular Channel Section



$$y_c = \left(\frac{q^2}{g} \right)^{1/3} \quad q = \text{discharge per unit width (B)} = \frac{Q}{B}$$

$$E_c = \frac{3}{2} y_c \quad F_r = \frac{V}{\sqrt{gy_a}}$$

(2) Triangular Channel Section



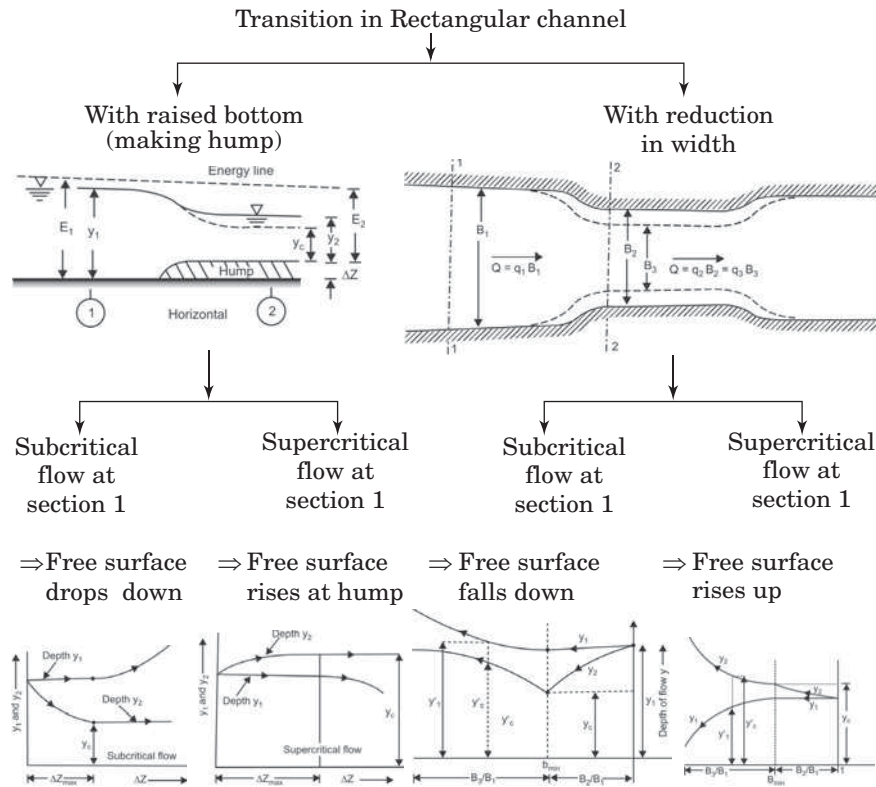
$$y_c = \left(\frac{2Q^2}{gm^2} \right)^{1/3} \quad Q = \text{Total discharge}$$

$$E_c = \frac{5}{4} y_c \quad F_r = \frac{\sqrt{2}V}{\sqrt{gy_e}} \quad F_r = \left(\frac{2Q^2}{gm^2 y_c^2} \right)^{1/2}$$

Note: For Parabolic Section $E_c = \frac{4}{3} y_c$.

Channel Transition: A transition is the portion of a channel with varying cross-section which connects one uniform flow channel to another uniform flow channel. Use of channel transition are metering of flow, dissipation of channel energy, reduction or increase of velocities, change in channel Section or alignment with minimum of energy dissipation. Assumptions made are

- (1) Channel bed is horizontal and frictionless ie TEL will be parallel to the channel bed
- (2) There is no loss of energy between two sections of the channel.



Venturi Flume: It is a structure in a channel which has a **Contracted Section** called throat, downstream of which follows a flared transition section designed to restore the stream to its original width.

Standing Wave flume or critical depth flume: It is a structure in a channel which has a **narrowed throat with a hump** at bottom. Downstream of which follows a flared section designed to restore the stream to original depth.

Parshall Flume: The converging section of the flume has a level floor, **the throat section has a downward sloping floor**, while the floor in the diverging section slopes upwards

Note: If y_1 and y_2 are alternate depths of horizontal frictionless rectangular channel then

$$y_c = \frac{2y_1^2 y_2^2}{y_1 + y_2} \quad \text{and} \quad E = \frac{y_1^2 + y_1 y_2 + y_2^2}{y_1 + y_2}$$

Gradually Varied Flow

4

It's a **Steady**, Non uniform flow, where depth of flow varies gradually from section to section along the length of the channel.

Assumptions in GVF:

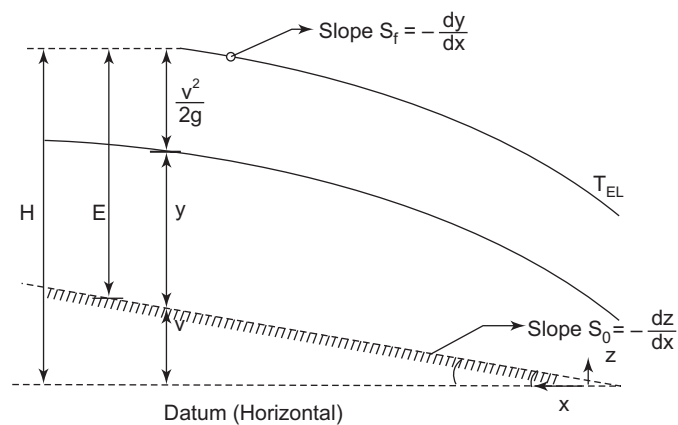
1. Energy Correction Factor (α) = 1
2. Pressure distribution is hydrostatic
3. Flow is Steady i.e, Constant discharge
4. Prismatic channel i.e, Slope and Shape constant
5. Bottom Slope of channel is very small i.e, HGL will lie at free surface.
6. Roughness coefficient is independent of depth.
7. Chezy's and Manning's equations are used to determine energy slope.

Differential Equation for GVF

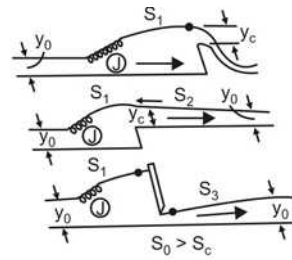
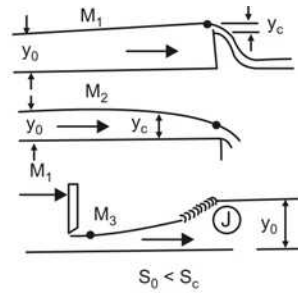
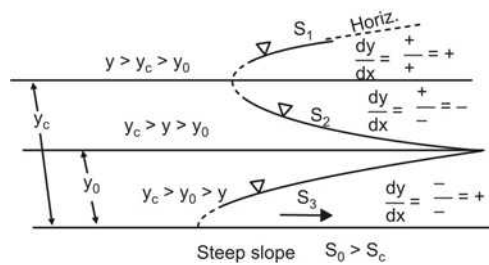
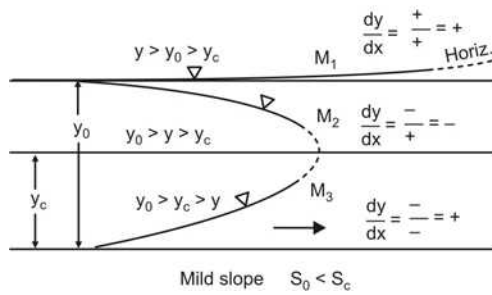
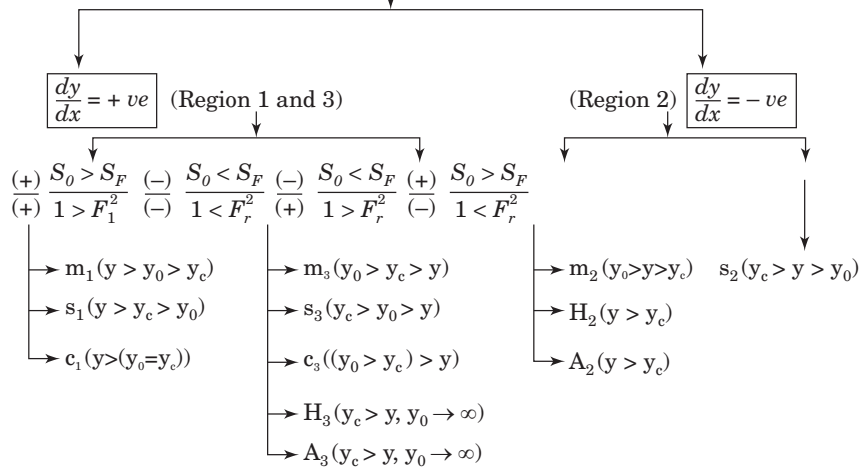
$$(1) \quad H = Z + E$$

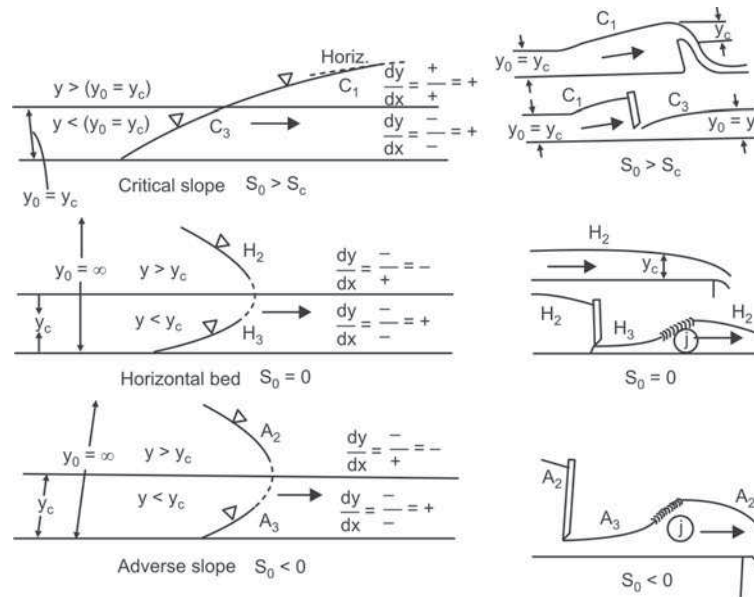
$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dE}{dx} \Rightarrow \boxed{\frac{dE}{dx} = S_0 - S_1}$$

$$(2) \quad H = Z + y + \frac{V^2}{2g}$$



Again differentiating wrt x $\frac{dy}{dx} = \frac{S_0 - S_F}{1 - F_1^2} F_r = \frac{Q^2 T}{g A^3}$





Note: Surface profile will approach normal depth asymptotically if depth of flow approaches to normal depth of flow. ($dy/dx = 0$)

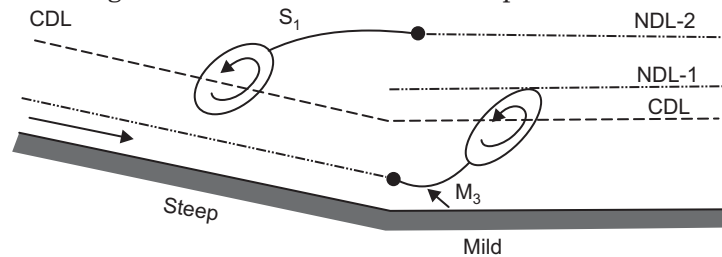
⇒ Surface profile will approach the critical depth line vertically if depth of flow approaches to critical depth of flow. ($dy/dx = S_0$)

⇒ Surface profile becomes horizontal as depth of flow becomes large ($dy/dx = 0$)

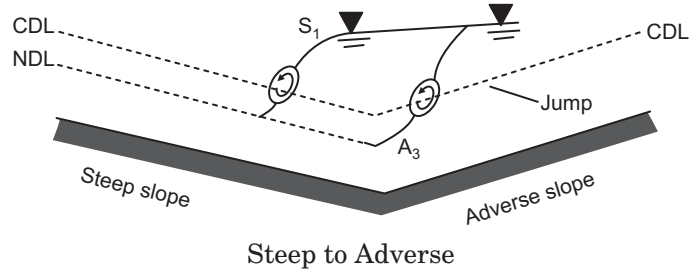
⇒ Surface profile meets the bed vertically if flow depth is reduced to zero ($dy/dx = 0$)

Break-in-Grade: when two channel sections have different bed slope, the section is called break in grade.

Steep to mild: In a limiting case of (i) and (ii) the jump may occur at the break in grade and there will be no GVF profile in such a case.



Either jump - S_1 or M_3 - jump
depending upon sequent depth requirement



Note: CDL is at constant height above the channel bed in both the slopes as critical depth y_c is independent of slope of channel.

$\Rightarrow \frac{dy}{dx} = +ve$ means depth of flow increases in flow direction ie Back water curve profile.

$\Rightarrow \frac{dy}{dx} = -ve$ means depth of flow decreases in flow direction ie Drow down water curve profile.

Computation of length of gradually varied flow profile:

- (1) Direct integration method – chow's method
- (2) Direct step method – Numerical method
- (3) Graphical method.

Direct step method:

$$\frac{dE}{dX} = s_0 - s_r \quad s_r = \frac{n^2 Q^2}{A^2 R^{4/3}}$$

$$\Delta X = \frac{\Delta E}{s_0 - s_f}$$

if y_1 and y_2 be depth of flow at section (1) and section (2) respectively then ΔX between them will be calculated as

Y(m)	Area (m ²)	R=A/P (m)	Velocity (m/sec)	Energy (m)	ΔE	S_f	$\bar{s}_f$	$s_0 - \bar{s}_f$	ΔX
y_1	A_1	R_1	V_1	E_1	$E_2 - E_1$	S_{f1}	$\bar{s}_f = \frac{s_{f1} + s_{f2}}{2}$	$s_0 - \bar{s}_f$	$\Delta X = \frac{\Delta E}{s_0 - \bar{s}_f}$
y_2	A_2	R_2	V_2	E_2		S_{f2}			

Rapidly Varied Flow: Hydraulic Jump

5

In rapidly varied flow, a sudden change in depth occurs at a particular stretch of a channel and the change from one depth to another takes place in a distance of very short length.

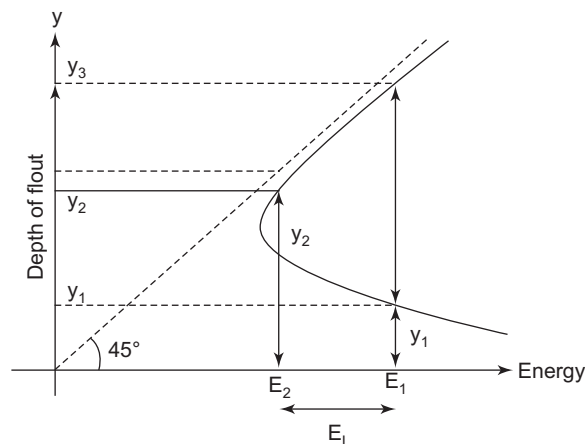
Hydraulic Jump is formed when water moving at super critical velocity in a relatively shallow stream strikes water having large depth and **sub critical velocity**.

Applications of hydraulic jump are as energy dissipator, to mix chemicals, desalination of sea water, to aerate polluted stream, to reduce the uplift pressure.

Sequent depth or conjugate depth:

During hydraulic jump, water generates considerable disturbances in the form of eddies and reverse flow rollers.

Because of this jump falls shorts of attaining alternate depth (y_3) and attain's another depth ($y_2 < y_3$) called conjugate depth.



Momentum equation for the jump:

$$\boxed{A\bar{y} + \frac{Q^2}{Ag} = \text{constant}} \quad \boxed{\frac{P + M}{\gamma} = \text{constant}}$$

Hydraulic jump in horizontal frictionless rectangular channel

$$(1) \frac{y^2}{2} + \frac{q^2}{gy} = \text{constant}$$

$$(2) \frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$(3) \frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8F_1^2} - 1 \right) \text{ where } F_1 = \frac{q^2}{gy_1^3}$$

$$(4) \text{Energy loss } E_L = \frac{(y_2 - y_1)^3}{4y_1 y_2}$$

(5) Relative Energy loss

$$\frac{E_L}{E_1} = \frac{\left(-3 + \sqrt{1 + 8F_1^2} \right)^3}{2 \left(2 + F_1^2 \right) \left(\sqrt{1 + 8F_1^2} - 1 \right)}$$

$$\frac{E_L}{y_1} = \frac{\left(\frac{y_2}{y_1} - 1 \right)^2}{4 \left(\frac{y_2}{y_1} \right)} \quad \frac{E_1}{y_1} = 1 + \frac{F_1^2}{2}$$

$$(6) \text{Efficiency of jump } \eta_{\text{jump}} = \frac{E_2}{E_1}$$

$$\eta_{\text{jump}} = 1 - \text{Relative energy loss}$$

$$(7) \text{Length of jump } L_j = 6.9 (y_2 - y_1)$$

$$(8) F_2^2 = \frac{8F_1^2}{\left(\sqrt{1 + 8F_1^2} - 1 \right)^3}$$

Types of Jump: It depends on the Froude's number of the **incoming flow** i.e. of upstream end

	Fr	E _L /E ₁	Water surface
Undular	1-1.7	≈ 0	Undulating
Weak	1.7-2.5	5 –18%	Small rollers form
Oscillating	2.5-4.5	18–45%	Water oscillates in random manner
Steady	4.5-9	45–70%	Roller and jump action
strong	≥ 9	≥ 70%	Very rough and choppy

Jump on sloping floor:

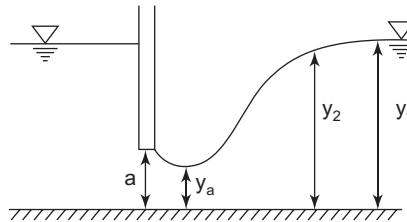
- (1) It requires more tail water depth than corresponding horizontal floor jumps.
- (2) Length of jump on sloping floor is longer
- (3) Energy loss will decrease on the sloping floor.

Location of jump:

y_0 = depth of flow at vena-contracta

y_2 = conjugate depth

y_t = tail water depth



- (a) $y_t = y_2$ Free hydraulic jump.
- (b) $y_t < y_2$ Free repelled jump.
- (c) $y_t > y_2$ Submerged or drowned jump.

Note: Energy loss is less in submerged jump.

Civil Engineering

9

Environmental Engineering

CONTENTS

Part-A

1. Water Demand, Its Source and Conveyance
2. Quality Parameters of Water
3. Treatment of Water
4. Distribution System

Part-B

5. Waste Water Characteristics
6. Disposal of Sewage Waste
7. Design of Sewerage System and Sewer Appurtenances
8. Sewage Treatment
9. Solid Waste Management
10. Air Pollution
11. Noise Pollution

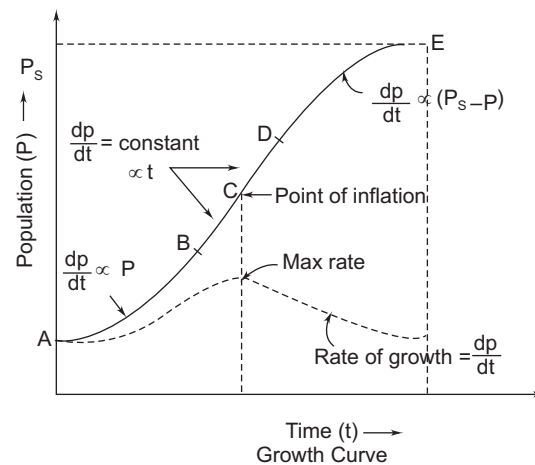
Water Demand, Its Source and Conveyance

1

Design Period: The number of years for which a provision is made in designing the capacities of various components of the water supply scheme.

Units	Design Period
Water treatment units	15 years
Service Reservoirs (overhead or ground level)	15 years
Pipe Connections to the several treatment units	30 years
Distribution system	30 years

Population Forecasting Methods:



where P_s = Saturation Population

(i) Arithmetic Increase Method:

$$P_n = P_o + n\bar{x}$$

P_n = Population after n decades from present.

P_o = Present population

n = No. of decades

$\bar{x}$ = Arithmetic mean of population Increase in known decades.

(ii) Geometric Increase Method:

$$P_n = P_c \left(1 + \frac{r}{100} \right)^n$$

P_o = Population at the end of last known census

r = Assumed growth rate (%)

$$r = \frac{r_1 + r_2 + \dots + r_n}{n} \text{ or } r = (r_1 \times r_2 \times \dots \times r_n)^{1/n}$$

(iii) Incremental Increase method (or method of varying Increment)

$$P_n = P_o + n\bar{x} + \frac{n(n+1)}{2}\bar{y}$$

$\bar{x}$ = Average Increase of population of known decades

$\bar{y}$ = Average of incremental increases of the known decades

(iv) The logistic curve method

$$P = \frac{P_s}{1 + m \log_e^{-1}(nt)}$$

P_s = Saherahan population

m = constant

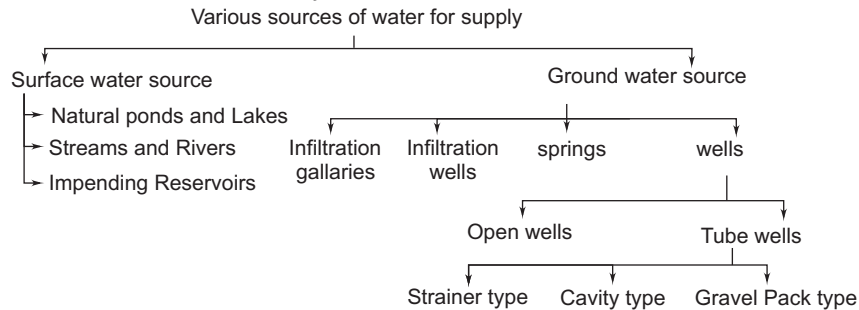
(v) *Decreasing rate of growth*: Applicable only when the rate of growth shows downward trend. In this method, the average decrease in percentage is worked out, and is then subtracted from the latest percentage increase for each successive decade.

(vi) *Comparative graphical method*: Cities of similar characteristics and conditions are selected which have grown in similar fashion in the past as that of city under consideration and their graph is plotted and then mean graph is plotted.

Note: Maximum daily demand + Fire demand = Coincident draft.

Design capacity of various components of water supply scheme.

- (i) Source of supply, pipe mains, pumps are to be designed for maximum daily.
- (ii) Distribution system (sensitive reservoir to water taps) for maximum hourly.



Joint in Water Supply Piping:

- (i) Spigot and socket joint: Also known as Bell-spigot joint. Where spigot end is inserted into the bell end.
- (ii) Collar joint: Recommended for joining R.C.C pipes and asbestos cement pipes.
- (iii) Expansion joint: Provided at suitable intervals to take into account the change in pipe length due to temperature variations.
- (iv) Flanged joint: Recommended for temporary work where the pipe line is to be dismantled after work or to be shifted.
- (v) Flexible joint: Where Settlement of pipe line can occur.
- (vi) Threaded joint: Recommended for connecting the GI pipes.
- (vii) Simplex joint: For joining asbestos cement pipes.

Pipe Appurtenances:

- (i) *Sluice Valve*: Placed at summits, and are provided to regulate the flow of water through the pipe and are essential to divide main line into several sections.
- (ii) *Air Valve*: Provided at summits to release air pressure.
- (iii) *Check valve or Reflux valve or Non-returning valve* : Provided in the pipe line which draws water from the pumps.
- (iv) *Relief valve or cut-off valve or Safety valve*: Helps in regulating the water hammer pressure.

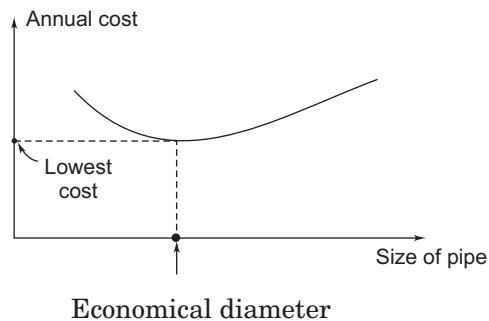
- (v) *Scour Valve or Blow-off valve or Drain valve*: Provided at dead end of the pipe line.
- (vi) *Foot Valve*: Prevent entry of debris into the pumping system and prevent back flow.
- (vii) *Butterfly Valve*: Regulate and Stop the flow specially in large size conduits.
- (viii) *Globe Valve*: Direction of Flow changes by 90° twice.
- (ix) *Ball valves or Ball Float Valves*: Used to maintain constant level in a service reservoir or elevated tank.
- (x) *Bib Cocks*: Used in small sized taps.
- (xi) *Fire hydrants*: Its an outlet provided in main water line for tapping water in case of fire.
- (xii) *Water Meter*: Device by which quantity of water flowing through a particular point is measured. It is of two types.
 - (a) *Displacement type or Positive displacement Meter*: For small flows
 - (b) *Velocity type or Interential meter*: For high flows.

Economical diameter of the pumping main:

$$D = (0.97 \text{ to } 1.22) \sqrt{Q}$$

D = Economical diameter (in meters)

Q = Discharge to be pumped (in m³/sec)



Quality Parameters of Water

2

Water impurities are classified as physical, chemical and biological impurities.

PHYSICAL WATER QUALITY PARAMETER

1. Suspended Solids: These are called as physical parameters where as dissolved solids are considered as chemical parameters.

Inorganic solids are non-biodegradable solids.

Note: Problem of SS comes only in surface water not in ground water.

Gravimetric SS are calculated by weighing them.

Total solids i.e., all solids (suspended or dissolved). are calculated by evaporating the sample and measuring the residue. Heating temperature is 104°C .

Dissolved solids (DS) = Total solids (TS) – Suspended solids (SS)

Organic fraction $\xrightarrow{600^{\circ}\text{C}}$ $\text{CO}_2 + \text{Water} + \text{Other Gases}$

Remaining solids are inorganic solids or fixed solids.

2. Turbidity: It is not direct quantitative measure of suspended solids.

Measurement of turbidity is done using the following:

- Turbidity rod
- Jackson's turbidimeter
 - Baylis turbidimeter
 - Nephelometer
- Turbidity which one milligram of finely divided silica produces in one litre of distilled water is taken as one unit.

Note: GOI manual gives turbidity in *NTU* i.e., Nephelometer turbidity unit. Acceptable limit is 1 and cause for rejection is 10.

Turbid raw water of natural source has turbidity greater than 25 ppm.

- Baylis and Nephelometer turbidimeters are based on colour matching techniques.
- Baylis turbidity meter light intensity is measured in the direction of the incident light only where as in nephelometer light intensity is measured at right angles to the incident ray.

NTU is based on **Scattering principle**.

If Formazine, a chemical, is used as base in place of SiO_2 . The turbidity unit is also sometimes called. FTU

3. Colour

- After suspended matter causing colour is removed by centrifugation, the colour obtained is called true colour.
- Water containing oxidised iron and manganese impart characteristic reddish or black colour.
- Organic compounds causing colour may exert chlorine demand.
- Measurement of colour is done by colour matching technique (tintometer).
- 1TCU is equal to colour produced by 1 mg per litre of platinum in the form of chloroplatinate ion.

Acceptable limit is **5 TCU** and cause for rejection is **25 TCU**.

3. Taste And Odour

- Taste and odour are caused by dissolved gases
- Algae secretes oily substances that may result in bad taste and odour.
- Odour is measured by an instrument known as **osmoscope**.
- Intensity of taste and odour is measured by **Threshold odour number(TON)**.
- TON allowed is between 1 – 3.
- $\text{TON} = \frac{A + B}{A}$ where A 4 mL is the volume of odourous water in mL B is the volume of odour free water required to produce a mixture in which odour is hardly detectable.

5. Temperature

An increase in 10°C, doubles the biological activity. The temperature should be between 10-25°C and greater than 25°C is objectional.

CHEMICAL PROPERTIES OF WATER

1. Total Dissolved Solids (TDS): TDS is made by determining the electrical conductivity of water.

$\Rightarrow (\text{Electrical conductivity in } \mu\text{MHo/cm at } 25^\circ\text{C}) \times 0.65 = \text{dissolved solid content in mg/l.}$

If we add organic matter.

• **Source of Total dissolved solids:**

Major source : Na, Ca, Mg, HCO_3^- , SO_4^{2-} , Cl^-

Minore source : Fe, K, CO_3^{2-} , NO_3^- , Fluoride Boron, Silica

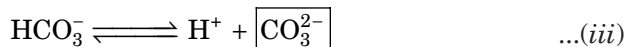
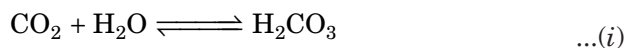
Acceptable limit of **TDS (mg/l) is 500** and cause for rejection is 2000.

2. Alkalinity

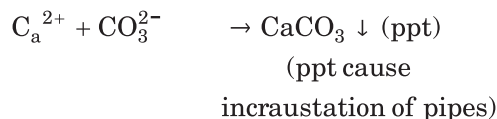
Quantity of ions in water that will react to neutralize hydrogen ions (H^+).

Common constituents of alkalinity are CO_3^{2-} , HCO_3^- , OH^- .

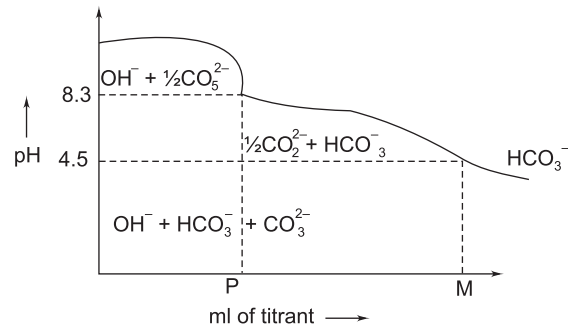
CO_3^{2-} is carbonate alkalinity, HCO_3^- is bicarbonate alkalinity OH^- is *caustic alkalinity*.



If algae is present in water, the water becomes alkaline ($pH = 9$ to 10).



- Alkalinity measurements are done by titrating the water with an acid. It is expressed in terms of mg/l as CaCO_3 .
- If 0.02 N H_2SO_4 is used in titration then 1 ml of acid will neutralize 1 mg of alkalinity as CaCO_3 .

Relative Quantity of Alkalinity Species are pH Dependent.

(Titration curve)

- If $P = M$ all alkalinity is caustic alkalinity.
- If $P = \frac{M}{2}$, all alkalinity is carbonate alkalinity
- If $P < \frac{M}{2}$, predominant species are carbonate and bicarbonate.
- If $P > \frac{M}{2}$, predominant species are carbonate and Hydroxide.
- If $P = 0$ total alkalinity is bicarbonate alkalinity.

3. pH

$\text{pH} = -\log_{10} [\text{H}^+]$ $[\text{H}^+]$ is in moles/litre.

pH is measured by potentiometer

7 – 8.5 is acceptable limit

Methyl orange is an acidic indicator.

Phenolphthalein is a basic indicator.

Note: Acidic water causes corrosion and alkaline water causes incrustation of pipe.

4. Hardness

Concentration of multivalent metallic cations in solution.

- Hardness can be divided in two parts i.e., carbonate hardness and non-carbonate hardness.
- HCO_3^- and CO_3^{2-} of calcium and magnesium cause carbonate hardness. It is also called temporary hardness.
- Sulphate, chloride and nitrate of calcium and magnesium gives permanent hardness. It is also called non-carbonate hardness.

Amount of Ca^{2+} and Mg^{2+} in water is determined by titration with versanate solution (EDTA method). Using **Eriochrome Black T** (EBT) as an indicator. EBT forms red colour and titration changes the colour to blue.

If $[\text{Ca}^{2+}]$ and $[\text{Mg}^{2+}]$ is known in mg/litre, total hardness would be equal to

$$\text{Total Hardness} = \frac{[\text{Mg}^{2+}] \text{ mg/l}}{\text{eq. wt of Mg}} \times \text{eq. wt. of CaCO}_3 + \frac{[\text{Ca}^{2+}] \text{ mg/l}}{\text{eq. wt of Ca}} \times \text{eq. wt. CaCO}_3$$

$$\therefore \text{Total Hardness} = [\text{Ca}^{2+}] \times \frac{50}{20} + [\text{Mg}^{2+}] \times \frac{50}{12}$$

Alkalinity and Hardness

Acceptable limit of total hardness = 200 mg/l and

Cause for rejection = 600 mg/l

Note : Sodium cation imports pseudo-hardness to water.

5. Chloride Content

Estimated by Mohr's method in which raw water is titrated with standard AgNO_3 Solution using K_2CrO_4 (Potassium chromate) as indicator.

6. Nitrogen Content

- It occurs in the form of

(a) Free ammonia $\longrightarrow$ Indicates recent pollution

(b) Organic ammonia (Albuminoid) $\longrightarrow$ indicates quantity of nitrogen before decomposition has started

(c) Nitrite $\longrightarrow$ indicates partly decomposed condition

(d) Nitrate $\longrightarrow$ indicates old pollution (fully oxidised)

Free ammonia should not be more than 0.15 mg/l

Organic ammonia should not be more than 0.3mg/l

Free ammonia + organic ammonia = **Kjedahl Nitrogen Ammonia**

- Nitrite permissible limit is zero.
- Nitrate is not harmful as it is fully oxidised. But too much of nitrate affects infants. Because it caused **blue baby disease** or **methemoglobinemia**. Nitrate concentration should not be more than 45mg/l

7. Phosphorus

- It facilitates rapid growth of aquatic plants.
- It interferes with water treatment like chemical coagulation.

8. Fluorides

- Up to 1 mg/l, it helps to prevent dental cavities
- Excess value (greater than 1.5 to 2 ppm) results in decolouration of teeth called mottling of teeth. (Infants are affected not adults).

9. Metals

- Ca, K, Na, Fe, Mn, Zn are non-toxic metals.
- Arsenic, Barium, Cadmium, Chromium, Cyanide, Lead & Mercury are toxic metals.

10. Dissolved Gas

- CH_4 explosive tendency
- H_2S bad taste and odour
- CO_2 biological activity, imparts bad taste and water becomes corrosive
- To determine oxygen deficiency of water 10% solution KMnO_4 is exposed to 27°C for 4 hours and the amount of oxygen absorbed is calculated.

Organics

Classified as biodegradable and non-biodegradable organics.

(a) Biodegradable

Reaction in the presence of oxygen is called **aerobic reaction** and reaction in the absence of oxygen is called **anaerobic reaction**

Amount of oxygen consumed during microbial utilization of organics is called BOD (biochemical Oxygen demand)

BOD after 5 days at 20°C is taken as standard BOD.

$\text{BOD}_5 = (\text{Initial Dissolved oxygen} - \text{Final dissolved oxygen}) \times \text{Dilution factor}$

Where, Dilution factor = $\frac{\text{final volume}}{\text{Sample volume}}$

(b) **Non-biodegradable**

- These organics decompose so slowly that they are called non-biodegradable.
- Measurement of non-biodegradable organics is done by COD i.e., chemical oxygen demand and also TOC i.e., total organic carbon

$$\begin{aligned}\text{Non-biodegradable organic} &= \text{COD} - \text{BOD}_u \\ \text{BOD}_u &= \text{ultimate BOD}\end{aligned}$$

- The usual routine tests are generally conducted to detect and count the presence of **coliforms** which in themselves are harmless aerobic lactose fermenters organisms.
- Coliform called bacteria coli (B-Coli, E-Coli)
- The tests for coliforms are :

(a) Membrane filter technique

(b) MPN test (Most Probable Number)

(c) Coliform index

Membrane Filter Technique

- Membrane is put in contact with nutrients (M-endo medium) that permits the growth of only **Coliform colony**. After incubation for 20 hrs number of visible colonies are counted. No visible colonies should be detectable in any 100 ml sample.

Most Probable Number (by Multiple Tube Fermentation Test)

- The presence of **acid** or **Carbon Dioxide** in the test tube will indicate presence of coliform bacteria.

Thomas equation

MPN/100 ml

$$= \frac{\text{Number of positive tube} \times 100}{\sqrt{(\text{ml of sample in negative tube}) \times (\text{ml of sample in all tube})}}$$

Coliform Index

It is defined as the reciprocal of smallest quantity of a sample which will give positive B-coli test MPN and B-Coli index are now obsolete.

Water Borne Diseases

Bacteria – Typhoid fever, Cholera, Bacillary Dysentery etc.,

Virus – Jaundice, Poliomyelitis,

Protozoa – Amoebic dysentery

Nuisance Bacteria : Iron Bacteria, Sulphur Bacteria

Treatment of Water

3

- Plain disinfection is sufficient for drinking purposes.
- For ground water containing excessive iron, dissolved carbon dioxide and odourous gases.

Aeration → Flocculation + Sedimentation → Rapid gravity → Disinfection → Supply

- If ground water contains only CO₂ and odourous gases, aeration and disinfection is sufficient.
- Surface water with turbidity less than 50 NTU
Plain Sedimentation + Slow sand filter + Disinfection
- Highly polluted surface water laden with algae

Prechloriation + Aeration + Flocculation and Sedimentation + Rapid gravity filtration + Disinfection (postchlorination) + Supply.

Screening

- Classified as **Coarse screen and fine screen.**
- In the form of bars spaced at 20 – 100 mm c/c.
- Kept inclined 3-6V : 1 H racking.

Microstrainer

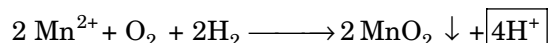
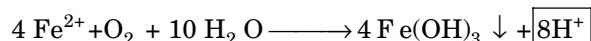
- Usefull for screening stored water, which do not contain a large amount of suspended matter.
- Ideal position of micro-strainer is earlier to rapid gravity or slow sand filters.

Pre-chlorination

- Practice of injecting chlorine into the raw water.
- Fairly high dose of chlorine (2-5 mg/l) is used.
- Kills algae and bacteria, reduces colour and slime formation.
- In case of clear ground water with high ammonia content, it is effective.
- Most advantageous when *extremely polluted clear raw water*.

AERATION

- Remove undesirable gases.
- It can also remove volatile liquid like phenols and humic acids.
- It removes iron and manganese.



- Increases the **acidity** of water.

Process of Aeration

- (a) **Spray nozzle** → removes 90% CO_2 and 99% H_2S .
- (b) **Cascade aerator** → Removes 20 – 45% CO_2 and 35% H_2S .
- (c) **Diffused air aerator** → Water absorbs oxygen from compressed air and colour, odour and taste are removed.
- (d) **Trickling bed or Tray tower:** Fe, Mn
 - To help oxidation, KMnO_4 may be mixed.

Algal Control

- Fairly alkaline water containing more concentration of nitrates and phosphates are prone to algal growth.
- Where organisms are more, a heavier dose of copper sulphate (2mg/l) or chlorine (3-5 mg/l) may be necessary.
- Rapid sand filters running without coagulants.

Pre-settlement Basins

Where water reaching main settling basins has suspended solids concentration more than 1000 mg/l by dry weight.

SEDIMENTATION

- To remove suspended solids.
- Classified into two categories:
 - (a) Plain sedimentation,
 - (b) Sedimentation with coagulation
- Sedimentation tanks are classified as **Quiescent type** (fill and draw type) and **continuous type**.

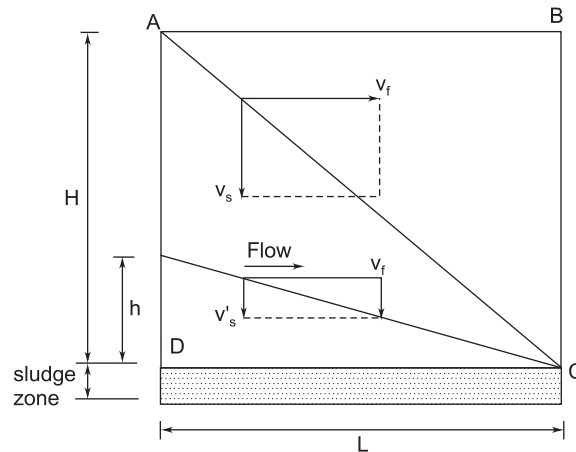
Quiescent type tank

- Detention time = 24 hr
- Period of cleaning = 8-12 hr. Minimum 3 numbers of tank, max. daily flow max daily flow = $1.8 \times \text{av. daily flow}$.
- Settling velocity

$$V_s = \frac{(\gamma_s - \gamma_w)d^2}{18\mu} \dots \text{applicable for } Re < 1 \text{ (stoke's law)}$$

$$d < 0.2 \text{ mm}$$

Discrete particle settling not for merging of particle i.e, not used in sedimentation with coagulation only for plain sedimentation.



$$\text{Time of horizontal flow} = \frac{\text{Length of tank}}{\text{Velocity of flow}} = \frac{L}{V_f}$$

$$\text{Velocity of flow, } V_f = \frac{Q}{BH}$$

$$t_D = \frac{\text{Volume of tank}}{\text{Discharge}}$$

$$V_s = \frac{Q}{BL} \text{ Over/flow rate.}$$

- Surface overflow rate can be thought of as settling velocity of that particle which if introduced at the top most point at inlet will reach the bottom most point at outlet.

- Percentage removal of these particle will be is given by

$$\frac{h}{H} = \frac{V_s'}{V_s}$$

Percentage removal of particle having settling velocity $V_s' = \frac{V_s'}{V_s} \times 100$.

Short Circuiting in Sedimentation Tank

- The deviation of actual flow of tank from the flow pattern of ideal tank.

Displacement efficiency (η_d)

$$= \frac{\text{Flow through period actual time needed to cross}}{\text{Theoretical Detention Time}} \times 100$$

Tube Settlers

- Settling efficiency of the sedimentation tank is primarily dependent upon the surface area of the tank and independent of its depth.
- Divide the tank height into compartments, to reduce the depth and to provide multiple surface area.

Data for Sedimentation Tanks

- Overflow rate: 15,000 – 30,000 litre/m²/day for plain sedimentation, 30,000 – 40,000 litre/m²/day for sedimentation with coagulation.
- Detention time is 3-4 hours for plain sedimentation tank and 2-2.5 hrs for sedimentation with coagulation.

$Q \times \text{detention time} = \text{volume of tank}$

$(V_f) = 0.3 \text{ m/minute.}$

$V_f \times \text{detention time} = \text{length of tank}$

- Width 10 to 12 m.

- Length

4 times the width.

Maximum daily flow = 1.8 × average daily flow.

- Depth of tank 3 m.

CIRCULAR SEDIMENTATION TANK**Volume of Circular tank**

$$V = D^2 (0.785 H + 0.011D)$$

$$Q \times t_d = \text{Volume}$$

$$\text{over flow rate} = \frac{\frac{Q}{\pi D^4}}{4}$$

$$\bullet \text{ Weir loading Rate} = \frac{Q}{\pi D}$$

SEDIMENTATION WITH COAGULATION

- A colloidal dispersion is defined as stable when the dispersion shows little or no tendency to aggregate.
- Aggregation of colloidal particles requires:
 - (a) particle destabilisation
 - (b) agglomeration
- “Zeta potential” is a measure of the stability of particle and indicates the potential, which would be required to penetrate the layer of ions surrounding the particle for destabilisation.
- The choice of the best coagulant for any particular water **Jar Test**.

Mechanism of Coagulation

(a) **Ionic Layer Compression:** The quantity of ions in water surrounding a colloid has an effect on reducing the repulsive force.

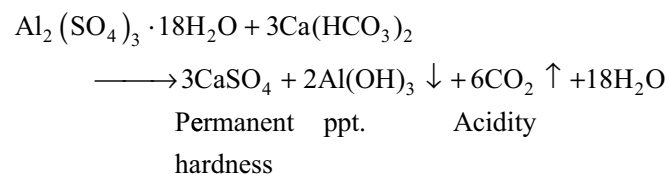
(b) **Adsorption and Charge Neutralisation:** Nature rather than quantity of iron is of prime importance.

(c) **Sweep Coagulation:** The aluminium hydroxide ($\text{Al}(\text{OH})_3$) formed when alum is added to water. Colloids become entrapped in the flocs as the flocs settle down.

(d) **Inter Particle Bridging:** Large molecules may be formed when Al or ferric sulphate dissociate in water (like $\text{Al}_7 (\text{OH})_{17}^{4+}$).

Common Coagulants added in Water**(a) Alum**

- $\text{Al}_2(\text{SO}_4)_3 \cdot 18\text{H}_2\text{O}$.

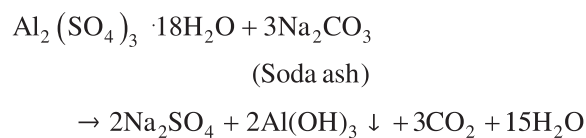
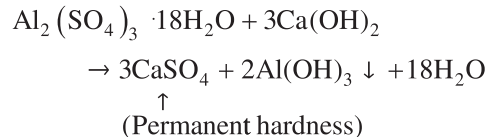


- Reaction introduces permanent hardness in water and water becomes corrosive.
- Normal dose is 10 – 30 mg/litre of water
- pH 6.5 to 8.5.

$\therefore$ 1 gm of alum gives = 0.234g = 0.24 gm of $\text{Al}(\text{OH})_3$ ppt

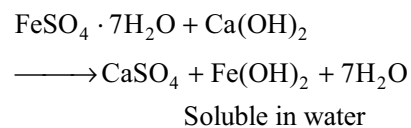
Alkalinity-Coagulation Relationships

Alkalinity is sufficient in water, lime $[\text{Ca}(\text{OH})_2]$ or soda ash (Na_2CO_3) may be added in water.

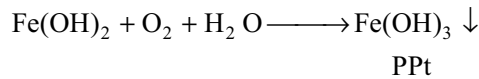
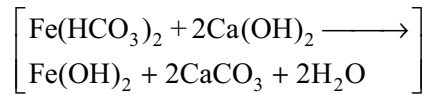
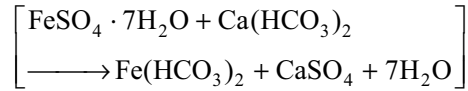
**(b) Copperas**

- $\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$

When lime is added first,

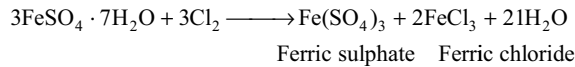


Copperas is added first,

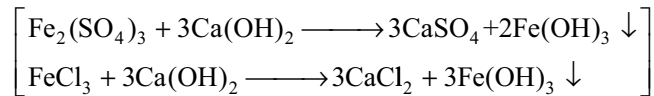


- pH : 8.5 and above
- Quantity 10 – 30 mg/litre

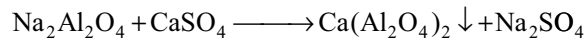
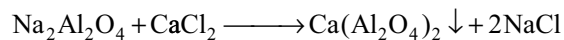
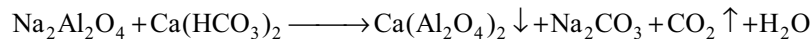
(c) **Chlorinated Copperas**



- Works in large pH range.
- Chlorinated copperas with lime

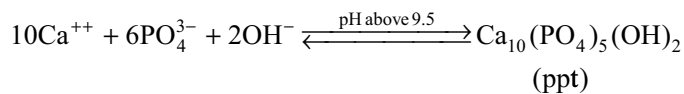


(d) **Sodium Aluminate** [$\text{Na}_2(\text{Al}_2\text{O}_4)$]



- Reduces the temporary and permanent hardness.
- Costlier than alum.
- pH range is 6 – 8.5.

(e) **Lime** where phosphorus removal is desired.



Coagulation Aids

Finally divided clay, bentonites and activated carbon.

1. High Turbidity Low, Alkalinity of Water

Low doses of coagulants

2. High Turbidity – High, Alkalinity of Water

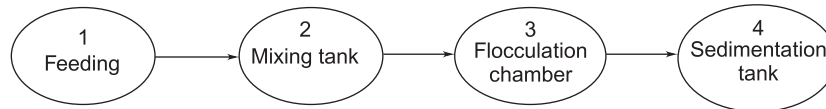
Higher coagulant dosage

3. Low Turbidity – High, Alkalinity of Water

High coagulant dosages

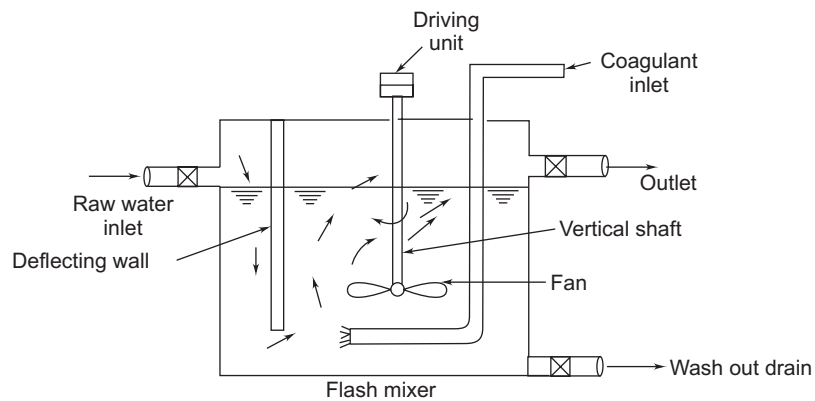
4. Low Turbidity – Low Alkalinity Water

Add both turbidity and alkalinity

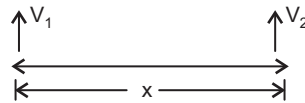


MIXING

Mechanical Mixer



- Temporal mean velocity gradient (G).



$$G = \frac{V_2 - V_1}{x} = f\left(\frac{P}{V}, \mu\right)$$

- Unit of G is sec^{-1}

$$G = \sqrt{\frac{P}{\mu V}}$$

where, P is power in watts

μ is dynamic viscosity

V is volume of tank in m^3

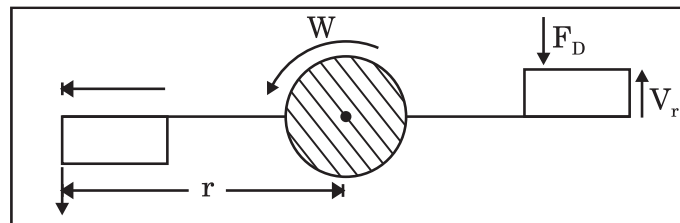
- Detention time 30 to 60 seconds.
- G is kept 300 per second or more.

FLOCCULATION

Slow mixing or agitation process in which destabilized colloidal particles are brought into intimate contact.

- Gt_d = It is the measure of conjugation opportunity.
- Gt_d adopted for alum is $(2 - 6) \times 10^4$ for FeCl_3 is $(1 - 1.5) \times 10^5$.
- Larger G and Smaller t_d will make Small and dense floc.
- Smaller G and larger t_d will make large and light floc.

Calculation of G



$$\text{Power} = F_D \times V_r$$

$$V_r = V_P - V_W$$

$$F_D = \frac{C_D P_W A_P V_r^2}{2}$$

$$G = \sqrt{\frac{C_D P_W A_P V_r^3}{2\mu V}}$$

Design Criteria

- Depth of tank 3 to 5 m.
- Detention time taken is 10 – 30 min.
- Peripheral velocity (V_p) 0.2 – 0.6 meter per sec.
- Gt_d adopted is 10,000 – 100,000.

FILTRATION

- Effective in controlling guinea worm disease.
- Filters are classified as:
 - (a) Slow sand filter
 - (b) Rapid sand filter
 - (c) Pressure filter

Theory of Filtration**(a) Mechanical Straining:**

Particles are removed in upper layers.

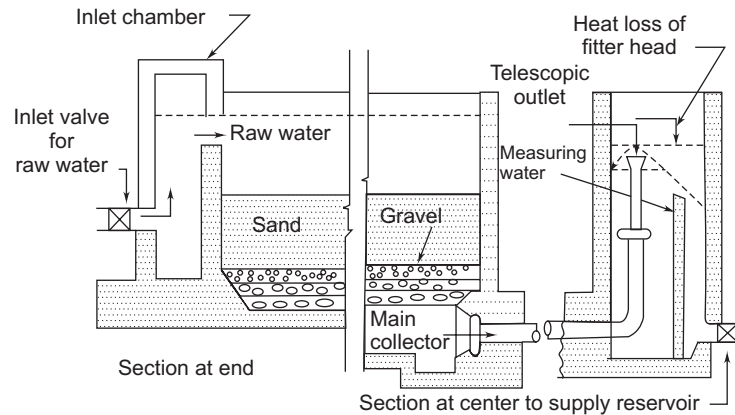
(b) Sedimentation: Particles finer than voids are removed by sedimentation.

(c) Biological Metabolism: Three zones of purification.

- The surface coating is known as “schmutzdecke”.
- The ‘autotropic’ zone existing a few millimeters.
- The “heterotrophic” zone, which extends around 300 mm into the bed.

(d) Electrolytic Changes

• Sand grains in filter and impurities in water carry opposite charges. Thus because of their interaction, chemical characteristics of water changes and it becomes pure.

SLOW SAND FILTER

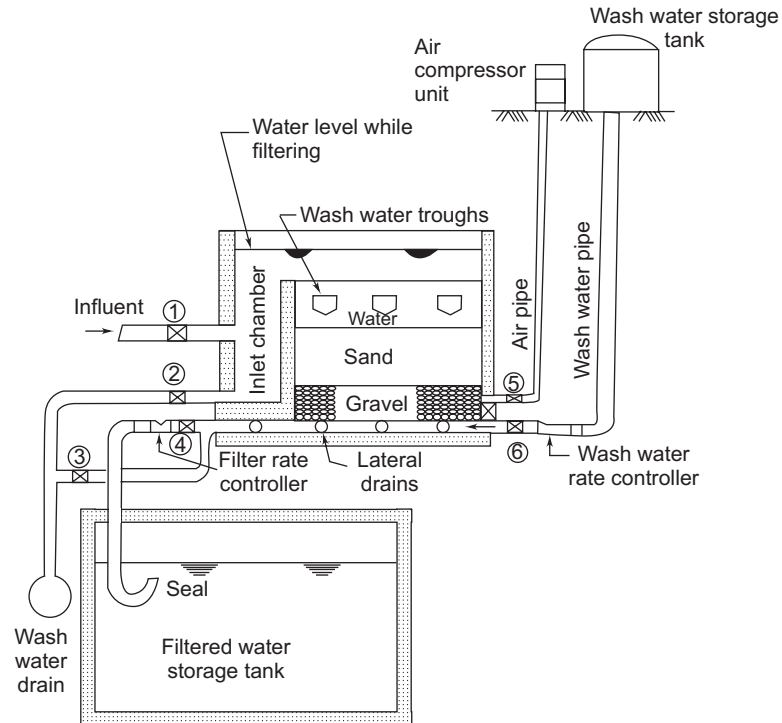
- Utilizes the effluent from plain sedimentation tank only.
- Depth of tank is 2.5 to 3.5m.
- Plan area required is 100 – 2000 m².
- D_{10} (of filter medium) = 0.2 – 0.3 mm.
- $\frac{D_{60}}{D_{10}} = 5$
- Depth of sand is 90 – 110 cm $\approx$ 1 m.
- Top 15 cm of sand layer would be finer.
- Top layer size — 3 — 6 mm
- Middle layer — 20 — 40 mm
- Bottom layer — 40 — 65 mm
- Design period of slow sand filter is 10 years.

Cleaning of Filter

- Top layer of sand is scraped and 1.5 to 3 cm of sand is removed
- Frequency of cleaning is 1 – 3 months.
- Rate of filtration is 100–200 l/hr/m² of filter area.
- Bacteria removal is 97-98%.
- Filter cannot be used if turbidity is greater than 50 ppm.
- Used for smaller plants in villages.
- Designed for maximum daily demand.

RAPID GRAVITY FILTER

- Particle more than and less than 1 μ m dia are efficiently removed.
- Removes suspended and colloidal matter.
- Remove microorganisms.



Back Washing

- Process takes 15 minutes
 - Washing period is 24 – 48 hrs.
 - Rate of washing is 15 to 90 cm rise per minutes
 - Area of tank is 10 – 100 m²/unit
 - Number of units required
- $$n = 1.22\sqrt{Q}, \text{ } Q \text{ is plant capacity in MLD.}$$
- Sand layer is 60 – 75 cm deep and D_{10} of sand is 0.45 – 0.7 mm.
 - Depth of water over sand layer = 1 – 2m.
 - Length/width = 1.25 to 1.33.

- $\frac{D_{60}}{D_{10}} = 1.3 \text{ to } 1.7$
- Max. loss of head = 2.5 to 3.0 meters.

Flow rate in rapid gravity filter 2-20 m/hr

∴ Head loss during back wash h

$$h = \frac{L \times (1 - n)(\gamma_s - \gamma_w)}{\gamma_w}$$

For the expanded length of bed.

$$h = \frac{L_{ex}(1 - n_{ex})(\gamma_s - \gamma_w)}{\gamma_w}$$

Porosity in the expanded condition

$$n_{ex} = \left(\frac{V_B}{V_t} \right)^{0.22}$$

V_B = Back wash velocity → 45 cm/min

V_t = Terminal settling velocity of the particle

Operational Troubles in Rapid Sand Filter

- (a) Air binding
- (b) Mud balls formations
- (c) Cracking of filter

Rapid sand filter is highly efficient in colour removal when used with coagulation sedimentation. Efficiency w.r.t. bacteria removal is 80 – 90%.

DUAL MEDIA FILTER AND MUTLI MEDIA FILTERS

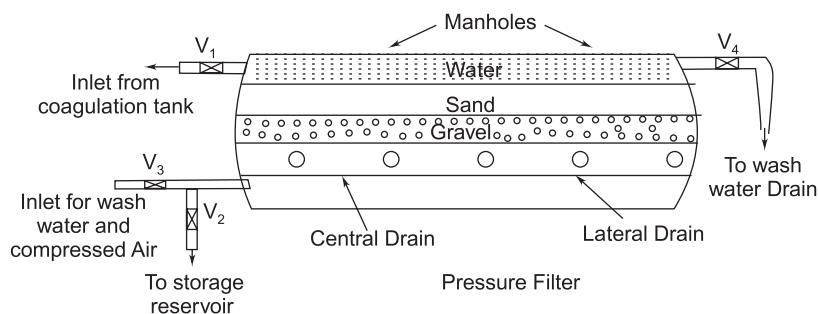
Larger anthracite grain (Sp. gr. 1.55) is provided is 30 – 60 cm depth of filter media and below this smaller sand grain (Sp. gr. 2.65) is provided.

• Multi Media Filters (MMF_s)

Use three or more materials for filter layers.

Multi-media filters are improved dual-media filters, with increased filter run times and better water quality.

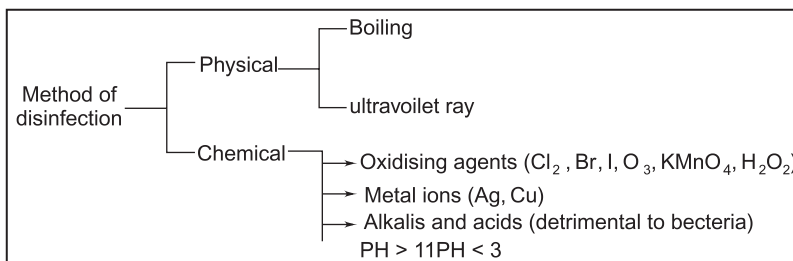
PRESSURE FILTERS



- Diameter of the tank is 1.5 – 3.0 m.
- Height or length is 3.5 to 8.0 m.
- Rate of filtration 6000 – 15000 lit/m²/hr.
- Used for clarifying softened water at industrial plants.

DISINFECTION

- Process of destruction or inactivation of harmful micro-organism in water either by physical process or chemical process.
- Sterilization process, all organism are killed by a physical phenomenon.



1. Minor Methods

(a) Treatment with excess lime : 14 – 40 ppm

(b) Treatment with ozone : 2-3 ppm

- Residual ozone is measured by orthotolidine test.
- It is less efficient than chlorine in killing bacteria.

(c) Treatment with F and Br

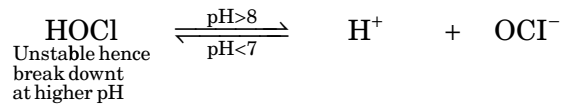
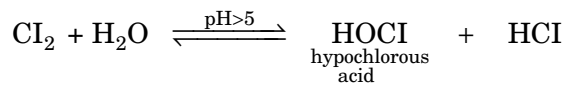
In the form of pills.

(d) Treatment with KMnO_4

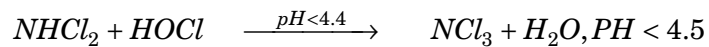
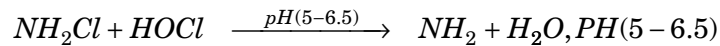
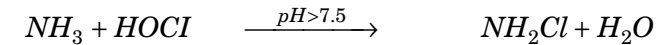
• KMnO_4 is mixed with water in a bucket and added to well. Pink colour forms because of this. If pink colour disappears, this implies that organic matter is present.

Major Methods

(a) Chlorination



HOCl is most destructive. It is 80% more effective than the OCI^- ion.



Forms in which Chlorine is Added

- (a) As free chlorine (liquid or gaseous form)
- (b) Hypochlorites (Bleaching powder)
- (c) Chloramines (ammonia + chlorine)
- (d) Chlorine dioxide (ClO_2)

Types of Chlorination

(a) Plain chlorination

- turbidity b/w 20-30 mg/l
- Dose is 0.5 mg/l

(b) Pre Chlorination

- Chlorine is added before filtration or rather before sedimentation and coagulation
- Doses required 0.1 to 0.5 mg/l

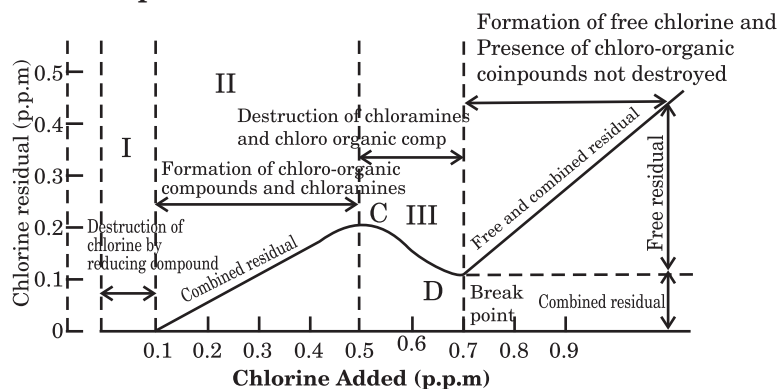
(c) Post Chlorination

- Dose of 0.1 to 0.2 mg/lit contact period of 20 min.

(d) Double Chlorination

- Prechlorination and post-chlorination combinedly.

(e) Breakpoint Chlorination



- Chlorine residue is tested by **DPD** test
- Chlorine is added beyond break point to ensure a residual of 0.2 – 0.3 mg/litre as free chlorine
- The difference of applied chlorine and residual chlorine beyond break point is called chlorine demand of water.

(f) Super Chlorination

- Excess chlorine (5 to 15 mg/l) is added in water during epidemic
- Dechlorinating agents are :

Sodium thiosulphate ($\text{Na}_2\text{S}_2\text{O}_3$) - cheapest of all

Activated carbon

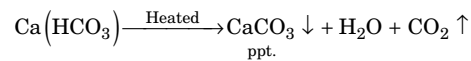
Sulphur dioxide (SO_2)

Testing of Chlorine Residue

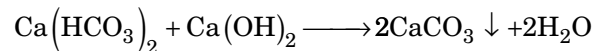
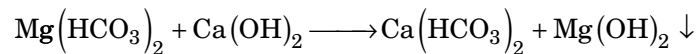
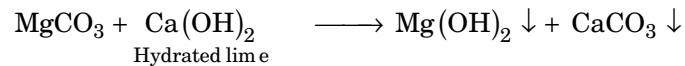
- (a) Orthotolidence test
- (b) DPD test
- (c) Chlorotex test
- (d) Starch iodide test

WATER SOFTENING**Removal of Temporary Hardness**

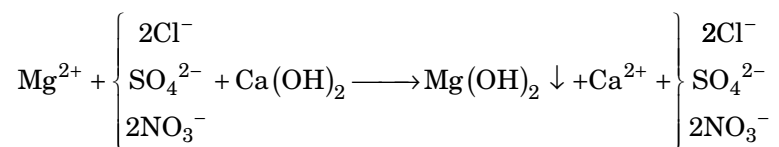
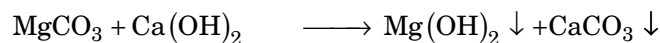
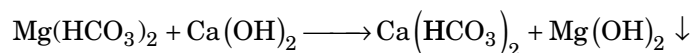
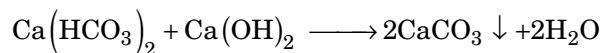
By simple boiling



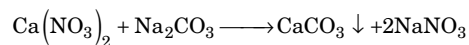
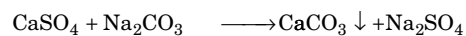
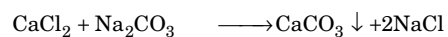
by **addition of lime.**



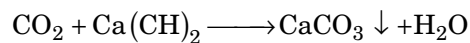
1 mole of MgCO_3 requires 1 mole of hydrated lime, whereas 1 mole of $\text{Mg}(\text{HCO}_3)_2$ requires 2 moles of lime.

Removal of Permanent Hardness (Water Softening)**1. Lime Soda Process**

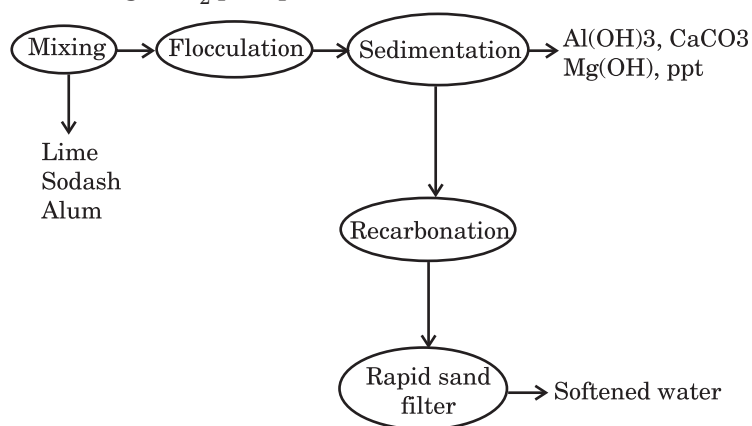
Further



Lastly



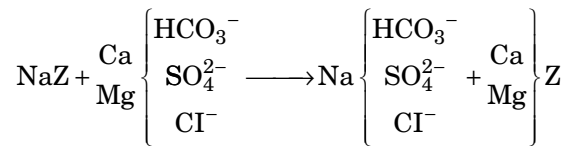
Optimum pH for CaCO_3 precipitation by lime addition is from 9 – 9.5 and for $\text{Mg}(\text{OH})_2$ precipitation is 11.0.



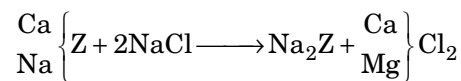
2. Base Exchange Process (Cation Exchange Process)

Zeolite is a natural or synthetic cation

Zeolite $\rightarrow \text{Na}_2\text{O} \cdot \text{Al}_2\text{O}_3 \cdot x\text{SiO}_3 \cdot y\text{H}_2\text{O}$ (green sand)



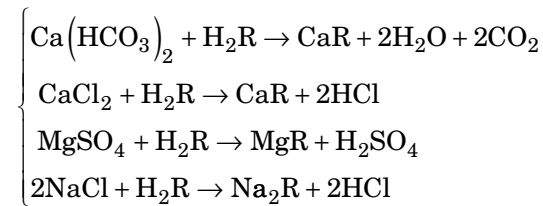
- Water will have zero hardness.
- Regeneration from CaZ_2 and NaZ



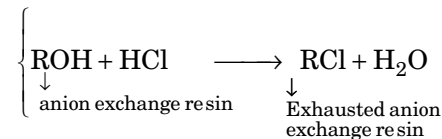
3. Demineralization Process

Removes all the minerals in water.

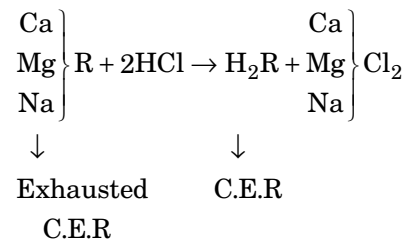
1st stage

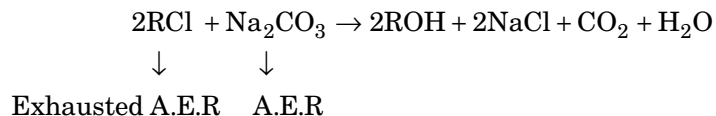


2nd stage



Regeneration



**Minor Treatments****(a) With activated carbon****(b) Treatment with Copper Sulphate ($\text{CaSO}_4 \cdot 7\text{H}_2\text{O}$)****(c) Removal of Iron and Maganese****(d) Fluoridation (Addition of Fluroine)****(e) Defluoridation (Removal of Fluorine)**

(i) Absorption by activated alumina (AA), commonly known as Prashanti Technology.

(ii) Nalgonda technique.

(iii) Ion exchange adsorption method.

(iv) Reverse osmosis process.

(f) Desalination**(h) Removal of Toxic Metals**

Distribution System

4

METHOD OF DISTRIBUTION

- (i) Gravitational system
- (ii) Direct pumping
- (iii) Combined system

System of Supply

- (i) Continuous supply
- (ii) Intermittent supply

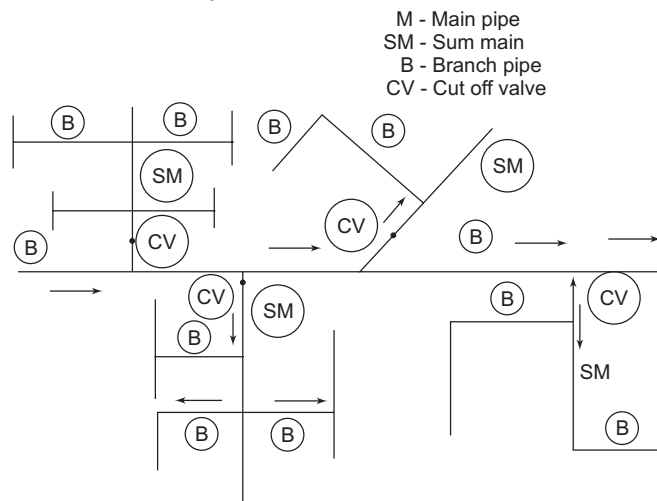
LAYOUTS OF DISTRIBUTION SYSTEM

• Pipes except the service connections are usually made of cast iron with some type of coating to avoid rusting where as for service connections galvanised iron pipes are used.

- (i) Dead end or tree system
- (ii) Grid system or reticular system
- (iii) Ring or circular system
- (iv) Radial system

Dead End System

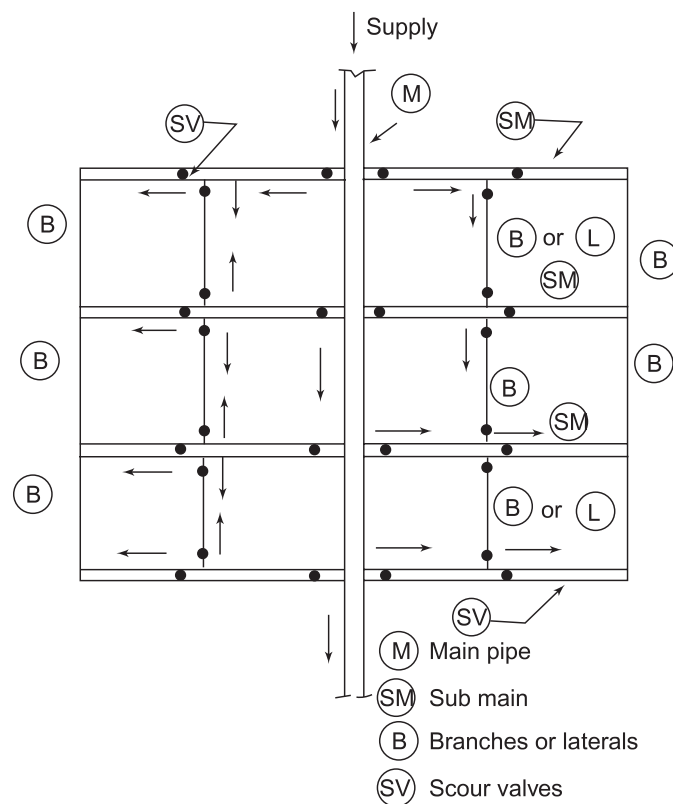
Also known as tree system.



- Water can reach at a particular point only through one route.
- Many dead ends which prevent free circulation of water.
- Stagnant water has to be removed periodically to provide service values

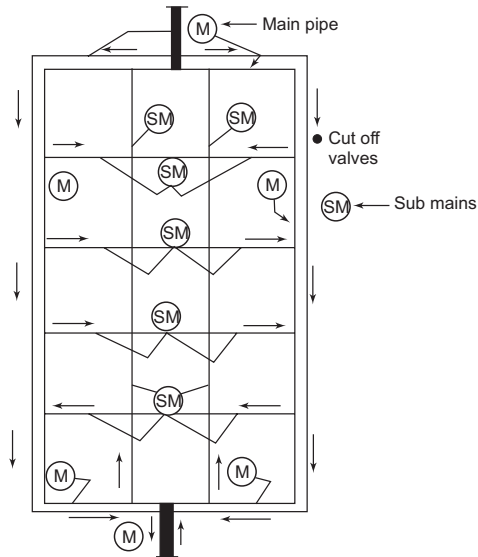
Grid System

- Also known as reticular system.



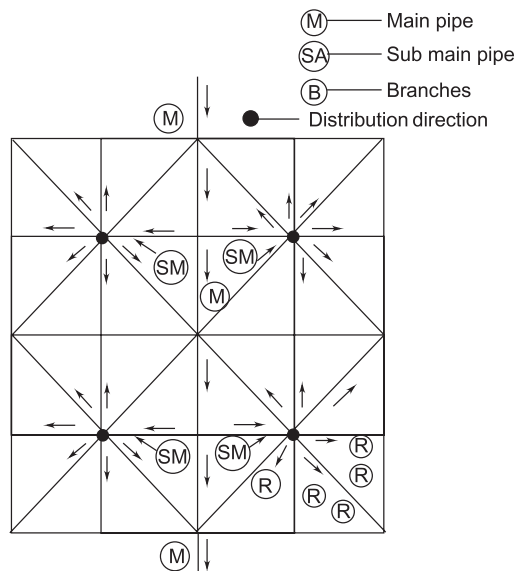
- At the time of fire, water can be diverted to the affected area by closing cut-off valves of other area.
 - Design is a bit difficult.
- Requires more length of pipe lines and a large number of sluice valve.

Ring System



- Also known as circular system.
- Consists of a main pipe all round the area.

Radial System



This method gives higher service head and efficient water distribution.

DETECTION OF LEAKAGE IN DISTRIBUTION SYSTEM

- (i) By direct observations.
- (ii) By using sounding rod: Stethoscope type instrument is used, also called aquaphone or a sonoscope.
- (iii) By plotting hydraulic gradient line and
- (iv) By using waste detecting meters.

PRESSURE IN THE DISTRIBUTION SYSTEM

Single storey buildings – 7m above ground level.

Two storey buildings – 12 m above ground level.

Three storey buildings – 17 m above ground level.

ANALYSIS OF NETWORK OF PIPES

• The Hazen– Williams is mostly used for computation of flow through pipes and following, two methods are used for analysis:

- (i) Equivalent pipe method.
- (ii) Hardy cross method.

Equivalent Pipe Method

- In this method different small loops are replaced by an imaginary single equivalent pipes having same discharge capacities and causing same head loss.
- Used in solving large network of pipes.

Hardy-Cross Method

A distribution of flow in the network is assumed and resulting head losses are balanced. In pipe network following two conditions are to be satisfied.

- (i) The algebraic sum of the pressure drops around a closed loop must be zero.
- (ii) The flow entering a junction must be equal to the flow leaving the same junction

Loss of head is $h_f = rQ^n$.

Steps

- Assume flow in each pipe satisfying continuity eq.
- Take clockwise flow as (+) ve
Anti clockwise flow as (–) ve

- Modification in discharge $\Delta Q = \frac{(-) \Sigma r Q^n}{\Sigma |r n Q^{n-1}|}$.
- ΔQ is added algebraically.

APPURTENANCES IN PLUMBING SYSTEM

1. Ferrules
2. Goose Neck
3. Service Pipe
4. Stop Cock
5. Water Meter

There are two types of meters

- (i) Velocity or inferential meters
- (ii) Positive or displacement meters

Velocity or Inferential Meters : Measure the horizontal velocity

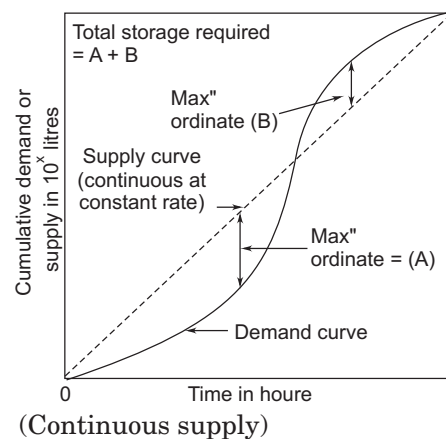
Measure large flow : Positive or Displacement Meters

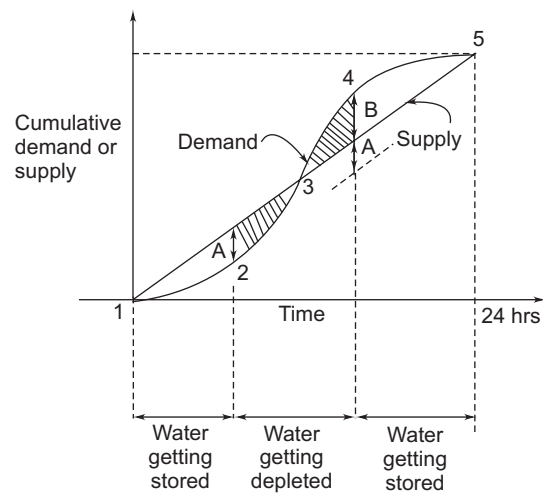
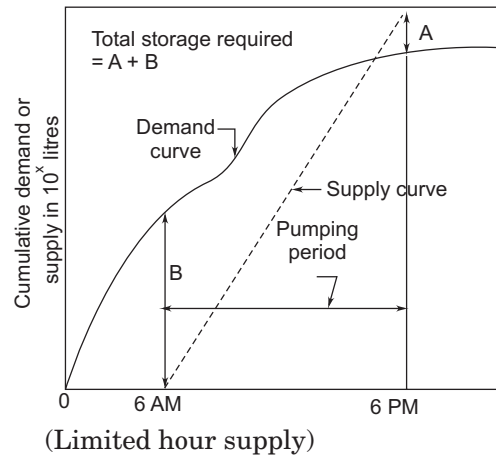
They work by the flow of water causing a piston to reciprocate within a cylinder

Design of Balancing Reservoir

(a) Mass Curve Method

A mass diagram is the plot of accumulated supply or demand versus time.





From (2 – 4) → Demand rate > Supply rate

→ Water getting depleted.

From (4 – 5 – 1 – 2) → Demand rate < Supply rate

→ Water getting stored.

From (4 – 5) → accumulation = B

From (1 – 2) → accumulation = A

From (2 – 3) → depletion = A

From (3 – 4) → depletion = B

⇒ Max. water that would be stored = $A + B$ (*i.e.*, From 4 – 2)

⇒ Max. water that would be depleted = $A + B$ (*i.e.*, From 2 – 4).

(b) **Analytical method:** In this method cumulative hourly demand and cumulative hourly supply are tabulated for all 24 hours.

Waste Water Characteristics

5

- Classified as industrial waste water or municipal waste water.

Important Waste Water Contaminants

Sl.No.	Contaminant	Source	Environmental significance
1.	Suspended solids	Domestic use, industrial wastes	Cause sludge deposits and anaerobic condition in aquatic environment
2.	Biodegradable organics	Domestic use, industrial waste	Cause biological degradation
3.	Pathogens	Domestic water	Transmit communicable diseases
4.	Nutrients	Domestic and industrial waste	Cause eutrophication
5.	Refractory organics	Industrial waste	Cause taste and odour problems

Physical Characteristics

Turbidity: Normally turbid.

Colour:

• Fresh waste water is gray or light brown and as organic compounds are broken down by bacteria, the dissolved oxygen in the waste water is reduced to zero and colour changes to black.

Odour:

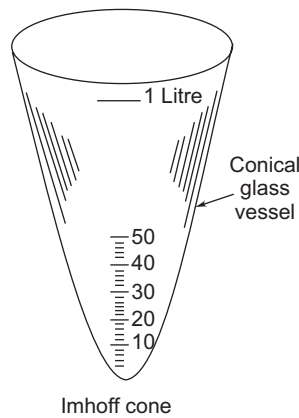
• Caused by gases produced by the anaerobic decomposition of organic matter.

Temperature:

Average temperatures of sewage in India is 20°C.

Chemical Characteristics

- Suspended solids, are those which remain floating in water.
- Dissolved solids are those which dissolve in waste water.
- Colloidal solids are finely divided solids remaining either in suspension or in solution.
- Settleable solids are that portion of solid matter which settles out, if the waste water is allowed to remain undisturbed for a period of 2 hours in imhoff cone.

**2. pH Value**

- The alkalinity of fresh waste water sample is alkaline but as time passes it becomes acidic.

3. Chloride Content

- Normal chloride content of domestic waste water is 120 mg/l.
- Measured by titrating the sample of waste water with standard silver nitrate solution, using potassium chromate as an indicator.

4. Nitrogen Content

- (a) Free ammonia or ammonia nitrogen (indicates recent pollution).
- (b) Albuminoid nitrogen or organic nitrogen (indicates quantity of nitrogen before decomposition has started).
- (c) Nitrites (indicates partly decomposed condition).
- (d) Nitrates [indicates old pollution (fully oxidised)].

5. Presence of Fats, Oils and Greases

- Determined by evaporating it and then mixing the residual solids left, with ether (hexane).

6. Sulphides, Sulphates and Hydrogen Sulphide Gas

- Formed due to the decomposition of various sulphur containing substances.

7. Dissolved Oxygen

- Temperature of sewage is more, than D.O. content will be less. Max quantity of D.O. that can remain mixed in water at a particular temperature is called *Saturation Dissolved Oxygen*.

- D.O. content of waste water is determined by the *Winkler's Method*.

8. Chemical Oxygen Demand (COD)

- Measure the content of organic matter of waste water, both biodegradable and non-biodegradable.

- Measured by using a strong chemical oxidising agent in an acidic medium.

- $\text{COD} - \text{BOD}_5 = \text{Nonbiodegradable organics}$.

9. Theoretical Oxygen Demand (ThOD)

For most practical cases, $\text{COD} = \text{ThOD}$ (taken)

However, generally

$\text{ThOD} > \text{COD} > \text{BOD} > \text{TOC}$

10. Total Organic Carbon (TOC)

Method of expressing the organic matter in terms of carbon content.

Bio-chemical Oxygen Demand

Measure of the quantity of oxygen required for oxidation of bio-degradable organic matter present in water sample by aerobic biochemical action.

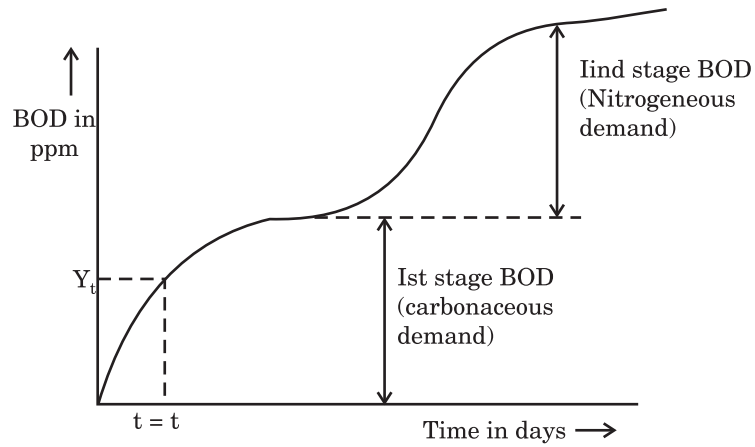
Oxygen demand of waste water is exerted by three classes of materials:

- (a) Carbonaceous organic materials.
- (b) Oxidisable nitrogen.
- (c) Chemical reducing compounds.

$\text{BOD or BOD}_5 = \text{D.O. consumed in the test by diluted sample}$

$$\times \left[\frac{\text{Vol. of the diluted sample}}{\text{Vol. of the undiluted sewage sample}} \right]$$

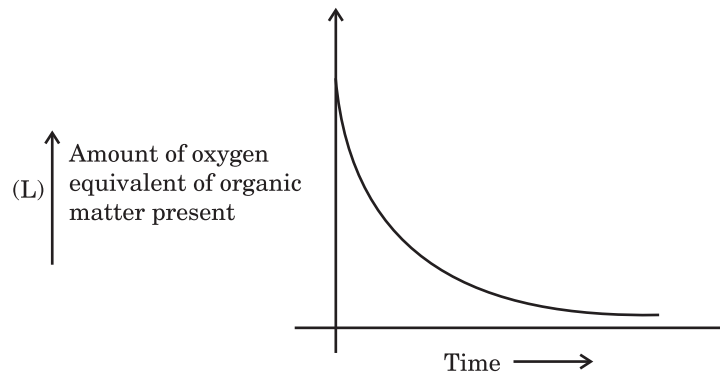
Note: Sample is diluted with dilution water so that sufficient oxygen is available during the incubation period of 5-days.



BOD Curve

Nitrification bacteria are autotrophs. Carbonaceous matter oxidising bacteria are hetrotropes.

Reaction Kinetics 1st order Reaction



L_t = amount of organic matter present at time t

$$\frac{dL_t}{dt} = -kL_t$$

$$L_t = L_0 e^{-kt}$$

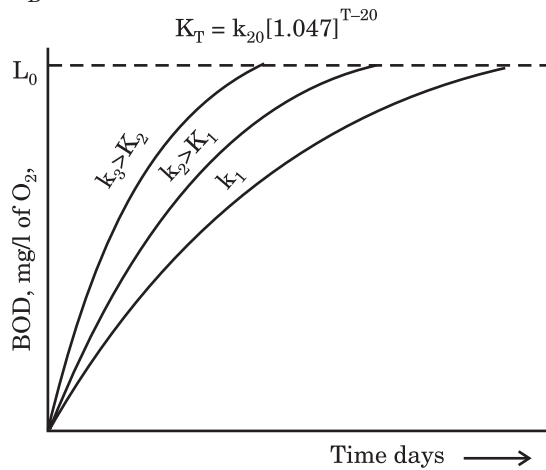
$$L_t = L_0 10^{-k_D t}$$

$$\text{BOD}_t = L_0(1 - 10^{-k_D t})$$

Unit of k_D is in terms of per day and it is temperature dependent.

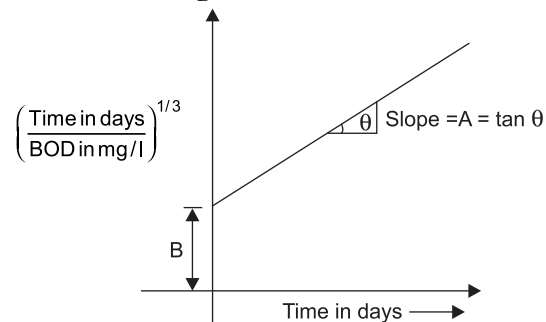
$$k_{D(T^\circ)} = k_{D_{20^\circ}} [1.047]^{T-20^\circ} \quad (\text{Vanthoff-Arrhenius equation})$$

k_D is also sometimes called deoxygenation constant.



BOD for municipal sewage is 200 – 500 mg/l.

Estimation of k_D



$$k_D = 2.61 \frac{A}{B} \text{ per day}$$

Population Equivalent

Average standard BOD of domestic sewage is 80 gms per person per day.

Relative Stability (S)

$$S = \frac{\text{O}_2 \text{ available in treated effluent}}{\text{Total O}_2 \text{ required for Ist stage BOD (i.e. BOD ultimate)}}$$

$$S = 100[1 - (0.794)^{t_{20}}] = 100[1 - (0.63)^{t_{37}}]$$

Note: A representative sample from a point where turbulence is thoroughly mixing up the sewage is called grab sample.

The seeded water is the water with seedign of mixed bacterial culture.

$$\text{BOD}_5 = \frac{(D_1 - D_2) - (B_1 - B_2)(1 - P)}{P}$$

where, D_1 = DO of diluted sample immediately after dilution mg/l

D_2 = DO of diluted sample after 5 days (120 hours) g/l

B_1 = DO of seeded control sample before incubation, mg/l

B_2 = DO of seeded control sample after 5 days incubation, mg/l

P = Decimal volumetric fraction of sample used

= Volumme of undiluted sample/volume of diluted sample

Disposal of Sewage Waste

6

BIS Standard for Disposal of Sewage

Parameter	Domestic sewage If discharge into surface water source	Industrial sewage**	
		Surface water	Public sewer
BOD ₅	20 mg/l	30 mg/l	500 mg/l
pH	—	5.5–9.0	5.5–9.0
Suspended solids	30 mg/l	100 mg/l*	600 mg/ml
Phenolic compounds	—	1 mg/l	5 mg/l
Cyanides	—	0.2 mg/l	2 mg/l

* Suspended organic matter contains less carbon so relaxation in norms.

** Contains refractory organics, which retard backval process here industrial sewage not mixed directly into public sewer.

Mechanism of Self Purification

1. Dilution and Dispersion

It is not a self-purification method.

$$C_{mix} = \frac{C_s Q_s + C_R Q_R}{Q_s + Q_R}$$

C_s = Concentration of material of sewage

C_R = Concentration of same material in river

Q_s and Q_R are discharges of sewage and river.

2. Sedimentation

- The suspended solid in the sewage will settle down at the bottom of the river and in due course they will be stabilized.

3. Sunlight

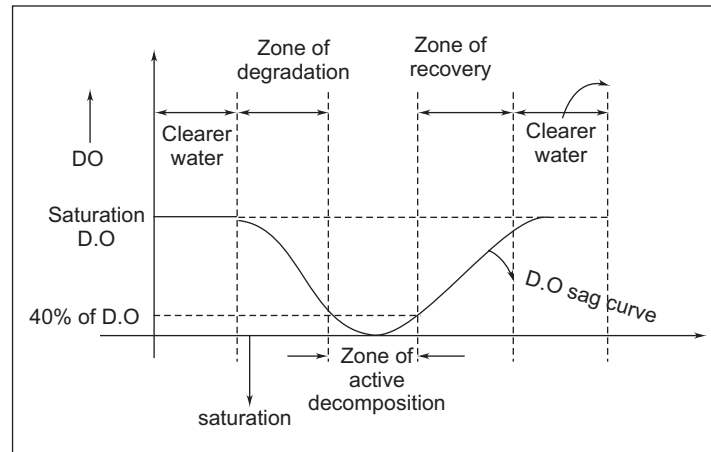
- Due to sunlight, in the process of photosynthesis, oxygen is released.

4. Oxidation

- Oxidation of organic matter occurs due to oxygen mixed in the river water.

5. Reduction

- By hydrolysis of organic matters settled at the bottom either chemically or biologically, the organic matters are stabilized.

Zone of Pollution in River Stream**Zone of Degradation**

- In this algae dies but the fish survives.
- DO reduces to upto 40% of saturation.

Zone of Active Decomposition

- It is a zone of heavy pollution.
- DO may even fall to zero.

Zone of Recovery

- D.O. content rises above 40% of the saturation value.

Zone of Clear Water

- DO will rise upto its saturation value, but the pathogens may remain.

Oxygen Deficit of a Polluted River Stream

Two mechanisms are known to contribute oxygen to surface water.

(a) Reaeration

(b) production of oxygen by algae photosynthesis

Reaeration

Oxygen deficit (D) = [Saturation D.O.] – [Actual D.O. (C)]

$$D = C_s - C$$

if r_g = rate of reaeration (Rate of oxygen addition)

$$r_g = K_r D \text{ (1st order reaction)}$$

K_r = Re oxygenation constant at base 'e'.

Rate of Oxygen Removal

y = BOD of stream added with sewage

$$\frac{dy}{dt} = \frac{dD}{dt}$$

$$y = L_0 - L_t$$

L_0 = ultimate BOD of mix.

L_t = oxygen equivalent of organic matter present at any time 't'

$$\frac{dy}{dt} = \frac{-dL_t}{dt}$$

K_d = Deoxygenation constant at base 'e'

r_d = Rate of de-oxygenation

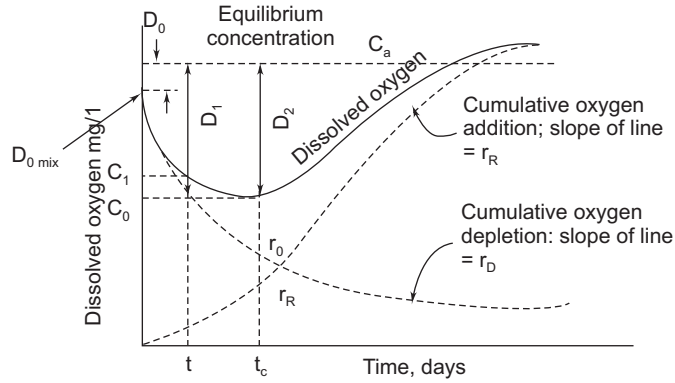
$$r_d = K_d \cdot L_t$$

The Oxygen Sag Curve

Net rate of oxygen deficit = Rate of re-aeration + Rate of de-oxygenation

$$\frac{dD}{dt} = K_d L_t - K_r D$$

Deficit is max. when rate of re-aeration = rate of de-oxygenation



Characteristics of the oxygen sag curve

D_0 = initial oxygen deficit

$$D = \frac{K_d L_0}{K_r - K_d} [e^{-K_d t} - e^{-K_r t}] + D_0 e^{-K_r t}$$

Streeter-Phelps equation.

$$D = \frac{K_D L_0}{K_R - K_D} [10^{-K_D t} - 10^{-K_R t}] + D_0 10^{-K_R t}$$

...(i)

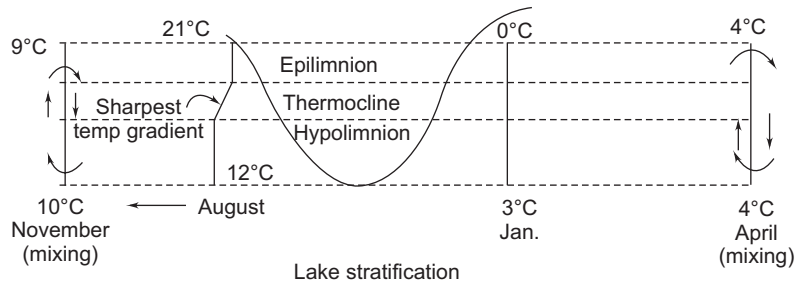
D_C = critical oxygen deficit

$$D_C = \frac{K_D L_0}{K_R} 10^{-K_D t_c}$$

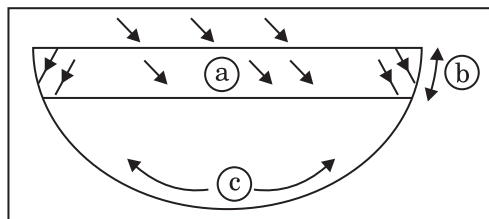
$$\left(\frac{L_0}{D_C f} \right)^{(f-1)} = f \left[1 - (f-1) \frac{D_0}{L_0} \right]$$

$f = \frac{K_R}{K_D}$ = self purification constant.

Lake Stratification



Biological Zones in Lakes



(a) Euphotic zone: Layer of lake through which sunlight can penetrate.

(b) Littoral zone: Near the shore where rooted plants grow is called littoral zone.

(c) Benthic zone: The bottom sediments in lake which contains bacteria.

Productivity of Lake

- Is a measure of algal growth.
- **Oligotrophic:** low level of productivity.
- **Mesotrophic:** Medium algal growth (medium productivity).
- **Eutrophic:** High productivity and algal growth.
- **Senescent:** Marshy, lake.

Eutrophication of Lake

- Natural process under which lakes get infested with algae and silt up gradually to become shallow and more productive.

Note: Once phosphorus is mixed in lake, only solution is to add lime to it and to dredge out the sediment at the bottom of lake.

BIS standard for waste water effluent to be discharged into sea.

BOD	–100 mg/l
COD	–250 mg/l
pH	–5.5 – 9.0 mg/l
SS	–100 mg/l
Fluoride	–15 mg/l
Oil and grease	–20 mg/l

Sewage farming

In case of sewage farming, stress is laid upon the use of sewage effluents for irrigating crops and increasing fertility of soil.

Design of Sewerage System and Sewer Appurtenances

7

Laying of Sewer

It consists of the following steps:

1. Marking of the Alignment
 - (a) By Reference Line
 - (b) By Sight Rail
2. Excavation of Trench
3. Bracing of the Trench
4. Dewatering of Trench
5. Laying and Joining of Pipes
6. Testing of Leakage
 - (a) By Water Test
 - (b) By Air Test

Assumptions in Sewer Design

- (i) The flow of waste water in sewer is steady and uniform.
- (ii) Design of sewer is based on peak flow discharge.
 - Manning's formula.

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

V = velocity of flow
R = hydraulic radius
S = slope of sewer

- Hazen William's formula.

$$V = 0.849 C R^{0.63} S^{0.54}$$

[C = Hazen-William coefficient]

Design Data

- Self cleansing velocity is the minimum velocity at which no solid gets deposited at the bottom of sewer.

Max hourly discharge = $3 \times$ average daily discharge

Max daily discharge = $2 \times$ average daily discharge

Min hourly discharge = $\frac{1}{3} \times$ average daily discharge

Min daily discharge = $\frac{2}{3} \times$ average daily discharge

Self cleansing velocity is given by **Shields formula**

$$V = \frac{1}{n} R^{1/6} [K_s (G_s - 1) d_p]^{1/2}$$

d_p = particle size

K_s = a dimension less constant

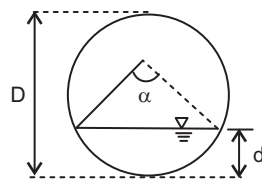
R = hydraulic radius of sewer

- Slope of sewer should be designed for min. permissible velocity at min flow.

Partial Flow Characteristics of Circular Sewer

Small letter : hydraulic elements under partial flow

Capital letter : the hydraulic elements under full flow condition then for a circular sewer.



$$\frac{d}{D} = \frac{1}{2} \left(1 - \cos \frac{\alpha}{2} \right)$$

$$\frac{a}{A} = \frac{\alpha}{360} - \frac{\sin \alpha}{2\pi}$$

$$\frac{p}{P} = \frac{\alpha}{360}$$

$$\frac{r}{R} = 1 - \frac{360 \sin \alpha}{2\pi\alpha} = \frac{a/A}{p/P}$$

$$\frac{v}{V} = \frac{N}{n} \left(\frac{r}{R} \right)^{2/3}$$

$$\frac{q}{Q} = \frac{v}{V} \times \frac{a}{A}$$

By knowing the conditions under which sewer runs full and by knowing two ratios of hydraulic elements, under partial and full flow conditions, third can be calculated analytically.

Note: 1. If Manning's coefficient 'n' is assumed constant with depth, then

$$\frac{v}{V} = \left(\frac{r}{R} \right)^{2/3}$$

2. For constant 'n', velocity of flow is max when $d/D = 0.81$.

3. For constant 'n', discharge is max when $\frac{d}{D} = 0.95$

4. $\frac{v}{V}$ decreases less sharply than $\frac{d}{D}$ below $\frac{1}{2}$ full depth (for constant n).

5. $\frac{q}{Q} = \frac{1}{2}$ at $\frac{1}{2}$ full flow

6. If $\frac{d}{D} \geq 0.5$, then $\frac{v}{V} \geq 1$

SEWER APPURTENANCES**(1) Manholes****Components of Manhole****(a) Access Shaft****(b) Working Chamber**

- Hight of chamber = 2m
- Size (rectangular) = 1.2×1.5 m
- If chamber is circular diameter = 1.2 m

Types of Manholes**(i) Straight – Trough Manholes**

Where there is a change in the size of sewer, the soffit or crown level of the two sewers should be the same.

(ii) Junction Manholes

Built at every junction of two or more seweres.

(iii) Drop Manholes: To connect the high level branch sewer to the low level main sewer by vertical dropping pipe.

(iv) Flushing Manhole: Where it is not possible to obtain self-cleansing velocities due to flatness of the gradient. It is essential that some form of flushing device be incorporated.

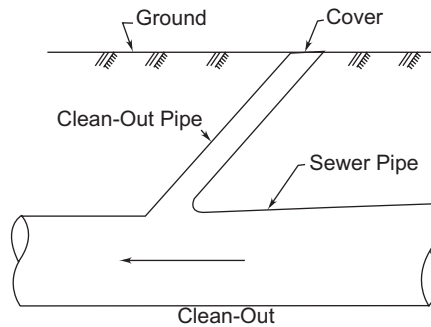
(2) Lamp Hole: Opening which is provided in a sewer line for lowering a lamp inside.

(3) Grease and Oil Trap**(4) Catch basin**

To allow the strom water to enter the sewer by eliminating the silt, grit, etc. at the bottom of the basin.

(5) Storm Water Inlet: Meant to admit the surface run off to the sewers storm water inlets having vertical openings.

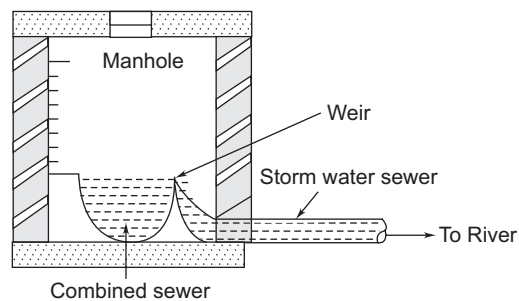
(6) Clean Outs:



(7) Inverted Siphons: Whenever a sewer pipe has to be dropped below the hydraulic gradient line for passing it beneath a valley. Any depression in the earth's surface or where it passes beneath some other obstructions in its path, it is known as an inverted siphon.

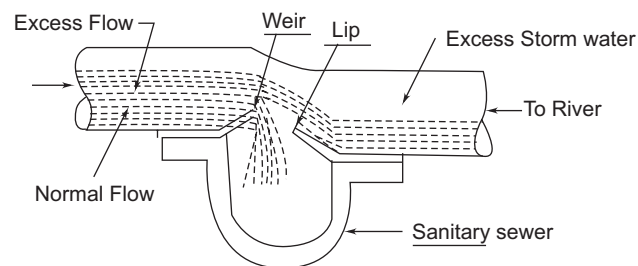
(8) Storm Water Regulator or Overflow Device

(i) Side Flow Weir

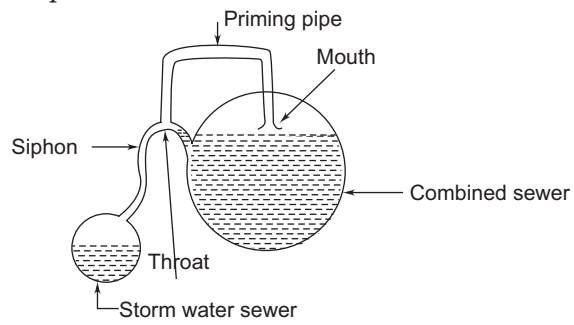


Overflow weir

(ii) Leaping Weir



(iii) **Siphon Spillway:** It is an automatic process, and works on the principle of siphonic action.



Siphon Spillway

Sewage Treatment

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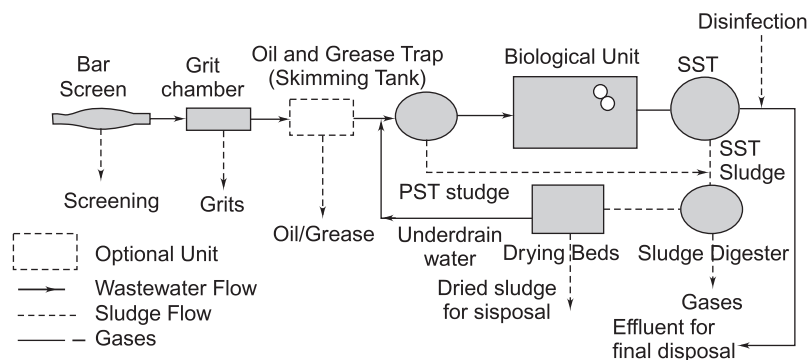
1. Treatment Methods

Unit Operations

The means of treatment in which the application of physical forces predominates are known as unit operations.

Unit process

The types of treatment in which the removal of contaminants is brought about by the addition of chemicals or the use of biological mass or microbial activities.



PST–Primary Settling Tank, SST–Secondary Settling Tank

Schematic flow diagram of a typical conventional treatment plant

Primary Treatments

1. Screening:

- Primary purpose of screen is to protect pumps and other mechanical equipments.
- Head loss through screen.

$$h_L \text{ (m)} = 0.0729 (V^2 - v^2)$$

V = velocity through opening of bar screen (in m/s)

v = approach velocity in upstream channel (in m/s)

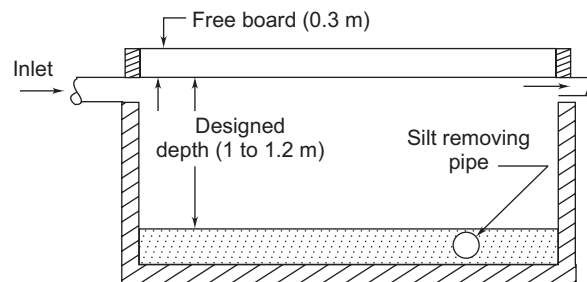
2. Comminution and Maceration

They are installed before fine screen.

3. Grit Chamber

Located either before or after sewage pumps. Separate non-decaying heavy inorganic material from sewage.

- Should not allow settlement of organic materials.
- Two channels are provided one for normal flow and other for peak flow.
- Only one section (channel) is required if velocity control devices has been used.



Design Criteria

- Removes particles of size ≥ 0.2 mm
- For 0.2 mm particle setting velocity = 0.025 m/s
- Surface over flow rate = $2160 \text{ m}^3/\text{m}^2/\text{day}$
- Horizontal critical flow velocity $V_C = K_C \sqrt{g(G_s - 1)d}$
 $K_C = 3 \text{ to } 4.5$
- Detention time is 40 – 60 sec.
- Depth is 1 – 1.5 m
- Free board is 0.3 m

4. Detritus Tank

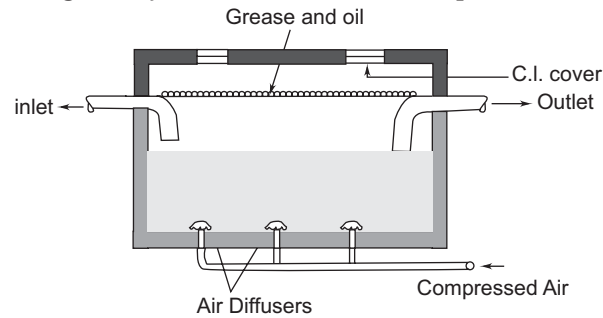
- To remove finer and inorganic particles.

5. Skimming Tank

- Removal of soaps, oil and greases.
- Provided before sedimentation tank.
- Compressed air is blown from below.

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- Chlorine gas may also be blown with compressed air.



Sedimentation

In primary sedimentation, organic suspended solids are settled.

Types of Settling

- Type 1. Discrete settling
- Type 2. Flocculent settling
- Type 3. Hindered or zone settling
- Type 4. Compression settling.

	PST	Over flowrate (m ³ /m ² /day)		Depth	Detention time
		Avt.	Peak		
1.	1°-settling only	25-30	50-60	2.5-3.5 m	2-2.5 hr
2.	1°-settling followed by secondary treatment	35-50	80-120	2.5-3.5 m	2-2.5 hr
3.	1°-settling with SAP	25-35	50-60	3.5-4.5	2-2.5 hr

$$\text{Surface area from average flow} = \frac{\text{Average discharge}}{\text{Average over flow rate}}$$

$$\text{Surface area from peak flow} = \frac{\text{Peak discharge}}{\text{peak overflow rate}}$$

Note: Sedimentation with Coagulation is not used generally in sewage treatment.

Secondary Treatment (Biological Treatment)

Secondary treatment is generally carried out aerobically.

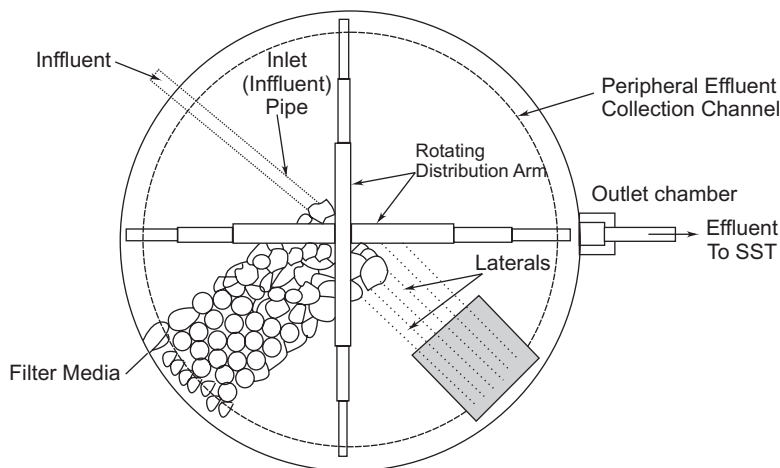
Units based on aerobic treatment are:

- Trickling filter
- Activated sludge process
- Oxidation pond etc.

Units based on annerobic treatments are:

- Septic tank
- Imhoff tank
- UASB Reactor (Upflow Anaerobic Sludge Blanket)

Standard Rate Trickling Filter



- 2 m deep circular bed, angular stone about 50 mm in size, supported on a tile floor
- Discharge under a head of 0.5-0.8 m

Operational Trouble in Standard Rate Filter

(a) Filter media ponding and clogging

(b) Excessive fly breeding

(c) Odour

	Standard	High rate	Super high rate
Hydraulic loading (in $\text{m}^3/\text{m}^2/\text{day}$)	1 – 4	10–40 (including recirculation)	40 – 200 (including recirculation)
Organic loading (in $\text{kg BOD}_5/\text{m}^3/\text{day}$)	– 0.08 – 0.32	0.32 – 1.0 (excluding recirculation)	06 – 0.8 (excluding recirculation)
Depth (m)	1.8 – 3.0	0.9 – 2.5 m	4.5 – 12 m
Recirculation ratio (Q_R/Q_0)	0	0.5 – 3.0	1 – 4

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Standard rate trickling filter, removes BOD upto 80-90%.

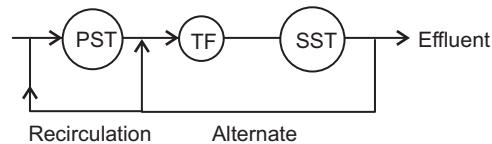
$$\eta = \frac{100}{1 + 0.0044\sqrt{u}}$$

u = organic loading in kg/hac-m/day

- Design of TF is done for average flow.

High Rate Trickling Filter

Recirculation is done by pumping the effluent of trickling filter to influent of trickling filter or to the primary sedimentation tank.



Design of High Rate TF

Surface area of trickling filter

$$= \frac{(Q_0 + Q_R)}{\text{Hydraulic loading (including recirculation)}}$$

$$\text{Volume of trickling filter} = \frac{Q_0 Y_1}{\text{Organic loading (excluding recirculation)}}$$

Q_0 = discharge from PST

Q_R = Recycled discharge

Y_0 = BOD of waste water

Y_1 = BOD of PST effluent

Y_2 = BOD of TF effluent

Efficiency of High Rate Trickling Filter

$$\eta_1 = \frac{100}{1 + 0.41 \sqrt{\frac{W_1}{V_1 F_1}}}$$

W_1 = BOD₅ applied to trickling filter in kg/day

$W_1 = Q_0 Y_1$ (excluding recirculation)

V_1 = Volume of trickling filter in m³

F_1 = Recirculation factor

$$F_1 = \frac{1 + R}{[1 + (1 - f)R]^2}, [R = \text{recirculation ratio} = \frac{Q_R}{Q_0}]$$

f = Treatability factor

$f = 0.9$ for sewage

$$F_1 = \frac{1 + R}{(1 + 0.1R)^2}$$

Efficiency of 2nd stage (η_2) is

$$\eta_2 = \frac{100}{1 + \frac{0.44}{1 - \eta_1} \sqrt{\frac{W_2}{V_2 F_2}}}$$

W_2 = BOD₅ applied on 2nd stage TF (excluding recirculation in kg/day)

V_2 = Volume of 2nd stage filter

F_2 = Recirculation factor for 2nd stage

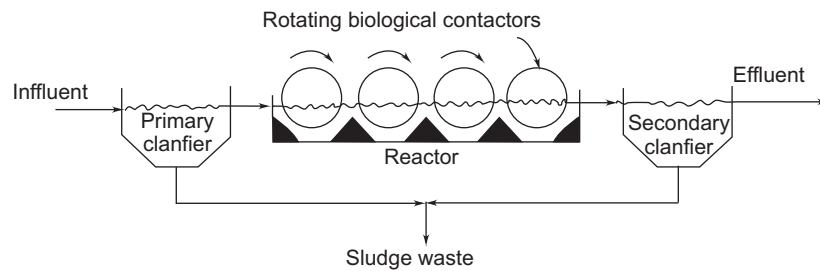
$$W_2 = \frac{W_1(100 - \eta_1)}{100}$$

Overall Efficiency in Terms of η_1 and η_2

$$\eta = \frac{W_1 - W_1 \left(1 - \frac{\eta_1}{100}\right) \left(1 - \frac{\eta_2}{100}\right)}{W_1} \times 100$$

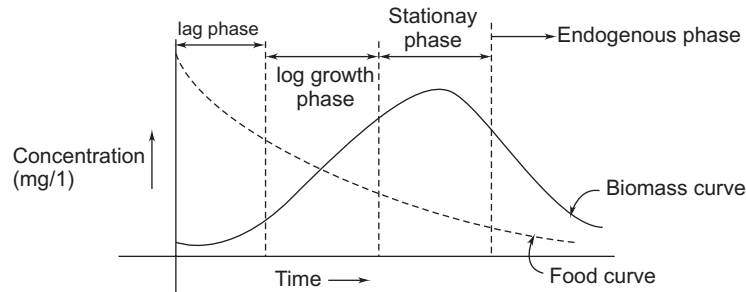
Rotating Biological Contractor (RBC)

- It is based on attached growth system.



- Film is moving (3-6 rpm).
- Sheared off mass is kept in suspension, by the turbulence created due to movement of discs.

Reaction Kinetics



- MLSS represents both living and dead bacteria.

$$\text{Rate of biomass growth} = \frac{dX}{dt}$$

$$\text{Rate of food consumption} = -\frac{dS}{dt} \quad \left(-\frac{dS}{dt} > \frac{dX}{dt} \right)$$

- During log-growth phase $\frac{dX}{dt} = kX$ [1st order equation]

$$k = \frac{k_0 S}{k_s + S} \quad (\text{Monod's equation})$$

k_0 = maximum growth rate constant

s = concentration of limiting food (in mg/l)

k_s = Half saturation constant

$$-y \frac{dS}{dt} = \frac{dX}{dt}$$

y is the fraction of food mass converted to biomass.

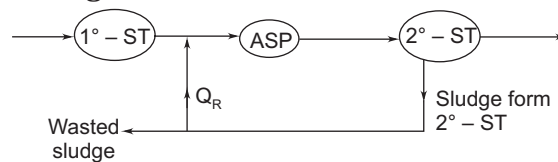
The equation for overall biological growth is

$$\frac{dX}{dt} = \left(\frac{k_0 S}{k_s + S} \right) X - k_d X$$

k_d = endogenous decay rate constant.

$$k_d = 0.06 \text{ per day}$$

Activated Sludge

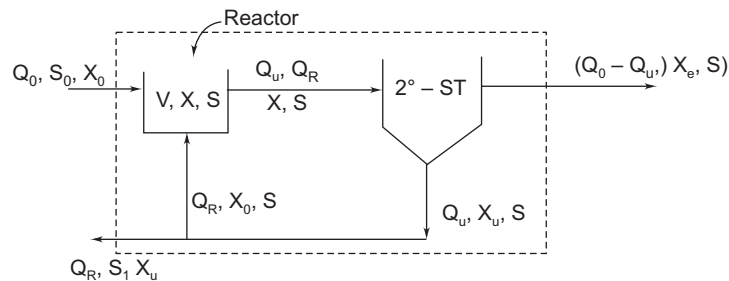


The settled sludge in secondary clarifier containing living micro-organisms is returned to the reactor to increase the available biomass and speed up the reaction.

The process can be:

- (i) completely mix process
- (ii) plug flow process

Completely Mix Process



Biomass in + Biomass growth = Biomass out

Food in – Food consumed = Food out

$$\frac{Q_u X_u}{VX} = \frac{Q_0 Y (S_0 - S)}{VX} - k_d$$

Design Parameters

1. Hydraulic retention time (θ) or Aeration time or detention time

$$\theta = \frac{\text{Volume of tank (v)}}{\text{Rate of flow in the tank excluding recirculation (Q}_0\text{)}}$$

2. Volumetric BOD loading

$$= \frac{\text{Mass of BOD applied}}{\text{Volume of aeration tank}}$$

3. Specific substrate utilization rate (U)

$$U = \frac{Q_0 (S_0 - S)}{VX}$$

$$4. \text{ F/M ratio} = \frac{Q_0 S_0}{VX} = \frac{\text{Food added}}{\text{Bacteria in system}}$$

5. Sludge age (θ_c)

$$\theta_c = \frac{\text{Mass of MLSS in the aeration tank}}{\text{Mass of MLSS leaving the system per day}}$$

$$\frac{1}{\theta_c} = UY - k_d$$

$$VX = \frac{YQ_0(S_0 - S)\theta_c}{1 + k_d\theta_c}$$

6. Sludge Volume Index (SVI)

- SVI represents the degree of concentration of sludge in the system
- It unit is **ml/gm**

$$\frac{Q_R}{Q_0} = \frac{X}{X_u - X} \quad \text{where } X \text{ is MLSS in tank (mg/l)}$$

$$X_u = \frac{10^6}{SVI}$$

$$\text{Recirculation Ratio} = \frac{Q_R}{Q_0} = \frac{X}{\frac{10^6}{SVI} - X}$$

$$SVI = \frac{V_{ob}(\text{ml/mg})}{X_{ob}(\text{l/mg})} = \frac{V_{ob}}{X_{ob}} \text{ ml/mg}$$

when 1 litre sample of mixed liquor is allowed to settle for 30 minutes and the settled sludge volume (V_{ob}) in ml. Concentration of suspended solids in the mixed liquor in mg/l be X_{ob} .

Design of Aeration Tank

Process type	Flow regime	MLLS mg/l	MLVSS MLSS	F/M	H.B.T. (hr.)	θ_c days	Q_r/Q	η	kg O ₂ required kg of BOD removed
Conventional	plug flow	1500 - 3000	0.8	0.3 - 0.4	4 - 6	5 - 8	0.25 - 0.5	85 - 92%	0.8 - 1.0
Completely mix	Completely mix	3000 - 4000	0.8	0.3 - 0.6	4 - 5	5 - 8	0.25 - 0.8	85 - 92%	0.8 - 1.0
Extended Aeration	Completely mix	3000 - 5000	0.8	0.1 - 0.18	12 - 24	10 - 15	0.5 - 1.0	95 - 98%	1.0 - 12

Excess Sludge Wasting

- Done to maintain steady level of MLSS in the tank and to maintain sludge age.

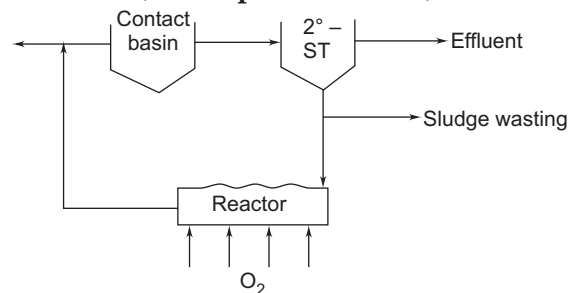
Nitrification

- This problem can be overcome by increasing the sludge wasting rate.
- Nitrification can lead to subsequent denitrification ($N_2 \uparrow$) in secondary tank causing sludge rising problem called blanket rising.

Sludge Bulking

- Sludge with poor settling characteristics

Contact Stabilization (Biosorption Process)



Completely Mix Process

- Sewage is maintained in log-growth phase by keeping high F/M ratio

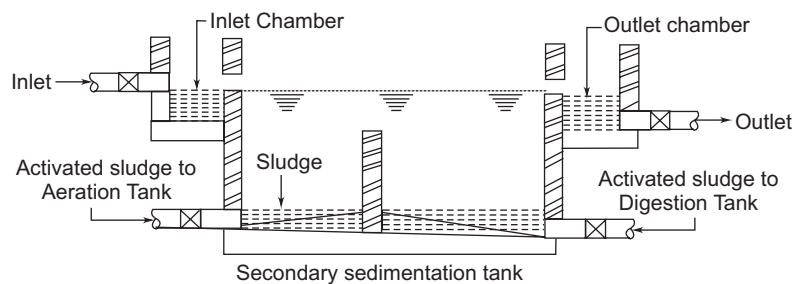
Extended Aeration

- F/M ratio is kept lower and detention time is kept more.

Oxidation Ditch

- An oxidation ditch is basically an extended aeration system of a modified activated sludge process.

Secondary Sedimentation Tank



- It is designed on the basis of:
 - (a) Solid loading rate
 - (b) Over flow rate

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	Overflow rate (m ³ /m ² /day)		Solid loading rate (kg/m ² /day)		Depth (m)	Detention time (hr)
	Avg.	Peak	Avg.	Peak		
2° ST for TF	15-25	40-50	70-120	190	2.5-3.5	1.5-2
2° ST for ASP excluding extended aeration	15-35	40-50	70-140	210	3.5-4.5	1.5-2
2° ST for extended aeration	8-15	25-35	25-120	170	3.5-4.5	1.5-2

$$\text{Area (Avg.)} = \frac{Q_0}{\text{Over flow rate}} \quad (\text{from overflow rate criteria})$$

$$\text{Area} = \frac{(Q_0 + Q_R)X}{\text{Solid loading rate}}, \quad X = \text{MLSS of aeration tank.}$$

Sludge Thickener

- To reduce the volume to be handled in sludge digester, sludge thickener is provided.
- Three types of thickening:
 1. Gravity thickening
 2. Air floating
 3. Centrifugation
- Designed for surface loading, rate of 20 – 25 m³/m²/day. The solid loading is 25 – 30 kg/m²/day.
- Depth is normally 3 to 4 m.
- Detention period is 4 hrs.

Sludge Digestion

- The sludge digestion serves both to reduce the volume of thickened sludge still further and to render the remaining solids inert and relatively pathogen free.

Aerobic Digestion

- Adopted only for biological sludge (2°-sludge) that does not contain primary sludge.
- An extension of extended aeration process.
- Process is energy consumption

Anaerobic Digestion

- The organisms are broadly classified as
- **Acid formers:** They consist of (facultative) and (anaerobic) bacteria and organisms which solublize the organic acids through hydrolysis.
- **Methane former:** They are strictly (anaerobic) and convert acids and alcohol along with hydrogen and carbon dioxide to methane.

Note: $V(100 - p) = V_1(100 - P_1)$

V = Volume of sludges at moisture content P

V_1 = Volume of sludge at moisture content P_1

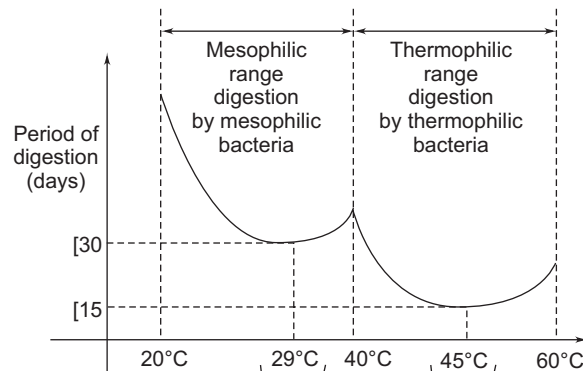
- When sludge is digested anaerobically the volume of original sludge is reduced to approx. $\frac{1}{3}$ of the original value.

Stages in Sludge Digestion Process

- Hydrolysis
- Fermentation (Acidogenesis)
- Methane formation (Methanogenesis)

Factors Affecting Sludge Digestion and their Control

(a) Temperature



(b) pH

- pH decreases due to overdosing of raw sewage, over withdrawal of digested sludge and sudden admission of industrial waste.

(c) Seeding with Digested Sludge

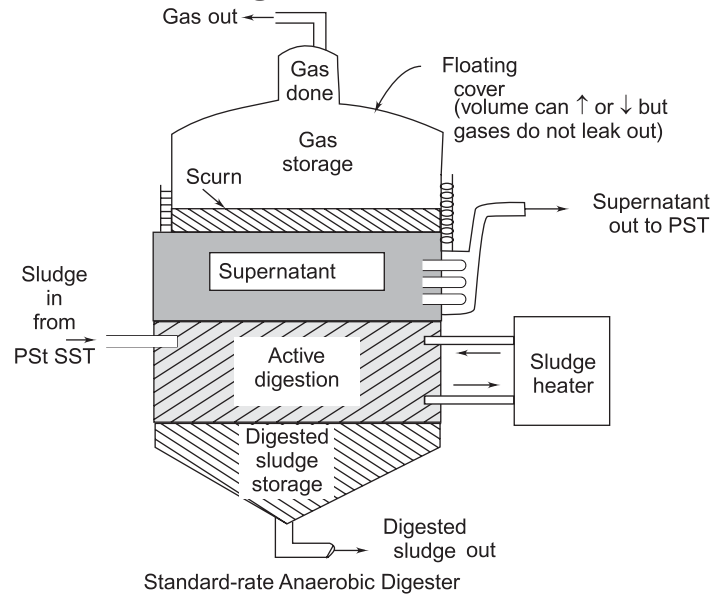
Helps in achieving quick balance condition.

(d) Mixing and Steering of Raw Sludge

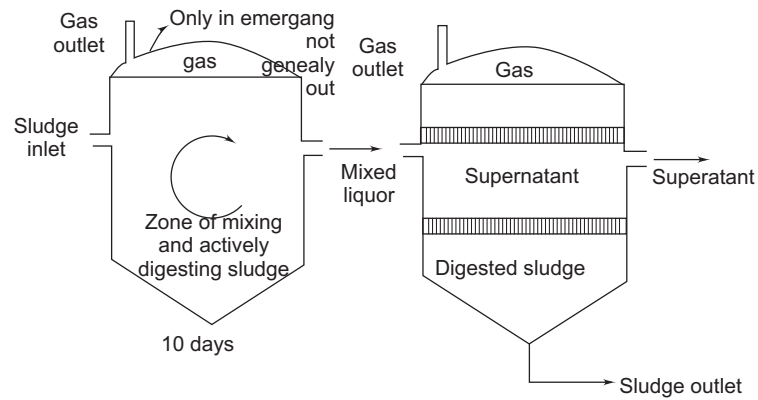
(e) Nuisance Organisms

Remedy is to add to precipitate iron sulphide.

Reactor for Anaerobic Digestion



High rate digestors



Design (Standard Rate Digester)

- Diameter 6 – 38 m
- Depth of tank 6 – 12m
- Lower slope 1 : 1 to 1 : 3
- Volume of digester

$$V = \left[V_1 - \frac{2}{3}(V_1 - V_2) \right] t$$

V = Volume of digester

V_1 = Volume of raw sludge

V_2 = Volume of digested sludge. It may be taken equal to volume of digested sludge.

t = digestion period

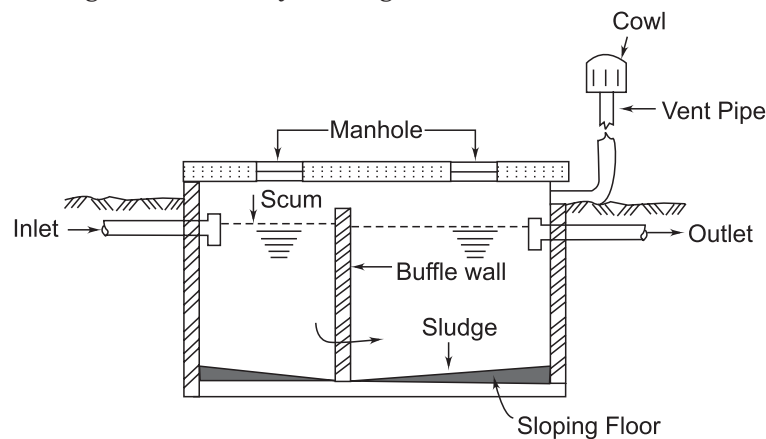
- Digestion period = 30 days

Design Parameters for Oxidation Pond

- Depth 1 – 1.5 m
- Detention time 15 – 30 days
- Pathogenic bacteria = 99.9%
- BOD removed = 80 – 90%
- Sludge accumulation is 2 – 5 cm/year
- Minimum depth of water 0.3 m

Septic Tank

- Designed as ordinary settling tank.



Septic Tank

Design Parameters of Septic Tank

- Flow of sewage 40 – 70 lpcd.
- Rate of accumulation = 30 lpc per year
- Detention time 12 – 36 hrs.
- Length/width ratio = 2 to 3

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- Depth 1.2 – 1.8 m

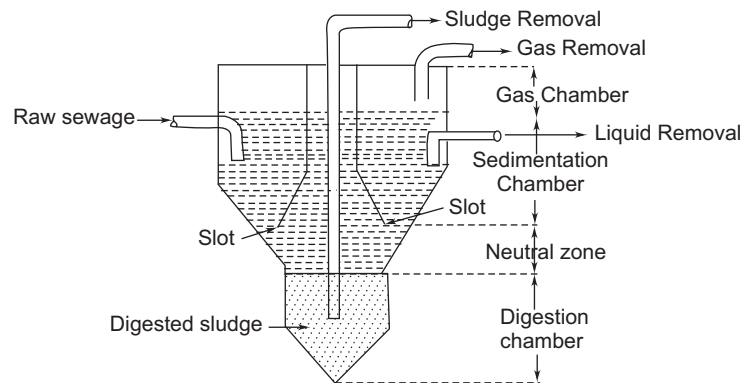
Cleaning period 6 month-1 year

- Volume of septic tank = (Sewage flow × Detention time) + (Sludge accumulation rate × cleaning period).
- Percolation rate is defined as the time in minute required for seepage of water through ground by 2.5 cm.

Dispersion Trench

$$\text{Absorption area required} = \frac{Q_0 \text{ (l/day)}}{\left(\frac{204}{\sqrt{t}}\right) \text{ l/m}^2 \text{ / day}}$$

Imhoff Tanks



High Rate Anaerobic Systems

- (a) Anaerobic Contact (AC) process
- (b) Anaerobic filters (AF)
- (c) Anaerobic fixed films (AFF) reactors
- (d) Upflow Anaerobic Sludge Blanket (UASB) reactor

Solid Waste Management

9

Type of solid waste

- (i) Municipal wastes
- (ii) Industrial wastes
- (iii) Hazardous wastes

Disposal of Solid Waste

Commonly employed methods for solid waste disposal, practised all over the world are:

1. Disposal of Refuse by Open Dumping

- Not an eco-friendly method
- Method is highly unacceptable

2. Disposal of Refuse by Sanitary Land Filling

Include: (1) site selection, (2) landfilling methods and operations, (3) occurrence of gases and leachate in landfills, and (4) movement and control of landfill gases and leachate.

3. Leachate in Landfills

- Leachate may be defined as liquid that has percolated through solid waste and has extracted dissolved or suspended materials from it.

4. Landfilling Method at Sanitary Landfill Sites

- Area method.
- Trench method.

5. Depression Method

6. Disposal of Refuse by Composting

- If the organic material excluding plastics, leather and rubber are separated from the solid wastes and are subjected to decomposition, either aerobically or anaerobically, the remaining end product is called compost and humus. The entire process involving both the separation and bacterial conversion of the organic solid wastes is known as *composting*.

Carbon-Nitrogen ratio

- Bacteria use nitrogen for building their cell structures (as proteins) and carbon for food (as energy).
- C/N is higher than the optimum, then nitrogen will be used up and carbon left over, thereby leaving the digestion of organic matter incomplete.
- C/N ratio is lower than the optimum, then the carbon will soon get exhausted and fermentation stop, leaving nitrogen in the digester which will combine with hydrogen to form ammonia (NH_3).
- Optimum C/N ratio is 30-50.

Indore Method

- Manual turning of piled up mass for its decomposition under **Aerobic** conditions.

Bangalore Method

- This method is widely adopted by municipal authorities throughout the country. Its an **Anaerobic** method.

Disposal of Refuse by Pulverization

- Refuse is pulverized in grinding machines, so as to reduce its volume.
- It is further disposed of by filling in trenches, or digested in open windrows or closed digestors.

Disposal of Refuse by Incineration and Thermal Pyrolysis**Incineration**

- Most effective methods of reducing the volume and weight.
- Estimation of oxygen requirement and heat balance are very vital.

Pyrolysis

- Combination of thermal cracking and condensation reactions in absence of oxygen.
- Also known as destructive distillation.

Note: Pyrolysis is an endothermic process – requiring continuous input of heat energy, while incineration is an exothermic process.

Air Pollution

10

Air pollutants can be classified as follows:

1. Natural contaminants.
2. Aerosols (particulates)
3. Gases and vapours

Natural Contaminants

From weeds, grasses and trees.

Aerosols

• Aerosols refer to the dispersion of solid or liquid particles of microscopic size in gaseous media, such as dust, smoke, or mist.

The following are the various aerosols.

- (i) **Dust**
- (ii) **Smoke**
- (iii) **Mists**
- (iv) **Fog**
- (v) **Fumes**

Gases

- (i) **Sulphur Dioxide**
- (ii) **Hydrogen Sulphide and Mercaptans**
- (iii) **Hydrogen Fluoride**
- (iv) **Oxides of Nitrogen**
- (v) **Carbon Monoxide**
- (vi) **Aldehydes**

Primary and Secondary Air Pollutants

Primary air pollutants are those emitted directly from identifiable sources.

Example :

1. Finer particles (less than 100 μ) in diameter).
2. Coarse particles (greater than 100 μ in diameter).
3. Sulphur compounds.
4. Oxides of nitrogen.
5. Carbon monoxide.
6. Halogen compounds.
7. Organic compounds.
8. Radioactive compounds.

Secondary air pollutants are produced by the interaction among two or more primary pollutants, or by reaction with normal atmospheric constituents, with or without photoactivation.

Examples

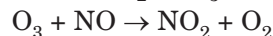
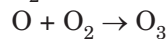
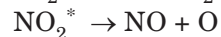
1. Ozone
2. Formaldehyde
3. PAN (peroxy acetyl nitrate)
4. Photochemical smog
5. Formation of acid mists (H_2SO_4)

$\text{Smog} = \text{Smoke} + \text{Fog}$

Note: Mercaptans are often added to natural or manufactured gas supplies so that leakage of gas will be noticed.

Photochemical Air Pollution

Nitrogen Dioxide Photochemical Reaction: Ultraviolet length energy is absorbed by NO_2 .



Sulphur Dioxide Photochemical Reaction: Ozone (O_3) may be formed in the atmosphere as a by product during photochemical oxidation of sulphur dioxide to sulphuric acid.

Factors Affecting Photochemical Reactions

- (i) Light intensity
- (ii) Hydrocarbon reactivity
- (iii) Ratio of hydrocarbons to nitric oxide
- (iv) Presence of light absorbers
- (v) Meteorological variables

Effects of Photochemical Smog

- 1. Eye irritation
- 2. Vegetation damage
- 3. Visibility reduction
- 4. Cracking of rubber
- 5. Fading of dyes

Global Implications of Air Pollution

- (a) Troposphere 00 to 11 km 15 to – 56°C
- (b) Stratosphere 11 to 50 km – 56 to – 02°C
- (c) Mesosphere 50 to 85 km – 02 to – 92°C
- (d) Thermosphere 85 to 500 km – 92 to 1200°C

- 1 percent argon (Ar), 0.03 percent carbon dioxide (CO₂)
- The layer of greatest interest in pollution control is the

troposphere**Acid Rain**

- Acid rain (or acid deposition) results when gaseous emissions of sulfur oxides (SO_x) and nitrogen oxides (NO_x) interact with water vapor and sunlight and are chemically converted to strong acidic compounds (H₂SO₄ and HNO₃).
- ‘clean’ rain is slightly acidic. If pH of rain is less than 5.6, it is called acid rain.

Global Warming

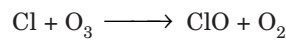
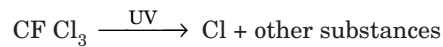
- The green house gases act like a thermal blanket surrounding the earth as due to the presence of green house gases the heat remains within the atmosphere and does not escape out of it.

The major green house gases are

- (i) Carbon dioxide (CO₂)
- (ii) Methane (CH₄)
- (iii) Nitrous oxide (N₂O)
- (iv) Chloroflouro carbon (CFC)

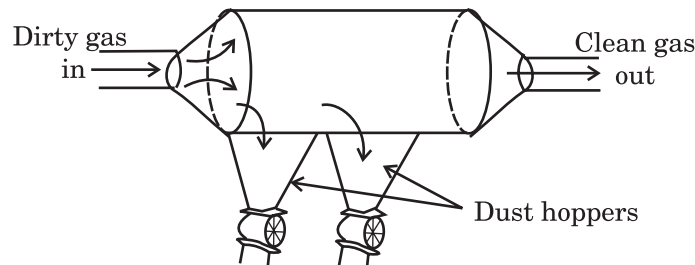
Ozone Layer Depletion

- Ozone layer is termed as *ozone umbrella*
- Primary reason for ozone layer depletion is CFC (Chlorofluoro carbon) or freons.
- Ozone is destroyed due to the photolytic reaction of CFC as shown below:



Control Devices for Particulates

(1) Gravitational Settling Chambers



Gravitational Settling Chambers

(2) Centrifugal Gas Collectors

Cyclone Collectors

- A cyclone collector consists of a cylindrical shell, conical base, dust hopper, and an inlet where the dust-laden gas enters tangentially.

Dynamic Precipitators

(3) Wet Scrubbers (or Collectors)

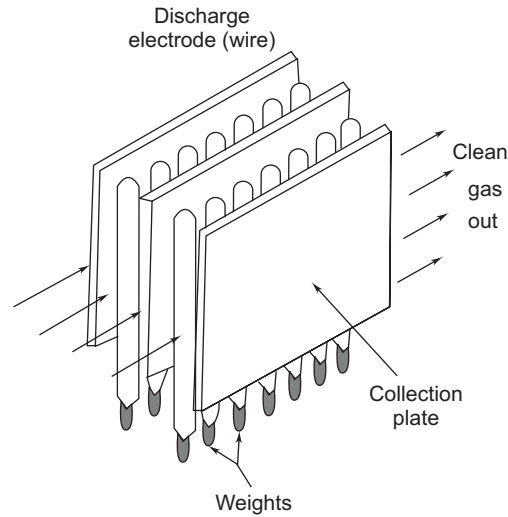
(a) Spray Towers

(b) Wet Cyclone Scrubbers

(c) Venturi Scrubbers

(4) Electrostatic Precipitators

- Extensively used in thermal power plants, pulp and paper industries, mining and metallurgical industries, iron and steel plants, chemical industries, etc.



High voltage electrostatic precipitators

(5) Fabric Filters**Control Devices for Gaseous Pollutants****1. Adsorption Unit**

- (i) Fixed bed Adsorbers
- (ii) Moving bed Adsorbers
- (iii) Fluidized bed Adsorbers

2. Absorption Units

- (i) Spray Towers
- (ii) Plate or Tray Towers
- (iii) Packed Towers

3. Condensation Units**4. Combustion (or Incineration) Units**

Note: Soot and carbon monoxide are by-products of combustion at low oxidation, while carbon dioxide is a by product of combustion in the presence of sufficient oxygen.

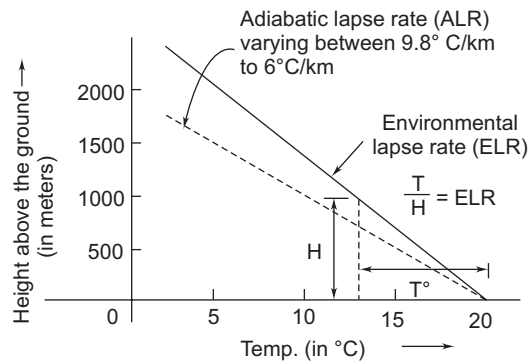
- (i) Direct-flame Combustion.
- (ii) Thermal Combustion:
- (iii) Catalytic Combustion:

Dispersion of Air Pollutants into the Atmosphere

Lapse Rate : In the troposphere, the temperature of the ambient (surrounding) air normally decreases with increase in the altitude (height). This rate of change of temperature is called lapse rate.

(i) Environmental Lapse Rate/Ambient Lapse Rate (ELR):

The lapse rate is known as the prevailing lapse rate, or the ambient lapse rate or the environmental lapse rate (ELR).

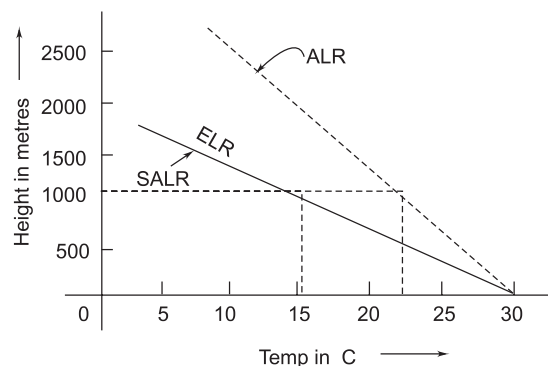


(ii) Adiabatic lapse Rate: Rate of decrease of temperature with height is called adiabatic lapse rate.

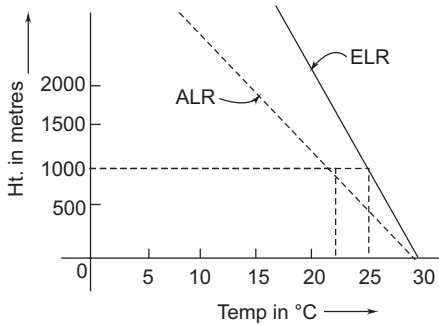
- Dry air expanding and cooling adiabatically cools at rate of 9.8°C per km and it is called dry adiabatic lapse rate.

- In saturated (wet) air, this rate is calculated to be 6°C per km, and is known as wet adiabatic lapse rate (due to release of latent heat or condensation of water vapour within saturated parcel of rising air).

(a) Super Adiabatic Lapse Rate

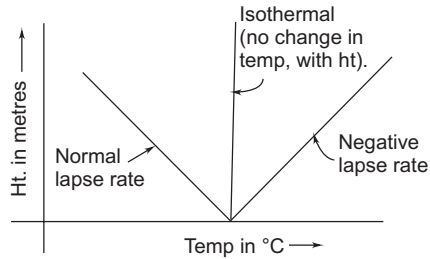


(b) Sub-adiabatic Lapse Rate



(c) **Neutral:** ELR and ALR are exactly equal.

Negative Lapse Rate and Inversion



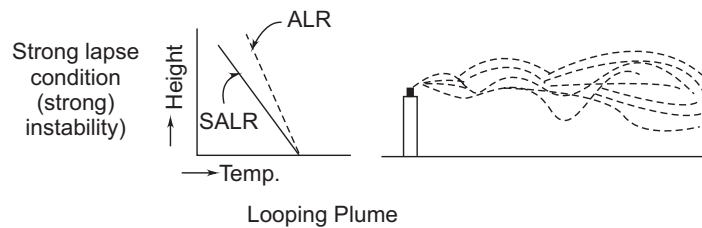
There are two types of inversion:

- (i) Radiation inversion
- (ii) Subsidence inversion

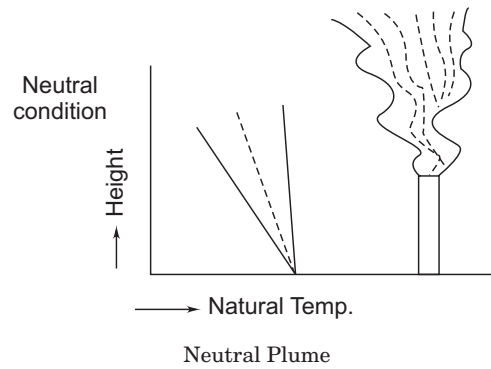
Plume Behaviour

Plume is defined by the path taken by continuous discharge of gaseous effluents emitted from a stack or chimney.

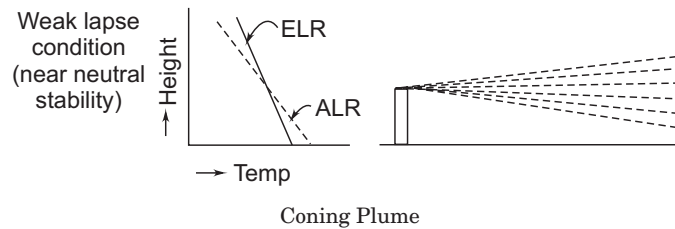
(i) Looping plume



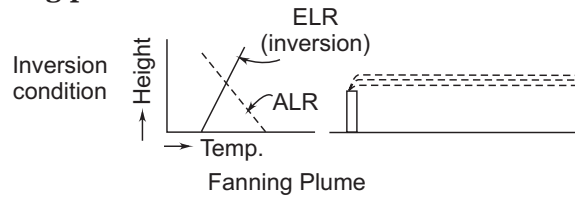
(ii) Neutral plume



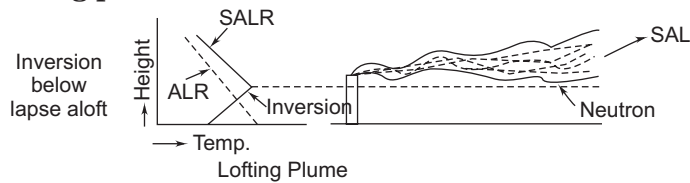
(iii) Coning plume



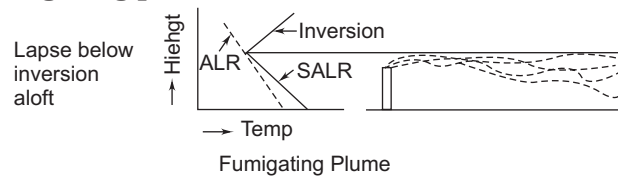
(iv) Fanning plume



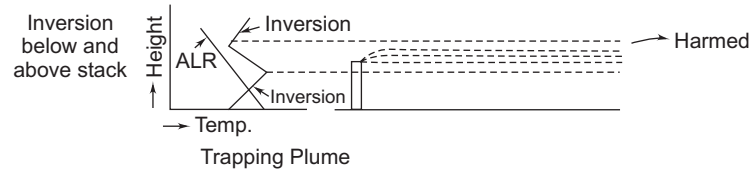
(v) Lofting plume



(vi) Fumigating plume



(vii) Trapping plume



Design of Stack Height

(i) For chimney emitting particulate matter:

$$h = 74 (Q_p)^{0.27} \text{ (height in metre)}$$

Q = particulate matter emission (tonnes/hour)

(ii) For chimney emitting SO_2

$$h = 14 (Q_s)^{1/3} \text{ (height in metre)}$$

Q_s = SO_2 emission (kg/hour)

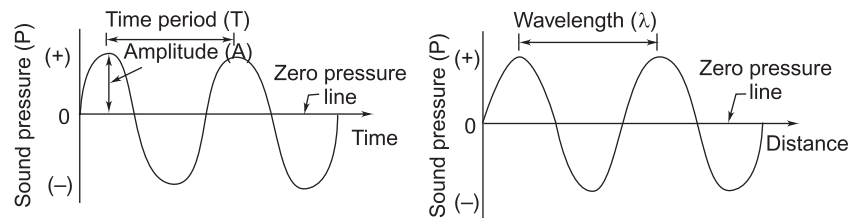
Unwanted sound pollutant which produces undesirable physiological and psychological effects in an individual.

Characteristics of Sound and Its Measurement

$$W = \frac{2\pi}{T}$$

$$T = \frac{1}{f}$$

$$\lambda = C \cdot \frac{1}{f}$$



Typical sinusoidal sound waves produced by alternating compression and rarefaction of air molecules

$$P_{\text{rms}} = \sqrt{p_{(t)}^2} = \sqrt{\frac{1}{T} \int_0^T p_{(t)}^2 dt}$$

$p(t)$ = Pressure at any time t

- Power of sound (W) is defined as the rate of doing work by a travelling sound wave in the direction of propagation of the wave.
- Sound intensity (I) is further defined as sound power averaged over the time, per unit area normal to the direction of propagation of the sound wave.

$$I = \frac{W}{a}$$

I = Intensity of sound wave in watt/m²

W = Power of sound wave in watts

a = A unit area $\perp$ to the direction of wave motion

$$I = \frac{P_{\text{rms}}^2}{\rho.C}$$

P_{rms} = r.m.s. sound pressure in pascals (Pa)

ρ = Density of air or medium in which sound wave is travelling in kg/m³

C = Velocity of sound wave in m/s.

Levels of Noise

$$L = \log_{10} \frac{Q}{Q_0} \text{ (bels)}$$

where, Q = Measured quantity of sound pressure, or sound power, or sound intensity

Q_0 = Reference standard quantity of sound pressure, or sound power, or sound intensity, as the case may be

L = Sound level in bels (B)

$$L = 10 - \log_{10} \frac{Q}{Q_0} \text{ (dB)}$$

Sound Pressure Level

$$\text{Sound pressure level} = L_p = 10 \times \log_{10} \left(\frac{p_{\text{rms}}}{20 \mu\text{Pa}} \right)^2$$

Sound Power Level

$$L_w = 10 \log_{10} \left(\frac{W}{10^{-12}} \right)$$

Sound Intensity Level

$$L_I = 10 \log_{10} \left(\frac{I}{10^{-12}} \right)$$

Note: If you consider 50 decibel noise and want to add another 50 decibel noise, it will make up only 53 decibel noise.

Averaging Sound Pressure Levels

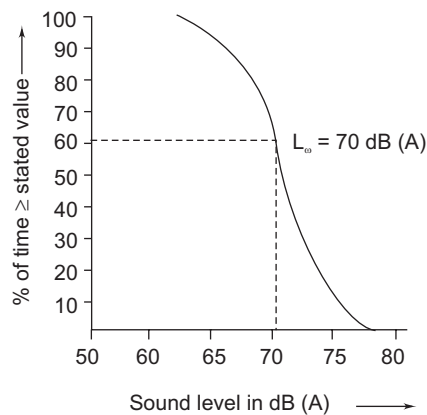
$$\bar{L}_p = 20 \log \frac{1}{N} \sum_{n=1}^{n=N} (10)^{L_n / 20}$$

where $\bar{L}_p$ = Average sound pressure level in dB re : 20 μ Pa
 N = Number of measurement readings
 L_n = nth sound pressure level in dB re. 20 μ Pa
 $n = 1, 2, 3, \dots, N$

Noise Rating Systems

The L_N Concept

L_N is a statistical measure indicating how frequently a particular sound level is exceeded.



Cumulative distribution curve

The L_{eq} Concept

The constant noise level, which, over a given time, expands the same amount of energy.

$$L_{eq} = 10 \log \sum_{i=1}^{i=n} (10)^{\frac{L_i}{10}} \times t_i$$

where n = Total number of sound samples
 L_i = The noise level of any i th sample
 t_i = Time duration of i th sample, expressed as fraction of total sample time.

S.No.	Acceptable outdoor noise level in residential areas	
	Location	Noise levels dB(A)
1.	Rural areas	25-35
2.	Suburban areas	30-40
3.	Urban residential areas	35-45
4.	Residential and business urban areas	40-50
5.	City areas	45-55
6.	Industrial areas	50-60

S.No.	Acceptable indoor noise level for various type of buildings	
	Location	Noise levels dB(A)
1.	Radio and TV studios	25-35
2.	Music rooms	30-35
3.	Hospitals, class rooms, auditoria	35-40
4.	Apartments, hotels, homes Conference rooms, small offices	35-40
5.	Court rooms, private offices, libraries	40-45
6.	Large public offices, banks, stores, etc.	45-50
7.	Restaurants	50-50

Civil Engineering

10

Hydrology

CONTENTS

1. General Aspects
2. Precipitation and Its Measurement
3. Abstractions from Precipitation
4. Runoff
5. Drought
6. Stream Flow Measurement
7. Hydrology
8. Floods
9. Flood Routing

General Aspects

1

Hydrology is the science of occurrence, circulation and distribution of water of the earth and its atmosphere.

Interception: Short term retention of rainfall by vegetation.

Infiltration: Movement of water into the soil of the earth's surface.

Percolation: Movement of water from one soil zone to lower soil zone.

Hydrological Budget Equation

Inflow – Outflow = storage

$$P - R - G - E - T = \Delta S$$

P = Precipitation

R = Runoff

G = Net Groundwater flow

E = Evaporation

T = Transpiration

ΔS = Net increase in storage

$$\text{Residence time} = \frac{\text{Volume of water in a phase}}{\text{Average slow rate in the phase}}$$

Instruments used in measurement	
1. Relative humidity	Psychrometer
2. Humidity	Hygrometer
3. Temp and Humidity	Thermohygrometer
4. Intensity of Radiation	Pyrheliometer
5. Wind speed	Anemometer
6. Rainfall depth	Ombrometer/Pluviometer
7. Transpiration	Phytometer

10.4 CIVIL ENGINEERING

8. Evapotranspiration	Lysimeter
9. Evaporation	Atmometer
10. Hydraulic conductivity	Permeameter
11. Infiltration capacity	Rainfall simulator

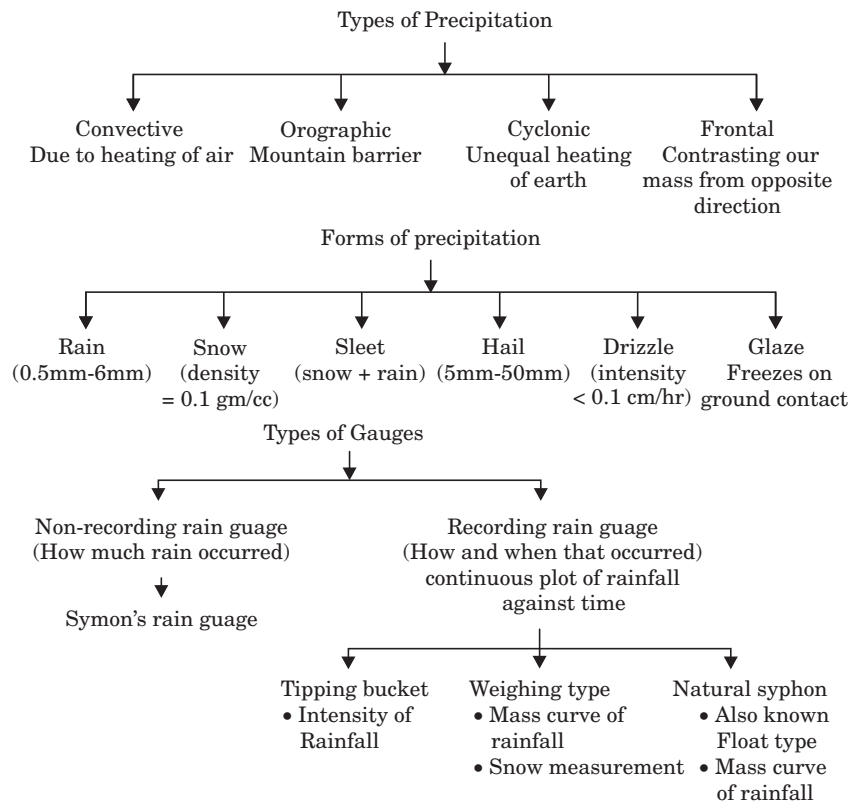
Isopleth: A line drawn on map along which the value of a particular phenomenon is uniform.

Name	Isopleth
1. Isobar	Pressure
2. Isobath	Depth in sea
3. Isobront	Thunderstorm
4. Isohaline	Salinity
5. Isohels	Sunshine
6. Isohyets	Rainfall
7. Isonif	Snowfall
8. Isoryme	Frost
9. Isotherm	Temperature
10. Isopleths	Evapotranspiration

Precipitation and Its Measurement

2

Fall of water in **various forms** on the earth from the clouds.



Rain gauge Network density = Area covered per gauge

As per I.S : 4987 – 1968	One station per
In Plains	520 km ²
Moderately Elevated Area	260 – 390 km ²
Hilly Area's	130 km ²

Note:

As per WMO, 10% of the rain guage stations should be self recording type.

Adequacy of Rain Guage Stations

$$1. \text{ Mean Rainfall } P_m = \frac{P_1 + P_2 + \dots + P_n}{n}$$

P_1, P_2 = rainfall recorded at stations 1 and 2 etc.

$$2. \text{ Standard deviation}$$

$$\sigma_{n-1} = \sqrt{\frac{(P_1 - P_m)^2 + (P_2 - P_m)^2 + \dots + (P_n - P_m)^2}{(n-1)}} \quad \begin{matrix} \text{if } n < 30 \text{ then } \sigma_{n-1} \\ \text{otherwise } \sigma_n \end{matrix}$$

$$3. \text{ Coefficient of variation } C_v = \frac{\sigma_{n-1}}{P_m}$$

$$4. \text{ Optimum number of stations } N = \left(\frac{C_u}{\varepsilon} \right)^2$$

ε = Allowable degree of error

Generally $\varepsilon < 0.1$

Normal Precipitation: Average Value of rainfall **30 yrs** data of particular date or month.

Average Annual Precipitation: Average Value of annual rainfall values for last **35 yrs**.

Estimation of Missing Data

1. Arithmetic mean method: If normal precipitation of the selected stations is **within** 10% of that of station with missing data

$$P_x = \frac{P_1 + P_2 + \dots + P_n}{n}$$

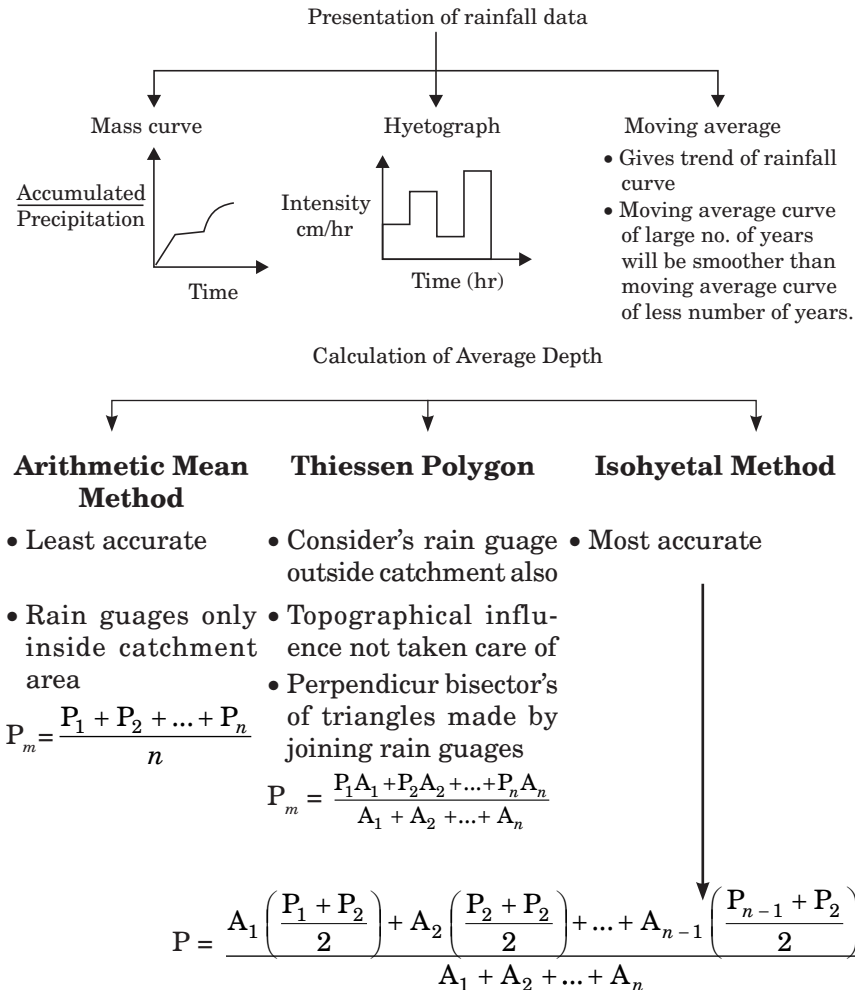
2. Normal ratio method: If normal precipitation of the selected station is **beyond** 10% of that of station with missing data.

$$\frac{P_x}{N_x} = \frac{1}{n} \left(\frac{P_1}{N_1} + \frac{P_2}{N_2} + \dots + \frac{P_n}{N_n} \right)$$

P = Precipitation

N = Normal precipitation

Inconsistency of Records: Corrected by double mass curve in which **previous records** are made consistent with present day environmental and land use conditions.



Depth Area Duration Curve

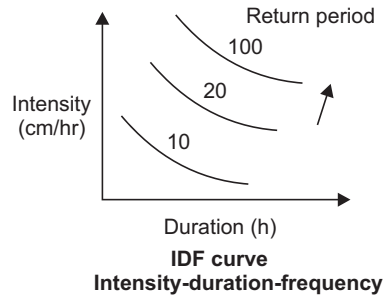
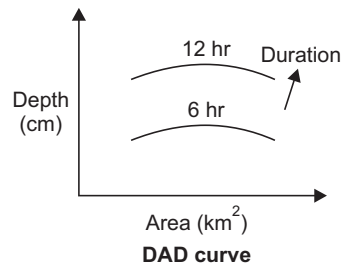
$$\bar{P} = P_o e^{-KA^n}$$

$\bar{P}$ = Average depth

P_o = Highest storm at storm centre

A = Area

Note :- Depth decrease with Increase in Area



Abstractions from Precipitation

3

Abstractions are Evaporation (E), Transpiration (T), Interception (I), Depression storage (DS) and Infiltration (IL).

Evaporation:

$$E_L = C (e_w - e_a) \text{ Dalton's law}$$

$$E_L = \text{Rate of Evaporation (mm/day)}$$

$$e_w = \text{Saturation vapour pressure}$$

$$e_a = \text{Actual vapour pressure}$$

Note:

1. Evaporation increases with decrease in atmospheric pressure.
2. Evaporation from sea water is less than fresh water due to salinity
3. Highest rate of evaporation from deep water bodies in winters

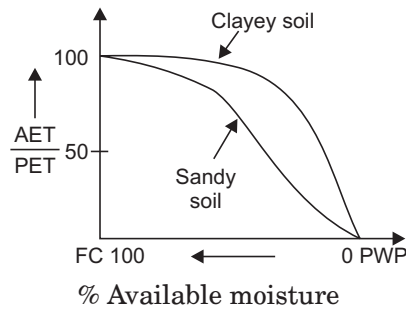
Measurement of Evaporation

Evaporimeter	Empirical Formulae	Analytical Methods
Concept of pan coefficient C_p	E_L	Water Budget
Lake evaporation	$= km (e_w - e_a) \left(\frac{1 + u_9}{16} \right)$	Equations (Hydrological continuity)
$= C_p \times \text{pan evaporation}$	$u_9 = \text{monthly mean wind velocity at 9m above the ground}$	Energy Balance (law of conservation of energy)
$C_p = 0.7$ class A Land pan	$km = 0.36$ Large deep water	Mass transfer (Theories of turbulent mass transfer)
$= 0.8$ ISI/US as floating pan	$km = 0.5$ Shallow, small water	
	$u_h = ch^{1/7}$	

Note:

Lake evaporation can be reduced by reducing the surface area, by providing mechanical cover or by chemical films of **cetyl Alcohol** (Hexa decanon) or **stearyl Alcohol** (octadecanol)

Evapotranspiration



AET → Actual Evapotranspiration

PET → Potential Evapotranspiration

PWP → Permanent wilting point

FC → Field capacity

- Estimation of **PET** is done by Penman's Equation and Blaney Criddle formula
- **Penman's Equation** is based on **energy balance** and **mass transfer**
- **Blaney-criddle formula**

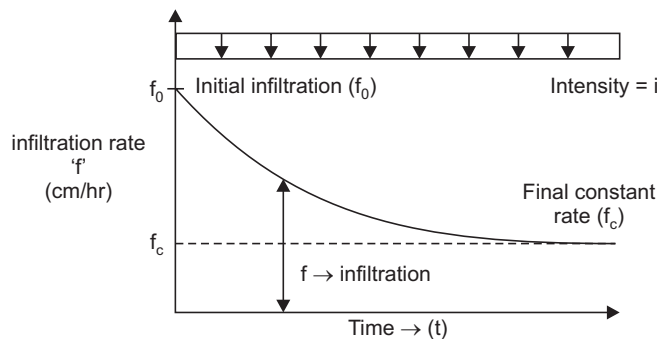
$$PET = 2.54 k \Sigma \frac{P_h T_f}{100}$$

K = Empirical coefficient

P_h = Monthly % of annual day time hrs

T_f = Mean monthly temp in °F

Horton's Infiltration Curve



$$f = f_c + (f_o - f_c)e^{-at}$$

$$f = f_c$$

$$f = i$$

$$i > f_c$$

$$i < f_c$$

Infiltration Indices

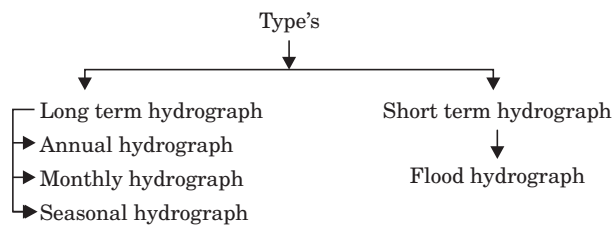
ϕ Index	W-Index
Average rainfall above which the rain fall volume is equal to run off volume	It excludes DS and Interception loss $W = \frac{IL}{t} = \frac{P - Q - S}{t}$

- (a) Rainfall Intensity – ϕ Index = Effective Rainfall Intensity
 (b) Rain fall Intensity – W-index = Effective Rain fall Intensity
 + Depression Storage + Interception

Runoff

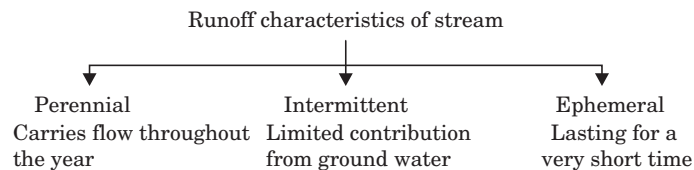
4

Hydrograph: Discharge in a stream plotted against time chronologically



Note:

Long term hydrograph's are used in calculating surface water potential of stream, Reservoir studies and drought studies.



Note:

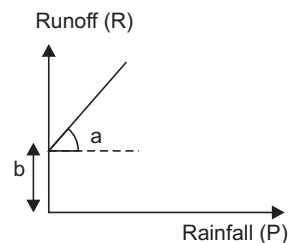
Influent Streams are those which raise the “Ground water table” and become ephemeral while **Effluent streams** are those in which water comes from nearby high “Ground water table”.

Rainfall Runoff Relationship

$$R = aP + b$$

$$a = \frac{N \sum PR - (\sum P)(\sum R)}{N \sum P^2 - (\sum P)^2}$$

$$b = \frac{\sum R - a \sum P}{N}$$



Coefficient of correlation

$$r = \frac{N \sum PR - (\sum P)(\sum R)}{((N \sum P^2 - (\sum P)^2)(N \sum R^2 - (\sum R)^2))}$$

Khosla's formula for **Monthly** Runoff:

Runoff = Rainfall – losses

Losses = 0.48 (mean monthly Temp (T_m))

It is indirectly based on **Water -budget equation**

Drought

5

$$\text{Index of wetness} = \frac{\text{Rainfall in a year}}{\text{Average Annual rainfall}} \times 100$$

$$\% \text{ Rain deficiency} = 100\% - \text{Index of wetness}$$

Droughts (Deficiency in precipitation)

Meteorological

As per IMD



Hydrological

Marked depletion of surface water and ground water

Agriculture

Extreme crop stress and wilting conditions

$$\text{Aridity Index} = \frac{\text{PET} - \text{AET}}{\text{PET}}$$

PET = Potential Evapotranspiration

AET = Actual Evapotranspiration

Decrease from normal precipitation	Classification	AI Anomaly	Classification
< 25%	No drought	0-25	Mild
26 - 50%	Moderate	26-50	Moderate
> 50%	Severe	>50	Severe

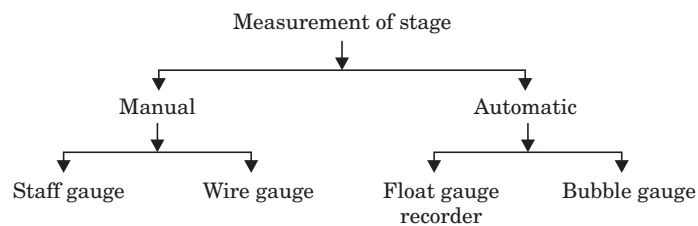
Note:

Drought prone area is an area in which probability of drought is between 0.2 to 0.4. If probability is greater than 0.4 then its chronically drought prone area.

Stream Flow Measurement

6

Stage : Height of the water surface in the river above some arbitrary datum



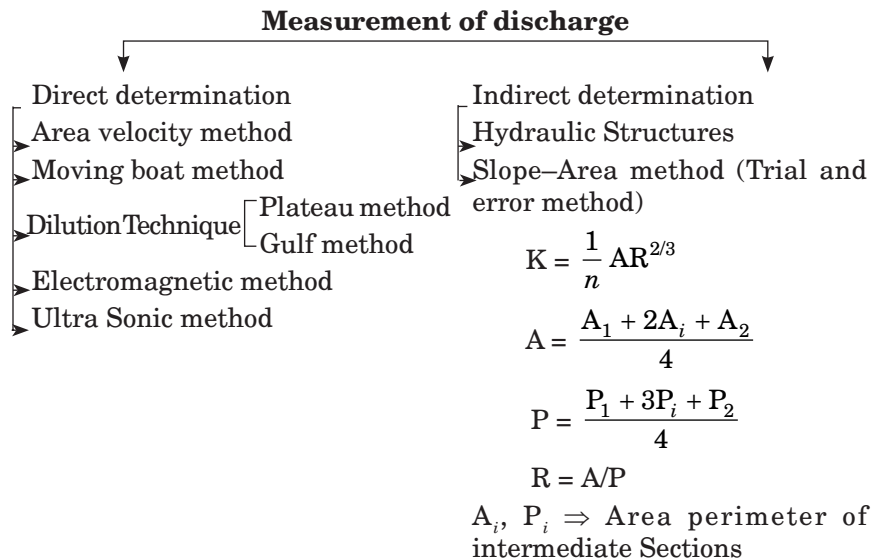
Current meter: Mechanical device to measure velocity of stream. It is calibrated in Towing Tank.

$$V = a N_s + b$$

N_s = Number of revolutions per sec

a, b = Current meter constants.

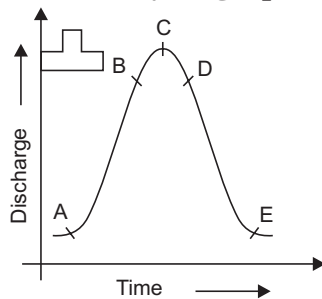
Velocity distribution in the vertical section across stream is **logarithmic**



Hydrology

7

Flood Hydrograph



AB → Rising limb → depends on duration, intensity and distribution of rainfall

BCD → Crest segment

DE → Recession Falling limb → depends on catchment characteristics

Shape of the flood hydrograph generally depends on (a) catchment shape (b) Size of catchment (c) Slope (d) drainage density (e) Rainfall intensity (f) Rainfall duration (g) Distribution of rainfall (h) Direction of storm movement

Unit Hydrograph: A hydrograph of **direct runoff** resulting from 1 cm of **effective rain fall** applied **uniformly** over a basin area at a **uniform rate** during a **specified period** of time (D-hr)

Assumptions of Unit hydrograph: Time Invariance and Linear Response

Limitations : Area between 2 km² to 5000 km², No large storage, precipitation in the form of rainfall only.

S-Curve hydrograph: Hydrograph of direct runoff resulting from a continuous effective rainfall of uniform intensity $\frac{1}{D}$ cm/hr. S-curve hydrograph attains an equilibrium discharge (Q_e) approximately at the end of base period of the unit hydrograph

$$Q_e = A \frac{1}{D} \text{ Km}^2 - \text{cm/hr} = 2.778 \frac{A}{D} \text{ m}^3/\text{sec}$$

A = Area (km²)

D = Duration (hr)

Synthetic unit hydrograph (SUH) : It is made for ungauged basin area's Basin parameters and unit hydrograph parameters of the ungauged, **hydrometeorologically homogenous** area are selected to basin parameter and unit hydrograph of gauged stations.

Three parameters for development of SUH;

(i) Basin time width T

(ii) Peak discharge Q_p

(iii) Lag time t_p

$$t_p = C_t (L \cdot L_{ca})^{0.3}$$

$$T = 72 + 3t_p$$

$$Q_p = \frac{2.78 C_p A}{t_p}$$

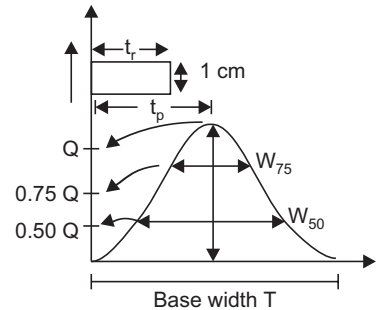
$$C_t = 1.35 \text{ to } 1.65$$

L = Basin length along the water course

L_{ca} = Centroid distance from gauging point

A = Area in Km^2

W_{75}, W_{50} = width of SUH at 75% and 50% of peak discharge



Instantaneous Unit hydrograph: Unit hydrograph of Infinitesimal duration

- Independent of rainfall characteristics and depends only on catchment characteristics

$$u(t) = \frac{1}{i} \frac{dS}{dt}$$

$u(t)$ = Ordinate of instantaneous hydrograph

i = Intensity of rainfall

S = Ordinate of S-curve

- Ordinate of IUH is the **slope of S-curve** of intensity 1cm/hr

Note:

Unit hydrograph → Sherman

Synthetic unit hydrograph → Snyder

Instantaneous unit hydrograph → Clark model based on Time Area Histogram

Floods

8

Standard Project flood (SPF): Flood from most severe combination of the meteorological and hydrological conditions excluding extremely rare combination.

Maximum Project flood (MPF): It includes extremely rare and catastrophic floods.

Probable maximum precipitation (PMP): Greatest or extreme rainfall of a given duration that is physically possible over a station.

Selecting Design Flood: Central Water Commission

Spillway for projects with Storage > 60 M m ³	PMP
Permanent barrage and minor dams with storage < 60 M m ³	SPF
Pick up wier's	Return period of 100 or 50 year's
Aqueducts → Waterways → Foundation free board	T = 50 years T = 100 years

Emperical Formula's for flood peak

(a) Dicken Formula used in Central and Northern parts of the country.

$$Q_P = C_D A^{3/4} \quad Q_P \rightarrow \text{m}^3/\text{sec} \quad C_D \rightarrow 6 \text{ to } 30 \quad A \rightarrow \text{km}^2$$

(b) Ryves Formula used in Tamil Nadu, Parts of Andhra Pradesh and Karnataka

$$Q_P = C_R A^{2/3} \quad C_R \rightarrow \text{Ryves coefficient}$$

(c) Inglis Formula used in western ghats of Maharashtra

$$Q_P = \frac{124 A}{\sqrt{A + 10.4}}$$

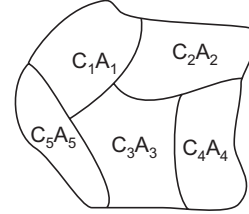
Rational Method

$$Q_p = C i A$$

$$C = \text{Runoff coefficient} = \frac{\text{Runoff}}{\text{Rainfall}}$$

$$C_{eq} = \frac{C_1 A_1 + C_2 A_2 + \dots + C_n A_n}{A_1 + A_2 + \dots + A_n}$$

i = Intensity of Rainfall, depends on time of concentration (t_c)
and Return period of rainfall $\left(T = \frac{1}{\text{Probability of exceedence}} \right)$

**Time of Concentration, by Kirpich Equation**

$$t_c = 0.01947 \left(\frac{L}{\sqrt{S}} \right)^{0.77}$$

L = Maximum length of travel by water

S = Slope of Catchment

Note:

Rational formula is found to be suitable for peak flow prediction in small catchment upto 50 km² in area.

Estimation of Design Flood for a particular return period: Gumble's method

- It is based on **extrapolation** for large return period

$$X_T = \bar{X} + K \sigma_{n-1}$$

X_T = Value of variate of return period T

$$\bar{X} = \frac{\sum x}{n}$$

n = Number of years of record

$$\sigma_{n-1} = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

K = Frequency factor

$$K = \frac{y_T - \bar{y}_n}{S_n}$$

$$y_T = \text{Reduced variate} \quad y_T = -\ln \ln \left(\frac{T}{T-1} \right)$$

$$\bar{y}_n = \text{Mean of reduced variate}$$

$$S_n = \text{Std. deviation of reduced variate}$$

$$\text{For } n > 50 \quad y_n = 0.577 \quad S_n = 1.2825$$

Confidence Limit

$$X_{1/2} = X_T + f(\alpha) S_e$$

$$f(\alpha) = \text{function of confidence probability '}\alpha\text{'}$$

α in percentage	50	68	80	90	95	99
$f(\alpha)$	0.674	1.0	1.282	1.645	1.96	2.58

$$S_e = \text{Probable error} = \frac{b \sigma_{n-1}}{\sqrt{n}}$$

where

$$b = \sqrt{1 + 1.3 k + 1.1 k^2}$$

$$k = \text{frequency factor}$$

Risk-Reliability

Probability of exceedence is p

$$\text{Return period } T = \frac{1}{p}$$

Probability of Non occurrence $q = 1 - p$

Risk $\rightarrow$ Probability of exceedence atleast once = $1 - q^n$.

Reliability $\rightarrow$ Probability of Non-occurrence = q^n .

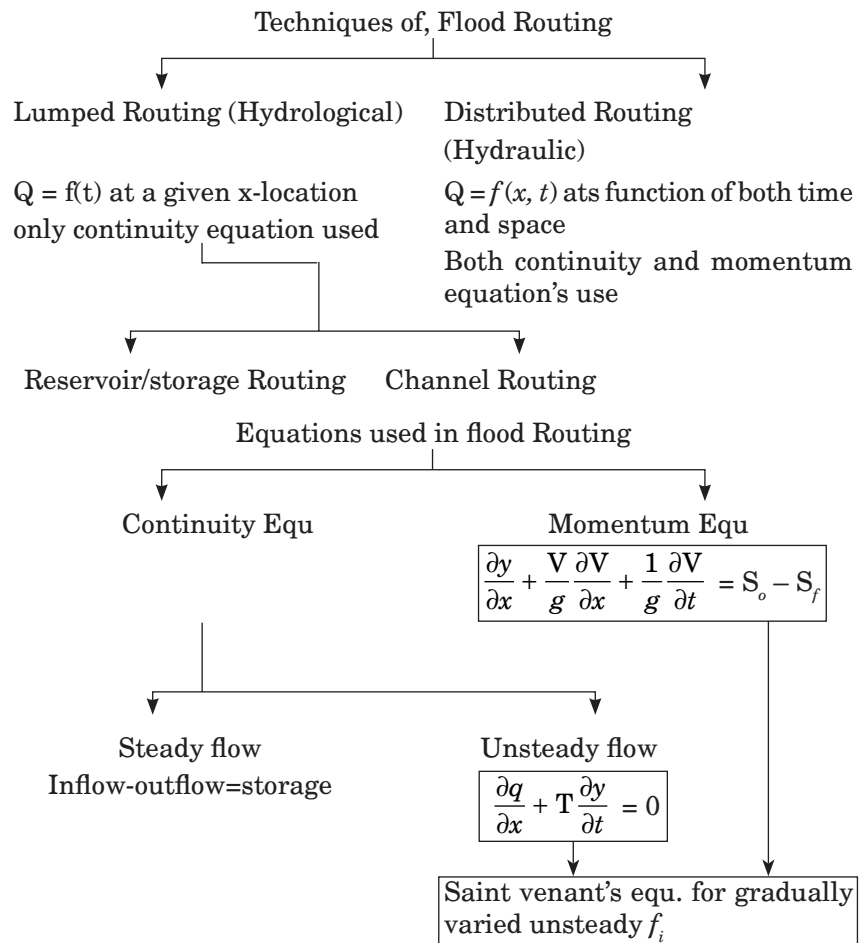
Probability of exceedence of m times in n years = ${}^n C_m p^m q^{n-m}$

Probability of exceedence of exactly 1 time in n years = ${}^n C_1 p q^{n-1}$

Flood Routing

9

It is the technique to determine the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections.



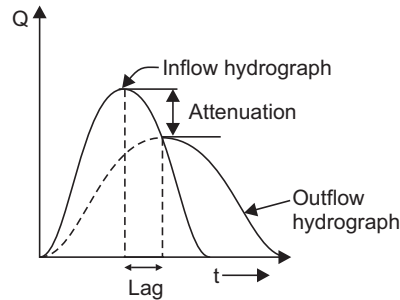
Reservoir Routing: It is done by

1. Standard 4th order Runge-kutta method (Numerical method)
2. Good rich method
3. Modified Pul's method

Graphical method

$$\left(\frac{I_1 + I_2}{2} \right) \Delta t - \left(\frac{Q_1 + Q_2}{2} \right) \Delta t = S_2 - S_1$$

In an uncontrolled outlet of a level pool routing following inflow and outflow hydrographs will be achieved



Attenuation: Reduction in peak

Lag: Time difference in two peaks

Note:

- The peak of outflow hydrographs will occur at the point of intersection of inflow and outflow hydrograph.
- Storage is max. at point of intersection.

Channel Routing: Storage is function of both inflow and outflow

$$S = K [x I^m + (1 - x)Q^m] \quad \text{Muskingum equation}$$

x = weighing factor, depends on shape of wedge

k = Storage time constant, has dimensions of time, and approximately equal to the time of travel of flood wave through channel reach.

I = Inflow Q = Outflow

$m = 0.6$, for artificial channel

$m = 1$ for natural channel.

$$S_2 - S_1 = k[x (I_2 - I_1) + (1 - x)] \quad \dots(1)$$

$$S_2 - S_1 = \left(\frac{I_2 + I_1}{2} \right) \Delta t - \left(\frac{Q_2 + Q_1}{2} \right) \Delta t \quad \dots(2)$$

From 1 and 2,

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

$$C_0 = \frac{-Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t} \quad C_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t} \quad C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_0 + C_1 + C_2 = 1 \quad 2Kx < \Delta t < Kx$$

Trap efficiency of Reservoir: Ability of reservoir to trap and retain incoming sediments.

$$\text{Trap efficiency} = f\left(\frac{\text{Capacity}}{\text{Inflow}}\right)$$

Civil Engineering

11

Soil Mechanics

CONTENTS

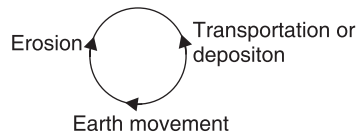
1. Origin of Soil
2. Properties of Soil (Soil-Water Relationship)
3. Index Properties of Soil
4. Classification of Soil
5. Clay Mineral and Soil-Structure
6. Compaction of Soil
7. Effective Stress
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9. Seepage Through Soil
10. Vertical Stress
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12. Shear Strength of Soil
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15. Cantilever Sheet Pile and Anchored Bulkhead
16. Shallow Foundation
17. Deep Foundation
18. Soil Exploration
19. Expansive Soil

Origin of Soil

1

Soil is an unconsolidated material, composed of soil particles, produced by disintegration of rocks.

Father of Soil Mechanics **Dr. Karl Terzaghi** coined the term “Soil Mechanics” in 1925.



The Geological Cycle

Erosion phase

Physical weathering

- Retains same composition of Parent rock
- Bulky particles
- Shape → Angular, Rounded, Flat and Elongated
- Single grain without bonding between them

Chemical weathering

- Change in mineral composition
- Plate like with high surface area
- Crystalline particle of colloidal size known as clay mineral

Transportation phase

Soil	Deposited by
Alluvial Soil	River
Marine Soil	Sea water
Locustrine Soil	Still water like lakes
Aeolian Soil	Wind
Glacial Soil	Ice

Note: Loess is an aelian soil.

Various types of Soil

Alluvial soil: Low density and liable to liquefaction in earthquake prone areas.

Black cotton soil: Residual deposits from basalt or trap rocks, contain clay mineral **Montmorillonite**.

Laterite soil: Iron oxide gives red or pink colour. Residual soil formed from basalt.

Desert soil: Uniform in gradation (eg. dune sand). It's Non-plastic and highly pervious.

Marine soil: Low shearing strength. Highly compressible, soft and Highly plastic.

Bentonite: Formed from volcanic ash, with high percentage of Montmorillonite.

Hard pans: Dense, well graded, cohesive aggregates of mineral particle. They do not disintegrate when submerged in water.

Loam: Mixture of sand, silt and clay.

Peat: Organic soil with fibrous aggregate's formed from vegetable matter in excess moisture (eg. in swamps), Highly compressible not suitable for foundation.

Indurated clay: Hardening of clay due to heat and pressure.

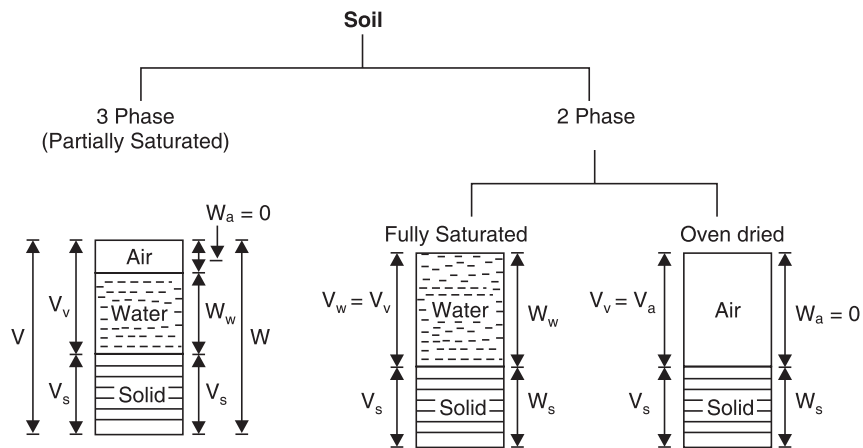
Note: Lithification is a process by which unconsolidated materials are converted into coherent solid rock as by compaction or cementation.

Properties of Soil (Soil-Water Relationship)

2

Phase diagram

Diagrammatic representation of the different phases in a soil mass.



Water Content

$$w = \frac{W_w}{W_s} \times 100$$

$$0 \leq w < \text{Infinity}$$

W_w = Gravity water + capillary water + Hygroscopic water

W_w can be removed by oven drying.

Void ratio

$$e = \frac{V_v}{V_s} \quad 0 < e < \text{Infinity}$$

Fine grained soil have higher Void ratio than coarse grained soil. While the size of void in coarse grained soil is greater than coarse grained soil.

Porosity

$$\eta = \frac{V_v}{V} \times 100 \quad 0 < \eta < 100$$

Void ratio serves as a useful parameter as compared to porosity void ratio is defined wrt volume of soil solid's which do not change even in compression.

Degree of Saturation

$$S = \frac{V_w}{V_v} \quad 0 \leq S \leq 100$$

for fully saturated soil $S = 100\%$

for fully dry soil $S = 0\%$

for partially saturated soil $0 < S < 100$

Air content

$$a_c = \frac{V_a}{V_v}$$

Percentage air Void

$$n_a = \frac{V_a}{V} \times 100$$

Percentage air void is always less than air content.

Bulk unit weight

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_a} \quad \text{units} \rightarrow \frac{\text{KN}}{\text{m}^3}, \frac{\text{N}}{\text{m}^3}, \frac{\text{kgf}}{\text{cm}^3}$$

Unit weight soil

$$\gamma_s = \frac{W_s}{V_s} \quad \gamma_s = G \gamma_w$$

where G = specific gravity of soil solid

Unit weight of water

$$\gamma_w = \frac{W_w}{V_w}$$

Value of γ_w changes with temperature but usually we take

$$\gamma_w = 9.81 \text{ KN/m}^3 \text{ at } 4^\circ\text{C}$$

Dry unit weight

$$\gamma_d = \frac{W_s}{V}$$

High value of γ_d indicates more compacted soil.

Saturated unit weight

$$\gamma_{\text{sat}} = \frac{\text{Wt. of Saturated Soil}}{\text{Volume of Soil}}$$

It's the unit weight of soil when all the air voids are filled with water

Submerged or Bouyant unit weight

$$\gamma_{\text{sub}} = \frac{(W_s)_{\text{sub}}}{V}$$

When the soil is below the water table then a Bouyant force acts on the soil solid and its saturated weight is decreased by unit weight of water.

$$\gamma_{\text{sub}} = \gamma_{\text{sat}} - \gamma_w$$

Note: Saturated soil may or may not be submerged but a submerged soil will always be saturated.

Specific gravity of solids

$$G = \frac{\gamma_s}{\gamma_w} \quad \text{No unit}$$

Also known as “Absolute specific gravity” or ‘grain specific gravity’.

Mass specific gravity of solid

$$G_m = \frac{\gamma_t}{\gamma_w} \quad \text{No unit} \quad G_m < G$$

Relative density

$$D_r = \frac{e_{\text{max}} - e_{\text{natural}}}{e_{\text{max}} - e_{\text{min}}}$$

$$= \frac{\gamma_{d \text{ max}}}{\gamma_{d \text{ natural}}} \left(\frac{\gamma_{d \text{ natural}} - \gamma_{d \text{ min}}}{\gamma_{d \text{ max}} - \gamma_{d \text{ min}}} \right)$$

This parameter (D_r) is generally used for sandy and gravelly soils

Relative density	Classification
0 – 15	Very loose
15 – 35	Loose
35 – 65	Medium dense
65 – 85	Dense
85 – 100	Very dense

Note: For uniformly graded coarse soil having perfectly spherical grain size when particles are arranged in

(a) Cubical array

$$e_{\max} = 91\%; \quad \eta_{\max} = 47.6\%$$

(b) Prismoidal array

$$e_{\min} = 35\%; \quad \eta_{\min} = 25.9\%$$

Some Important Relationships

$$1. W_s = \frac{W}{1 + w}$$

$$2. V_s = \frac{V}{1 + e}$$

$$3. \gamma_d = \frac{\gamma_t}{1 + w}$$

$$4. e = \frac{\eta}{1 - \eta}$$

$$5. \eta = \frac{e}{1 + e}$$

$$6. Se = wG$$

$$7. \gamma_t = \frac{G + Se}{1 + e}$$

$$8. \gamma_{\text{sat}} = \frac{G + e}{1 + e} \gamma_w$$

$$9. \gamma_d = \frac{G}{1 + e} \gamma_w$$

$$10. \gamma_{\text{submerged}} = \frac{G - 1}{1 + e} \gamma_w \quad 11. \gamma_d = \frac{(1 - \eta_a) G_s \gamma_w}{1 + w G_s}$$

$$12. S = \frac{w}{\frac{\gamma_w}{\gamma_t} (1 + w) - \frac{1}{G_s}}$$

Methods of determination of water content

- Over drying method:** Soil sample is derived in controlled temperature (105 – 110°C) for 24 hrs in laboratory. Above 110°C, **water of crystallisation** will be lost

$$w = \frac{w_2 - w_3}{w_3 - w_1}$$

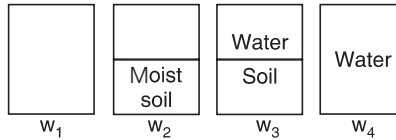
$$w_1 = \text{wt. of container}$$

$$w_2 = \text{wt. of container} + \text{wt. of moist soil}$$

$$w_3 = \text{wt. of container} + \text{of dry soil.}$$

- Torsion balance moisture meter method:** For rapid and accurate determination of water content. Since drying and weighing occur simultaneously, the method is useful for soils which quickly reabsorb moisture after drying.

3. Pycnometer method:



$$w = \left[\left(\frac{w_2 - w_1}{w_3 - w_4} \right) \left(\frac{G - 1}{G} \right) - 1 \right]$$

- This method is more suitable for cohesionless soil as the removal of entrapped air from cohesive soil is difficult. Hence w_3 cannot be measured accurately.

4. Sand bath method:

- Rapid field method, Hence, not accurate.
- Sand bath is a large, open vessel containing sand filled to a depth of 3 cm or more.
- Same formula of oven drying method.

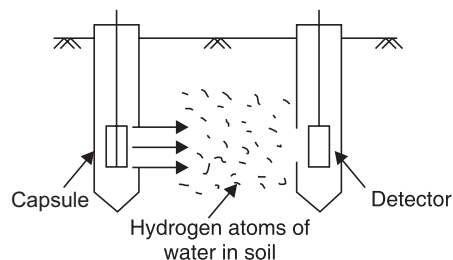
5. Calcium Carbide method

- Quick method but not so accurate.
- $\text{CaC}_2 + 2\text{H}_2\text{O} \rightarrow \text{C}_2\text{H}_2 \uparrow + \text{Ca(OH)}_2$
- Scale is calibrated to give the water content based on total weight (w) of the soil. So, actual water content (based on soil solid) is to be recalculated.

$$\text{Reading given} = w_t = \frac{w_w}{W} = \frac{w_w}{W_s + W_w}$$

$$\text{So } W_w = \frac{w_t}{1 - w_t}$$

6. Radiation method:

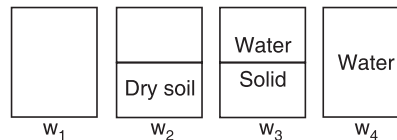


- Gives water content in in-situ condition
- Loss of energy by radio-active material (cobalt 60) due to scattering of neutrons by hydrogen atoms of soil while travelling from capsule to detector is proportional to water content.

7. Alcohol method:

- Quick method
- Not to be used with soils containing calcium compound or organic content.

Determination of specific gravity of soil solid (G)



- Pycnometer method
- SP. gravity values are reported at 27°C
- G can also be determined indirectly by using shrinkage limit.

$$G_T = \frac{W_2 - W_1}{(W_4 - W_1) - (W_3 - W_2)}$$

To remember, Rewrite as

$$G_T = \frac{W_2 - W_1}{(W_4 - W_3) - (W_1 - W_2)}$$

$$\Rightarrow G = \frac{(W_1 - W_2)}{(W_1 - W_2) - (W_4 - W_3)}$$

$$G_{27^\circ\text{C}} = \frac{G_T \times \text{unit wt. of water at } T^\circ\text{C}}{\text{unit wt. of water at } 27^\circ\text{C}}$$

Note: Pycnometer method is used for determination of water content as well as specific gravity. In water content determination, w_2 is the moist soil while in determination of specific gravity w_2 is dry soil.

Determination of Unit Weight

1. Core cutter method

- Field method suitable for, fine grained and clayey soil.
- Not suitable for stoney, gravelly soil and dry soil.

$$\gamma_t = \frac{W_2 - W_1}{V}$$

W_2 = wt. of core + soil

W_1 = wt. of core

V = volume of core

$$\gamma_d = \frac{\gamma_t}{1 + w}$$

w = water content

2. Water displacement method

- Suitable for cohesive soils only

$$V = V_w - \left(\frac{W_2 - W_1}{\gamma_p} \right)$$

W_1 = wt. of soil sample

W_2 = wt. of soil sample coated with parathin *wax*.

V_w = volume of water displaced by w_2

γ_p = unit wt. of paraffin *wax*

3. Sand replacement method

- field method
- used for gravelly, sandy and dry soil

4. Water ballon method

- volume of the pit is measured by covering the pit with plastic sheet and then filling it with water.
- wt. of water thus calculated is equal to volume of soil excavated.

5. Radiation method

- Bulk density of in situ soil.
- Quick and convenient.

Index Properties of Soil

3

Properties which help to access the **engineering behaviour** of soil and which assist in determining its **classification** accurately are termed as index property:



Grain Shape

- Sand and gravel have bulky grains of angular or rounded shape.
- Higher the angularity higher will be shearing strength.

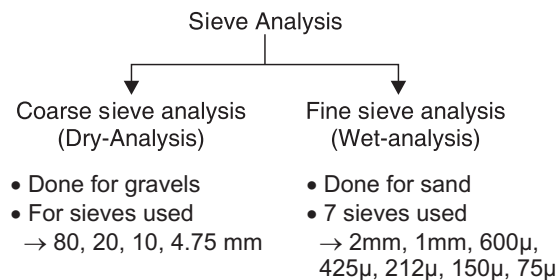
- Sphercity $S = \frac{D_e}{L}$

D_e = dia. of equivalent spherical particles

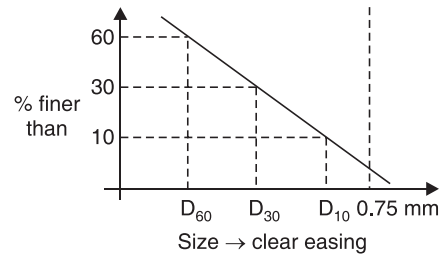
L = Length of particles

where $D_e = \left(\frac{6V}{\pi} \right)^{1/3}$

Grain Size



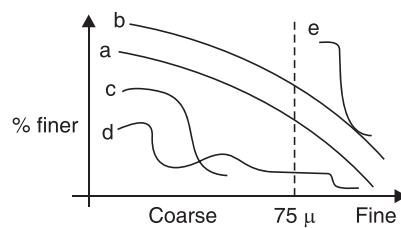
Grain Size Distribution Curves.



D_{10} = effective size of particles i.e. particles which if present alone will cause the same effect as caused by the soil.

D_{60} = Size of the particle such that 60% particle are finer than this size

$$D_{60} > D_{30} > D_{10}$$



$a \rightarrow$ well graded

a and $b \rightarrow$ similarly graded

$c \rightarrow$ poorly/uniformly graded coarse

$d \rightarrow$ Gap graded

$e \rightarrow$ Poorly graded fine soil.

Note: If there is a kink in the graph, then it shows the mixture of soil of two different geological formations.

Coefficient of uniformity $C_u = \frac{D_{60}}{D_{10}}$

Coefficient of curvature $C_c = \frac{D_{30}^2}{D_{60} \times D_{10}}$

Sedimentation Analysis,

Stoke's Law: $V_t = \frac{(\gamma_s - \gamma_l)d^2}{18\mu}$

V_t = terminal velocity

$d \rightarrow$ diameter of the grain

11.14 CIVIL ENGINEERING

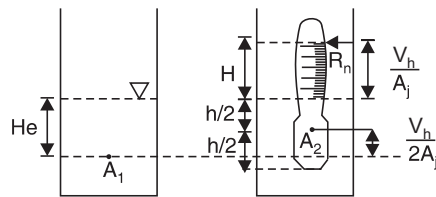
- Range of diameter of particle for stoke's law validity = **0.2 mm to 0.0002 mm**
- If particle size is greater than 0.2 mm, then **turbulent motion** will occur and if particle size is smaller than 0.0002 mm then **Brownian** motion will occur.

$$\frac{h}{t} \propto d^2 \Rightarrow \frac{d_1}{d_2} = \sqrt{\frac{h_1}{t_1} \times \frac{t_2}{h_2}}$$

h = height through which particle falls in t -secs.

Hydrometer Analysis

$$H_e = H + \frac{1}{2} \left(h - \frac{V_h}{A_j} \right)$$



V_h = Volume of hydrometer

A_j = Area of jar

h = length of bulb in hydrometer

Here, H corresponds to reading R_H

Specific gravity of soil suspension at depth H_e

$$G = 1 + \frac{R_h}{1000}$$

$$\frac{(\gamma_s - \gamma_w)d^2}{18\mu} = \frac{H_e}{t} = \frac{H + \frac{1}{2} \left(h - \frac{V_h}{A_j} \right)}{t}$$

Correction's in hydrometer Analysis

Meniscus correction → Always positive (C_m)

Deflocculating Agent correction → Always negative (C_d)

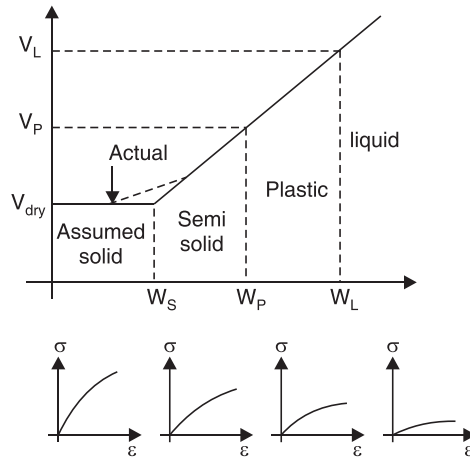
Temperature correction → If temp. is more than its positive otherwise negative (C_t)

Composite correction

$$C = C_m - C_d - C_t$$

Consistency limits

It's the relative ease with which a soil can be deformed. Atterberg classified the consistency in 4-stages → solid, semi-solid, plastic, liquid.



V_L = Volume at liquid limit

V_P = Volume at Plastic limit

V_{dry} = Volume at shrinkage limit

W_L = Water content at liquid limit

W_P = Water content at plastic limit

W_S = Water content at shrinkage limit.

$$\frac{V_L - V_P}{W_L - W_P} = \frac{V_P - V_{dry}}{W_P - W_S}$$

Note: Naturally occurring water content is generally between W_L and W_P

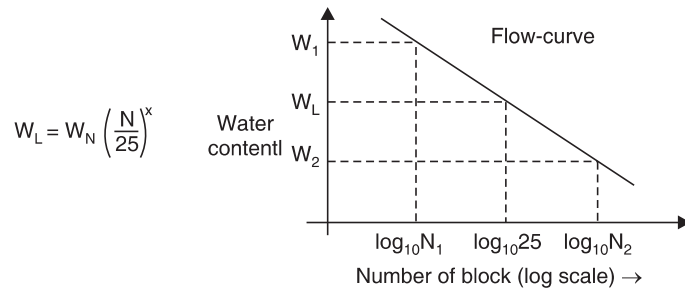
● Volume of soil does not decrease when water content is reduced beyond shrinkage limit.

Liquid limit

Minimum water content at which soil has tendency to flow.

All soils at liquid limit have similar shear strength.

Found out by two tools → (a) Casagrande's (b) Cone penetration.
Water content at which 25 blows close the 2mm groove cut in soil sample placed in Casagrande's bowl is called liquid limit

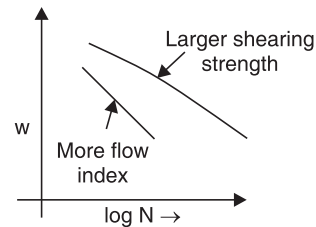


Flow Index (I_f): Slope of flow curve is called flow index

$$I_f = \frac{W_1 - W_2}{\log_{10} N_2 - \log_{10} N_1}$$

$$I_f = \frac{W_1 - W_2}{\log \frac{N_2}{N_1}}$$

$$\text{Flow Index} \propto \frac{1}{\text{Shear Strength}}$$



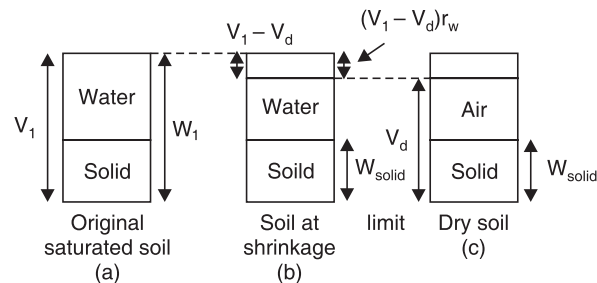
Plastic limit

Minimum water content at which soil is in plastic stage.

At plastic limit, a soil when rolled into a thread of 3 mm starts to crumble shear strength at the plastic limit is about 100 times at the liquid limit.

Shrinking limit

Minimum water content at which soil is completely saturated or the maximum water content at which further reduction in water content does not cause any reduction in the volume of soil sample.



$$\text{Water content at shrinkage limit} = \frac{(W_1 - W_{\text{solid}}) - (V_1 - V_d)\gamma_w}{W_{\text{solid}}}$$

(From figure (b))

$$G = \frac{W_{\text{solid}}}{V_1\gamma_w - (W_1 - W_{\text{solid}})}$$

Shrinkage ratio: Volume change in soil above shrinkage limit expressed as a percentage of dry soil per unit change in water content above shrinkage limit.

$$\text{S.R.} = \frac{\left(\frac{V_1 - V_2}{V_d}\right) \times 100}{W_1 - W_2} \quad \text{S.R.} = \frac{r_d}{r_w}$$

Note: Shrinkage ratio is the mass specific gravity of the soil in dry state.

$$\text{Volumetric Shrinkage} = \frac{V_1 - V_d}{V_d} \times 100$$

Plasticity Index:

$$I_p = W_L - W_P$$

I_p	Consistency
0	Non Plastic
< 7	Low Plastic
7–17	Medium Plastic
> 17	Highly Plastic

$$I_s = W_P - W_S$$

Consistency Index

$$I_c = \frac{W_L - W}{W_L - W_P}$$

Liquidity Index

$$I_L = \frac{W - W_P}{W_L - W_P} \quad I_c + I_L = 1$$

Shrinkage Index

Solid	Semi solid	Plastic	liquid
$I_L < 0$	$I_L < 0$	$0 < I_L \leq 1$	$I_L > 1$
$I_c > 1$	$I_c > 1$	$0 < I_c < 1$	$I_c < 0$

 W_S
 W_P
 W_L

Consistency	I_c	I_L
Very stiff	> 1	< 0
Stiff	1–0.75	0–0.25
Medium Stiff	0.75–0.5	0.25–0.5
Soft	0.5–0.25	0.5–0.75
Very Soft	0.25–0	0.75–1
liquid State	< 0	> 1

→ Brittle failure

→ Range of plastic failure

11.18 CIVIL ENGINEERING

Toughness Index

$$I_t = \frac{I_P}{I_f} \quad I_t = \frac{I_P}{I_f} = \log \frac{S_p}{S_f}$$

S_p = Shear strength at plastic limit

S_e = Shear Strength at liquid limit

Note: For $I_t < 1$, soil is friable i.e., get's crushed at plastic limit.

Thixotropy

It is the increase in strength of soil with **passage of time** due to tendency of clay soil to regain its **chemical equilibrium** with the **reorientation** of water molecules in the **adsorbed layer**.

During Pile driving in clayey soil, frictional resistance by soil increases due to property of thixotropy.

Sensitivity (S_t): Degree of disturbance due to remoulding

$$S_t = \frac{\text{Unconfined Compressive Strength due to undisturbed soil}}{\text{Confined compressive strength due to remoulded soil}}$$

$$S_t = \frac{q_u \text{ (undisturbed)}}{q_u \text{ (remoulded)}}$$

1	Insentive (General and Coarse sand)
2–4	Normal
4–8	Sensitive (Silt and Clay)
8–16	Extra
> 16	Quick

Note: Stiff clay having figures and crack have $S_t \leq 1$.

Activity Number (A_c):

$$A_c = \frac{I_P}{\% \text{ of clay size particles}}$$

i.e. size $< 2\phi$

< 0.75	Inactive
0.75 – 1.25	Normal
> 1.25	Active

Note : More activity means more change in volume.

Classification of Soil

4

Soil classification is done on the basis of index properties like grain size distribution and plasticity. Three important systems of classification are

1. **The Unified Soil Classification System (USCS):** Developed by casagrande for the use in airfield construction. Coarse grained soils were classified on the basis of grain size distribution while fine grained soils on the basis of plasticity characteristics.

2. **American Association of Stale Highway and Transportation Official (AASHTO):**

Soil are classified into 8 groups from A_1 to A_7 with A_8 for muck or peat

Group index: $GI = 0.2 a + 0.005 ac + 0.01 bd$

where $a = \% \text{ passing through } 75 \mu \text{ sieve} - 35$ $1 \leq a \leq 40$

$b = \% \text{ passing through } 75 \mu \text{ sieve} - 15$ $1 \leq b \leq 40$

$c = \text{liquid limit} - 40$ $1 \leq c \leq 20$

$d = \text{plasticity index} - 10$ $1 \leq d \leq 20$

Group index **0 indicates good subgrade material** while group index of **20 indicates very poor subgrade material**.

3. **Indian standard soil classification system:** It's given by A-casagrande

Coarse grained soil

Classified on the basis of

Particle size, fineness, C_u , C_u

Fine grained soil

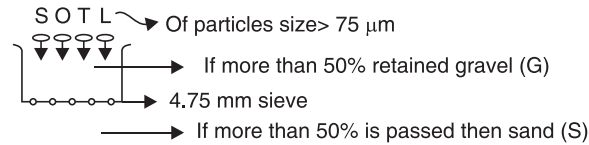
classified on the basis of

Compressibility, liquid limit, plasticity index.

Coarse grained soil classification based on grain size (mm)

Boulder	Cobble	Coarse grained soil					Fine Grained soil	
		Gravel		Sand				
		coarse	fine	coarse	medium	fine	silt	clay
>300	300-80	80-20	20-4.75	4.75-2.0	2-0.425	0.425-0.075	0.075-0.002	<0.002

Note: Fine sand is a coarse grained soil

Coarse grained soil classification**Case 1:** When fines i.e. particles less than $75 \mu\text{m}$ are less than 5%

Well graded gravel (GW)	Poorly graded gravel (GP)
$C_u > 4, 1 < C_c < 3$	Either or both of these are not satisfied
Well graded sand (SW)	Poorly graded sand (SP)
$C_u > 6, 1 < C_c < 3$	Either or both of these are not satisfied

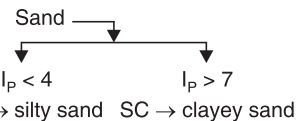
Case 2: When fines are between 5–12 %

- Dual symbol is used
- Divided into 8 parts based on gradation and fines
- For Eq:
 GW-GM → Well graded gravel with silt as fine
 SP-SC Poorly graded sand with clay as fines.

Gradation	Finer
GW	GC
GP	GM
SW	SC
SP	SM

Case 3: When fines are greater than 12%

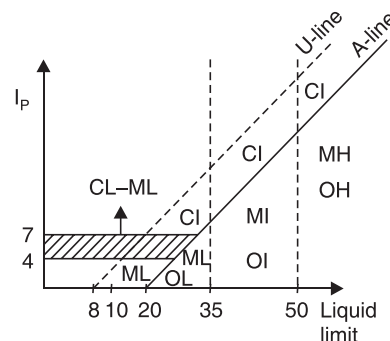
Gravels

**Note:** If I_p between 4 – 7 dual symbols will be used.**Fine grained soil classification**A-line: $I_p = 0.73 (W_L - 20)$ V-line: $I_p = 0.9 (W_L - 8)$ $W_L < 35$,CL → Low plastic **inorganic** clay

ML → Low plastic silt

OL → Low plastic **organic** clay $35 < W_L < 50$,

CI → Intermediate plastic inorganic clay



MI → Intermediate plastic silt

OI → Intermediate plastic organic clay

$W_L > 50$

CH → Highly plastic inorganic clay

MH → Highly plastic silt

OH → Highly plastic organic clay.

Note: Fine grained soil in indian standard soil classification are sub divided into low, medium, high on the basis of compressibility while in unified soil classification system it has only two categories of low and high compressibility.

Clay Mineral and Soil-Structure

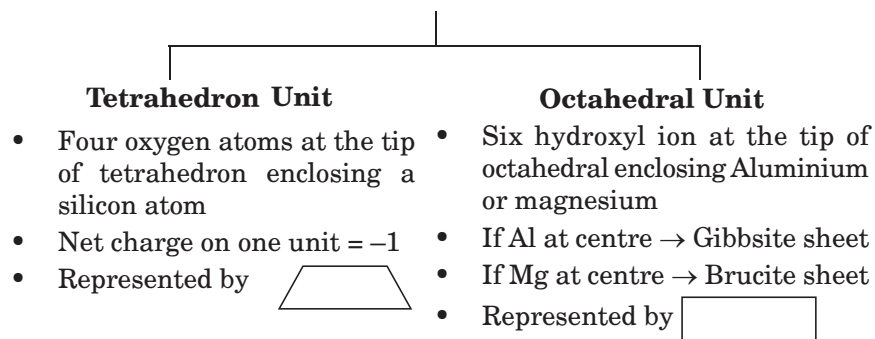
5

Properties of coarse grained soil are hardly effected by minerological composition because they are formed due to physical forces and retain the minerological composition of parent rock while properties of fine grain soils like clay depend to a large extent on the nature and characteristics of minerals present.

$$\text{Specific Surface} = \frac{\text{Surface Area}}{\text{Mass or Volume}}$$

Soil Structure

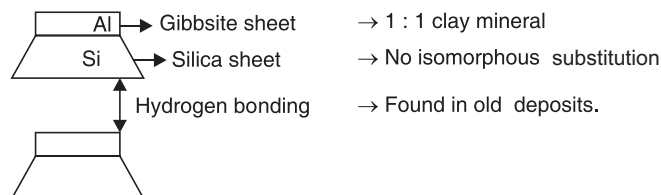
Atomic sturchure of clay mineral



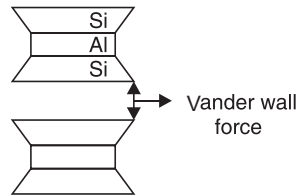
Isomorphic Substitution: Substitution of metallic Ion with another metallic Ion of lower valency but same physical size. for eg Si^{+4} replaced by Al^{+3} in a tetrahedral unit.

Various Clay Minerals

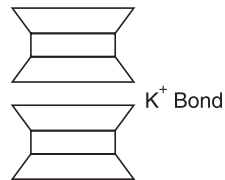
1. Kaolinite:



Note: Halloysite when air dried gets converted to Kaolinite and both Halloysite and Kaolinite are used for making chinaware.

2. Montmorillonite:

- 2 : 1 clay mineral
- Highly plastic with little internal friction.
- Common in residual soil derived from volcanic ash.
- Bentonite is a montmorillonite clay.
- Found in Black cotton soil.

3. Illite:

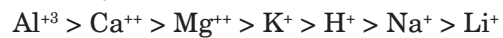
- 2 : 1 clay mineral
- Substantial Isomorphous substitution
- Ionic bonding

Properties of clay minerals

	Grain size	Base exchange capacity	I _p	Dry strength	Activity
Kaolinite	Maximum	Min	Min	Min	Min
Illite	↑	↓	↓	↓	↓
Montmorillonite	Minimum	Max	Max	Max	Max

Cation Exchange capacity: Ability at clay particle to absorb ions on its surface or edge is called base or carbon exchange capacity. It is based on size of particles and mineral structure.

Replacement ability of various cations



Note: Sodium clay is stabilised by using lime by using cation exchange property only.

Compaction of Soil

6

Compaction	Consolidation
Reduction in volume of air voids at a given water content	Volume reduction due to expulsion of pore water from voids
Partially saturated soil	Completely saturated soil
Instantaneous phenomenon	Time dependent phenomenon
Specific compaction Technique	By static load placed on soil

Note: Cohesionless soils are compacted by Vibrations while cohesive soils are compared are compacted by application of static pressure.

Proctor Test

A definite relationship is established between the degree of dry density and soil moisture content. Optimum moisture content (OMC) is the water content at which a particular soil attains maximum dry density (MDD).

- Compactive effort (energy per unit volume) for curve 2 is more than curve 1.

Note: On increasing the compactive effort curve shifts backwards and upwards i.e. OMC decreases and MDD increases.

$$\text{Compactive effort } E = \frac{NnWh}{V}$$

$N \rightarrow$ No. of blows per layer

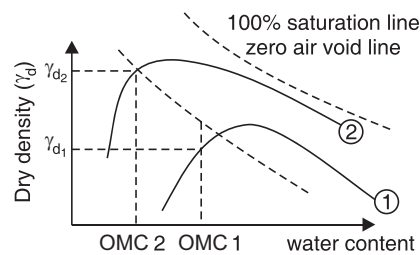
$n \rightarrow$ no. of layers

$W \rightarrow$ weight of hammer

$h \rightarrow$ height of fall

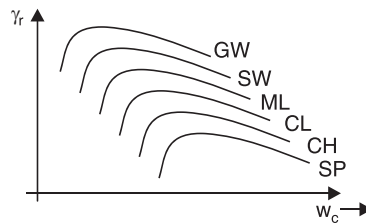
$V \rightarrow$ volume of mould

Note: Zero air void line cannot be practically achieved, as all air voids cannot be ever removed.

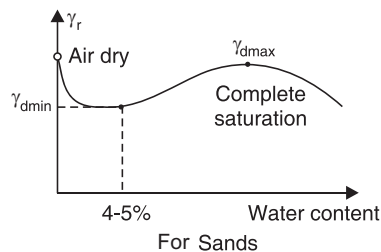


Compaction Tests:

Standard Proctor tests (Light compaction test)		Modified Proctor test (Heavy compaction test)	Indian std. light Compaction test	Indian std. heavy Compaction test
Volume of mould	942 cc	942 cc	1000 cc	1000 cc
No. of layers	3	5	3	5
No. of blows/layer	25	25	25	25
Height of free fall	12 inches	18 inches	310 mm	450 mm
Wt. of hammer	2.495 kg (5.5 lb)	4.54 kg (10 lb)	2.6 kg	4.9 kg

Compaction Curve for different Soils:

- Well graded soil can be compacted more than poorly graded soil
- Order of MDD on basis of grain size only
Gravel (G) > Sand (S) > SILT (M) > Clay (C)
- Low plastic soil (L) can achieve higher γ_d than highly plastic (H) soils.



Note: In compaction of sand, initially γ_d decreases due to bulking of sand at nearly 4-5% water content. Then γ_d increases and reaches to maximum at complete saturation.

Suitability of Compaction Equipment

Type of Equipment	Suitable of Soil Type	Nature of project
Rammers or tampers	All soils	In confined areas such as fills behind retaining walls, basement walls, etc. Trench fills.
Smooth wheeled rollers	Crushed rocks, gravels, sands	Road construction, etc.
Pneumatic tyred rollers	Sands, gravels silts, clayey soils, not suitable for uniformaly graded soils	Base, sub-base and embankment compaction for highways, airfields, etc. Earth dams
Sheepsfoot rollers	Clayey soil, Pure clay	Core of earth dams
Vibratory rollers	Sands	Embankments for oil storage tanks, etc.

Comparison of dry of optimum with wet of optimum compaction

Property	Dry of optimum	Wet of optimum
Structure after compaction	Flocculated (random)	Dispersed (oriented)
Water deficiency	More	Less
Permeability	More, isotropic	Less, anisotropic
Compressibility		
at low stress	Low	Higher
at high stress	High	Lower
Swellability	High	Low
Shrinkage	Low	High
Stress-strain behaviour	Brittle: high peak higher elastic modulus	Ductile: no peak, lower elastic modulus
Construction pore water pressure	Low	High
Strength (undrained) as moulded, after saturation	High somewhat higher if swelling prevented	Much lower Low
Sensitivity	more	Low

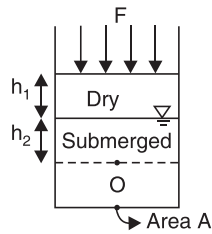
Selection of Compaction water content

Project	Compaction Water Content	Reason
Core of an earth dam	Wet of optimum	To reduce permeability and prevent cracking in core
Homogeneous embankment	Dry of optimum	To have a stronger soil and to prevent build-up of high pore water pressure
Subgrade of pavement	Wet of optimum	To limit volume changes

Effective Stress

7

Effective stress concept is developed by **Terzaghi** and applied to **Fully Saturated** soils only.



$$\bar{\sigma} = \sigma - \mu$$

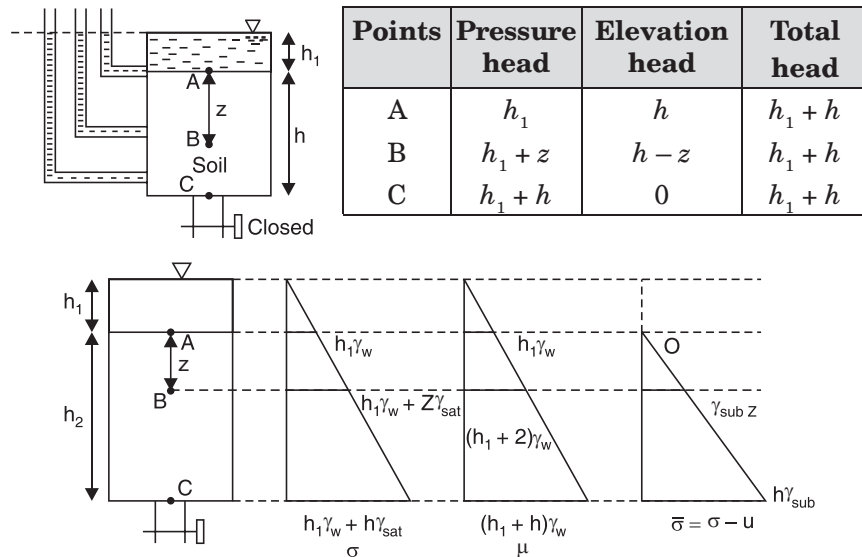
σ = Effective stress
 σ = Total stress
 μ = Pore water pressure

$$\sigma_0 = \frac{F}{A} + \gamma_d h_1 + \gamma_{sat} h_2 \quad \bar{\sigma}_0 = \frac{F}{A} + \gamma_d h_1 + \gamma_{sat} h_2$$

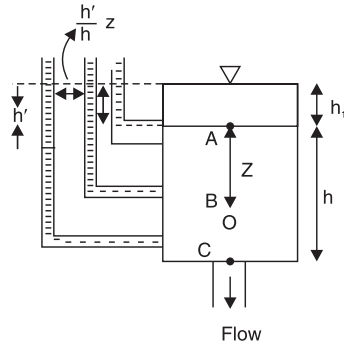
Note: Effective stress is not a physical parameter hence cannot be measured.

Analysis of Seepage force on effective stress

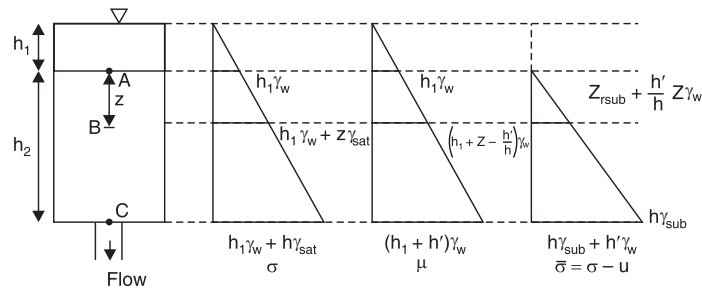
1. Hydrostatic condition (no flow condition)



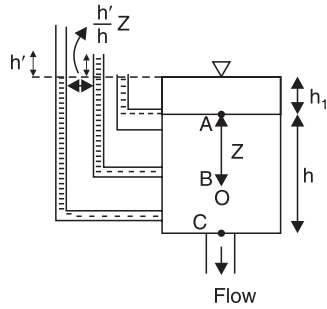
2. Downward flow condition



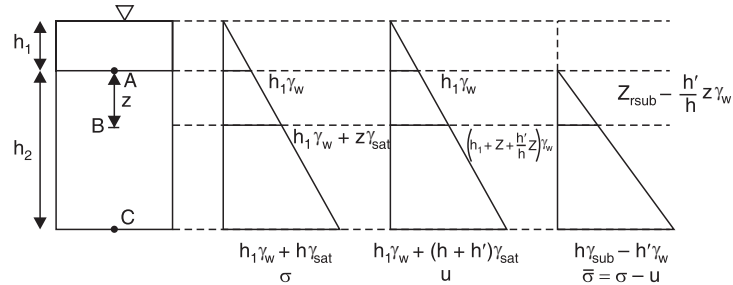
Po- ints	Pressure head	Eleva- tion head	Total head
A	h_1	h	$h_1 + h$
B	$h_1 + z - \frac{h'}{h} z$	$h - z$	$h_1 + h - \frac{h'}{h} z$
C	$h_1 + h - h'$	0	$h_1 + h - h'$



3. Upward flow condition



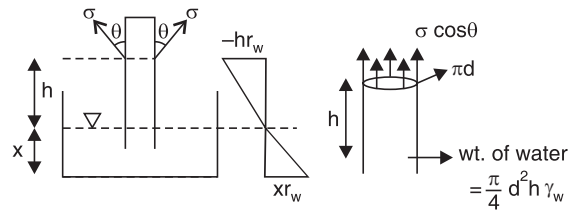
Po- ints	Pressure head	Eleva- tion head	Total head
A	h_1	h	$h_1 + h$
B	$h_1 + z + \frac{h'}{h} z$	$h - z$	$h_1 + h + \frac{h'}{h} z$
C	$h_1 + h + h'$	0	$h_1 + h + h'$



Capillarity and Permeability

8

Water which is held above the water table by phenomenon of surface tension is called capillary water. Due to effect of capillary rise there is increase in unit weight of the soil up to the height of capillary rise



$$\pi d \sigma \cos \theta = \frac{\pi}{4} d^2 h \gamma_w$$

$$h = \frac{4 \sigma \cos \theta}{\gamma_w d}$$

For $\theta = 0^\circ$ $h = \frac{4 \sigma}{\gamma_w d}$

For pure glass,

$$\sigma = 73 \text{ N/m},$$

$$r_w = 9.8 \text{ kN/m}^3 \quad h = \frac{0.03}{d_{mm}}$$

d = dia of tube in mm

h = height of capillary rise in m

$$h = \frac{0.03}{0.2 D_{10}}$$

D_{10} = Effective size of particle in mm

Other empirical formula:

$$h_{cm} = \frac{C}{e D_{10 \text{ cm}}}$$

c = empirical constant

$$= 0.1 - 0.5 \text{ cm}^2$$

Note: Capillary moisture in fine sand and silt allows unsupported excavation to be made because of stability it provides by virtue of **induced shear** strength.

- Bulking of sand also occurs due to capillary. It produces **apparent cohesion** which holds the particles in clusters, enclosing honeycombs.

Quick sand condition

In case of **upward seepage flow**, if the upward seepage force becomes equal to the buoyant weight of soil the effective stress in the soil becomes zero.

Critical hydraulic gradient

$$i_{cr} = \frac{\gamma_{sub}}{\gamma_w} = \frac{G - 1}{1 + e}$$

Note: Quick sand condition occurs in sand but not in clay because in clay cohesion exists.

$$\text{Factor of safety} = \frac{i_{cr}}{i_e}$$

i_e = exit hydraulic gradient.

Permeability: It is the ease with which water can flow through any medium.

Darcy's law: In one dimensional flow, discharge through fully saturated soil is given by

$$q = K i A \text{ or } V = K i \quad i = \frac{\Delta h}{l}$$

q = discharge

A = cross sectional area of the soil corresponding to flow ' q '

i = hydraulic gradient

Δh = loss of head in length ' L '

K = coefficient of permeability

V = Discharge velocity or superficial velocity

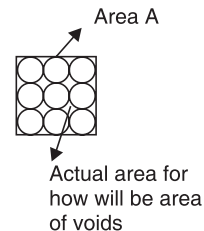
$$(a) V_s = \frac{V}{\eta} \quad V_s > V$$

η = porosity (< 1)

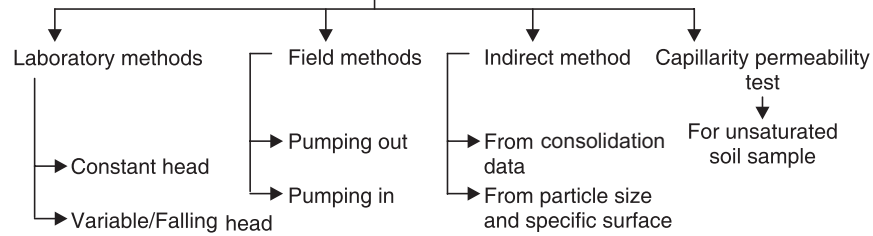
V_s = Actual velocity or seepage velocity corresponding to area of voids in cross-sectional area A

$$(b) K_p = \frac{K}{\eta}$$

where K_p = coefficient of percolation.

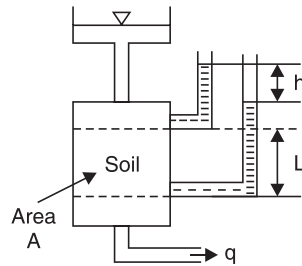


Determination of coefficient of Permeability



Constant head permeability test

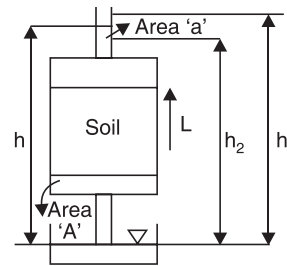
- Coarse grained soil



$$K = \frac{qL}{Ah}$$

Falling head permeability method

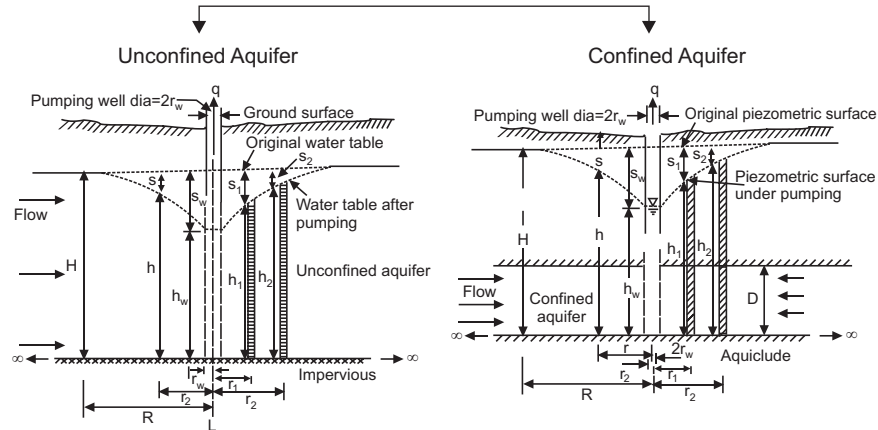
- Fine grained soil.



$$K = \frac{2.303 a L}{A t} \log_{10} \left(\frac{h_1}{h_2} \right)$$

Note: Reliability of laboratory method depends on the extent to which the test specimen represents the original ground conditions.

Pumping out test: Method useful for homogenous coarse grained soil.



$$K = \frac{2.303 q}{\pi (h_2^2 - h_1^2)} \log_{10} \left(\frac{r_2}{r_1} \right) \quad K_2 = \frac{2.303 q}{2\pi D(h_2 - h_1)} \log_{10} \left(\frac{r_2}{r_1} \right)$$

Note: Radius of influence, is the circle over which the effect of pumping is observed. It gradually merges asymptotically to the water table

$$R = 3000 d \sqrt{K} \quad \text{Sichardt's formula}$$

R = radius of influence in m

d = draw down in the well in m

K = coefficient of permeability (m/sec)

Pumping in test: More economical but less reliable than pumping out test as it gives coefficient of permeability of stratum which is close to the hole. It is also of two types.



From consolidation equation:

$$K = c_v m_v \gamma_w$$

c_v = coefficient of consolidation

m_v = coefficient of volume compressibility

γ_w = unit wt. of water.

From particle size and specific surface



$$K = \frac{1}{CS^2} \frac{r_w}{\mu} \frac{e^3}{1+e}$$

C = Shape factor coefficient

S = Surface area per unit volume

$$S = \frac{6}{D} = \frac{6}{\sqrt{ab}}$$

D → dia of particle
or size b/w

a_{mm} and b_{mm} .

$$K = CD_{10}^2$$

$$K \rightarrow \text{cm/sec}$$

$$D_{10} \rightarrow \text{mm}$$

or

$$K = 100 D_{10}^2$$

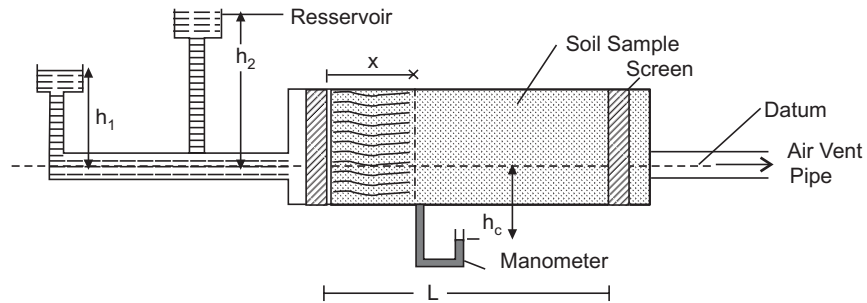
$$K \rightarrow \text{cm/sec}$$

$$D_{10} \rightarrow \text{cm}$$

$$\log_{10} (KS^2) = a + b\eta$$

$$\eta \rightarrow \text{porosity}$$

$$a, b \rightarrow \text{constants}$$

Capillarity permeability test:

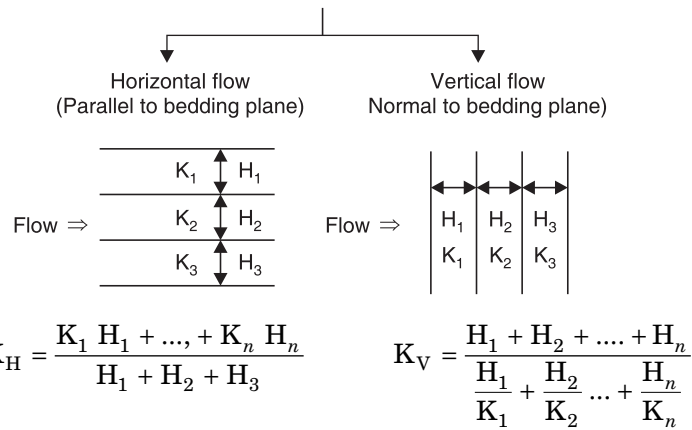
$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2K}{S\eta} (h_1 + h_c)$$

where, S = degree of saturation

η = porosity

h_c = capillary head

water move from x_1 to x_2 in time t_1 to t_2

Permeability of Stratified Soil

Seepage Through Soil

9

Laplace Equation in two dimensional Flow

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Potential function $\phi = KH$

where, H = Total head

K = Permeability coefficient

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

Laplace equation for homogenous **isotropic** soil (in 2D)

where $\phi = K_x H$ and $\phi = K_y H$ and $K_x = K_y$

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0$$

Laplace equation for **Anisotropic** soil (in 2D) where $\phi_x = K_x H$ and $\phi = K_y H$ and $K_x \neq K_y$

Note: For Anisotropic soil, the section is transformed with x -distance changed to $x \sqrt{\frac{K_y}{K_x}}$ while keeping the vertical dimension constant.

Coefficient of permeability of transformed section $K' = \sqrt{K_x K_y}$ (in 2D).

Calculation of discharge through a flow net

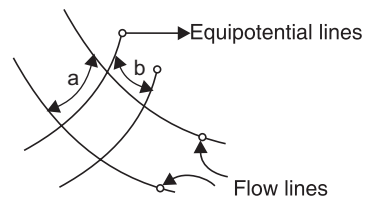
$$q = KH \frac{N_f}{N_d} \left(\frac{a}{b} \right)$$

q = flow per unit width perpendicular to the pressure plane of section

H = Total head loss

N_f = No. of flow channels

N_d = No. of equipotential drops



$$N_f = N_\psi - 1 \quad N_\psi = \text{No. of flow lines}$$

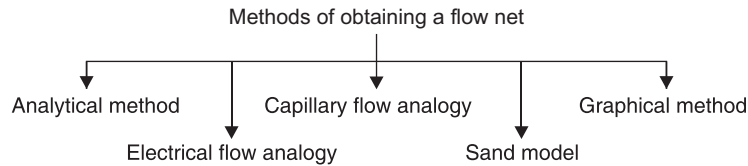
$$N_D = N_\phi - 1 \quad N_\phi = \text{No. of equipotential lines}$$

$$\frac{N_f}{N_D} = \text{Shape factor}$$

$a, b \rightarrow$ dimensions of an elementary square hence ratio of a/b is generally 1

Note: Shape factor (N_f/N_D) is the function of boundary conditions **only**, and will change only when extent of flow is changed. It will not change even if U/S and D/S water levels are interchanged and direction of flow is reversed.

Keeping the boundary conditions same, if the value of N_f is changed then the value of N_D will also be automatically changed in such a way to keep the value of shape factor constant.



Phreatic Line: It is the top flow line which follows the path of base parabola. It is a stream line. The pressure on this line is atmospheric and below this line is hydrostatic.

(a) **Phreatic line with filter**

$$x + S = \sqrt{x^2 + y^2}$$

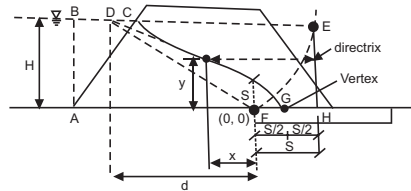
At $x = 0, y = S$

$$\frac{dy}{dx} = \frac{S}{\sqrt{S^2 + 2xS}}$$

At $x = 0 \quad \frac{dy}{dx} = 1$

At F $\frac{dy}{dx} = 1$ and $y = S$ then $q = KS$

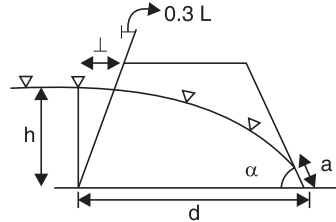
Hence, here q is found out without knowing the complete flownet.



(b) Phreatic line with out filter(i) For $\alpha < 30^\circ$

$$q = Ka \sin^2 \alpha$$

$$a = \frac{d}{\cos \alpha} - \sqrt{\frac{d^2}{\cos^2 \alpha} - \frac{h^2}{\sin^2 \alpha}}$$

(ii) For $\alpha > 30^\circ$

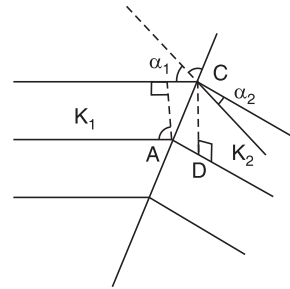
$$q = Ka \sin \alpha \tan \alpha$$

$$a = \sqrt{d^2 + h^2} - \sqrt{d^2 - h^2 \cot^2 \alpha}$$

Flow through Non-homogenous section

$$\frac{K_1}{K_2} = \frac{\tan \alpha_1}{\tan \alpha_2}$$

If $K_1 > K_2$ then $\alpha_1 > \alpha_2$, the flow gets deflected towards the normal otherwise vice a versa.

**Filter specifications by Terzaghi**

1. **Upper limit** of grain size to ensure **no significant invasion** of particles

$$\frac{D_{15}(\text{filter})}{D_{85}(\text{protected material})} < 5$$

2. **Lower limit** to ensure **sufficient head loss** without excessive seepage pressure

$$4 < \frac{D_{15}(\text{filter})}{D_{15}(\text{protected})} < 20$$

3. Additional guideline

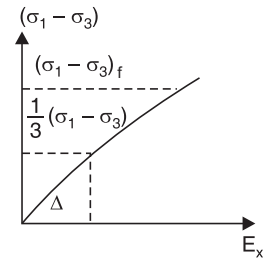
$$\frac{D_{50}(\text{filter})}{D_{50}(\text{protected material})} < 25$$

Vertical Stressess

10

Modulus of Elasticity of Soil: Determined by **Triaxial** test. It is taken as the **secant** modulus (1/2 to 1/3) of the peak stress. Sometimes initial **tangent modulus** or tangent modulus is also used.

Note: In undrained condition, value of poisson's ratio is 0.5 while in drained condition it's less than 0.5.



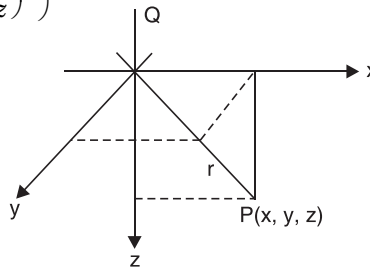
Vertical Stress due to concentrated load

Boussineq's
(For isotropic soil)

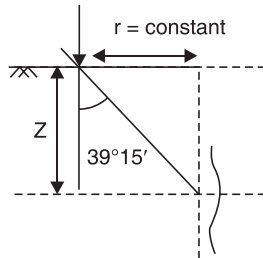
$$\sigma_{z_b} = \frac{3}{2\pi} \frac{Q}{z^2} \frac{1}{\left(1 + \left(\frac{r}{z}\right)^2\right)^{5/2}}$$

Westergaard's Equation
(For An-Isotropic soil)
Poisson's ratio assumed zero.

$$\sigma_{z_w} = \frac{1}{\pi} \frac{Q}{z^2} \frac{1}{\left(1 + 2\left(\frac{r}{z}\right)^2\right)^{3/2}}$$



Note: $\frac{r}{z} < 1.5$ $\sigma_{z_b} > \sigma_{z_w}$; $\frac{r}{z} > 1.5$ $\sigma_{z_b} < \sigma_{z_w}$

Bossinesq's result

 when z is constant

$$\sigma_{z \max} = 0.0888 \frac{Q}{r^2}$$

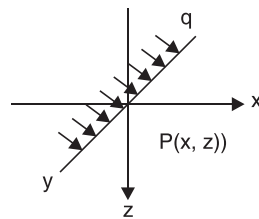
 when r is constant

$$\sigma_{z \max} = 0.1332 \frac{Q}{z^2}$$

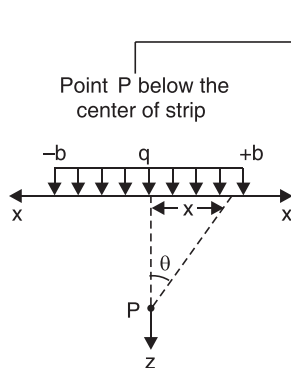
Note: Westergaard's result are more close to the field conditions but Boussinesq results are used for calculation because they provide conservative results.

Vertical stress due to line load

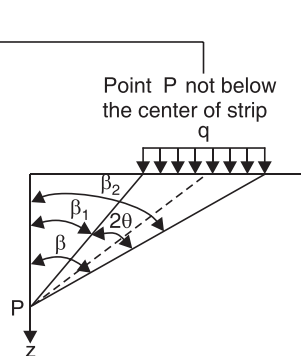
$$\sigma_z = \frac{2q}{\pi z} \left(\frac{1}{1 + \left(\frac{x}{z} \right)^2} \right)^2$$


 $\Rightarrow$ At $x = 0$ at depth z

$$\sigma_z = \frac{2}{\pi} \frac{q}{z}$$

Vertical stress due to strip load


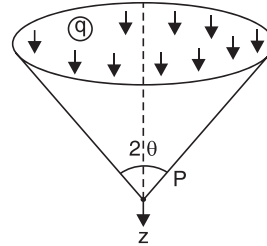
$$\sigma_z = \frac{q}{\pi} (2\theta + \sin 2\theta)$$



$$\sigma_z = \frac{q}{\pi} (2\theta + \sin 2\theta \cos 2\phi)$$

Vertical stress due to a circular area

$$\sigma_z = q (1 - \cos^3 \theta)$$



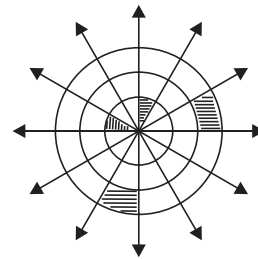
Vertical stress below the corner of a rectangle

$$\sigma_z = \frac{q}{2\pi} \left[\frac{mn}{\sqrt{m^2 + n^2 + 1}} \times \frac{m^2 + n^2 + 2}{m^2 + n^2 + m^2 n^2 + 1} + \sin^{-1} \left(\frac{mn}{\sqrt{m^2 + n^2 + m^2 n^2 + 1}} \right) \right]$$

where $m = \frac{B}{z}$ $n = \frac{L}{z}$

Note: The value of m and n can be interchanged in equation.

Newmark's Influence chart: Newmark developed the influence chart based on **Boussinesq's** equation to compute **vertical** stress, **horizontal** and **shear** stress due to uniformly loaded area of any shape (regular or irregular) **below** any point, **inside** or **outside** the loaded area



$$\sigma_z = \frac{1}{m \times n} \times q \times N$$

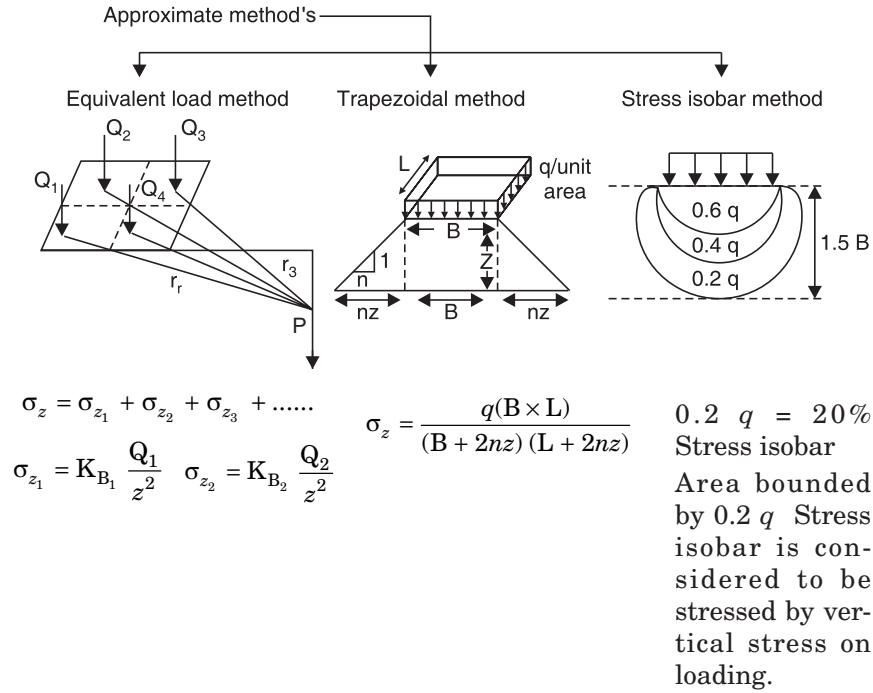
m = No. of concentric circles (normally 10)

n = No. of radial lines (normally 20)

q = Intensity of load

N = Equivalent no. of areas

All the area units will have equal influence at the center whether the area is inside or outside the loaded area, it will have the same influence at the center of the chart.



Compressibility and Consolidation

11

Settlement

Gradual sinking of the structure due to compression of the soil below

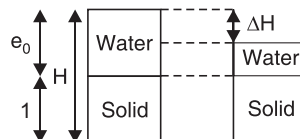
$$S_t = S_{\text{immediate}} + S_{1^{\circ}\text{-consolidation}} + S_{2^{\circ}\text{-consolidation}}$$

Due to **expulsion of air** or compression of pore air

Due to expulsion of **excess** pore water pressure generated due to increases in **Total stress**.

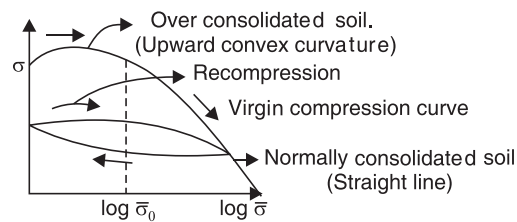
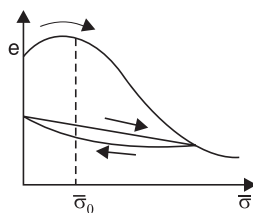
Due to gradual re-adjustment of clay particles into more stable configuration under **constant effective stress**.

Note: one dimensional consolidation (ie zero lateral strain) is measured by oedometer test/consolidation test.



$$\frac{\Delta H}{H} = \frac{\Delta V}{V} = \frac{\Delta e}{1 + e_0}$$

Compressibility characteristics



$$\frac{\bar{\sigma}_0}{\sigma} = \text{Over consolidation ratio (OCR)}$$

$$\bar{\sigma}_0 = \text{Pre consolidation stress}$$

If existing effective stress $\bar{\sigma} > \bar{\sigma}_0$, then soil is normally consolidated

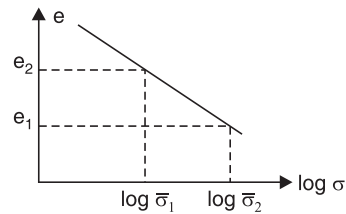
ie OCR = 1

If $\bar{\sigma} < \bar{\sigma}_0$, then over consolidated soil an i.e. OCR > 1

Coefficients in Compressibility of clay

1. Compression index c_c

$$c_c = \frac{e_1 - e_2}{\log_{10} \bar{\sigma}_2 - \log_{10} \bar{\sigma}_1} = \frac{\Delta e}{\log_{10} \left(\frac{\bar{\sigma}_2}{\bar{\sigma}_1} \right)}$$



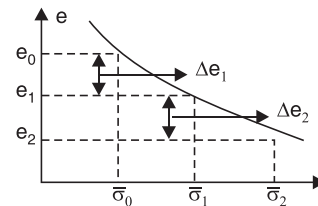
- (a) $c_c = 0.009 (w_L - 10)$ undisturbed soil of medium sensitivity,
 w_L = liquid limit %
- (b) $c_c = 0.007 (w_L - 7)$ Remoulded soil of low sensitivity
- (c) $c_c = 0.4 (e_o - 0.25)$ undisturbed soil of medium sensitivity
- (d) $c_c = 1.15 (e_o - 0.35)$ Remoulded soil of low sensitivity.

Note: c_c has a **constant value** for a given type of soil and is not a function of effective stress.

2. Coefficient of Compressibility (a_v)

$$a_v = \frac{\Delta e}{\Delta \bar{\sigma}}$$

$\Rightarrow a_v$ decreases with the increase in effective stress.



3. Coefficient of volume compressibility (m_v)

$$m_v = \frac{\text{Volume change per unit volume}}{\text{Increase in effective stress}}$$

$$m_v = \frac{a_v}{1 + e_0} \quad e_0 = \text{Initial void ratio}$$

$\Rightarrow$ value of m_v depends on stress range and is not constant for a particular soil.

$$\Rightarrow \text{compression modulus } E_c = \frac{1}{m_v}$$

Computation of Primary Settlement

H_0 = Depth in clay below water table only

ΔH = Change in depth (settlement)

e_0 = Initial void ratio.

$$1. \frac{\Delta H}{H} = \frac{\Delta e}{1 + e_0}$$

$$2. \Delta H = m_v \Delta \sigma H_0$$

$$3. \Delta H = \frac{c_c H_0}{1 + e_0} \log \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right)$$

$$4. \text{Over consolidated soil} \quad \bar{\sigma}_0 + \Delta \sigma < \bar{\sigma}_c$$

$$\Delta H = \frac{c_r H_0}{1 + e_0} \log \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}} \right)$$

Normally consolidated soil $\bar{\sigma}_0 + \Delta \bar{\sigma} > \bar{\sigma}_c$

$$\Delta H = \frac{c_r H_0}{1 + e_0} \log \left(\frac{\bar{\sigma}_c}{\bar{\sigma}_0} \right) + \frac{c_c H_0'}{1 + e_0'} \log \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_c} \right)$$

Computation of Secondary Settlement:

$$\Delta H = \frac{c_\alpha H_0}{1 + e_0} \log_{10} \frac{t_2}{t_1}$$

e_0 = void ratio at the end of 1° settlement

H_0 = height at the end of 1° settlement.

$$c_\alpha = \frac{\Delta e}{\log(t_2/t_1)} = \frac{\Delta e}{\Delta \log t}$$

$$c_\alpha \text{ is 4-6\% of the value of } \frac{c_c}{1 + e_0}$$

Terzaghi's one dimensional consolidation equation:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

u = excess pore pressure

$\frac{\partial u}{\partial t}$ = Rate of change of pore pressure with time

where $C_v = \frac{K}{m_v \gamma_w}$ C_v = coefficient of consolidation

$\frac{\partial u}{\partial z}$ = Rate of change of pore pressure with depth

Time factor (T_v):

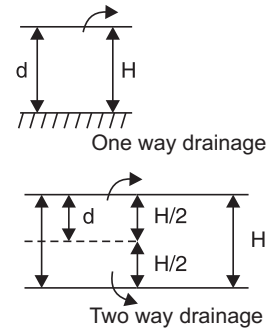
$$T_v = \frac{C_v t}{d^2}$$

C_v = coefficient of consolidation in cm^2/sec

d = length of drainage path

$d = H$ in one way drainage

$d = \frac{H}{2}$ in two way drainage



$$(a) \ u \leq 60\% \quad T_v = \frac{\pi}{4} (u)^2 \quad u = \text{Avg. degree of consolidation}$$

$$(b) \ u > 60\% \quad T_v = 1.781 - 0.933 \log (100 - u)$$

$$T_{50} = 0.196$$

$$T_{90} = 0.848$$

Degree of consolidation

$$(a) \quad U = \frac{\Delta h}{\Delta H} \times 100$$

Δh = settlement at any stage

ΔH = settlement at end of consolidation.

$$(b) \quad U = \frac{e_o - e}{e_o - e_f}$$

e_o = initial void ratio

e = void ratio of any stage

e_f = final void ratio

$$(c) \quad U = \frac{U_i - U}{U_i - U_f}$$

U_i = Initial pore water pressure

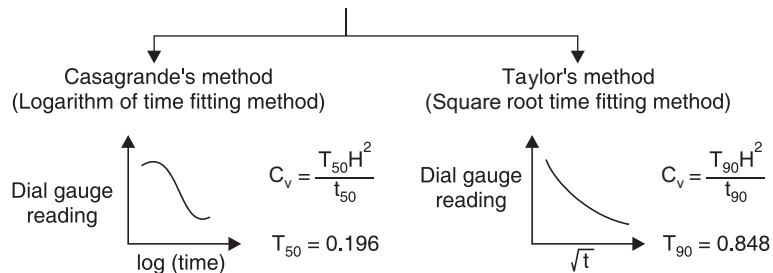
U = Pore water pressure at any stage

$$(d) \quad U = \frac{U_i - U_z}{U_i}$$

U_f = Pore water pressure at the end of stress increment

U_z = excess pore pressure at any depth Z .

Determination of coefficient of consolidation (C_v)



Note: C_v is inversely proportional to liquid limit (w_L) where as c_c is directly proportional to liquid limit.

Value of C_v decreases with increases in plasticity.

Compression Ratio's

$$(a) \text{ Initial compression ratio } r_0 = \frac{R_i - R_0}{R_i - R_f}$$

$$(b) 1^\circ - \text{consolidation ratio } r_p = \frac{R_o - R_{100}}{R_i - R_f}$$

$$(c) 2^\circ - \text{consolidation ratio } r_s = 1 - (r_0 + r_p)$$

$$r_s = \frac{R_{100} - R_f}{R_i - R_f}$$

R_i = initial dial gauge reading

R_0 = Dial gauge reading for beginning of 1° - consolidation

R_{100} = Dial gauge reading for completion of 1° - consolidation

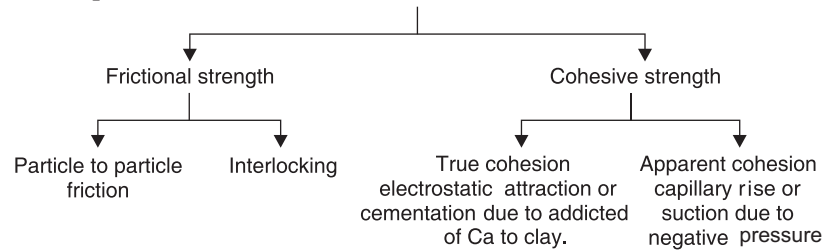
R_f = final dial gauge reading

Shear Strength of Soil

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It is the capacity of the soil to resist shearing stress. It is defined as the maximum value of shear stress that can be mobilised within a soil mass.

It depends on two factors



Note: Whatever be the nature of loading on soil, failure on soil occurs by shearing, it never occurs by crushing of particles.

Mohr's hypothesis: Shear stress on failure plane at failure reaches a value which is a unique function of normal stress on that plane.

$$\tau_{ff} = f(\sigma_{ff})$$

τ_{ff} = Shear stress on failure plane at failure

σ_{ff} = Normal stress on failure plane at failure.

Coulomb's hypothesis:

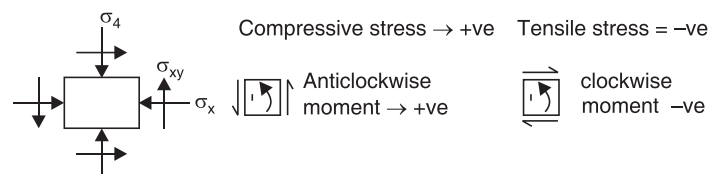
$$\tau_f = C + \sigma \tan \phi$$

$$\tau_f = C' + \bar{\sigma} \tan \phi'$$

where C , ϕ are total stress parameters and C' , ϕ' are effective stress parameters.

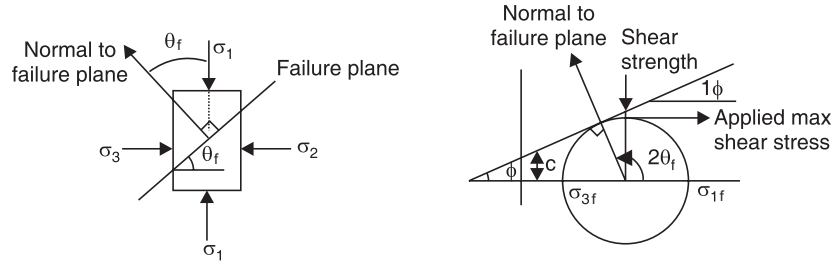
Note: Shear strength parameter's C , ϕ , C' , ϕ' are not the inherent properties of soil. They are related to the type of test and the condition under which these are measured.

Sign Convention for drawing Mohr's circle



Relation between of ϕ , ϕ , σ_{1f} and ϕ_{3f}

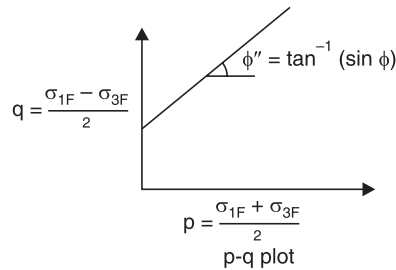
Based on Mohr's Colomb's failure envelope



$$\theta_f = 45 + \frac{\phi}{2}$$

i.e. failure plane makes an angle of $(45 + \phi/2)$ degree with major principal plane

$$\frac{\sigma_{1f} - \sigma_{3f}}{2} = \frac{\sigma_{1f} + \sigma_{3f}}{2} \sin \phi + C \cos \phi$$



$$\sigma_{1f} = \sigma_{3f} \tan^2 \left(45 + \frac{\phi}{2} \right) + 2C \tan \left(45 + \frac{\phi}{2} \right)$$

$$\sigma_{3f} = \sigma_{1f} \tan^2 \left(45 - \frac{\phi}{2} \right) - 2C \tan \left(45 - \frac{\phi}{2} \right)$$

Stability analysis based on drainage conditions

Drained Condition: If the rate of loading is such that water in the pores of soil gets sufficient time to drain out, the condition of loading is called drained condition.

Undrained Condition: The rate of loading is rapid such that the water in the pores does not get sufficient time to drain out, that condition of loading is called undrained condition.

Note: Both drained and undrained conditions depends on rate of loading and type of soil. Hence these conditions are actually relative.

(a) Shear strength under drained condition

- Effective stress approach is used
- Drained analysis is used to evaluate long term stability.

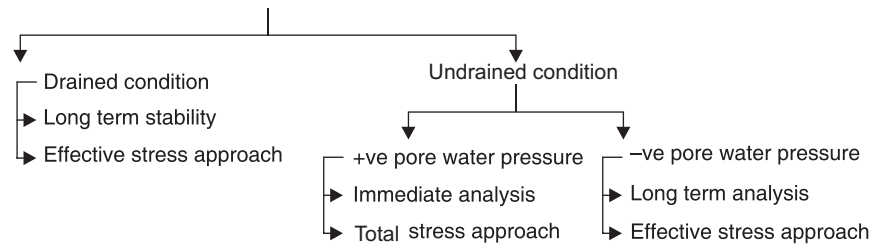
(b) Shear strength under undrained condition

(i) +ve excess pore water pressure develops

- Total stress approach is used
- Shear strength evaluated at the end of construction period

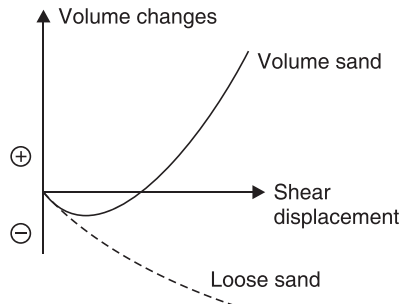
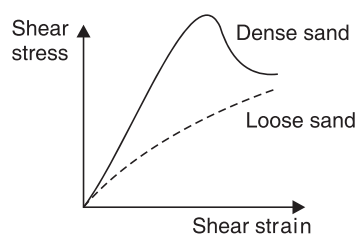
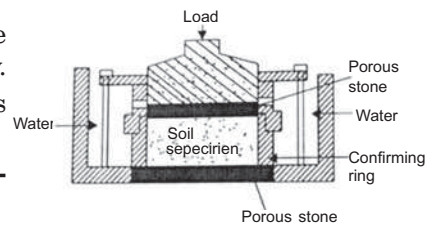
(ii) -ve excess pore water pressure develops

- Effective stress approach is used
- Long term stability is analysed



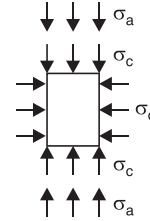
Direct shear test

- Good for free draining soil like sand and gravel not used for clay.
- Effective stress and total stress are same
- Shear normally applied at **constant rate of strain**
- Shear and vertical deformations are measured during test using dial gauge



Triaxial test

- Suitable for all types of soil
- Pore water pressure can be measured under undrained condition.
- Volume change can also be measured under drained condition.



$\sigma_3 = \sigma_c = \text{confining pressure}$

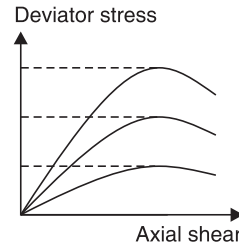
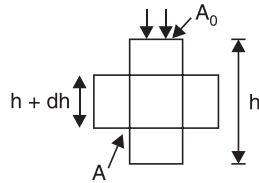
$\sigma_1 = \sigma_c + \sigma_a$ $\sigma_a = \text{deviator stress.}$

$$\sigma_c = \bar{\sigma}_{V_0} \left(\frac{1 + 2K_0}{3} \right)$$

$K_0 = \text{Coefficient of earth pressure at rest} = 1 - \sin \phi$

$\bar{\sigma}_{V_0} = \text{Vertical effective stress in the field.}$

$$\sigma_a = \sigma_1 - \sigma_3 = \frac{\text{Axial load}}{\text{Corrected area}}$$



$$A(h + dh) = V = V_0 + dV$$

$$A = \frac{A_0(1 - \epsilon_v)}{(1 - \epsilon_a)}$$

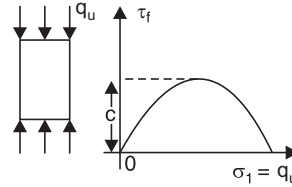
For undrained test $\epsilon_v = 0$

$$\text{So } A = \frac{A_0}{1 - \epsilon_a}, \quad \epsilon_a = \frac{dh}{h}$$

Note: At any time in triaxial test either pore water line is open and drainage line is closed or vice-versa, i.e. either pore water pressure measurement will be made under undrained condition or volume change is measured under drained condition.

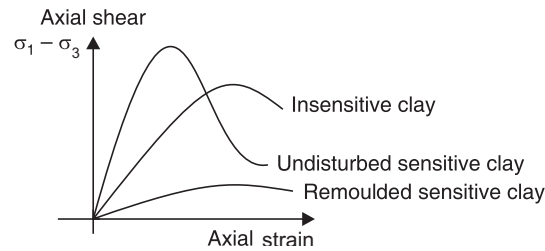
Unconfined Compression Test (UC Test)

- For fully saturated soil. (otherwise $\phi \neq 0$)
- No confining pressure is applied ($\sigma_c = 0$)
- Used to test cohesive soil, in non-cohesive soil sample cannot be prepared without confining pressure.



q_u = unconfined compressive strength

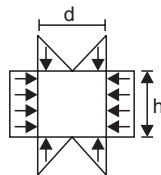
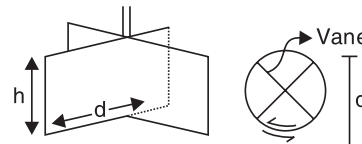
$$q_u = \frac{P}{A_f} = \frac{P}{\left(\frac{A_0}{1 - \epsilon_a} \right)}$$



- Used for rapid assesment of **consistency**.
- Also used for finding **sensitivity** of clay soil, other than fissured clay.

Vane Shear test

- For plastic cohesive soil which is very sensitive
- Maximum torque applied is the total shear



$$\tau_f = \frac{T}{\pi d^2 \left(\frac{h}{2} + \frac{d}{6} \right)}$$

[when both top and bottom end shear the soil]

$$\tau_f = \frac{T}{\pi d^2 \left(\frac{h'}{2} + \frac{d}{12} \right)}$$

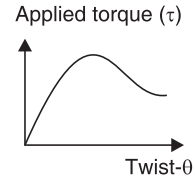
[when top end does not shear's the soil n']

$h' \rightarrow$ height of vane used in shearing.

Here, τ_f = Shear strength = C_u (undrained cohesion)

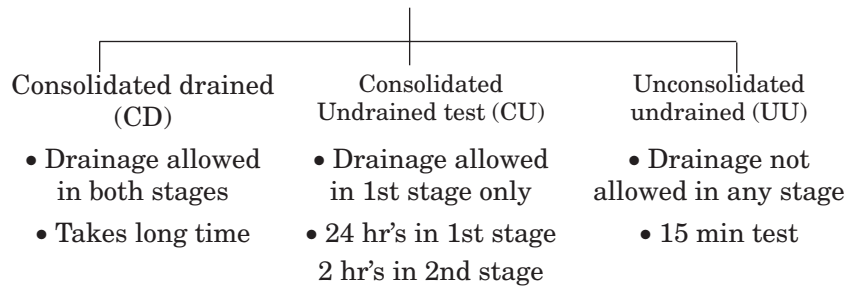
$$C_u = \frac{q_u}{2}$$

$$\text{Sensitivity} = \frac{C_{u \text{ undisturbed}}}{C_{u \text{ remoulded}}}$$



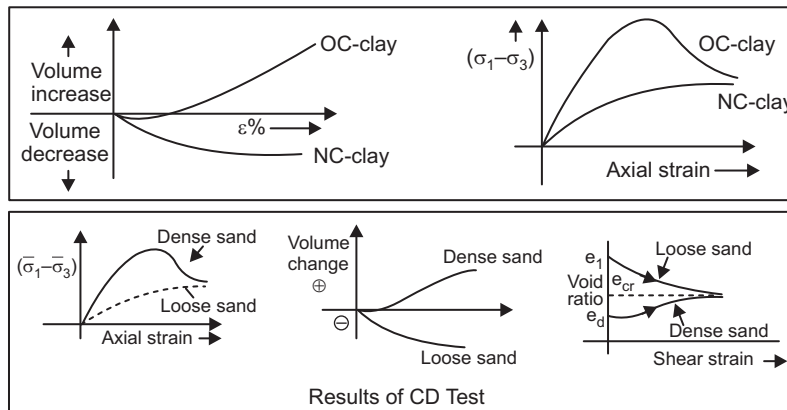
	Lab size	Field size
Height of Vane (H)	20 mm	10–20 cm
Dia of Vane (D)	17 mm	10 cm
Thickness of Vane (t)	0.5–0.1 mm	2–3 cm

Type of Triaxial Tests



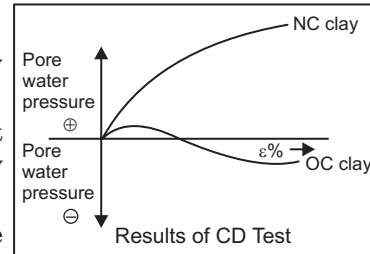
Consolidated Drained Test

- Used in Analysis of gradual loading condition
- To check long term stability of embankment which has been in existence since long

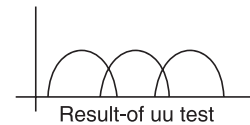


Consolidated Undrained Test

- Sudden unloading such as draw down or dewatering
- Unloading of an embankment that has lived some of its life (i.e. already consolidated)
- Gives both total stress and effective stress parameters

**Unconsolidated Undrained test**

- Test is suitable for soil of low permeability or when loading is very fast.
- Total stress analysis is performed as it is an undrained test.
- Used in sudden loading such as rapid construction
- Short term stability under pore water pressure.



Note: $\frac{C_u}{\bar{\sigma}_\tau} = 0.11 + 0.0037 I_p \%$

For NC clays, shear strength depends on plasticity Index.

Soil Liquefaction: The phenomenon in which **Saturated loose sand** in **undrained** condition under rapid loading develops **positive pore water pressure** which ultimately reduces the **effective stress to zero**, is called liquefaction. It occurs at high frequency of Vibration e.g. during pile driving, vibration of machine, explosive blasting and earthquake shock.

Pore Pressure Coefficients: It is used to express the response of pore water pressure to change in total stress under undrained conditions and enable the initial value of pore water pressure to be determined.

$$\Delta u = B[\Delta\sigma_3 + \Delta(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$\Delta U_1 = B\Delta\sigma_3$$

$$\Delta U_2 = AB (\Delta\sigma_1 - \Delta\sigma_3)$$

$$B = \frac{\Delta U_1}{\Delta\sigma_3}$$

$$\Delta U_1 = \text{Change in pore pressure due to increase in cell pressure}$$

$$\Delta U_2 = \text{Change in pore pressure due to increase in deviator stress.}$$

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- For saturated soil $B = 1$
- For dry soil $B = 0$
- B varies with the stress range
- A can be found from CD test
- B can be found from first stage in UU test and AB from its second stage.
- A is also not a constant, depends on type of soil, stress condition etc.
- For a given soil A depends on strain, sample distribution, anisotropy and OCR.

Value of A	Type of Soil
– 0.5 to 0	Heavily over consolidated Soil
< 0	Over consolidated soil
0.5–1	Normally consolidated soil
2–3	Loose saturated time sand

Stability of Slopes

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Assumption's in analysis of slope stability

- (a) Slope–stability problem is a two dimensional problem.
- (b) Shear parameters of the soil are constant along any possible slip surface.
- (c) In problems involving seepage of water, flow net can be constructed and the seepage force can be determined.

Stability of Infinite Slope

$$F.O.S = \frac{\tau_f}{\tau}$$

$$F.O.S = \frac{C + \sigma \tan \phi}{\gamma z \cos \beta \sin \beta}$$

$$(a) F = \frac{\tan \phi}{\tan \beta}$$

when $C = 0$

$$\sigma = \gamma z \cos^2 \beta$$

$$(b) F = \frac{C + \gamma z \cos^2 \beta \tan \phi}{\gamma z \cos \beta \sin \beta}$$

(c) $F = 1$, and $Z = H_c$ then

$$H_c = \frac{C}{\gamma \cos^2 \beta (\tan \beta - \sin \phi)}$$

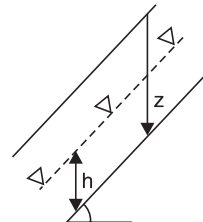
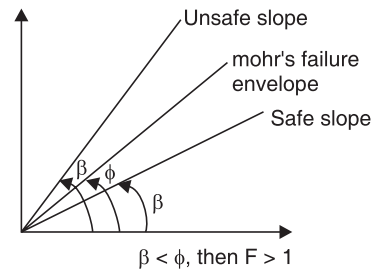
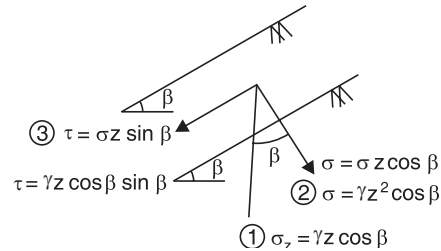
$$(d) S_n = \frac{C}{\gamma H_c} = \cos^2 \beta (\tan \beta - \sin \phi)$$

Hence, S_n is a dimensionless Quantity

- (e) Seepage taking place and W.T. is parallel to the slope in cohesionless soil.

$$F_s = \left[1 - \left(\frac{\gamma_w}{\gamma} \right) \left(\frac{h}{z} \right) \right] \frac{\tan \phi'}{\tan \beta}$$

ϕ' = effective friction angle



γ = avg. total unit weight of the soil above the slip surface upto the ground level.

$$\gamma = \frac{\gamma_1 h_1 + \gamma_2 h_2}{h_1 + h_2}$$

(f) If water table is at ground level

$$F_s = \frac{\gamma'}{\gamma_{\text{sat}}} \frac{\tan \phi}{\tan \beta} \approx F_s = \frac{1}{2} \frac{\tan \phi}{\tan \beta}$$

(g) Infinite slope of purely cohesive soil.

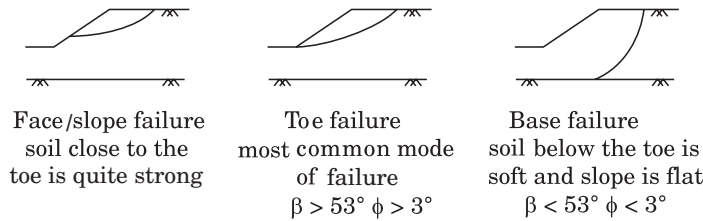
$$F_c = \frac{C}{\gamma z \sin \beta \cos \beta}$$

(h) C – ϕ soil in infinite slope

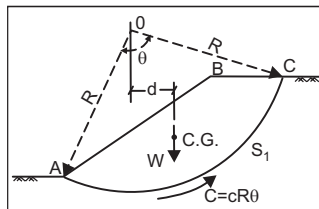
$$F_s = \frac{C}{\gamma z \sin \beta \cos \beta} + \frac{\tan \phi}{\tan \beta}$$

Stability of Finite Slope

(a) **Swedish Circle Method:** Surface of sliding is assumed as “arc of circle”

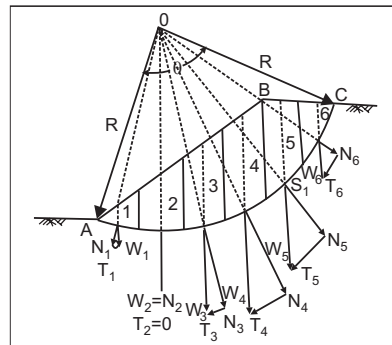


Purely cohesive soil



$$F = \frac{CR^2\theta}{Wd}$$

cohesive frictional soil



$$F = \frac{CR\theta + \sum N \tan \phi}{\sum T}$$

- (b) **Friction Circle Method:** This method is based on the assumption that the resultant force R on the rupture surface is **tangential** to a circle of radius $r = R \sin \phi$ which is concentric with the trial slip circle.

- friction is assumed to be fully mobilised
- Total stress approach is used

$$F_c = \frac{C}{C_m}$$

$$F_\phi = \frac{\tan \phi}{\tan \beta} = \frac{\tan \phi}{\tan \phi_m}$$

- (c) **Taylor's stability number:** It's a **dimensionless** parameter. It is obtained for factor of safety wrt cohesion while the factor of safety wrt friction F_ϕ is assumed to be unity.

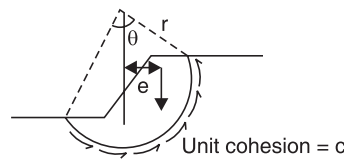
$$S_n = \frac{C}{\gamma H_C} = \frac{C}{\gamma F_C H}$$

In case of saturated slope replace γ by γ_{sat}° while in submerged slope replace it by γ' .

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \phi$$

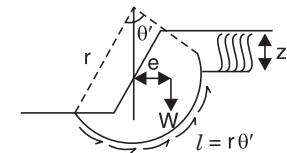
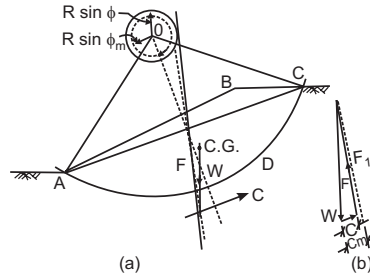
ϕ_w = weight friction angle.

- (d) **Fellinius Methods:** For purely cohesive soil



$$F = \frac{Cr^2\theta}{We}$$

r = radius of rupture curve



$$F = \frac{Cr^2\theta'}{We}$$

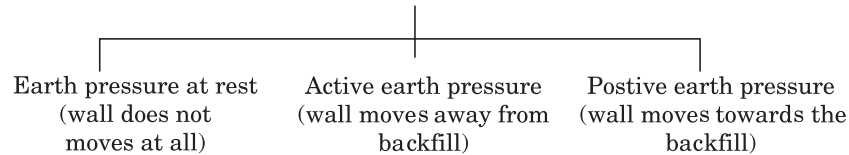
$$Z_c = \frac{2C}{\gamma}$$

Earth pressure and Retaining walls

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Earth pressure is the lateral force exerted by the soil on any structure retaining that soil. There are several types of retaining structure's like retaining walls (gravity type, cantilever type, counter fort type), bracings in cuts, abutment of a bridge, sheet pile/anchored sheet pile.

Type's of lateral earth pressure



1. Earth pressure at rest

$$\Rightarrow \sigma_x = \sigma_y \text{ and } \epsilon_x = 0$$

$$\text{So } \sigma_x = \frac{\mu}{1 - \mu} \sigma_z$$

$$\sigma_x = K_0 \sigma_z$$

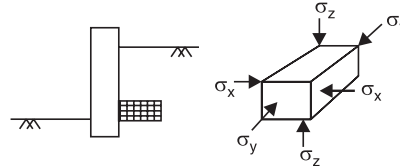
$$\text{Hence, } K_0 = \frac{\mu}{1 - \mu}$$

$$\Rightarrow \text{For cohesion less soil } (C = 0) K_0 = 1 - \sin \phi$$

$$\Rightarrow \text{For normally consolidated soil (N. C. Soil)}$$

$$K_0 = 0.19 + 0.233 \log (I_p)$$

$$\Rightarrow \text{For over consolidated soil (OC soil) } K_{o(oc)} = K_{o(NC)} \sqrt{O.C.R}$$



where OCR = over consolidation ratio.

Type of soil	K_0
Dense sand	0.4 – 0.5
Loose sand	0.45 – 0.5
Mechanically compacted	0.8 – 1.0
N.C clays	0.5 – 0.6
O.C clays	1.0 – 4.0

Note: K_0 for dens sand is lesser than that of loose sand.

For NC clays $K_0 < 1$

For OC clays $K_0 > 1$

Active earth pressure and passive earth pressure

Active earth pressure	Passive earth pressure
⇒ Failure plane is inclined at $45 + \phi/2$ with the horizontal ⇒ Very little movement is required to mobilise the active pressure $\Delta H = 0.2\%$ of H Dense sands $\Delta H = 0.5\%$ of H loose sands $\Delta H = 0.4\%$ of H clay's ⇒ length of failure block $= H \cot \left(45 + \frac{\phi}{2} \right)$ $\Rightarrow K_q = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left(45 + \frac{\phi}{2} \right)$	⇒ Failure plane is inclined at $45 - \phi/2$ with the horizontal ⇒ much higher movement is required to mobilise the pressure $\Delta H = 2\%$ of H Dense sands $\Delta H = 15\%$ of H loose sands ⇒ length of failure block = $H \cot \left(45 - \frac{\phi}{2} \right)$ $\Rightarrow k_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left(45 + \frac{\phi}{2} \right)$

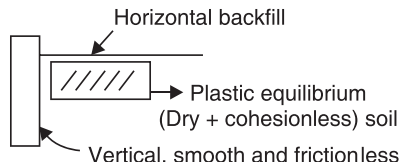
Earth pressure theories

1. Rankine theory (1857)
2. Coulomb's theory (1776)

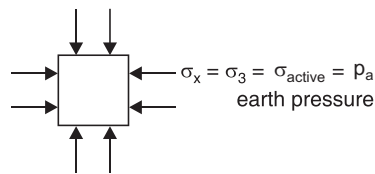
Note: Coulomb's theory came before Rankine's theory and was even more versatile as it considered friction between wall and the soil but it is still not used because this theory does not satisfies the static equilibrium condition occuring in nature. The three forces do not meet at a common point when sliding surface is assumed to be planer.

Rankine's theory **over estimates the active earth pressure** and **under estimates the passive earth pressure** and as retaining walls are designed for active earth pressure hence, Rankine theory gives more conservative results.

1. **Rankine's theory.** Originally Rankine's theory was only for cohesionless soil but later it was extended to cohesive as well as submerged soil.



(a) Active earth pressure



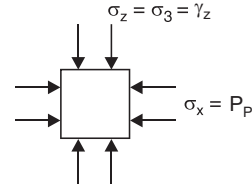
$$p_a = k_a \gamma z - 2c\sqrt{k_a}$$

$$\text{where } k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left(45 - \frac{\phi}{2} \right)$$

(b) Passive earth pressure

$$P_p = k_p \gamma z + 2c\sqrt{k_p}$$

$$\text{where } k_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left(45 + \frac{\phi}{2} \right)$$

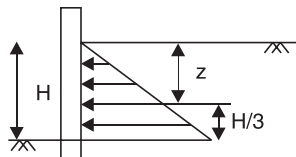


Note: By compacting soil active earth pressure decreases where as passive earth pressure increases

$$k_a = \frac{1}{k_p}$$

Various cases of earth pressures

1. **Cohesionless soil on a vertical smooth wall**



$$p_{az} = k_a \gamma z$$

$$p_{ap} = k_p \gamma z$$

$$F_a = \frac{k_a \gamma H^2}{2} \quad F_p = \frac{k_p \gamma H^2}{2}$$

(6) Soil with Inclined back fill

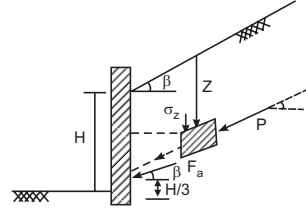
$$\sigma_z = \gamma z \cos \beta$$

$$p_a = k_a \gamma z \cos \beta$$

$$K_a = \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$$

$$K_p = \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}$$

$$F_a = \frac{k_a \gamma H^2}{2} \cos \beta$$

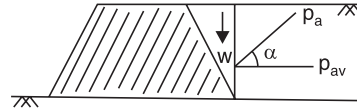


7. Horizontal back fill with inclined wall

$$P_a = \sqrt{W^2 + p_{av}^2}$$

$$p_{av} = \frac{1}{2} k_a \gamma H^2$$

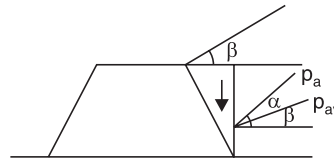
$$\tan \alpha = \frac{W}{p_{av}}$$



8. Inclined back fill with inclined wall

$$P_a = \sqrt{P_{av}^2 + W^2 + 2P_{av} W \sin \beta}$$

$$\tan \alpha = \frac{W + P_{av} \sin \beta}{P_{av} \cos \beta}$$

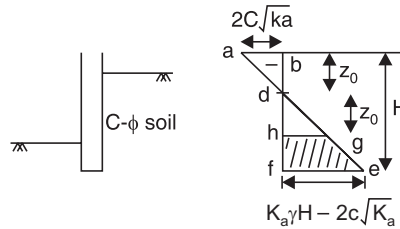


9. Active earth pressure on cohesive soil.

$$H_C = 2 z_0 = \frac{4C}{\gamma \sqrt{K_a}}$$

No-contact loss = Active earth pressure corresponds to area efgh

After contact loss = Active earth pressure corresponds to area fde

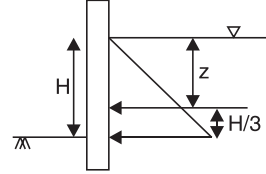


2. Submerged cohesionless soil on vertical smooth wall

$$p_{az} = k_a \gamma_{\text{sub}} Z + \gamma_w Z$$

$$p_{pz} = K_P \gamma_{\text{sub}} Z + \gamma_w Z$$

$$F_a = \frac{K_a \gamma_{\text{sub}} H^2}{2} + \frac{\gamma_w H^2}{2}$$



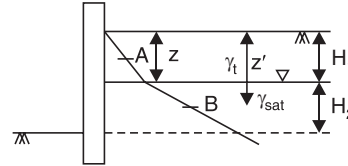
3. Partially submerged cohesionless soil on vertical smooth wall

For point A

$$p_a = k_a \gamma_t z$$

For point B

$$p_a = K_a \underbrace{(\gamma_t H_1 - \gamma_{\text{sub}} (Z' - H_1))}_{\text{Effective stress}} + \gamma_w (Z' - H_1)$$



Note: water pressure due hydrostatic condition is same in all directions hence hydrostatic pressure is not multiplied by k_a

4. Back fill with two different soils (ϕ_1 and ϕ_2)

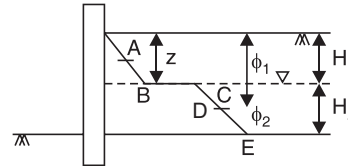
$$\text{A: } p_a = K_{a1} \gamma_2 Z$$

$$\text{B: } p_a = K_{a1} \gamma_1 H_1$$

$$\text{C: } p_a = K_{a2} \gamma_1 H_1$$

$$\text{D: } p_a = K_{a2} (\gamma_1 H_1 + \gamma_2 (Z' - H_1))$$

$$\text{E: } p_a = K_{a2} (\gamma_1 H_1 + \gamma_2 H_2)$$



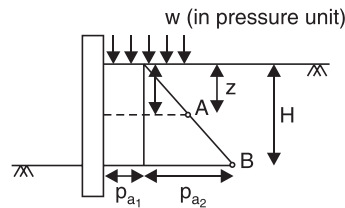
5. Soil with surcharge load

$$p_a = p_{a1} + p_{a2}$$

$$\text{A: } p_{a1} = k_a w$$

$$p_{a2} = k_a \gamma z$$

$$\text{B: } p_{a1} = k_a w \quad p_{a2} = k_a \gamma H$$



$$F = K_{a1} w H + \frac{k_a \gamma H^2}{2}$$

For No contact loss

$$F_a = \left[\frac{K_a \gamma Z^2}{2} - 2C \sqrt{K_a} Z + \frac{2C^2}{\gamma} \right] - \left[\frac{1}{2} \times \frac{2C}{\gamma \sqrt{K_a}} \times 2C \sqrt{K_a} \right]$$

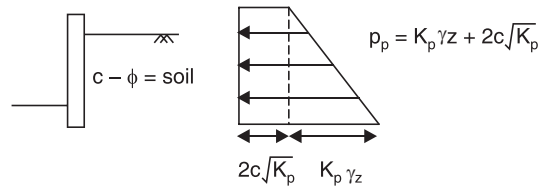
$$F_a = \frac{K_a \gamma Z^2}{2} - 2C \sqrt{K_a} Z$$

After contact loss

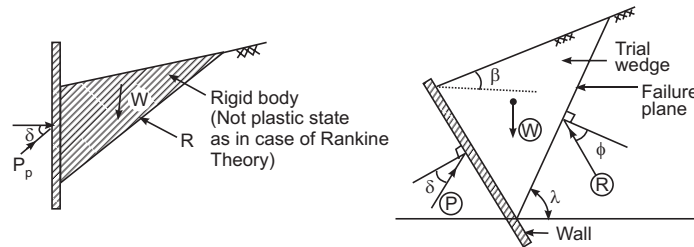
$$F_a = \frac{K_a \gamma Z^2}{2} - 2C \sqrt{K_a} Z + \frac{2C^2}{\gamma}$$

Note: As there is no net earth pressure upto a depth of $2Z_0$ we can make unbraced cut in clayey soil upto depth of $2Z_0$, i.e. $\frac{4C}{\gamma \sqrt{K_a}}$

10. Passive earth pressure on cohesive soil:



Coulomb's theory of earth pressure:



Here sliding wedge is assumed as a at an angle λ from horizontal rigid body.

Forces acting on trial wedge will be W , R , P whose directions will be known. The position and line also of action of earth pressure will also be known in advance. By assuming various trial wedges at different trial angle λ the value of P will be calculated.

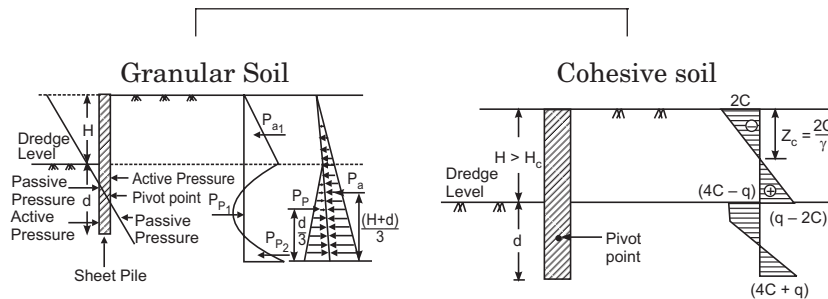
Highest value of P is for active earth pressure while lowers value of P is for passive earth pressure.

Cantilever Sheet Pile and Anchored Bulkhead

15

A sheet pile wall is an earth and water retaining structure which behaves as a fixed vertical cantilevers in resisting lateral earth pressure

Cantilever sheet pile



$$P_p \cdot \frac{d}{3} = P_a \frac{(H+d)}{3} \text{ without FOS}$$

$$\frac{P_p}{\text{FOS}} \times \frac{d}{3} = P_a \frac{(H+d)}{3} \text{ with FOS}$$

$$P_p = \frac{1}{2} K_a \gamma d^2$$

$$P_p = \frac{1}{2} K_a \gamma (H+d)^2$$

$$q = \gamma H$$

At depth H ,

$$P_a = q - 2C$$

$$P_p = 2C$$

Resultant at depth H ,

$$P_p - P_a$$

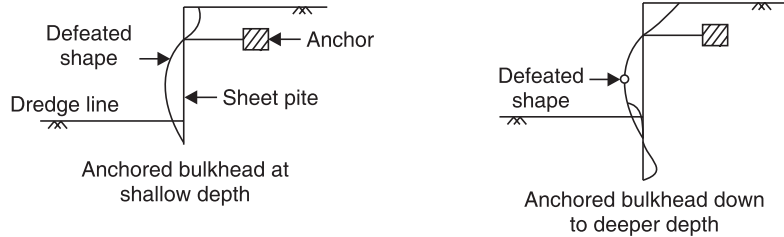
$$P_p - P_a = 4C - q$$

Resultant earth pressure at $H + d$

$$P_p - P_a = 4C \pm q$$

Anchored Bulkhead

If the wall height is large, support against lateral pressure is provided by embedment in the ground as well as by tie rods near the top. This type of earth retaining structure is called an anchored bulkhead.

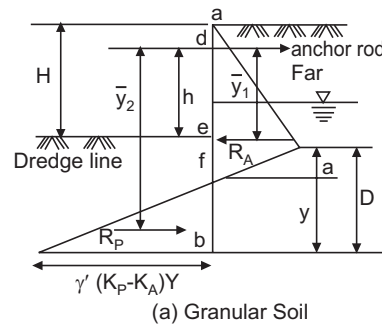


(a) Anchored bulkhead in granular soil.

F_{ar} = Force in anchor rod

R_A = Resultant active earth pressure acting at $\bar{y}_1$ below the anchor rod level.

R_p = Resultant passive earth pressure acting at $\bar{y}_2$ below the anchor rod.



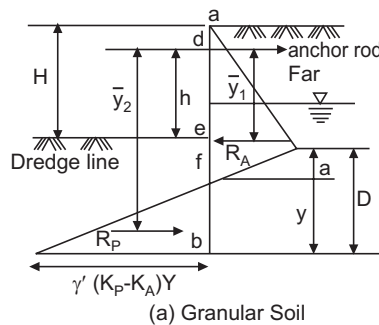
For equilibrium

$$F_{ar} + R_p - R_a = 0$$

Depth 'a' to the point of zero pressure

$$a = \frac{P_{AC}}{\gamma' (K_p - K_a)}$$

(b) Anchored bulkhead in cohesive soil.



Coffer Dam: It is a temporary structure constructed usually in river lake etc to keep the working area dry for construction of other structure.

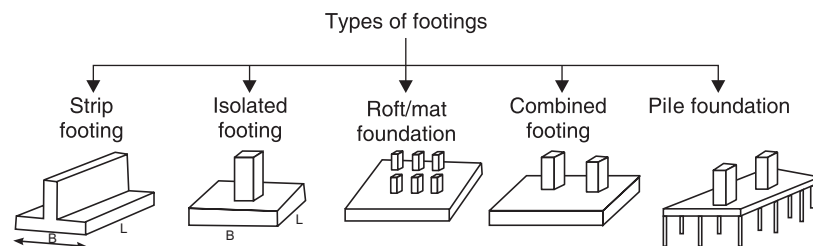
After the construction of coffer dam area is dewatered by pumping.

Types of coffer dam: Earth embankments, cantilever sheet pile, double wall coffer dam, braced coffer dams.

Shallow Foundation

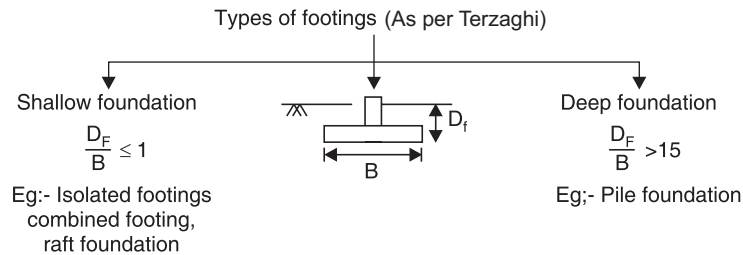
16

Footings are the lower most supporting part of the structure known as sub-structure and are last structural elements through which load is transferred to foundation comprising soil/rock.



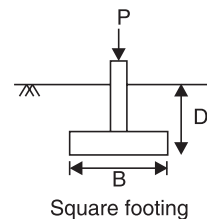
General requirement for foundation

- (a) Foundation must be safe against shear failure.
- (b) Settlement of the foundation should be within permissible limits.
- (c) Foundation should be located at such depth, such that its performance is not affected by seasonal volume changes.



Basic Definitions

1. **Bearing Capacity:** The load carrying capacity of foundation soil or rock which enables it to bear and transmit loads from the structure.
2. **Gross Pressure or Gross Loading Intensity (q_g):** It is the total pressure at the base of the footing due to weight of the super-structure,



self weight of the footing and weight of the earth fill.

$$q_g = \frac{P}{B^2} + \gamma D_f$$

- 3. Net Pressure Intensity:** Its the excess of gross pressure to over burden pressure.

$$q_{\text{net}} = q_g - \gamma D_f$$

For safe design

$$\frac{P}{B^2} \leq q_{\text{net-allowable}}$$

Note: If excavation is not backfilled (i.e. in case of basements)

$$q_{\text{net}} = \frac{P}{B^2} - \gamma D_f \quad \text{and} \quad \frac{P}{B^2} - \gamma D_f \leq q_{\text{net-allowable}}$$

In this case load carrying capacity of the soil is increased. If the raft is constructed at the depth such that $\frac{P}{B^2} - \gamma D_f = 0$, then soil is called

upon not to resist any load. Such as raft is called fully **compensated raft** or floating raft.

- 4. Ultimate bearing capacity (q_u):** The maximum gross intensity of loading that the soil can support before it fails in **shear** is called ultimate bearing capacity.
- 5. Net ultimate bearing capacity:** It is the minimum net pressure causing shear failure of soil.

$$q_{\text{nu}} = q_u - \gamma D_f = \frac{P}{B^2}$$

- 6. Net Safe bearing Capacity:**

$$q_{\text{ns}} = \frac{q_{\text{nu}}}{\text{FOS}}$$

where FOS of 2-3 is adopted.

- 7. Gross safe bearing Capacity:**

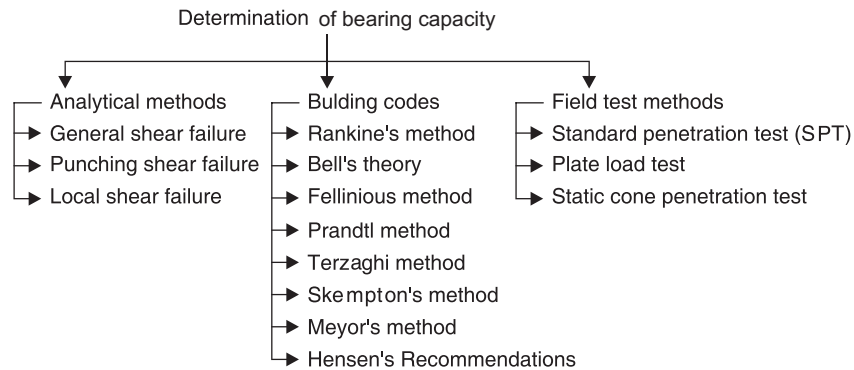
$$q_s = \frac{q_{\text{nu}}}{\text{FOS}} + \gamma D_f$$

- 8. Safe bearing Pressure:** Maximum net intensity of loading that can be allowed on soil without the **settlement** exceeding the permissible value.

Note: No factor of Safety is used when dealing with settlement.

9. Allowable bearing pressure: Maximum net intensity of loading that can be imposed on the soil with no possibility of **shear** failure or the possibility of **excessive** settlement.

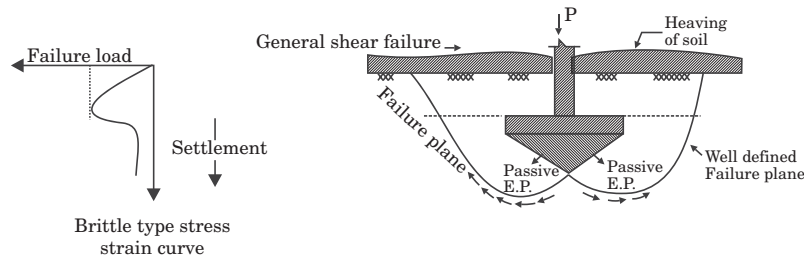
Note: It is smaller of Net safe bearing capacity and safe bearing pressure.



Analytical Method's

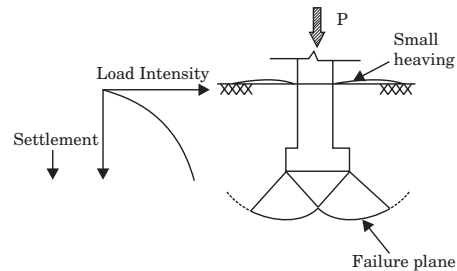
Based on shear failure criteria

1. General Shear Failure:



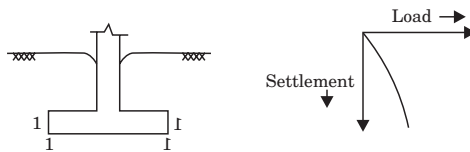
- Brittle type shear-stress curve
- Over consolidated clay with low compressibility
- Well defined failure pattern
- Failure due to tilting of foundation
- Occurs in soil with relative density $> 70\%$
- Occurs after plastic equilibrium state is reached.

2. Local Shear Failure:



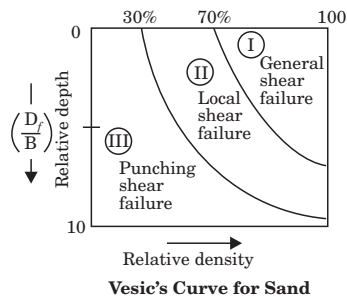
- Partial development of state of plastic equilibrium
- Slight heaving of soil adjacent to foundation
- Foundation doesn't tilt
- Failure is not sudden
- Plastic stress-strain curve
- Occurs in loose sand with relative density 30–70%

3. Punching Shear Failure:



- Shearing in vertical direction around the edge
- No heaving of adjacent soil
- No tilting of foundation
- Very loose sand with relative density less than 30%
- Deep foundations generally fail by punching only

Parameter	General shear failure	Local Shear failure
1. Friction angle(ϕ)	$> 36^\circ$	$< 28^\circ$
2. Strain at failure	$\leq 5^\circ$	$\geq 15^\circ$
3. SPT number	> 30	< 5
4. Relative density	$> 17\%$	$< 20\%$
5. Void ratio	< 0.55	> 0.75
6. Unconfined Compressive Strength	$> 100 \text{ KN/m}^2$	$< 80 \text{ KN/m}^2$



$$q_u - \gamma D_f \tan^4 \left(45 + \frac{\phi}{2} \right)$$

The diagram illustrates a pile foundation system. A pile is shown with a diameter D_f . Below the pile, a rectangular area is subjected to a uniform downward load q_u . This area is divided into two parts, labeled 1 and 2, which are subjected to horizontal stresses σ_h .

$$q_u = \mathbf{C}\mathbf{N}_c + \gamma\mathbf{D}_f\mathbf{N}_q$$

The diagram shows a semi-circular failure surface of radius b below a horizontal ground surface. A linearly increasing load q_{ul} is applied to the ground surface, starting from zero at the left edge of the failure zone and increasing to a maximum value q_{ul} at the right edge. The failure zone is bounded by a vertical line on the left and a vertical line on the right, separated by a distance b . The center of the semi-circle is at the bottom. A dashed line represents the failure surface. A point O is marked at the center of the critical circle, which is the center of the semi-circle. The angle between the horizontal line and the line connecting the origin of the load to the center of the critical circle is $23^{\circ}12'$. The ground surface is indicated by a horizontal line with cross-hatching on both sides of the failure zone.

$$q_{ult} = \frac{Wlr + CR}{b \cdot lo}$$

$$q_u = \text{CN}_c + r\text{D}_f \text{N}_q + 0.5 \text{ Br N}_r \quad \text{for strip footing}$$

- General shear failure governed by Mohr's criteria
- Strip footing at shallow depth ($D/B \leq 1$)
- Shear resistance of soil between ground surface and footing base is neglected

Zone I → Zone of elastic Equilibrium

Zone II → Radial Shear Zone

For C-soil → circular shape

C – ϕ soil → logarithmic shape

Zone III → Passive plastic Equilibrium with $\left(45 - \frac{\phi}{2}\right)$ angle with the Horizontal.

For strip footing,

$$q_u = CN_c + \gamma D_f N_q + 0.5 B \gamma N_r$$

$$q_{nu} = \underbrace{CN_c}_{\text{Cohesion}} + \underbrace{(\gamma D_f - 1)N_q}_{\text{Overburden}} + \underbrace{0.5 B \gamma N_r}_{\text{Soil in Shearing Zone}}$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_q = \frac{a^2}{2 \cos^2 (45 + \phi/2)}$$

$$a = e^{(3\pi/4 - \phi/2) \tan \phi}$$

$$N_r = \frac{1}{2} \tan \phi \left[\frac{K_{p\gamma}}{\cos^2 \phi} - 1 \right]$$

Note: N_c , N_q , N_r are the function's of ϕ only. Even K_p (passive earth pressure coefficient) is also function of ϕ . Hence the bearing capacity increases as the value of ϕ increases.

$$\text{For clayey soil } (\phi = 0) \quad N_c = 5.7 \quad N_q = 1 \quad N_r = 0$$

Modification in Terzaghi's Equation

1. For Source footing

$$q_{nu} = 1.3 CN_c + q (N_q - 1) + 0.4 B \gamma N_r$$

2. For Circular footing

$$q_{nu} = 1.3 CN_c + q (N_q - 1) + 0.3 B \gamma N_r$$

3. For rectangular footing

$$q_{nu} = \left(1 + \frac{0.3 B}{L}\right) CN_c + q(N_q - 1) + \left(1 - \frac{0.2 B}{L}\right) (0.5 B \gamma N_r)$$

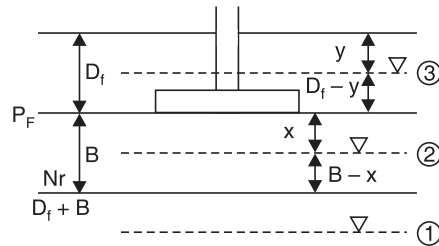
4. For local shear failure ($\phi < 29^\circ$)

$$C_m \rightarrow \frac{2}{3} C \quad (\tan \phi)_m = \frac{2}{3} \tan \phi$$

From C_m and ϕ_m find N'_C, N'_q, N'_r

$$q_{nu} = \frac{2}{3} C N'_C + q(N'_q - 1) + 0.5 B \gamma N'_r$$

5. For water table, in strip footing



- (a) For (1) $\rightarrow$ water table below $D_f + B$

$$q_{nu} = C N'_C + \gamma_t D_f (N'_q - 1) + 0.5 B \gamma_t N'_r$$

- (b) For (2) $\rightarrow$ water table between D_f and $D_f + B$

$$q_{nu} = C N'_C + \gamma_t D_f (N'_q - 1) + 0.5 (x \gamma_t + (B - x) \gamma_{sub}) N'_r$$

- (c) For (3) water table between 0 and D_f

$$q_{nu} = C N'_C + [\gamma_t y + (D_f - y) \gamma_{sub}] (N'_q - 1) + 0.5 B \gamma_{sub} N'_r$$

Another way of modification due to water table

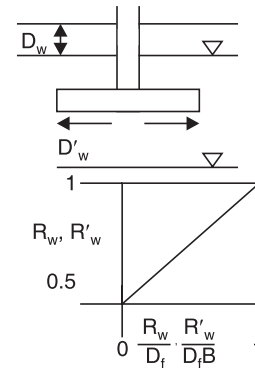
$$q_{nu} = C N'_C + D_f r_t (N'_q - 1) R_w + 0.5 B r_t N'_r R'_w$$

$$R_w = 0.5 \left(1 + \frac{D_w}{D_f} \right)$$

when $0 < \frac{D_w}{B} \leq 1$

$$R'_w = 0.5 \left(1 + \frac{D'_w}{B} \right)$$

when $0 < \frac{D'_w}{B} < 1$



6. **Skempton's Method** (C-soil)

$\Rightarrow$ Applicable only for saturated clay

$\Rightarrow$ Applicable to deep foundations also.

$$q_{nu} = CN_c$$

'C' can be found out from

- (i) U-U test
- (ii) Uncombined compression test
- (iii) Vane shear test

For value of NC

(a) Strip footing

$$N_c = 5 \left(1 + 0.2 \frac{D_f}{B} \right), \frac{D_f}{B} < 2.5 \quad \text{for } \frac{D_f}{B} \geq 2.5 \quad N_c = 7.5$$

(b) Source/circular footing

$$N_c = 6 \left(1 + 0.2 \frac{D_f}{B} \right), \frac{D_f}{B} < 2.5 \quad \text{For } \frac{D_f}{B} \geq 2.5 \quad N_c = 9$$

(c) For rectangular footing

$$N_c = 5 \left(1 + 0.2 \frac{D_f}{B} \right) \left(1 + 0.2 \frac{B}{L} \right), \frac{D_f}{B} < 2.5$$

$$\text{For } \frac{D_f}{B} \geq 2.5 \quad N_c = 7.5 \left(1 + \frac{0.2 B}{L} \right)$$

7. Meyerhoff's Method (C-φ soil)

Applicable for both shallow as well as deep foundation.

Note: Failure surface is assumed to go above the foundation level.

$$q_u = CN_c S_c d_c i_c + q N_q S_q d_q i_q + 0.5 B \gamma N_\gamma S_\gamma d_\gamma i_\gamma$$

where S, d, i are empirical correlation factors for shape, depth, inclination. For φ-soil $N_c = 5.14$ $N_q = 1$ $N_\gamma = 0$

8. Hensen's Recommendations (φ-soil)

$$q_u = CN_c S_c d_c + q N_q S_q d_q i_q + 0.5 B \gamma N_\gamma S_\gamma d_\gamma i_\gamma$$

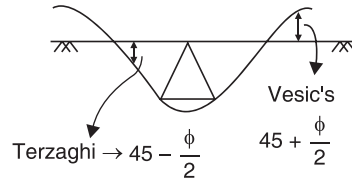
$$\text{For } \phi = 0; \quad q_{nu} = CN_c (1 + S_c + d_c - i_c)$$

9. Vesic's Bearing Capacity:

Failure surface assumed by Vesic is similar to Terzaghi but the angle of inclination of failure surface with the horizontal is

$$45 + \frac{\phi}{2} \text{ rather than } 45 - \frac{\phi}{2} \text{ as}$$

given by Terzaghi.



10. **I.S. Code Method** (IS 6403–1981):

$$q_{nu} = CN_c (S_c d_c i_c) + q(N_q - 1) (S_q d_q i_q) + 0.5B\gamma N_\gamma (S_\gamma d_\gamma i_\gamma) W'$$

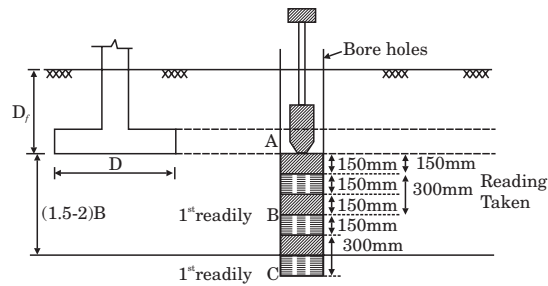
$N_c, N_q, N_\gamma \rightarrow$ From Vedic's equation

$W' \rightarrow$ water table correction factor

$\Rightarrow W' = 1$ if water table below $D_f + B$

$\Rightarrow W' = 0.5$ if water table at D_f

$\Rightarrow$ Interpolation if water table between D_f and $D_f + B$

Field Tests1. **Standard Penetration Test**

$\Rightarrow$ For Granular soils only

$\Rightarrow$ N-value is determined at selected number of bore holes and avg. value of corrected N is calculated for the depth from $D_f + (1.5 - 2)B$.

$\Rightarrow$ Any value greater than 50% of the avg. value is discarded and new avg. value is found out.

$\Rightarrow$

N	ϕ	Relative Density
< 4	25-30	0
4-10	27-32	15

$\Rightarrow$ Over burden Correction

$$N_1 = N_0 \left(\frac{350}{\bar{\sigma} + 70} \right)$$

$N_0 =$ observed SPT value

$N_1 =$ Corrected N value of overburden

$$\bar{\sigma} \leq 280 \text{ KN/m}^2 \quad \text{So } N_1 > N_0$$

$\Rightarrow$ Dilatancy correction required only in saturated fine sand or silt

$\Rightarrow$ More significant in case of fine dense sand ($N_1 > 15$) as it has tendency to dilate under rapid loading and –ve pore pressure will develop.

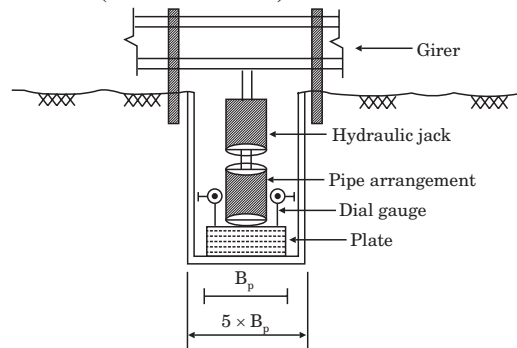
$$N_2 = 15 + \frac{1}{2}(N_1 - 15)$$

⇒ **Teng's Formula** for q_{nu} in granular soil.

$$q_{nu} = \frac{1}{6} (3N^2 BR_w' + 5(100 + N^2) D_f R_w). \text{ Continuous or strip footing}$$

$$q_{nu} = \frac{1}{3} (N^2 BR_w' + 3(100 + N^2) D_f R_w) \text{ Square or circular footing}$$

2. Plate load test: (IS 1888-1992)



It is used to calculate

- (a) Ultimate bearing capacity
- (b) Allowable bearing capacity
- (c) Safe settlement of foundation

⇒ Significant only for cohesionless soil

For Clayey soil

$$\frac{q_{uf}}{S_f} = \frac{q_{up}}{S_p}$$

For Granular Soil

$$\frac{q_{uf}}{q_{up}} = \frac{B_f}{B_p}$$

$$\frac{S_f}{S_p} = \left[\frac{B_f}{B_p} \left(\frac{B_p + 0.3}{B_f + 0.3} \right) \right]^2 \quad B_f, B_p \text{ in metres}$$

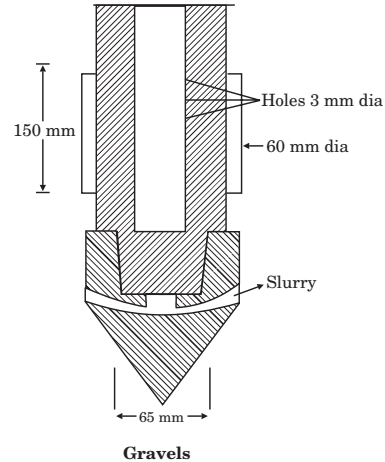
Depth Correction

$$S_f(\text{correction}) = S_f \left(\frac{1}{1 + \frac{D_2}{B_f}} \right)^{0.5}$$

D_2 = depth of foundation from the level at which plate load test is carried out.

3. State cone penetration test (CPT):

- ⇒ Particularly for soft clays, silts and fine to medium sand deposits.
- ⇒ Continuous record of soil resistance
- ⇒ Cone area 10 cm², Apex angle 60°
- ⇒ Rate of pushing cone 20 mm/sec upto depth of 100 mm.



Allowable Bearing Capacity ($Q_{A \text{ net}}$)

It is the **minimum** of bearing strengths obtained from shear failure criteria and settlement criteria. Empirical relations for Q_A are:

1. Peak Henson's Formula

$$q_{a(\text{net})} = 0.44 \text{ NS } C_w$$

$$C_w = 0.5 \left(1 + \frac{D_w}{D_f + B} \right)$$

N = corrected SPT N-value

S = Permissible settlement (mm)

C_w = Water table correction factor

D_w = depth of WT below ground

2. Teng's formula:

$$q_{a(\text{net})} = 1.4(N-3) \left(\frac{B+0.3}{2B} \right)^2 \cdot S \cdot C_w \cdot C_D$$

C_D = depth correction factor

$$C_w = 0.5 \left(1 + \frac{D_w}{B} \right)$$

$$C_D = \left(1 + \frac{D_f}{B} \right) \leq 2$$

3. I.S. Method for Raft:

$$q_{\text{net}}(\text{safe}) = 0.88 \text{ N S } C_w$$

$C_w \rightarrow$ from Peek henson's

Settlement of foundation:

$$S = S_{\text{immediate}} + S_{1^{\circ}} + S_{2^{\circ}}$$

$$\text{Where } S_{\text{immediate}} = \frac{q_n B(1 - \mu^2)}{E_s} \times I_f$$

$S_{\text{immediate}} \Rightarrow$ Elastic settlement for both sandy and clayey soil
 μ = Poisson's ratio

E_s = modular of elasticity

I_f = Influence factor = $f(\text{shape, rigidity of structure})$

E_s can be calculated from Triaxial tests or field tests.

$$S_{\text{rigid immediate}} = S_{\text{flexible immediate}} \times 0.8$$

Deflected Shape of Footings

	Clayey Soil	Granular soil
Flexible Footing Settlement varies Pressure constant		
Rigid Footing Settlement constant Pressure varies		

Permissible Settlements in Shallow Foundation

	Total settlement (mm)	Differential Settlement (mm)	Angular distortion (mm)
Isolated footing on clay	65	0.0015L	1/666
Isolated footing on sand	40	0.0015L	1/666
Raft on clay	65-100	0.0021L	1/500
Raft on sand	40-65	0.0021L	1/500

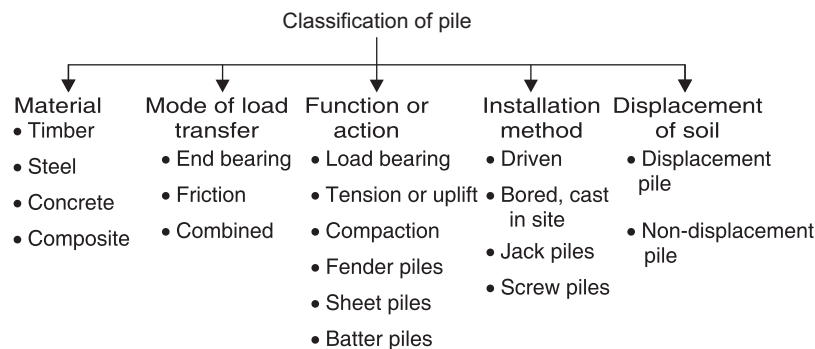
Deep Foundation

17

When the soil at shallow depth is poor and the load to be transmitted is heavy then the depth of the foundation has to be increased till the suitable soil strata is met, such foundation are called deep foundations.

Pile: Small dia shaft which is driven or bored into ground.

Piers and wells: large diameter shafts constructed by excavation and sunk to required depth.



End bearing piles: Used in stiff clay. Dense sand.

Friction pile: Used in soft soil, clay.

Tension or uplift piles: anchor structures subjected to **hydrostatic pressure or overturning moment**.

Compaction pile: compact loose granular soil

Anchor pile: Anchor against **horizontal pull** from water or sheet piling.

Fender pile: protect water-front structures against **impact from ships** and other floating objects.

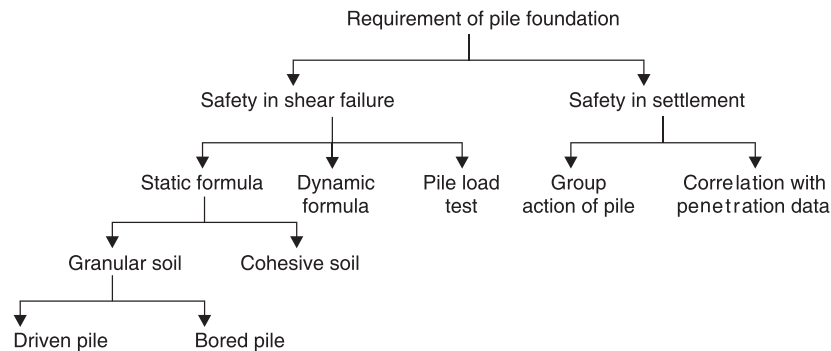
Sheet piles: Used as cut-offs to reduce seepage and uplift in hydraulic structure.

Batter piles: Resist horizontal and inclined forces, especially in water front structure.

Displacement piles: It displaces the pile laterally or upward. In loose sand, a pile densifies the sand upto a distance of 3.5 times the diameter of pile measured from the centre of pile. In case of clays, large displacement piles remould the soil to a distance of 2 times the diameter of pile.

Non-displacement piles: Are bored piles. Such piles are formed in the soil by boring or excavation and then these voids are filled with concrete sides are either supported by casing or by Bentonite slurry.

Note: Driven concrete piles are generally of diameter upto 500 mm. But bored piles may be even 2-3 m.



1. Static formula

$$Q_u = Q_{pu} + Q_f$$

Q_u = Ultimate load

Q_{pu} = Ultimate point load

Q_f = Ultimate skin friction.

$$Q_{pu} = q_{pu} \cdot A_b$$

$$Q_f = F_s \cdot A_s$$

q_{pu} = Unit point bearing resistance

A_b = Area of base

F_s = Unit skin friction resistance

A_s = Surface area of pile in contact with soil.

So $Q_u = q_{pu} A_b + F_s A_s$

For C - ϕ soil $q_{pu} = CN_c + \gamma D_f N_q + 0.5 B \gamma N_\gamma$

Neglect $0.5 B \gamma N_\gamma$ wrt. γD_f as $B \ll D_f$

Hence, $q_{pu} = CN_c + \gamma D_f N_q$

for C - soil $q_{pu} = CN_c$

for ϕ - soil $q_{pu} = \gamma D_f N_q$

(a) static formula in driven granular soil.

$$Q_u = (\gamma D_f N_q) A_b + F_s A_s$$

where $F_s = K \sigma_{avg} \tan \delta$

For loose sand to from loose to medium sand $D_f = 15$ (dia of pile)

For dense sand $D_f = 20$ (dia of pile)

For values of K and S

Pile material	δ	Value of K	
		Loose	Dense
Steel	20°	0.5	1
Concrete	3/4 ϕ	1	2
Timber	2/3 ϕ	1.5	4.0

Note: As per IS code $\delta = \phi$ and $K = 1 - 3$ for driven piles in loose to medium sand

When there are several layers of soil then

$$Q_p = K \tan \delta (\sigma_{avg1} \cdot A_1 + \sigma_{avg2} A_2 + \dots + \sigma_{avgn} A_n)$$

(b) Static formula in bored granular soil.

Point bearing of bored cast in situ piles = $\frac{1}{2} \times$ point bearing resist-

ance of driven piles.

Due to boring, the value of K becomes very small and taken as 0.5. Rest all calculations as above.

Note: Maximum upper limit of frictional stress (f_s) is = 110 KN/m²

Maximum upper limit of point bearing stress.

(a) 11000 KN/m² → silica sand

(b) 5000 KN/m² → calcareous sand

(c) Static formula for piles in clay.

$$Q_u = (C_{ub} N_c) A_b + (\alpha C_u) A_s$$

C_{ub} = Undrained cohesion at the base of the pile

C_u = Undrained cohesion in the embedded length of pile.

$N_c = 9$ (By **skempton's**).

α = Depends on adhesion between soil and pile called adhesion factor

$\alpha = 0.1 \rightarrow$ for very loose clays

$\alpha = 0.3 \rightarrow$ for very stiff clays

Note: Smaller the undrained strength, softer is the consistency and greater is the tendency to adhere to the pile.

2. Dynamic formula: It is based on resistance to penetration hence used in driven piles only

Energy Imparted = Work done in pile driving

$$Q_u \times S = W \times H$$

(a) Engineering News Formula

$$Q_{\text{allowable}} = \frac{WH}{\text{FOS} (S + C)}$$

W = load in Kg

H = Height of fall in cm

S = Settlement/blow in cm

= It is corresponding to last 5-blows of drop hammer also called, as real set per blow.

= last 20 blows of steam hammer.

C = Emperical factor

= 2.5 cm for drop hammer

= 0.25 cm for single acting steam hammer.

FOS = 6

$$\text{Another form: } Q_{\text{KN}} = \frac{166.64 E_{\text{KJ}}}{S + 2.54}$$

S = Settlement/Avg penetration for last 100 mm of driving per blow. Minimum permissible value of S = 1.25 mm.

(b) Modified Hilly Formula:

$$\text{Ultimate Driving Resistance (R)} = \frac{Wh\eta}{S + \frac{C}{2}}$$

W = Weight of hammer (Tonnes)

h = Ht of fall (cm)

S = Final set per blow (last – one – cm)

C = Total elastic compression per blow ie of soil

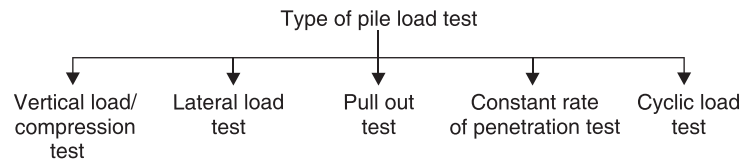
+ pile + Dolly

η = Efficiency of blow.

3. Pile load test: (IS 2911) It is the only direct method for determining the allowable loads on the piles and, is considered to be most reliable as it is an insitu test.



Note: As per IS code, for more than 200 piles there should be a minimum of two initial test where as routine test is done on 0.5% to 2% of total number of piles.

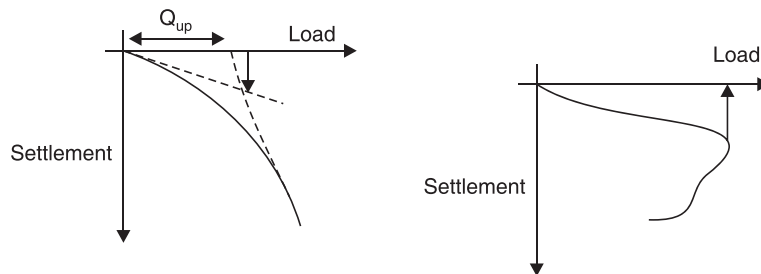


Cyclic load: test is carried out to determine **skin friction and end bearing separately** for a pile load on a single pile. It is generally an initial test.

Note: Test pile is a pile which is especially bored for the purpose of conducting test and will not be the part of foundation in future.

While working pile is a pile which is a part of foundation and is being used for the purpose of testing at present.

Ultimate load will be calculated from the load settlement curve



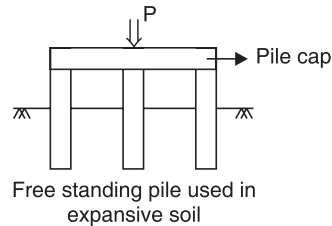
Allowable load on single pile: Will be the minimum of the following cases.

- (i) 50% of the ultimate load at which total settlement is equal to the 1/10 of the pile diameter.
- (ii) 2/3rd of the load at which total settlement is 12 mm.
- (iii) 2/3rd of the load at which net settlement is 6 mm (total settlement – elastic settlement)

Group Action of Piles

When piles are driven there is uncertainty regarding vertical installation of piles.

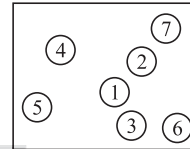
Hence minimum number of piles driven are 3. where in case of bored piles single pile can also be used.



Note: To void tightening of ground, pile in sand should begin at centre and then moved out-ward. (numbering in while piles to be driven)

$$\text{Group efficiency } \eta = \frac{Q_{ug}}{n Q_u}$$

Q_{ug} = ultimate load capacity of pile group
 Q_u = ultimate load capacity of single pile
 n = No. of piles.



Note: Minimum spacing between piles according to I.S code

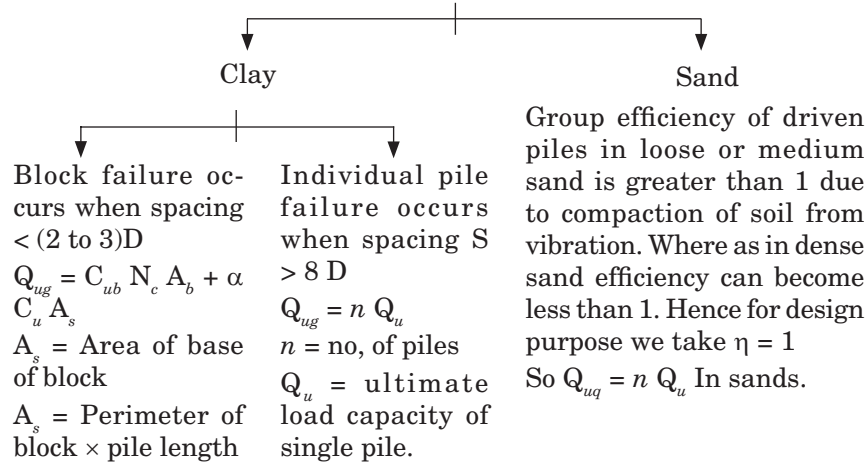
For loose sand or back filled soil = $2 \times \text{Diameter}$

For point bearing piles = $2.5 \times \text{Diameter}$

For friction piles = $3 \times \text{Diameter}$.

In case of non-circular piles, diameter of the circumscribed circle is taken as diameter

Ultimate bearing capacity of pile group.



Safe load capacity

$$Q_s = \left[\frac{\text{minimum of } [Q_{ug}, n Q_u]}{\text{F.O.S}} \right]$$

Converse labarre pile group efficiency

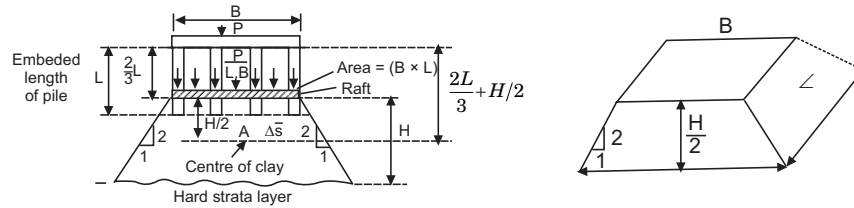
$$\eta_g = 1 - \frac{\theta}{90} \left[\frac{(n-1)m + (m-1)n}{mn} \right]$$

$$\tan \theta = \frac{d}{s}$$

 d = diameter of pile s = centre to centre spacing. m = no. of rows n = no. of piles in a row.**Settlement of pile groups**

As the zone of influence of pile group is generally more than the individual pile so settlement of pile group is generally greater than the settlement of individual pile to same loading. (ie same load per pile)

1. Settlement of pile group in clays

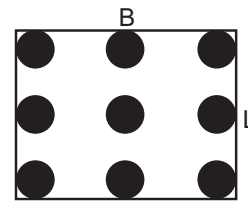


(a) When pile is in uniform clay deposit

$$\Delta H = \frac{C_c}{1 + e_o} H \log \left(\frac{\bar{\sigma}_o + \Delta \bar{\sigma}}{\bar{\sigma}_o} \right)$$

$$\bar{\sigma}_o = \left(\frac{2}{3} L + \frac{H}{2} \right) \gamma_t$$

$$\Delta \bar{\sigma}_o = \frac{P}{\left(B + \frac{H}{2} \right) \left(L + \frac{H}{2} \right)}$$



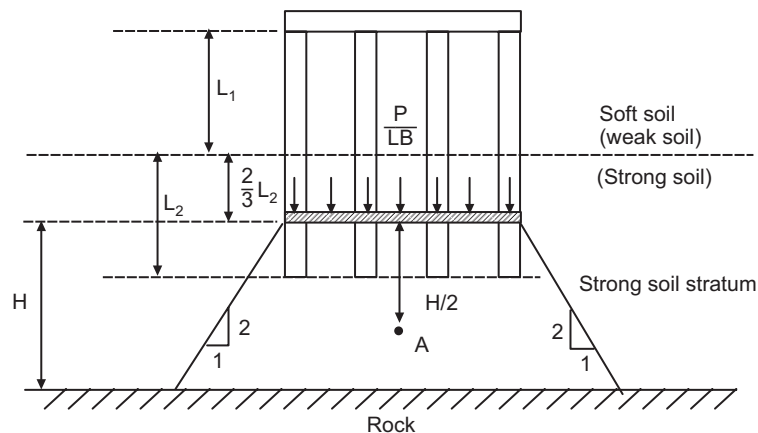
(b) When piles are driven into strong stratum through an overlaying weak stratum

Here depth H is measured from $\frac{2}{3} L_2$ to the bottom solid surface

where L_2 is the depth of embedment in strong soil.

$$\Delta H = \frac{C_c}{1 + e_o} H \log \left(\frac{\bar{\sigma}_o + \Delta \bar{\sigma}}{\bar{\sigma}_o} \right)$$

where $\bar{\sigma}_o$ is measured at point A (mid depth of H)

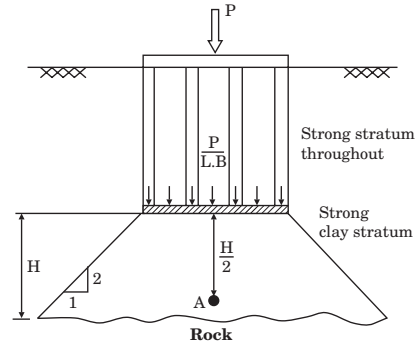


(c) In case of bored piles or end bearing piles resting on firm stratum

Here H is measured from the bottom of piles to the bottom hard strata

$$\Delta H = \frac{C_c}{1 + e_o} H \log \left(\frac{\bar{\sigma}_o + \Delta \bar{\sigma}}{\bar{\sigma}_o} \right)$$

$\bar{\sigma}_o$ = effective stress at point A



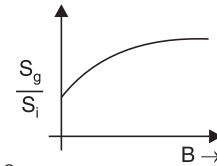
2. Settlement of pile groups in sand

$$\text{Group settlement ratio} = \frac{S_g}{S_i} = \left(\frac{4B + 2.7}{B + 3.6} \right)^2$$

S_g = Group settlement at the same load of pile group.

B = Size of pile group in meter.

S_i = Settlement of individual pile calculated from the pile load test



The field and laboratory studies carried out for obtaining the necessary information about the soil characteristics including the position of ground water-table is called soil-exploration.

Preliminary Stage

Geological study at site and site Reconnaissance.

Detailed Stage

In detailed site investigation, nature, sequence and thickness of various subsoil layers, their lateral variations, physical properties, position of ground water etc. are determined

Boring and its methods

Making and advancing of bore holes is called boring

Auger Boring

Done in partially saturated sands, silts and medium to stiff clays

Highly disturbed sample for identification only

Small depth of exploration e.g. Highways and borrow pits

Wash Boring

Not done in hard soils, rocks and soil containing boulders.

Disturbed samples are obtained

Rotary Boring

Mud rotary Boring

Core drilling (least disturbed samples)

Percussion Boring

Only method to be used in bouldery and gravelly stratum

Heavy drilling bit is dropped and raised.

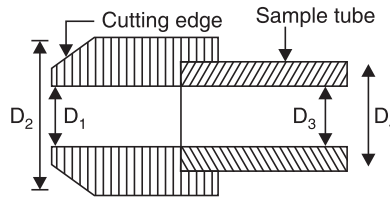
Soil Samples

Disturbed sample are those in which natural soil structure gets modified or destroyed during the sampling operation.

Undisturbed samples are those in which original soil structure is preserved as well as mineral properties have not undergone any change. These are needed in natural water content, density, specific gravity, grain size distribution, Atterberg's limits, coefficient of permeability, consolidation parameters, shear strength parameters.

(a) Inside clearance

$$C_i = \frac{D_3 - D_1}{D_1} \times 100 \quad 1-3\%$$



(b) Outside Clearance

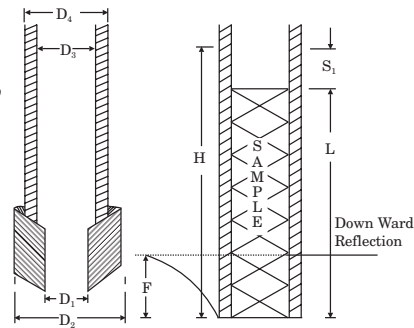
$$C_o = \frac{D_2 - D_4}{D_4} \times 100 \quad 0-2\%$$

(c) Area ratio

$$A_r = \frac{D_2^2 - D_1^2}{D_1^2} \times 100$$

< 20 For stiff clays

< 10 For sensitive clays



$$\text{Recovery Ratio} = \frac{L}{H}$$

$$= \frac{\text{length of sample before withdrawal}}{\text{Penetration of the sampler in the soil mass}}$$

$$\text{Gross Recovery Ratio} = \frac{L_1}{H}$$

L_1 = Gross length of sample, equal to the distance from the top of the sample to the cutting edge irrespective of whether the lower part of the sample is lost or otherwise.

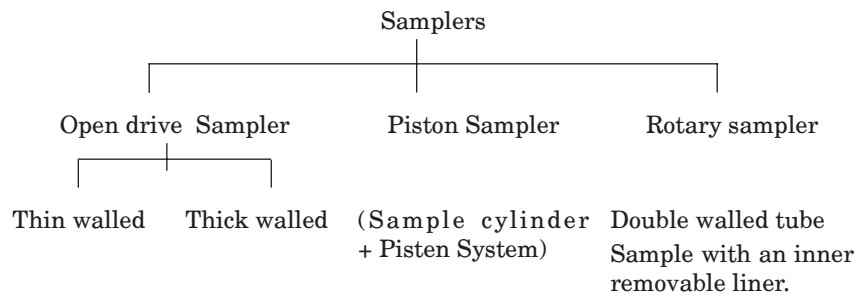
$$\text{True Recovery Ratio} = \frac{L_1}{H - F}$$

F = depth of downward Reflection curve

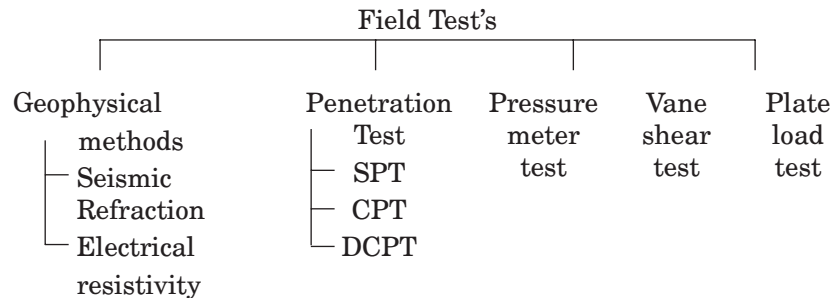
If Recovery Ratio = 1, Good recovery

< 1, compressed

> 1, Soil is swelled



Depth of Exploration: Unless a bed rock is encountered, boring should be carried out to such a depth that the net increase in stress in the soil mass due to superimposed external load is less than 10%. In case of square loaded area isobar of 10% intensity of loading is extended to a depth twice the width of foundation.



Seismic refraction method

$$D = \frac{d}{2} \sqrt{\frac{V_2 - V_1}{V_2 + V_1}}$$

U = depth between two stratum

d = distance between source and geophone

V_1 and V_2 = Sesmic velocities of lower and upper stratum.

Electric Resistivity Method

$$\delta = \frac{2\pi DV}{I}$$

δ = Apparent resistivity

D = distance between spikes in cm

V = potential difference in volts

I = Current in Amphere

δ = Resistance in Ohm-cm

Note: SPT is carried out in a bore hole where as CPT and DCPT are carried without bore holes.

Soil which has tendency to increase in volume in presence of water and decrease in volume in absence of water are called as expansive or swelling soils.

Identification of Expansive Soil

1. **Free Swell Test:** 10 cm³ of dry soil sample passed through 425 mm sieve, into a 100 cm³ graduated cylinder filled with water. Volume observed after 24 hours.

$$\text{Free Swell \%} = \frac{\text{Final Volume} - \text{Initial Volume}}{\text{Initial Volume}} \times 100$$

Mineral present in Clay	Free Swell%
Montmorillonite	1200–2000
Kaolinite	80
Illite	30–80

2. **Differential Free Swell Test:** Two samples of 10 gm, passing through 425 mm sieve into a 50 cm³ cylinder one containing kerosene (non polar liquid) while other carrying distilled water. Volume observed after 24 hours.

$$\text{DFS} = \frac{\text{Soil volume in water} - \text{Soil volume in kerosene}}{\text{Soil volume in kerosene}} \times 100$$

Degree of expansiveness	DFS%
Low	Less than 20
Moderate	20–35
High	35–50
Very High	Greater than 50

3. **Plasticity Index, Shrinkage limit and colloidal content:**

- Higher the plasticity index, higher will be swelling potential.
- Lower shrinkage limit means soil will start swelling at low water content.
- Higher the colloidal content, higher will be the expansion possibility.

4. **Swelling test:** Two identical undisturbed samples are obtained at a moisture content that is expected at construction time. Both specimen's are dried to a moisture content below shrinkage limit. Volume of one of them is measured. While the other specimen is first loaded to a pressure equal to that of the expected structural load and then allowed free access to water. The volume of other is measured.

5. **Swelling Pressure Test:** Pressure required to be applied over a swelling soil specimen to prevent its expansion when it comes in contact with water. This test is conducted in the odometer.

Note: If the swelling pressure is less than 20 KN/m^3 there is less expansiveness and shallow foundation can be used. Some soils as bentonite may have swelling pressure of the order of 200 KN/m^3 .

Design of Foundation on Swelling Soil

1. Structure are made strong and rigid to with stand the effect of swelling.
2. Structure are made flexible so that they change their shape according to swelling of soil.
3. Deep foundations such as belled piers and under reamed piles are constructed to isolate the foundation from the swelling effect of soil.
4. By providing an impervious apron surrounding the structure.

Elimination of Possibility of Swelling

1. Providing external loads greater than swelling pressure.
2. Pre-wetting soil mass to a moisture content equal to equilibrium moisture content.
3. Lime stablisation reduces the liquid limit and plastic limit hence it can also be used.

Civil Engineering

12

Irrigation

CONTENTS

1. Irrigation and its Methods
2. Soil-moisture Plant Relationship
3. Water Requirement of Crops
4. Canal-Design
5. Earthen-Dams
6. Canal-Irrigation
7. Water-Logging
8. Gravity Dams

Irrigation and its Methods

1

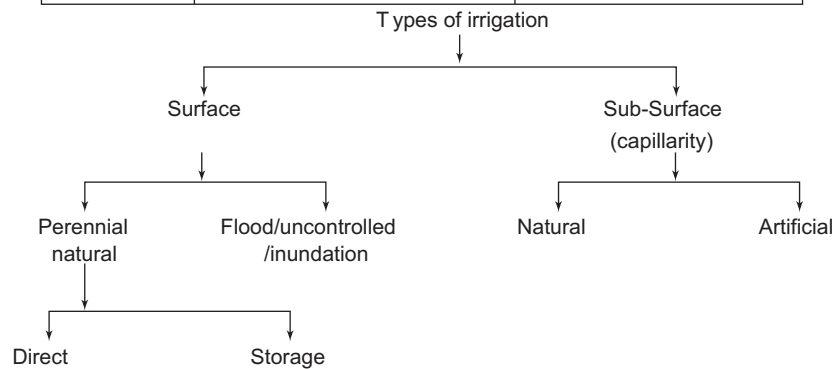
Irrigation is the science of artificial application of water to the land, in accordance with the crop requirements throughout the crop period for full-fledged nourishment of the crops.

Crop yield expressed in quintal/ha or tonnes/ha.

Productivity is expressed as crop yield per mm of water applied.

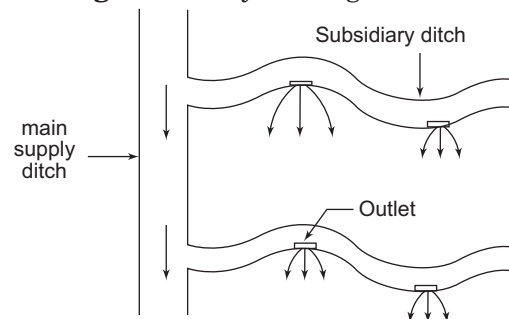
Types of irrigation projects

Projects	Irrigation Potential CCA	Cost of project
Major	> 10,000 ha	> 5 crore
Medium	2000 – 10,000 ha	0.5 – 5 crore
Small	< 2000 ha	0.25 – 0.5 crore



Methods of irrigation

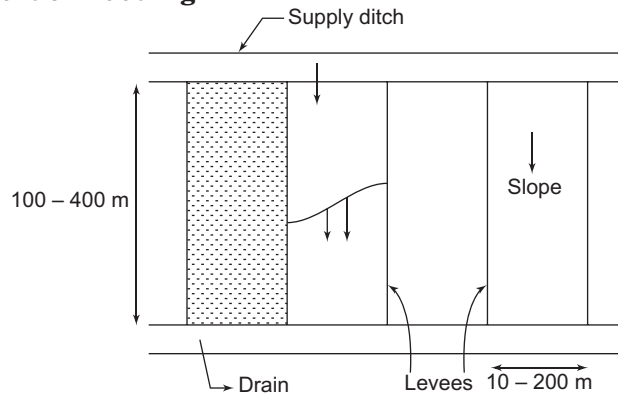
(1) Free flooding or ordinary flooding or wild flooding



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- used for rolling land where borders, checks, basins and furrows are not feasible.

(2) Border flooding



$$t = 2.303 \frac{y}{f} \log \frac{Q}{Q - fA}$$

Q = Discharge through supply ditch

A = Area of land strip to be irrigated

y = depth of water flowing over the border Strip.

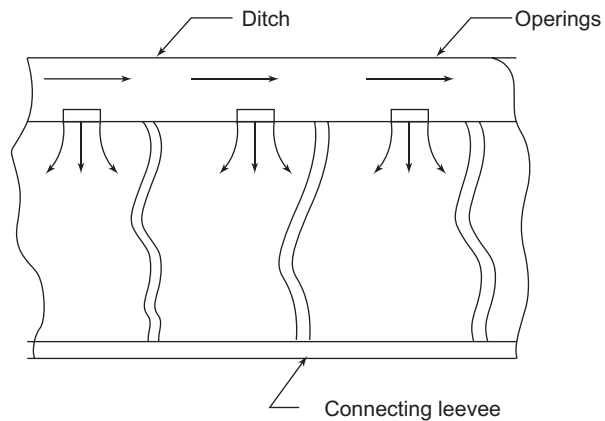
f = Rate of infiltration of soil.

t = Time required to cover the given Area A

$$A_{\max} = \frac{Q}{f}$$

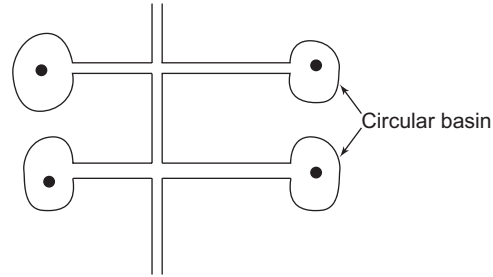
Note : Surface flow will stop after $A_{\max}$ and deep percolation will start.

(3) Check flooding



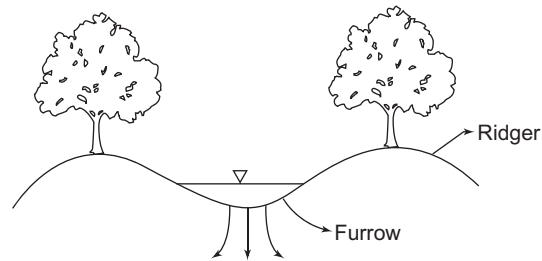
- Close growing crops like jowar & paddy.
- Deep homogeneous loam or clay soil with medium infiltration rate.
- Suitable for both permeable and less permeable soil.

(4) Basin flooding



- Special type of check flooding and adopted specially for orchard trees.
- Basin shape may be irregular, square, rectangular, circular.
- Not suitable for coarse sand.

(5) Furrow irrigation



- Less evaporation
- Less wastage of land
- Wide range of natural slopes
- Preferred in flat or gentle slopes.

(6) Sprinkler irrigation

- Not suitable for soil with low infiltration rates (eg clay).
- Best suited for very light soil.
- Generally used for Tea, Coffee, Not at all used in Rice or Jute.

Note : For rice and jute, standing water is used

(7) Drip irrigation

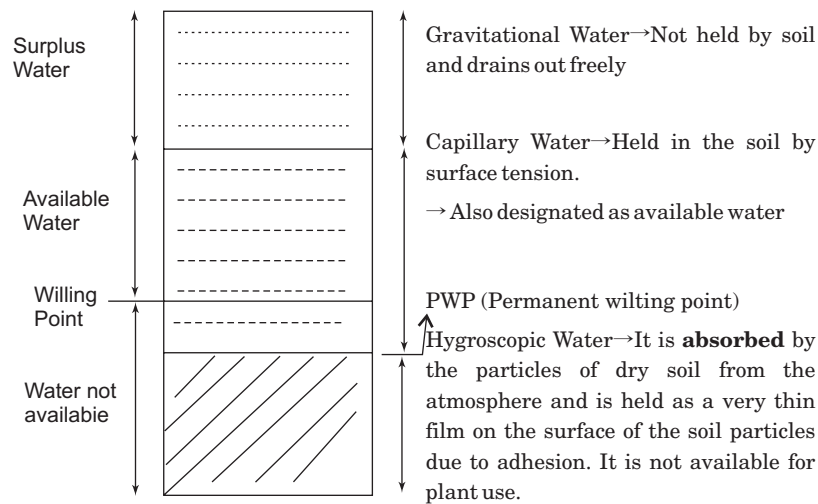
- Best suited for row crops and orchards like grapes, tomatoes, cabbage etc.
- Water and fertilizer is slowly and directly applied to the root zone of the plants.
- Achieved with the help of specially designed drippers and emitters.

Note : Sprinkler and drip irrigation systems falls under a category known as pressurised irrigation systems.

Soil-moisture Plant Relationship

2

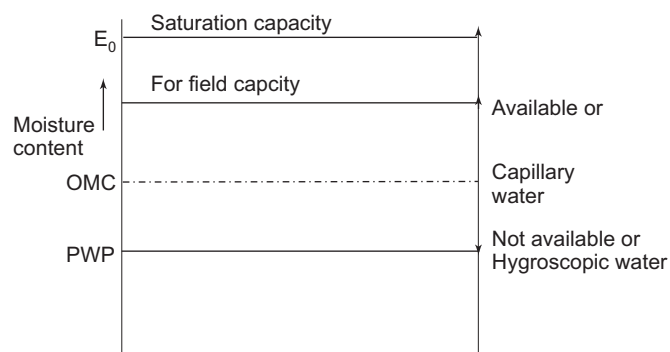
Classification of Soil water



Soil moisture tension: Tenacity with which water is retained in the soil, measured as force per unit area. Generally expressed in atmosphere.

$$\text{Soil moisture Stress} = \text{Soil moisture tension} + \text{Osmotic Pressure}$$

The Osmotic pressure of the soil solution must be maintained as low as possible by controlled leaching.



Saturation capacity → Max water holding capacity of the soil.

Field capacity → Max amount of moisture which can be held against gravity.

Soil moisture tension at permanent wilting point ranges from 7 to 32 atmospheres.

Depth of water stored in the root zone (d_w)

Field capacity = $\frac{\text{wt. of water retained in certain volume of soil}}{\text{wt. of same volume of dry soil}} \times 100$

$$d_w = \frac{\gamma_d}{\gamma_w} \times d \times F$$

F = Field capacity

d = depth of root zone

γ_d = dry unit wt. of soil

γ_w = unit wt. of water

Note:- γ_d is the unit weight of dried soil sample, not of soil solids.

Available moisture depth to plant, $d'_w = \frac{\gamma_d}{\gamma_w} \times d$ (F.C-PWP)

Readily available moisture depth to plant, $d''_w = \frac{\gamma_d}{\gamma_w} \times d$ (F.C-Readily available moisture)

$$\text{Porosity} = \text{Field capacity} \times \frac{\gamma_d}{\gamma_w}$$

Irrigation Water Quality:

(1) Total concentration of soluble salts

$$\text{Salinity concentration } (C_s) = \frac{\text{Conc. of salt} \times \text{Qty. of water applied}}{\text{Qty. of water applied} + \text{Amount of Rainfall-Qty. of water used}}$$

- Generally expressed in ppm or mg/l.
- Indirectly measured by determination of electrical conductivity expressed in micro mho's per centimetre

Electrical Conductivity	Class	Uses
<250	Low	For all crops
250-750	Medium	Used after leaching
750-2250	High	High salt tolerant plants
>2250	Very high	Not suitable

(2) Proportion of Sodium ions to other Cations

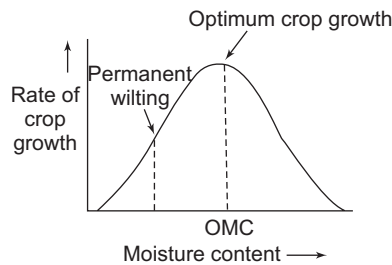
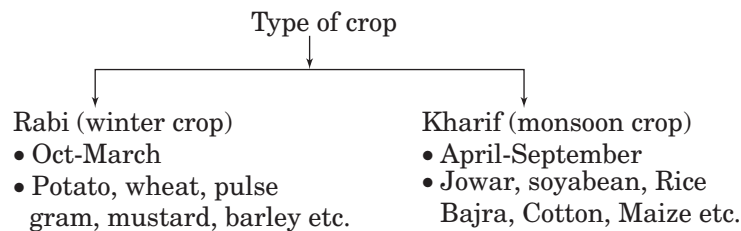
$$\text{Sodium Adsorption Ratio, SAR} = \frac{\text{Na}^+}{\sqrt{\frac{\text{Ca}^{++} + \text{Mg}^{++}}{2}}}$$

SAR	Type of water	Uses
0-10	Low sodium water (S_1)	For all crops
10-18	Medium sodium water (S_2)	Only in coarsed grained soil
18-26	High sodium water (S_3)	Used only after leaching
>26	Very high sodium water (S_4)	Not suitable

Note : High conc. of bi-carbonate ions may result in precipitation of calcium and magnesium bicarbonates, which will relatively increase the sodium concentration, and will become hazardous.

Water Requirement of Crops

3



Crop period: Time b/w sowing of crop and its harvesting.

Base period: Time b/w first watering and last watering done before harvesting.

Crop period > Base period

Duty: Area of land in hectares that can be irrigated when one cumec of water is supplied throughout entire base period. Expressed in hectare per cumec.

Delta: Total depth of water applied over an irrigated land at different watering throughout entire base period. Denoted by and expressed in cm or m.

$$\Delta = \frac{8.64}{D} B$$

$\Delta \rightarrow$ meters

$B \rightarrow$ Base period in days

$D \rightarrow$ hec/m^3

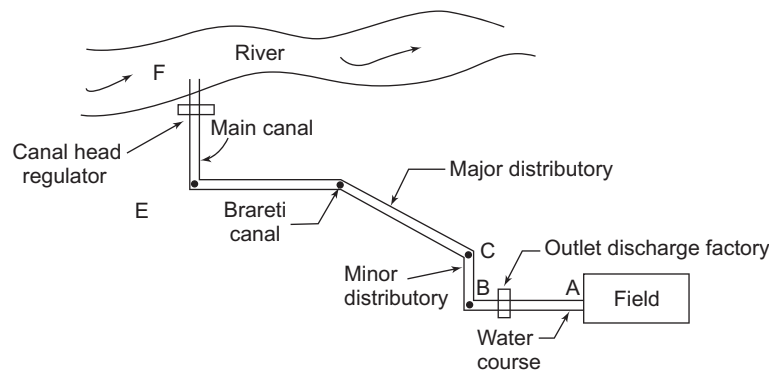
$$\text{Duty}_A > \text{Duty}_B > D_C > D_D \dots > D_F$$

If duty at A = D

And there are 20% losses

then,

Duty at F = 0.8D



Commanded area: Area to be irrigated by canal system.

Gross command area (GCA): Total area that can be irrigated if unlimited supply of water is available.

Culturable command area (CCA): That part of GCA which is fit for cultivation.

Intensity of irrigation: Percentage of CCA that is proposed to be cultivated annually.

Note: Intensity can be even greater than 100%. For Eg. If intensity of Rabi crop is 70% and that of Kharif crop is 50% then, Total intensity is 70+50 = 120%

Crop-Ratio: Ratio of area's of land irrigated in Rabi and Kharif season. It is to be selected such that discharge through canal remains uniform.

Paleo Irrigation: Watering done prior to sowing of crop.

Kor Watering (Kor depth or Kor period): First watering after the plants have grown few centimetres (similarly for other's)

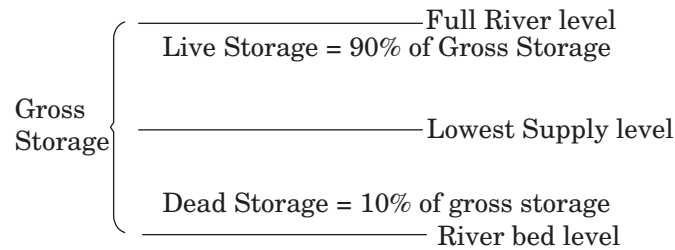
Outlet Factor: Duty of water at the head of field channel, also called outlet discharge factor.

Capacity Factor: Ratio of mean supply discharge of canal to maximum discharge capacity

Duty on capacity: Duty at the head of canal.

Time Factor: No. of days canal actually run to the total number of days of the watering period.

Cash crops: Crops that cannot be consumed directly by the cultivators. Crops like Jute, Tea, Cotton, Sugarcane, tobacco are excluded from the list of cash crops.



Irrigation Efficiencies

1. Water Conveyance Efficiency (η_c)

$$\eta_c = \frac{W_f}{W_r} \times 100$$

W_f = water delivered to field
 W_r = water diverted from River to canal

It accounts for losses in conveyance system.

2. Water Application Efficiency (η_a)

$$\eta_a = \frac{W_s}{W_f} \times 100$$

W_s = water stored in the root zone
 W_f = water delivered to field

It accounts for loss due to surface runoff and deep-percolation.

3. Water Use Efficiency (η_u)

$$\eta_u = \frac{W_u}{W_f} \times 100$$

W_u = water used beneficially including leaching

4. Water Storage Efficiency (η_s)

$$\eta_s = \frac{W_s}{W_a} \times 100$$

W_s = water stored in the root zone during irrigation

Where, W_n = Field capacity – Available moisture

5. Water distribution Efficiency (η_d)

$$\eta_d = \left(1 - \frac{y}{d}\right) \times 100$$

y = average numerical deviation in water depth.
 d = average depth of water stored in root zone.

6. Consumptive use Efficiency (η_{cu})

$$\eta_{cu} = \frac{W_{cu}}{W_d} \times 100$$

W_{cu} = consumptive use of water
 W_d = net amount of water depleted from root zone

Irrigation requirements of crops

1. Consumptive Irrigation requirement (CIR)

$$CIR = E_t - R_e$$

E_t = Evapotranspiration
 R_e = Effective Rainfall

2. Net Irrigation requirement (NIR)

LR = Leaching requirement

$$NIR = CIR + LR + PSR + NEW$$

PSR = Pre Sowing requirement

NSR = Nursery water requirement

3. Field Irrigation Requirement (FIR)

$$FIR = \frac{NIR}{\eta_a}$$

η_a = Application Efficiency

4. Gross Irrigation Requirement (GIR)

$$GIR = \frac{FIR}{\eta_c}$$

η_c = conveyance efficiency

Note: $GIR > FIR > NIR > CIR$

Canal-Design

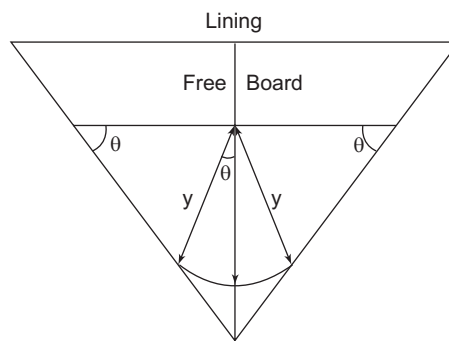
4

Design of lined canals

(1) Triangular section (For $Q < 150 \text{ m}^3/\text{sec}$)

$$A = y^2 (\theta + \cot \theta)$$

$$P = 2y (\theta + \cot \theta)$$

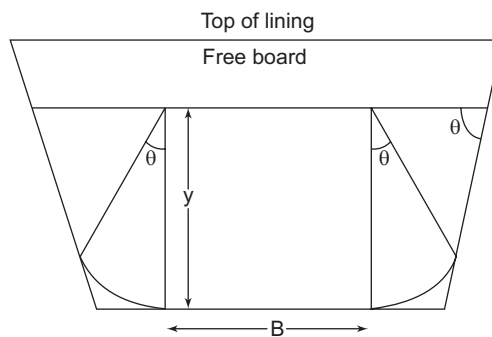


$$R = \frac{A}{P} = \frac{y}{2}$$

(2) Trapezoidal section (For $Q > 150 \text{ m}^3/\text{sec}$)

$$A = y (B + y\theta + y \cot \theta)$$

$$P = B + 2y (\theta + \cot \theta)$$



Economic justification of lining of canal:Benefit cost ratio ≥ 1

$$\text{Average Annual Benefits} = mR_1 + PR_2$$

 $m \rightarrow$ cumec of water saved due to lining $R_1 \rightarrow$ irrigation water sold to farmer's at ₹ R_1 /cumec $P \rightarrow$ % of saving achieved in maintenance cost by lining $R_2 \rightarrow$ Rate of maintenance cost in Rupees per year

$$\text{Annual cost of lining} = \frac{c}{y} + \frac{c}{2} \frac{r}{100}$$

 $c =$ Total initial investment $y =$ service life in years $r =$ rate of annual simple interest

$$\frac{\text{Benefit}}{\text{cost}} = \frac{mR_1 + PR_2}{\frac{c}{y} + \frac{c}{2} \left(\frac{r}{100} \right)} \geq 1$$

Design of unlined canal

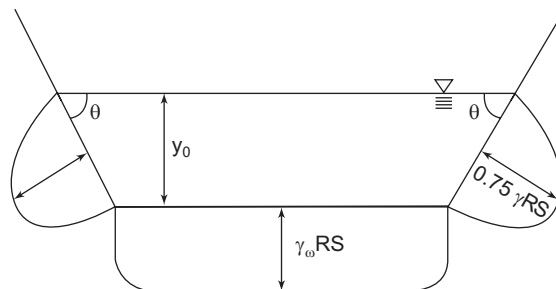
$$\tau_0 = \gamma RS$$

Where, $\tau_0 =$ Tractive force at the bottom of the channel

$$\frac{\tau'_0}{\tau_0} = 0.75$$

 $\tau'_0 =$ Average tractive force at the channel side

$$\tau_c = 0.056\gamma_\omega d (G_s - 1)$$



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τ_c = critical tractive stress at channel bottom

G_s = specific gravity of sediment ≈ 2.667

$$\tau'_c = \tau_c \sqrt{1 - \frac{\sin^2 \theta}{\sin^2 \phi}}$$

θ = side slope angle

ϕ = angle of repose of soil

Note: For no sediment movement from

(a) Channel bottom $\tau_0 \leq \tau_c$ i.e $d \geq 11$ RS

(b) Channel sides $\tau'_0 \leq \tau'_c$

Kennedy's theory

(1) $V_0 = 0.55 m y^{0.64}$ where

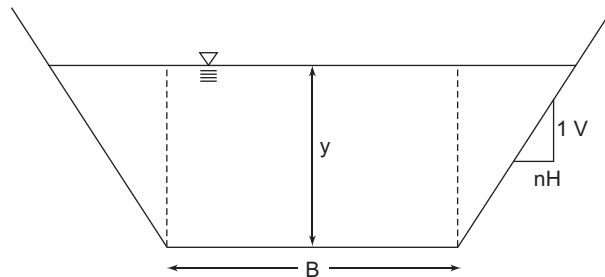
Critical velocity ratio, $m = \frac{V}{V_0} = \frac{\text{Actual mean velocity}}{\text{critical velocity}}$

$m = 1$, standard particles

$m = 1$ to 1.2, coarser sediments

$m = 0.7$ to 1, finer particles

(2)



$$A = (B + ny) y \quad \dots(1)$$

$$A = \frac{Q}{V_0} \quad \dots(2)$$

Get B from (1) and (2)

$$(3) P = B + 2y\sqrt{n^2 + 1}$$

$$(4) R = \frac{A}{P} \quad R = \text{Hydraulic mean depth}$$

$$(5) \quad V = C\sqrt{RS} \text{ where } C = \frac{\frac{1}{n} + \left(23 + \frac{0.00155}{S}\right)}{\frac{1}{n} + \left(23 + \frac{0.00155}{S}\right) \frac{n}{\sqrt{R}}}$$

or

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

(6) If $V \approx V_0$ then ok, otherwise repeat.

Lacey's theory:

He gave three regime concept (a) True (b) Initial (c) Final

- Equations given by Lacey are based on final regime concept.
- Total no of independent equations of Lacey's regime theory are 3.

$$(1) \quad v = \left(\frac{Qf^2}{140} \right)^{1/6} \text{ where, silt factor } f = 1.76 \sqrt{d_{\text{mm}}}$$

$$(2) \quad R = \frac{5}{2} \frac{V^2}{f}$$

$$(3) \quad A = \frac{Q}{V}$$

$$(4) \quad P = 4.75\sqrt{Q}$$

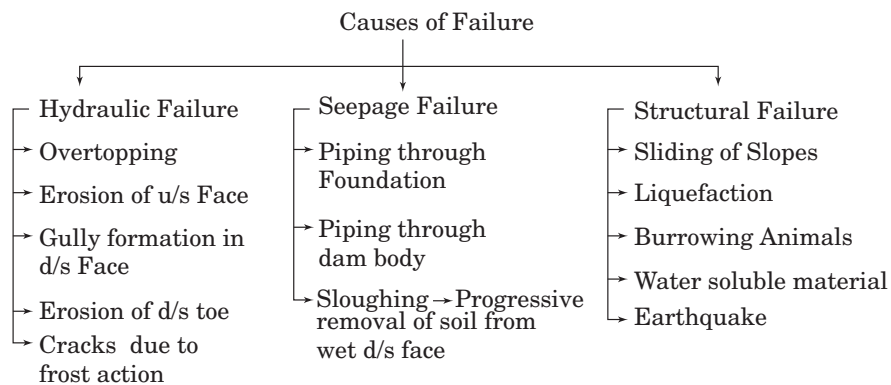
$$(5) \quad S = \frac{f^{5/3}}{3340Q^{1/6}}$$

$$(6) \quad \text{Lacey's regime scour depth} = 1.35 \left(\frac{q^2}{f} \right)^{1/3}$$

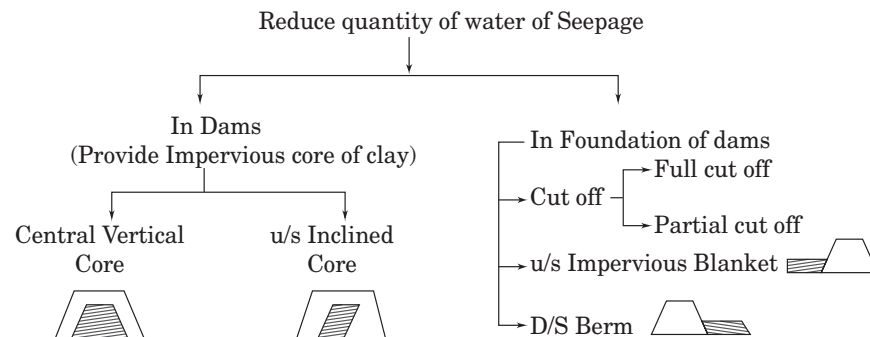
Earthen-Dams

5

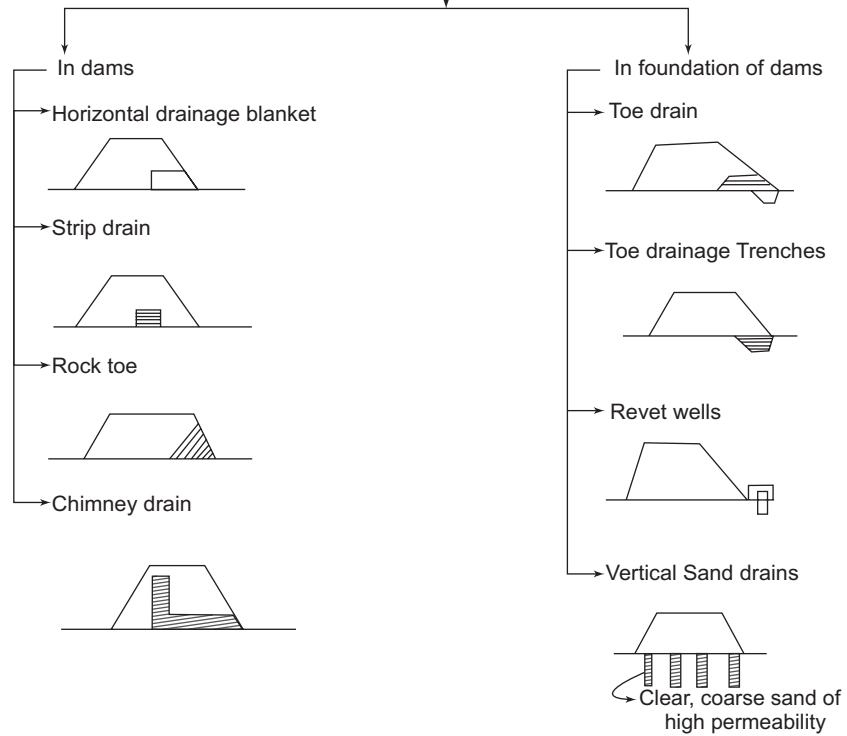
Built with natural material with very less processing and use of primitive equipments.



Measures to control seepage through earth dam



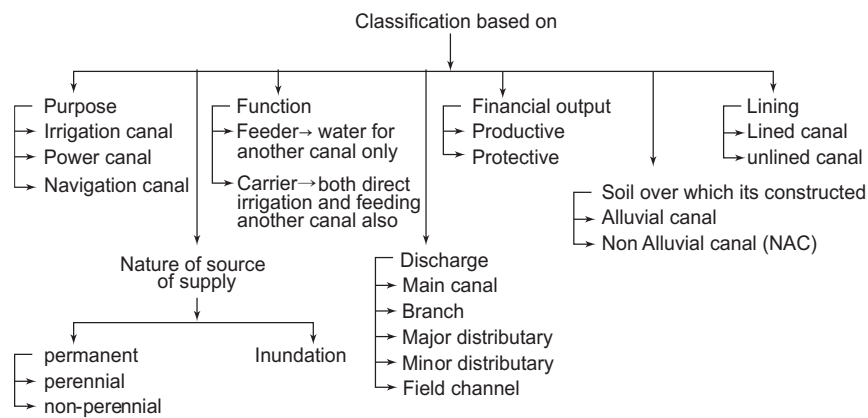
(a) Provide Safe drainage to Seeping water



Canal-Irrigation

6

An artificial channel constructed to carry water from a river, tank or reservoir for various purposes.



Note: Indra Gandhi Canal is a feeder canal

On the Basis of Canal Alignment, it can also be classified as

(1) Ridge or Watershed → Aligned alongridge → Irrigate areas on both sides → No drainage intersects → Cross-drainage works not required	(2) Contour Canal → Aligned parallel to contour → Irrigate only on one side → Cross drainage works are required → Generally in hilly areas	(3) Side - Slope Canal → Aligned at right angles in the contours → Parallel to natural drainage → Cross drainage works not required

Water-Logging

7

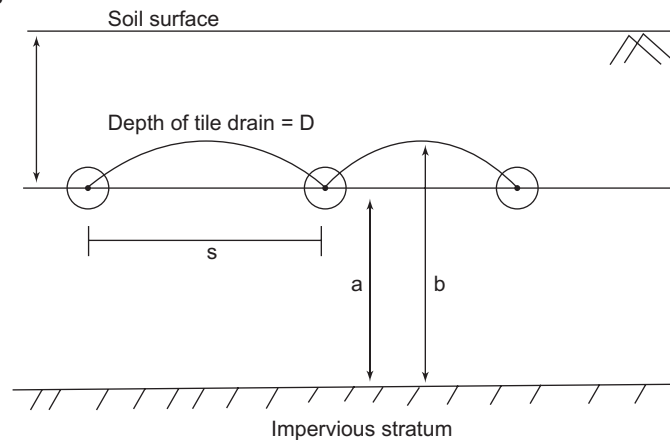
War bandhi: System of equitable water distribution by turns, according to predetermined schedule.

Mulching: Spreading of extraneous material on the surface of soil to increase water filtration.

Reclamation: Process by which an uncultivable land is made fit for cultivation.

Leaching: Process in which land is flooded with water. So that alkali salts get dissolved in water and percolate to join the water table.

Spacing between the tile drains



$$S = \frac{4k}{q} (b^2 - a^2)$$

k = Permeability coefficient (m/sec)

q = discharge per unit length

b = height of water table above impervious layer

a = height of centre of drain above impervious layer

Note: For Reclamation of Alkaline soil, Gypsum as well as leaching is done while for Acidic soil, limestone is used.

Leaching requirement (L.R.)

$$\text{L.R.} = \frac{\text{Depth of water drained out per unit area}}{\text{Depth of water applied per unit area}} = \frac{D_d}{D_a}$$

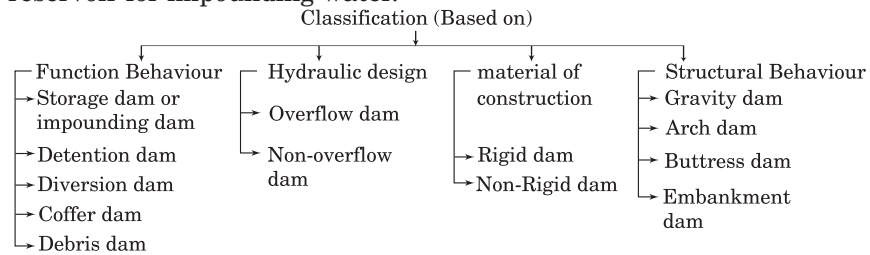
$$\text{L.R.} = \frac{\text{Salt content of irrigation water}}{\text{Salt content of drained water}} = \frac{C_i}{C_d}$$

$$\text{L.R.} = \frac{\text{Electrical conductivity of irrigation water}}{\text{Electrical conductivity of drained water}} = \frac{E.C_i}{E.C_d}$$

Gravity Dams

8

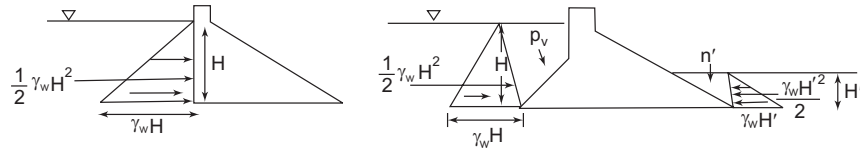
A dam is a barrier constructed across river in order to create a reservoir for impounding water.



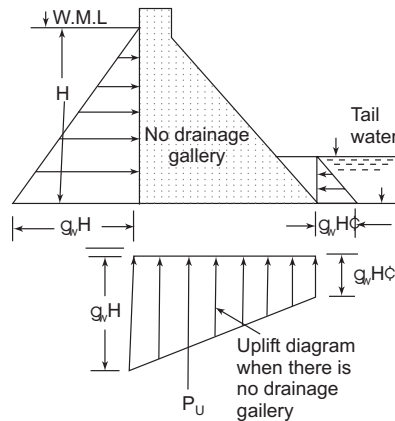
Gravity dam: It is a solid masonry or concrete structure with an approximate triangular cross-section, so that external forces exerted on it are resisted by its own weight.

Forces acting on gravity dam

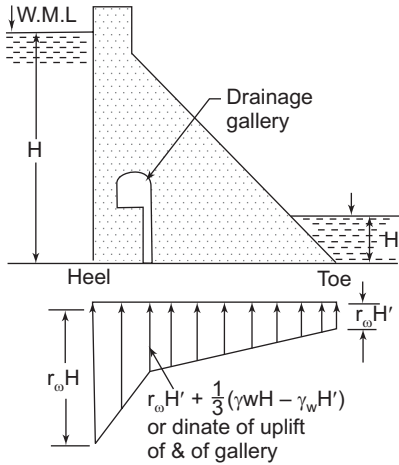
(1) Water Pressure



(2) Uplift Pressure



(a) When there is no drainage gallery



(b) when drainage gallery is present

(3) Earthquake Forces: India is divided into 4 seismic zones, zone I, zone III, zone IV & zone V.

Zone V is most serious zone.

(i) Effect due to vertical acceleration (α_v)

$$\text{Net effective wt. of the dam reduces to} = W = \frac{W}{g} \alpha_v$$

$$= W(1 - K_v) \quad \text{where } \alpha_v = K_v g$$

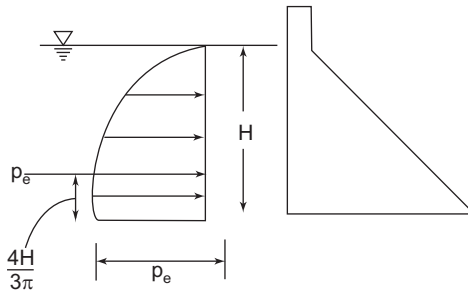
Vertical acceleration acting downward reduces weight of the dam.

(ii) Effect due to horizontal acceleration (α_h)

$$(a) \text{ Horizontal Inertia force } F_H = \left(\frac{W}{g} \right) \alpha_H$$

(b) Hydrodynamic pressure

$$P_e = 0.555 K_h \gamma_w H$$



Acts at a height of $\frac{4H}{3\pi}$ from the base

Moment about base $M_e = P_e \left(\frac{4H}{3\pi} \right)$

$$M_e = 0.42 P_e H$$

(4) **Silt Pressure** $P_{\sin} = \frac{1}{2} \gamma_{\text{sub}} h^2 K_a$ $K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$

If the u/s face is inclined, the vertical weight of the silt supported on the slope also acts as vertical force.

(5) **Wave Pressure**

$$\begin{aligned} h_w &= 0.032\sqrt{V.F} + 0.763 - 0.271(F)^{3/4} \quad F < 32 \text{ km} \\ h_w &= 0.032\sqrt{V.F} \quad V > 32 \text{ km} \end{aligned}$$

h_w = height of wave (meter)

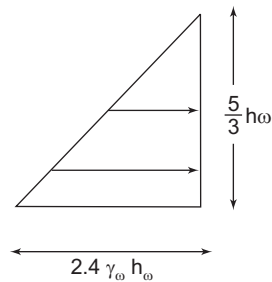
V = Wind Velocity (Km/hr)

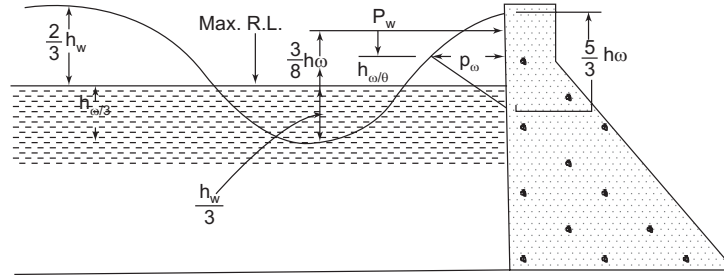
F = Fetch (Km)

$\text{Max. } P_w = 2.4 \gamma_w h_w$ acting at height of $\frac{h_w}{8}$ from still water

Total force due to wave action

$$P_w = \frac{1}{2} (2.4 \gamma_w h_w) \left(\frac{5}{3} h_w \right)$$





$$P_w = 19.62h_w^2 \text{ KN/m}$$

P_w at a distance of $\frac{3}{8}h_w$ from reservoir surface.

(6) Ice Pressure: Pressure thrust on the face of the dam due to expanding or melting of ice.

(7) Weight of the dam: $W = \gamma_c V$

γ_c = unit weight of concrete

V = Volume of dam body per unit length.

Criteria of Structural Stability and modes of failure of Gravity dam

(1) Overturning about Toe

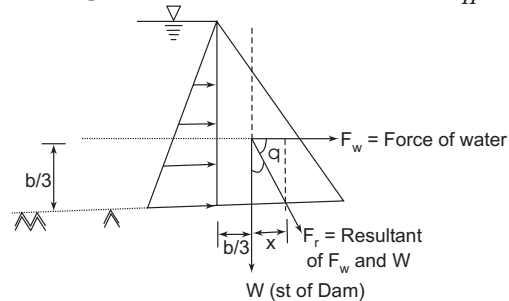
$$F_R = \sqrt{F_H^2 + F_v^2}$$

$$e = \frac{B}{2} - \bar{x}, \bar{x} = \text{distance of } \overline{F_R} \text{ from toe}$$

$$F_S = \frac{M_R}{M_O} > 1.5$$

M_R = Restoring moment about toe (due to ΣF_v)

M_O = Overturning moment about toe (due to ΣF_H)



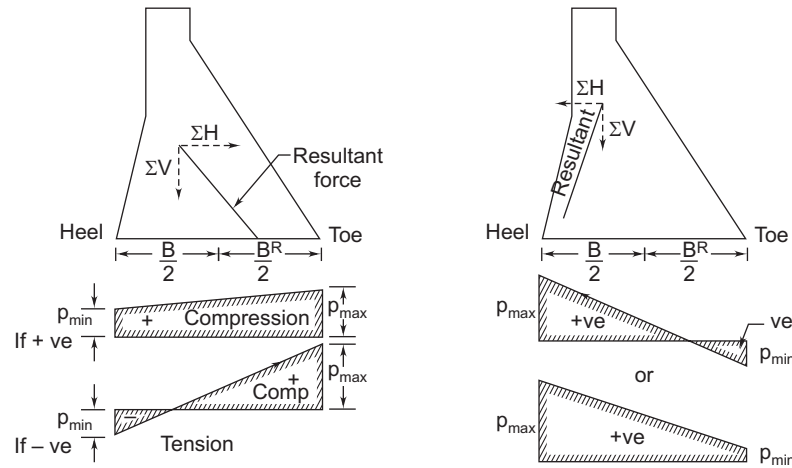
(2) Compression or crushing failure

$$\sigma_{\max/\min} = \frac{\Sigma V}{B} \left(1 \pm \frac{6e}{B} \right)$$

ΣV = Total vertical force

B = Base width

e = eccentricity of resultant force from the centre of the base.



(3) Tension failure: max. permissible tensile stress for high concrete gravity dams under loading conditions may be taken as 500 KN/m^2 . For No tension at the base of the dam, $\sigma_{\min} = 0$

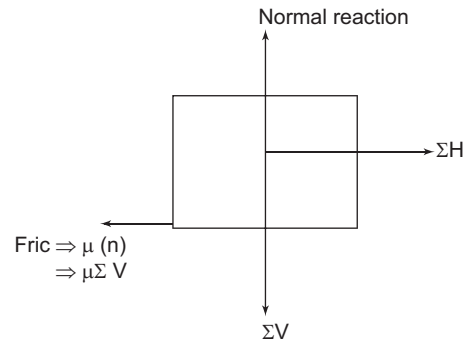
$$1 - \frac{6e}{B} = 0 \Rightarrow e = \frac{B}{6}$$

Middle third rule: For No tension at base, max. permissible value of eccentricity on either side is $B/6$.

(4) Failure due to sliding

$$\text{FOS}_{\text{sliding}} = \frac{\mu \Sigma F_v}{\Sigma H}$$

$$\frac{\Sigma H}{\Sigma V} = \text{sliding factor}$$



$$FOS = \frac{\mu}{\text{sliding factor}} > 1$$

Note: If Shear resistance of joint is also considered, then equ. of FOS against sliding is measured by shear friction factor (SFF)

$$S.F.F = \frac{\mu \Sigma v + Bq}{\Sigma H} \quad \begin{array}{l} B = \text{width of dam (metre)} \\ q = \text{Permissible Shear (KN/m}^2\text{)} \end{array}$$

Principal and Shear Stresses

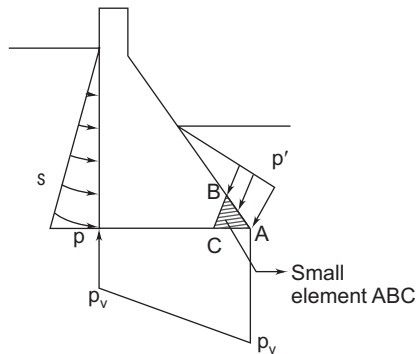
(1) **Principal Stress** put $\Sigma v = 0$

$$p' ds \sin \alpha + \sigma dr \cos \alpha = p_v db$$

where,

$$ds = db \sin$$

$$dr = db \cos$$



So,

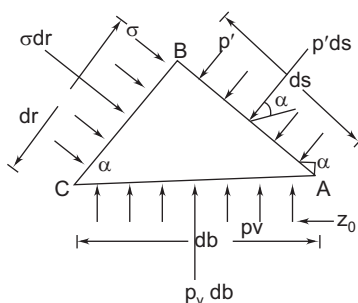
$$p' \sin^2 \alpha + \sigma dr \cos^2 \alpha = p_v$$

Resolving

$$\sigma = p_v \sec^2 \alpha - p' \tan^2 \alpha$$

(a) For No tail water, $p' = 0$

$$\sigma_{\max} = p_v \sec^2 \alpha$$



(b) If hydrodynamic pressure p'_e then

$$\sigma = p_v \sec^2 \alpha - (p' - p_e) \tan^2 \alpha$$

(2) **Shear Stress** put $\Sigma H = 0$

$$\sigma dr \sin \alpha - p' ds \cos \alpha = \tau_0 db$$

Substituting and Resolving, $\tau_0 = (p_v - p') \tan \alpha$

(a) For No tail water $P^1 = 0$ $\tau_0 = p_v \tan \alpha$

(b) If hydrodynamic pressure p'_e , then $\tau_0 = [p_v - (p' - p'_e)] \tan \alpha$

Elementary Profile of a gravity dam

(1) For No tension at base, when reservoir is full

$$B \geq \frac{H}{\sqrt{G - C}}$$

G = Specific gravity of material of dam

C = uplift coefficient

when uplift is not considered, $C = 0$ $B \geq \frac{H}{\sqrt{G}}$

(2) For No sliding $B \geq \frac{H}{\mu(G - C)}$

Max. stress at toe, $P_V = \gamma_w H (G - C)$

Min stress at heel $P_V = 0$

Principal Stress near toe $\sigma = \gamma_w H (G - C + 1)$

Shear Stress near toe $\tau_0 = \gamma_w H \sqrt{G - C}$

Note: If max represents the allowable stress of the dam material then maximum height of the dam

$$M_{\max} = \frac{\sigma_{\max}}{\gamma_w (G + 1)}$$

If Height of dam is less than $H_{\max}$, then its **low gravity dam**.
And if Height is greater than $H_{\max}$ then its **High gravity dam**.

Civil Engineering

13

Highway

CONTENTS

1. Introduction to Highway
2. Geometric Design
3. Horizontal Alignment
4. Vertical Alignment
5. Traffic Engineering: Characteristics
6. Traffic Engineering Studies
7. Traffic Control Regulation
8. Highway Material
9. Pavement Design
10. Flexible Pavement Design
11. Design of Rigid Pavement
12. Highway Maintenance

Introduction to Highway

1

Note:

Roman Roads → Tresaguet construction → Met calf construction
→ Telford constructions → Macadam construction.

John Macadam (1756 – 1836) was the first person to put forward a concept which suggested that heavy foundation stones are not required at all at the bottom layer

Development of highway planning in India: After first world war (28/07/1914 to 11/11/1918) as more motor vehicles came on road necessity of proper road planing came into light. So a **Road Development committee** headed by **Mr. M.R. Jayakar** was appointed by the government.

Recommendations

- (a) 20 years long term planning programmes.
- (b) Establishment of Indian Road Congress (IRC) in 1934 for holding of periodic road conferences to discuss about road construction and development.
- (c) Introduction of central road fund in 1929, which imposed duty on motor spirit, vehicle taxation, license fee etc.
- (d) Central Road Research Institute (CRRI) in 1950 (near Apollo metro station – New Delhi) to carry out research and development work.

Important year's of highway development

- 1. Nov. 1927 → Jayankar committee formed.
- 2. Feb. 1928 → Recommendations by Jayankar committee
- 3. 1929 → central road fund
- 4. 1934 → Indian road congress
- 5. 1939 → motor vehicle act
- 6. 1943 – 63 Nagpur road plan (finished in 1961)

7. 1950 Central Road Research Institute
8. 1956 National highway act
9. 1961 – 81 Bombay road plan
10. 1981 – 01 Lucknow road plan

NAGPUR PLAN (1943 – 63)

1. Roads are divided into four classes :
 - (a) National Highways
 - (b) Provincial and state highways
 - (c) District roads
 - (d) Village roads
2. Recommendations for star and grid pattern
3. Road density of 16 Km per 100 Km Square
4. Completed in 1961

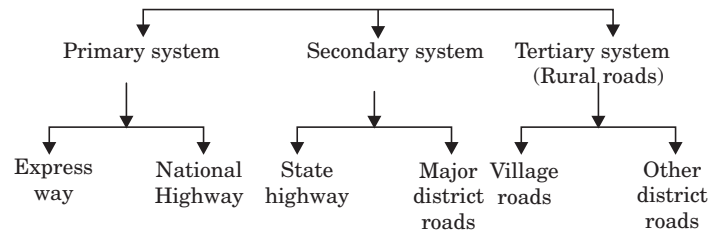
BOMBAY PLAN (1961 – 81)

1. Road density of 32 km per 100 km square, 40% of length would be surfaced
2. Construction of 1600 km of Expressway included in plan
3. Highway financing from those also who get indirect benefits. Like cess in land revenue, toll projects and tax on diesel oil.
4. Traffic Engineering cell in each state.

LUCKNOW PLAN (1981 – 01)

- The first two plans were not the part of total transportation plan of the country.
- The first two plans didn't meet the needs of freight and passenger movement by road.

1. The roads were classified as



2. Road density of 82 km per 100 km square

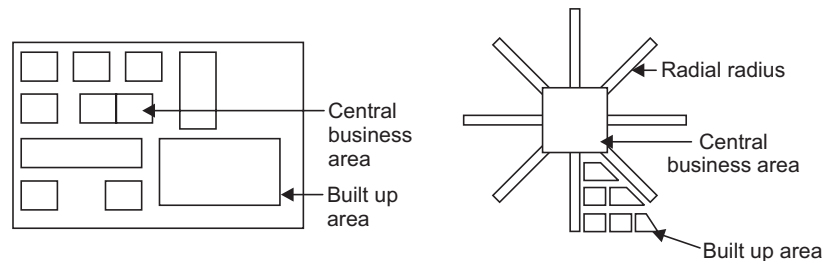
3. Expressway's to be constructed on major traffic corridors.
4. National Highway should form square grid of 100 km $\times$ 100 km.
5. Energy conservation, environmental quality of roads and road safety measures were also given due importance in this plan.

Comparison of Nagpur plan and Bombay plan

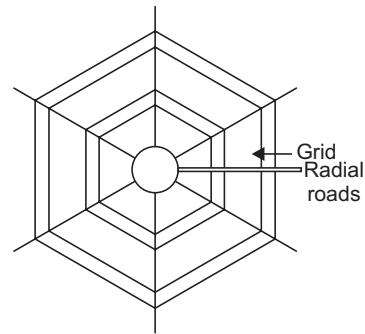
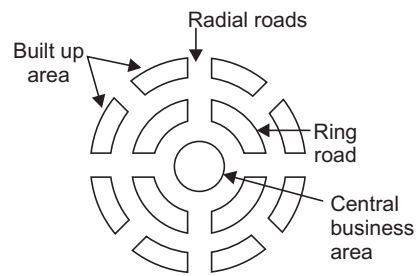
1. Nagpur plan gave **two** formulae, one for metalled roads and other for unmetalled roads while Bombay plan gave **five** different formulae for each class of road *i.e.* NH, SH, MDR, ODR, and VR.
2. Nagpur road plan divides area into agricultural and Non-agricultural area while Bombay road plan divides it into three parts developed and agricultural area, semi-developed area, undeveloped and uncultivated area.
3. Nagpur road plan formula does not takes into account the towns with very large population. They divided the population in only **two** groups 2001–5000 and above 5000 While Bombay road plan divides it into nine groups.
4. Length of railways deducted in calculating the highway length during Nagpur plan while in Bombay plan it is emphasised to develop highway system independently so in Bombay road plan it is not deducted.
5. A 15% increase in calculated road length of both categories for development of agriculture and industry during next 20 years was decided in Nagpur road plan. While in Bombay road plan this allowance was kept as 5%.
6. Bombay road plan provided 1600 Km of Expressway out of proposed National Highways.

ROAD PATTERNS

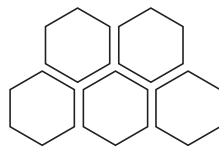
- (a) Rectangular or block pattern (b) Radial or star and block pattern



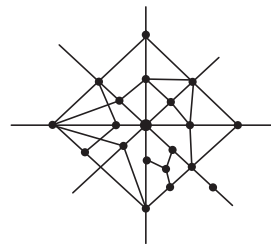
(c) Radial or star and circular pattern (d) Radial or star and grid pattern



(e) Hexagonal pattern



(f) Minimum travel pattern



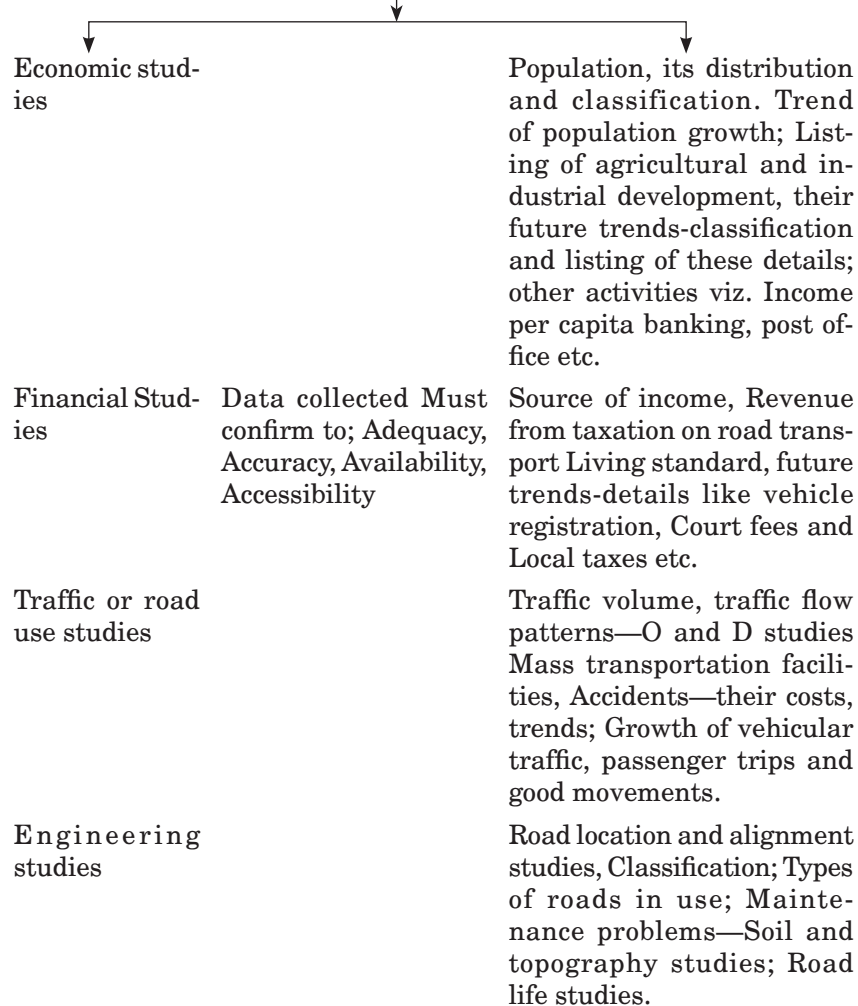
Note:

Chandigarh → Rectangular and block pattern

New Delhi → Radial and circular pattern at cannought place

Nagpur road plan formulated star and grid pattern.

Studies in planning Survey

Planning Surveys

Highway Alignment: Factors controlling highway alignment are.

- (a) **Obligatory points:** Control points governing the alignment, which deviate the highway from its shortest or earliest path
- (b) **Traffic:** Origin and destination studies should be carried out and desire lines to be drawn, New roads should then be aligned or per desire line, traffic flow pattern and future trends.

- (c) **Geometric design:** Horizontal alignment of the road to be adjusted based on minimum radius of curvature and transition curves. Vertical alignments are based on visibility, passenger comforts etc.
- (d) **Economy:** Initial cost of construction can be decreased if high embankment and deep cuttings are avoided and Highways are aligned as per balanced cutting and filling of earth,
- (e) **Other considerations:** Road alignments are sometimes even based on political constituencies. Other considerations include drainage, hydrological features etc.

Engineering Surveys for Highway Locations

1. **Map study:** To have an idea of several alternate routes, locations of valleys, lakes, ponds, bridges, contours etc.
2. **Reconnaissance:** A field survey of land along the proposed alternative routes of the map in the field is studied. All relevant details not available on map are collected and noted down.
3. **Preliminary survey:** To compare different proposals, to workout the cost and quantity of earthwork and to finalise the best alignment. Its procedure involves following steps.
 - (a) Primary traverse
 - (b) Topographical features
 - (c) Levelling works
 - (d) Drainage and hydrological data studies
 - (e) Soil survey
 - (f) Material survey
 - (g) Traffic survey
 - (h) Determination of final centre line.
4. **Final location and detailed survey:** Marking of centre line of field and then a detailed survey is carried out for collecting necessary information for the preparation of plans and construction details of highway project. Temporary bench marks are fixed at intervals of about 250 meters and at all drainage and under pass structure.

Geometric Design

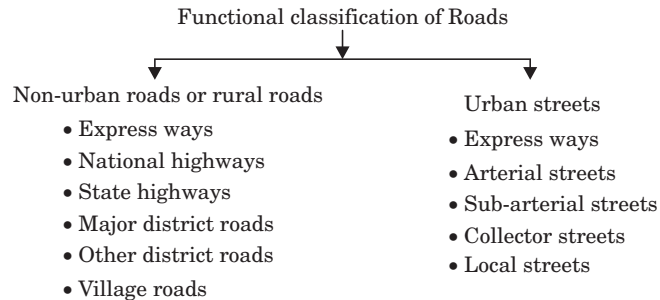
2

Designing of visible dimensions (both in horizontal and vertical plane) of highway is called geometric design.

Factors controlling geometric design:

- (i) Road user characteristics
- (ii) Vehicle characteristics
- (iii) Safety requirements
- (iv) Environmental considerations
- (v) Economy
- (vi) Topography
- (vii) Functional classification
- (viii) Traffic volume and composition
- (ix) Design speed.

- The total time required for **perception, intellection, emotion and volition** (PIEV) that is from the instant the object comes in the line of sight of the driver to the instant he arrives at a decision under **normal circumstances** is called **reaction time**. As per IRC, its **2.5 sees** (90th percentile reaction time).
- **Design Vehicles** are those whose dimensions and weight are adopted for determining the elements of highway design. Max permissible width of vehicle is **2.44 m**.
- The Max. speed at which vehicles can continuously travel safely under favourable conditions is called design speed.
- **Ruling design speed** is the speed which is used as guiding criteria for correlating various geometric design features.
- Where site conditions, cost etc. do not permit ruling design speed then the speed is decreased to **minimum design speed**.
- Topography of the land, through which the road passes is known as terrain.



Terrain	% Cross-slope
Plain	0-10
Rolling	10-25
Mountainous	25-60
Steep	> 60

Cross-sectional elements

- Friction:** As per IRC: coefficient of longitudinal friction is 0.35–0.4 depending on speed where as that of lateral friction is 0.15. Lack of adequate friction causes slipping or skidding
Slipping: Wheel revolves more than corresponding longitudinal movement.
Skidding: When longitudinal movement on road is more than wheels revolution.
- Unevenness:** Unevenness index is the cumulative measure of vertical undulation of the pavement surface recorded per unit horizontal length of the road. A value less than **1500 mm per km** is considered **good**. Pavement surface condition is measured by **Bump-Indicator** in terms of unevenness index.
- Light reflection:** Concrete roads have better visibility and less glare.
- Drainage:** Both geometry and texture of pavement surface should help in draining out water from the surface in less time.

Camber or cross fall: It's the rising of the middle of the road surface in the transverse direction to drain off rain water from road surface.

- Camber is measured in 1 in n or $n\%$
- For cement concrete pavement. Camber is in straight line otherwise parabolic.
- For providing the desired amount and shape of camber, **templates** or **camber boards** are used.

Surface type	Heavy rain	Light rain
Concrete/Bituminous	2%	1.7%
Gravel/WBM	3%	2.5%
Earthen	4%	3.0%

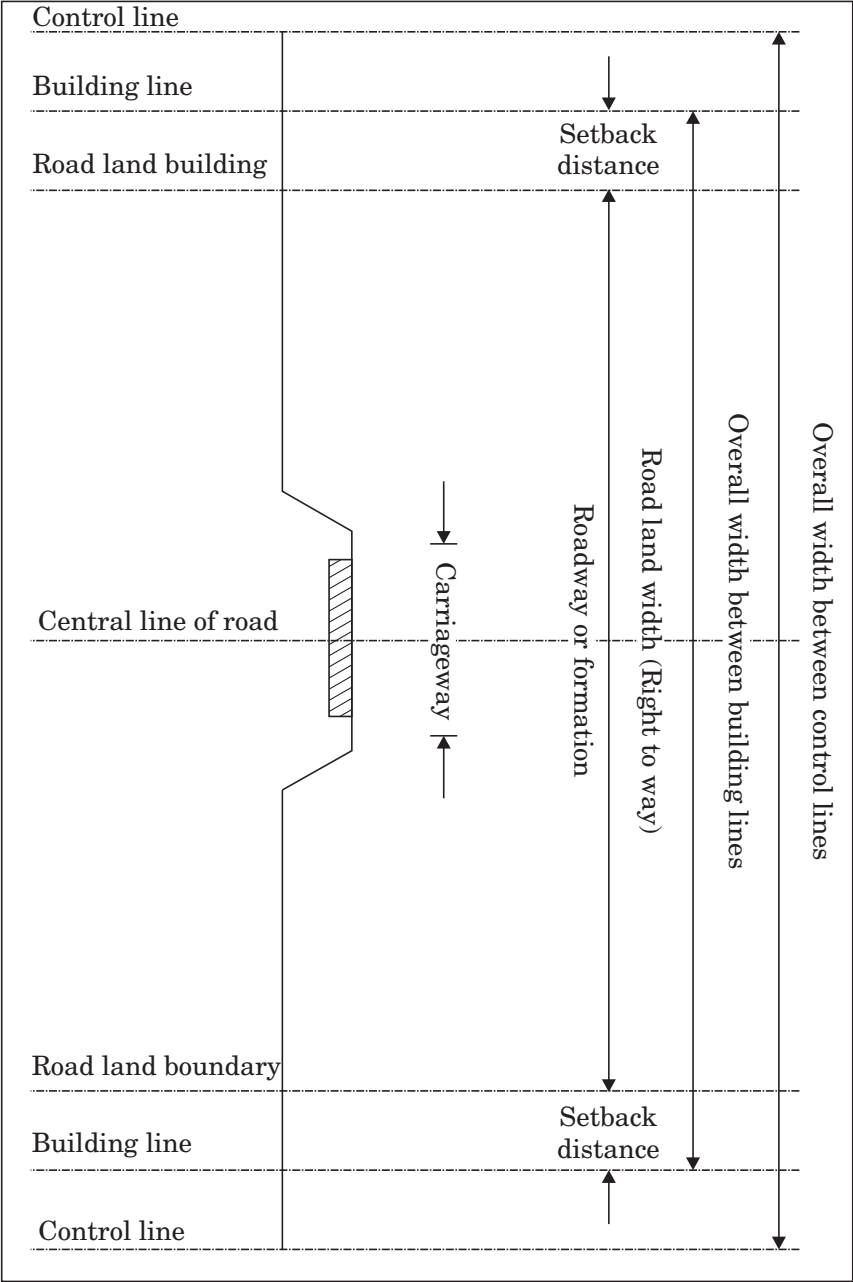
Width of Carriage way: It is decided on the basis of capacity which depend on traffic lane and number of lanes.

IRC Specification for Carriage way width

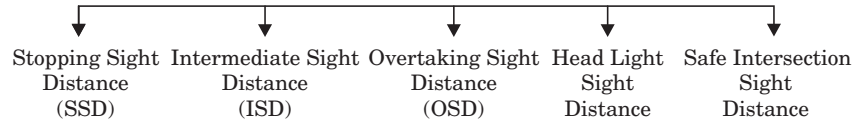
Single lane	3.75
Two lane, no kerbs	7.0
Two lane, raised kerbs	7.5
Intermediate carriage	5.5
Multi-lane	3.5

Median or Traffic Separators: It prevents head on collision between vehicles moving in opposite directions on adjacent lanes. It helps to channelize traffic and segregate slow traffic to protect pedestrians.

As per IRC, minimum desirable width for median is for rural highways and can be reduced to 3 m if land is restricted.



Sight Distance: The distance from which any types of obstruction on the road length could be visible to the driver from some distance ahead is called Sight Distance.



(i) **Stopping Sight Distance:** It is the minimum sight distance available on a highway at any spot having sufficient length to enable the driver to stop a vehicle travelling at a design speed safely without collision with any obstruction.

S.S.D = lag distance + Braking distance

v = design speed, m/sec

t = reaction time (2.5 sees)

g = acceleration due to gravity (m/sec²)

f = coefficient of longitudinal friction (0.35)

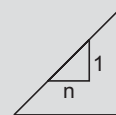
$$\text{S.S.D} = vt + \frac{v^2}{2gf}$$

Speed (km/hr)	< 30	40	50	60	> 80
f	0.40	0.38	0.37	0.36	0.35

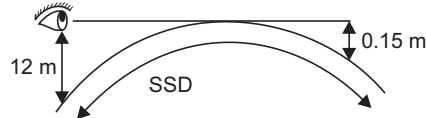
Note:

When vehicle is moving up (+) or down (–) the grade (n) then

$$\text{S.S.D} = vt + \frac{v^2}{2g \left(f \pm \frac{n}{100} \right)}$$

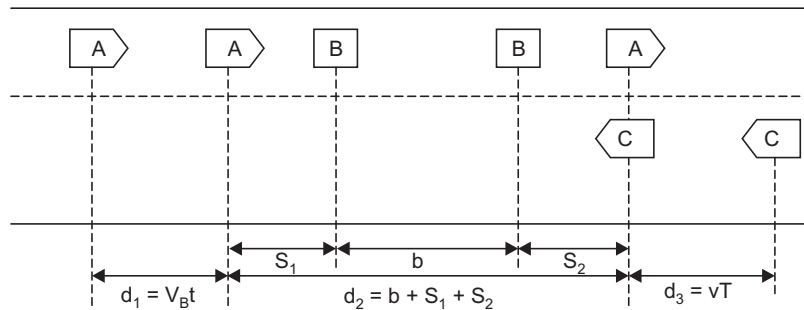


- Effect of grade should be considered for divided highway but not for undivided highway.
- Single lane roads on which two way traffic is permitted, stopping distance is **twice** the minimum stopping distance to stop both vehicles coming from opposite direction.
- For S.S.D on vertical curves height of driver eye is taken as **1.2 m** and that of obstruction as **0.15 m**.



(ii) **Intermediate Sight Distance (ISD):** It is taken as **twice** the stopping sight distance.

(iii) **Overtaking Sight Distance (OSD):** Minimum distance open to the vision of the driver of a vehicle intending to overtake the slow vehicle ahead safely against the traffic in the opposite direction.



Minimum O.S.D = $d_1 + d_2 + d_3$

- For overtaking vehicle B, vehicle A will reduce its speed to that of vehicle B (V_B) for a reaction time (t) of nearly 2 secs to decide or confirm to overtake B.

So $d_1 = V_B t$

- Let 'T' be the time taken by A to overtake B by travelling distance d_2 . In that very time vehicle C, which is moving at design speed $V_C = v$ will travel distance d_3 so $d_3 = vT$
- Now as in time T, vehicle A accelerates and overtakes B by travelling d_2 , in that very time vehicle B travels distance b at constant velocity V_1 so $b = v_b T$ and $d_2 = v_b t + \frac{1}{2} a T^2$ where 'a' is the acceleration of

vehicle A.

So $v_b T + \frac{1}{2} a T^2 = b + S_1 + S_2$

If $S_1 = S_2 = S = 0.7 V_B + 6$

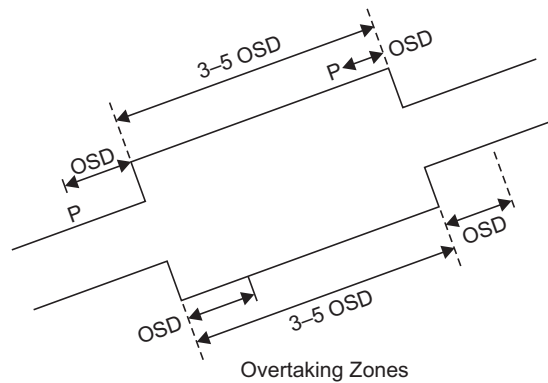
and $v_b T = b$ then $2S = \frac{1}{2} a T^2$

$$T = \sqrt{\frac{4S}{a}}$$

Note:

It V_B is not given then its $(V - 16)$ km/hr where V is the design speed in km/hr or $(v - 4.5)$ m/sec where v is design speed in (m/sec).

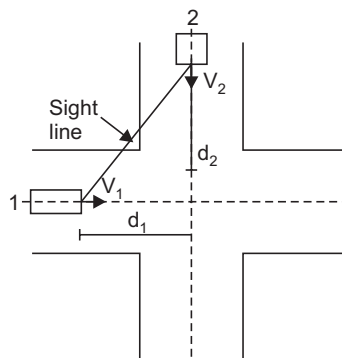
- When no vehicle is expected from opposite direction ie on divided highways and on roads with one way traffic regulation.
OSD = $d_1 + d_2$
- As per IRC, on divided highways with four or more lanes, only sight distance should be greater than stopping sight distance.
- No separate design's are there for highway on grades.



(iv) Sight distance at intersection:

$$\text{S.S.D for 1} = d_1 = v_1 t$$

$$\text{S.S.D for 2} = d_2 = v_2 t$$



Horizontal Alignment

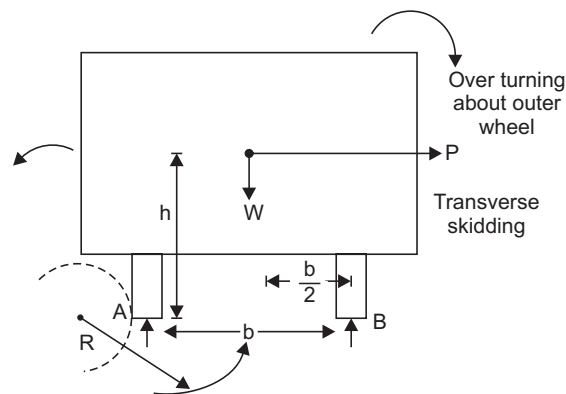
3

It includes the radius of circular curve, design of super elevation, extra widening at horizontal curves, design of transition curve and set back distance.

Design speed

Type	Plain	Rolling	Hilly	Steep
NH & SH	100–80	85–65	50–40	40–30
MDR	80–65	65–30	40–30	30–20
ODR	65–50	50–40	30–25	25–20
VR	50–40	40–35	25–20	25–20

The presence of horizontal curve imparts **centrifugal force P** which is a reactive force **acting outward** on a vehicle negotiating it.

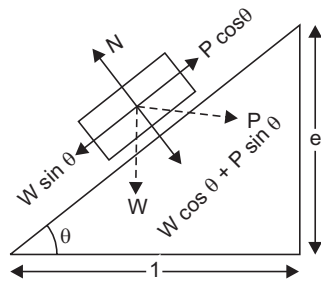


$$\text{So centrifugal force} = P = \frac{mv^2}{R} \Rightarrow \frac{P}{W} = \frac{v^2}{Rg}.$$

$\frac{P}{W}$ is centrifugal ratio or **impact factor**.

- To prevent overturning about outer wheel $\frac{b}{2h} \geq \frac{v^2}{gR}$.
- To prevent transverse skidding $f > \frac{v^2}{gR}$.

Super-elevation: Rising of the outer edge of the road with respect to inner edge in order to contract the effect of centrifugal force is called Super elevation.



$$\frac{v^2}{gR} = \frac{\tan \theta + f}{1 - f \tan \theta}$$

For $\theta < 40$

$$1 - f \tan \theta \approx 1, \tan \theta = e$$

So
$$\frac{v^2}{gR} = e + f$$

Note:

Equilibrium super elevation is when $f = 0$. It results in situation in which pressure on outer and inner wheels become equal.

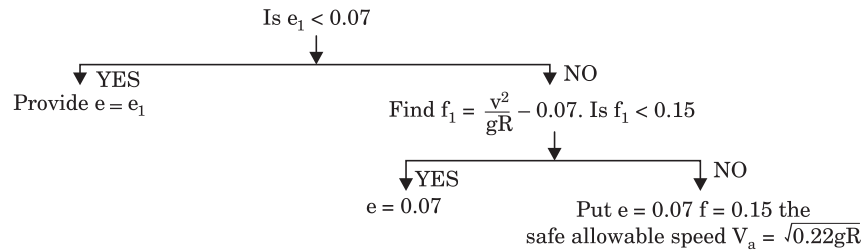
As $f = 0$ so $e = \frac{v^2}{gR}$

If $e = 0$ then $f = \frac{v^2}{gR}$

For fast moving vehicles roads are designed assuming minimum friction and maximum super elevation while for slow moving vehicles roads are designed assuming maximum friction and minimum super elevation.

Design of Super-elevation

Basic equation $e + f = \frac{v^2}{gR}$; put $f = 0$ and get $e_1 = \frac{(0.75v)^2}{gR}$



Note:

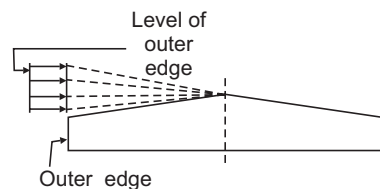
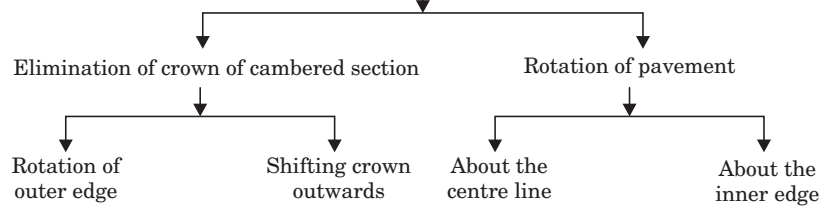
Max super elevation for plain and rolling terrain, hilly terrain bound by snow = 0.07

For hilly terrain not bound by snow its = 0.10

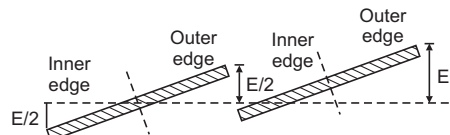
For Urban roads is = 0.04

Minimum super elevation for drainage purpose = 2–4%

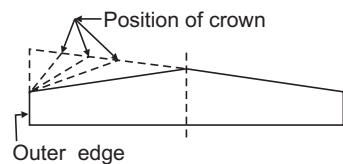
Attainment of super-elevation (done in transition curve)



(a) Outer edge rotated about the crown.



Method (1), rotating about centre line
Method (2), rotating about the inner edge.



(b) Crown shifted outwards
(diagonal crown method).

Note:

In rotating the pavement about the centre line, the vertical profile remains unchanged and there is advantage in balancing the earth work while in rotating the pavement about the inner edge leads to drainage problem as well as centre of the pavement is raised resulting in altered vertical alignment.

Ruling Radius of Horizontal curve: It is the minimum possible radius of the circular curve on which a vehicle moving at **design speed** can pass the curve safely. For R_{ruling} maximum value of super elevation (7% or 10%) and coefficient of friction (0.15) are adopted

$$R_{\text{ruling}} = \frac{v^2}{g(e + f)}$$

Absolute Minimum radius of horizontal curve: It is the minimum possible radius of the circular curve on which a vehicle moving at **minimum possible speed** (designed for that highway) can pass the curve safely

$$R_{\text{absmin}} = \frac{v^2 \min}{g(e + f)} \quad e, f \text{ are maximum}$$

Note:

Radius of the curve to be adopted should always be greater than ruling minimum and even in most adverse case it should be greater than absolute minimum.

Extra Widening (We): It is the additional width of carriage way that is required on a curved section of a road over and above that required on a straight alignment. It has two components.

Mechanical Widening

When a vehicle negotiates on horizontal curve then the rear wheel follows a path of shorter radius than the front wheel its called **off-tracking**

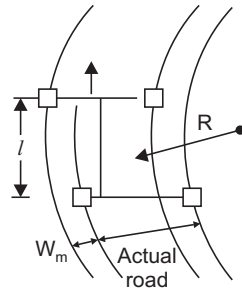
$$W_m = \frac{nl^2}{2R}$$

Psychological widening

This is done due to the tendency of the drivers to drive close to the edges of the pavements on curves.

$$W_p = \frac{v}{2.64\sqrt{R}} = \frac{V}{9.5\sqrt{R}}$$

$$W_e = \frac{nl^2}{2R} + \frac{v}{2.64\sqrt{R}}$$



$l \rightarrow$ length of wheel base (6.1 m)

$n \rightarrow$ no. of lanes.

Note:

On single lane road, its sufficient to provide mechanical widening only.

- If $R > 300$ m, no extra widening will be required.
- For multi-lane roads, the pavement widening may be calculated by adding half the widening for two lane roads to each line.

Horizontal transition curve: It is provided to change the horizontal alignment from straight to circular curve gradually and has a radius which decreases from infinity at the straight end (target point) to the desired radius of the circular curve at the other end (point of tangency)

Note:

IRC recommends **Spiral** Curve for horizontal transition.

Length of horizontal transition curve

1. Rate of change of centrifugal acceleration (driver's comfort criteria)

$$L_{S_1} = \frac{v^3}{cR} \quad v \rightarrow \text{m/sec}, R \rightarrow m$$

$$C = \frac{80}{75 + V} \quad V \rightarrow \text{km/hr}, 0.5 \leq C \leq 0.8, C \rightarrow \text{m/sec}^3$$

2. Rate of introduction of super elevation (passenger's comfort criteria)

Plain and rolling Terrain (Rate (N) = 1 in 150) Sleep and Hilly Terrain (Rate (N) = 1 in 60)

$$L_{S_2} = 35 \frac{v^2}{R} = 2.7 \frac{V^2}{R}$$

$$L_{S_2} = 12.96 \frac{v^2}{R} = \frac{V^2}{R}$$

3. Method of introduction of super elevation

Rotation of Outer edge wrt centreline Rotation of pavement about inner edge

$$L_{S_3} = \frac{e(W + We)N}{2}$$

$$L_{S_3} = eN (W + We)$$

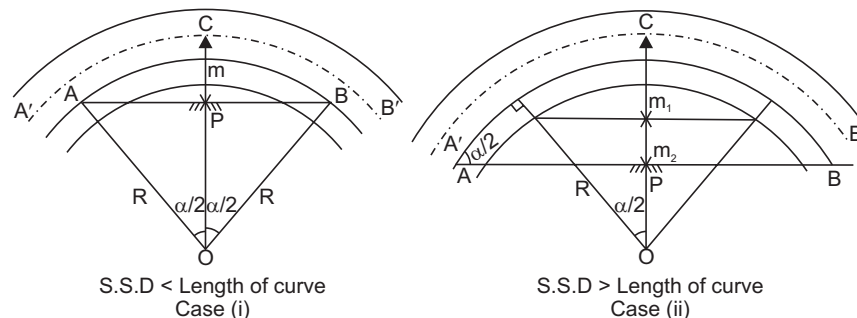
$$\text{Length of transition curve } L_s = \max (L_{S_1}, L_{S_2}, L_{S_3})$$

Note:

When a transition curve is introduced between straight and circular curve then the circular curve has to be shifted inwards so that transition curve meets the circular curve tangentially. This is called shift.

$$\text{Shift 's'} = \frac{L_s^2}{24R}$$

Set Back distance: It's the clearance distance required from the **centre line** of the horizontal curve to an obstruction on the **inner side** of the curve to provide adequate sight distance at a horizontal curve.

Step's to find any set back distance (m)

13.22 CIVIL ENGINEERING

1. Draw the centre line of curve A'CB' (as set back distance is to be measured from here).
2. Calculate S.S.D and draw it on the inner lane of the road *i.e.*, AB
3. Join AB from the straight line.
4. Then the distance CP is the setback distance, where P is the point of obstruction.

In case (i) $m = R - R \cos \frac{\alpha}{2}$

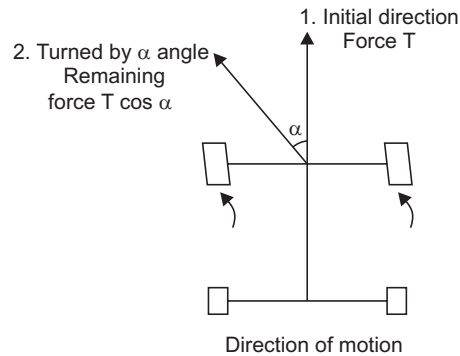
In case (ii) $m = m_1 + m_2$

$$m_1 = R - R \cos \frac{\alpha}{2}$$

$$m_2 = \frac{(SSD - L_C)}{2} \sin \left(\frac{\alpha}{2} \right)$$

Curve Resistance: It is the loss of tractive force when a **rear wheel** driven vehicle negotiates on a **horizontal curve**.

$$\text{Curve resistance} = \text{Remaining force} = T - T \cos \alpha$$



Note:

There is no curve resistance in case of front wheel driven vehicle.

Vertical Alignment

4

It is a road profile in which elevation is on vertical axis and horizontal distance is along the centre line of the road on horizontal axis.

Gradient is the rate of rise or fall along the length of the road with respect to the horizontal.

IRC specifications for Road Gradient is different terrains (%)

Terrain	Ruling	Limiting	Exceptional
Plain/Rolling	3.3	5	6.7
Hilly	5	6	7
Steep	6	7	8

- **Ruling gradients** is the maximum gradient with which designer attempts to **design** the vertical profile of the road.
- **Limiting gradient** is to be adopted when ruling gradient results is very high **increase is cost** of construction due to cutting and filling.
- **Exceptional gradient** are given at **unavoidable situations**. They are limited to a stretch of 100 m in a single run.
- **Critical length** of the grade is the maximum length of the **ascending gradient** which a loaded truck can operate without reduction in speed or more than **25 km/hr**.

Note:

A minimum gradient is also to be provided for drainage purpose for concrete drains it is 1 in 500, for open soil drains its 1 in 200.

Grade compensation: Reduction is vertical gradient due to the horizontal curve, which is intended to off set the extra tractive effort involved at the curve.

$$\text{Grade compensation} = \frac{30 + R}{R} \%, R \rightarrow \text{Radius of curve in m}$$

$$\text{Maximum value} = \frac{75}{R} \%$$

13.24 CIVIL ENGINEERING

Note:

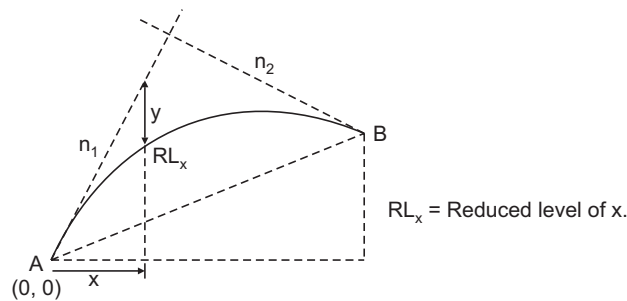
No compensation required if grade is flatter than 4%

Summit-Curve: These are vertical curves with **convexity upwards**. Its design is governed by criteria of **sight distance**. Generally **parabolic curves** are used as summit criteria.

Note:

Circular curves offer equal sight distance at every point on the curve.

- (i) Equation of curve $y = \frac{Nx^2}{2L}$ where $N = n_1 - n_2$
- (ii) Radius of curvatures = $\frac{L}{N}$
- (iii) Highest point on summit curve at $x = \frac{n_1 L}{N}$
- (iv) $RL_x = RL_A + n_1 x - \frac{N}{2L} x^2$



Note:

Highest point on the summit curve always lie on the side of flatter gradient

- (v) Length of summit curve (L)

SSD is given

OSD/ISD is given

$$L > SSD \quad L = \frac{NS^2}{4.4}$$

$$L > OSD/ISD \quad L = \frac{NS^2}{9.6}$$

$$L < SSD \quad L = 2S - \frac{4.4}{N}$$

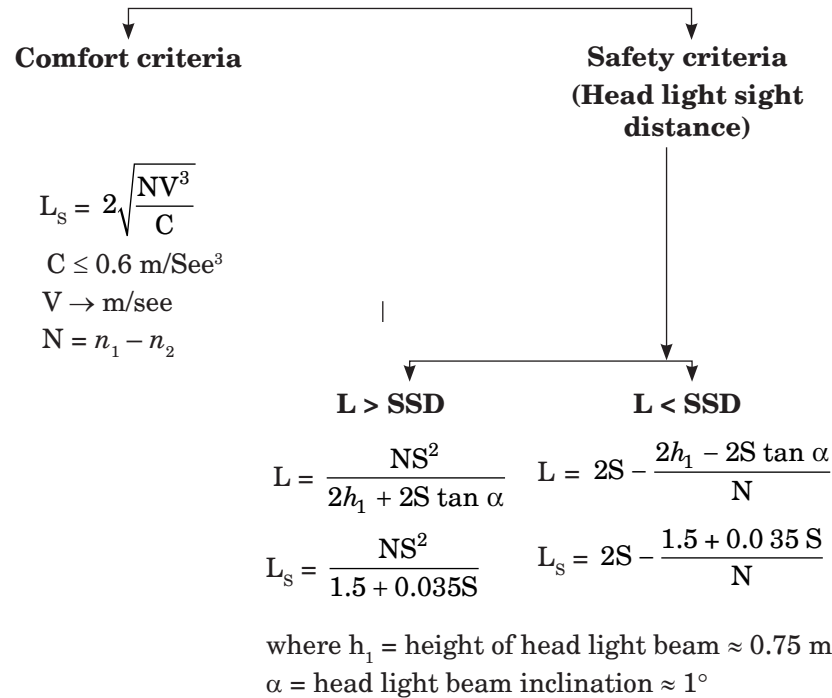
$$L < OSD/ISD \quad L = 2S - \frac{9.6}{N}$$

Note:

In case of very small deviations the length required sometimes comes out to be negative indicating that there is no problem of sight restriction at the summit curve.

Valley Curves : Also known as sag curves. There are the vertical curves having **convexity downwards**. Its design is governed by **comfort criteria** as well as **safety criteria**. Generally **cubic parabola** is preferred for vertical valley curve.

Length of valley curve ($L \rightarrow$ max of both criteria's)

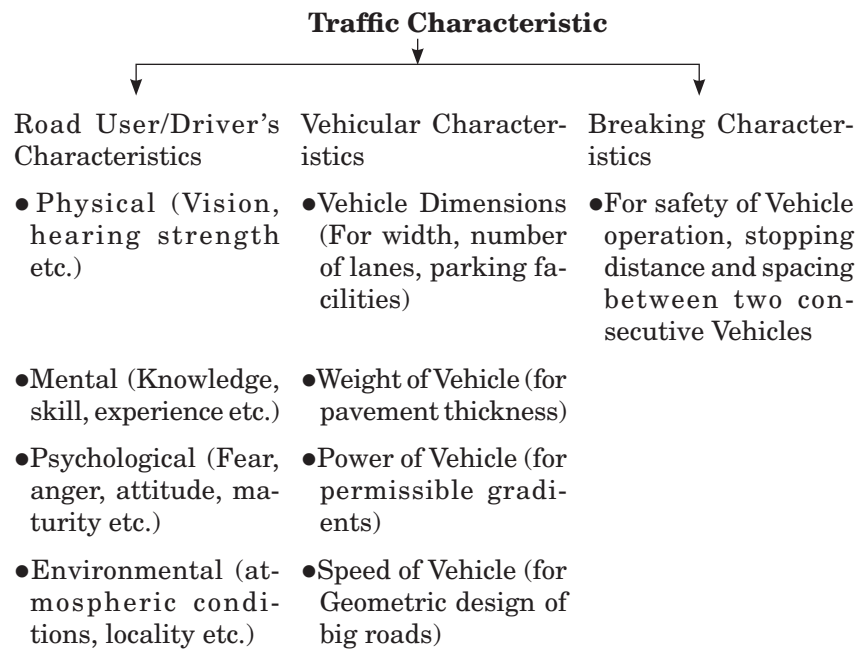

Note:

While calculating S.S.D we neglect the effect of grade as it will be both in up and down grade.

Traffic Engineering: Characteristics

5

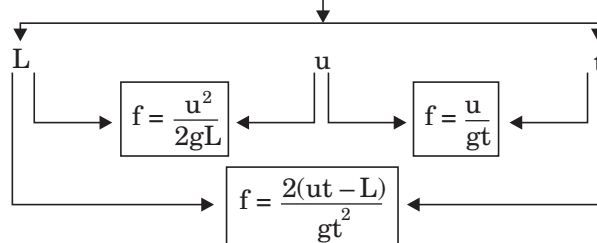
To achieve free and rapid flow of traffic with least number of accidents



Braking Test: At least **2 of the 3 measurements are needed**

1. Braking distance L (m)
2. Initial speed u (m/sec)
3. Duration of brake application t (sec)

To find coefficient of longitudinal friction 'f'



Traffic Engineering Studies

6

Traffic surveys for collecting traffic data is also called census.

1. **Traffic Volume Study:** It is the study of no. of vehicles crossing a section of road per unit time. Measured in vehicle/day or vehicles per hour.

$$q = \frac{n \times 3600}{T} \text{ Vph}$$

n = no. of vehicles passing in time T sec.

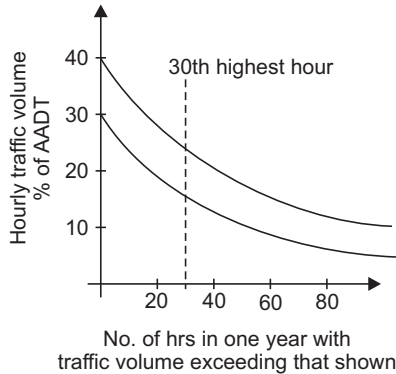
q = equivalent hourly flow.

Conducted by toll plaza ticketing, Registration offices, statistical approach, interview, check post, modern global positioning studies.

Traffic volume count is done with the help of mechanical counters or manually using pneumatic tube, multi-pen recorder.

Presentation of Traffic Volume Data

- (i) **Annual Average Daily Traffic (AADT):** Count the number of vehicles, passing a point for a year than divide it by 365. It includes **seasonal variations** also
- (ii) **Average Daily Traffic (ADT):** Count the number of vehicles passing a point for a period of 'D' days and then divide it by 'D' where D is the no. of days greater than 7 and less than 365. It includes **Weekend Variations** also.
- (iii) **Trend chart:** Showing trend of volume over years.
- (iv) **Traffic flow map along the route:** Thickness of line along the route represents the volume along the route.
- (v) **Volume flow diagrams:** are drawn at intersections indicating the show of volume in different directions at intersections.
- (vi) **Thirteenth highest hourly volume:** It is the hourly volume that will be exceeded only 29 times in a year and all other hourly volumes of the year will be less than this value.

**Note:**

If the roads are designed as per 30th highest hourly volume then the % probability of congestion in a year will be

$$\frac{29 \times 100}{24 \times 365} = 0.331\%$$

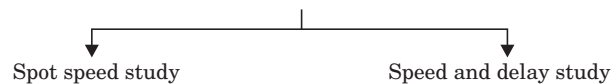
(vii) **Periodic Volume Counts:** These are used to calculate expansion factors

$$\text{Hourly expansion factor} = \frac{\text{Total Volume for 24hr period}}{\text{Volume for particular hour}}$$

$$\text{Daily expansion factor} = \frac{\text{Average total volume for a week}}{\text{Average volume for particular day}}$$

$$\text{Monthly expansion factor} = \frac{\text{AADT}}{\text{ADT of that month}}$$

2. Speed studies: Speed of vehicles vary with time and space.



Spot speed: It is measured using Enoscope, pressure contact tubes, loop deflector and Doppler radar. Average of spot speed is done in two way.

Time mean speed (V_t)	Space means speed (V_s)
Arithmetic mean of speed of vehicles passing a point on a highway during an interval of time.	Average speed of vehicles over a certain road length at any time. It is the harmonic mean of the speed.
$V_t = \frac{\sum V_i}{n} \quad \text{or} \quad V_t = \frac{\sum q_i V_i'}{\sum q_i}$	$V_s = \frac{n}{\sum \frac{1}{V_i}} \quad \text{or} \quad V_s = \frac{\sum q_i}{\sum \left(\frac{q_i}{V_i'} \right)}$

V_i = observed instantaneous speed of i^{th} vehicle (km/hr)

n = No. of vehicles observed

V_i' = mean velocity of vehicles in the speed range.

q_i = No. of vehicles in the i^{th} speed range.

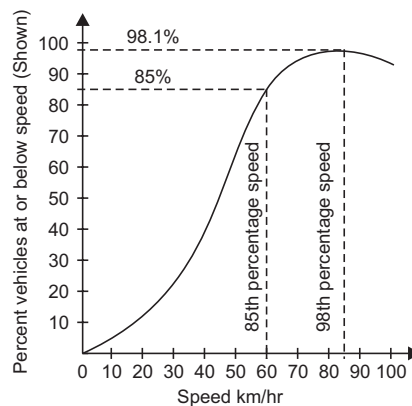
Running speed (excludes stop delays)

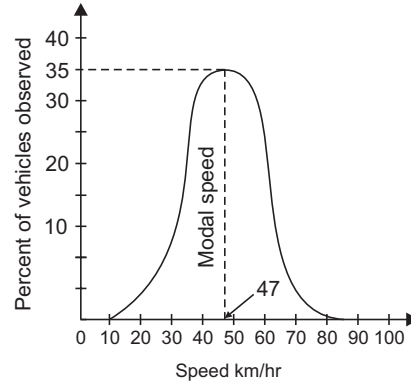
$$= \frac{\text{Length of travel}}{\text{Total time in which vehicle was running}}$$

$$\text{Journey speed (includes stop delay)} = \frac{\text{Length of travel}}{\text{Total Journey time}}$$

Note:

- $V_t > V_s$ as Arithmetic mean > Harmonic mean
- Running speed > Journey speed
- Reciprocal of V_s gives average travel time of all vehicles.
- Spot speed study cannot be used to find density of vehicles





Presentation of spot speed data

98th percentile speed $\Rightarrow$ Design speed

85th percentile speed $\Rightarrow$ Safe speed

15th percentile speed $\Rightarrow$ Lower speed

Modal speed is the speed with which largest number of vehicles move.

Speed and delay studies: are carried out over a long distance and hence it is possible to determine the traffic density. It helps in planning and taking remedial measures to tackle delay at specific locations.

Various methods for carrying out speed and delay study are

- (i) Floating car method
- (ii) Licence plate method
- (iii) Interview technique
- (iv) Elevated observation
- (v) Photographic technique

Floating car methods: Four observers are required

$$\bar{t} = t_w - \frac{y}{q}$$

$$q = \frac{x + y}{t_a + t_w}$$

q = Traffic volume in N–S-direction.

$\bar{t}$ = Average journey time in NS direction

t_w = Journey time of vehicle in NS direction (with traffic).

t_a = Journey time of vehicle in SN direction (against traffic).

x = Number of vehicles in NS direction when test vehicle is moving in SN direction.

y = Number of vehicles (overtaking -overtaken) when vehicle moving in NS direction

Running time = $\bar{t}$ – stop delay

$$\text{Running speed} = \frac{l}{\text{Running time}}$$

3. Origin and destination studies: These studies are helpful in planning new highways and improving new existing services. It is also used in planning **mass rapid transit system**. (MRTS)

Method of collection of data

- (i) **Road side interview method:** Quick collection of data.
- (ii) **Licence plate method:** For small area's like business centres.
- (iii) **Return post card method:** For heavy traffic
- (iv) **Tag on car method:** Continuously moving heavy traffic
- (v) **Home interview method:** Care should be taken while selecting the sample size
- (vi) **Work spot interview method:** By interviews in offices factories etc. O and D data is presented in several forms 1. O and D table 2. Desire lines. 3. pie-chart. 4. Contour lines.

Note:

Desire lines are straight lines connecting the origin points with destinations. The width of such desire lines is drawn proportional to the number of trips in both directions.

4. Traffic capacity studies

Traffic volume (q): Number of vehicles passing through a point during specified unit of time expressed in vehicle/hour or vehicle/day.

Traffic density (k): Number of vehicles occupying a unit length of a lane or roadway at a given instant expressed in vehicle/kilometre.

$$q = kV \quad \text{where } V = \text{space mean speed}$$

Time headway (hi): It is the time interval between the passes of rear bumper of successive vehicle at a point

$$\text{Traffic volume (Veh/hr)} = \frac{1}{\text{Average Time headway(hr/veh)}}$$

Space headway (S_i): Distance between the rear bumper of successive vehicle

$$\text{Density per lane} = \frac{1}{\text{Average space headway}}$$

Traffic Capacity: Maximum no. of vehicles in a lane or a road that can pass a given point in unit time. Expressed in vehicles/hour/lane.

Basic capacity (Theoretical capacity) for nearly ideal roadway and traffic conditions	Possible capacity Under prevailing roadway and traffic conditions.	Practical capacity (Design capacity) It varies from zero to basic capacity.

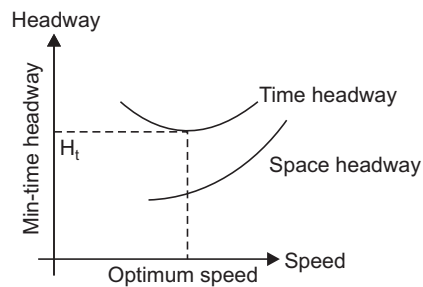
Maximum theoretical capacity 'c':

$$C = \frac{1000 V}{S} \quad C = \frac{3600}{H_t}$$

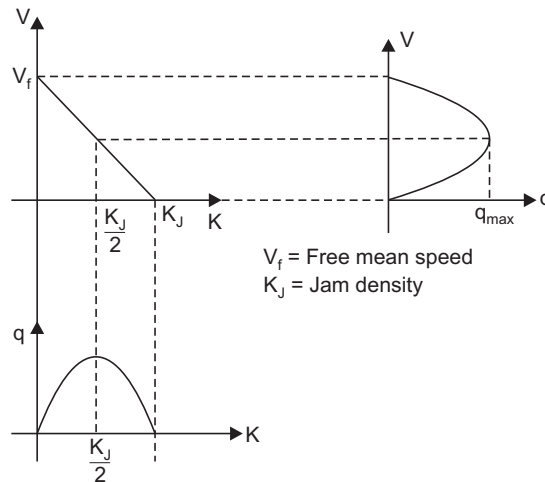
V = space mean speed (km/hr)

S = Minimum space headway (m)

H_t = Minimum time headway (Sec)



Relation between speed, density and volume (Given by Green-Shield)



$$V = V_f - \left(\frac{V_f}{K_J} \right) K$$

$$q = (V_f) K - \left(\frac{V_f}{K_J} \right) K^2$$

$$q = (K_J) V - \left(\frac{K_f}{V_f} \right) V^2$$

$$q_{\max} = \frac{V_f K_J}{4}$$

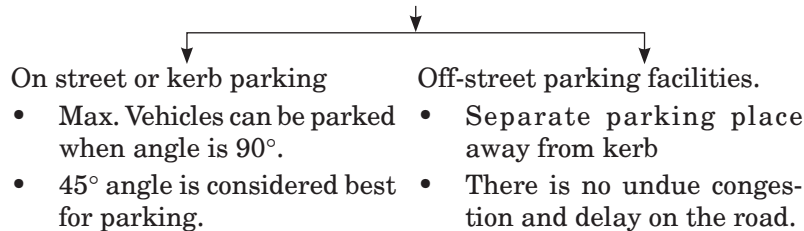
Note:

Highest traffic density will occur when the vehicles are practically standing still on a given route but in this case traffic volume will tend to zero.

Capacity is the quantitative measure where as level of service is the qualitative measure of How. There are six level of services from free flow (A) to forced Flow(F).

Passanger car unit (PCV): It is the ratio of capacity of roadway when there are passanger cars only to the capacity of same roadway when there are vehicles of that class only. For motor cycle its 0.5, for tempo, auto rickshaw its 1.0 and for large Bullock cart its 8.

5. Parking study: It is covered using video recording



6. Accident Studies: To find out the reason and cause behind accident and to take preventive measures. Various records are kept as.

(i) Location files: Identification of points of high accident.

(ii) Spot maps: Accident spots are marked by pins and symbols on map.

(iii) Condition diagram: All important physical conditions at accidental area.

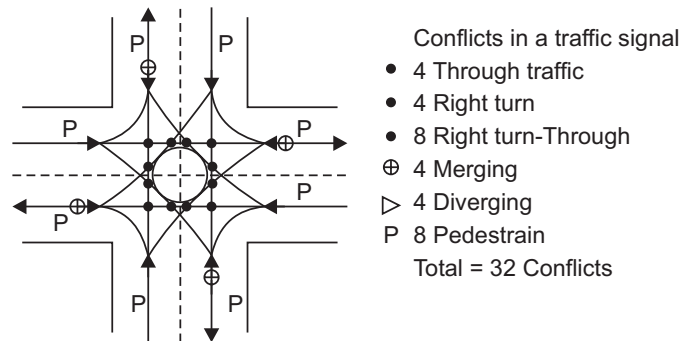
(iv) Collision diagram: Approximate path of vehicles and pedestrians involved in accident.

Traffic Control Regulation

7

Intersection: It is an area where two or more roads join or cross. At traffic intersection change in direction of movement may occur.

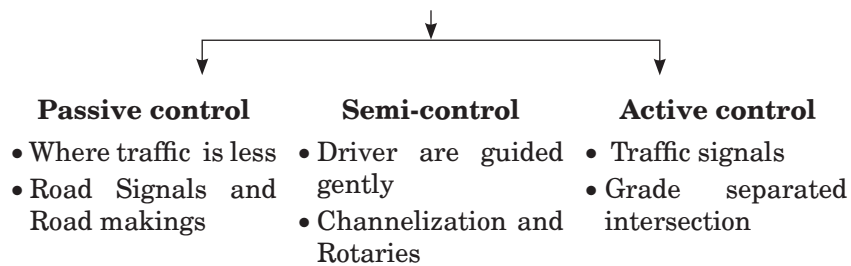
Crossing conflicts are major conflicts where as merging and diverging conflicts are minor conflicts.



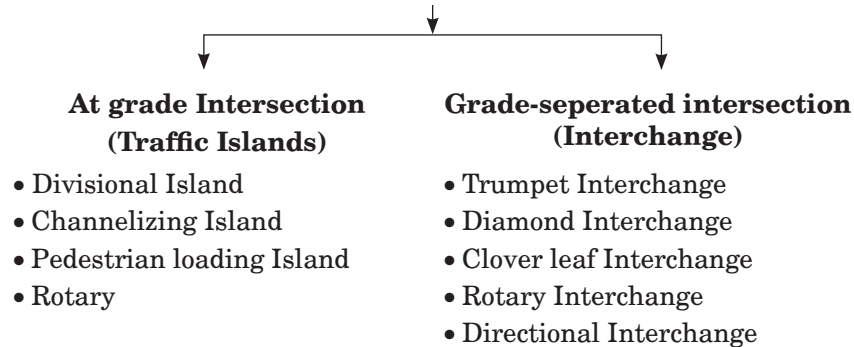
Number of potential conflicts:

- (a) Both roads are two way = 24
- (b) One road is two way, other one-one way = 11
- (c) Both roads are one way = 6

Intersection Control



Types of Intersection



AT GRADE INTERSECTIONS

Divisional Islands Separate the opposing flow of traffic on a highway with four or more lanes. By dividing the highway into two one way roads the head on collisions are eliminated.

Channelizing Islands are used to guide the traffic into proper channel through the intersection area.

Pedestrian loading Islands are provided at regular bus stops and similar places for the protection of passengers. During crossing a multilane highway pedestrian refuge island after two or three lanes is desirable.

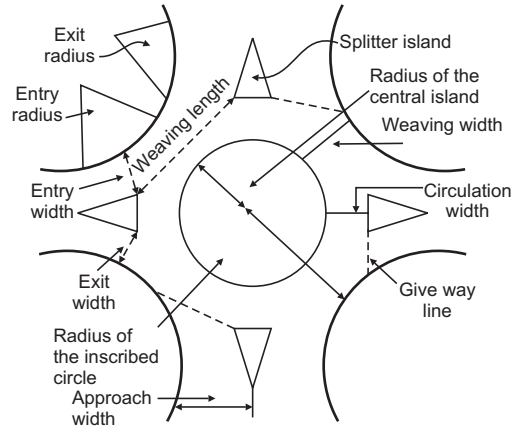
Rotary Island is a large central island of a rotary intersection, this island is much large than the central island of channelized intersection. Here vehicles from converging arms are forced to move round in island in one direction and weave out of the rotary movement into their desired directions,

Guide for Selection of Rotaries

- (i) Relatively equal traffic on roads which are approaching rotary.
- (ii) Suitable when number of approach roads is more than four.
- (iii) Beneficial when right turning traffic is very high.
- (iv) Upper limit → 3000 vehicles/hour, Lower limit → 500 vehicles/hour.

Design Elements

- (a) Design speed for Urban areas/Restricted location is 30 km/hr while that of Rural areas is 40 km/hr.



Design of Rotary

(b) Radius at entry

Rural areas $\rightarrow 20 - 35$ m

Urban areas $\rightarrow 15 - 25$ m

(c) Radius at exit = 1.5 to 2 times radius at entry

(d) Radius of central Island = $\frac{4}{3} \times$ Radius of entry

(e) Minimum weaving length

For Rural areas = 45 m

For urban areas = 30 m

(f) Minimum width of carriage way be at least 5 m with necessary widening to account for the curvature of road.

(g) Width of non-weaving section = width of widest single only into the rotary

(h) Width of weaving section

$$W_{\text{weaving}} = \frac{e_1 + e_2}{2} + 3.5 \quad \begin{array}{l} e_1 = \text{entry width (m)} \\ e_2 = \text{exit width (m)} \end{array}$$

(i) Idealised entry angle is 60° and exit angle is 30°

(j) Capacity of the rotary

$$Q = \frac{280 w \left(1 + \frac{e}{w}\right) \left(1 - \frac{p}{3}\right)}{\left(1 + \frac{w}{l}\right)}$$

Q = practical capacity *i.e.* PCU/
hour

w = width of weaving section

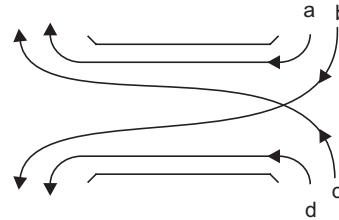
$$e = \frac{e_1 + e_2}{2}$$

e_1 = entry width

e_2 = exit width

l = length of weaving section

p = proportion of weaving traffic



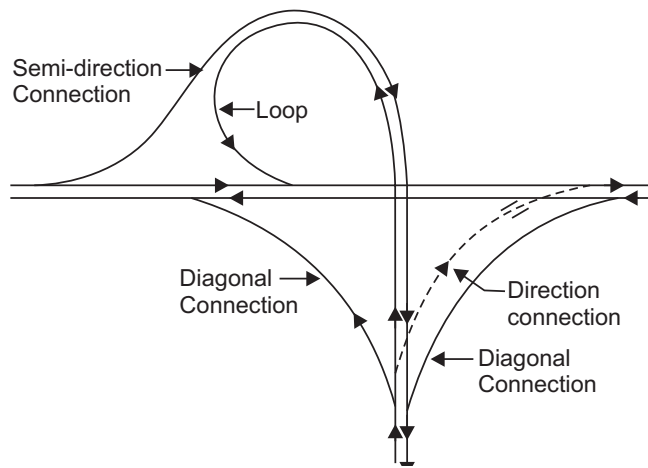
$$p = \frac{b + c}{a + b + c + d}$$

Note:

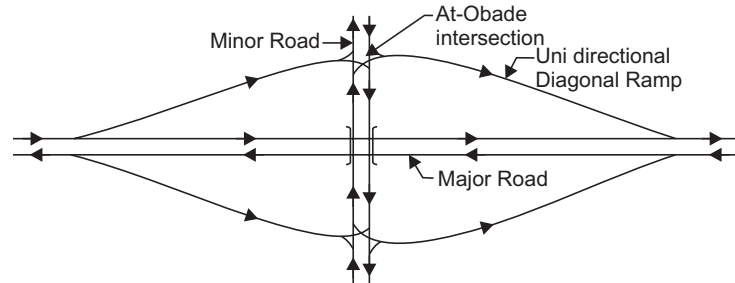
p will be calculated at all the weaving section and highest value will be adopted.

GRADE SEPARATED INTERSECTION

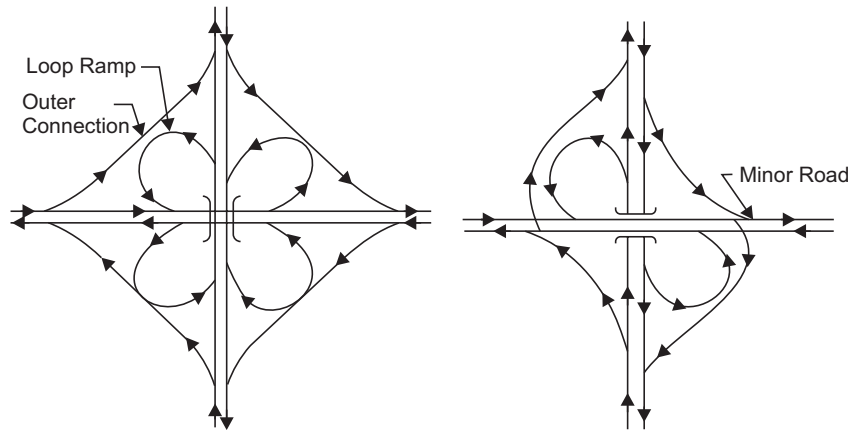
- It permits the cross flow of traffic at different levels without interruptions.
- A grade separation is a crossing of a highway and a railway, two highway or a pedestrian walkway and a highway at different levels.



Trumpet Interchange



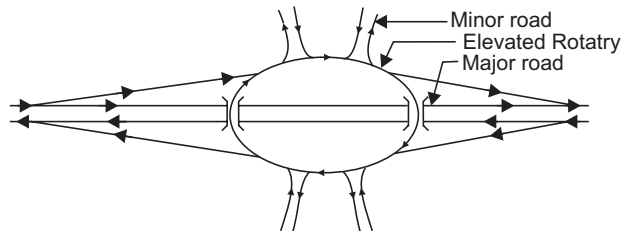
Diamond Interchange



Full Cloverleaf

Partial Cloverleaf

- Diamond interchange is the simplest of the 4 leg interchange.
- Diamond interchange is ideal for both urban and rural traffic but the right turning traffic has to cover extra travel distance.
- For crossing of two major roads of equal importance in rural areas clover leaf interchange is preferred.

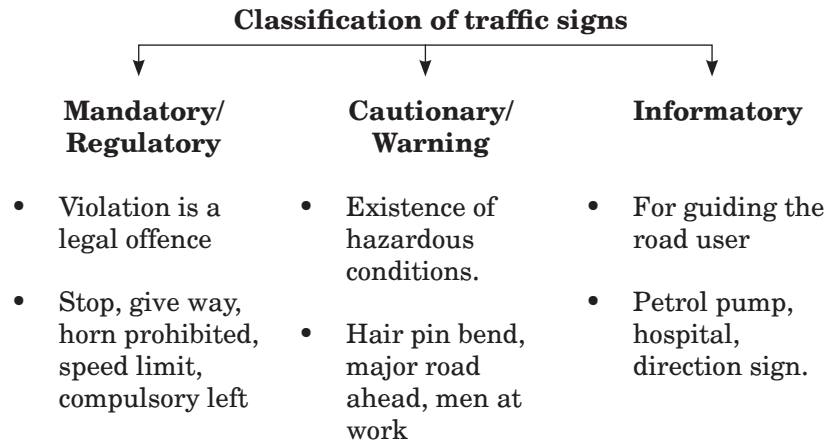


Rotary Interchange

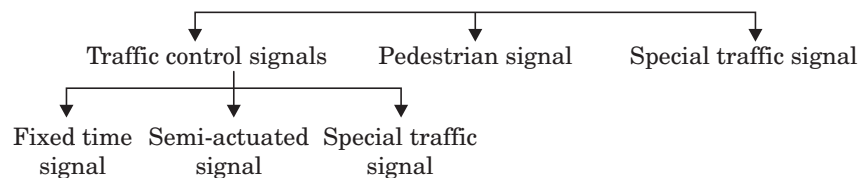
- Rotary interchange is useful where a number of roads intersect at the interchange and sufficient land is available.
- Directional interchange requires more than one structure or a 3 level structure. These are operationally more efficient but most expensive.

Traffic Control Devices

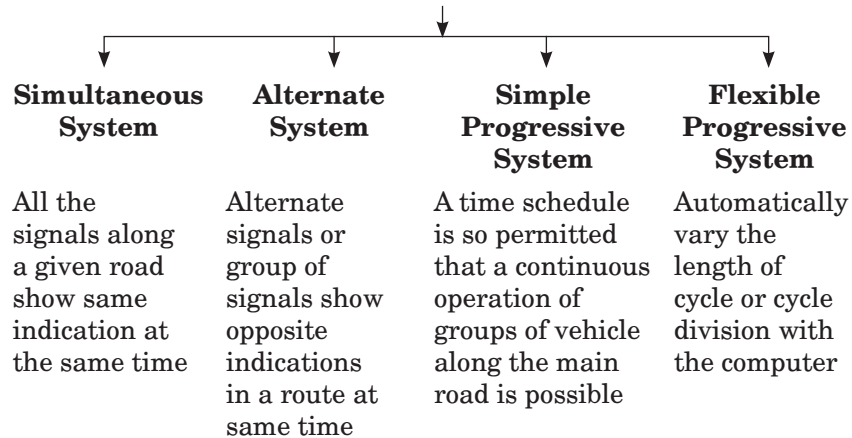
All the traffic signals, signs, pavement marking or other devices placed or created with the approval of a traffic authority having the necessary jurisdiction to regulate, warn or guide traffic.



Traffic signal: These control devices alternately direct the traffic to stop and proceed at intersection using red and green traffic lights signals automatically



Co-ordination of Traffic Signal System



Note:

Simple progressive system is considered to be the best.

Various methods of signal design

(a) Trial cycle method	(b) Webster method
(c) Approximate method	(d) IRC method

Webster Method:

Optimum cycle time (Sec)

$$= \frac{1.5 L + 5}{1 - Y}$$

$$L = nt_L + R$$

$$Y = \sum_{i=1}^n \frac{q_i}{S_i}$$

$$S_i = \frac{3600}{\text{time-head way (secs)}}$$

n = No. of phase

t_L = Start up loss time

R = All red time

q_i = Critical lane volume for phase (max. volume per lane)

S_i = Saturation flow for that phase

Road width	3 m	3.5 m	4 m	4.5 m	5 m	5.5 m
$S \left(\frac{\text{PCU}}{\text{hr}} \right)$	1850	1890	1950	2250	2350	2990

Note:

Y should not be greater than 1.

Webster method is the most rational design method.

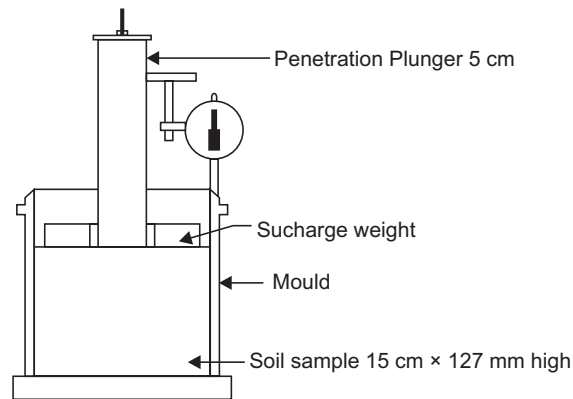
- **IRC method** is an **approximate method**, where optimum cycle time is checked by Webster method.
- Pedestrian walking speed is taken as 1.2 m/sec with initial walk period of 7 secs.
- Minimum green time for vehicular traffic is taken as 16 secs.
- Cycle time should be in multiple of 5 secs.

Highway Material

8

1. Pavement Material Soil: Various tests are performed on soil to check its shearing strength bearing strength and penetration value.

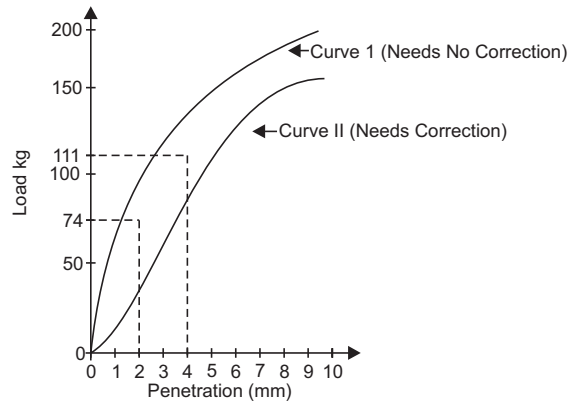
(a) California bearing ratio (CBR) test: Developed by California division of Highway for classifying and evaluating soil sub grade and base course materials for flexible pavements.



- Empirical test, which measures the strength of material and its **not** the true representation of resilient modulus.
- Diameter of piston = 50 mm
- Rate of standard penetration = 1.25 mm/min.
- The pressure upto a penetration of 12.5 mm and its ratio to the bearing value of Standard Crushed Rock is termed as CBR
- 4 days soaked remoulded sample is used
- Average value of 3 test specimens is reported as CBR.

$$\text{CBR} = \frac{\text{load carried by specimen}}{\text{load carried by standard specimen}} \times 100$$

- Load carried by std specimen at
2.5 mm → 1370 kg (70 kg/cm²)
5 mm → 2055 kg (105 kg/cm²)



- If value of CBR at 2.5 mm > value of CBR at 5 mm, adopt CBR value of 2.5 while if value of CBR at 2.5 mm < value of CBR at 5 mm. Repeat the test and even then if value of CBR at 5 mm comes greater then adopt the greater value.

Typical CBR values of soil and their Ratings.

Soil	Range of CBR values	Rating
Clay	2–5	Very poor subgrade
Silt	5–8	Poor subgrade
Sand	8–20	Fair to good subgrade
Gravel	20–30	Excellent subgrade
	30–60	Good sub-base
	60–80	Good base
	80 Std crushed stones	Best base

(b) Plate bearing test: To evaluate the support capability of soil. It is applicable in the design of both **rigid** and **flexible** pavement. It is a **field** test.

- Diameter of plate = 75 cm
- Seating load = 320 kg

$$K = \frac{P}{0.125} \text{ kg/cm}^2/\text{cm}$$

where K = Modulus of subgrade reaction.

P = Loading pressure corresponding to the settlement of 1.25 mm.

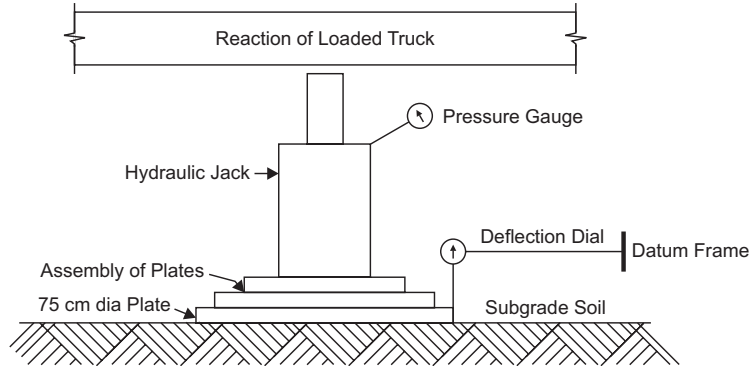
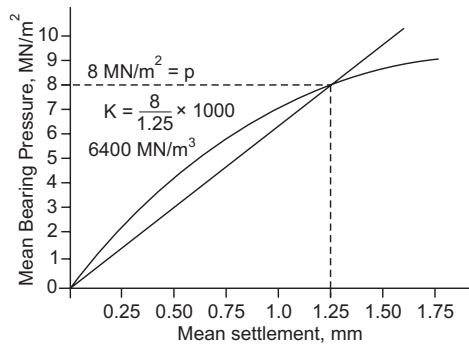


Plate Bearing test



Mean Settlement

$$K_{75} = K_{30} \times \frac{30}{75}$$

K_{75} = K when 75 cm diameter plate is used.

K_{30} = K when 30 cm diameter plate is used.

$$K_{\text{soaked}} = \frac{K_{\text{unsoaked}}}{P_{\text{unsoaked}}} \times P_{\text{soaked}}$$

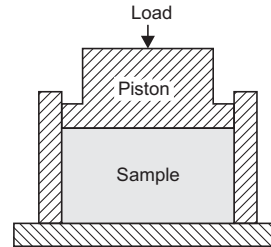
K_{soaked} = K in soaked condition

K_{unsoaked} = K in unsoaked condition.

2. **Pavement Material Aggregate:** By volume aggregate generally accounts for 92 to 96 percent of Bituminous concrete and about 70 to 80 percent of Portland cement concrete. Various desirable properties of aggregate are strength, Hardness, Toughness, Shape of Aggregates, Adhesion, Durability and freedom from deleterious particles.

(a) Crushing test: Gives **strength** of Aggregates

- Aggregates passing from 12.5 mm and retained on 10 mm.
- Filled in cylinder of 11.5 cm diameter and 18 cm height in three layers.
- Each layer tamped 25 times and weighted W_1 .
- Then sample is subjected to 40 tonnes of load at the rate of 4 tones per minute.
- Crushed aggregate passed through **2.36 mm sieve** and passing material weighs W_2 .



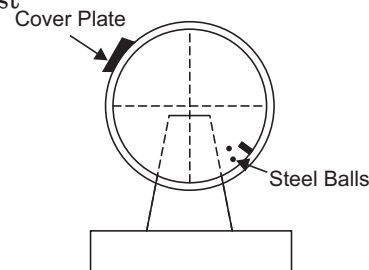
Crushing test setup

$$\text{Aggregate crushing value} = \frac{W_2}{W_1} \times 100$$

(b) Abrasion test: Gives **hardness** of Aggregates

- Loss Angles abrasion test
- Devel abrasion test
- Dorry abrasion test.

- Diameter of drum = 700 mm,
- length = 520 mm
- Cast Iron balls of 48 mm diameter and 340–445 gm weight.



Loss Angeles abrasion test setup

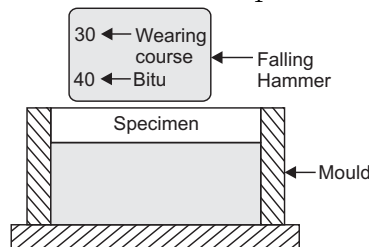
- Aggregate quantity used = 5–10 kg
- Speed of rotation = 30–33 rpm
- Material is sieved through **1.7 mm sieve** and % is thus calculated.

(c) Impact test: Gives **toughness** of Aggregates

- Aggregate passing 12.5 mm and retained on 10 mm sieve.
- Filled in cylinder of 10.2 mm diameter and 5 cm height in 3 layers.
- Each layer tamped 25 times and weighted W .
- 15 blows from metal hammer of 13.5–14 kg from 38 cm.

- Crushed aggregate passed through 2.36 mm sieve and passing material weighs W_2 .

$$\text{Aggregate Impact Value} = \frac{W_2}{W_1} \times 100$$



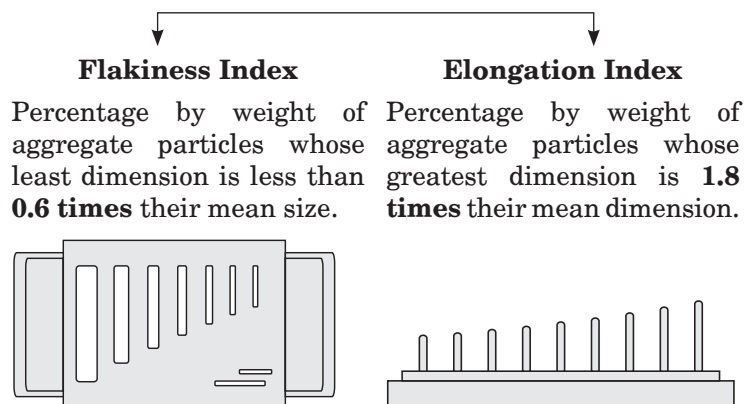
Note:

$$\text{Coefficient of hardness} = 20 - \frac{\text{loss of weight in grams}}{3}$$

(d) Soundness test: Gives resistance of aggregate to weathering action (*i.e.*, **Durability**)

- Aggregates of specified size are subjected to cycle of alternate wetting in a saturated solution of either **sodium sulphate** or **magnesium sulphate** for 16–18 hours and then dried in oven at 105–110°C to a constant weight. After 5 cycles, loss in weight is determined by serving out under sized particles.
- Loss in weight should be less than 12% when sodium sulphate is used and 18% when magnesium sulphate is used.

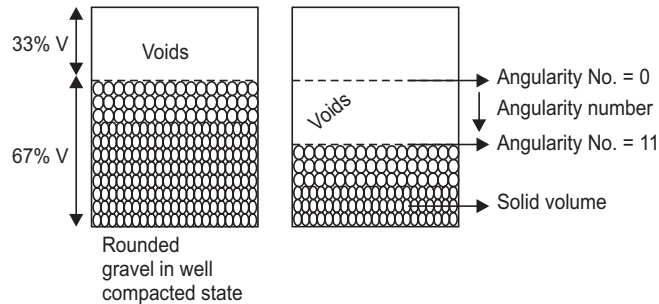
(e) Shape test: Gives idea of **workability** and **stability** of mix.



(f) Angularity Number: Measures the voids in excess of 33%.

$$\text{Angularity Number} = 67 - \% \text{ Solid volume.}$$

- Higher the angularity no, more angular is the aggregate.
- It varies from 0 to 11.



(g) Specific gravity and water absorption:

$$G_{\text{app}} = \frac{M_D/V_N}{D_W}$$

G_{app} = Apparent specific gravity

M_D = Dry mass of aggregate

D_W = Density of water

$$G_{\text{Bulk}} = \frac{M_D/V_B}{D_W}$$

V_N = Net volume of aggregate excluding volume of adsorbed water.

V_B = Total or Bulk volume of aggregate including volume of adsorbed water

- Difference between the apparent and bulk specific gravities is the water permeable voids of the aggregates.
- The specific gravity of aggregates is normally between 2.5 to 2.9 and water absorption values range from 0.1 – 2.0 percent.

(h) Bitumen adhesion test: Gives **stripping value** of aggregates.

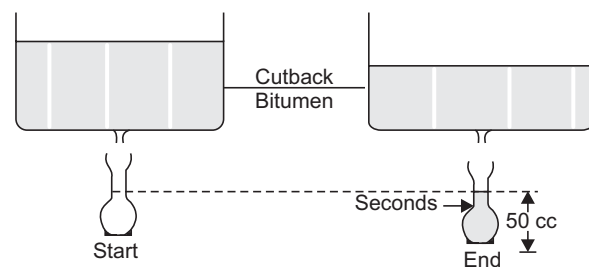
- Adhesion problem occurs only when the aggregate is wet or cold.
- As per IRC, **static immersion test** is performed. In this test an aggregate fully coated with binder is immersed in water maintained at 40°C temperature for 24 hours

Note:

IRC has specified maximum stripping value as 5%.

3. Pavement Material Bitumen: It is considered flexible material from structural point of view. Most common source of bitumen is through **Petroleum crude**. Indian crude does not yield good bitumen for rural work except **Digboi** bitumen of Assam.

(a) Viscosity test: Done by STV (Standard Tar Viscometer). Say bolt Furol, Redwood and Engler.

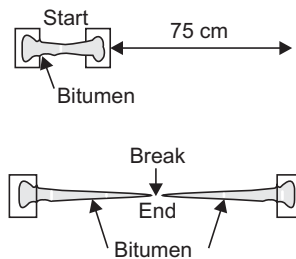


The viscosity of liquid bitumen is measured by **Efflux Viscometer**. Viscosity is indirectly measured as the time required to pass a measured quantity (50 to 200 ml) of bitumen through the orifice (of diameter 3–10 mm) at a constant temperature.

Furol Viscosity is standardised test.

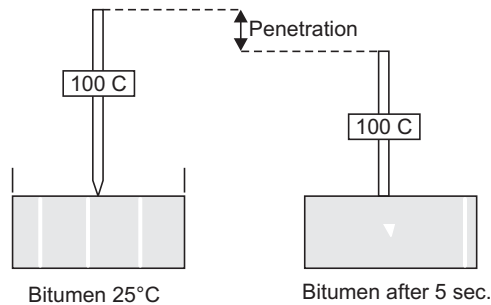
(b) Ductility test: Ability to **deform** under load

- Measured by stretching a standard briquette of bitumen having cross-sectional area of 1 cm^2 at a temperature of 27°C and rate of pull being 5 cm/min .



- As per indian standards, minimum ductility value is 75 cm. for grades 45 and above.

(c) **Penetration test: Resistance to flow** or measurement of **hardness** of bitumen.



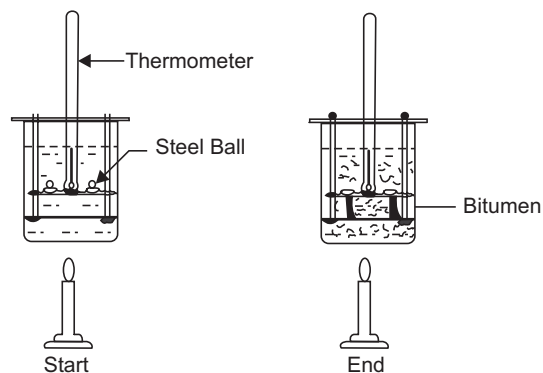
- Distance penetrated in sample by a standard pointed needle at 27°C when a 100 gm of load is applied for 5 secs.
- Unit of penetration is 1/10 mm.
- 80/100 means penetration of 8–10 mm.

Note:

Penetration test is not required in tar.

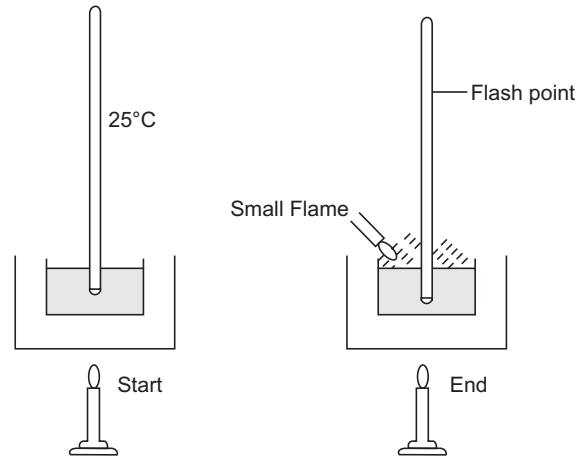
(d) **Softening point:** It is done to ensure safety.

- Done by **ring and ball** apparatus.
- Softening point is the temperature at which bitumen attains a particular degree of softness under standardised test conditions.
- It indicates the temperature at which bitumen passes from solid to liquid consistency.



(e) Flash and Fire Point

Flash point is the lowest temperature in degree celsius at which the application of a test flame causes the vapour from bitumen to **catch fire momentarily** in the form of flash.



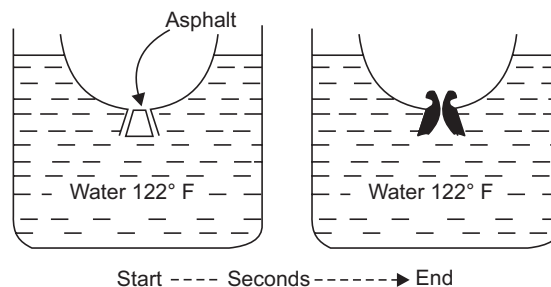
Fire point is the lowest temperature in degree celsius at which the application of test flame causes the bitumen to **ignite and burn** atleast 5 secs under specified test conditions.

(f) Specific gravity: Determined by Pycnometer.

- At 27°C it 1.00 for pure bitumen its range is 0.97 to 1.02.
- Tar has specific gravity more than bitumen (1.10 to 1.25)

(g) Float test: For viscosity of bitumen

- It is time in secs required for a small plug of chilled bitumen which is held in an open mould attached to the bottom of a saucer to become sufficiently fluid, when the saucer is floated in water at 50°C.



Note:

Type of Bitumen	Use
Penetration grade 30/40	Areas where temperature difference range is less than 25°C, Roads with high volume of traffic.
Penetration grade 60/70	Superior type of roads with high traffic in normal summer temperature.
Cutback bitumen	Cold weather condition.
Emulsion	Wet conditions (Rainy season) maintenance work, soil stabilisation in deserts.

Cut back Bitumen: Bitumen whose viscosity is reduced by adding volatile diluents.

Types of cutback

Rapid curing (RC)	Medium curing (MC)	Slow curing (SC)
Penetration value 80/120	Good wetting property	Blending bitumen with high boiling point gas or oil.
Eg: Petroleum	Eg: Kerosene, light diesel oil	

Note:

- The cutbacks are also designated by numbers. The bigger the number, the thicker or viscous it will be.
- MC – 2 will be thicker than MC – 1 but RC – 5 and SC – 5 will have same viscosity.

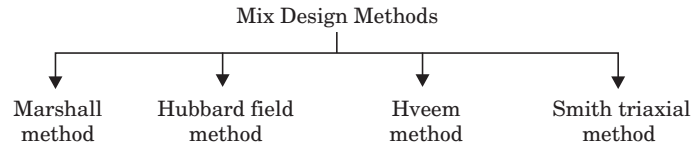
Tar: Produced by **carbonization of coal**. RT – 1 is used for surface painting while RT – 5 is used for grouting.

Note:

Tar is soluble in Toluene while Bitumen is soluble in carbon disulphide and carbon tetra chloride.

Tar is more temperature susceptible, shows greater viscosity variations with temperature.

Bitumen is more water resistant.



Marshall Mix Design: This **wet** mix design determines the optimum moisture content. **Stability test** and **flow test** provides the performance prediction measure for the marshall mix design methods.

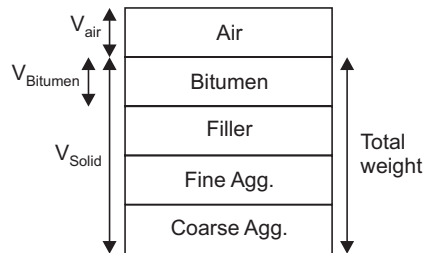
Stability: Maximum load carried by the specimen at a standard temperature of 60°C and expressed in kg.

Flow: It is measured as determination in units of 0.25 mm between that load and maximum load during stability test.

Properties of the Mix

1. Theoretical specific gravity (G_t)

$$G_t = \frac{\text{Total weight}}{(\text{volume of solid}) \gamma_w}$$



2. Bulk specific gravity (G_m)

$$G_m = \frac{\text{Total weight}}{(\text{Total volume}) \gamma_w}$$

$$= \frac{\text{Weight in Air}}{\text{Weight in Air} - \text{Weight in water}}$$

Note:

$G_t > G_m$ always.

3. Percentage Air voids (V_v):

$$V_v = \frac{\text{Volume of air}}{\text{Total volume}} \times 100$$

4. Percentage volume of bitumen (V_b):

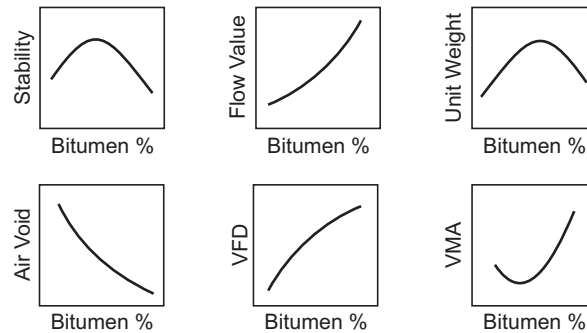
$$V_b = \frac{\text{Volume of bitumen}}{\text{Total volume}} \times 100$$

5. Voids in mineral aggregate (VMA):

$$\text{VMA} = \frac{\text{Volume of (Air + Bitumen)}}{\text{Total volume}} \times 100$$

6. Voids filled with Bitumen (VFB):

$$\text{VFB} = \frac{\text{Volume of Bitumen}}{\text{Volume of (Air + Bitumen)}} \times 100$$



Marshall graphical plots

Note:

Optimum binder content will be the average value of bitumen obtained from the graphs of stability, (bulk specific gravity and percentage air voids.)

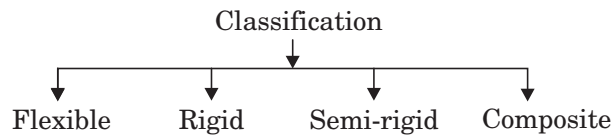
Marshall Mix design specification

Test property	Value
Stability	340 kg
Flow value	8–16 units
Percentage air voids	3–5
VFB	75–85

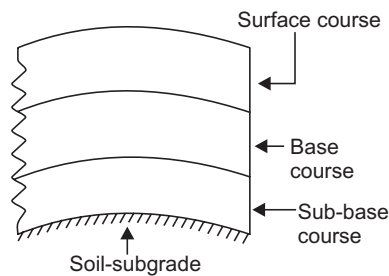
Pavement Design

9

Pavement: Load bearing and load distributing component of a road.

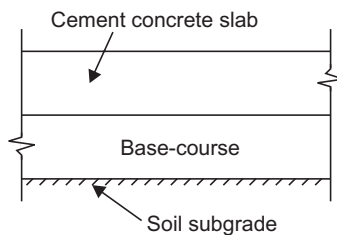


- In flexible pavement compressive stresses are transmitted to the lower layer by **grain to grain** transfer through the point of contact.



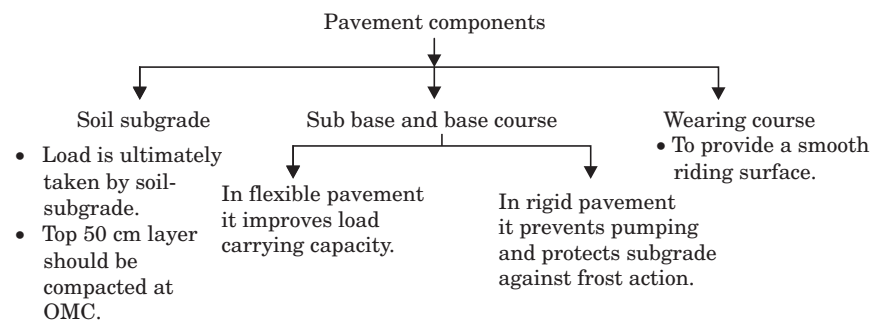
Flexible pavement

- In flexible pavement if the lower layer of the soil subgrade is undulated, the flexible pavement surface also gets undulated.
- **Bituminous concrete** is one of the best flexible layer materials.
- In rigid pavement (which is constructed as concrete slab) strength depends on its **flexural strength** or beam action of the slab for withstanding the wheel load.



Rigid-pavement

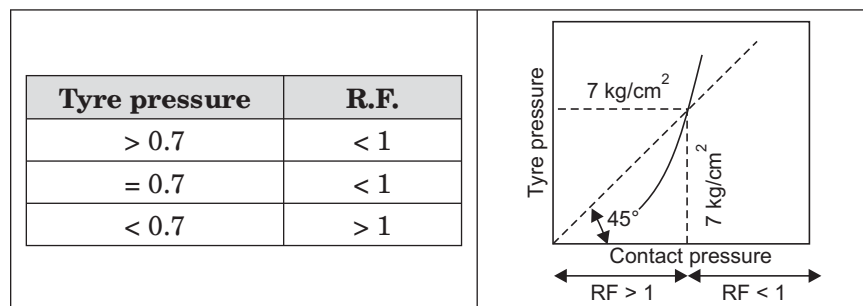
- The critical condition of stress in the rigid pavement is the maximum flexural stress occurring in the slab due to wheel load and the temperature changes.
- A semi-rigid pavement derives its strength both by load spreading and flexural action. It is made by using materials like pozzolonic concrete (lime-flyash-aggregate mix), lean cement concrete or soil cement etc.
- Composite pavement has layers made up of materials having different properties. Eg: A lean concrete base having a roller compacted concrete slab over it and bituminous concrete surfacing at the top.



Contact Pressure and Tyre pressure:

$$\text{Rigidity factor} = \frac{\text{Contact Pressure}}{\text{Tyre Pressure}}$$

$$\text{Contact pressure} = \frac{\text{Wheel load}}{\text{Area of imprint}}$$



Note:

- When there is high pressure in tyre's then tyre comes in tension. And obviously then contact pressure will be less than tyre pressure.
- Tyre pressure is a important for upper layers while contact pressure is important for bottom layers.

Traffic Fore cost

$$N = 365 P(1 + r)^x \frac{[(1 + r)^n - 1]}{r}$$

N = **Cummulative** number of **commercial** vehicles during **design** life.

r = rate of increase of traffic expressed in **fraction** *i.e.* 0.075 **per year**.

p = Number of **commercial** vehicles per day at **last count**.

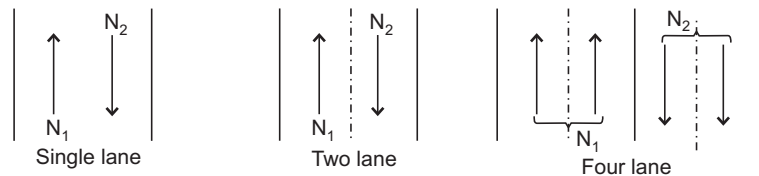
x = Construction period in **year's**.

n = Design life in years.

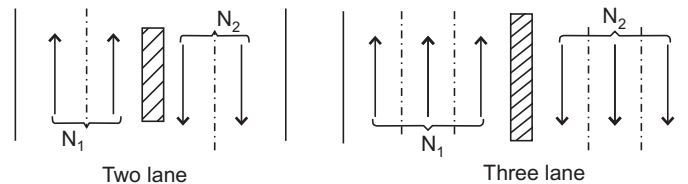
Here, $P(1 + r)^x = A$, where A can be given as. Initial design traffic in Vehicle per day in the year of completion of construction.

Note:

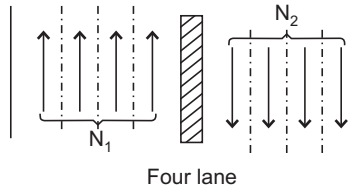
- This value of N is then modified by multiplying it with lateral distribution factor (LDF) and Vehicle damage factor (V.D.F.)

Lateral Distribution Factor (L.D.F.)**1. Single carriage way**

$$LDF = 1 (N_1 + N_2) \quad LDF = 0.50 (N_1 + N_2) \quad LDF = 0.40 (N_1 + N_2)$$

2. Dual carriage way

$$LDF = 0.75 (\max (N_1, N_2)) \quad LDF = 0.6 (\max (N_1, N_2))$$



Four lane

$$\text{LDF} = 0.45 (\max (N_1, N_2))$$

Vehicle Damage Factor (VDF): It is equivalent number of standard axles per commercial vehicle. It is a multiplier to convert the number of commercial vehicles of different axle loads and axle configuration to the number of standard axle load repetitions.

For eg: If 500 Vehicles is equivalent in 4000 standard axles then

$$\text{VDF} = \frac{4000}{500} = 8$$

Load consideration in Design

Fixed traffic approach
(Equivalent Single Wheel
Load-ESWL)

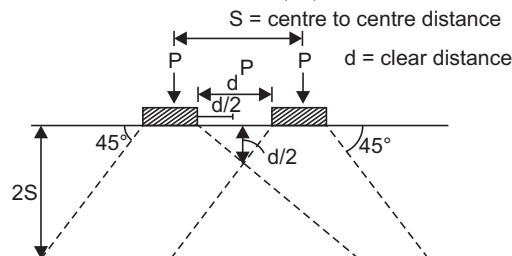
Variable traffic and variable
vehicle approach

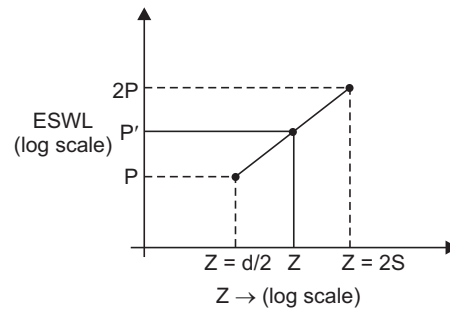
Fixed Vehicle Approach
(Equivalent Axle Load
Factor-EALF)

Equivalent Single Wheel Load

Load on a single tyre which will cause an equivalent magnitude of pre-selected parameters at a given location to that resulting from the multiple wheel load at same location.

$$\log P' = \log P + \left[\frac{\log 2P - \log P}{\log 2S - \log \left(\frac{d}{2} \right)} \right] \times \left[\log Z - \log \left(\frac{d}{2} \right) \right]$$



**Note:**

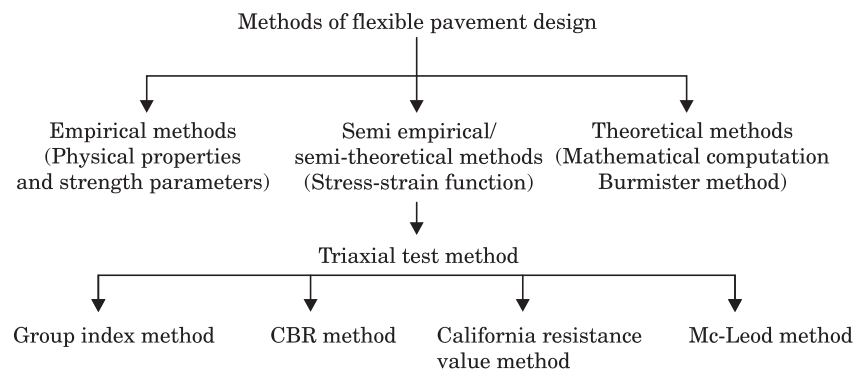
This approach is used in **Airport pavement Design**.

Equivalent axle load Factor (EALF): It defines the damage caused to the pavement by one application of the axle load under consideration relative to the damage caused by single application of standard axle load of 80 kN.

$$\text{EALF of a axle load} = \left(\frac{\text{axle load}}{\text{standard axle load}} \right)^4$$

$$\text{Eg: 240 kN axle load has EALF} = \left(\frac{240}{80} \right)^4 = 81$$

Methods of Flexible Pavement Design



- 1. Group Index Method:** Arbitrary index assigned to the soil types in numerical equations posed on percentage fineness, liquid limit and plasticity index.

$$G.I = 0.2a + 0.005ac + 0.01bd$$

For $a, b, c, d \rightarrow$ Refer soil classification.

Note:

- Thickness of surface course and base course is function of both Traffic volume and group index while that of Sub-base course depends only on Group Index.
- Quality of material is not considered in this method as it suggests the same thickness for poor or good quality material.

- 2. California bearing Ratio method:**

Pavement thickness,
$$t = \sqrt{\left(\frac{1.75}{\text{CBR}}\right) - \left(\frac{A}{\pi}\right)}$$

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Applicable only when CBR < 12%

t = thickness in cm

P = wheel load in kg

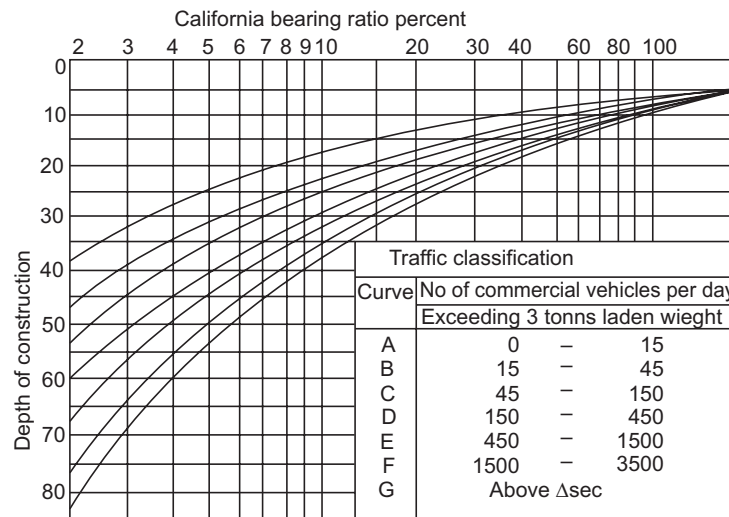
CBR = in percentage

A = Area of contact; cm^2

where

$A = \pi r^2$

r = radius of contact area



Note:

- There are 7 curves from A to G in CBR design chart by IRC 37.
- For finding the thickness of pavement directly from the chart only two things are needed.
 - (a) No. of commercial vehicles per day exceeding 3 tonnes.
 - (b) CBR % of soil-subgrade over which the pavement is to be made.
- In CBR method also, it gives total thickness required over the subgrade irrespective of the quality of material used in component layers.

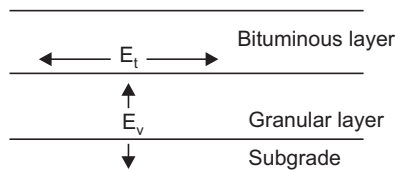
IRC 37, Recommendations for CBR test

1. **Remoulded** soil should be tested in laboratory after soaking it for **4 days**.
2. Soil should be compacted at OMC to proctor density.

3. At least **3 samples** should be tested.
4. Top 50 cm subgrade should be compacted to atleast **95 to 100%** of proctor density.
5. Annual rate of increase in vehicles is taken as **7.5%**.
6. The design thickness is considered applicable for single axle loads upto 8200 kg and tandem axle loads upto 14500 kg.

IRC recommendations for flexible pavement design:

1. It considers **fatigue cracking** and **Rutting** as major flexible pavement failures.
2. Fatigue cracking in bituminous layers looks like **crocodile** cracking and Rutting is the permanent deformation caused due to deformation in subgrade.
3. Fatigue is caused due to tensile strain (E_t) at the bottom of bituminous layer while Rutting is caused due to vertical strain (E_v) at the subgrade top.



4. For fatigue cracking, cracking in 20% of area has been considered for traffic upto 30 MSa and 10% beyond 30 MSa.
5. For Rutting, the limiting value recommended as 20 mm in 20% of length upto 30 MSa and in 10% length for traffic beyond 30 MSa.

$$N_R = 4.1656 \times 10^{-8} \left(\frac{1}{E_v} \right)^{4.5337}$$

$$N_f = 2.21 \times 10^{-4} \left(\frac{1}{E_t} \right)^{3.89} \left(\frac{1}{E} \right)^{0.854}$$

N_R = Number of cumulative standard axles to produce rutting of 20 mm.

N_F = Number of cumulative standard axles to produce 20% cracked surface area.

E = Elastic modulus of bituminous surfacing.

6. IRC gives design chart for pavement thickness depending on CBR of subgrade (**Range 2 to 10%**) and design traffic in MSa (**Range 1 to 150 MSa**)

7. If adequate data is not available then by IRC, “**Guidelines for traffic prediction on Rural highways**” an average annual growth rate of **7.5%** may be adopted.
8. If annual growth rate data for commercial vehicles is not available or it is less than 5%, a growth rate of **5%** may be adopted (**IRC : SP 84 – 2009**)
9. Commercial vehicles are the vehicles having gross vehicle weight of 30 kN.
10. Design life for **Expressways** and **Urban roads** may be **20 years** or more. For **National Highway** and **State highways** its minimum of **15 years** for other category roads its 10 to 15 years.
11. Conversion of different axle load repetitions into equivalent standard axle load repetitions. Can be done as

$$(a) \text{ Single axle with single wheel on either side} = \left(\frac{\text{Axle load in kN}}{65} \right)^4$$

$$(b) \text{ Single axle with dual wheel on either side} = \left(\frac{\text{Axle load in kN}}{80} \right)^4$$

$$(c) \text{ Tandem axle with dual wheels on either side} = \left(\frac{\text{Axle load in kN}}{148} \right)^4$$

$$(d) \text{ Tridem axle with dual wheel on either side} = \left(\frac{\text{Axle load in kN}}{224} \right)^4$$

3. California Resistance Value Method

$$\text{Thickness of pavement } t \text{ (cm)} = \frac{(K)(TI)(90 - R)}{C^{1/5}}$$

K = Numerical constant = 0.166

R = Stabilometer value

C = Cohesionometer value

Traffic Index TI = $1.35 (\text{EWL})^{0.11}$

EWL = Equivalent wheel load

EWL = $\sum (\text{EWL constant}) \times (\text{AADT})$

Note:

$$\frac{t^1}{t^2} = \left(\frac{C_2}{C_1} \right)^{1/5}$$

Number of Axle	EWL Constants	AADT
2	330	N_1
3	1070	N_2
4	2460	N_3
5	4620	N_4

Then $EWL = 330 N_1 + 1070 N_2 + 2460 N_3 + 4620 N_4$

4. M_C - Leod Method: Thickness of gravel base required

$$t \text{ (cm)} = K \log_{10} (P/S)$$

P = Gross wheel load, kg

S = Total subgrade support, kg

K = Base course constant, depends on loaded area

5. Tri-Axial test method:

Single layer system, Pavement thickness

$$T = \sqrt{\frac{3PXY}{2\pi E_S \Delta} - a^2}$$

Two layer system, pavement thickness

$$T = \left\{ \sqrt{\left(\frac{3PXY}{2\pi E_S \Delta} \right)^2 - a^2} \right\} \left\{ \frac{E_S}{E_P} \right\}^{1/3}$$

P = Wheel load, (kg)

E_S, E_P = modulus of elasticity of subgrade and pavement (kg/cm²)

X = Traffic coefficient

Δ = Design deflection (0.25 cm)

Y = Saturation coefficient

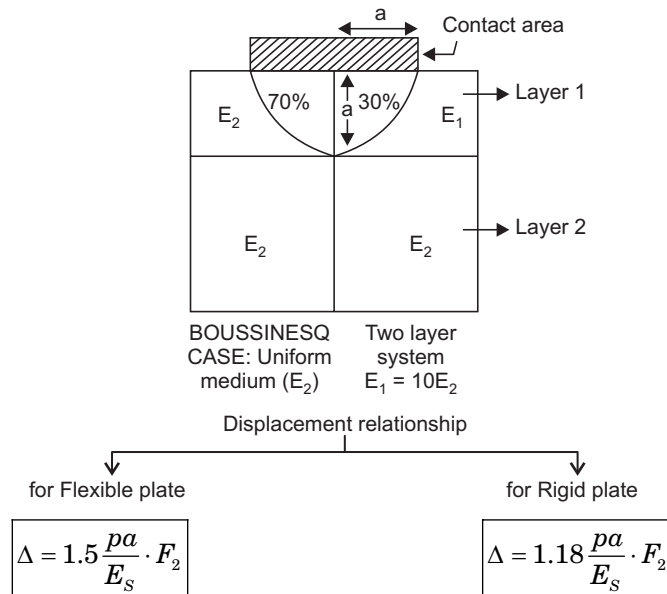
a = radius of contact area (cm)

Note:

- $\left(\frac{E_S}{E_P} \right)^{1/3}$ is known as stiffness factor.
- The relation between pavement layers of thickness t_1 and t_2 with elastic modulus E_1 and E_2 is given by $\frac{t_1}{t_2} = \left(\frac{E_2}{E_1} \right)^{1/3}$.

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6. **Burmister method:** It is a **layered system analysis** in which the pavement is composed of flexible layers of materials placed in such a way that the elastic modules of top layer of is highest.
- Length and width of section (*i.e.*, horizontal dimensions) are considered infinite but depth or thickness (*i.e.*, vertical dimension) is considered finite.
 - Vertical stress on subgrade are reduced from 70 to 30 percent by introducing a pavement layer of thickness equal to radius of the load, having elastic modules 10 times to that of subgrade soils elastic modulus.



D = Design deflection

p = contact pressure at road surface

E_s = modulus of elasticity of soil subgrade

a = Radius of contact area.

F_2 = Deflection factor depends on $\frac{E_s}{E_p}$ and $\frac{h}{a}$;
 h = depth of layer 1.

Design of Rigid Pavement

11

Modulus of sub-grade reaction (K)

Cement concrete pavement rests on soil foundation which can be treated as a spring having a spring constant K.

$$K = \frac{p}{0.125} \text{ kg/cm}^3$$

As per IRC $K_{25 \text{ cm}} = 0.5 K_{30 \text{ cm}}$
 where p = pressure in plate bearing test (kg/cm²)
 corresponding to 1.25 mm settlement.

Radius of Relative Stiffness: It is the relative stiffness of slab to sub-grade.

$$l \text{ (cm)} = \left[\frac{Eh^3}{12 K(1 - \mu^2)} \right]^{1/4}$$

E = modulus of elasticity of cement concrete in kg/cm²
 (3×10^5 kg/cm²)

h = slab thickness in cm

K = modulus of sub-grade reaction

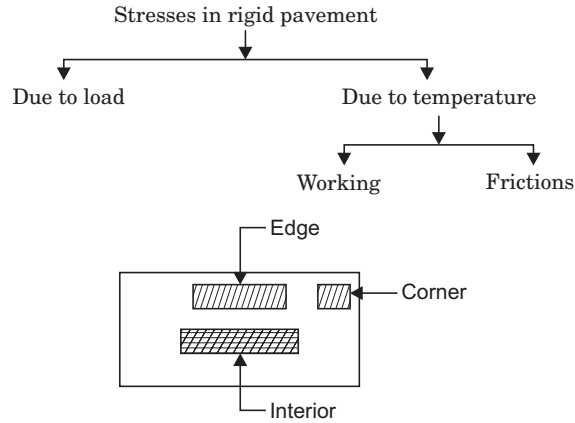
μ = concrete's poisson ratio = 0.15

Equivalent Radius of Resisting Section: It is the radius of small area of the pavement which is resisting the bending moment of a plate due to loading. As per **Westergard**.

$$b \text{ (cm)} = \begin{cases} \sqrt{1.6a^2 + h^2} - 0.675h, & a < 1.724h \\ a, & \text{otherwise} \end{cases}$$

a = radius of wheel load distribution (cm)

h = Slab thickness (cm)



(a) Stresses due to load (by westergard)

$$\sigma_{\text{corner}} \text{ (kg/cm}^2\text{)} = \frac{3p}{h^2} \left(1 - \left(\frac{a\sqrt{2}}{l} \right)^{0.6} \right)$$

$$\sigma_{\text{edge}} \text{ (kg/cm}^2\text{)} = \frac{0.572p}{h^2} \left(4 \log_{10} \left(\frac{l}{b} \right) + 0.359 \right)$$

$$\sigma_{\text{internal}} \text{ (kg/cm}^2\text{)} = \frac{0.316p}{h^2} \left(4 \log_{10} \left(\frac{l}{b} \right) + 1.069 \right)$$

Note:

σ_{corner} is tensile stress at top of slab while σ_{edge} and σ_{internal} are tensile, stresses at the slab bottom.

(b) Stresses due to change in temperature: It is of two types

(i) Warping stresses: (Due to **daily variation** in temperature)

$$\sigma_{t \text{ corner}} = \frac{E \alpha t}{3(1-\mu)} \sqrt{\frac{a}{l}}$$

$$\sigma_{t \text{ edge}} = \max \left\{ \frac{C_x E \alpha t}{2}, \frac{C_y E \alpha t}{2} \right\}$$

$$\sigma_{t \text{ internal}} = \frac{E \alpha t}{2} \left(\frac{C_x + \mu C_y}{1 - \mu^2} \right)$$

α = thermal expansion coefficient of concrete

t = temp difference between top and bottom of slab.

C_x, C_y = coefficient depending upon $\frac{L_x}{l}, \frac{L_y}{l}$ respectively.

Note:

During Night, tensile stresses develop at the top of slab and Vice Versa for day.

(ii) Frictional stresses: Stresses developed due to **seasonal variation** of temperature and no temperature gradient is considered across the thickness.

Note:

In winter's tension develops in the slab while in summer compression develops in the slab.

Critical combination of stresses

	Corner	Edge	Interior
Wheel load	Maximum	Intermediate	Minimum
Temperature	Minimum	Intermediate	Maximum

Note:

Designing is done using edge region stress and checking is done for corner region.

Combinations

(a) Summer and Mid-day (At edge)

$$\sigma_{\text{load}} + \sigma_{\text{warping}} - \sigma_{\text{friction}}$$

(b) Winter and Mid-day (At edge)

$$\sigma_{\text{load}} + \sigma_{\text{warping}} + \sigma_{\text{friction}}$$

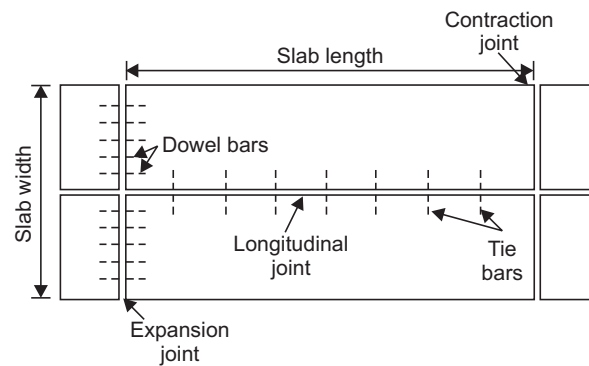
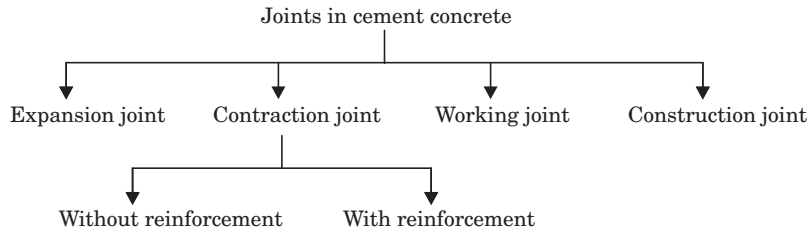
(c) Mid-Night (At Corner)

$$\sigma_{\text{load}} + \sigma_{\text{warping}}$$

Note:

At day time, critical stresses develop at edge's while at night they develop at corner.

Generally maximum stress occur at summer mid-day.



Location of Joints

1. **Expansion:** Provided to prevent expansion due to **rise in temperature** with respect to construction temperature.
 - Provided at 50 – 60 m spacing if construction is in winter and 90 to 120 m if construction is done in summers.
 - For rough interface the spacing is generally kept at 140 m.
 - Approximate gap width is 20 to 25 mm.
 - For load transference across the transverse joint, **dowel bars** are placed.

$$L = \frac{\delta}{2 \alpha \Delta T}$$

L = maximum spacing between expansion joint.

δ = Gap of expansion joint.

α = coefficient of thermal expansion.

ΔT = rise in temperature.

2. **Contraction Joint:** To provide control due to **Shrinkage and moisture Variation**.

(a) When no reinforcement has been provided

$$L = \frac{2\sigma_{st}}{f \gamma_{conc.}}$$

σ_{st} = permissible tensile strength in concrete.

f = Coefficient of friction between concrete and base.

$\gamma_{conc.}$ = Unit weight of concrete.

Note: For $f = 1.5 \gamma_{conc.}$ 24 kN/m³, then $\sigma_{st} = 0.8 \text{ kg/cm}^2$
Maximum spacing without reinforcement is **4.5 m**.

(b) When reinforcement is provided in the slab.

$$L = \frac{2\sigma_{st} A_{st}}{B h \gamma_{conc.} f}$$

A_{st} = Area of steel in complete width of slab.

h = depth of slab.

3. Warping Joints: These are provided to relieve stresses due to warping. These are known as hinged joints. They are rarely needed.

4. Construction Joints: These are provided at the end of a day's work or when work is stopped unexpectedly due to interruption for more than 30 minutes.

Longitudinal Joint: These are provided along the length of the pavement to reduce the warping stress. These are provided with **tie bars**.

Note:

Tie bars are not designed as load transfer device but ensures that two slabs remain firm together.

Length of tie bar = 2 development length (L_d)

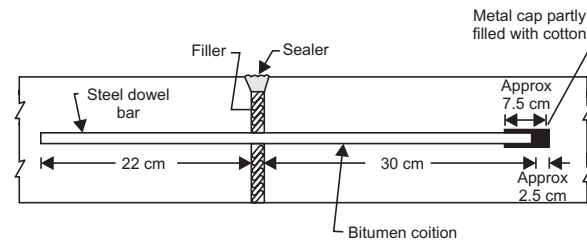
$$L_d = \frac{\sigma_{st} \phi}{4 Z_{bd}} \quad \phi = \text{dia of bar (nearly 10 mm)}$$

Z_{bd} = bond stress between steel and concrete.

σ_{st} = permissible tensile stress in steel.

Dowel bar: Load transfer device across transverse joint and they keep the two slab at same height.

- Length between 40 cm to 73 cm, nearly **60 cm**.
- Diameter of bar varies between 20 mm to 30 mm.
- Spacing between dowel bars is kept at 30 cm.



Expansion joint with Dowel Bar

- Mild steel round bars are bonded at one end while the other side is kept free.
- Stresses in dowel bar are given by **Bradbury Analysis**.
- Stresses in dowel bars are

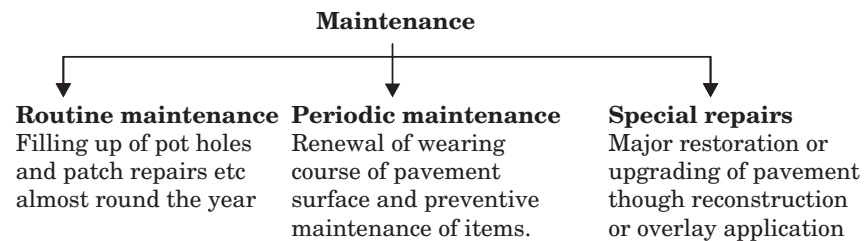
Shear	Bending	Bearing
$P_s = \sigma_s \times \frac{\lambda d^2}{4}$	$P_{\text{bend}} = \sigma_s \times \frac{\sigma_b \times 2d^3}{L + 8.8\delta}$	$P_{\text{bearing}} = \frac{\sigma_{br} L^2 d}{12.5(L + 1.5\delta)}$

L = embedded length of dowel bar (cm)

δ = Gap of joint (cm)

Note: Load transfer capacity of the dowel bar is assumed to be 40% of the wheel load.

The distance on either side of the load position upto which the group of dowel bars are effective in load transfer is taken as $1.8 l$ where l = radius of relative stiffness.



Defects of Flexible pavement

1. Surface defects

- (a) **Fatty surface:** Bituminous binder moves upward
- (b) **Hungry surface:** Loss of aggregates
- (c) **Smooth surface:** Very low skid resistance
- (d) **Streaking:** Alternate lean and heavy lines of bitumen

2. Cracks

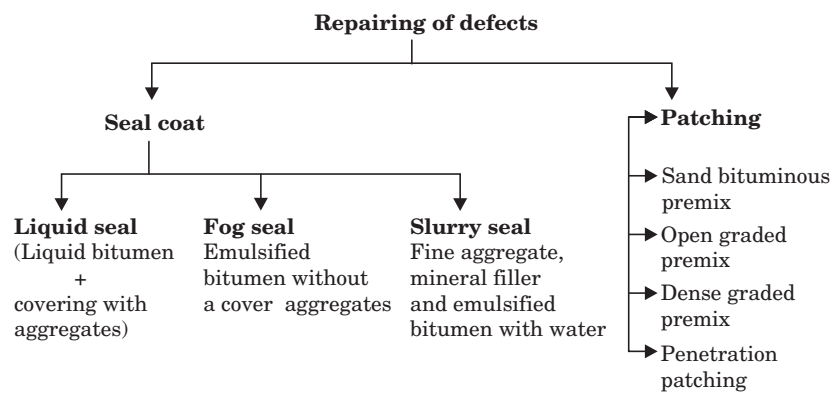
- (a) **Hair line cracks:** Short and fine cracks at short interval
- (b) **Alligator or map cracking:** Random deep cracks
- (c) Longitudinal cracks
- (d) **Edge cracks:** At 0.3 – 0.5 m from inside edge
- (e) Shrinkage or Transverse cracks
- (f) Reflection or Sympathetic cracks – over joints

3. Disintegration

- (a) **Stripping:** Separation due to poor bitumen adhesion.
- (b) **Ravelling:** Progressive disintegration of the surface
- (c) **Pot hole:** Bowl shaped holes extending into base course
- (d) Edge breaking
- (e) **Loss of aggregate:** Some portion of aggregate remains intact, while from other places it is lost.

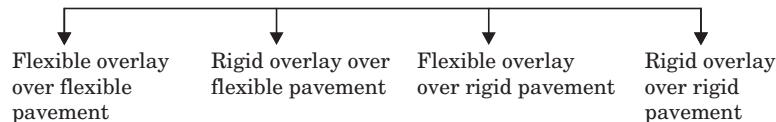
4. Deformation:

- (a) **Slippage:** Relative movement between surface layer and layer beneath.
- (b) **Rutting:** Longitudinal depressions on the surface.
- (c) **Corrugation:** Regular undulations.
- (d) **Shoving:** Localised bulging due to plastic movement with in layer.
- (e) **Shallow depressions:** Size nearly 25 mm.

**Defects of rigid pavements**

1. Scaling of cement concrete
2. Shrinkage Cracks
3. Spalling of joints
4. Warping cracks
5. Mud pumping
6. Structural cracks.

Overlay: Strengthening may be done by providing additional thickness of the pavement, which is called overlay.

**1. Flexible overlay over flexible pavement**

$$h_o = h_d - h_e$$

h_o = overlay thickness required (cm)

h_d = Design thickness required (cm)

h_e = Total thickness of existing pavement (cm)

Benkelman Beam Deflection method:

Characteristic deflection (D_c) $\rightarrow D_c = D + t\sigma$

Where D = design deflection

σ = Standard deflection

t = depends on % of deflection values to be covered in design.

Over lay thickness design

$$h_o = \frac{R}{0.434} \log_{10} \frac{D_c}{D_a} \quad \text{Ruiz's Equation.}$$

h_o = thickness of Bituminous overlay in cm

R = Deflection reduction factor depending on the overlay material.

$$h_o = 550 \log_{10} \frac{D_c}{D_a} \quad \text{As per 1 RC}$$

h_o = Thickness of granular or WBM overlay in mm.

$D_c = \bar{D} + 6$, after applying the corrections for pavement temperature and subgrade moisture,

$D_a = 1.00, 1.25$ and 1.50 mm if the projected design traffic A is 1500 to 4500, 450 to 1500, and 150 to 450 respectively

2. Rigid overlay over rigid pavement

$$h_o = (h_d^n - h_e^n)^{1/n}$$

h_o = Rigid overlay thickness

h_d = Design thickness

h_e = existing pavement thickness.

3. Flexible overlay over rigid pavement

$$h_f = 2.5 (fh_d - h_e)$$

$$h_b = 1.66 (fh_d - h_e)$$

h_f = flexible overlay thickness

h_e = Existing rigid pavement thickness

h_d = Design thickness of rigid pavement

f = factor which depend upon modulus of existing pavement.

h_b = Thickness of bituminous overlay.

Civil Engineering

14

Surveying

CONTENTS

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2. Linear Measurement
3. Compass-Surveying
4. Theodolite
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7. Tacheometry
8. Trigonometric Levelling
9. Photogrammetry
10. Theory of Errors
11. Curves
12. Field Astronomy
13. Measurement of Area and Volume
14. Plane-Table Surveying
15. Contouring

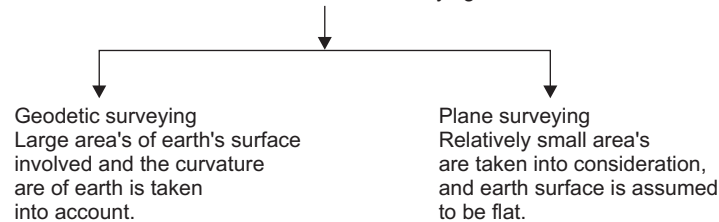
Fundamentals of Surveying

1

Surveying is the method of making measurement of the relative positions of natural and man-made features on earth surface and the presentation of this information either graphically or numerically.

Both Plan and Map are the graphical representations of the features on horizontal plane but plan is a large scale representation while map is a small scale representation.

Basic classification of surveying

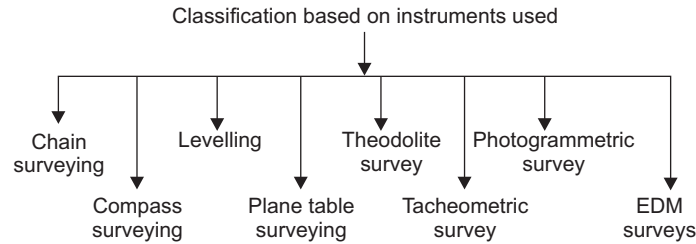


Note: Geodetic surveying is done by Department of National Survey of India. It is done for fixing widely spaced control point, which may afterwards be used as necessary control points.

Classification of Surveying Based on Purpose

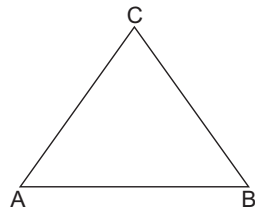
- (i) **Topographical Survey:** Conducted to obtain data and to make a map indicating inequalities of land surface.
- (ii) **Engineering Survey:** Used for design and construction of new routes e.g. roads and railways. Also used to calculate the area and volumes of land and data for setting out curves for route alignment.
- (iii) **Cadastral Survey:** Done to produce plans of property boundaries for legal purpose.
- (iv) **Hydrographic Survey:** Conducted on or near water bodies. Marine survey is also one of its type.
- (v) **Astronomic Survey:** Conducted for determination of latitudes, longitudes, azimuths, local time etc. at various places on the earth by observing heavenly bodies.

- (vi) **Geological Survey:** Conducted to obtain data of different strata of earth's surface for the purpose of geological studies.

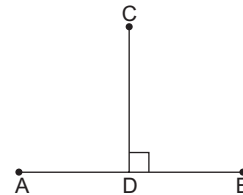


Principles of Surveying

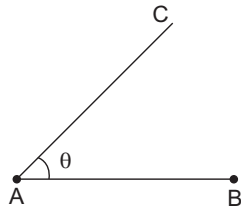
- (i) **Work from Whole to Part:** So as to localise the error and prevent their accumulation.
- (ii) **Locate a Point by Atleast two Measurements:** Locating at point C.



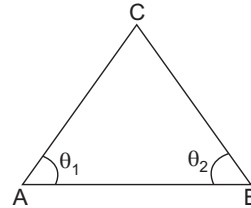
(a) By measuring AC and BC



(b) By measuring CD at right angle to AB while AD and BD is known.



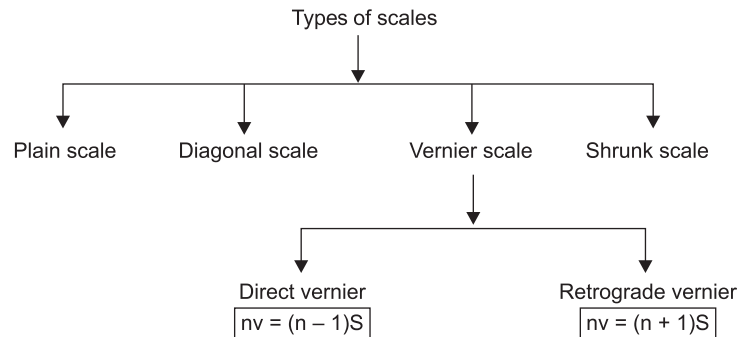
(c) By measuring AC and angle θ



(d) By measuring θ_1 and θ_2

Scale of a map: It is the fixed ratio that every distance on the plan bears with corresponding distance on the ground.

$$\text{Representative fraction (RF)} = \frac{\text{Map distance}}{\text{Ground distance}}$$



- (i) **Plain Scale:** Used to measure two dimensions only, like units and length.
- (ii) **Diagonal Scale:** It is possible to measure in three dimensions such as meters, decimeter and centimeters; unit, tenth, hundreds.
- (iii) (a) **Direct Vernier:** $(n - 1)$ divisions of main scale is equal to n divisions of vernier scale. In this Vernier Scale moves in same direction as main scale.
- (b) **Retrograde Vernier:** $(n + 1)$ divisions of main scale is equal to n divisions of vernier scale. In this vernier scale moves in opposite direction to that of main scale

$$\text{Least count of Vernier Scale} = \frac{s}{n}$$

S = Value of one smallest division of main scale

n = no. of division on the vernier

V = Value of one smallest division of Vernier Scale

- (iv) **Shrunk Scale**

$$\text{Shrunk scale} = \text{Original scale} \times \text{Shrinkage factor (SF)}$$

$$\text{Shrinkage Ratio (SR) or (SF)} = \frac{\text{Shrunk length}}{\text{Original length}} = \frac{\text{Shrunk scale}}{\text{Original scale}} = \frac{\text{Shrunk RF}}{\text{Original RF}}$$

$$\text{Correct distance on map in case of original scale} = \frac{\text{Measured distance on Map}}{\text{Shrinkage Ratio}}$$

$$\text{Correct area on map in case of original scale} = \frac{\text{Measured distance on map using planimeter}}{(\text{Shrinkage Ratio})^2}$$

Error due to use of Wrong Scale

$$(i) \text{ Correct length} = \frac{\text{RF of wrong scale}}{\text{RF of correct scale}} \times \text{Measured length}$$

$$(ii) \text{ Correct area} = \left(\frac{\text{RF of wrong scale}}{\text{RF of correct scale}} \right)^2 \times \text{Measured area}$$

Precision and Accuracy

Precision is the degree of fineness and care with which any physical measurement is made. It represents the set of observations that are closely grouped together and have small deviations from true value.

Accuracy is the degree of perfection obtained. For a set of measurements to be accurate, its value should be close to true value.

Note: A good draughts man can plot a length to within 0.25 mm.

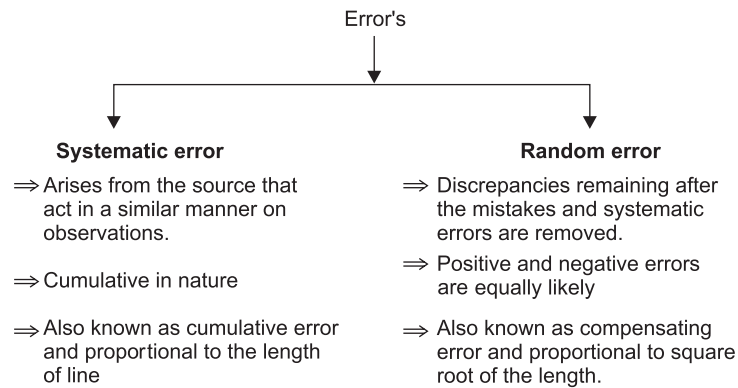
Mistakes and Error

Mistakes are caused by misunderstanding of the problem, carelessness or poor judgement and these can be corrected only if discovered.

Error's arise due to physical conditions of the instrument like temperature at the time of measurement and human eye's limitations.

Error = Measured value – True value
Correction = True value – Measured value

Hence Correction = – Error



Linear Measurement

2

Also known as chain surveying or tape or offset surveying. Chain surveying is suitable only for areas of small extent on open ground.

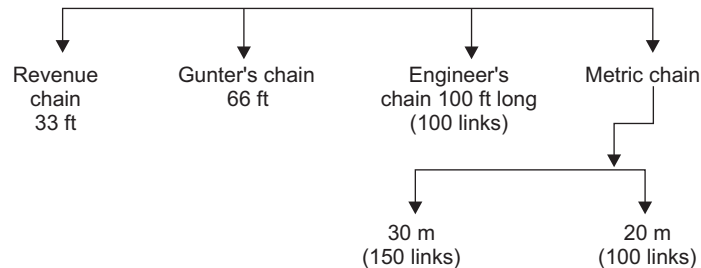
Terms Used in Large Survey Area

- (i) **Main station:** It is a point in chain survey where two triangle sides meet.
- (ii) **Main survey line:** Chain line joining two main survey stations.
- (iii) **Tie station or subsidiary station:** Station on survey line joining main stations.
- (iv) **Base line:** Longest survey line from which direction of all other survey lines are fixed.
- (v) **Proof line or check line:** Provided to check the accuracy of the field work.
- (vi) **Offset:** Distance of object from the survey line.
- (vii) **Chainage:** Distance measured along the main survey line in direction of progress of work.

Well Conditioned Triangle: Triangle in which all intersections of lines are clear. Angle's between the lines for clear intersections should be 30° to 120° . An **equilateral triangle** is the most appropriate well conditioned triangle.

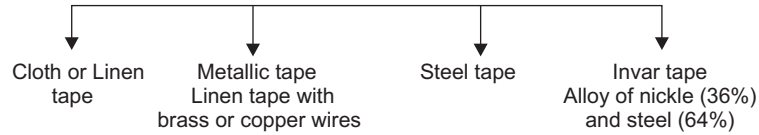
Equipments Used for Measuring Lines

- (i) **Surveying chain:** Used where very high accuracy is not required.



Note: As per IS specifications, every metre length of the chain should individually be accurate to within $\pm 2\text{mm}$ when measured under a tension of 80 Newton.

(ii) Tapes:



(iii) Pegs: To mark definite points on ground temporarily.

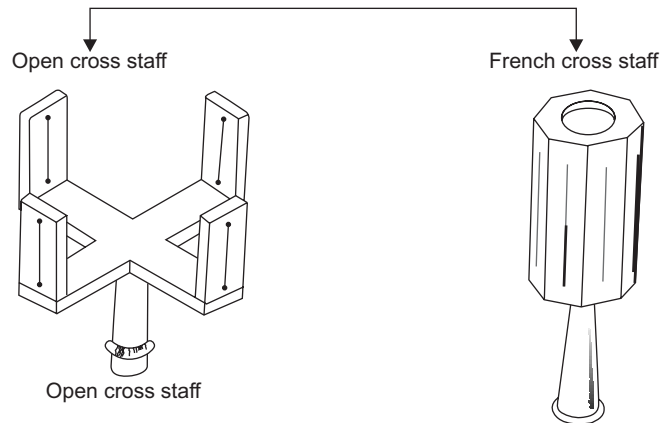
(iv) Arrows: It provides a check over the length of line as entered in the field notes.

(v) Ranging rods: To locate intermediate points such that these points lie on straight lines joining the end stations.

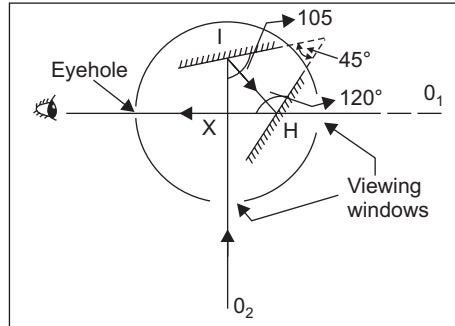
(vi) Offset rod: Similar to ranging rod with a sout open hook at the top.

Equipments for Measurement Right angles

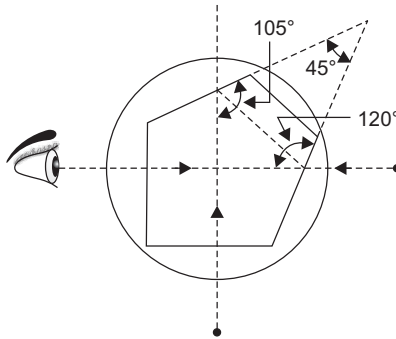
(i) Cross staff



(ii) Optical Square: Pocket instrument, more convenient and accurate than a cross staff.

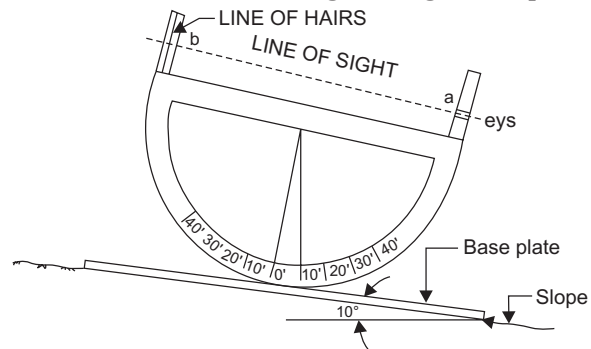


- (iii) Prism Square: It has two reflecting surfaces fixed at 45° . No adjustment is required in it.

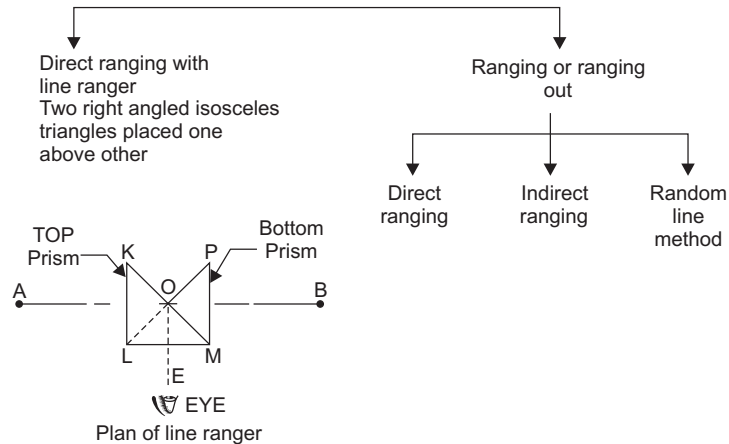


Equipment for Measuring Ground Slope

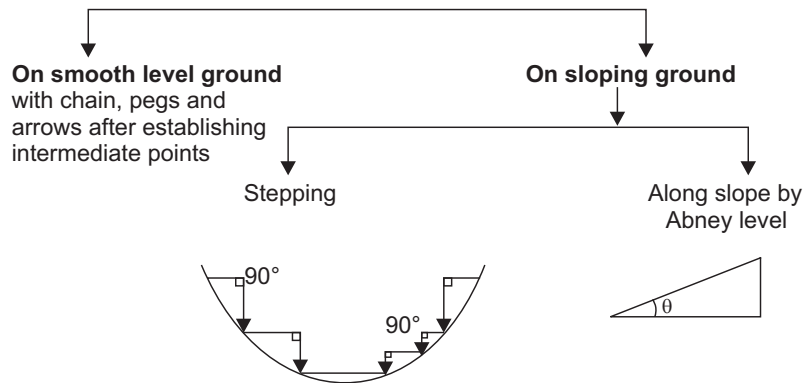
- (i) Simple clinometer: For measuring the angle of slope of the ground.



Equipments for Establishing Intermediate points



Linear Measurement with Chain



Correction in Chain Measurement

(i) Standardisation Correction

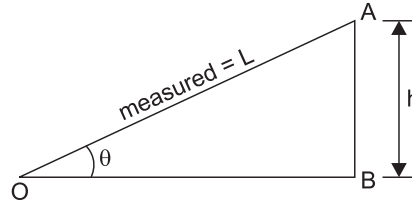
$$\text{Correct length} = \text{Actual length of chain} \times \frac{\text{Length reported to be measured}}{\text{Nominal length of chain}}$$

$$\text{Correction} = \text{True length} - \text{measured length}$$

(ii) Slope Correction

(a) when slope θ is measured

$$\text{Correction} = L \cos \theta - L \Rightarrow -L (1 - \cos \theta)$$



To be calculated = $L \cos \theta$

(b) When AB is measured

$$\text{Correction} = \frac{-h^2}{2L}$$

Note: Slope correction is always negative.

Corrections in Tape Correction

- (i) Standardisation correction: Same as above
- (ii) Slope correction: Same as above.
- (iii) Pull correction is (C_p):

$$C_p = \left(\frac{P - P_s}{AE} \right) L$$

P_s = Applied pull

P_s = Standard pull

L = Length of tape

A = Area of cross section

E = Modulus of elasticity

= 2.1×10^5 N/mm² for steel tape

= 1.54×10^5 N/mm² for Invar tape

Note: If pull is more than standard pull correction will be positive.

(iv) Temperature Correction (C_t):

$$C_t = \alpha (T_m - T_o) L$$

T_m = Mean temperature

T_o = Standardisation temperature

α = Thermal coefficient

(v) Sag Correction (C_s)

$$C_s = \frac{-w^2 l^3}{24 p^2}$$

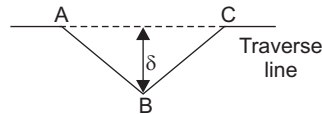
w = weight of tape per unit length in N/m

l = length of tape suspended between supports

p = applied pull

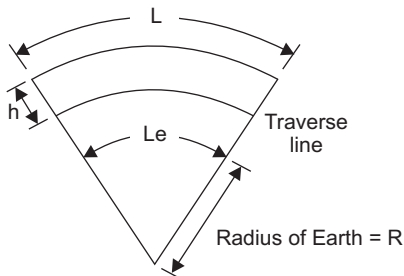
(vi) Misalignment correction (C_m)

$$C_m = - \left[\frac{\delta^2}{2AB} + \frac{\delta^2}{2BC} \right]$$



(vii) Mean sea level correction

$$C_n = \frac{-Lh}{R}$$



Order of field work in chain surveying

- (i) Reconnaissance
- (ii) Marking of stations
- (iii) Running survey lines
- (iv) Taking offsets

Compass-Surveying

3

The method of establishing control points by taking linear and angular measurements is called traversing.



Closed Traverse: It starts from a station and closes either on the same station or another station whose location is already known.

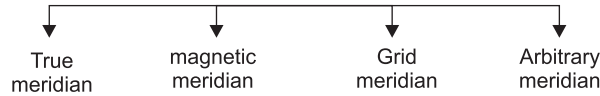
Open Traverse: An open traverse starts from a station and closes on another station whose location is neither known or established. Generally used in areas for road, railway line etc.

Note: Error's in closed traverse can easily be detected, adjusted and balanced while in open traverse it is not possible hence it should be avoided.

Difference Between Traverse Surveying & Chain Surveying

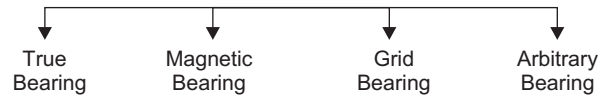
Chain Surveying	Traverse Surveying
1. Requires only linear measurements hence relatively easier	Requires both linear and angular measurements. Hence requires trained personnel.
2. The framework consists of triangles.	Frame work consists of a series of connected lines forming an open or closed polygon. As the directions of lines are measured, the polygon can be plotted without construction triangles.
3. Check lines are required in chain Surveying	Check lines are not required because accuracy of the framework can be checked by the methods of adjustments
4. Used when area to be surveyed is small, ground is flat and accuracy required is low	Used when area involved is large, and better accuracy is required.

Meridian: The direction of line is defined by the horizontal angle which the line makes with the reference angle line. That reference line is called meridian.



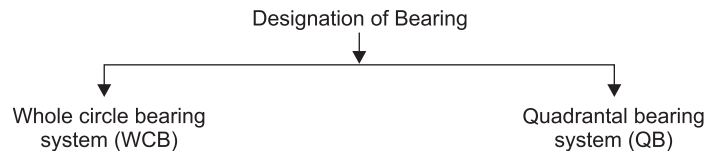
- (i) **True Meridian:** It represents the true north-south direction at the place. It is determined by astronomical observations of the Sun or Stars. It is fixed at a point.
- (ii) **Magnetic Meridian:** It is the direction indicated by a freely suspended, balanced magnetic needle at that point.
- (iii) **Grid Meridian:** For survey of a country, the true Meridian passing through the central place is sometimes taken as a reference meridian for the whole country. Such reference meridian is grid meridian.
- (iv) **Arbitrary Meridian:** It is the meridian which is taken in any convenient, arbitrary direction.

Bearing: It is the horizontal angle which the line makes with the meridian.



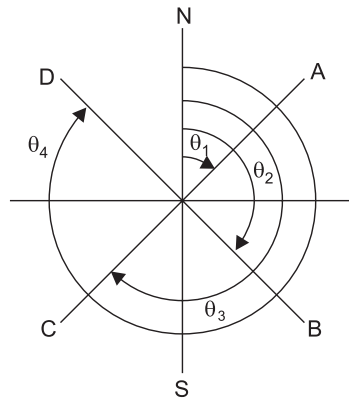
Note: True bearing of a line (also called Azimuth) does not change with time and can be re-established even after hundreds of years.

Magnetic bearings are used for small, less important surveys.

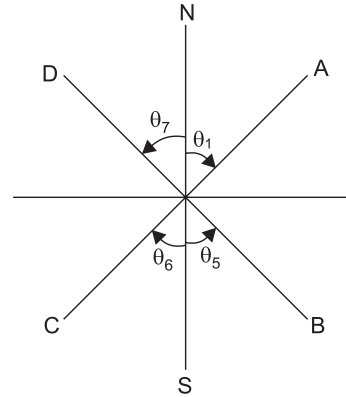


Whole Circle Bearing System: It is the horizontal angle between the line and the north end of the reference meridian in a clockwise direction. It varies between 0° to 360° .

Quadrantal Bearing System: It is the acute angle which the line makes with the meridian. It is measured from North point or South point whichever is closer. It is always less than 90° .



Whole circle Bearing (WCB)



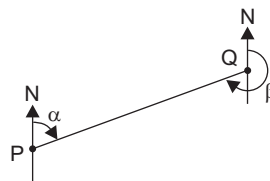
Quadrantal Bearing (QB)

Reduced Bearing: When WCB exceeds 90° it is reduced to the corresponding angle less than 90° .

WCB	RB	Quadrant
$0^\circ - 90^\circ$	WCB	NE
$90^\circ - 180^\circ$	$180 - \text{WCB}$	SE
$180^\circ - 270^\circ$	$\text{WCB} - 180$	SW
$270^\circ - 360^\circ$	$360 - \text{WCB}$	NW

Fore Bearing (FB): The bearing of line in the direction of the progress of survey.

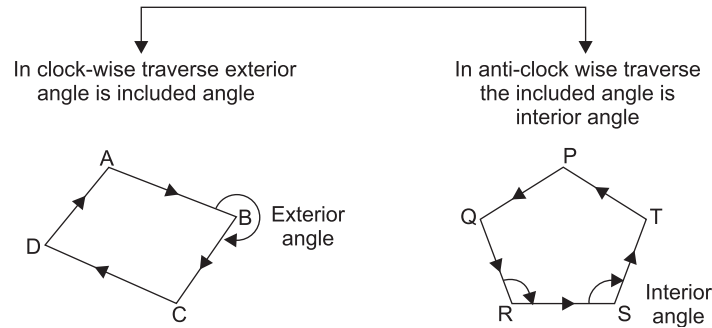
Back Bearing (BB): The bearing of line in the opposite direction of the progress of survey



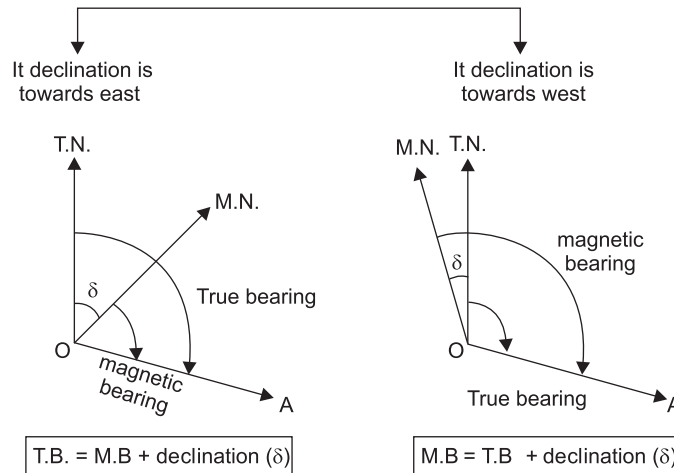
Line	FB	BB
PQ	α	β
QP	β	α

$$F \cdot B = B \cdot B \pm 180$$

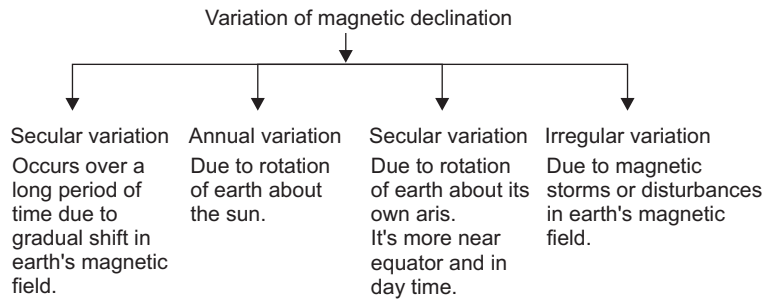
Included Angle: When two line meet at a point, the angle between them is included angle. Angle measured in the clockwise direction from the preceding line of a traverse to the forward line.



Magnetic Declination: The horizontal angle which the magnetic meridian makes with the true meridian is known as declination or magnetic declination at that point.



- (i) **Isogonic Line:** Lines passing through the points on earth surface at which the declination is the same at a given time. They radiate from North and South pole and follow irregular paths.
- (ii) **Agonic Lines:** Lines passing through points of zero declination. True meridian and magnetic meridian coincides with each other.



Local Attraction: It is the attraction of the magnetic needle to a local magnetic field other than earth's magnetic field.

$$\text{Sum of external included angles} = (2N + 4) \times 90^\circ$$

where N = No. of sides of the closed traverse.

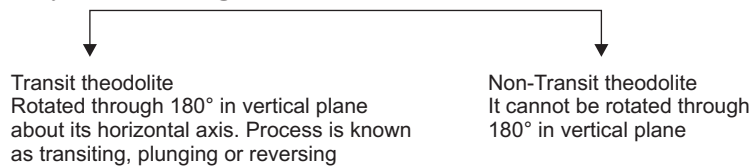
Dip: The angle made by the lines of magnetic force with the earth's surface is called dip. Magnetic needle becomes horizontal at equator but becomes vertical at magnetic poles.

Note: In Southern hemisphere, the South end of the needle dips down.
= Lines joining points of same dip are called Isoclinic while of zero declination are called Aclinic.

Theodolite

4

It's an instrument used for measuring horizontal and vertical angles in surveying. It is also used in prolonging a line, measuring distances indirectly and levelling.



Note: Size of theodolite is defined by lower graduated circle.

Basic Definitions

- (i) Centering: Process of setting up the instrument exactly over the station mark.
- (ii) Horizontal axis: Axis about which telescope is rotated in vertical plane.
- (iii) Vertical axis: Axis about which telescope can be rotated in horizontal plane.
- (iv) Line of sight: Imaginary line passing through the intersection of the cross-hair of the diaphragm and the optical centre of the objective.
- (v) Line of collimation: When line of sight comes in horizontal plane.
- (vi) Axis of level tube: Line tangential to the longitudinal curve of the level tube at its centre.
- (vii) Face right: When vertical circle is on the right hand side of observer.
- (viii) Face left: When vertical circle is on the left hand side of observer.
- (ix) Telescope Normal: When vertical circle on left side and bubble is up.
- (x) Telescope Inverted: When vertical circle on right side and bubble is down.
- (xi) Swinging: Revolving the telescope in the horizontal plane about its vertical axis.

- (xii) Changing face: Operation of bringing the telescope from face left to face right condition and vice-versa.
Errors eliminated by changing face are:
 - (a) Line of collimation not perpendicular to the horizontal axis.
 - (b) Horizontal axis not perpendicular to the vertical axis.
 - (c) Line of collimation not being parallel to the axis of the altitude level.
- (xii) Double Sighting: In this process horizontal or vertical angle is measured twice, once in telescope normal condition and other in the inverted condition.
- (xiv) Lining in: It is the process of establishing intermediate points with the help of theodolite on a given straight line whose ends are intervisible.
- (xv) Balancing in: It is the process of establishing intermediate points with the help of theodolite on a given straight line whose ends are not intervisible.

Temporary Adjustments of a Theodolite (in Order)

- (i) Setting, (ii) Centering, (iii) Levelling, (iv) Focussing the eye piece, (v) Focussing the objective.

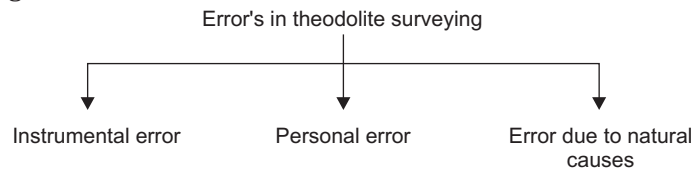
Permanent Adjustment of a Theodolite (in order)

- (i) Plate level test
- (ii) Cross-hair ring test
- (iii) Collimation in Azimuth test
- (iv) Spire test
- (v) Bubble tube adjustment
- (vi) Verticle arc test

Comparision Between Method of Repetition and Reiteration

- (i) Repetition method is preferred for single angle measurement
- (ii) Reiteration method is preferred in triangulation, where number of angles are taken at one station only.
- (iii) Following error's are eliminated in repetition method:
 - (a) Error due to eccentricity of Verniers and centres is eliminated by reading both the Verniers.
 - (b) Error due to line of sight and horizontal axis being out of adjustment is eliminated by taking both face reading.

- (c) Error due to inaccurate graduations on the main scale is eliminated by taking the reading on the different parts of the circle.
- (d) Error due to inaccurate bisection of signal is eliminated by taking more number of observations.
- (e) Other errors are also minimised by dividing the cumulative angle value with the number of observation.

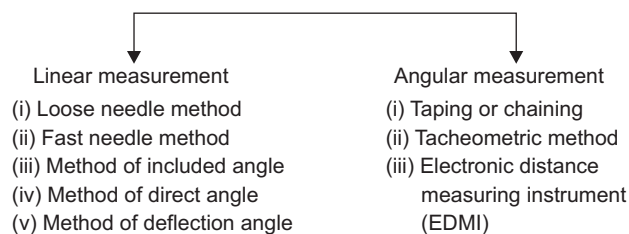


- (i) Line of sight not being perpendicular to horizontal axis.
- (ii) Horizontal axis not being perpendicular axis.
- (iii) Spire test is done to check if horizontal and vertical axis are being perpendicular or not.
- (iv) Vertical index error is the error when there is imperfect adjustment of the vertical vernier circle and the verniers do not read zero.
- (v) Error due to imperfect graduations on circle.

Traversing

5

A traverse is a series of connected lines whose length and direction are measured in field. In traversing two types of measurements are needed:



Angle Misclosure: It is equal to the difference between actual sum of measured angles and the theoretical sum of included angles:

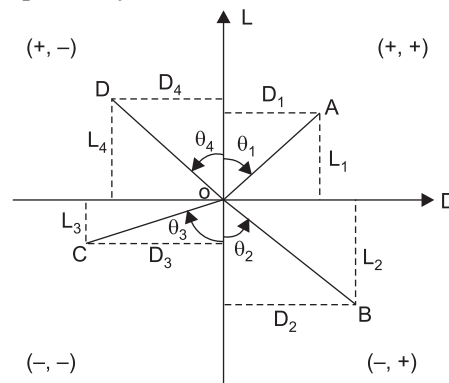
$$\text{Sum of internal angles of a traverse} = (2N - 4) \times 90^\circ$$

$$\text{Permissible angle misclosure} = K\sqrt{N}$$

where N = Number of sides of traverse

K = Depends on theodolite least count, accuracy desired and no. of repetitions. Generally taken as $20''$.

Latitude and Departures: The orthographic projection of a line on meridian and on the axis perpendicular to the meridian is called latitude and departure respectively.



Line	Latitude	Departure
OA	$+L_1 \cos \theta_1$	$+L_1 \sin \theta_1$
OB	$-L_2 \cos \theta_2$	$+L_2 \sin \theta_2$
OC	$-L_3 \cos \theta_3$	$-L_3 \sin \theta_3$
OD	$+L_4 \cos \theta_4$	$-L_4 \sin \theta_4$

Checks in Traverse

Closed Traverse

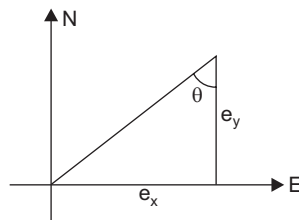
Open Traverse

(i) Algebraic sum of latitude = 0 $\Sigma L = 0$ (ii) Algebraic sum of Departure = 0 $\Sigma D = 0$	(i) Algebraic sum of latitude = Difference in the latitude of final and initial control points. (ii) Algebraic sum of Departure = Difference in the departures of final and initial control points.

If in closed traverse $\Sigma L \neq 0$ so $\Sigma L = e_y$ and $\Sigma D \neq 0$ so $\Sigma D = e_x$

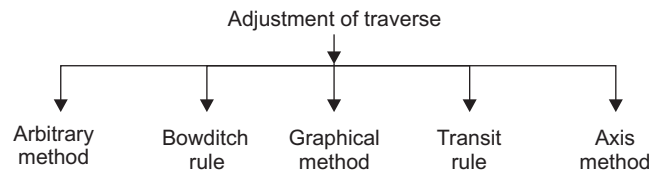
then Error of closure $e = \sqrt{e_x^2 + e_y^2}$

Direction of closing error $\tan \theta = \frac{e_x}{e_y}$



$$\text{Relative error of closure} = \frac{\text{Error of closure}}{\text{Perimeter of traverse}} = \frac{e}{p}$$

Note: Relative error of closure is also known as relative accuracy or degree of accuracy.



- (i) Arbitrary method: It is based on the discretion of the surveyor. Based on field conditions.
- (ii) Bowditch rule: Used where lengths are changed less and angles are changed more.

Error in latitude of any line

$$= \text{Total error in latitude } (e_y) \times \left(\frac{\text{Length of that line}}{\text{Traverse perimeter}} \right)$$

Similarly for departure of that line

- (iii) Graphical method: It's based on Bowditch rule used for theodolite traverse with low accuracy.
- (iv) Transit rule: Used where Angular measurements are more precise than the linear measurement.

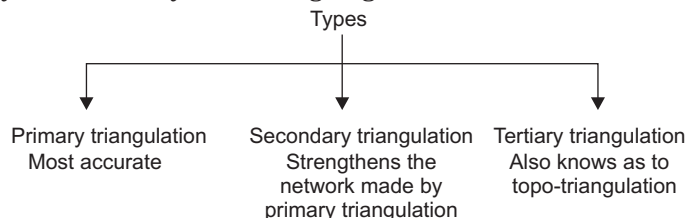
Error in latitude of any line

$$= \frac{\text{Total error in latitude } (e_y) \times \text{Numerical value of the latitude of that line}}{\text{Arithmetic sum of all latitudes}}$$

Similarly for departure of that line.

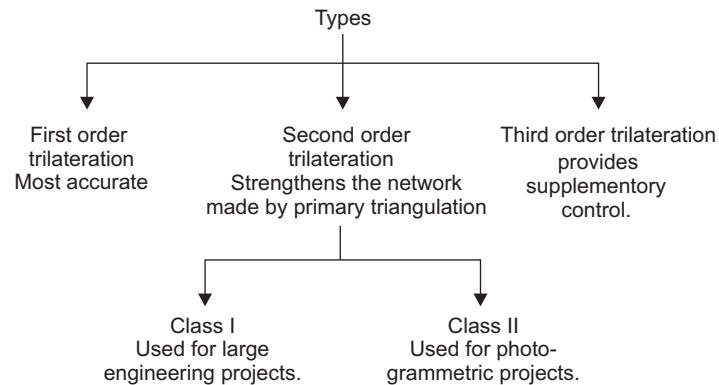
- (v) Axis method: Corrections are applied only to length and angles are measured very precisely.

Triangulation: It is a network of triangles which are used for control network in which only one base line is measured and remaining sides are only calculated by measuring angles.



Note: Accuracy of shape is measured in terms of strength of figures and its value depends upon No. of observed directions. No. of geometric conditions and magnitude of distance angles A and B.

Trilateration: All sides of each triangles are measured in the field itself. Angles are indirectly computed from the lengths of the sides of triangle.

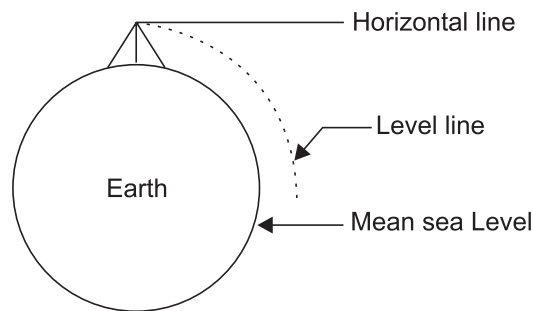


Levelling

6

A level line is the one that is at a constant height relative to mean sea level. It is a curved line.

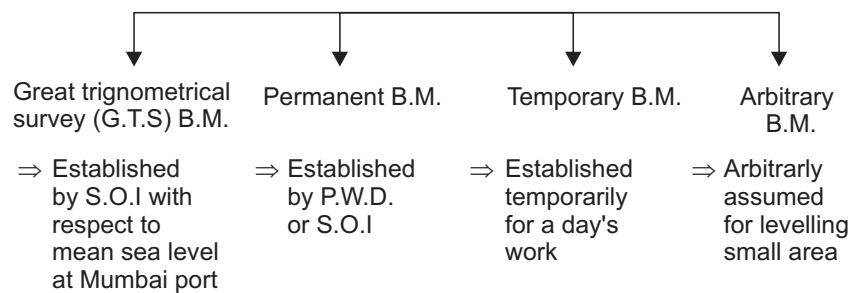
The value of mean sea level at datum is obtained by averaging the elevations of high and low tides (for a period of nearly 19 years).

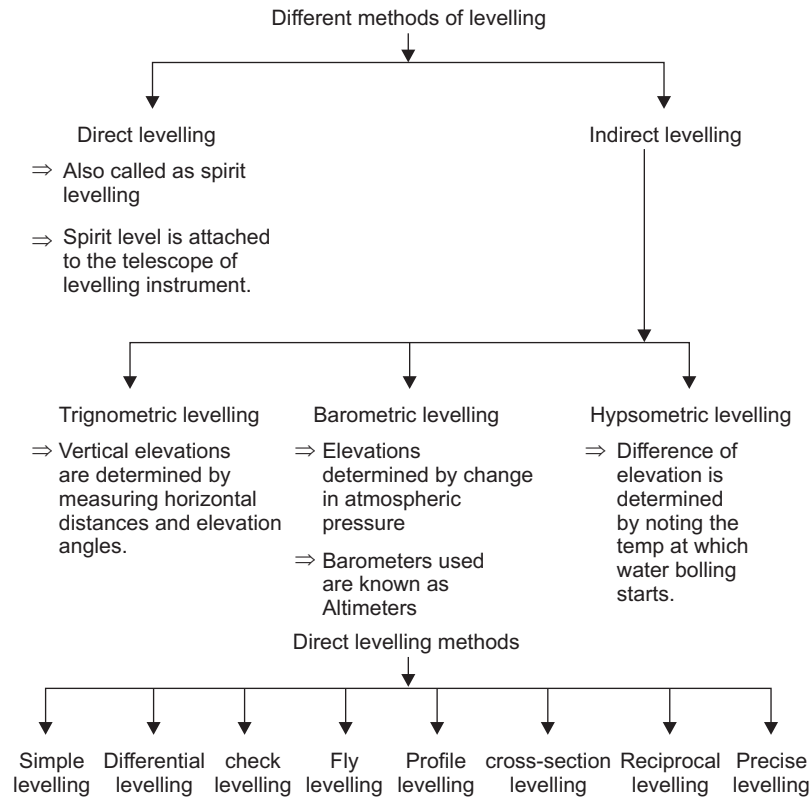


Elevations are measured along the gravity and it is the vertical distance above or below the datum surface.

Altitude is the vertical distance of the point above mean sea level.

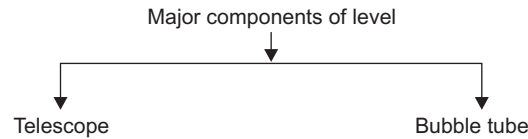
Bench Marks (B.M.): It is a fixed point of known elevation above the datum. Any point whose elevation is definitely known can be used as bench mark.





- (i) **Simple Levelling** is used for determining the difference of elevations of two points which are visible from a single position of the instrument.
- (ii) **Differential Levelling** is used when points are situated quite apart. Also called as compound levelling.
- (iii) **Check Levelling** is done for checking the already obtained elevations.
- (iv) **Fly Levelling** is done where rapidly but low precision is required. Generally used in reconnaissance.
- (v) **Profile Levelling** is done for the purpose of determining the elevations of the ground surface along a fixed line. Also called as longitudinal levelling.
- (vi) **Cross-section Levelling** is done to determine the difference of ground surface along the lines perpendicular to the centre line.
- (vii) **Reciprocal Levelling** is done where points are situated quite apart and its not possible to set up the instrument mid way between the points.

(viii) **Precise Levelling** is done where high accuracy is desired.

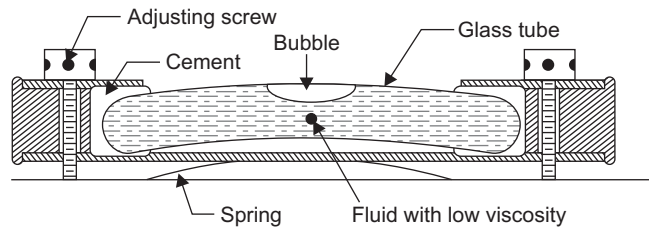


Optical Defects of Lens

- (i) **Chromatic Aberration:** Occurs due to dispersion of light. White light is split into different colour components.
- (ii) **Spherical Aberration:** Occurs as the rays incident on the edge of the lens are refracted more than rays incident on the centre of the lens.

Note: An achromatic combination (convex lens of crown glass and concave lens flint glass) is free from chromatic aberration while an aplantic combination of lens (two identical plano convex lenses) is free from spherical aberration.

Level Tube: Sensitivity of level tube is expressed in terms of the angle in seconds subtended at the centre by an arc of the level tube equal to one division of the tube.

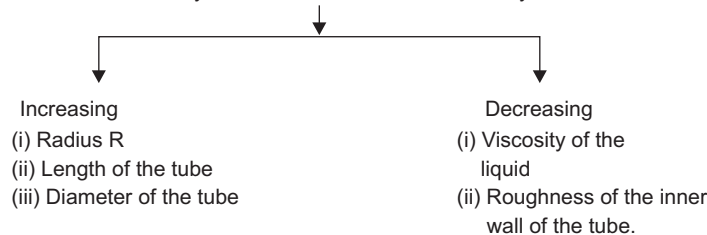


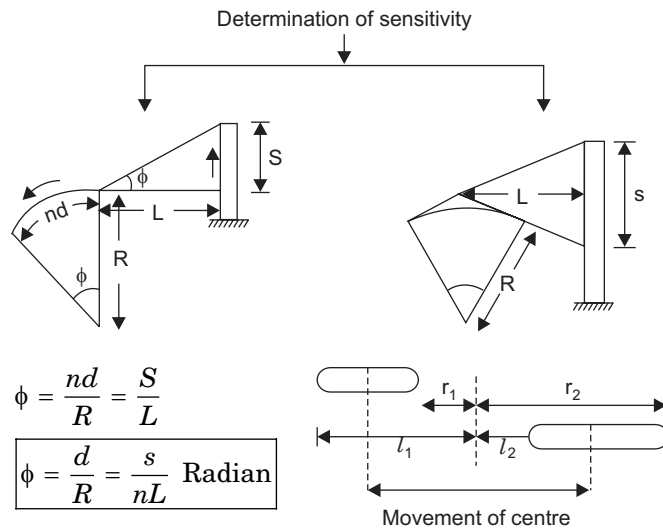
$$\text{Sensitivity} = \frac{412.53}{R}$$

where, R = Radius of upper surface of tube in meters

Graduations are assumed as 2 mm.

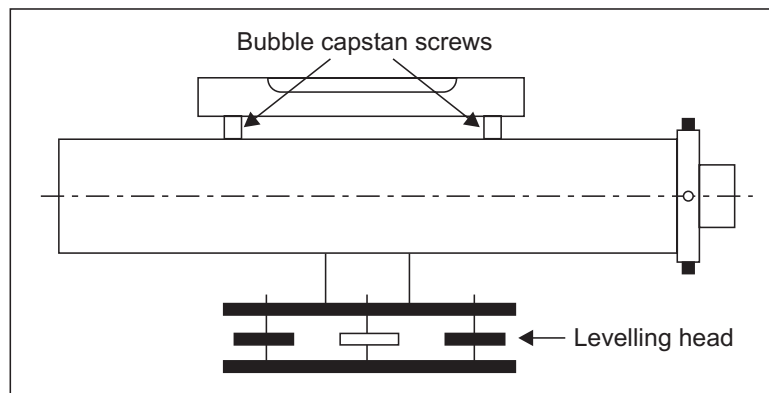
Sensitivity of level tube can be **increased** by



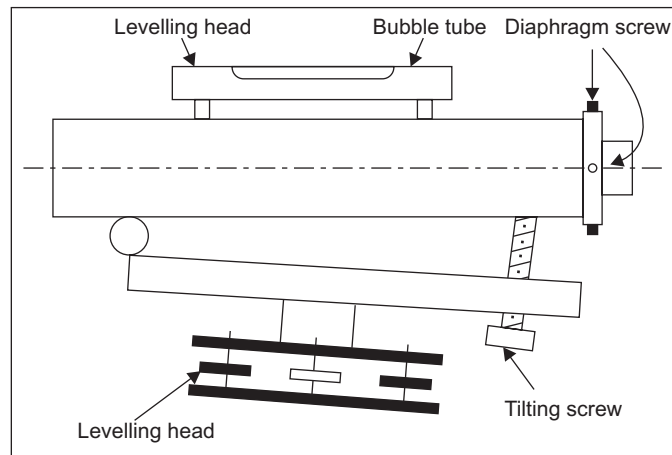


n = No. of division in movement of centre, then apply same formula

Dumpy level



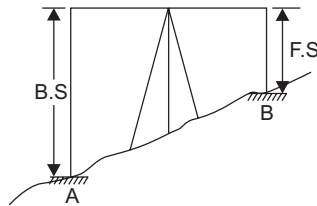
Tilting Level



- (i) Dumpy level is less accurate and consumes large time in levelling hence generally not preferred nowadays.
- (ii) In tilting level, the telescope is not rigidly fixed to the vertical spindle but is capable of a slight tilt in the vertical plane about an axis placed just below the telescope. Hence, line of collimation need not to be perpendicular to the vertical axis.

Procedure of Levelling

R.L. of A = may be B.M. or 100 (assumed)



Height of instrument = RL of A + Back sight (BS)

RL of B = Height of instrument – Fore sight (FS)

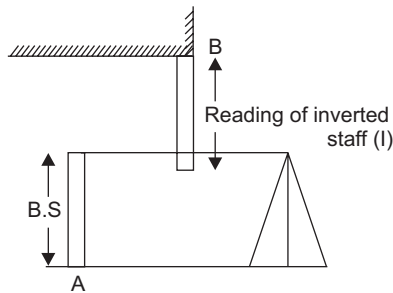
$$RL \text{ of } B = RL \text{ of } A + BS - FS$$

Note: First reading made on a point of known reduced level is always a Back sight reading.

Readings are booked in the level book by two methods—Rise and fall method, Height of instrument method.

Inverted Staff: When the point whose elevation is to be found is much above the line of collimation, the staff is placed inverted with its zero end touching the point.

$$RL \text{ of soffit (B)} = RL \text{ of A} + BS \text{ at A} + I$$

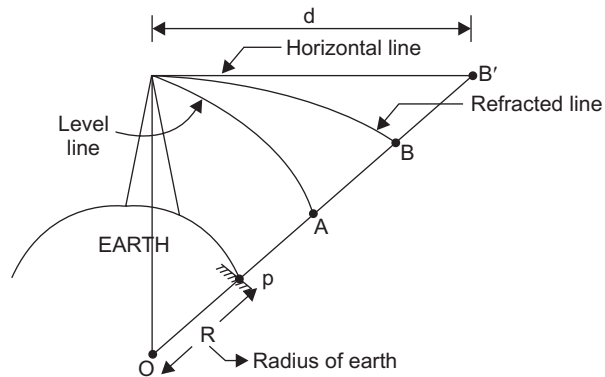


Effect of Curvature of Earth and Refraction

Actual readings needed = PA

Theoretical reading observed = PB'

Practically the reading which is taken including the refraction effect = PB



$$\text{Curvature correction} = AB' = 0.0785 d^2$$

$$\text{Refraction correction} = BB' = \frac{1}{7} AB' = 0.0112 d^2$$

$$PA = PB + BB' - AB' \text{ where } d = \text{Is in kilometers}$$

Note: Curvature correction (AB') is negative while retraction correction (BB') is positive. So total correction needed is $-0.0673d^2$

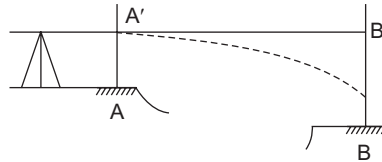
$$PA = PB - 0.0673 d^2$$

Distance of Visible Horizon: $d = 3.85\sqrt{h}$ h is in meter and d in km.

Reciprocal Levelling: It eliminates the error's due to curvature of earth, refraction and collimation.

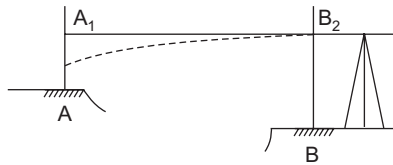
Apparent difference when instrument near $A = AA_1 - BB_1 = X$

Apparent difference when instrument near $B = AA_2 - BB_2 = Y$



$$\text{True difference in level} = \frac{1}{2}(x + y)$$

$$\text{Error in level} = \frac{1}{2}(x - y)$$



If this error in level is total error 'e' then

$$\text{Collimation error} = e - 0.0673 d^2$$

Tacheometry

7

It is an optical distance measurement method. Generally used for rough or steep grounds where accuracy is less while chaining (or tapeing).

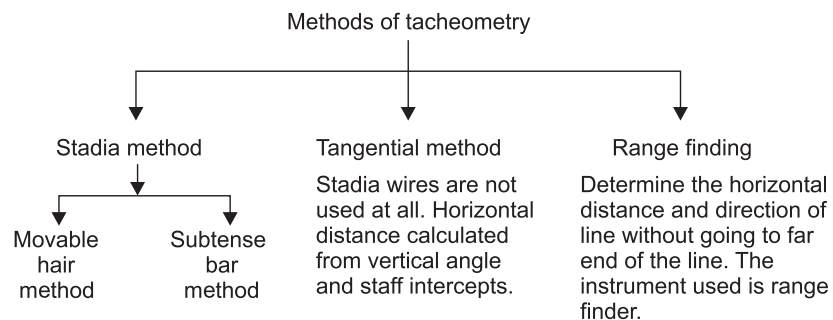
Tacheometer: Transit theodolite fitted with Stadia diaphragm.

- (i) Telescope is fitted with an anallactic lens.
- (ii) Multiplying constant (K) should be 100.
- (iii) Additive constant (c) should be zero.
- (iv) Eyepiece has high magnification power.

Subtense Theodolite: Special diaphragm is used in tacheometer, whose stadia hairs can be raised or lowered by a micrometer screw.

Stadia Rod: It is a 5-15 m long graduated rod in decimals of metre. Also known as **vertical stave**.

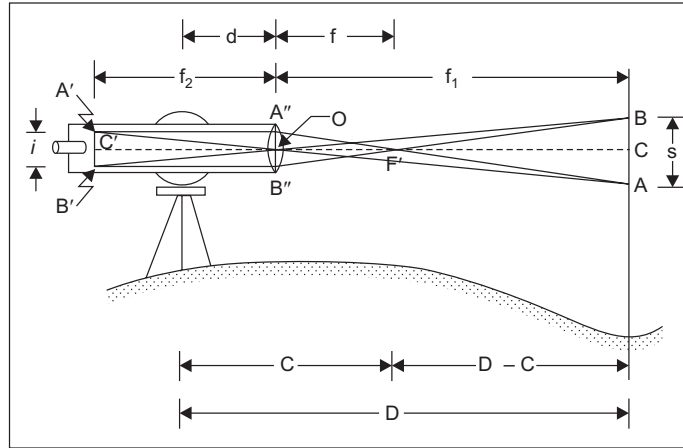
Subtense Bar: Used in measuring both horizontal and vertical distances in places where chaining is impossible. It is also known as **horizontal stave**.



Principal of Stadia Method

$$D = KS + C$$

D = Horizontal distance between staff and vertical axis of tacheometer



S = Staff intercept

$$K = \frac{f}{i} \quad [C = f + d] \quad \text{where} \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

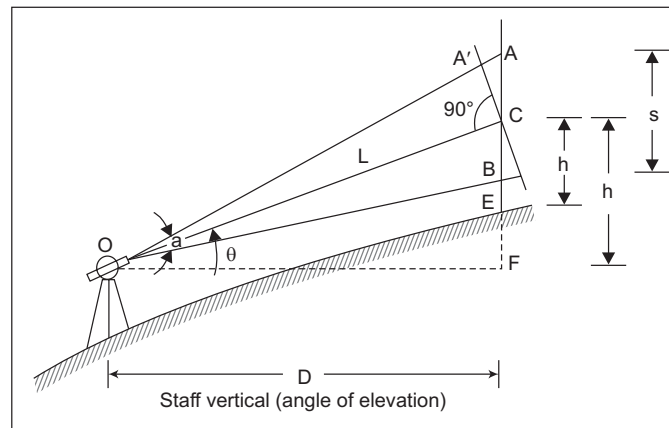
Multiplying constant K , generally taken as 100

Additive constant C , ranges from 0.25 to 0.35 and taken as 0.

d = horizontal distance between optical center O and vertical axis of tacheometer.

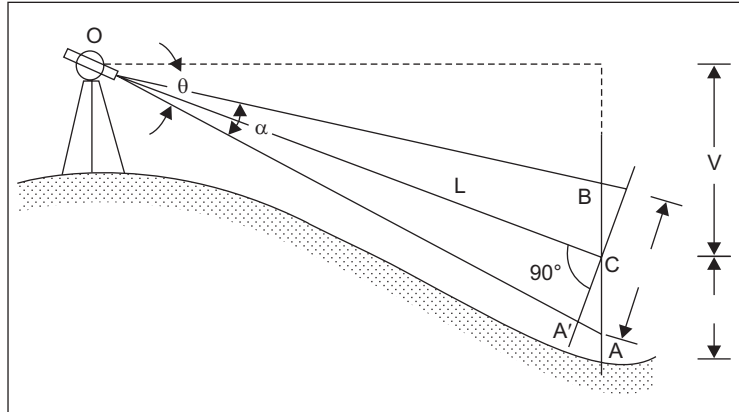
Distance and Elevation Formula for Inclined Sights

(i) When staff is vertical



Angle of elevation

Elevation of staff station when elevation angle is $\theta = HI + V - h$



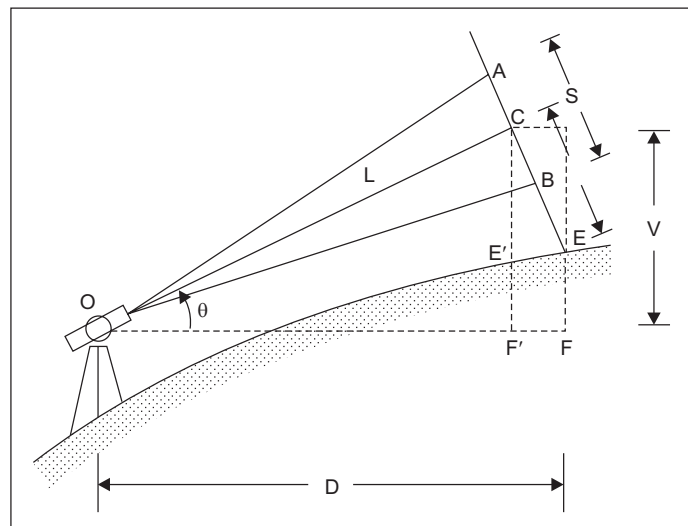
Angle of depression

Elevation of Staff Station when depression angle is $\theta = HI - V - h$

where, $V = L \sin \theta = (KS \cos \theta + C) \sin \theta$

$D = L \cos \theta = (KS \cos \theta + C) \cos \theta$

(ii) When staff is normal

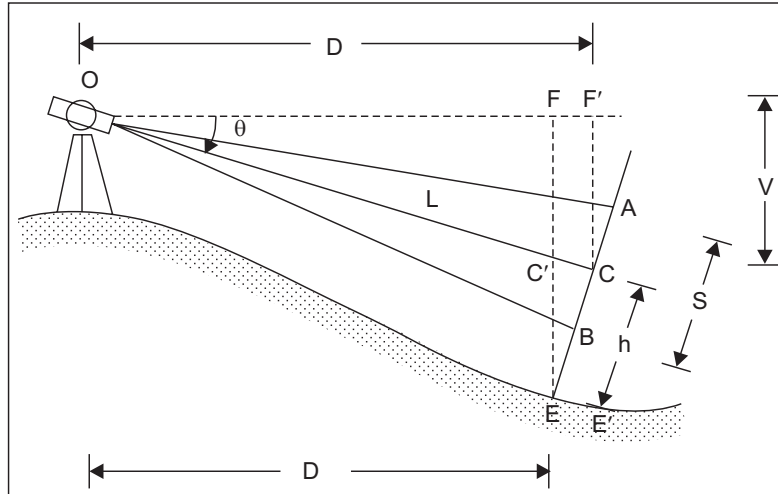


Angle of elevation

Elevation of staff station = $HI + V - h \cos \theta$

where $V = L \sin \theta = (KS + C) \sin \theta$

$D = (KS + C) \cos \theta + h \sin \theta$



Angle of depression

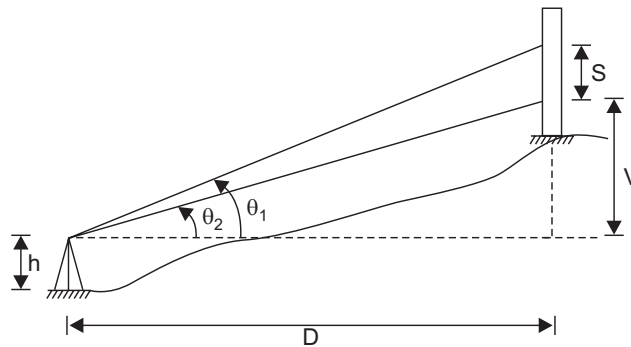
Elevation of staff station = $HI - V - h \cos \theta$

where $V = L \sin \theta = (KS + C) \sin \theta$

$D = (KS + C) \cos \theta - h \sin \theta$

Tangential Method of Tacheometry

(i) Two angles of elevation

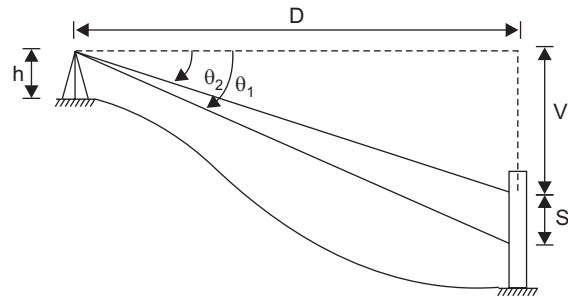


$$D = \frac{S}{\tan \theta_1 - \tan \theta_2}$$

$$V = D \tan \theta_2$$

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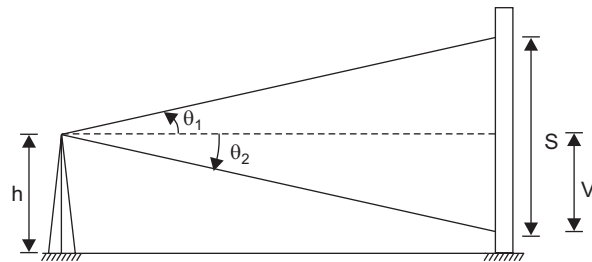
(ii) Two angles of depression



$$D = \frac{S}{\tan \theta_1 - \tan \theta_2}$$

$$V = D \tan \theta_2$$

(iii) One angle of elevation and one angle of depression



$$D = \frac{S}{\tan \theta_1 + \tan \theta_2}$$

$$V = D \tan \theta_2$$

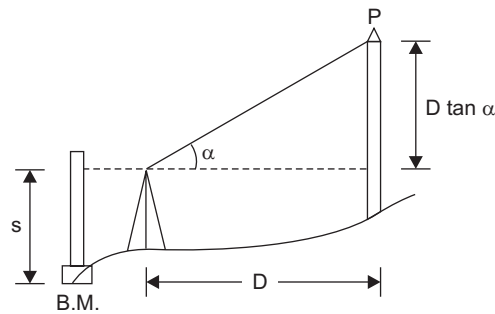
Note: Subtense bar is an instrument used for measuring horizontal distances.

Trigonometric Levelling

8

It's an indirect levelling method in which relative elevation of various points are determined from the vertical angle and horizontal distances.

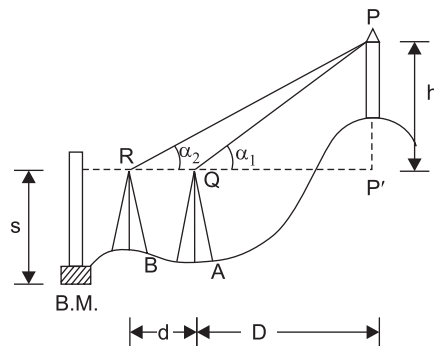
- (i) Determination of top level of a high object when base is accessible



$$RL \text{ of Point } P = RL \text{ of B.M.} + \text{Staff reading } (S) + D \tan \alpha$$

- (ii) Determination of top level of a high object when base is not accessible

- (a) Line of collimation of station A and station B are at same level



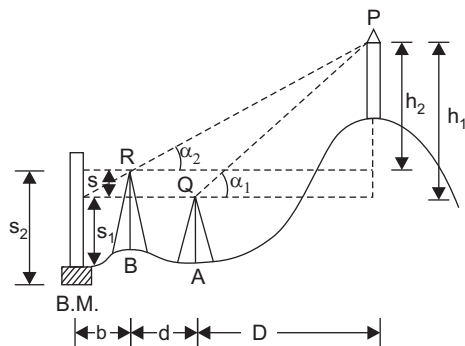
$$h = \frac{d \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} = \frac{d \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}$$

$$RL \text{ of point } P = RL \text{ of BM} + \text{Staff reading } (S) + h$$

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Note: It is very difficult to set up the line of colimation at B exactly at the same level at A .

(b) Instrument axis at B higher than that of A

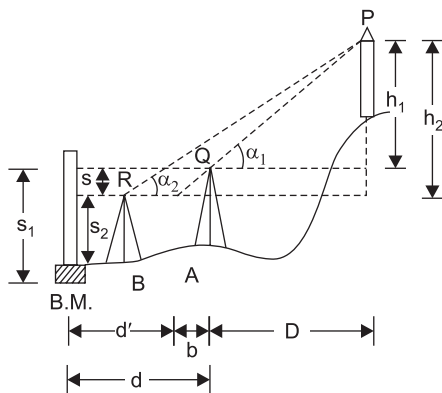


$$RL \text{ of point } P = RL \text{ of } BM + \text{Staff reading } (s_1) + h_1$$

$$h_1 = \frac{(d + s \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)}$$

$$s = s_2 - s_1$$

(c) Instrument axis at B lower than that of A

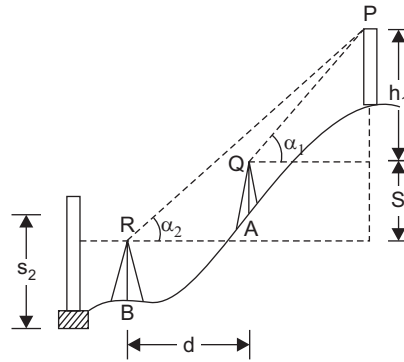


$$RL \text{ of point } P = RL \text{ of } BM + \text{Staff reading } (s_1) + h_1$$

$$h_1 = \frac{(d - s \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin(\alpha_1 - \alpha_2)}$$

$$s = s_1 - s_2$$

(d) Instrument axis have large difference of level



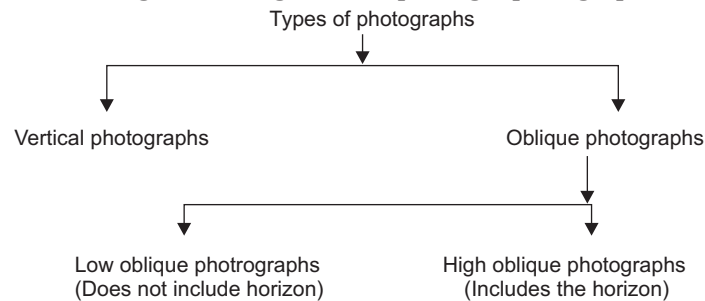
$$RL \text{ of point } P = RL \text{ of } BM + s_2 + s + h_1$$

$$h_1 = \frac{(d - s \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)}$$

Photogrammetry

9

It is the science of obtaining information about physical objects through process of recording, measuring and interpreting of photographs of the area.

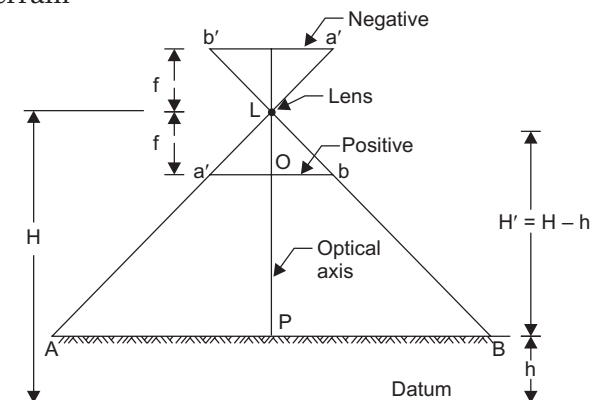


Comparators are used for accurate measurement of distance on photograph Mono-comparator is used for measurement on one photograph at a time while stereo comparators are used for measurement on an overlapping stereo pair of photographs.

Note: Map is an orthographic projection while an aerial photograph is a central projection.

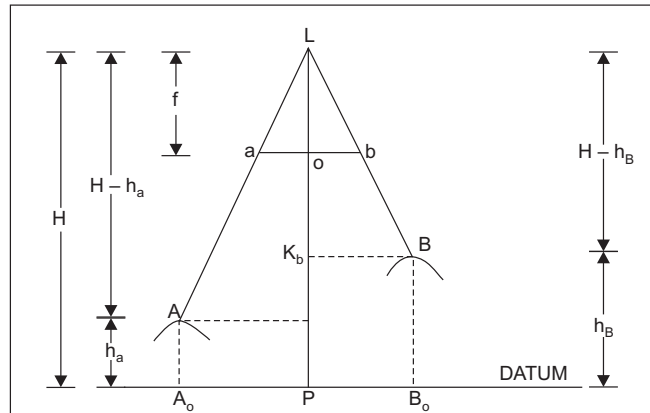
Scale of a Vertical Photograph

(a) Flat terrain



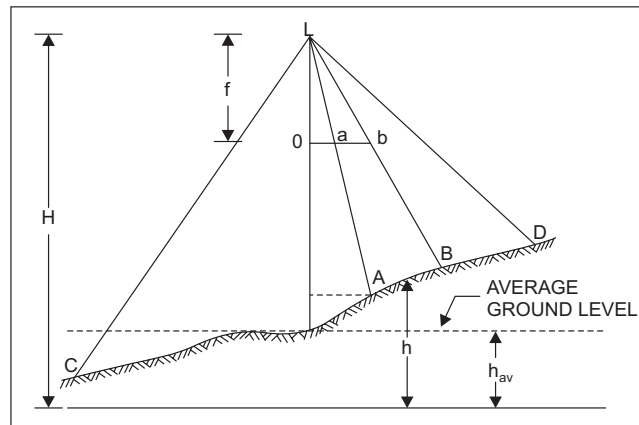
$$S = \frac{f}{H - h}$$

(b) Datum scale



$$S = \frac{f}{H}$$

(c) Average scale



$$S_{av} = \frac{f}{H - h_{avg}}$$

Ground coordinates

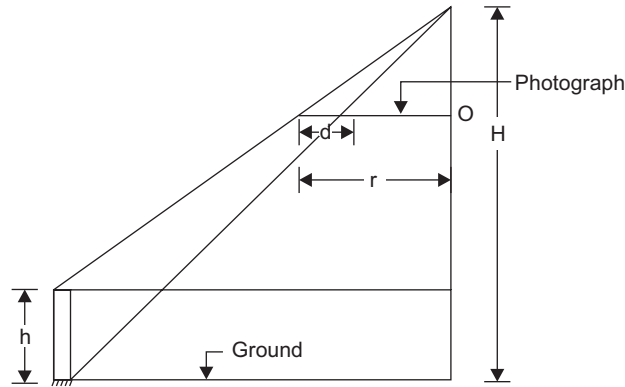
(X_A, Y_A) : Ground co-ordinates H = flying height

(x_a, y_a) : Photo co-ordinates h_A = elevation of point A above datum

f = focal length of lens

$$\left[X_A = x_a \left(\frac{H - h_a}{f} \right) \right] \left[Y_A = y_a \left(\frac{H - h_a}{f} \right) \right]$$

Relief Displacements in a Vertical Photograph



d = relief displacement

h = height of the object above datum

H = flying height above the datum

r = radial distance of the image of the top of object

$$d = \frac{rh}{H}$$

Calculation of Flying Height of a Vertical Photograph

(i) When focal length is ' f ' and scale of photograph is ' s ' then

$$H' = \frac{f}{s} \quad H' = \text{flying height above ground}$$

(ii) When photo coordinates are (x_a, y_a) , and (x_b, y_b) , focal length is f , elevation of points above ground is h_a & h_b respectively then distance between points on the photograph will be

$$AB = \sqrt{\left[\frac{x_b}{f}(H - h_b) - \frac{x_a}{f}(H - h_a) \right]^2 + \left[\frac{y_b}{f}(H - h_b) - \frac{y_a}{f}(H - h_a) \right]^2}$$

Required Number of Photographs

Ground Area: A Area covered by one photograph: a

longitudinal overlap/end lap = p_l

side lap = p_s

Length of photograph in direction of flight = l

Width of photograph normal to the direction of flight = w

Scale of photograph = s

$$(i) \quad N = \frac{AS^2}{(1 - p_l)(1 - p_s)wl}$$

$$(ii) \quad N = \left[\frac{L_1}{(1 - p_l) \frac{l}{s}} + 1 \right] \left[\frac{W_1}{(1 - p_w) \frac{w}{s}} + 1 \right]$$

L_1 = Length of ground to be covered

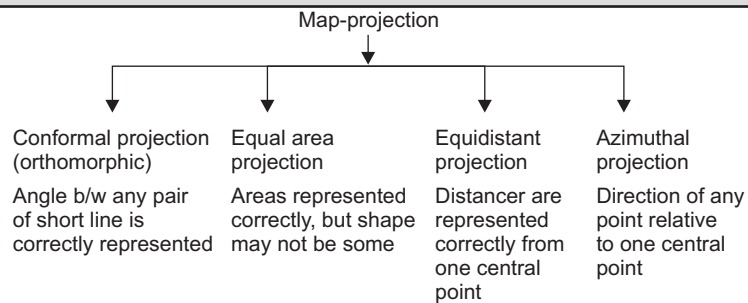
W_1 = Width of ground to be covered.

Crab and Drift

Crab of a photograph is the **angle** formed between the flight line and edges of the photograph in the direction of flight.

Drift is the **lateral shifting** of the photograph. It is generally due to wind. The crab is generally introduced to avoid draft.

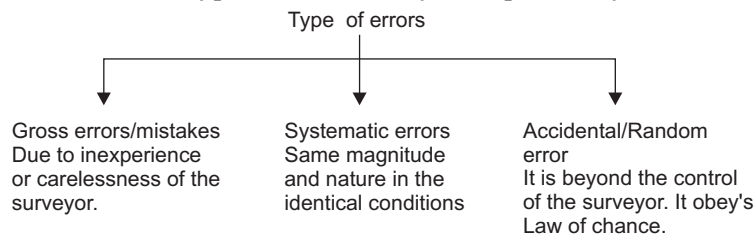
Note: In tilted photographs, three angles completely define the angular orientation of that photograph. These are tilt (t), swing (s) and azimuth (α).



Theory of Errors

10

True value of the quantities (angles and distances) is impossible to determine as some type of errors always creep in every measurement.

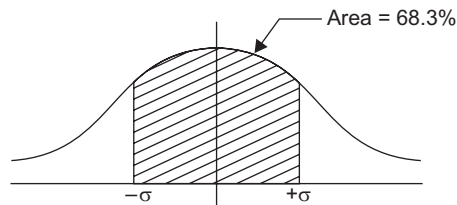


Note: In this chapter it is preassumed that mistakes and systematic errors have been eliminated and only accidental errors are left to be eliminated.

Laws of Accidental Error

It follows normal probability distribution curve also known as Gaussian distribution

Error = Measured value – True value
Residual = Measured value – Most probable value



Most probable value is the value which is close to true value than any other value.

y = frequency of occurrence of the residual or variation

$$y = \frac{1}{\sqrt{2\pi}\sigma} e^{-0.5(v/\sigma)^2}$$

v = residual

σ = standard deviation

$$\sigma = \sqrt{\frac{\sum v^2}{n-1}}$$

Note: Standard deviation is also known as root mean square error of a measurement.

Smaller the value of standard deviation, the greater is the precision.

Indices of Precision for Observations of Equal Weight

(i) Standard deviation (σ)

$$\sigma = \sqrt{\frac{\sum v^2}{n-1}}$$

For accessing the precision of a set of observations.

(ii) Variance (v)

$$V = \frac{\sum v^2}{n-1} = \sigma^2$$

Used as a measure of dispersion or spread.

(iii) Standard error of the mean (σ_m)

$$\sigma_m = \pm \sqrt{\frac{\sum v^2}{n(n-1)}} = \pm \frac{\sigma}{\sqrt{n}}$$

(iv) Standard error of single observation (σ_1)

$$\sigma_1 = \sqrt{\frac{\sum v^2}{n-1}}$$

(v) Most probable error in single observation (e)

$$e = \pm 0.6745\sigma$$

(vi) Most probable error of the mean (e_m)

$$e_m = \pm 0.6745 \sigma_m$$

(vii) Maximum error: If any measurement deviates from mean by more than $\pm 3.29\sigma$ then that measurement is considered as mistake and it can be rejected.

(viii) Different percentage error:

$$(a) \text{ 90\% error } (E_{90}) = \pm 1.645 \sigma$$

$$(b) \text{ 95\% error } (E_{95}) = \pm 1.96 \sigma$$

$$(c) \text{ 95.5\% error } (E_{95.5}) = \pm 2.0 \sigma$$

$$(d) \text{ 97.7\% error } (E_{97.7}) = \pm 3.0 \sigma$$

Definition of Weight: It is a measure of relative trust worthiness of the set of observations. The greater the precision of an observation, the greater will be its weight.

Rules of Weight Allocation

- (i) The weights are taken inversely proportional to the variance or square of standard errors (or probable errors).

$$\frac{w_1}{w_2} = \frac{v_2}{v_1}$$

- (ii) The weights of the quantities measured in similar conditions are assigned in direct proportion to the number of times (n) the quantity is measured.

$$\frac{w_1}{w_2} = \frac{n_1}{n_2}$$

- (iii) Weights are sometimes allocated by personal judgement depending on field prevailing and environmental conditions i.e. lower weights allocated to observations in difficult terrain.

- (iv) Weight of level line is taken as inversely proportional to the length (L) of the route. Thus,

$$\frac{w_1}{w_2} = \frac{L_2}{L_1}$$

Laws of Weight

- (i) If the no. of observations ($a, b, c \dots$) of a particular quantity have unit weight (i.e. 1) then the value of quantity will be the arithmetic mean of observation.

Observation	Weight	Value = $\frac{a + b + c}{3}$
a	1	
b	1	
c	1	

- (ii) The weight of the weighted arithmetic mean is equal to the sum of individual weights. (i.e. $w_1 + w_2 + w_3$)

Observation	Weight	Value = $\frac{aw_1 + bw_2 + cw_3}{w_1 + w_2 + w_3}$
a	w_1	
b	w_2	
c	w_3	

- (iii) Weight of observation a is w_1 and weight of observation b is w_2 .

$$\text{Weight of } a \pm b = \frac{1}{\frac{1}{w_1} + \frac{1}{w_2}} = \frac{w_1 w_2}{w_1 + w_2}$$

- (iv) Weight of observation a is w_1 . If the observation is multiplied by factor K then

$$\text{Weight of } Ka = \frac{w_1}{K^2}$$

- (v) If the equation is multiplied by its own weight, then the weight of resulting equation is equal to the reciprocal of the weight of that equation.

$$\text{Weight of } a + b \text{ is } w_1$$

$$\text{Weight of } w_1 (a + b) \text{ will be } \frac{1}{w_1}.$$

- (vi) Weight of an equation remains unchanged if all signs of equations are changed

$$\text{Weight of } a + b \text{ is } w_1$$

$$\text{Weight of } -(a + b) \text{ will be } w_1 \text{ only.}$$

- (vi) Weight of an equation remain unchanged if the equation is added or subtrated from a constant.

$$\text{Weight of } a + b \text{ is } w_1$$

$$\text{Weight of } 180 - (a + b) \text{ or constant } \pm (a + b) \text{ will be } w_1$$

Indices of Precision for Observations of Different Weights

- (i) Standard deviation of weighted observations (σ_w)

$$\sigma_w = \pm \sqrt{\frac{\Sigma(wv^2)}{n - 1}}$$

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(ii) Standard error of the mean of the weighted observations $(\sigma_m)_w$ is

$$(\sigma_m)_w = \pm \frac{\sigma_w}{\sqrt{\sum w}}$$

(iii) Standard error of the single observations of weight w_1

$$(\sigma_1)_w = \pm \frac{\sigma_w}{\sqrt{w_i}}$$

(iv) Most probable error of the single observations of weight w_1

$$E_w = \pm 0.6745 (\sigma_1)_w$$

(v) Most probable error of the mean $(E_m)_w$

$$(E_m)_w = \pm 0.6745 (\sigma_m)_w$$

Propagation of Standard Errors

If $A = f(x, y, z)$ then total error in A , $\delta_A = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y + \frac{\partial f}{\partial z} \delta z + \dots$

The standard error in A due to standard errors e_x, e_y, e_z in x, y, z etc. is obtained from the following relation:

$$e_A^2 = \left(\frac{\partial f}{\partial x} e_x \right)^2 + \left(\frac{\partial f}{\partial y} e_y \right)^2 + \left(\frac{\partial f}{\partial z} e_z \right)^2 + \dots$$

(i) Addition or subtraction of Quantities

$A(f) = a + b$ where e_a and e_b are Std. errors in a and b respectively

$$e_A = \pm \sqrt{e_a^2 + e_b^2} \quad e_A = \text{error in observation } A$$

(ii) Product of Quantities

$$A(f) = a \times b \times c$$

$$e_A = \pm abc \left[\left(\frac{e_a}{a} \right)^2 + \left(\frac{e_b}{b} \right)^2 + \left(\frac{e_c}{c} \right)^2 \right]^{1/2}$$

(iii) Quotient of Quantities

$$A(f) = \frac{a}{b} \quad e_A = \pm \frac{a}{b} \left[\left(\frac{e_a}{a} \right)^2 + \left(\frac{e_b}{b} \right)^2 \right]^{1/2}$$

(iv) Power of a quantity

$$A(f) = a^n \quad e_A = \pm na^{n-1}e_a$$

(v) Root of a quantity

$$A(f) = a^{1/n} \quad e_A = \pm \left(\frac{a^{1/n}}{n} \frac{e_a}{a} \right)$$

(vi) Addition of a constant to a quantity

$$A(f) = a \pm k \quad e_A = e_a$$

(vii) Product of a constant

$$A(f) = Ka \quad e_A = \pm ke_a$$

Theory of Least Square: The most probable value is that value which makes the sum of squares of the residual to a **minimum** value.

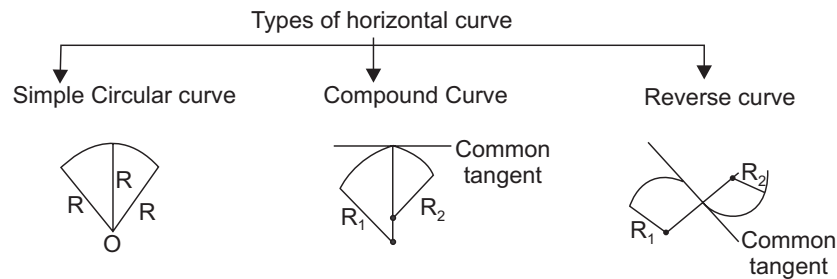
- (i) If the observations have equal weight, then $\Sigma v^2 = \text{minimum}$
- (ii) If the measurements are of unequal weights, then $\Sigma w(v^2) = \text{minimum}$

Methods of least square adjustments are

- (i) Normal equation method
- (ii) Method of correlates or condition equations method

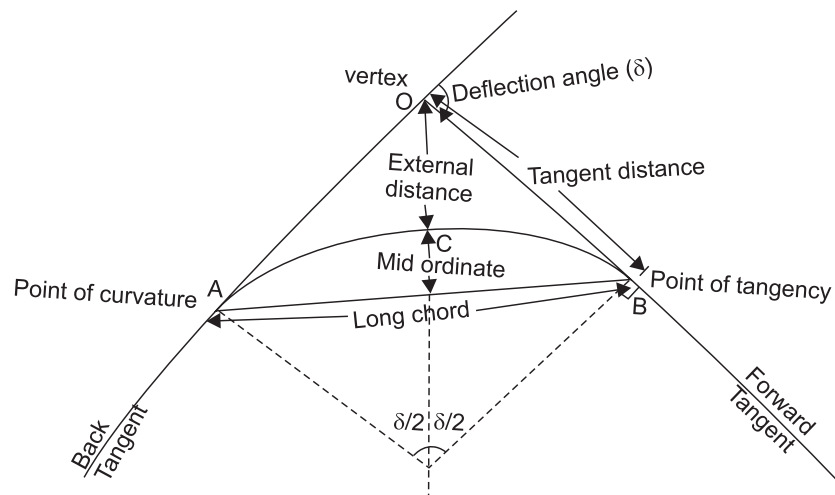
A horizontal curve is provided at the point where two straight lines intersect in the horizontal plane.

A vertical curve is provided at the point where two straight lines at different gradient intersect in the vertical plane.

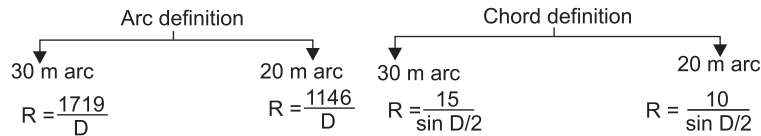


Note: Reverse curves are provided on the routes when two straight lines are parallel or when angle between them is very small. These are very frequently used in hilly roads.

Horizontal Circular Curve



Degree of a Curve: It is the angle subtended at the centre by **an arc or chord** of a specified length. Arc definition is used in case of highways while chord definition is used in case of railways.



Elements of a Simple Circular Curve

(i) Length of curve (l): Length of ACB

$$l = \left(\frac{2\pi R}{360} \right) \delta$$

(ii) Tangent length Tangent distance:

$$T = R \tan \left(\frac{\delta}{2} \right)$$

(iii) Change of tangent points:

(a) Chainage of A = Chainage of Vertex – Tangent distance

(b) Chainage of B = Chainage of A + length of curve

Note: Chainage of $B \neq$ Chainage of Vertex + Tangent distance

(iv) Length of long chord (L)

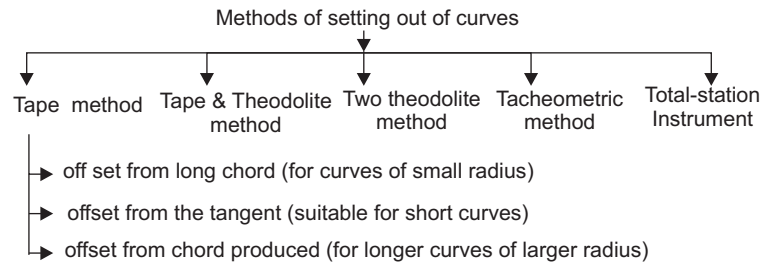
$$L = 2R \sin \left(\frac{\delta}{2} \right)$$

(v) External distance or Apex distance (E):

$$E = R \left(\sec \frac{\delta}{2} - 1 \right)$$

(vi) Mid-ordinate (m)

$$M = R \left(1 - \cos \frac{\delta}{2} \right)$$



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- (i) Tape method: Only chain or tape is used and no angular measurements are made.
- (ii) Tape and theodolite method: Linear and angular measurements are made.
- (iii) Two theodolite method: No linear measurements are made, curve is plotted by angular measurements only. It is very accurate.
- (iv) Tacheometric method: Angular and linear measurements are done with the help of tacheometer.
- (v) Total station method: Curve is set out by the co-ordinates at various points.

Note: In horizontal curves to avoid inconvenience to slow moving vehicles, the maximum value of the centrifugal ratio for roads is specified as $1/4$ and for railway tracks its $1/8$.

Bernoulli's Lemniscate: It is special type of transition curve which is generally used when angle of deflection is very large. As it is symmetrical, so it can even be used when curve is transitional throughout without a circular curve between the two transition curves. It is objectionable in Railways but allowed on highways.

- On 21st March, sun crosses the equator from South to North and it is called first point of Aries. On this day, day and night are equal. Also called as vernal equinox.

On 21st/22nd September, sun crosses the equator from South to North and it is called first point of Libra. Also called as Autumnal equinox.

- Angular distance of any plane North or South of equator is called Latitude (θ).
- Measured angle between pole to zenith point for any place is called Colatitude (c).

$$c + \theta = 90^\circ$$

- Angle between meridian of a place from a fixed meridian is called longitude.
- Angular distance from horizon, measured on a vertical circle passing through the body is called Altitude (α).
- Angular distance of body from Zenith is called Co-Altitude (z) or Zenith distance.

$$\alpha + z = 90^\circ$$

- Angle between vertical circle passing through the body from observer's meridian is called Azimuth (A).
- Angular distance of a body from the plane of celestial equator, measured along declination circle is called Declination (δ).
- Angular distance of heavenly body from pole is called co-declination (δ)

$$\delta + \delta = 90^\circ$$

- Angle between observer's meridian and declination circle passing through the body is called Hour Angle (H). It is measured from South in Westward direction.
- Angular distance measured Eastward from first point of Aries is called right ascension (R.A.). It is angle between hour circle passing from body to hour circle passing from first point of Aries, measured in East direction.

Terrestrial Latitude and Longitude

- (i) Axis of the earth: Axis joining the North and South pole of the earth.
- (ii) Meridian: Any great circle whose plane passes through the axis of earth.
- (iii) Equator: Great circle perpendicular to axis of the earth.
- (iv) Latitude : Vertical angle above or below equator.
- (v) Longitude: Horizontal angle between great circle passing through place and standard meridian.

Note: Prime meridian or standard meridian is a meridian passing through Greenwich.

- (vi) Parallel of latitude: Parallel of latitude through a point is a small circle passing through that point and parallel to equator.

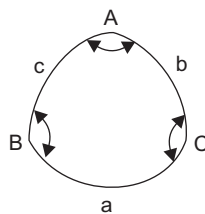
$$1 \text{ Nautical mile} = 1.852 \text{ km}$$

1 Nautical mile is the distance on arc of great circle by 1 minute angle at the centre of earth.

Celestial Latitude and Longitude System

Celestial latitude is the vertical angle measured above or below arc of ecliptic while celestial longitude is the horizontal angle measured from first point of Aries to the east.

Spherical triangle: If a, b, c are sides of spherical triangle then A is the angle formed at centre of sphere by arc BC or A is the angle between two great circle passing from AB and AC .

Properties of Spherical Triangle

- (i) Any angle is less than π .
- (ii) $\pi < (A + B + C) \leq 3\pi$
- (iii) Sum of any two sides $>$ third side.
- (iv) If sum of any two sides is equal to two right angle (π) then the sum of angles opposite them is also equal to π .

(v) The smaller angle is always opposite the smaller side.

(vi) Area of spherical triangle = $\frac{\pi R^2}{180} \times E$ where $E = (A + B + C) - 180$.

Time: Interval lapsed between any two instants. Several time measurements used by astronomers are:

(i) **Sidereal Time:** Hour angle of first point of Aries (r) measured westward at any instant is called sidereal time of that instant.

The interval of time which elapses since the upper transit of first point of Aries over observer's meridian is known as local sidereal time of the place i.e. L.S.T.

$$\begin{aligned} \text{LST} &= \text{Right Ascension (RA) of the observer's meridian} \\ &= \text{RA of a star} + \text{HA of star} \end{aligned}$$

(ii) **Apparent Solar Time:** Measurement of time based on daily apparent motion of the sun round the earth.

The interval of time which lapses between two successive lower transit of centre of sun over meridian of the place is called apparent Solar day.

(iii) **Mean Solar Day:** Interval of time between two successive lower transit of mean sun.

The instant when the mean sun crosses the local meridian at its upper transit is known as local mean noon. While the hour angle of the mean sun recorded westward from 0 to 24 hours is known as local mean time.

(iv) **Standard Time:** It's the time of the watch. Standard meridian of a country is generally selected such that it lies at an exact number of hours from Greenwich but Indian standard meridian which passes from Allahabad is at 6 hours 30 minutes East of Greenwich.

Standards, time (ST) = LMT + Difference of longitude converted to time

$$\begin{aligned} \text{Equation of time} &= \text{Apparent solar time} \\ &\quad - \text{Mean solar time} \end{aligned}$$

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Conversion of Time: $1^\circ = 4$ minutes, as $360^\circ = 24$ hrs.

(i) Local time to Std. time.

$$\text{LMT} = \text{MT} + \text{Difference in longitude.}$$

(ii) Local time to Greenwich time

$$\text{LMT} = \text{GMT} \pm \text{Longitude of place}$$

(iii) Local apparent time = local mean time + equation of time

(iv) Mean solar time = Sidereal time – retardation

(v) Sidereal time = Mean solar time + Acceleration

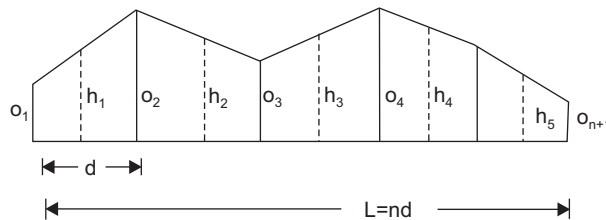
1 Tropical year = 365.24422 mean solar days

1 side real year = 366.2422 sidereal days

Measurement of Area and Volume

13

Measurement of Area



$h_1, h_2 \dots h_n$ are offsets at mid point of each division.

(i) Mid-ordinate rule

$$\text{Area} = (h_1 + h_2 + \dots + h_n) d$$

(ii) Average ordinate rule

$$\text{Area} = \frac{o_1 + o_2 + \dots + o_{n+1}}{n+1} \times nd; \quad \text{Area} = \frac{nd \sum o}{n+1}$$

(iii) Trapezoidal rule: $A = d \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + \dots + o_{n-1} \right)$

where o_1 & o_n are end ordinates.

Trapezoidal rule is more accurate than the above two.

(iv) Simpson's one-third rule

$$A = \frac{d}{3} [(o_1 + o_n) + 4(o_2 + o_4 + \dots + o_{n-1}) + 2(o_3 + o_5 + \dots + o_{n-2})]$$

(a) Short length of boundaries between two adjacent ordinates is a **parabolic arch**.

(b) It is applicable only when there are odd number of offsets.

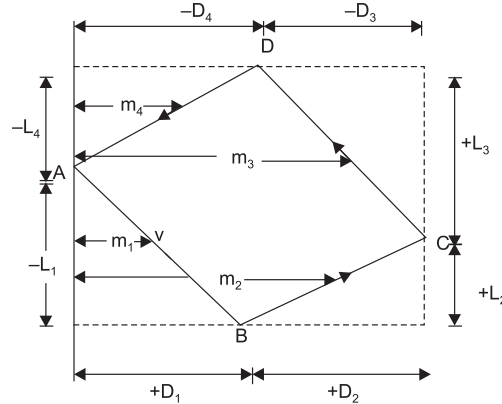
(c) If there are even no. of offset's then area upto the second last segment is calculated by Simpson's rule and for the last segment trapezoidal rule is used.

(v) Area of meridian distance method

$$\text{Area} = \Sigma \Delta L.m$$

Or

$$\text{Area} = (-L_1 \times m_1) + (L_2 \times m_2) + (L_3 \times m_3) + L_4 \times m_4$$



Here, 'm' is a meridian distance

$$m_1 = \frac{D_1}{2} \quad m_2 = m_1 + \frac{D_1}{2} + \frac{D_2}{2}$$

$$m_3 = m_2 + \frac{D_2}{2} - \frac{D_3}{2} \quad m_4 = \frac{D_4}{2}$$

Meridian distance: Distance of mid-point of a line w.r.t. fixed meridian.

(vi) Area by double meridian distance method (D.M.D.)

$$\text{Area} = \frac{1}{2} \Sigma ML$$

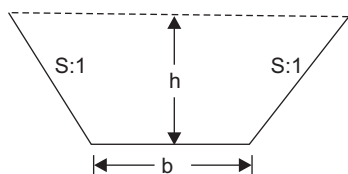
$$\text{Area} = \frac{1}{2} (-L_1 m_1 + L_2 m_2 + L_3 m_3 - L_4 m_4)$$

Here 'm' is double meridian distance

$$m_1 = D_1 \quad m_2 = m_1 + (D_1 + D_2) \quad m_3 = m_2 + D_2 - D_3 \quad m_4 = D_4$$

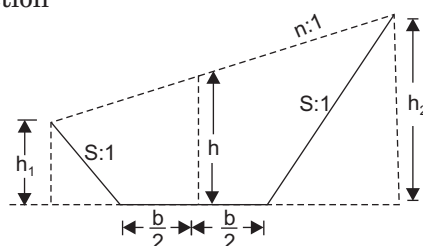
Measurement of Area of Cross-section

(i) Level Section



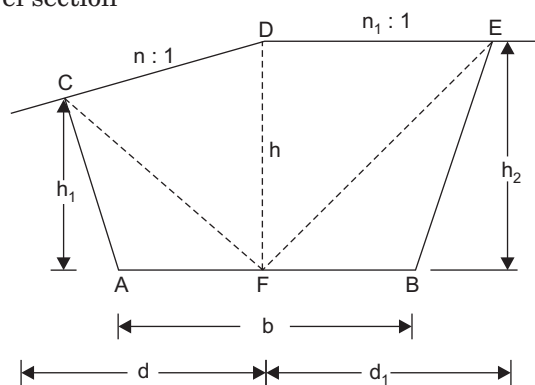
$$\text{Area} = (b + Sh)h$$

(ii) Two-level section



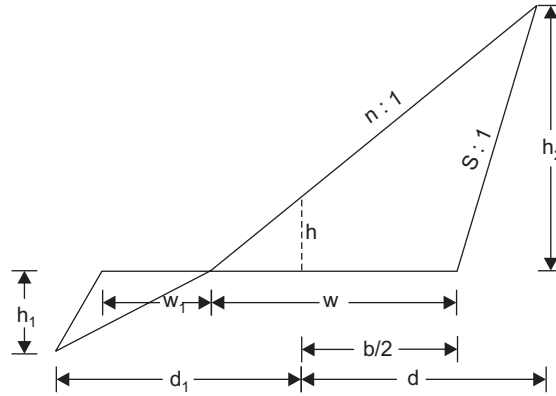
$$\text{Area} = \frac{s \left(\frac{b}{2} \right)^2 + n^2 (bh + sh^2)}{n^2 - s^2}$$

(iii) Three level section



$$\text{Area} = \frac{b}{4} (h_1 + h_2) + \frac{h}{2} (d + d_1)$$

(iv) Side hill two level section



$$\text{Excavation Area} = \frac{\left(\frac{b}{2} + nh\right)^2}{2(n-s)}$$

$$\text{Enbankment Area} = \frac{\left(\frac{b}{2} - nh\right)^2}{2(n-s)}$$

Volume Measurement

(i) Trapezoidal formula

$$V = \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \dots + A_{n-1} \right] L$$

$A_1, A_2, A_3 \dots A_n$ are the end consecutive areas and L is the **distance between them**.

(ii) Prismoidal formula (Simpson's rule is used)

$$V = \frac{L}{3} ((A_1 + A_n) + 4(A_2 + A_4 + \dots + A_{n-1}) + 2(A_3 + A_5 + \dots + A_{n-2}))$$

Note: In case if only prismoidal rule is to be applied, then the area halfway between the sections is interpolated by averaging the dimensions of the end sections and not by averaging the end areas.

Planimeter is used to find the areas of irregular shapes which are plotted to scale. Types of planimeter are Amisler polar planimeter (commonly used) and the rolling planimeter.

It is an instrument used for surveying by a graphical method in which the field work and plotting are done simultaneously. This graphical method of producing topographic maps is known as cartographic surveying.

Principle Used: Unknown point of interest can be established by measuring its directions from known points.

Accessories Used:

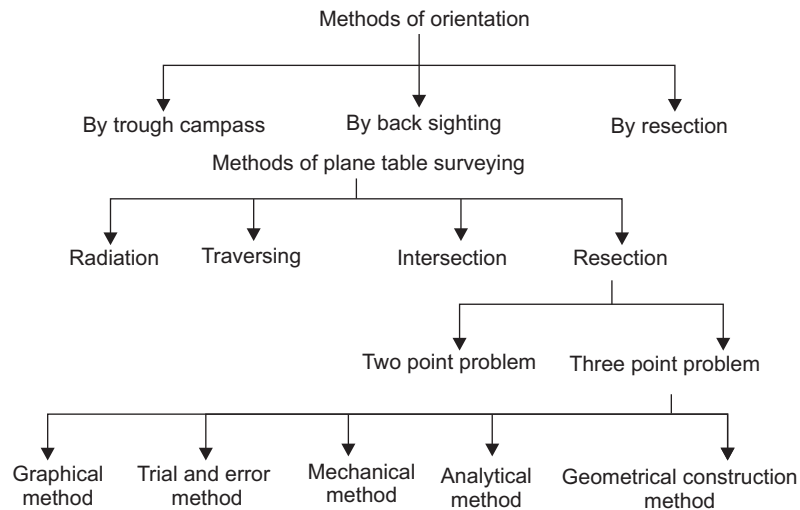
- (i) Plane Table Board
- (ii) Tripod: To support the plane table board
- (iii) Alidade: To set direction
- (iv) Trough compass: To locate North-South direction
- (v) Spirit level/level tube: To make plane table board horizontal.
- (vi) Plumbing fork: For centering of plane table.
- (vii) Indian clinometer: To determine the difference of elevations of two points by measuring the inclination of line of sight.
- (viii) Drawing sheet.

Temporary Adjustments in Plane Table

- (i) Surface of board should be perpendicular to the vertical axis of the instrument.
- (ii) The fiducial edge of the alidade should be a straight line.
- (iii) The two vanes (object vane & eye vane) should be perpendicular to the base of the alidade.

Setting up the Plane Table

- (i) Centering: Process of bringing the plotted station point exactly over the ground station.
- (ii) Levelling: Operation of bringing the plane table in a horizontal plane.
- (iii) Orientation: Process of keeping the plane table parallel to the position it occupied at the first station.



- (i) **Radiation:** Suitable when area to be surveyed is small and all the required stations to be plotted are clearly visible and accessible from the instrument station.
- (ii) **Traversing:** Generally used when a narrow strip of terrain is to be surveyed. It is used for both open and close traverses.
- (iii) **Intersection:** It is used when the base stations are visible as well as accessible while the other stations are only visible but not accessible.
- (iv) It is employed when the plane table occupies a position not yet plotted on the drawing sheet.

Two Point/Three Point Problem: It consist of locating the position of plane table station on the drawing sheet by observation of two/three well defined points which are visible as well as already plotted on the plane table.

Note: Three point problem is better than two point problem because its simple and accurate.

Contouring

15

Contour: Its an imaginary line passing through the points of equal elevation on the earth surface.

When contours are drawn under water, they are termed as **Submarine contours, fathoms or bathymetric curves**.

Contour Interval: Vertical distance between consecutive contour. It is desirable to have constant contour interval through the map. It depends on following factors:

- (i) Scale of map
- (ii) Purpose of map
- (iii) Nature of the country
- (iv) Time
- (v) Funds

Characteristics of Contour

- (i) A zero meter contour line represents the coastline. When no vlaue is represented, it indicates a flat terrain.
- (ii) Two contours intersect eachother only in case of overhanging cliff or a cave penetrating a hill side.
- (iii) Equally spaced contours represent uniform slope.
- (iv) A watershed or edge line contour crosses the valley contour at right angle.
- (v) Direction of steepest slope is along the shortest distance between the contours.
- (vi) Two contour lines having same elevations cannot unite and continue as one line.
- (vii) A set of close contour with higher figures inside and lower figure outside represent a hill while with those having lower figures inside and higher figures outside represent lakes or depressions.

Application: Determination of intervisibility, Route location, Drainage area, Site of structures, Capacity of reservoir, earthwork estimates.

Note: A very steep slope is scrap and a high scrap is known as crag.

Civil Engineering

15

Railway

CONTENTS

1. Rail, Railway and Rail joints
2. Sleepers and Track fasteners
3. Ballast, Sub-grade and Track Alignment
4. Track Stresses & Creeps of Rail
5. Geometric Design of the Track
6. Points and Crossing
7. Track Junctions
8. Railway Station and Station Yard
9. Signalling and Control Systems
10. Traction and Tractive Resistance

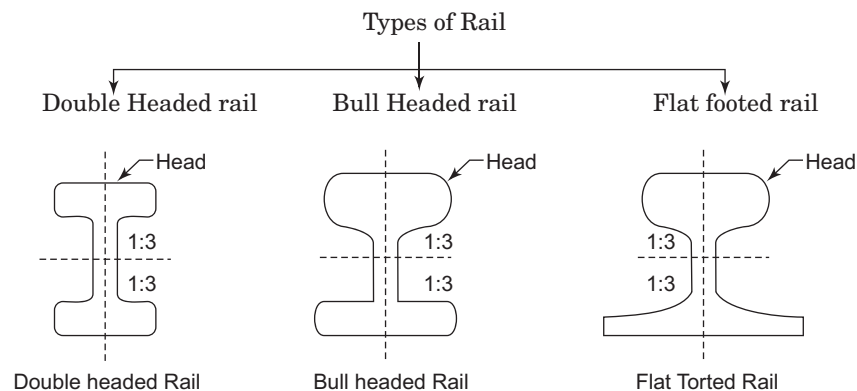
Rail, Railway and Rail Joints

1

Rail: These are steel girders for the purpose of carrying axle load made up of **high carbon steel** which convert **moving wheel loads of train into point load**, which then acts on sleepers.

Requirement of Rails:

- (i) Rails are tested by falling weight test.
- (ii) Rails are manufactured by open heart or duplex process.
- (iii) Maximum wear of head allowed is 10mm
- (iv) Minimum tensile strength needed 72 kg/m^2 .



Note: 52 kg rail (i.e. 52kg/m F.F) is suitable upto 130 kmph and 60 kg rail is suitable upto speed of 160 kmph.

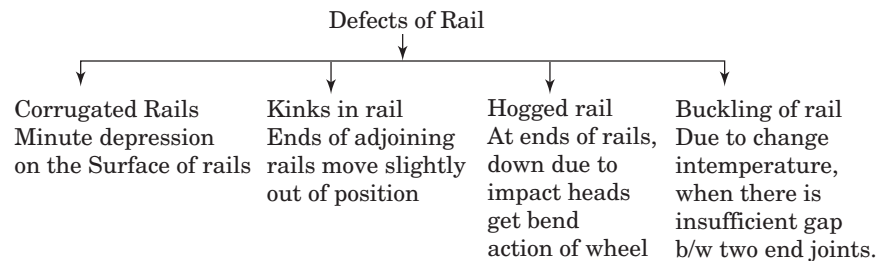
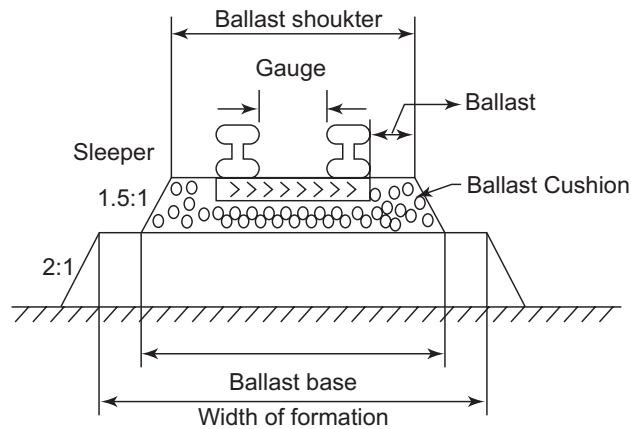
Length of Rail: Rails of larger length are always preferred as they will have less no. of joints. Rail length of 12.8m for BG tracks and rail length of 11.8 gm for MG tracks are used in Indian railways.

Note :
$$\frac{\text{wt. of Rail}}{\text{wt. of locomotive}} \approx \frac{1}{510}$$

when wear of head exceeds 5% of total weight, the rail is to be replaced.

Permanent way OR Railway Track

- ⇒ Sometimes temporary tracks are laid down for material transportation. Hence, word permanent way is coined for railway track.
- ⇒ It is **Semi-elastic in** nature due to packing of ballast cushion.



Ganges in Railway Track: It's the clear distance between inner faces of two track rails.

Broad Gauge = 1.676m

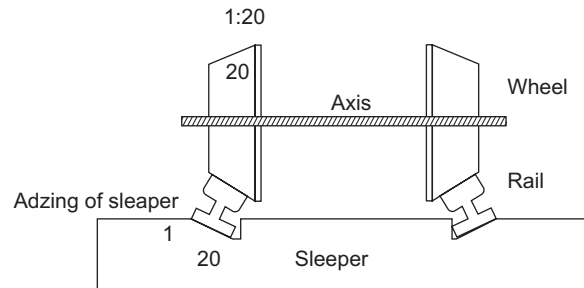
Narrow Gauge = 0.762m

Meter Gauge = 1.0m

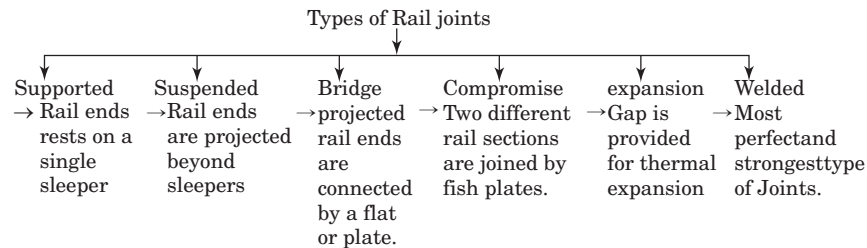
Standard gauge = 1.435m (Delhi metro)

Coning of wheels: Wheels of the train are made at a slope of 1:20. Thus is known as coning of wheels.

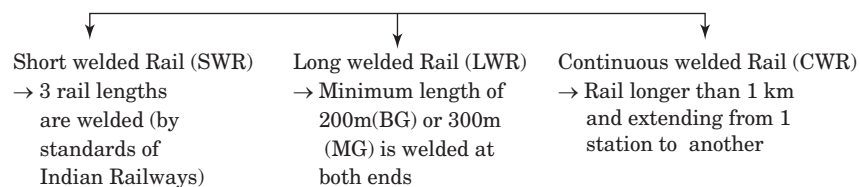
Adzing of sleepers: For effective use of coning of wheels, the rails are also laid at the slope of 1 in 20 on the sleepers.



Rail Joints: Are needed to hold together the adjoining ends of the rail. They are the weakest part of the track.



Welded Rails: Rails are welded to provide sufficient restraint at the ends of rail and better degree of fixity of rail to the sleeper so that the stresses produced are resisted by sleeper fasteners.



Length of welded Rail:

$$L = (n - 1)S$$

$$n = \frac{\alpha T E_s A_s}{R}$$

S = sleeper spacing

n = No. of sleepers required

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α = coefficient of thermal expansion

T = rise in temperature

E_s = modulus of elasticity

A_s = cross. sectional area of rail

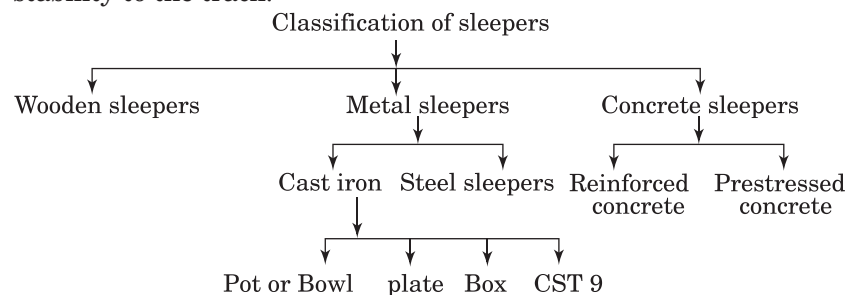
R = resisting force/sleeper

Breathing length: minimum length of rail required to be welded at the end of track, so the portion of rail between welded rail does not undergo any thermal expansion or contraction.

Sleepers and Track Fasteners

2

Sleepers are the members which support the rail and are laid transverse to it. They act as elastic medium for providing longitudinal and lateral stability to the track.



⇒ Wooden sleepers are best sleepers but they have maximum life of 12-15 years.

⇒ C.I sleepers are used more than steel sleepers as they are less prone to corrosion.

⇒ C.I sleepers can be used with every type of ballast but are not suitable for track circuiting.

⇒ CST 9 sleeper has inverted triangular pot on either side of the rail section and a plate with the projecting rib and a box on top of the plate.

⇒ Steel sleepers are light in weight, require less no. of fasteners but get easily rusted.

⇒ Concrete sleepers have high track modulus, hence used for developing high speed tracks.

Note: Serviceable portion of the spiked killed wooden sleepers is cut and used with tie bars in station yards is known as check sleepers.

Composite Sleeper Index (C.S.I): used to measure the mechanical strength of timber.

$$CSI = \frac{S + 10 H}{20}$$

S = strength Index H = Hardness Index

Both Index are measured at 12% moisture content

Track fasteners:

- (i) Fish plates: used to join two rail joints.
- (ii) Spiker: hold's rail on wooden sleepers.
- (iii) Bolts: used where sleepers rest on steel girders.
- (iv) Chair: support bull headed rails on sleepers.
- (v) Keys : fix rails to chairs on metal sleepers
- (vi) Bearing plates: used below F.F rails to distribute load over wooden sleeper.

Sleeper density: No. of sleepers per rail length. Its $M + x$

Where

M = rail length (13 for BG)

x = varies b/w 3 to 7

for BG sleeper density is $M+5$ i.e. 18 sleepers/rail.

Squaring of Sleeper: Adjusting ballast under the sleepers to space them parallel to each other.

Ballast, Sub-grade and Track Alignment

3

Ballast: It's high Quality crushed stone with desired specifications placed immediately beneath the sleepers.

Ballast Cushion: Depth of ballast below the bottom of the sleeper, normally measured under the rail seat. On the curved track super-elevation is maintained by ballast cushion.

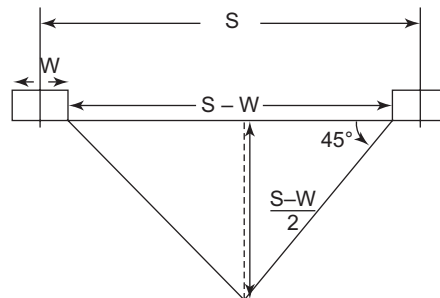
Types of ballast

- (i) Broken stone: Best material as ballast, has maximum stability.
- (ii) Gravel or River pebble: Smooth and round so poor packing and interlocking.
- (iii) Sand: Good drainage, provides silent track, blown off due to vibrations
- (iv) Asher or girders: Excellent drainage property, Excellent ballast material for station yards.

Minimum depth of Ballast layer = $D_{\min}$

$$D_{\min} = \frac{S - W}{2}$$

Depth of ballast-Section



Packing: Process of ramming the ballast underneath the sleeper.

Boxing: Loosely filled ballast above the packed layer.

Subgrade: Naturally occurring soil prepared to receive the ballast.

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The ideal material for subgrade is soil containing gravel, sand, clay and silt in equal proportion with moisture content just above the plastic limit.

Subgrades can be improved by

- (i) adding mixture of sand, silt and clay
- (ii) providing blanket of non-cohesive soil over poor cohesive soil.
- (iii) injecting mixture of cement and sand under pressure.
- (iv) providing concrete mats to stabilise poor subgrade.

Survey works for alignment of track.

- (i) Traffic survey
- (ii) Reconnaissance survey
- (iii) Preliminary survey
- (iv) Detailed or location survey

Track Stresses & Creeps of Rail

4

Track Modulus (u): Load per unit length of the rail reoccurred to produce unit deformation or depression in the track.

Note: Elasto – plastic theory is used to define tack modulus

Stresses on the rail: Torsional stresses are developed due to eccentric vertical loads while max shear stress below the contact surface of rail and diesel locomotive is 36.25 kg/mm^2

Hammer blow effect: Alternate lifting and sudden pressing is called hammer blow

$$\text{Hammer blow} = 0.14 \times \frac{M}{g} \times (2\pi n)^2 \sin \theta$$

Where, M = Net over weight in kg

r = Crank pin diameter in m

n = Number of revolutions of wheel per sec

θ = Crank angle

Steam Effect: The vertical component of pressure of steam acting on piston is given in F.P.S units as

$$P_v = \left(\frac{\pi}{4} d^2 \right) P \left(\frac{r \sin \theta \pm h}{L} \right)$$

Where, L = Length of connecting rod in inches

d = diameter of piston in inches

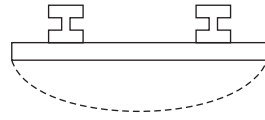
H = height of cross head above the centre line of driving wheel in inches

θ = Crank angle.

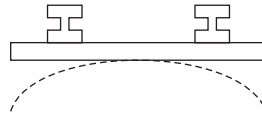
Inertia of reciprocating forces

$$F_v = \frac{m}{g} r (2\pi n)^2 \left(\cos \theta + \frac{r}{L} \cos 2\theta \right) \left(\frac{r \sin \theta \pm h}{L} \right)$$

Stresses on sleeper



Deflection under newly
Constructed sleepers



Depressions at ends due to
repeated action of loads.

Method of calculating longitudinal bending stress in rail.

$$(i) \quad X_i = 42.33 \sqrt{\frac{I}{\mu}}$$

x_i = distance from the load to the point of contraflexure of the rail in cm

I = vertical moment of inertia of rail section in cm^4

μ = Track modulus in kg/cm^2

$$(ii) \quad \sigma_{\text{comp}} = \frac{m_0}{Z_{\text{comp}}} \text{ tonnes/cm}^2 \quad \sigma_{\text{tension}} = \frac{m_0}{Z_{\text{tension}}} \text{ tonnes/cm}^2$$

m_0 = B.M in tonne cm immediately under an isolated load p tonne on one rail.

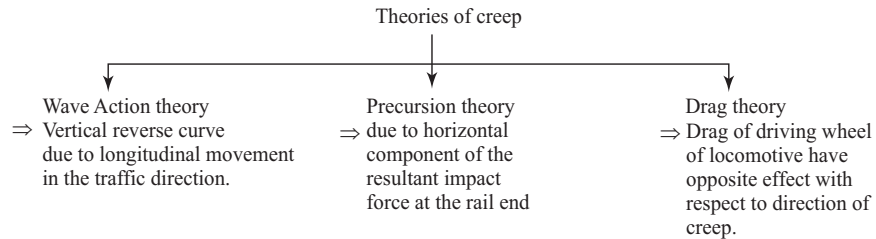
$Z_{\text{comp}}/Z_{\text{tension}}$ = section modulus of rail in compression/tension

$\sigma_{\text{comp}}/\sigma_{\text{tension}}$ = consequent compressive/tensile stress in the rail head/foot under the load p in tonne per square cm

$$(iii) \quad d = \frac{9.25p}{4\sqrt{I\mu^3}} \quad d = \text{deflection of track in cm}$$

P = load on one rail in tonnes.

Creep of the rail: It's the longitudinal movement of rail with respect to sleepers in a track.



Prevention of creep:

- (i) Pulling back rails to original position
- (ii) Using steel sleepers
- (iii) By providing sufficient crib ballast

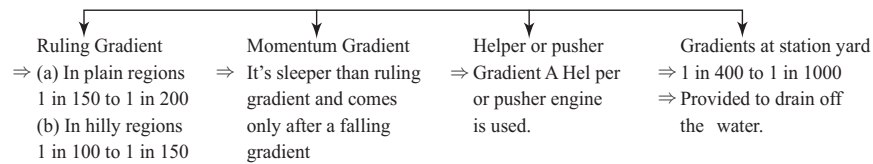
Factors governing creep of the rail:

- (i) Alignment of track: observed greater on curves
- (ii) Grade of track: more creep in downward steep gradients.
- (iii) Type of rails: old rails have more creep
- (iv) More creep in the direction of heaviest traffic.

Geometric Design of the Track

5

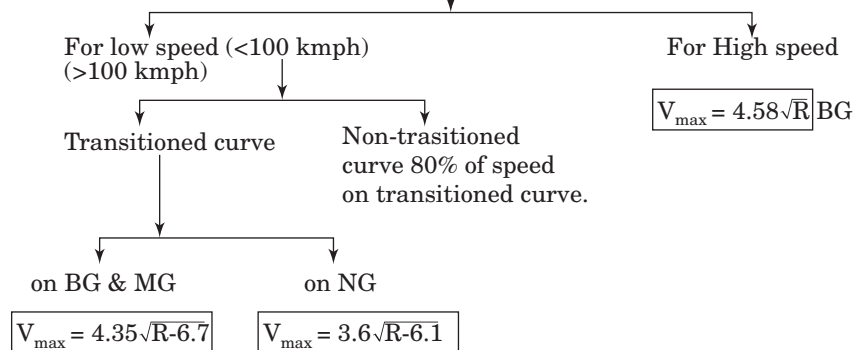
Gradient: Any rise or fall in track level is called gradient



Grade compensation: Due to curvature on the grade, the gradients on the curves are to be reduced to reduce the resistance in motion of train.

	% per degree of curve
BG	0.04
MG	0.03
NG	0.02

Safe speed of train as per Martin's Formula



Degree of Curve:

For 30m chain	$\frac{1720}{R}$

For 20m chain

$$\frac{1150}{R}$$

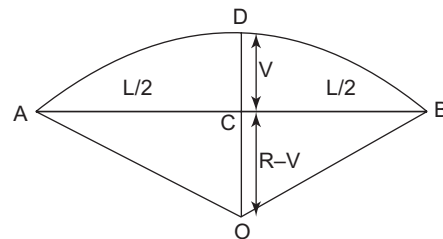
As per Indian Railways,

	Maximum degree of curve	Minimum radius
BG	10°	175m
MG	100°	109m
NG	50°	44m

Versine of curve: used to check the accuracy of the curve

$$\text{Versine} = V = \frac{L^2}{8R} \quad V = \text{cm}$$

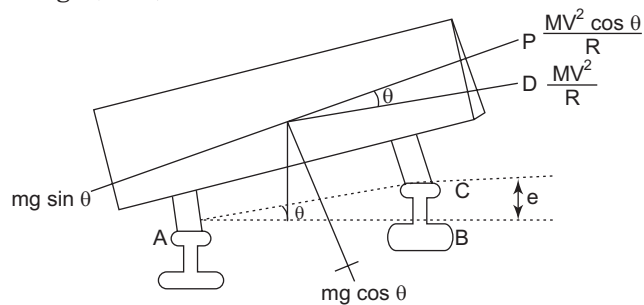
$$L, R = \text{m}$$


Super elevation or cant

$$\tan \theta = \frac{e}{G} = \frac{v^2}{gR}$$

$$e = \frac{Gv^2}{127R} \quad \begin{array}{l} V \text{ in kmph} \\ R \text{ in m} \end{array}$$

G → Gauge (in m)



Equilibrium Speed

Maximum limit of super elevation

Track	Speed<120kmph	>120 kmph
BG	16.5 cm	18.5 cm
MG	10 cm	×
NG	7.6 cm	×

Equilibrium speed

When sanction speed > 50 kmph

When sanction speed < 50 kmph

$$V_{equ} = \min \left\{ \begin{array}{l} \frac{3}{4} V_{\max} \\ \text{safe speed by martin} \end{array} \right. \quad V_{equ} = \min \left\{ \begin{array}{l} 50 \text{ kmph} \\ \text{safe speed by martin} \end{array} \right.$$

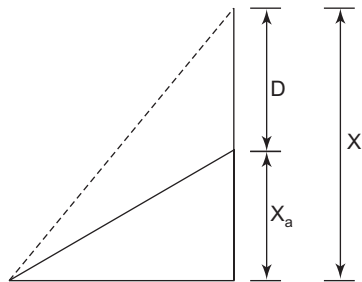
$$\text{Weighted average speed} = \frac{\sum N_i V_i}{\sum N_i} \quad N_i = \text{Number of train's having speed } V_i$$

Can't Deficiency: For trains running with higher speed than equilibrium speed actual can't requirement is more than provided. This shortage is known as can't deficiency.

$$\text{can't deficiency} = X_1 - X_A$$

X_1 = can't required for higher speed train

X_A = Actual can't provided as per average speed

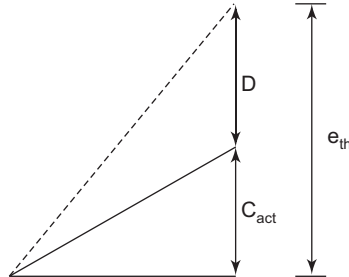


$$e_{th} = e_{act} + D$$

e_{th} = theoretical can't

e_{act} = Actual can't

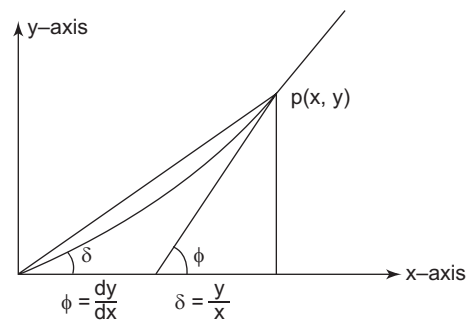
D = can't deficiency


Limit on D_{max}

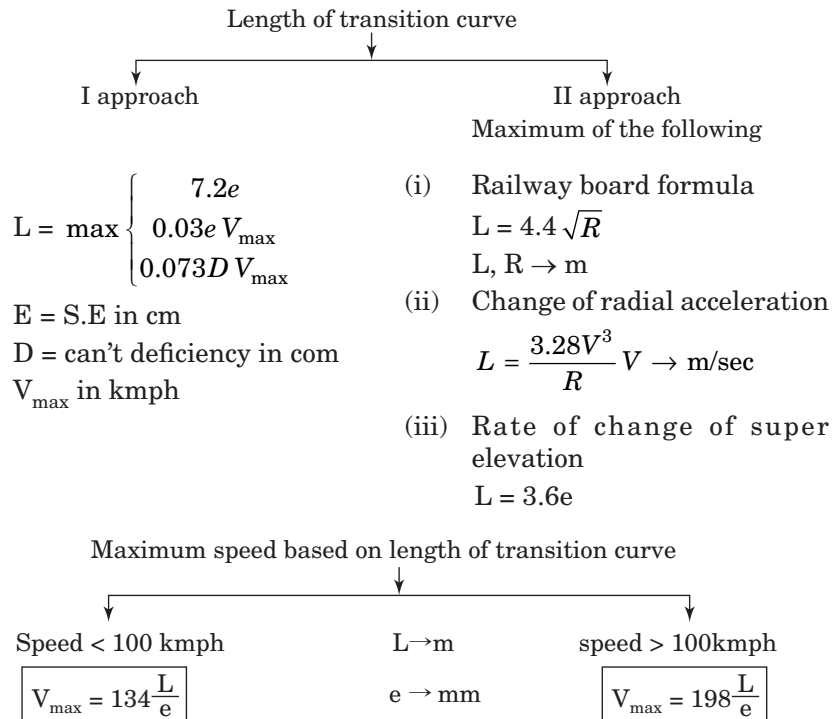
	B.G track		M.G	N.G
	< 100kmph	> 100kmph		
D_{max}	7.6 cm	10 cm	5.1 cm	3.8 cm

Transition Curve: cubic parabola is used as transition curve.

- (i) Equation of deflection $y = \frac{x^3}{6RL}$
- (ii) Spiral angle $\phi = \tan^{-1} \left(\frac{x^2}{2RL} \right)$
- (iii) Deflection angle $\delta = \tan^{-1} \left(\frac{1}{3} \tan \phi \right)$
- (iv) Shift $s = \frac{L^2}{24R}$



Note: Transition curve are early set out by offset method.
Cubic parabola is also known as Froude's curve.



Gauge widening on curves

$$W_e = \frac{13(B + L)^2}{R}$$

B- Rigid wheel base in meters

For BG = 6m For MG = 4.88m

R-Radius of curve (in m)

$$L = 0.02 \sqrt{h^2 + Dh}$$

L = Lap of flange

h = depth of wheel flange below rails in cm

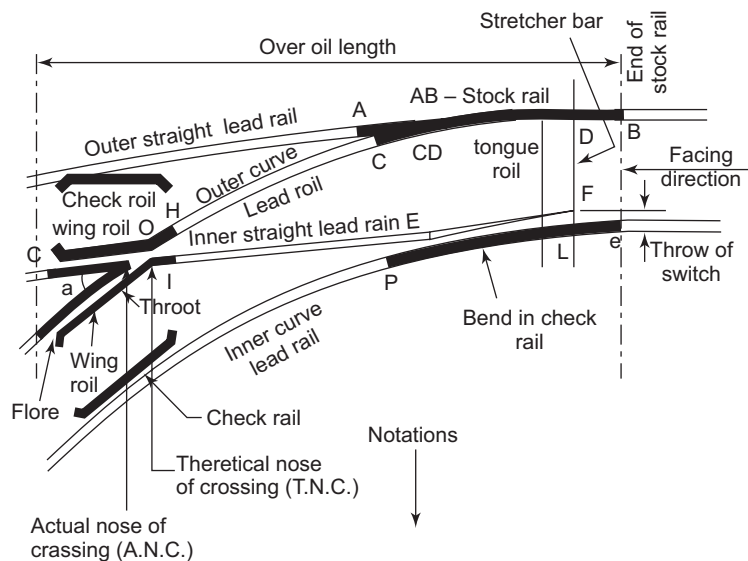
D = Diameter of wheel in cm

W_c = Widening of gauge in cm

Points and Crossing

6

Turnout: It's the combination of points and crossing which enables a back either a branch line or siding to take off from main track.



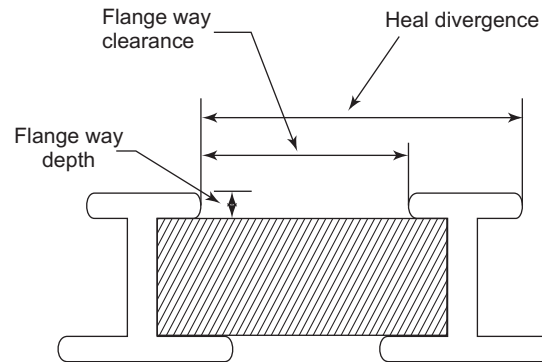
Turn out consists of

- (i) 2 points or switches
- (ii) 2 pair of stock rails
- (iii) A pair of check rails.
- (iv) An acute angle crossing
- (v) 4 lead rails
- (vi) A pair of stock rail

Important Points of Switch:

- (i) Heel divergence: Distance b/w running faces of stock rail and tongue rail at the heel of switch.

- (ii) Flange way clearance: Distance b/w adjacent faces of tongue rail and stock rail at the heel of switch.



- (iii) Flange way depth: Vertical distance b/w top of rail to heel block.
 (iv) Throw of switch: max distance by which toe of tongue rail moves sideways.
 (v) Switch angle: Angle b/w running faces of stock rail and tongue rail when tongue rail touches the stock rail.

$$\alpha = \frac{\text{Heel divergence}}{\text{Length of tongue rail}}$$

When thickness of tongue rail at toe, $t = 0$

$$\sin \alpha = \frac{d}{S_2}$$

When thickness of tongue rail at toe = t

$$\sin \alpha = \frac{d - t}{S_1}$$

d = heel divergence

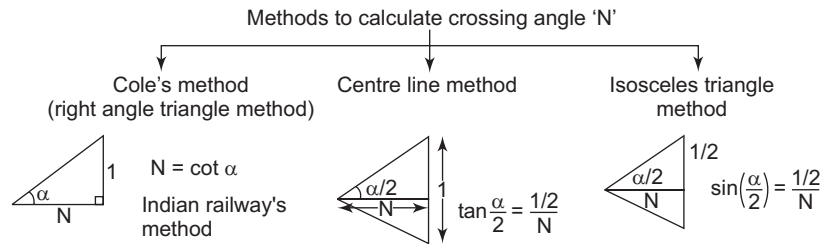
S_1 = Actual length of tongue rail

S_2 = theoretical length of tongue rail.

Crossing angle:

$$\text{No. of crossing (N)} = \frac{\text{The speed at the leg of crossing}}{\text{The length of crossing T.N.C.}}$$

T.N.C is the theoretical nose of crossing

**Design of Turnout:**

- (i) Curve lead: Distance measured along stock rail b/w TNC and toe of switch
- (ii) Switch lead: Distance between heel of the switch and toe of the switch
- (iii) Lead: Distance between TNC and heel measured along stock rail

$$\boxed{\text{Curve lead} = \text{lead} + \text{switch lead}}$$

Method (a):

$$(i) \quad CL = G \cot \frac{\alpha}{2}$$

$$CL = \sqrt{2R_0 G}$$

$$CL \approx 2 GN$$

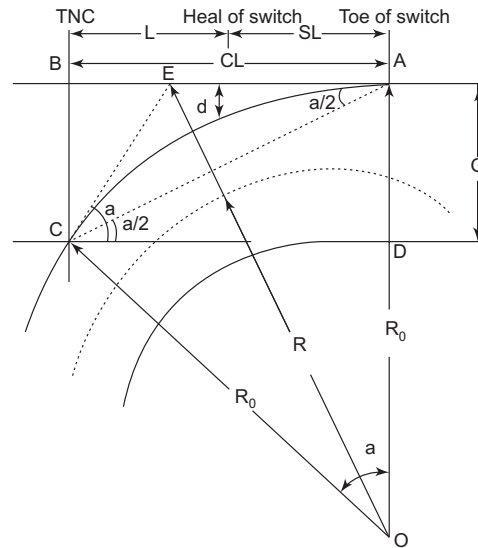
(ii) Central Radius R

$$r = R_0 - \frac{G}{2} \quad R_0 = G + 2GN^2$$

Indian Railways

$$R_0 = 1.5G + 2GN^2$$

$$R_0 = CL \operatorname{cosec} \alpha$$

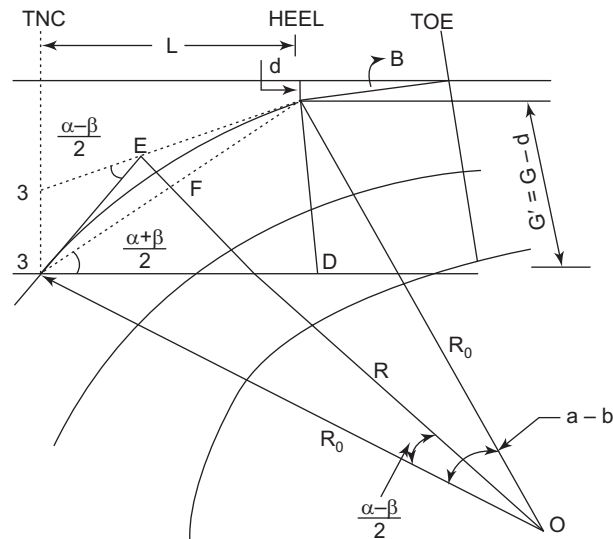


$$(iii) SL = \sqrt{2R_0d}$$

$$(iv) L = CL - SL$$

$$(v) d = \frac{SL^2}{2R_0}$$

Method (b)



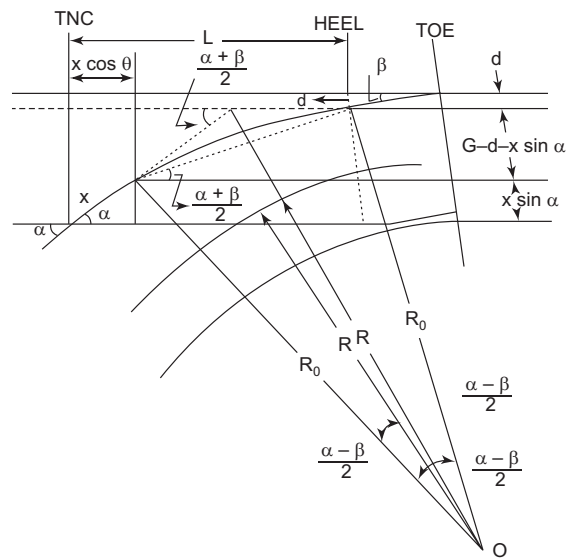
$$(i) L = (G - d) \cot \left(\frac{\alpha + \beta}{2} \right)$$

$$(ii) R = R_0 - \frac{G}{2} r_0 = \frac{G - d}{\cos \beta - \cos \alpha}$$

Method (c)

$$(i) L = x \cos \alpha + (G - d - x \sin \alpha) \cot \left(\frac{\alpha + \beta}{2} \right)$$

$$(ii) R_0 = \frac{G - d - x \sin \alpha}{\cos \beta - \cos \alpha} \quad R = R_0 - \frac{G}{2}$$



Types of switches

Stab switch
No separate tongue
rail is provided

Split switch
Tongue rail is combined
with stock rail

Loose heel
type

Fixed heel
type

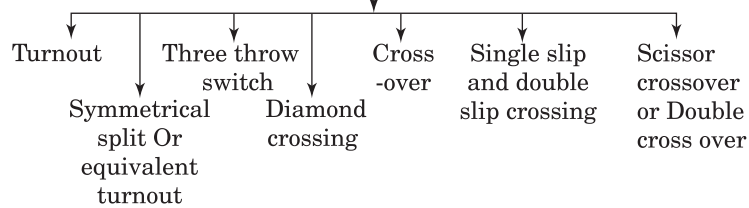
Note: Check rails are provided on opposite sides of the crossing for guiding the wheel while wing rails help in channelizing the wheels in their route.

Track Junctions

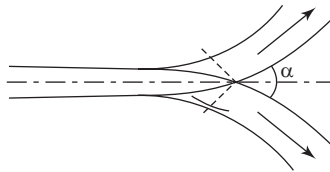
7

→ It's used to transfer trains from one track to another.

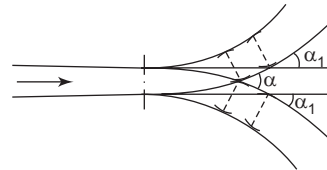
Types of track Junctions



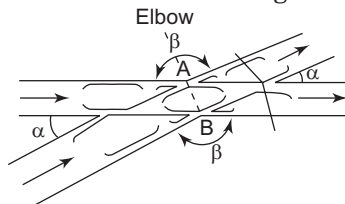
Symmetrical split or equivalent turnout



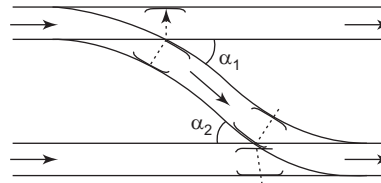
Three throw switch



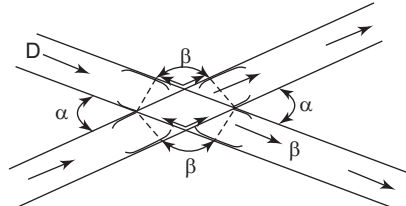
Diamond crossing



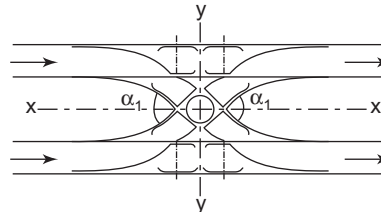
Cross over



Single/double slip crossing



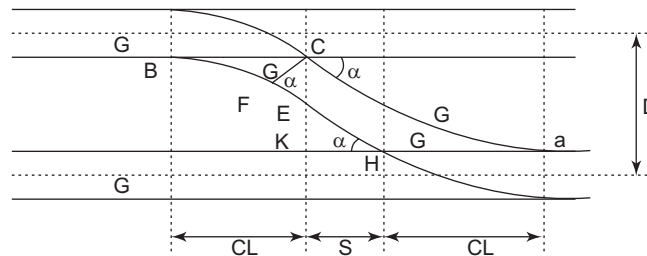
Scissor or double cross over



- (i) Diamonds crossings are avoided on curves
- (ii) Cross-over are useful when trains are approaching from one direction only.
- (iii) In single slip tracks can be changed from one direction only while in double tracks can be changed from either direction.
- (iv) Scissor cross-over enables the train to change track from either direction.

Design of cross-over

Type 1: Two turn out provided on two tracks joint with straight position between two turnoffs.



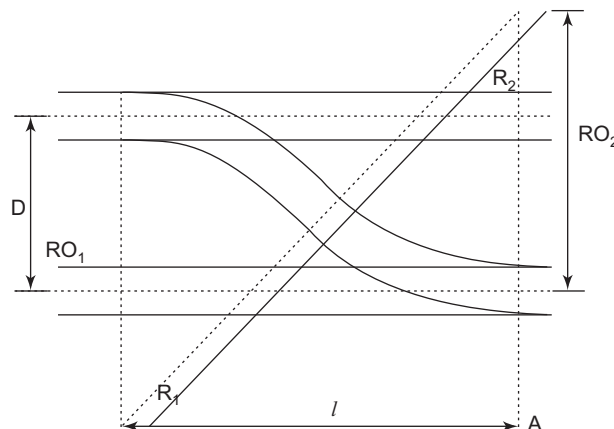
$$N = \cot$$

$$\text{Length of one turnout} = CL = 2GN$$

$$\text{Length of straight portion } S = (d - G)N - G\sqrt{1 + N^2}$$

$$\text{Overall length of turnout} = 4NG + (D - G)N - G\sqrt{1 + N^2}$$

Type 2: Cross over with curved path between two turnouts having different crossing angles.



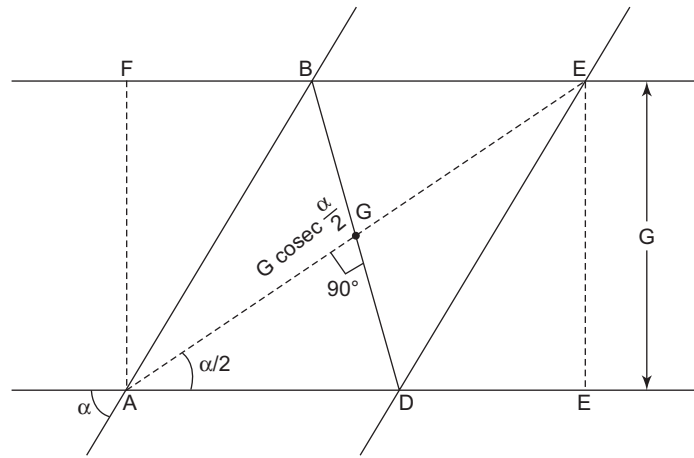
15.26 CIVIL ENGINEERING

$$RO_1 = 1.5G + 1.5GN_1^2 \quad RO_2 = 1.5G + 1.5GN_2^2$$

$$R_1 = RO_1 - \frac{G}{2} \quad R_2 = RO_2 - \frac{G}{2}$$

$$\text{Overall length } L = \sqrt{(O_1O_2)^2 - (A_2A)^2}$$

Design of diamond crossing



$$(i) \quad AB = BC = CD = DA = G \operatorname{cosec} \frac{\alpha}{2}$$

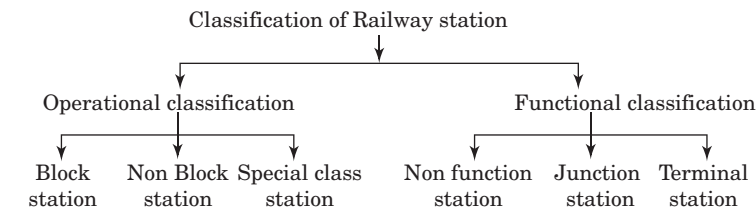
$$(ii) \quad DE = BF = G \cot \alpha$$

$$(iii) \quad AC = G \operatorname{cosec} \frac{\alpha}{2}$$

$$(iv) \quad BD = G \sec \frac{\alpha}{2}$$

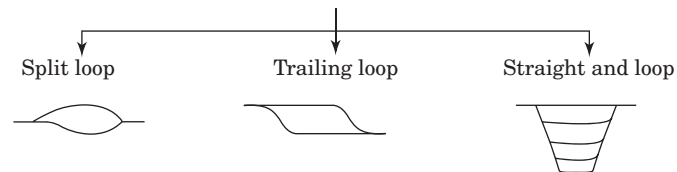
Railway Station and Station Yard

8

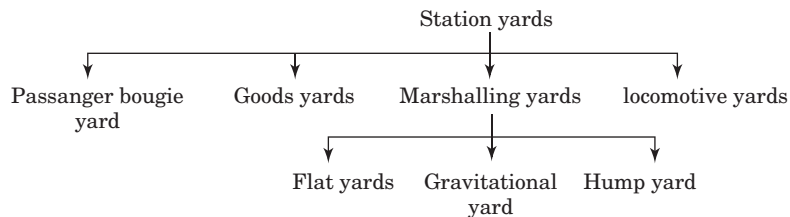


Operational classification Functional classification

Loop: When branch from main line again terminates at the same line.



Sidings: When a branch line from main line or a loop line terminates at a dead end with a buffer stop or sand hump.



Note: Marshalling yard is considered as “the heart that pumps the flow of commence along the tracks and it may too, without the internal vigilance become the grave yard of wagons”.

Other equipments of station yard

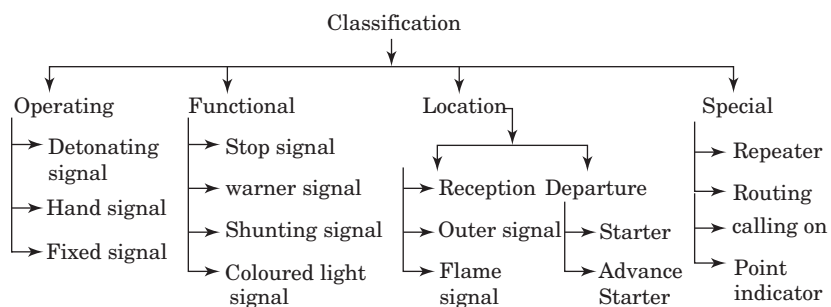
- (i) Engine shed or loco shed: Purpose is to clean, repair and maintain locomotives

- (ii) Ash pits: To receive ashes from locomotive boilers.
- (iii) Drop pits: To remove wheels of engine
- (iv) Triangles and turn tables: Changing the direction of engine
- (v) Traverser: To transfer wagons one at a time to and fro from parallel tracks without necessity of shunting.
- (vi) scotch blocks: To separate all the sidings and shunting lines through running lines.
- (vii) Fouling marks: Points fixed between a pair of converging tracks to indicate a foul or possibility of collision of vehicles beyond these points.
- (viii) Sand Hump or Snag dead end: A hump of sand with a rising gradient at the dead end of siding is provided.

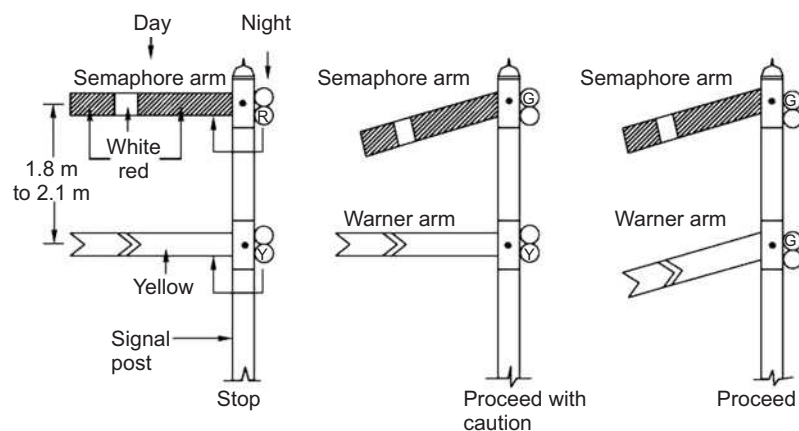
Signalling and Control Systems

9

These are systems, devices and means by which trains are operated efficiently



- (i) Detonating signals are used in foggy and cloudy weather. These are placed on rails which explode with when train passes over them.
- ii) Stop/semaphore signal: Under normal position its arm is horizontal and it indicates stop or danger indication.
- (iii) Warner signal: A semaphore signal at entrance is combined with a warner system.



- (iv) Shunting signals are used in station yards in shunting operation.
- (v) Calling on signals are very much useful with repair work, it permits to the train to proceed with caution.
- (vi) Repeater or co-acting signal's are used where driver's vision is obstructed. A duplicate arm of smaller size is placed at a suitable position on the same post.
- (vii) Routing signals are provided where no of lines exist at a station taking off different locations from main line.

Note: Absolute block system or space interval system is extensively used in India.

Traction and Tractive Resistance

10

Tractive effort: Pull applied by engine on driving wheel

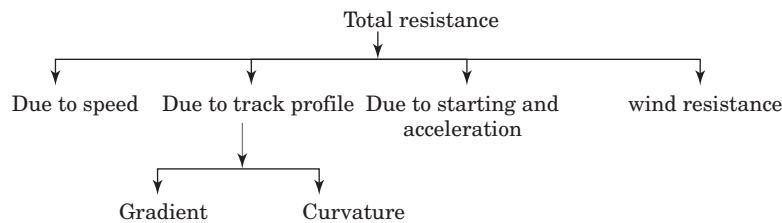
Hauling Capacity: Maximum value of frictional force due to driving wheels

$$\text{Hauling Capacity (H.C.)} = \mu n w$$

μ = friction coefficient

n = no of driving wheels

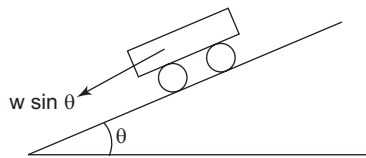
w = wt. of one pair of driving wheel



(i) Due to speed

$$R_T = 0.0016 w + 0.00008 wv + 0.0000006 wv^2$$

w = wt. of train in tonnes V = Speed of trains in kmph



(ii) Due to track profile

(a) Gradient $R_g = w \tan \theta$

(b) Curvature $R_c = 0.0004 WD \rightarrow$ For BG

0.0003 WD $\rightarrow$ For MG where D – degree of curve

0.0002 WD $\rightarrow$ For NG

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(iii) Due to starting and acceleration

(a) starting $R_s = 0.15 W_1 + 0.005 W_2$

W_1 = wt. of locomotive(tonnes)

W_2 = wt. of Wagons (tonnes)

(b) Acceleration $R_a = 0.28 W \left(\frac{V_2 - V_1}{t} \right)$

$V_2, V_1 \rightarrow$ In kmph

$t \rightarrow$ In secs

(iv) Wind resistance

$$R_w = 0.000017 a V_w^2$$

a = exposed area of train in m^2

V_w = Speed of wind in kmph

Note: For moving train

Tractive Resistance > Hauling capacity > Total resistance

Civil Engineering

16

Airport, Dock, Harbour and Tunnel Engineering

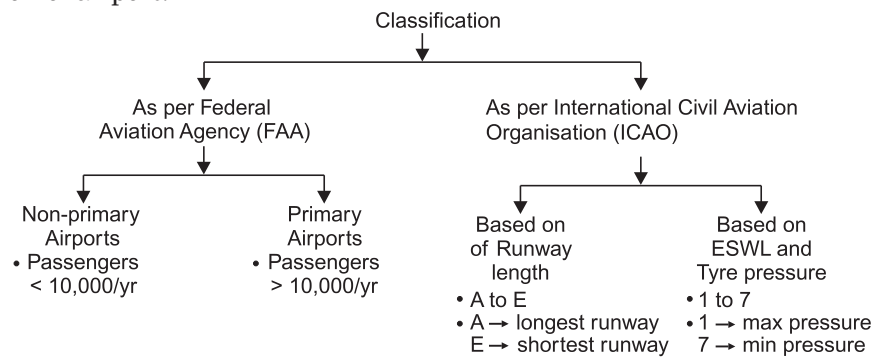
CONTENTS

1. Airport
2. Airport Planning
3. Runway Design
4. Airport Capacity
5. Taxiway Design
6. Water-Transportation
7. Tides And Shore Protection Works
8. Breakwaters
9. Dock
10. Tunnel Engineering

Airport

1

Airport is an area which is to be regularly used for landing and take off of airport.



Aircraft Components

(i) Engine:

(a) **Piston Engine:** For moderate speeds at low altitudes

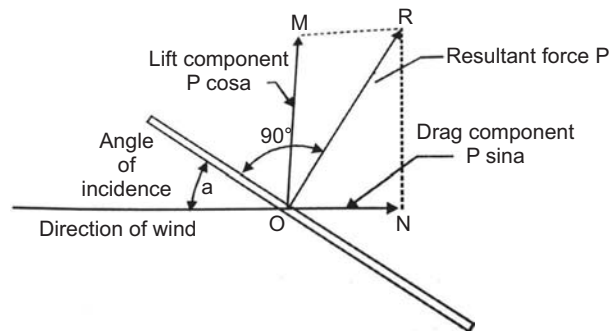
(b) **Turbo Jet:** For Higher altitude and greater temperature difference.

(c) **Turbo Prop:** At low as well as high altitude.

(d) **Ram Jet:** No moving parts, operates at high speed

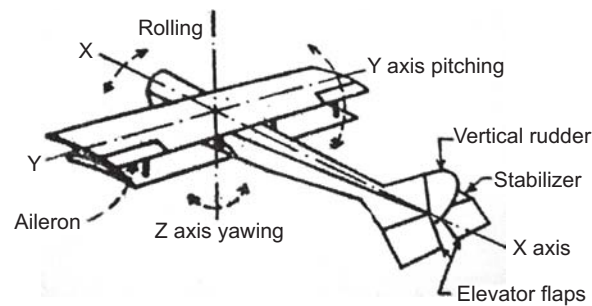
(e) **Rocket Engine:** No limit on altitude

(ii) Wings:



(iii) **Three Controls:**

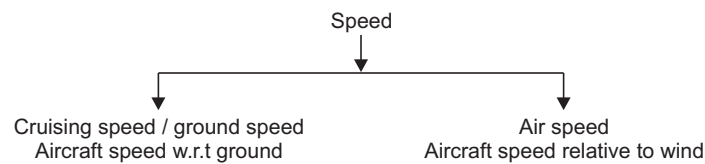
Controller	Used for
Elevator	Pitching
Rudder	Yawing
Aileron	Rolling



(iv) **Fuselage:** It's the main body part of aircraft

Minimum turning radius: Distance of farther wing tip from the centre line of rotation.

Minimum circular radius: Minimum radius with which the aircraft can turn in space.



Maximum Ramp weight > Maximum take off weight
> Maximum landing weight

Jet blast: Cement concrete pavement is provided at least at the touch down portion as the bituminous pavement gets affected by the jet blasts.

Noise: Noise foot print recommended is 1 km² for 90 PN dB and 3 km² for 80 PN dB (Perceived Noise decibel)

Airport Planning

2

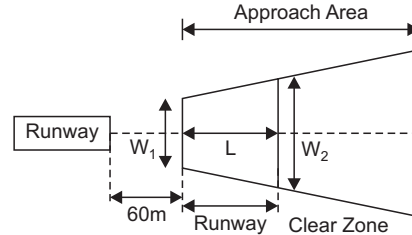
Factors affecting Airport site Selection: Airport use, proximity to other airports, Topography of area, obstruction and visibility, wind (direction, duration and intensity), Noise, Future development, Drainage and soil conditions.

Survey's for site selection: Traffic survey, metrological survey, Topographical survey, soil survey, material survey. Drainage survey.

Note: Most desirable soil with natural drainage conditions contains large amount of **sand and gravels**.

Zoning laws: Laws made in order to prevent future development of obstructions in the vicinity of airport and turning and taking off direction of aircraft.

Approach zone: Wide clearance area required on either side of runway along the direction of landing or take off of airport.



	w_1	w_2	L
Non-Instrument Runway	150 m	270 m	600 m
Instrument Runway	300 m	525 m	750 m

Note: Instrument Runway is equipped with device permitting the landing under condition of poor visibility.

Clear Zone: The inner most portion of approach zone, which is the most critical portion from obstruction point of view.

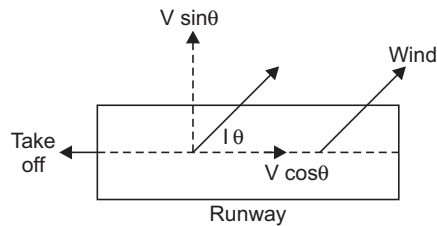
Turning Zone: The area of airport other than approach zone which is used for turning operation of aircraft. Any object located 4.5 km from runway reference point shall be considered as an obstruction for aircraft turning operation if its height is more than 51 m.

Runway Design

3

Runway orientation: It is oriented such that the direction of wind is opposite to the direction of landing and takeoff. If landing and take off operation are done along the wind direction then longer runway will be required.

Cross-wind component:

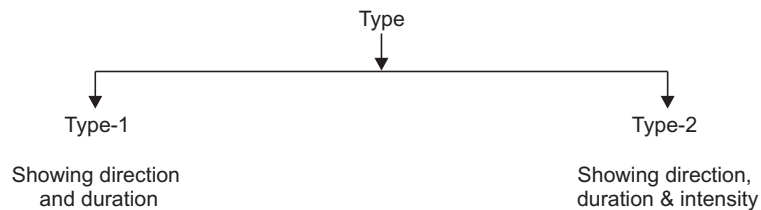


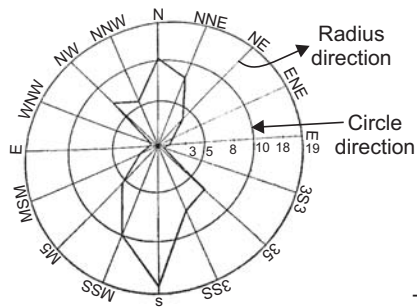
$V \sin \theta$ = Cross wind component

Type of aircraft	Permissible limit of cross wind component
Small aircraft	15 kmph
Mixed traffic	25 kmph
Big aircraft	35 kmph

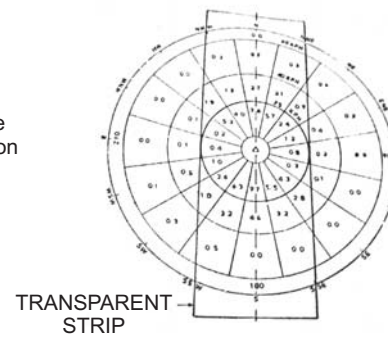
Wind Coverage: The percentage of time in a year during which cross wind component is within the permissible limits.

Wind rose diagram: It's the graphical representation of direction, duration and intensity of wind. The wind data of at least 5 years and preferably 10 years is needed for airport design.





Radius → wind direction
Circle → Duration of wind



Radius → wind direction
Circle → wind intensity
Values entered → wind coverage

Note: Calm period is the percentage of time during which intensity is less than 6.4 kmph.

Basic Runway length

Length of runway under following **assumed** conditions

- (i) Airport at sea level
- (ii) Temperature at airport is 15°C (standard)
- (iii) Runway is levelled in longitudinal direction.
- (iv) No wind blowing on runway.
- (v) Aircraft is loaded at its full capacity.
- (vi) There is no wind blowing enroute to destination
- (vii) Enroute temperature is standard.

Corrections in runway length

(i) **correction for elevation:**

$$L' = L \times \frac{7}{100} \times \frac{E}{300}$$

$$L_1 = L + L'$$

E → elevation from MSL (m)

L → Basic runway length

L' → Correction due to elevation

L_1 → Corrected length

(ii) Correction for temperature:

Standard temp = $15 - 0.0065 (E)$

Reference temp = $T_a + 1/3 (T_m - T_a)$

T_a = Monthly mean of average daily temperature of hottest month

T_m = Monthly mean of maximum daily temperature.

ΔT = Reference temp – Standard temp.

$$L'' = L_1 \Delta T \times \frac{1}{100}$$

$$L_2 = L_1 + L''$$

L'' = Correction due to temperature

L_2 = Corrected length after temperature correction.

Check: $\frac{L_2 - L}{L} \times 100 < 35$

(iii) Correction for Gradient:

$$L''' = L_2 \times G \times \frac{20}{100}$$

$$L_3 = L_2 + L'''$$

G = Gradient in%

L''' = Correction due to gradient

L_3 = Final runway length

Note: ICAO does not recommend any gradient correction

Geometric design of Runway

(i) Runway length: Actual runway length (L_3) depends upon elevation, temperature and gradient.

(ii) Runway width: Recommended range is 10 m to 45 m. Typical aircraft traffic is concentrated at central 24 m.

(iii) Width of safety area: Runway width + shoulder on either side + area cleared, graded and drained.

As per ICAO, for Non Instrumental runway minimum width of safety area

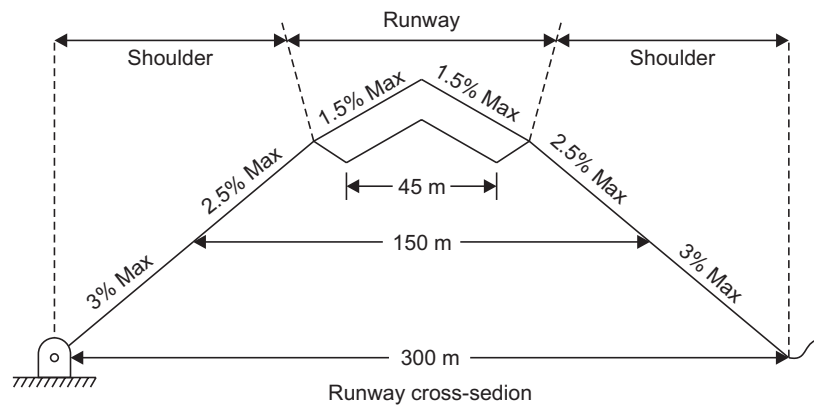
(a) for A, B, C type = 150 m

(b) for D and E type = 78 m

For instrumental runway its 300 m

- (iv) **Length of safety area:** 60 m beyond runway at both ends
- (v) **Transverse gradient:** for proper drainage. AS per ICAO
 - (a) For A, B, C type = Max. Value 1.5%
 - (b) For D and E type = Max. Value 2%
 While minimum value is 0.5% in both above cases

Note: ICAO recommends that upto 75 m from centre its maximum gradient can be 5%

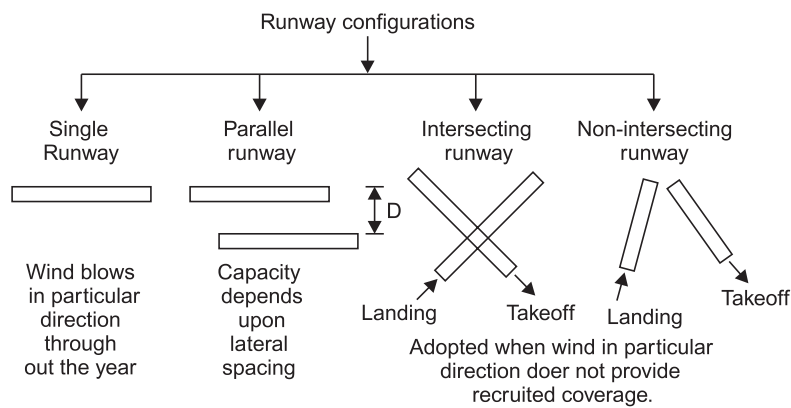


- (vi) **Longitudinal gradient**
 - (a) For A, B, C type = 1.5% max.
 - (b) For D, E type = 2% max.
- (vii) **Effective gradient**
 - (a) For A, B, C type = 1% max.
 - (b) For D, E type = 2% max.
- (vii) **Rate of change of longitudinal gradient**
 - (a) For A and B type = 0.1% per 30 m length
 - (b) For C type = 0.2% per 30 m length
 - (c) For D, E type = 0.4% per 30 m length
- (viii) **Sight distance:** for A, B, C type of airport any two points having 3 m above surface (2.1m above surface for D and E type) of runway should be mutually visible from a distance equal to half of runway length.

Airport Capacity

4

Airport Capacity: No. of movements (landing or taking off operation) of aircraft which the airport can safely handle within an hour.



Runway Capacity: (operations/hr or operations/year)

It's the ability of a runway system to accommodate aircraft landing takeoffs. Factors affecting runway capacity are: Air traffic control, demand characteristics, Environmental factors, layout and design of runway system

Gate Capacity: It's an ability of specified no of gates to accommodate aircraft loading and unloading operations under conditions of continuous demand where as gate may be defined as aircraft parking space adjacent to terminal building for loading and unloading of passengers.

Gate Capacity for a single gate = No of aircrafts/minute/gate

$$\text{Capacity}_{\text{single gate}} (C_{Sg}) = \frac{1}{\text{Weighted service time}}$$

Capacity of all gates $C = G C_{Sg}$

$$\text{Capacity of gate system} = \min \text{ all } \left[\frac{G_i}{T_i M_i} \right]$$

G_i = No of gates that can accommodate aircraft of class i

T_i = Mean gate occupancy time of aircraft of class i

M_i = Fraction of aircraft of class i demanding service.

Taxiway Capacity: It directly affects the runway and gate capacity. It depends on runway operation rate, aircraft mix and location of exit taxiway.

Taxiway Design

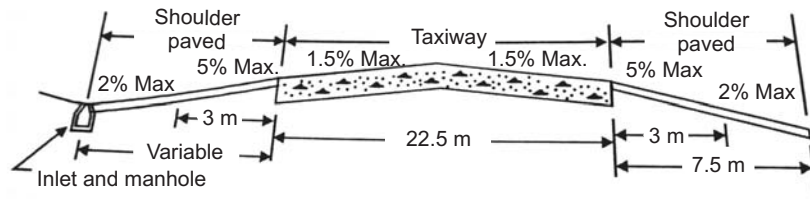
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It provides access to the aircraft from runway to loading apron or service hanger and back.

Factor's effecting geometric design of taxiway:

(i) **Length of taxiway:** As short as possible

(ii) **Width of taxiway:** 22.5 m to 7.5 m



(iii) **Longitudinal gradient:**

(a) for A, B type: max 1.5%

(b) for C, D, E type: max 3%

(iv) **Transverse gradient:** for A and B type its max value is 1.5% and minimum is 0.5%

(v) **Rate of change of longitudinal gradient**

(a) for A, B, C type = 1% in 30%

(b) for D, E type = 1.2% in 30%

(vi) **Sight distance:**

(a) for A, B, C type: Surface should be visible from 3 m height at 300 m distance

(b) for D, E type: Surface should be visible from 2.1 m height at 250 m distance

(vii) **Turning radius**

(a) Radius of the curve

$$R = \frac{V^2}{125 f}$$

R = radius (m)

V = velocity (kmph)

f = Transverse friction coefficient (0.13)

(b) By Horonjeff's equation

$$R = \frac{0.388 W^2}{\frac{T}{2} - S}$$

W = wheel base of aircraft in m (16 m – 18 m)

T = width of taxiway pavement (std = 22.5 m)

S = distance *b/w* midway point of the main gear and edge of taxiway pavement in m

$$S = 6 + \frac{\text{Tread of main loading gear}}{2}$$

(c) Absolute minimum turning radius regardless of speed

1. for subsonic jet = 120 m

2. for supersonic jet = 180 m

Minimum radius of taxiway will be the maximum of a , b , c discussed above

Exit taxiway

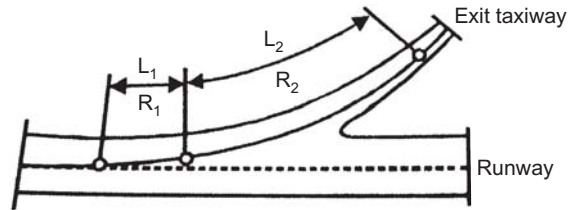
Taxiway located at various points such that landing aircraft leaves the runway as early as possible. Location of taxiway depends upon

- (i) Number of exit taxiway
- (ii) Exit speed
- (iii) Weather condition
- (iv) Topographic feature.

Design of exit taxiway: At high turn off speeds of 65 kmph to 95 kmph, a compound curve is necessary to minimise the tire wear on the nose gear so, the main radius curve R_2 should be preceded by a larger radius curve R_1 as shown. Aircraft path approximates a **spiral**

$$L_1 = \frac{V^3}{45.5 C R_2} \quad C \approx 0.39$$

$$\text{Stopping distance} = \frac{V^2}{25.5 d} \quad d = \text{deceleration}$$



Apron: Paved area for parking of aircraft as well as for loading and unloading the passenger and cargo

Size of apron depends upon

- (i) **Size of loading area:** i.e gate size
- (ii) **Number of gate position:** It's the runway capacity in unit of movement per hour

$$\text{Number of gate position} = \frac{\text{Capacity of Runway}}{60 \times 2} \times \text{Average gate occupancy time}$$

(iii) **Aircraft parking system:**

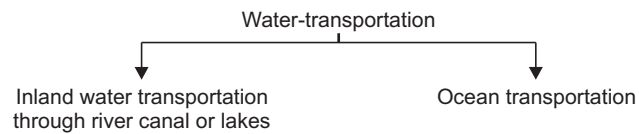
- (a) Frontal system
- (b) Open area system
- (c) Finger system
- (d) Satellite system

Note: Holding apron is the storage area for waiting of aircraft before take off generally provided at busy airports.

Hanger: It's the covered area for repair and servicing the aircraft.

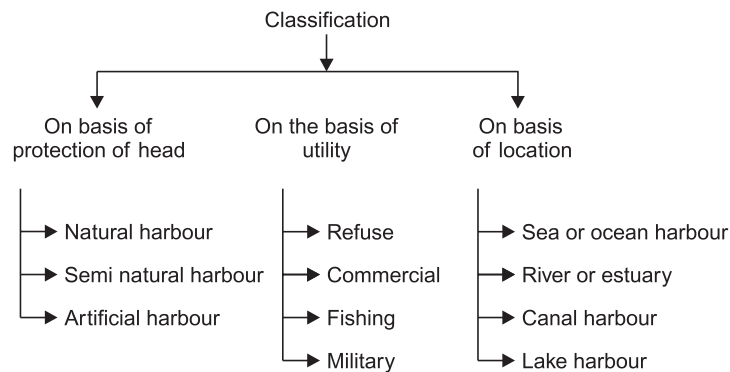
Water-Transportation

6



Note: Water transportation has higher load carrying capacity but it's slow and risky (due to storms and hurricanes)

Harbour: A partially enclosed protected water area to provide safe and suitable accommodation for vessels, seeking refuge, supplies, refuelling, repair or transfer of cargo.



Selection of site

- (i) An easy accessibility via wide and straight approach.
- (ii) Wide entrance for manoeuvring.
- (iii) Deep water bay.
- (iv) Rock bottom locations should be avoided.

Harbour layout elements

- (i) **Harbour Size:** Minimum harbour area is equal to the space required for the dock plus the turning basin in front of them.

(ii) Harbour depth

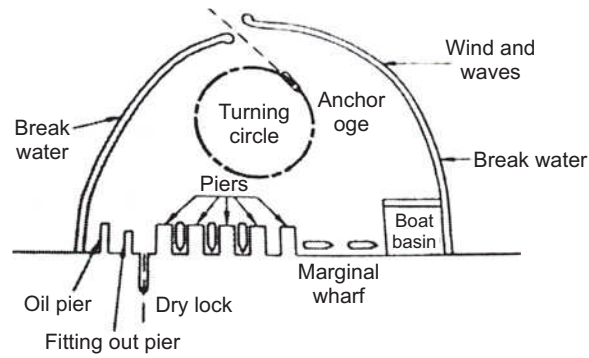
$$D = D' + D'' + \frac{H}{3}$$

D = Channel depth

D' = Draft of the largest ship

D'' = Allowance for squat of the moving ship

H = Height of the storm wave, crest to trough.

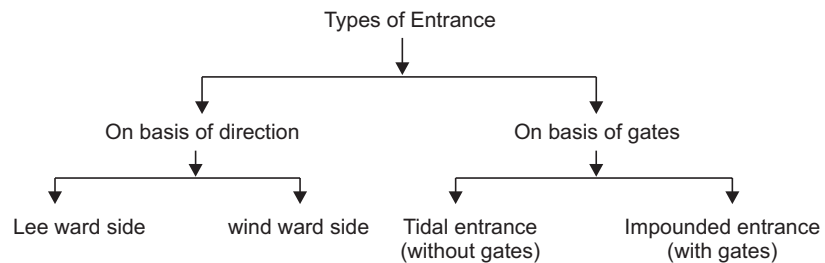


max . harbour depth = Loaded draft + 1.2 m (when bottom is soft)

max . harbour depth = Loaded draft + 1.8 m (when bottom is rock)

Note: Loaded draft = Submerged depth of ship.

- (iii) Turning basin:** It's minimum radius is twice the length of ship.
- (iv) Anchorage Area:** Area of the circle whose radius is = $(4 \times \text{depth of water}) + (\text{length of ship}) + (\text{safe clearance to adjacent ship})$
- (v) Harbour entrance channel:** It connects the harbour to the deep sea. The radius of the curve should be larger than five times the length of largest expected ship.



Sounding

Measurement of depth below the water surface. Instruments used are

- (i) **Fathometer (Echo-Sounding):** Used where depth of water is too much. Main parts of an echo sounding apparatus are
 - (a) Transmitting and receiving oscillators
 - (b) Recorder unit.
 - (c) Transmitter/power unit.
- (ii) **Sextant:** To find the distance of an observer in the sea. Also used to take simultaneous observations of two moving objects.
- (iii) **Station Pointer:** It's a three-armed protractor and consists of graduated circle with a fixed arm and two movable arms on either side.
- (iv) **Hi-fix:** A high precision light weight electronic position fixing system.

Tides And Shore Protection Works

7

Tide: A natural phenomenon by which level of ocean water undergoes periodic rise and fall.

Note: The influence of moon is greater than sun.

Mean sea level is the based on analysis of tidal sea level fluctuations over 19 years.

Lunar day: Time taken by moon to complete one revolution around earth.

Note: Lunar day is 50 minutes longer than solar day.

Spring tides: Highest tides when the generating forces due to moon and sun are additives.

Neap tides: When lines connecting the earth with sun and earth with moon form a right angle

Tidal range: Level difference between high level to low level of water for a particular tide.

$$\text{Tidal force due to sun} = 0.45 \times \text{Tidal force due to moon}$$

Wind: It's speed is expressed in **knots** (1043 kmph). It's the length of one minute of area of a meridian which varies with different latitudes. It causes waves in open sea.

Wave: It is generated by transfer of energy from air which is moving over the water surface.

(i) $\text{Height of wave (m)} = 0.34 \sqrt{F}$ By Stevenson

F = Fetch (surface giving rise to waves) in km

(ii) $\text{Length of wave (m)} = \frac{t^2}{2\pi} g = 1.56 t^2$ By Bertin's formulae

t = time period (in see)

Note: Distance between adjacent crest of a wave gives its length.

- (iii) Significant wave height (H_s) = Average height of 1/3 highest waves.
- (iv) Average height (H_A) = $0.6 H_s$
- (v) Highest wave $H_{\max} = 1.87 H_s$

Note: Clapotis: It is a phenomenon by which waves strikes a vertical break water in deep water, it is reflected back and on meeting another advancing wave of similar amplitude will merge and rise vertical in a wall of water.

Shore protection works:

- (i) **OFF shore break water:** Rubble mound type structure, effective for intercepting littoral movement.
- (ii) **Seawall, bulkheads and revetments:** Structures **parallel** to shoreline which develop a demarcating line between land and sea. Revetment is protective pavement supported by an earthwork.
- (iii) **Sand dunes:** Formation along coast prevent movement of tides into the area.
- (iv) **Protective or spending beaches:** Grant protection to adjacent uplands.
- (v) **Groynes:** Structures or projections from the bank for trapping of littoral drift

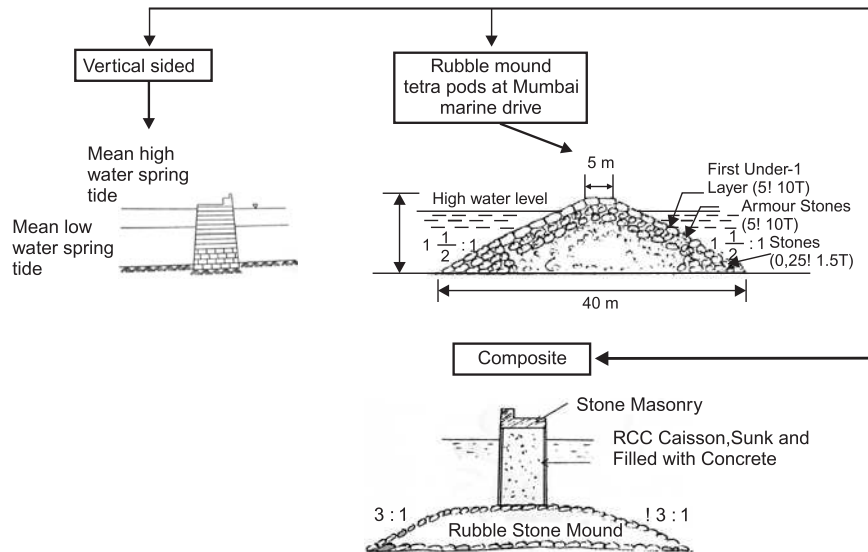
Note: Littoral drift is the drifting of sand in a zigzag line.

Breakwaters

8

It is a structure constructed with the purpose of protecting an area from the effect of sea waves so that ships can safely be moored in that area.

Break water height is kept equivalent to 1.2 to 1.25 times the height of the wave expected above the high water level.



Fenders: It is the form of cushion provided on jetty face to **absorb the impact of the ship** and protect them from damage. It also prevents the ship paint from being damaged because of relative motion between dock and ship.

Wharf: These are platforms at which vessels take on and discharge passenger and cargo. They are the docks that are **parallel** to the shore. Wharfs which are built parallel with the shore are called quays.

Open type wharf's

- Provide little resistance to water movement
- Decks are supported by piles or cylinder

Solid type wharf's

- Provide adequate resistance to the impact of mooring vessels
- Decks are supported by sheet piles cells, bulkhead, cribs, caisson and quay walls

Note: A break water which substitutes a quay wall and which can be used as a platform for loading and unloading cargo in calm water.

Dolphins: Used for tying up ships and for transferring cargo between ships moored along both sides of dolphin

Mooring dolphins

- Not designed for impact of ship

Breasting dolphins

- Designed to take impact of ship

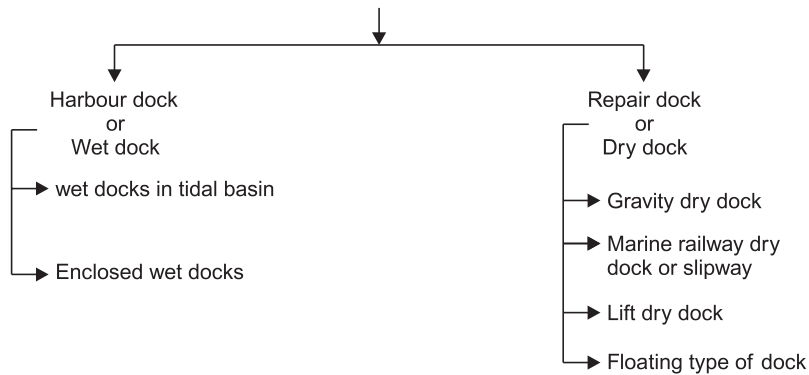
Catwalks: Provide access to and between dolphins.

Jetties: Built roughly perpendicular to the shore for maintaining an entrance channel.

Dock

9

Dock is artificial enclosure for the reception of ships.

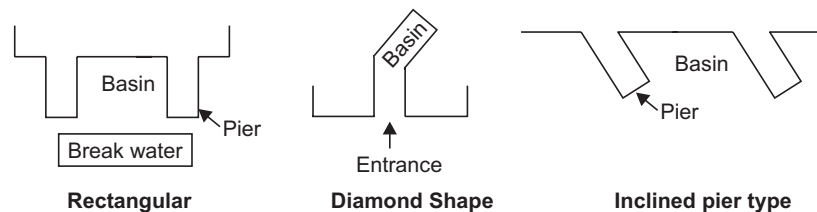


Harbour/Wet dock

For berthing of vessels to facilitate loading and unloading of passengers.

- (i) Wet docks in tidal basin, are ports on the open sea coast protected by an outlying breakwater. Lock gates for entrance are not provided hence lot of fluctuations in water level.
- (ii) In Enclosed wet docks water level is maintained by providing locks and lock gates so that cargo handing becomes convenient. It is costly.

Shapes of dock and basin



Repair/Dry dock

There are used for repair of vessels.

- (i) **Gravity dry dock:** Operates by admitting a vessel into the chamber, close the gate and then pumping out the water.
- (ii) **Marine railway dry dock or slip:** It consists of inclined path of timber or stone upon which a series of rails are fixed and they run up from a sufficient depth of water to a required height above the high water level.
- (iii) **Lift dry dock:** Platforms are lowered into and raised from water by hydraulic power applied through series of cylinders.
- (iv) **Floating type of dry dock:** Its a floating vessel which can lift a ship out of water and retain it above water by means of its own buoyancy.

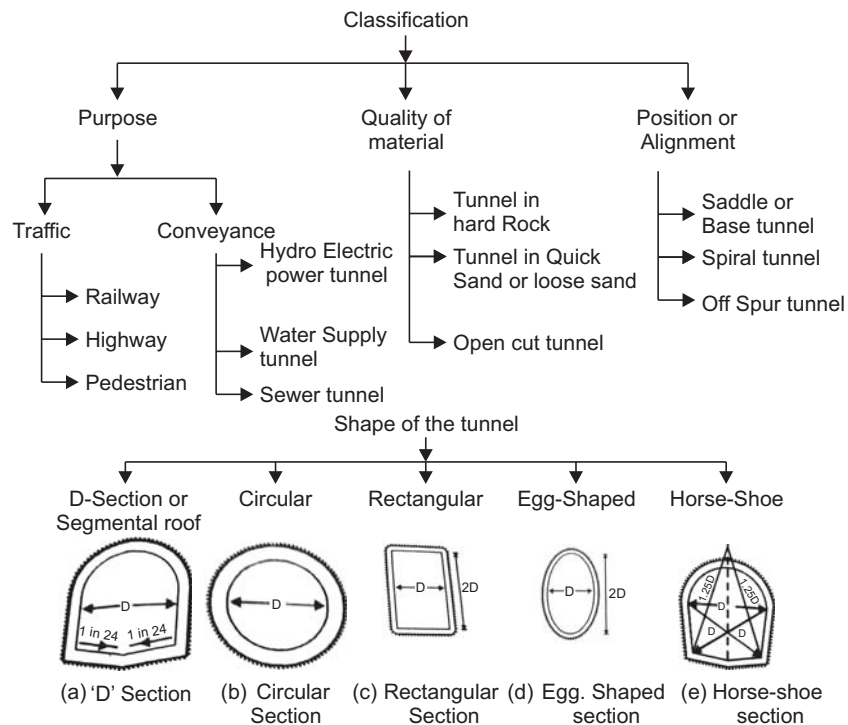
Tunnel Engineering

10

It is an engineering structure, artificial gallery, passage or roadway beneath the ground, under the bed of a stream or through a hill or mountain.

Advantages of tunnel

- (i) For carrying public utilities like water, gas, railway lines or roads etc.
- (ii) Aerial warfare and bombing of cities have given intangible values to tunnel.
- (iii) Lesser maintenance and operating cost.
- (iv) Cost of hauling is decreased due to lighter grades, possible in tunnels.



- (i) **In Rock tunnels**, the risk of failure or collapse caused by external pressure from water or loose or unstable soil conditions on tunnel lining is practically non-existent and it is then convenient to have a section with an arched roof and straight sides which is called segmental or D-section.
- (ii) **Circular section** is best suited for materials without cohesion (**Quick sands**) where the pressure acts normal to the line of profile.
- (iii) Rectangular section is suitable only in case of hard rocks.
- (iv) **Egg shaped** section is used in **sewers** as it maintains required self cleansing velocity.
- (v) Horse shoe shape is best suited for traffic purposes.

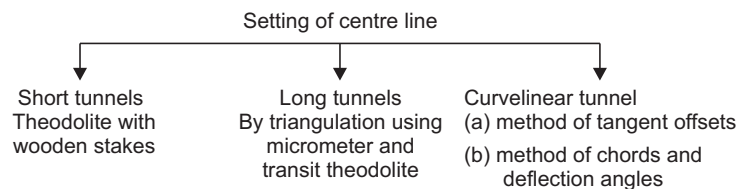
Investigations in tunnel construction

Information collected are

- (i) Origin of soil mass.
- (ii) Hydrology in surrounding proposed tunnel site.
- (iii) Presence of foul gases.
- (iv) Temperature of soil near by.
- (v) Location of faults, folds etc.
- (vi) Bearing capacity of soil mass at proposed site.

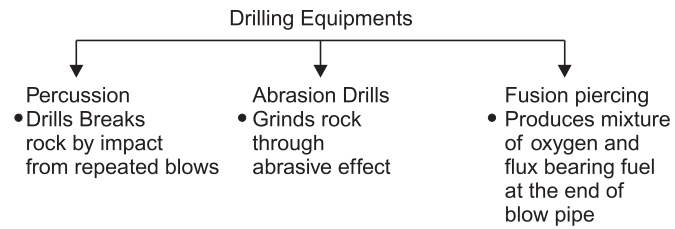
Operations Involved in Survey

- (i) Locating centre line on ground.
- (ii) Providing centre line to inside tunnel.
- (iii) Providing required grade at the bottom of tunnel.
- (iv) Checking tunnel cross-section.



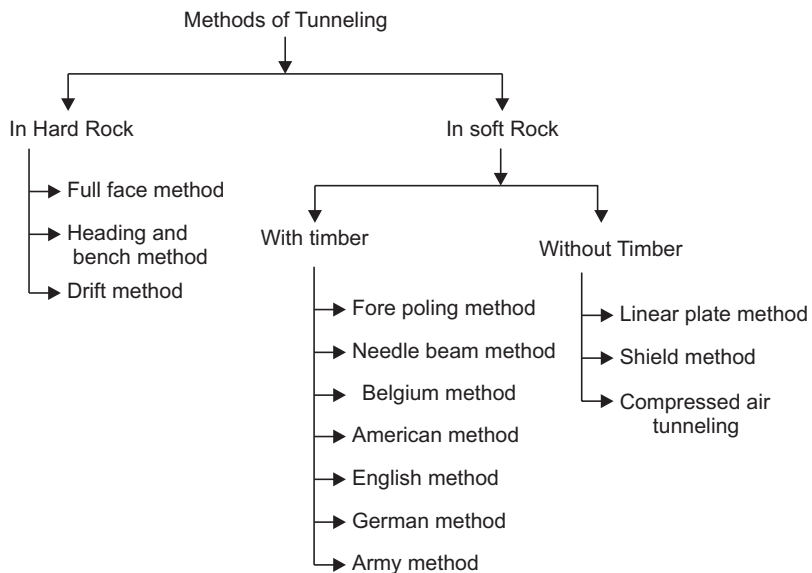
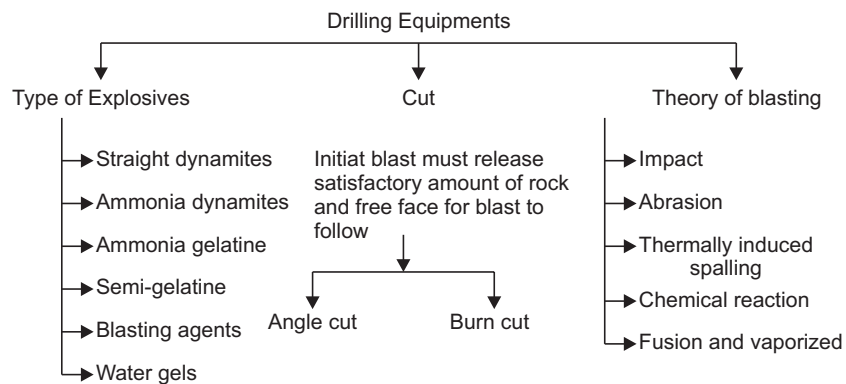
Shafts

There are vertical wells sunk along the centre line of tunnel to permit excavation, removal of excavated material and to provide adequate ventilation during construction.



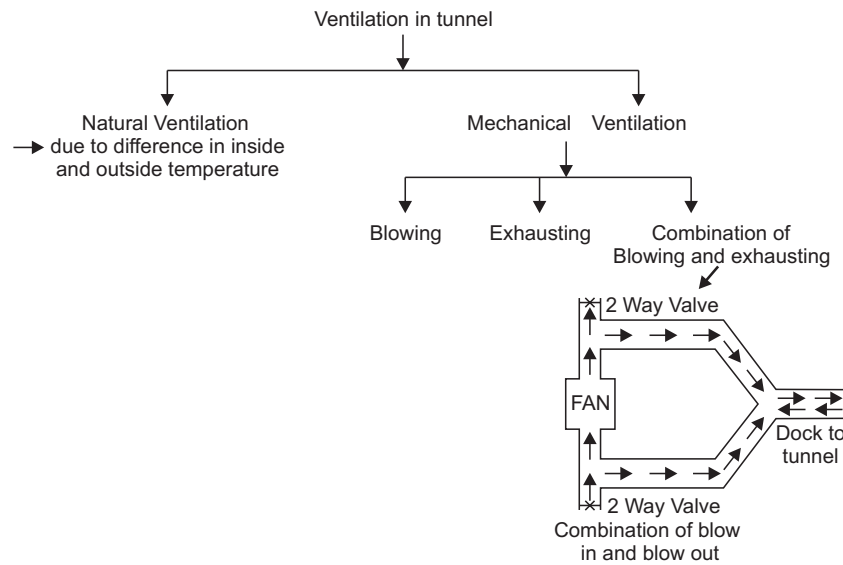
Blasting

Operation performed to loosen rock so that it may be excavated or removed from its existing position.



Hauling of Muck

It's the operation of loading and removing excavated or hosted materials and then dumping it at predetermined sites. It is done to make enough working room and final cleaning of bottom to place inverts. It is either done by hands or machines.



Note: A light intensity of **260 lumens per square meter** in the working area is considered satisfactory in tunnelling operations.