A Handbook on

Civil Engineering

Revised & Updated

Contains well illustrated formulae & key theory concepts

For

ESE, GATE, PSUs

& OTHER COMPETITIVE EXAMS

MADE EASY
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A Handbook on Civil Engineering

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Director's Message

B. Singh (Ex. IES)

During the current age of international competition in Science and Technology, the Indian participation through skilled technical professionals have been challenging to the world. Constant efforts and desire to achieve top positions are still required.

I feel every candidate has ability to succeed but competitive environment and quality guidance is required to achieve high level goals. At MADE EASY, we help you to discover your hidden talent and success quotient to achieve your ultimate goals. In my opinion CSE, ESE, GATE & PSU's exams are tool to enter in to main stream of Nation serving. The real application of knowledge and talent starts, after you enter in to the working system. Here in MADE EASY you are also trained to become winner in your life and achieve job satisfaction.

MADE EASY alumni have shared their winning stories of success and expressed their gratitude towards quality guidance of MADE EASY. Our students have not only secured All India First Ranks in ESE, GATE and PSU entrance examinations but also secured top positions in their career profiles. Now, I invite you to become alumni of MADE EASY to explore and achieve ultimate goal of your life. I promise to provide you quality guidance with competitive environment which is far advanced and ahead than the reach of other institutions. You will get the guidance, support and inspiration that you need to reach the peak of your career.

I have true desire to serve Society and Nation by way of making easy path of the education for the people of India.

After a long experience of teaching in Civil Engineering over the period of time MADE EASY team realised that there is a need of good *Handbook* which can provide the crux of Civil Engineering in a concise form to the student to brush up the formulae and important concepts required for ESE, GATE, PSUs and other competitive examinations. This *handbook* contains all the formulae and important theoretical aspects of Civil Engineering. It provides much needed revision aid and study guidance before examinations.

B. Singh (Ex. IES)
CMD, MADE EASY Group

A Handbook on

Civil Engineering

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Civil Engineering

1

Strength of Materials

CONTENTS

Properties of Metals, Stress and Strain

1

Important Mechanical Properties

- **Elasticity**
  It is the property by virtue of which a material deformed under the load is *enabled* to return to its original dimension when the load is removed.

> Remember
> If body regains *completely* its original shape then it is called *perfectly* elastic body
> *Elastic limit* marks the *partial* break down of elasticity beyond which removal of load result in a degree of *permanent deformation*.
> Steel, Aluminium, Copper, may be considered to be perfectly elastic *within certain limit*.

- **Plasticity**
  The characteristics of the material by which it undergoes *inelastic strain* beyond those at the *elastic limit* is known as plasticity.

> Remember
> This property is particularly useful in operation of *pressing* and *forging*.
> When large deformation occurs in a *ductile* material loaded in *plastic* region, the material is said to undergo *plastic flow*.

- **Ductility**
  It is the property which permits a material to be drawn out *longitudinally* to a reduced section, under the action of *tensile force*.

> Remember
> A ductile material must posses a high degree of plasticity and strength.
> Ductile material must have *low* degree of elasticity.
> This is useful in *wire drawing*.

- **Brittleness**
  It is lack of ductility. Brittleness implies that it can *not* be drawn out by tension to smaller section

> Remember
> In brittle material failure take place under load *without* significant deformation.

> Ordinary *Glass* is nearly *ideal* brittle material.
> Cast iron, *concrete* and ceramic material are brittle material.

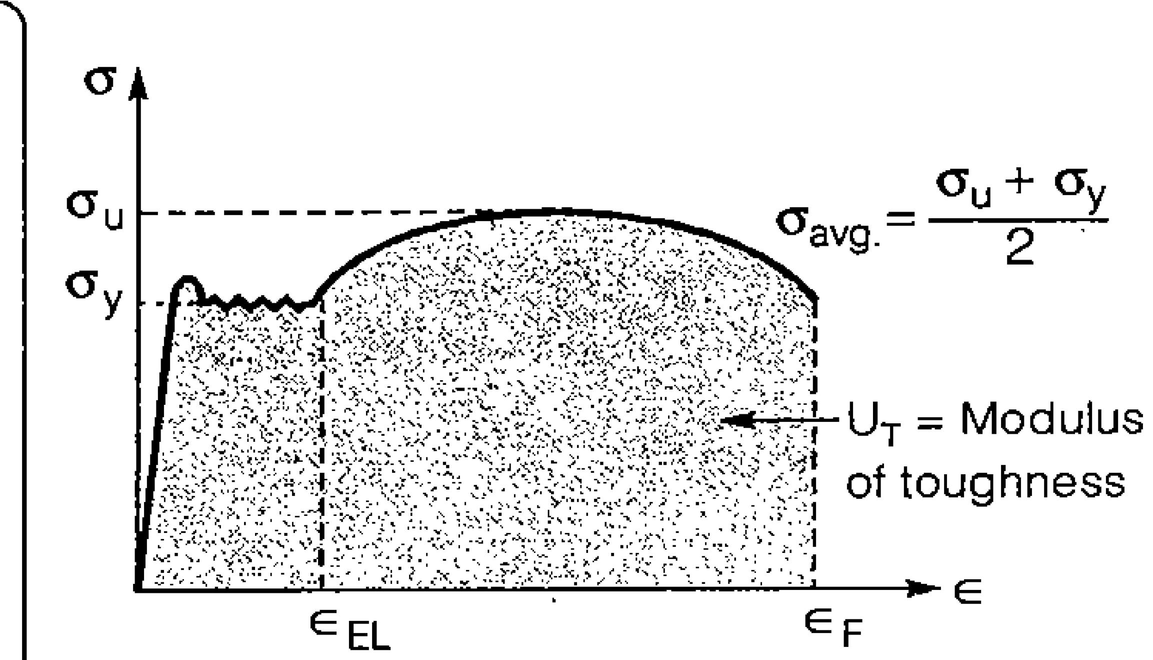
- **Malleability**
  It is the property of a material which permits the material to be *extended* in *all direction* without rupture.

> Remember
> A malleable material posses a *high degree* of plasticity, but *not* necessarily *great strength*.

- **Toughness**
  It is the property of material which enables it to absorb energy *without fracture*.

  Modulus of toughness U_T = shaded area = $\left(\dfrac{\sigma_u + \sigma_y}{2}\right)\epsilon_f$

> Remember
> It is desirable in material which is subjected to *cyclic* or *shock loading*.
> It is represented by area under *stress-strain* curve of material upto fracture.
> *Bend test* used for common comparative test of toughness.



- **Hardness**
  It is the ability of a material to resist *indentation* or *surface abrasion*.

> Remember
> Brinnell hardness test is used to check hardness.
>
> $$\text{Brinnell hardness number} = \frac{P}{\dfrac{\pi D}{2}\left[D - \sqrt{D^2 - d^2}\right]}$$
>
> where, P = Standard load
> D = Diameter of steel ball (mm)
> d = Diameter of indent (mm)

- **Strength**
  This property enables material to resist fracture under load.

This is most important property from *design* point of view. Load required to cause fracture, divided by area of test specimen, is termed as ***ultimate strength***.

- **Creep**
  Creep is a permanent deformation which is recorded with passage of time at constant loading. It is plastic deformation (permanent and non-recoverable) in nature.
  Note: The temperature at which creep is uncontrolable is called *Homologous Temperature.*
- **Fatigue**
  Due to cyclic or reverse cyclic loading fracture failure may occur if total accumulated strain energy exceeds the toughness. Fatigue causes rough fracture surface even in ductile metals.
- **Resilience**
  It is the total elastic strain energy which can be stored in the given volume of metal and can be released after unloading.
  It is equal to area under load deflection curve within ***elastic limit.***

Stress and strain

Stress (N/mm²)

It is the resistance offered by the body to deformation

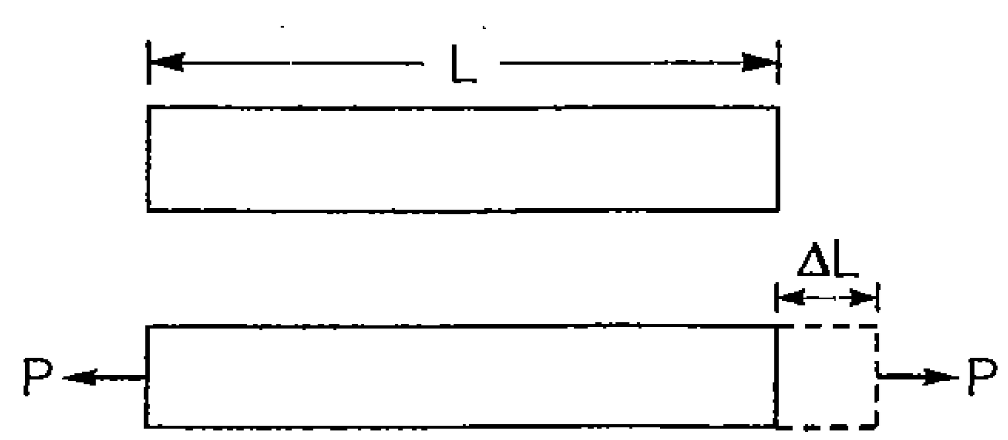
- $\text{Nominal stress (Engineering stress)} = \dfrac{\text{Load}}{\text{Original Area}}$

- $\text{Actual/True stress} = \dfrac{\text{Load}}{\text{Changed (Actual) Area}}$

Strain

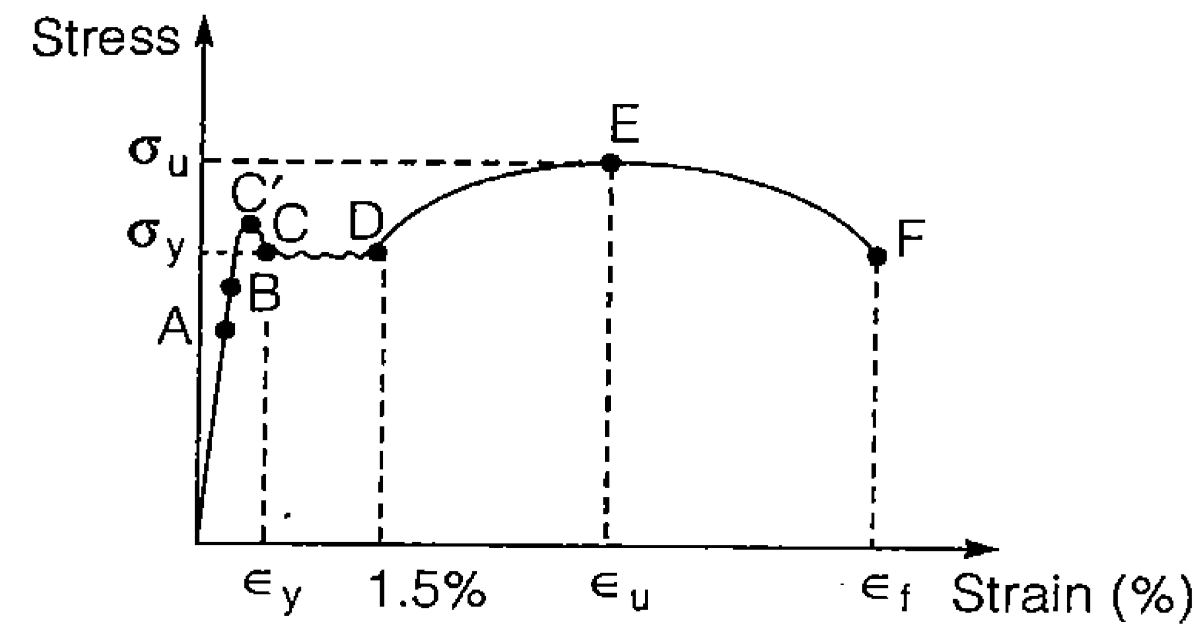
Deformation per unit length in the direction of deformation is known as strain.

$$\text{Strain} = \frac{\Delta L}{L}$$



It is a ***dimensionless*** quantity.

Engineering Stress-Strain curve of mild steel for tension under static-loading



OA - Straight line (proportional region, *Hooke's law is valid*)
OB - Elastic region
BC - Elasto plastic region
CD - Perfectly plastic region
DE - Strain hardening
EF - Necking region
A - Limit of proportionality
B - Elastic limit
C - Lower yield point
F - Fracture point
C′ - Upper yield point
D - Strain hardening starts
E - Ultimate point or maximum stress point

- **Limit of Proportionality**
  It is the stress at which the stress-strain curve ***ceases*** to be a straight line.

Hooke's law is valid upto proportional limit.

- **Elastic Limit**
  It is the point on the stress-strain curve upto which the materials remains elastic.

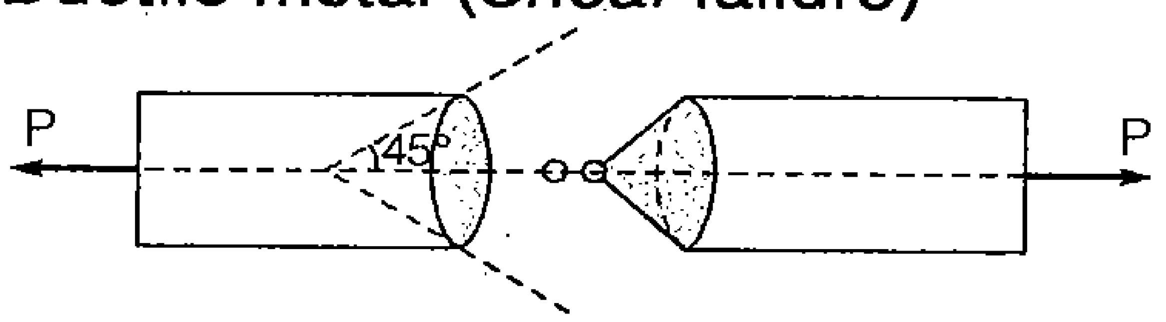
Upto this point there is ***no permanent*** deformation after removal of load.

- **Plastic Range**
  It is the region of the stress-strain curve between the elastic limit and point of rupture.
- **Yield Point**
  This point is just beyond the elastic limit, at which the specimen undergoes an appreciable increase in length ***without*** further increase in the load.
- **Rupture Strength**
  It is the stress corresponding to the failure point 'F' of the stress-strain curve.
- **Proof Stress**
  It is the stress necessary to cause a ***permanent extension*** equal to defined percentage of gauge length.

Slope of OA = Modulus of elasticity (*Young's Modulus*).
It is constant of proportionality which is defined as the intensity of stress that causes unit strain.
Plastic strain is 10 to 15 times elastic strain.
Fracture strain (ϵ_f) depends on ***percentage carbon*** in steel.
When carbon percentage increases then fracture strain decreases and yield stress increases.

Type of Tension failure in Metal

A. Ductile metal (*Shear* failure)

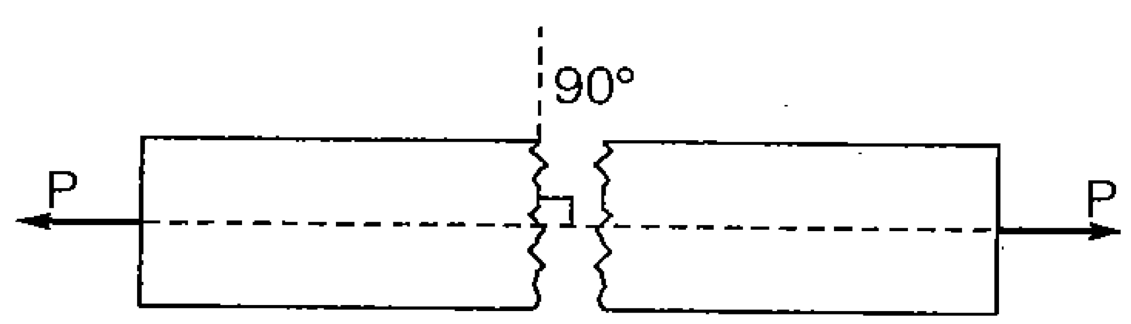


Failure plane is at ***45°***

Cup-cone fracture

Shear strength < Tensile strength ≤ Compressive strength

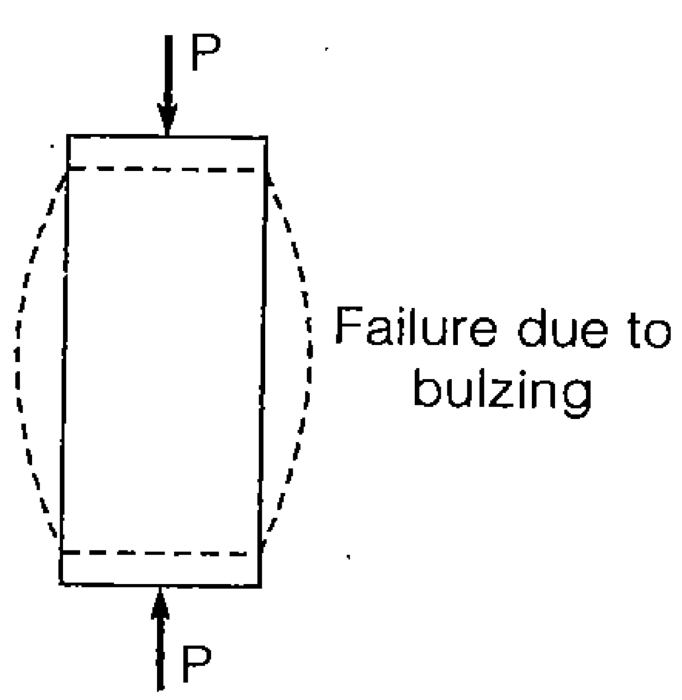
B. Brittle metal



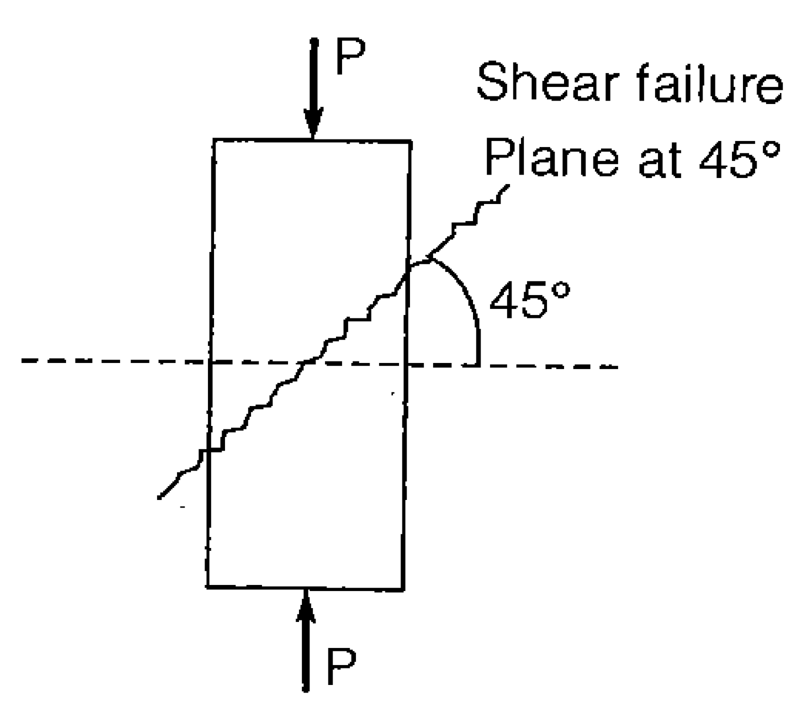
Failure plane at ***90°*** with longitudinal direction
Necking is not formed and failure is due to tension failure.
Tensile strength < Shear strength < Compressive strength

Type of failure in compression

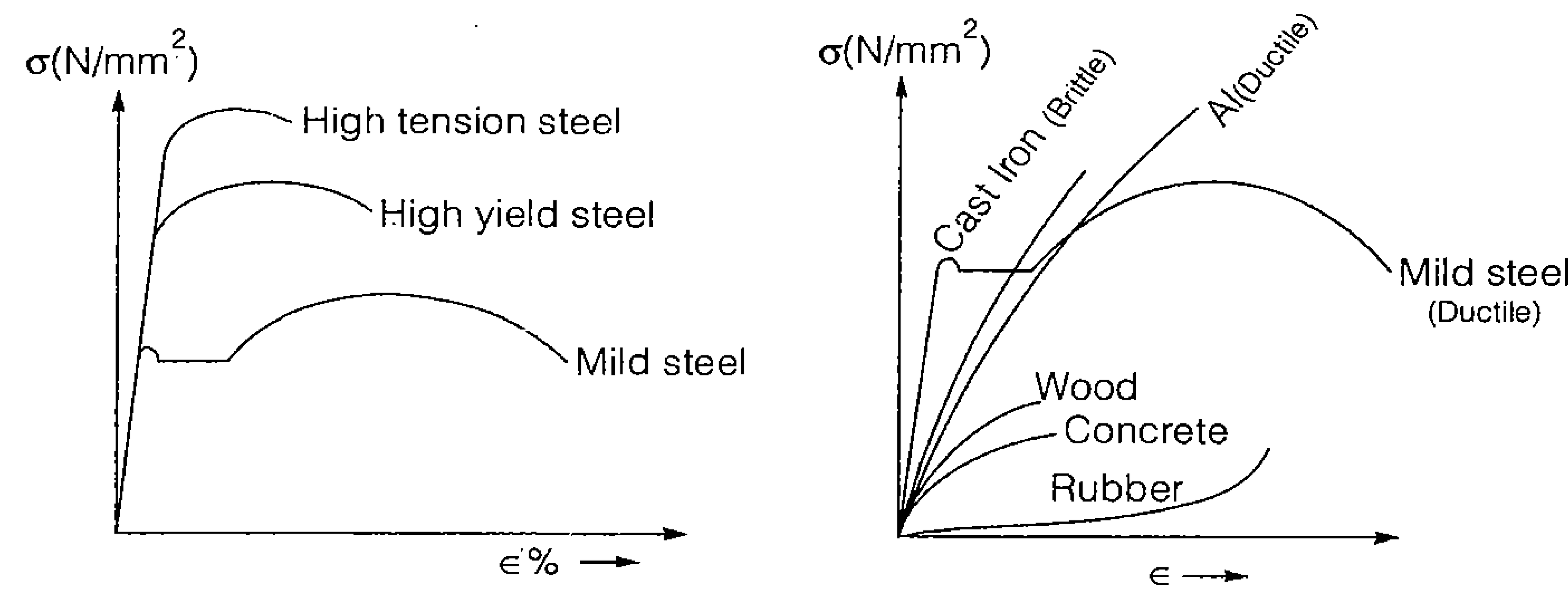
A. Ductile material



B. Brittle material



Stress-Strain Diagram for Various type of Steel/Material



All grades of steel have same young's modulus but different yield stress.

Ductile material

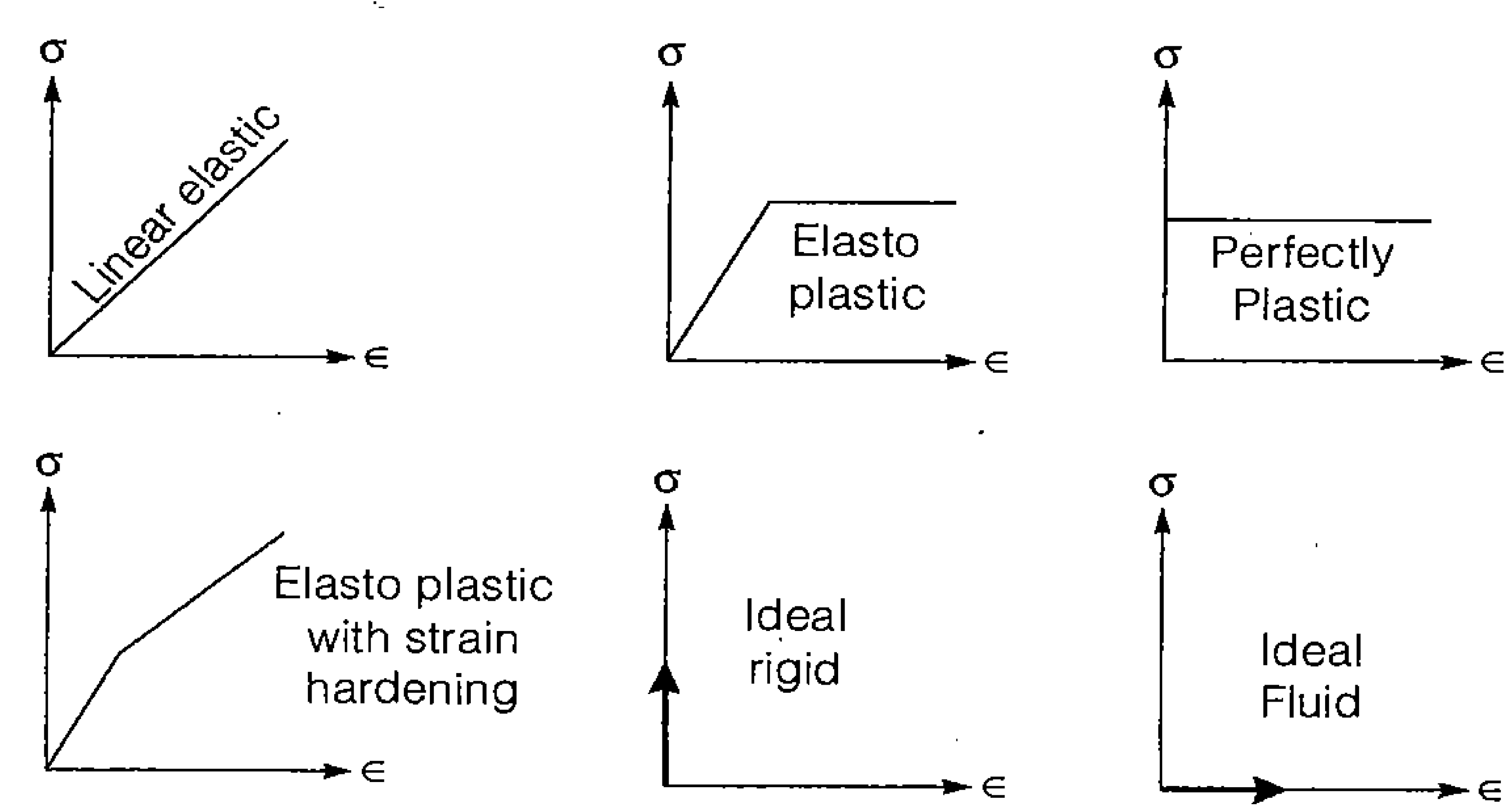
If post elastic strain is greater than 5%, it is called ductile material.
It undergoes large permanent strains before failure,
Large reduction in area before fracture
e.g. ***lead***, mild steel, copper

Brittle Material

If post elastic strain is less than 5%. It is called brittle material.

It fails with only little elongation after the proportional limit is exceeded.
Very less reduction in area before fracture, e.g. Bronze, ***Rubber***, Glass

Behaviour of Various Material



Where σ = Stress, ∈ = Strain

'Mild steel' is ***more*** elastic than ***'Rubber'***.

Hooke's Law

When a material behaves elastically and exhibits a linear relationship between stress and strain, it is called linearly elastic. For such materials stress (σ) is directly proportional to strain (ϵ).

$\boxed{\sigma \propto \epsilon} \rightarrow \boxed{\sigma = E.\epsilon}$

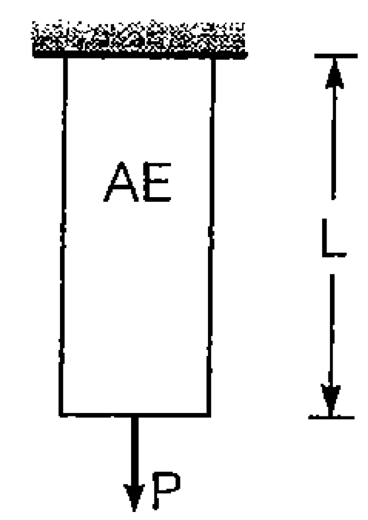
where, σ = Stress
ϵ = Strain
E = Young modulus of elasticity

- $E_{cast\ iron} \approx \frac{1}{2} E_{steel}$.
- $E_{Aluminium} \approx \frac{1}{3} E_{steel}$

Axial elongation (Δ) of prismatic bar due to external load

$$\Delta = \frac{PL}{AE}$$

Here, P = Load applied
L = Length of bar
A = Area of bar
E = Young modulus



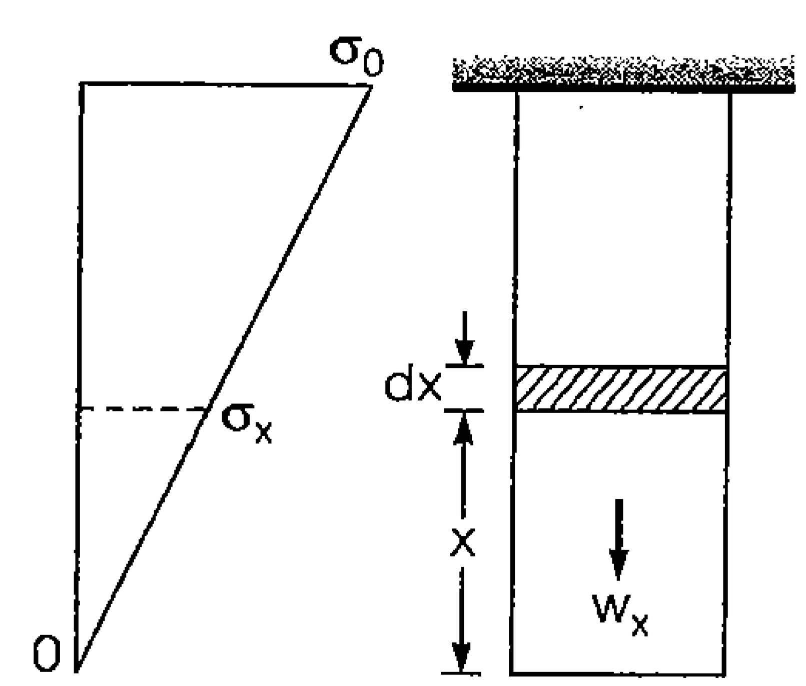
$$\Delta = \frac{P}{\frac{EA}{L}} = \frac{P}{K}$$

K = AE/L = Axial stiffness of bar
AE = Axial rigidity
EI/L = Flexural stiffness
EI = Flexural rigidity

Deflection of bar (Δ) due to self-weight

A. Prismatic bar

$$\Delta = \frac{WL}{2AE} = \frac{\gamma L^2}{2E}$$



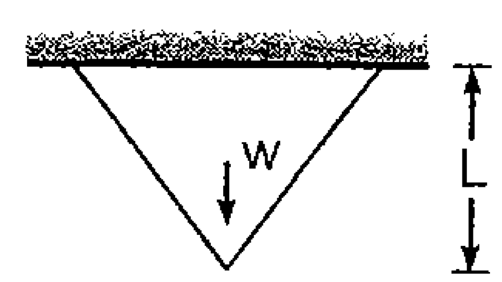
Stress diagram

Here, W = Total Self weight

B. Conical bar

$$\Delta = \frac{\gamma L^2}{6E} = \frac{1}{3} \times \text{Deflection of prismatic bar of same length}$$

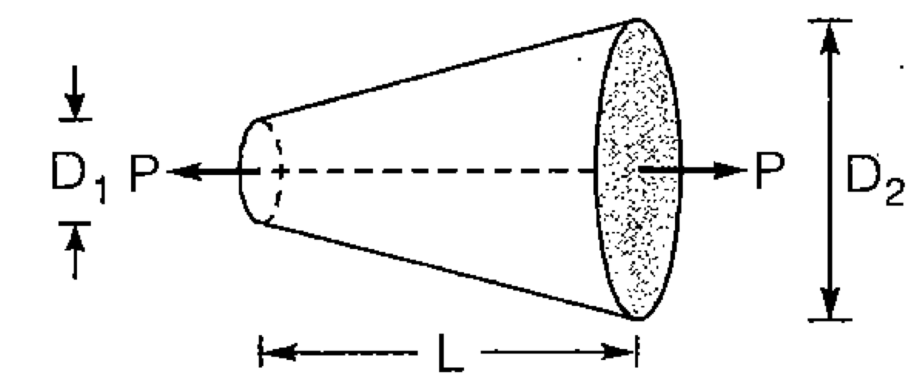
Here, γ = Specific weight
L = Length of bar
E = Young's modulus



Deflection (Δ) of Tapered Bar

A. Circular tapering bar

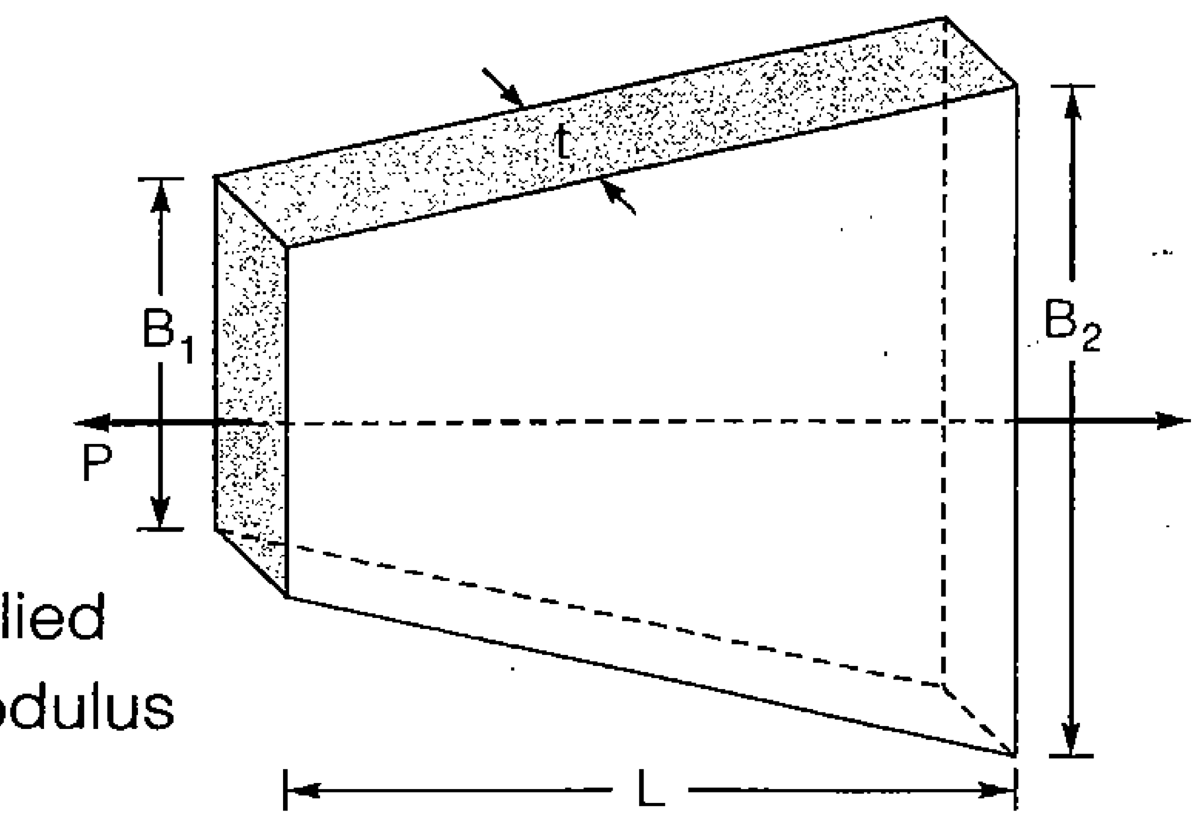
$$\Delta = \frac{4PL}{\pi E D_1 D_2}$$



where, P = Load applied
L = Length of bar
D_1 and D_2 are Diameter as shown in fig.

B. Rectangular tapering bar

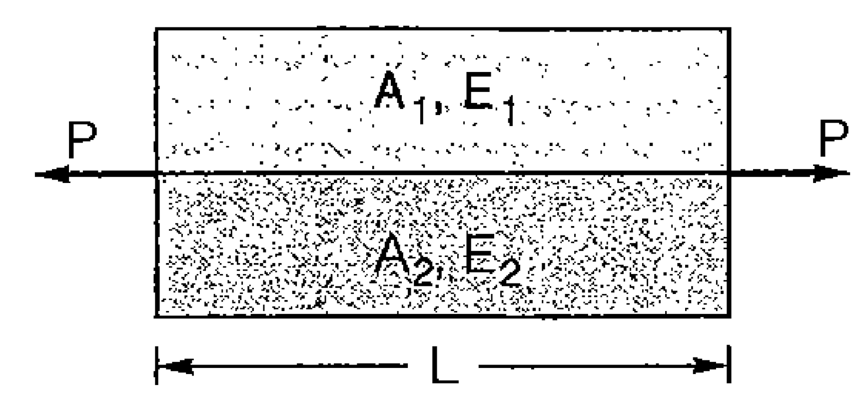
$$\Delta = \frac{PL \log_e\left(\frac{B_2}{B_1}\right)}{E \cdot t(B_2 - B_1)}$$



where, t = thickness
P = Load applied
E = Young modulus

Equivalent Young's Modulus of Parallel Composite Bar

$$E_{equivalent} = \frac{A_1 E_1 + A_2 E_2}{A_1 + A_2}$$



where, A_1 = Area of first bar
A_2 = Area of second bar
E_1 = Young's modulus of first bar
E_2 = Young's modulus of second bar

• Elastic Constants

Elastic constants are those factor which determine the deformation produced by a given stress system acting on material.

- Modulus of elasticity (E) = $\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

  Modulus of rigidity (G) = $\frac{\text{Shear stress}}{\text{Shear strain}}$

  Bulk modulus (K) = $\frac{\text{Direct stress}}{\text{Volumetric strain}}$

Poisson's Ratio (μ)

$$\mu = \frac{-(\text{Lateral strain})}{\text{Longitudinal Strain}} \qquad \mu = \frac{|\text{Lateral strain}|}{\text{Longitudinal strain}}$$

Under uniaxial loading

$0 \le \mu \le 0.5$

$\mu = 0$ for cork

$\mu = 0.5$ For perfectly plastic body ***(Rubber)***

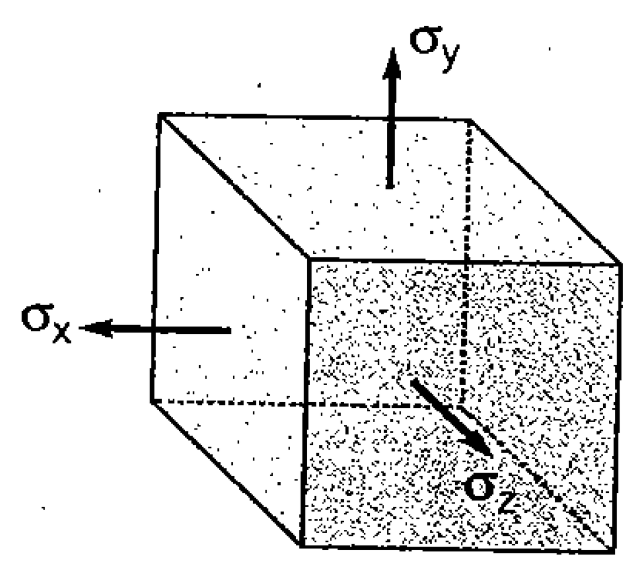
$\mu = 0.25$ to 0.42 for elastic metals

$\mu = 0.1$ to 0.2 for concrete

$\mu = 0.286$ mild steel

μ is greater for ductile metals than for brittle metals.

Volumetric Strain under Tri-Axial Loading



where, σ_x = Stress in x-direction

σ_y = Stress in y-direction

σ_z = Stress in z-direction

ϵ_v = Volumetric strain

$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z$$

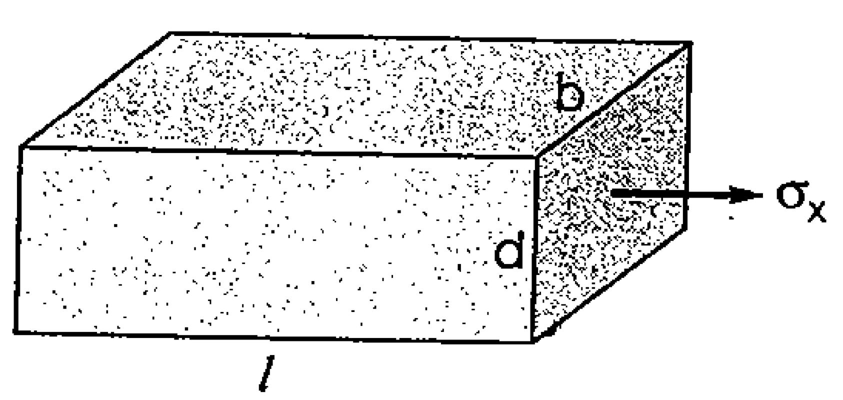
$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

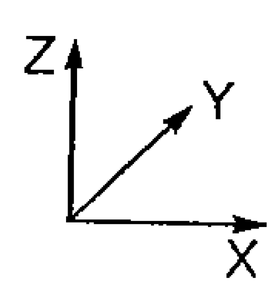
Under hydrostatic loading

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\therefore \quad \epsilon_V = \frac{3\sigma}{E}(1-2\mu)$$

Uni-axial Loading on Rectangular Parallelopipe



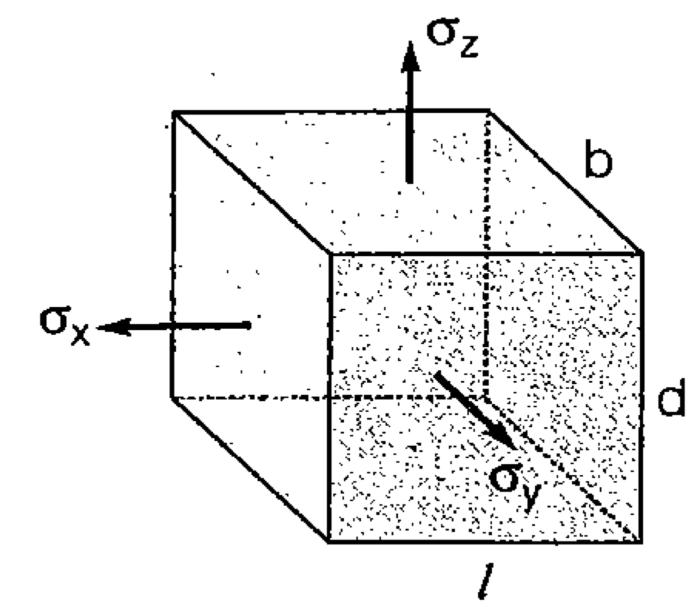


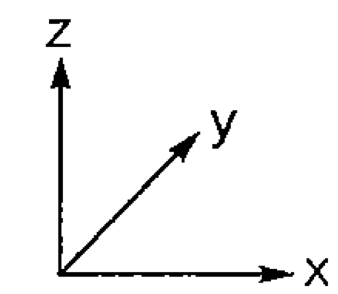
$$\epsilon_x = \frac{\Delta l}{l} = \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{\Delta b}{b} = -\frac{\mu\sigma_x}{E} \qquad \epsilon_z = \frac{\Delta d}{d} = -\frac{\mu\sigma_x}{E}$$

Here, ϵ_x, ϵ_y and ϵ_z are strain in x, y and z directions respectively.
Δl, Δb and Δd are change in length, width and depth respectively.
l, b and d are original length, width and depth respectively.

Triaxial loading on Rectangular Parallelopipe





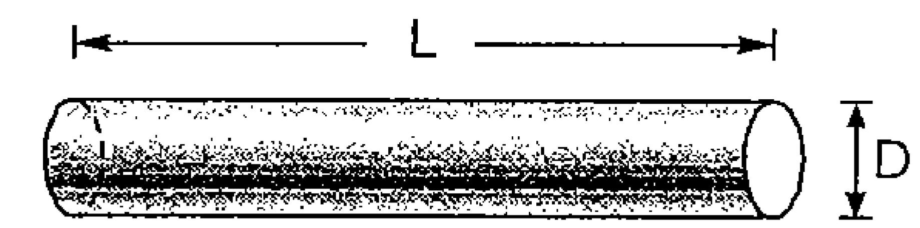
$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} = \frac{\delta l}{l}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} = \frac{\delta b}{b}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} = \frac{\delta d}{d}$$

Sign convention: Tensile is positive, and Compressive is negative.

Volumetric Strain of Cylindrical Bar



$$\epsilon_v = \text{Longitudinal Strain} + (2 \times \text{Diametric strain})$$

Volumetric Strain of Sphere

$$\epsilon_V = 3 \times \text{Diametric strain}$$

Matrix Representation of Stress and Strain

3-D stress matrix

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

3-D strain matrix

$$\begin{bmatrix} \epsilon_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{xz}}{2} \\ \frac{\phi_{yx}}{2} & \epsilon_{yy} & \frac{\phi_{yz}}{2} \\ \frac{\phi_{zx}}{2} & \frac{\phi_{zy}}{2} & \epsilon_{zz} \end{bmatrix}$$

Relation between E, G, K, μ

- $E = 3K(1-2\mu)$
- $E = 2G(1+\mu)$
- $E = \frac{9KG}{3K+G}$
- $\mu = \frac{3K-2G}{6K+2G}$

Here, E = Young's modulus, G = shear modulus
K = Bulk modulus, μ = Poisson ratio

Material	Number of Independent elastic constant
Homogeneous & Isotropic	2
Orthotropic (Wood)	9
Anisotropic	21

Strain Energy

It is the ability of material to absorb energy when it is strained

$$U = \frac{1}{2} P \times \delta = \frac{1}{2} T \times \theta$$

Here, P = Applied load
δ = Elongation due to applied load
T = Applied torque
θ = Angle of twist due to applied torque

- **Resilience:** Ability of a material to absorb energy in the ***elastic region*** when it is strained.

$$= \text{Area under P-}\delta\text{ curve} = \frac{1}{2} P \times \delta$$

- **Proof Resilience:** ***Maximum*** energy absorbing capacity of a material in the ***elastic region*** is called proof resilience.

$$= \text{Area under P-}\delta\text{ curve} = \frac{1}{2} P_{EL} \times \delta_{EL}$$

Here P_{EL} = Load at elastic limit
δ_{EL} = Elongation upto elastic limit

$$\text{Modulus of Resilience} = \frac{\text{Proof Resilience}}{\text{Volume}} = \frac{\sigma_{EL}^2}{2E}$$

Here σ_{EL} = Strain at elastic limit
E = Modulus of elasticity

Thermal Stress and Strain

$$\sigma_{Th\text{-}stress} = E \alpha T$$

$$\Delta = L \alpha T$$

$$\text{Strain} = \frac{L \alpha T}{L} = \alpha T$$

$$\alpha_{steel} = \alpha_{concrete} = 12 \times 10^{-6} /^\circ C$$

$$\alpha_{Aluminium} > \alpha_{Brass} > \alpha_{Copper} > \alpha_{Steel}$$

where, σ = Thermal stress
α = Coefficient of thermal expansion
T = Temperature change
Δ = Change in length

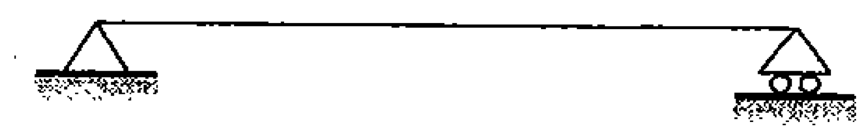
Remember: When bar is ***free*** to expand then there will be ***no thermal*** stress due to change in temperature.

■■■

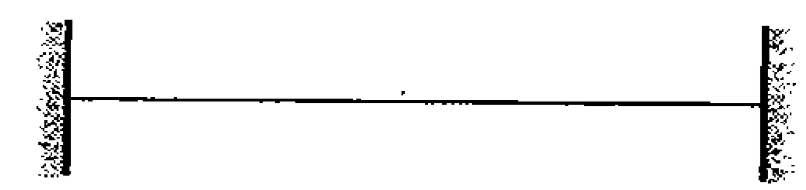
2 Shear Force and Bending Moment

Types of Beam

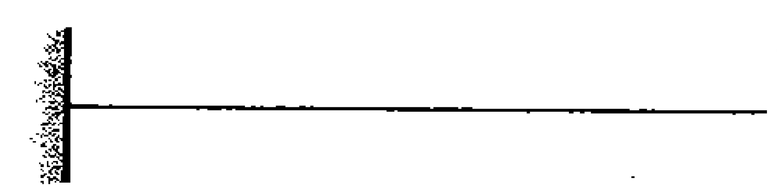
- **Simply Supported Beam**
  If the ends of a beam are made to rest freely on supports it is call simply (freely) supported beam.



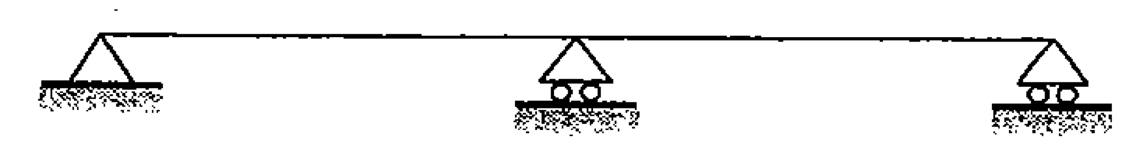
- **Fixed Beam**
  If a beam is fixed at both ends it is called fixed beam its another n is encastre or built-in beam.



- **Cantilever Beam**
  If a beam is fixed at one end while other end is free, it is called cantil beam.



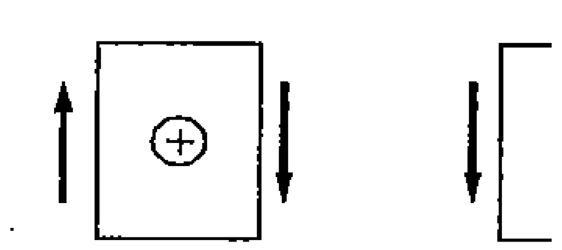
- **Continuous Beam**
  If more than two supports are provided to beam, it is called continu beam.



Shear Force

It is the internal resistance developed at any section to maintain ***body equilibrium of either*** left or right part of the section.

- **Sign Convention**
  Shear force having an upward direction to the left hand side of ***section*** or downward direction to the right hand side of section will be taken positive and vice-versa.



Remember: It may be horizontal or vertical.
Shear force at any section is ***algebraic*** sum of all transvers forces ***either*** from left or right of that section.

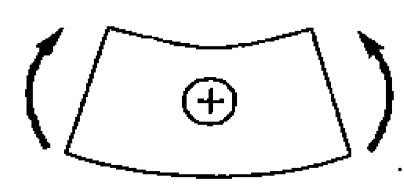
Bending Moment

Bending moment at any section is the internal reaction due to all the ***transverse*** force ***either*** from left side or from right side of that section.

> **Remember**
> It is equal to ***algebraic*** sum of moments at that section either from left or from right side of that section.
> Bending moment is different from twisting moment.

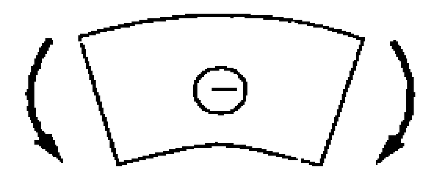
- Sign convention of Bending moment

A bending moment causing ***concavity upward*** will be taken as positive and called ***sagging*** bending moment.



Sagging

A bending moment causing ***convexity upward*** will be taken as ***negative*** and will be called a ***hogging*** bending moment.



Hogging

Relationship Between Bending Moment (M), Shear Force (S) and Loading Rate (w)

- Rate of change of shear force is equal to load

$$\boxed{\frac{dS}{dx} = w}$$

Here, w = Load per unit length

> **Remember** Negative slope represents downward loading.

- Rate of change of bending moment ***along the length*** of beam is equal to shear force.

$$\boxed{\frac{dM}{dx} = S_x}$$

> **Remember**
> At hinge, bending moment will be zero.
> Bending moment is maximum or minimum when shear force is zero or changes sign at a section.
> If degree of loading curve = n then
> degree of shear force curve = n + 1
> and degree of bending moment curve = n + 2
> Point of contra-flexture/inflexion is that point where bending moment ***changes its sign***.

■■■

Principal Stress/ Principal Strain

3

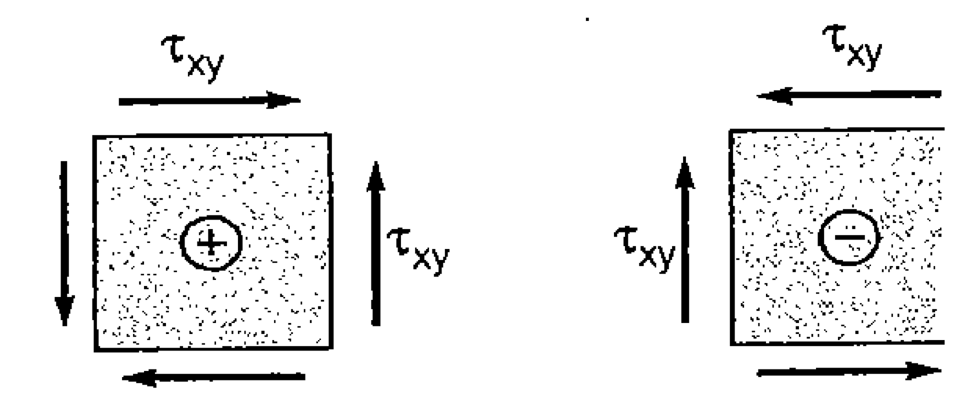
Principal Stress

Principal stress are maximum or minimum ***normal*** stress which may developed on a loaded body.

> **Remember** The plane of principal stress carry ***zero shear stress***.

Sign Conventions

- ***Tensile*** stress is considered ***positive*** and ***compressive*** stress is ***negative***.
- Angle 'θ' is considered ***positive*** if it is in ***anti-clockwise direction***.



Positive shear stress Negative shea

- ***Shear*** stress acting on a positive face of an element is consid positive if it is acts in positive direction of one of the coordi axes and negative if acts in the negative direction of the a Similarly on a negative face of an element is positive if it a negative direction of the axes and negative if it act in the pos direction.

> **Remember** Normally the reference plane taken are major principal plane vertical plane.

Analytical Method of Analysis

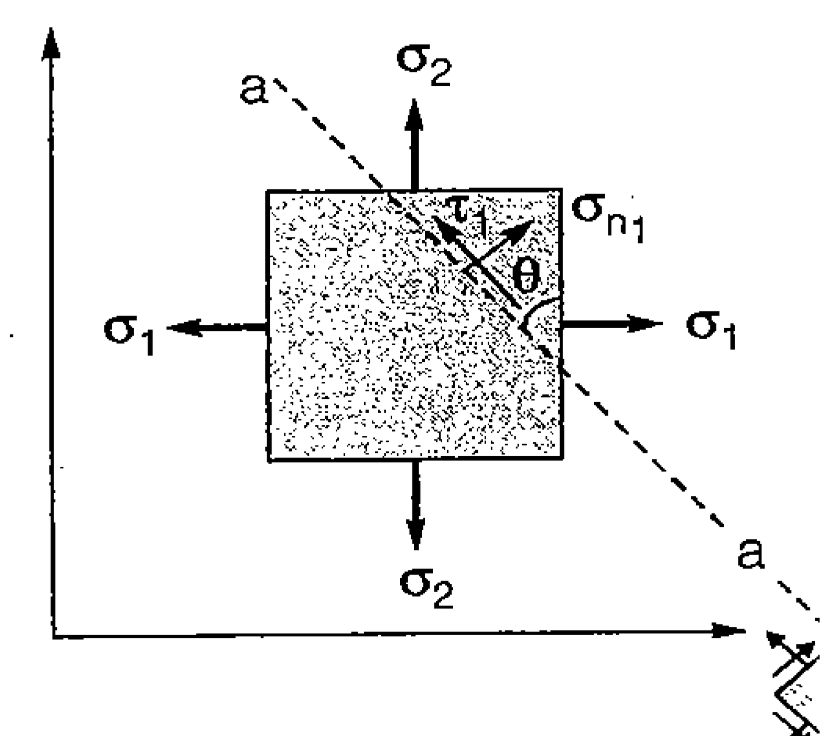
(i) If σ_1 and σ_2 are given ***principal*** stress as shown in figure, then no and shear stress on plane a-a which is inclined at angle 'θ' major principal plane ($\sigma_1 > \sigma_2$)

$$\therefore \quad \sigma_{n_1} = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$$

$$\sigma_{n_1} = \frac{\sigma_1 + \sigma_2}{2} + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cdot\cos 2\theta$$

$$\sigma_{n_2} = \frac{\sigma_1 + \sigma_2}{2} - \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cdot\cos 2\theta$$

$$\tau_1 = \tau_2 = -\left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta$$



Remember

$$\sigma_{n_1} + \sigma_{n_2} = \sigma_1 + \sigma_2 = \text{constant}$$

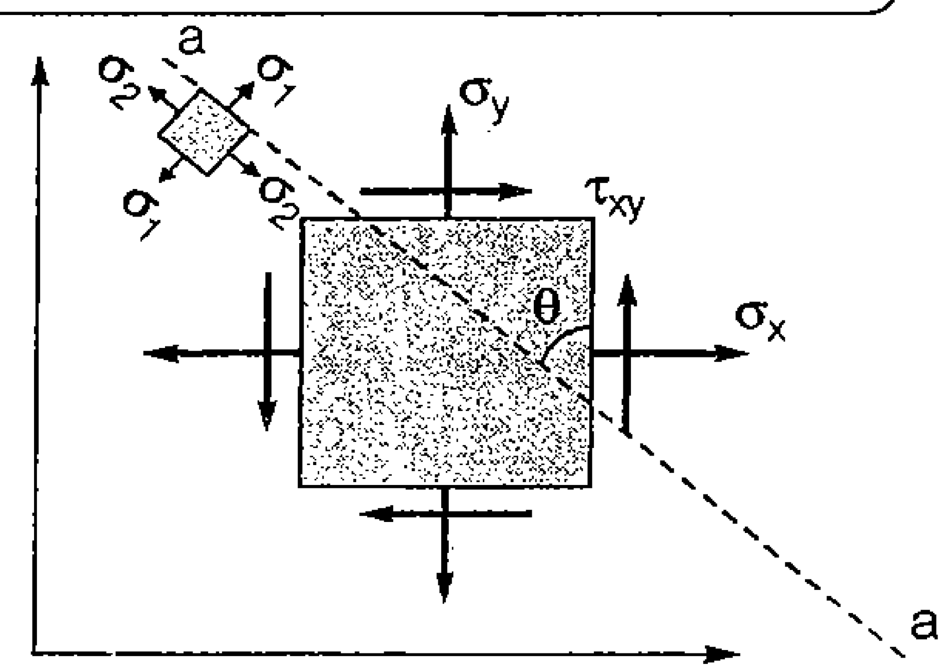
If $\theta = 45°$ or $135°$ then,

$$\tau_1 = \tau_2 = \tau_{max} = -\left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

On the plane of τ_{max}.

$$\sigma_{n_1} = \sigma_{n_2} = \frac{\sigma_1 + \sigma_2}{2}$$

(ii) If σ_x and σ_y are normal stress on vertical and horizontal plane respectively and this plane is accompanied by shear stress τ_{xy} then normal stress and shear stress on plane a-a, which is inclined at an angle θ from plane of σ_x.



$$\sigma'_{1(a-a)} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cdot \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma'_{2(a-a)} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right) \cdot \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{(a-a)} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

Remember

If θ occupies a position such that $\tau_{(a-a)}$ becomes zero, then such a plane is called principal plane and σ_1 and σ_2 become principal stress.

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

θ_p = Angle of principal plane

(iii) If σ_x, σ_y and τ_{xy} are given and we have to find out principal stresses

$$\sigma_1/\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

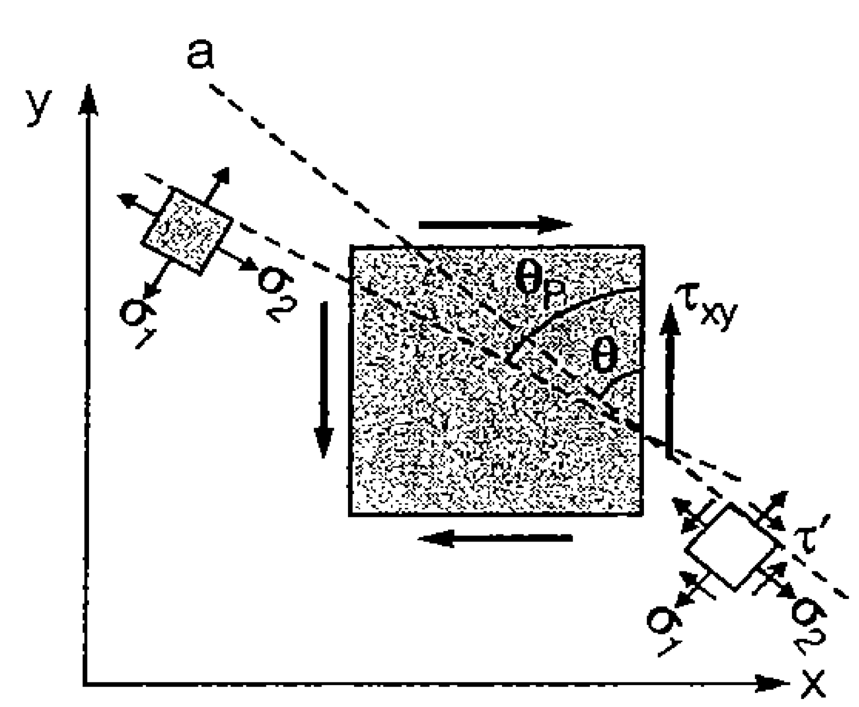
(iv) Pure shear case

Thus normal stress on plane a – a

$$\sigma_{1(a-a)} = \tau_{xy} \sin 2\theta$$

$$\sigma_{2(a-a)} = -\tau_{xy} \sin 2\theta$$

$$\tau' = \tau_{xy} \cos 2\theta$$



Remember

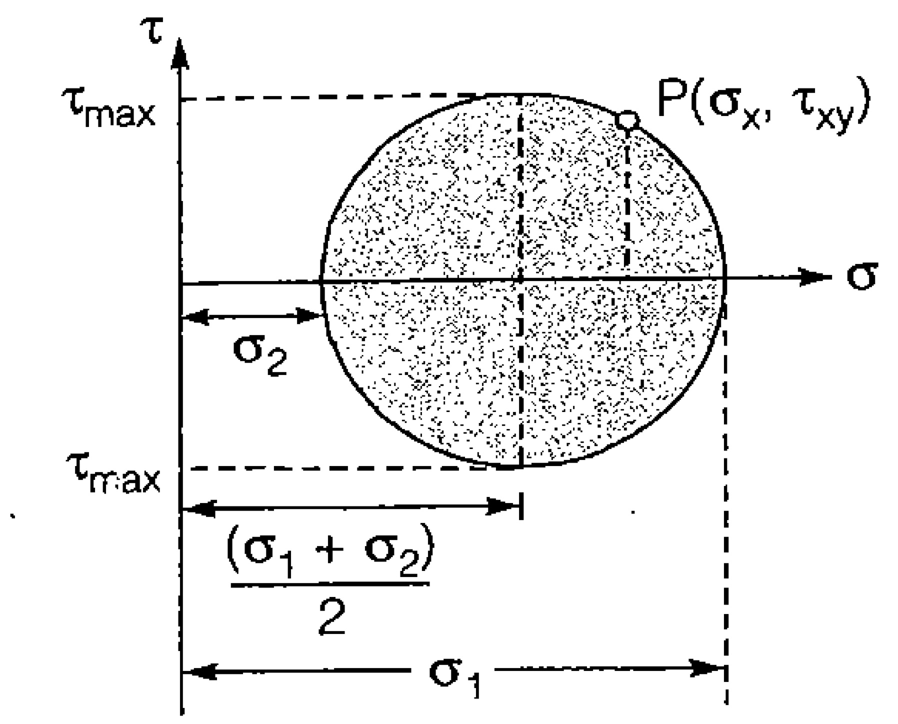
$$\sigma_n + \sigma_n' = \sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_x' + \sigma_y' = \text{constant}$$

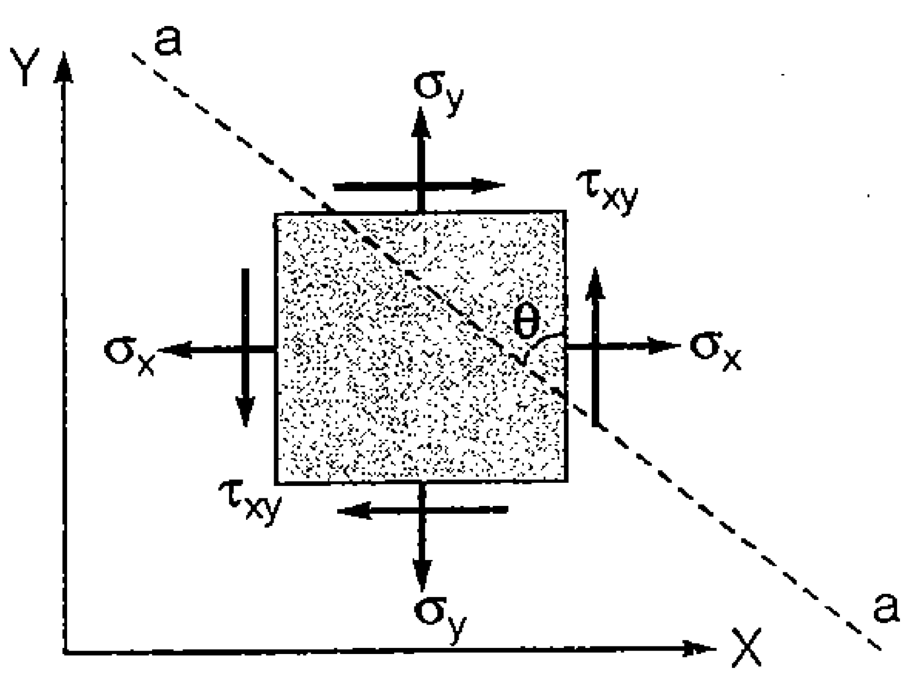
$$\epsilon_x + \epsilon_y = \epsilon_1 + \epsilon_2 = \text{constant}$$

$$I_x + I_y = I_x' + I_y' = \text{constant}$$

Graphical Method of Analysis/Mohr's Circle

Mohr's circle is the locus of points representing magnitude of ***normal*** and ***shear stress*** at various plane in a given stress element.





$$\sigma_1/\sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

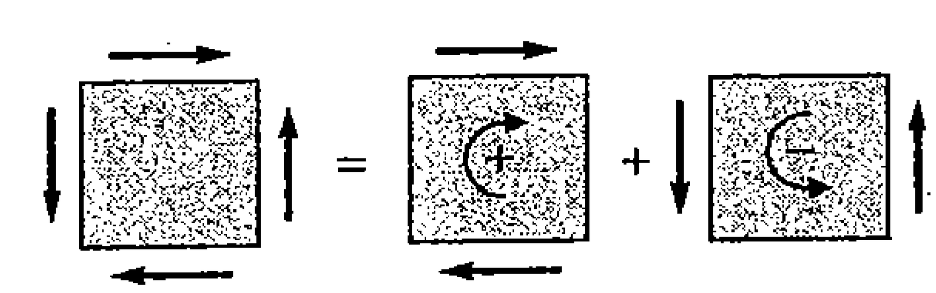
$$a = \frac{\sigma_1 + \sigma_2}{2}$$

- Radius of Mohr's circle

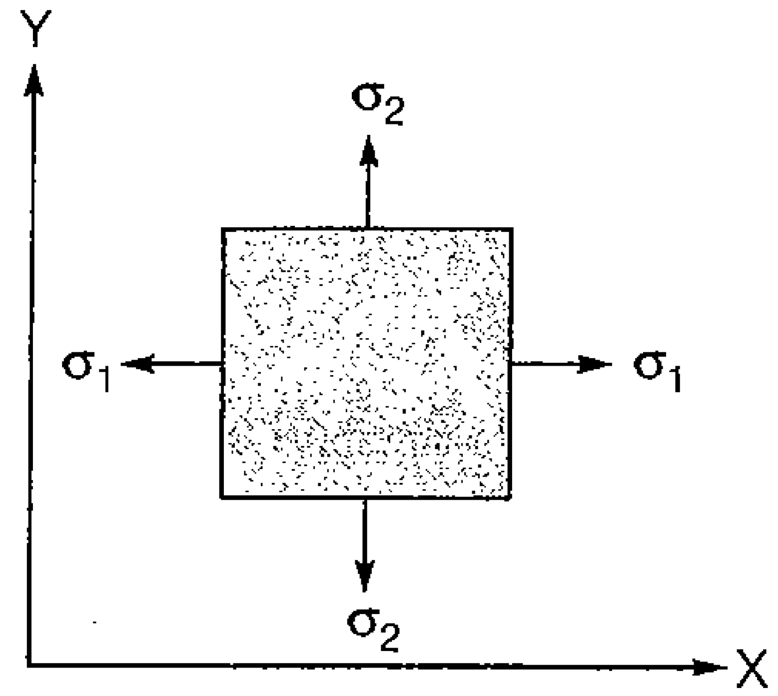
$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

Remember

- Radius of Mohr circle represent the value of maximum shear stress.
- Normal stress on the plane of maximum shear stress is represented by coordinate of centre of Mohr circle.
- Mohr circle reduce to ***a point*** in case of ***hydrostatic*** loading and ***zero*** shear. In case of ***pure shear***, centre will fall at ***origin***.
- If shear stress causes clockwise couple at the centre of element then it will be plotted above σ_{axis} (+ve) and vise-versa.



Analysis of Strain



ϵ_1 = Major principal strain = $\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$

ϵ_2 = Minor principal strain = $\frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}$

$\sigma_1 = \frac{E}{1-\mu^2}[\epsilon_1 + \mu\epsilon_2]$, $\sigma_2 = \frac{E}{1-\mu^2}[\epsilon_2 + \mu\epsilon_1]$

Symbol has usual meanings.

- **Total strain energy per unit volume**

$$U = \frac{1}{2}\sigma_1\epsilon_1 + \frac{1}{2}\sigma_2\epsilon_2$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^3 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \text{ for 3D case}$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right] \text{for 2D case}$$

Remember

- Plane stress does not lead to plain strains.
- For strain analysis formulas, put $\frac{\phi_{xy}}{2}$ is place of τ_{xy} every where in stress formulas.

- Max shear stress = $\frac{1}{2}$ (difference of principal stress)
  Max shear strain = difference of principal strains
- **For shear:** Radius of Mohr circle = τ_{max}

  For strain: Radius of Mohr circle = $\frac{\phi_{max}}{2}$

■■■

4 Theory of Failure

Maximum principal stress theory (Rankines theory)

According to this theory, permanent set takes place under a state of complex stress, when the value of maximum principal stress is equal to that of yield point stress as found in a simple tensile test.

For design criterion, the maximum principal stress (σ_1) must not exceed the working stress 'σ_y' for the material

$\boxed{\sigma_{1,2} \le \sigma_y}$ For no failure.

$\boxed{\sigma_{1,2} \le \frac{\sigma_y}{FOS}}$ For design.

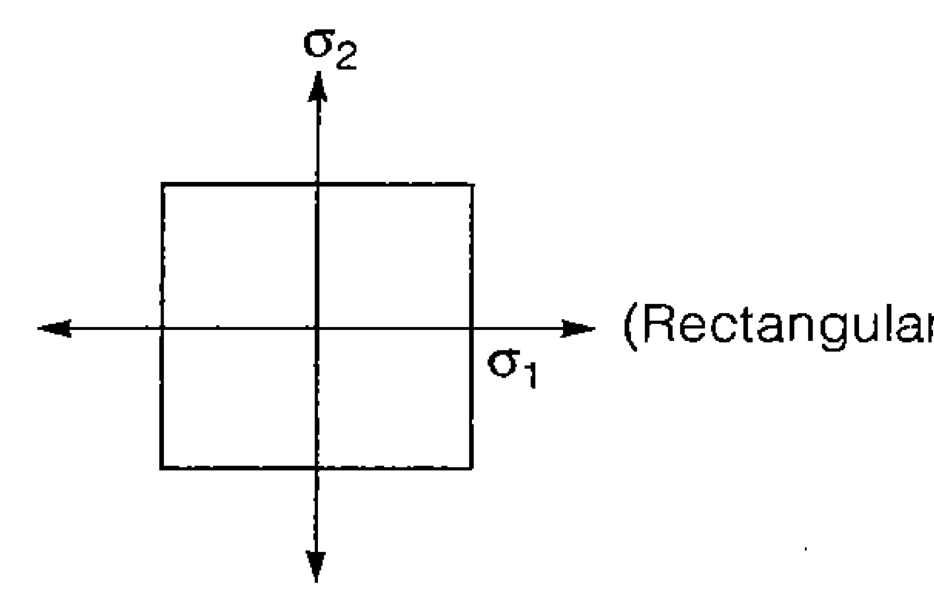
Note: For no shear failure $\tau \le 0.57\sigma_y$

Graphical representation

Remember

For brittle material, which do not fail by yielding but fail by brittle fracture, this theory gives satisfactory result.

The graph is always square even for different values of σ_1 and σ_2.



Maximum principal strain theory (ST. Venant's theory)

According to this theory, a ductile material begins to yield when the maximum principal strain reaches the strain at which yielding occurs in simple tension

$\boxed{\epsilon_{1,2} \le \frac{\sigma_y}{E}}$ For no failure in uni-axial loading.

$\boxed{\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} \le \frac{\sigma_y}{E}}$ For no failure in tri-axial loading.

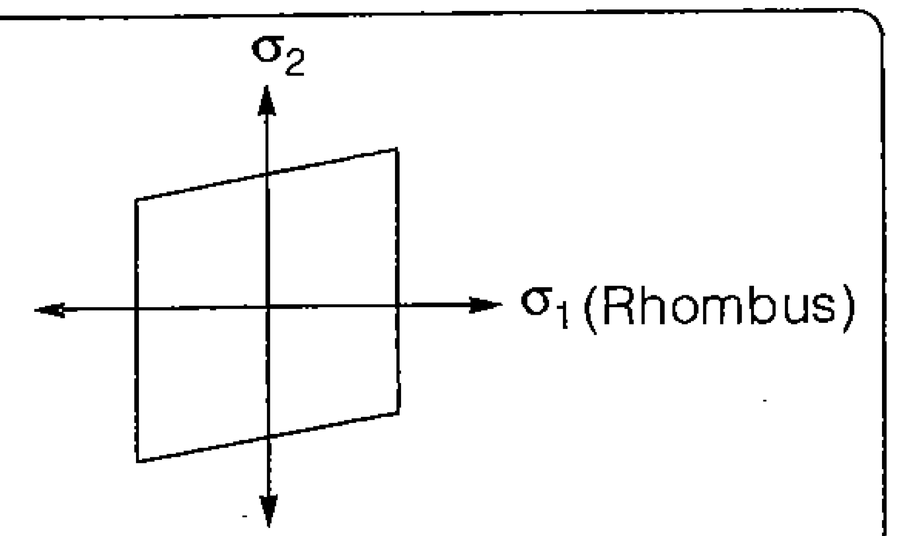
$\boxed{\sigma_1 - \mu\sigma_2 - \mu\sigma_3 \le \left(\frac{\sigma_y}{FOS}\right)}$ For design. Here, ϵ = Principal strain

σ_1, σ_2 and σ_3 = Principal stresses

Graphical Representation

Remember

This theory over estimate the elastic strength of ductile material.



Maximum shear stress theory (Guest & Tresca's theory)

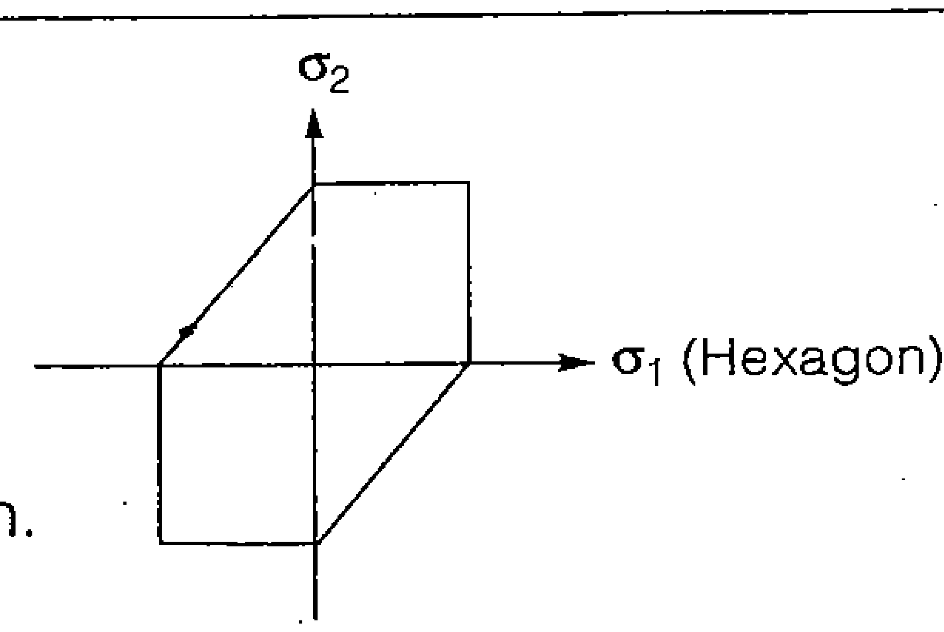
According to this theory, failure of a specimen subjected to any combination of loads when the maximum shearing stress at any point reaches the failure value equal to that developed at the yielding in an axial tensile or compressive test of the same material.

Graphical Representation

Remember

$$\tau_{max} \le \frac{\sigma_y}{2}$$ For no failure.

$$\sigma_1 - \sigma_2 \le \left(\frac{\sigma_y}{FOS}\right)$$ For design.



σ_1 and σ_2 are maximum and minimum principal stresses respectively.

Here, τ_{max} = Maximum shear stress

σ_y = Permissible stress

This theory is well justified for ductile materials.

Maximum strain energy theory (Haigh's theory)

According to this theory, a body under complex stress fails when the total strain energy on the body is equal to the strain energy at elastic limit in simple tension.

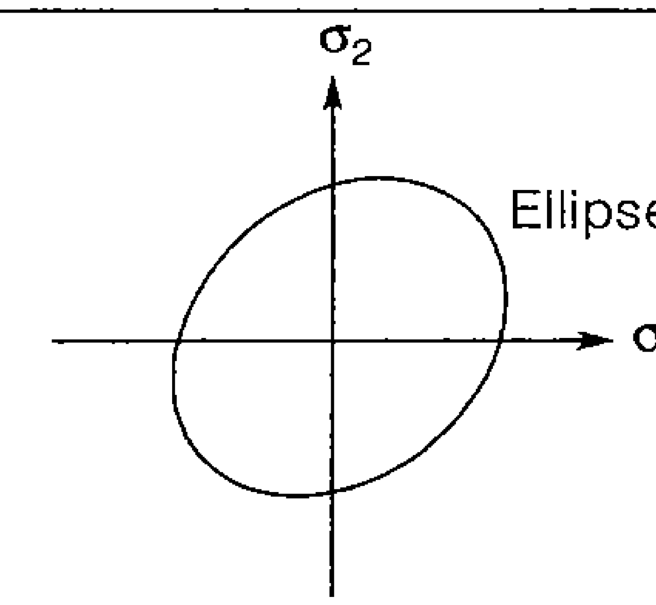
Graphical Representation

Remember

$$\left\{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right\} \le \sigma_y^2$$ for no failure.

$$\left\{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right\} \le \left(\frac{\sigma_y}{FOS}\right)^2$$ for design.

This theory does not apply to brittle material for which elastic limit stress in tension and in compression are quite different.

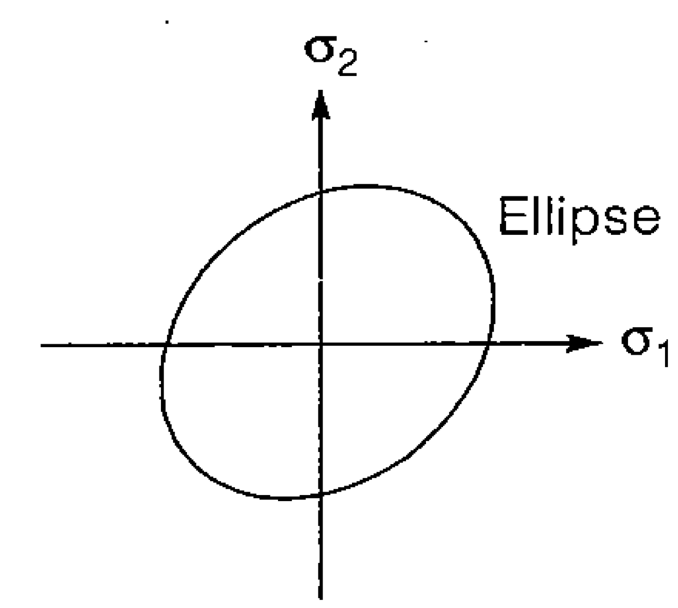


Maximum shear strain energy/Distortion energy theory/ Mises-Henky theory

It states that inelastic action at any point in a body, under any combination of stress begins, when the strain energy of distortion per unit volume absorbed at the point is equal to the strain energy of distortion absorbed per unit volume at any point in a bar stressed to the elastic limit under the state of uniaxial stress as occurs in a simple tension/compression test.

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \le \sigma_y^2$$ For no failure.

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \le \left(\frac{\sigma_y}{FOS}\right)^2$$ For design.



Remember

- It can not be applied for material under hydrostatic pressure.
- All theories will give same results if loading is uniaxial.

■■■

5

Deflection of Beams

For design purpose, a beam should be designed in such a way that it has adequate stiffness so that the deflections are within permissible limits.

Stiffness of beam is inversely proportional to deflection.

Methods of Determining Deflection of Beam

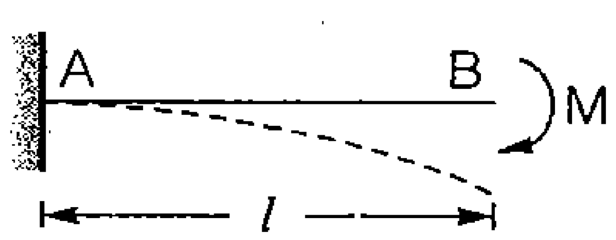
- Double integration method.
- Moment area method
- Strain energy method
- Conjugate beam method

Deflection of Beam Under Different Loading/Support Condition

- Notation used

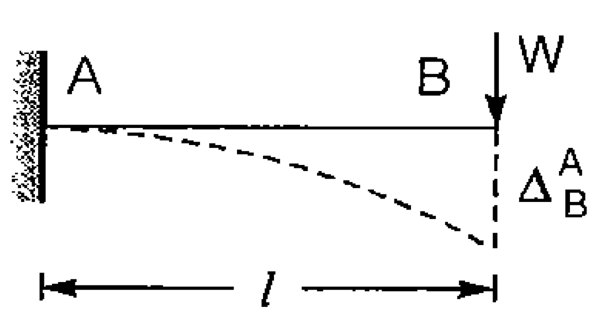
θ_B^A = Slope at B w.r.t A

Δ_B^A = Deflection at B w.r.t A

- 

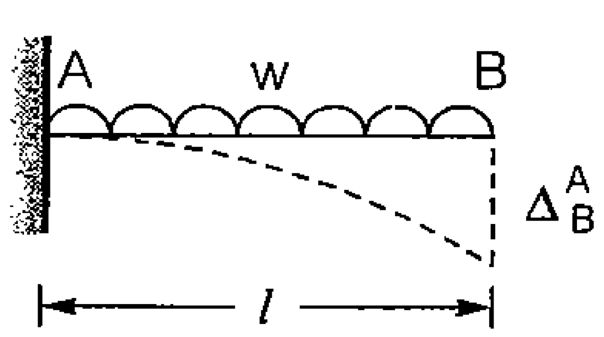
Deflection $(\Delta_B^A) = \dfrac{Ml^2}{2\,EI}$

Slope $(\theta_B^A) = \dfrac{Ml}{EI}$

- 

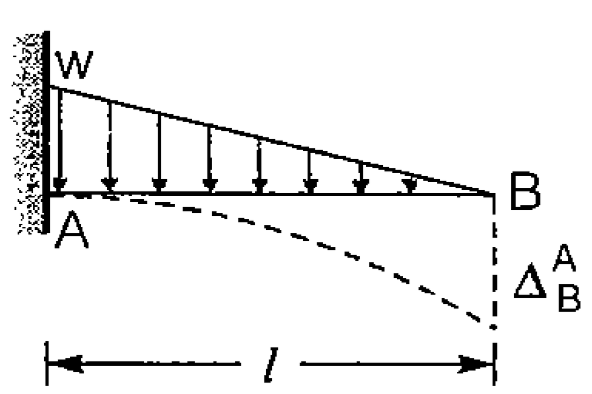
Deflection $(\Delta_B^A) = \dfrac{Wl^3}{3\,EI}$

Slope $(\theta_B^A) = \dfrac{Wl^2}{2\,EI}$

- 

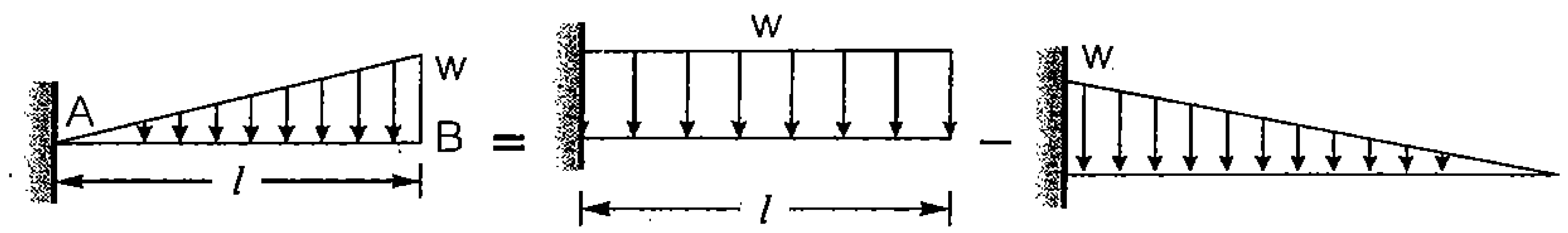
Deflection $(\Delta_B^A) = \dfrac{wl^4}{8EI}$

Slope $(\theta_B^A) = \dfrac{Wl^3}{6\,EI}$

- 

Deflection $(\Delta_B^A) = \dfrac{wl^4}{30EI}$

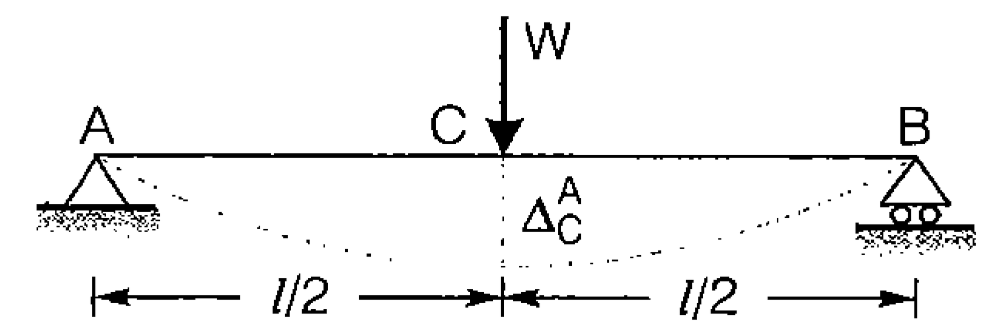
Slope $(\theta_B^A) = \dfrac{Wl^3}{24\,EI}$

- 

Deflection $(\Delta_B^A) = \dfrac{Wl^4}{8\,EI} - \dfrac{Wl^4}{30EI}$

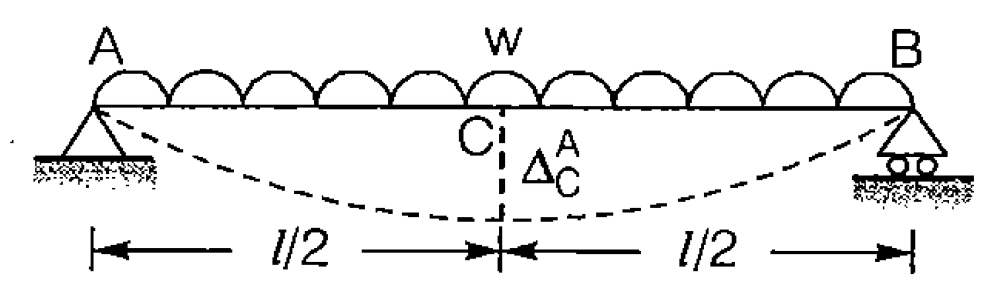
Slope $(\theta_B^A) = \dfrac{Wl^3}{6EI} - \dfrac{Wl^3}{24EI}$

- Simply supported beam



Deflection $(\Delta_C^A) = \dfrac{Wl^3}{48\,EI}$

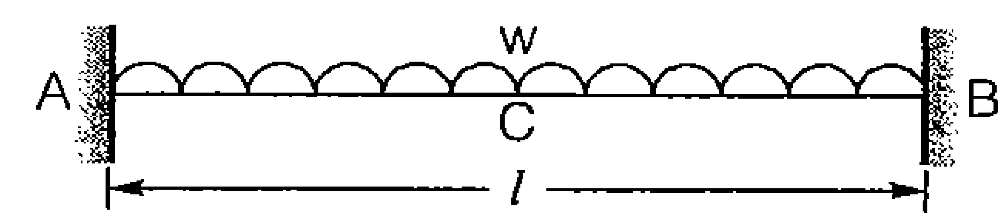
Slope $(\theta_C^A) = \dfrac{Wl^2}{16\,EI}$

- 

Deflection $(\Delta_C^A) = \dfrac{5\,wl^4}{384\,EI}$

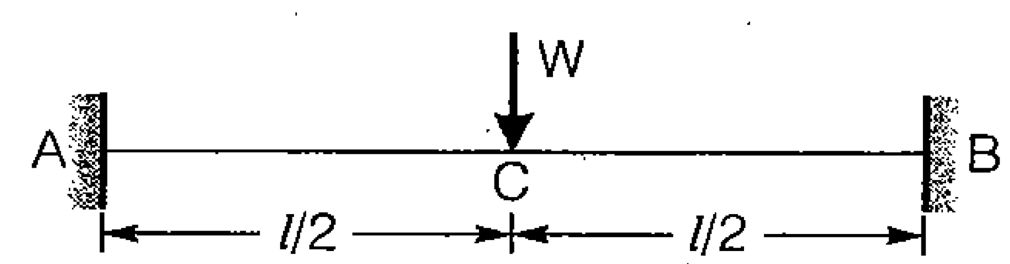
Slope $(\theta_C^A) = \dfrac{Wl^3}{24\,EI}$

- Fixed Beam



$\theta_A = \theta_B = 0$

Deflection$(\Delta_c) = \Delta_{max} = \dfrac{Pl^3}{192\,EI} = \dfrac{1}{4} \times \Delta_{max}$ in SS beam



$\theta_A = \theta_B = 0$

Deflection$(\Delta_c) = \Delta_{max} = \dfrac{Wl^4}{384\,EI} = \dfrac{1}{5} \times \Delta_{max}$ in SS beam

■■■

6 Pressure Vessels

Types of Pressure Vessels

Pressure vessels are mainly of two type:

(i) Thin shells

If the thickness of the wall of the shell is less than 1/10 to 1/15 of its diameter, then shell is called thin shells.

$$t < \frac{D_i}{10} \text{ to } \frac{D_i}{15}$$

> **Remember** For thin shell, it is assumed that the normal stresses, which may be ***either*** tensile or compressive are ***uniformly*** distributed through the thickness of wall.

(ii) Thick Shells

If the thickness of the wall of the shell is greater than 1/10 to 1/15 of its diameter, then shell is called thick shells.

$$t > \frac{D_i}{10} \text{ to } \frac{D_i}{15}$$

Nature of stress in thin cylindrical shell subjected to internal pressure

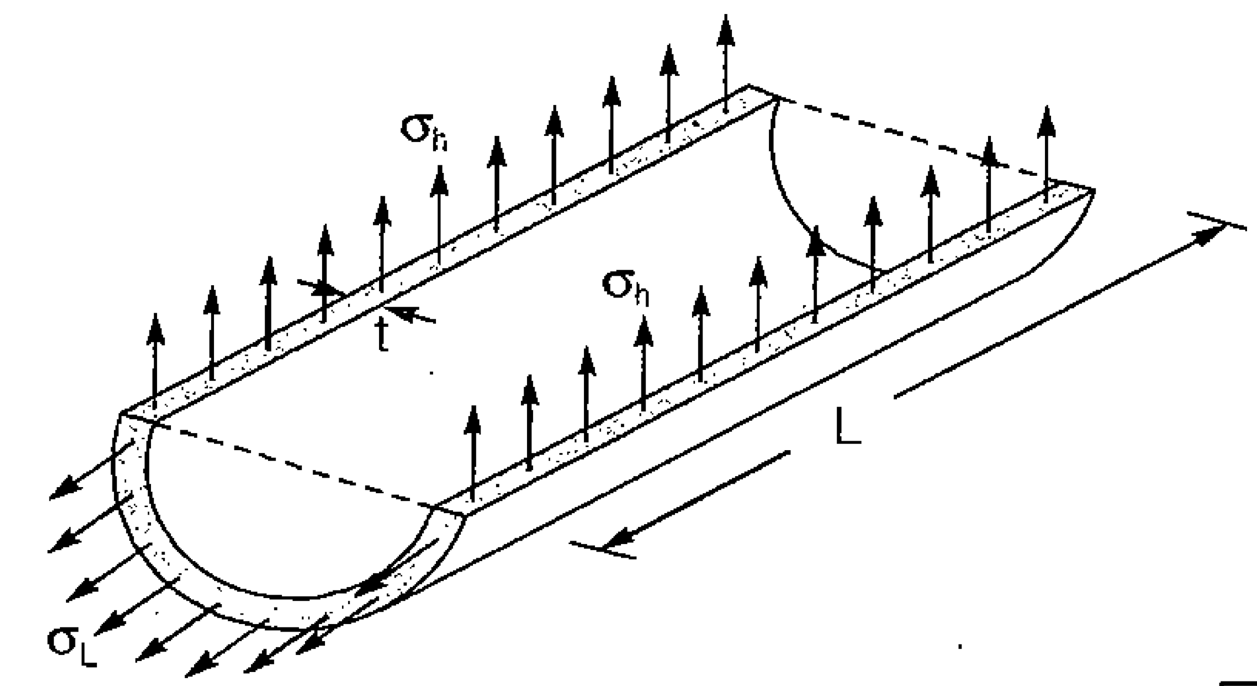
(i) Hoop stress /circumferential stress will be tensile in nature.

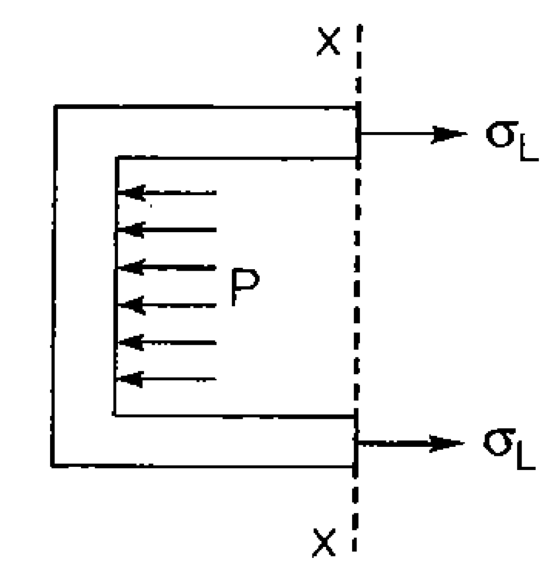
(ii) Longitudinal stress/axial stress will be tensile in nature.

(iii) Radial stress will be compressive in nature.

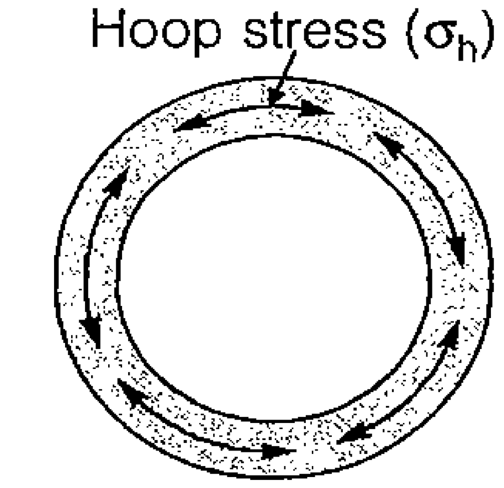
> **Remember** Radial compressive stress varies from a value at the inner surface equal to pressure 'p' to the atmospheric pressure at the outside surface.
>
> If internal pressure in thin cylinders is low, the radial stress is negligible compared with axial stress and hoop stress. This ***radial stress*** is neglected.

Analysis of thin cylinder





- Longitudinal Stress $\sigma_L = \dfrac{pd}{4t}$
- Hoop Stress $\sigma_h = \dfrac{pd}{2t}$
- Longitudinal Strain $\epsilon_L = \dfrac{pd}{4tE}(1-2\mu)$



- Hoop Strain $\epsilon_h = \dfrac{pd}{4tE}(2-\mu)$

Here, p = Pressure of fluid, t = Thickness of cylinder
d = Inside diameter, μ = Poisson's ratio

- Ratio of Hoop Strain to Longitudinal Strain $\dfrac{\epsilon_h}{\epsilon_L} = \dfrac{2-\mu}{1-2\mu}$
- Volumetric Strain (ϵ_v) of Cylinder $\epsilon_v = \dfrac{pd}{4tE}(5-4\mu)$
- Max shear stress in the plane of metal (x-y plane) or $\sigma_h - \sigma_L$ plane

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_h - \sigma_L}{2} \qquad \tau_{max} = \frac{PD}{8t}$$

- Absolute max shear stress

$$\tau_{obs.max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{PD/2t - 0}{2} = \frac{PD}{4t}$$

Remember

If fluid is compressible, volumetric strain will be

$$\epsilon_V = \frac{pd}{4t\,E}(5-4\mu) + \frac{p}{K}$$

k = Bulk modulus of fluid

P = Pressure of fluid

Minimum thickness of cylinder required for a given pressure 'P' and diameter 'd' is $t \geq \frac{pd}{2\sigma}$

Analysis of thin sphere

- Hoop stress/longitudinal stress $\sigma_L = \sigma_h = \frac{pd}{4t}$
- Hoop strain/longitudinal strain $\epsilon_L = \epsilon_h = \frac{pd}{4t\,E}(1-\mu)$
- Volumetric strain of sphere $\epsilon_V = \frac{3pd}{4t\,E}(1-\mu)$

Remember

Thickness ratio for cylindrical shell (t_c) and sphere (t_s), for ***same strain*** in both side.

$$\frac{t_c}{t_s} = \frac{2-\mu}{1-\mu}$$

Thickness ratio for cylindrical shell (t_c) and sphere (t_s), for ***same maximum stress*** in both side.

$$\frac{t_c}{t_s} = 2$$

Auto frittage is used for prestressing the cylinder.

Wire winding is done for ***strenghting thin shell. Compounding*** is done for ***thick*** shell cylinders.

Analysis of Thick Cylinders/Lame's Theorem

- Lame's Assumption
  (i) Material of shell is homogeneous, isotropic and linear elastic.
  (ii) Plane section of cylinder, perpendicular to longitudinal axis remains plane under pressure.

- Lame's equations

(i) Hoop stress: $\sigma_x = \frac{B}{x^2} + A\,(\text{tensile})$

(ii) Radial stress: $P_x = \frac{B}{x^2} - A\,(\text{compressive})$

Where, B and A are Lame's constant

- **Subjected to internal pressure**

(i) At $x = R_i$, $\sigma_h = \frac{P\left[R_o^2 + R_i^2\right]}{R_o^2 - R_i^2}$ (ii) At $x = R_o$, $\sigma_h = \frac{2PR_i^2}{R_o^2 - R_i^2}$

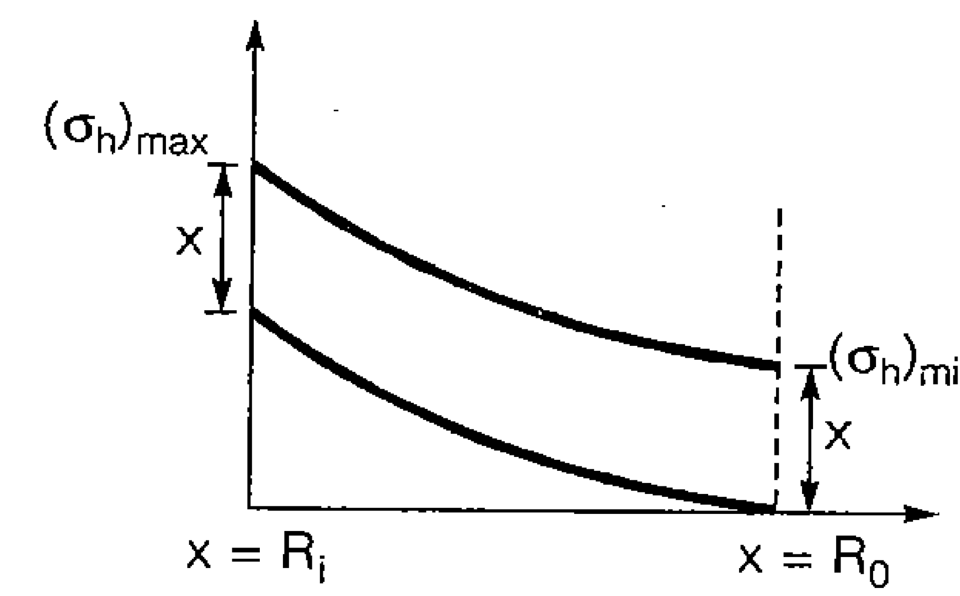
- **Subjected to external pressure**

(i) At $x = R_i$, $\sigma_h = \frac{-2PR_o^2}{R_o^2 - R_i^2}$ (ii) At $x = R_o$, $\sigma_h = \frac{-P\left[R_o^2 + R_i^2\right]}{\left[R_o^2 - R_i^2\right]}$

where, R_i = Inner radius

R_0 = Outlet radius

$$(\sigma_h)_{max} = p + (\sigma_h)_{min}$$



$$\sigma_x - P_x = \left(\frac{B}{x^2} + A\right) - \left(\frac{B}{x^2} - A\right) = 2A$$

Remember: Radial and hoop compression vary ***hyperbolically.***

Analysis of Thick spheres

- Lame's equation:

$$\sigma_x = \frac{2B}{x^3} + A\,(\text{Tensile}) \qquad P_x = \frac{2B}{x^3} - A\,(\text{compressive})$$

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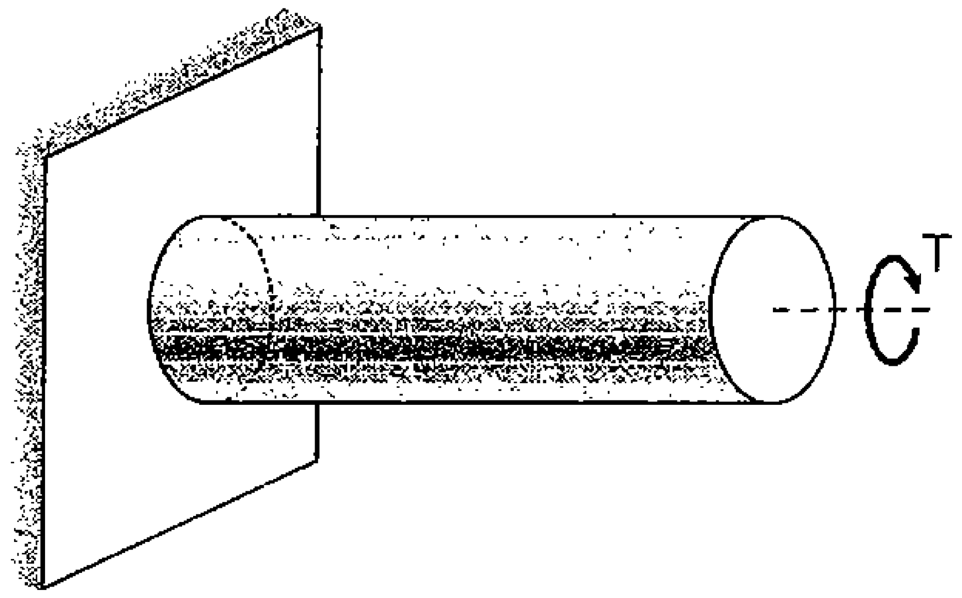
7

Torsion of Shaft

Torsion

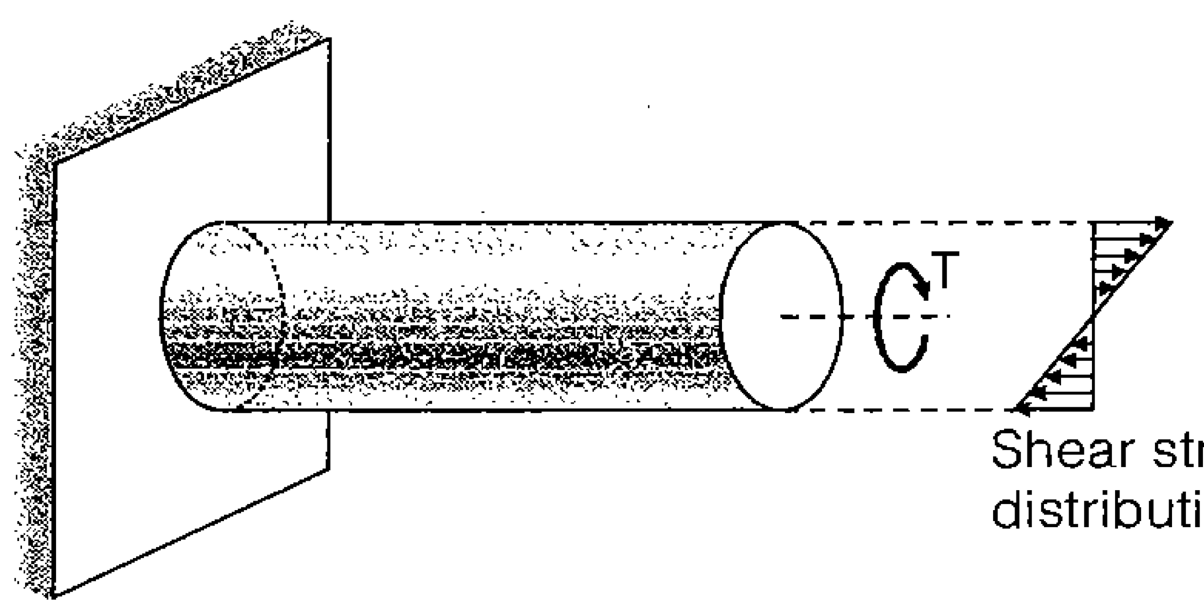
Torsion means twisting of a structural member when it is loaded by ***couple*** that produces rotation about ***longitudinal*** axis.



> Remember
>
> Torsion causes rotation of all the fibre about ***longitudinal/polar*** axis.
> Force required for torsion is ***normal*** to ***longitudinal*** axis having certain ***eccentricity*** from centroid.
> If no ***shear force*** and no ***bending moment*** is present in structural member then it will be a case of ***pure*** torsion.
> For pure torsion, shaft is ***prismatic.***
> If torque applied in non-circular section then ***warping*** will occur.
> A plane section ***before*** twisting remains plane ***after*** twisting.

Equation of Torsion

$$\boxed{\frac{\tau}{r} = \frac{T}{J} = \frac{G \cdot \theta}{L}}$$

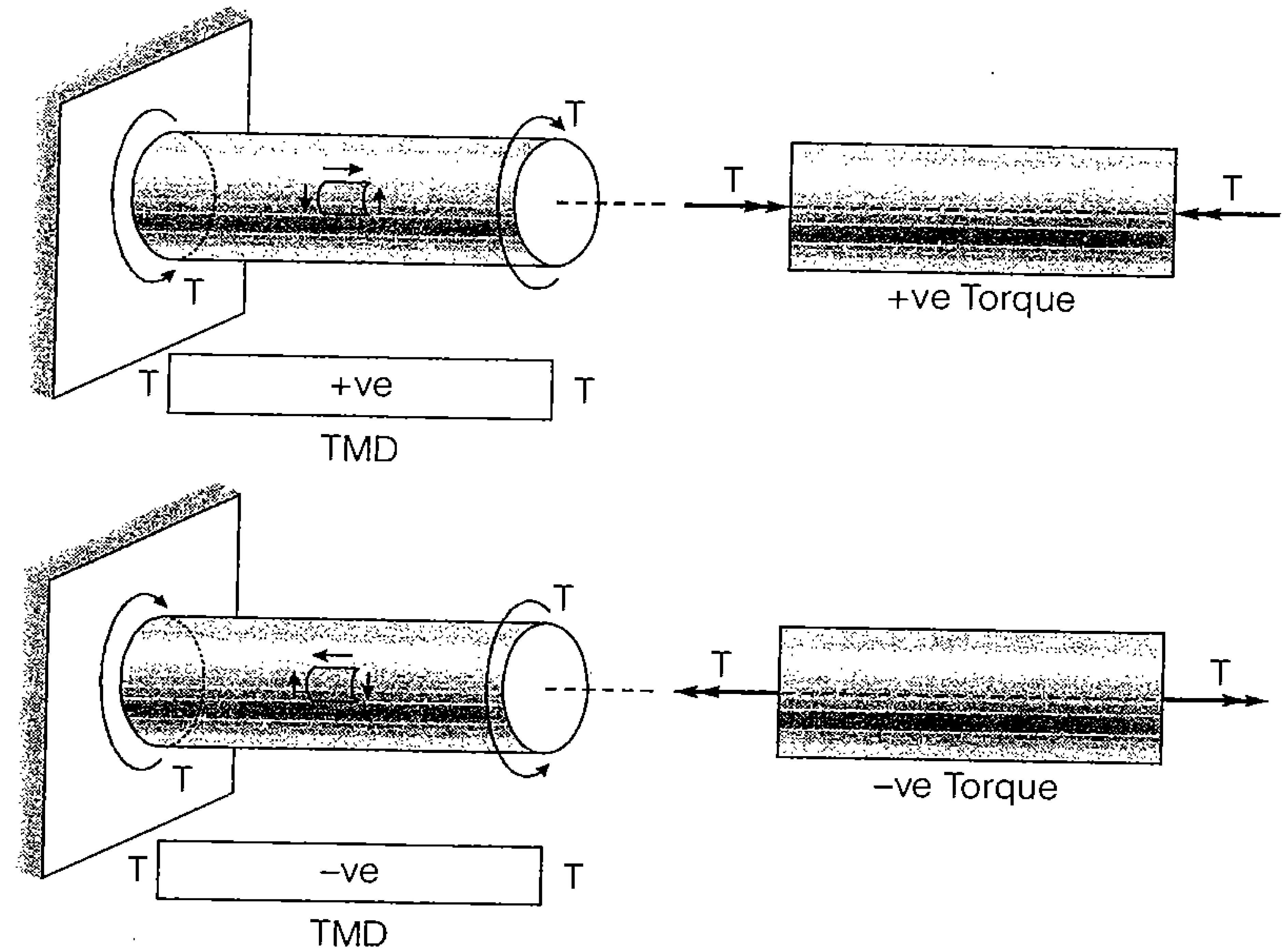


Here,
- τ = Shear stress
- r = Distance from centre of shaft
- D = Diameter of shaft
- T = Torque
- J = Polar moment of inertia
- G = Shear modulus
- θ = Angle of twist
- L = Length of shaft

> Remember
>
> Shear stress should be maximum at extreme fibre of shaft.
> Angle of twist is maximum at the ***free end*** of shaft.

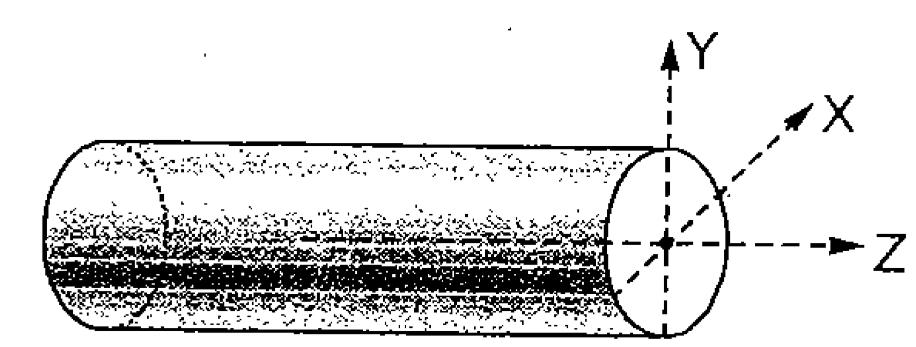
Sign Convention

Sign convention of torque can be explained by right hand thumb rule. A positive torque is that in which there is tightening effect of nut on the bolt. From either side of the cross-section. If torque is applied in the direction of right hand fingers than right hand thumbs direction represents movement of the nut.



TMD stands for Torsion moment diagram.
T = Torque

Moment of Inertia About Polar Axis



(i) **For Solid Circular Section**

$$\boxed{J = I_z = I_x + I_y = \frac{\pi}{64}d^4 + \frac{\pi}{64} \cdot d^4 = \frac{\pi}{32}d^4}$$

- Polar section modulus/torsional strength (Z_p)

$$\boxed{\frac{\tau}{r} = \frac{T}{J}}$$

$$\tau_{(max)} = \frac{T}{J/r_{max}}$$

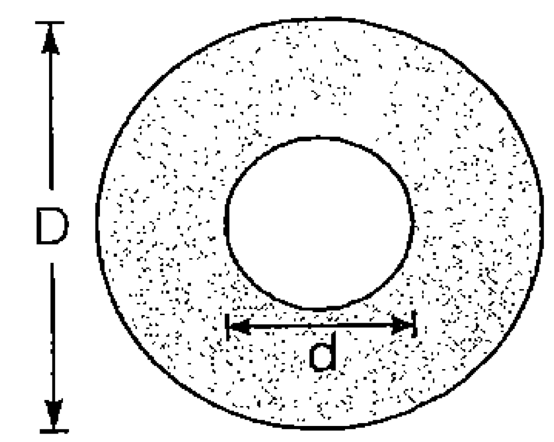
$$\tau_{max} = \frac{T}{Z_p}$$

$$Z_p = \frac{J}{r_{max}} = \frac{\pi}{16}D^3$$

$$\tau_{max} = \frac{16T}{\pi d^3}$$

(ii) For Hollow Circular Section

$$\boxed{I_{z\,(Hollow)} = \frac{\pi}{32}(D^4 - d^4)}$$

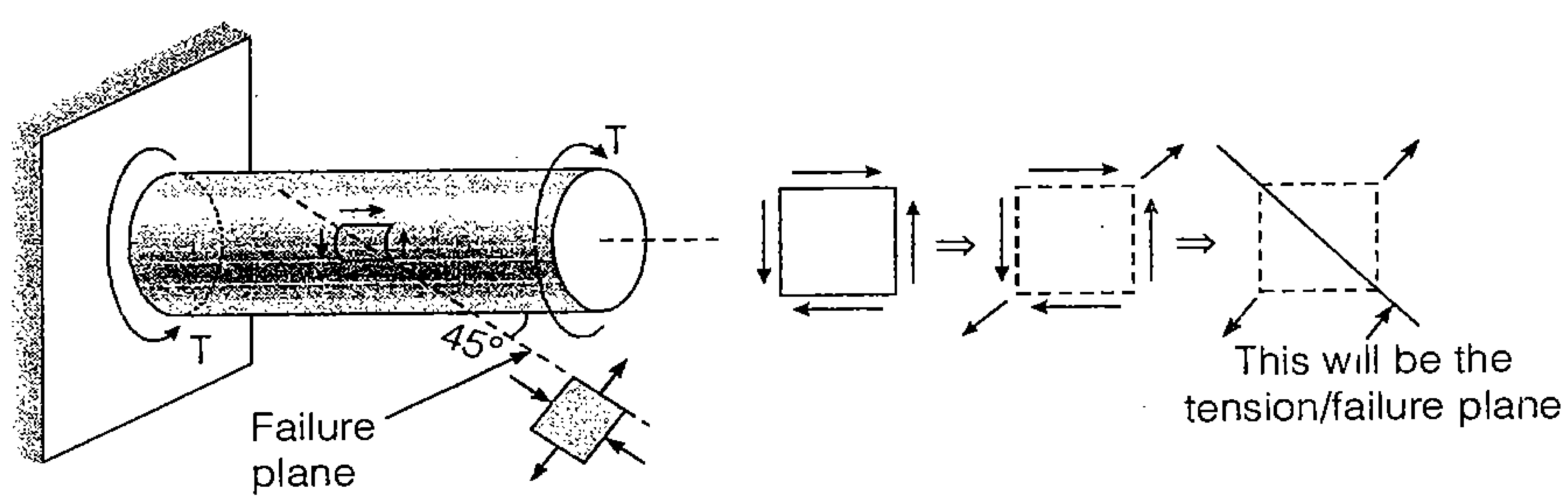


D = Outside diameter
d = Inside diameter

- Polar section modulus/torsional strength (Z_p)

$$\boxed{J = \frac{\pi}{32}D^4\,(1-K^4)} \quad \boxed{Z_p = \frac{\pi}{16}\cdot D^3\,(1-K^4)} \quad \boxed{K = \frac{d}{D}}$$

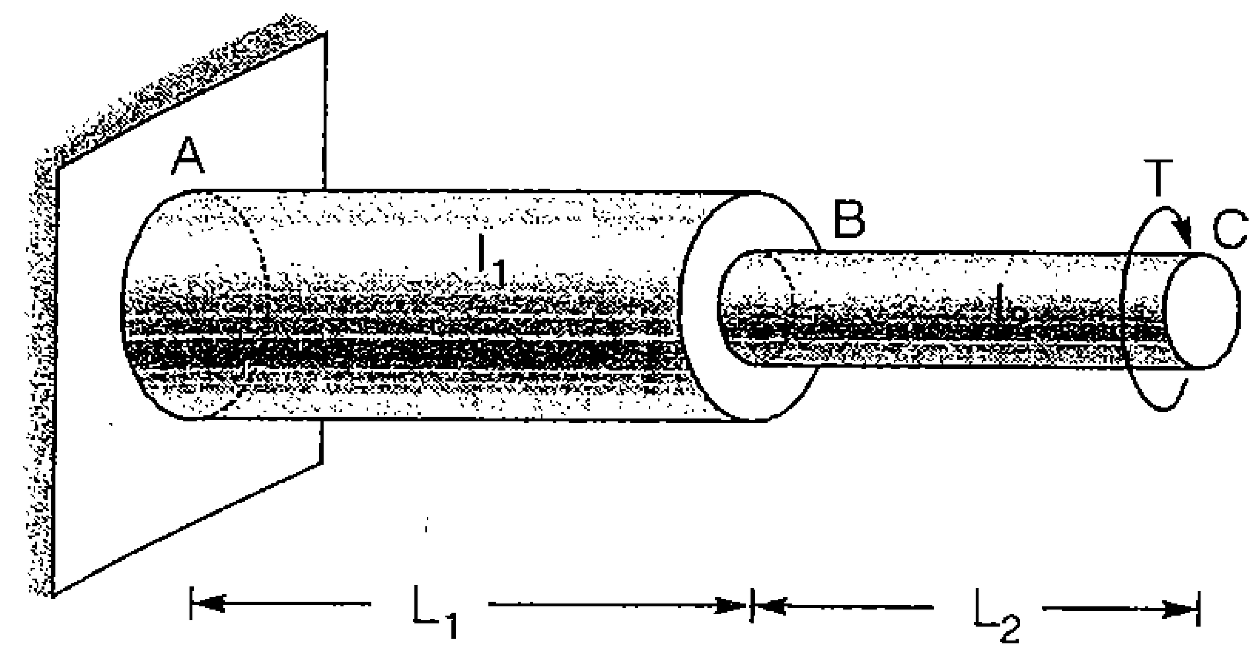
How to find out tension (failure) plane



- In ductile metals in pure torsion failure is due to shear stress in the direction of τ_{max}. Which is at 90° with the longitudinal axis and failure plane is smooth plane.
- Brittle mettle fails in tension so in brittle mettle failure plane will be a rough helical plane at 45° from longitudinal axis.

Compound Shaft

- Series Connection



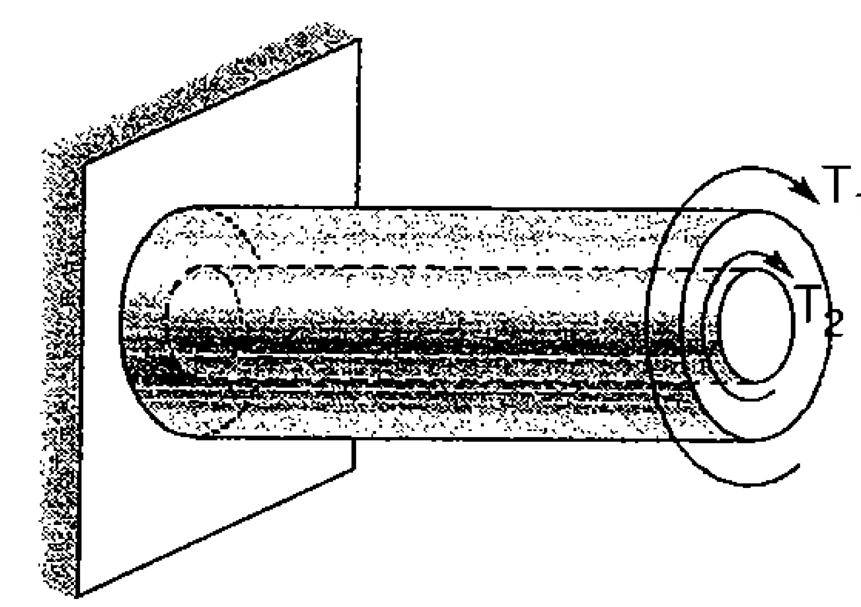
θ_{AB} = Angular deformation of 1st shaft
θ_{BC} = Angular deformation of 2nd shaft
θ_{AC} = Total angular deformation of free end of shaft from the fixed end
= $\theta_{AB} + \theta_{BC}$

$$\boxed{\theta_{AC} = \frac{TL_1}{G_1J_1} + \frac{TL_2}{G_2J_2}}$$

> **Remember**
> Torque will be ***same*** on both shafts.
> Total angular deformation of free end of shaft from the fixed end will be equal to ***sum*** of angular deformation of first shaft and angular deformation of second shaft.

- Parallel Connection

θ_1 = Angular deflection of first shaft
θ_2 = Angular deflection of second shaft
T_1 = Torque on first shaft
T_2 = Torque on second shaft



$$\boxed{\theta_1 = \theta_2} \quad \boxed{\frac{T_1L}{G_1J_1} = \frac{T_2L}{G_2J_2}}$$

Strain energy (U) stored in shaft due to torsion

$$\boxed{U = \frac{1}{2}T\cdot\theta = \frac{1}{2}\frac{T^2L}{G\cdot J} = \frac{\tau_{max}^2}{4G} \times \text{Volume of shaft}}$$

Here, G = Shear modulus
T = Torque
J = Moment of inertia about polar axis

> **Remember**
> Ratio of strain energy for solid and hollow shaft subjected to same torque if outside diameter of both shaft is equal.
>
> $$\boxed{\frac{U_{Hollow}}{U_{solid}} = \frac{D^2 + d^2}{D^2}}$$ d = Inside diameter of hollow shaft

Remember

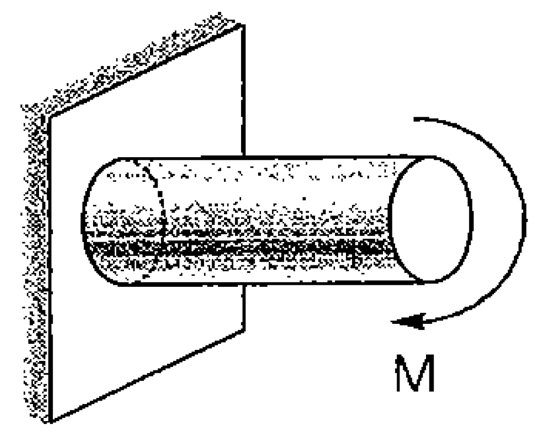
Ratio of torque in case of hollow and solid shaft subjected to *same maximum shear stress*.

$$\frac{T_{Hollow}}{T_{Solid}} = \frac{D^4 - d^4}{D^4}$$

Effect of pure bending on shaft

$$\sigma = \frac{32\,M}{\pi D^3}$$

σ = *Principal* stress
D = Diameter of shaft
M = Bending moment

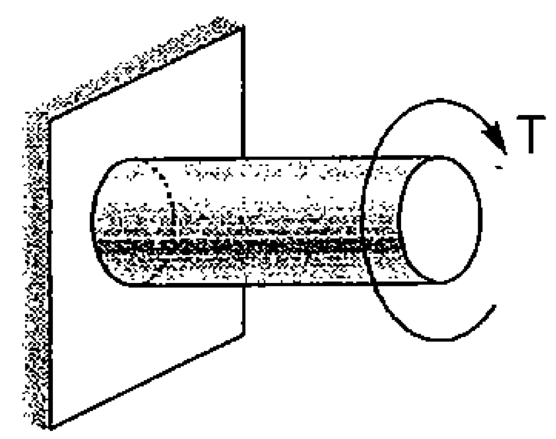


Pure bending is different from pure torsion.

Effect of Pure Torsion on Shaft

$$\tau_{max} = \frac{16T}{\pi D^3}$$

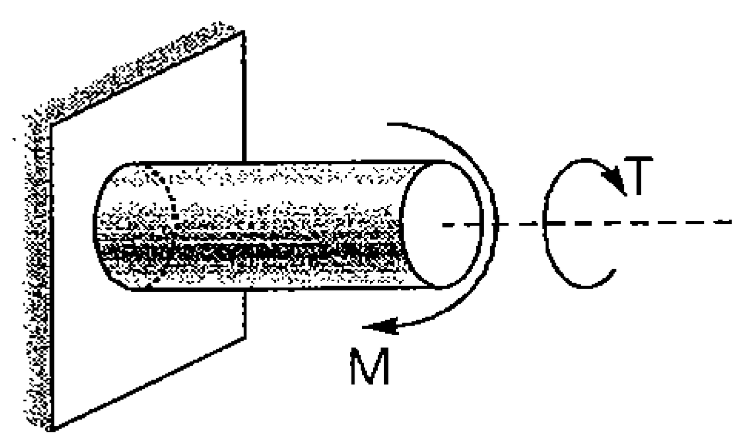
τ_{max} = Maximum *shear* stress
T = Torque
D = Diameter of shaft



Combined effect of bending and torsion

$$\text{Principal Stress} = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

$$\text{Maximum shear stress} = \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$$

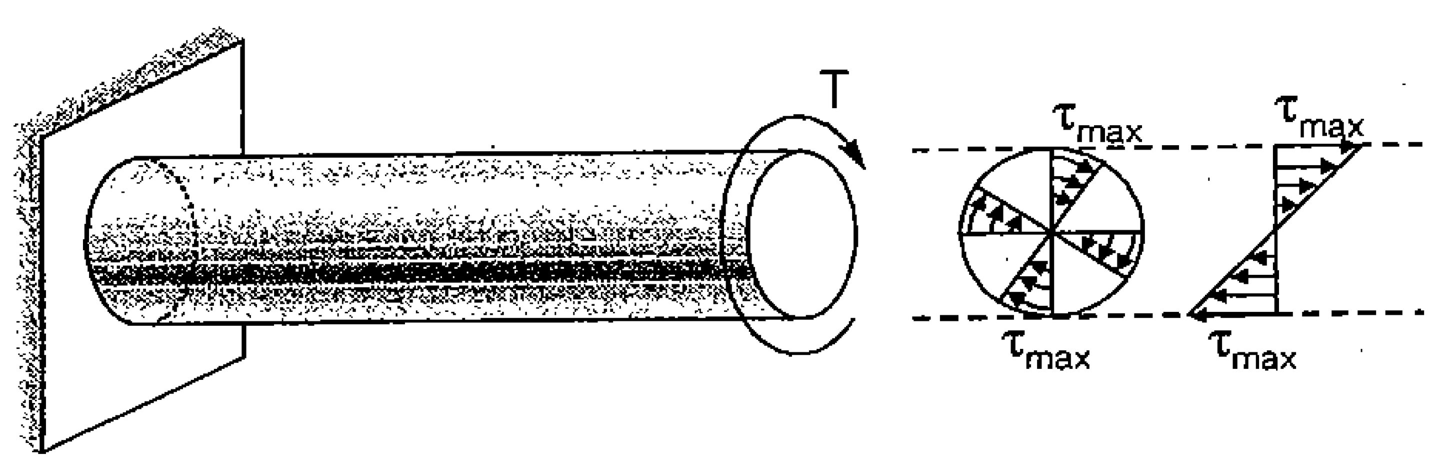


Equivalent *bending moment* = $\frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$

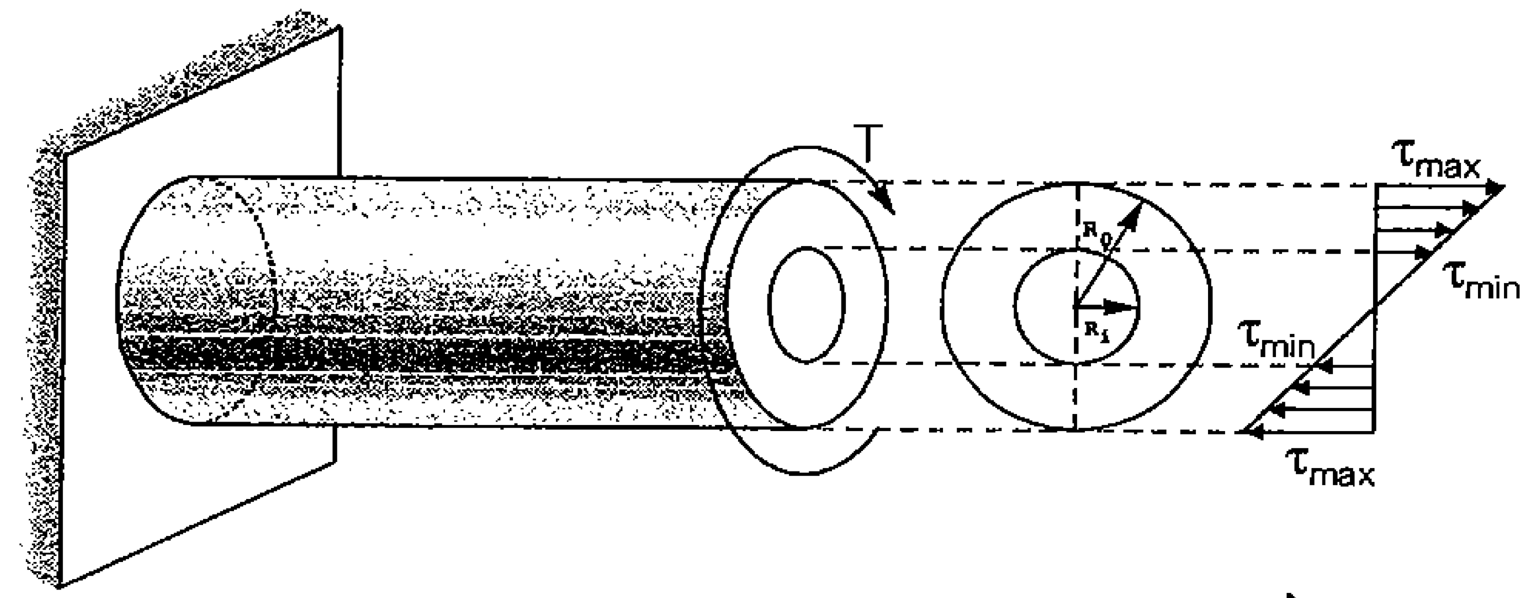
Equivalent torque = $\sqrt{T^2 + M^2}$

Shear Stress Distribution

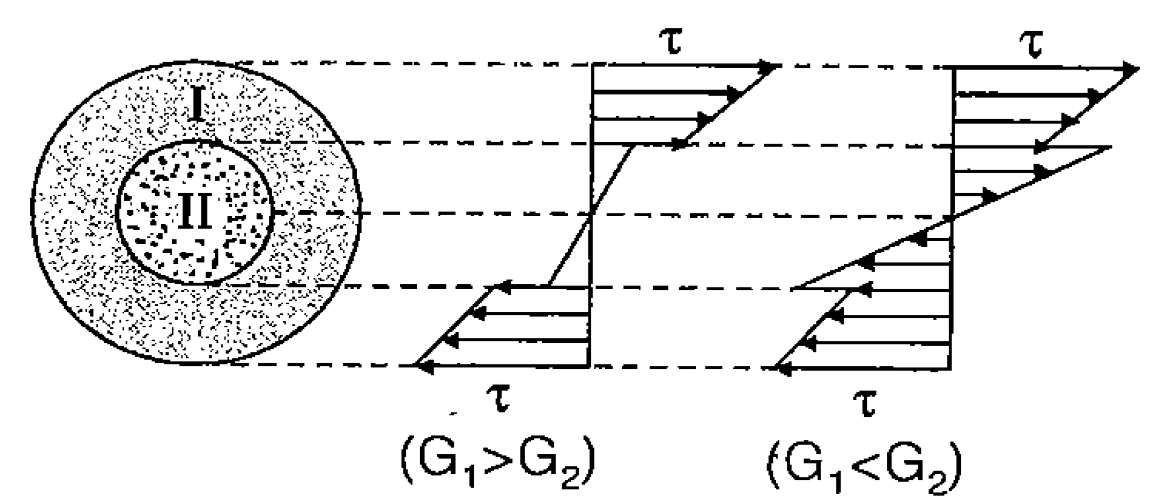
1. Solid Circular Section



2. Hollow Circular Section



3. Composite Circular Section



4. **Thin Tubular Section:** In view of small thickness shear stress is assumed to be uniform.

Power Transmitted in the Shaft

$$P = \frac{2\pi NT}{60}\,\text{Watt} \qquad P = \frac{2\pi NT}{60{,}000}\,\text{kW} \qquad \omega = \frac{2\pi N}{60}$$

N = No. of revolution per minute i.e., rpm, T = Torque in Nm
ω = Angular speed in rad/sec

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Shear Centre

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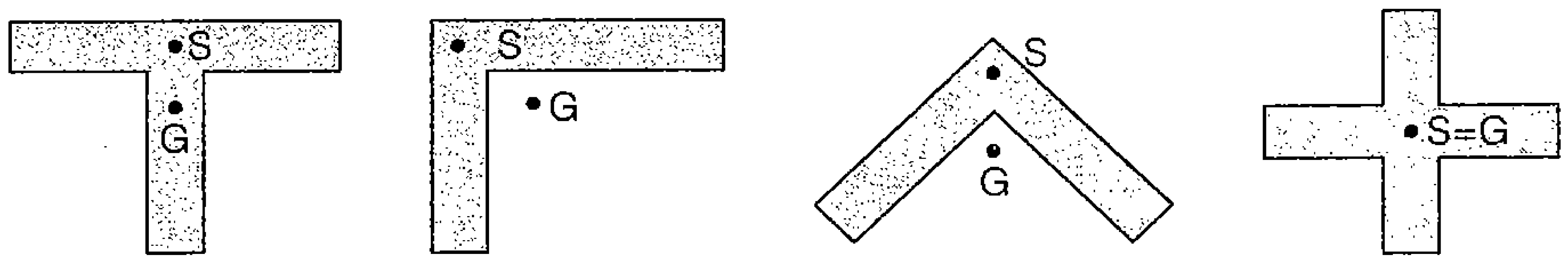
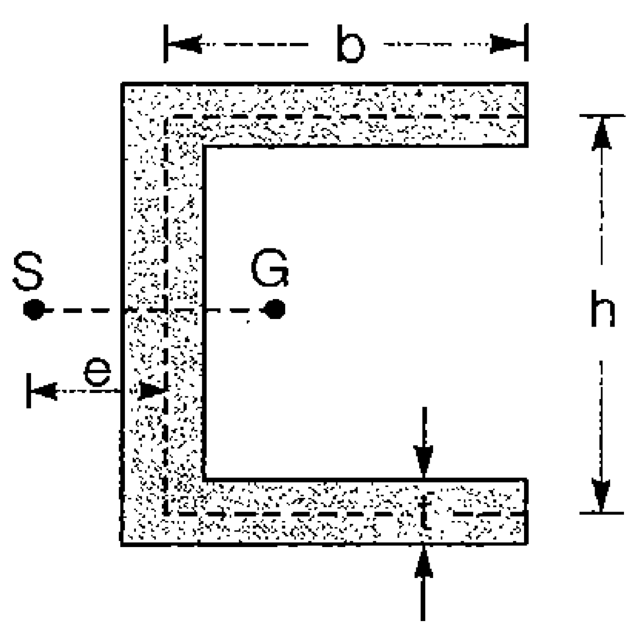
- Shear centre is a point from which a concentrated load passes then there will be only bending and no twisting. It is also called centre of flexure. It is that point through which the resultant of shear passes.
- Shear centre always lies on the axis of symmetry if exists.



S = shear centre, G = Centre of gravity

Distance of shear centre for important sections

1. Channel Section

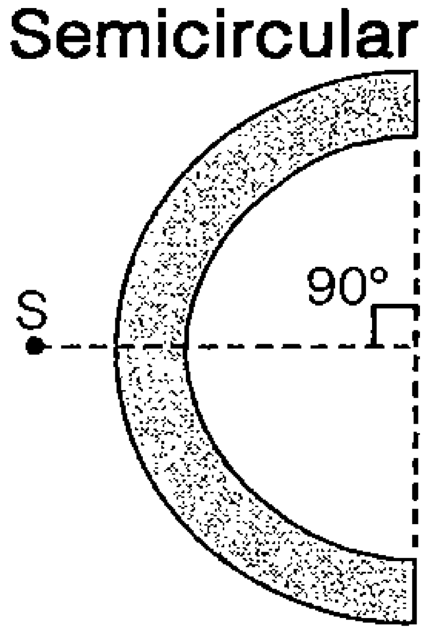


$$e = \frac{b^2h^2t}{4I}$$

t = thickness

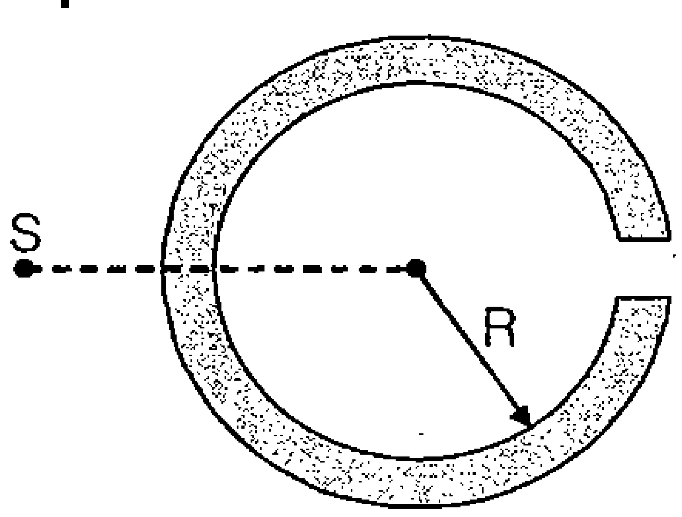
Where, I is MOI about symmetrical axis.

2. Semicircular Section



$$e = \frac{2R}{\pi/2} = \frac{4R}{\pi}$$ (> R)

3. Open Circular Slit



$e = 2R$ (> R)

> *If slit is closed then shear centre will coincide will CG*

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Columns

9

STRUT

Structural member subjected to axial compressive load is called strut.

- **Column:** Vertical structural member fixed at both ends and subjected to axial compressive load is called column.

Buckling Failure : Euler's Theory

- Assumptions in Euler's Theory
  (i) Axis of column is *perfectly straight* when *unloaded.*
  (ii) Load passes through axis
  (iii) Stress in structure are within *elastic limit.*
  (iv) Flexural rigidity is constant.
  (v) Material is isotropic, homogeneous and linear elastic.
  (vi) Column is long and prismatic and it fails only in buckling.
- Limitation of Euler's Formula
  (i) There is always *crookedness* in the column and the load may not be exactly axial.
  (ii) This formula does not take into account the axial stress and the buckling load given by this formula may be much more than the actual buckling load.

$$P_e = \frac{\pi^2 EI_{min}}{l_e^2}$$

P_e = Buckling load
I_{min} = Min. Moment of inertia about centroidal axis
l_e = ***Effective*** length

> It is applicable for long column. Effect of crushing is neglected.

	Column	Fails in
1.	Short column	Crushing
2.	Long column	Buckling
3.	Intermediate column	Combined Crushing and Buckling

Euler's load for different column with different end Condition

End condition	Both end hinged	One end fixed other free	Both end fixed	One end fixed and other hinged
Effective length(l_e)	L	2L	$\frac{L}{2}$	$\frac{L}{\sqrt{2}}$

Slenderness Ratio (λ)

Slenderness ratio of a compression member is defined as the ratio of its effective length to least radius of gyration.

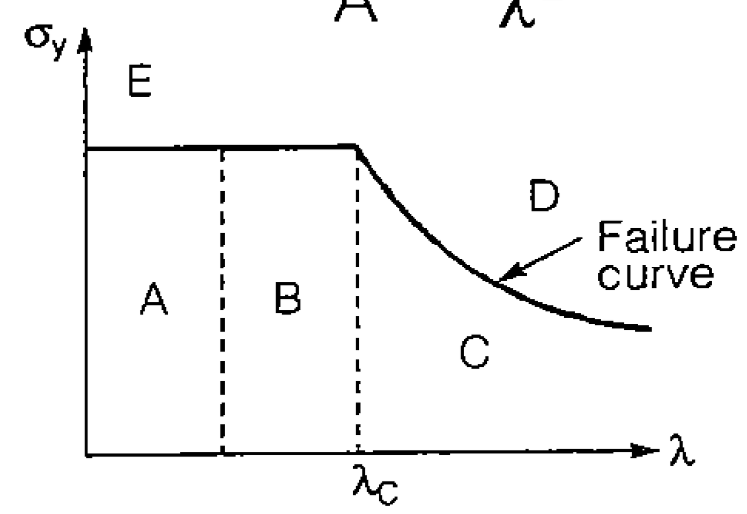
$$\lambda = \frac{L_e}{r_{min}}$$

$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$

L_e = Effective length

r_{min} = Least radius of gyration

$\therefore$ Buckling stress $(\sigma_b) = \frac{P_e}{A} = \frac{\pi^2 E}{\lambda^2}$



For validity of Euler's theory

$\sigma_e \le \sigma_y$

$$\Rightarrow \lambda \ge \sqrt{\frac{\pi^2 E}{\sigma_y}}$$

$\lambda_c = 90$ for Mild steel

Here, σ_y = Permissible stress
D = Unsafe long column
A = Safe short column
λ_c = Critical slenderness ratio
C = Safe long column
B = Intermediate safe column
E = Unsafe short column

Rankine's Formula

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_e}$$

Rankine load = P_R

Crushing load = $P_C = \sigma_c \times A$

Buckling load = $P_e = \frac{\pi^2 EI_{min}}{L_e^2}$, $P_e = \frac{\pi^2 EA}{\lambda^2}$

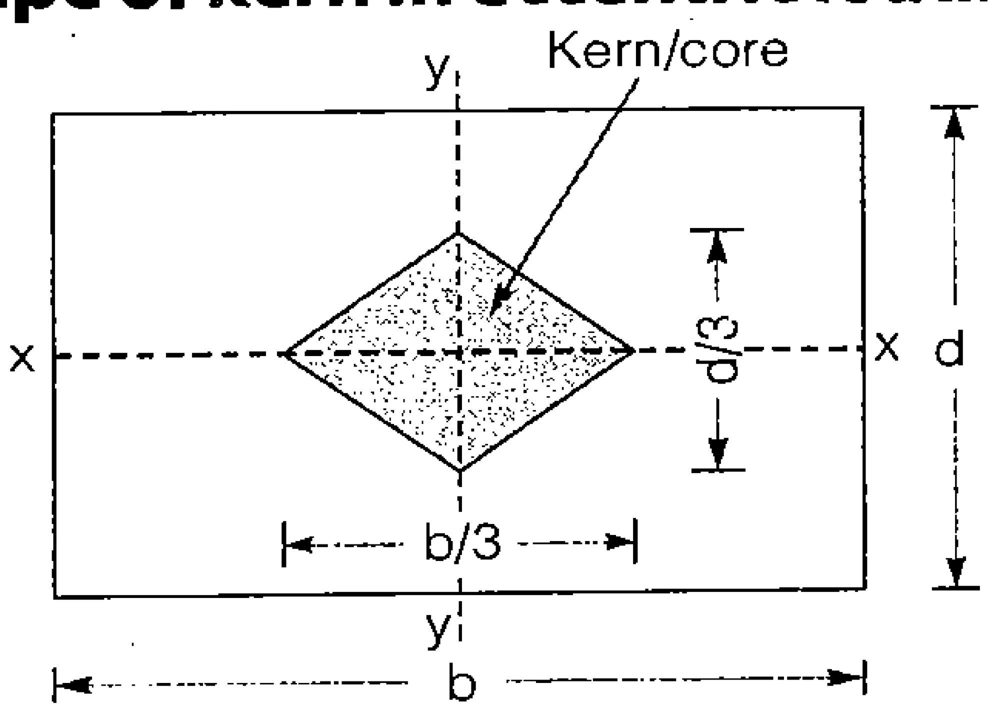
$$\therefore \quad P_r = \frac{A\sigma_c}{1+\left(\frac{\sigma_c}{\pi^2 E}\right)\lambda^2} \rightarrow P_r = \frac{\sigma_c A}{1+\alpha\lambda^2}$$

Here, A = Area of column

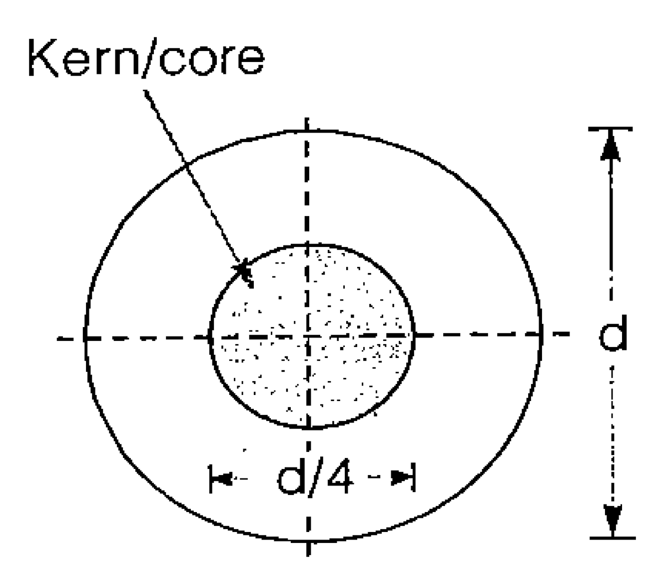
$\alpha = \frac{\sigma_c}{\pi^2 E}$ = Rankine's constant

> This formula is applicable to any column.
> Effect of both crushing and buckling is considered in this formula.

Shape of kern in eccentric loadings



Rectangular Column



Circular Column

Shape of kern for rectangular and I-section is Rhombus and for square section shape is square for circular section shape is circular.

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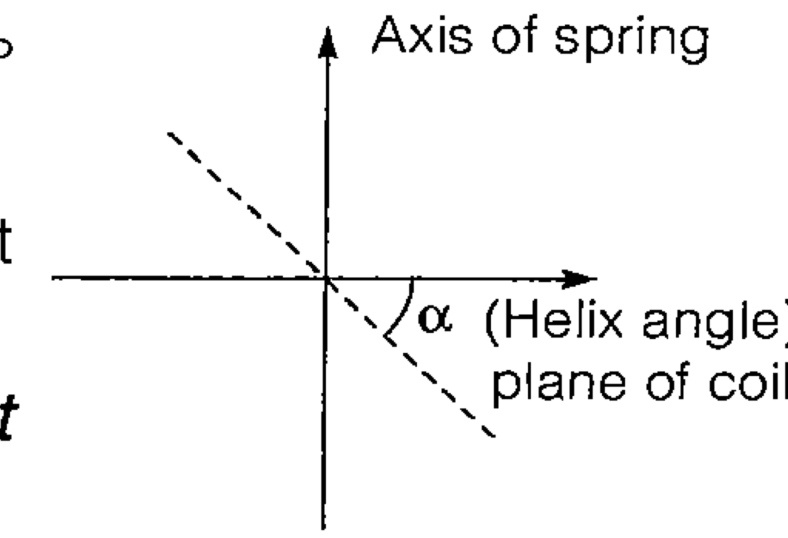
10 Springs

Spring are used to absorb energy and restore it slowly or rapidly.

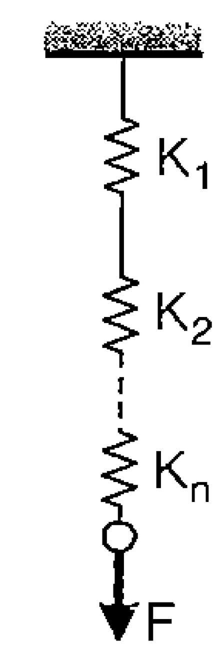
Type of spring on the basis of helix angle

- If helix angle is ***less than*** or ***equal*** to 10° then it is called closed coil spring.
- If helix angle is ***greater than*** 10° then it is called open coil spring.
- The best form of spring absorbs ***greatest amount*** of energy for a given stress.
- Spring stores energy in the form of ***resilience***.

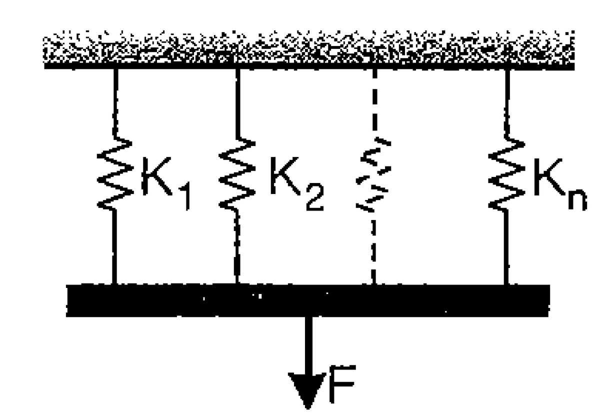


Series and parallel arrangement of springs/Equivalent spring constant (k_{eq})

- In Series: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots \frac{1}{k_n}$



- In parallel: $k_{eq} = k_1 + k_2 + \cdots k_n$



> - Stiffness of spring is inversely proportional to number of coils in the spring. Therefore when a spring is cut into two parts its stiffness become ***double*** for every ***individual*** part.
> - Springs are added just like as capacitors in electronics.Both does the same work i.e., absorbs energy.

Closed coil helical spring under axial pull

(i) $\tau_{max} = \frac{16PR}{\pi d^3}$

(ii) Strain energy stored in spring

$$U = \frac{T^2L}{2GI_P} = \frac{32P^2R^3n}{Gd^4}$$

(iii) Axial deflection under load P

$$\frac{\partial U}{\partial P} = \Delta = \frac{64PR^3n}{Gd^4}$$

(iv) Coefficient of stiffness of spring (k)

$$k = \frac{P}{\Delta} = \frac{Gd^4}{64R^3n} \qquad k \propto \frac{1}{n}$$

Remember

Spring index (C) = $\frac{D}{d}$

Wahl's factor is considered to consider the effect of direct shear stress and curvature effect.

Strain energy stored in spring (U)

$$U = \frac{1}{2} T \cdot \theta$$

T = torque applied

θ = angular deflection

Wahl's correction factor (k_w)/Stress concentration factor (k_c)

$$k_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad k_c = \frac{4C-1}{4C-4}$$

Here, C = Spring index

Remember

The ***average*** value of modulus of rigidity for ***steel*** used for spring equal to ***79300 MPa***.

Shot peening, result in raising the ***fatigue*** life of spring because it leave the surface in compression.

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A Handbook on
Civil Engineering

2

Structural Analysis

CONTENTS

Static and Kinematic Indeterminacy

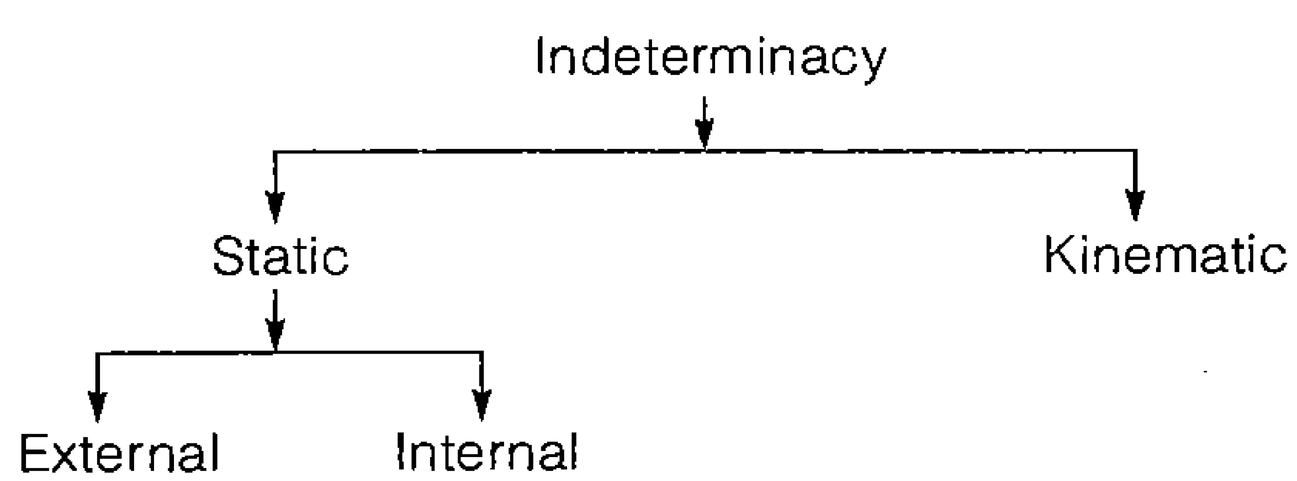
1

Statically Determinate Structures

Conditions of equilibrium are sufficient to analyse the structure. Bending moment and shear force is independent of the cross-sectional area of the components and flexural rigidity of the members. No stresses are caused due to temperature change. No stresses are caused due to lack of fit or differential settlement.

Statically Indeterminate Structures

Additional compatibility conditions are required. Bending moment and shear force depends upon the cross-sectional area and flexural rigidity of the members. Stresses are caused due to temperature variation. Stresses are caused due to lack of fit or differential settlement.



Static Indeterminacy

If a structure can not be analyzed for external and internal reactions using static equilibrium conditions alone then such a structure is called indeterminate structure.

(i) $\boxed{D_s = D_{se} + D_{si}}$

where, D_s = Degree of static indeterminacy
D_{se} = External static indeterminacy
D_{si} = Internal static indeterminacy

- **External static indeterminacy:**
It is related with the support system of the structure and it is equal to number of external reaction components in addition to number of static equilibrium equations.

(ii) $\boxed{D_{se} = r_e - 3}$ For 2D

$\boxed{D_{se} = r_e - 6}$ For 3D

where, r_e = total external reactions

- **Internal static indeterminacy:**
It refers to the geometric stability of the structure. If after knowing the external reactions it is not possible to determine all internal forces/internal reactions using static equilibrium equations alone then the structure is said to be internally indeterminate.
For geometric stability sufficient number of members are required to preserve the shape of rigid body without excessive deformation.

(iii) $\boxed{D_{si} = 3C - r_r}$ ... For 2D

$\boxed{D_{si} = 6C - r_r}$ ... For 3D

where, C = number of closed loops.
and r_r = released reaction.

(iv) $\boxed{r_r = \Sigma(m_{j'} - 1)}$ ... For 2D

$\boxed{r_r = 3\Sigma(m_{j'} - 1)}$ ... For 3D

where $m_{j'}$ = number of member connecting with J′ number of joints.
and J′ = number of hybrid joint.

(v) $\boxed{D_s = m + r_e - 2j}$ ... For 2D truss

$\boxed{D_{se} = r_e - 3}$ & $\boxed{D_{si} = m - (2j - 3)}$

(vi) $\boxed{D_s = m + r_e - 3j}$ ... For 3D truss

$\boxed{D_{se} = r_e - 6}$ & $\boxed{D_{si} = m - (3j - 6)}$

(vii) $\boxed{D_s = 3m + r_e - 3j - r_r}$ ... 2D Rigid frame.

(viii) $\boxed{D_s = 6m + r_e - 6j - r_r}$ ... 3D rigid frame.

(ix) $\boxed{D_s = (r_e - 6) + (6C - r_r)}$ ... 3D rigid frame

Kinematic Indeterminacy

If the number of unknown displacement components are greater than the number of compatibility equations, for these structures additional equations based on equilibrium must be written in order to obtain sufficient number of equations for the determination of all the unknown displacement components. The number of these additional equations necessary is known as degree of kinematic indeterminacy or degree of freedom of the structure.

A fixed beam is kinematically determinate and a simply supported beam is kinematically indeterminate.

(i) Each joint of plane pin jointed frame has 2 degree of freedom.

(ii) Each joint of space pin jointed frame has 3 degree of freedom.

(iii) Each joint of plane rigid jointed frame has 3 degree of freedom

(iv) Each joint of space rigid jointed frame has 6 degree of freedom.

Degree of kinematic indeterminacy is given by:

(i) $\boxed{D_k = 3j - r_e}$ ...For 2D Rigid frame when all members are axially extensible.

(ii) $\boxed{D_k = 3j - r_e - m}$ ...For 2D Rigid frame if 'm' members are axially rigid/inextensible.

(iii) $\boxed{D_k = 3(j + j') - r_e - m + r_r}$ ... For 2D Rigid frame when J′ = Number of Hybrid joints is available.

(iv) $\boxed{D_k = 6(j + j') - r_e - m + r_r}$ ...For 3D Rigid frame.

(v) $\boxed{D_k = 2(j + j') - r_e - m + r_r}$ ... For 2D Pin jointed truss.

(vi) $\boxed{D_k = 3(j + j') - r_e - m + r_r}$ ... For 3D Pin jointed truss.

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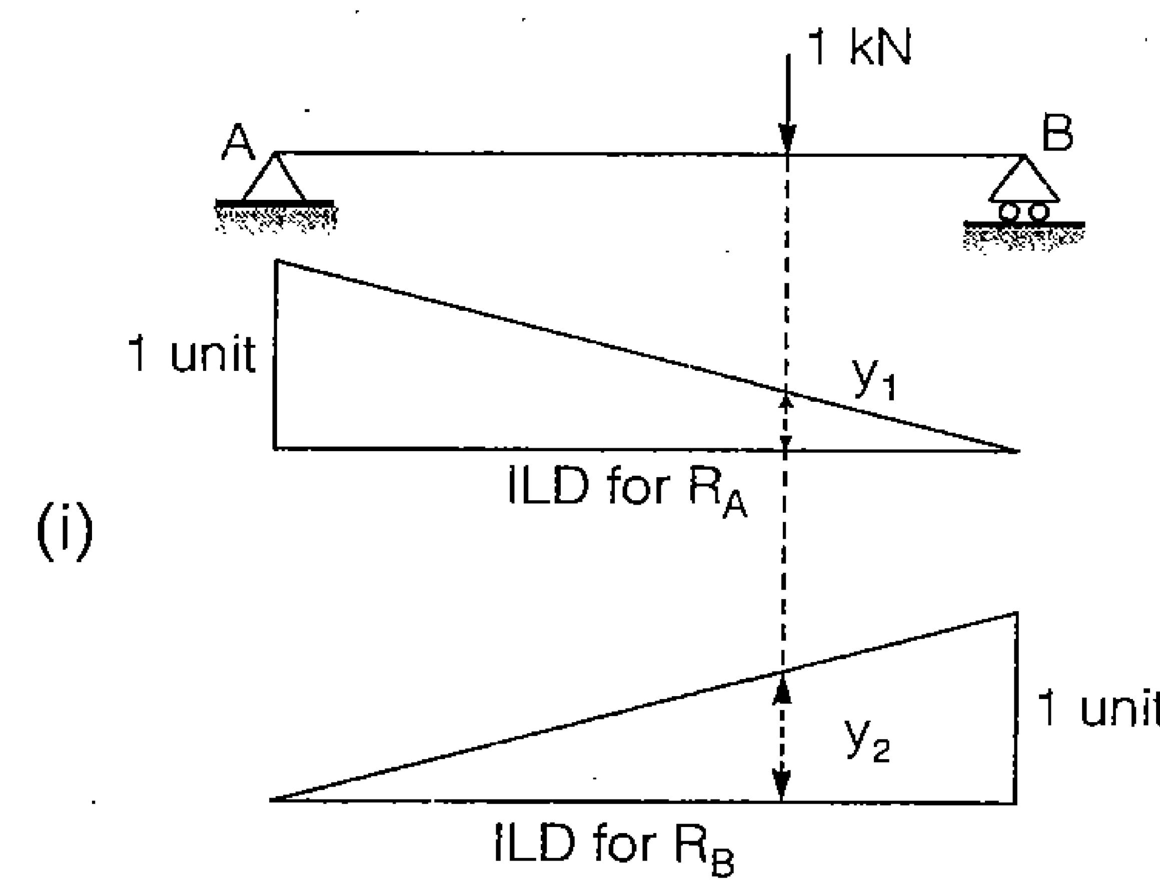
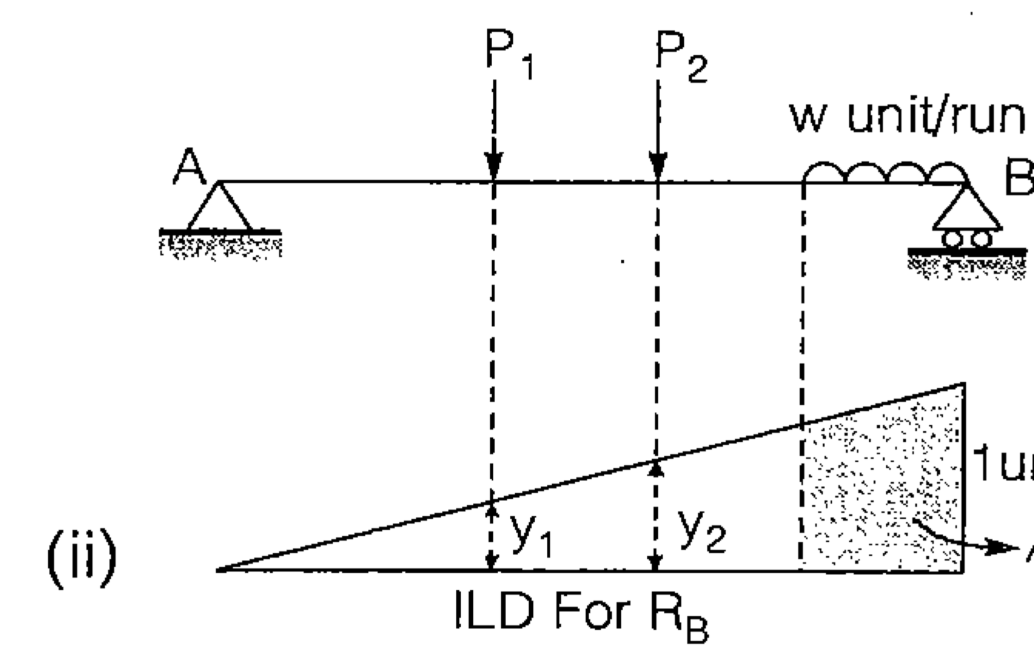
Influence Line Diagram and Rolling Loads 2

Influence Line Diagram represents variation in the values of a particul stress function such as reactions, SF, BM, axial force, slope or deflectio etc., when a unit concentrated load moves from one end to the other end span.

Simply Supported Beam

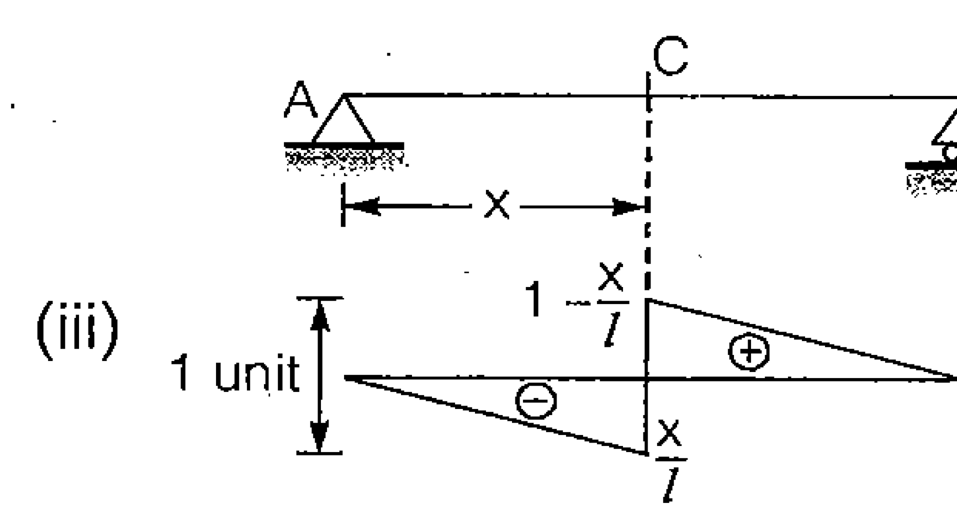
The ordinate of influence line at any section (say y_1) represents magnituc of that stress function when unit concentrated load is placed at that sectio



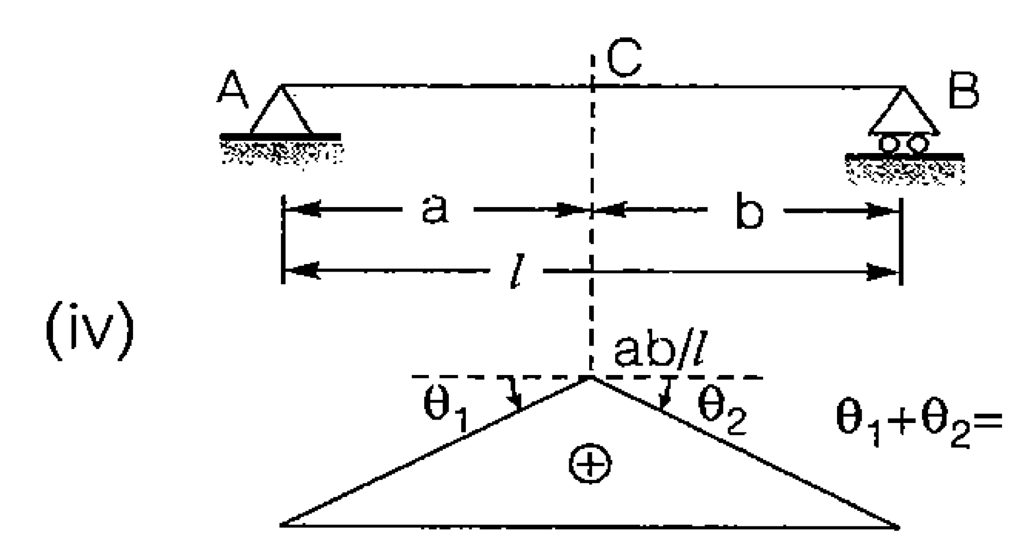


Total vertical reactions RB due to given load system is

$$\boxed{R_B = P_1Y_1 + P_2Y_2 + wA}$$

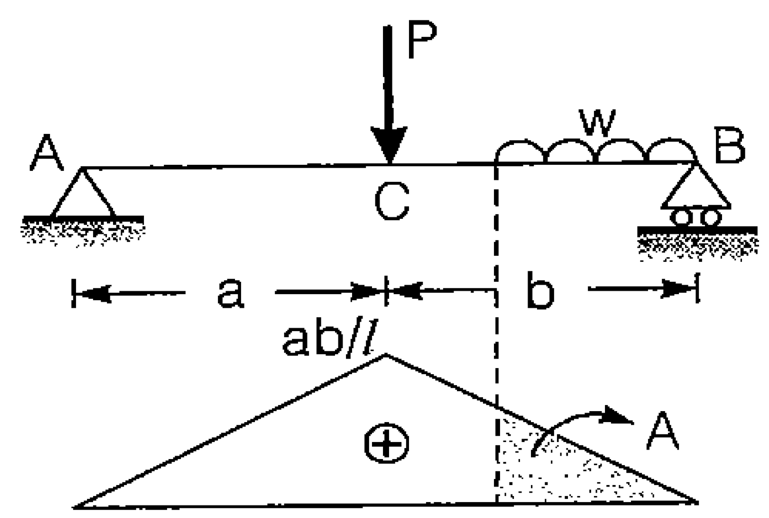


ILD for shear force at C



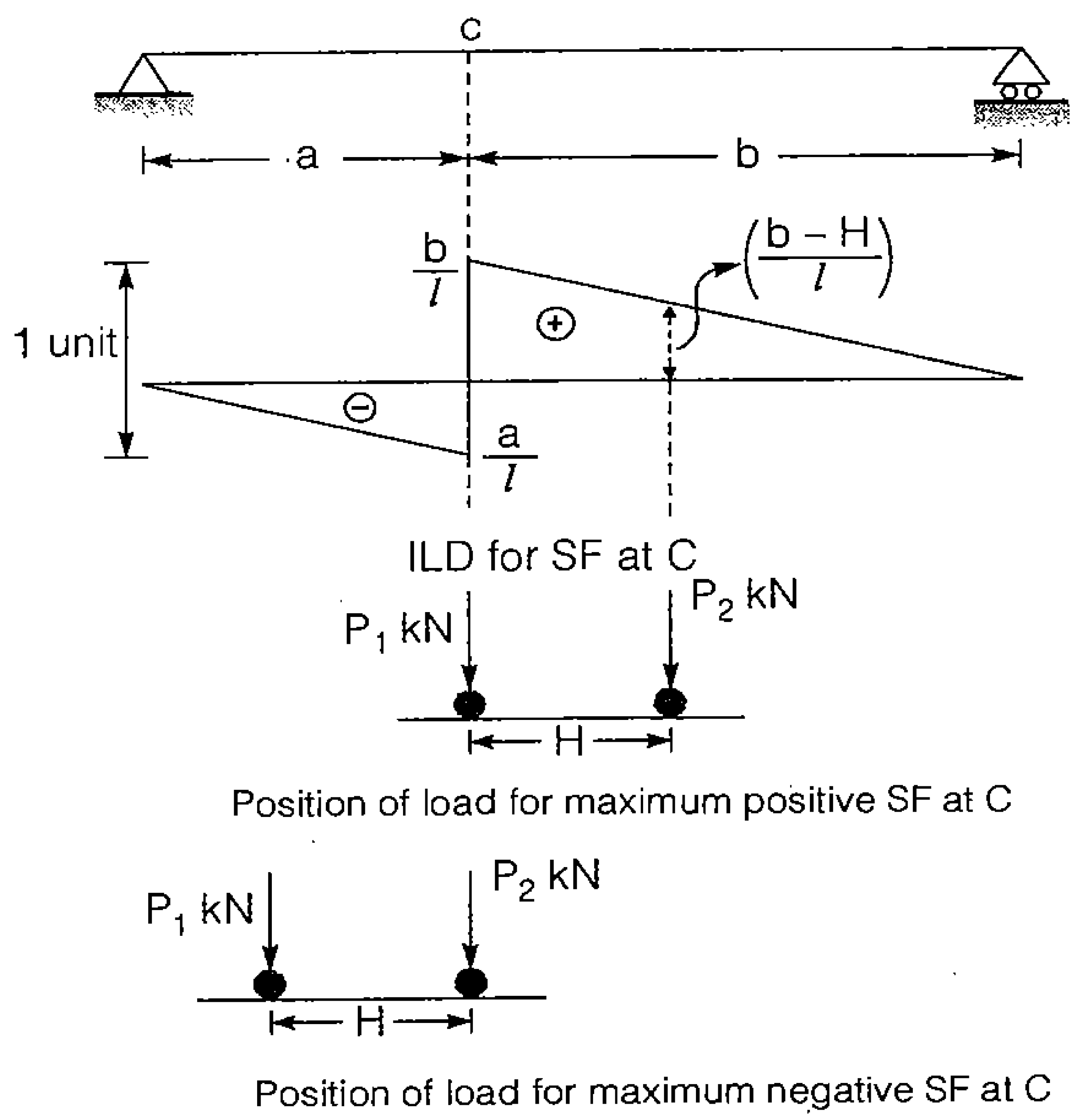
ILD for Bending Moment at C

(v)



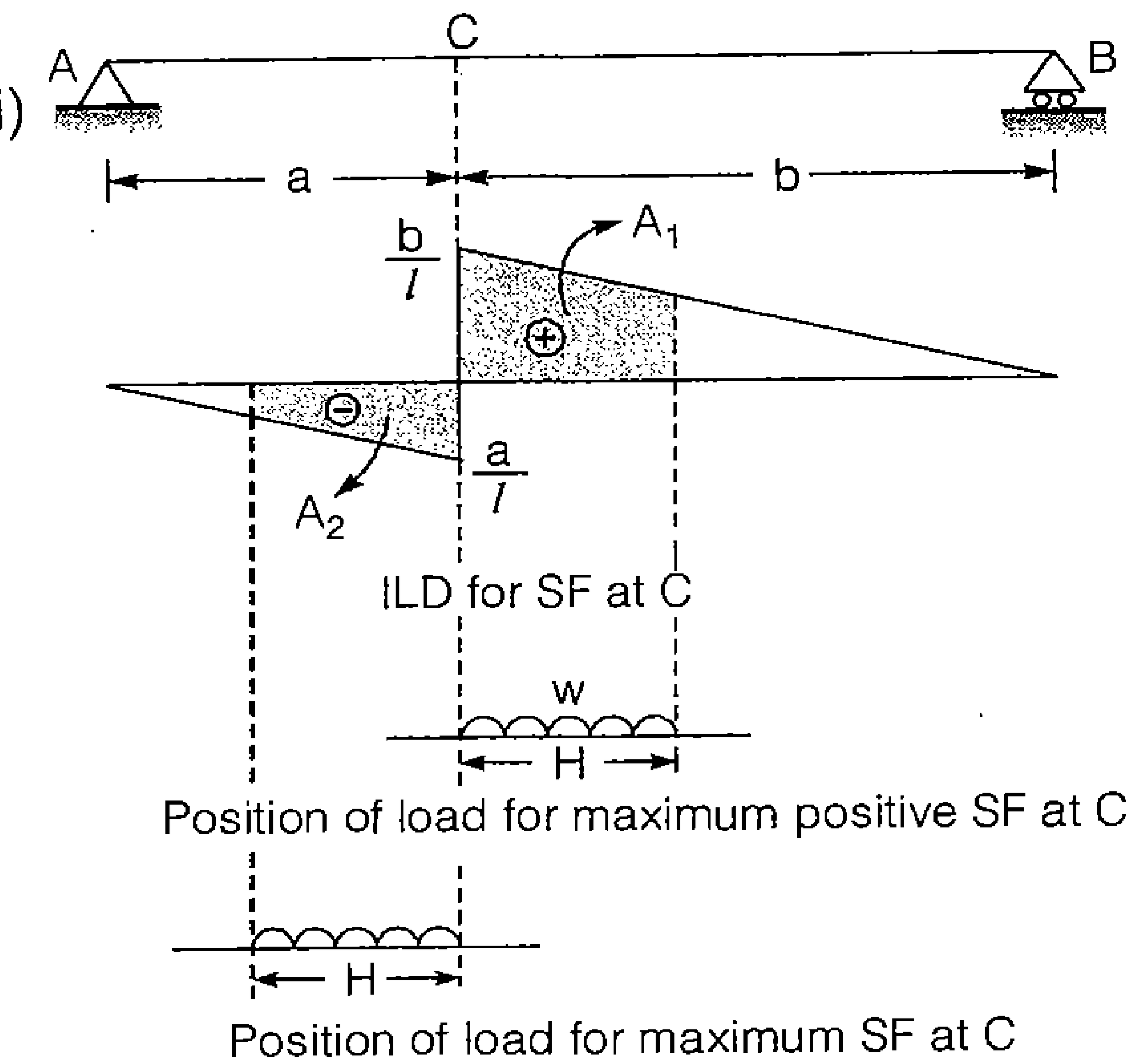
$$\boxed{BM_{max} \text{ at C} = P \cdot \frac{ab}{l} + wA}$$

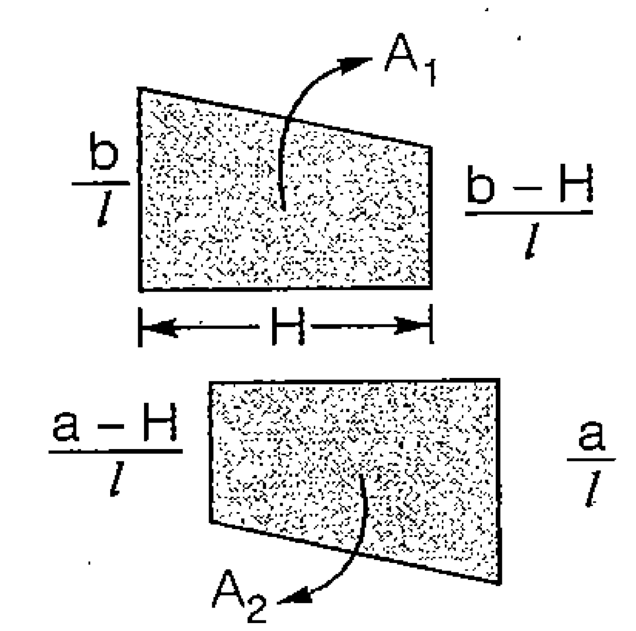
(vi)



Maximum + ve SF at C $= P_1\left(\frac{b}{l}\right) + P_2\left(\frac{b-H}{l}\right)$
Maximum – ve S.F at C $= -P_1\left(\frac{a}{l}\right) - P_2\left(\frac{a-H}{l}\right)$

(vii)

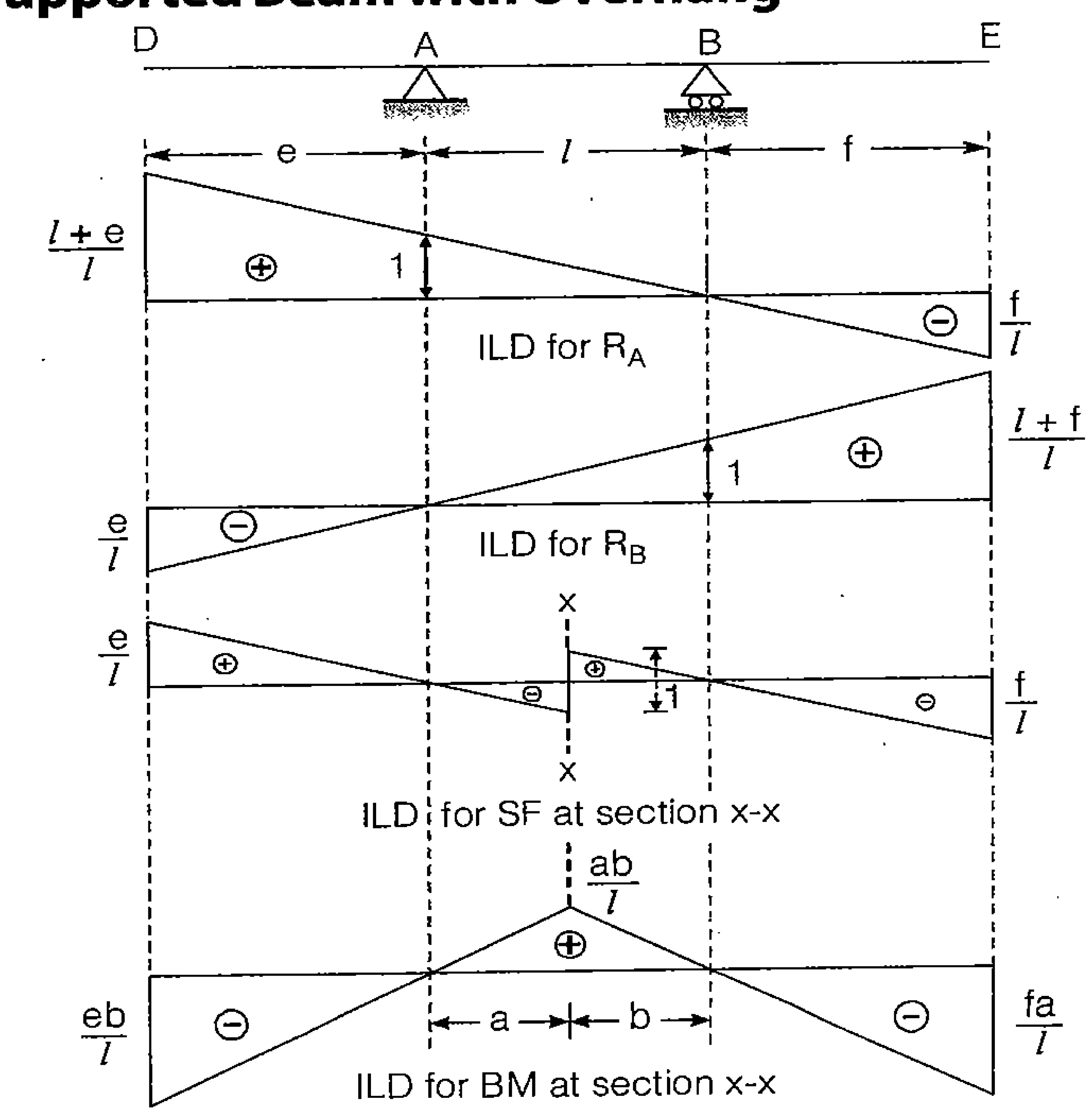




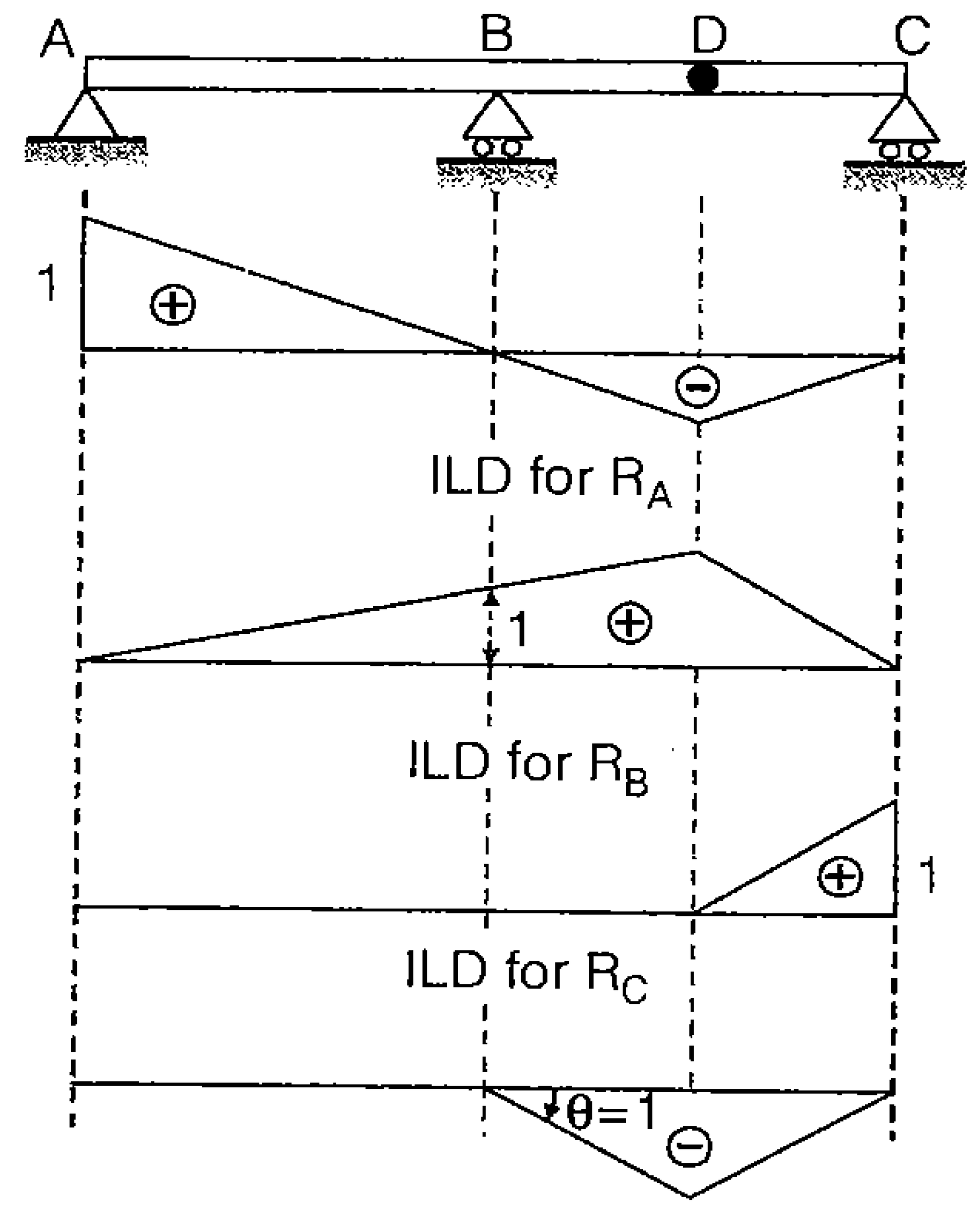
$$\boxed{\text{Maximum positive SF at C} = wA_1}$$

$$\boxed{\text{Maximum negative SF at C} = wA_2}$$

Simply Supported Beam with Overhang



Simply Supported Beam Carrying Internal Hinge

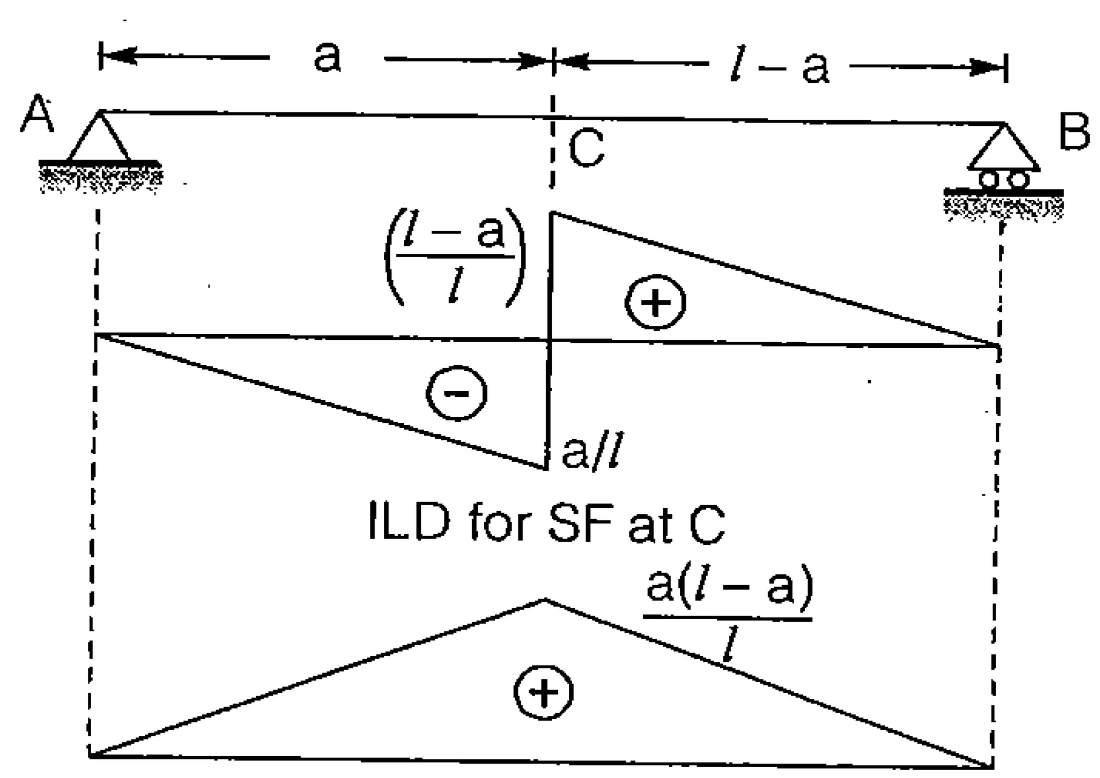


ILD for BM at B

- The ordinate of ILD for reaction and shear force is dimensionless. Whereas ordinate of BM in ILD has dimension of length.
- ILD for reaction, shear force and BM is linear for determinate structures and will be curved for indeterminate structures.

Effect of rolling loads

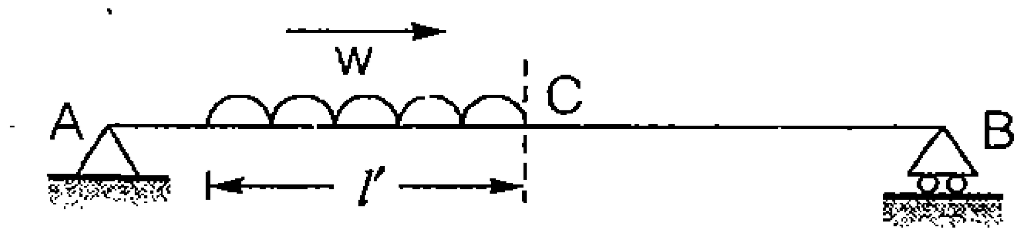
To find maximum SF and BM at section C when a UDL passes over the girder from let to right having length less then length of girder.



ILD for BM at C

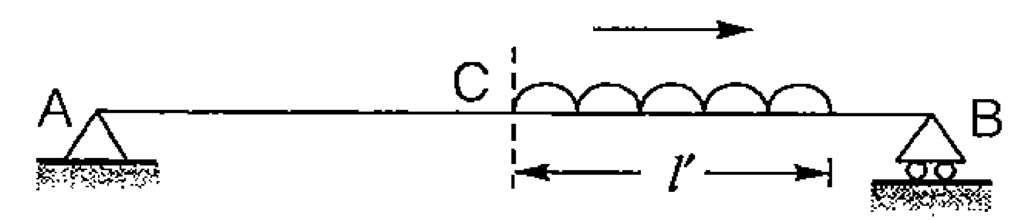
Length of load is l' (<L)

1. The maximum negative shear force at section C will be, when head of the load is just to the left of section and loaded part is AC.



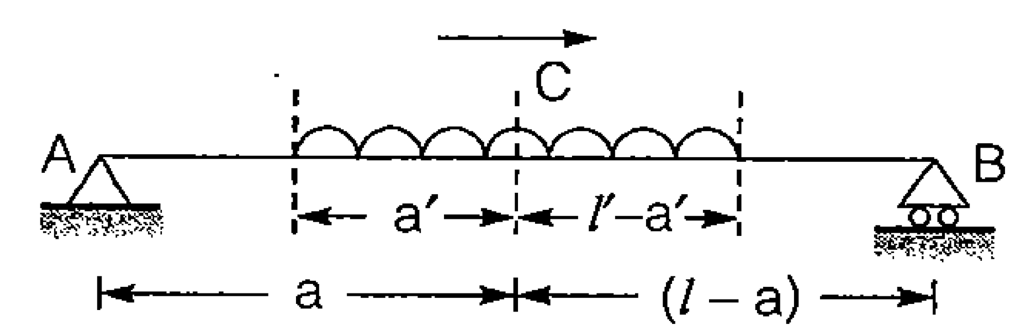
maximum negative shear force at C = w × area of ILD below loading

2. Maximum positive shear force at C will occur when tail of load is just to the right of C.



3. Maximum BM at C will occur when

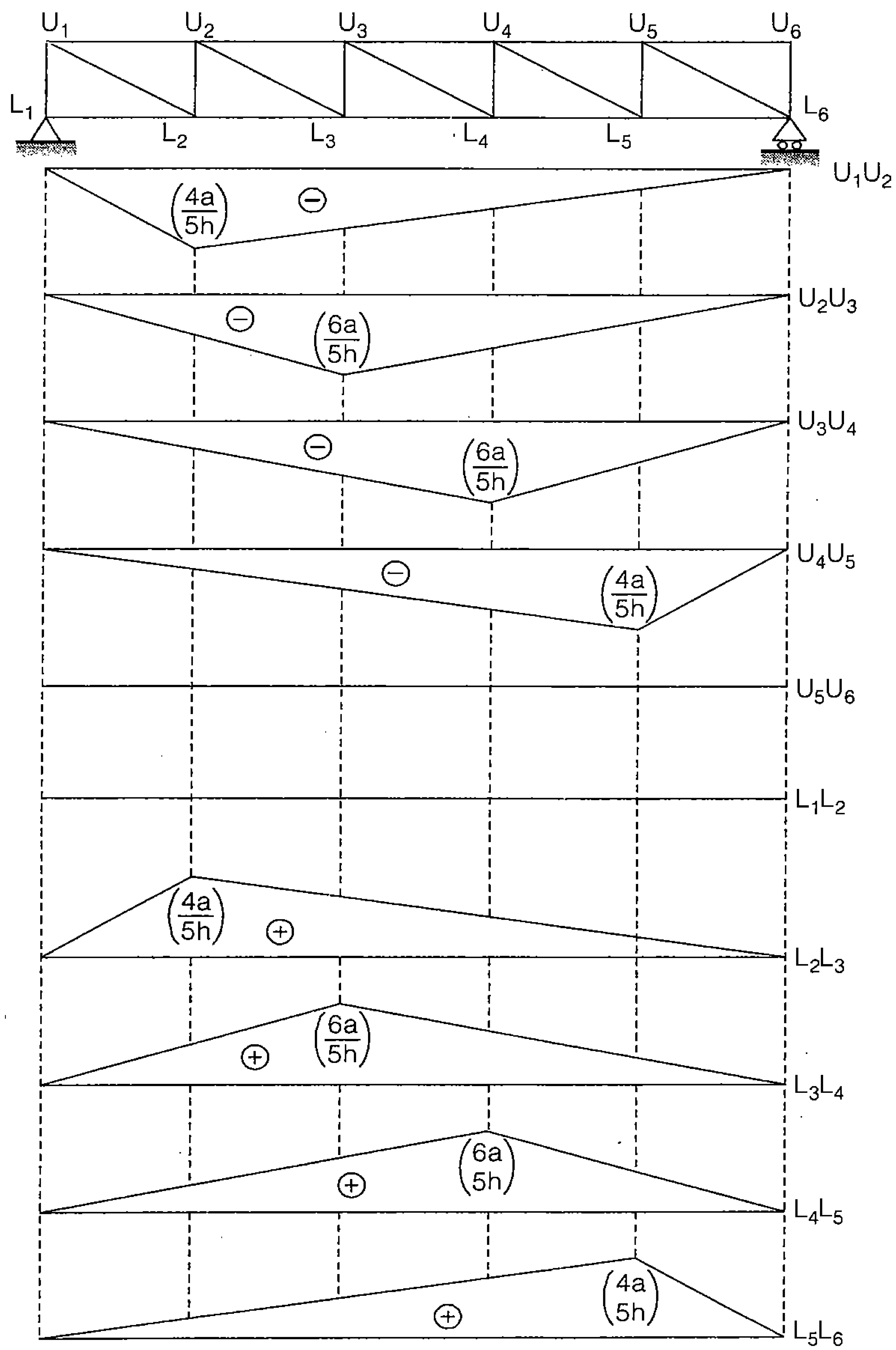
average loading just to the left of C = average loading just to the right of C. It means section C will devide load in the same ratio as it devides to the span.



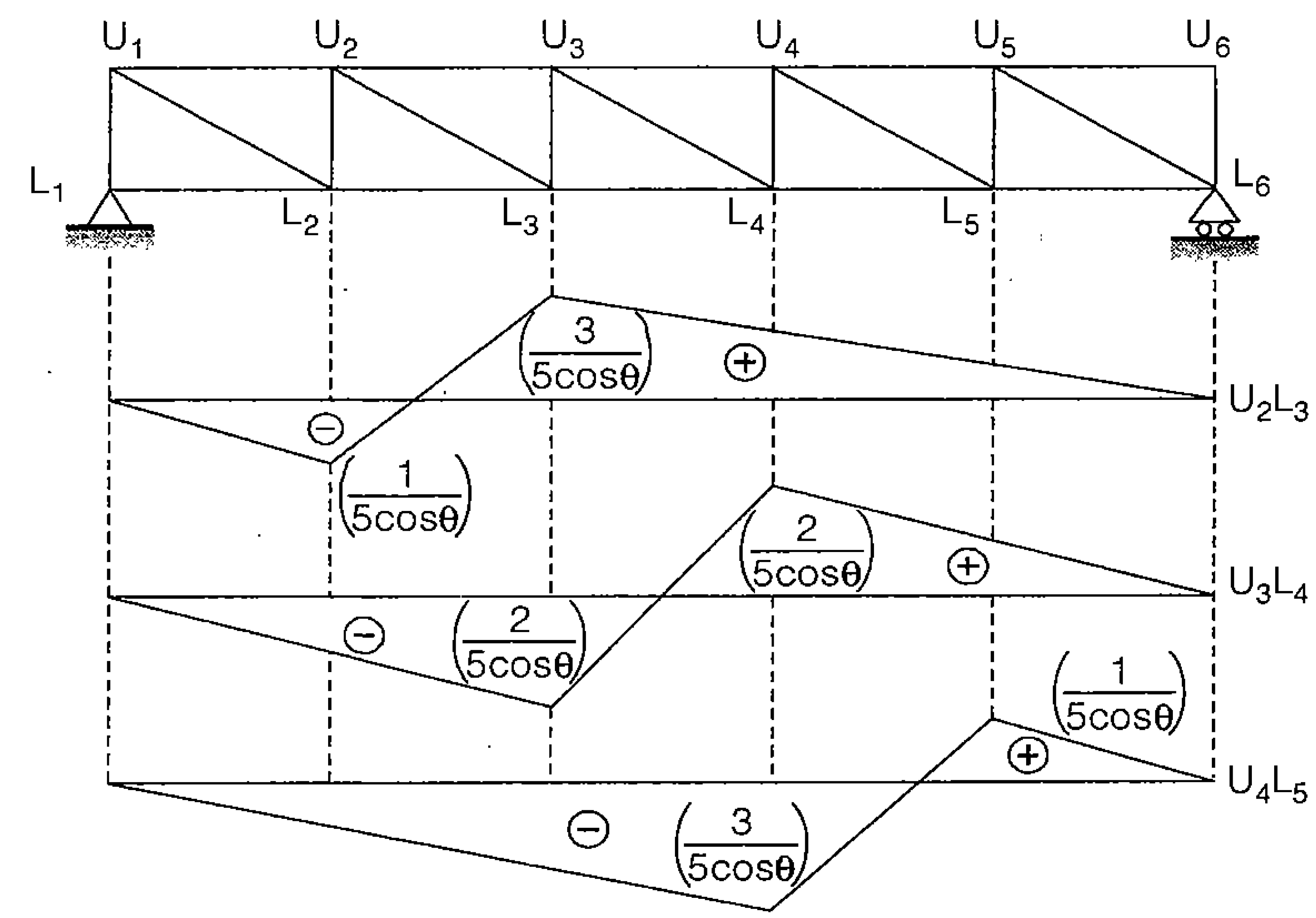
$$\left(\frac{a'}{a}\right) = \left(\frac{l' - a'}{l - a}\right)$$

- To obtain absolute max BM due to UDL average loading to the left of span = average loading to the right of span. It means load should be placed symmetrically at the centre.
- Absolute max SF will occur either at left support A or at right support B when load is adjacent to the support.

Influence line diagram for truss members



ILD for Inclined members



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Arches

3

Three Hinged Arches

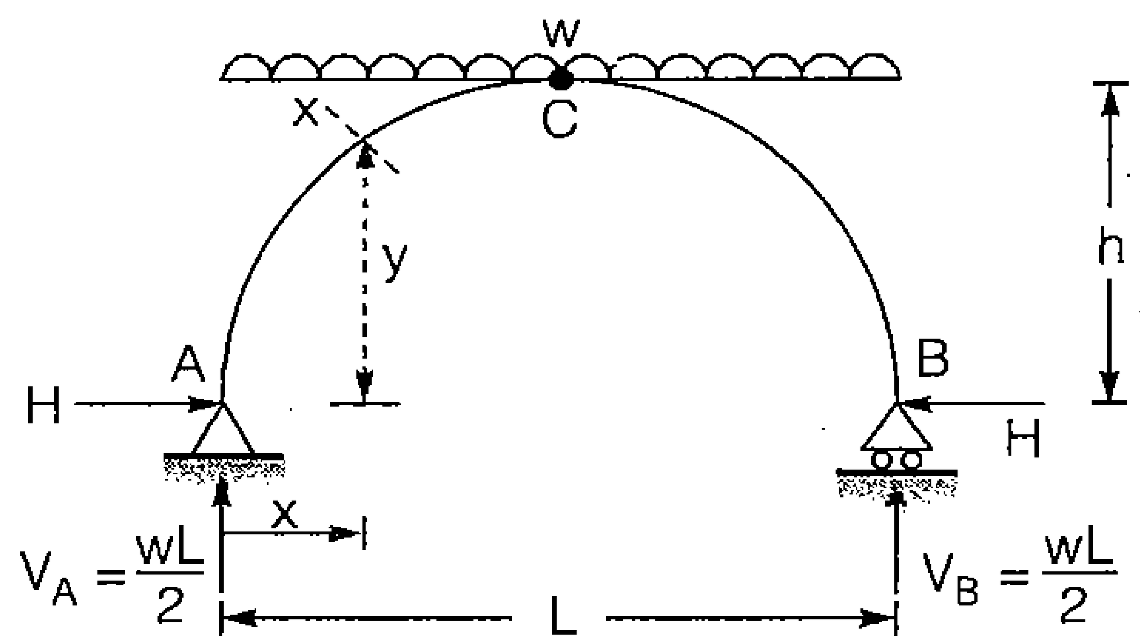
(i) Three Hinged Parabolic Arch of Span L and rise 'h' carrying a UDL over the whole span.

$D_s = 0$

$BM_c = 0$

$H = \frac{wl^2}{8h}$

$M_x = V_A x - \frac{wx^2}{2} - Hy$



where, H = Horizontal thrust

V_A = Vertical reaction at A = $\frac{wl}{2}$

$\left(V_A x - \frac{wx^2}{2}\right)$ = Simply supported beam moment i.e., moment caused by vertical reactions.

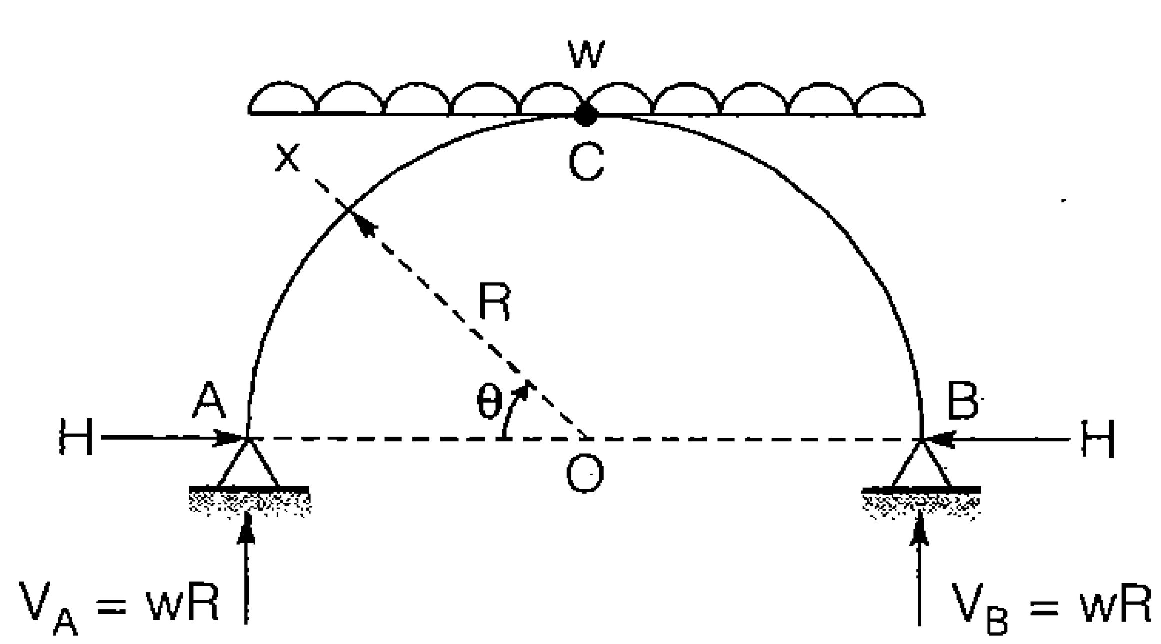
Hy = H-moment

D_s = Degree of static indeterminacy

BM_c = Bending Moment at C.

(ii) Three Hinged Semicircular Arch of Radius R carrying a UDL over the whole span.

$H = \frac{wR}{2}$

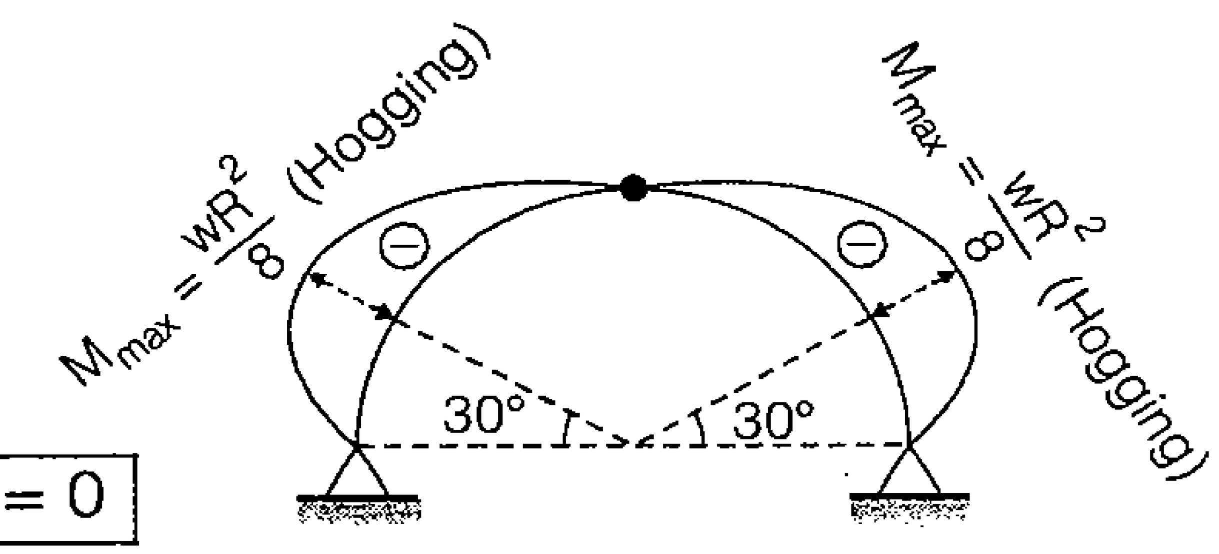


$M_x = \frac{-wR^2}{2}[\sin\theta - \sin^2\theta]$

$M_{max} = \frac{-wR^2}{8}$

$BM_C = 0$

Point of contraflexure = 0



(iii) Three Hinged Parabolic Arch Having Abutments at Different Levels

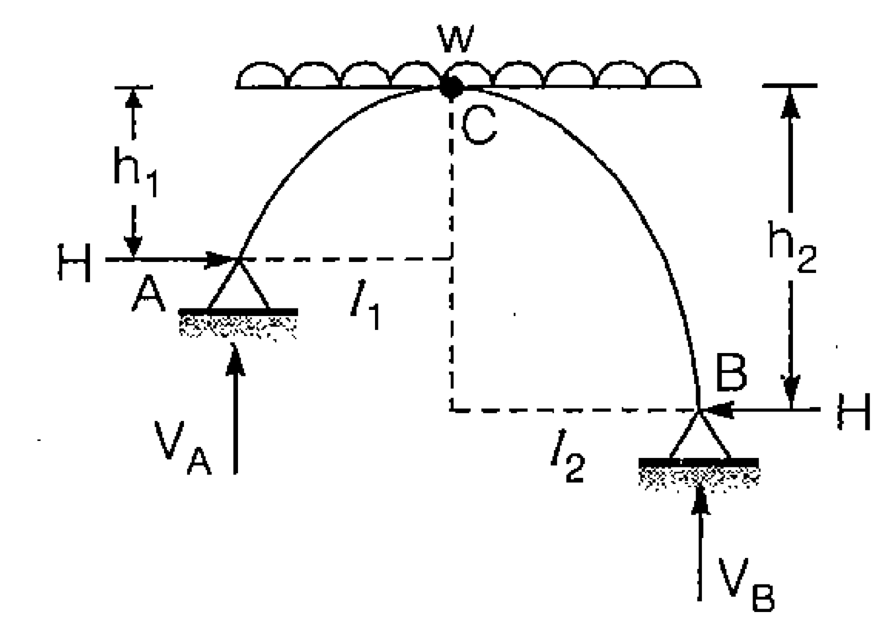
(a) When it is subjected to UDL over whole span.

$H_A = H_B = \frac{wl^2}{2\left(\sqrt{h_1} + \sqrt{h_2}\right)}$

$l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}$

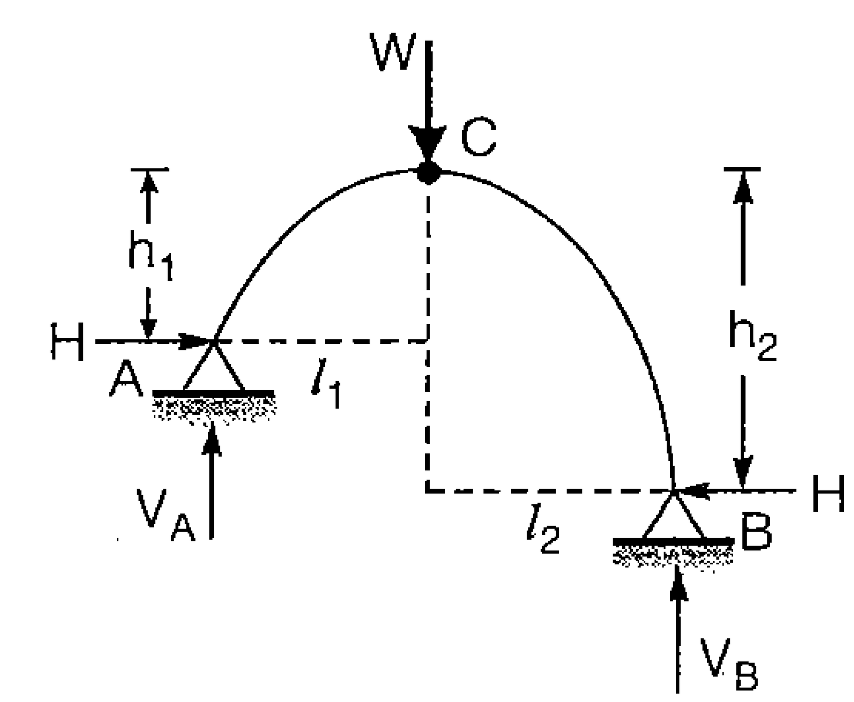
$l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}$

$BM_c = 0$



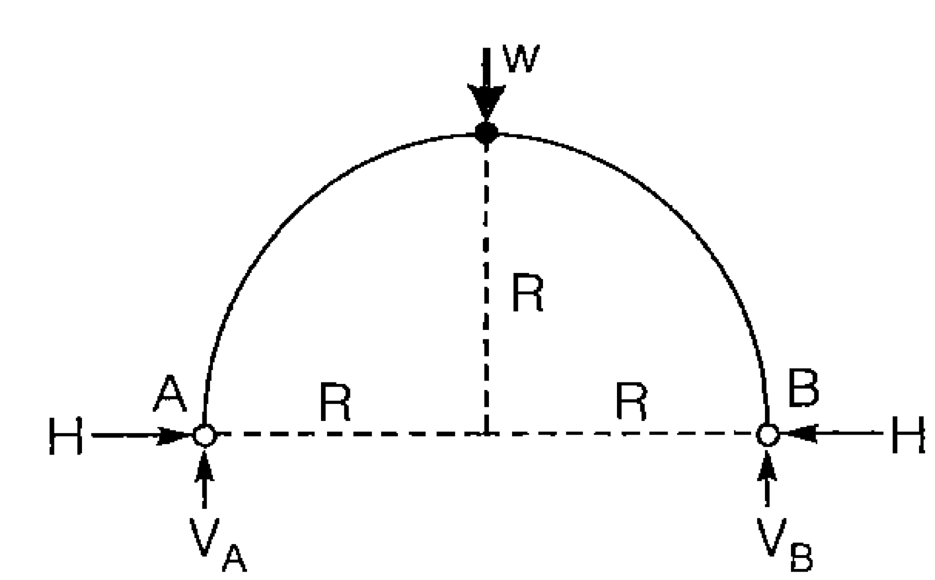
(b) When it is subjected to concentrated load W at crown.

$H = \frac{wl}{\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$



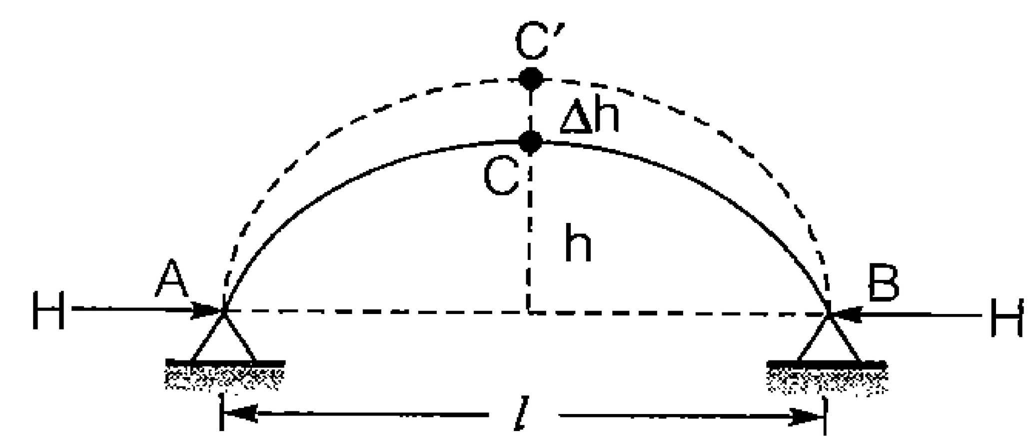
(iii) Three Hinged Semicircular Arch Carrying Concentrated Load W at Crown

$H = V_A = V_B = \frac{w}{2}$



Temperature Effect on Three Hinged Arches

(i) $\Delta h = \left(\frac{l^2 + 4h^2}{4h}\right)\alpha T$



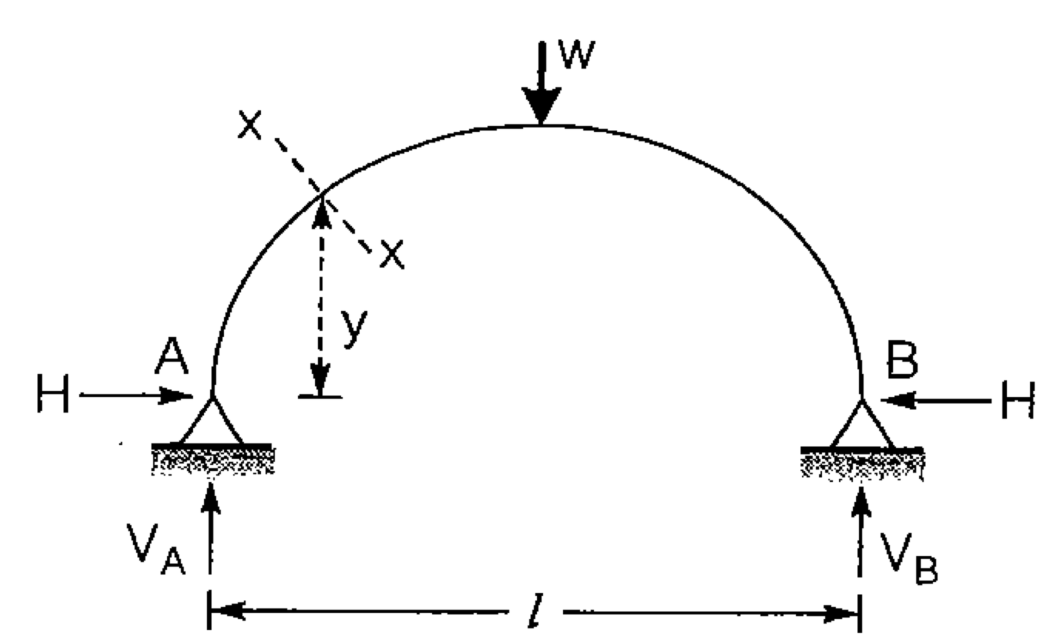
where, Δh = free rise in crown height
l = length of arch
h = rise of arch
α = coefficient of thermal expansion
T = rise in temperature in °C

(ii) $H \propto \frac{1}{h}$ where, H = horizontal thrust
and h = rise of arch.

(iii) $\% \text{ Decrease in horizontal thrust} = \frac{\delta h}{h} \times 100$

Two Hinged Arches

$$H = \frac{\int My \frac{ds}{EI}}{\int \frac{y^2 ds}{EI}}$$



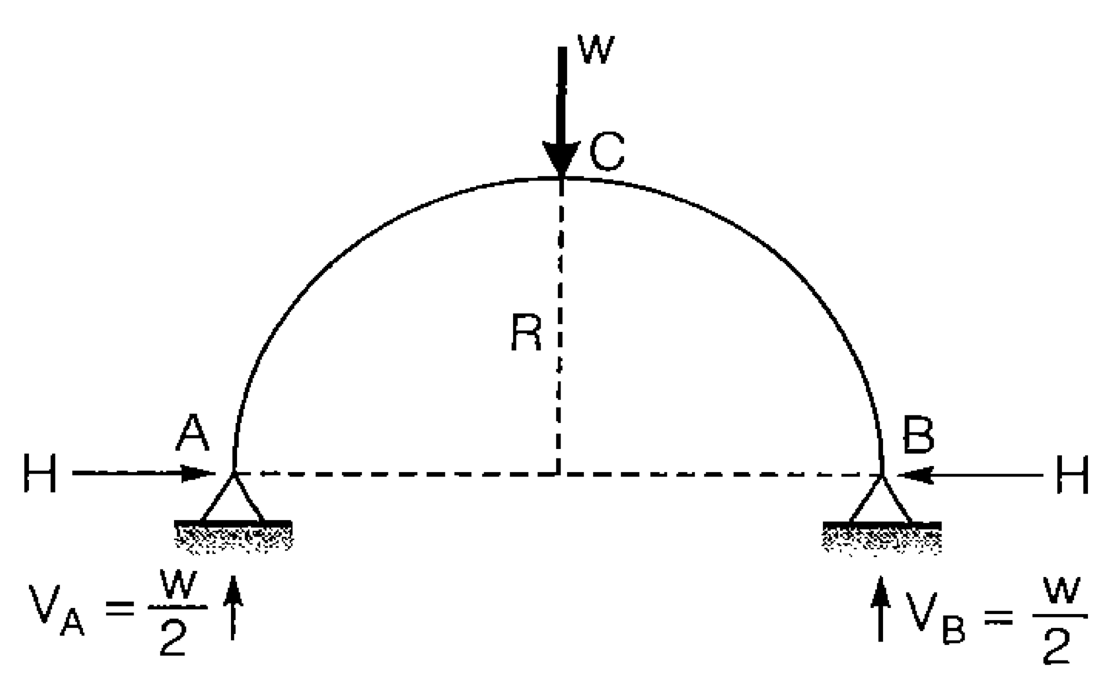
Two hinged arch of any shape

$D_s = 1$

where, M = Simply supported Beam moment caused by vertical forces.

(i) Two hinged semicircular arch of radius R carrying a concentrated load 'w' at the crown.

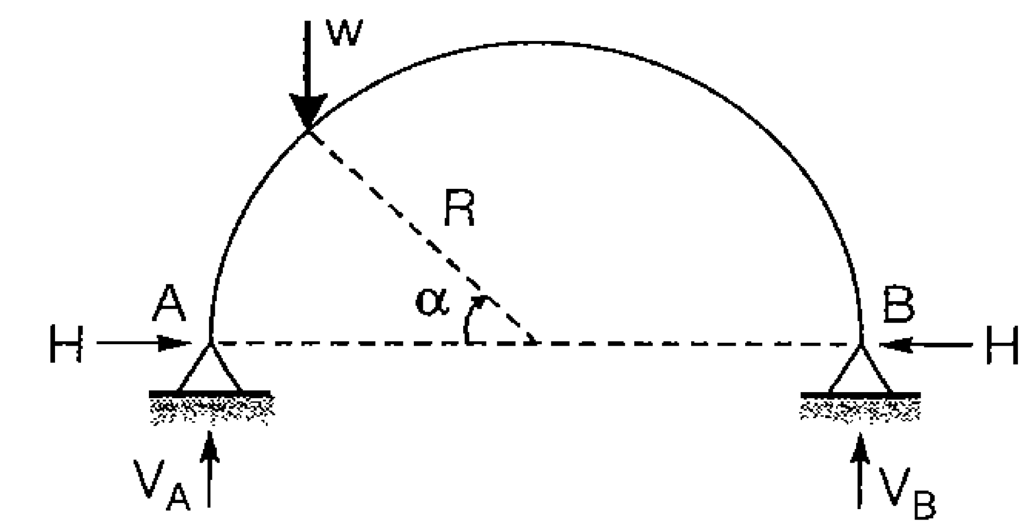
$H = \frac{w}{\pi}$



Two hinged circular arch

(ii) Two hinged semicircular arch of radius R carrying a load w at a section, the radius vector corresponding to which makes an angle α with the horizontal.

$H = \frac{w}{\pi}\sin^2\alpha$

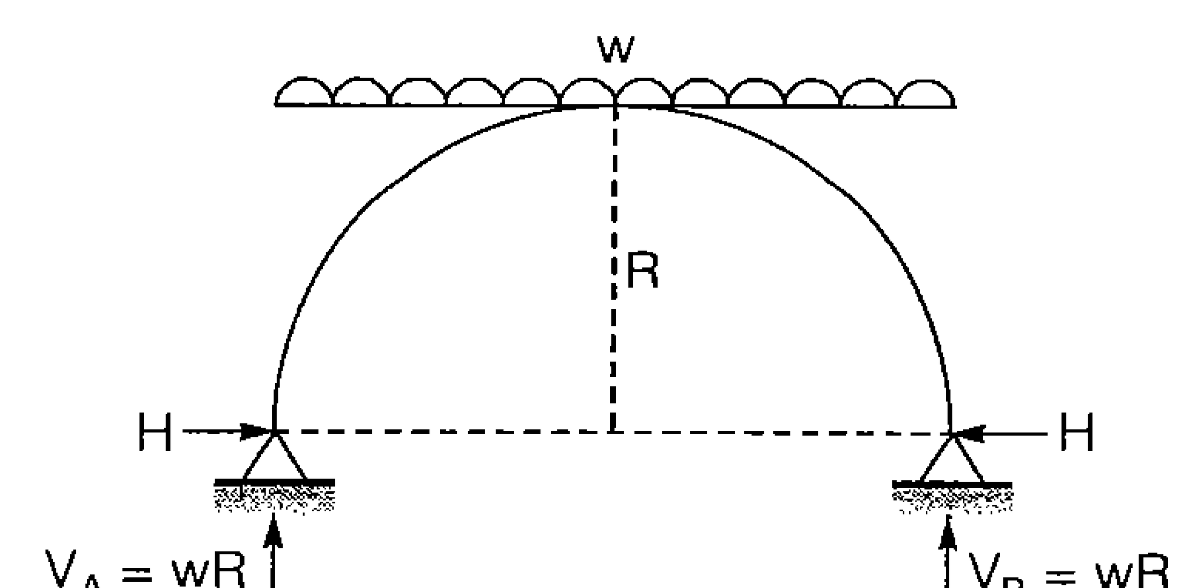


Two hinged circular arch

$H_{\text{Three hinged semicircular arch}} > H_{\text{Two hinged semicircular arch}}$

(iii) A two hinged semicircular arch of radius R carrying a UDL w per unit length over the whole span.

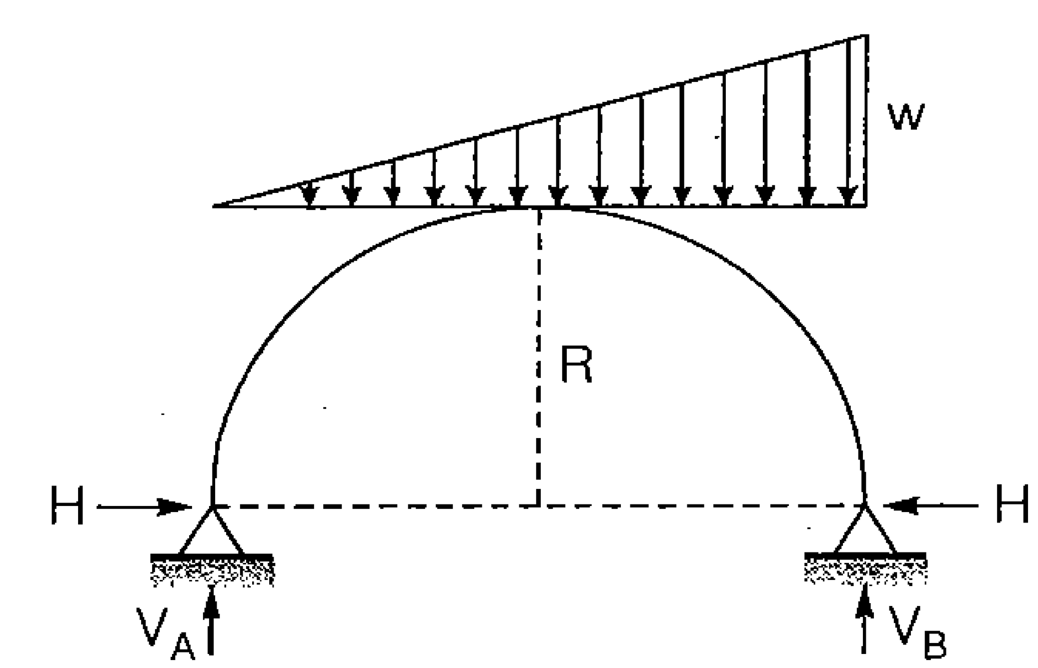
$H = \frac{4}{3} \cdot \frac{wR}{\pi}$



Two hinged semicircular arch

(iv) A two hinged semicircular arch of radius R carrying a distributed load uniformly varying from zero at the left end to w per unit run at the right end.

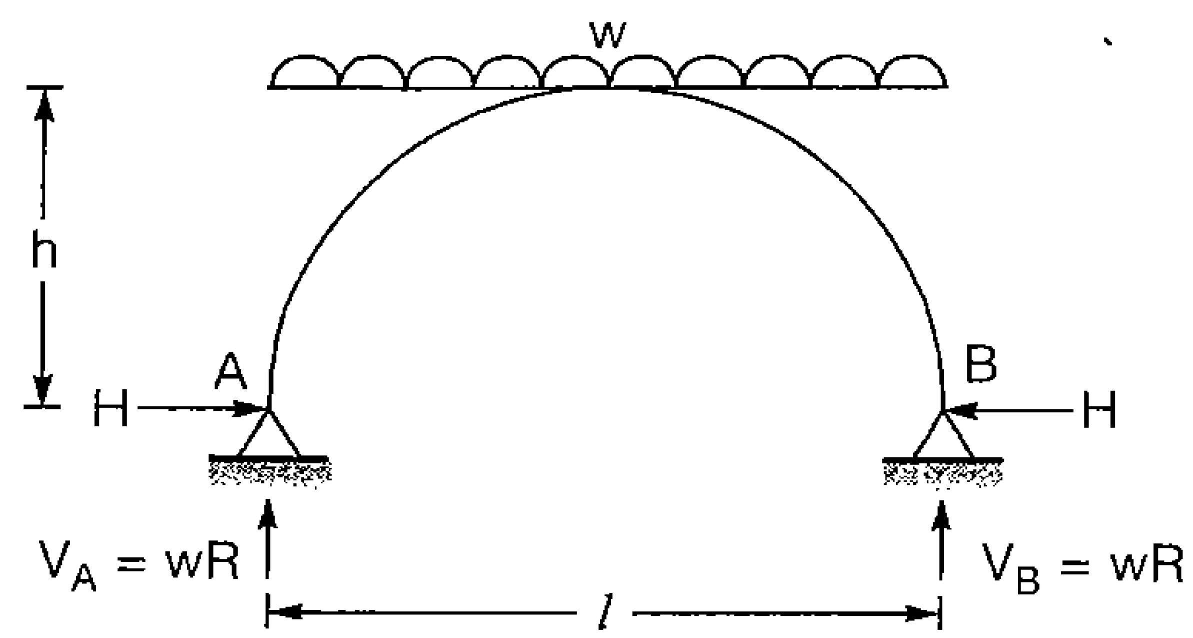
$H = \frac{2}{3} \cdot \frac{wR}{\pi}$



Two hinged semicircular arch

(v) A two hinged parabolic arch carries a UDL of w per unit run on entire span. If the span of the arch is L and its rise is h.

$$H = \frac{wl^2}{8h}$$

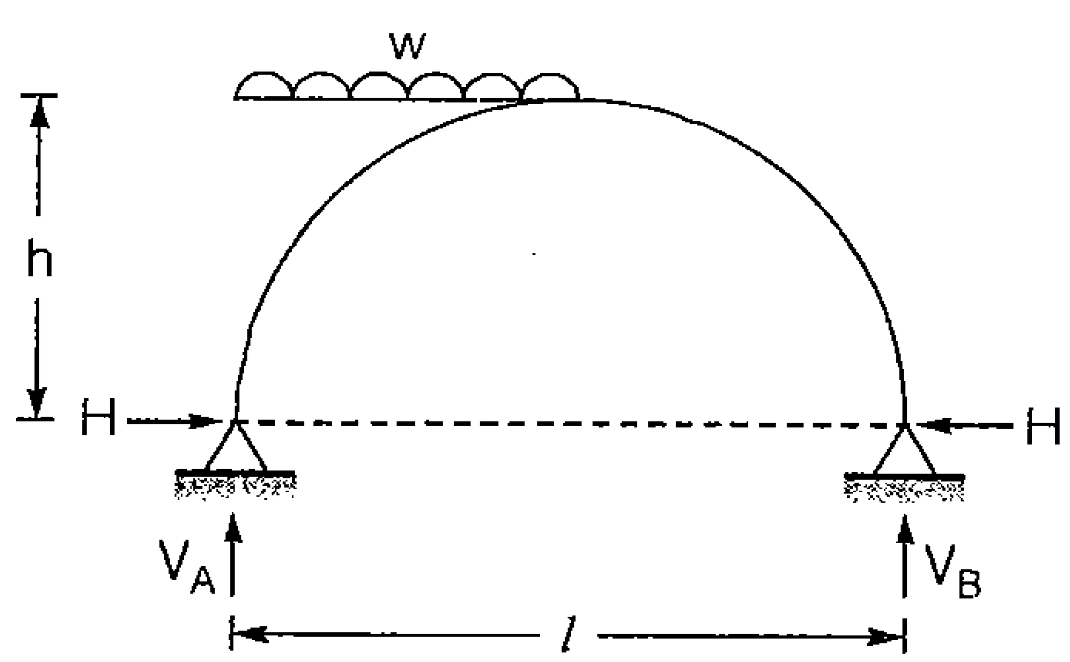


Two hinged parabolic arch

Remember: Horizontal thrust for two hinged parabolic arch is equal to horizontal thrust for three hinged parabolic arch carrying UDL over entire span.

(vi) When half of the parabolic arch is loaded by UDL, then the horizontal reaction at support is given by

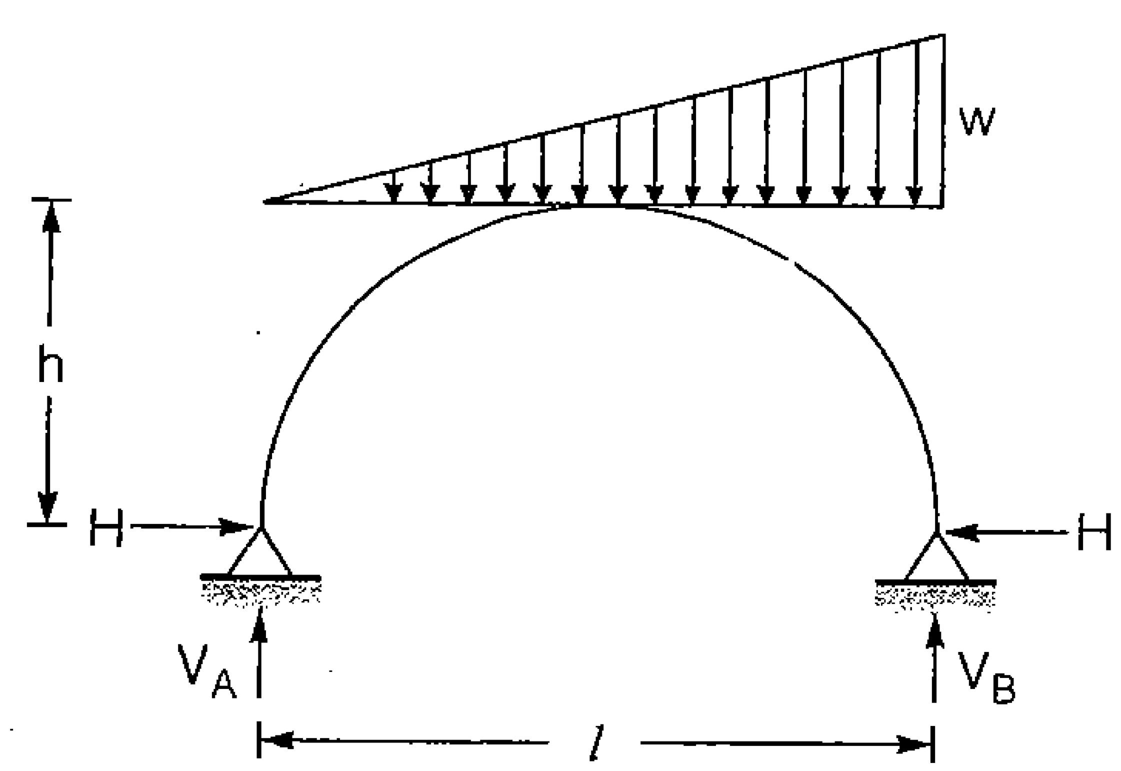
$$H = \frac{wl^2}{16h}$$



Two hinged parabolic arch

(vii) When two hinged parabolic arch carries varying UDL, from zero to w the horizontal thrust is given by

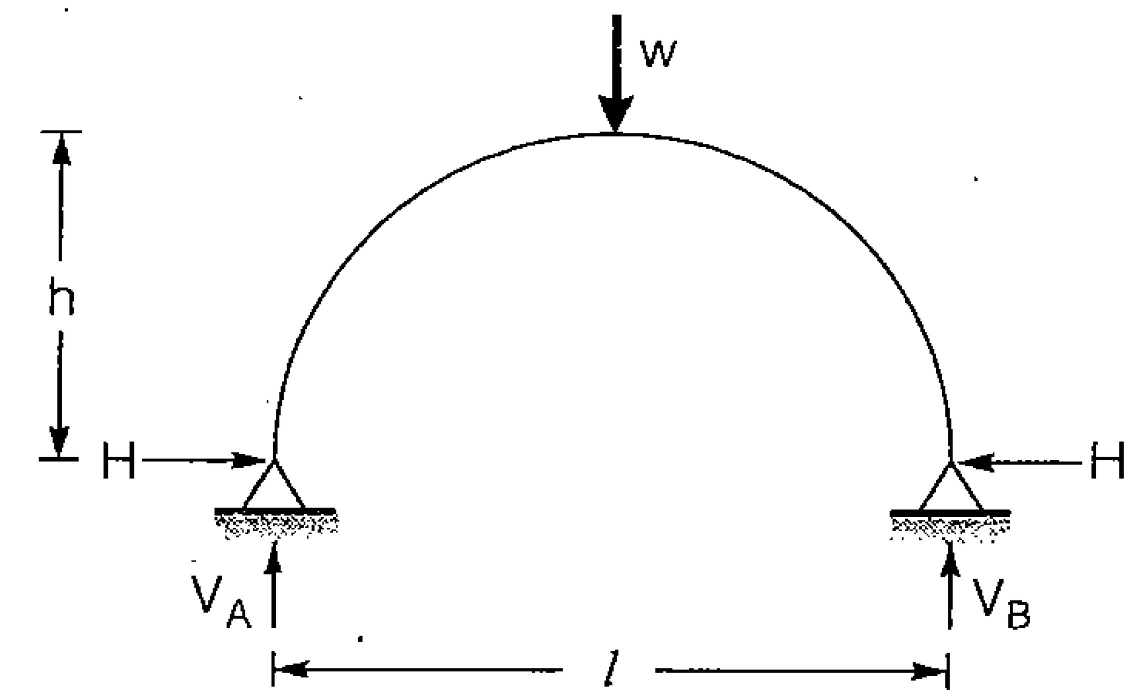
$$H = \frac{wl^2}{16h}$$



Two hinged parabolic arch

(viii) A two hinged parabolic arch of span l and rise h carries a concentrated load w at the crown.

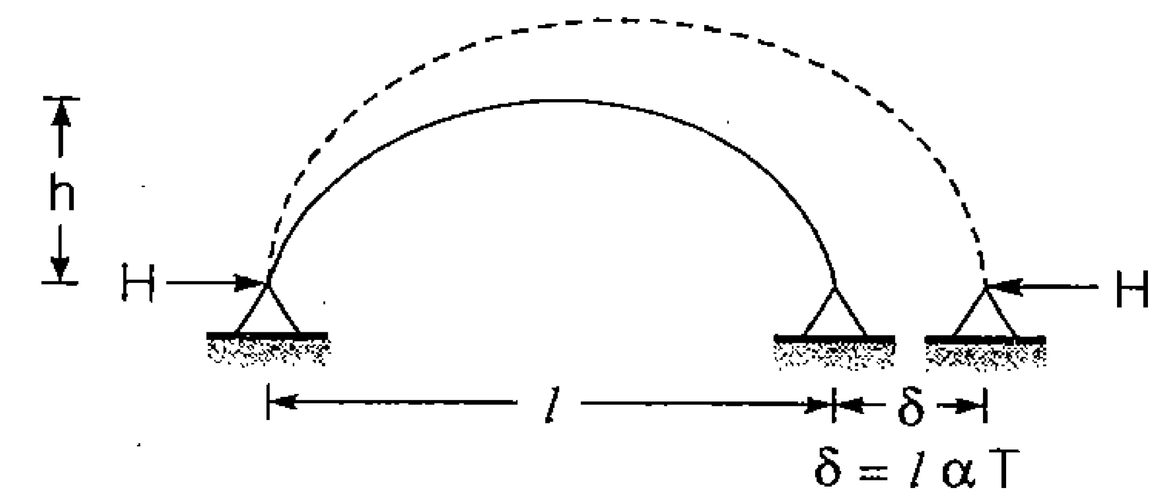
$$H = \frac{25}{128}\frac{wl}{h}$$



Two hinged parabolic arch

Temperature Effect on Two Hinged Arches

$$H = \frac{l\alpha T}{\int \frac{y^2 ds}{EI}}$$



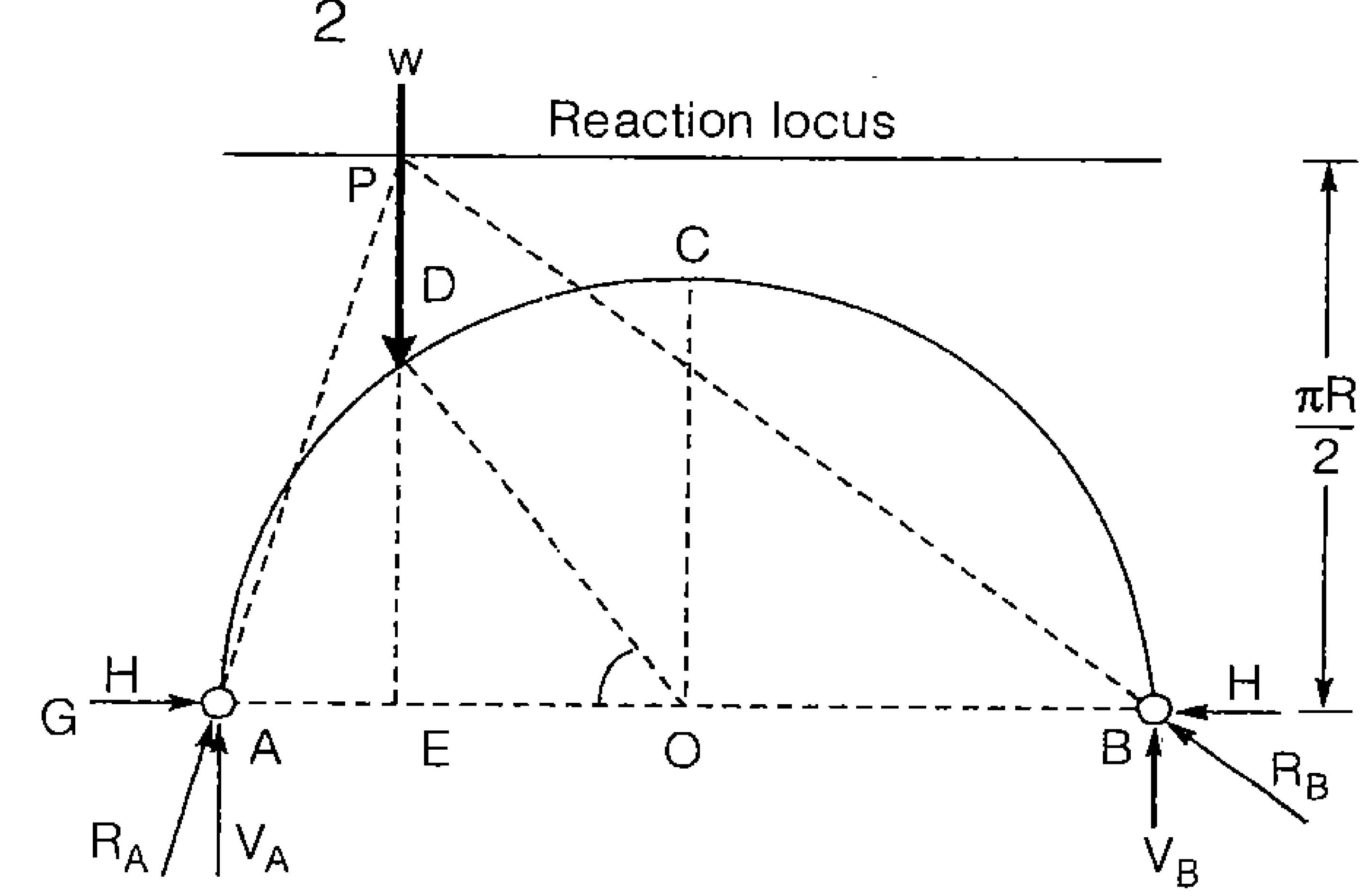
(i) $H = \dfrac{4EI\alpha T}{\pi R^2}$ where H = Horizontal thrust for two hinged semicircular arch due to rise in temperature by T °C.

(ii) $H = \dfrac{15}{8}\dfrac{EI_0\alpha T}{h^2}$ where I_0 = Moment of inertia of the arch at crown.
H = Horizontal thrust for two hinged parabolic arch due to rise in temperature T °C.

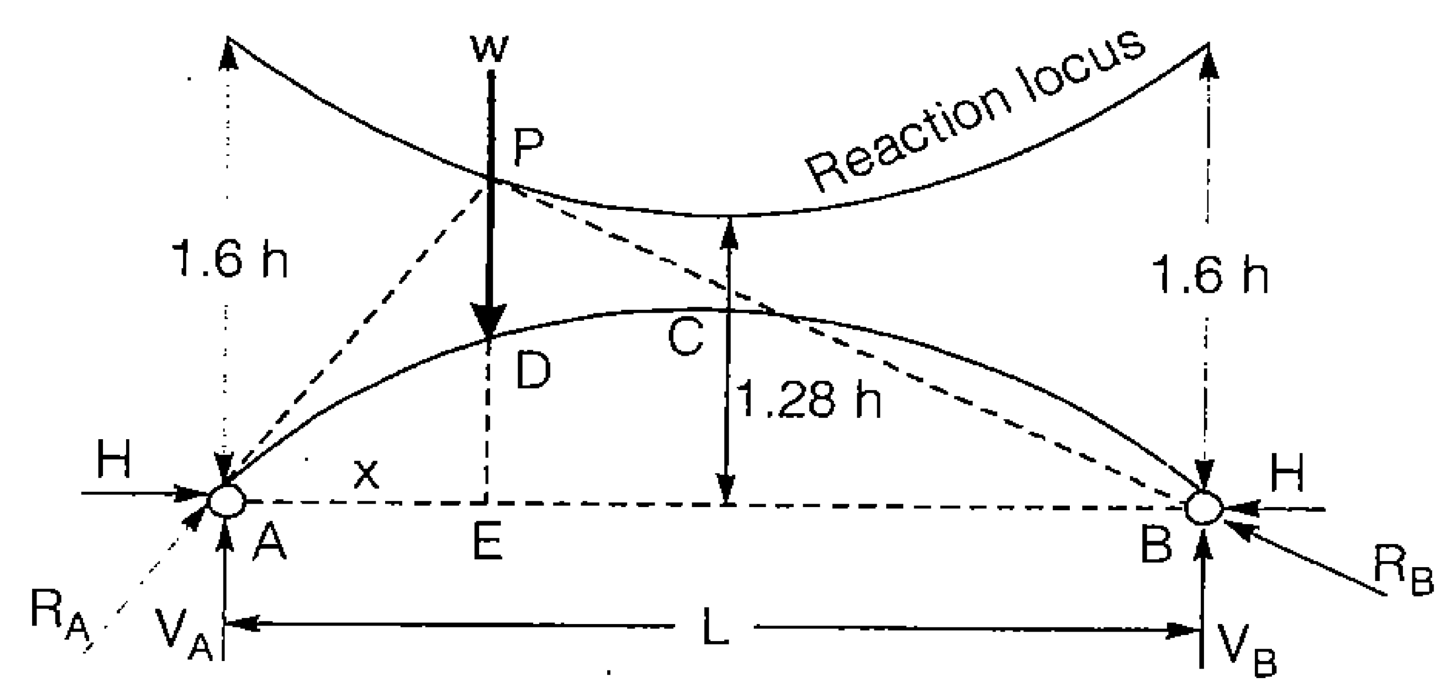
Reaction Locus for a Two Hinged Arch

(a) Two Hinged Semicircular Arch

Reaction locus is straight line parallel to the line joining abutments and height at $\frac{\pi R}{2}$.



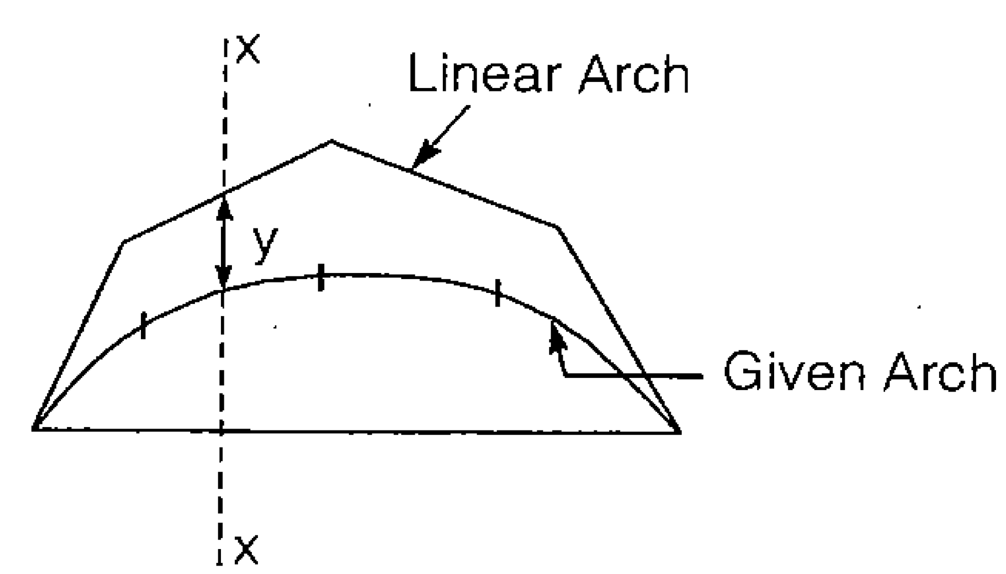
(b) Two Hinged Parabolic Arch



$$y = PE = \frac{1.6hL^2}{L^2 + Lx - x^2}$$

Eddy's Theorem

$$M_x \; \alpha \; y$$



where, M_x = BM at any section

y = distance between given arch and linear arch

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4 Methods of Structural Analysis

Methods of Structural Analysis

$D_S < D_K$

Force Method/Flexibility Method/Compatibility Method

Examples:

(i) Virtual work/Unit load method
(ii) Method of consistent deformation
(iii) Three moment theorem
(iv) Column analogy method
(v) Elastic centre method
(vi) Castigliano's theorem of minimum strain energy
(vii) Maxwell-Mohr equation

$D_K < D_S$

Displacement Method/Stiffness Method/Equilibrium Method

Examples:

(i) Slope deflection method
(ii) Moment distribution method
(iii) Minimum potential energy method

Difference between Force Method and Displacement Method

Force Method	Displacement Method
1.Unknowns are taken redundants forces/reactions.	1.Unknowns are taken displacement
2.To find unknown forces or redundants compatibility equations are written.	2. To find unknown displacement joint equlibrium conditions are written.
3.The number of compatibility equations needed is equal to degree of static indeterminacy.	3.The number of equlibrium conditions needed is equal to degree of kinematic indeterminacy.

STRAIN ENERGY METHOD

- Strain energy stored due to axial load

$$U_i = \int \frac{P^2 dx}{2AE}$$

where, P = Axial load
dx = Elemental length
AE = Axial rigidity

- Strain energy stored due to bending

$$U_i = \int \frac{M_x^2 ds}{2EI}$$

where, M_x = Bending moment at section x-x
ds = Elemental length
EI = Flexural rigidity

or E = Modulus of elasticity
I = Moment of inertia

- **Strain energy stored due to shear**

$$U_i = \int \frac{q^2}{2G} \cdot dv$$

where, q = Shear stress
G = Modulus of rigidity
dv = Elemental volume

- **Strain energy stored due to shear force**

$$U_i = \int \frac{S^2 ds}{2GA_S}$$

where, A_s = Area of shear
S = Shear force
G = Modulus of rigidity
ds = Elemental length.

- **Strain energy stored due to torsion**

$$U_i = \int \frac{T^2 dx}{2GI_P}$$

where, T = Torque acting on circular bar.
dx = Elemental length
G = Modulus of rigidity
I_P = Polar moment of inertia

- **Strain energy stored in terms of maximum shear stress,**

$$U_i = \frac{\tau_{max}^2}{4G} \cdot V$$

where, τ_{max} = Maximum shear stress at the surface of rod under twisting.
G = Modulus of rigidity
V = Volume

- **Strain energy stored in hollow circular shaft is,**

$$U_i = \int \frac{\tau_{max}^2}{4G} \cdot V \cdot \left(\frac{D^2 + d^2}{D^2}\right)$$

where, D = External dia of hollow circular shafts
d = Internal dia of hollow circular shaft
τ_{max} = Maximum shear stress

- **Castigliano'ss first theorem**

$$\frac{\partial U}{\partial \Delta} = P \quad \& \quad \frac{\partial U}{\partial \theta} = M$$

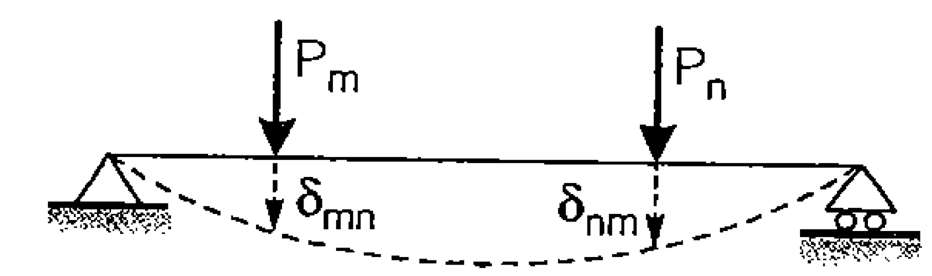
where, U = Total strain energy
Δ = Displacement in the direction of force P.
θ = Rotation in the direction of moment M.

- **Castiglianos Second Theorem**

$$\frac{\partial U}{\partial P} = \Delta, \quad \frac{\partial U}{\partial M} = \theta$$

- **Betti's Law**

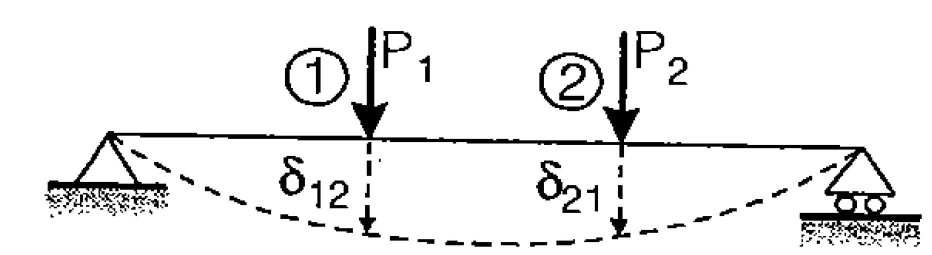
$$\Sigma P_m \delta_{mn} = \Sigma P_n \delta_{nm}$$



where, P_m = Load applied in the direction m.
P_n = Load applied in the direction n.
δ_{mn} = Deflection in the direction 'm' due to load applied in the direction 'n'.
δ_{nm} = Deflection in the direction 'n' due to load applied in the direction 'm'.

- **Maxwells Reciprocal Theorem**

$$\delta_{21} = \delta_{12}$$

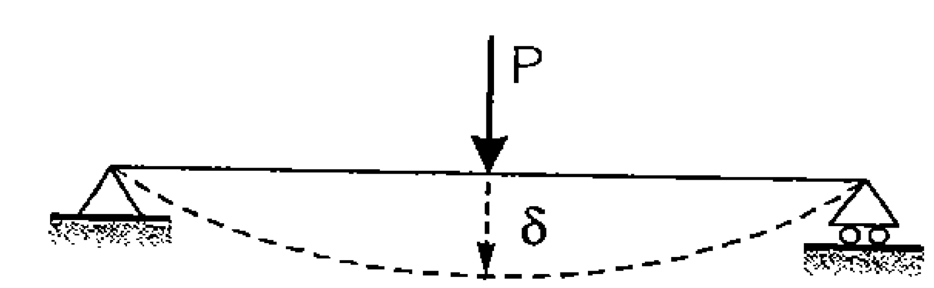


where, δ_{21} = deflection in the direction ② due to applied load in the direction ①.
δ_{12} = Deflection in the direction ① due to applied load in the direction ②.

Remember

- Total real external work done

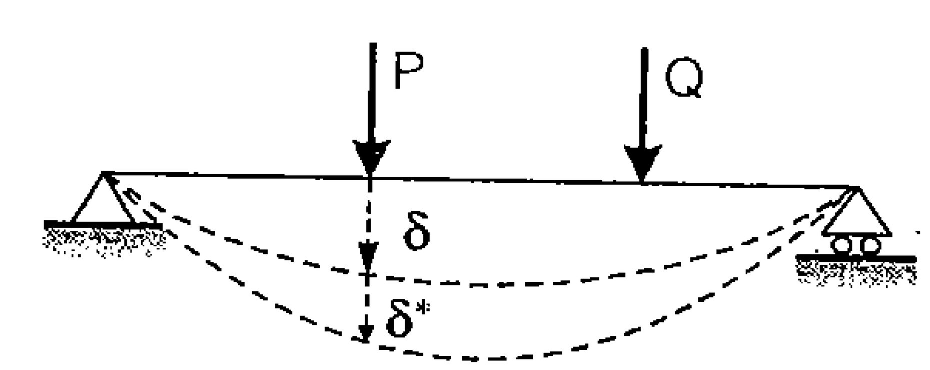
$$W_e = \frac{1}{2} P\delta$$



where, δ = Deflection caused by a static force P in the direction of P.

- Total external virtual work done,

$$W_e^* = P \cdot \delta^*$$

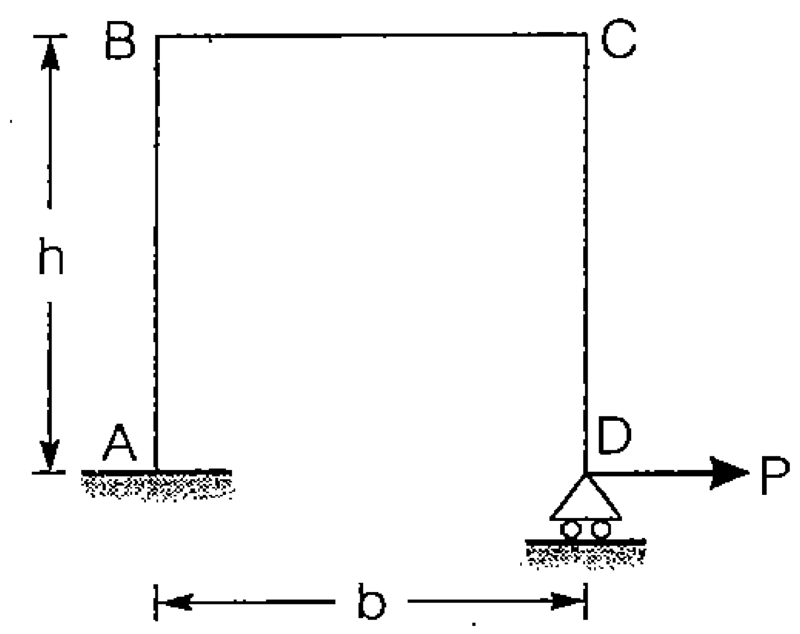


where, δ* = Deflection caused by any other force system say Q.

- **Standard Cases of Deflection**

(i) For the portal frame shown in the figure below horizontal deflection at D due to load P, assuming all members have same flexural rigidity is given by

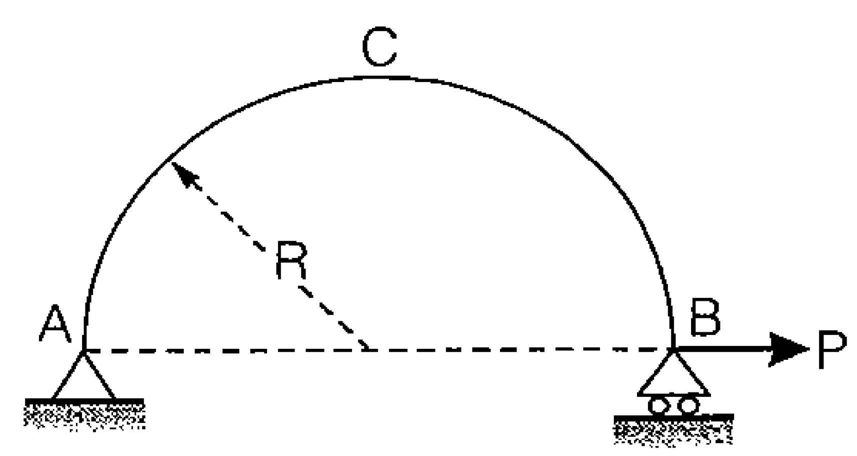
$$\delta = \frac{Ph^2(2h+3b)}{3EI}$$



where, δ = Deflection of D in the direction of load P.

(ii) Semicircular arch whose one end is hinged and other supported on roller carried a load P as shown in the figure.

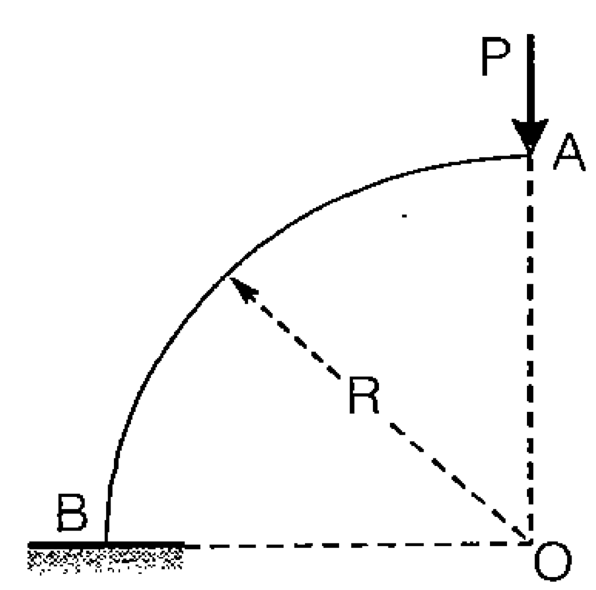
$$\delta = \frac{\pi}{2} \cdot \frac{PR^3}{EI}$$



where, δ = Deflection at B in the direction of load P.

(iii) A quadrantal ring AB of radius r support a concentrated load P as shown

$$\delta_v = \frac{\pi}{4} \cdot \frac{PR^3}{EI} \qquad \delta_H = \frac{1}{2} \cdot \frac{PR^3}{EL}$$

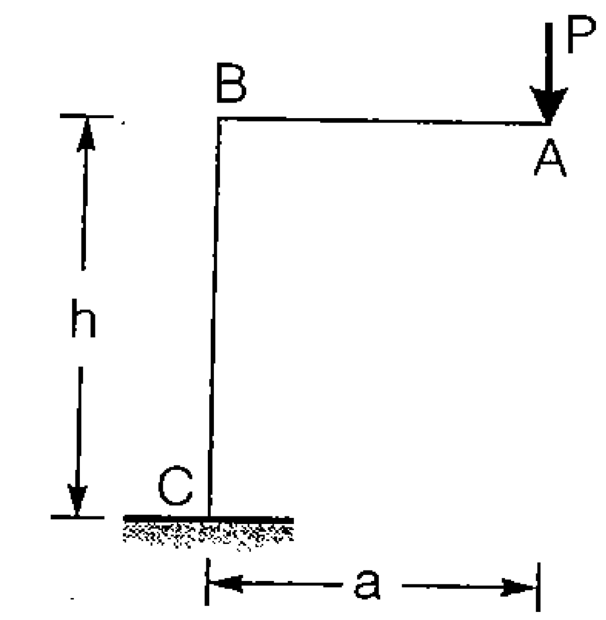


where, δ_v = Vertical deflection (deflection in the direction of load P) at end A

δ_H = Horizontal deflection of end A.

(iv) A portal frame as shown in figure carries a load P at A

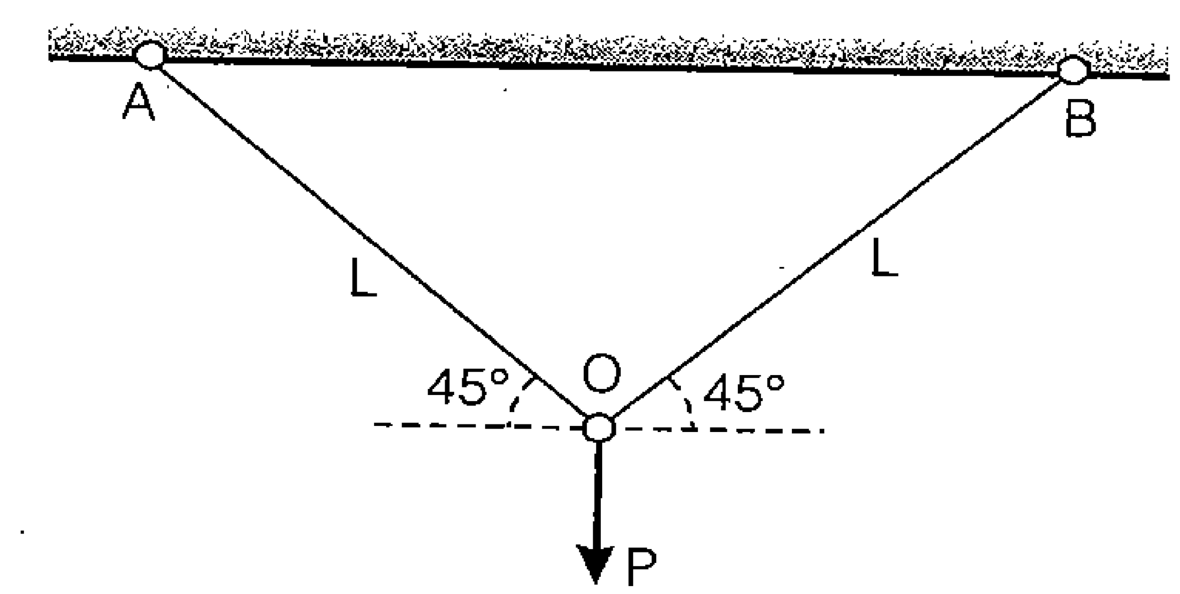
$$\delta_H = \frac{Pah^2}{2EI} \quad \& \quad \delta_v = \frac{Pa^2(a+3h)}{2EI}$$



where, δ_H & δ_v is horizontal & vertical deflection at end A respectively.

(v) Figure shows two identical wires OA and OB each of area A and inclined at 45° from horizontal. A load P is supported at O

$$\delta_v = \frac{PL}{AE}$$



where, δ_v = Vertical deflection at 'O'.

moment distribution method (Hardy Cross method)

- **Stiffness:** It is the force/moment required to be applied at a joint so as to produce unit deflection/rotation at that joint.

$$k = \frac{F}{\Delta} \text{ or } \frac{M}{\theta}$$

where, K = Stiffness

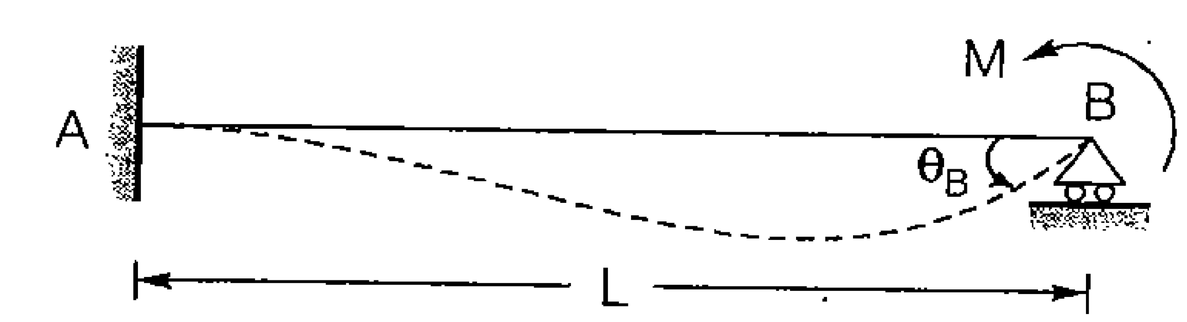
F = Force required to produce deflection Δ

M = Moment required to produce rotation θ.

- **Stiffness of beam**

(i) Stiffness of member BA when farther end A is fixed.

$$K = \frac{4EI}{L}$$



where, K = Stiffness of BA at joint B. When farther end is fixed.

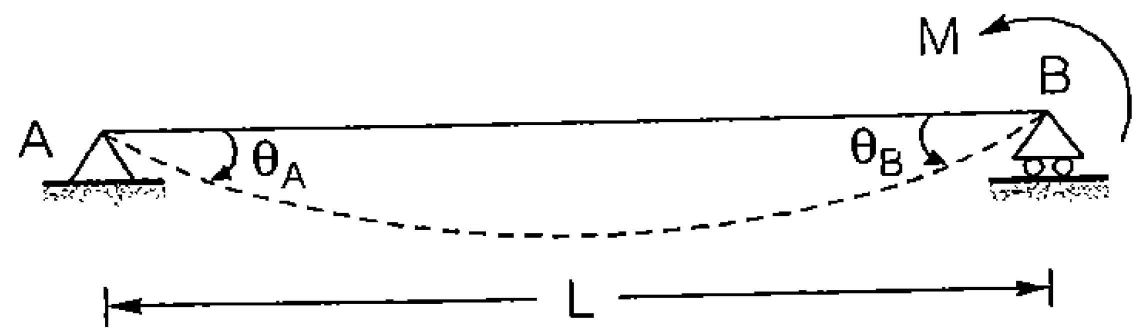
EI = Flexural rigidity

L = Length of the beam

M = Moment at B.

(ii) Stiffness of member BA when farther end A is hinged

$$K = \frac{3EI}{L}$$



where, K = Stiffness of BA at joint B. When farther end is hinged

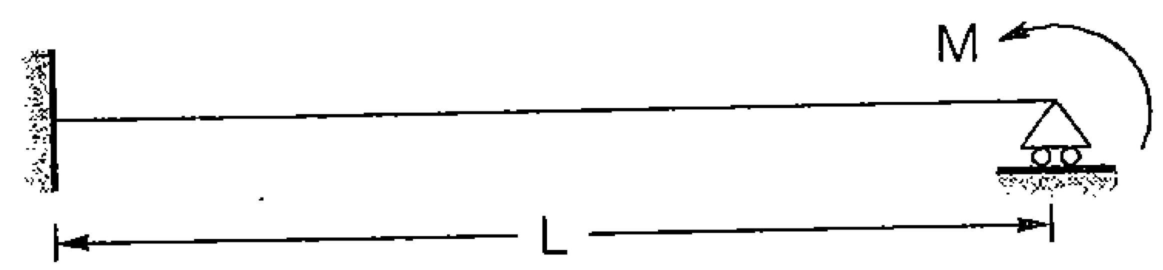
- **Carry over factor:**

$$\text{Carry over factor} = \frac{\text{Carry over moment}}{\text{Applied moment}}$$

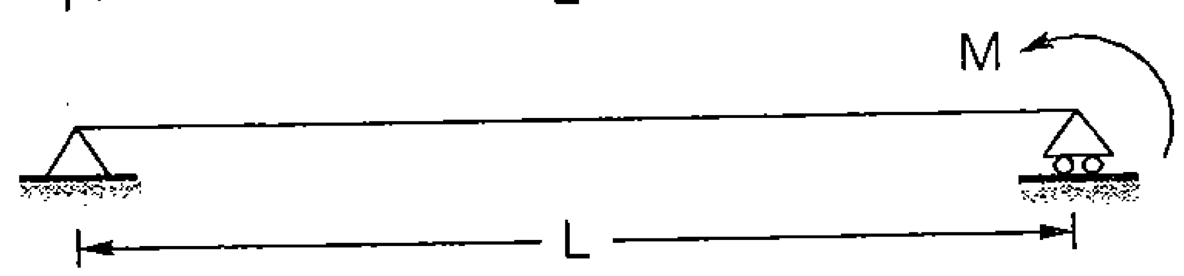
COF may greater than, equal to or less than 1.

- **Standard Cases:**

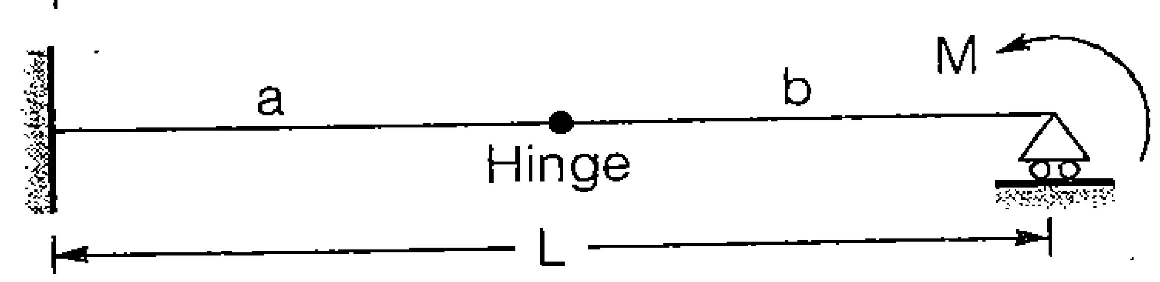
(i) $COF = \frac{1}{2}$



(ii) $COF = 0$



(iii) $COF = \frac{a}{b}$



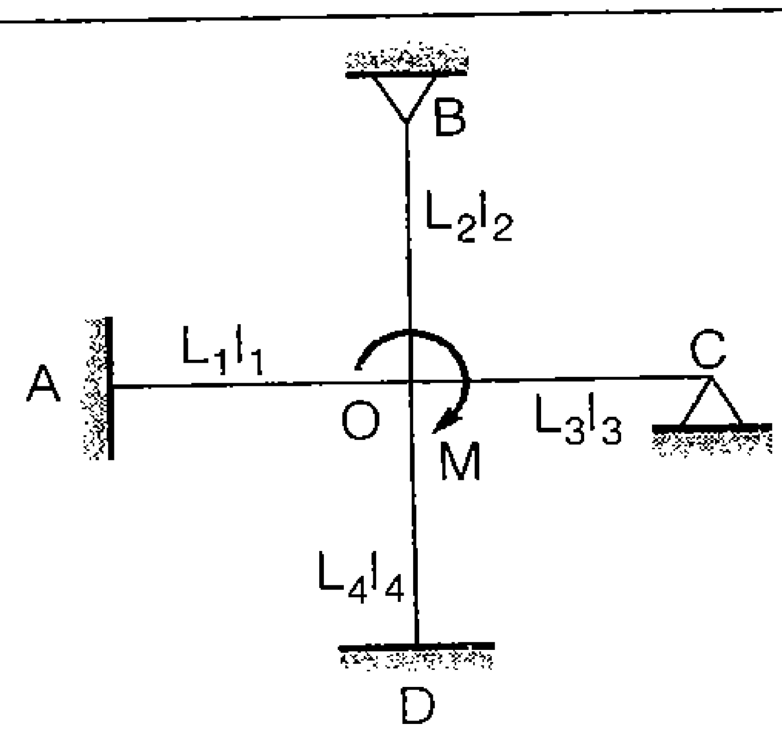
- **Distribution Factor (D.F):**

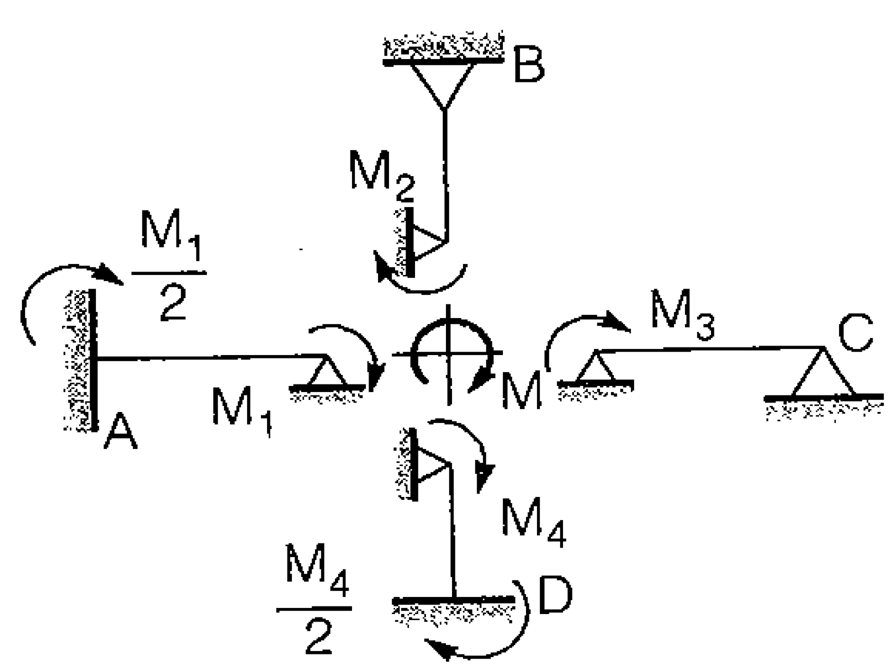
$$DF = \frac{\text{Stiffness of a member}}{\text{Sum of stiffness of all members at that joint}}$$

or

$$DF = \frac{\text{Relative stiffness of a member}}{\text{Sum of relative stiffness of all member at that joint}}$$

> Summation of DF for all member at a joint is one.

- 



M_1 = M(DF of OA) M_2 = M(DF of OB)
M_3 = M(DF of OC) M_4 = M(DF of OD)

where M_1, M_2, M_3 and M_4 are moment induced in member OA, OB, OC and OD respectively.

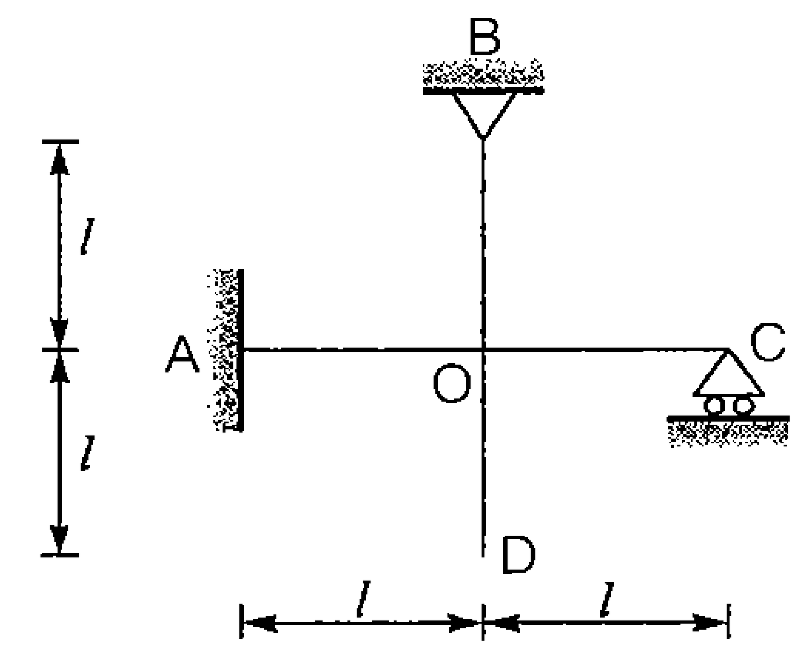
- **Relative Stiffness:**
  (i) When farther end is fixed.

$$\text{Relative stiffness for member} = \frac{I}{L}$$

  (ii) When farther end is hinged.

$$\text{Relative stiffness for member} = \frac{3I}{4L}$$

where, I = MOI and L = Length of Beam

- 

Stiffness of OA = $\frac{4EI}{l}$

Stiffness of OB = $\frac{3EI}{l}$

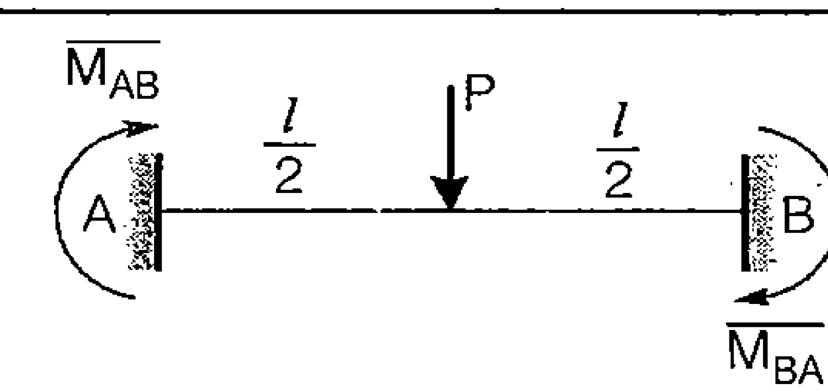
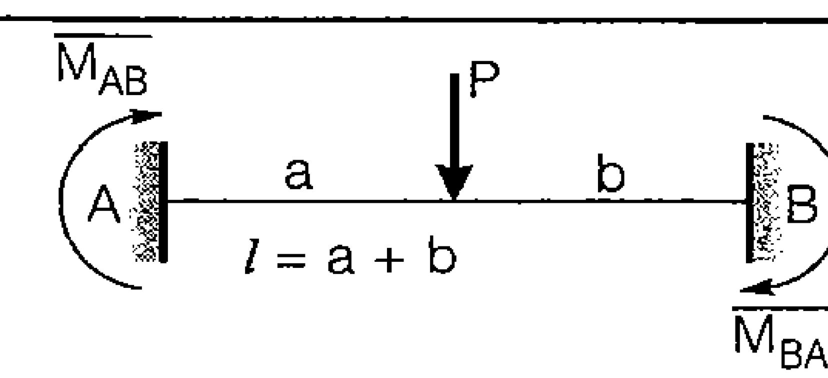
Stiffness of OC = $\frac{3EI}{l}$

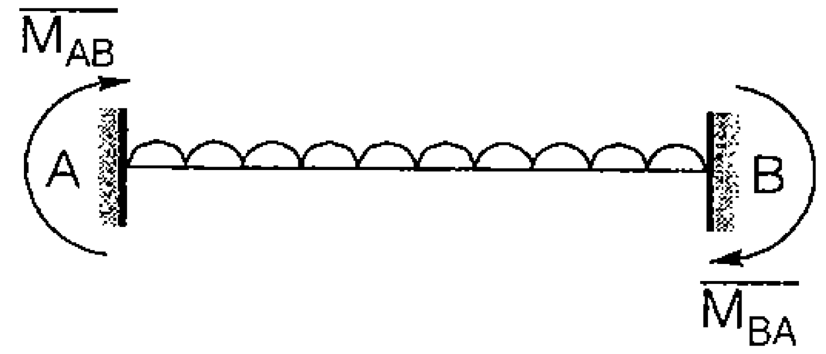
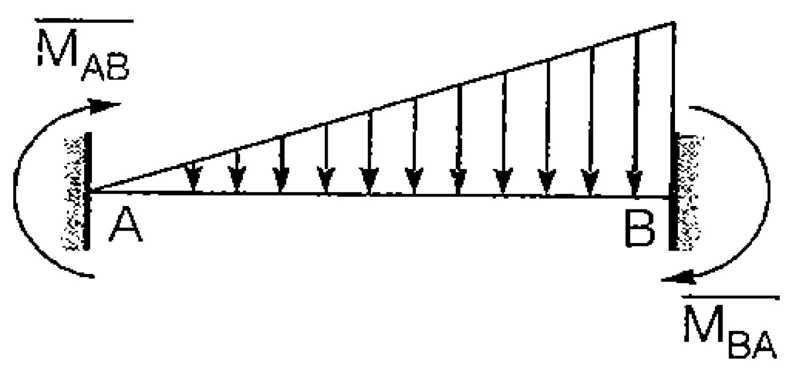
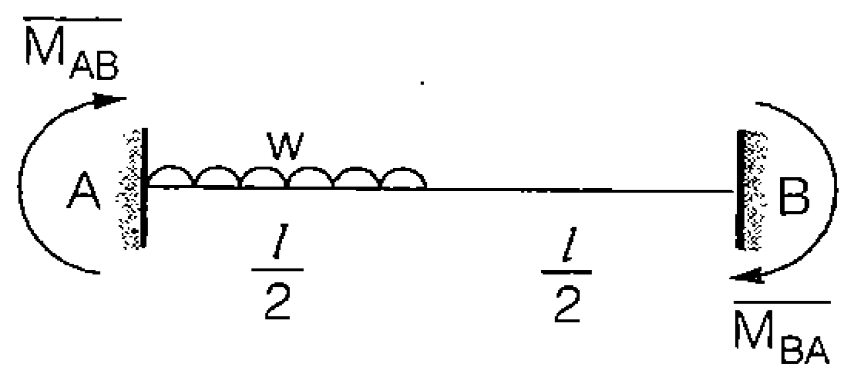
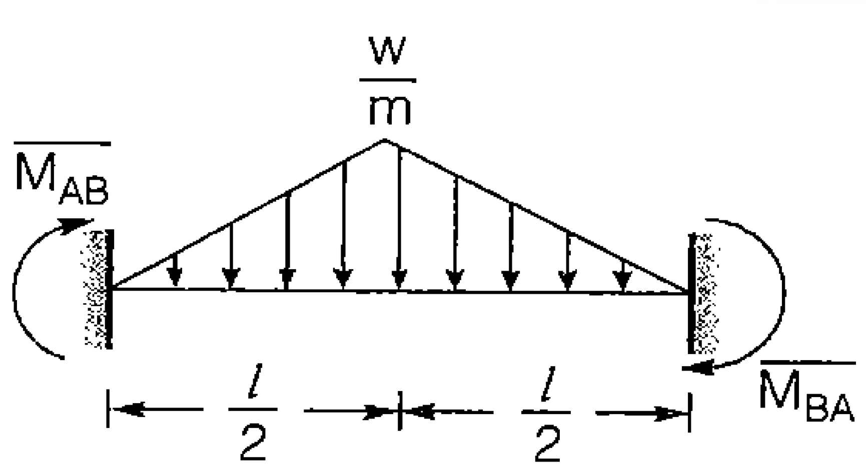
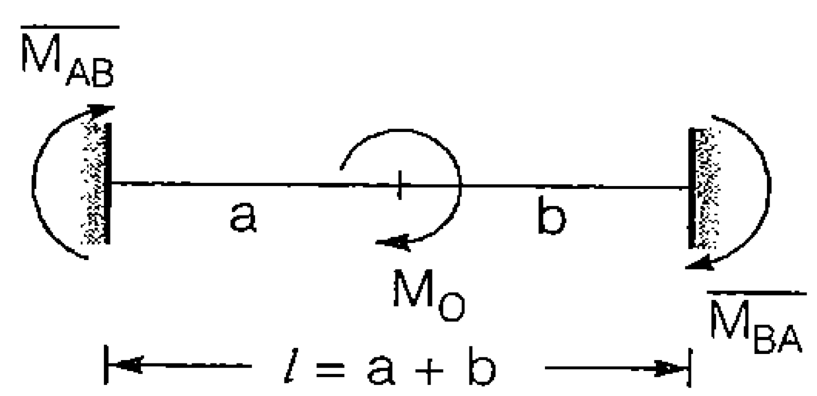
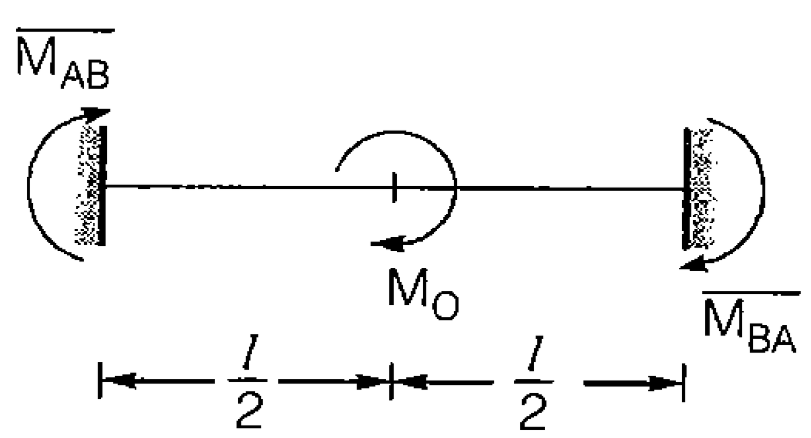
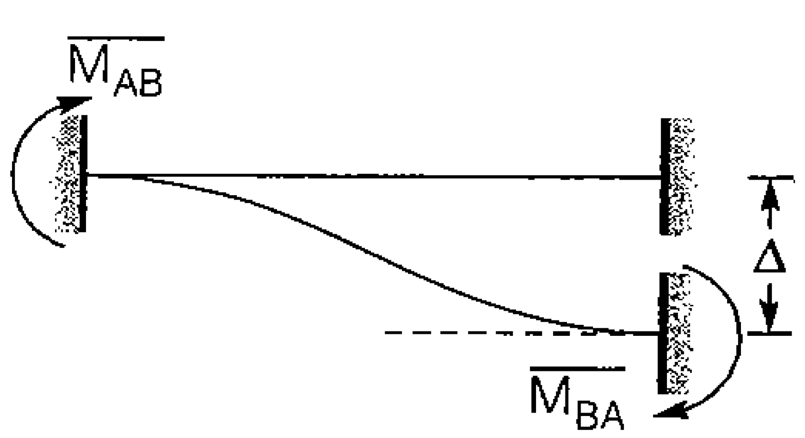
Stiffness of OD = 0

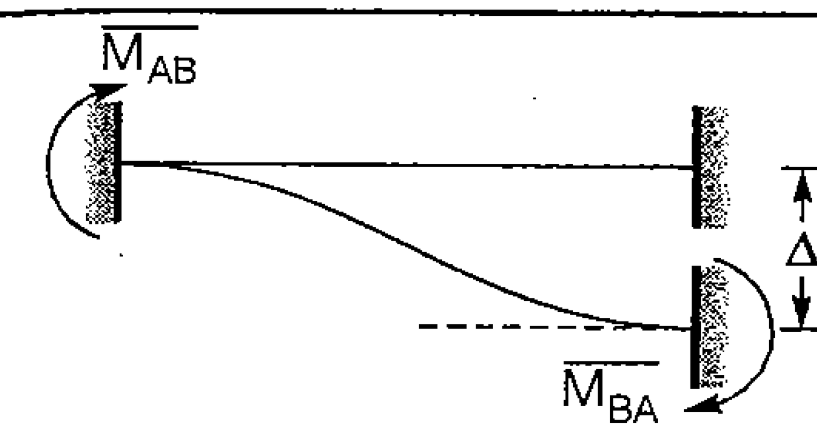
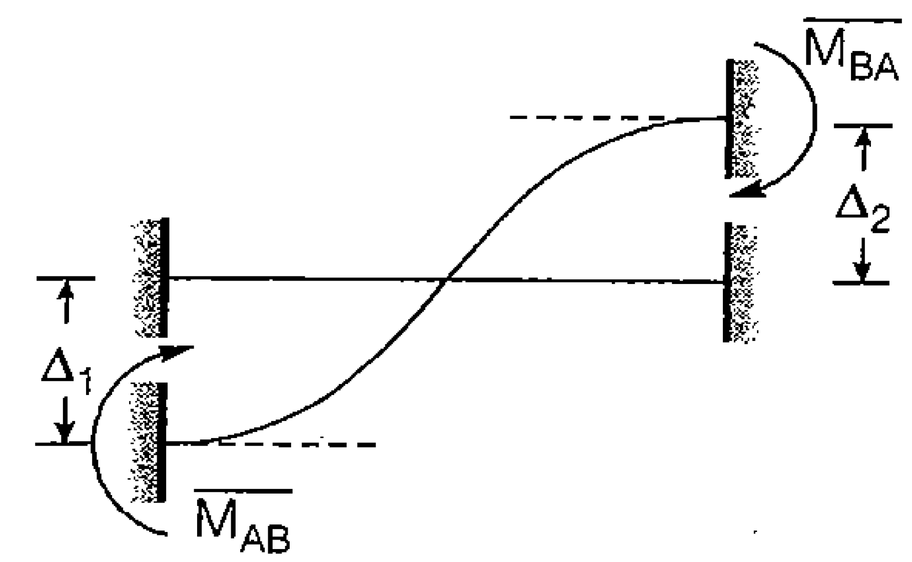
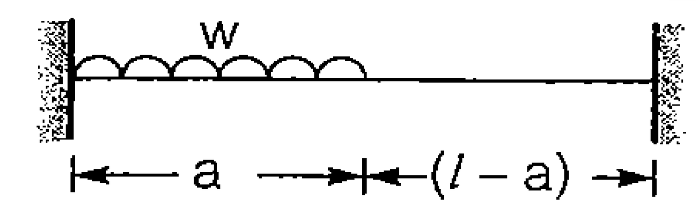
- **Fixed End Moments**

Sign Convention

+ve → Sagging
–ve → Hogging
and All clockwise moment → +ve
and All Anti clockwise moment → –ve
Span length is l

	$\overline{M_{AB}}$	$\overline{M_{BA}}$
	$\frac{-Pl}{8}$	$\frac{Pl}{8}$
	$\frac{-Pab^2}{l^2}$	$\frac{Pa^2b}{l^2}$

	$\frac{-wl^2}{12}$	$\frac{wl^2}{12}$
	$\frac{-wl^2}{30}$	$\frac{wl^2}{20}$
	$\frac{-11}{192}wl^2$	$\frac{5}{192}wl^2$
	$\frac{-5}{96}wl^2$	$\frac{5}{96}wl^2$
	$\frac{M_O b(3a-l)}{L^2}$	$\frac{M_O a(3b-l)}{L^2}$
	$\frac{M_O}{4}$	$\frac{M_O}{4}$
	$\frac{-6EI\Delta}{l^2}$	$\frac{-6EI\Delta}{l^2}$

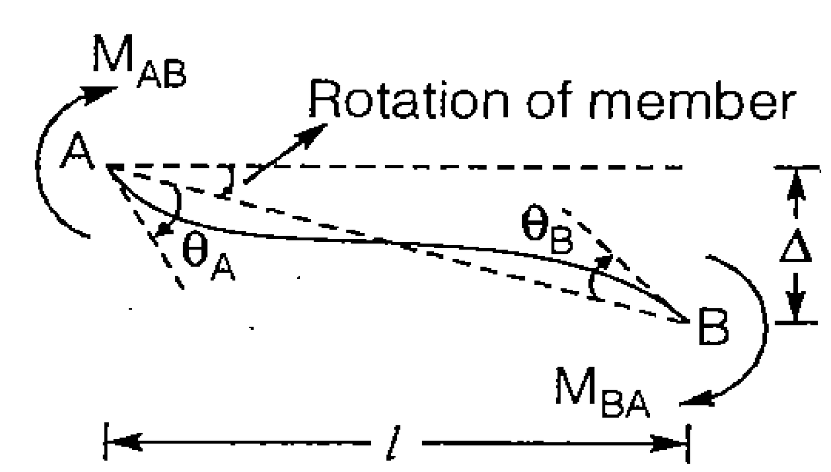
	0	$\frac{-3EI\Delta}{l^2}$
	$\frac{6EI(\Delta_1+\Delta_2)}{l^2}$	$\frac{6EI(\Delta_1+\Delta_2)}{l^2}$
	$\frac{wa^2}{12l^2}(6l^2 - 8l6.a + 3a^2)$	$\frac{wa^2}{12l^2}(4l - 3a)$

Slope deflection Method (G.A. Maney Method)

In this method, joints are considered rigid. It means joints rotate as a whole and the angles between the tangents to the elastic curve meeting at that joint do not change due to rotation. The basic unknowns are joint displacements (θ and Δ).

To find θ and Δ, joints equilibrium conditions and shear equations are established. The forces (moments) are found using force displacement relations. Which are called slope deflection equations.

- Slope Deflection Equation
  (i) The slope deflection equation at the end A for member AB can be written as:



$$M_{AB} = \overline{M_{AB}} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

(ii) The slope deflection equation at the end B for member BA can be written as:

$$M_{BA} = \overline{M_{BA}} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

where, L = Length of beam, EI = Flexural Rigidity

$\overline{M_{AB}}$ & $\overline{M_{BA}}$ are fixed end moments at A & B respectively.

M_{AB} & M_{BA} are final moments at A & B respectively.

θ_A and θ_B are rotation of joint A & B respectively.

Δ = Settlement of support.

- **Sign Convention**

  $+ M \rightarrow$ Clockwise

  $- M \rightarrow$ Anti-clockwise

  $+ \theta \rightarrow$ Clockwise

  $- \theta \rightarrow$ Anti-clockwise

  $\Delta \rightarrow$ +ve, if it produces clockwise rotation to the member & vice-versa.

The number of joint equilibrium conditions will be equal to number of 'θ' components & number of shear equations will be equal to number of 'Δ' components.

■■■

5 Trusses

Degree of Static Indeterminacy

(i) $D_s = m + r_e - 2j$ where, D_s = Degree of static indeterminacy

m = Number of members, r_e = Total external reactions

j = Total number of joints

(ii) $D_s = 0 \Rightarrow$ Truss is determinate

If $D_{se} = +1$ & $D_{si} = -1$ then $D_s = 0$ at specified point.

(iii) $D_s > 0 \Rightarrow$ Truss is indeterminate or redundant.

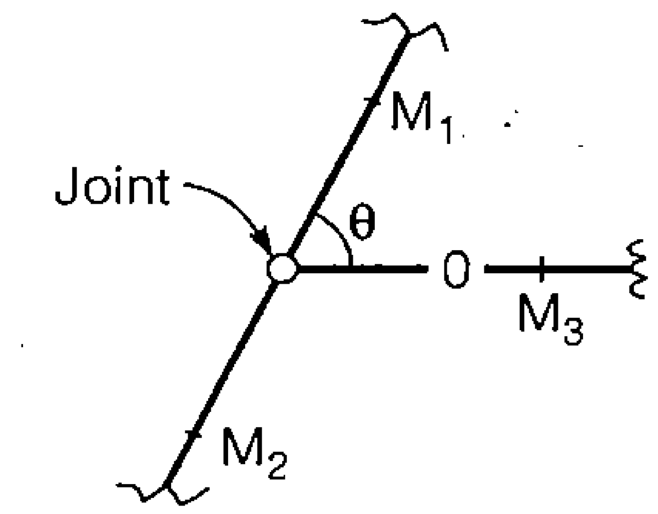
Truss Member Carrying Zero forces

(i) M_1, M_2, M_3 meet at a joint

M_1 & M_2 are collinear.

$\Rightarrow M_3$ carries zero force.

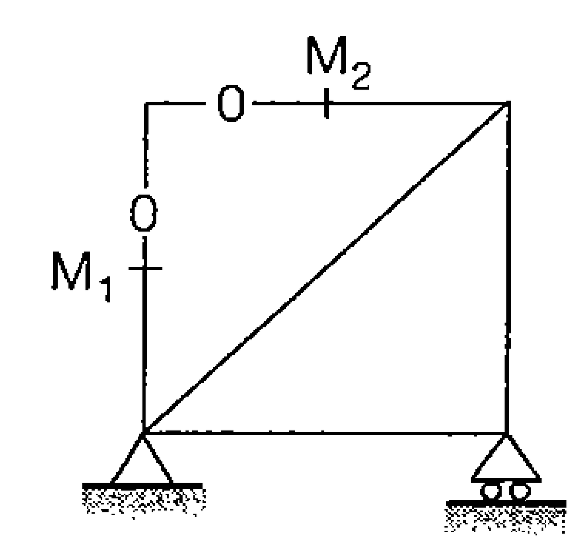
where M_1, M_2, M_3 represents member.



(ii) M_1 & M_2 are non collinear

and $F_{ext} = 0$

$\Rightarrow M_1$ & M_2 carries zero force.



Indeterminate Truss

(i) Final force in the truss member

$$S = P + kX \quad \text{and} \quad X = \frac{-\Sigma \frac{PkL}{AE}}{\Sigma \frac{k^2L}{AE}}$$

sign convn $\rightarrow$ +ve for tension, – ve for compression.

where, S = Final force in the truss member

K = Force in the member when unit load is applied in the redundant member

L = Length of the member
A = Area of the member
E = Modulus of elasticity
P = Force in the member when truss become determinate after removing one of the member.
P = Zero for redundant member.

> Remember
> It $D_{se} = +1$ then reaction is taken as redundant
> It $D_{si} = +1$ then member is taken as redundant.

Lack of Fit in Truss

$$\frac{\partial U}{\partial X} = \Delta \qquad \text{where, } U = \Sigma\frac{Q^2L}{2AE}$$

Q = Force induce in the member due to that member which is 'Δ' too short or 'Δ' too long is pulled by force 'X'.

Deflection of Truss

$$y_c = \Sigma k\left[L\alpha T + \frac{PL}{AE}\right]$$

where, y_c = Deflection of truss due to effect of loading & temp. both.

If effect of temperature is neglected then

$$y_c = \frac{\Sigma PKL}{AE}$$

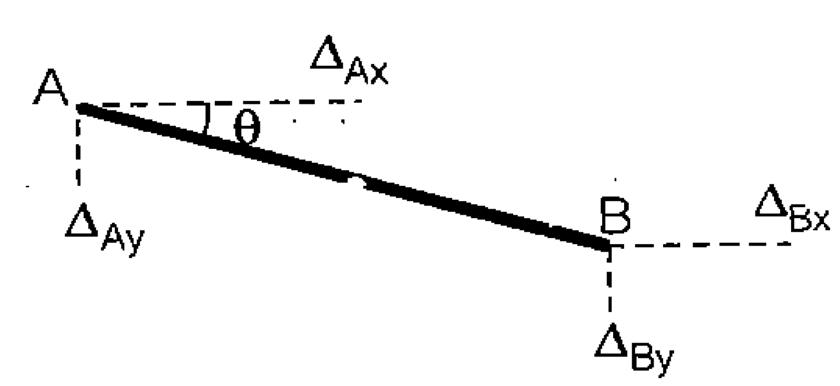
α = Coefficient of thermal expansion
T = Change in temperature

$T = +ve$ it temperature is increased

$T = -ve$ it temperature is decreased.

P & K have same meaning as mentioned above.

Stiffness Method for Truss



$$\Delta_{AB} = \Delta_B - \Delta_A$$

$$P_{AB} = \frac{AE}{L}[(\Delta_{Bx} - \Delta_{Ax})\cos\theta + (\Delta_{By} - \Delta_{Ay})\sin\theta]$$

where, Δ_{AB} = Axial deflection of member AB.
P_{AB} = Force in member AB (Axial force)

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6 Matrix Method of Structural Analysis

Flexibility

The flexibility of a structure is defined as the displacement caused a unit force.

$$f = \frac{\delta}{P} \quad \text{or} \quad f = \frac{\theta}{M}$$

where, f = Flexibility, δ = Displacement
P = Force, θ = Rotation, M = Mom

Stiffness

The stiffness is defined as the force required to produce a u displacement.

$$k = \frac{P}{\delta} \quad \text{or} \quad k = \frac{M}{\theta}$$

where, k = stiffness

$$f = \frac{1}{k}, \quad f_{xy} = f_{yx} \;\&\; k_{xy} = k_{yx}$$

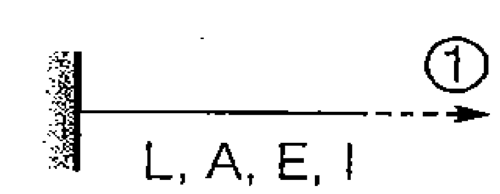
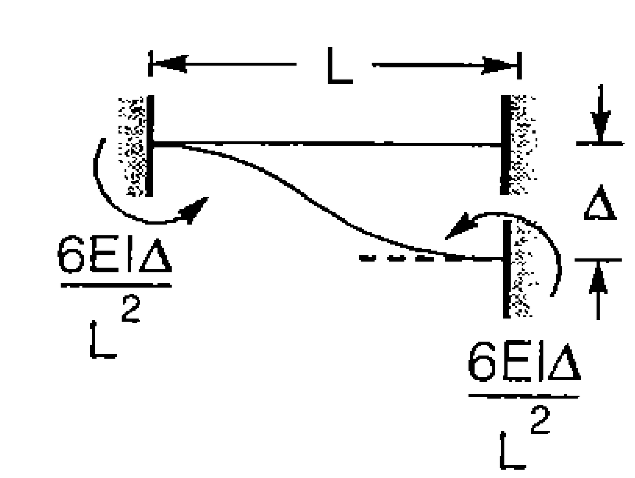
} From Maxwell Reciprocal Theorem

where,

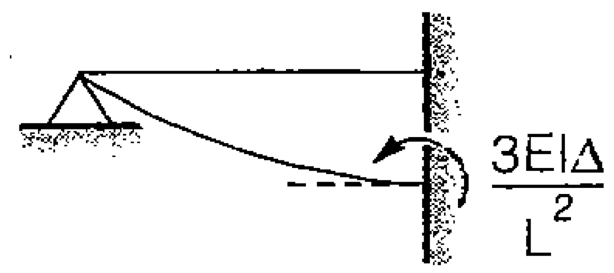
f_{xy} = Deflection in direction x due to unit force in direction y.
x: Direction of displacement; y: Direction of unit force.

k_{xy} = Force in direction x due to unit displacement in direction y.
x: Direction of force; y: Direction of unit displacement.

Results

Type of Displacement	Diagram	Flexibility	Stiffnes
(i) Axial		$\frac{L}{AE}$	$\frac{AE}{L}$
(ii) Transverse Displacement			
(a) with far end fixed		$\frac{L^3}{12EI}$	$\frac{12EI}{L^3}$

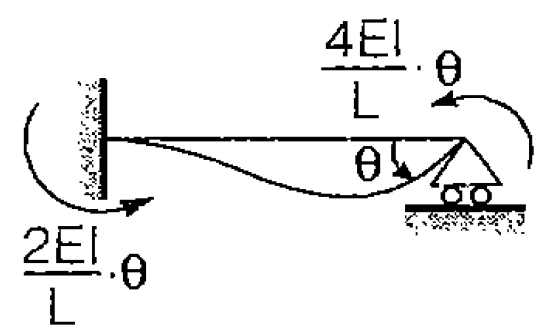
(b) with far end hinged



$\frac{L^3}{3EI}$ $\frac{3EI}{L^3}$

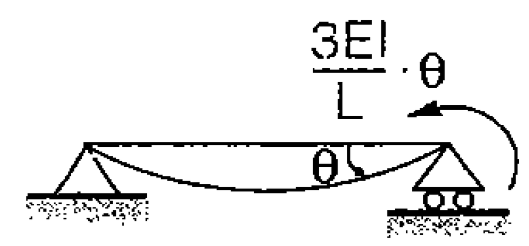
(iii) Flexural Displacement

(a) with far end end fixed



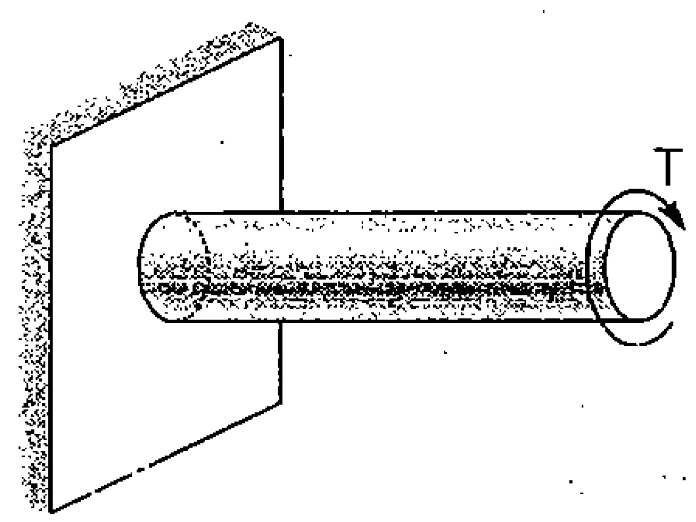
$\frac{L}{4EI}$ $\frac{4EI}{L}$

(b) with far end hinged



$\frac{L}{3EI}$ $\frac{3EI}{L}$

(iv) Torsional Displacement



$\frac{L}{GI_P}$ $\frac{GI_P}{L}$

> **Remember:** It is not necessary that all members of flexibility or stiffness matrix will be homogeneous.

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A Handbook on **Civil Engineering**

3

RCC & Prestressed Concrete

CONTENTS

1 Introduction

Concrete

1. Modulus of elasticity of concrete

$$E_c = 5000\sqrt{f_{ck}}$$

where f_{ck} = characteristic strength of concrete

2. Tensile strength of concrete in flexure

$$f_{cr} = 0.7\sqrt{f_{ck}}$$

Remember: Characteristic strength of concrete is the value of strength of concrete below which not more than 5% of test results are expected to fall.

3. Permissible value of strength in concrete

Grade	Direct tensile strength (f_{ckt})	Compression		Bondstress (τ_{bd})	
		Direct (σ_{cc})	Bending (σ_{cbc})	WSM	LSM
M15	2	4	5	0.6	1
M20	2.8	5	7	0.8	1.2
M25	3.2	6	8.5	0.9	1.4
M30	3.6	8	10.0	1.0	1.5
M35	4.0	9	11.5	1.1	1.7
M40	4.4	10	13.0	1.2	1.9

- τ_{bd} given in table is only for plain mild steel bar in tension.
- τ_{bd} value should be increased by 60% for deformed bars both in LSM and WSM.
- For bars in compression the value should be increased by 25%.

Steel

1. Young's modulus of all type of steel is 2×10^5 N/mm^2.

- Type of steel
    1. Mild steel → Fe 250
       Here, 250 is the characteristic strength of mild steel bars.
       Also, $f_y = 250$ N/mm^2

2. HYSD bars
   Fe 415
   Fe 500
3. Permissible stresses in steel

Permissible Stresses in Steel Reinforcement

S. No.	Type of Stress in Steel Reinforcement	Permissible stresses in N/mm^2	
		Mild steel bars (Fe250)	High yield strength deformed bars (Fe 415)
(1)	(2)	(3)	(4)
(i)	Tension (σ_{st} or σ_{sv})		
	(a) Up to and including 20 mm	140	230
	(b) Over 20 mm	130	230
(ii)	Compression in column	130	190
(iii)	Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account	The calculated compressive stress in the surrounding concrete multiplied by 1.5 times the modular ratio or σ_{xc} whichever is lower	
(iv)	Compression in bars in a beam or slab where the compressive resistance of the concrete is not taken into account:		
	(a) Up to and including 20 mm	140	190
	(b) Over 20 mm	130	190

NOTE

1. For high yield strength deformed bars of Grade Fe 500 the permissible stress in direct tension and flexural tension shall be 0.55 f_y. The permissible stresses for shear and compression reinforcement shall be as for Grade Fe 415.
2. For welded wire fabric, the permissible value in tension σ_{yt}, is 230 N/mm^2.
3. For the purpose of this standard, the yield stress of steels for which there is no clearly defined yield point should be taken to be 0.2 percent proof stress.

■■■

Working Stress Method 2

Modulur Ratio

$$m = \frac{E_s}{E_c}$$

where, m = Modular ratio
E_s = Modulus of elasticity of steel
E_c = Modulus of elasticity of concrete

Equivalent Area of Concrete

$$A_c = m A_s$$

Here, A_c = Area of concrete
A_s = Area of steel

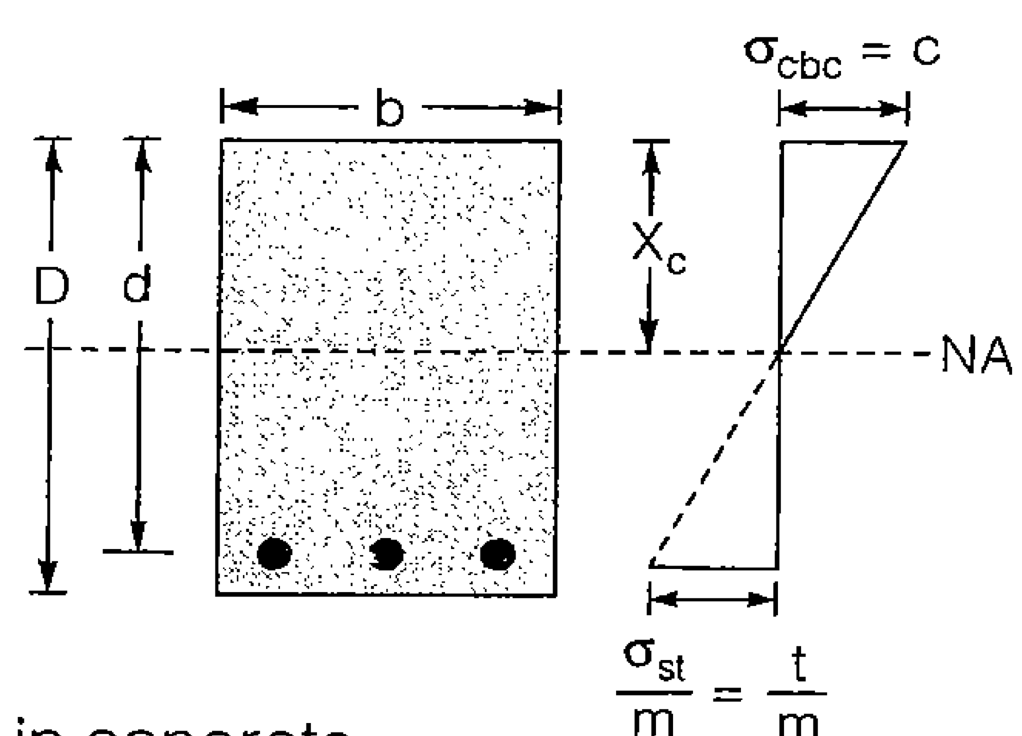
Stress in Concrete

$$p_c = \frac{p_s}{m}$$

where, p_c = stress in concrete
p_s = stress in steel

Critical Depth of Neutral Axis (X_c)

$$X_c = \left(\frac{mc}{t+mc}\right)d$$



Here, D = Overall depth
d = Effective depth
σ_{cbc} = c = permissible stress in concrete
σ_{st} = t = permissible stress in steel

SINGLE REINFORCED RECTANGULAR SECTION

Actual depth of Neutral axis (X_a)

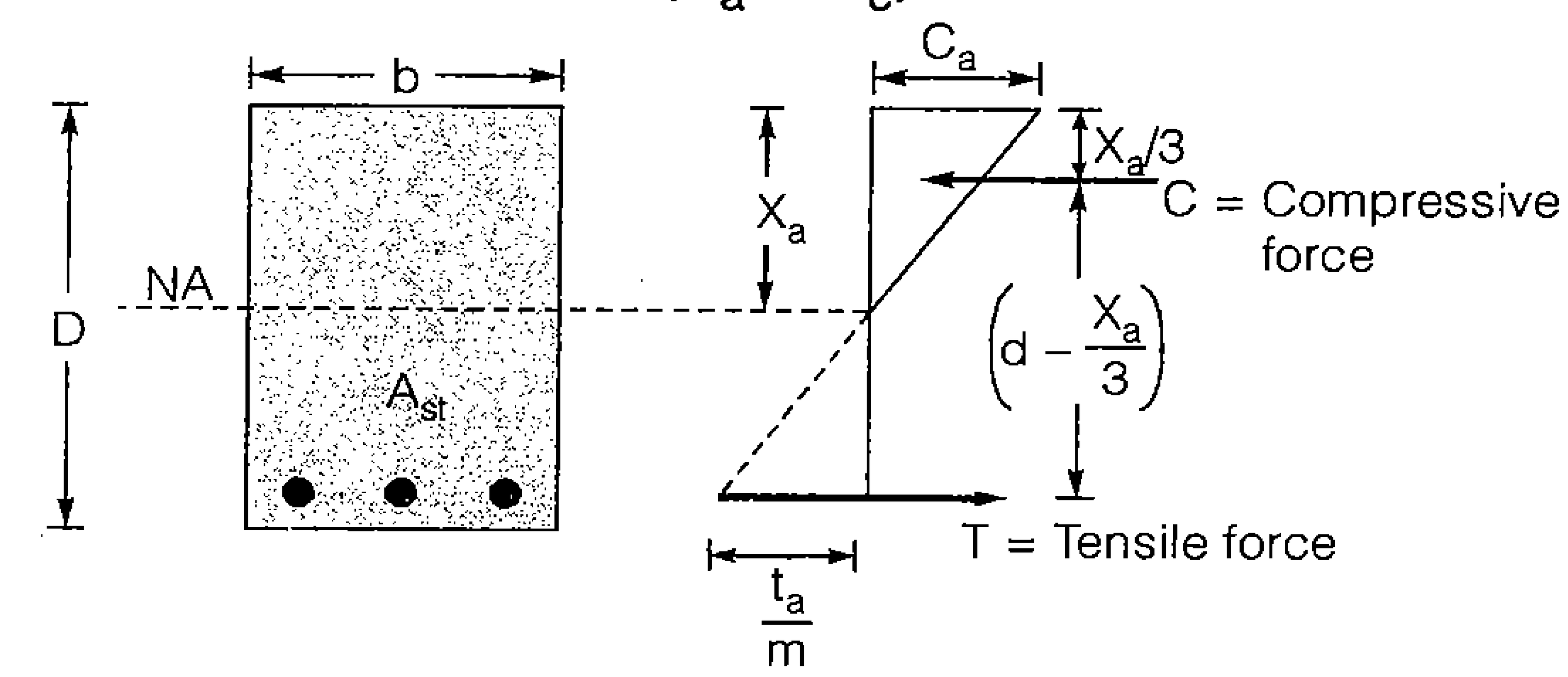
> **Remember:** In working stress method actual depth of neutral axis is calculated by equating moment of Area on both side of Neutral axis.

$$\frac{B X_a^2}{2} = m A_{st}(d - X_a)$$

- Special case:
  - (i) when $X_a = X_c$ for balanced section.
  - (ii) when $X_a > X_c$ for over reinforced section.
  - (iii) when $X_a < X_c$ for under reinforced section.

Moment of resistance (M_r)

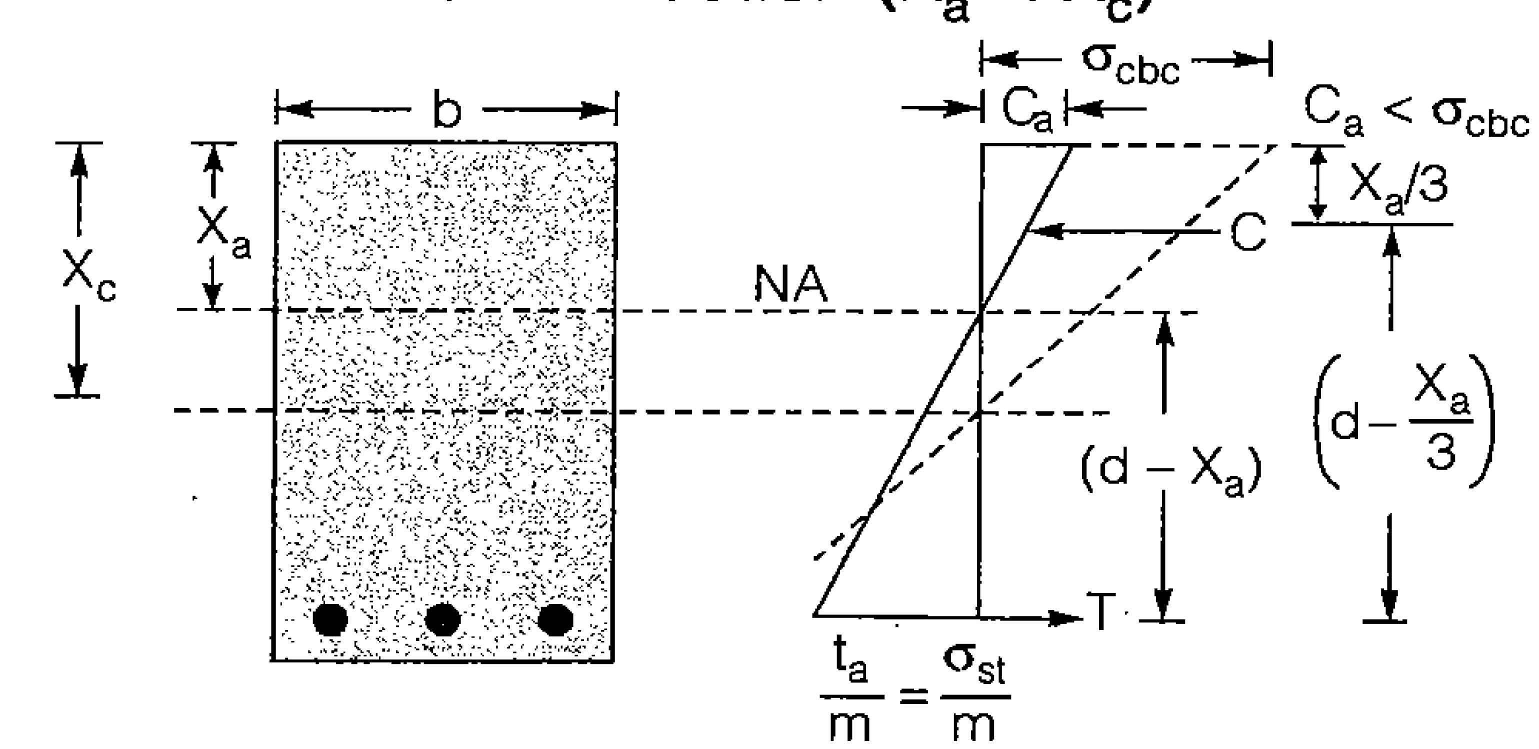
(i) For balanced section ($X_a = X_c$)



$$M_r = bX_a \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{X_a}{3}\right) \quad \text{or} \quad M_r = \sigma_{st} \cdot A_{st}\left(d - \frac{X_a}{3}\right)$$

$$\left(d - \frac{X_a}{3}\right) = \text{Lever Arm}$$

(ii) For under reinforced section ($X_a < X_c$)



$$M_r = BX_a \cdot \frac{C_a}{2}\left(d - \frac{X_a}{3}\right) \quad \text{or} \quad M_r = \sigma_{st} \cdot A_{st}\left(d - \frac{X_a}{3}\right)$$

$$C_a = \frac{\sigma_{st} \cdot X_a}{m(d - X_a)}$$

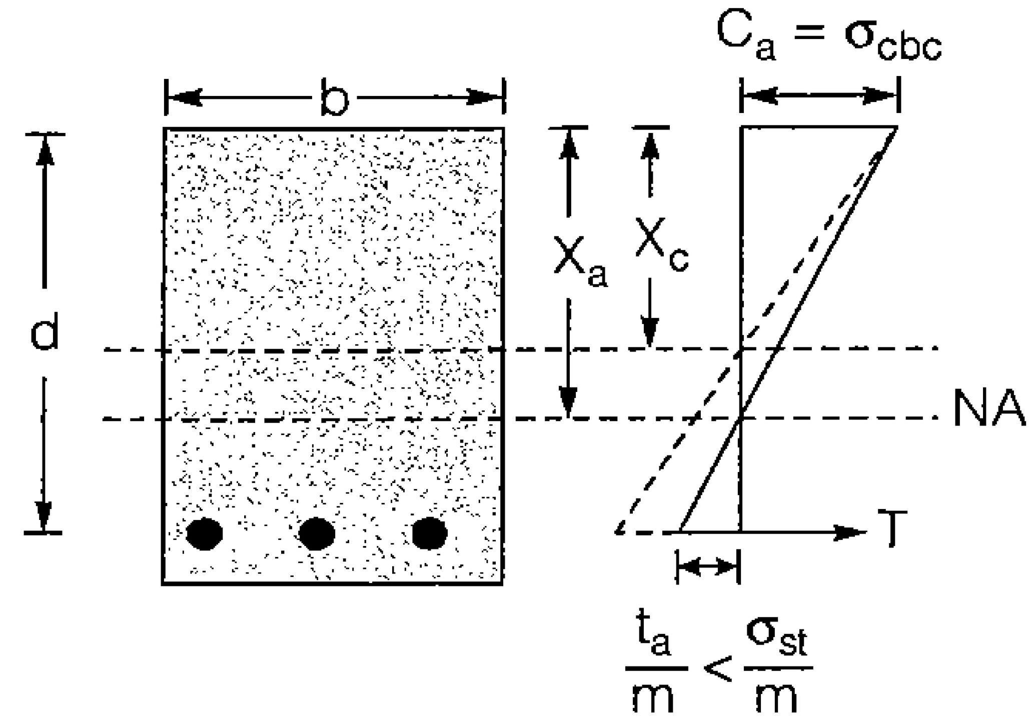
Here, $C_a < \sigma_{cbc}$

(iii) For over-reinforced section ($X_a > X_c$)

$$\frac{\sigma_{cbc}}{X_a} = \frac{t_a/m}{d - X_a}$$

$$M_r = bX_a \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{X_a}{3}\right)$$

$$M_r = t_a A_{st}\left(d - \frac{X_a}{3}\right)$$



Section Stress diagram

Doubly reinforced section

DOUBLY REINFORCED RECTANGULAR SECTION

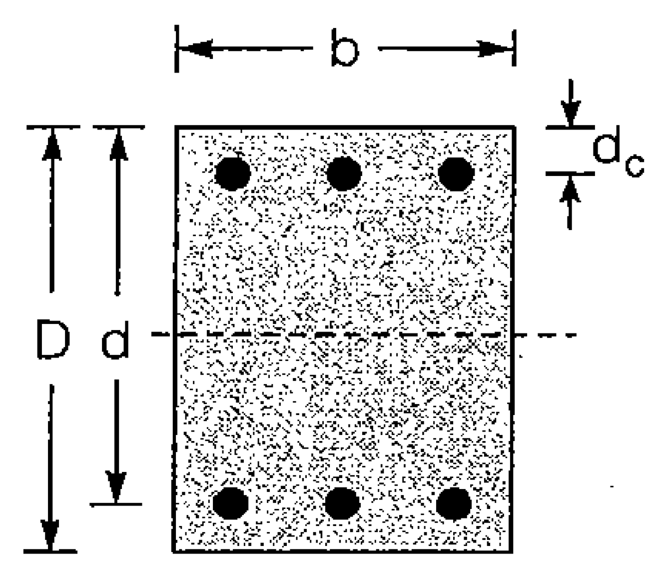
Critical depth of Neutral axis, (X_c)

$$X_c = \frac{mc}{t + mc} \cdot d$$

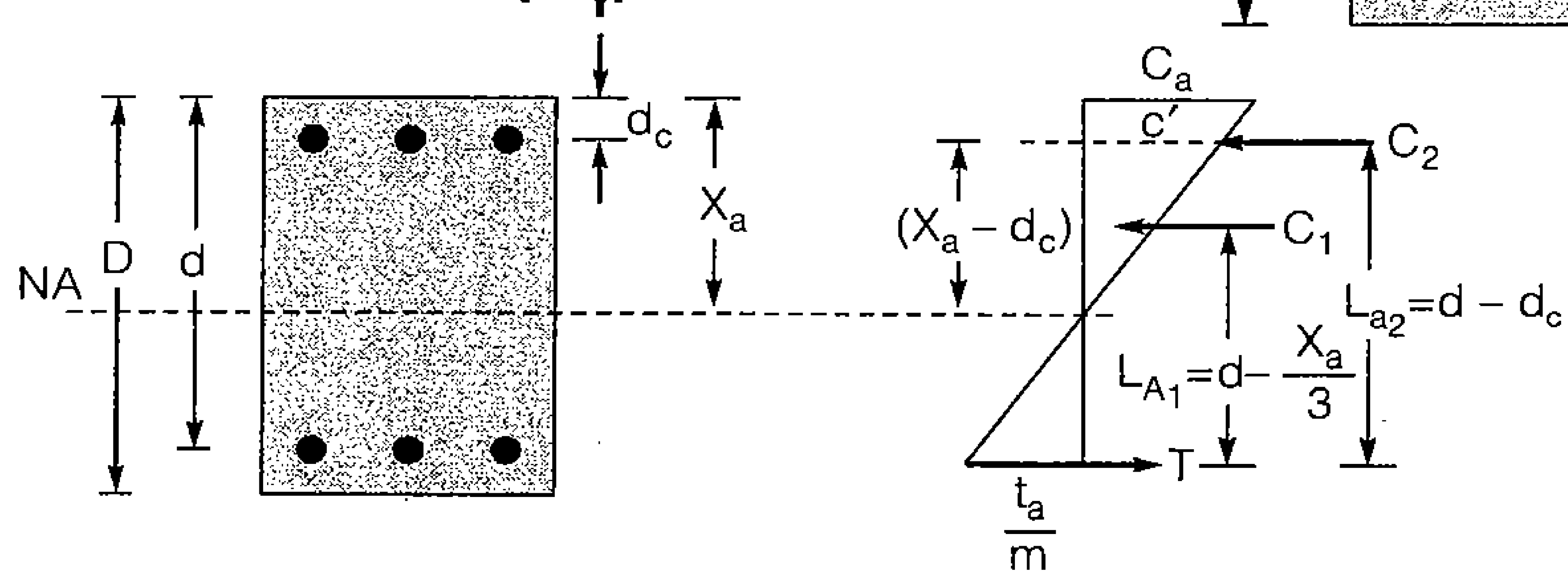
Actual depth of Neutral axis, (X_a)

$$\frac{bX_a^2}{2} + (1.5m - 1)\,A_{sc}\,(X_a - d_c) = m\,A_{st}(d - X_a)$$

Here, X_a = Actual depth of Neutral axis



Moment of resistance (M_r)



Section Stress diagram

$$M_r = C_1(L_{A1}) + C_2 L_{A2}$$

$$\Downarrow$$

$$M_r = bX_a \frac{C_a}{2}\left(d - \frac{X_a}{3}\right) + (1.5\,m - 1)\,A_{sc} \cdot c'(d - d_c)$$

Design Steps

(i) $M_r = Q\,b\,d^2$ → (ii) $Q = \frac{1}{2} c \cdot j \cdot k$ → (iii) $k = \frac{mc}{t + mc}$

(iv) $j = \left(1 - \frac{k}{3}\right)$ → (v) $m = \frac{280}{3\,\sigma_{cbc}}$ → (vi) $X_a = k \cdot d$

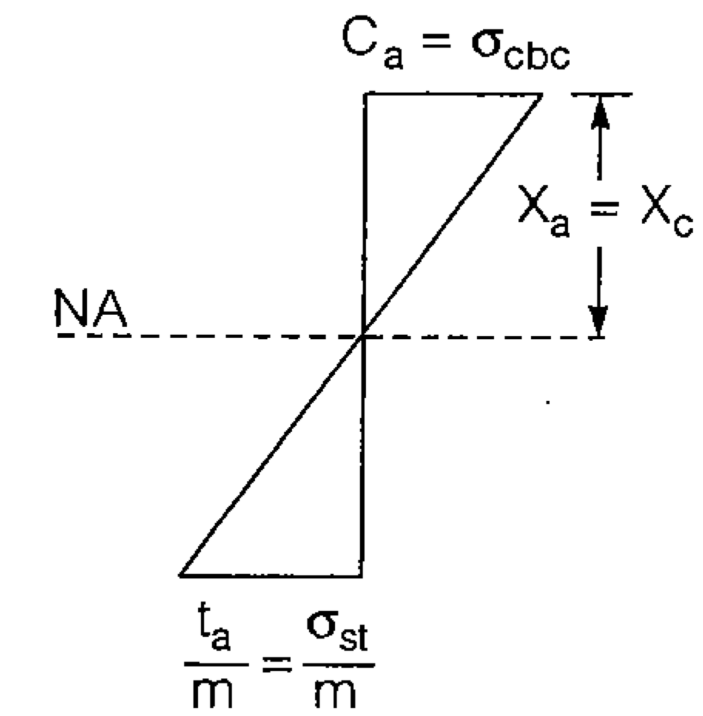
- Special case:

(i) Balanced section

$X_a = X_c$

$\sigma_a = \sigma_{cbc}$

$t_a = \sigma_{st}$

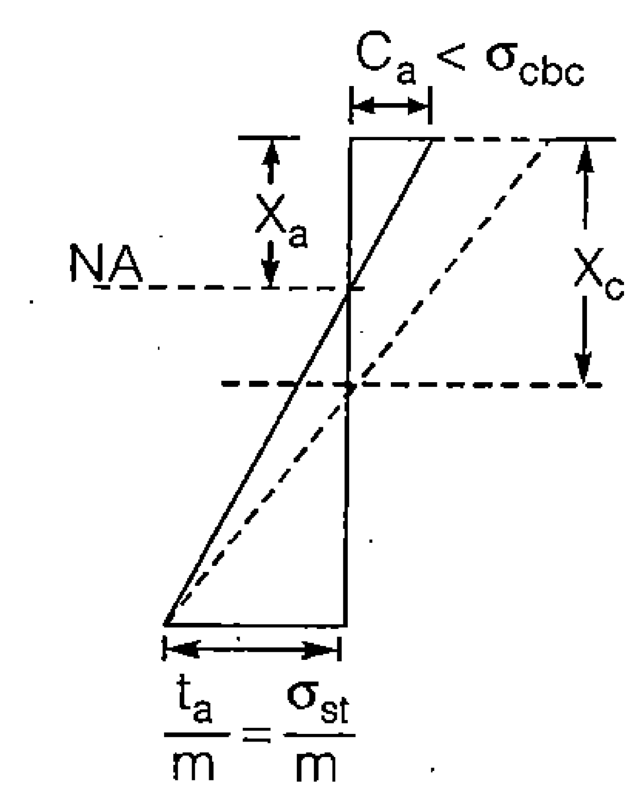


(ii) Under reinforced section

$X_a < X_c$

$C_a < \sigma_{cbc}$

$t_a = \sigma_{st}$

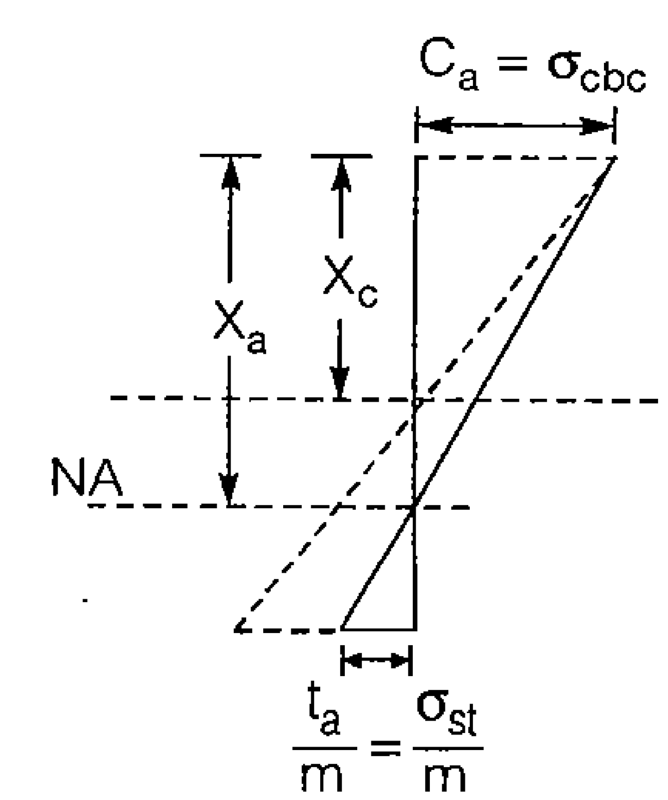


(iii) Over reinforced section

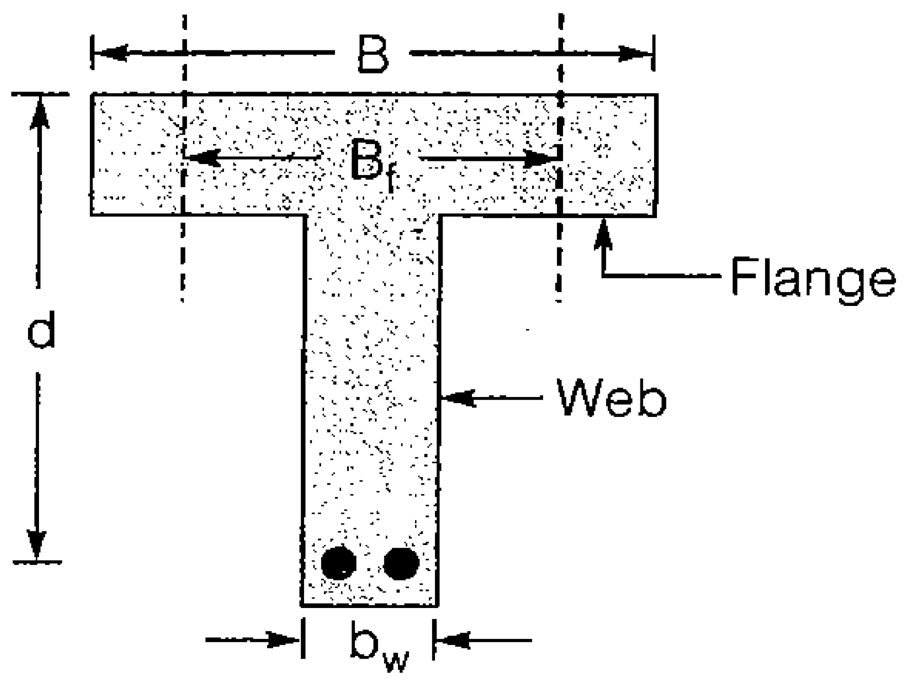
$X_a > X_c$

$C_a = \sigma_{cbc}$

$t_a < \sigma_{st}$



SINGLY REINFORCED T-SECTION



where, B_f = Effective width of flange
b_w = Width of web
d = Effective depth

Effective width of flange

(a) For beam casted monolithic with slab

$$B_f = \text{Minimum}\begin{cases}\left(\dfrac{l_0}{6} + b_w + 6d_f\right)\\ \text{or}\\ b_w + \dfrac{l_1}{2} + \dfrac{l_2}{2}\end{cases}$$

(b) For Isolated T-beam

$$B_f = \frac{l_0}{\left(\frac{l_0}{B} + 4\right)} + b_w$$

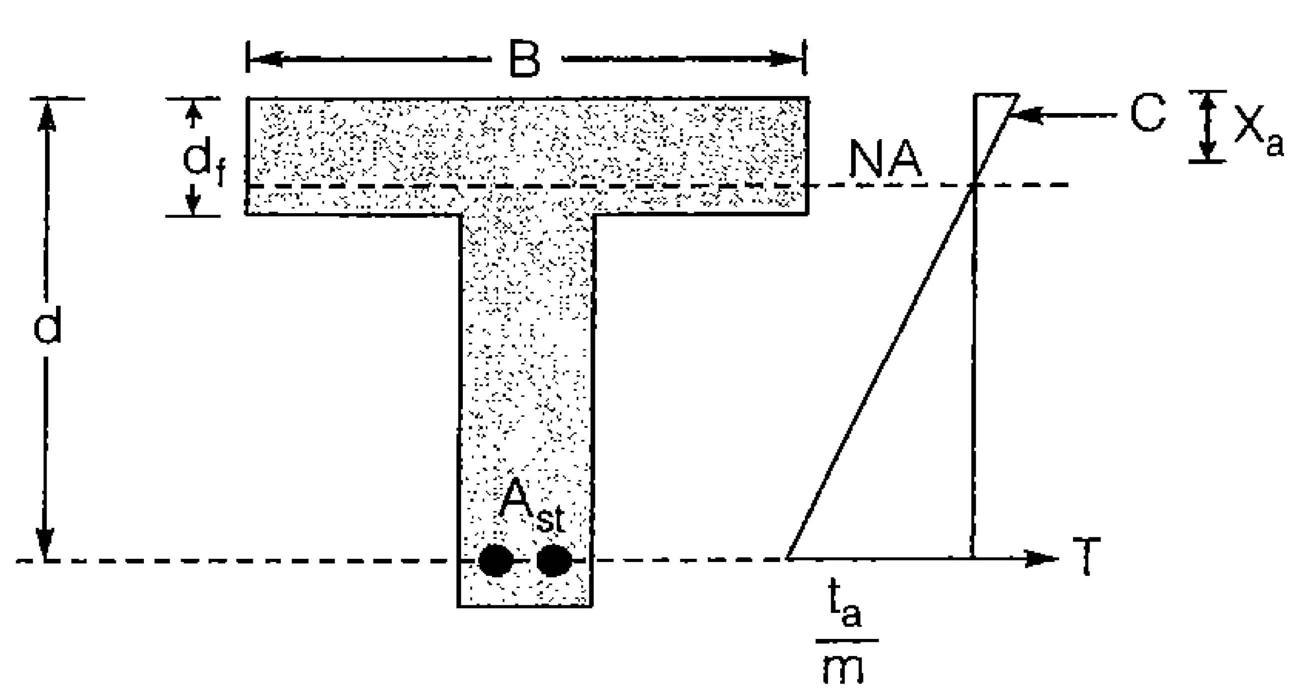
l_0 = Distance between points of zero moments in the beam
B = Total width of flange
b_w = Width of web

Critical depth of Neutral axis (X_c)

$$X_c = \left(\frac{mc}{t + mc}\right)d$$

- When Neutral axis is in flange area
  (i) Actual depth of Neutral axis

$$\frac{BX_a^2}{2} = mA_{st}(d - X_a)$$



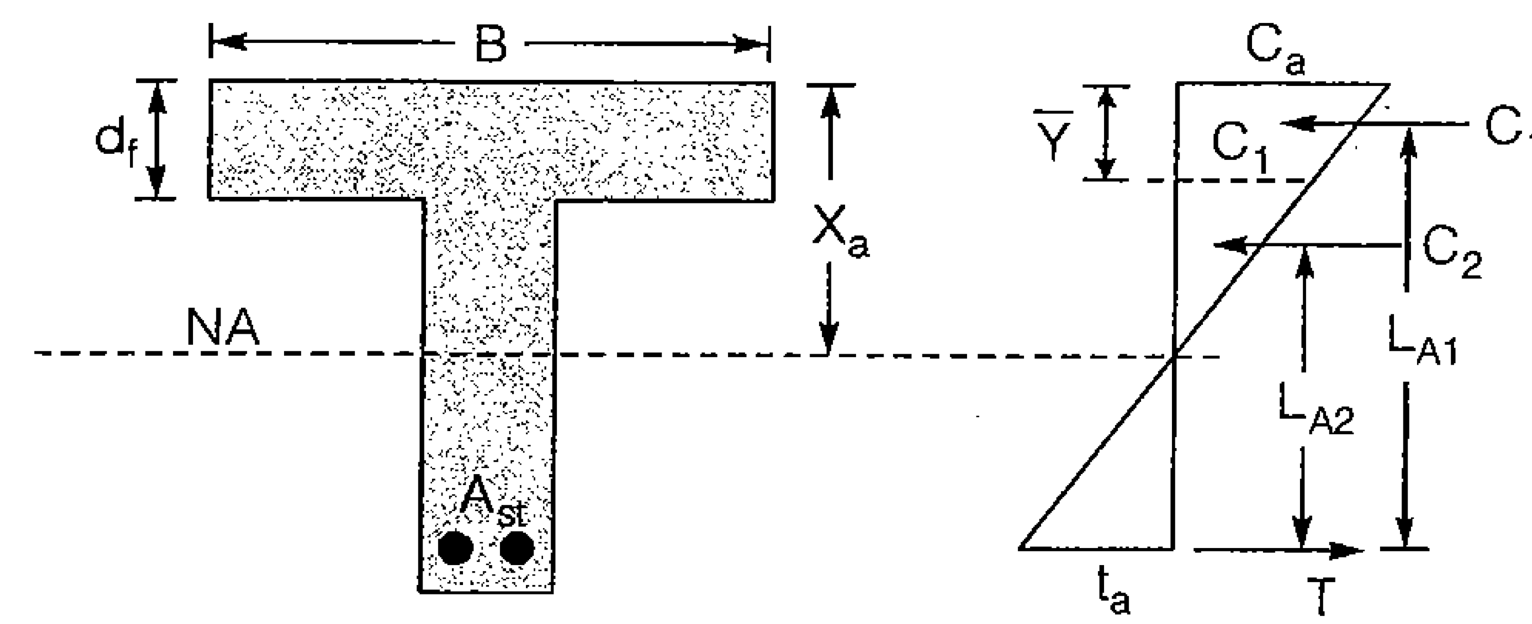
Here, X_a = Actual depth of Neutral axis

(ii) Moment of resistance (M_r)

$$M_r = BX_a \cdot \frac{C_a}{2}\left(d - \frac{X_a}{3}\right), \quad M_r = t_a A_{st}\left(d - \frac{X_a}{3}\right)$$

$X_a = X_c$	Balanced section	$C_a = \sigma_{cbc}$	$t_a = \sigma_{st}$
$X_a < X_c$	Under reinforced section	$C_a < \sigma_{cbc}$	$t_a = \sigma_{st}$
$X_a > X_c$	Over reinforced section	$C_a = \sigma_{cbc}$	$t_a < \sigma_{st}$

- When Neutral axis is in web area



Section　　Stress diagram

(i) For actual depth of neutral axis

$$B_f d_f \cdot \left(X_a - \frac{d_f}{2}\right) = mA_{st}(d - X_a)$$ By neglecting web area

$$B_f d_f \cdot \left(X_a - \frac{d_f}{2}\right) + bw\,\frac{(X_a - d_f)^2}{2} = mA_{st}(d - X_a)$$ By considering web area

(ii) Moment of resistance (M_r)

$$M_r = B_f \cdot d_f\left(\frac{C_a + C_1}{2}\right)\left[d - \left(\frac{C_a + 2C_1}{C_a + C_1}\right)\frac{d_f}{3}\right] + b_w(X_a - d_f)\cdot\frac{C_1}{2}\left[d - d_f - \frac{(X_a - d_f)}{3}\right]$$

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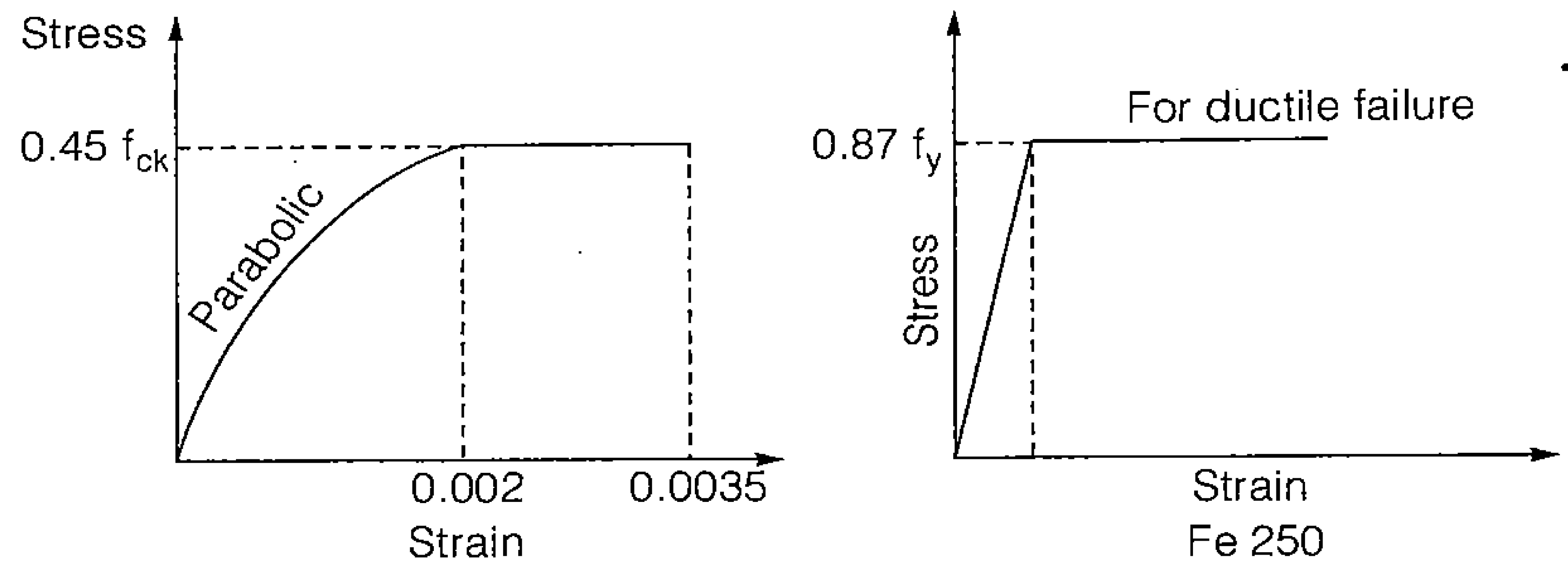
3

Limit State Method

Design stress strain curve at ultimate state



- Design value of strength

  For concrete

$$f_d = \frac{f}{\gamma_{mc}} = \frac{0.67 f_{ck}}{1.5} = 0.45 f_{ck}$$

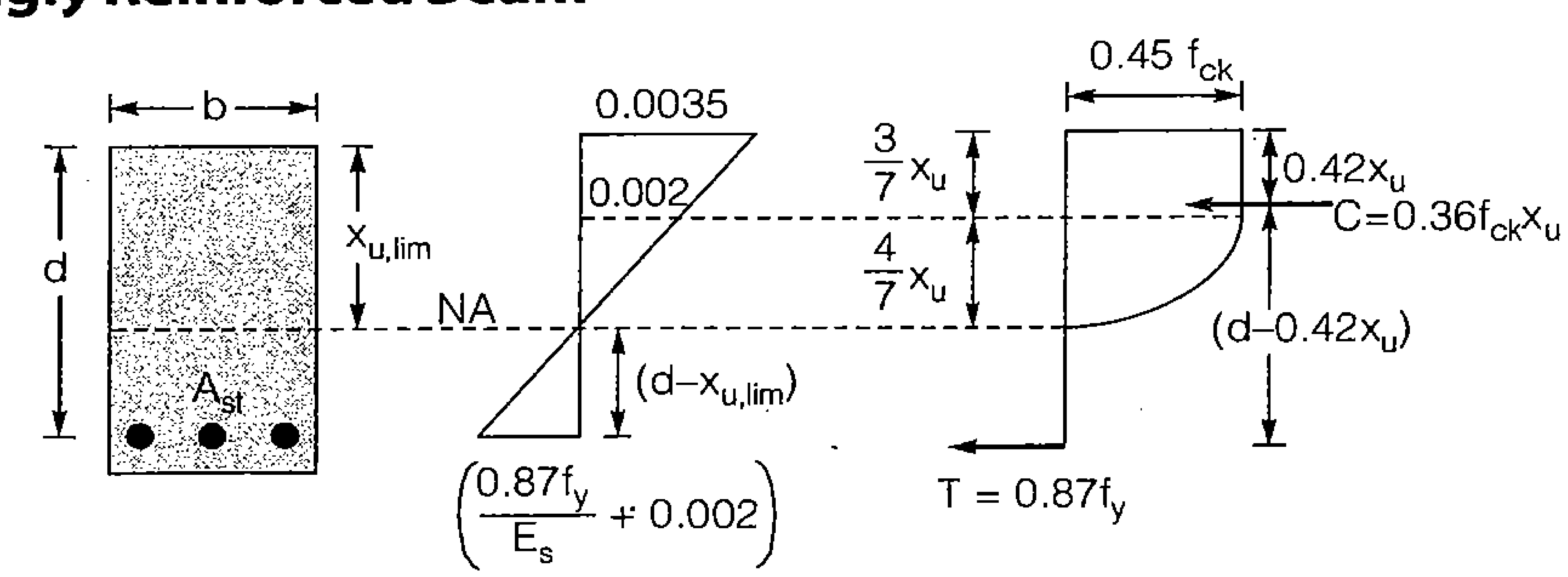
where, γ_{mc} = Partial factor of safety for concrete = 1.5

f_d = design value of strength

For steel

$$f_d = \frac{f_y}{1.15} = 0.87 f_y$$

Singly Reinforced Beam



Section | Strain Diagram | Stress Diagram

1. Limiting depth of neutral axis ($x_{u,lim}$)

$$X_{u,lim} = \frac{700}{0.87 f_y + 1100} \times d$$

for

Steel	$X_{u,lim}$
Fe 250	0.53 d
Fe 415	0.48 d
Fe 500	0.46 d

Here d = effective depth of beam

2. Actual depth of neutral axis (X_u)

$$C = T \Rightarrow \quad X_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

3. Lever arm = $d - 0.42\, X_u$
4. Ultimate moment of resistance

$$M_u = 0.36 f_{ck} b X_u (d - 0.42 X_u) \quad \text{or} \quad M_u = 0.87 f_y A_{st} (d - 0.42 X_u)$$

Remember: In LSM, actual depth of NA is found by equating total compressive and tensile force.

- Some special cases

1. When $X_u < X_{u,lim}$

   It is an under-reinforced section

$$M_u = 0.36 f_{ck} b X_u (d - 0.42 X_u)$$

and $$M_u = 0.87 f_y A_{st} (d - 0.42 X_u)$$

2. When $X_u = X_{u,lim}$

   It is balanced section

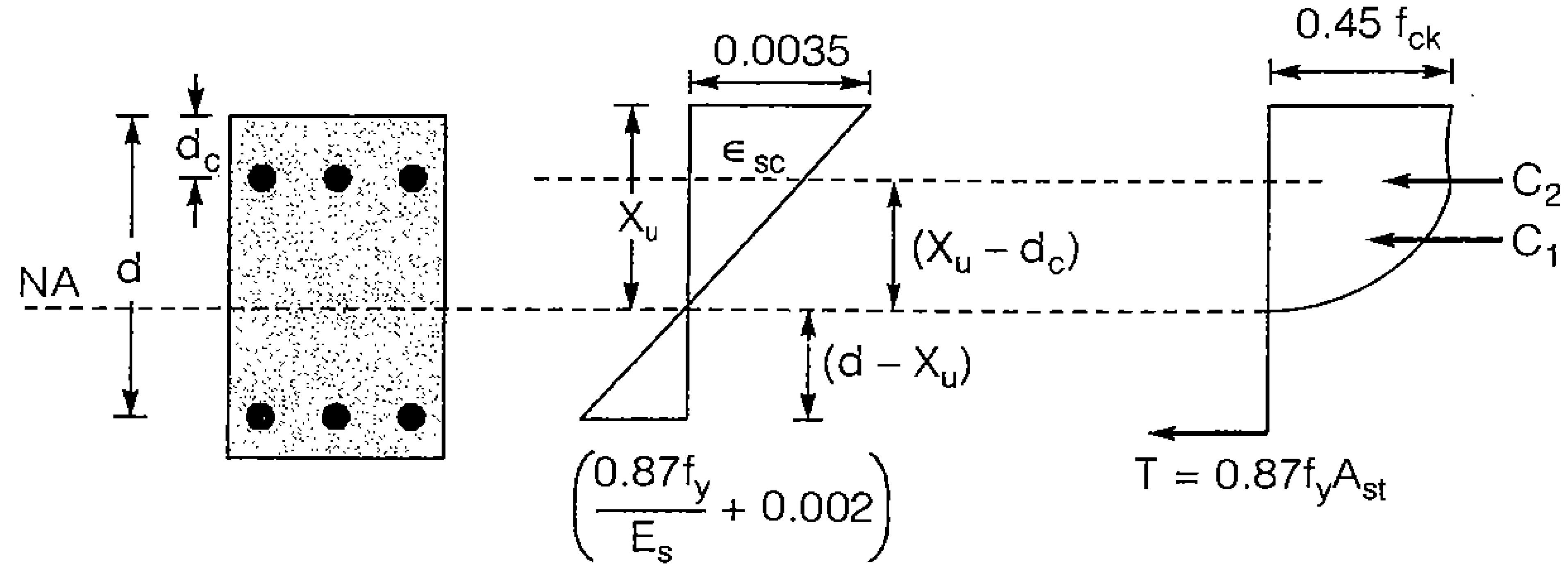
$$M_u = 0.36 f_{ck} b X_{u,lim} (d - 0.42 X_{u,lim})$$

$$M_u = 0.87 f_y A_{st} (d - 0.42 X_{u,lim})$$

3. When $X_u > X_{u,lim}$

   It is over reinforced section. In this case, keep X_u limited to $X_{u,lim}$ and moment of resistance of the section shall be limited to limiting moment of resistance, ($M_{u,lim}$).

Doubly Reinforced Section



Section Strain Diagram Stress Diagram

1. Limiting depth of neutral axis

$$\boxed{X_{u,lim} = \frac{700}{0.87f_y + 1100} \times d}$$

2. For actual depth of neutral axis (X_u)

$$\boxed{C = T} \Rightarrow \boxed{C_1 + C_2 = T}$$
$$\downarrow$$
$$\boxed{0.36f_{ck}bX_u + (f_{sc} - 0.45f_{ck})A_{sc} = 0.87f_yA_{st}}$$

3. Ultimate moment of resistance

$$\boxed{M_u = 0.36f_{ck}bX_u(d - 0.42X_u) + (f_{sc} - 0.45f_{ck})A_{sc}(d - d_c)}$$

where f_{sc} = stress in compression steel and it is calculated by strain at the location of compression steel (f_{sc})

T-Beam

1. Effective width of flange

$$\boxed{\text{Discussed in WSM}}$$

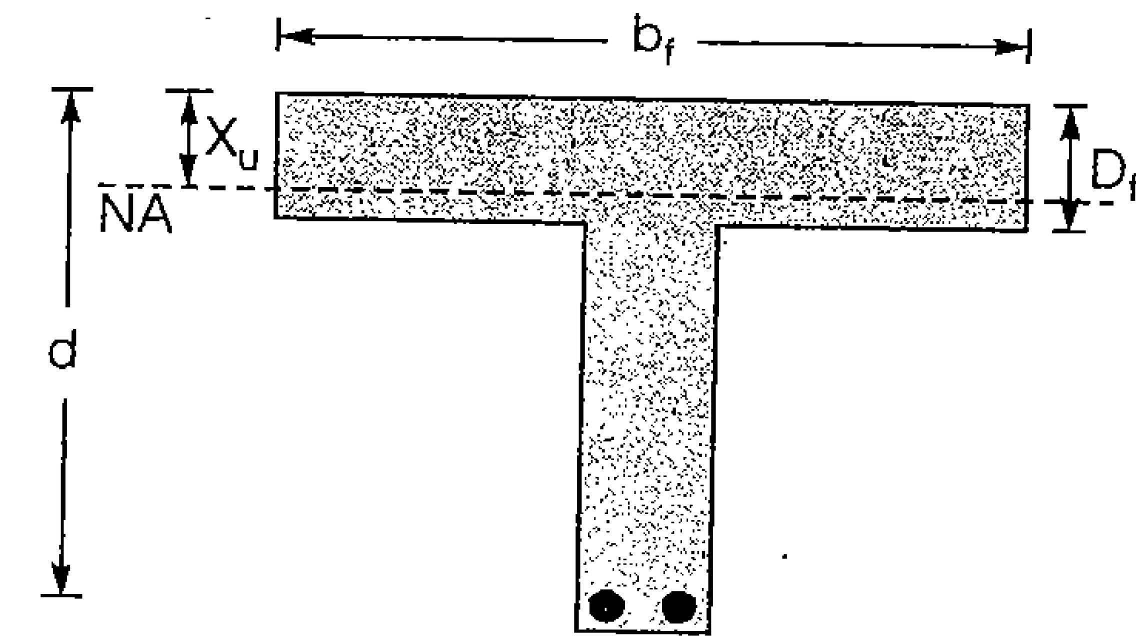
2. Limiting depth of neutral axis

$$\boxed{X_{u,lim} = \frac{700}{0.87f_y + 1100} \times d}$$

- **Singly reinforced T-Beam**

Case-1: When NA is in flange area

i.e., $\boxed{X_u < D_f}$



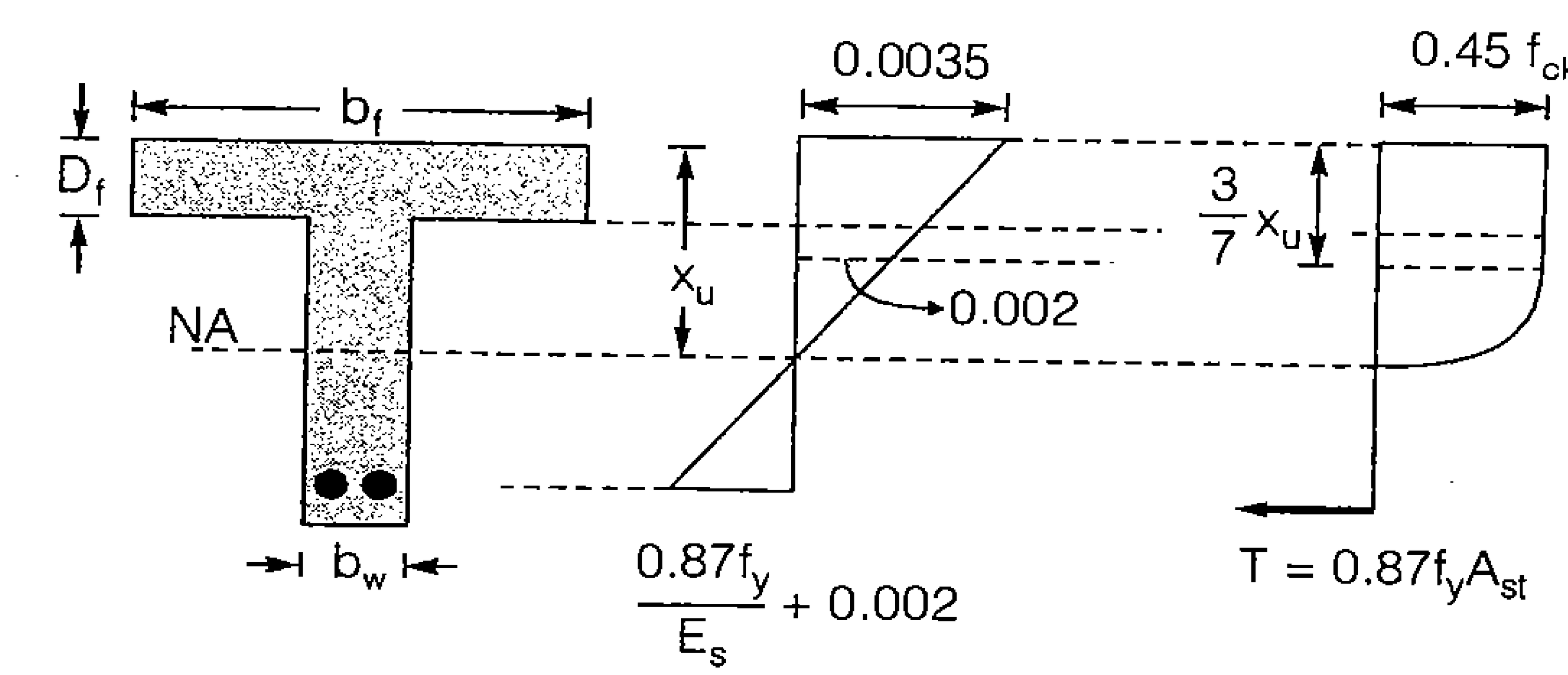
(a) for X_u

$$\boxed{X_u = \frac{0.87f_yA_{st}}{0.36f_{ck}b_f} < D_f}$$

(b) Ultimate moment of resistance

$$\boxed{M_u = 0.36f_{ck}b_fX_u(d - 0.42X_u)} \quad \textbf{or} \quad \boxed{M_u = 0.87f_yA_{st}(d - 0.42X_u)}$$

Case-2: When NA is in web area ($X_u > D_f$)



Section Strain Diagram Stress Diagram

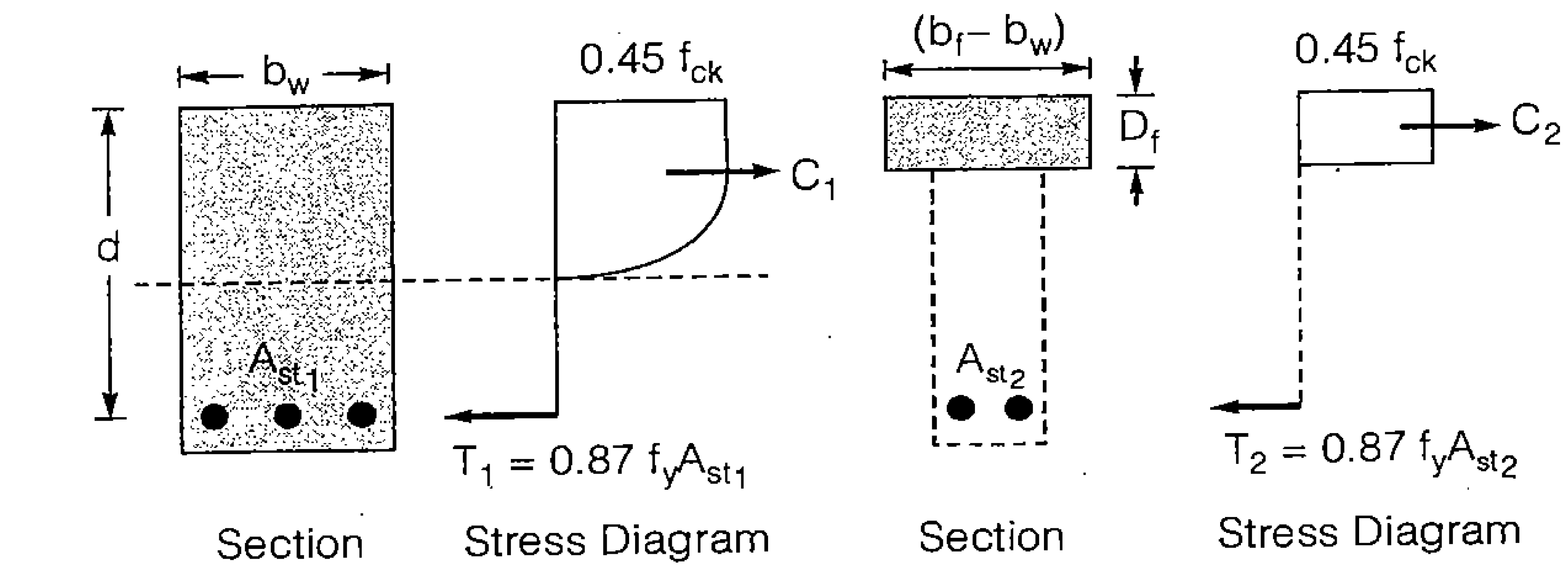
Case (a) when $X_u > D_f$ and $D_f < \frac{3}{7}X_u$

i.e., depth of flange is less than the depth of rectangular portion of stress diagram.

1. For actual depth of neutral ais

$$\boxed{0.36f_{ck}b_wX_u + 0.45f_{ck}(b_f - b_w)D_f = 0.87f_yA_{st}}$$

2. Ultimate moment of resistance



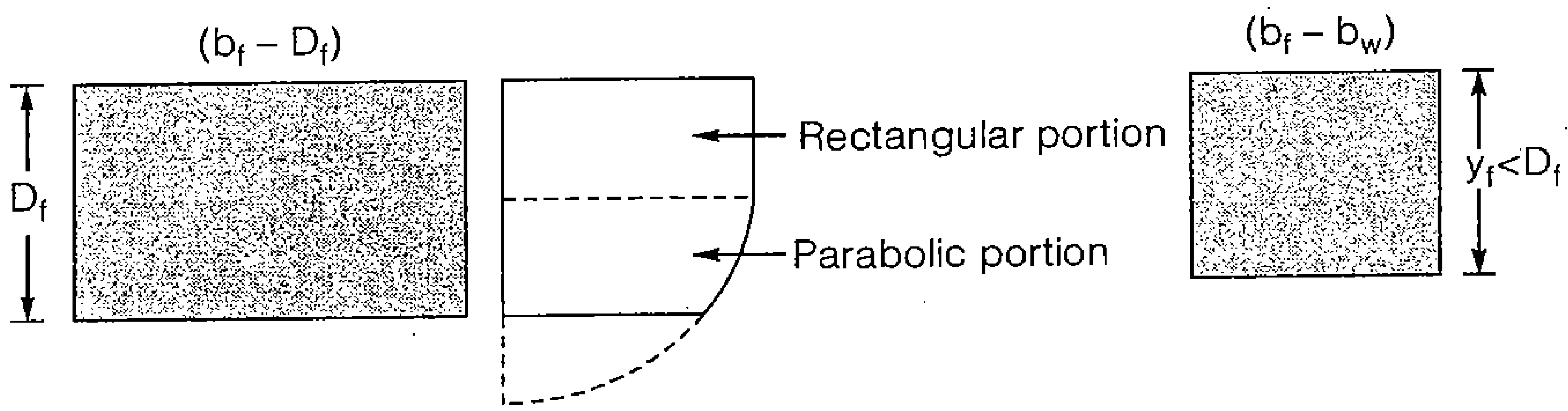
$$M_u = 0.36f_{ck}b_wX_u(d - 0.42X_u) + 0.45f_{ck}(b_f - b_w)D_f\left(d - \frac{D_f}{2}\right)$$

$$M_u = 0.87f_yA_{st_1}(d - 0.42X_u) + 0.87f_yA_{st_2}\left(d - \frac{D_f}{2}\right)$$

$$A_{st_1} = \frac{0.36f_{ck}b_wX_u}{0.87f_y}, \quad A_{st_2} = \frac{0.45f_{ck}(b_f - b_w)D_f}{0.87f_y}$$

- **Special Case (2):** When $X_u > D_f$ and $D_f > \frac{3}{7}X_u$

  i.e., depth of flange is more than depth of rectangular portion of stress diagram.



As per IS 456: 2000

$(b_f - b_w)D_f$ portion of flange is converted into $(b_f - b_w)y_f$ section for which stress is taken constant throughout the section is 0.45 f_{ck}.

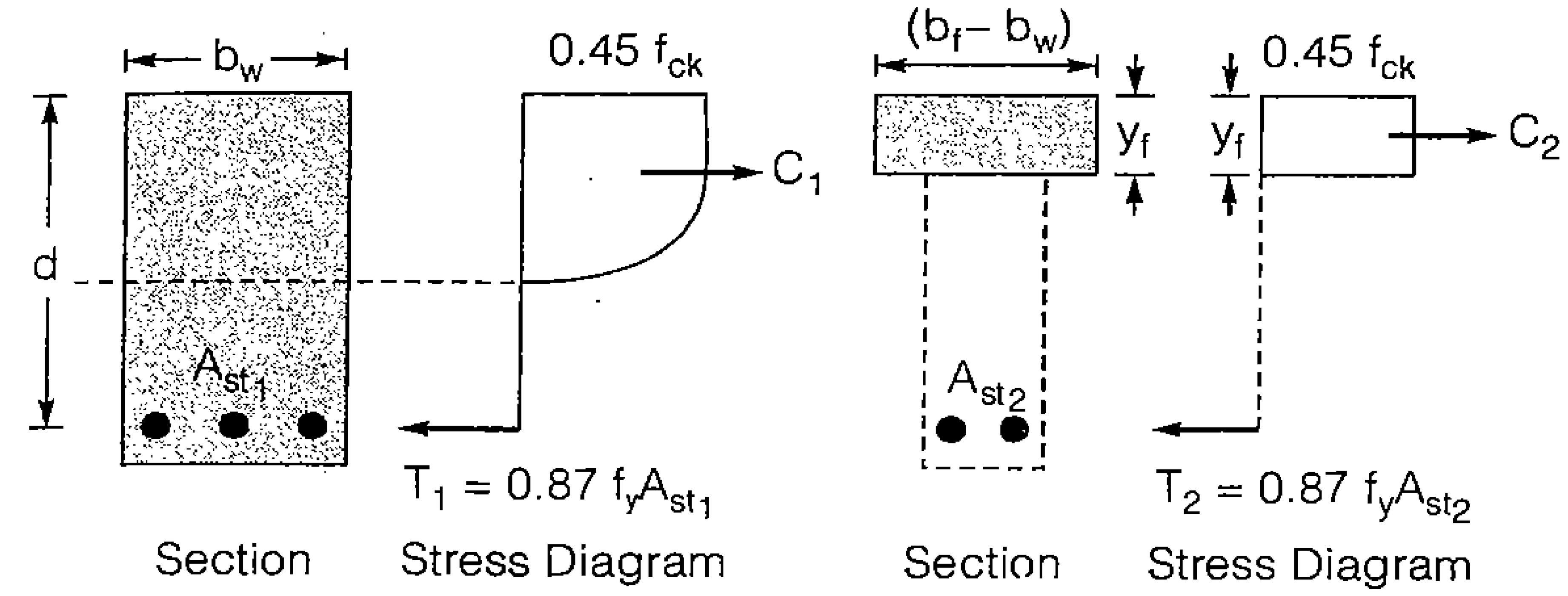
As per IS 456: 2000

$$y_f = 0.15X_u + 0.65D_f < D_f$$

1. For actual depth of neutral axis

$$0.36f_{ck}b_wX_u + 0.45f_{ck}(b_f - b_w)y_f = 0.87f_yA_{st_1} + 0.87f_yA_{st_2}$$

or $$0.36f_{ck}b_wX_u + 0.45f_{ck}(b_f - b_w)y_f = 0.87f_yA_{st}$$



$$M_u = 0.36f_{ck}b_wX_u(d - 0.42X_u) + 0.45f_{ck}(b_f - b_w)y_f\left(d - \frac{y_f}{2}\right)$$

$$M_u = 0.87f_yA_{st_1}(d - 0.42X_u) + 0.87f_yA_{st_2}\left(d - \frac{y_f}{2}\right)$$

$$A_{st_1} = \frac{0.36f_{ck}b_wX_u}{0.87f_y} \quad \text{and} \quad A_{st_2} = \frac{0.45f_{ck}(b_f - b_w)y_f}{0.87f_y}$$

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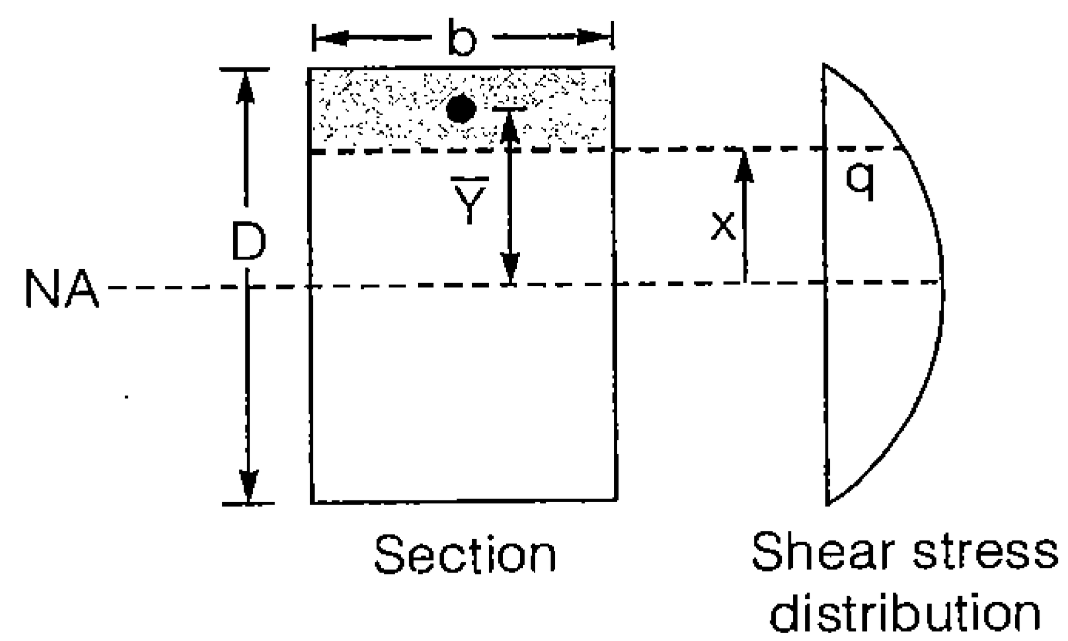
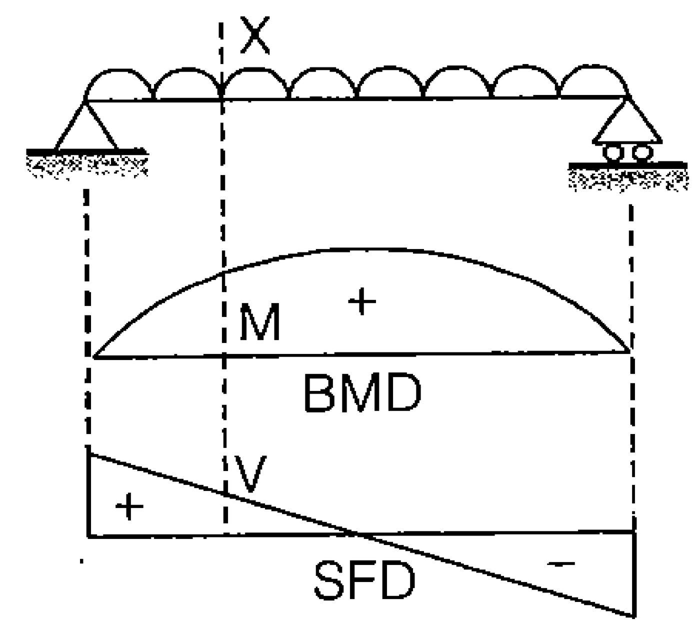
4 Shear

Shear stress

(a) For Homogeneous beam

$$q = \frac{V}{Ib} \cdot A\overline{Y}$$





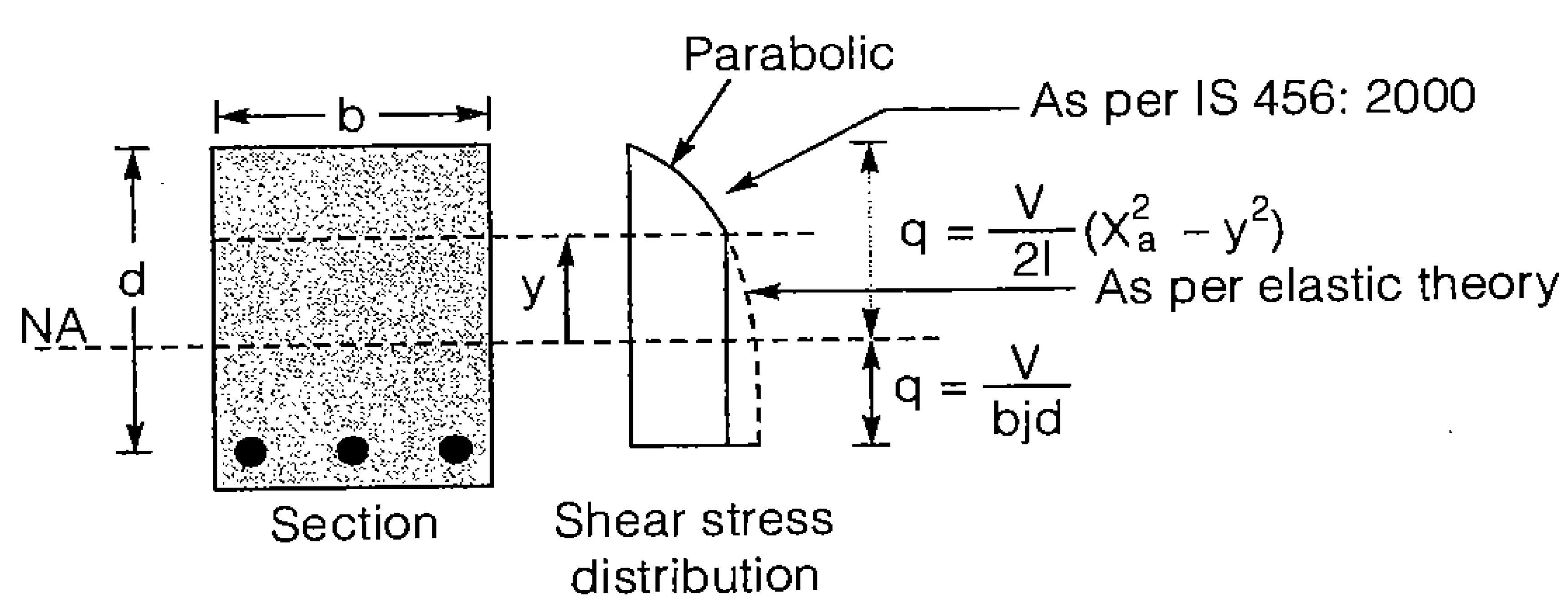
where, q = shear stress at any section

V = shear force at any section

$A\overline{Y}$ = Moment of area of section above the point of consideration

I = Moment of inertia of section = $\frac{bD^3}{12}$

(b) For Reinforced concrete beam



(i) Shear stress above NA

$$q = \frac{V}{2I} \cdot (X_a^2 - y^2) \qquad q_{max} = \frac{V}{2I} \cdot X_a^2 \text{ at } y = 0$$

(ii) Shear stress below NA $q = \frac{V}{bjd}$

As per IS 456 : 2000

Nominal shear stress, $\tau_v = \frac{V}{bd}$

Remember The maximum shear stress $q = \frac{V}{bjd}$ obtained from elastic theory, is greater than the nominal shear stress (or Average shear stress) τ, suggested by IS 456: 2000.

Design shear strength of concrete (τ_c) without shear reinforcement as per IS 456: 2000

τ_c depends on
(i) Grade of concrete
(ii) Percentage of steel,

$$p = \frac{A_{st}}{bd} \times 100$$

where, A_{st} = Area of steel
b = Width of the Beam
d = Effective depth of the beam

p	WSM		LSM	
	M 20	M 25	M 20	M 25
0 ≤ 0.15	0.18	0.19	0.28	0.29
0.25	0.22	0.23	0.36	0.36
0.50	0.30	0.31	0.48	0.49
0.75	0.35	0.36	0.56	0.57
1.00	0.39	0.40	0.62	0.64

- **Maximum shear stress $\tau_{c,max}$ with shear reinforcement is**

	M15	M20	M25	M30	M35	M40 & above
LSM	2.5	2.8	3.1	3.5	3.7	4.0
WSM	1.6	1.8	1.9	2.2	2.3	2.5

$$\tau_v \ngtr \tau_{c\,max}$$

- **Minimum shear reinforcement (As per IS 456: 2000)**

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y}$$ This is valid for both WSM and LSM

or

$$S_v \leq \frac{2.175 f_y A_{sv}}{b}$$

where, A_{sv} = Area of shear reinforcement
S_v = Spacing of shear reinforcement

- **Spacing of shear reinforcement**

  Maximum spacing is minimum of (i), (ii) and (iii)

  (i) $$S_v = \frac{2.175\, f_y\, A_{sv}}{b}$$

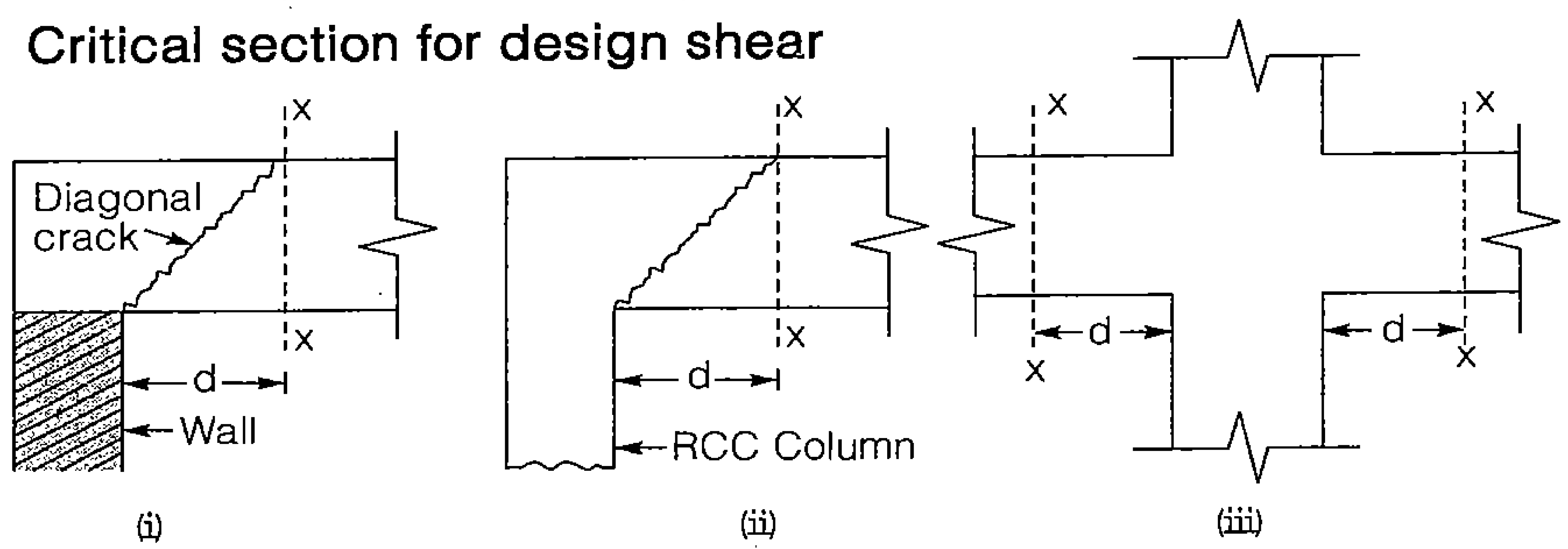
  (ii) 300 mm

  (iii) 0.75d → For vertical stirrups

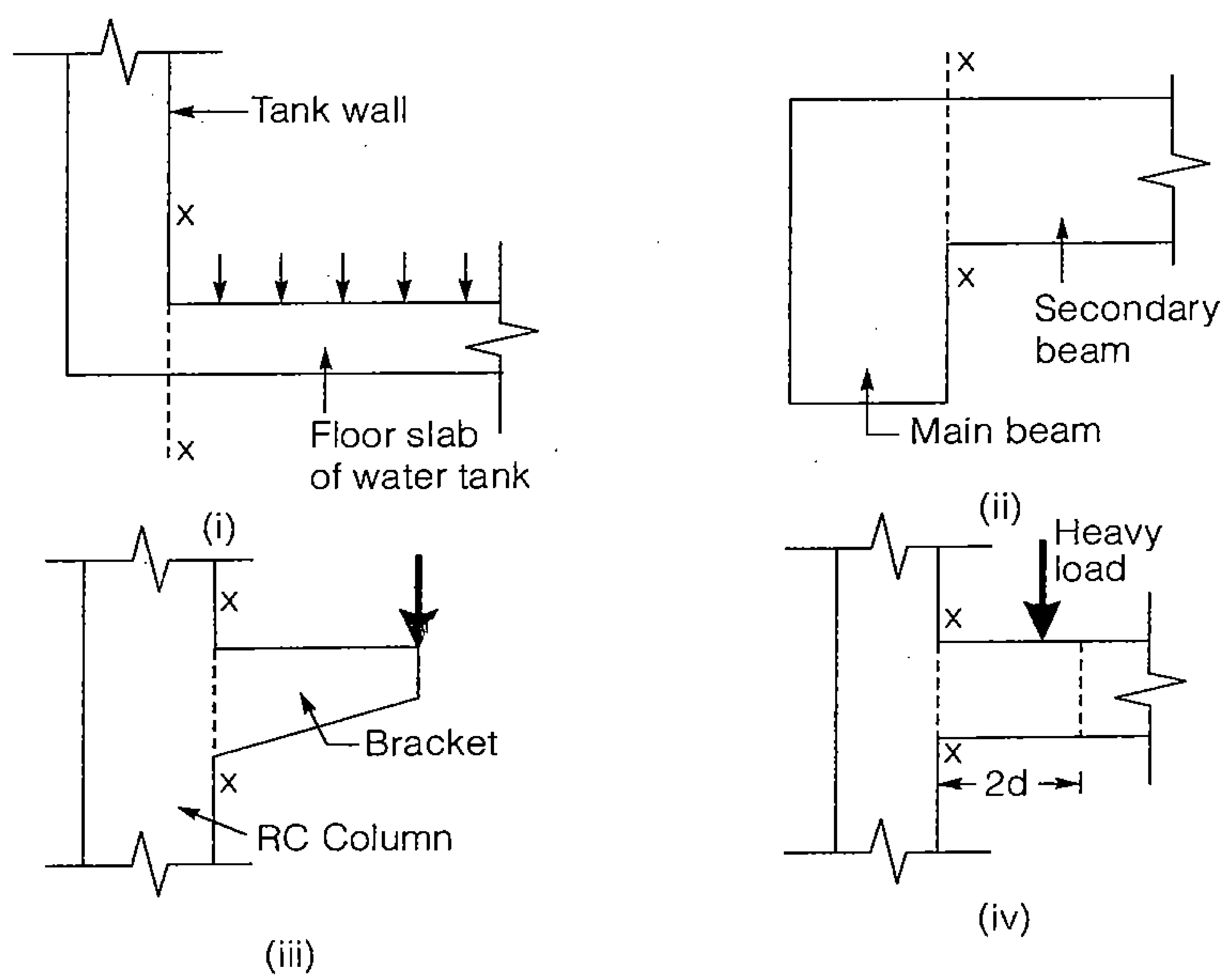
  d → For inclined stirrups

  where, d = effective depth of the section

- **Critical section for design shear**



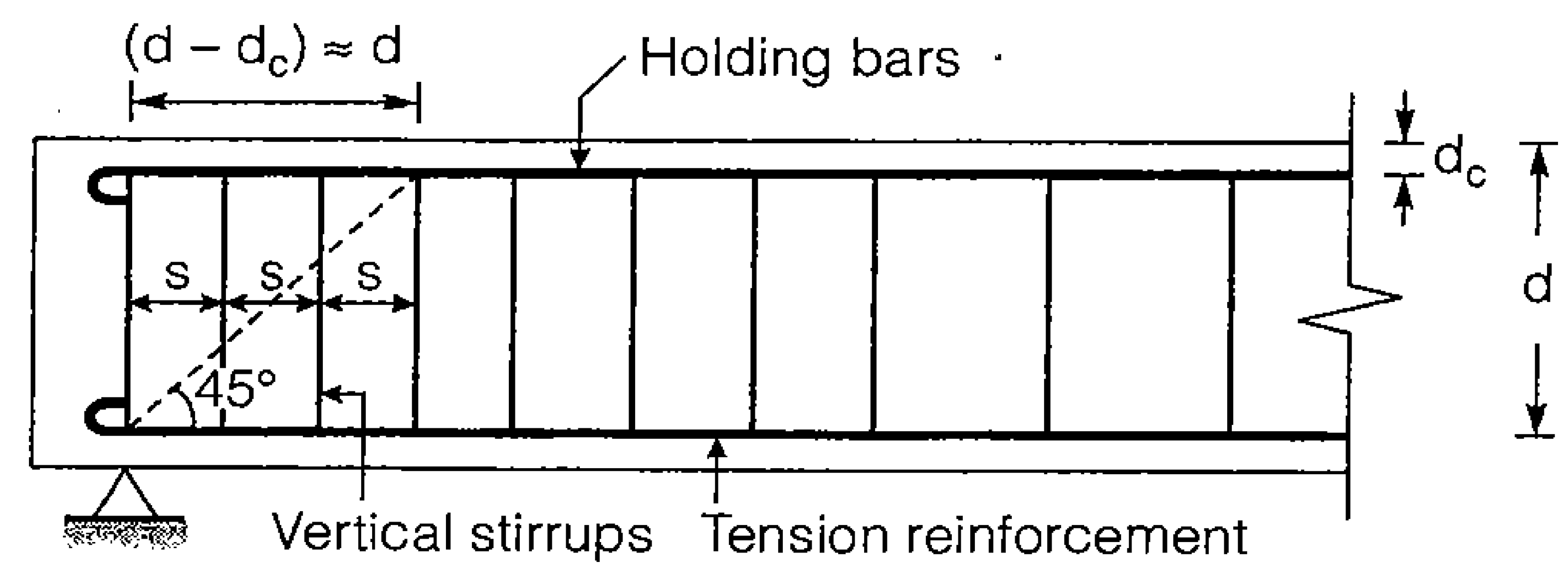
(a) Critical section X-X at d from the face of the support



(b) Critical section X-X at the face of the support

The above provisions are applicable for beams generally carrying uniformly distributed load or where the principal load is located beyond 2d from the face of the support.

- **Vertical stirrups:**



Shear force V_s will be
Resisted by shear
Reinforcement provided in 'd' length of the beam,

$$V_s = \left(\frac{d}{S_v}\right) A_{sv} \cdot \sigma_{sv} \quad \text{for WSM}$$

where, A_{sv} = Cross-sectional area of stirrups

S_v = Centre to centre spacing of stirrups

$$V_{su} = \left(\frac{d}{S_v}\right) A_{sv} (0.87\, f_y) \quad \text{for LSM}$$

- **Inclined stirrups : or a series of bars bent-up at different cross-section:**

$$V_s = A_{sv} \cdot \sigma_{sv} \cdot (\sin\alpha + \cos\alpha)\left(\frac{d}{S_v}\right) \quad \text{for WSM}$$

$$V_{su} = A_{sv} \cdot (0.87\, f_y)(\sin\alpha + \cos\alpha)\left(\frac{d}{S_v}\right) \quad \text{LSM}$$

- **Bent up Bars:**

  Single or a group of bent up bars are provided at distance $\sqrt{2}a = \sqrt{2}jd$ from support in such a way that $< ACB = 45°$, $< CAB = < CBA = 67\frac{1}{2}°$.

  Generally bar should not be bent up beyond a distance $l/4$ from the support. Where l = length of span.

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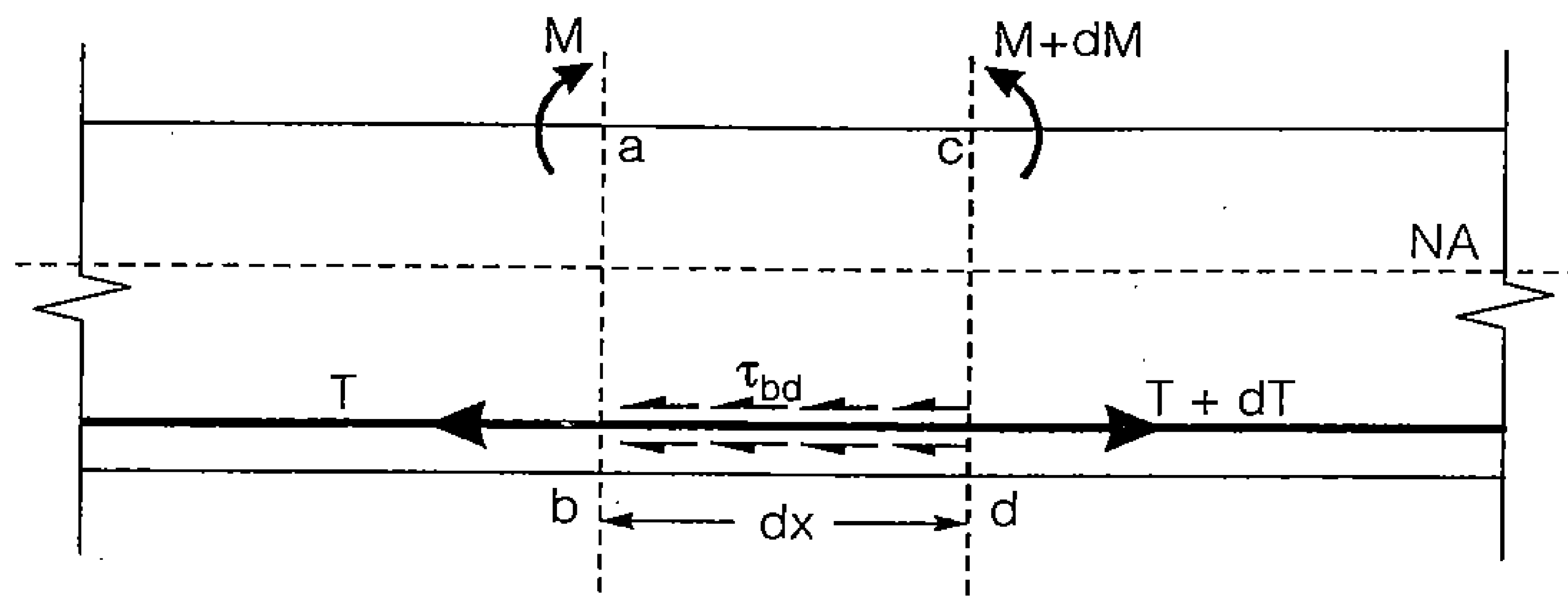
Bond, Anchorage and Development Length

5

Bond stress (τ_{bd})

$$\tau_{bd} = \frac{V}{\Sigma p\, jd}$$



where V = Shear force at any section

d = Effective depth of the section

Σp = Sum of all perimeter of reinforcement

$= n \cdot \pi (\phi)$

n = Number of reinforcement

ϕ = diameter of reinforcement

Permissible bond stress

As per IS 456 : 2000

	M15	M20	M25	M30	M35	M40
WSM	0.6	0.8	0.9	1.0	1.1	1.2
LSM	—	1.2	1.4	1.5	1.7	1.9

These value of bond stress is for plain bar in tension.

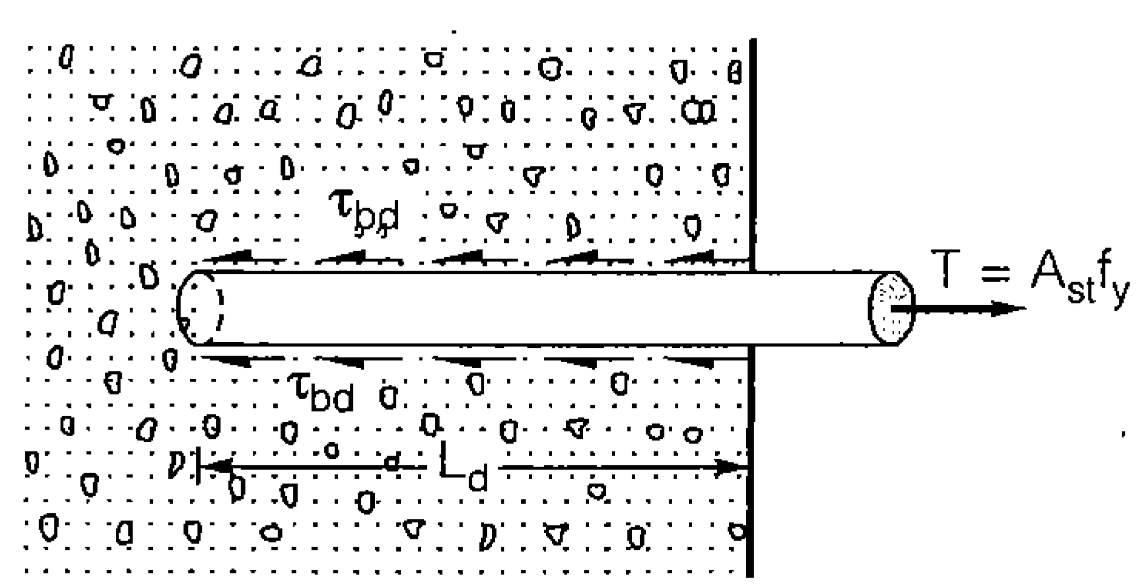
For deformed bar the above value should be increased by 60%.

For bar in compression the above value should be increased by 25%.

Development length (L_d)

$$L_d = \frac{\phi\, \sigma_{st}}{4 \cdot \tau_{bd}}$$ For WSM

$$L_d = \frac{\phi \cdot 0.87\, f_y}{4 \cdot \tau_{bd}}$$ For LSM



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Torsion

6

Equivalent shear force

$$V_{eq} = V + \frac{1.6\,T}{b}$$ For WSM

$$V_{u\,eq} = V_u + \frac{1.6\,T_u}{b}$$ For LSM

where V = Shear force

T = Torsional moment

b = Width of the section

- Nominal shear stress

$$\tau_v = \frac{V_{eq}}{bd} \ngtr \tau_{c\max}$$ For WSM

$$\tau_{vu} = \frac{V_{ueq}}{bd} \ngtr \tau_{c,\max}$$ For LSM

Equivalent moment

$$M_{eq} = M + \frac{T}{1.7}\left[1 + \frac{D}{b}\right]$$ For WSM

$$M_{u\,eq} = M_u + \frac{T_u}{1.7}\left[1 + \frac{D}{b}\right]$$ For LSM

where, M = Bending moment

D = Overall depth of the section

d = Effective depth

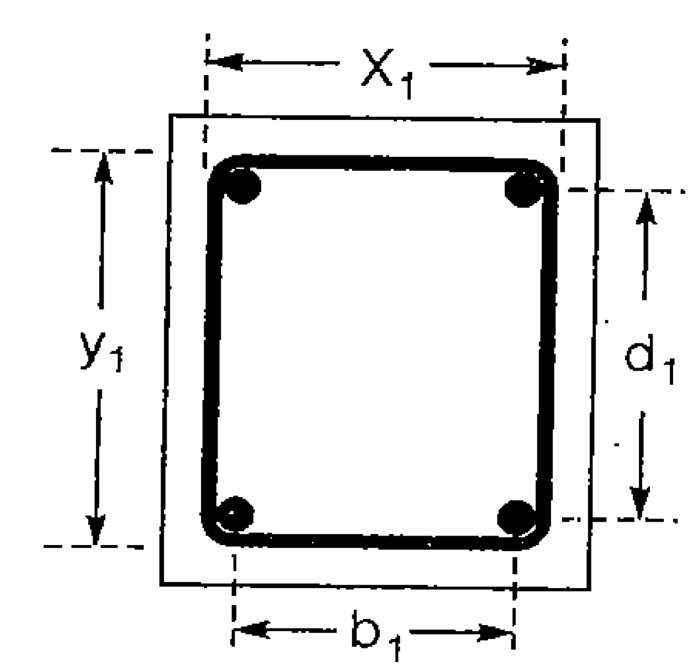
Transverse reinforcement

As per IS 456: 2000

$$A_{Sv} = \frac{S_v}{d_1 \sigma_{S_v}}\left[\frac{T}{b_1} + \frac{V}{2.5}\right]$$

Also, $$S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d_1}{V_s}$$

where, $$V_s = \frac{T}{b_1} + \frac{V}{2.5}$$



Here, T = Torsional moment

S_v = Spacing of the stirrup reinforcement

b_1 = Centre to centre distance between corner bars in the direction of width

d_1 = Centre to centre distance between corner bars in the direction of depth of member

b = Breadth of member

σ_{sv} = Permissible tensile stress in shear reinforcement

Maximum spacing for Transverse reinforcement

(i) x_1 (ii) $\frac{x_1 + y_1}{4}$ (iii) 300 mm

> **Remember**: When a beam is subjected to torsion, if depth of the beam is more than 450 mm or for beam not subjected to torsion if the depth of web exceeds 750 mm then side face reinforcement equal to 0.1% of cross-sectional area and is equally distributed on both faces of the beam.

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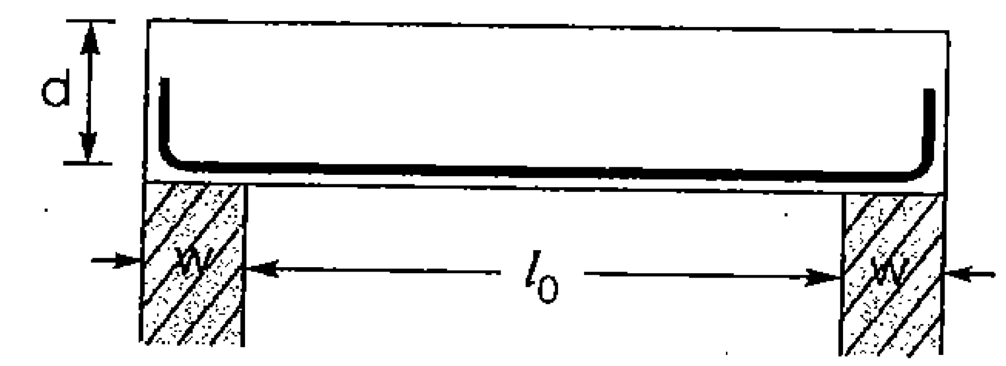
7 Beams and Slabs

Effective span

A. Simply supported beams and slabs (l_{eff})



$$l_{eff} = \text{minimum}\begin{cases} l_0 + w \\ l_0 + d \end{cases}$$

Here, l_0 = clear span

w = width of support

d = depth of beam or slab

B. For continuous beam

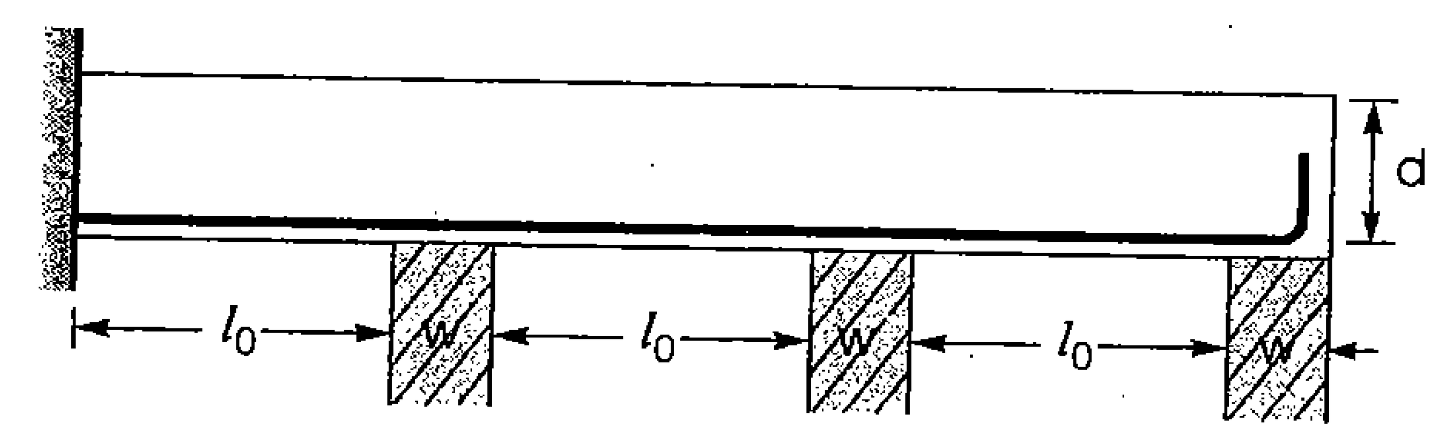
(i) If width of support $< \frac{1}{12}$ of clear span

$$l_{eff} = \text{minimum}\begin{cases} l_0 + w \\ l_0 + d \end{cases}$$

(ii) If width of support $> \frac{1}{12}$ of clear span

(a) When one end fixed other end continuous or both end continuous.

$$l_{eff} = l_0$$

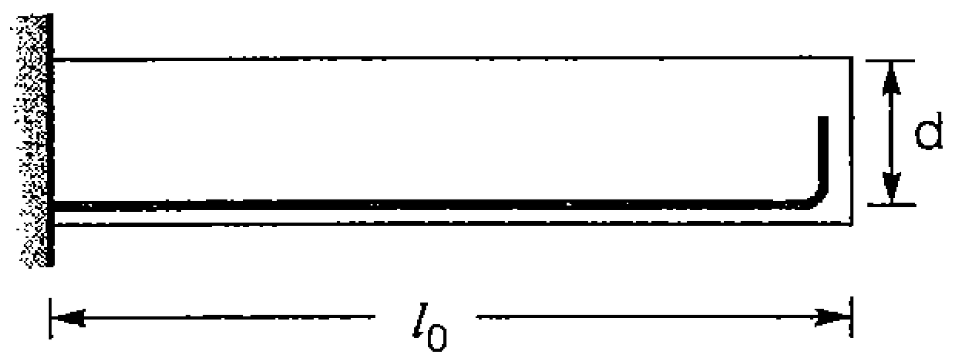


(b) When one end continuous and other end simply supported.

$$l_{eff} = \text{Minimum}\begin{cases} l_0 + w/2 \\ l_0 + d/2 \end{cases}$$

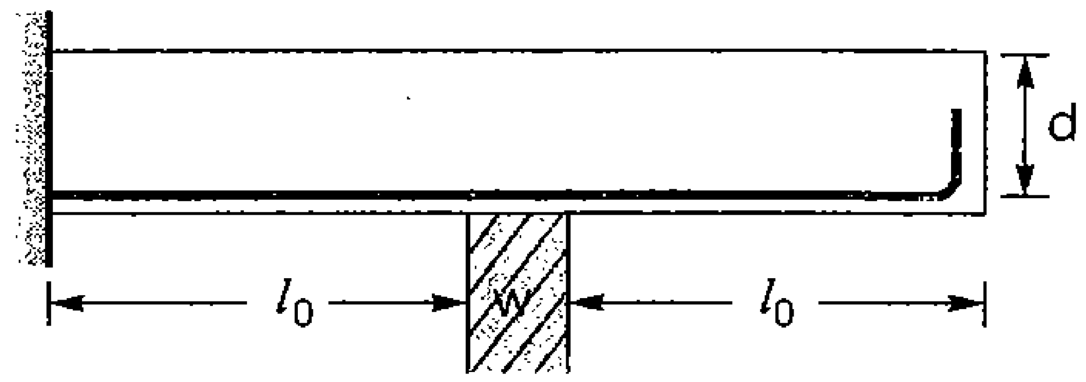
C. Cantilever

(i)



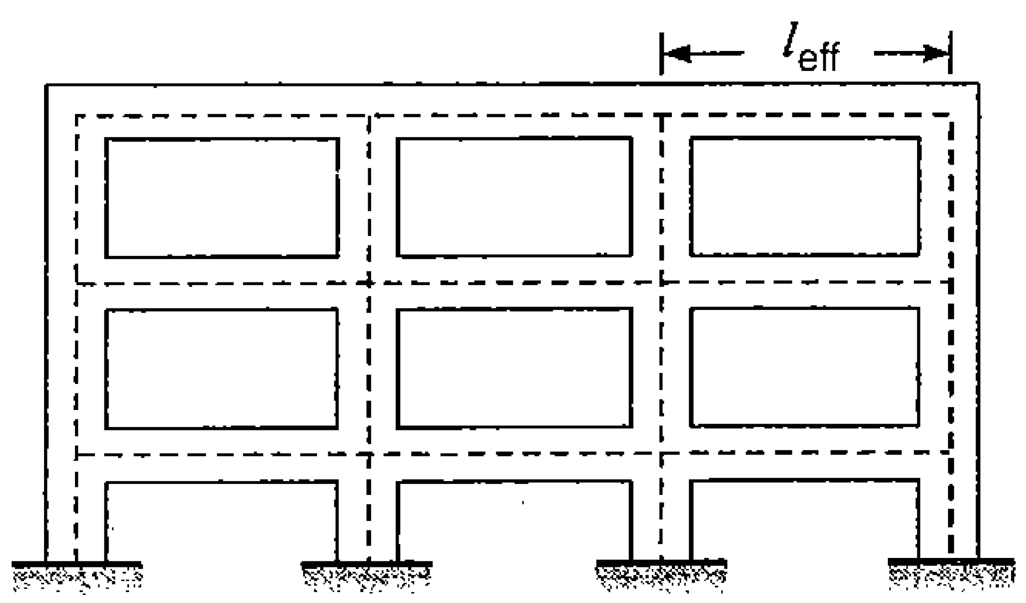
$$l_{eff} = l_0 + \frac{d}{2}$$

(ii)



$$l_{eff} = \left(l_0 + \frac{w}{2}\right)$$

D. Frames



l_{eff} = Centre to centre distance

Control of deflection

(i) This is one of the most important check for limit state of serviceability.

(a) The final deflection due to all loads including the effect of temperature, creep and shrinkage and measured from as cast level of the support of floors, roofs and other horizontal members should not normally exceed $\frac{\text{span}}{250}$.

(b) The deflection including the effect of temperature, creep and shrinkage occurring after erection of partition and application of finishes should not normally exceed $\frac{\text{span}}{350}$ or 20 mm whichever is less.

(ii) The vertical deflection limit may generally be satisfied if

(a) Basic span to effective depth ratio for span upto 10 m is

Types of Beams:	$\frac{\text{span}}{\text{effective depth}}$
For cantilever	→ 7
For simply supported	→ 20
For continuous	→ 26

(b) For span > 10 m effective depth = $\frac{(\text{span})^2}{10 \times A}$

where 'A' is span to effective depth ratio for span upto 10 m.

(c) Depending upon the tension reinforcement the value 'A' can be modify by multiplying a factor called modification factor (MF_1)

$$\text{effective depth} = \frac{\text{span}}{A \times MF_1}$$

where $f_s = 0.58\, f_y \times \frac{\text{Area of steel required}}{\text{Area of steel provided}}$

(d) Depending upon area of compression reinforcement, value (A) can be further modified using a modification factor (MF_2)

$$\text{effective depth} = \frac{\text{span}}{A \times MF_1 \times MF_2}$$

(e) For flanged beam: A reduction factor is used.

(f) Deflection check for two way slab:

Support Condition	Span/overall depth	
	Mild Steel	Fe415/Fe500
Simply supported	35	28
Continuous	40	32

Slenderness limit

1. **For simply supported or continuous beams**

$$l_0 \not> \text{minimum}\begin{cases} 60b \\ 250\dfrac{b^2}{d} \end{cases}$$

where, l_0 = Clear span
b = Width of the section
and, d = Effective depth

2. **For cantilever beam** $l_0 \not> \text{minimum}\begin{cases} 25b \\ 100\dfrac{b^2}{d} \end{cases}$

(i) Minimum tension reinforcement $\boxed{\dfrac{A_{st}}{bd} = \dfrac{0.85}{f_y}}$

(ii) Maximum tension reinforcement = 0.04 bD

(iii) Maximum compression reinforcement = 0.04 bD

where, D = overall depth of the section

(iv) Where, D > 750 mm, side face reinforcement is provided and it is equal to 0.1% of gross cross-section area (b×D). It is provided equally on both face.

(v) Maximum spacing of side face reinforcement is 300 mm.

(vi) Maximum size of reinforcement for slab/beam is 1/8 of total thickness of the member

(vii) Nominal cover for different members

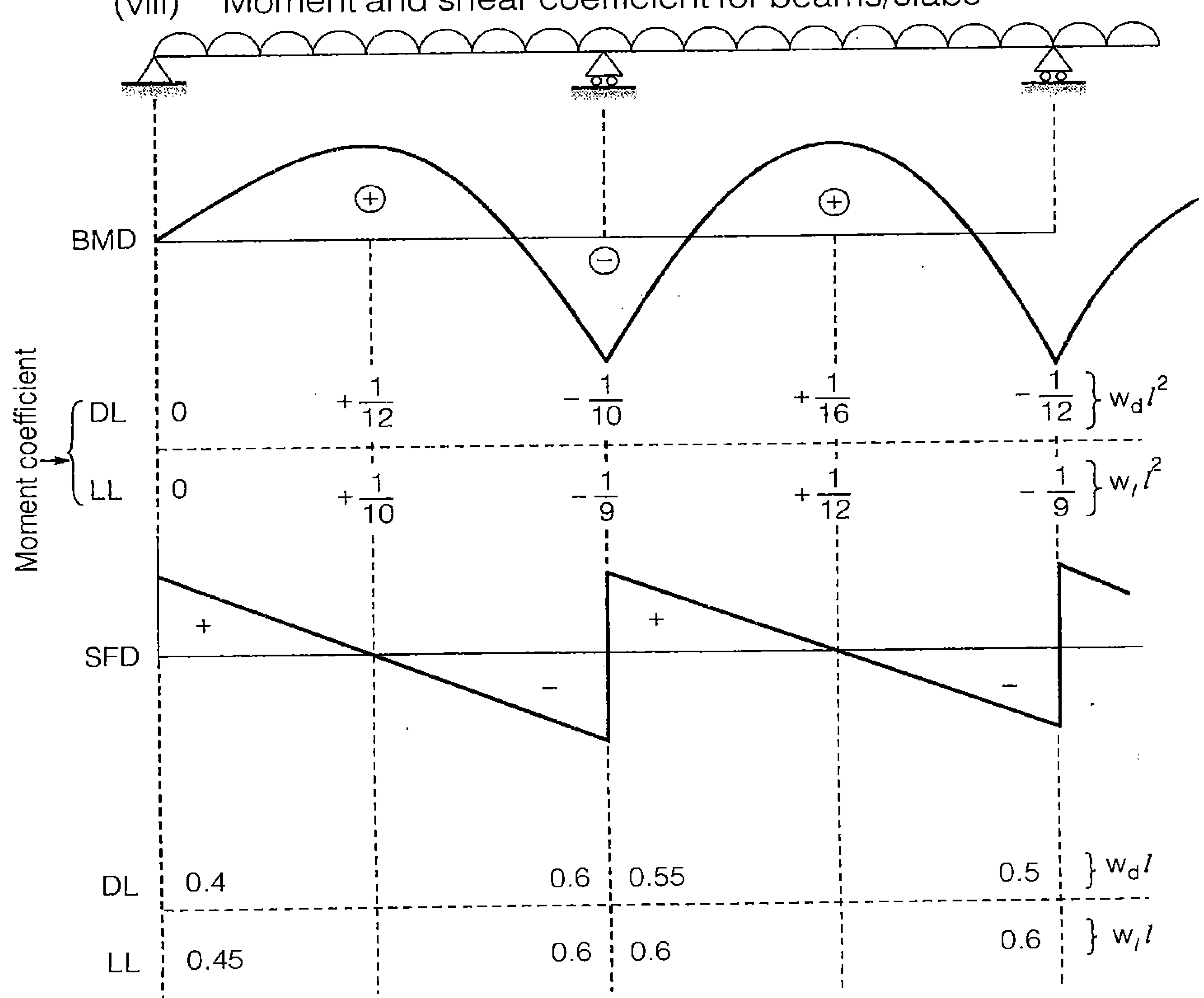
Beams → 25 mm

Slab → 20 to 30 mm

Column → 40 mm

Foundations → 50 mm

(viii) Moment and shear coefficient for beams/slabs



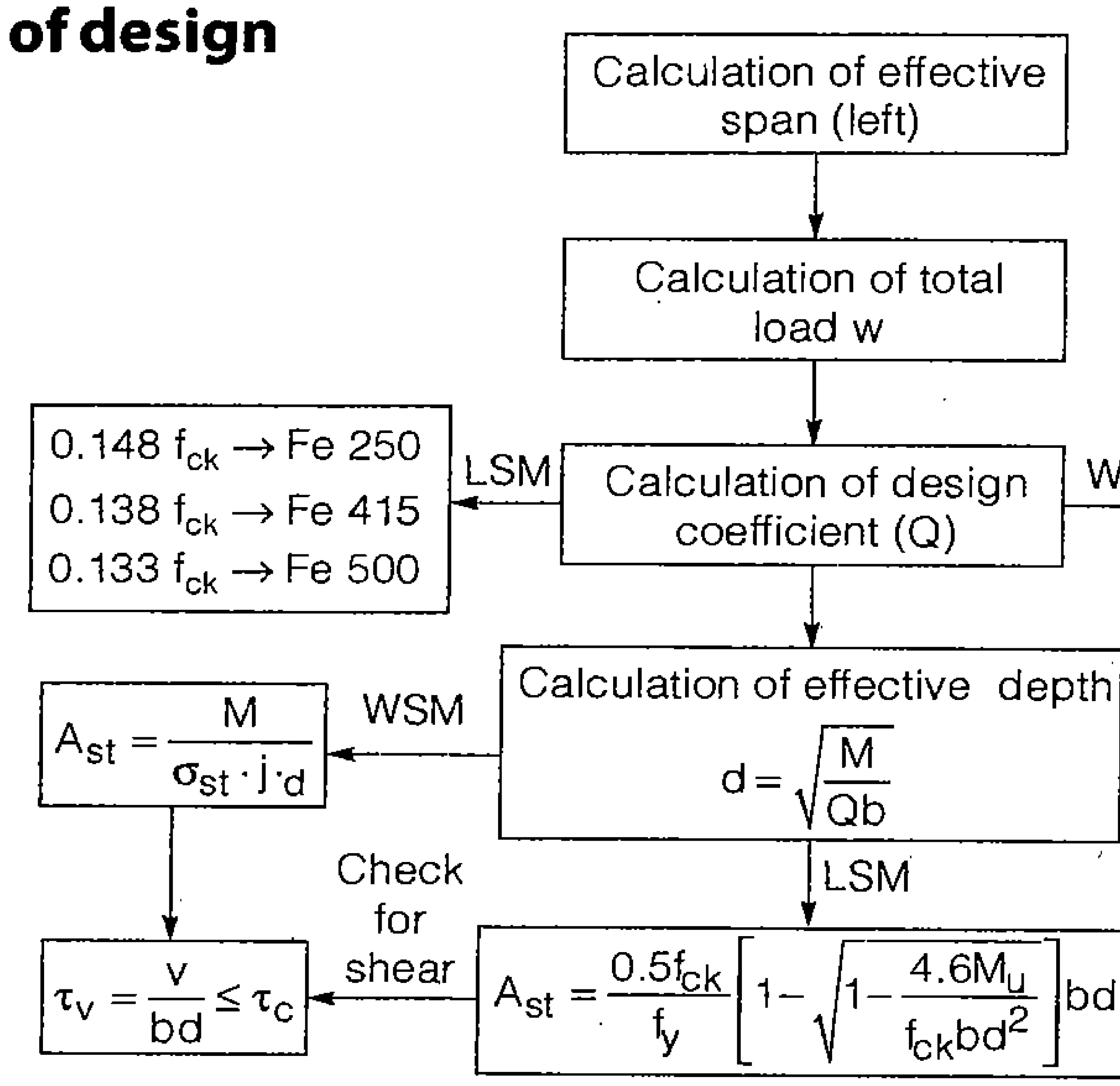
One way slab

(i) $\boxed{\dfrac{l_y}{l_x} > 2}$ where, l_y = length of longer span

l_x = length of shorter span

(ii) Slab is supported only on two edges.

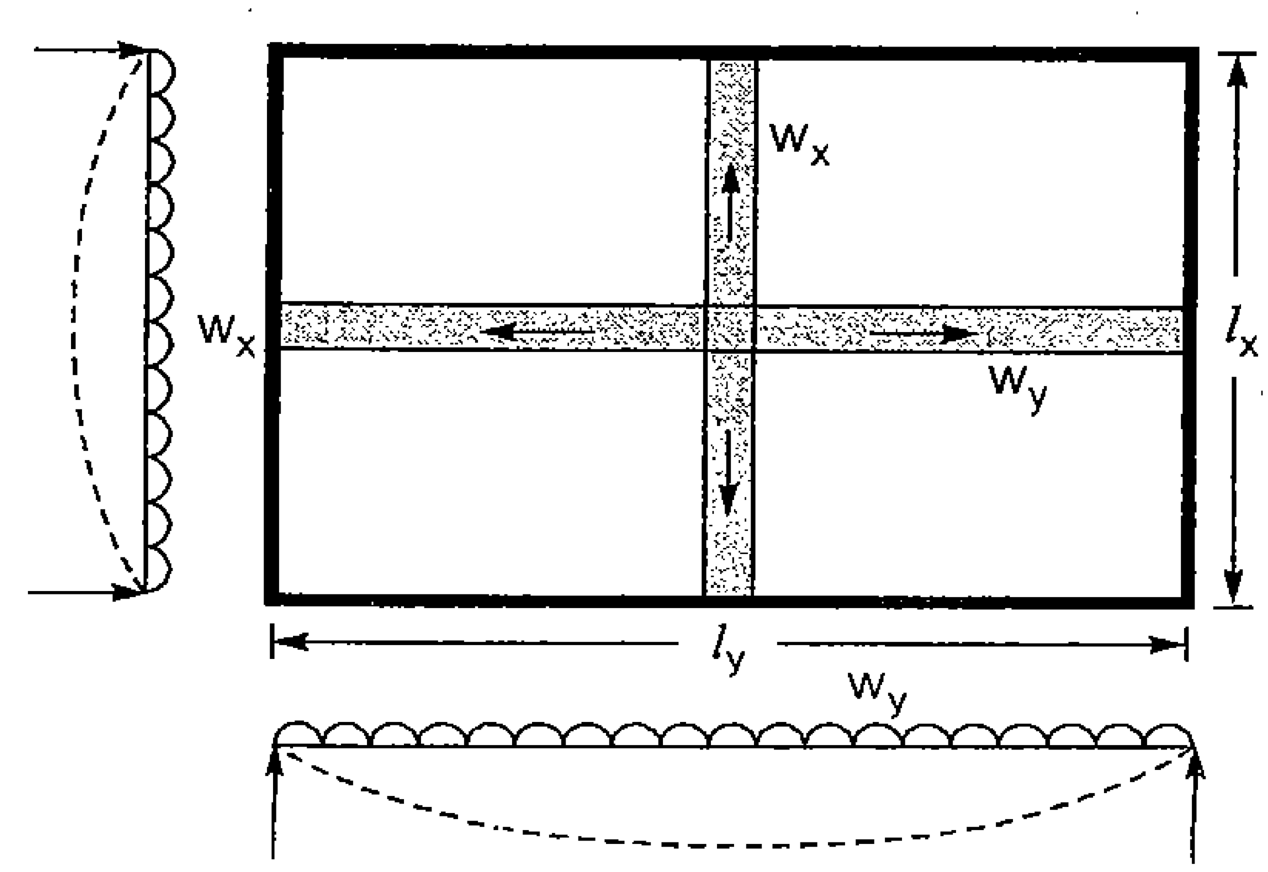
Steps of design



Two way slab

(i) $\boxed{\dfrac{l_y}{l_x} \le 2}$ (ii) Slab is supported on all edges.

- **Design of two way slab**
  1. Grasoff Rankine method
     - It is used for corners not held down position.
     - It is purely simply supported case.



(i) $w_y = \left(\frac{1}{1+r^4}\right) w$ $w_x = \left(\frac{r^4}{1+r^4}\right) w$

(ii) Moment in x-direction (M_x) $M_x = \frac{w_x l_x^2}{8}$

Moment in y-direction (M_y) $M_y = \frac{w_y l_y^2}{8}$

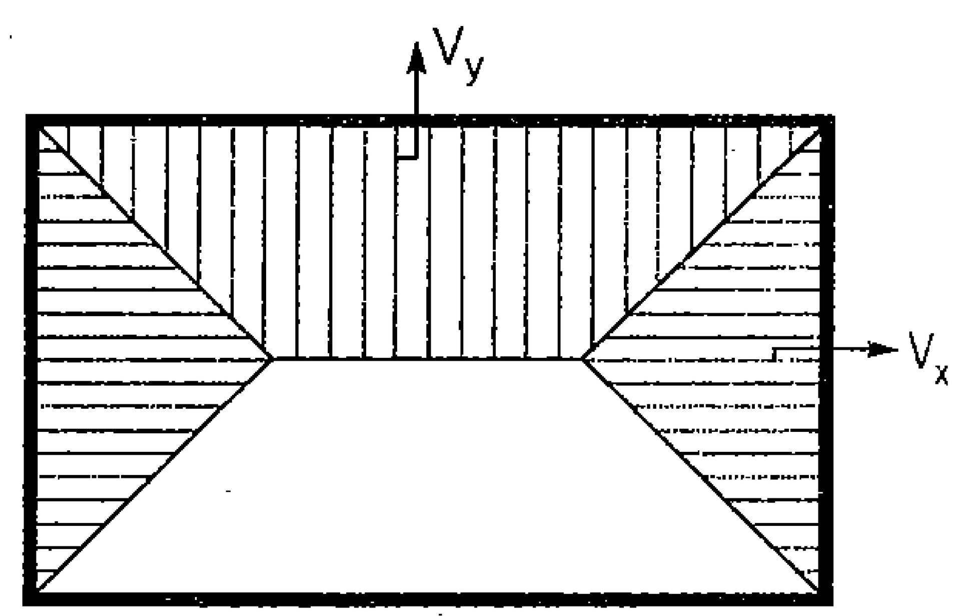
(iii) Shear force

At shorter edge (V_x)

$$V_x = \frac{1}{3} \cdot w\, l_x$$

At longer edge (V_y)

$$V_y = \left(\frac{r}{2+r}\right) w.l_x$$



(Load Distribution)

2. Design of slab with corner held down position
   (a) Pigeauds method:

$$M_x = r_x' \cdot \frac{w l_x^2}{8} \qquad M_y = r_y' \cdot \frac{w l_y^2}{8}$$

where, the values of r_x' and r_y' are read from table

   (c) I.S. code method

$$M_x = \alpha_x\, w\, l_y^2 \qquad M_y = \alpha_y\, w\, l_x^2$$

The values of α_x and α_y read from table (page 91, IS : 456-2000)

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8 Columns

Working Stress Method

- Slenderness ratio (λ)

$$\lambda = \frac{\text{effective length}}{\text{least lateral dimension}}$$

if $\lambda > 12$ then the column is long.

- Load carrying capacity for short column

$$P = \sigma_{sc} A_{sc} + \sigma_{cc} A_c$$

where, A_c = Area of concrete, $A_c = A_g - A_{sc}$

σ_{sc} = Stress in compression steel

σ_{cc} = Stress in concrete

A_g = Total gross cross-sectional area

A_{sc} = Area of compression steel

- Load carrying capacity for long column

$$P = C_r(\sigma_{sc} A_{sc} + \sigma_{cc} A_c)$$

where, C_r = Reduction factor

$$C_r = 1.25 - \frac{l_{eff}}{48B}$$

or

$$C_r = 1.25 - \frac{l_{eff}}{160\, i_{min}}$$

where, l_{eff} = Effective length of column

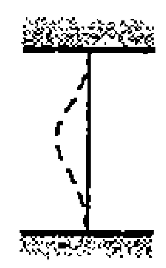
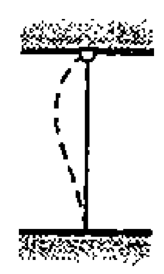
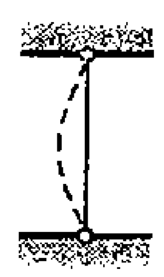
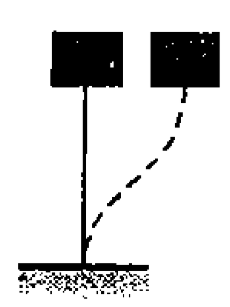
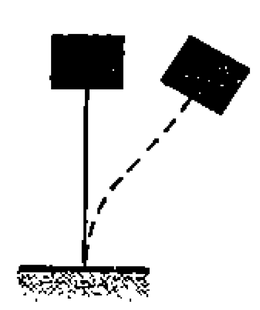
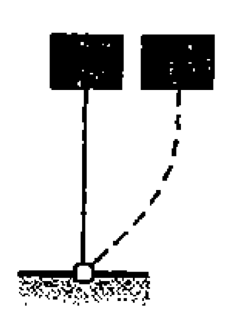
B = Least lateral dimension

i_{min} = Least radius of gyration and $i_{min} = \sqrt{\frac{I}{A}}$

where I = Moment of inertia and A = Cross-sectional area

- Effective length of column

Effective Length of Compression Members

Degree of End Restraint of compression members (i)	Symbol (ii)	Theoretical value of Effective Length (iii)	Recommended value of Effective Length (iv)
Effectively held in position and restrained against rotation in both ends		0.50 l	0.65 l
Effectively held in position at both ends, restrained against rotation at one end		0.70 l	0.80 l
Effectively held in position at both ends, but not restrained against rotation		1.00 l	1.00 l
Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position		1.00 l	1.20 l
Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		—	1.50 l
Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position		2.00 l	2.00 l
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end.		2.00 l	2.00 l

- **Column with helical reinforcement**

  Strength of the column is increased by 5%

  $$P = 1.05(\sigma_{sc} A_{sc} + \sigma_{cc} A_c)$$ for short column

  $$P = 1.05\,C_r(\sigma_{sc} A_{sc} + \sigma_{cc} A_c)$$ for long column

> Helical reinforcement is provided only for circular columns.

- **Longitudinal reinforcement**
  - (a) Minimum area of steel = 0.8% of the gross area of column
  - (b) Maximum area of steel
    - (i) when bars are not lapped A_{max} = 6% of the gross area of column
    - (ii) when bars are lapped A_{max} = 4% of the gross area of column
- **Minimum number of bars for reinforcement**

  For rectangular column → 4

  For circular column → 6
- **Minimum diameter of bar = 12 mm**
- **Maximum distance between longitudinal bar = 300 mm**
- **Pedestal:** It is a short length whose effective length is not more than 3 times of least lateral dimension.
- **Transverse reinforcement (Ties)**

  $$\phi = \text{maximum}\begin{cases}\frac{1}{4}\cdot\phi_{main}\\ 6\text{ mm}\end{cases}$$

  where, ϕ_{main} = dia of main logitudnal bar

  ϕ = dia of bar for transverse reinforcement
- **Pitch (p)**

  $$\phi = \text{minimum}\begin{cases}\text{least lateral dimension}\\ 16\,\phi_{min}\\ 300\text{ mm}\end{cases}$$

  where, ϕ_{min} = minimum dia of main longitudnal bar
- **Helical reinforcement**
  - (i) Diameters of helical reinforcement is selected such that

  $$0.36\left[\frac{A_g}{A_c}-1\right]\frac{f_{ck}}{f_y} \le \frac{V_h}{V_c}$$

(ii) Pitch of helical reinforcement: (p)

(a) $p \ngtr 75$ mm (b) $p \ngtr \frac{1}{6} dc$ (c) $p \nless 3\,\phi_h$ (d) $p \nless 25$ mm

where, d_c = Core diameter = $d_g - 2 \times$ clear cover to helical reinforcement

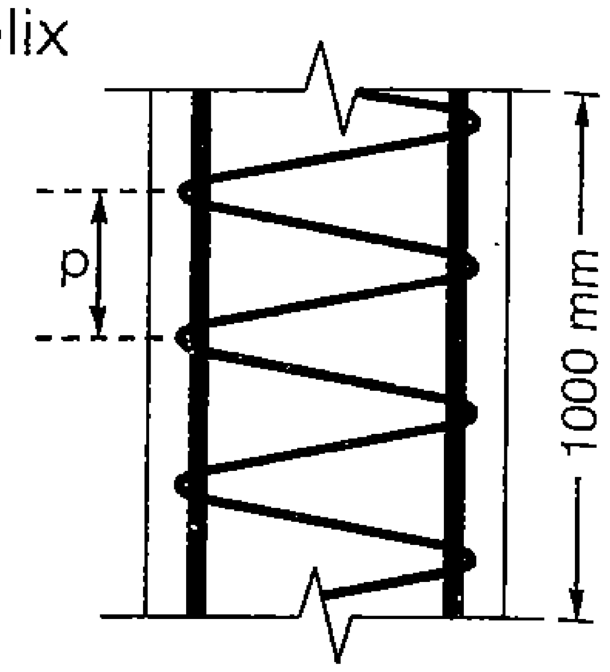
A_G = Gross area = $\frac{\pi}{4}(d_g)^2$

d_g = Gross diameter

V_h = Volume of helical reinforcement in unit length of column

ϕ_h = Diameter of steel bar forming the helix

$$V_h = \left(\frac{1000}{p}\right)(\pi d_h)\frac{\pi}{4}(\phi_h)^2$$

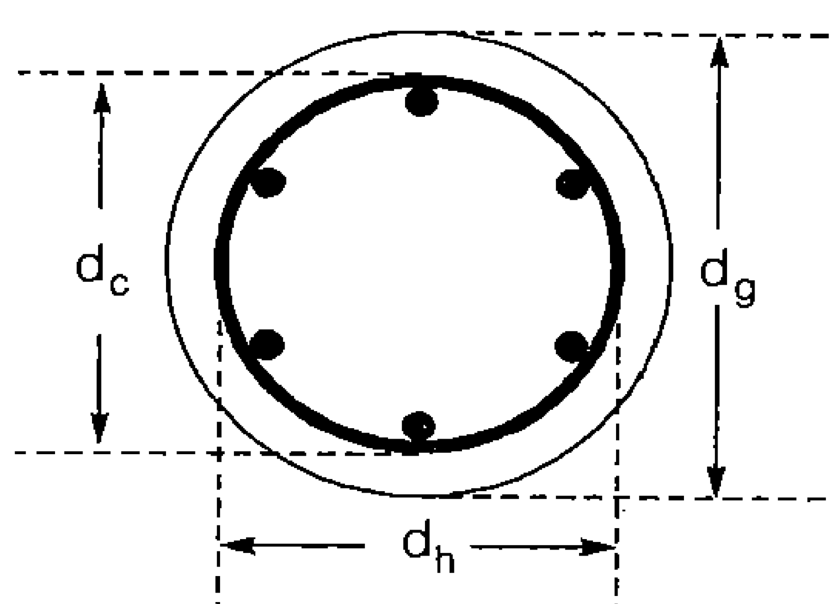


$$A_c = \frac{\pi}{4}(d_c)^2 \qquad V_c = A_c \times 1$$

d_h = centre to centre dia of helix

$= d_g - 2$ clear cover $- \phi_h$

ϕ_h = diameter of the steel bar forming the helix



- **Some others IS recommendations**

(a) Slenderness limit

(i) Unsupported length between end restrains $\ngtr$ 60 times least lateral dimension.

(ii) If in any given plane one end of column is unrestrained than its unsupported length $\ngtr \frac{100B^2}{D}$.

(b) All column should be designed for a minimum eccentricity of

$$e_{min} = \text{maximum}\begin{cases}\frac{l}{500} + \frac{'B' \text{ or } 'D'}{30} \\ 20 \text{ mm}\end{cases}$$

limit state method

1. **Slenderness ratio (λ)**

if $$\lambda = \frac{\text{effective length}}{\text{least lateral dimension}}$$

$\lambda < 12$ Short column

2. **Eccentricity**

$$e_{min} = \text{maximum}\begin{cases}\frac{l}{500} + \frac{B \text{ or } D}{20} \\ 30 \text{ mm}\end{cases}$$

If $e_{min} \le 0.05\,D$ then it is a short axially loaded column.

$$P_u = 0.4\,f_{ck}\,A_c + 0.67\,f_y\,A_{sc}$$

where, P_u = axial load on the column

3. **Short axially loaded column with helical reinforcement**

$$P_u = 1.05\,(0.4\,f_{ck}\,A_c + 0.67\,f_y\,A_{sc})$$

4. **Some others IS code Recommendations**

(a) Slenderness limit

(i) Unsupported length between end restrains $\ngtr$ 60 times least lateral dimension.

(ii) If in any given plane one end of column is unrestrained than its unsupported length $\ngtr \frac{100B^2}{D}$,

(b) All column should be designed for a minimum eccentricity of

$$e_{min} = \text{maximum}\begin{cases}\frac{l}{500} + \frac{B}{30} \\ 20 \text{ mm}\end{cases}$$

- **Concentrically Loaded Columns**

where e = 0, i.e, the column is truly axially loaded.

$$P_u = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

This formula is also used for member subjected to combined axial load and bi-axial bending and also used when e > 0.05D.

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9

Footings

Isolated Footings

Footings are structural elements that transfer loads coming from the superstructure to the earth. If these loads are to be properly transmitted, foundations must provide adequate safety against sliding and overturning.

Theoretically speaking, isolated footings must be designed for both axial load and moment but practically isolated footings are designed only for axial loads.

Foundations may be broadly classified under two heads: *shallow foundation* and *deep foundation*.

According to Terzaghi, a foundation is shallow if its depth is equal to or less than it width. In the case of deep foundation, the depth is greater than the width. Apart from deep strip, rectangular or square foundations, other common forms of deep foundations are; pier foundations, pile foundation and well foundation. The shallow foundations are of the following types: Spread footing, strap footing, combined footing and mat or raft footing.

Spread footings: A spread footing or simple footing, is a type of shallow foundation used to transmit the load of an isolated column, or that of a wall, on the subsoil. In the case of wall, the footing is continuous while in the case of column, it is isolated.

Combined footings: A spread footing which supports two or more columns is termed as a combined footing. Such a footing is provided when the individual footings are either very near to each other, or overlap. Combined footings may either be rectangular or trapezoidal.

Strap or Cantilever footings: A strap footing consists of spread footings of two columns connected by a strap beam. The strap beam does not remain in contact with soil, and thus does not transfer any pressure to the soil.

Mat or Raft foundation: A mat or raft is a combined footing that covers the entire area beneath a structure and supports all the walls and columns. When the available soil pressure is low or the building loads

are heavy, the use of spread footings would cover more than one-half of the area and it may prove more economical to use mat or raft foundation.

Pile foundation: Pile foundation is a deep foundation used where the top soil is relatively weak. Piles transfer the load to a lower stratum of greater bearing capacity, by way of end bearing, or to the intermediate soil though skin friction. This is more common type of deep foundation generally used for buildings where a group of piles transfer the load of the super-structure to the sub-soil.

Design of Isolated Footing

Rectangular footing

Given values

1. Load = P or P_u
2. Bearing capacity of soil = q_u
3. Size of column
4. Grade of concrete and steel

Design Steps

(i) Size of foundation

Load from column = P

Add weight of foundation (P_F) = 0.1P

∴ Total load P_T = 1.1P (even for limit state method use unfactored load for calculation of area)

∴ Area of footing, $$A = \frac{P_T}{q_u} = \frac{\text{Total load}}{\text{Bearing capacity of soil}}$$

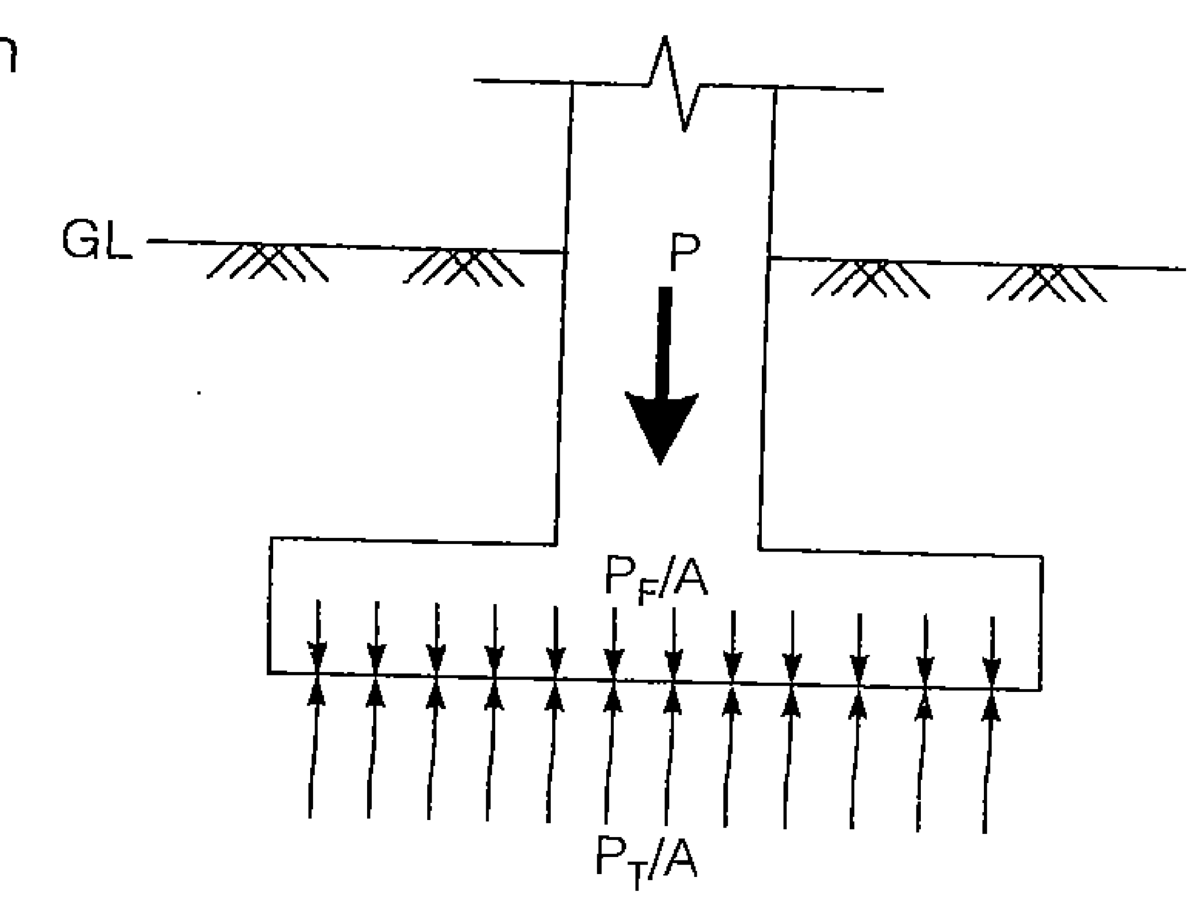
Choose L and B such that A = L × B

Net soil pressure,

$$w = \frac{P}{A} = \frac{\text{Load from column without self weight}}{\text{Area provided}}$$

Net soil pressure over foundation

$$w = \frac{P_T - P_F}{A} = \frac{P}{A}$$



For LSM

Net soil pressure $$(w_u) = \frac{1.5P}{A}$$

(ii) Check for bending moment

Critical section for bending moment is at the face of the column

Consider 1 m strip of foundation

Bending about x-x

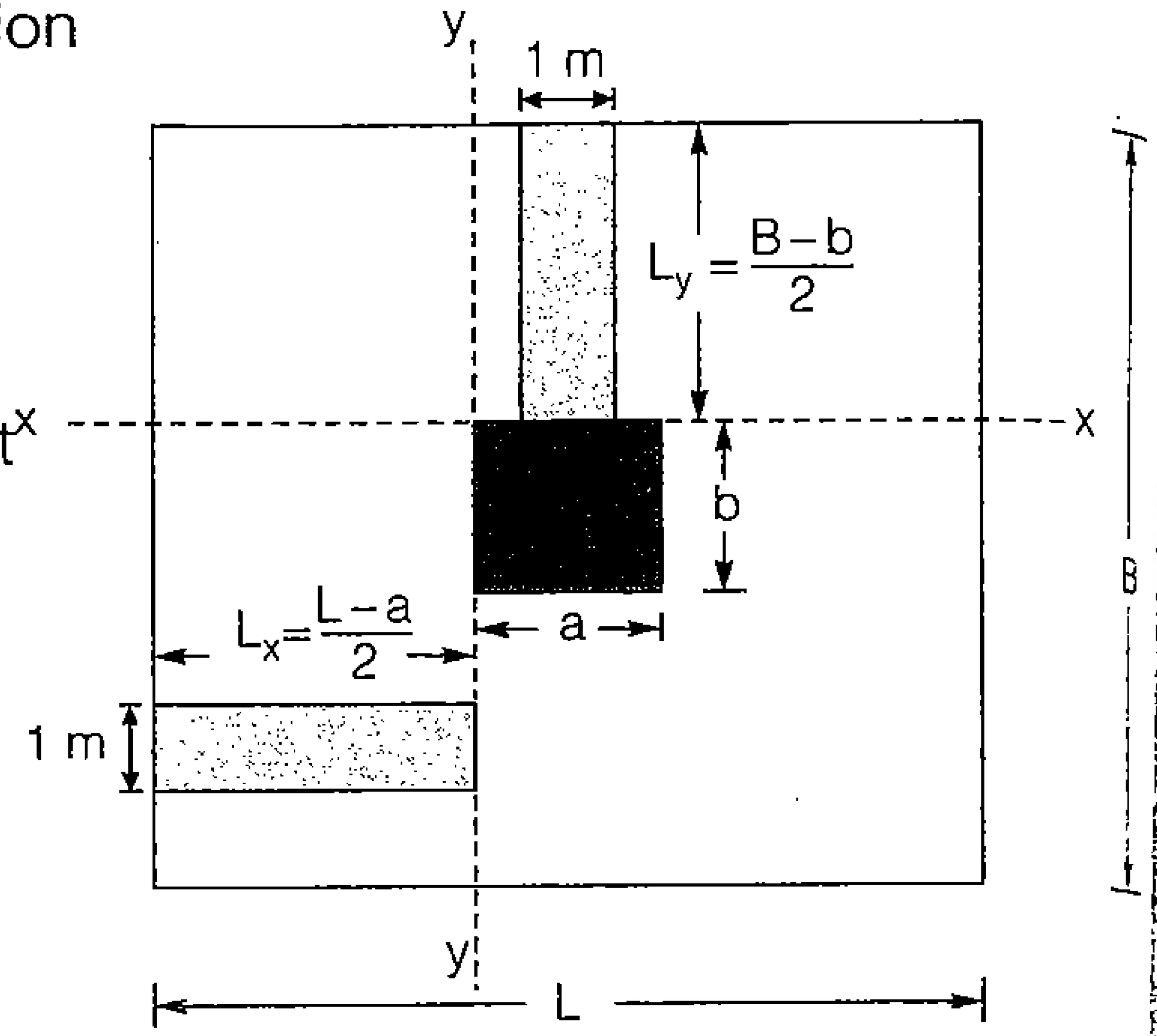
$$L_y = \frac{B-b}{2}$$

∴ Maximum bending moment

$$\frac{w}{2} \times \left(\frac{B-b}{2}\right)^2 = \frac{w(B-b)^2}{8}$$

Similarly bending about y-y

$$L_x = \frac{L-a}{2}$$



Maximum BM $$M_y = \frac{w}{2} \times \left(\frac{L-a}{2}\right)^2 = \frac{w(L-a)^2}{8}$$

> **NOTE:** Use $w = w_u = 1.5\,w$ for Limit State Method.

(iii) Depth required $$d = \sqrt{\frac{M_{max}}{Qb}}$$ Where, b = 1000 mm

For LSM $$d = \sqrt{\frac{M_{umax}}{Qb}}$$

(iv) Check for single shear (one-way shear)

Critical section for one-way shear is at distance 'd' from the face of the column.

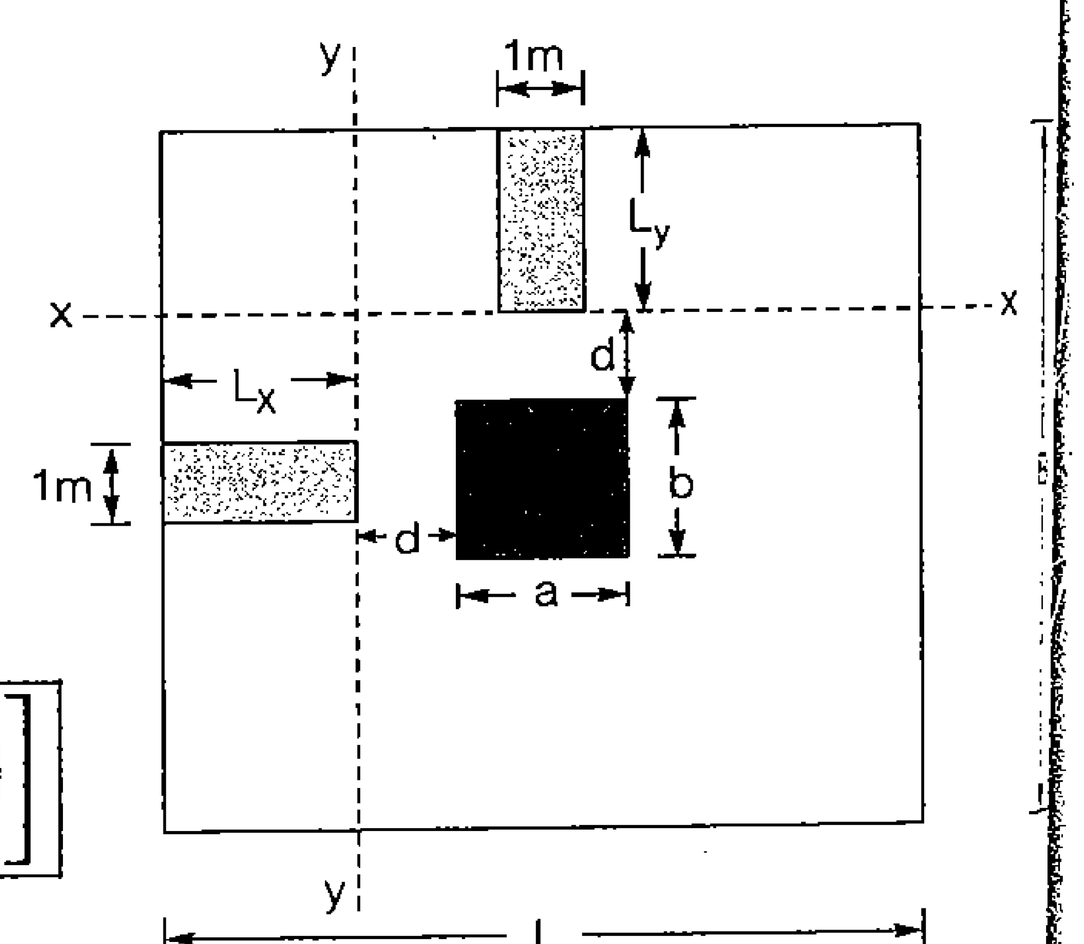
Shear at x-x

Overhang,

$$L_y = \left[\left(\frac{B-b}{2}\right) - d\right]$$

Shear force,

$$V_x = w.1 \times L_y = w\left[\left(\frac{B-b}{2}\right) - d\right]$$



Similarly Shear force at y-y

Overhang,

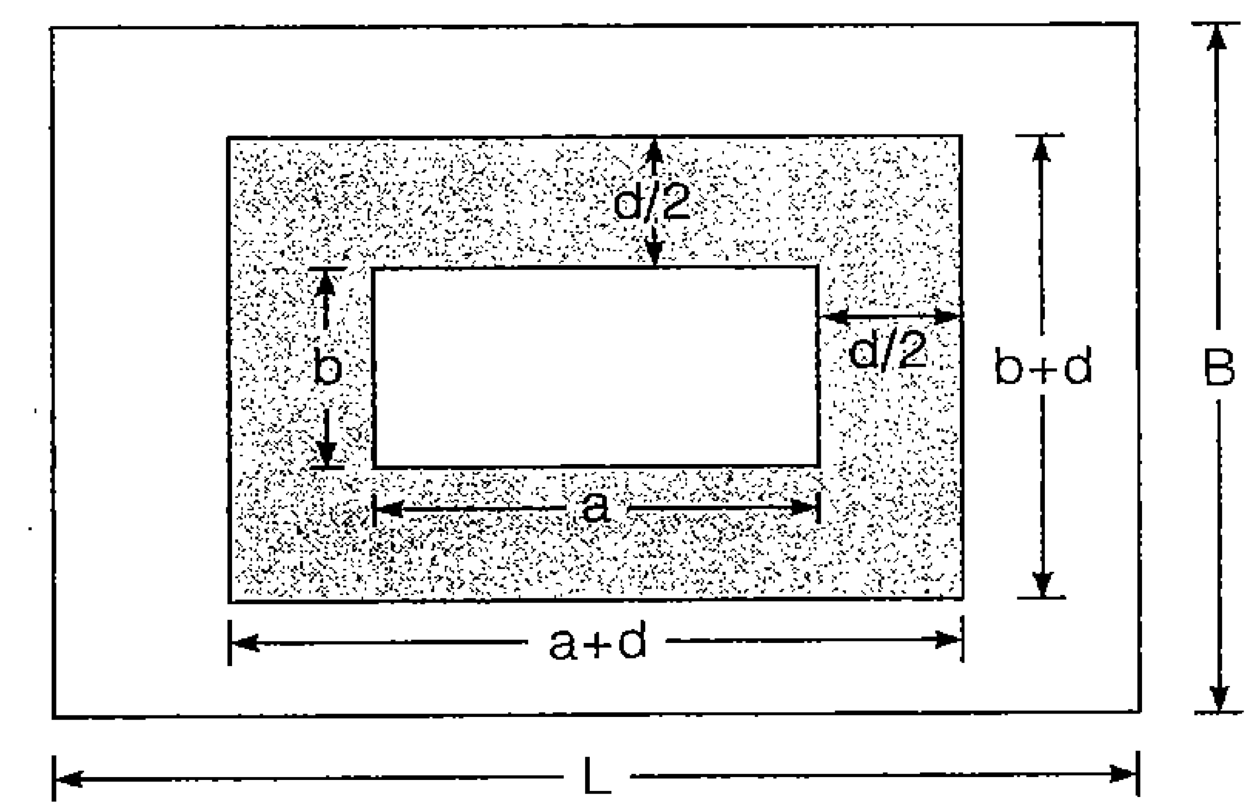
$$L_x = \left[\left(\frac{L-a}{2}\right) - d\right] \qquad V_y = w.1 \times L_x = w\left[\left(\frac{L-a}{2}\right) - d\right]$$

Find out maximum of V_x and V_y

Nominal shear stress $$\tau_v = \frac{V_{max}}{Bd} < \tau_c$$

where, τ_c is permissible design shear strength of concrete in N/mm² as given in IS 456 : 2000

Footing should be always safe in shear. No shear reinforcement is provided.



(v) Check for two-way (Punching Shear).

Critical section for punching shear also called two-way shear is at distance 'd/2' from face of the column all around.

Net punching force

$$P_{net} = P - w(a+d)(b+d)$$

Punching shear stress developed = $\dfrac{\text{Net punching force}}{\text{cross-section area of resisting section}}$

∴ Cross-section area = perimeter × depth

Now perimeter = 2[(a+ d) + (b+ d)], Depth = d

∴ Punching shear stress $$\tau_{vp} = \frac{P - w(a+d)(b+d)}{2[(a+d)+(b+d)] \times d}$$

Above developed stress should be less than the permissible punching shear stress

Permissible punching shear stress

$$\tau_{cp} = k_s \times 0.16\sqrt{f_{ck}}$$ (working stress method)

$$\tau_{cp} = k_s \times 0.25\sqrt{f_{ck}}$$ (Limit state method)

$k_s = (0.5 + \beta_c)$ but not greater than 1, β_c being the ratio of short side to long side of the column

(vi) Area of steel for longer span

The area of steel A_{st} of long bars parallel to direction L is calculated as under

For M_y moment

$$A_{st} = \frac{M_y}{\sigma_{st} jd} \quad \text{(Working stress method)}$$

$$A_{st} = \frac{M_{uy}}{0.87 f_y (d - 0.42x_u)} \quad \text{(Limit state method)}$$

This reinforcement is equally distributed over entire width B.

> **NOTE** Area of steel calculated above is for 1 m, width. Calculate this area for width B and distribute uniformly over entire width

For total width B of footing, total area of steel = B × A_{st}

(vii) Area of steel for shorter span

The area of steel A_{st} of short bars parallel to direction B is calculated as under

$$A_{st} = \frac{M_x}{\sigma_{st} jd} = \frac{M_{ux}}{0.87 f_y (d - 0.42x_u)} \quad \text{for 1 m}$$

For total length L of footing, total area of steel = L × A_{st}

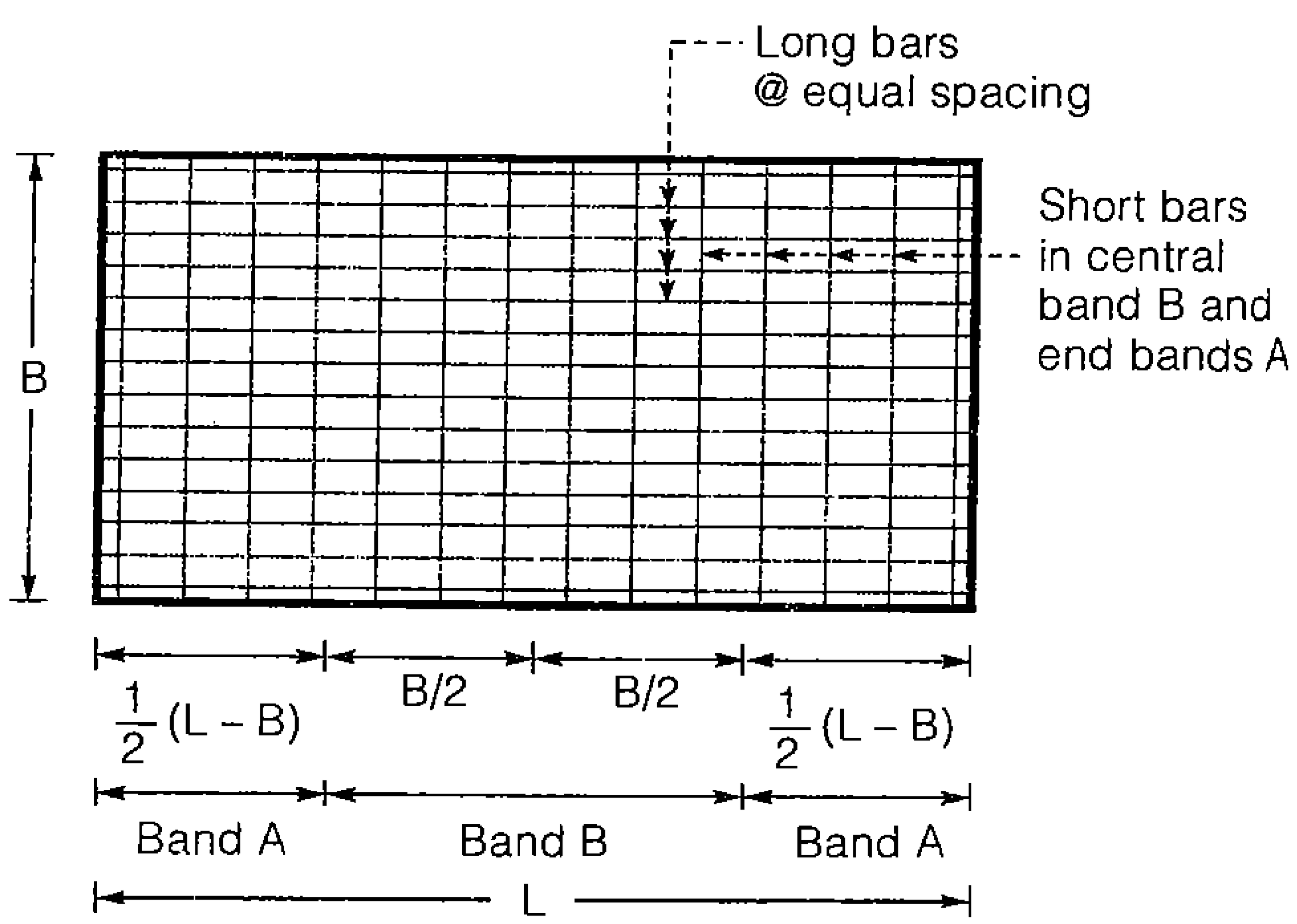
This area is provided in two distinct band widths:

- The central band B of width B, and
- The end bands A, each of width $\frac{1}{2}(L - B)$

The reinforcement in central band width = $\frac{2}{\beta + 1}$ × total reinforcement in short direction.

Where, β = ratio of long side to the short side of the footing

The remainder of reinforcement shall be uniformly distributed in outer portions of the footing.



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Prestress Concrete 10

Prestress Concrete is one in which there have been introduced internal stresses of such magnitude and distribution that stresses resulting from given external loading is counter balanced to a desired degree.

> **Remember** In case of prestress concrete very high strain steel and high strain concrete is used.

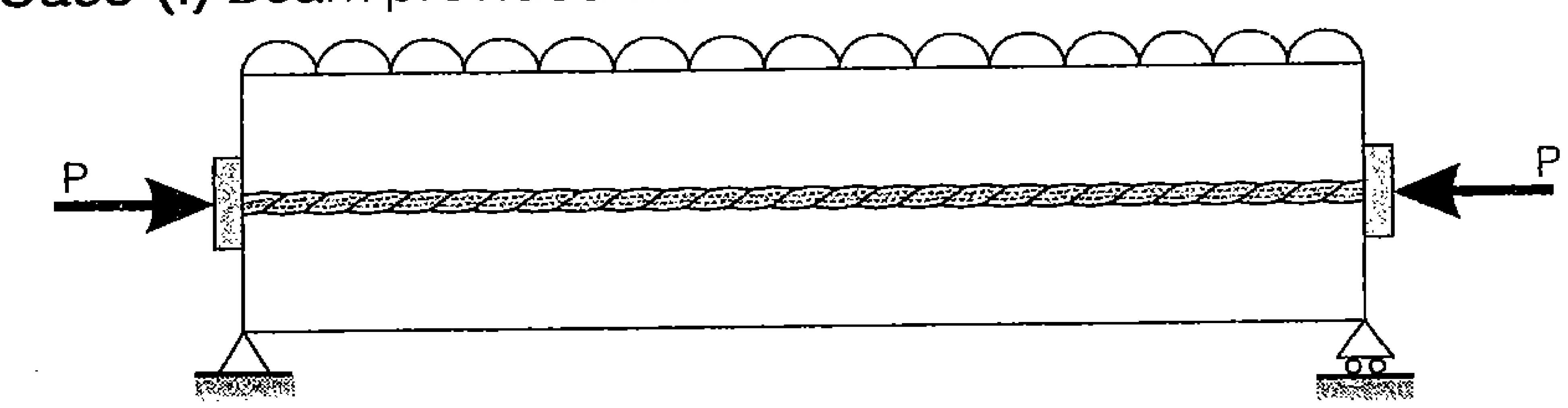
Analysis of prestress and Bending stress

- Assumptions
  - (i) Concrete is homogeneous elastic material.
  - (ii) Within the range of working stresses, both concrete and steel behave elastically and Hooke's law is valid.
  - (iii) A transverse plane section before bending remain plain after bending.
- Following are the three concepts of analysis
  - (a) Stress concept analysis
  - (b) Strength concept analysis
  - (c) Load balancing method

Stress concept Method

Following are the two cases for analysis.

Case-(i) Beam provided with a concentric tendon:



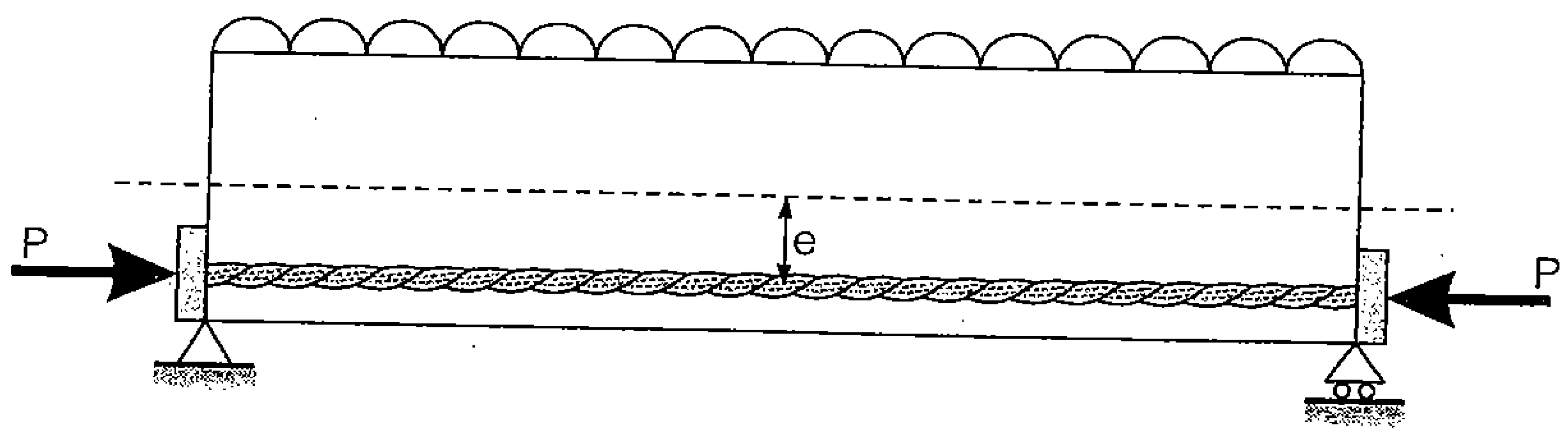
Let, P prestressing force applied by the tendon. Due to this prestressing force, the direct compressive force induced is given by, $f_a = \frac{P}{A}$.

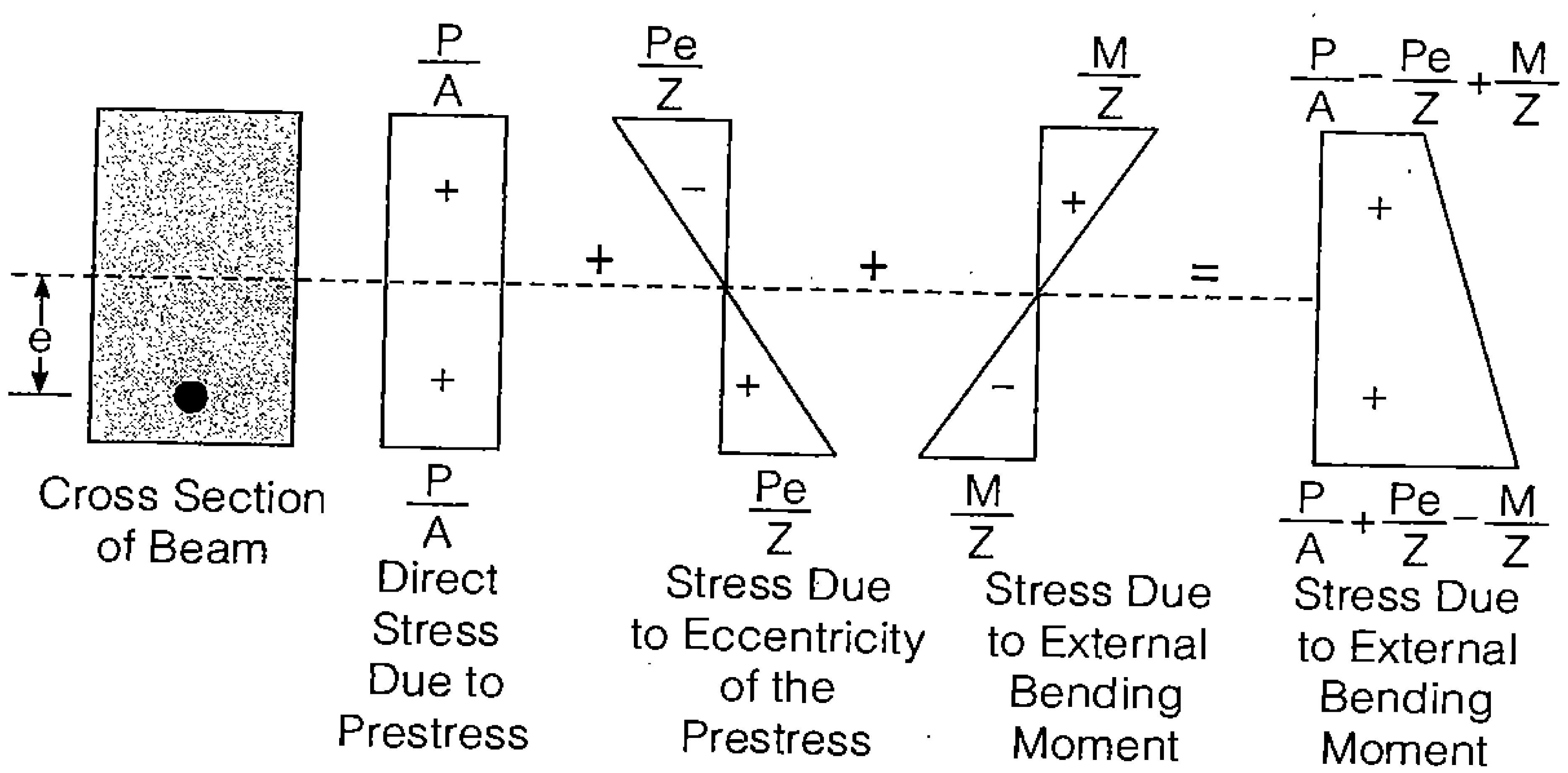
If due to dead load & external loads, the bending moment at the section is M, then the extreme stresses at the section due to bending moment alone is $f_b = \pm\frac{M}{Z}$.

Hence final stress at the extreme top edge = $\frac{P}{A}+\frac{M}{Z}$

and stress at the extreme bottom edge = $\frac{P}{A}-\frac{M}{Z}$

- **Case-(ii):** Beams with eccentrics tendon:





(i) Direct stresses due to prestressing force = $+\frac{P}{A}$

(ii) Extreme stresses due to eccentricity of the prestressing force = $\mp\frac{P.e}{Z}$

(iii) Extreme stresses due to bending moment = $\pm\frac{M}{Z}$.

- **Final stresses**

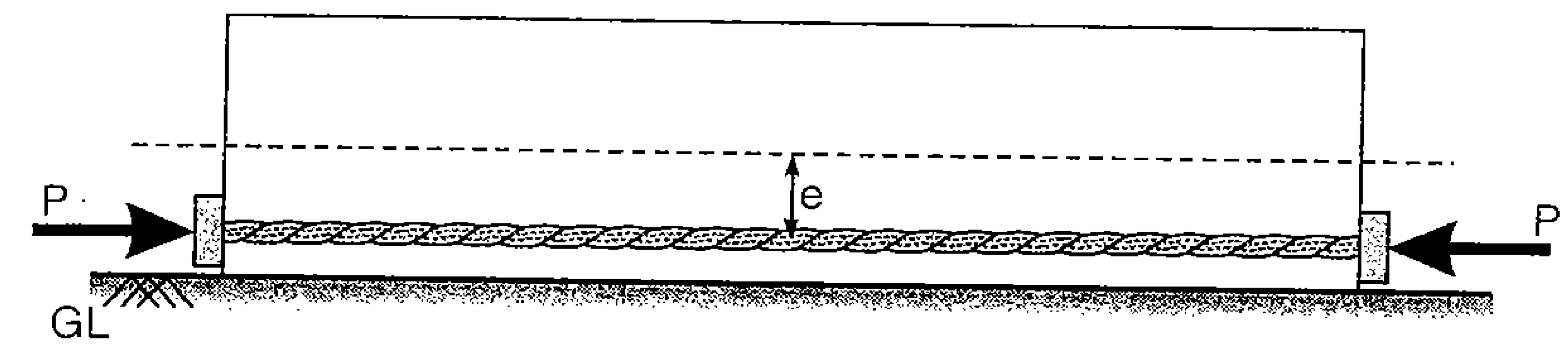
Stress at top fibre = $\frac{P}{A}-\frac{P.e}{Z}+\frac{M}{Z}$

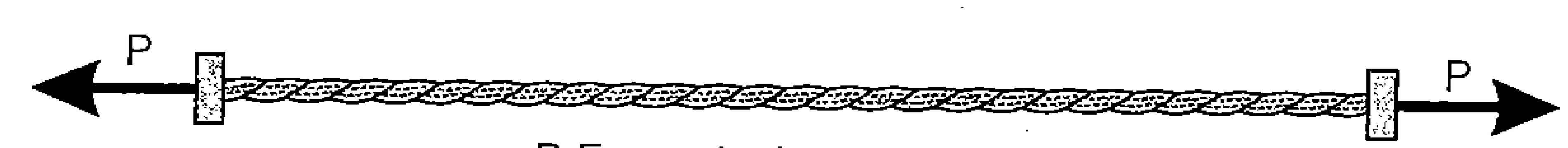
Stress at bottom fibre = $\frac{P}{A}+\frac{P.e}{Z}-\frac{M}{Z}$

By providing an eccentricity to the tendon, a hogging moment (P.e.) is developed which will produce stresses, which will counteract the stresses due to external bending moment.

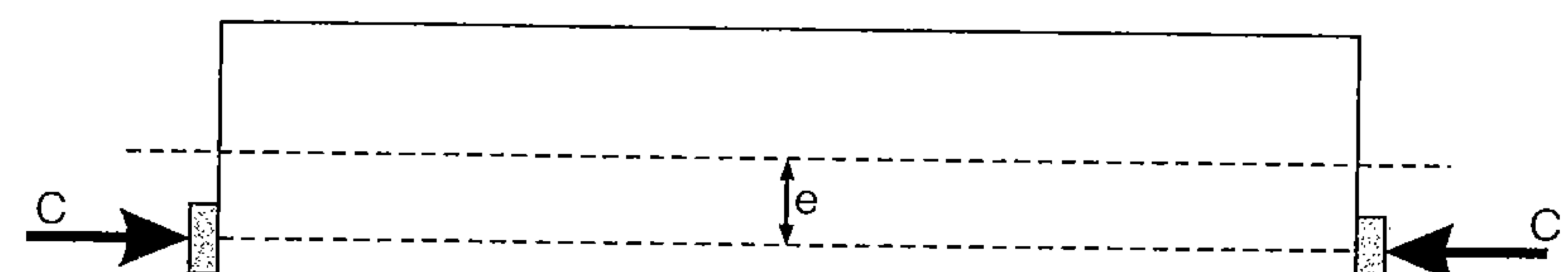
Strength Concept method

Consider a beam of length *l* provided with a tendon at an eccentricity e. Suppose the beam is lying on the ground i.e. the beam is not subjected to any external load. Hence there is no external bending moment on the beam.





P-Force Acting on tendon



C-Force acting on Concrete

The following equal forces are existing.

(i) The P-force which is the tension in the tendon.

(ii) The C-force which is the compressive force acting on the concrete. Stresses in concrete are produced entirely due to C-force.

In the absence of any external bending moment the C-force and P-force act at the same level. Line of action of P-force is called the *P-line*. The *P-line* is nothing but the tendon line itself. The line of action of the C-force is called the *C-line* or *Pressure line*. Hence in the absence of any external bending moment the *P-line* and the *C-line* coincide.

Suppose the beam is subjected to a bending moment M, then the C-line will be shifted from the P-line by a distance 'a' called lever arm.

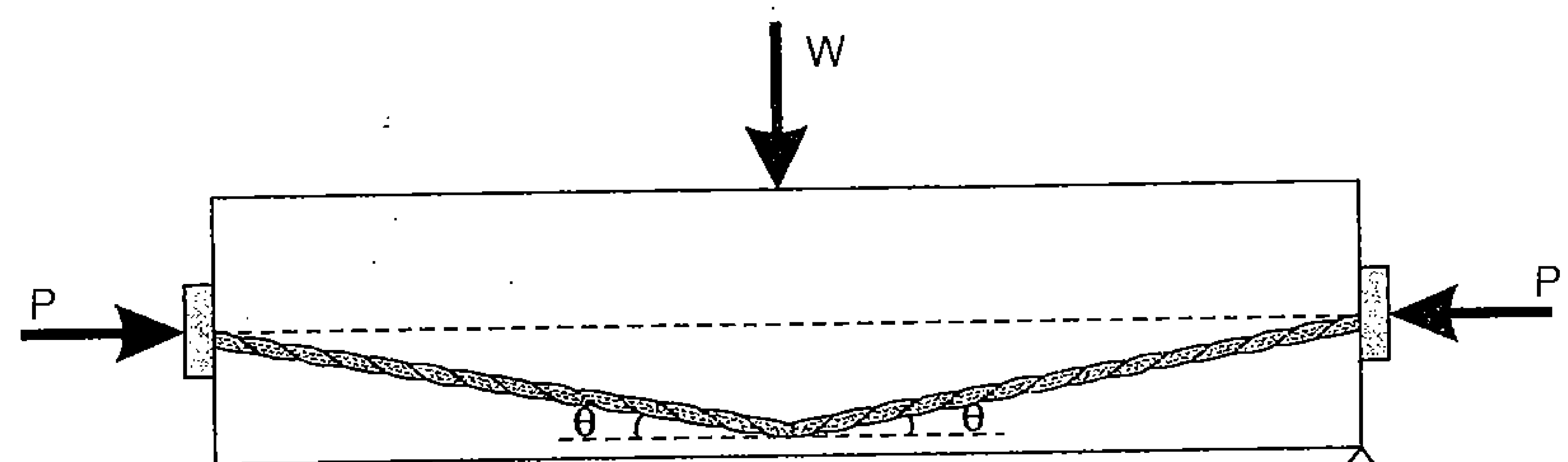
$$\boxed{a = \frac{M}{P} = \frac{M}{C}}$$

Extreme stresses in concrete are given by

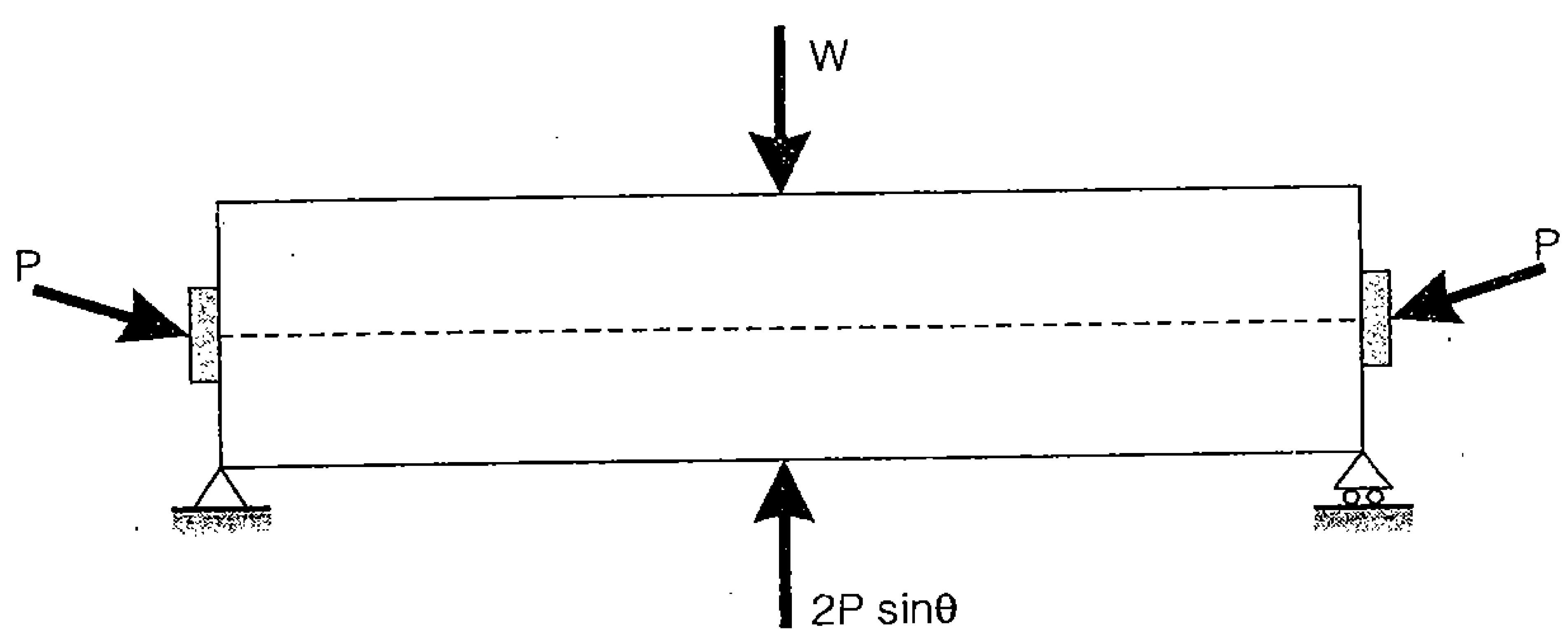
$$\boxed{= \frac{C}{A} \pm \frac{C \times \text{eccentricity of } C}{Z}}$$

Load Balancing Concept

- **Prestressed Beam with Bent Tendon**



BEAM WITH BENT TENDON



By providing bent tendons, the tendons will exert an upward pressure on the concrete beam and will therefore counter act a part of the external downward loading.

Considering the concrete as a free body. We find an upward force 2P sin θ.

The net downward load at the centre will be (W-2P sinθ).

The axial longitudinal force provided by the tendon = P cosθ = P {since θ is small}

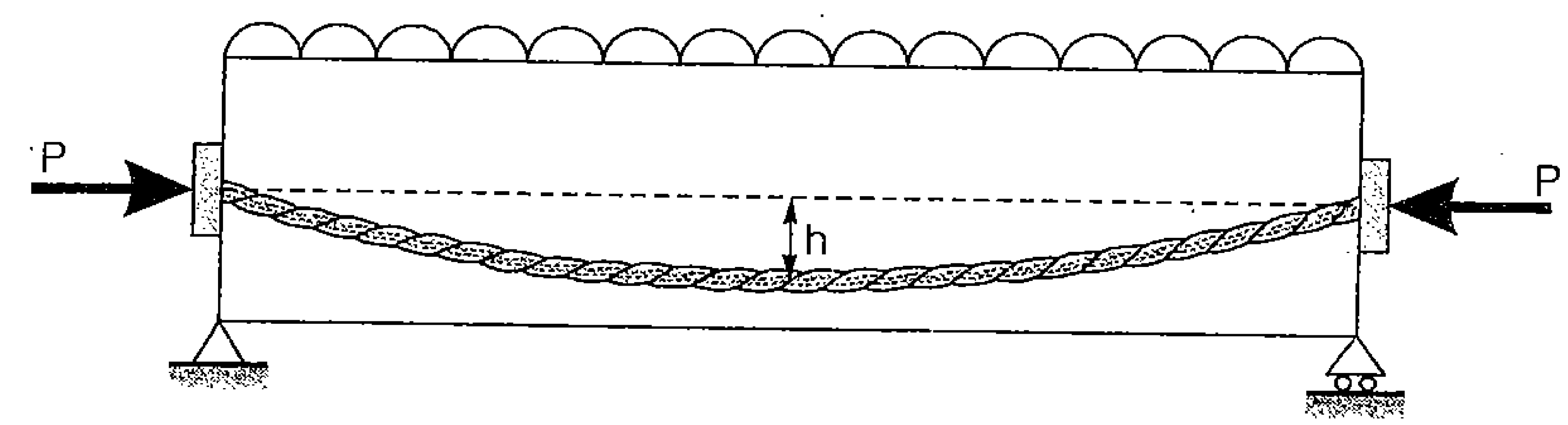
Direct stress on the section $= \frac{P\cos\theta}{A} = \frac{P}{A}$

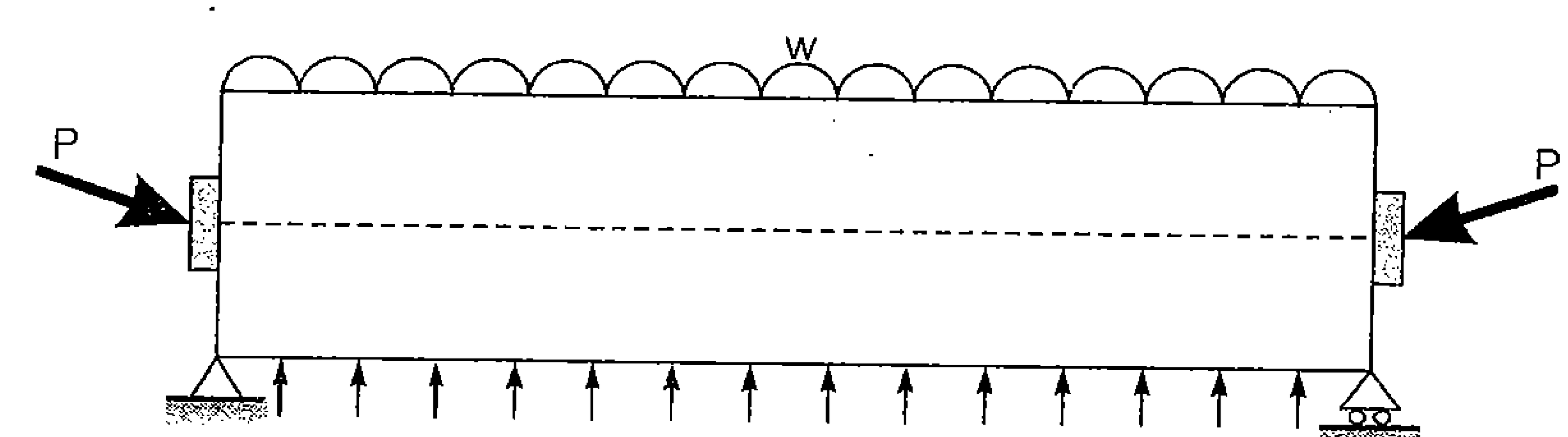
Net BM, $\boxed{M = \frac{(W - 2P\sin\theta)l}{4} + \frac{wl^2}{8}}$

Where, w = dead load per unit length of the beam. Extreme fibre stress $= \frac{P}{A} \pm \frac{M}{Z}$

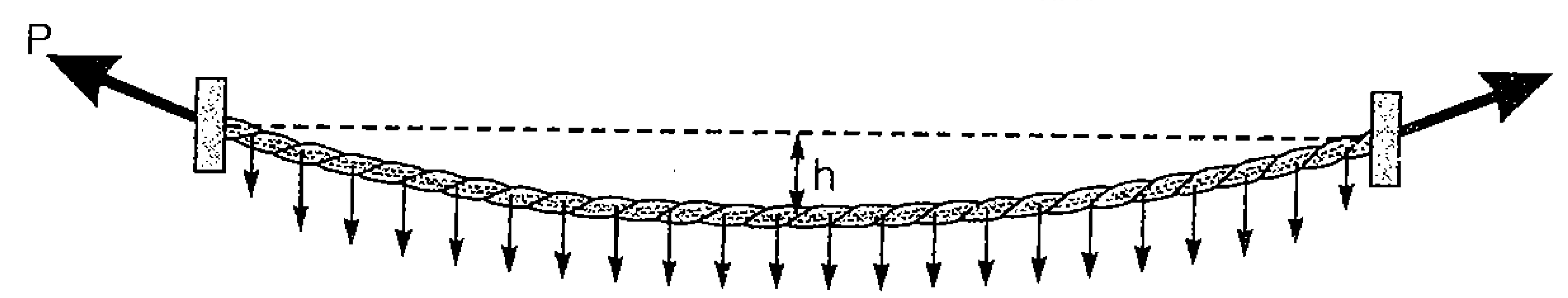
It may be realized that the profile of the tendon should follow the shape of the bending moment diagram for the given external loads in order it may offer considerable and effective upward forces. For e.g., if the loading on the beam is a uniformly distributed load, the tendon may be provided along a parabolic profile.

- **Tendon with Parabolic Profile**





w_c per unit run
Loads Transmitted to Beam



w_c per unit run
Loads Transmitted to Tendon

Let l be the span of the beam and h be the dip of the cable.
The cable will exert an upward udl = w_c/m on the beam, but the cable will be subjected to downward udl of w_c per unit run.
Let V and H are vertical and horizontal components of P.

$$V = \frac{w_c . l}{2}$$

The cable is an absolutely flexible member, therefore BM at every section of cable is zero. Hence BM at the centre of the cable is

$$\frac{w_c . l}{2} \times \frac{l}{2} - w_c . \frac{l}{2} . \frac{l}{4} - H.h = 0 \qquad H = \frac{w_c l^2}{8h}$$

Since dip of the cable is very small, we can make approximation $\cos \alpha = 1$ and $P \cos \alpha = P$

Now consider the beam, it is subjected to

(i) External load w per unit length

(ii) Upward udl transmitted by the cable = w_c per unit length.

Net UDL = $w - w_c$

$$\text{Net BM at the centre} = \frac{(w - w_c)}{8} l^2$$

$$\text{Extreme stresses} = \frac{P}{A} \pm \frac{\text{Net BM}}{Z}$$

Losses of Prestress

The steel wires of a prestressed concrete member do not retain all the preliminary prestress. A certain amount of loss of prestress always takes place.

Losses may be classified as follows:

Types of Losses	Losses in pretensioned member	Losses in posttensioned member
1. Loss of prestress during tensioning process due to friction.		
(a) Loss due to length effect.	No Loss	$P_o kx$
(b) Loss due to curvature effect.	No Loss	$P_o \mu\alpha$
(c) Loss due to both length and curvature effect.	No Loss	$P_o (kx + \mu\alpha)$ Here, P_o = Prestressing force at the jacking end. K = Wobble friction factor 15×10^{-4} per meter < K < 50×10^{-4} per meter. α = Cumulative angle in radians through which tangent to the cable profile has turned between any two points under consideration. μ = Coefficient of friction in curves = 0.25 to 0.55.
2. Loss of prestress at the anchoring stage.	No Loss	$\frac{\Delta l}{l} . E_s$ Here Δl = effective slip of the wire. l = Length of the tendon E_s = Young's modulus for tendon wires.
3. Loss of prestress occuring Subsequently.		

(a) Loss of stress due to shrinkage of concrete.	$(3\times10^{-4})E_s$ Here E_s = Youngs modulus for tendon wire.	$\frac{2\times10^{-4}}{\log_{10}(T+2)}\cdot E_s$ Here T = Age of concrete at the time of transfer of stress (in days).
(b) Loss of stress due to creep to concrete	$\phi.m.f_c$ Here m = Modular ratio = E_s/E_c f_c = Original prestress in concrete at the level of steel.	$\phi.m.f_c$
(c) Loss of stress due to elastic shortening of concrete.	$m.f_c$ Here f_c = Initial stress in concrete at the level of steel.	zero it all the bars are tensioned at same time $\frac{\Delta l}{l}\cdot E_s$ for subsequent tensioning
(d) Loss of stress due to creep of steel or loss due to stress relaxations.	1 to 5% of initial prestress	1 to 5% of initial prestress

■■■

A Handbook on **Civil Engineering**

4

Design of Steel Structures

CONTENTS

1 Structural Fasteners

Riveting

The size of the rivet is the diameter of the shank.

(a) Gross dia of rivet or dia of hole

$$\boxed{d' = d + 1.5\,\text{mm}} \quad \text{for } d \le 25 \text{ mm}$$

and $$\boxed{d' = d + 2.0\,\text{mm}} \quad \text{for } d > 25 \text{ mm}$$

where d = Nominal dia of rivet

d' = Gross dia of rivet or dia of hole.

> **NOTE:** For strength calculation effective diameter is taken into account. This is based on the assumption that rivet fills the hole completely.

(b) Unwins formula

$$\boxed{d_{mm} = 6.05\sqrt{t_{mm}}}$$

where, d_{mm} = dia of rivet in mm

t_{mm} = thickness of plate in mm.

Bolted Joints

Bolts may be used in place of rivets for structure not subjected to vibrations. The following types of bolts are used in structures:

(i) Black bolts

- Hexagonal black bolts are commonly used in steel works.
- They are made from low or medium carbon steels.
- They are designated as black bolts $M \times d \times l$ where d = diameter, and l = length of the bolts.

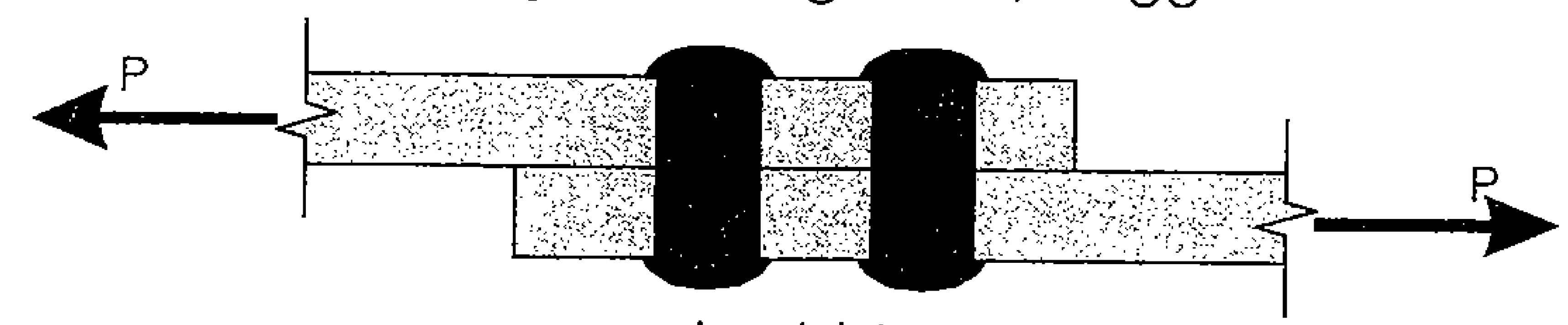
(ii) Precision and Semi Precision Bolts

- They are also known as close tolerance bolts.
- Sometimes to prevent excessive slip, close tolerance bolts are provided in holes of 0.15 to 0.2 mm oversize. This may cause difficulty in alignment and delay in the progress of work.

- **Types of Riveted and Bolted Joints**
  There are two types of riveted or bolted joints.

(i) Lap joint

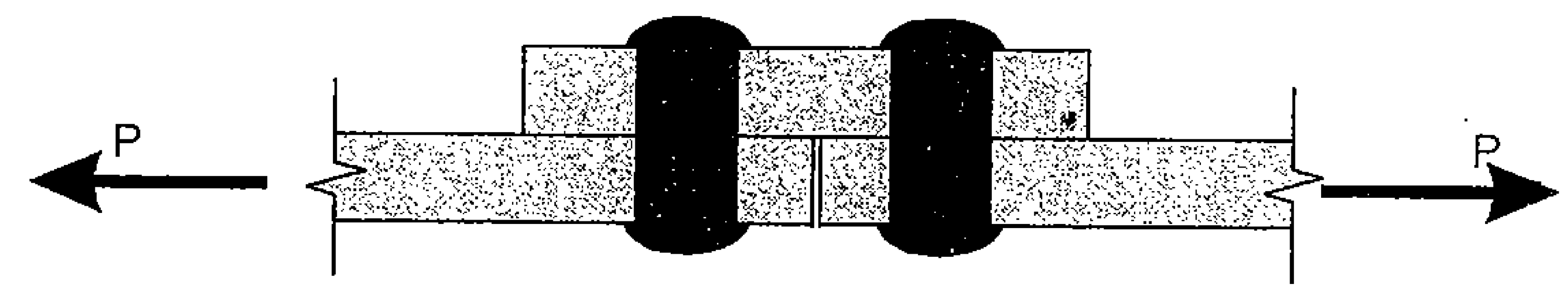
- The lap joint is that in which the plates to be connected overlap each other.
- The lap joint may have single-row, staggered or chain riveting.



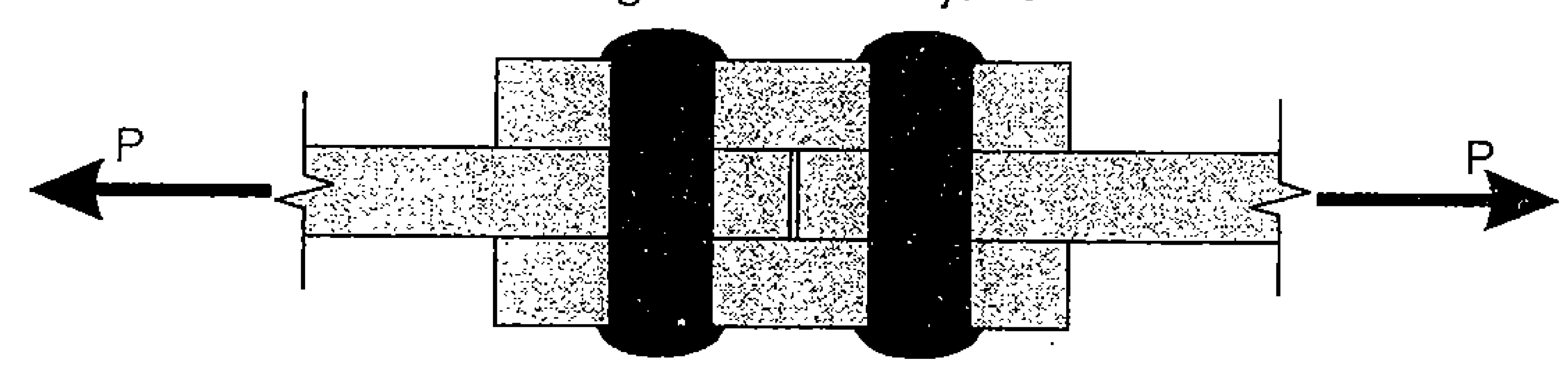
Lap joint

(ii) Butt Joint

- The butt joint is that in which the plates to be connected butt against each other and the connection is made by providing a cover plate on one or both sides of joint.



Single-cover butt joint



Double-cover butt joint

- The butt joint may have a single row or staggered or chain riveting.

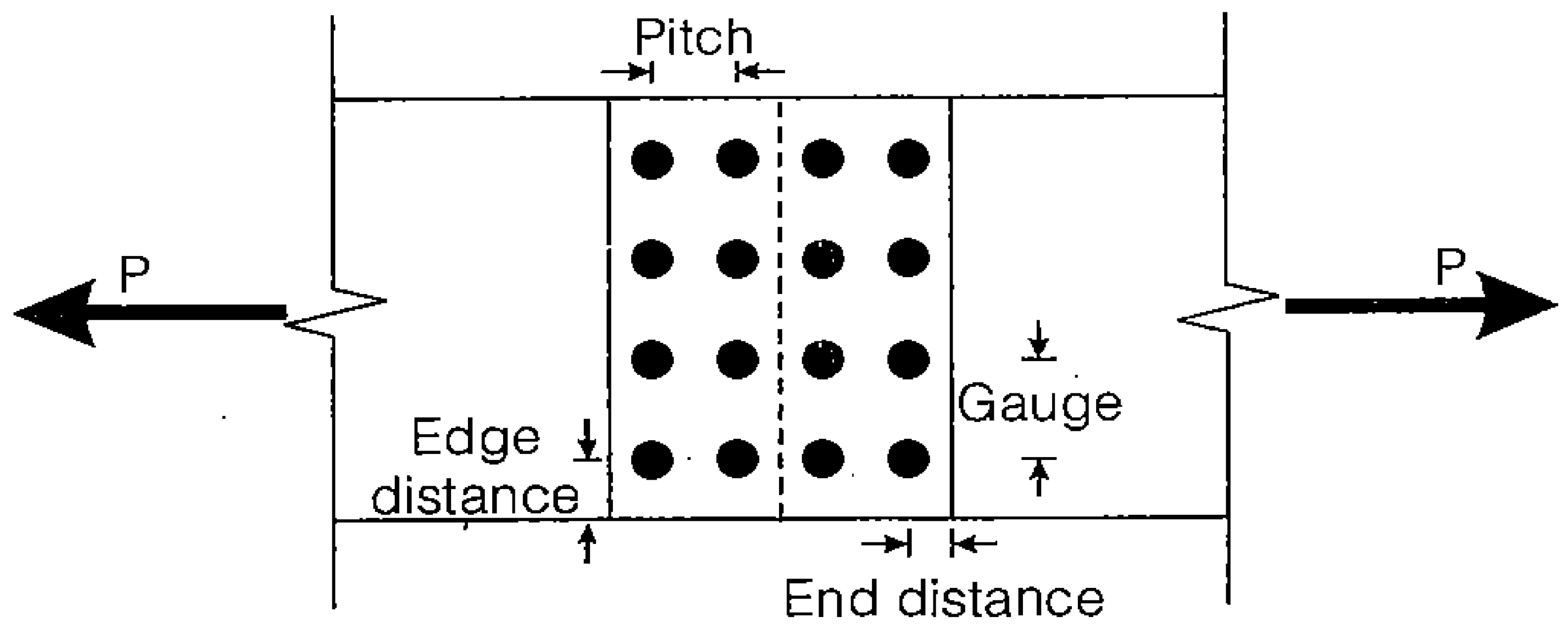
> **Remember**
>
> **(i) Nominal diameter (d):** The diameter of the shank of a rivet before riveting, is called the nominal diameter. For a bolt, the diameter of the unthreaded portion of the shank is called its nominal diameter.
>
> **(ii) Effective diameter or gross diameter:** The effective or gross diameter of a rivet is equal to the diameter of the hole it fills after riveting. For a bolt, the nominal diameter is same as the gross diameter.
>
> **(iii) Net area:** The net area of a bolt is the area at the root of the thread.
>
> **(iv) Gauge:** A row of rivets parallel to the direction of force is called a gauge line. The normal distance between two adjacent gauge line is called the gauge.

(v) **Edge distance:** It is the distance between the edge of a member or cover plate and the centre of the nearest rivet hole.

(vi) **Proof load :** Initial tension in HSFG bolts is known as proof load of the bolt.

(vii) **Slip Factor:** Coefficient of friction in friction type joint is known as slip factor.

(viii) **Pitch :** The distance between centres of any two adjacent rivets parallel to the direction of force is called pitch. Diagonal pitch is the distance between centres of any two adjacent rivets in the diagonal direction is called diagonal pitch.



Failure of Riveted/Bolted Joints

(i) **By Tearing of Plate between rivets**

Strength of tearing per pitch length

$$P_t = (p - d')t \times f_t$$

where, f_f = Permissible tensile stress in plates
t = Thickness of plate
d' = Dia of hole (gross dia of rivet)
p = Pitch

(ii) **Strength of rivet in single shear** $P_s = \frac{\pi}{4}(d')^2.f_s$

(iii) **Strength of rivet in double shear**

$$P_s = 2 \times \frac{\pi}{4}.d'^2.f_s$$

where, f_s = allowable shear stress in rivets
d' = dia of hole.

(iv) **Failure due to bearing or crushing of rivet or plates**

Strength of rivet in bearing

$P_b = f_b.d'.t$ where, f_b = bearing strength of rivet.

NOTE
Shearing strength of joint is simply the sum of shearing strength of individual rivets.
Bearing strength of joint is simply sum of bearing strength of individual rivets in the joints.

Efficiency of Joints (η)

$$\eta = \frac{\text{Minimum}\{P_s, P_b, P_t\}}{P}$$

where, P_s = Strength of joint in shear
P_b = Strength of joint in bearing
P_t = Strength of joint in tearing
P = Strength of plate in tearing when no deduction has been made for rivet holes
$= p.t.f_t$

- Rivet value $R_V = \text{minimum}\begin{cases} P_s \\ P_b \end{cases}$
- Number of rivet, $n = \frac{\text{Force}}{R_v}$

- **IS 800 : 1984 Recommendation**
Maximum permissible stress in rivets & bolts

Type of fastener	Axial tension, σ_{at} (MPa)	Shear, τ_{vf} (MPa)	Bearing, σ_{pf} (MPa)
(i) Power driven			
(a) Shop rivets	100	100	300
(b) Field rivets	90	90	270
(ii) Hand driven rivets	80	80	250
(iii) Close tolerance and turned bolts	120	100	300
(iv) Bolts in clearance holes	120	80	250

- Rivet diameter, Pitch

Minimum pitch	2.5 times of nominal diameter of the rivet
Maximum pitch for	
(i) any two adjacent rivets (including tacking rivets)	32 t or 300 mm, whichever is less
(ii) rivets lying in a line parallel to the force in the member:	
(a) in tension	16 t or 200 mm. whichever is less
(b) in compression	12 t or 200 mm, whichever is less

where t = thickness of thinner outside plate

Permissible Stresses

Cases	Permissible stress
Axial tension and compression	$0.60\ f_y$
In bending	$0.66 f_y$
In bearing(ex-at support)	$0.75 f_y$
In shear	max. permissible avg. = $0.40 f_y$ max. permissible = $0.45\ f_y$

Max Permissible Deflections

(a) Max permissible horizontal and verticle deflection $= \dfrac{\text{Span}}{325}$(WSM)

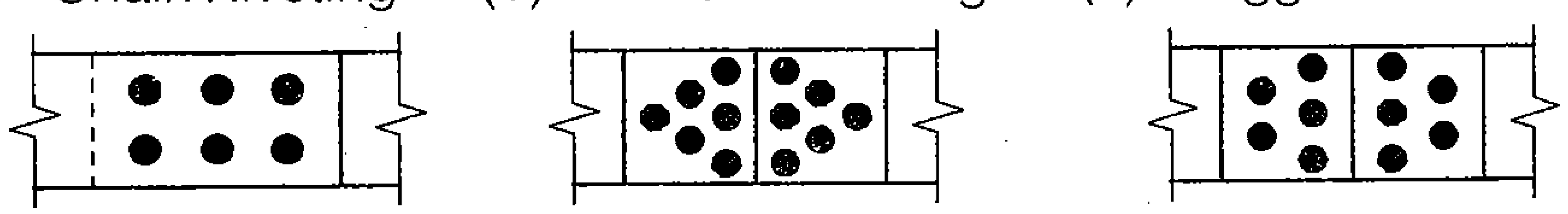
(b) Max permissible deflection when supported elements are susseptible to cracking $= \dfrac{\text{Span}}{360}$(LSM)

(c) Max permissible deflection when supported element are not susseptible to cracking $= \dfrac{\text{Span}}{300}$(LSM)

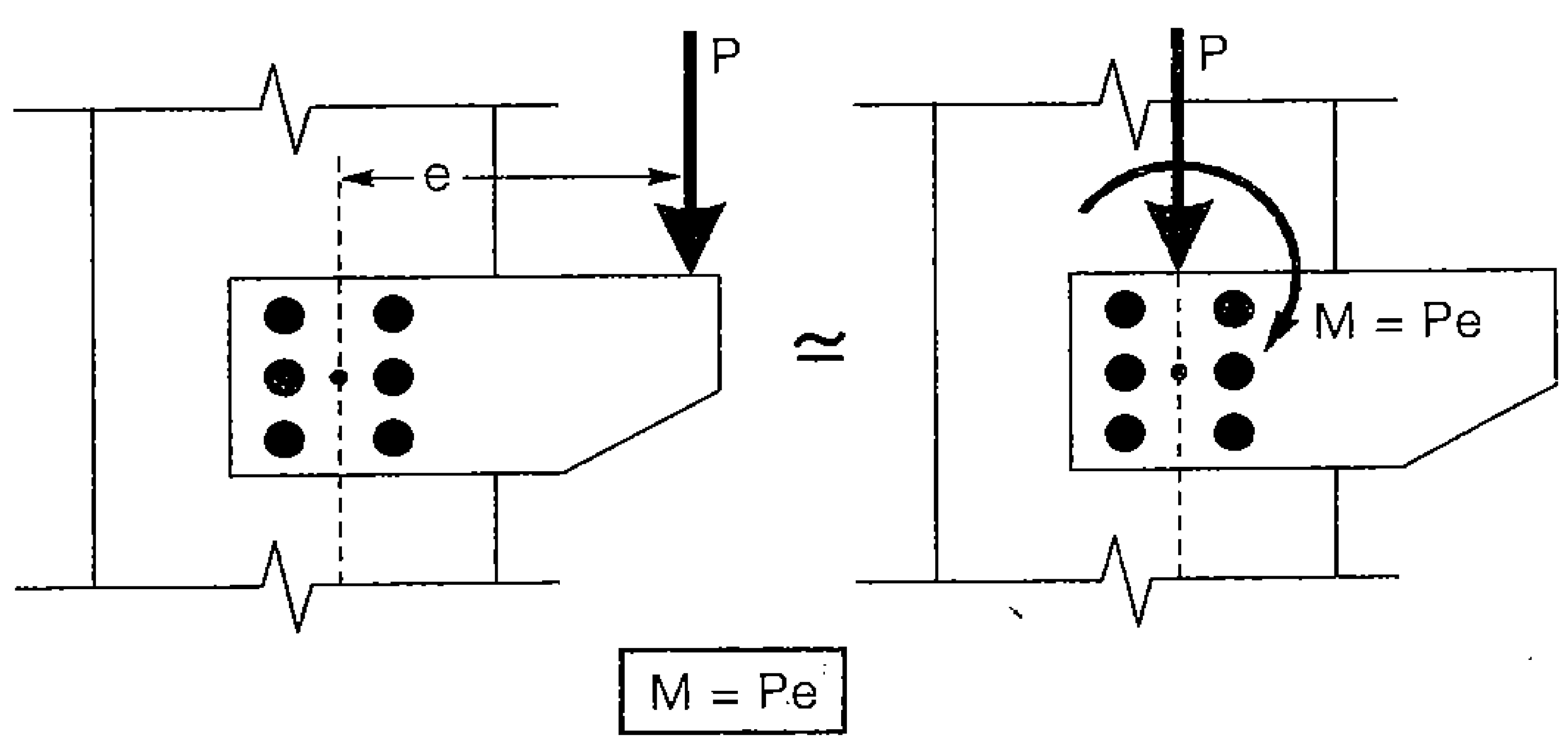
> **Remember**
> 1. When wind and earthquake loads are considered permissible stresses in steel structures are increased by 33.33% and in rivets and welds are increased by 25%.
> 2. By providing proper edge distance, we can prevent shear failure, splitting failure and bearing failure of plates.

Arrangement of Rivets

(a) Chain Riveting (b) Diamond Riveting (c) Staggered Riveting

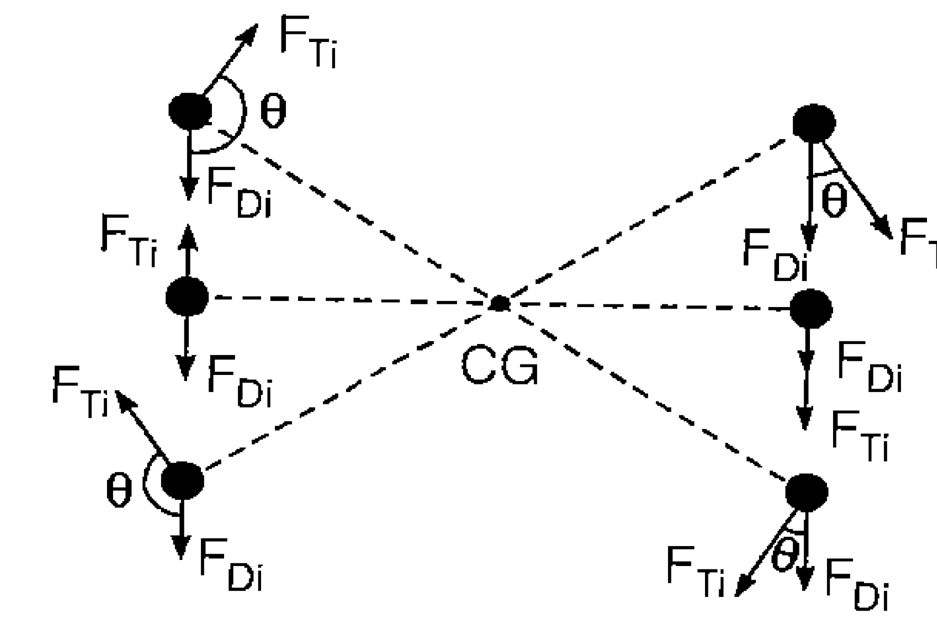


Eccentric Connections



(i) $F_{Di} = \dfrac{P.A_i}{\Sigma A_i}$

(ii) $F_{Ti} = \dfrac{Per_i}{\Sigma A_i r_i^2} A_i$



Special Case: When all the rivets are of same diameter then.

(a) $F_{Di} = \dfrac{P}{n}$ (b) $F_{Ti} = \dfrac{Per_i}{\Sigma r^2}$ or $F_{Ti} = \dfrac{Mr_i}{\Sigma r^2}$

(iii) $Fr_i = \sqrt{(F_{Di})^2 + (F_{Ti})^2 + 2F_D.F_T \cos\theta} \le R_v$

where, F_{Di} = Direct force in i^{th} rivet.
F_{Ti} = Force in i^{th} rivet due to torsional moment
r_i = Distance of i^{th} rivet from CG
A_i = Area of i^{th} rivet $= \dfrac{\pi}{4}(d_i)^2$
F_{Di} = Always acts in the direction of applied load P.
F_{Ti} = Always acts perpendicular to the line joining CG of rivet group and the rivet under consideration.
F_{ri} = Resultant force in i^{th} rivet.

***Note:** Most critical rivet is one for which θ is minimum and r is maximum.*

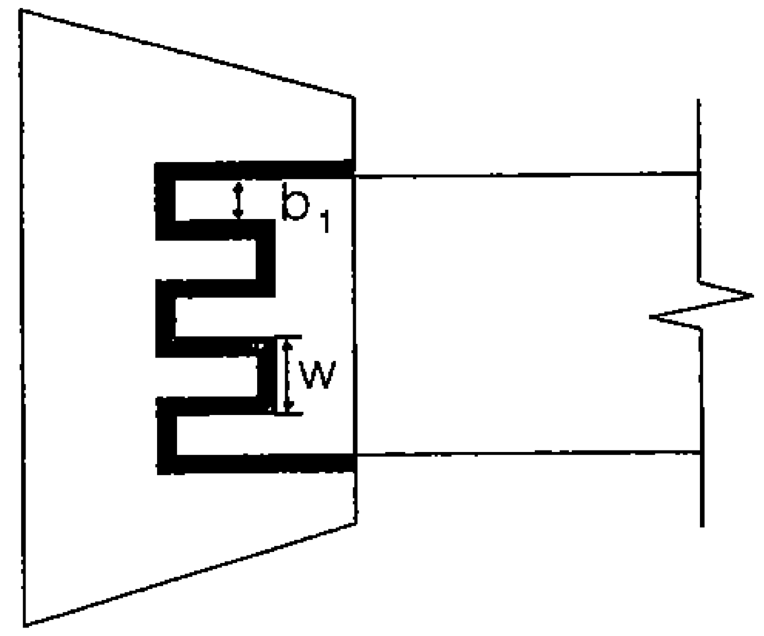
Angle b/w fusion faces	Value of k
60°-90°	0.70
91°-100°	0.65
101°-106°	0.60
107°-113°	0.55
114°-120°	0.50

- **Minimum size of weld**
  It depends upon thickness of thicker plate

Thickness of thicker plate	Minimum size
0-10 mm	3 mm
11-20 mm	5 mm
21-32 mm	6 mm
>32 mm	8 mm

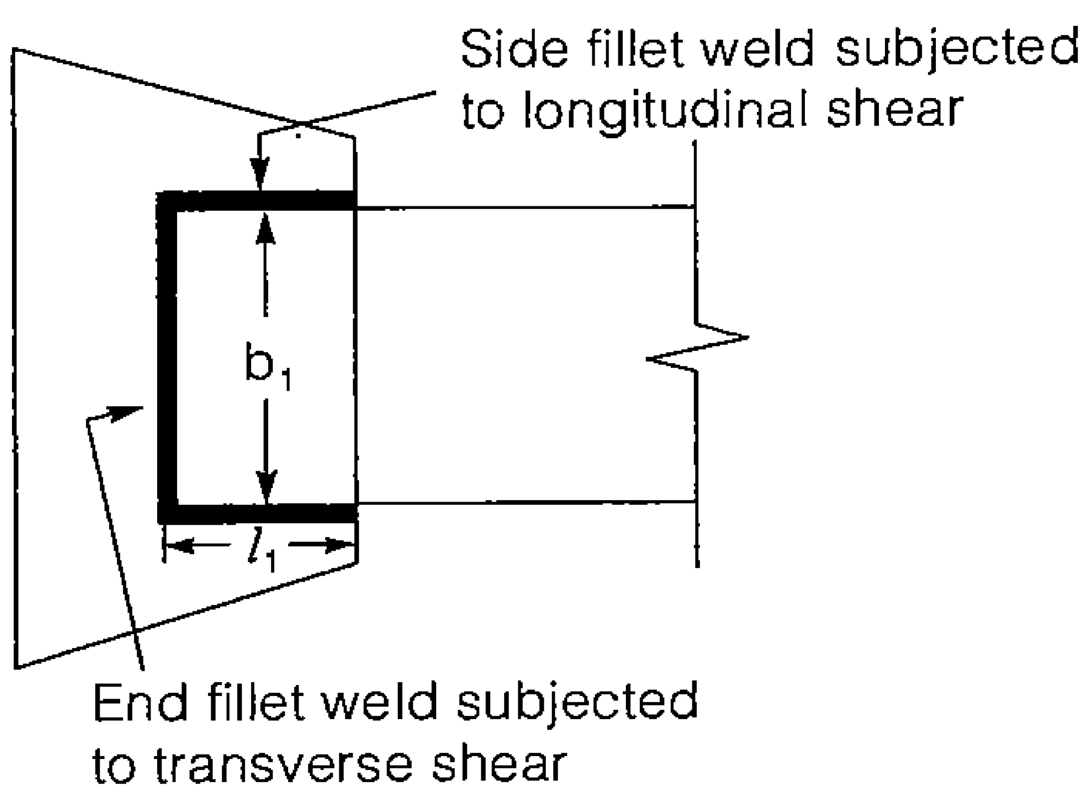
Max clear spacing between effective length of weld
In compression zone = 12t or 200 mm (minimum).
In tension zone = 16 t or 200 mm (minimum)

- Slot weld



$b_1 \nless 2t$
$w \nless 3t$ or 25 mm

- Side fillet weld



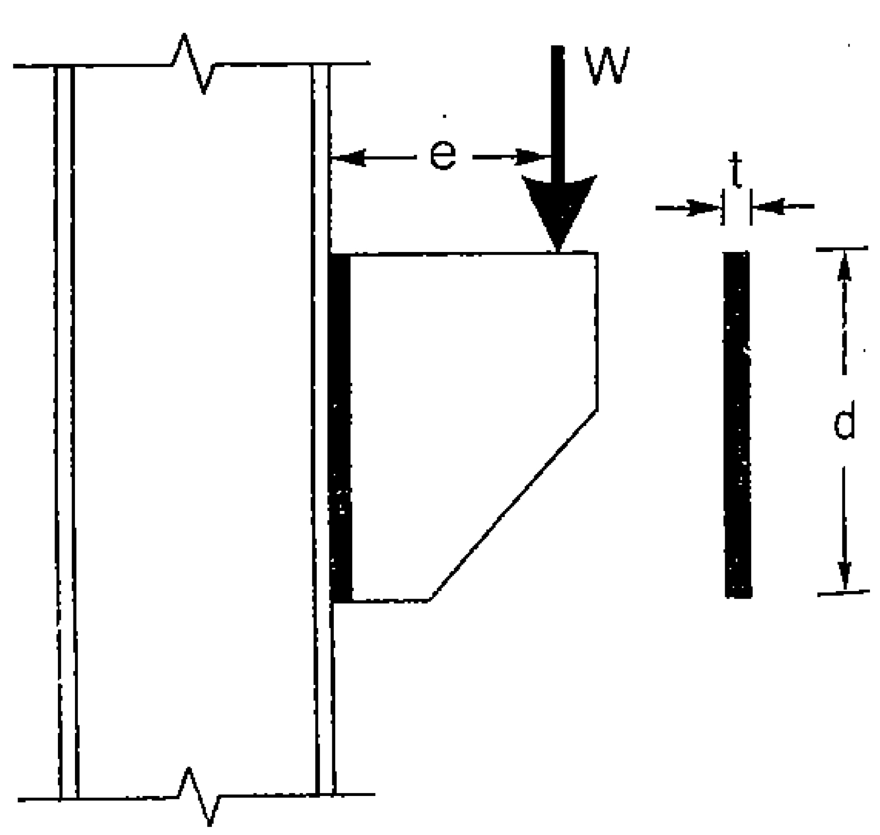
(a) $l_1 \nless b_1$
(b) $b_1 \ngtr 16t$ to make stress distribution uniform
(c) if $b_1 > 16t$ use end fillet weld.

Welded Connection

- **Permissible Stresses**
  (a) Tensions and compression on section through the throat of butt weld = 150 N/mm²
  (b) Shear on section through the throat of butt or fillet weld = 108 N/mm² ≅ 110 N/mm²
  Throat thickness t = k × size of weld

- **Butt-welded Joint Loaded Eccentrically**
  - Let thickness of weld throat = t, and length of weld = d
  - Shear stress at weld, $P_s = \dfrac{W}{d \times t}$

  Where t = thickness of weld throat and d = length of weld.



- Tensile or compressive stress due to bending at extreme fibre,

$$P_b = \frac{6M}{t \times d^2}$$ For the safety of joint the interaction equation.

$$\left[\left(\frac{P_s}{\text{Permissible shear stress in weld}}\right)^2 + \left(\frac{P_b}{\text{Permissible tensile stress in weld}}\right)^2\right] \le 1$$

- **Equivalency Method**

$$\sqrt{P_b^2 + (3P_s)^2} \le 0.9f_y$$ (based on max distortion energy theory)

Permissible bending stress for flanged section = 165 N/mm² = $0.67f_y$
For solid section (■, ●, ▲) permissible bending stress is 185 N/mm²

FILLET-WELDED JOINT LOADED Eccentrically

There can be two cases:
(i) Load not lying in the plane of the weld
(ii) Load lying in the plane of the weld

(i) Load not lying in the plane of the weld :

- Let thickness of weld throat = t and total length of weld = 2 × d
- Vertical shear stress at weld,

$$p_s = \frac{W}{2d \times t}$$

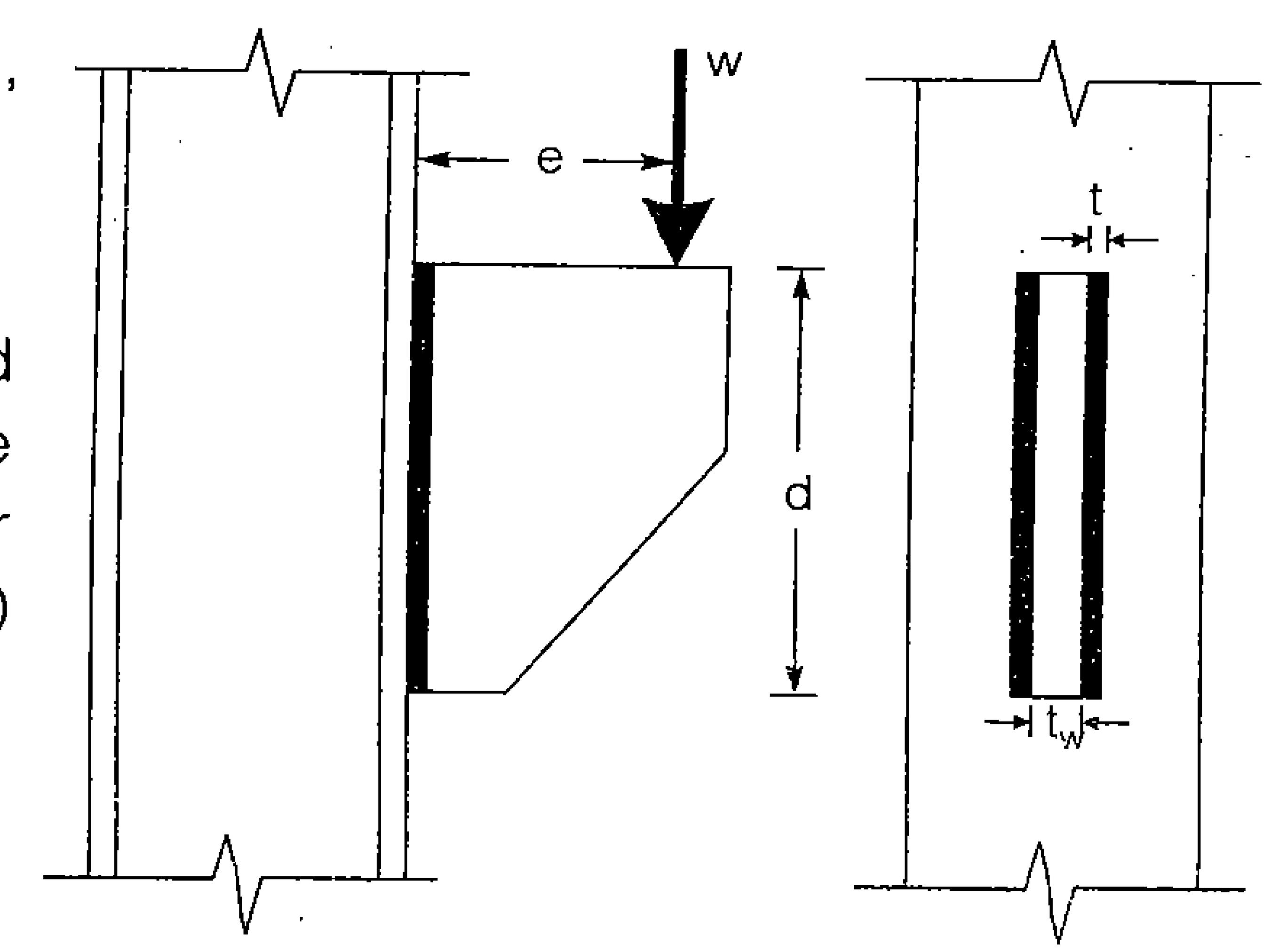
- Horizontal shear stress due to bending at extreme fibre,

$$p_b = \frac{M}{I} \times y = \frac{(W \times e) \times d/2}{\dfrac{2 \times t \times d^3}{12}} = \frac{3We}{td^2}$$

- Resultant stress,

$$p_r = \sqrt{p_s^2 + p_b^2}$$

- The value of p_r should not exceed the permissible shear stress p_q (= 108 MPa) in the weld.



- For design of this connection, the depth of weld may be estimated approximately by $d = \sqrt{\dfrac{6 \times W \times e}{2 \times t \times p_b}}$

(ii) **Load lying in the plane of the weld:** Consider a bracket connection to the flange of a column by a fillet weld as shown in figure

- Vertical shear stress at weld, $p_s = \dfrac{W}{l \times t}$

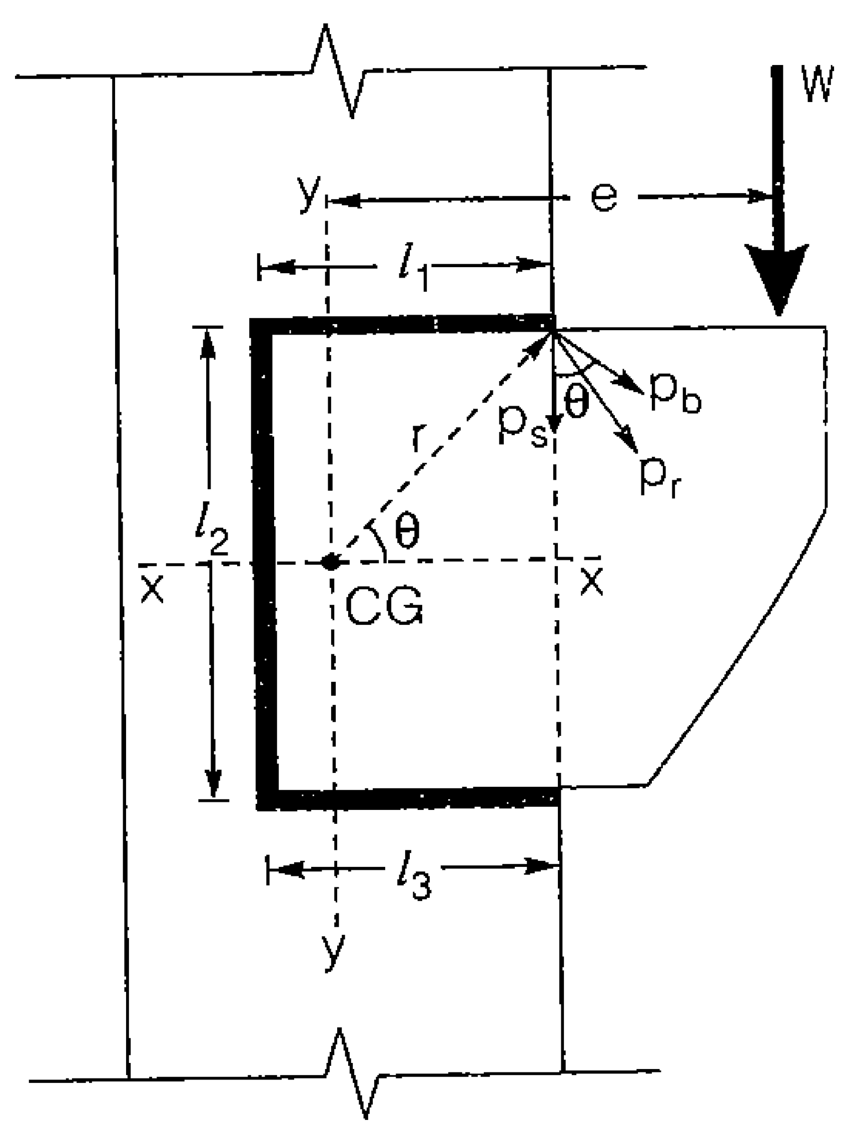
  where,
  $l(l_1 + l_2 + l_3)$ = the length of weld
  and t = thickness of the throat

- Torsional stress due to moment, at any point in the weld,

  $p_b = \dfrac{T \times r}{I_p}$

  where,
  T = torsional moment = W × e
  r = distance of the point from cg of weld section
  I_p = polar moment of inertia of the weld group
  $= I_x + I_y$

- The resultant stress, $p_r = \sqrt{p_s^2 + p_b^2 + 2p_s p_b \cos\theta}$
- For safety, $p_r \not> $ permissible stress in fillet weld, i.e. 108 MPa.
- The resultant stress p_r will be maximum at a point where r is maximum and q is minimum.



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2 Tension Member

Introduction

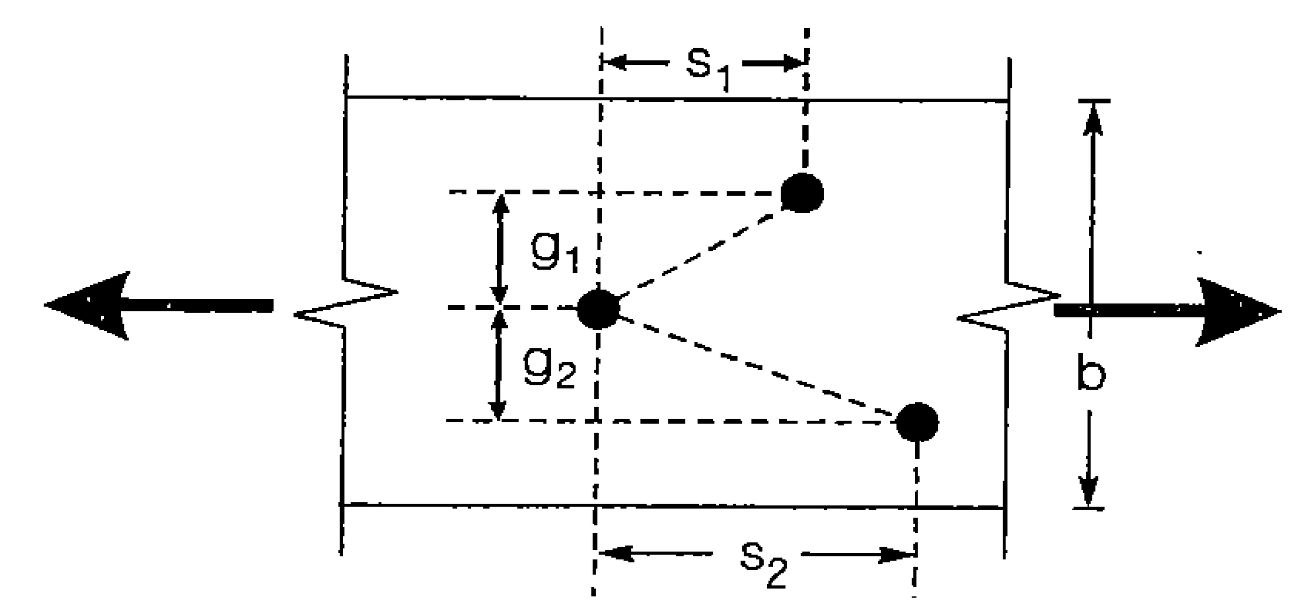
1. Tension member has no stability problem.
2. In tension member net section will be effective whereas in compression member gross section is effective.

Types of member	Max. Slenderness Ratio
1. A tension member in which reversal of direct stress due to loads other then wind or earthquake forces.	180
2. A member normally acting as a tie in roof truss or bracing system. But subjected to possible reversal of stress resulting from the action of wind or earthquake forces.	350

NET SECTIONAL AREA

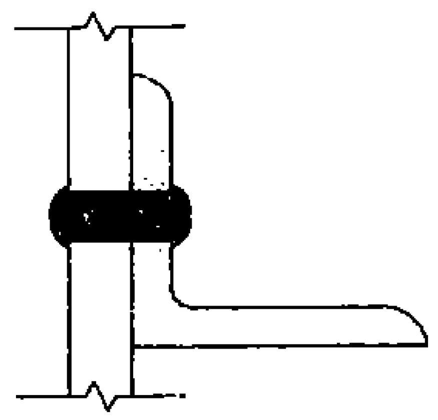
(i) For Plate $\text{Net Area} = (b \times t) - nd't + \left(\dfrac{s_1^2}{4g_1} + \dfrac{s_2^2}{4g_2}\right)t$

where,
s_1 = Distance between two consecutive rivets in the direction of load, also called pitch.
g_1 = Distance between two consecutive rivets perpendicular to the direction of load also called gauge.
b = Width of the plate
n = Number rivets at the section
t = Thickness of the plate
d′ = Gross diameter of the rivet



(ii) Single angle connected by one leg only.

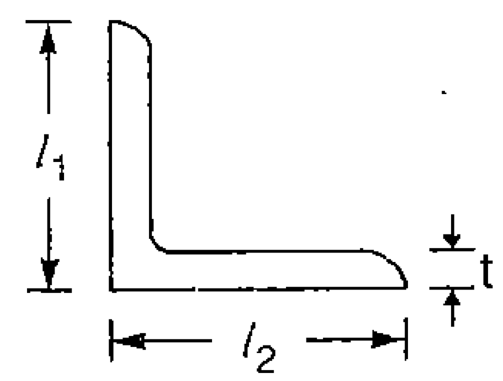
(a) $A_{net} = A_1 + kA_2$



where, A_1 = Net cross-section of area of the connected leg.

A_2 = Gross cross-sectional area of unconnected leg. (out stand)

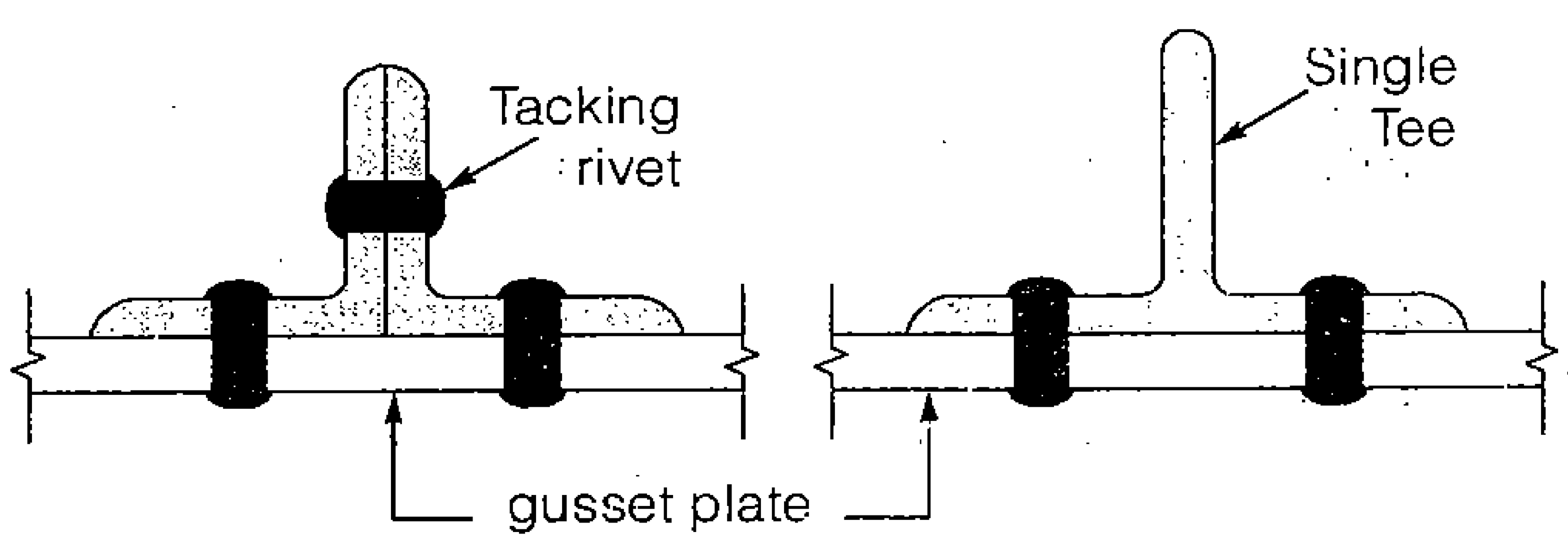
(b) $k = \dfrac{3A_1}{3A_1 + A_2}$



(c) $A_1 = \left(l_1 - \dfrac{t}{2}\right)t$

(d) $A_2 = \left(l_2 - \dfrac{t}{2}\right)t$ (e) $A_{net} = (l_1 + l_2 - t)\,t$

(iii) For pair of angle placed back to back (or a signal tee) connected by only one leg of each angle (or by the flange of a tee) to the same side of a gusset plate : or It the two angles are tagged along a-a.



(a) $A_{net} = A_1 + kA_2$

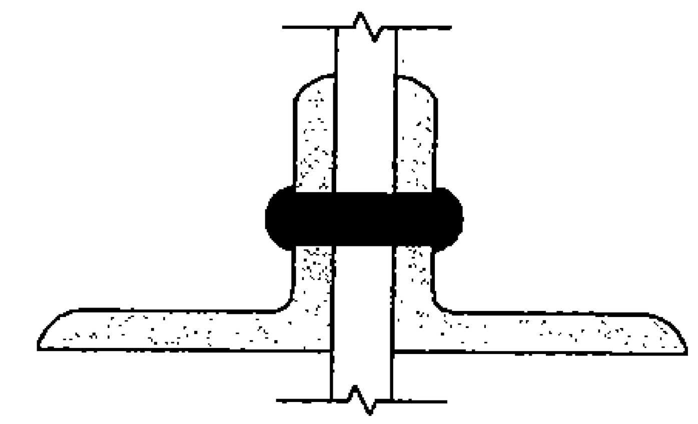
(b) $k = \dfrac{5A_1}{5A_1 + A_2}$ where, A_1 = Area of connected leg

A_2 = Area of outstand (unconnected leg)

(c) The area of a web of tee = Thickness of web × (depth − thickness of flange)

(d) The outstand legs of the pair of angles should be tacked by rivets of a pitch not exceeding 1 m.

(iv) If two angles are placed back to back and connected to both sides of the gusset plate. Then



$A_{net} = A_1 + A_2$ (k = 1) when tack riveted.

If not tack riveted then both will be considered seperately and case (ii) will be followed $k = \dfrac{3A_1}{3A_1 + A_2}$

Permissible Stress in Design

- The direct stress in axial tension on the effective net area should not exceed σ_{at}

  where $\sigma_{at} = 0.6 f_y$

  and f_y = minimum yield stress of steel in MPa
- Allowable stress σ_{at} in Axial Tension for steel Conforming to **IS : 226-1975**.

Form	Thickness/ Diameter	σ_{at} (MPa)
1. Plates, angles, tees and I-beams, channels and flats.	Upto and including 20 mm 20 to 40 mm over 40 mm	150 144 138
2. Bars (round, square and hexagonal).	Upto & including 20 mm over 20 mm	150 144

Lug Angle

The lug angle is a short length of an angle section used at a joint to connect the outstanding leg of a member, thereby reducing the length of the joint. When lug angle is used k = 1.

■■■

3 Compression Member

Strength of an Axially Loaded Compression Member

- The maximum axial compressive load P

$$\boxed{P = \sigma_{ac} \times A}$$

where, P = axial compressive load (N)

σ_{ac} = permissible stress in axial compression (MPa)

A = gross-sectional area of the member (mm^2)

- IS : 800-1984 uses the Merchant Rankine formula for σ_{ac} which is given as

$$\boxed{\sigma_{ac} = 0.6 \times \frac{f_{cc} \times f_y}{\left[f_{cc}^n + f_y^n\right]^{1/n}}}$$

where, f_{cc} = elastic critical stress in compression $= \frac{\pi^2 \times E}{\lambda^2}$

λ = slenderness ratio $= \frac{l}{r}$

l = effective length of the compression member.

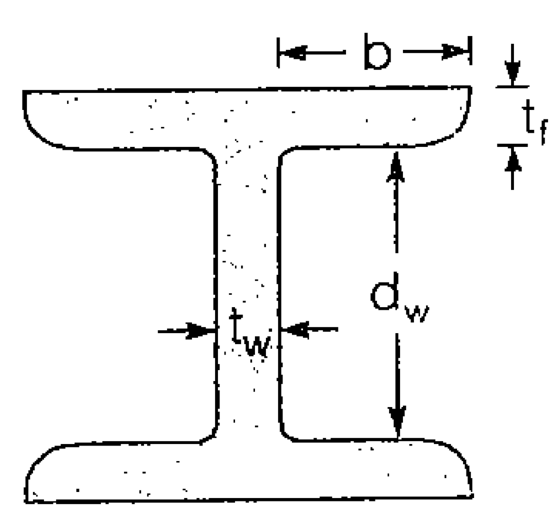
r = appropriate radius of gyration of the member (minimum value)

E = modulus of elasticity of steel = 2×10^5 MPa

n = a factor assumed as 1.4

Remember

- According to **IS 800: 1984**, the direct stress in compression on the gross cross-sectional area of axially loaded compression member shall not exceed $0.6f_y$, nor the permissible σ_{ac} value calculated using the above formula.
- The critical stress at which the plate buckles is inversely propational to $(b/t)^2$.
- To prevent the buckling of flange plate and web plate.



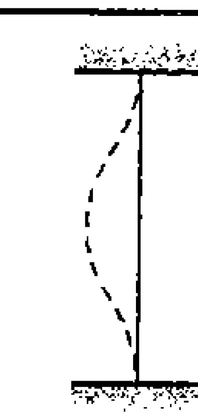
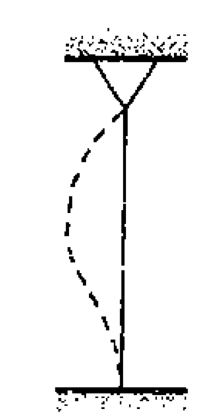
$$\frac{b}{t_f} \not> 16, \quad \frac{d_w}{t_w} \not> 50$$

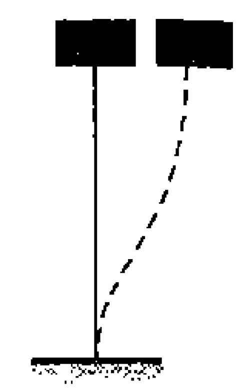
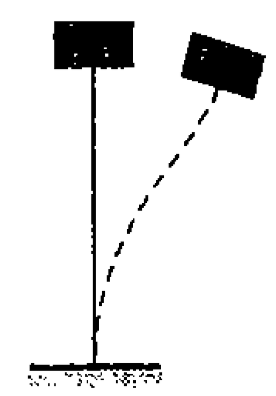
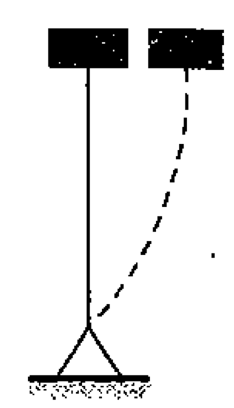
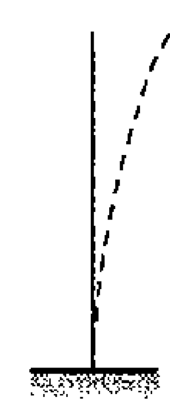
- Maximum Slenderness Ratio (Clause 3.7.1 IS:800-1984)

Sl. No.	Type of member	Max. slenderness ratio
1.	A member carrying compressive loads resulting from dead load and superimposed loads	180
2.	A member subjected to compressive loads resulting from wind/earthquake forces provided the deformation of such members does not adversely affect the stress in any part of the structure	250
3.	A member normally carrying tension but subjected to reversal of stress due to wind or earthquake forces	350

Effective Length

Table: (Effective length of compression members of constant dimensions (Clause 5.2.2 IS : 800-1984)

Sl. No.	*Degree of end restraint of compression member*	*Recommended value of effective Length*	*Symbol*
1.	Effectively held in position and restrained against rotation at both ends	0.65L	
2.	Effectively held in position at both ends restrained against rotation at one end	0.80L	
3.	Effectively held in position at both ends, but not restrained against rotation	1.00L	

4.	Effectively held in position and restrained against rotation at one end, and at the other end restrained against rotation but not held in position.	1.20L	
5.	Effectively held in position and restrained against rotation at one end, and at the other end partially restrained against rotation but not held in position	1.50L	
6.	Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position	2.00L	
7.	Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end	2.00L	

NOTE: For battened columns, the effective length shall be increased by 10%

Angle struts

- The slenderness ratio ($\lambda = l/r$) should not exceed the values given in Table).

Table: Angle Struts (Clauses 5.5, IS : 800 -1984)

Sl. No.	Type	End Connections	Effective length	Allowable stress	Slenderness Ratio
1.	Single angle discontinuous	(i) One rivet or bolt at each end	l = L	$0.8\sigma_{ac}$	180
		(ii) Two or more rivets or bolts or welding at each end	l = 0.85L	σ_{ac}	-
2.	Double angle, tacked, discontinuous	(i) Connected on same side of gusset plate (a) One rivet or bolt at each end	l = L	$0.8\sigma_{ac}$	$\frac{l}{r} \not> 180$
		(b) Two or more rivets, bolts or welding at each end	l = 0.85L	σ_{ac}	-
		(ii) Connected on both Sides of gusset plate by two or more rivets, bolts or welding	l = 0.7 to 0.85L depending on rigidity of joint	σ_{ac}	-
3.	Single or double angle continuous	One or more rivet, bolt or welding	l = 0.7L to 1.0L depending on end rigidity	σ_{ac}	-

Built-up Compression Member

Tacking Rivets

- The slenderness ratio of each member between the connections should not be greater than 40 nor greater than 0.6 times the most unfavorable slenderness ratio of the whole strut. In no case should the spacing of tacking rivets in a line exceed 600 mm for such members, i.e. two angles, channels or tees placed back-to-back.
- For other types of built-up compression members, say where cover-plates are used, the pitch of tacking rivets should not exceed 32 t or 300 mm, whichever is less, where *t* is the thickness of the thinner outside plate. When plates are exposed to the weather, the pitch should not exceed 16 *t* or 200 mm whichever is less.
- The diameter of the connecting rivets should not be less than the minimum diameter given below.

Thickness of member	Minimum diameter of rivets
Up to 10 mm	16 mm
Over 10 mm to 16 mm	20 mm
Over 10 mm	22 mm

Design of Compression Members

The following steps are followed for designing an axially loaded compression member:

(i) Assume some value of permissible compressive stress σ_{ac} and calculate the approximate gross sectional area A required

$$A_{approx} = \frac{\text{Axial compressive load}}{\text{Assumed permissible stress}}$$

For single-angle-channel-or I-section (low loads) 80 MPa and for built-up sections (heavy loads) 110 MPa may be assumed initially as permissible compressive stress.

(ii) Choose a trial section having area $\simeq A_{approx}$.

(iii) Determine the actual permissible stress corresponding to maximum slenderness ratio l/r of the trial section.

(iv) Calculate the safe load to be carried by trial section by multiplying, the actual permissible stress by the area of the trial section.

If the safe load is equal to or slightly more than the actual load, the trial section is suitable for selection. Otherwise the above steps should be repeated.

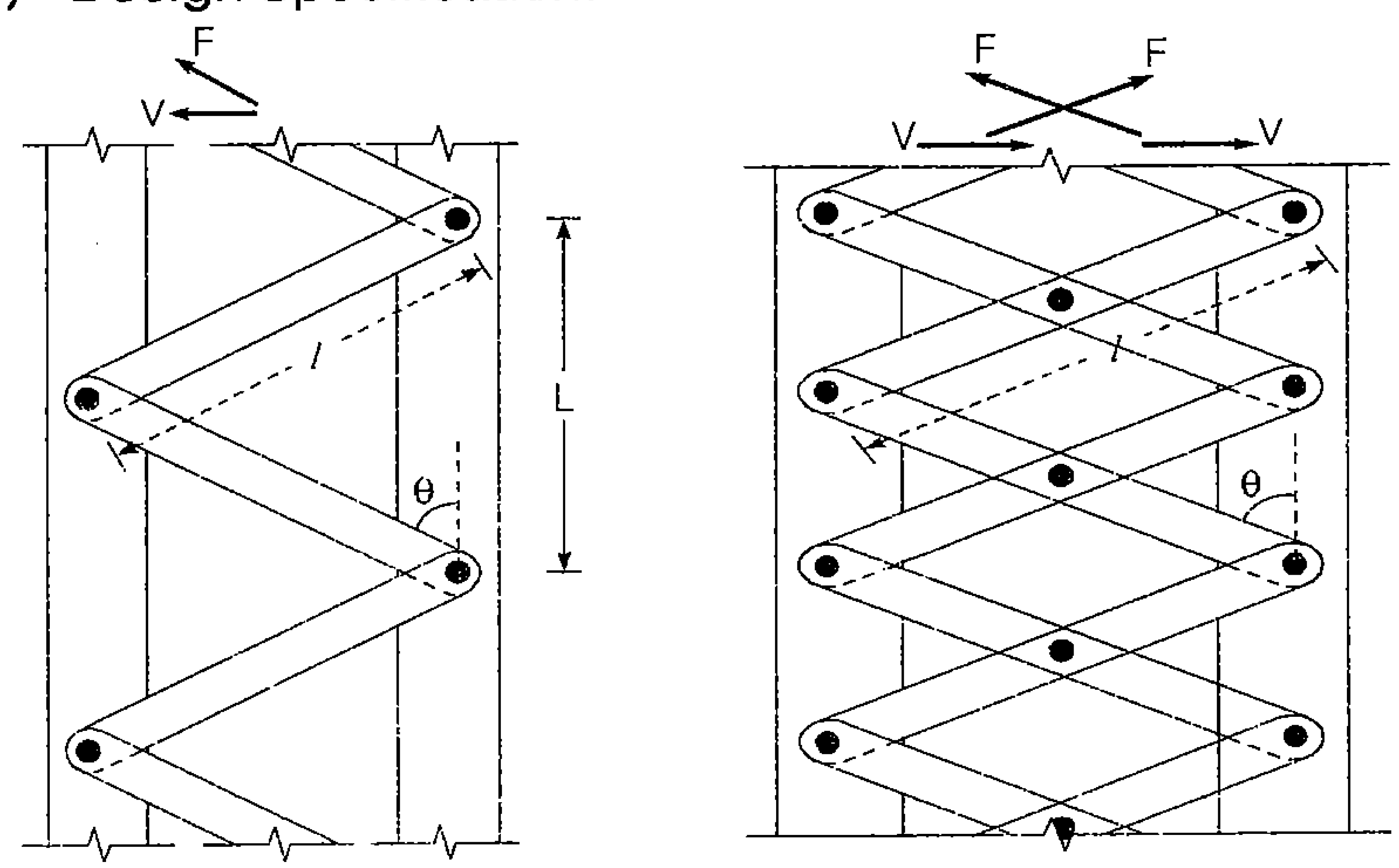
(v) Check the slenderness ratio.

Lacings

(a) General requirements:

1. Radius of gyration about the axis ⊥ to the plane of lacing $\nless$ radius of gyration about the axis in the plane of lacing
2. The lacing system should not be varied throughout the length of the strut as far as practicable.
3. The single-laced systems on opposite sides of the main components should preferably be in the same direction so that one be the shadow of the other.

(b) Design Specification:



(a) Single lacing (b) Double lacing

1. The angle of inclination of the lacing with the longitudinal axis of the column should be between 40° to 70°.

2. The slenderness ratio l_e/r of the lacing bars should not exceed 145. The effective length l_e of the lacing bars should be taken as follows:

Type of lacing	Effective length l_e
Single lacing, riveted at ends	Length between inner end rivets on lacing bar (= l, as shown in Fig. 17)
Double lacing, riveted at ends and at intersection	0.7 times length between inner end rivets on lacing bars (= $0.7 \times l$)
Welded lacing	0.7 times distance between inner ends of effective lengths of welds at ends ($0.7 \times l$)

> **Remember** Lacing is generally preferred in case of eccentric loads. Battening is normally used for axially loaded columns and where the components are not far apart.

3. For local Buckling Criteria

$$\frac{L}{r^c_{min}} \not> 50$$
$$\not> 0.7\ \lambda_{whole\ section}$$

where, L = distance between the centres of connections of the lattice bars to each component as shown in fig.

r^c_{min} = minimum radius of gyration of the components of compression member

4. Minimum width of lacing bars in riveted construction should be as follows:

Nominal rivet diameter (mm)	22	20	18	16
Width of lacing bars (mm)	65	60	55	50

5. Minimum thickness of lacing bars:

$t \nless l/40$ for single lacing

$\nless l/60$ for double lacing riveted or welded at intersection

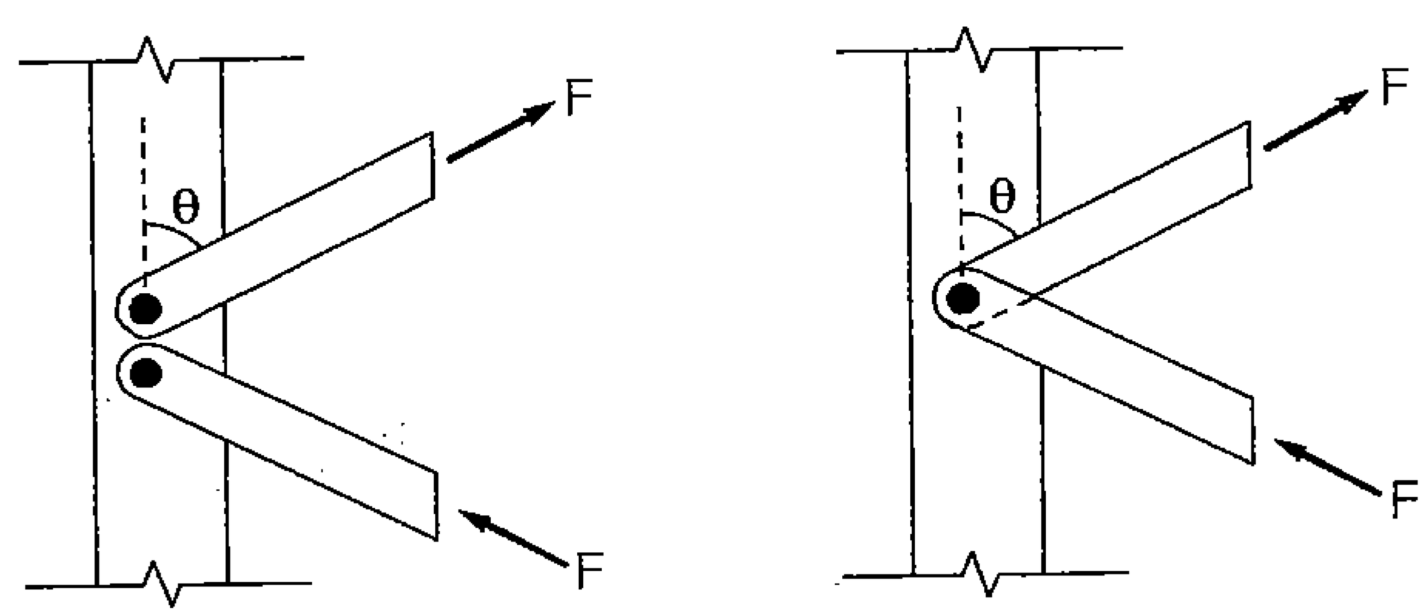
where, l = length between inner end rivets as shown in fig.

6. The lacing of compression members should be designed to resist a transverse shear, V = 2.5% of axial force in the member.
   - For single lacing system on two parallel faces, the force (compressive or tensile) in each bar, $\boxed{F = \frac{V}{2\sin\theta}}$
   - For double lacing system on two parallel planes, the force (compressive or tensile) in each bar, $\boxed{F = \frac{V}{4\sin\theta}}$
   - If the flat lacing bars of width b and thickness t have rivets of diameter d then,

   $$\text{Compressive stress in each bar} = \frac{\text{force}}{\text{gross area}} = \frac{F}{b \times t} \not> \sigma_{ac}$$

   - $$\text{Tensile stress in each bar} = \frac{\text{force}}{\text{net area}} = \frac{F}{(b-d)\times t} \not> \sigma_{at}$$

7. End Connections:
   - ***Riveted connection:*** Riveted connections may be made in two ways as shown in Fig. (a) and (b).



(a) (b)

For case (a),

$$\boxed{\text{Number of rivets required} = \frac{F}{\text{Rivet value}}}$$

Force case (b),

- $$\text{Number of rivets required} = \frac{2F\cos\theta}{\text{Rivet value}}$$

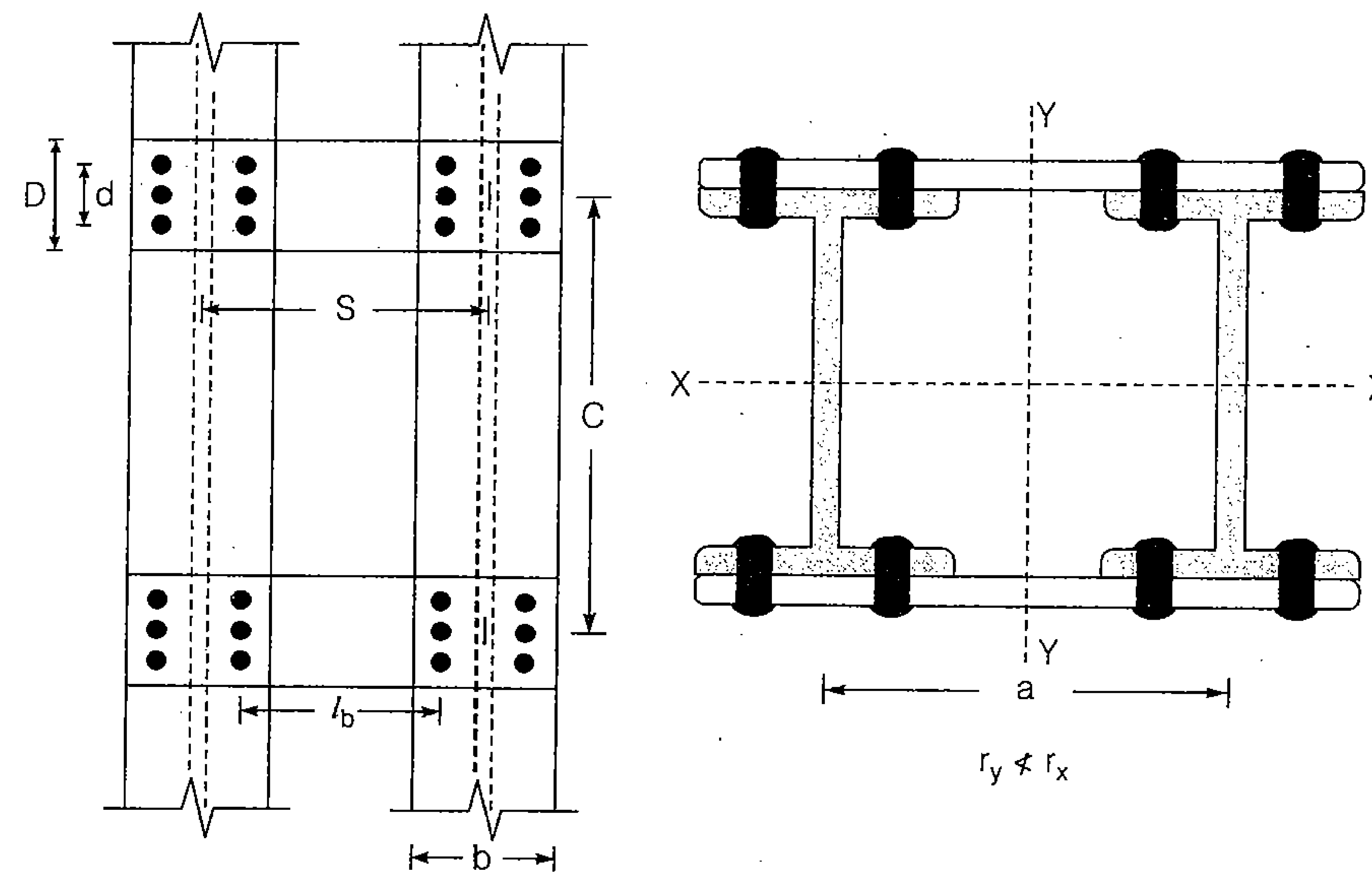
- **Welded connections**

Lap joint : Overlap ≮(1/4) times thickness of bar or member, whichever is less.

Butt joint : Full penetration butt weld or fillet weld on each side. Lacing bar should be placed opposite to flange or stiffening member of main member.

Battens

(a) General Requirements:



1. $\boxed{r_y \not< r_x}$
2. The number of battens should be such that the member is divided into not less than three parts longitudinally.

(b) Design Specifications:

1. Spacing of battens C, from centre to centre of end fastening should be such that the slenderness ratio of the lesser main component,

   $\frac{C}{r^c_{mm}} \not> 50$, or 0.7 times the slenderness ratio of the compression member as a whole about $x - x$ axis (parallel to battens), which is less

   where C = spacing of battens as shown in fig.

   r^c_{min} = minimum radius of gyration of components.

2. $d > \left(\frac{3}{4}\right)a$ for intermediate battens,

$d >$ a for end battens

and $d > 2 \times b$ for any batten.

where d = effective depth of batten,

a = centroid distance of members,

b = width of member in the plane of batten

3. Thickness of battens,

$t > \frac{l_b}{50}$ where, l_b = distance between innermost connecting line of rivets or welds.

4. $V = \frac{2.5}{100}P$ and P = total axial load on the comp. member.

- Transverse shear V is divided equally between the parallel planes of battens. Battens and their connections to main components resist simultaneously a longitudinal shear.

$V_1 = \frac{V \times C}{N \times S}$ and a moment, $M = \frac{V \times C}{2N}$

where, C = spacing of battens

N = number of parallel planes of battens

S = minimum transverse distance between centroids of rivet group or welding.

- Check for longitudinal shear stress,

$\frac{V_1}{D \times t} \not> \tau_{va}$ where, t_{va} = permissible average shear stress = 100 MPa for steel of IS : 226-1975

D = overall depth of battens,

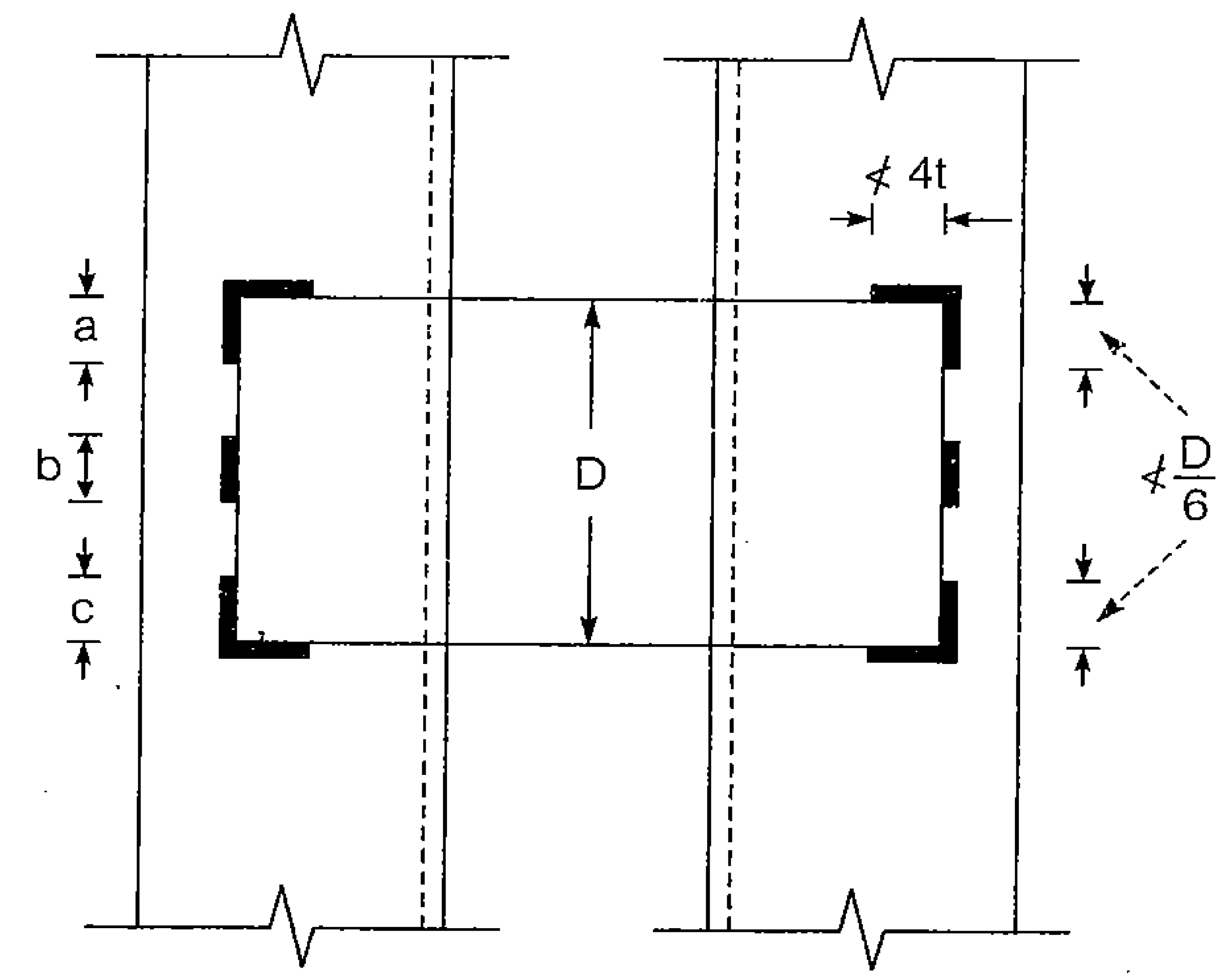
t = thickness of battens.

- Check for bending stress,

$\frac{M}{Z} = \frac{M}{\frac{1}{6}D \times t^2} \not> \sigma_{bc\,or\,bt}$ where, $\sigma_{bc}\sigma_{bt}$ = permissible bending compressive or tensile) stress = 165 MPa for steel of IS : 226-1975

5. End connections:
   - Design the end connections to resist the longitudinal shear force V_1 and the moment M as calculated in steep 4 above.

- For welded connections Lap $\not<$ 4t Where t is thickness of plate
- Total length of weld at end of edge of batten $\not<$ D/2
- Length of weld at each edge of batten $\not<$ 1/3 total length of weld required
- Return weld along transverse axis of column $\not>$ 4t where, t and D are the thickness and overall depth of the battens respectively.



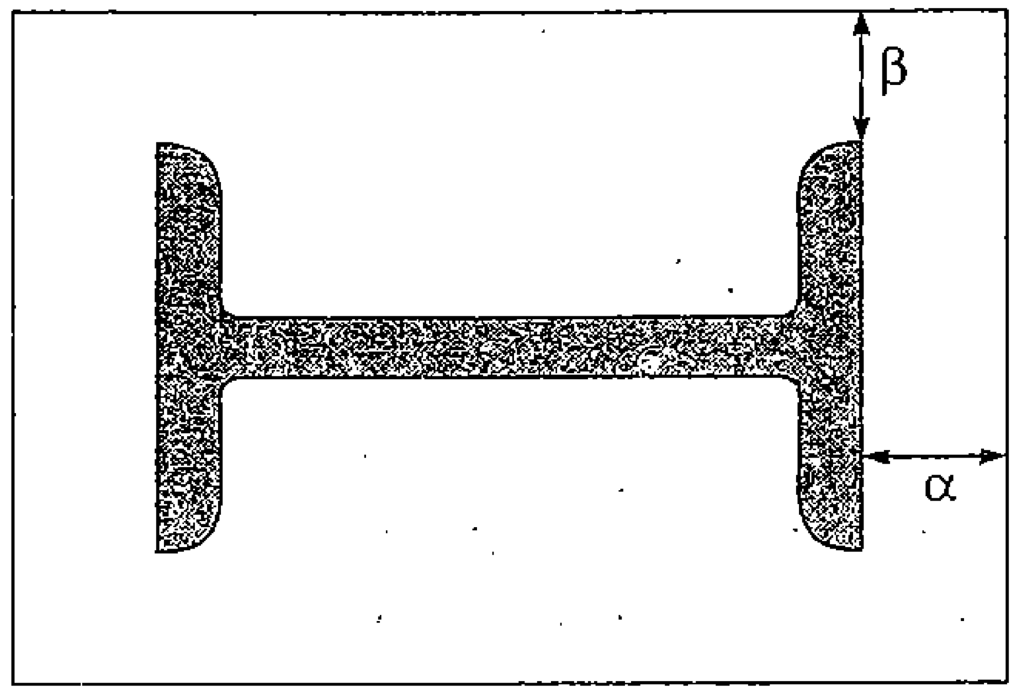
$a + b + c \not< \frac{D}{2}$ t = thickness of batten

Slab Base

- Sufficient fastenings are provided to retain the column securely on the base plate and resist all moments and forces (except direct compression in the column.) arising during transit, unloading and erection.

- $$\text{Area of slab base} = \frac{\text{axial load in the column}}{\text{permissible compressive stress in concrete}}$$
- The thickness of a rectangular slab base as per IS 800: 1984.

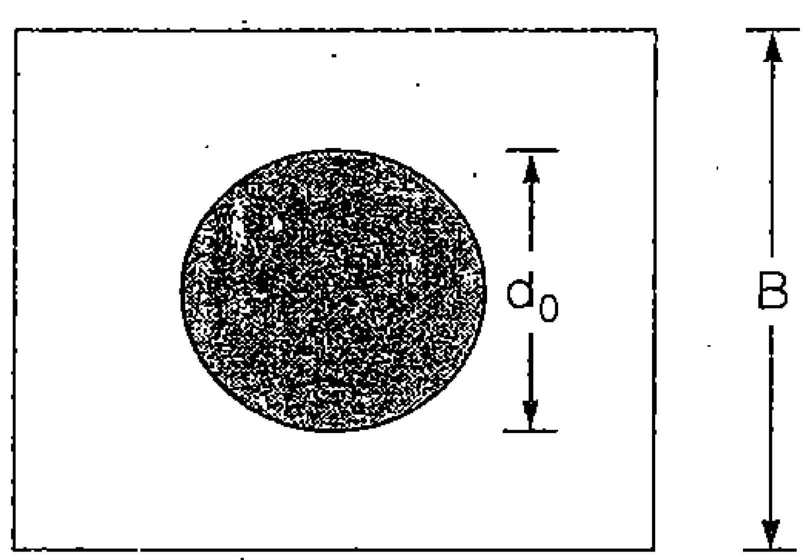
$$t = \sqrt{\frac{3w}{\sigma_{bs}}\left(a^2 - \frac{b^2}{4}\right)}$$



where t = the slab thickness (mm)
w = the pressure or loading on the underside of the base (MPa)
a = the greater projection of the plate beyond the column (mm) = max. (α, β).
b = the lesser projection of the plate beyond the column (mm) = min. (α, β)
σ_{bs} = the permissible bending stress in slab bases
= 165 MPa for flanged beams
= 185 MPa for solid beams

- The thickness of a square slab base plate under a solid round column.

$$t = 10\sqrt{\frac{90\,W}{16\sigma_{bs}} \times \frac{B}{(B - d_0)}}$$



W = the total axial load (kN)
B = the length of the side of cap or base (mm)
d_0 = the diameter of the reduced end (if any) of the column (mm).

- The cap or base plate should not be less 1.5 (d_0 + 75) mm in length or diameter.

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Beams

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A beam is designed to resist maximum bending moment and is checked fo shear stress and deflection, and also for web crippling and web buckling.

Design for Bending

- The maximum permissible compressive or tensile bending stres
σ_{bc} or σ_{bt} = 0.66 f_y
where f_y = yield stress of steel
The permissible bending stress (compressive or tensile) σ_{bc} or σ as per IS: 226-1975 is as follows:

Nominal plate thickness	Yield stress f_y (MPa)	$s_{bc} = s_{bt}$ (MPa)
Angle, tee, I, channel and flat sections	250	165
Up to and including 20 mm		
Over 20 mm up to and including 40 mm	240	158.4
Over 40 mm	230	151.8

Effective Length of Compression Flange

(i) Effective Length of Compression Flange :

End Connections	Effective length, *l*
(i) each end restrained against torsion.	
(a) ends of compression flange unrestrained for lateral bending	*l* = span
(b) ends of compression flange partially restrained for lateral bending	*l* = 0.85 × span
(c) ends of compression flange fully restrained for lateral bending	*l* = 0.7 × span

(ii) Cantilever beams of projecting length L,	
(a) Built-in at the support, free at end	*l* = 0.85L
(b) Built-in at the support, restrained against torsion at the end by continuous construction.	*l* = 0.75L
(c) Built - in at the support, restrained against lateral deflection and torsion at the free end	

by continuous cross members over several beams	$l = 0.5L$
(d) Continuous and unrestrained against torsion at the support and free at the end	$l = 3L$
(e) Continuous and partially restrained against torsion at the support and free at end	$l = 2L$
(f) Continuous at the support, restrained against torsion at the support and free at the end	$l = L$

NOTE The above values are increased by 20% if the ends of beam are not restrained against torsion.

If there is a degree of fixity at the end, the effective length should be multiplied by $\frac{0.5}{0.85}$ in (b) and (c) above and by $\frac{0.75}{0.85}$ in (d), (e) and (f) above.

(ii) Check for Shear

- Max permissible, shear stress

$$\boxed{\tau_{vm} = 0.45 f_y}$$

- For design purpose, the above condition is deemed to be satisfied if the average shear stress in an unstiffened member calculated on the cross section of web does not exceed the value

$$\boxed{\tau_{va} = 0.4 f_y}$$

(iii) Check for Deflection

- The maximum deflection $\ngtr \frac{l}{325}$ of the span in general.

(iv) Check for web crippling and web buckling:

Built-up Beams

(i) Symmetrical built-up beams

- Area of each cover plate

$$\boxed{A_p = \frac{Z - Z_1}{d}}$$

where, Z_1 = Section modulus of rolled I section available,

d = depth of beam

(ii) Unsymmetrical built-up beam

- The area of cover plates $\boxed{A_p = \frac{1.2 \times (Z - Z_1)}{d}}$

Gantry Girders

(i) The gantry girders are subjected to unsymmetrical bending due lateral thrust.

(ii) The deflection of gantry girders under dead and imposed loads shou not exceed the following values as per IS : 800-1984.

(a) Where cranes are manually operated	$\frac{L}{500}$
(b) Where electric overhead travelling crane are operated upto 50t	$\frac{L}{750}$
(c) Where electric overhead travelling cranes are operated, over 50t	$\frac{L}{1000}$
(d) Other moving loads such as charging cars etc.	$\frac{L}{600}$

Where, L = span of crane runway girder

Beam Column

- Members subjected to axial compression and bending are proportiona to satisfy the Eq.(1)

$$\boxed{\frac{\sigma_{ac,cal}}{\sigma_{ac}} + \frac{C_{mx} \times \sigma_{bcx,cal}}{\left[1 - \frac{\sigma_{ac,cal}}{0.6 f_{ccx}}\right]\sigma_{bcx}} + \frac{C_{my} \times \sigma_{bcy,cal}}{\left[1 - \frac{\sigma_{ac,cal}}{0.6 f_{ccy}}\right]} \le 1.0} \quad \ldots(i$$

However if the ratio $\frac{\sigma_{ac,cal}}{\sigma_{ac}}$ is less than 0.15, Eq (ii) may be used in lie of Eq. (i)

$$\boxed{\frac{\sigma_{ac,cal}}{\sigma_{ac}} + \frac{\sigma_{bcx,cal}}{b_{bcx}} + \frac{\sigma_{bcy,cal}}{\sigma_{bcy}} \le 1.0} \quad \ldots(ii$$

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5 Plate Girders

- Economic depth of the girder

$$D = 1.1\sqrt{\frac{M}{\sigma_{bt} \times t_w}}$$

- A self weight may be assumed to begin with the design

$$w = \frac{W}{300}\text{kN/m}$$

Design of Web

- Average shear stress in the web $\tau_{va,cal} = \frac{V}{d_w \times t_w} \ngtr$ permissible average shear stress, τ_{va}

Web stiffeners

- IS : 800-1984 recommends the provision of web stiffeners as follows:

  (i) $\frac{d_1}{t_w} \le$ lesser of $\frac{816}{\sqrt{\tau_{va,cal}}}$ and $\frac{1344}{\sqrt{f_y}}$ and 85. No stiffener is required.

  (ii) $\frac{d_2}{t_w} \le$ lesser of $\frac{3200}{\sqrt{f_y}}$ and 200. Vertical stiffeners are provided.

  (iii) $\frac{d_2}{t_w} \le$ lesser of $\frac{4000}{\sqrt{f_y}}$ and 250.

  Vertical stiffeners and one horizontal stiffener at a distance from the compression flange equal to two-fifths of the distance from the compression flange to the neutral axis are provided.

  (iv) $\frac{d_2}{t_w} \le$ lesser of $\frac{6400}{\sqrt{f_y}}$ or 400.

  where $d_2 = 2 \times$ clear distance from compression flange angles or plate or tongue plate to the neutral axis.

- In no case should the greater clear dimension of a web panel should exceed $270t_w$ nor the lesser clear dimension of the same panel should exceed $180t_w$.

- The term $\left(A_f + \frac{A_w}{6}\right)$ is called the effective flange area.

Permissible Bending Stress

- The maximum compressive stress $\sigma_{bc,cal}$ is calculated on gross flange area, i.e.,

  $\sigma_{bc,cal} = \frac{M \times D/2}{I_{gross}} \ngtr$ permissible bending stress in compression, σ_{bc}.

- The maximum tensile stress $\sigma_{bt,cal}$ is calculated on the net flange area i.e.,

  $\sigma_{bt,cal} = \frac{M \times D/2}{I_{gross}} \times \frac{\text{gross flange area}}{\text{net flange area}} \ngtr$ permissible bending stress in tension, σ_{bt}

Curtailment of Flange Plates

- Length of the plate to be curtailed

$$l_n = l\sqrt{\frac{A_1 + A_2 + A_3 + \ldots + A_n}{A_f + A_{we}}}$$

Where, l = span
n = no. of plates to be curtailed counting 1, 2, 3,... from outer plate.
A_{we} = effective web area

Web Stiffeners

- Unless the outer edge of each stiffener is continuously stiffened, the outstand of all stiffeners from the web should not be more than $\frac{256 \times t}{\sqrt{f_y}}$ (=16t for steel sections and 12t for flats where t is the thickness of the section or flat.
- Where vertical stiffeners are required, they should be provided throughout the length of the girder at a distance not greater than $1.5d_1$ and not less than $0.33\ d_1$.
- When horizontal stiffeners area provided d_1 should be taken as the clear distance between the horizontal stiffener and tension flange (farthest flange) ignoring fillets.
- The moment of inertia I of a pair of vertical stiffener about the centre of web or a single stiffener about the face of the web should be,

$$I \ge \frac{1.5 \times d_1^3 \times t^3}{c^2}$$

where, t = the min. required thickness of web.
c = the max. permitted clear distance between vertical stiffener for thickness t.

- Sometimes vertical stiffeners are subjected to external forces and therefore the moment of inertia of the stiffener should be increased as described below.
  (a) Bending moment on stiffener due to eccentricity of vertical loading with respect to vertical axis of the web.

  Increase of $I = \frac{150M \times D^2}{E \times t_w} cm^4$

  (b) Lateral loading on stiffener:

  Increase of $I = \frac{0.3V \times D^3}{E \times t_w} cm^4$

- For first horizontal stiffener at 2/5th of the distance between compression flange and neutral axis, from the compression flange:

$$I \geq 4C \times t^3$$

Where I = moment of inertia of a pair of horizontal stiffeners about the centre of the web or a single stiffener about the face of the web

t = the minimum thickness of web required

c = actual distance between vertical stiffeners

- For second horizontal stiffener at the neutral axis,

$$I \geq d_2 \times t^3$$

Stiffeners are connected to web to withstand a shearing force not less than $\frac{125 \times t_w^2}{h}$ kN/m, where h = outstand of stiffener in mm.

Load Bearing Stiffeners

- Bearing stiffeners are provided at the points of concentrated loads and at supports.
- Where these stiffeners are to provide restraint against torsion of the plate girder at the ends,

$$I \not< \frac{D^3 \times T}{250} \times \frac{R}{W}$$

■■■

6 Industrial Roofs

- If loads from purlins, false ceiling etc. are applied in between the nodes, then principal rafters or main ties are designed for combined stresses from bending and axial load.

Purlins and Grits

- Angle, channel, I and Z sections are used for purlins and grits to support the cladding.
- IS : 800 - 1984 provides general design procedure for angle purlins conforming to steel grades Fe 410-O, Fe 410-S, Fe 410 -W and roof slopes not exceeding 30° based on a minimum live load of 750 N/m² if the following requirements are fulfilled:
  (i) Width of angle leg in the plane perpendicular to the roof covering $\geq \frac{L}{45}$

  (ii) Width of angle leg in the plane parallel to the roof covering $\geq \frac{L}{60}$

  (iii) Maximum bending moment in the purlin,

$$M = \frac{w \times L^2}{10}$$

  Where w = uniformly distributed load per unit length on purlin including wind load

  L = span of purlin

  (iv) The bending moment about minor axis may be neglected and the angle purlin may be designed for the above moment.

$$\therefore \quad Z_x \text{required} = \frac{M}{\sigma_{bc}} = \frac{w \times L^2}{10 \times 165} mm^3$$

■■■

7

Plastic Analysis

Load Factor

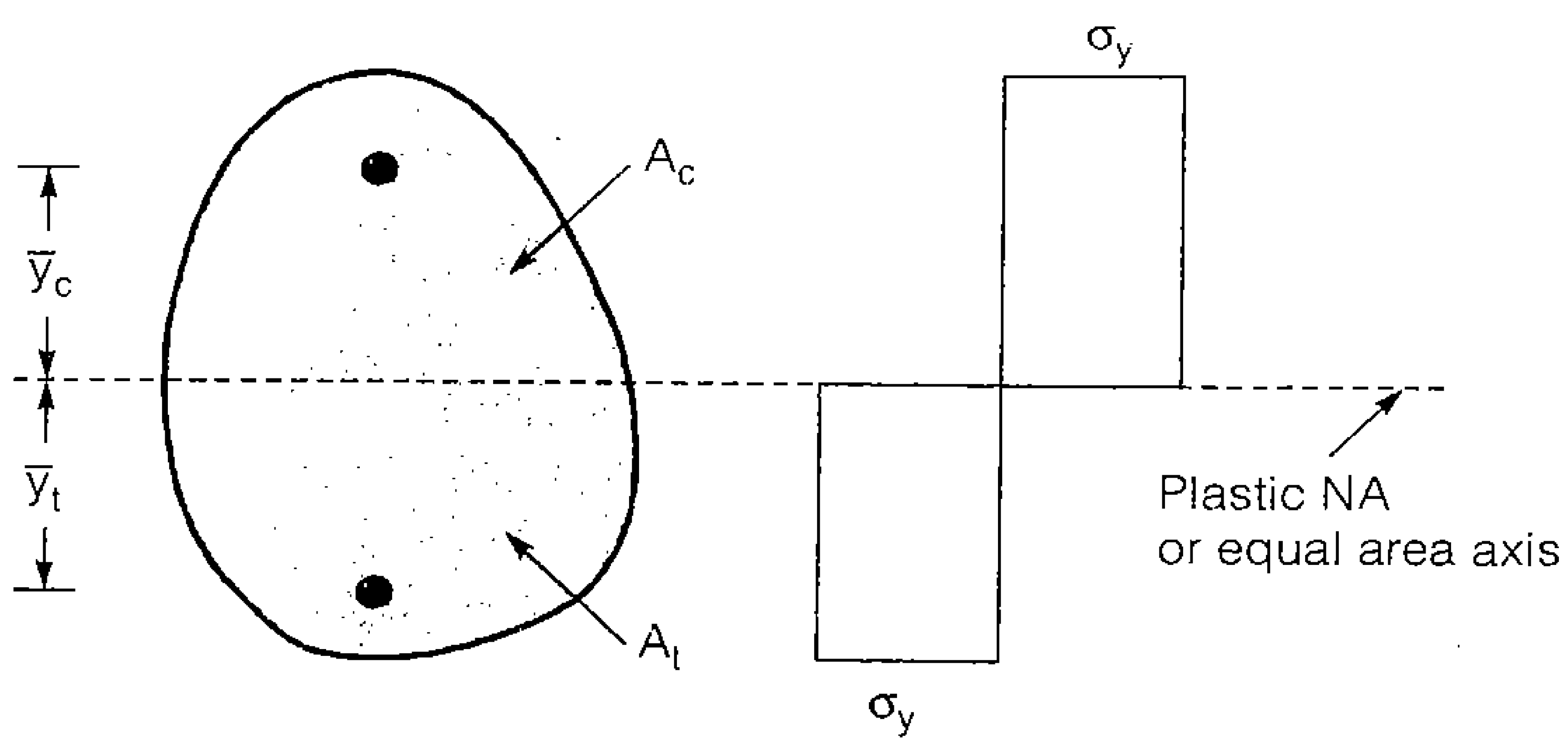
- The load factor (λ)

$$\lambda = \frac{\text{Collapse load}}{\text{Service load}} = \frac{P_c}{P}$$

Saving in material

$$\%\text{ saving in material} = \left[1 - \frac{\text{Area required by plastic theory}}{\text{Area required by elastic theory}}\right] \times 100$$

Plastic Sections Modulus



- Plastic Section Modulus

$$Z_p = A_c\bar{y}_c + A_t\bar{y}_t$$

> **Remember**
> Total compressive force = Total tensile force
> Note that in plastic analysis the Neutral axis divides the cross-section into two equal halves whereas in elastic analysis NA. Passes through C.G.
> i.e., $A_c = A_t = A/2$, where A is total area

Shape Factor (α)

$$\alpha = \frac{M_p}{M_y} = \frac{Z_p}{Z_y}$$ Where Z_y is elastic section modulus

$$\text{Load factors } (\lambda) = \frac{P_c}{P}$$

$$\text{Load Factor} = \text{Factor of safety} \times \text{Shape factor} \qquad \lambda = FS \times \alpha$$

Shape factors for different shapes

Section	Shape Factor (α)
1. Rectangular section	1.5
2. (a) Triangular section (vertex upward)	2.34
(b) Triangular section (vertex horizontal)	2.00
3. Solid circular section	1.7
4. Hollow circular section (K = Ratio of inner diameter to outer diameter)	$1.7 \times \frac{(1-K^3)}{(1-K^4)}$
5. Thin circular ring solid	1.27
6. (a) Diamond section (Rhombus)	2.00
(b) Thin hollow rhombus	1.50
7. I-section	
(a) About strong axis	≈ 1.12
(b) About weak axis	≈ 1.55
8. T-section	≈ 1.90 to 1.95

Reserve Strength (Ψ)

- It is the ratio of the ultimate load W_u to the load at first yield W_y of the structure.

$$\psi = \frac{W_u}{W_y}$$

Length of Plastic hinge (L_p)

- It is the length of the beam over which the moment is greater than the yield moment (M_y).

(a) For simply supported beam carrying concentrated load, length of plastic hinge is given by

$$L_P = \frac{L}{3} \text{ (for rectangular section)} \qquad L_P = \frac{L}{8} \text{ (for I section)}$$

$$L_P = L\left[1-\frac{1}{\alpha}\right]$$

(b) For simply support beam carrying UDL length of plastic hinge is given by

$$L_P = L\sqrt{1-\frac{1}{\alpha}}$$

Where α is shape factor and L is total length of the beam.

- The length of plastic hinge depends on loading and geometry.

Collapse Loads

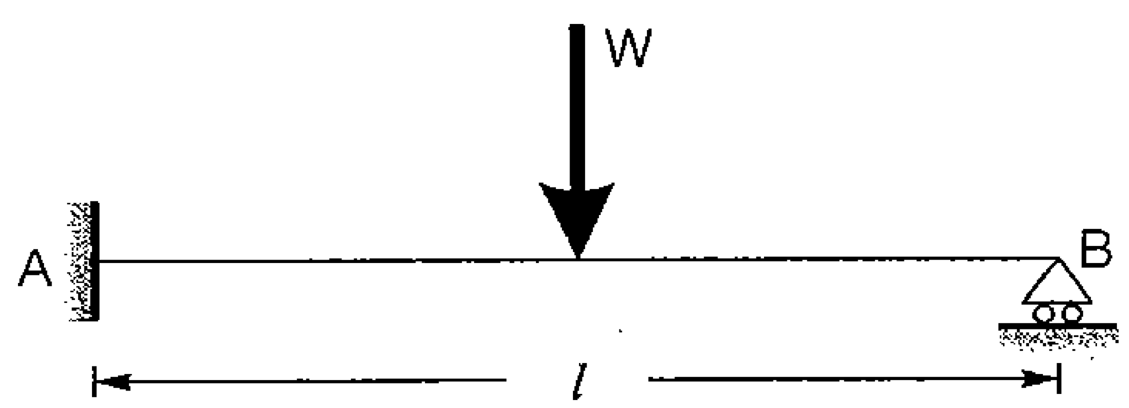
1. Simply supported beam with concentrated load at the center

$$w_c = \frac{4M_p}{l}$$

2. Simply supported beam with uniformly distributed load

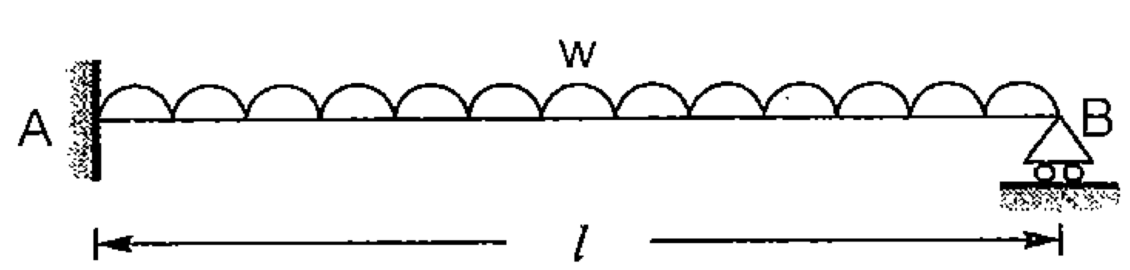
$$w_c = \frac{16\,M_p}{l}$$

3. Propped cantilever with concentrated load at the center



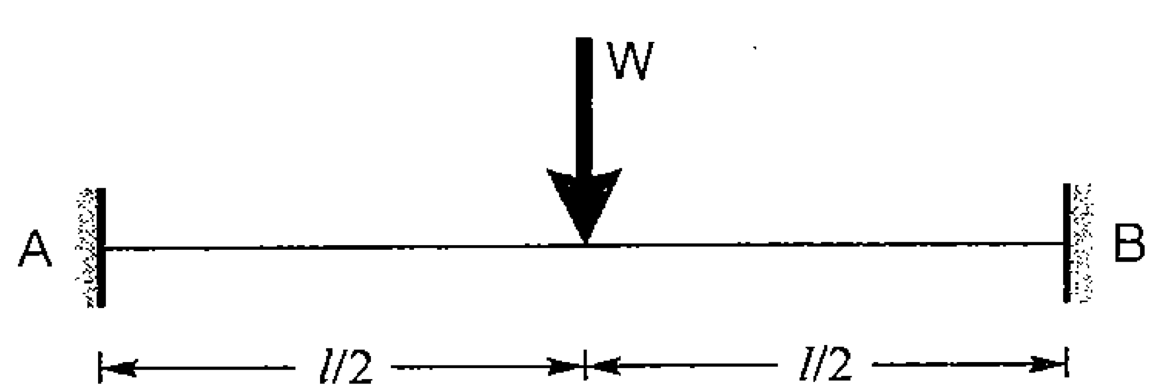
$$W_c = \frac{6\,M_p}{l}$$

4. Propped cantilever with uniformly distributed load



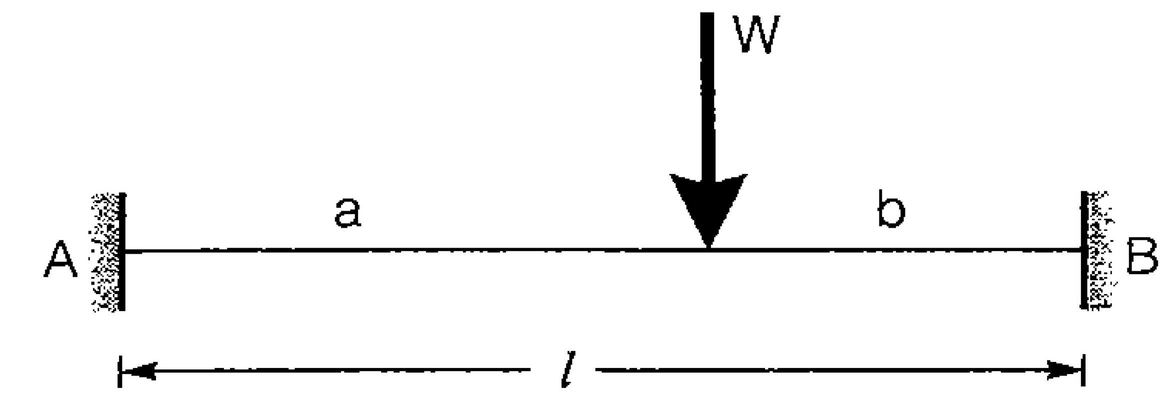
$$w_c = \frac{11.656\,M_p}{l}$$

5. Fixed beam with concentrated load at the center



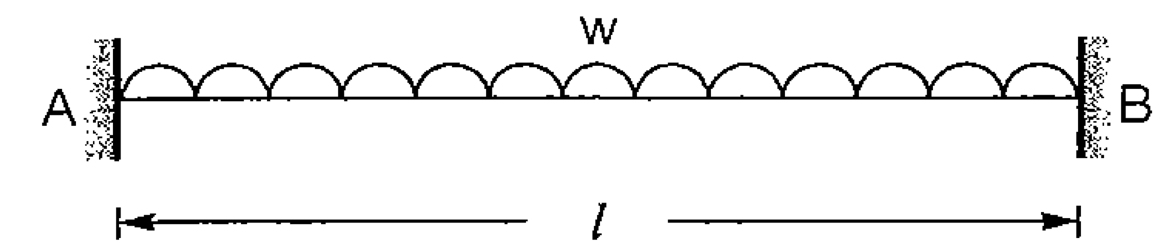
$$W_c = \frac{8\,M_p}{l}$$

6. Fixed beam with eccentric loading



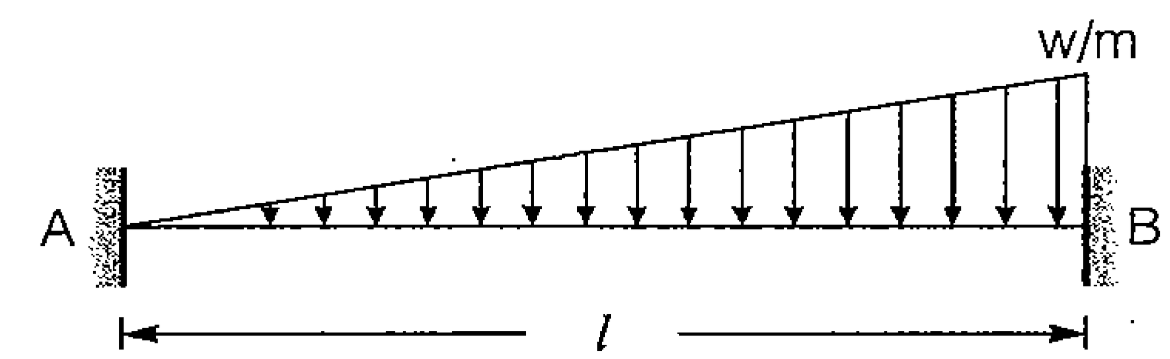
$$W_c = \frac{2l}{ab}M_p$$

7. Fixed beam with uniformly distributed load



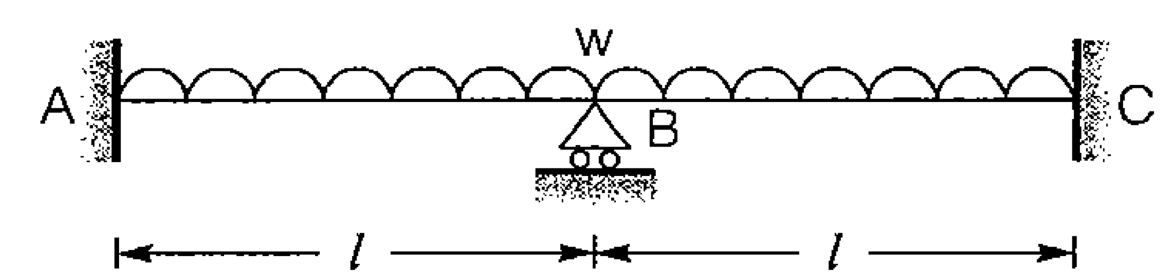
$$W_c = \frac{16M_p}{l^2}$$

8. Fixed beam with hydrostatic loading



$$w_c = \frac{18\sqrt{3}\,M_p}{l^2}$$

9. Continuous beam with uniformly distributed load



$$W_c = \frac{11.656\,M_p}{l^2}$$

The positions of the plastic hinges are one at the support B and one on each side of the central support at a distance of 0.414 l from A & C.

Remember

- **Upper bound theorem**
  It satisfies equilibrium and mechanism condition. $P \geq P_u$
- **Lower bound theorem**
  It satisfies equilibrium and yield condition. $P \leq P_u$

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A Handbook on

Civil Engineering

5

CPM & PERT

CONTENTS

PERT

1

PERT stands for programme evaluation and review technique.

1. Three time estimates are made.
2. It is based on probabilistic approach.
3. Time is directly proportional to time hence minimum cost will be achieved corresponding to minimum completion time.
4. Suitable for new type of projects.
5. Each activity follows β distribution.

(i) **Expected completion time of an Activity : (t_E)**

$$t_E = \frac{t_0 + 4t_m + t_p}{6}$$

Where, t_0 = Optimistic time
t_p = Pessimistic time
t_m = Most likely time

(ii) **Standard deviation of an Activity (σ)** $\sigma = \frac{t_p - t_0}{6}$

(iii) **Variance of an activity : (σ^2)** $\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$

(iv) **Central limit theorem :**

(a) The mean time of the project as a whole is

$t_E = t_{E_1} + t_{E_2} + \ldots\ldots$ along the critical path.

Probability of completion of project in time t_E is 50%.

(b) The standard deviation of the project as a whole is

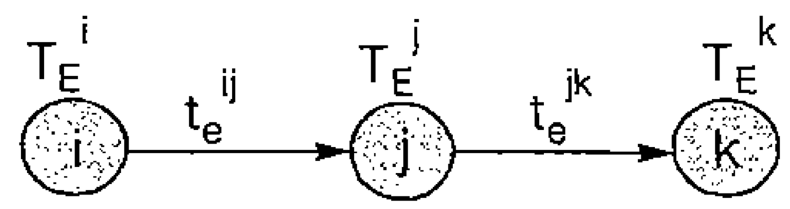
$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots\ldots}$ along the critical path.

- **Critical Path:** The time wise longest path is called critical path. In this path any type of delay in any event will cause delay to the project. These are shown by double line or dark lines in a network.
  An event is critical if its slack is zero.

Event Time

(i) **Earliest expected event occurring time (T_E)**

$T_E^j = T_E^i + t_e^{ij}$ when there is only one path.



Where, t_e^{ij} = Expected completion time of an activity i – j

$$T_E^j = (T_E^i + t_e^{ij})_{max}$$ .... when there are more than one path.

Where T_E^i = Earliest expected time of event i.

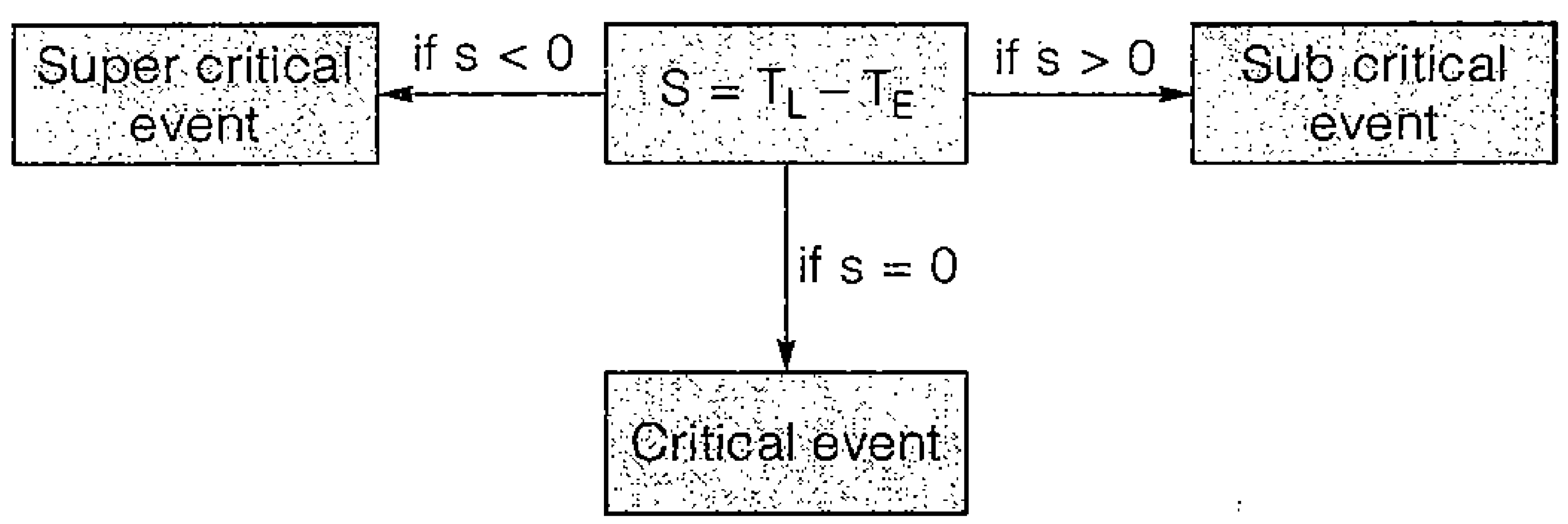
T_E^j = Earliest expected time of event j.

(ii) Latest allowable occurrence time (T_L):

$$T_L^i = T_L^j - t_e^{ij}$$ when there is only one path.

$$T_L^i = (T_L^j - t_e^{ij})_{min}$$ when there are more than one path.

(iii) Slack (s): This is the time by which an event may be delayed without affecting the completion time of the project.



Remember If there are more than 1 critical paths then variance and SD of the project along each critical path may be different. Under such circumstances that path requires more attention along which SD or variance is greater. Hence the variance or SD of the project should be taken the largest value among all the paths.

- Probability Factor (z)

$$z = \frac{T_s - T_E}{\sigma}$$

Where, T_S = Given scheduled completion time of the project

T_E = Expected completion time of the project.

σ = Standard deviation

z	P
0	50%
+1	84.13%
+2	97.72%
+3	99.87%

z	P
0	50%
–1	15.87%
–2	2.28%
–3	0.13%

■■■

CPM

2

Definitions

CPM stands for critical path method. This is based on deterministic approach in which only one time estimate is made for activity completion.

1. A network diagram in CPM is activitiy oriented.
2. Cost is the most important criteria. Minimum cost is found corresponding to optimum time.
3. There is only single time estimate for each acitvity.
4. The probability of completion of activity in this estimated duration is 100%.
5. It is based on deterministic approach.
6. Suitable for repetitive type of work.
7. Normal distribution is followed.

Activity times

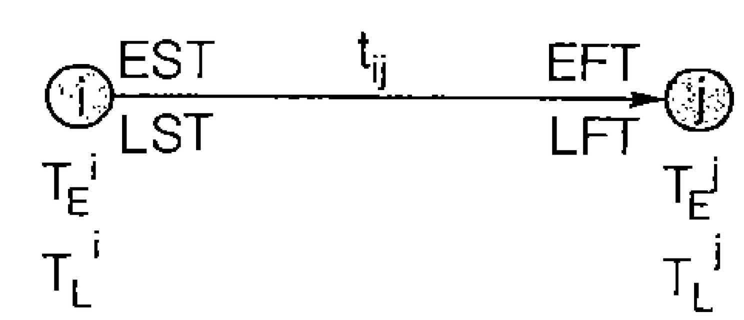
(i) **Earliest start time**

$$EST = T_E^i$$

(ii) **Earliest finish time**

EFT = EST + Activity time

$$EFT = T_E^i + t_{ij}$$



(iii) **Latest finish time**

LFT = T_L of head event

$$LFT = T_L^j$$

(iv) **Latest start time**

LST = LFT – t_{ij}

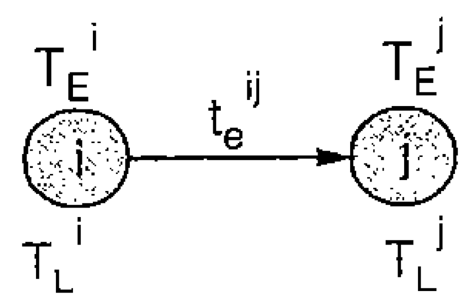
$$LST = T_L^j - t_{ij}$$

Float

Float denotes the range within which activity time or its finish time may fluctuate without effecting the completion of the project.

(i) **Total Float (F_T) :**

$$F_T = LST - EST \quad \text{or} \quad F_T = LFT - EFT$$



$$\boxed{F_T = T_L^j - T_E^i - t_e^{ij}}$$

(ii) **Free Float (F_F) :**

$$\boxed{F_T = T_E^j - T_E^i - t_e^{ij}} \text{ or } \boxed{F_F = F_T - S_j}$$

where S_j = Head event slack

(iii) **Independent Float (F_{ID}) :**

$$\boxed{F_{ID} = T_E^j - T_L^i - t_e^{ij}} \quad \boxed{F_{ID} = F_F - S_i} \quad \boxed{F_{ID} = F_T - S_i - S_i}$$

where S_i = Tail event slack

$\boxed{F_T = 0}$ - for Critical path $\boxed{F_T > 0}$ - for Subcritical path

$\boxed{F_T < 0}$ - for Supercritical path

(iv) **Interfering float (F_{IN})**
It is the another name of head event slack.

$$\boxed{F_{IN} = S_j = F_T - F_F}$$

Remember

Trick to memorise float formula:

	j	i	ij	
F_T	L	E	t_{ij}	$\Rightarrow$ $F_T = T_L^j - T_E^i - t_{ij}$
F_F	E	E	t_{ij}	$\Rightarrow$ $F_F = T_E^j - T_E^i - t_{ij}$
F_{ID}	E	L	t_{ij}	$\Rightarrow$ $F_{ID} = T_E^j - T_L^i - t_{ij}$

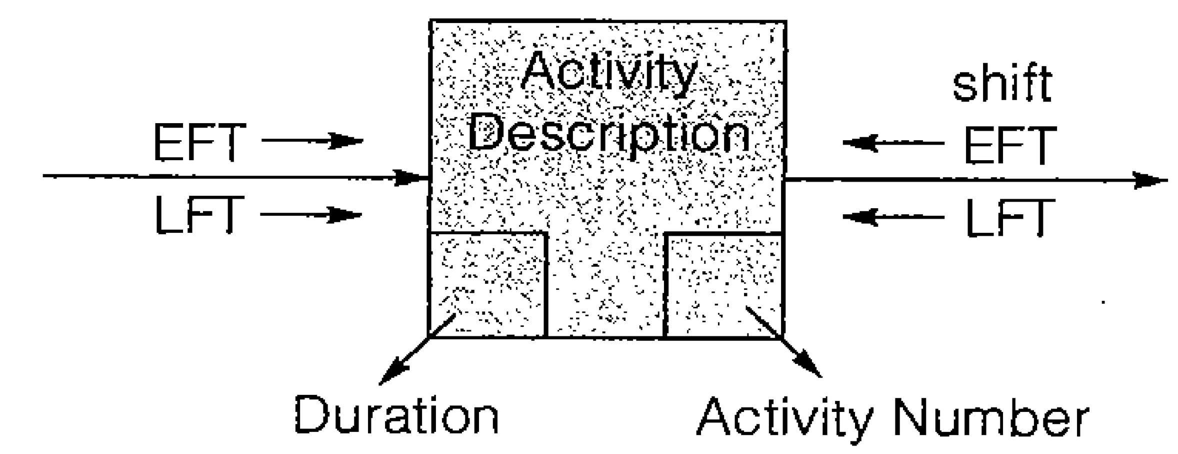
CPM systems

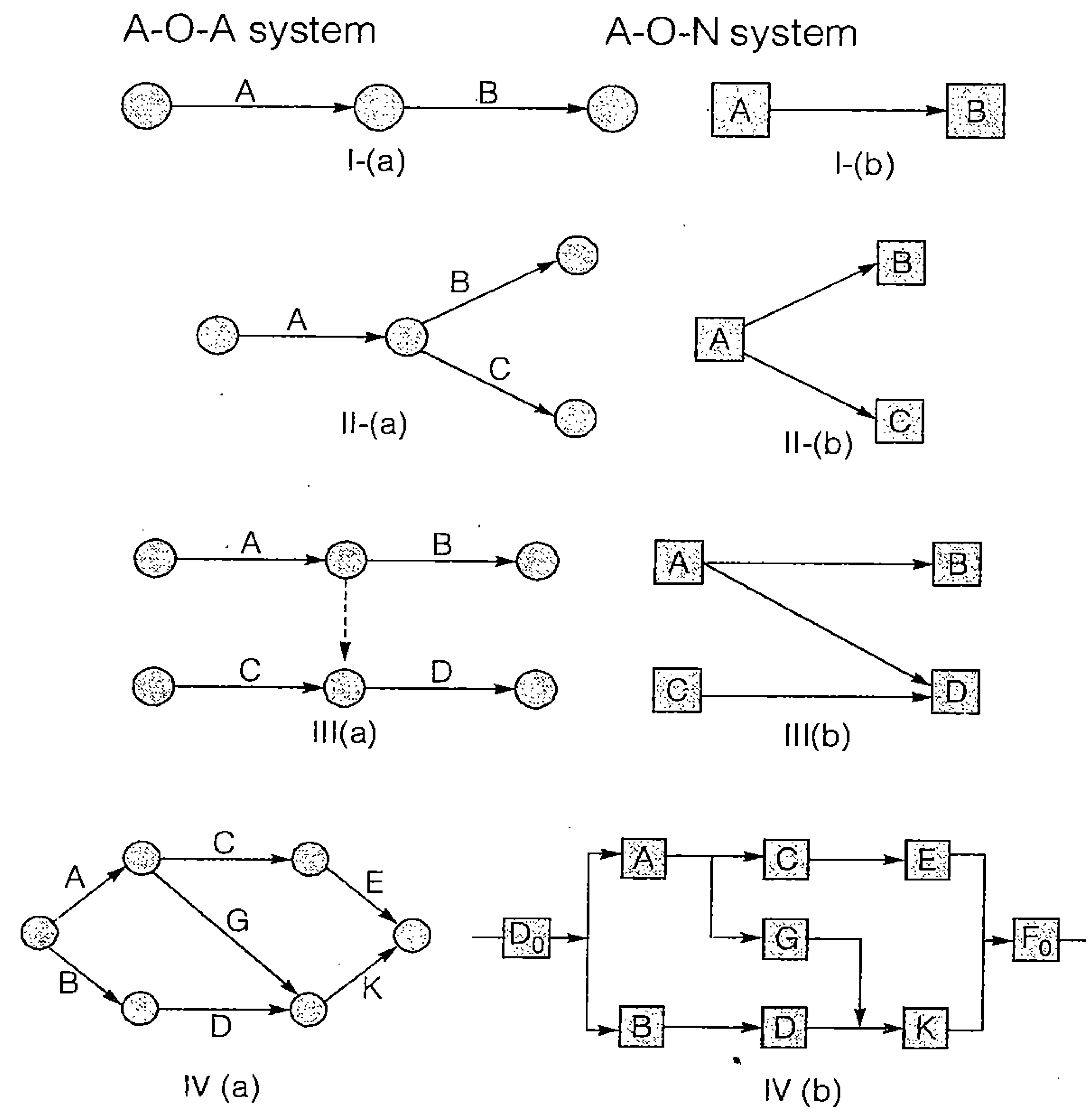
Mainly two systems are used in CPM analysis:

1. **A-O-A system** (Activity on arrow system)
   An activity is graphically represented by an arrow.
   The tail end and head end of arrow represent start and finish of an activity respectively.
2. **A-O-N System** (Activity on node system or precedence diagram).
   Activity is represented by circle or node. Events have no places.
   Arrows are used only to show the dependency relationship between activity nodes.

When two or more activities start parallel then an activity called DEBUT (D_O) is provided at the beginning. Like wise a finish activity (F_O) is provided at the end when more than one activities finish parallel. Activity D & F have zero duration.

Remember





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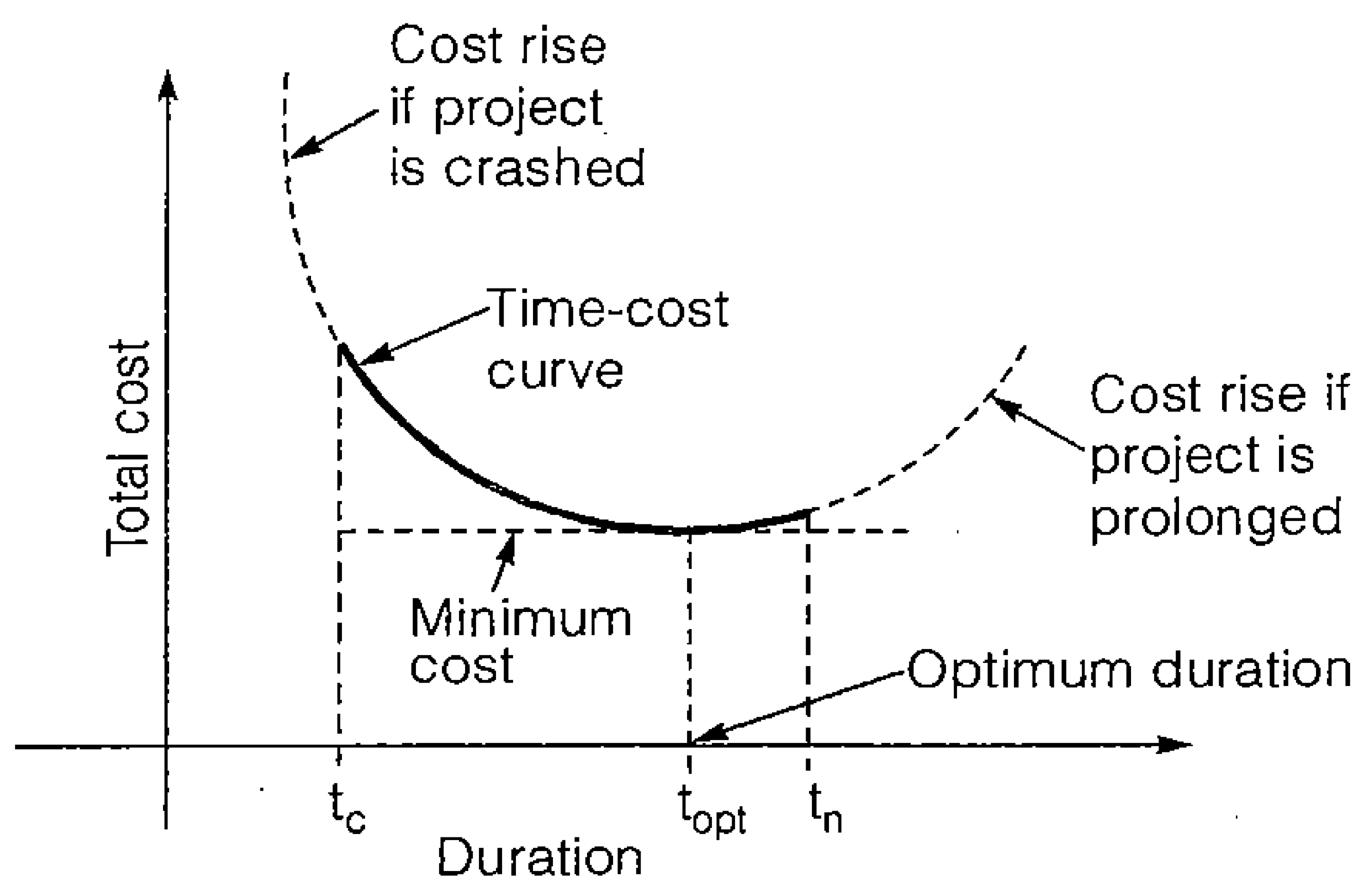
3 Crashing of Networks

Cost Model Analysis

In whole of CPM Cost Model, we will be assuming that project duration is reduced by deploying more resources on critical activities.

PROJECT COST

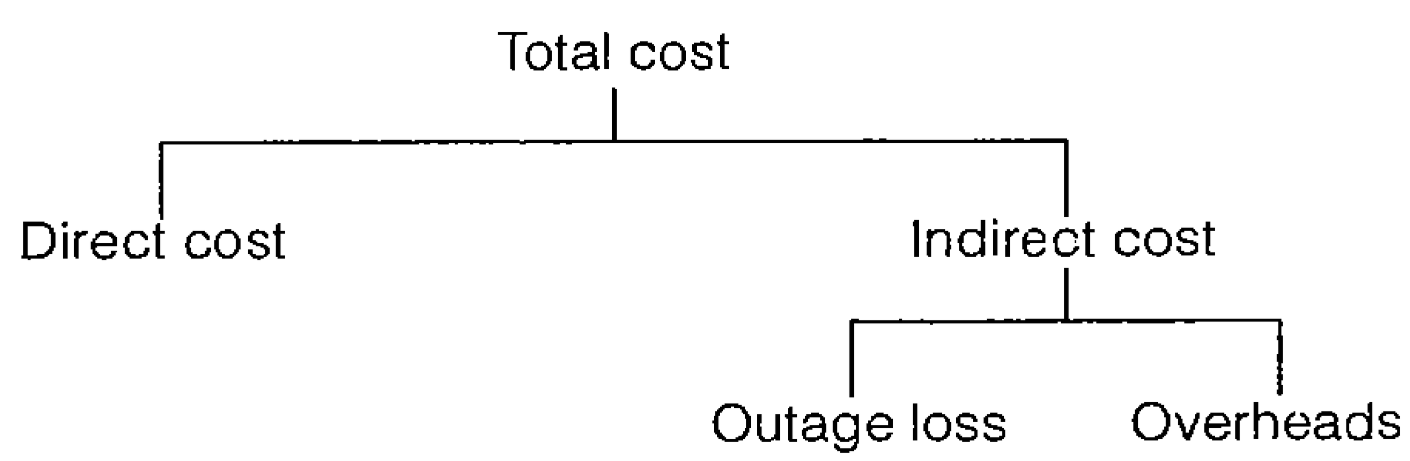


Total Project cost is the sum of two separate cost:

(a) the direct cost for accomplishing the work, and

(b) the indirect cost related to the control or direction of that work, financial overhead, lost production, and the hike.

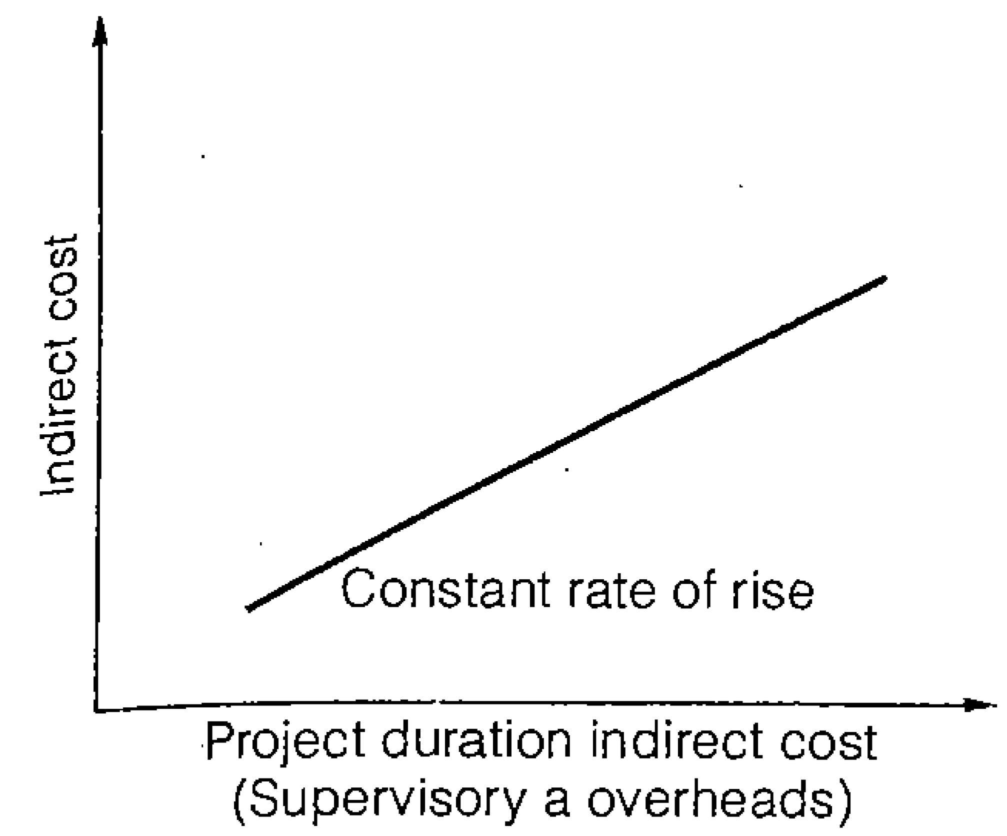
The components of the total cost are depicted in Fig.

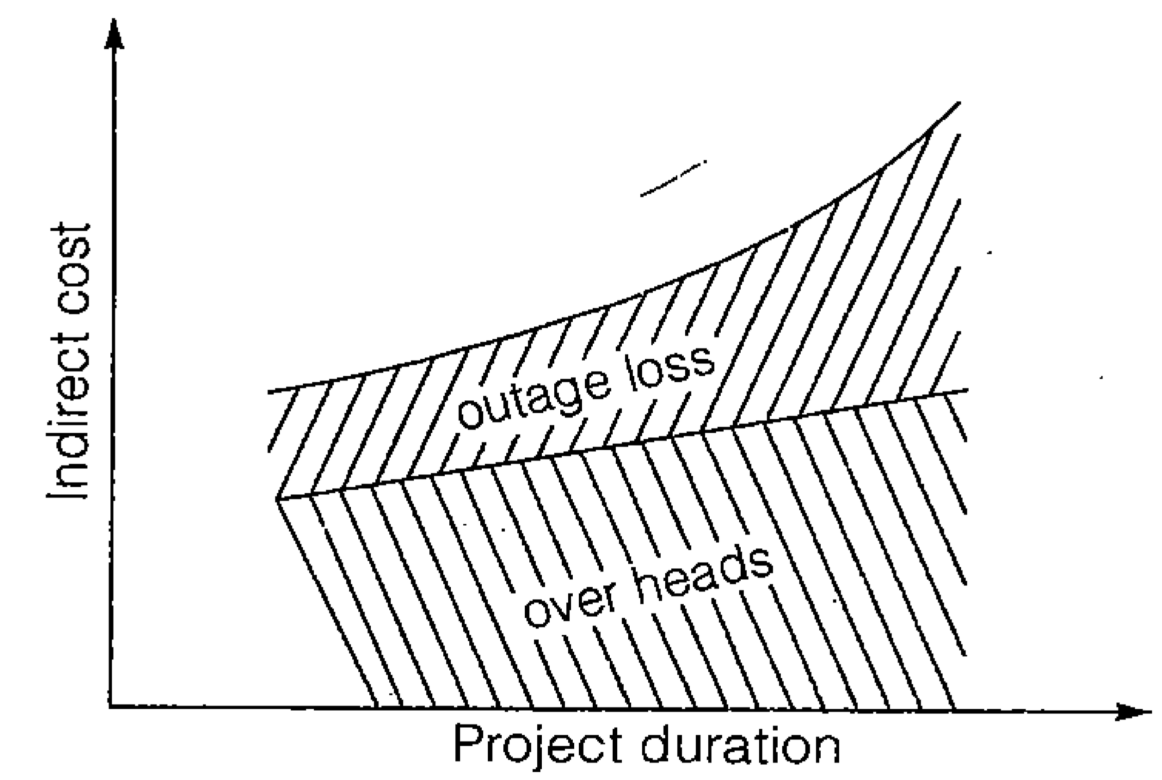


Components of project cost

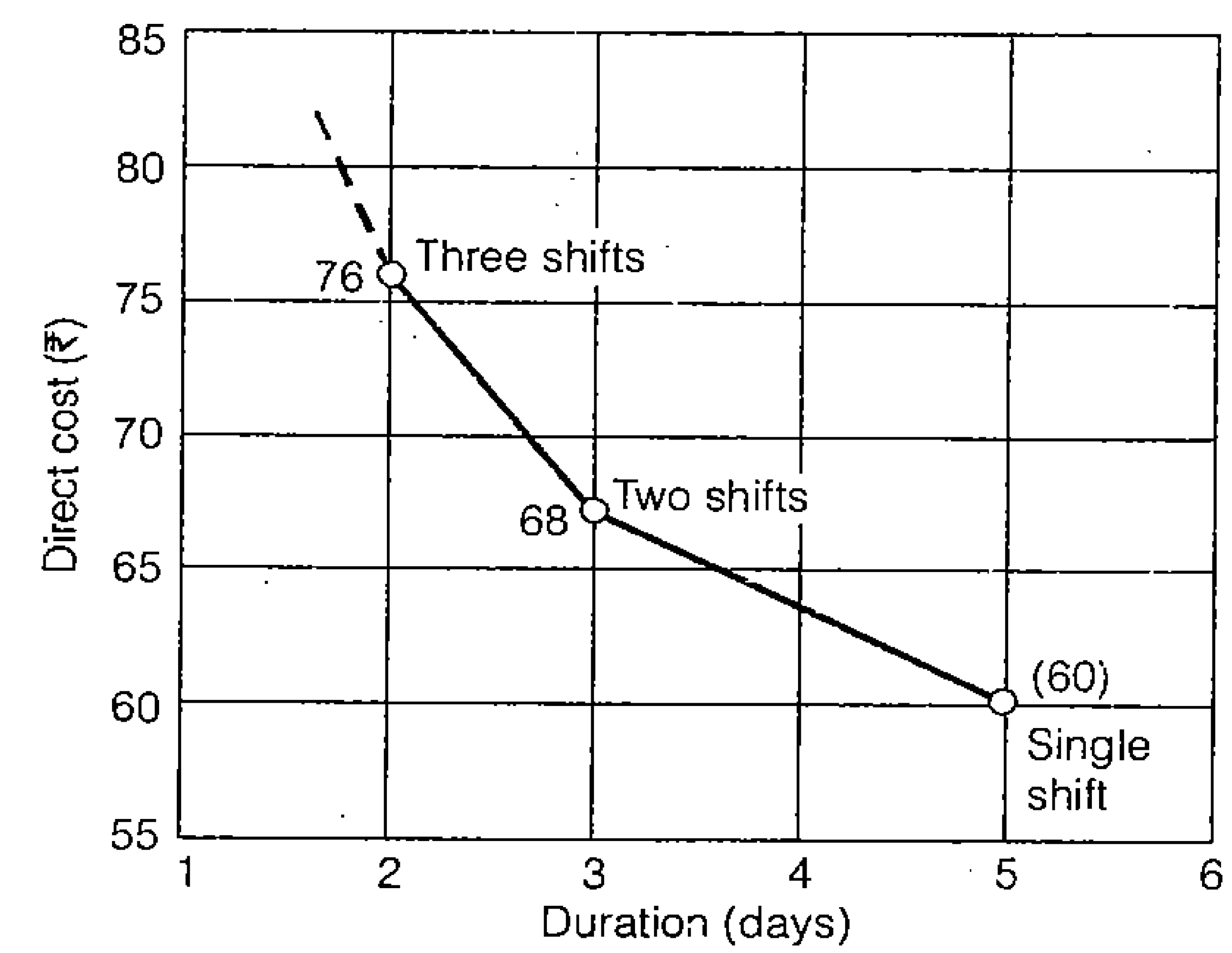
- **Indirect Project Cost:** Indirect costs on a project are those expenditures which cannot be apportioned or clearly allocated to the individual activities of a project, but are assessed as a whole.

  Indirect cost rises with increased duration.
  The total indirect cost curve will thus be curved.

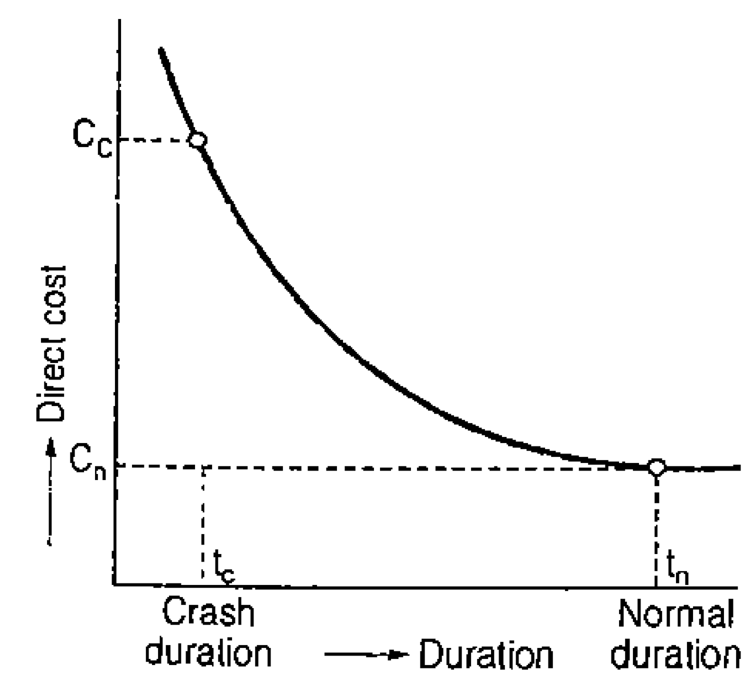




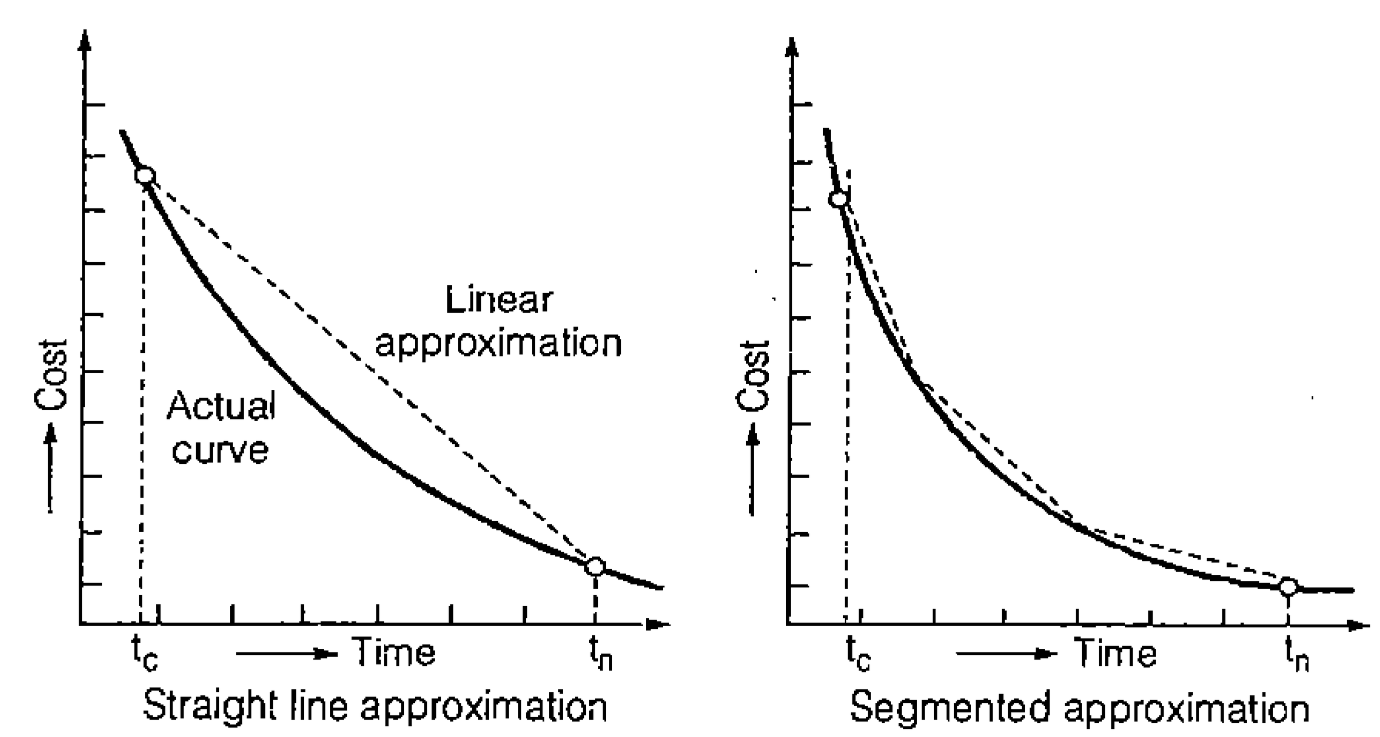
- **Direct Project Cost:** These include labour cost, material cost, equipment cost etc.
- **Normal time (t_n) :** Normal time is the standard time that an estimator would usually allow for an activity.
- **Crash time (t_c) :** Crash time is the minimum possible time in which an activity can be completed, by employing extra resources. Crash time is that time, beyond which the activity cannot be shortened by any amount of increase in resources.



- **Normal cost (C_n) :** This is direct cost required to complete the activity in normal time duration.
- **Crash cost (C_c) :** This is the direct cost corresponding to the completion of the activity within crash time.



Generalized direct cost-time curve



Direct cost curve approximation

- The straight line or segmented approximation of the direct cost curve is helpful in carrying out the project cost analysis. In such analysis, the cost slope is used.
- **Cost Slope:** The cost slope is the slope of the direct cost curve, approximated as straight line. It is defined as follows:

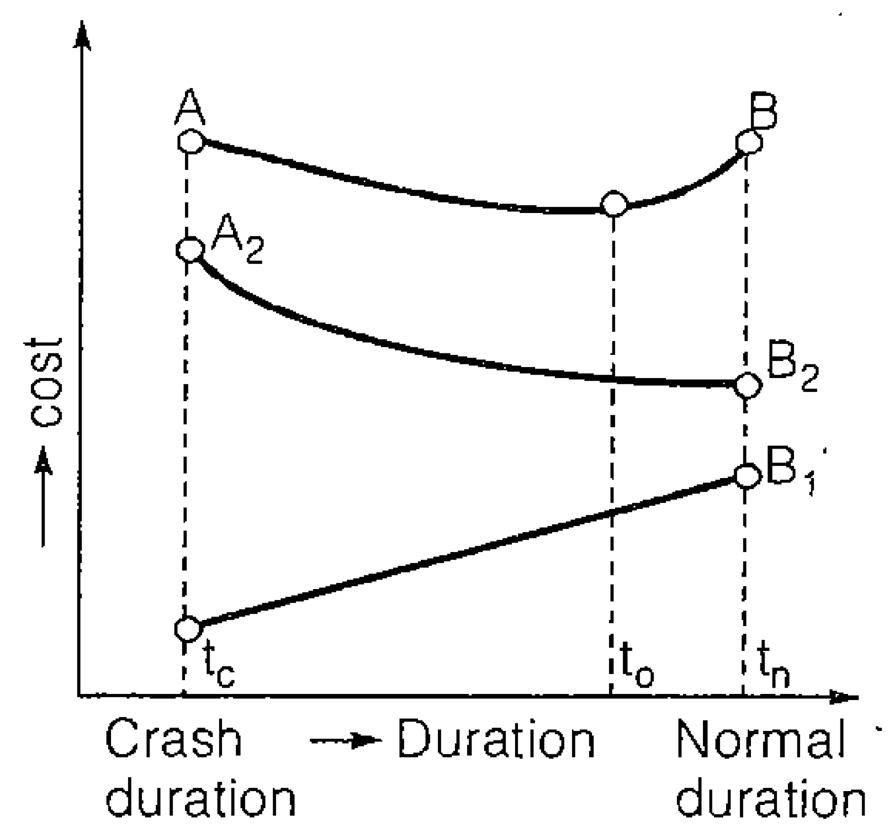
$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

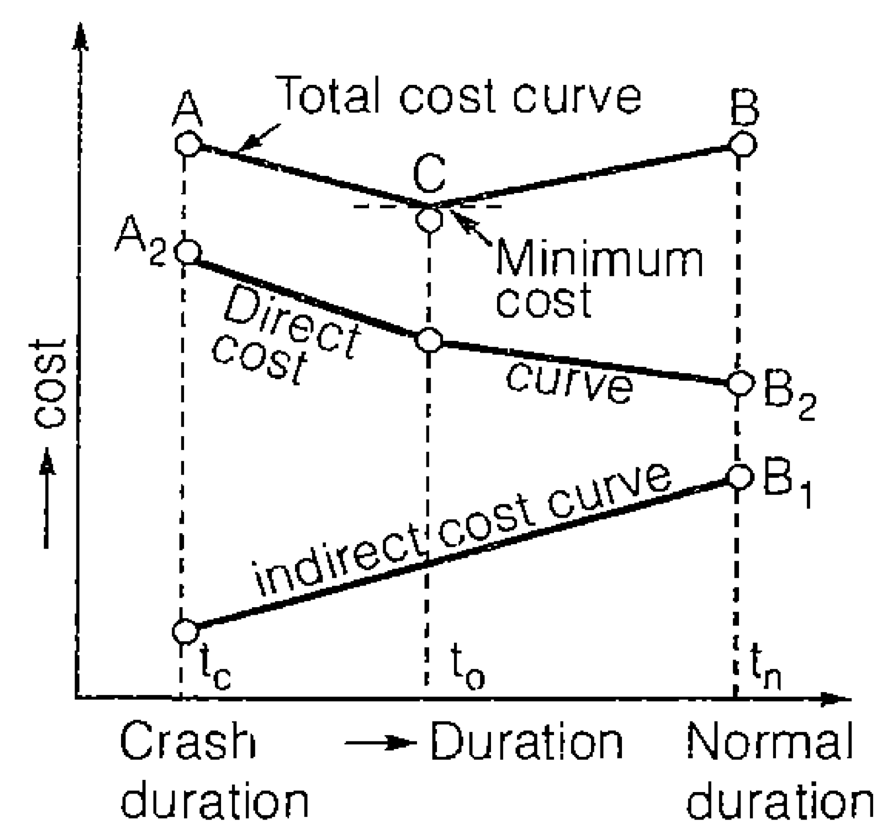
or

$$CS = \frac{C_c - C_n}{t_n - t_c} = \frac{\Delta c}{\Delta t}$$

- **Total Project Cost and Optimum Duration:** The total project cost is the sum of the direct cost and indirect costs.

  We find that the minimum total cost is obtained at some duration known as the optimum duration. The corresponding cost is known as the minimum cost. If the project duration is increased, total cost will increase, while if project duration is decreased to the crash value, project cost will be the highest.





Remember

Steps in crashing:

1. Estimate project duration and find the critical path.
2. Find cost slope of all activities.
3. The critical activity having minimum cost slope is crashed in 1st stage. The next stage crashing will involve activity having second lower cost slope in critical path.
4. Total cost of project at this stage is calculated.
5. Step 3 and 4 are repeated till all activities of project are crashed along critical paths, corresponding time is crash time of project.
6. It is to be noted that only critical activities should be crashed.

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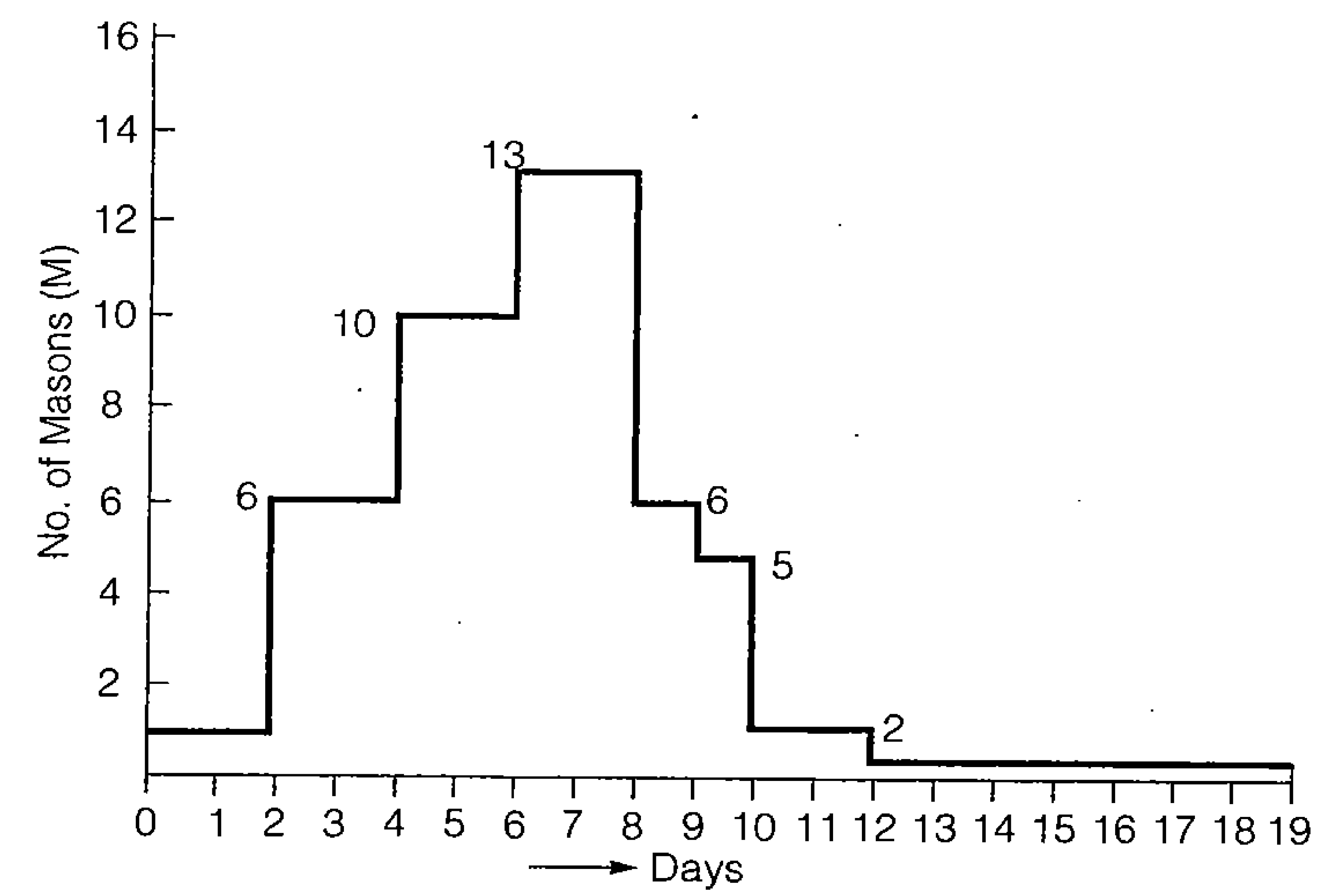
4 Resource Allocation & CPM Updating

- 'Resource' is a physical variable required for completion of activities. It can be material, manpower, machinery, space, money or time resources.
- "Resource allocation" is deciding the resources to each activity.
- The diagram which shows variation in the requirement of resources with time is called *"Resource usage profile" or "Histogram".*



These are helpful to know requirement of resources at different time in different activities.

Method to Achieve Resource Allocation

1. **Resource Smoothening**
   In resource smoothening the total project duration is not changed but the activites having floats are rescheduled so that a uniform demand for the resources is achieved. Thus in this case there is constraint on project duration. There is however no constraint on the resources.
2. **Resource Levelling**
   In resource levelling, the activities are so rescheduled that the maximum or peak resources requirement does not cross the limit of available resources. In resheduling the available floats are used first. If by doing

so, the resources demand is more than the available resources, the duration of some of the activities is increased so that the resources requirements for these activities is decreased. Thus in this process, the project duration initially planned might be increased.

> NOTE: Resource smoothening and resource levelling both are trial and error methods.

CPM Updating

- The process of reviewing the progress of project execution and redrafting the network according to latest requirements is called "updating".
- During redrafting scheduled dates are revised. New critical path may appear hence project priorities may change.
- Crashing of new critical activities may be required to make project on schedule.
- Updating is necessary to compensate for deviations in actual execution of works and original plans.
- During the process of the updating neither activities are deleted nor new activities added.

> Remember
>
> **When to update:**
> 1. Updating should be more frequent for shorter duration projects.
> 2. For larger duration projects frequency should be increased as project is nearing completion.
> 3. Whenever major change in the duration of any of activity occurs then updating should be done.
> 4. If change in estimated duration occurs, updating is essential.

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5 Engineering Economy

INTEREST AND INTEREST FORMULAE

- **Interest**

  The term interest is used to designate a rental for the use of money.

- **Simple Interest**

  When the total interest earned or charged is directly proportional to the principal involved, the interest rate and the no. of interest periods for which the principal is committed, the interest is called simple

$$I = (P)(n)(i)$$

where I = total interest
P = principal amount rent or borrowed
n = number of years (interest periods)
i = interest rate per year (per interest period)

- **Compound Interest**

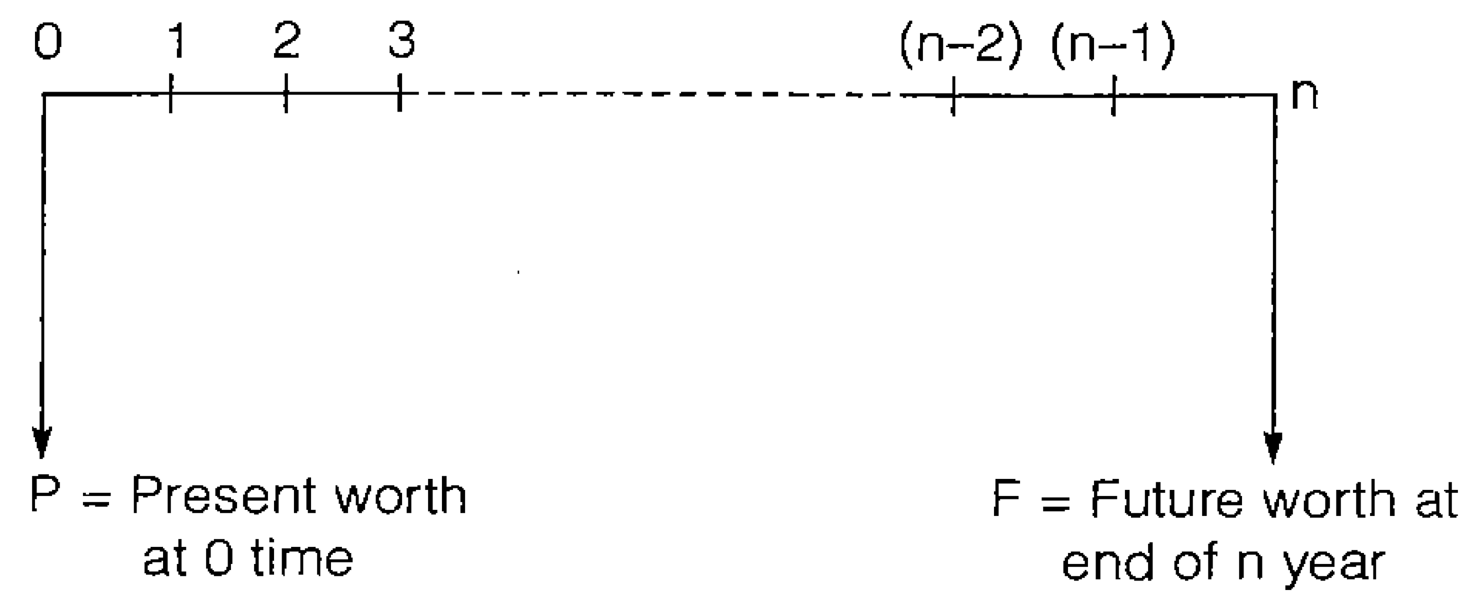
  Whenever the interest charge for any interest period (a year, for example) is based on the remaining principal amount plus any accumulated interest charges upto the beginning of that period, the interest is said to be compound.

- **Notations and Cash Flow**

  i = the annual interest rate
  n = the number of annual interest periods
  P = Present sum of money i.e. Present worth at zero time
  F = future sum of money i.e. Future worth equal to the compound amount at the end of n years
  A = a single payment, in a series of n equal payments, made at the end of each annual interest period
  G = uniform period-by-period increase or decrease in amount (the arithmetic gradient)

Cash Flow Diagram

The graphic presentation of each value plotted at appropriate time is called a cash flow diagram. The normal conventions for cash flow diagrams are as follows;

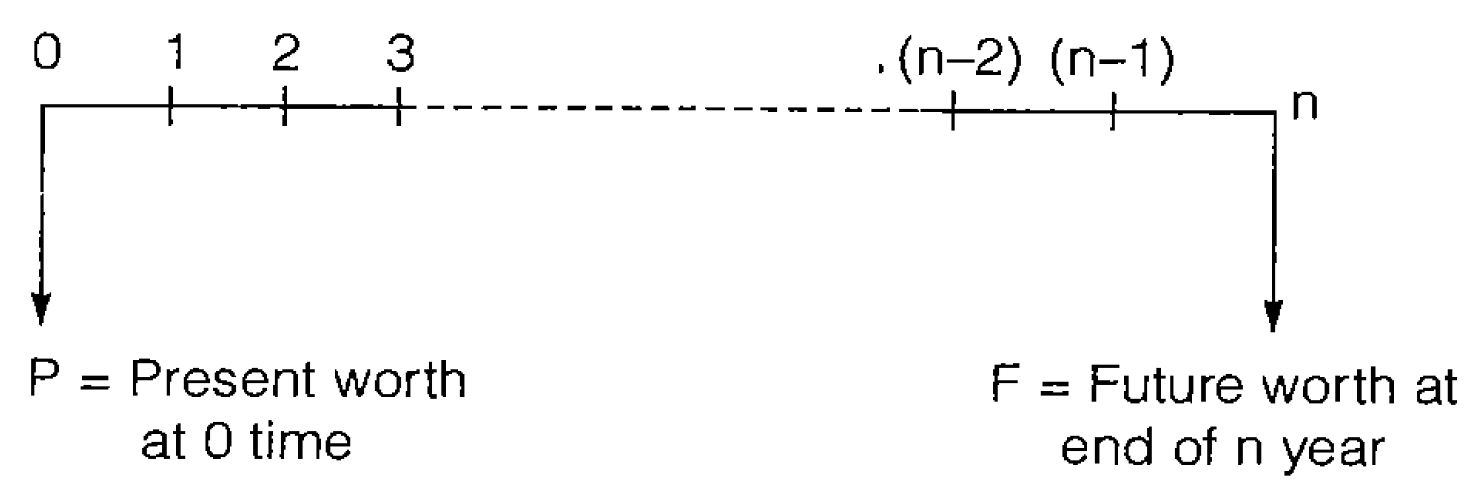


(a) The horizontal line is a time scale with progression of time moving from left to right. The value indicated on time scale (viz., 0, 1,2,...n) indicates the end of the respective period.

(b) The arrows signify cash flow, normally downward arrows represent disbursement or costs and upward arrows represent receipts or benefits.

Interest formulae for single payment series

Figure shows a cash flow diagram involving a present single sum (P), a future single sum (F), separated by n years with interest rate i per year;



- **Case I.**
  Finding F when P is given.
  At the end of n yrs,

$$\boxed{F = P(1+i)^n}$$

$$\boxed{\frac{F}{P} = (1+i)^n}$$

The quantity $(1 + i)^n$ is commonly called "single payment compound amount factor" indicated by the functional notations as $\left(\frac{F}{P}, i, n\right)$

$\therefore \left(\frac{F}{P}, i, n\right)$ = Single payment compound amount factor = $(1 + i)^n$.

- **Case II.**
  Finding P when F is given
  From equation $F = P(1 + i)^n$, solving this for P gives the relation

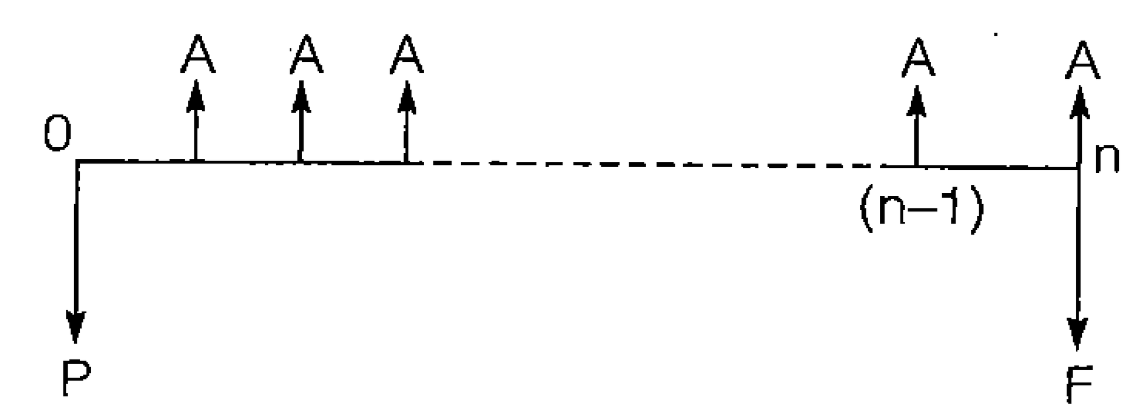
$$\boxed{P = F\left[\frac{1}{(1+i)^n}\right]}$$

The quantity $\frac{1}{(1+i)^n}$ is called **"single payment present worth factor"** indicated by the functional symbol as $\left(\frac{P}{F}, i, n\right)$

$\therefore \left(\frac{P}{F}, i, n\right)$ = Single payment present worth factor = $\frac{1}{(1+i)^n}$

Interest formulae for equal payment series

Fig. shows a general cash flow diagram involving a series of uniform (equal) payments, each of amount A, occurring at the end of each year with interest rate i per year.



- **Case III.**
  Finding P when A is given
  If A exists at end of each year for n years with i rate of interest, the present worth P is obtained by summing the present worth of each payment of amount A

$$\boxed{P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \ldots + \frac{A}{(1+i)^n}}$$

$$\boxed{P = A\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots + \frac{1}{(1+i)^n}\right]}$$

The series in the bracket is in the geometric progression whose first term (a) $\frac{1}{(1+i)}$, geometric ratio (r) is $\frac{1}{(1+i)}$.

Sum is given by a $\left(\frac{1-r^n}{1-r}\right)$

$$\therefore \quad \boxed{P = A\frac{1}{(1+i)}\left[\frac{1-\left(\frac{1}{1+i}\right)^3}{1-\left(\frac{1}{1+i}\right)}\right] = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$$

The quantity $\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ is called "uniform series present worth factor" indicated by the function notation as $\left[\frac{P}{A}, i, n\right]$

$\left[\frac{P}{A}, i, n\right]$ = uniform (equal) series present worth factor = $\frac{(1+i)^n - 1}{i(1+i)^n}$

- **Case IV.**
  Finding A when P is given

$$\boxed{A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]}$$

The quantity in the bracket is called "capital recovery factor" using the functional symbol as $\left(\frac{P}{A}, i, n\right)$.

- **Case V.**
  Finding F when A is given

$$\boxed{P = \frac{F}{(1+i)^n} = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$$

$$\therefore \quad \boxed{F = A\left[\frac{(1+i)^n - 1}{i}\right]}$$

The factor in the bracket is called "equal payment series compound amount factor" given by fundamental notation as $\left(\frac{F}{A}, i, n\right)$

- **Case VI.**
  Finding A when F is given

$$\boxed{A = F\left[\frac{i}{(1+i)^n - 1}\right]}$$

The quantity in the bracket $\left[\frac{i}{(1+i)^n - 1}\right]$ is called "sinking fund factor" using functional notation as $\left(\frac{A}{F}, i, n\right)$

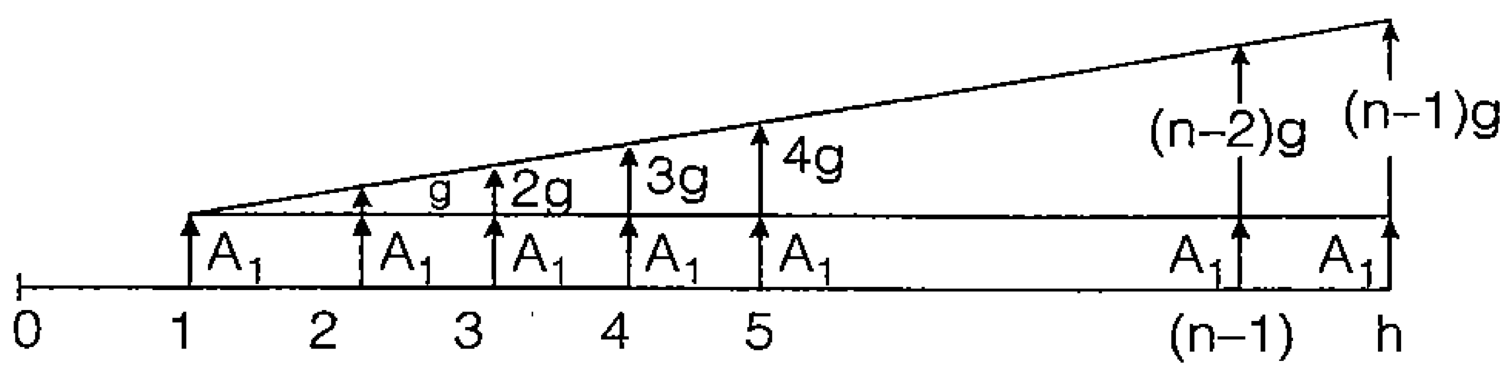
Hence, $\boxed{A = F\left(\frac{F}{A}, i, n\right)}$.

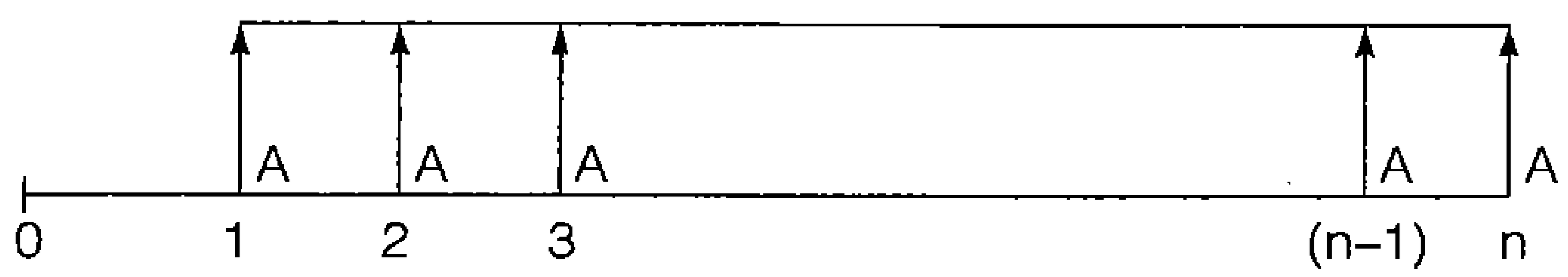
Table COMPOUND INTEREST FACTORS

	To find	Given	Factor by which given is multiplied	Factor Name	Functional notaiton
Singe Payment	F	P	$(1+i)^n$	Single payment compound amount factor	$\left(\frac{F}{P}, i, n\right)$
	P	F	$\frac{1}{(1+i)^n}$	Single payment present worth factor	$\left(\frac{P}{F}, i, n\right)$
Uniform Series Payment	P	A	$\frac{(1+i)^n - 1}{i(1+i)^n}$	Uniform series present worth factor	$\left(\frac{P}{A}, i, n\right)$
	A	P	$\frac{i(1+i)^n}{(1+i)^n - 1}$	Capital recovery factor	$\left(\frac{A}{P}, i, n\right)$
	F	A	$\frac{(1+i)^n - 1}{i}$	Uniform series compound amount factor	$\left(\frac{F}{A}, i, n\right)$
	A	A	$\frac{i}{(1+i)^n - 1}$	Sinking fund factor	$\left(\frac{A}{F}, i, n\right)$

Interest formulae for uniform gradient payment series

Some economic analysis problems involve receipts or disbursements that are projected to increase or decrease by a uniform amount each period, thus contributing an arithmetic series. In general, a uniformly increasing series of payment for n interest periods may be expressed as A_1, A_1+g, A_1+2g, A_1+3g.....$A_1+(n-1)g$ as shown in Fig. Where A_1 denotes the first year end payment in the series and 'g' the annual change in magnitude called gradient amounts.





A_1 = payment at the end of the first year
g = annual change in gradient
n = number of years
A = the equivalent annual payment of the series
A_2 = the equivalent annual payment of the gradient series {0, g, 2g, ... (n – 1)g} at the end of successive years.

$$\boxed{A = A_1 + A_2}$$

where $A_2 = F\left(\frac{A}{F}, i, n\right) = \left[\frac{i}{(1+i)^n - 1}\right]$ and F is the future amount equivalent to the gradient series. The future amount equivalent to the gradient series can be derived from the table as follows,

Table: Gradient Series and an equivalent Set of Series

End of year	Gradient Series	Set of series equivalent to the Gradient series
1	0	0
2	g	g
3	2g	g + g
4	3g	g + g + g
.	.	.
.	.	.
.	.	.
n – 1	(n – 2)g	g + g + g + ...+g
n	(n – 1)g	g + g + g + ...g + g

$$\boxed{F = g\left[\frac{F}{A}, i, (n-1)\right] + g\left[\frac{F}{A}, i, (n-2)\right] + \ldots + g\left[\frac{F}{A}, i, 2\right] + g\left[\frac{F}{A}, i, 1\right]}$$

$$= g\left[\frac{(1+i)^{n-1} - 1}{i}\right] + g\left[\frac{(1+i)^{n-2} - 1}{i}\right] + \ldots$$

$$+ g\left[\frac{(1+i)^2 - 1}{i}\right] + g\left[\frac{(1+i)^1 - 1}{i}\right]$$

$$= \frac{g}{i}\left[\begin{matrix}(1+i)^{n-1} + (1+i)^{n-2} + \ldots + (1+i)^2 \\ +(1+i)^1 - (n-1)1\end{matrix}\right]$$

$$= \frac{g}{i}\left[\begin{matrix}(1+i)^{n-1} + (1+i)^{n-2} + \ldots + (1+i)^2 \\ +(1+i)^1 + 1(1+i)^0\end{matrix}\right] - \frac{ng}{i}$$

The terms in the brackets consist n terms, 1st term being $(1 + i)^0$ and ratio being $(1 + i)$ of geometric progression.

$$= \frac{g}{i}\left[(1+i)^0 \frac{1-(1+i)^n}{1-(1+i)}\right] - \frac{ng}{i} = \frac{g}{i}\left[\frac{(1+i)^n - 1}{i}\right] - \frac{ng}{i}$$

$$A_2 = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= \frac{g}{i}\left[\frac{(1+i)^n - 1}{i}\right]\left[\frac{i}{(1+i)^n - 1}\right] - \frac{n}{g}\left[\frac{i}{(1+i)^n - 1}\right]$$

$$A_2 = \frac{g}{i} - \frac{ng}{i}\left[\frac{i}{(1+i)^n - 1}\right]$$

$$\boxed{A_2 = g\left[\frac{1}{i} - \frac{n}{i}\left(\frac{A}{f}, i, n\right)\right]}$$

The resulting factor $\left[\frac{1}{i} - \frac{n}{i}\left(\frac{A}{F}, i, n\right)\right]$ is called gradient factor for annual compounding interest and will be designated $\left(\frac{A}{G}, i, n\right)$

∴ A_2 = Equivalent annual cost of set of gradient series

$$= g\left(\frac{A}{G},i,n\right)$$

Nominal and Effective Interest Rates

$$\text{Effective interest rate} = \left(1+\frac{r}{c}\right)^c - 1$$

where r is nominal interest rate and c is no. of interest periods per year. If the compounding is annual then r = i.

DEPRECIATION

Depreciation is defined as the loss in value of an asset with the passage of time. The main purpose of the depreciation is to provide for the recovery of capital that has been involved in the possession of the physical property.

- **Salvage Value (or Resale Value)**
  It is the value of the property at the end of its utility period without being dismantled. Salvage value implies that the property has further utility.

- **Scrap Value**
  The value of a property realized when it become absolutely useless except for sale as junk is its scrap value.

- **Book Value**
  It is defined as the value of the property shown in the account books in that particular year i.e. the original cost less total depreciation till that year.

Methods for Calculating Depreciation

There are several methods of calculating depreciation. The following notation have been use;

C_i = Initial cost of an asset at zero time or original cost (This will include the cost of asset +transporting charge + installation and other charges spent initially)

C_S = Salvage value (or scrape value) to be estimated at the end of utility period or scrap cost

n = the life of the asset

B_m = Book value at the end of period 'm'

(i) **Straight Line Method**
In this method, the property is assumed to lose value by a constant amount every year. At the end of the life, the salvage value (or scrap value) is let.

$$D_m = \frac{C_i - C_S}{n} \qquad D_m = D_1 = D_2 = D_3 = D_n \qquad B_m = C_1 - m\left(\frac{C_i - C_s}{n}\right)$$

This method is recommended for all the equipments/assets which constant demand and do not face any obsolescence during their useful life. It is widely used in the case of all civil engineering appliances and applications.

(ii) **Declining Balance Method (or Constant Percentage Method)**
In this method, the property is assumed to lose value annually at a constant percentage of its book value.

Fixed Declining Balance $$\text{FDB} = 1 - \left(\frac{C_s}{C_i}\right)^{1/n}$$

(iii) **Fixed Double Declining Balance Method**
In this method also, the property is assumed to lose value annually by fixed factor of the book value.
FDDB = Fixed factor for double declining balance method.
FDDB is taken as double the straight line rate. i.e. FDDB = $\frac{2}{n}$

$$\text{FDDB} = \left(1 - \frac{C_s}{C_i}\right)^{1/n}$$

(iv) **Sum of the Years Digit Method**
In this method, the digits corresponding to the number of each years of life are listed in reverse order. The general expression for the annual depreciation for any year (m) when the life is n years is expressed as

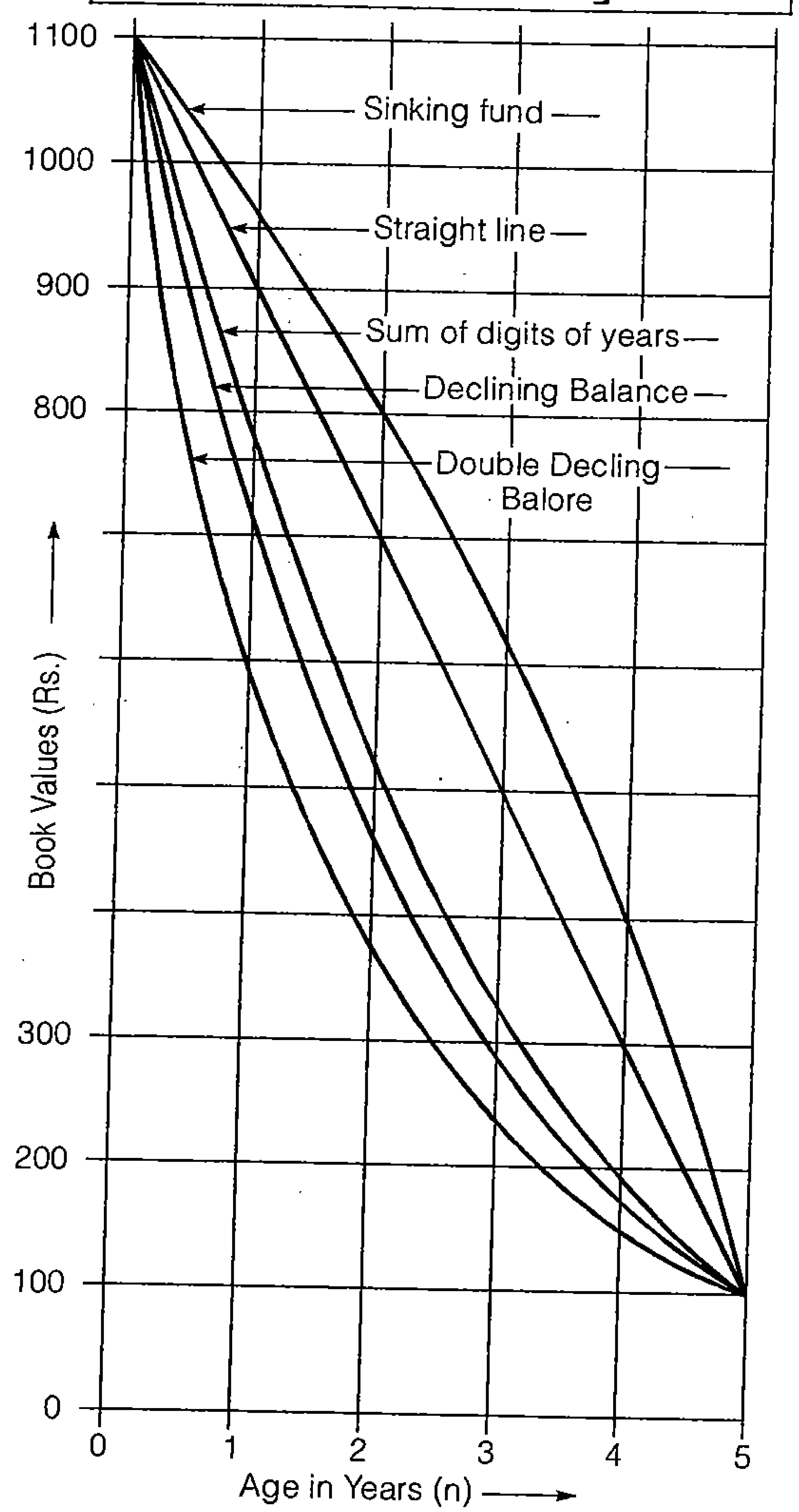
$$D_m = (C_i - C_s)\left[\frac{(n-m+1)}{\frac{n(n+1)}{2}}\right]$$

m = No. of years of which depreciation is calculated

(iv) Sinking Fund Method

Sinking fund depreciation model assumes that the value of an asset decreases at an increasing rate. Equal amount (D) is assumed to be deposited into a sinking fund at the end of each year of the assets life.

$$D = [C_i - C_S]\left[\frac{i}{(1+i)^n - 1}\right][1+i]^{m-1}$$



DEPLETION

$$\text{Depletion for a year} = \frac{\text{Cost of property}}{\text{No. of units in the property}} \times \text{Units sold during the year}$$

■■■

6 Engineering Fundamentals of Equipments

Rolling Resistance

Rolling Resistance R (kg/tonne)

$$R = \frac{P}{W}$$

where P = Required tractive force, kg
W = Gross weight of vehicle, tonnes

The low cable is fitted with a dynamometer which measures the average tension in the cable.

- **Grade Resistance**
  The effect of grade is to increase, for a plus slope, or to decrease, for a minus slope, the required tractive effort by 10 kg per gross tonne of weight for each 1 per cent of grade.
  It is the physical property which is affected by the type of equipment or the construction or the type of road.
- **Coefficient of Traction**
  The coefficient of traction is affected by:
  (i) Weight on the driving wheels or tracks
  (ii) Gripping action of the wheel or track i.e. type of tread on the tyre or design of the grouser of tracks
  (iii) Ground conditions
- **Drawbar Pull**
  Effective drawbar pull = Available drawbar pull ± Rolling resistance on a level haul surface ± grade resistance for upgrade/downgrade slope
- **Rimpull**
  It defines the tractive force between the rubber tires of driving wheels and the surface on which the tires operate.
  It is expressed in kg and is calculated as per following conditions:
  (i) If the coefficient of traction is high enough such that slippage is eliminated, then

$$\text{Maximum rimpull} = \frac{375 \times \text{HP} \times \text{Efficiency}}{\text{Speed in mph}}$$

  (ii) If the coefficient of traction is such that slippage starts before its rated capacity, then

Maximum rimpull = Total pressure between driving wheels and the surface × coefficient of traction.

- **Effect of temperature and pressure on IC Engine**

The rated horse power of an IC engine is the power tested under standard conditions of temperature and pressure.

If the test is carried out under conditions different from the standard, the horse power may be determined by using the following formula:

$$H_c = H_o \frac{P_s}{P_o}\sqrt{\frac{T_o}{T_s}}$$

where,

H_c = Corrected horse power for standard condition.

H_o = Observed horse power as determined for test

P_s = Standard barometric pressure (760 mm of Hg)

P_o = Observed barometric pressure in mm of Hg at time of test

T_o = Absolute observed temperature, (equal to 273°C + Observed temperature in °C)

T_s = Absolute temperature for standard condition, (Equal to 273° + 15.5°)

Depreciation Cost

$$\text{Annual depreciation} = \frac{\text{Initial value} - \text{Salvage value}}{\text{Useful life of equipment (in years)}}$$

Initial value = Price of equipment + transportation cost + loading and unloading charge + Installation charge

- **Investment Cost**

Annual Investment Cost = 10-20% of the average annual cost of the equipment

The average annual cost of the equipment may be found out in following ways :

(i) When there is no salvage value of the equipment

$$P_{av} = \frac{P + \frac{P}{n}}{2} \qquad P_{av} = \frac{P(n+1)}{2n}$$

where, P = Total initial cost

P_{av} = Average annual cost

n = life in years

(ii) When there is salvage value of the equipment;

$$P_{av} = P + \left(\frac{P-s}{n}\right) + s \qquad P_{av} = \frac{P(n+1) + s(n-1)}{2n}$$

where, P = Total original cost

P_{av} = Average annual cost

n = Life in years

s = Salvage value

- **Fuel Consumption Cost**

Fuel consumption in litre per hour

(i) For a gasoline engine = operating factor × rated HP × 0.30

(ii) For a diesel engine = operating factor × rated HP × 0.20

Operating factor = Engine factor × Time Factor

- **Lubricating oil cost**

An empirical formula may be used to estimate the quantity of lubricating oil;

$$q = \frac{\text{rated HP} \times \text{Operating factor} \times 0.003 \text{ kg per HP-hr}}{0.74 \text{ kg per liter}} + \frac{C}{t}$$

where;

q = Quantity of consumed lubricating oil in litre/hr

C = Capacity of crankcase of engine in litre

t = Number of hours between changes

The operating factor may be assumed as 0.6 when sufficient data is not available for the purpose.

■■■

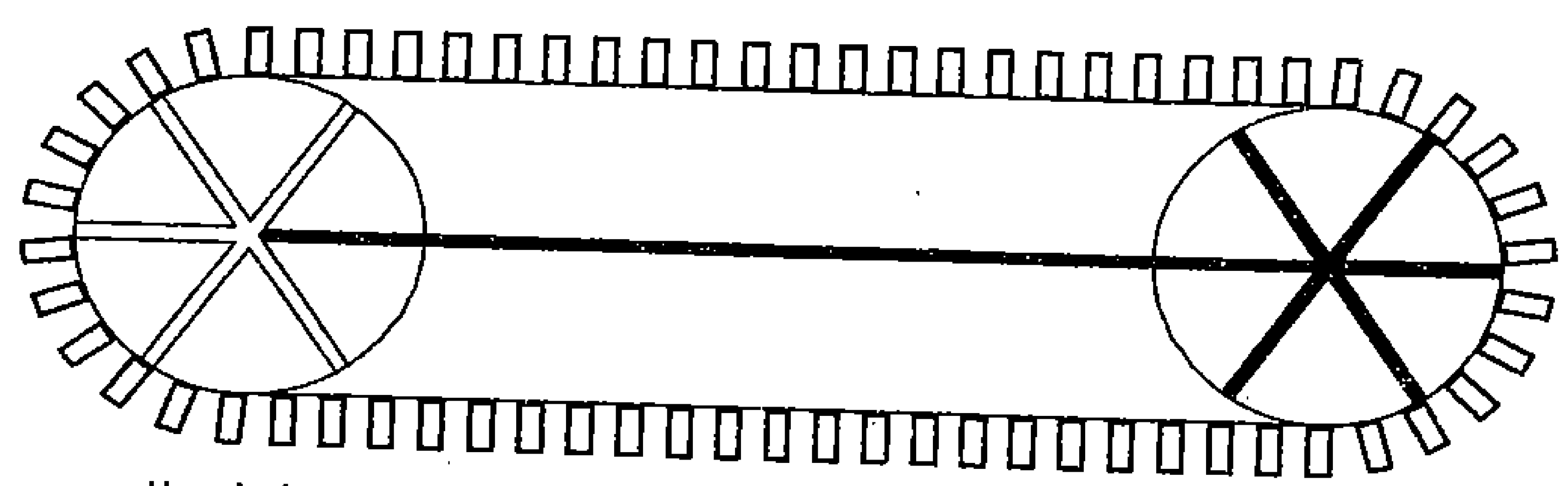
Earthwork Equipments 7

Tractor

The primary purpose of a tractor is to pull or push loads, and it may be used also as a mount for many types of equipment such as bulldozer, shovel, dragline, hoe, trenchers, etc.

■ Crawler Tractor

- A crawler track is an endless chain consisting of steel links made of steel plates connected together by pins and bushings as shown in figure.



- Generally, it is used for moving heavy units on rough surface having poor traction.
- The optimum pull that a crawler tractor can provide depends upon its weight and is equal to the coefficient of traction (depending upon road surfaces) multiplied by the weight of unit, regardless of the power supplied by the engine.
- It is suited for short haul say 60 to 150 m.

■ Wheel Tractor

- One of the basic advantages of a wheel tractor compared with a crawler tractor lies in its higher speed.
- However, in order to attain a higher speed, a wheel tractor must sacrifice its pulling effort.
- As the speed is increased through the selection of higher gears. The rimpull will be decreased in approximately the same proportion.

Comparison Between Crawler Tractor and Wheel Tractor

The following factors should be considered when comparing crawler tractors with wheel tractors:

1. **Traction:** Coefficient of traction for a crawler tractor is upto 0.9 where it is up to 0.6 for a wheel tractor.

2. **Useful Rimpull:** Since useful rimpull = machine weight × coefficient of traction, therefore, a crawler tractor negotiates very heavy loads whereas a wheel tractor is useful for light loads.
3. **Speed:** A wheel tractor possesses speed up to 3 to 4 times higher than a crawler tractor. Where haul distance is considerable and/or jobs are scattered at different locations, the wheel tractor can be used more efficiently as to be compared with crawler tractor.
4. **Maneuverability:** A wheel tractor has steering wheel which is easy to operate and control while a crawler tractor is provided with stick control which is not easy to control.
5. **Compaction:** Ground pressure of wheel tractors vary from 1.25 kg/cm^2 (0.125 N/mm^2) to 1.50 kg/cm^2 (0.150 N/mm^2) whereas the same for crawler tractor stands from 0.85 kg/cm^2 (0.085 N/mm^2) to 1.00 kg/cm^2 (0.1 N/mm^2), hence crawler tractors can be effectively used on loose or muddy soil
6. **Cost:** Crawler tractors are more costly initially than wheel tractors.
7. **Operation and maintenance cost:** Operation, maintenance and repair cost is less in wheel tractor as compared to crawler tractor.
8. The tar and concrete pavements are liable to damage by crawler tractors while wheel tractors are liable to slip over smooth footing.

DOZERS

- Dozers are very efficient excavating tools for short haul applications upto 100 m.
- A dozer is a tractor-power unit that has a blade attached to the machine's front. It is designed to provide tractive power for drawbar work.

- Types of Dozers

  The dozers are classified on the following basis of the mounting i.e

  (i) crawler-mounted dozer

  (ii) wheel-Mounted dozer

- Comparison of wheel dozer and crawler dozer

Wheel Dozer	Crawler Dozer
1. Good on firm soils and concrete and abrasive soils that have no sharp-edged pieces	1. Can work on a variety of soils; sharp-edged pieces not as destructive to dozer though fine sand will increase running gear wear

2. Best for level and downhill work.	2. Can work over almost any terrain
3. Wet weather causing soft & slick surface conditions will slow or stop operation	3. Can work on soft ground and over mud-slick surfaces;
4. The concentrated wheel load will provide compaction & kneading action to ground surface	4. Will exert very low ground pressures with special low ground pressure undercarriage and track configuration
5. Good for long travel distances	5. Good for short work distances
6. Best in handling loose soils.	6. Can handle tight soils.
7. Fast return speeds, 8-26 mph	7. Slow return speeds, 5-10 mph
8. Can only handle moderate blade loads	8. Can push large blade loads

The output of Dozers

Output of a bulldozer in bank measure vol/hr

$$= \frac{\text{Rated mold board capacity in loose volume}}{\text{Swell factor}}$$

$$\times \frac{\text{Actual operating time in minutes per hour}}{\text{Time required per trip in minutes}}$$

whereas,

time required per trip in minutes or cycle time in minutes is given by

$$= \frac{D}{F} + \frac{D}{R} + G$$

D = Haul distance in metres

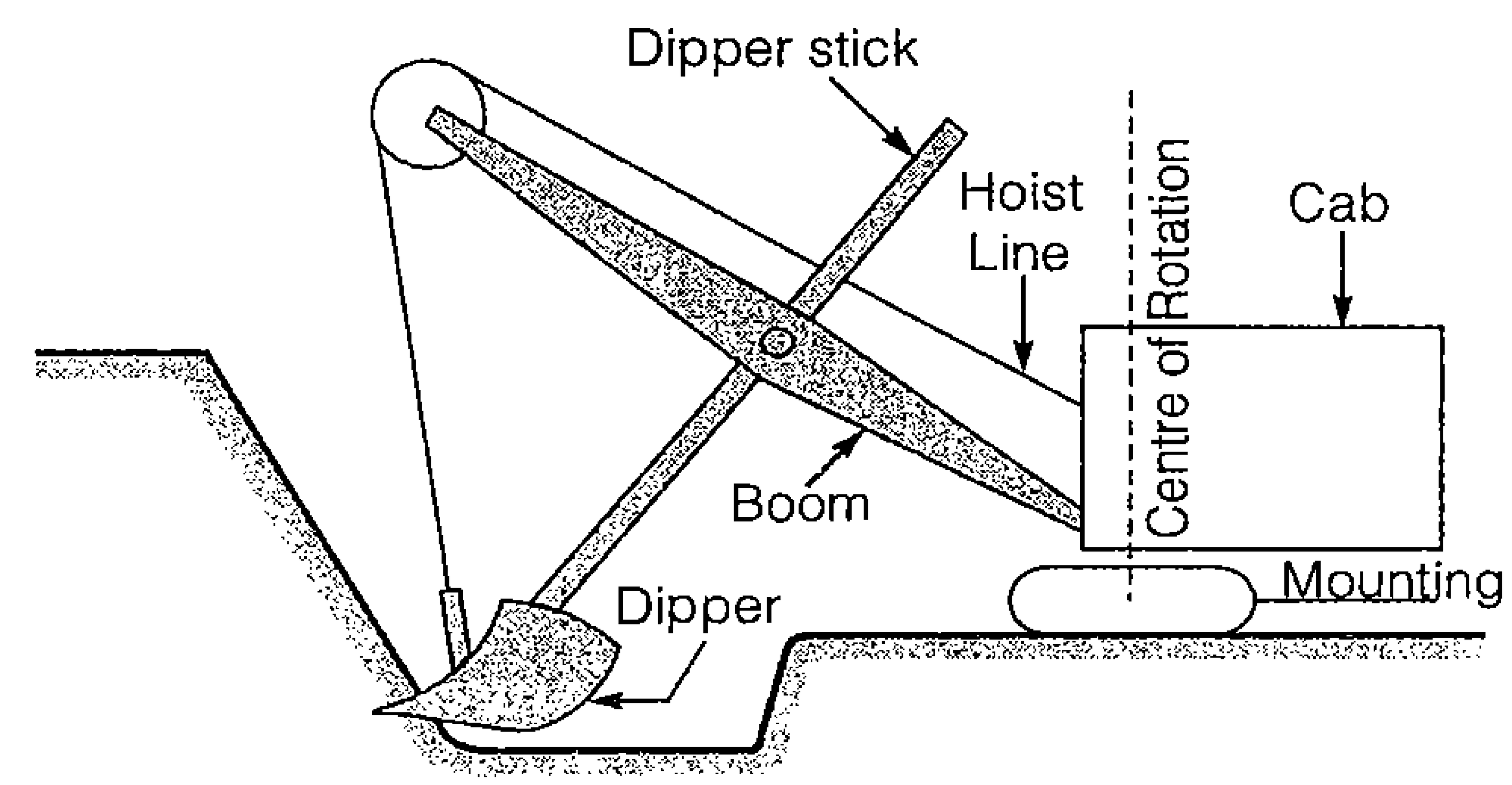
F = Forward speed in meters/minute

R = Reverse speed in meters/minute

G = Gear shifting time in minutes (0.15 minute to 0.30 minutes)

Power Shovel

- Power shovels are used predominantly for hard digging above track level and for loading units.
- They are capable of excavating all classes of earth, except the solid rock without prior loosening.
- The size of a shovel is indicated by the size of bucket of dipper, expressed in cubic meters and varies from 0.375 m^3 to 5 m^3.



Output of Power Shovel

- The output of power shovel is expressed in cubic meter per hour based on bank-measure volume.
- If no allowance is made for any lost time, then

$$\text{Output of shovel} = \text{Bank measure capacity of dipper} \times \frac{3600}{\text{Cycle time in seconds}} (m^3/hr)$$

In general cases,

$$\text{Output of shovel} = \frac{\text{Loose volume of dipper}}{1 + \text{swell friction}} \times \frac{\text{actual time in sec./hr}}{\text{cycle time in sec.}} \times \text{efficiency}$$

expressed in cubic meter/hr.

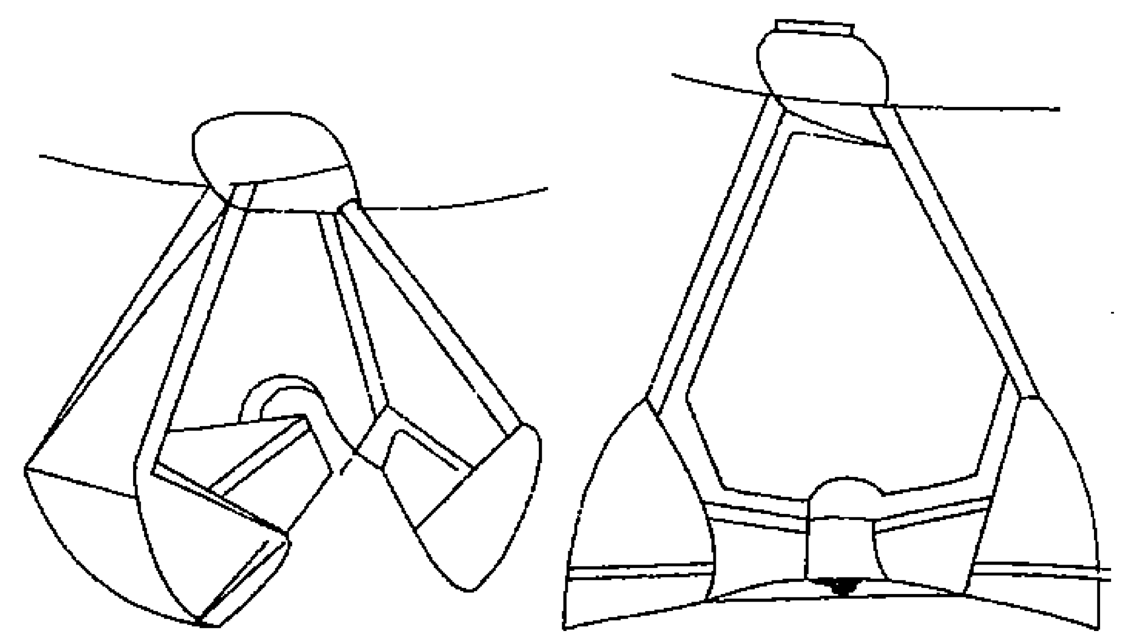
Dragline

- Since the basic character of the machine is dragging the bucket against the material to be dug, it is called as dragline.
- A dragline has distinct advantages compared with a power shovel because of its long light boom.

Clamshells

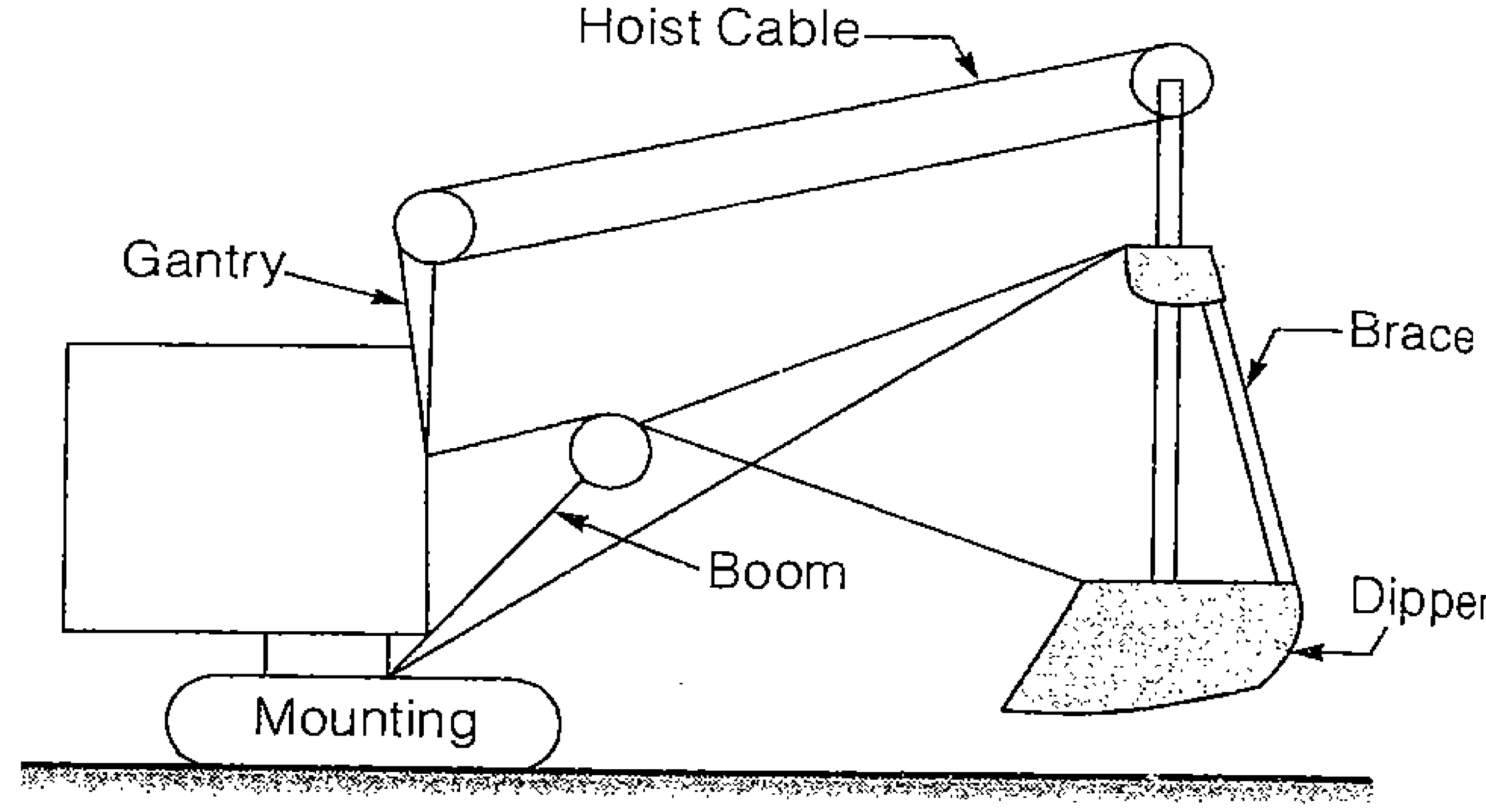
- Clamshell is a machine having most of the characteristics of dragline and crane in common. Digging is done like a dragline and once the bucket is filled, it works like a crane.
- The clamshell bucket is designed to excavate material in a vertical direction.

- It works like an inverted jaw with a biting motion.
- The clamshell is capable of working at, above and below ground level.
- The clamshell machine consists of a clamshell bucket hung from a lattice-boom crane.



Hoe

Hoe is an excavating equipment of the power-shovel group. Since the digging mechanism resembles to an ordinary garden hoe, it is named as Hoe. But it is referred by several names such as hoe, backhoe, back shovel and pull shovel.



Hoe Production

The basic production formula for a hoe used as an excavator is Hoe (excavation) production

$$\frac{3600\text{ sec} \times Q \times F}{t} \times \frac{E}{60-\text{min. hour}} \times \frac{1}{\text{Volume correction}}$$

where, Q = heaped bucket capacity in lcy
F = bucket fill factor for hoe buckets
t = cycle time in seconds
E = Efficiency (in minutes per hour)

Volume correction for loose volume to bank volume, $\frac{1}{1+\text{swell factor}}$;

for loose volume to tons, $\frac{\text{loose units weight, } lb}{2{,}000\ lb/\text{ton}}$

Scraper

Scraper is a machine which can scrape the ground and load it simultaneously, transport it over the required distance, dump at the desired place and then spread the dumped material over the required area in required level and return to the pit for the next cycle.

- **Output of a Scraper**

The output of a scraper is the bank measure volume per hour (cubic meter/hr)

$$\text{Output} = \text{Optimum loose volume per trip} \times S \times \frac{60}{t} \times \text{Efficiency}$$

where, S = Swell factor depending upon type of soil
t = Cycle time per trip in minutes
t = Fixed time (Loading + Dumping and turning + Accelerating + Decelerating) + Haul time + Return time in minutes.

Compaction Equipment

- **Types of Compacting Equipment**

Applying energy to a soil by one or more of these methods will cause compaction :

1. Impact – sharp blow
2. Pressure – static weight
3. Vibration – shaking
4. Kneading – manipulation or rearranging

1. **Sheepsfoot Roller**
   - The sheepsfoot roller is suitable for compacting all fine-grained materials, but is generally not suitable for use on cohesionless granular materials.
2. **Tamping Rollers**
   - Tamping foot compactors are high speed, self-propelled, non-vibratory rollers.
   - A tamping foot roller is effective on all soils except clean sand.
3. **Vibrating Compactors**
   - Vibration has two measurements – amplitude and frequency.
   - The amplitude controls the effective area, or depth to which the vibration is transmitted into the soil.
4. **Smooth Drum Vibratory Soil Compactors**
   - The smooth drum compactors, whether single-or dual-drum models, generate three compactive forces :
     (1) Pressure (2) Impact (3) Vibration

- These rollers are most effective on granular materials, with particle sizes ranging from large rocks to fine sand.
- They can be used on semicohesive soils with up to about 10% of the material having a PI of 5 or greater.

5. Padded Drum Vibratory Soil Compactors
   - These rollers are effective on soils with up to 50% of the material having a PI of 5 or greater.
   - These units are designed specifically for trench work or for working in confined areas.
6. Pneumatic-Tired Rollers
   - These are surface rollers that apply the principle of kneading action to effect compaction below the surface.
   - These units are frequently used to proof roll roadway subgrades and airfields bases and on earth-fill dams.
7. Pneumatic-Tired Rollers with Variable Inflation Pressures
   - The first passes are made with relatively low tire pressures.
   - As the soil is compacted, the tire pressure is increased to suit the particular conditions of the soil.
8. Towed Impact Compactors
   - These compactors have used three, four and five-sided drums.
   - As the compactor is towed, the drum rotates, lifting itself up on edge, and then falls back to earth.
   - These compactors can be used on a wide range of materials viz. rock, sand, gravel, silt, and clay.
9. Compaction Wheels
   - To avoid the hazards of having to have men working in trenches, a compaction wheel attached to an excavator boom is often used to achieve compaction when backfilling utility trenches.
10. Manually Operated Vibratory-Plate Compactors
    - A self-propelled vibratory-plate compactor used for consolidating soils and asphalt concrete in locations where large units are not practical.
11. Manually Operated Rammer Compactors
    - Small compactors such as the self-propelled vibratory-plate or the rammer will provide adequate compaction if:
      1. Lift thickness is small (usually 3 to 4 in.)
      2. Moisture content is carefully controlled, and
      3. Coverages are sufficient

Roller Production Estimation

The production formula for a compactor in compacted cubic yards per hour =

$$\frac{16.3 \times W \times S \times L \times \text{Efficiency}}{n}$$

where,

W = compacted width per roller pass in feet
S = average roller speed in miles per hour
L = compacted lift thickness in inches
n = number of roller passes required to achieve the required density.

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Concreting Equipments

8

Batching Plants

Batching plants are available in 3 categories :

1. Manual
2. Semiautomatic
3. Fully automatic

Typical batching tolerances

Ingredient	Individualbatchesand cumulative batches with a tare compensated control	Cumulative batchers without a tare compensated control
Cement and other Cementitious materials	± 1% of the required weight of materials being weighed or ± 0.3%of scale capacity, whichever is greater.	±1% of the required cumulative weight of materials being weighed or ± 0.3% of scale capacity, whichever is greater.
Aggregates	± 2% of the required weight of material being weighed or ± 0.3% of scale capacity, whichever is greater.	± 1% of the required cumulative weight of materials being weighed or ± 0.3% of scale capacity, whichever is greater.
Water	± 1% of the required weight of material being weighed or ± 0.3% of scale capacity, whichever is greater.	
Admixtures	± 3% of the required weight of material being weighed or ± 0.3% of scale capacity,or ± the minimum dosage rate per100 lb (45.4 kg) of cement, whichever is greater.	± 3% of the required cumulative weight of material being weighed or ± 0.3% of scale capacity, or ± the minimum dosage rate per 100 lb (45.4 kg) of cement as it applies toeach admixture, whichever is greater.

Mixer

Machine used for the purpose is called concrete mixer which may be batch type or continuous type.

Batch mixer – mixes and discharges each load of materials separately.

Continuous mixer – produces concrete continuously without any break so long as it is in operation. It is not much in use.

- **Batch mixer may be**
  (i) Tilting type and
  (ii) Non-tilting type
- **Tilting Type Mixer**
  Basic features of the tilting type mixer are as follows:
  (i) It is commonly used on large construction projects.
  (ii) It can be used for bigger sizes of particles even bigger than 7.5 cm.
  (iii) In this material is rolled down.
  (iv) It imparts more satisfactory results for dry concrete.
  Basic features of this non-tilting type mixer are:
  (i) It is suitable for small works.
  (ii) It is recommended for use when material size is not greater then 7.5 cm.
  (iii) Materials are lifted, rolled and then dropped
- **Size of Tilting/Non-Tilting Mixer**
  As per IS : 1791 – 1961, mixers have been designated as :
  Tilting type – 100T, 140T, 200T
  Non-tilting type – 140 NT, 200 NT, 280NT, 400NT, 800 NT
  where T stands for tilting type, and
  NT stands for non-tilting type
  Number 100, 140, 200 indicates the nominal volume of mixed concrete in liters.
  These mixers may be available in standard size of 0.375, 0.50, 1.50, 2.25 and 3 cum capacities.

Compaction Equipment

- **Tamping**
  Tamping is the process of compacting the concrete manually-A rod or hand tool is inserted to the full depth and moved up and down such that air bubbles go out of concrete. It is compacted in layer of 15 cm in case of RCC and 30 cm in case of mass concrete.
- **Vibrators**
  (a) Internal (Needle) vibrator
  - It is known as needle immersion or poker vibrator and it consists of a power unit and long flexible tube at the end of which a vibrating head is attached.
  - It is very effective for mass concreting.

(b) External (Form) Vibrator
- It is used only when the section is thin and heavily reinforced where penetration of needle vibrator is not possible.

(c) Surface Vibrator
- It is also called screed or pan vibrator.
- It vibrates the concrete from the surface at the time when striking off the concrete i.e. screeding is being carried out.
- It is effective only if the depth is up to 15 to 20 cm.

(d) Vibrating Table
- It is used when stiff and harsh mix is used in prefabricated elements and for laboratory specimens only.

Special method of vibrations

(a) Vibropressing
- Compaction is done by applying external pressure on top and vibrating from bottom.
- It is useful in production of flag stone for paving and concrete road kerbs.

(b) Shock (Jolting)
- Concrete in the mould is subjected to jolting action with the help of cams.
- It is useful for precast concrete products.

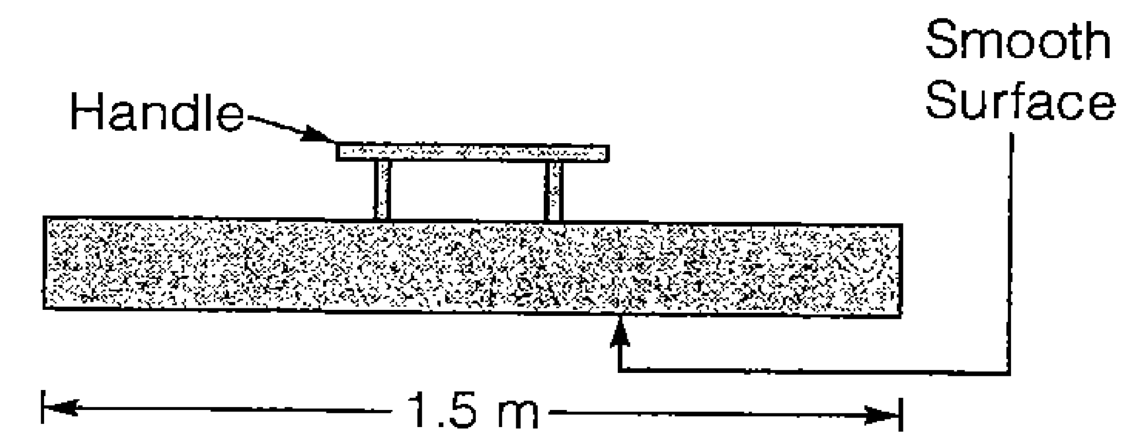
(c) Centrifugation
- Concrete in the mould is being spun at very high speed which expels the water from the mix and compaction proceeds.
- Here the compaction is done by centrifugal forces, it results in very water tight structures like pipes for water supply and sewage disposal.

(d) Airjets
- Air is driven out from concrete through air jets and the particles come closer and compaction is done.

Finishing Concrete

- Screeding
  The process of striking off the excess concrete to bring the top surface up to proper grade is called screeding.

- Floating
  Floating consists in removing the irregularities on the surface of concrete which are left after screeding.
  This is done by a wooden float.



- Trowelling
  It is final operation of finishing.
  It should be done after all excess water has evaporated.
  Troweling with steel float in conical shape gives a very smooth finish.
- Curing of Concrete
  Curing is defined as the process of maintaining humidity and moisture at favourable temperature of freshly placed concrete during some definite period following placing, casting and finishing to attain full hydration of the cement.
- Object of Curing
  The basic object of curing is to attain the maximum strength of concrete. In addition to it, other benefits are listed below :
  (i) It increases durability and impermeability of concrete.
  (ii) It reduces shrinkage of concrete.
  (iii) It improves wear resisting and weather resisting qualities.

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Hoisting Equipments

9

Hoisting is the operation of lifting the load. Hence equipments used for hoisting purpose lift the load from the place, hold it in suspension during transfer from one location to the other and finally place it on the desired location.

Types of Hoisting Equipments

(i) Pulley

Pulley and sheave are used for lifting rough surfaced and heavy objects. Both chains and wire ropes are used for this purpose.

(ii) Chain Hoists

- It is used for lifting loads up to 50 tonnes.
- The system consists of hand chain and the load chain. The pull applied through the hand chain is transmitted to the load chain with a multiplication factor of over 20.

(iii) Winch

- A winch is a combination of gears (spur and pinion), clutches and brakes. The operation is controlled through a series of levers.
- It is commonly used in lifting the railway gates.

(iv) Cranes

- Cranes are most widely used equipment as an independent unit. Lifting capacity varies from ½ tonne to 500 tonnes.

Major Crane types

Construction cranes are generally classified into two major families:

(i) Mobile cranes and

(ii) Tower cranes.

Mobile Cranes

■ Crawler Cranes

- The full revolving superstructure of this type is mounted on a pair of continuous, parallel crawler tracks.
- The crawlers provide the crane with travel capability around the job site.

- The crawler tracks provide such a large ground contact area that soil failure under these machines is only a problem when operating on soils having a low bearing capacity.

■ Telescoping Boom Truck Mounted Cranes

- Most of these units can travel on the public highways between projects under their own power with a minimum of dismantling.
- These machines, however, have higher initial cost per rated lift capability. If a job requires crane utilization for a few hours to a couple of days, a telescoping truck crane should be given first consideration because of its ease of movement and setup.

■ Lattice Boom Truck-Mounted Cranes

- The lattice-boom truck crane has a full revolving superstructure mounted on a multiaxle truckcarrier.
- The lattice-boom structure is lightweight.
- The disadvantage of these units is the time and effort required to disassemble them for transport.

■ Heavy Lift Cranes

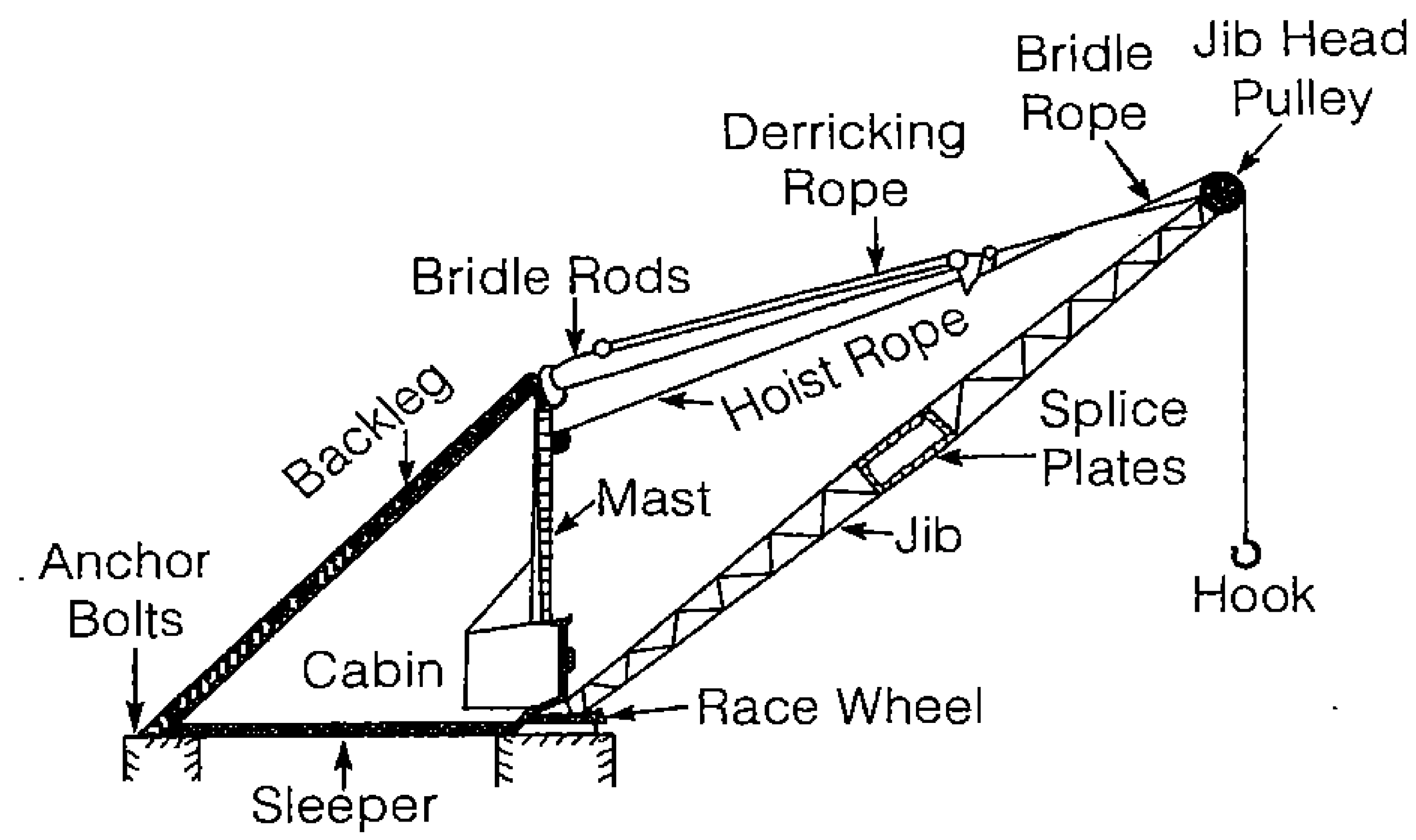
- These are machines that provide lift capacities in the 600-through 2000-short-ton range.
- These cranes consist of a boom and counterweight, each mounted on independent crawlers that are coupled by a stinger.

■ Tower Cranes

- Tower cranes provide high lifting height and good working radius, while taking up a very limited area.
- These advantages are achieved at the expense of low lifting capacity and limited mobility, as compared to mobile cranes.
  1. Top-slewing (fixed tower) tower cranes have fixed towers and a swing circle mounted at the top, allowing only the jib, tower top, and operator cabin to rotate.
  2. Bottom slewing (slewing tower) tower cranes have the swing circle located at the base, and both the tower and jib assembly rotate relative to the base.

■ Derrick Crane

(a) **Derrick Crane:** The power is supplied by a diesel engine or by an electric motor. It consists of a mast supported by a number of guys.



Power driven Scotch Derrick Crane

(b) **Whirler Crane:** It combines the advantage of long boom of derrick crane and mobility of the mobile crane.

(c) **Gantry Crane:** A gantry crane or overhead crane is a must in factories and workshops. It consists of bridge and crab. Two main girders are fixed at the end of the bridge. The chief advantage lies in the three way movement of load.

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A Handbook on
Civil Engineering

6

Building Materials

CONTENTS

Cement

1

Introduction

- Cement is an extremely ground material having adhesive and cohesive properties which provide a binding medium for the discrete ingredients.
- The processes used for the manufacture of cement can be classified as dry and wet.
- In the wet process, the limestone brought from the quarries is first crushed to smaller fragments. Then, it is taken to a ball or tube mill where it is mixed with clay or shale as the case may be and ground to a fine consistency of slurry with addition of water. The slurry is stored in tanks under constant agitation and fed into huge firebrick lined rotary kilns.
- In the dry process the raw materials are ground, mixed and fed to the rotary kiln in the dry state.

Chemical Composition

- The identification of the major complex compounds is largely based on R.H. Bogue's work and hence these are called Bogue's compounds.

Constituents	Percentage	Average
Lime (CaO)	60 to 67%	63
Silica (SiO_2)	17 to 25%	20
Alumina (Al_2O_3)	3 to 8%	6
Iron oxide (Fe_2O_3)	0.5 to 6%	3
Magnesia (MgO)	0.1 to 4%	2
Sulphur Trioxide (SO_3)	1 to 3%	1.5
Soda and Potash ($Na_2O + K_2O$)	0.5 to 1.3%	1

- **Bogue's Compounds**

Name	Chemical formula	Percentage
Tricalcium Silicate (C_3S)	$3CaO.SiO_2$	30-50
Dicalcium Silicate (C_2S)	$2 CaO.SiO_2$	20-45
Tricalcium Aluminate (C_3A)	$3CaO.Al_2O_3$	8-12
Tetracalcium Alumino Ferrite (C_4AF)	$4 CaO.Al_2O_3.Fe_2O_3$	6-10

Type of Cements

(i) Ordinary Portland Cement
(ii) Rapid Hardening Cement – **IS : 8041-1990**

(iii) Extra Rapid Hardening Cement
(iv) Low Heat Portland Cement – **IS : 12600-1989**
(v) Portland Slag Cement – **IS : 455-1989**
(vi) Portland Pozzolana Cement – **IS : 1489-1991(Part 1 and 2)**
(vii) Sulphate Resisting Portland Cement – **IS : 12330-1988**
(viii) White Portland Cement – **IS : 8042-1989**
(ix) Coloured Portland Cement – **IS : 8042-1989**
(x) Hydrophobic Cement – **IS : 8043-1991**
(xi) High Alumina Cement – **IS : 6452-1989**
(xii) Super Sulphated Cement – **IS : 6909-1990**
(xiii) Special Cements
  (a) Masonry Cement (b) Air Entraining Cement
  (c) Expansive Cement (d) Oil Well Cement

FIELD TESTS FOR CEMENTs

- **Colour:** Grey colour with a light greenish shade.
- **Physical Properties:** Cement should feel smooth when touched between fingers.
- If hand is inserted in a bag or heap of cement, it should feel cool.
- If a small quantity of cement is thrown in a bucket of water, it should sink and should not float on the surface.
- **Presence of lumps:** Cement should be free from lumps.
- **Permissible Limits for Impurities in Water**

Impurity	Permissible Limits
Organic	200 mg/L
Inorganic	3000 mg/L
Sulphates (SO_3)	400 mg/L
Chlorides (Cl)	2000 mg/L for plain concrete work, 500 mg/L for reinforced concrete work
Suspended matter	2000 mg/L

Laboratory Tests For Cements

1. **Chemical Composition Test**
   - Ratio of percentage of lime to percentage of silica, alumina and iron oxide known as Lime Saturation Factor (LSF), when calculated by the formula $\dfrac{CaO-0.7SO_3}{(2.8SiO_2+1.2Al_2O_3+0.65Fe_2O_3)}$ shall not be greater than 1.02 and not less than 0.66.

- Ratio of percentage of alumina (Al_2O_3) to that of iron oxide (Fe_2O_3) shall not be less than 0.66
- Weight of insoluble residue shall not be more than 4 per cent.
- Weight of Magnesia shall not be more than 6 per cent.
- Total loss on ignition shall not be more than 5 per cent.
- Total sulphur content calculated as sulphuric anhydride shall not be more than 2.5% when C_3A is 5% or less and shall not be more than 3% when C_3A is more than 5%

2. Normal Consistency Test
   - The normal (standard) consistency of a cement paste is defined as that consistency which will permit a Vicat plunger having 10 mm diameter and 50 mm length to penetrate a depth of 33 to 35 mm from the top (or 5 to 7 mm from the bottom) of the mould.
   - **Vicat Apparatus**: Vicat apparatus assembly consists of a plunger 300 gm in weight with a length of 50 mm and diameter of 10 mm and a mould which is 40 mm deep and 80 mm in diameter.
3. Initial Setting Time Test
   - Initial setting time should not be less than 30 minutes for OPC and 60 minutes for low heat cement.
4. Final Setting Time Test
   - The final setting time should not be more than 10 hours.
5. Soundness Test
   - The soundness of cement is determined either by 'Le Chatelier's method' or by means of a 'Autoclave' test.
   - No satisfactory test is available for deduction of soundness due to excess of calcium sulphate. But its content can be easily determined by chemical analysis.
     - Le Chatelier's Method
     - Autoclave Test
6. Strength Test

(a) Compressive Strength Test
- Three cubes are tested for compressive strength at 1 day, 3 day, 7 day and 28 day where the period of testing being reckoned from the completion of vibration.
- The compressive strength shall be the average of the strengths of the three cubes for each period respectively.
- The compressive strength of 33 grade OPC at 3 day, 7 day and 28 day is 16 MPa, 22 MPa and 33 MPa respectively.

(b) Tensile Strength Test
- Six briquettes are tested and average tensile strength is calculated.
- Load is applied steadily and uniformly, starting from zero and increasing at the rate of 0.7 N/mm^2 in 12 seconds.
- OPC should have a tensile strength of not less than 2 MPa and 2.5 MPa after 3 and 7 days respectively.
- Generally tensile strength is 10-15% of compressive strength.

7. **Fineness Test:** There are three methods for testing fineness viz.

(a) Sieve Method
- 100 gm of cement sample is taken and air set lumps, if any, in the sample are broken with fingers.
- The sample is placed on a 90 micron sieve and continuously sieved for 15 minutes.
- The residue should not exceed the limits specified below:

	Type of cement	%Residue by weight
(i)	Ordinary Portland cement	10
(ii)	Rapid hardening cement	5
(iii)	Portland pozzolana cement	5

(b) Air Permeability Method
- Fineness of cement is represented by specific surface i.e. total surface area in cm^2 per gram of cement.

(c) Wagner Turbidimeter Test
- The cement is dispersed uniformly in a rectangular glass tank filled with kerosene.
- Parallel light rays are passed through the solution which strike the sensitivity plate of a photoelectric cell.

8. Heat of Hydration Test
   - The apparatus used to determine the heat of hydration of cement is known as calorimeter.
   - The heat of hydration for low heat Portland cement should not be more than 66 and 75 cal/gm for 7 and 28 days respectively.
9. Specific Gravity Test
   - The specific gravity of cement is obtained by using Le Chatelier's flask.

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Mortar

2

- Building mortar is defined as a mixture of cement, sand and water.
- Mortar is similar to concrete but it does not contain coarse aggregate.
- Mortar are used for filling joints as a binder in stone and brick masonry.

Bulking of Sand

- In the case of aggregates there is another effect of the presence of moisture viz. bulking which is an increase in the volume of a given mass of sand (fine aggregate) caused by the films of water pushing the sand particle apart. For a moisture content of about 5-8% this increase of volume may be as much as 20-40% depending upon the grading of sand.
- Finer the materials more will be the increase in volume for a given moisture content.

Classification of Mortars

Mortars are classified on the basis of the following:

(i) Bulk density
(ii) Kind of binding materials
(iii) Nature of application
(iv) Special mortars

Properties of Good Mortar Mix and Mortar

The important properties of a good mortar mix are mobility, placeability and water retention.

- Mobility
  - It is used to indicate the consistency of mortar mix which may range from stiff to fluid.
  - The mobility of mortar mix depends on the compositions of mortar and the mortar mixes to be used for masonry work are made sufficiently mobile.
- Placeability
  - The placeability of mortar mix should be such that a strong bond is developed with the surface of the bed.

Properties of a Good Mortar

- It should be capable of developing good adhesion with the building units such as bricks, stones etc.

- It should be capable of developing the designed stresses.
- It should be cheap.
- It should be durable.
- It should be easily workable.
- It should set quickly so that speed in construction may be achieved.

Uses of Mortar

- To bind the building units such as bricks, stones.
- To carry out pointing and plaster work on exposed surfaces of masonry.
- To form an even and soft bedding layer for building units.
- To form joints of pipes.
- To hide the open joints of brickwork and stonework.
- To improve the general appearance of structure.

Functions of Sand in Mortar

1. Bulk 2. Setting 3. Shrinkage 4. Strength.

Tests for Mortars

1. **Adhesiveness to Building Units:** Mortar is placed to join them so as to form a horizontal joint. If size of bricks is 19 cm × 9 cm × 9 cm, a horizontal joint of 9 cm × 9 cm = 81 cm^2 will be formed. Ultimate adhesive strength of mortar per cm^2 area is obtained by dividing maximum load with 81 cm^2 area.
2. **Crushing Strength:** Brick masonry or stone masonry laid in mortar to be tested are crushed in compression machine. The load at which the masonry crushes gives the crushing strength
3. **Tensile Strength:** The briquettes are tested in a tension testing machine. Cross-sectional area of central portion is 38 mm × 38 mm or 1444 mm^2 or 14.44 cm^2.

GUNITING

- The guniting is the most effective process of repairing concrete work which has been damaged due to inferior work or other reasons. It is also used for providing an impervious layer.
- Gunite is a mixture of cement and sand, the usual proportion being 1 : 3. A cement gun is used to deposit this mixture on the concrete surface under a pressure of about 2 to 3 kg/cm^2.
- The surface to be treated is cleaned and washed. The nozzle of gun is generally kept at a distance of about 75 cm to 85 cm from the surface to be treated and velocity of nozzle varies from 120 to 160 m/sec.

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Lime

3

Some Basic Definitions

1. **Calcination:** The heating of limestone to redness in contact with air is known as the calcination.
2. **Hydraulicity:** It is the property of lime by which it sets or hardens in damp places, water or thick masonry walls where there is no free circulation of air.
3. **Quick Lime:** The lime which is obtained by the calcination of comparatively pure limestone is known as the quick lime or caustic lime. It is capable of slaking with water and has no affinity for carbonic acid.
   - Its chemical composition is (CaO) oxide of calcium and it has great affinity for moisture.
   - The quick lime as it comes out from kilns is known as the lump lime.
4. **Setting:** The process of hardening of lime after it has been converted into paste form is known as the setting. It is quite different from mere drying.
5. **Slaked Lime:** The product obtained by slaking of quick lime is known as the slaked lime or hydrate of lime. It is in the form of white powder and its chemical composition is $Ca(OH)_2$ or hydrated oxide of calcium.

$$\underset{\text{Quick Lime}}{CaO} + \underset{\text{Water}}{H_2O} \longrightarrow \underset{\text{(Hydrated Lime)}}{Ca(OH)_2} + \text{Heat}$$

6. **Slaking:** When water is added to the quick lime in sufficient quantity a chemical reaction takes place.
   - Due to this chemical reaction the quick lime cracks, swell and falls into a powder form which is the calcium hydrate $Ca(OH)_2$ and it is known as the hydrated lime.
   - This process is known as the slaking.

Classification of Limes

(i) Fat Lime (ii) Hydraulic lime (iii) Poor lime or lean lime

1. **Fat Lime:** This lime is also known as the high calcium lime. Pure Lime, rich lime or white lime. It is popularly known as the fat lime as it slakes vigorously and its volume is increased to about 2-2.5 times the volume that of quick lime. The percentage of impurities in such limestone is less than 5%.
2. **Hydraulic Lime:** This lime is also known as the water lime as it sets under water. It contains clay and some amount of ferrous oxide. Depending upon the percentage of clay present the hydraulic lime is divided into following three types.
   - Feebly hydraulic lime
   - Moderately hydraulic lime
   - Eminently hydraulic lime

   The Hydraulic lime can set under water and in thick walls where there is no free circulation of air.
3. **Poor Lime:** This lime is also known as the impure or lean lime. It contains more than 30% of clay. It slakes very slowly.

Impurities in Limestones

1. **Magnesium carbonate**
   - The magnesium limestones are hard, heavy and compact in texture.
   - The magnesium limestones display irregular properties of calcination, slaking and hardening.
   - Upto 5% of magnesium oxide imparts excellent hydraulic properties to the lime.
2. **Clay**
   - It is mainly responsible for the hydraulic properties of lime.
   - The percentage of clay to produce hydraulicity in lime stone usually varies from 10 to 30.
   - Limes containing 3-5 per cent of clay do not display any hydraulic property and do not set and harden under water.
3. **Silica:** In its free form it has a detrimental effect of the properties of lime.
4. **Iron compounds**
   - Iron occurs in small proportions as oxides, carbonates and sulphides.
   - Pyrite or iron sulphide is regarded to be highly undesirable.
   - For hydraulic limes 2-5 per cent of iron oxide is necessary.
5. **Sulphates:** Sulphates if present slow down the slaking action and increase the setting rate of limes.
6. **Alkalis:** When pure lime is required the alkalis are undesirable. However, up to 5 per cent of alkalis in hydraulic lime do not have any ill effect.

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Concrete

4

Corrosion of steel in concrete

The term corrosion is used to indicate the conversion of metals by natural agencies into various compounds. The term rusting is used to refer corrosion of ferrous metals.

Theories of Corrosion

1. Chemical Action Theory

$Fe + O + 2CO_2 + H_2O = Fe(HCO_3)_2$ ...(i)

$2Fe(HCO_3)_2 + O = 2Fe(OH)CO_3 + 2CO_2 + H_2O$ ... (ii)

$Fe(OH)CO_3 + H_2O = Fe(OH)_3 + CO_2$ ...(iii)

- The combined action of oxygen, carbon dioxide and moisture on steel results in soluble ferrous bicarbonate $Fe(HCO_3)_2$. This ferrous bicarbonate is then oxidized to basic ferric carbonate $2Fe(OH)CO_3$. This basic ferric carbonate is converted into hydrated ferric oxide $Fe(OH)_3$ (final product) and carbon dioxide is liberated.

2. Electrolytic Theory: According to this theory, metal contains anodic and cathodic areas and these areas, when connected by electrolytes such as water, moisture, aqueous solutions, etc. cause corrosion.

Causes of Corrosion

(i) Congested reinforcement in small concrete sections.
(ii) Excessive water-cement ratio.
(iii) Improper construction methods.
(iv) Inadequate design procedure.
(v) Insufficient cover to steel from exposed concrete surface.
(vi) Presence of moisture in concrete.
(vii) Presence of salts.

Effect of Corrosion

Important effect of corrosion is the formation of cracks and these cracks usually progress or advance most rapidly where shearing stresses are the greatest and where slipping occurs due to loss of bond.

Water-cement Ratio

Important properties of water to be used for cement concrete are:

- Content of organic solids not more than 0.02%.

- Content of inorganic solids not more than 0.30%
- Content of sulphates not less than 0.05%.
- Content of sulphate alkali chlorides not more than 10%.
- Turbidity not more than 2000 ppm.
- Acid not more than 10,000 ppm.
- pH should be between 4.5 to 8.5.

Bleeding of Concrete

If excess water in the mix comes up at the surface causing small pores through the mass of concrete, it is called bleeding.

Segregation

It is caused when coarse aggregate is separated out from the finer materials resulting in large voids, less durability and less strength.

Some rules-of-thumb are developed for deciding the quantity of water in concrete.

- Weight of water = 28% of the weight of the cement + 4% of the weight of total aggregate.
- Weight of water = 30 % of the weight of the cement + 5% of the weight of total aggregate.

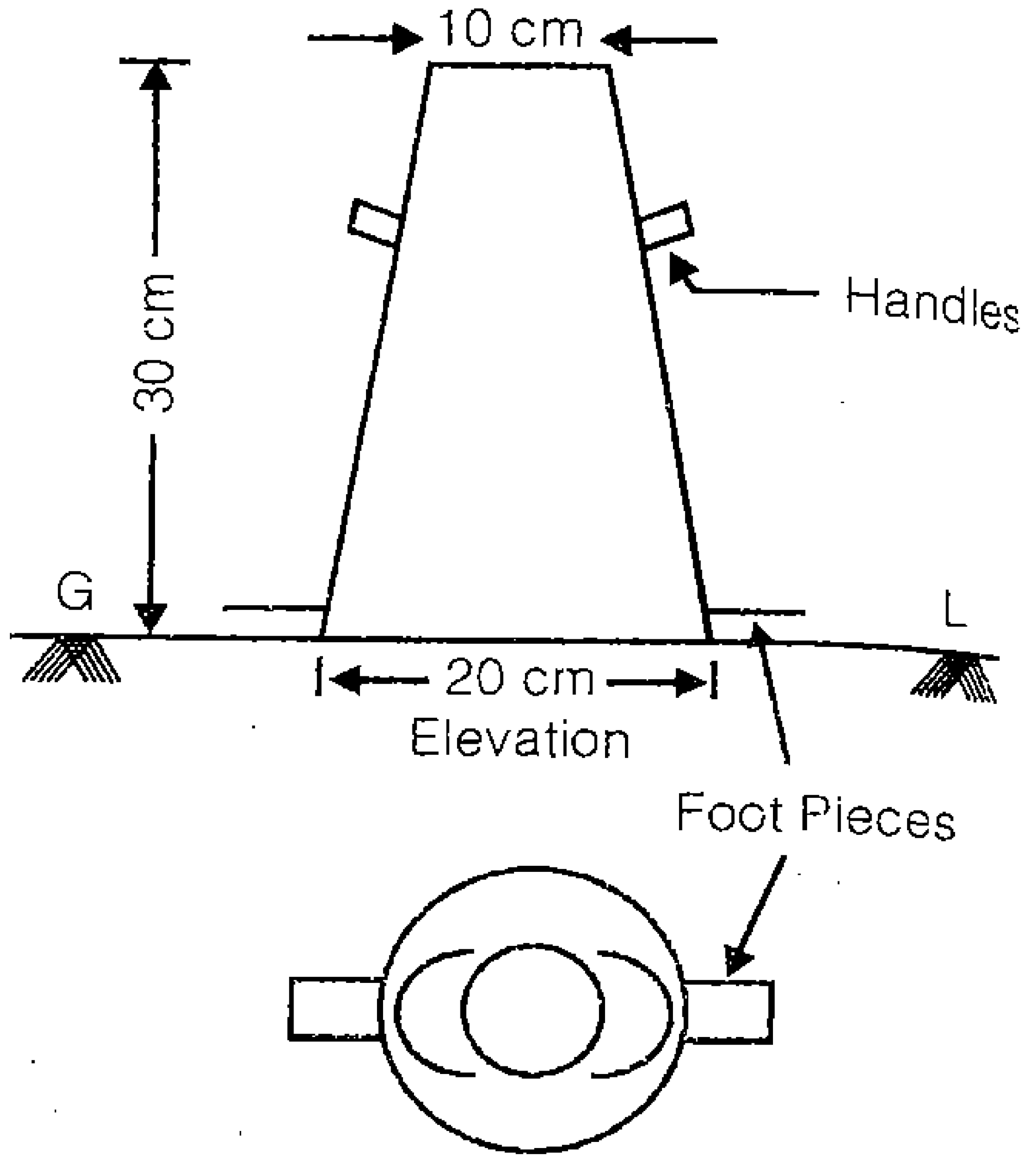
Workability of Concrete

- Workability is the amount of work to produce full compaction.
- The important facts in connection with workability are:
  (i) If more water is added to attain the required degree of workmanship, it results into concrete of low strength and poor durability.
  (ii) If the strength of concrete is not to be affected, the degree of workability can be obtained:
    - by slightly changing the proportions of fine and coarse aggregates, in case the concrete mixture is too wet; and
    - by adding a small quantity of water cement paste in the proportion of original mix, in case the concrete mixture is too dry.
  (iii) The workability of concrete is also affected by the maximum size of the coarse aggregates to be used in the mixture.

> *Remember* In order to measure the workability of concrete mixture, various tests are developed. Tests such as flow test, Vee-Bee test and compaction factor test are used in great extent in laboratory. Slump test, which is commonly used in the field, is briefly described below. Test for workability

Slump Test

- Slump test is the most commonly used method of measuring consistency of concrete which can be employed either in laboratory or at site of work.
- It is not a suitable method for very wet or very dry concrete and stiff mix.
- It does not measure all factors contributing to workability.
- The diameter of the rod is 16 mm and its length is 60 cm. The strokes to be given for ramming vary from 20 to 30.



Plan Slump Cone

Recommended Slumps of Concrete

No.	Type of concrete	Slump
1.	Concrete for road construction	20 to 40 mm
2.	Beams and slabs	50 to 100 mm
3.	Normal RCC work	80 to 150 mm
4.	Mass concrete	25 to 50 mm
5.	Concrete to be vibrated	10 to 25 mm
6.	Impermeable work	75 to 120 m

Workability, Slump and Compacting Factor of Concretes with 20 mm or 40 mm Maximum Size of Aggregate

Degree of workability	*Slump mm*	*Compacting factor Small apparatus*	*Compacting factor Large apparatus*	*Use for which concrete is suitable*
Very Low	–	0.78	0.80	Roads vibrated by power-operated machines.
Low	25-75	0.85	0.87	Roads vibrated by hand-operated machines.
Medium	50-100	0.92	0.935	At the less workable end of this group, manually compacted flat slabs using crushed aggregates.
High	100-150	0.95	0.96	For sections with congested reinforcement.
Very High	–	–	–	Flow table test is more suitable.

Compaction Factor Test

- In the compaction factor test the degree of workability of concrete is measured in terms of internal energy required to compact the concrete thoroughly.
- A compaction factor of 0.95 represents flowing concrete having high workability; 0.92 plastic concrete having medium workability; 0.85 stiff plastic concrete having low workability and a compaction factor of 0.75 represents stiff concrete having very low workability.
- The compacting factor test is designed primarily for use in the laboratory but it can also be used in the field.
- The degree of compaction called the compacting factor is measured by the density ratio i.e., the ratio of the density actually achieved in the test to density of same concrete fully compacted.

Vee-Bee Test

- This is carried out in such a manner that the specimen concrete in the test receives more or less same treatment in respect of the method of placing as it would in actual execution of the work. This test is preferred for finding workability of stiff concrete mix having very low workability.
- In this test a Vee-Bee time of 5 to 3 seconds represent stiff plastic concrete having medium workability, 10 to 15 seconds represents stiff concrete of low workability and Vee-Bee time to 18 to 10 seconds represent very stiff concrete having very low workability.

Vee Bee Consistometer

- This is a good laboratory test to measure indirectly the workability of concrete.
- This test consists of a vibrating table, metal pot, a sheet metal concrete a standard iron rod.
- The time required for the shape of concrete to change from slump concrete shape to cylindrical shape in second is known as Vee Bee Degree.
- This method is very suitable for very dry concrete whose slump value can not be measured by slump test but the vibration is too vigorous for concrete with a slump greater than about 50 mm.

Flow Test

- This is a laboratory test which gives an indication of the quality of concrete with respect to consistency cohesiveness and the proneness to segregation.

- The spread or the flow of the concrete is measured and this flow is related to workability.

$$\text{Flow percent} = \frac{\text{Spread diams in cm} - 25}{25} \times 100$$

The value could range anything from 0-150%.
It can be realized that the compacting factor test measures the inherent characteristics of the concrete which relates very close to the workability requirements of concrete and as such it is one of the good test to depict the workability of concrete.

Estimating Yield of Concrete

- A rule-of-thumb as given below, may be used to find out the approximate yield of concrete from a given concrete mix.
- If the proportion of concrete is a : b : c, i.e., if a parts of cement, b parts of sand and c parts of coarse aggregates are mixed by volume, the resulting concrete will have a volume of 2/3 (a + b + c).
- Let w, a, b and c be absolute volumes of water, cement, fine aggregate and coarse aggregate respectively. Then, w + a + b + c = 1.

$$\text{Absolute volume} = \frac{\text{Weight of the materials}}{\text{Apparent sp. gr.} \times \text{Unit wt. of water}}$$

Methods for Proportioning Concrete Mixes

1. **Minimum voids method**
   The quantity of sand used should be such that it completely fills the voids of the coarse aggregate and similarly the quantity of cement used should be such that it fills the voids of sand. However in actual practices the quantity of sand used in the mix is kept 10% more than the voids in the coarse aggregate and the quantity of cement is taken 15 % more than the voids in the sand.
2. **Maximum density method**
   Method of minimum voids was later improved by Fuller. For maximum density of mix.
   He gave following expression.

$$P = 100\left(\frac{d}{D}\right)^{1/2}$$

D = Maximum size of aggregate.

P = % by weight of matter finer than diameter d.

3. **Abram's water-cement ratio law**
   - This law states that for any given conditions of test the strength of workable concrete mix is dependent only on the water cement ratio. It means that if the concrete is fully compacted, the strength is not affected by aggregate shape, type or surface texture or the aggregate grading. According to this law, the strength of mix increases with decrease in water content.
   - In terms of crushing strength after 7 days curing $P_7 = \frac{984}{7^x}\ \text{kg/cm}^2$
     where P_7 is cylinder crushing strength in kg/cm^2 and x is water cement ratio by volume.
   - In terms of crushing strength after 28 days curing $P_{28} = \frac{984}{4^x}\ \text{kg/cm}^2$
     where P_{28} is cylinder crushing strength after 28 days curing.
   - Strength of concrete increases with age.

Months	Age factor
1	1.00
3	1.10
6	1.15
12	1.20

MIX DESIGN

- When the task of deciding the proportion of the constituents of concrete is accomplished by use of certain established relationships (which are based on inferences drawn from large number of experiments) the concrete thus produced is termed as Design mix concrete.
- When the proportions of cement, aggregate and water are adopted based on arbitrary standard the concrete produced is termed as Nominal mix concrete.
- Nominal mix concrete is used in works where the quality control requirement for design mixes are difficult to be implemented. Nominal mix concrete can be produced by taking cement, fine and coarse aggregate in the ratio of 1 : n : 2n for normal work. However, the ratio of the coarse aggregate to fine aggregate can vary from 1.5 : 2.5 :1 in situations where denser or more workable concrete is to be produced.

AGGREGATES SIZE

- For RCC work the maximum size of aggregates is limited to 20-25 mm.

- For a concrete of given workability rounded aggregates require least water cement ratio. Particle shape is very important since the water cement ratio governs greatly the strength of concrete.
  Coarse aggregates > 4.75 mm size.
  Fine aggregates < 4.75 mm size.

Fineness modulus

- The fineness modulus of an aggregate is an index number which is roughly proportional to the average size of the particles in the aggregate. The coarser the aggregate, the higher the fineness modulus.
- Fineness modulus is obtained by adding the % of the weight of the material retained on the total 10 number of IS sieves (between 80 mm to 150 μm) and dividing it by 100.

Aggregate	Fineness modulus
Coarse aggregate	6 to 8.5 (in general 6.93)
Fine aggregate	2 to 3.5 (in general 3.05)
Mixed aggregate	4.7 to 7.0
Fine sand	2.2 to 2.6
Medium sand	2.6 to 2.9
Coarse sand	2.9 to 3.2

VIBRATORS

Following are the four types of vibrators:

1. **Internal Vibrators:** These vibrators consist of a metal rod which is inserted in fresh concrete. Skilled and experienced men should handle internal vibrators. These vibrators are more efficient than other types of vibrators.
2. **Surface Vibrators:** These vibrators are mounted on platform or screeds. They are used to finish concrete surfaces such as bridge floors, road slabs, station platform, etc.
3. **Form Vibrators:** These vibrators are attached to the formwork and external centering of walls, columns, etc. The vibrating action is conveyed to concrete through the formwork during transmission of vibrations. Hence they are not generally used. But they are very much helpful for concrete sections which are too thin for the use of internal vibrators.
4. **Vibrating Tables:** These vibrators are widely used for making precast products.
5. **Period of Curing:** The curing period is about 7 to 14 days.

Water-proofing Cement Concrete

- Cement concrete to a certain extent may be made impermeable to water by using hydrophobic cement.

Following are the three methods adopted for water-proofing of RCC flat roofs:

1. **Finishing:** For ordinary building of cheap construction, finishing of roof surface is done at the time of laying cement concrete. The finishing of flat roof is carried out in cement mortar of proportion 1:4, i.e., one part of cement to four parts of sand by volume.
2. **Bedding Concrete and Flooring:** In this method, the surface of RCC slab is kept rough and on this surface, a layer of concrete is laid. The concrete may be brickbats lime concrete (1:2:4) or brickbats cement concrete (1:8:14). The thickness of the concrete layer is about 10 cm.
3. **Mastic Asphalt and Jute Cloth:** In this method, a layer of hot mastic asphalt is laid on the roof surface. Jute cloth is spread over this year.

Lightweight Concrete

The bulk density of ordinary concrete is about 2300 kg/m^3. Concrete having bulk density between 500 to 1800 kg/m^3 is known as lightweight concrete and it is prepared from the following materials:

1. **Binding material :** Ordinary Portland cement and its varieties can be used as binding material.
2. **Aggregates :** For lightweight concrete, loose porous materials are used as aggregates.
3. **Steel :** Lightweight concrete is highly porous and hence, it leads to corrosion of reinforcement.

- The volume of one 50 kg bag of cement is 34.5 liters.
- About 19 liters (38 per cent) of water is required to hydrate 50 kg cement in sealed container.
- The ordinary Portland cement when tested for its compressive strength at 28 days according to the method described by the standard specification, yields a minimum compressive strength of 43 N/mm^2 is called 43 grade cement.
- Too fine cement is susceptible to air set and deteriorates earlier.
- The setting time of cement can be controlled by varying the quantity of gypsum in cement.
- Sometimes with the addition of water to cement, a premature set occurs within 5 minutes. This is called false set and is due to the presence of anhydrous gypsum which is formed due to grinding of gypsum with too hot clinker. This is not to be worried. Continuous

mixing aggregates, cement and water will break the false set without harming any property of the concrete.

- The ability of cement to maintain a constant volume is known as soundness of cement.
- The ability of aggregate to resist excessive changes in volume due to changes in physical conditions is known as soundness of aggregate. In laboratory, it is found by determining resistance of aggregate to disintegration by saturated solution of sodium sulphate or magnesium sulphate.
- Concrete with increased workability without adding more water is called flowing concrete.
- M20 is the designation of concrete mix. Letter M refers to the mix and number 20 refers to the characteristic strength of 15 cm cube after 28 days equal to 20 N/mm^2.
- **Bleeding of Concrete :** is said to occur when unreacted water in the mix tends to rise to the surface of freshly placed concrete due to sedimentation of constituents of concrete. This produces continuous capillary pores which provides a clear straight access to chemicals and deleterious materials in concrete and lowers the strength and workability of concrete.
- $$\text{Compaction factor} = \frac{\text{mass of partially compacted concrete}}{\text{mass of fully compacted concrete}}$$
- The normal tensile stress in concrete when cracking occurs in a flexure test is known as modulus of rupture of concrete. It is measured by performing a flexure test on un-reinforced concrete beam of specified size and span considering concrete to be homogeneous.

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5 Bricks

COMPOSITION OF GOOD BRICK EARTH

Following are the constituents of brick earth:

1. **Alumina**
   It is the chief constituent of every kind of clay. A good brick earth should contain about 20 to 30 per cent of alumina. This constituent imparts plasticity to earth so that it can be moulded.
2. **Silica**
   - A good brick earth should contain about 50 to 60 per cent of silica. Presence of this constituent prevents cracking, shrinking and warping of raw bricks. It thus imparts uniform shape to the bricks.
   - Excess of silica destroys the cohesion between particles and bricks become brittle.
3. **Lime**
   - It should be present in a finely powdered state and not in lump.
   - Lime prevents shrinkage of raw bricks. Sand alone is infusible. But it slightly fuses at kiln temperature in presence of lime.
   - Excess of lime causes the brick to melt and hence, its shape is lost. Lumps of lime are converted into quick lime after burning and this quicklime slakes and expands in presence of moisture.
4. **Oxide of Iron**
   - About 5 to 6 per cent is desirable in good brick earth. It helps lime to fuse sand. It also imparts red colour to bricks.
   - Excess of oxide of iron makes the bricks dark blue or blackish.
5. **Magnesia**
   A small quantity of magnesia in brick earth imparts yellow tint colour to bricks and decreases shrinkage. But excess of magnesia leads to the decay of bricks.

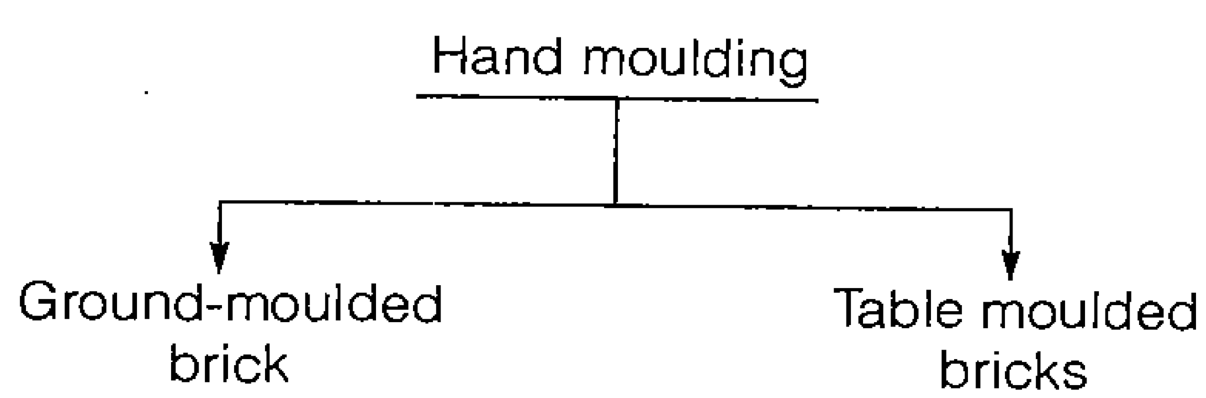
Harmful Ingredients in Brick Earth

1. **Lime**
   - It causes unsoundness in brick if present in excess amounts.
2. **Iron pyrites**
   - If iron pyrites are present in brick earth, bricks are crystallized and disintegrated during burning.

3. **Alkalies**
These are mainly in the form of soda and potash.
4. **Pebbles**
The presence of pebbles or grits of any kind is undesirable in brick earth because it will not allow the clay to be mixed uniformly and thoroughly which will result in weak and porous bricks.
5. **Organic Matter**
Presence of organic matter in brick earth assists in burning. But if such matter is not completely burnt, bricks become porous.

Manufacture of bricks

1. **Preparation of clay**
Clay of bricks is prepared in the following order:
(i) Unsoiling (ii) Digging
(iii) Cleaning (iv) Weathering
(v) Blending (vi) Tempering
2. **Moulding**
   - Hand Moulding



   - Machine Moulding
     - Plastic Clay Machine
     - Dry Clay Machines
3. **Drying**
4. **Burning:** Burning of bricks is done either in clamps or in kilns.
   (a) Clamps: Its shape in plan is generally trapezoidal. Floor of clamp is prepared in such a way that short end is slightly in the excavation and wider end is raised at an angle of about 15° from ground level.
   (b) Kilns: A kiln is a large over which is used to burn bricks. The kiln which are used in the manufacture of bricks are of the following two types.
   (i) Intermittent kilns (ii) Continuous kilns
   (i) Intermittent Kiln: This may be over ground or under ground they are classified in two ways: (a) Intermittent up-drought kilns, (b) Intermittent down-drought kilns

Comparison between bull's trench kiln and Hoffman's kiln

No. Item	Bull's trench kiln	Hoffman's kiln
1. Burning capacity	About 3 Lakhs in 12 days	About 40 lakhs in one season
2. Continuity of working	It stops functioning during monsoon as it is not provided with a permanent roof	It functions all the year with a permanent roof
3. Cost of fuel	High as consumption of fuel is more	Low
4. Drying space	It requires more space	It requires less space
5. Initial cost	Low	High
6. Nature	It is semi-continuous in loose sense	It is continuous is nature
7. Popularity	More popular because of less initial cost	Less popular because of high initial cost
8. Quality of bricks	Percentage of good quality brick is small	Percentage of good quality bricks is more.

Comparison between clamp Burning and kiln Burning

No. Item	Clamp-burning	Kiln burning
1. Capacity	About 20000-100000	avg. 25000
2. Cost of fuel	Low as grass, cow dung, litter may be used	High because of coal dust is to be used
3. Initial cost	Very low as no structures are to be built	More as permanent structures are to be constructed
4. Quality of bricks	The percentage of good quality bricks is small about 60%	Percentage of good quality bricks is high 90%
5. Regulation of fire	It is not possible to control or regulate fire during the process of burning	The fire is under control throughout the process of burning
6. Skilled supervision	Not necessary through out the process of burning	The continuous skilled supervision is necessary
7. Structure	Temporary structure	Permanent structure
8. Suitability	For small scale	For large scale
9. Time of burning and cooling	It requires about 2-6 months.	Actual burning times is 24 hr. and 12 days are required for cooling of bricks.

TESTS FOR BRICKS

1. **Absorption**
   - A brick is taken and it is weighed dry. It is then immersed in water for a period of 16 hours.
   - Then weight again and the difference in weight should not, in any case, exceed
     (a) 20 per cent of weight of dry brick for first class bricks.

(b) 22.5 per cent for second class bricks.
(c) 25 per cent for third class bricks.

2. Crushing strength
   - Minimum crushing strength for first class bricks $\not< 10\ N/mm^2$ and for second class bricks $\not< 7.5\ N/mm^2$
3. Hardness
   In this test, a scratch is made on brick surface with the help of a finger nail. If no impression is left on the surface, brick is treated to be sufficiently hard.
4. Presence of soluble salts
   - Soluble salts, if present in bricks, will cause efflorescence on the surface of bricks.
   - It is immersed in water for 24 hours. It is then taken out and allowed to dry in shade. Absence of grey or white deposits on its surface indicates absence of soluble salts.
   - If the white deposits cover about 10% surface, the efflorescence is said to be slight.
   - When white deposit cover about 50% of surface thin it is said to be moderate.
   - If grey or white deposits are found on more than 50% of surface, the efflorescence becomes heavy and it is treated as serious.
5. Shape and Size
   - Its shape should be truly rectangular with sharp edges.
   - 20 bricks are randomly selected of standard size ($19 \times 9 \times 9$ cm) for good quality bricks, the results should be within the following permissible limits:

     Length – 368 cm to 392 cm
     Width – 174 cm to 186 cm
     Height – 174 to 186 cm
6. Soundness
   - In this test, two bricks are taken and they are struck with each other.
   - Bricks should not break and a clear ringing sound should be produced.
7. Structure
   - It should be homogeneous, compact and free from any defects such as holes, lumps, etc.
   - High duty fire-clays can resist temperature range of 1482°C to 1648°C; medium duty fire-clays can resist temperature range of 1315°C to 1482°C and low duty fire-clays can resist temperature up to 870°C only.

Quality of Good Bricks

- The bricks should be table-moulded, well burnt in kilns, copper-coloured free from cracks and with sharp and square edges.
- The bricks should be uniform in shape and should be of standard size.
- The bricks should give a clear metallic ringing sound when struck with each other.
- The bricks when broken or fractured should show a bright homogeneous and uniform compact structure free from voids.
- The brick should be sufficiently hard. No impression should be left on brick surface, when it is scratched with finger hail.
- The bricks should not break into pieces when dropped flat on hard ground from a height of about one meter.
- The bricks, when soaked in water for 24 hour should not show deposits of white salts when allowed to dry in shade.
- No brick should have the crushing strength below $5.50\ N/MM^2$.

Classification of Bricks

The bricks can broadly be divided into two categories:

1. **Unburnt Bricks:** The unburnt or sun dried bricks are dried with the help of heat received from sun after the process of moulding. These bricks can only be used in the construction of temporary and cheap structures. Such bricks should not be used at places exposed to heavy rains.
2. **Burnt Bricks:** These are classified in four categories:

   (i) First Class Bricks
   - These bricks are table-moulded and of standard shape and they are burnt in kilns.
   - The surfaces and edges of the bricks are sharp square smooth and straight.
   - First class bricks have all qualities of good bricks.
   - These bricks are used for superior work of permanent nature

   (ii) Second Class Bricks
   - These bricks are ground moulded and they are burnt in kilns.
   - The surface of these bricks is some what rough and shape is also slightly irregular.
   - These bricks are commonly used at places where bricks work is to be provided with a coat of plaster.

   (iii) Third Class Bricks
   - These are ground moulded and they are burnt in clamps.

- These bricks are not hard and they have rough surface with irregular and distorted edges.
- These bricks gives dull sound when struck together.
- They are used for unimportant and temporary structures.

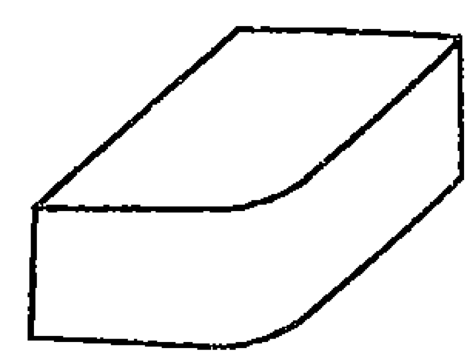
SIZE AND WEIGHT OF BRICKS

For India

- Standard size of bricks is 19 cm × 9 cm × 9 cm
- Nominal size (with mortar) is 20 cm × 10 × 10 cm.
- The commonly adopted nominal size of traditional bricks is 23 cm × 11.4 cm × 7.6 cm.
- It is found that the weight of 1 m^3 of bricks earth is about 1800 kg. Hence the average weight of a brick will be about 3 to 3.50 kg.

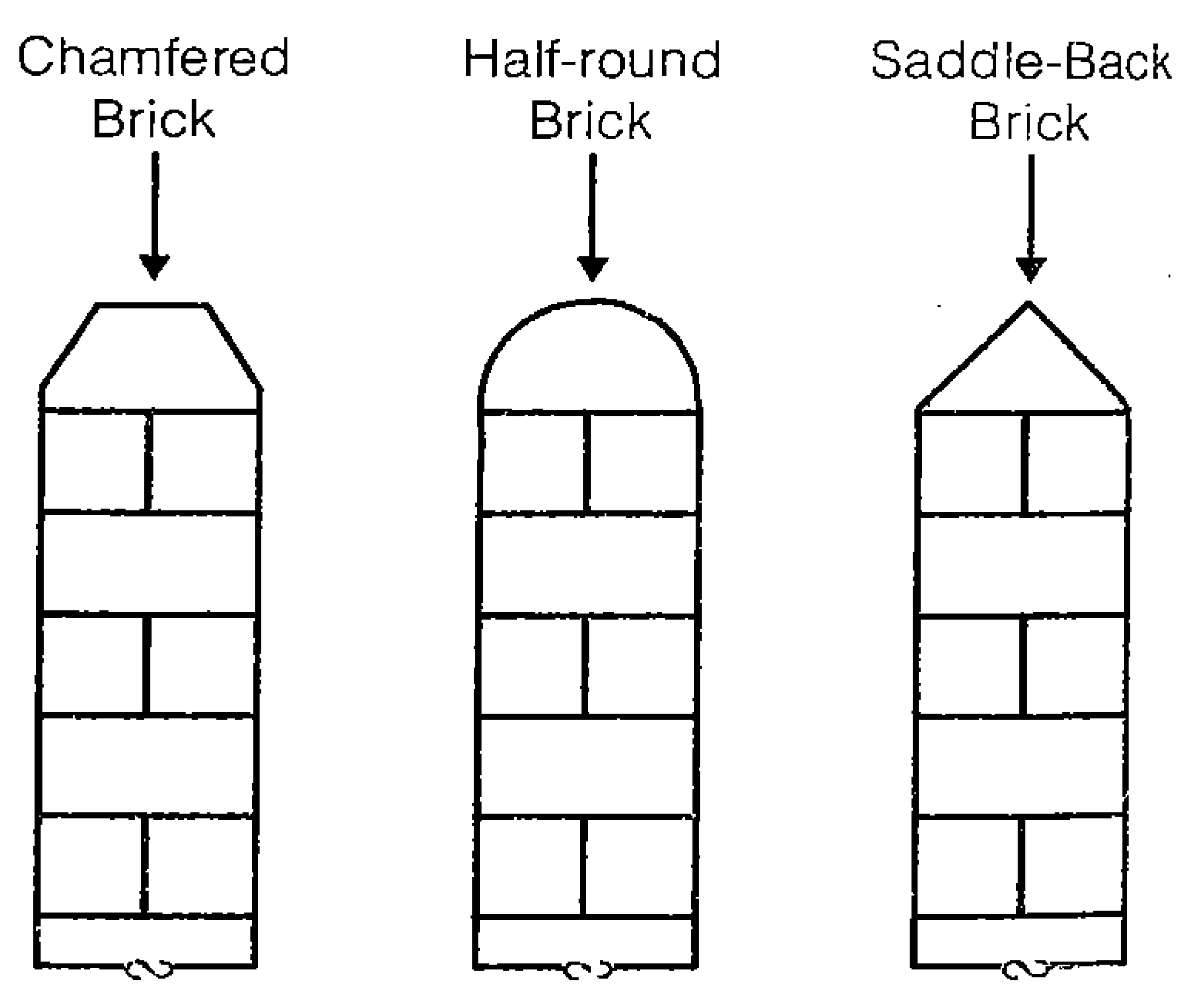
Shape of Bricks

1. Bullnose Brick
   - A brick moulded with a rounded angle is termed as a bullnose. It is used for a rounded quoin.
   - A connection which is formed when a wall takes a turn is known as quoin.



Bullnose Brick

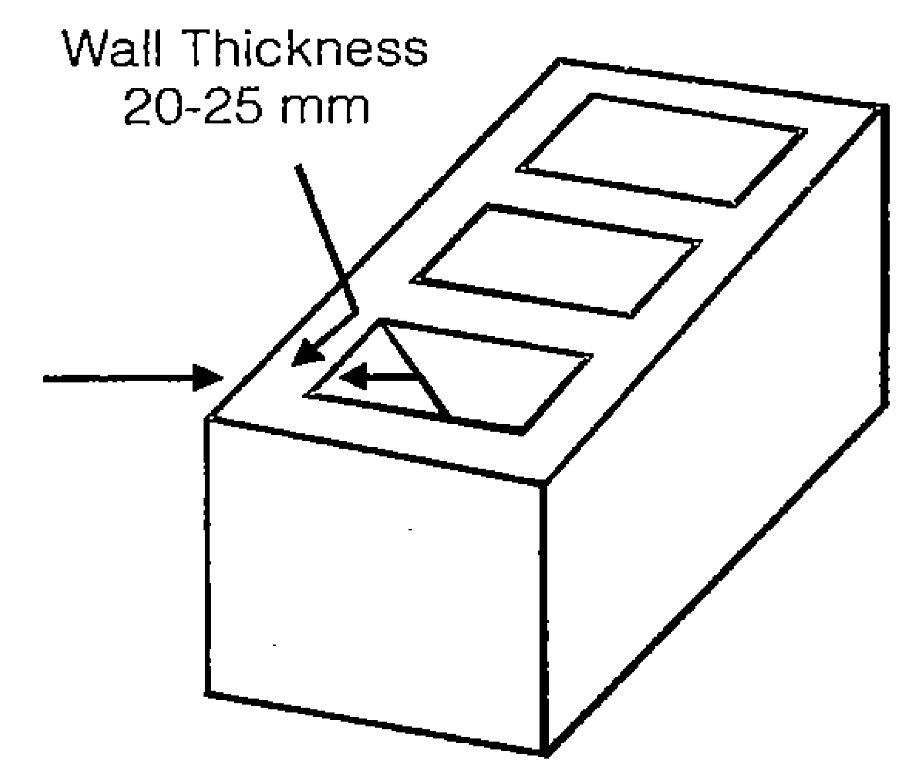
2. Channel Bricks
   - These bricks are moulded to the shape of a gutter or a channel and they are very often glazed.
   - These bricks are used to function as drain.
3. Coping bricks
   - These bricks are made to suit the thickness of walls on which coping is to be provided.
   - Such bricks take various forms such as chamfered half-round or saddle-back.



Brick Copings

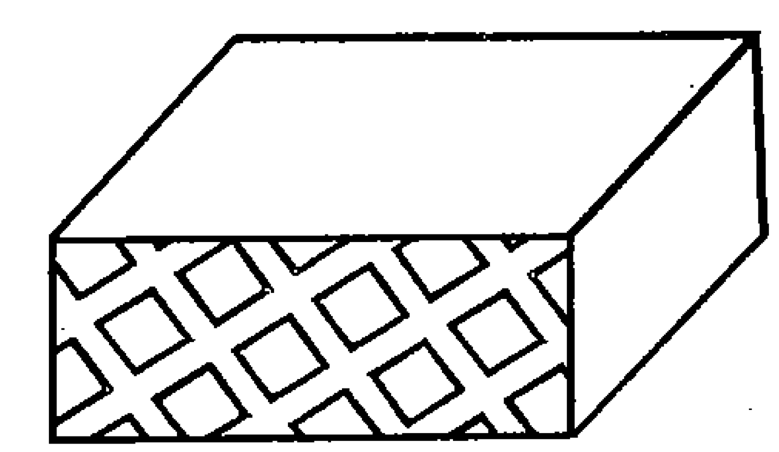
4. Cownose Bricks
   - A brick moulded with a double bullnose on end is known as cownose.

5. Curved Sector Bricks
   - These bricks are in the form of curved sector and they are used in the construction of circular brick masonry pillars, brick chimneys.
   - The perforation may be circular, square, rectangular or any other regular shape in cross-section.
   - The water absorption after immersion for 24 hour in water should not exceed 15% by water.
   - compressive strength of perforated bricks should not be less than 7 N/mm^2 on gross area.
6. Hollow Bricks
   - These are also known as cellular or cavity bricks. Such bricks have wall thickness of about 20 mm to 25 mm. They are prepared from special homogeneous clay. They are light in weight about one third the weight of the ordinary bricks of the same size. The use of such bricks leads to speedy construction. They also reduce the transmission of heat, sound and damp. They are used in the construction of brick partitioning.



Hollow Brick

7. Paving bricks
   - These bricks are prepared from clay containing a higher percentage of iron. Excess iron vitrifies the bricks at a low temperature. Such bricks resist better the abrasive action of traffic. Paving bricks may be plain or checkered.



Chequered Brick

8. Perforated Bricks
   - Perforated bricks are used in the construction of brick panels for lightweight structures and multi-storeyed framed structures.

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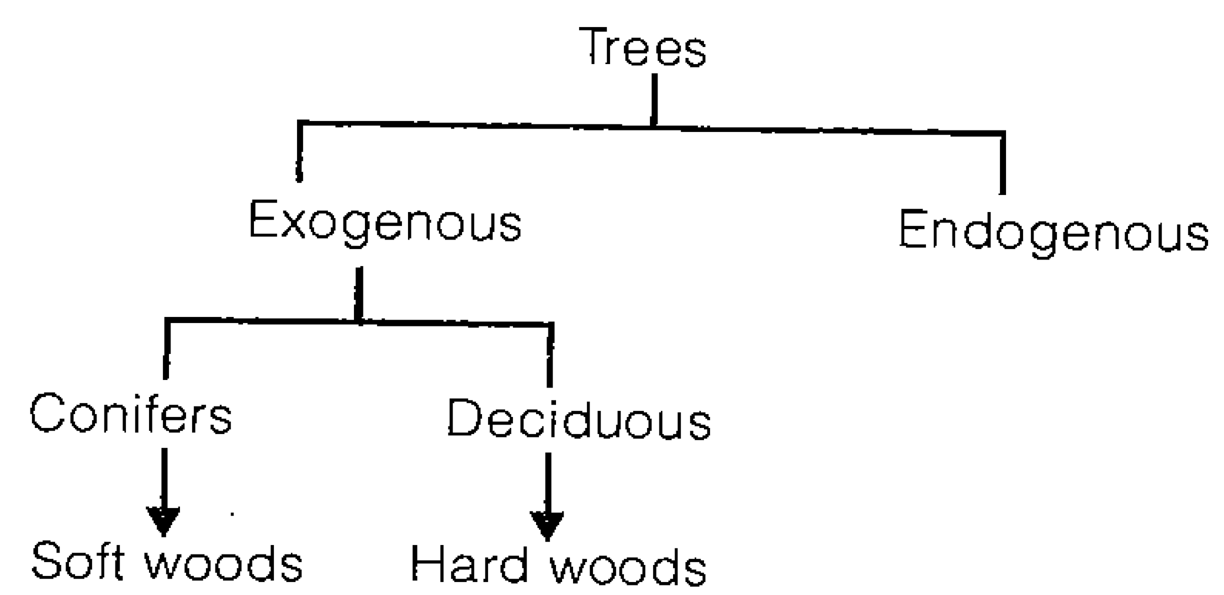
Timber

6

Classification of trees

- Trees are classified according to their mode of growth. Following is the classification of trees:



1. Exogenous Trees
   - Conifers are also known as evergreen trees and leaves of these do not fall till new ones are grown. As these trees bear cone-shaped fruits, they are given the name conifers. These trees yield soft woods.
   - Deciduous trees are also known as broadleaf trees and leaves of these trees fall in autumn and new ones appear in spring season. Timber for engineering purposes is mostly derived from deciduous trees. These trees yield hard woods.

Comparison of Soft Wood and Hard Wood

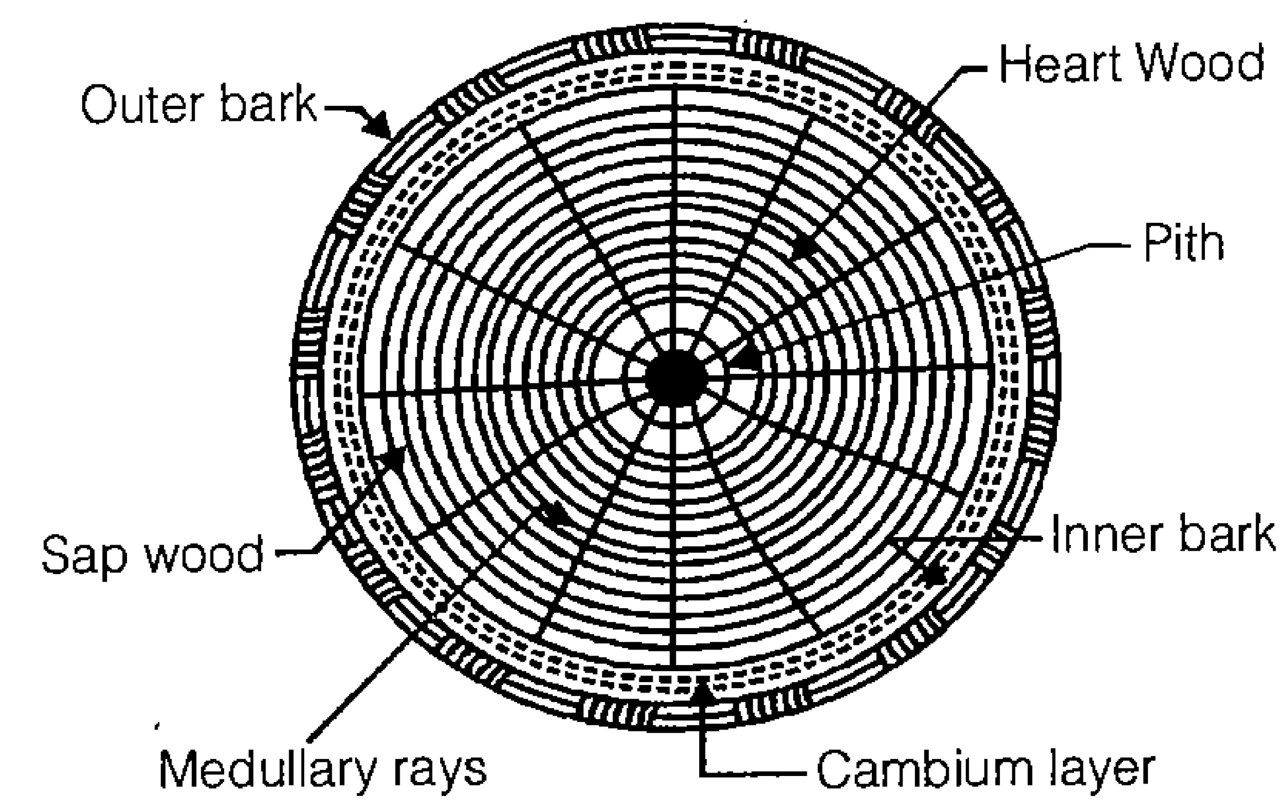
No.	Item	Soft Woods	Hard Woods
1.	Annual rings	Distinct	Indistinct
2.	Colour	Light	Dark
3.	Fire resistance	Poor	More
4.	Medullary rays	Indistinct	Distinct
5.	Strength	Strong for direct pull and weak for resisting thrust of shear	Equally strong for resisting tension, compression and shear
6.	Structure	Resinous and split easily	Non-resinous and close-grained
7.	Weight	Light	Heavy

2. Endogenous Trees
   These trees grow inwards and fibrous mass is seen in their longitudinal sections. Timber from these trees has very limited engineering applications. Examples of endogenous trees are bamboo, cane, palm, etc.

STRUCTURE OF A TREE

From the visibility aspect, the structure of a tree can be divided into *two* categories:

1. **Macrostructure**
   (i) **Pith:** The innermost central portion or core of the tree is called the pith or medulla.
   (ii) **Heart Wood:** The inner annual rings surrounding the pith is known as heart wood. It is usually dark in colour.



Cross-section of an exogenous tree

   (iii) **Sap Wood:** The outer annual rings between heart wood and cambium layer is known as sap wood.
   (iv) **Cambium Layer:** The thin layer of sap between sap wood and inner bark is known as cambium layer.
   (v) **Inner Bark:** It gives protection of cambium layer from any injury.
   (vi) **Outer Bark:** It consists of cells of wood fibre and is also known as cortex.
   (vii) **Medullary Rays:** The thin radial fibres extending from pith to cambium layer are known as *medullary rays*.
2. **Microstructure**
   - Wood consists of living and dead cells of various sizes and shapes.
   - A living cell consists of four parts, namely (i) membrane, (ii) protoplasm (iii) sap (iv) core. Cell membrane consists mainly of cellular tissue and cellulose. Protoplasm is a granular, transparent, viscous vegetable protein composed of carbon, hydrogen, oxygen,

nitrogen and sulphur. Core of cell differs from protoplasm merely by the presence of phosphorus and it is generally oval.

- **Age of trees for felling:** The age of good trees for felling varies from 50 to 100 years.
- **Season for felling:** In autumn and spring, sap is in vigorous motion and hence, felling of trees in these seasons should be avoided. For hilly areas, mid-summer would be the proper season for felling as there is heavy rainfall in winter. For plain areas, mid-winter would be the proper season for felling as in summer, water contained in sap would be easily evaporated and it will lead to the formation of cracks.

DEFECTS IN TIMBER

Defects occurring in timber are grouped into the following five divisions.

1. **Defect Due to Conversion**
   (i) Chip mark (ii) Diagonal grain
   (iii) Torn grain (iv) Wane
2. **Defects Due to Fungi**
   (i) Blue Stain (ii) Brown Rot
   (iii) Dry Rot (iv) Heart rot
   (v) Sap Stain (vi) Wet Rot
   (vii) White Rot
3. **Defects Due to Insects**
   (i) Beetles (ii) Marine Borers
   (iii) Termites
4. **Defects Due to Natural Forces**
   (i) Burls (ii) Callus
   (iii) Chemical stain (iv) Coarse grain
   (v) Dead wood (vi) Druxiness
   (vii) Foxiness (viii) Knots
   (ix) Rind galls (x) Shakes
   (xi) Twisted fibres (xii) Upsets
   (xiii) Water stain (xiv) Wind cracks
5. **Defects Due to Seasoning**
   Following defects occur in seasoning process of wood.
   (a) Bow (b) Case-hardening
   (c) Check (d) Collapse
   (e) Cup (f) Honey-combing
   (g) Radial shakes (h) Split
   (i) Twist (j) Warp

PRESERVATION OF TIMBER

Preservation of timber is carried out to achieve the following three objects:

- To increase the life of timber structures
- To make the timber structures durable, and
- To protect the timber structures from the attack of destroying agencies such as fungi, insects, etc.

Requirements of a Good Preservative

- It should allow decorative treatment on timber after being applied over timber surface.
- It should be capable of covering a large area with small quantity.
- It should be cheap and easily available.
- It should be free from unpleasant smell.
- Its penetrating power into wood fibres should be high. It is necessary for the preservative to be effective to penetrate at least for a depth of 6 mm to 25 mm.

Types of Preservatives

1. **Ascu Treatment**
   - Ascu is special preservative which is developed at the Forest Research Institute, Dehradun. Its composition is as follows.
   - X- Part by weight of hydrated arsenic pentoxide, ($As_2O_5.2H_2O$).
   - Y- Part by weight of blue vitriol or copper sulphate, ($CuSO_4.5H_2O$).
   - Z-Part by weight of potassium dichromate ($K_2Cr_2O_7$) or sodium dichromate ($Na_2Cr_2O_7.2H_2O$)
   - This preservative gives timber protection against the attack of white ants. The surface treated with this preservative can be painted, polished, varnished or waxed.
2. **Chemical Salts**
   - These are water-borne preservatives and they are mostly salts dissolved in water. The usual salts used are copper sulphate, mercury chloride, sodium fluoride and zinc chloride.
   - These preservatives are odourless and non-inflammable.
3. **Coal Tar**
   Its cheapness and effective resistance. Coal tar is fire-resistant.
4. **Creosote oil**
   In this case, timber surface is coated with creosote oil.
   - Creosote oil is one of the best antiseptic. It is a black or brown liquid, weakly affected by water, neither volatile nor hygroscopic,

harmless to wood or metal, inflammable, with an unpleasant odour and having low wood-penetrating ability to the extent of 1 mm to 2 mm only.

- Creosote oil should not be used for interior surfaces of dwelling houses, foodstuff-storage premises, in underground installations and near inflammable surfaces.

5. Oil Paints
   - The timber surface is coated with 2 or 3 coats of oil paint.
   - The wood should be seasoned, otherwise sap will be confined and if will lead to the decay of timber.
   - The oil paints preserve timber from moisture and make it durable.
6. Solignum Paints
   - These paints preserve timber from white ants as they are highly toxic in nature.
   - They can be mixed with colour pigment and applied in hot state with the help of brush.
   - The timber surface may therefore be given the desired colour or appearance.

Methods for Preservation

There are six Methods Adopted for Preservation of Timber:

1. Brushing
   - The solution prepared from preservative is applied on timber surface by good quality of brushes.
   - This is the simplest method and it is generally adopted for seasoned timber.
   - The crocks should be filled up before the application of preservative.
2. Charring
   - The surface to be charred is kept wet for about half an hour and it is then burnt up to a depth of about 15 mm over a wood fire.
   - The charred portion is then cooled with water.
   - Due to burning, a layer of coal is formed on the surface.
   - This layer is not affected by moisture and it is not attacked by white ants, fungi.
   - The disadvantage of this method are:
     (i) The charred surface becomes black in appearance and hence it cannot be used for exterior work.
     (ii) There is some loss of strength of timber as the cross-section is reduced due to charring.

3. Dipping and Steeping
   - In this method, the timber to be given preservative treatment is dipped or soaked for a short period in the solution of preservative.
   - This method gives slightly better penetration of preservative than in case of brushing or spraying.
4. Hot and Cold Open Tank Treatment
   - In this method, the timber is submerged in a tank containing solution of preservative which is heated for a few hours at temperature of 85°C-95°C.
   - Tank is then allowed to cool down gradually while the timber is still submerged in the tank.
   - This method is effective in giving protection to the sap wood.
5. Injecting Under Pressure
   - This method proves to be essential for treating non-durable timbers which are to be used as places where there is danger of attack by fungi and insects.
6. Spraying
   - In this method the solution of preservative is filled in a spraying pistol and it is then applied on timber surface under pressure.
   - This method is also quite effective and it is superior than brushing.

FIRE RESISTANCE OF TIMBER

1. Application of Special Chemicals
   - It is found that two coats of solution of borax or sodium arsenate with strength of 2 per cent are quite effective in rendering the timber fire-resistant.
   - When the temperature rises, they either melt or give off gases which hinder or forbid combustion.
2. Sir Abel's Process

   In this process, timber surface is cleaned and it is coated with a dilute solution of sodium silicate. A cream-like paste of slaked fat lime is then applied and finally, a concentrated solution of silicate of soda is applied on the timber surface.

SEASONING OF TIMBER

1. Objects of Seasoning
   - To allow timber to burn readily, if used as fuel.
   - To decrease the weight of timber and thereby to lower the cost of transport and handling.
   - To make timber safe from the attack of fungi and insects.

- To reduce the tendency of timber to crack, shrink and warp.
- To make timber fit for receiving treatment of paints, preservatives, varnishes
- To import hardness, stiffness, strength and better electrical resistance to timber.

2. Methods of Seasoning

(a) Natural Seasoning

In this method, the seasoning of timber is carried out by natural air and hence it is alsc sometimes referred to as air seasoning.

Advantage

(i) Depending upon the climatic conditions, the moisture content of ward can be brought down to about 10-20%

(ii) It does not require skilled supervision

(iii) This method of seasoning timber is cheap and simple.

(iv) It is uneconomical to provide artificial seasoning to timber sections thicker than 100 mm, as such sections dry very slowly.

Disadvantage

(i) As the process depends on the natural air, it sometimes becomes difficult to control it

(ii) The drying of different surface may not be even and uniform.

(iii) If ends of thick sections of timber are not projected by suitable moistureproof coating, there are chances for end splitting.

(b) Artificial Seasoning

- Following are the reasons for adopting the artificial seasoning to the natural seasoning.
  (a) The defects such as shrinkage, cracking and warping are minimized.
  (b) The drying is controlled and there are practically no chances for the attack of fungi and insects.
  (c) The drying of different surface is even and uniform.
  (d) It considerably reduces the period of seasoning.
  (e) There is better control of circulation of air, humidity and temperature.

(i) Boiling

In this method of artificial seasoning, timber is immersed in water and water is then boiled. But it affects the elasticity and strength of wood.

(ii) Chemical seasoning

This is also known as salt seasoning. In this method, timber is immersed in a solution of suitable salt. It is then taken out and seasoned in the ordinary way.

(iii) Electrical seasoning

- In this method, use is made of high frequency alternating currents.
- This is the most rapid method of seasoning.
- Due to high cost this method is unecomonical.

(iv) Kiln Seasoning

- In this method, drying of timber is carried out inside an airtight chamber or oven.

(v) Water Seasoning

- Timber pieces are immersed wholly in water, preferably in running water of a stream. Care should be taken to see that timber is not partly immersed.
- Timber is taken out after a period of about 2 to 4 weeks. During this period, sap contained in timber is washed away by water.

■■■

A Handbook on

Civil Engineering

7

Soil Mechanics

CONTENTS

Properties of Soils 1

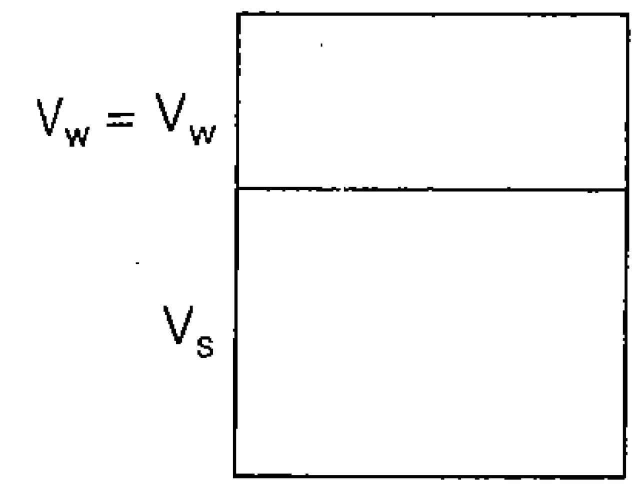
Phase Diagram

Soil mass is in general a three phase system composed of solid, liquid and gaseous matter in a blended form with each other.

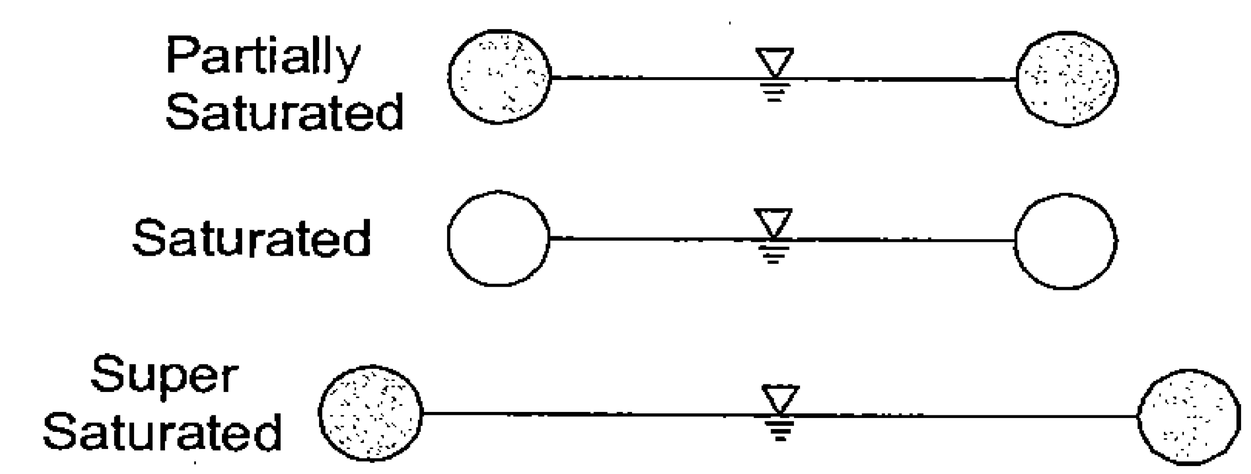
But in phase diagram - for understanding, these three matters are shown separately.

> NOTE: If voids absent in a system, it won't be termed soil e.g. Granite (Aquifuge), rock.

In supersaturated, state with change in water content volume of voids changes hence volume of soil changes.



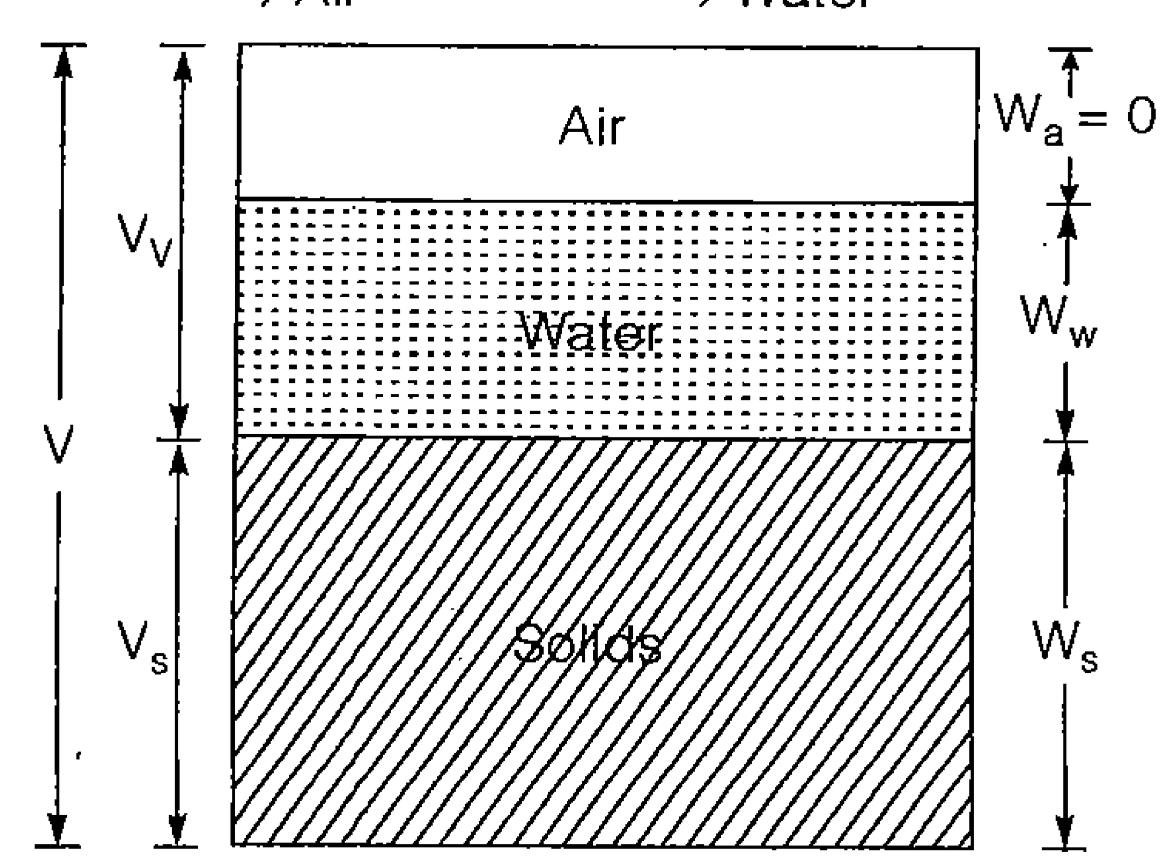
2 phase (super saturated)



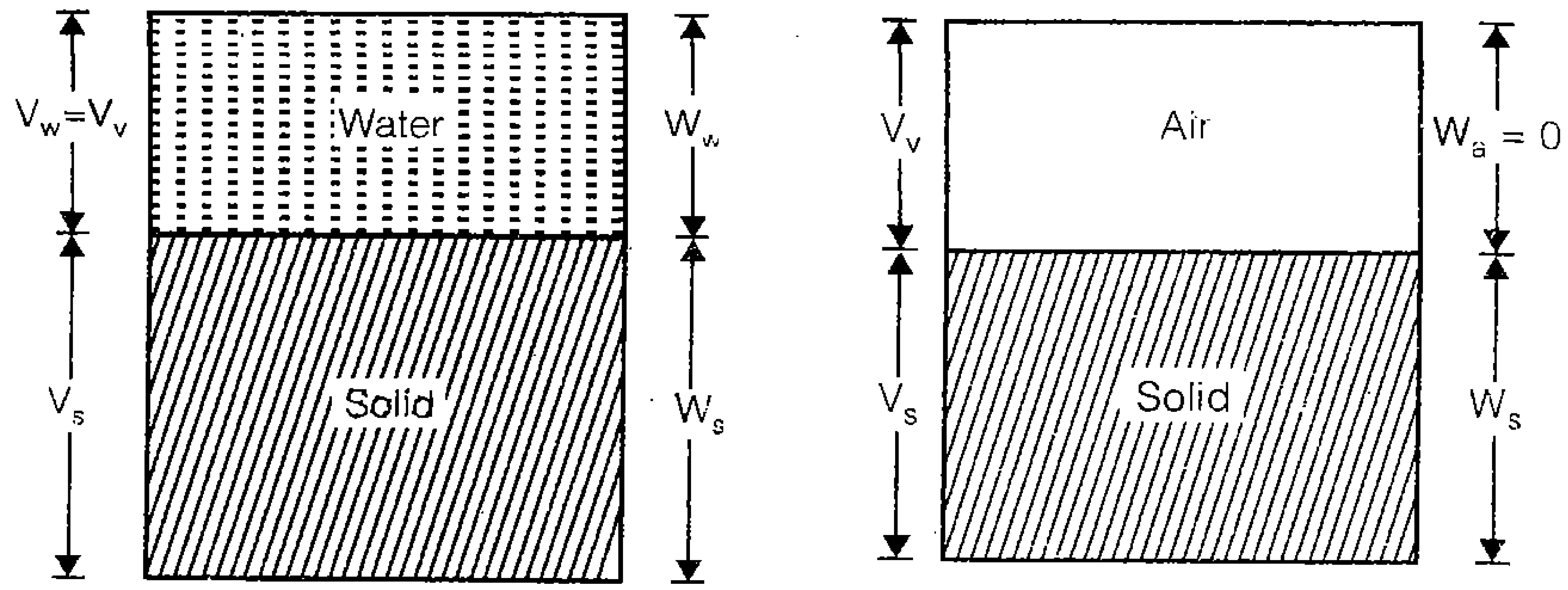
The diagrammatic representation of the different phases in a soil mass is called the "phase diagram".

Soil

- 3 Phase
  - Partially saturated
  - $0 < S < 1$
  - → Solids
  - → Water
  - → Air
- 2 Phase
  - Oven dried: $S = 0$ → Solid → Air
  - Fully saturated: $S = 1$ → Solid → Water
  - Super saturated: $S = 1$ → Solid → Water



3-Phase: (Partially Saturated)



2-Phase (Saturated) 2-Phase (Dry)

Water content

$$\boxed{w = \frac{W_w}{W_s} \times 100}$$

where, W_w = Weight of water
W_s = Weight of solids

There can be no upper limit to water content, i.e., $\boxed{w \geq 0}$

> **NOTE** Water content in soil represents (gravity water + Capilary water + Hygroscopic water) which can be removed on ovendrying except structural water.

Void ratio

$$\boxed{e = \frac{V_v}{V_s}}$$

where, V_v = Volume of voids
V_s = Volume of solids.

Though size of individual void of coarse grained soil is more, total volume of voids in fine grained soil is more.

In general $\boxed{e > 0}$ i.e., no upper limit for void ratio.

Porosity

$$\boxed{n = \frac{V_v}{V} \times 100}$$

where, V_v = Volume of voids
V = Total volume of soil

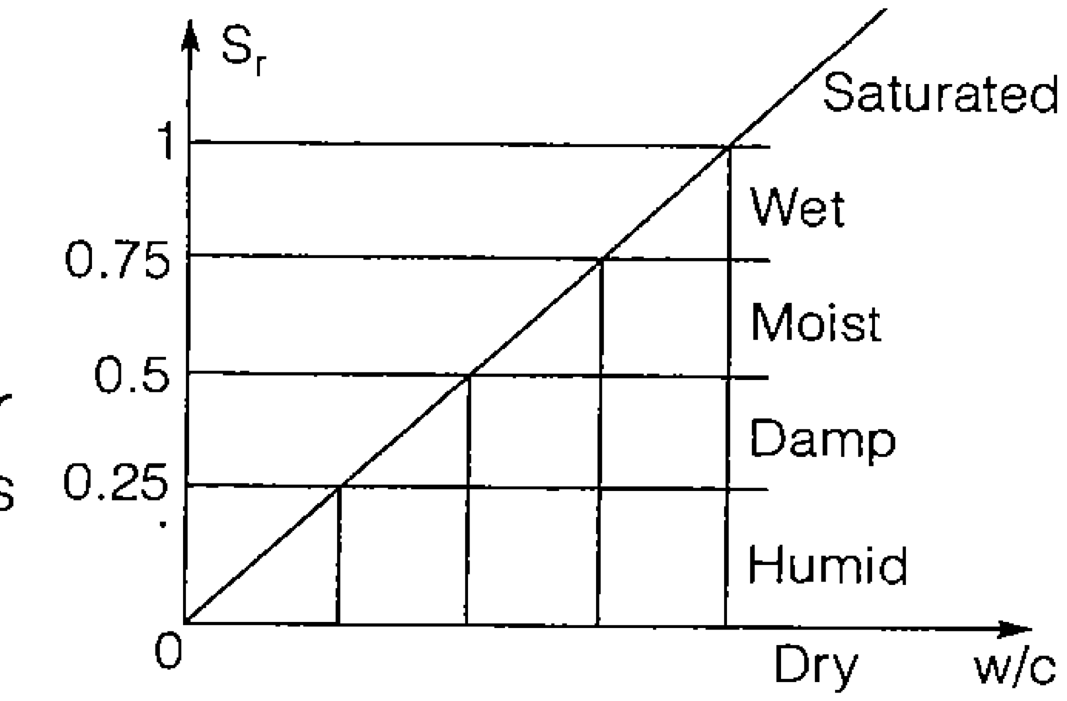
Porosity cannot exceed 100% i.e.,

$$\boxed{0 < n < 100}$$

Void ratio is more important engineering property.

> **NOTE** In comparison to porosity, void ratio is more frequently used because volume of solids remains same, whereas total volume changes.

Degree of Saturation



$$\boxed{S = \frac{V_w}{V_v} \times 100}$$

where, V_w = volume of water
V_v = Volume of voids

$\boxed{0 \leq S \leq 100}$

for perfectly dry soil : S = O

for Fully saturated soil : S = 100%

Air Content

$$\boxed{a_c = \frac{V_a}{V_v} = 1 - S}$$ V_a = Volume of air

$$\boxed{S_r + a_c = 1}$$

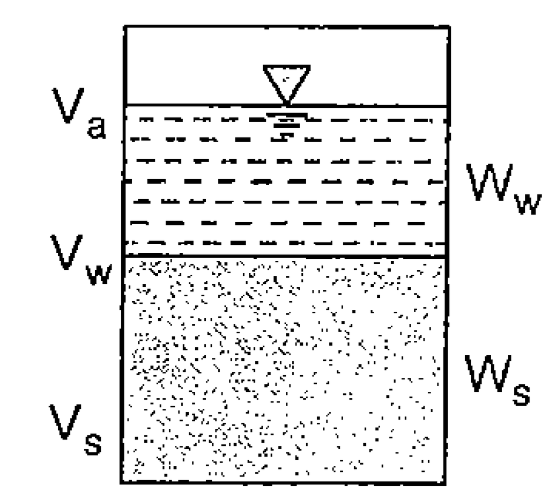
% Air Void

$$\%n_a = \frac{\text{Volume of air}}{\text{Total volume}} \times 100 = \boxed{\frac{V_a}{V} \times 100} \quad \boxed{n_a = n.a_c}$$

> **NOTE** Engineering significance of both ac and na is equivalent as both involve variable parameters.

Unit Weight

(a) Bulk unit weight



$$\boxed{\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_a}}$$

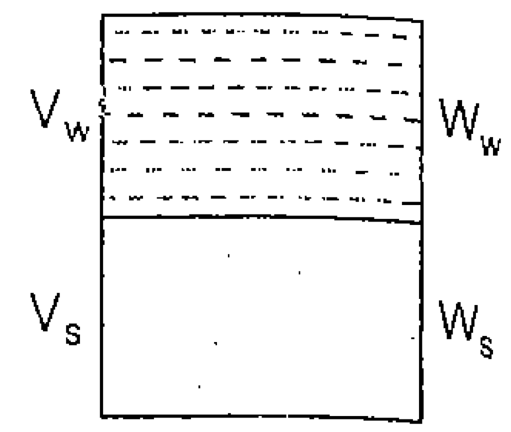
Thus Bulk unit weight is total weight per unit volume.

(b) **Dry Unit Weight** is the weight of soil solids per unit volume.

$$\boxed{\gamma_d = \frac{W_s}{V}}$$

- Dry unit weight is used as a **measure of denseness** of soil. More dry unit weight means more compacted soil.

(c) **Saturated unit weight:** It is the ratio of total weight of fully saturated soil sample to its total volume.



$$\gamma_{sat} = \frac{W_{sat}}{V}$$

(d) **Submerged unit weight or Buoyant unit weight (γ'):** It is the submerged weight of soil solids per unit volume.

$$\gamma' = \gamma_{sat} - \gamma_w$$

γ_{sat} = unit wt. of saturated soil
γ_w = unit wt. of water

(e) **Unit wt. of solids :** $\gamma_s = \dfrac{W_s}{V_s}$

NOTE: $\gamma_s > \gamma_{sat} > \gamma_{bulk} > \gamma_{dry} > \gamma_{submerged}$

- γ' is roughly 1/2 of saturated unit weight.

Specific Gravity

True/Absolute Special Gravity, G

- Specific gravity of soil solids (G) is the ratio of the weight of a given volume of solids to the weight of an equivalent volume of water at 4°C.

$$G = \frac{W_s}{V_s \cdot \gamma_w} = \frac{\gamma_s}{\gamma_w}$$

G = 2.6 to 2.75 for inorganic solids
= 1.2 to 1.4 for organic solids

NOTE:
(i) G α Mineral content (like Iron + Mica) $\propto \dfrac{1}{\text{Amount of organic content}}$

(ii) $G_{\text{fine grained}} > G_{\text{coarse grained soil}}$

- **Apparent or mass specific gravity (G_m):** Mass specific gravity is the specific gravity of the soil mass and is defined as the ratio of the total weight of a given mass of soil to the weight of an equivalent volume of water.

$$G_m = \frac{W}{V \cdot \gamma_w} = \frac{\gamma \text{ or } \gamma_d \text{ or } \gamma_{sat}}{\gamma_w}$$

where, γ is bulk unit wt. of soil
$\gamma = \gamma_{sat}$ for saturated soil mass
$\gamma = \gamma_d$ = for dry soil mass

$$G_m < G$$

In India, G is reported at 27 °C,

$$G_{T°C} = G_{27°C}\left(\frac{\gamma_w \text{ at } 27°C}{\gamma_w \text{ at } T°C}\right)$$

Relative density (I_D)

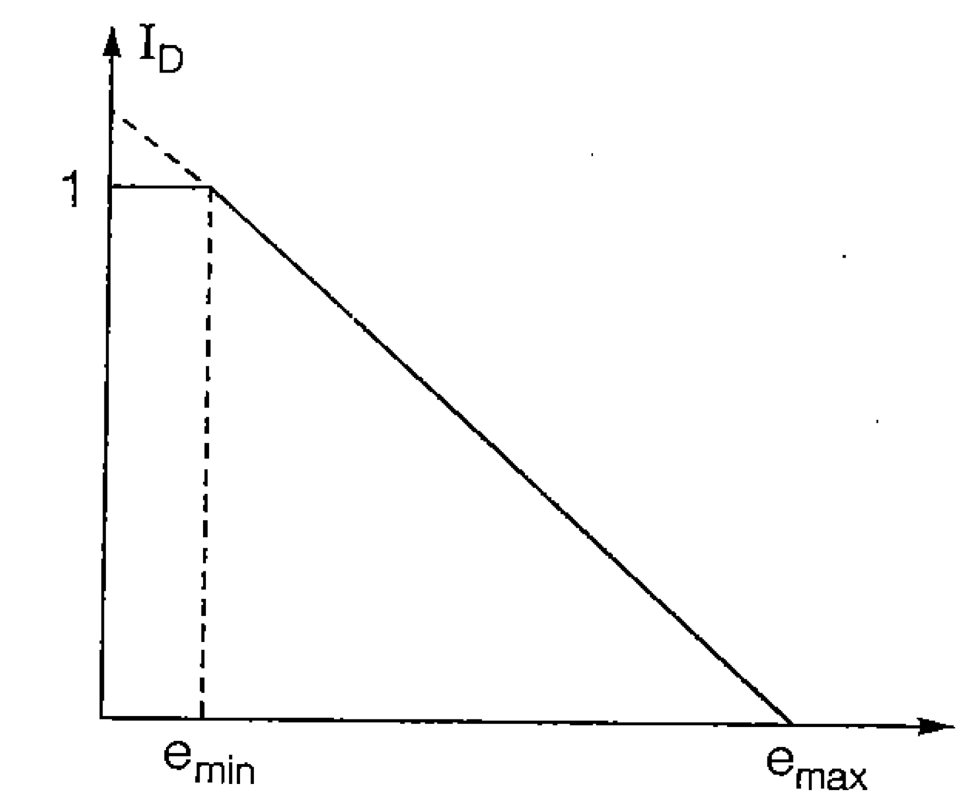
To compare degree of denseness of two soils.

$$I_D \propto \text{Shear strength} \propto \frac{1}{\text{Compressibility}}$$

$$\%I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

$$\%I_D = \frac{\dfrac{1}{\gamma_{dmin}} - \dfrac{1}{\gamma_d}}{\dfrac{1}{\gamma_{dmin}} - \dfrac{1}{\gamma_{dmax}}} \times 100$$

$\%I_D$	Discription
0 – 15	Very loose soil
15 – 30	Loose soil
30 – 65	Medium soil
65 – 85	Dense soil
85 – 100	Very dence soil



NOTE: For uniformly graded coarse soil having perfectly spherical grain of same size.

(a) When particles are arranged in cubical array
$e_{max} = 91\%$, $n_{max} = 47.6\%$

(b) When particles are arranged in prismoidal array (Rhomohedral array)
$e_{min} = 35\%$, $n_{min} = 25.9\%$

Relative Compaction	Relative Density
Indicate : Degree of denseness of cohesive + cohesionless soil $R_c = \dfrac{\gamma_D}{\gamma_{Dmax}}$	**Indicate :** Degree of denseness of natural cohesionless soil

Some Important Relationships

(i) Relation between γ_d, γ

$$\gamma_d = \frac{\gamma}{1 + w}$$

(ii) $V_s = \frac{V}{1+e}$ (iii) $W_s = \frac{W}{1+w}$

(ii) Relation between e and n

$$n = \frac{e}{1+e} \quad \text{or} \quad e = \frac{n}{1-n}$$

(iii) Relation between e, w, G and S :

$$Se = w.G$$

(iv) Bulk unit weight (γ) in terms of G, e, w and γ_w
γ, G, e S_r, γ_w

$$\gamma = \frac{(G + eS_r)\gamma_w}{1+e}$$

$$\gamma = \frac{G\gamma_w (1+w)}{(1+e)} \qquad \{Srxe = wG\}$$

(v) Saturated unit weight (γsat.) in terms of G, e & γ_w

$$S_r = 1 \qquad \gamma_{sat} = \left[\frac{G+e}{1+e}\right]\cdot\gamma_w$$

(vi) Dry unit weight (γ_d) in terms of G, e and γ_w

$$S_r = 0 \qquad \gamma_d = \frac{G\gamma_w}{1+e} = \frac{G\gamma_w}{1+\frac{wG}{S}} = \frac{(1-\eta_a)G\gamma_w}{1+wG}$$

(vii) Submerged unit weight (γ') in terms of G,e and γ_w

$$\gamma' = \gamma_{sat} - \gamma_w = \gamma' = \left(\frac{G-1}{1+e}\right)\cdot\gamma_w$$

(ix) Relation between degree of saturation (s) w and G

$$S = \frac{w}{\frac{\gamma_w}{\gamma}(1+w) - \frac{1}{G}}$$

> **NOTE** Index properties are used for identification and classification of soils, ex. water content, consistency limits, insitu density, particle size distribution, sensitivity, activity.

Methods for determination of water content

(i) Oven Drying Method

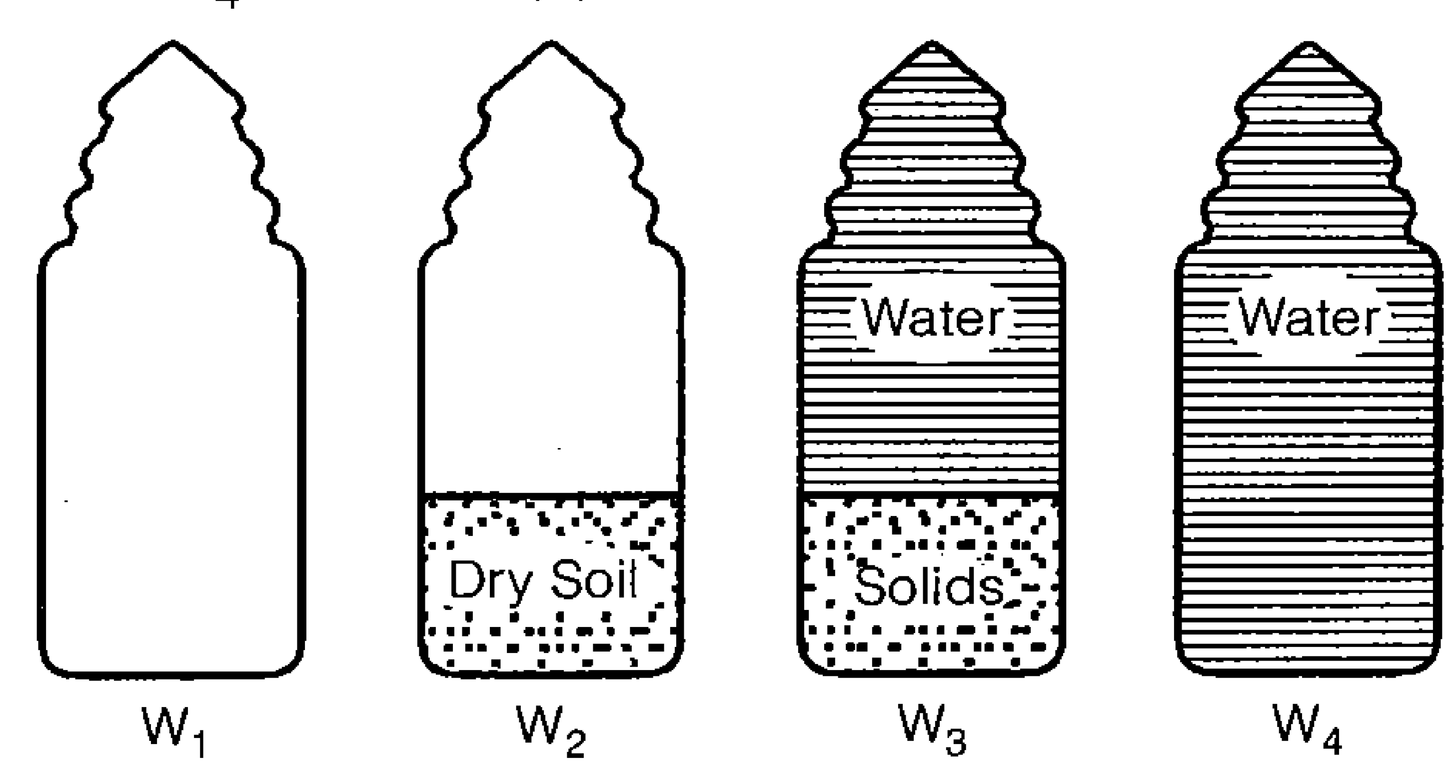
- Simplest and most accurate method
- Soil sample is dried in a controlled temperature (105-110°C)
- For organic soils, temperature is about 60°C.
  Soil having gyprum, temperature ≯ 80°C
- Sample is dried for 24 hrs.
- For sandy soils, complete drying can be achieved in 4 to 6 hrs.
- Water content is calculated as:

$$w = \frac{W_2 - W_3}{W_3 - W_1} \times 100\%$$

where, W_1 = weight of container
W_2 = weight of container + moist sample
W_3 = weight of container + dried sample
Weight of water = $W_2 - W_3$
Weight of solids = $W_3 - W_1$

(ii) Pycnometer Method

- quick method
- capacity of pycnometer = 900 m*l*.
- this method is more suitable for cohesionless soils.
- **used when specific gravity of soil solids is known**
- Let W_1 = Wt. of empty dried pycnometer bottle
  W_2 = Wt. of pycnometer + Soil
  W_3 = Wt. of pycnometer + Soil + Water
  W_4 = Wt. of pycnometer + Water.



$$w = \left[\frac{(W_2 - W_1)}{(W_3 - W_4)}\cdot\left(\frac{G-1}{G}\right) - 1\right] \times 100\%$$

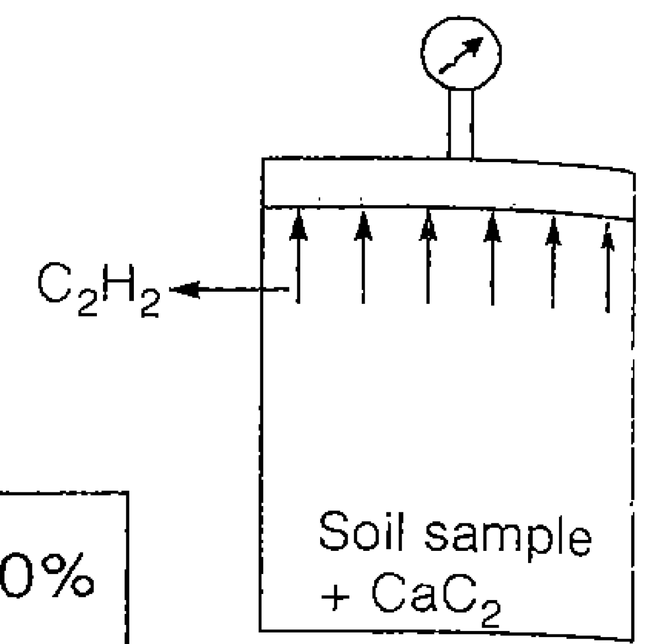
(W_1, W_2, W_3 and W_4 are in anticlockwise order)

(iii) Calcium Carbide Method/Rapid Moisture Meter Method Field Method

- Quick method (requires 5 to 7 minutes) ; but may not give accurate results.
- The reaction involved is
  $CaC_2 + 2H_2O \rightarrow C_2H_2\uparrow + Ca(OH)_2$
- Soil sample weights 4-6 gms.
- The gauge reads water content with respect to total mass of soil. i.e.,

$$w_r = \frac{W_w}{(W_s)_{wet}}$$

(In this equipment pressure calliberated against water content with respect to total mass)



- Actual water content $w = \frac{w_r}{1 - w_r} \times 100\%$

w_r is moisture content recorded, expressed as fraction of moist wt. of solid.
w is actual water content.

(iv) Sand Bath Method (Field Method)

- quick, field method
- used when electric oven is not available.
- soil sample is put in a container & dried by placing it in a sand bath, which is heated on kerosene store.
- water content is determined by using same formula as in oven drying method.

(v) Torsion Balance Moisture Meter Method

- quick method for use in laboratory.
- Infrared radiations are used for drying sample.
- **Principle:** The torsion wire is prestressed accurately to an extent equal to 100% of the scale reading. Then the sample is evenly distributed on the balance pan to counteract the prestressed torsion and the scale is brought back to zero. As the sample dries, the loss in weight is continuously balanced by the rotation of a drum calibrated directly to read moisture% on wet basis.

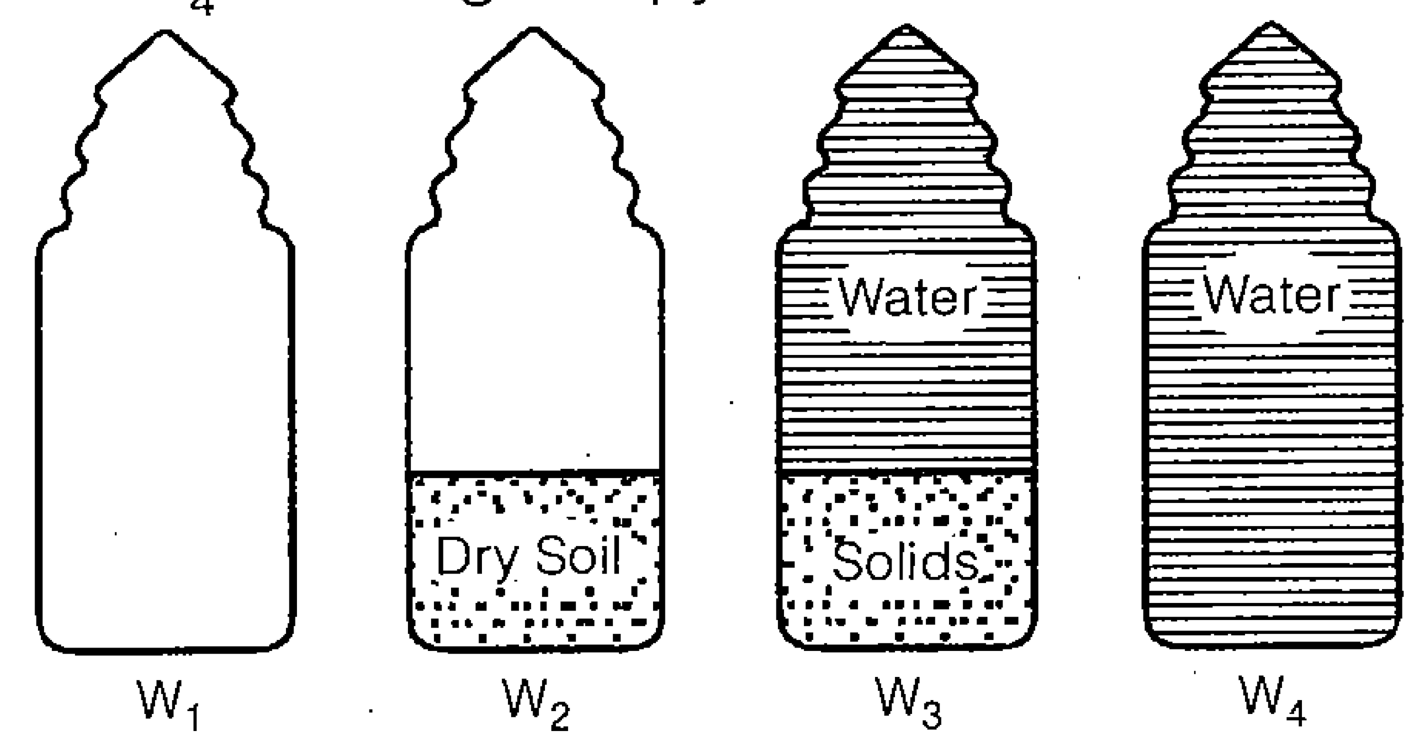
(vi) Alcohol Method

- It is a quick method adopted in field.
- Should not be used for organic soil and soils containing calcium compound.

Determination of Specific gravity of soil solids

- Pycnometer method is used.
- Instead of pycnometer, Density bottle (50 ml) OR Flask (500 ml) can also be used.

Let, W_1 = Weight of empty pycnometer
W_2 = Weight of pycnometer + soil sample (oven dried)
W_3 = Weight of pycnometer + soil solids + water
W_4 = Weight of pycnometer + water



$$G = \frac{W_2 - W_1}{(W_2 - W_1) - (W_3 - W_4)} \Rightarrow G = \frac{W_s}{W_s - W_3 + W_4}$$

NOTE

1. Specific gravity values are generally reported at 27°C (in India)
2. If T°C is the test temperature then Sp.Gr. at 27°C is given by,

$$G_{27°C} = G_{T°C} \times \frac{\text{unit wt. of water at } T°C}{\text{unit wt. of water at } 27°C}$$

3. If kerosene (better wetting agent) is used instead of water then,

$$G = \frac{W_s}{W_s - W_3 + W_4} \times K \quad [K = \text{Sp. gr. of Kerosene}]$$

4. G can also be determined indirectly by using shrinkage limit.

Methods for the determination of insitu unit weight

(a) Core-Cutter Method

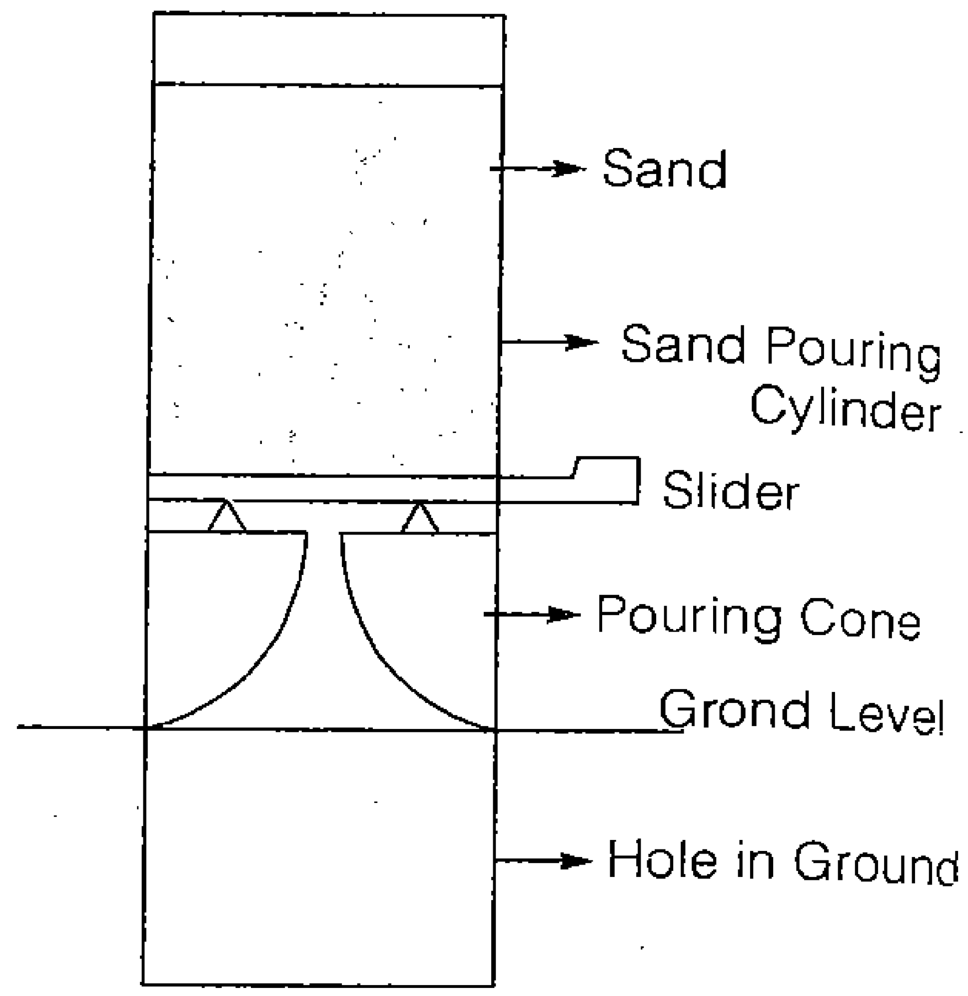
- Used in case of cohesive soils.
- Cannot be used in case of hard and gravelly soils.
- Method consists of driving a core-cutter (Volume = 1000 cc) into the soil and removing it, the cutter filled with soil is weighed. Volume of cutter is known from its dimensions and in situ unit weight is obtained by dividing soil weight by volume of cutter.

$$\gamma = \frac{W}{V}$$

- If water content is known in laboratory, the dry unit weight can also be computed.

$$\boxed{\gamma_d = \frac{\gamma}{1+w}}$$

(b) Sand Replacement Method



- Used in case of hard and gravelly soils.
- A hole in ground is made. The excavated soil is weighed. The volume of hole is determined by replacing it with sand. Insitu unit weight is obtained by dividing weight of excavated soil with volume of hole.

(c) Water Displacement Method

- Suitable for cohesive soils only, where it is possible to have a lump sample.
- A regular shape, well trimmed sample is weighed. (W_1). It is coated with paraffin wax & again weighed (W_2). The sample is now placed in a metal container filled with water upto the brim. Let the volume of displaced water be V_w. Then volume of uncoated specimen is calculated as,

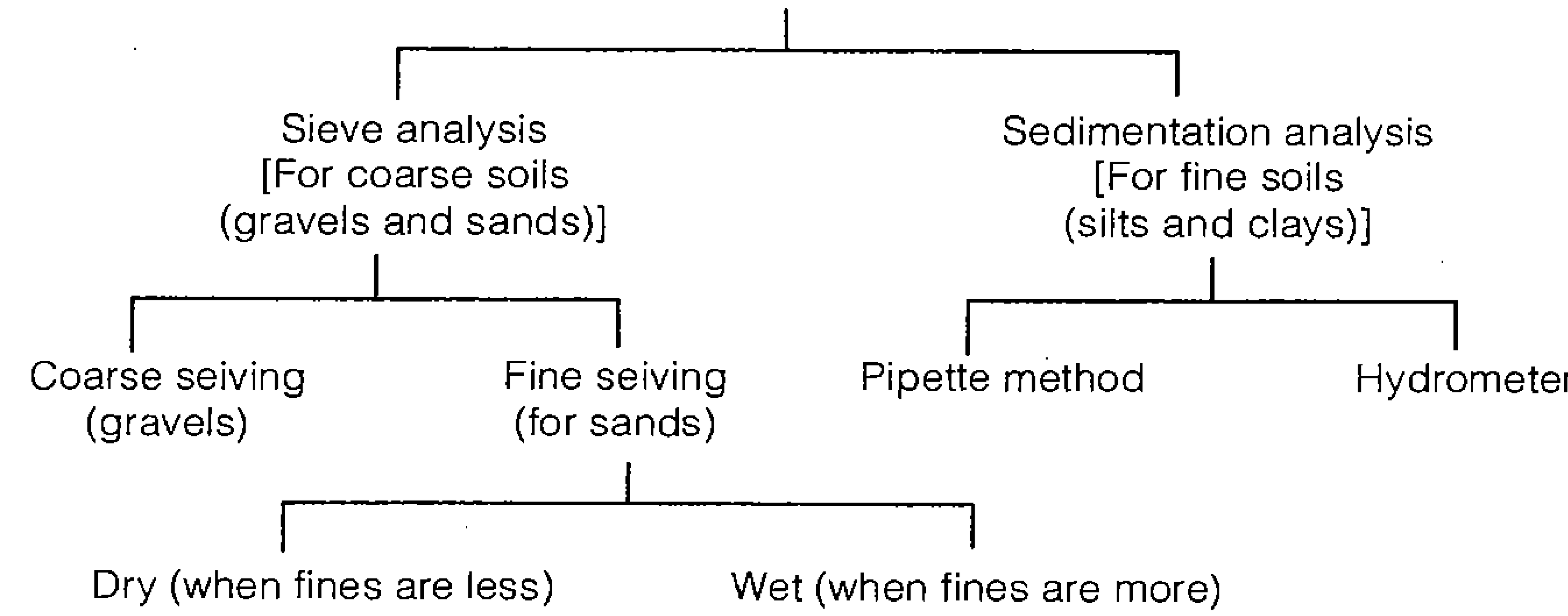
$$\boxed{V = V_w - \left(\frac{W_2 - W_1}{\gamma_p}\right)}$$

where, γ_p = unit wt. of paraffin wax

Thus, bulk unit wt. of soil $\gamma = \frac{W_1}{V}$

- **Sands + Gravels :** Bulky grains
  Bulk grains classified as - angular, Subangular, Sub rounded, rounded, well rounded
  Higher angularity ∝ Higher Shear Strength
- **Clay Minerals :** Flaky grains

Grain size distribution



- **Sieve Analysis : (For Coarse Grained Soils)**
  The fraction retained on 4.75 mm sieve is called the gravel fraction which is subjected to coarse sieve analysis.
  The material passing 4.75 mm sieve is further subjected to fine sieve analysis if it is sand or to a combined sieve and sedimentation analysis if silt and clay sizes are also present.
- **Coarse Sieves:** 4.75 mm, 10 mm, 20 mm, 80 mm.
- **Fine Sieves :** 75 μ, 150 μ, 212 μ, 425 μ, 600 μ, 1 mm, 2mm.
- **Concept of "Percentage finer"**
  % retained on a particular sieve

$$= \frac{\text{Weight of soil retained on that sieve}}{\text{Total weight of soil taken}} \times 100$$

Cumulative % retained = sum of % retained on all sieves of larger sizes and the % retained on that particular sieve.
"Percentage finer" than the sieve under reference = 100% - Cumulative % retained.

- **Sedimentation Analysis**
  According to stokes law, the terminal velocity is given by,

$$\boxed{V = \frac{g}{18} \cdot \frac{\rho_s - \rho_w}{\mu} \cdot D^2}$$

ρ_s = density of grains (g/cm^3)
ρ_s = density of water (g/cm^3)
μ = viscosity of water
g = acceleration due to gravity (cm/s^2)
D = Diameter of grain (cm)

If 'h' the height through which particle falls in time 't', then

$$\boxed{\frac{h}{t} = k.D^2} \qquad \therefore \boxed{\frac{D_1}{D_2} = \sqrt{\frac{h_1}{h_2} \cdot \frac{t_2}{t_1}}}$$

NOTE

- Stokes law is applicable for spheres of diameter between 0.2 mm and 0.0002 mm.
- Spheres of diameter larger than 0.2 mm falling through water cause turbulence, whereas, for spheres with diameter less than 0.0002 mm, Brownian motion takes place and the velocity of settlement is too small for accurate measurement.

- **Pipette Method**

In this method, the weight of solids per cc of suspension is determined directly by collecting 10 cc of soil suspension from a specified sampling depth.

If m_d = dry mass (obtained after drying the sample) then, mass present in unit vol. of pipette

$$\frac{m_d}{\text{Vol.of pipette }(V_p)} = \frac{m_d}{10\text{ ml. }(V_p)}$$

If M_d = total mass of soil dissolved in total volume of water (V)

then mass/unit volume = $\frac{M_d}{V}$

Therefore, % finer is given by = %

$$N = \frac{m_d/V_p}{M_d/V}$$

If m is the mass of dispersing agent dissolved in the total volume V, then actual % finer,

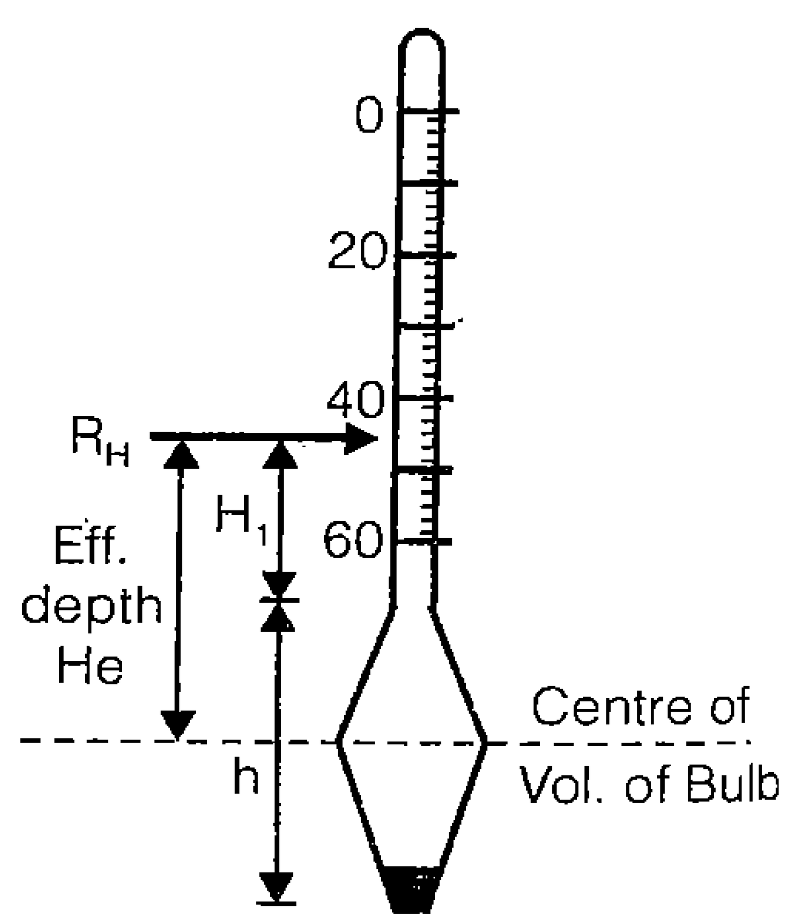
$$\%N = \frac{\frac{m_d}{V_p} - \frac{m}{V}}{\frac{M_d}{V}} \times 100$$

- **Hydrometer Method**

In this method the weight of solids present at any time is calculated indirectly by reading the density of soil suspension.

- **Calibration of Hydrometer**

Establishing a relation between the hydrometer reading R_H and effective depth (H_e).



The effective depth is the distance from the surface of the soil suspension to the level at which the density of soil suspension is being measured.

Effective depth is calculated as

$$H_e = H_1 + \frac{1}{2}\left(h - \frac{V_H}{A_j}\right)$$

where, H_1 = distance (cm) between any hydrometer reading and neck.
h = length of hydrometer bulb
V_H = volume of hydrometer bulb
A_J = area of the cross section of the Jar.

Reading of Hydrometer is related to sp. gr. or density of soil suspension as :

$$G_{SS} = 1 + \frac{R_H}{1000}$$

Thus a reading of R_H = 25 means, G_{ss} = 1.025 and a reading of R_H = −25 means, G_{ss} = 0.975

% finer is given as :

$$N = \frac{G}{G-1}\cdot\gamma_w\cdot\frac{V}{W}\cdot\frac{R_H}{10}\%$$

where, G = sp. gr. of soil solids
R_H = final corrected value of hydrometer
V = Total volume of soil suspension
W = weight of soil mass dissolved.

- **Corrections to Hydrometer Reading**

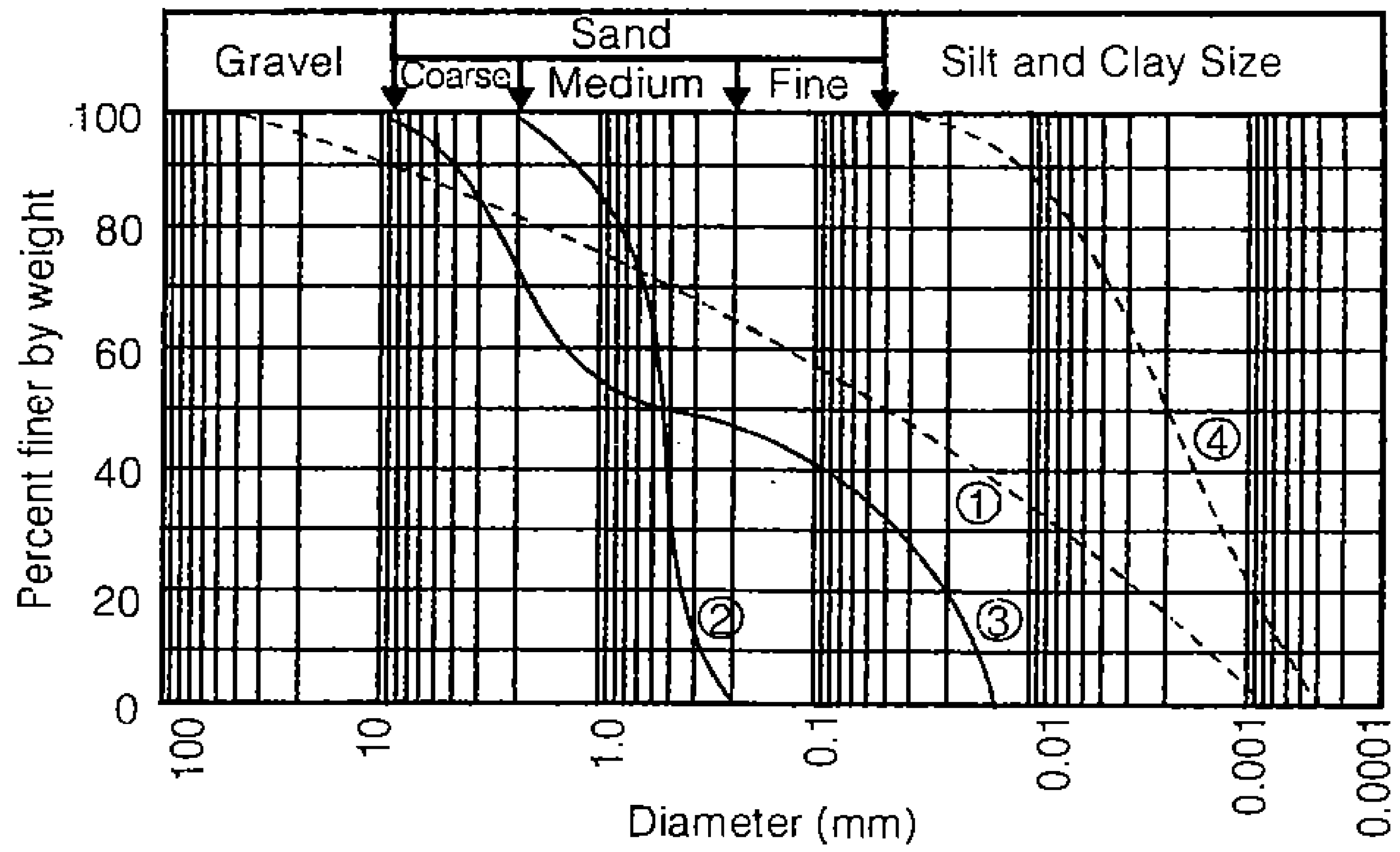
(i) Meniscus correction : (C_m)
Hydrometer reading is always corresponding to the upper level of meniscus.
Therefore, meniscus correction is always positive (+ C_m).

(ii) Temperature correction : (C_t)
Hydrometers are generally calibrated at 27°C. If the test temperature is above the standard (27°C) the correction is added and, if below, it is subtracted.

(iii) Dispersing/Deflocculating agent correction: (C_d)
The correction due to rise in specific gravity of the suspension on account of the addition of the deflocculating agent is called Dispersing agent correction (C_d).
C_d is always negative.
The corrected hydrometer reading is given by

$$(R_H) = R_H + C_m \pm C_t - C_d$$

- **Grain Size Distribution Curves**



Curve-1: Well graded soil : good representation of grain sizes over a wide range and its gradation curve is smooth.

Curve-2: Poorly graded soil / Uniform gradation:

It is either an excess or a deficiency of certain particle sizes or has most of the particles about the same size.

Curve-3: Gap graded soil : In this case some of the particle sizes are missing.

Curve-4: Predominantly coarse soil.

Curve-5: Predominantly fine soil.

> NOTE: If slope of the curve is steep, soil is poorly graded.
> If slope is inclined, soil is well graded.

The diameter D_{10} corresponds to 10% of the sample finer in weight on the Grain size distribution curve. This diameter D_{10} is called effective size.

> NOTE: For a soil mass if effectgive size is X mm. Then it means a soil having spherical particles of X mm size will have same effect as the above soil with D_{10} = X mm

Similarly, D_{30} and D_{60} are grain dia. (mm) corresponding to 30% fine and 60% finer.

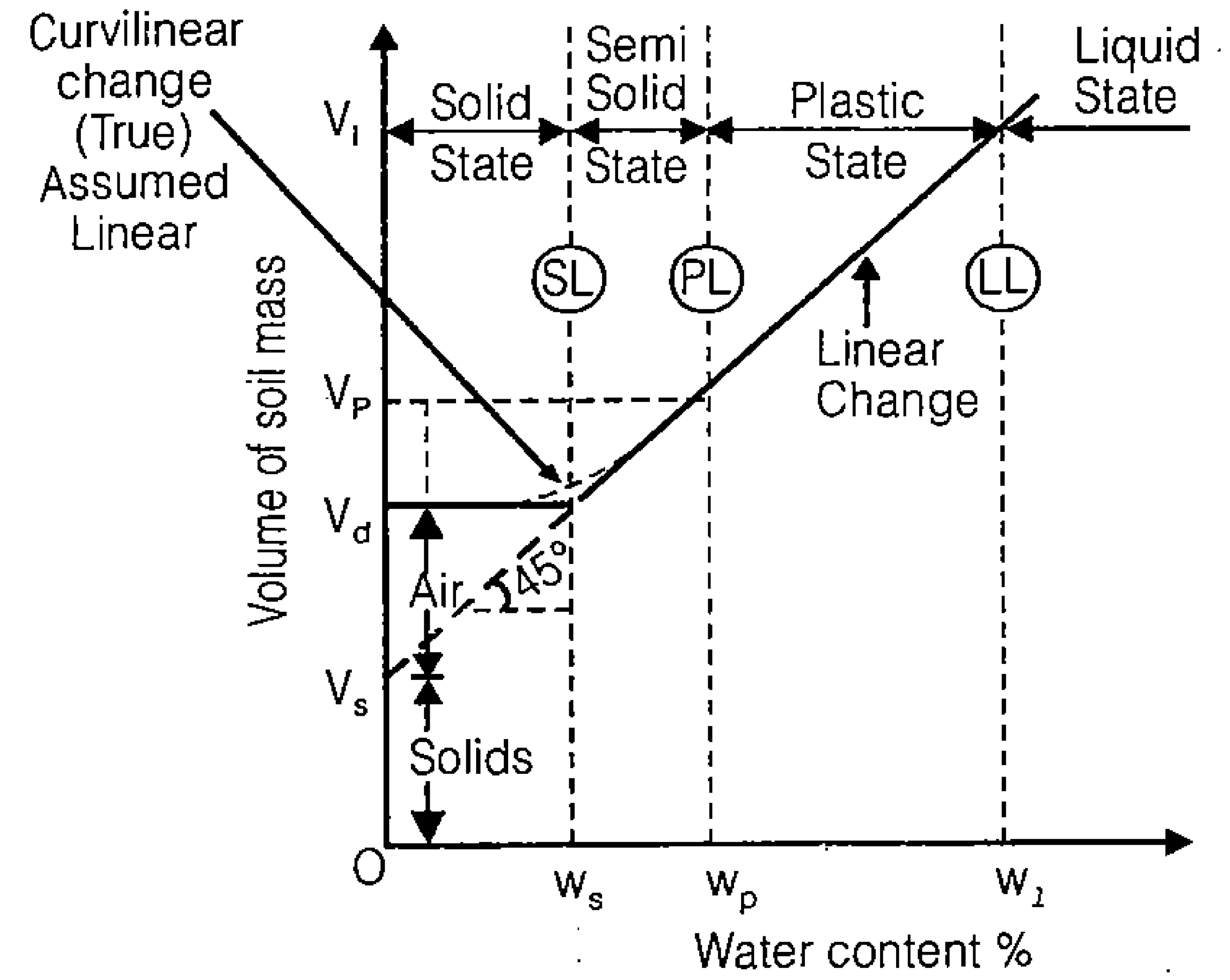
The shape parameters related to these are :

(a) **Coefficient of Uniformity** $C_u = \dfrac{D_{60}}{D_{10}}$

(b) **Coefficient of Curvature** $C_c = \dfrac{D_{30}^2}{D_{10} \times D_{60}}$

- for a soil to be well graded :
  [$1 < C_c < 3$] and [$Cu > 4$] for gravels ;
  [$C_u > 6$] for sands.
- $Cu \approx 1$ for uniform soils/poorly graded soils.

Consistency of clays : Atterberg limits



LL = w_l = liquid limit
PL = w_p = Plastic limit

SL = w_s = Shrinkage limit
V_1 = Volume of soil mass at LL
V_P = Volume of soil mass at PL
V_d = Volume of soil mass at SL
V_S = Volume of solids

- **Plasticity Index (I_p):** It is the range of moisture content over which a soil exhibits plasticity.

$I_p = w_L - w_p$

w_L = water content at LL
w_p = water content at PL

If PL ≥ LL, I_p is reported as zero.

Soil classification related to plasticity Index:

I_p(%)	Soil description
0	Non plastic
1 to 5	Slight Plastic
5 to 10	Low Plastic
10 to 20	Medium Plastic
20 to 40	Highly plastic
> 40	Very Highly plastic

- **Relative Consistency or Consistency-index (I_c)** : to study insitu behaviour saturated fine grained soil at its natural water content

$I_C = \dfrac{w_L - w_N}{I_p}$

$\therefore$ For $w_N = w_L \Rightarrow I_C = 0$
For $w_N = w_P \Rightarrow I_C = 1$

If $I_C < 0$, the natural water content of soil (w_N) is greater than w_L and the soil mass behaves like a liquid, but only upon disturbance.

If $I_C > 1$, soil is in semi solid state and will be very hard or stiff.

- **Liquidity Index (I_L)**

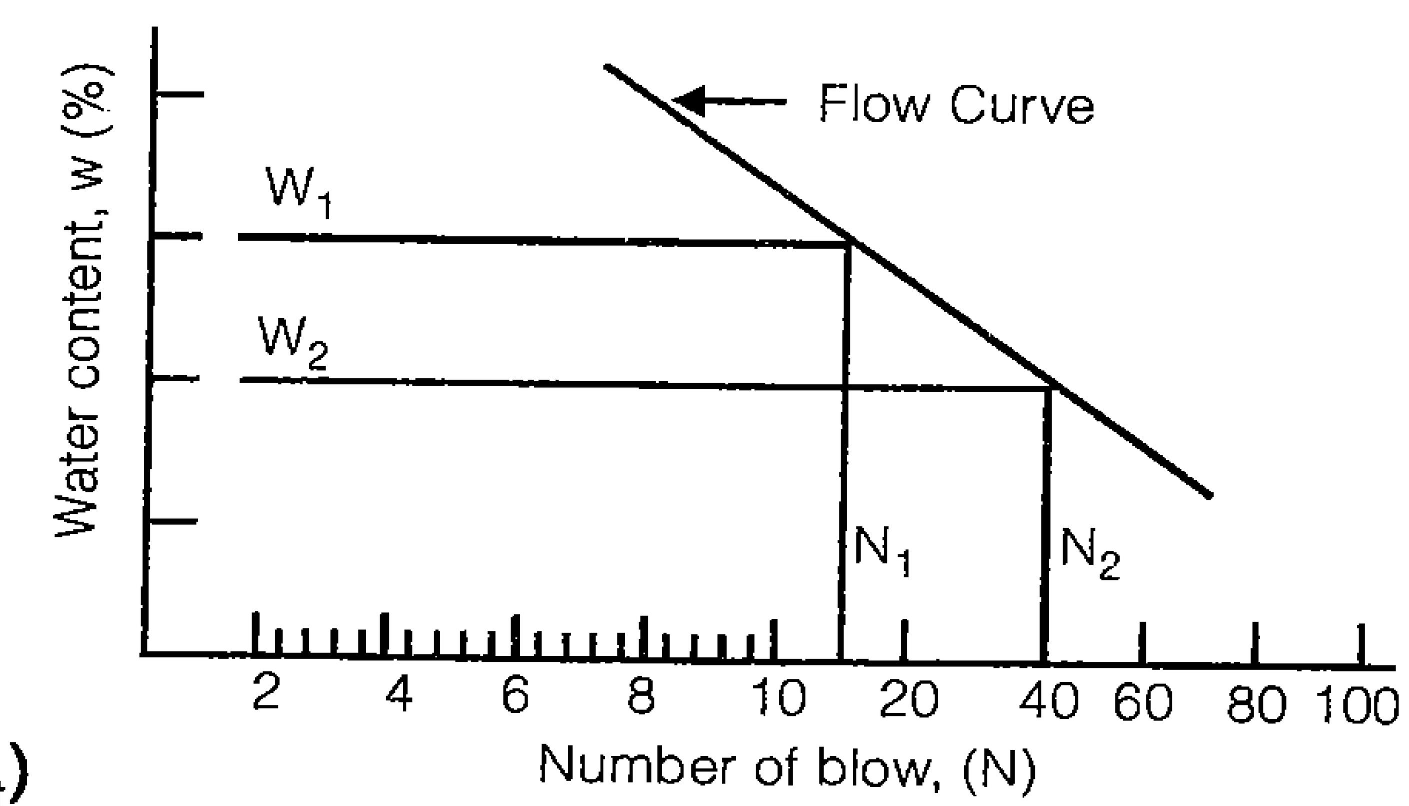
$$I_L = \frac{w_N - w_P}{I_P}$$

For a soil in plastic state I_L varies from 0 to 1.

Consist.	Description	I_C	I_L
Liquid	Liquid	<0	>1
Plastic	Very soft	0 – 0.25	0.75 – 1.00
	Soft	0.25 – 0.5	0.50 – 0.75
	Medium stiff	0.50 – 0.75	0.25 – 0.50
	Stiff	0.75–1.00	0.0 – 0.25
Semi-solid	Very stiff OR Hard	> 1	< 0
Solid	Hard OR Very Hard	> 1	< 0

- **Flow Index (I_f)**

$$I_f = \frac{w_1 - w_2}{\log_{10}(N_2/N_1)}$$



- **Toughness Index (I_t)**

$$I_T = \frac{I_P}{I_F}$$

For most of the soils : $0 < I_T < 3$

When $I_T < 1$, the soil is friable (easily crushed) at the plastic limit.

- **Shrinkage Ratio (SR)**

$$SR = \frac{\frac{V_1 - V_2}{V_d} \times 100}{w_1 - w_2}$$

where, V_1 = volume of soil mass at water content w_1%.

V_2 = volume of soil mass at water content w_2%.

V_d = volume of dry soil mass

Now, at SL, $w_2 = w_s$ and $V_2 = V_d$

$$\therefore \quad SR = \frac{\left(\frac{V_1 - V_d}{V_d} \times 100\right)}{(w_1 - w_s)}$$

If w_1 & w_2 are expressed as ratio,

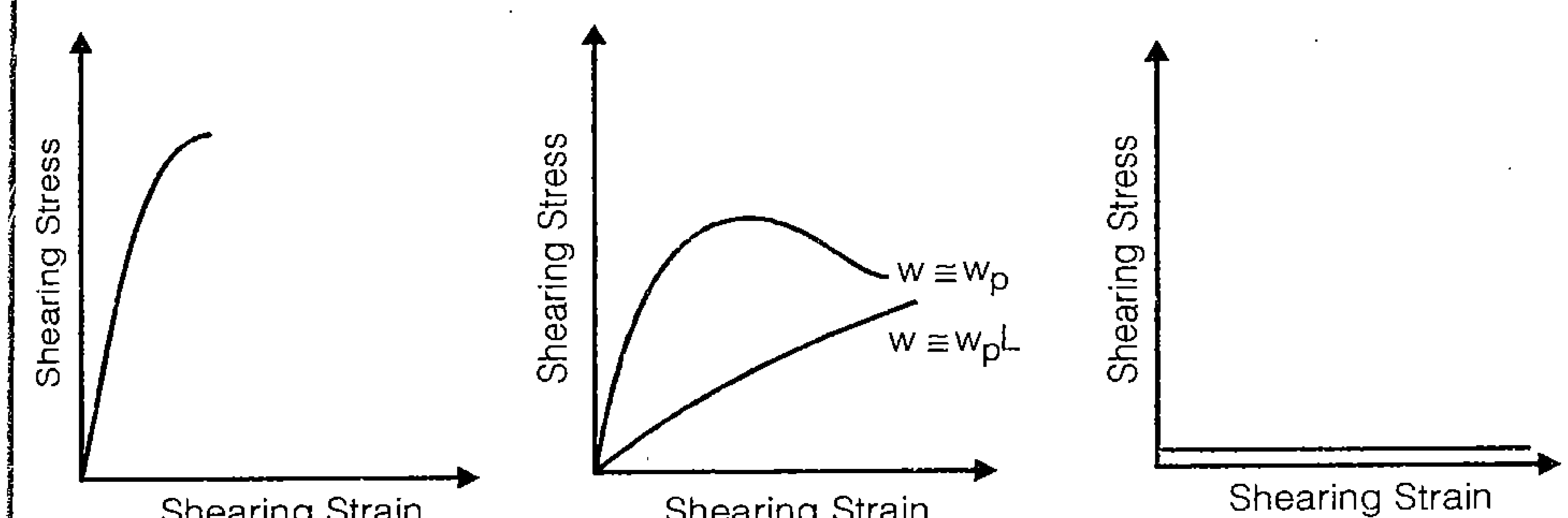
$$SR = \frac{(V_1 - V_2)/V_d}{w_1 - w_2}$$

But, $w_1 - w_2 = \dfrac{(V_1 - V_2)/\gamma_w}{W_s}$

$$\therefore \quad SR = \frac{W_s}{V_d} \cdot \frac{1}{\gamma_w} = \frac{\gamma_d}{\gamma_w}$$

Properties	Relationship	Governing Parameters
Plasticity	∝	Plasticity Index
Better Foundation Material upon Remoulding	∝	Consistency Index
Compressibility	∝	Liquid Limit
Rate of loss in shear strength with increase in water content	∝	Flow Index
Strength of Plastic Limit	∝	Toughness Index

Stress-strain curve for different consistency states



(a) $I_L < 0$ (brittle) (b) $0 < I_L < 1$ (Plastic) (c) $I_L > 1$ (Liquid)

- **Unconfined Compressive Strength (q_u)**

Defined as the load per unit area at which an unconfined prismatic or cylindrical specimen of standard dimensions of a soil fails in a simple compression test.

$q_u = 2 \times$ shear strength of a clay soil (under undrained condition).

q_u is related to consistency of clays as :

Consistency	q_u (KN/m²)	(Kg/cm²)
very soft	< 25	< 0.25
soft	25-50	0.25-0.50
Medium	50-100	0.50-1.0
Stiff	100-200	1.0-2.0
Very stiff	200-400	2.0-4.0
Hard	> 400	> 4.0

- **Sensitivity (S_t):** It is defined as the ratio of the unconfined compressive strength of an undisturbed specimen of the soil to the unconfined compressive strength of a specimen of the same soil after remolding at unaltered water content.

$$S_t = \frac{(q_u)\text{undisturbed}}{(q_u)\text{remoulded}}$$

$S_t \leq 1$: in case of stiff clay having cracks and fissures.

Soil classification based on sensitivity :

Sensitivity	Classification
1	No loss in strength on remoulding.
2-4	Soil is normal sensitive
4-8	Sensitive
8-15	Extra-Sensitive
>15	Quick

- **Thixotropy:** It is the property of certain clays by virtue of which they regain, if left alone for a time, a part of the strength lost due to remoulding, at unaltered moisture content.
  Higher the sensitivity, larger the thixotropic hardening.
- **Activity** $\text{Activity of clay} = \frac{\text{Plasticity index}}{\%\text{by weight finer than } 2\mu}$

Activity based classification of clays

Activity	Classification
< 0.75	Inactive
0.75-1.25	Normal
>1.25	Active

Volume change during swelling or shinkage = f(I_P and % clay) or Actity

NOTE: Black cotton soil is very active.

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Classification of Soils 2

USCS

It is adopted by IS code. It was given by A-Casagrande. It uses particle size distribution for coarse soils and plasticity for fine soils.

Classification of Soils

Object:

Sorting soils into groups showing similar behaviour based on index property, Generally used property are

(i) Grand Size Distribution (ii) Plasticity

Depending upon Intended use different classification systems have evolved:

1. **Unifield Soil Classification System (USCS)**

Given by Casajsande

Intended for use in Airfield, Construction

Major Soils Groups	Soil Type	Prefix	Classification Parameters
Coarse Grained	Gravel	G	Grain size distribution
	Sand	S	
Fine Grained	Silt	M	Plasticity characteristics
	Clay	C	
Organic		O	Percentage of organic matter and particles of decomposed vegetation
Peat		Pt	

Note : ISCS is a modified USCS system.

2. **AASHTo Classification System**

For Highway Construction

Soil Classified into 8 groups divided into subgroups based on group index, GI.

GI value ranges between

O (Good Subgrade Material) to 20 (Poor Subgrade Material)

NOTE: Higher GI, less desirability of soil as highway material within that subgroup.

3. Indian Standrad Soil Classification System (ISSCS)

% Fineness : % of soil fraction passing thorugh 75 P sieve.

(i) % Fineness < 50% = Soil contain mainly Coarse Grained fraction otherwise Fine grained fraction

(ii) Fraction retained over 75 P is undergone with plasticity studies, i.e. $W_L + I_P$ identifies.

Classification chart for Coarse Grained Soil i.e. when % fineness, Less than 50%

	Gravel (G)		Sand (S)		Remarks
	Well graded (W) (If) $C_u > 4$, $C_c = 1$ to 3	Poorly Graded (P) (else)	Well graded (W) (If) $C_u > 6$, $C_c = 1$ to 3	Poorly Graded (P) (else)	
(i) p < 5%	GW	GP	SW	SP	Gradation only govern the properties
(ii) p – 5 to 12%	GW – GC or GW – GM	GP – GC or GP – GM	SW – SC or SW – SM	GP – GC or GP – GH	Dual symbol check presence of either clay (C) or silt (M), using plasticity chart
(ii) p > 12 %	$I_P < 7\%$ GM	$I_P > 7\%$ GC	$I_P < 4\%$ SM	$I_P > 7\%$ SC	Presence of fine particles dominate the soil characteristics

For I_P 4 – 7%, Dual Symbols used

where P - % passing through 75μ or micron sieve.

ISSCS

Coarse grain classification
Based on
1. Particle size
2. Cu
3. Cc
4. Fineness

Fine grain classification
Based on
1. Plasticity (PI)
2. Liquid limit/compressibility

Clay (size)	Silt (size)	Fine	Medium	Coarse	Fine	Coarse	Cobble	Boulder
		Sand			Gravel			
0.002 mm	0.075 mm	0.425 mm	2.00 mm	4.75 mm	20 mm	80 mm	300 mm	

Classification of soils based on grain size.

- **On the basis of fineness coarse grain soils are further classified**

Case-I: When fineness is < 5%

1. GW – Well graded gravel
   Cu > 4
   1 < Cc < 3
   Fineness < 5%
2. GP – Poorly graded gravel
   Above values of Cu and Cc are not satisfied.
3. SW – Well graded sand
   Cu > 6
   1 < Cc < 3
4. SP – Poorly graded sand/uniformly graded sand
   Cu and Cc are not in range.

Case-II: If fineness is 5% to 12% the dual symbol are used.

1. GW – GC well graded gravel containing clay.
   Fineness – 5 to 12%
   Clay > Silt
   Gravel > Sand
   Cu > 4; 1 ≤ Cc ≤ 3
2. GW – GM Well graded gravel containing silt
   Cu > 4
   1 ≤ Cu ≤ 3
   Silt > Clay
   Gravel > Sand
3. SW – SC Well graded sand containing clay
   Sand > Gravel
   Clay > Silt

Cu > 6
1 ≤ Cc ≤ 3
Fineness 5 to 12%

4. SW – SM Well graded sand containing silt
Sand > gravel
Silt > Clay
Cu > 6
1 ≤ Cu ≤ 3
Fineness 5 to 12%

For poorly graded soils like GP – GC, GP, GM, SP-SC SP-SM the values of Cu and Cc are not satisfied.

Case-III: When fineness is more than 12%

GC: Clayey gravel
Gravel > Sand
Clay > Silt $I_P > 7\%$

GM: Silty gravel
Sand < Gravel
Clay < Silt $I_P < 4\%$

SC: Clayey silt
Sand > Gravel
Siktl <Clay $I_P > 7\%$

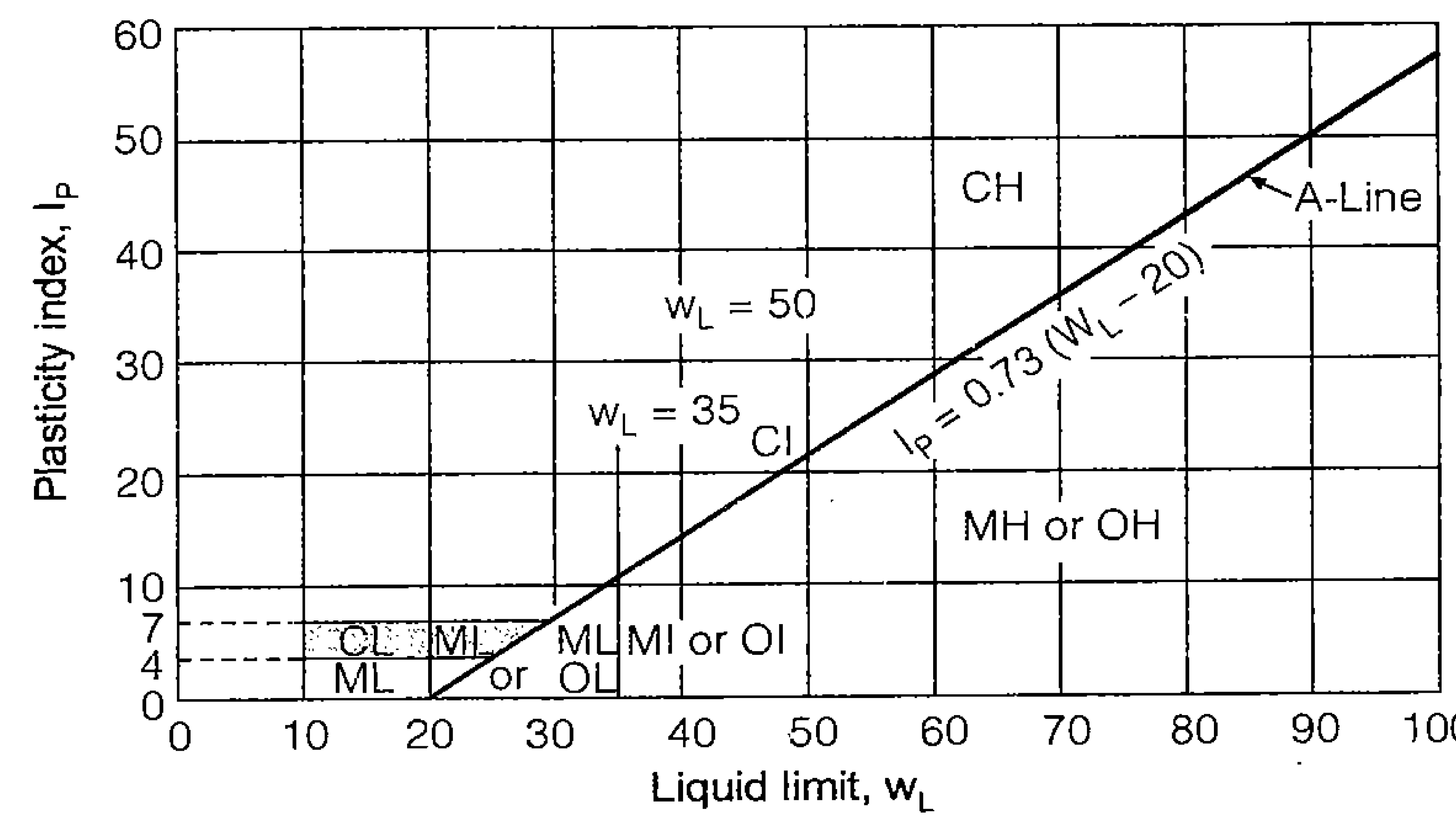
SM: Silty sand
Sand > Gravel
Silt > Clay $I_P < 4\%$

Note : For I_P between 4 and 7, Dual Symbols are used.

Classification of Fine Soils

1. Silts (0.002 mm to 0.075 mm)
   - Coarse 0.02 to 0.075 mm
   - Medium 0.01 to 0.02 mm
   - Fine 0.002 to 0.01 mm
2. Clay → <0.002 mm
   (i) Low plastic soils (LL < 35%)
   CL → Low plastic inorganic clay
   ML → Low plastic silt
   OL → Low plastic organic clay
   (ii) Medium plastic soils (35% < LL 50%)
   CI → Medium plastic inorganic clay
   MI → Medium plastic silt
   OI → Medium plastic organic clay

(iii) High plastic soils (LL > 50%)
CH → High plastic inorganic clay
MH → High plastic silt
OH → High plastic organic clay



Equation of A-line $I_P = 0.73 (W_L - 20)$
Equation of U-line $I_P = 0.9 (W_L - 8)$

	Clay mineral	Properties
1.	Kaolinite mineral	Hydrogen bond is there which is strongest bond. Ex. China clay
2.	Illite mineral	Ionic bond. Medium change in volume due to moisture change.
3.	Mont morillonite	Water bond which is weakest bond. Max change in volume due to moisture change. Ex. Black soils & Bentonite soils

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Permeability

3

Permeability of Soil

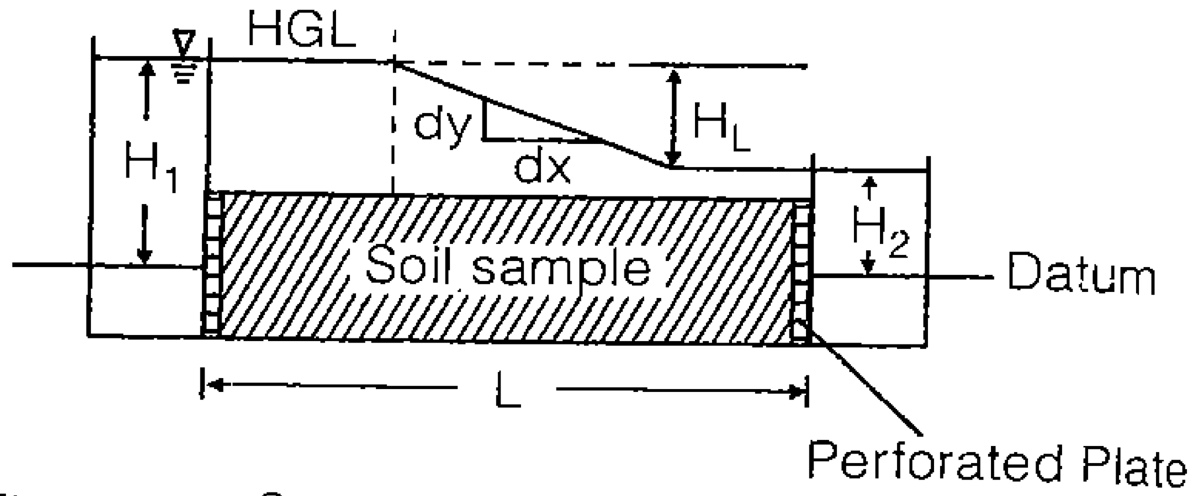
The permeability of a soil is a property which describes quantitatively, the ease with which water flows through that soil.

Darcy's Law

Darcy established that the flow occurring per unit time is directly proportional to the head causing flow and the area of cross-section of the soil sample but is inversely proportional to the length of the sample.

(i) Rate of flow (q)

$$q \propto \frac{\Delta h}{L}A \rightarrow q = KiA$$



Where, q = rate of flow in m^3/sec.
K = Coefficient of permeability in m/s
i = Hydraulic gradient
A = Area of cross-section of sample

$$i = \frac{H_L}{L}$$

where, H_L = Head loss = $(H_1 - H_2)$

$$i = \tan\theta = \frac{dy}{dx}$$

(ii) Seepage velocity

$$V_s = \frac{V}{n}$$

where, V_s = Seepage velocity (m/sec)
n = Porosity & V = discharge velocity (m/s)

(iii) Coefficiency of percolation

$$K_P = \frac{K}{n}$$

where, K_p = coefficiency of percolation
and n = Porosity.

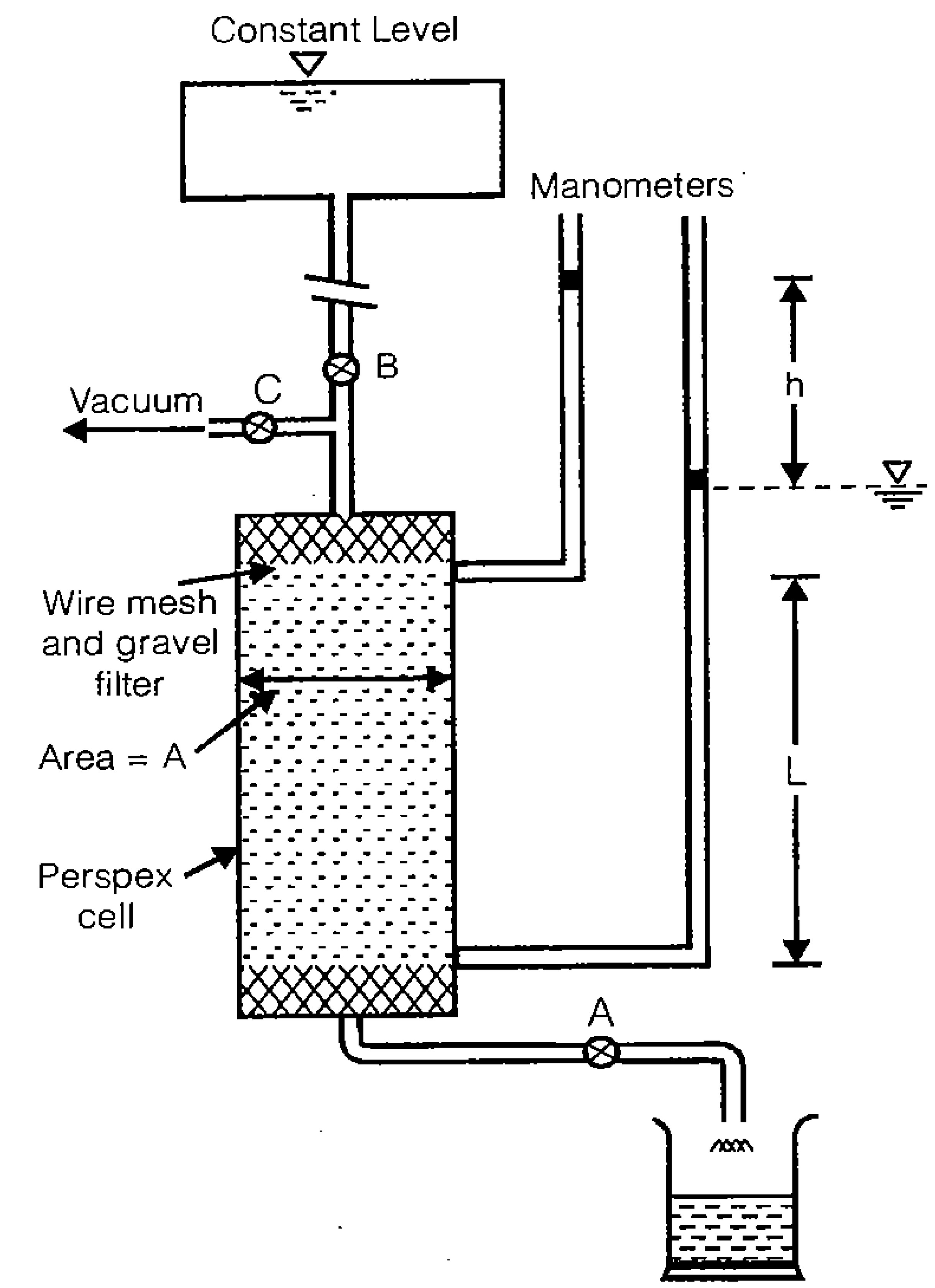
Constant Head Permeability Test

$$K = \frac{QL}{tH_LA}$$

where, Q = Volume of water collected in time t in m^3.

Constant Head Permeability test is useful for coarse grain soil and it is a laboratory method.

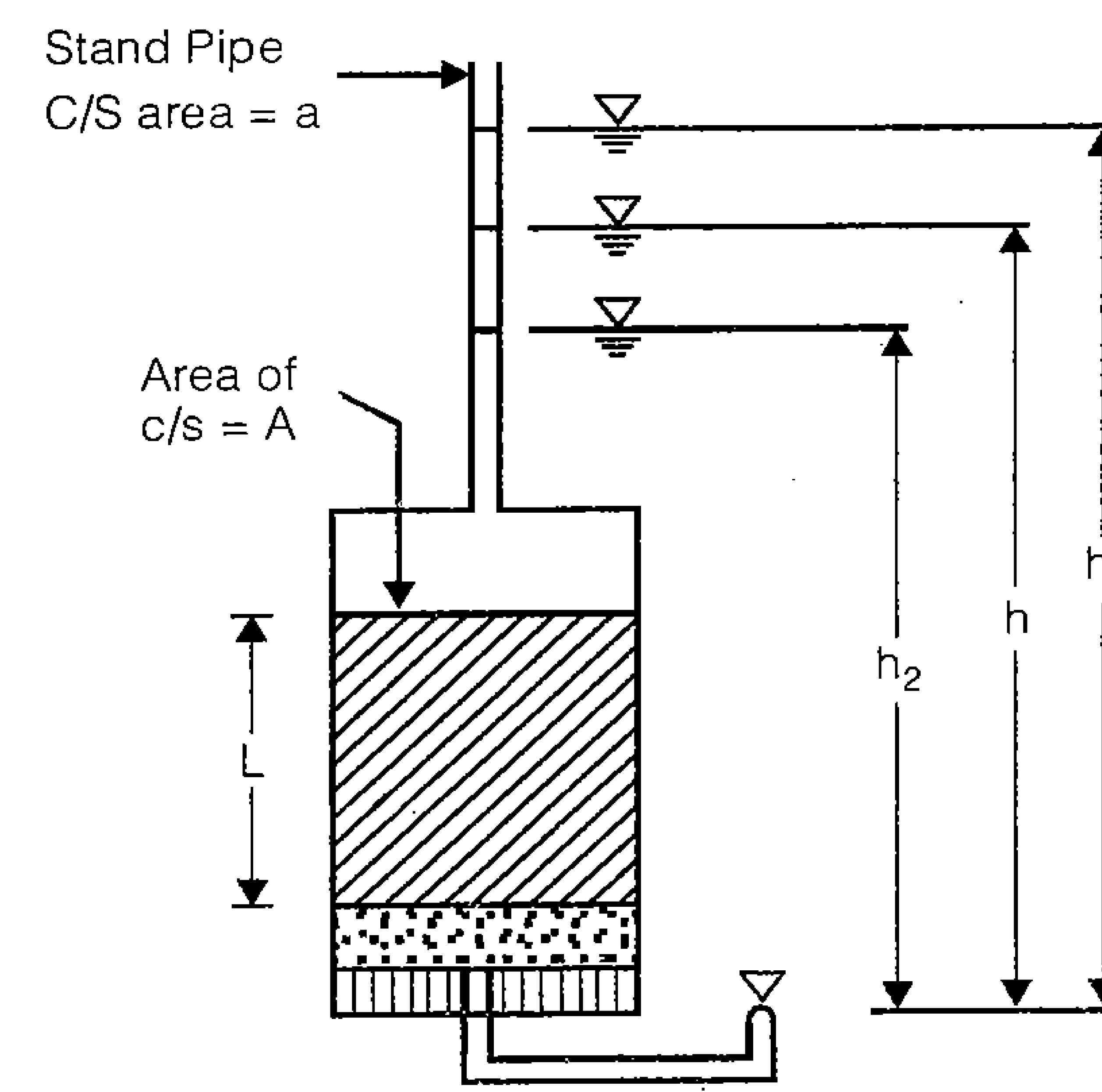
$$K \alpha \frac{1}{\mu}, \quad \mu \alpha \frac{1}{T} \rightarrow K \alpha T$$



where,
K = Coefficient of permeability
μ = Coefficient of dynamic viscosity
T = Temperature

Falling Head Permeability Test OR Variable Head Permeability Test

$$K = \frac{2.303aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right)$$



where,
a = Area of tube in m^2
A = Area of sample in m^2
t = time in 'sec'
L = length in 'm'
h_1 = level of upstream edge at $t = 0$
h_2 = level of upstream edge after time 't'.

Remember: Falling head permeability test is useful for fine grained soil and it is a laboratory method.

Kozney–Karman Equation

$$K = \frac{1}{C}\cdot\frac{1}{S^2}\cdot\frac{\gamma_w}{\mu}\cdot\frac{e^3}{1+e}$$

where, C = Shape coefficient, ~5mm for spherical particle

S = Specific surface area = $\frac{\text{Area}}{\text{Volume}}$

For spherical particle.

$$S = \frac{4\pi R^2}{\frac{4}{3}\pi R^3} = \frac{6}{\text{Diameter}}$$

R = Radius of spherical particle.

$$S = \frac{6}{\sqrt{ab}}$$

When particles are not spherical and of variable size. If these particles passes through sieve of size 'a' and retain on sieve of size 'b'.

e = void ratio

μ = dynamic viscosity, in (N–s/m²)

γ_w = unit weight of water in kN/m³

$$\frac{k_1}{k_2} = \frac{e_1^2}{e_2^2}$$

Allen Hazen Equation

$$K = C.D_{10}^2$$

Where, D_{10} = Effective size in cm. k is in cm/s
C = 100 to 150

NOTE: It is valid for particle size of soil 0.1 mm to 3 mm. It is valid for sand.

Lloudens Equation

$$\log_{10} KS^2 = a + b.n$$

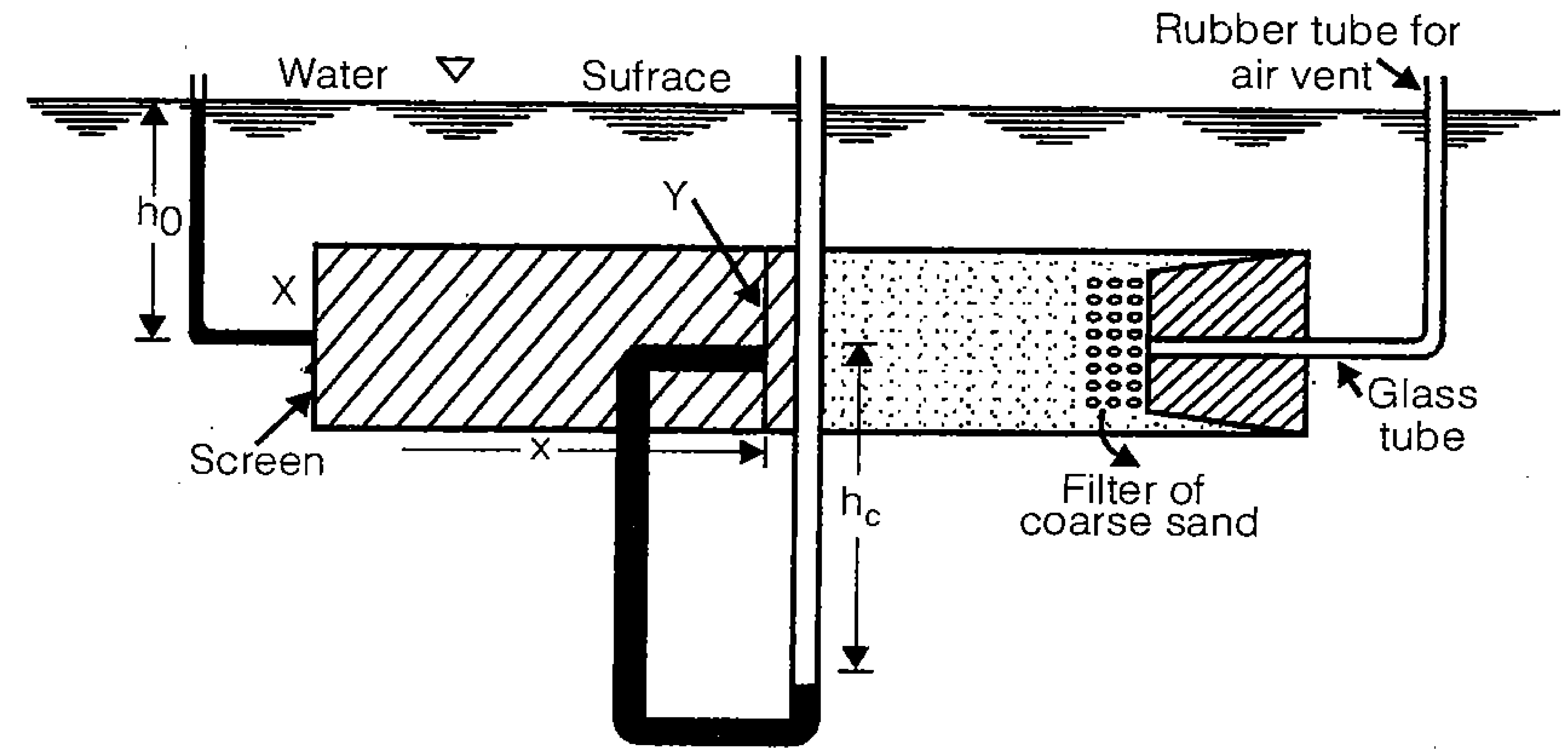
where, S = Specific surface area
n = Porosity.

a and b are constant.

Consolidation equation $K = C_v m_v \gamma_w$

where, C_v = Coefficient of consolidation in cm²/sec
m_v = Coefficient of volume Compressibility in cm²/N

Capillary Permeability Test



$$i = \frac{h_0 + h_c}{x}$$

where, S = Degree of saturation
K = Coefficient of permeability of partially saturated soil.

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2K}{S.n}[h_{o_1} + h_c]$$

where h_c = remains constant (but not known as depends upon soil)
h_{o_1} = head under first set of observation,
n = porosity, h_c = capillary height

Another set of data gives,

$$\frac{x_2'^2 - x_1'^2}{t_2' - t_1'} = \frac{2K}{S.n}[h_{o_2} + h_c]$$

h_{o_2} = head under second set of observation

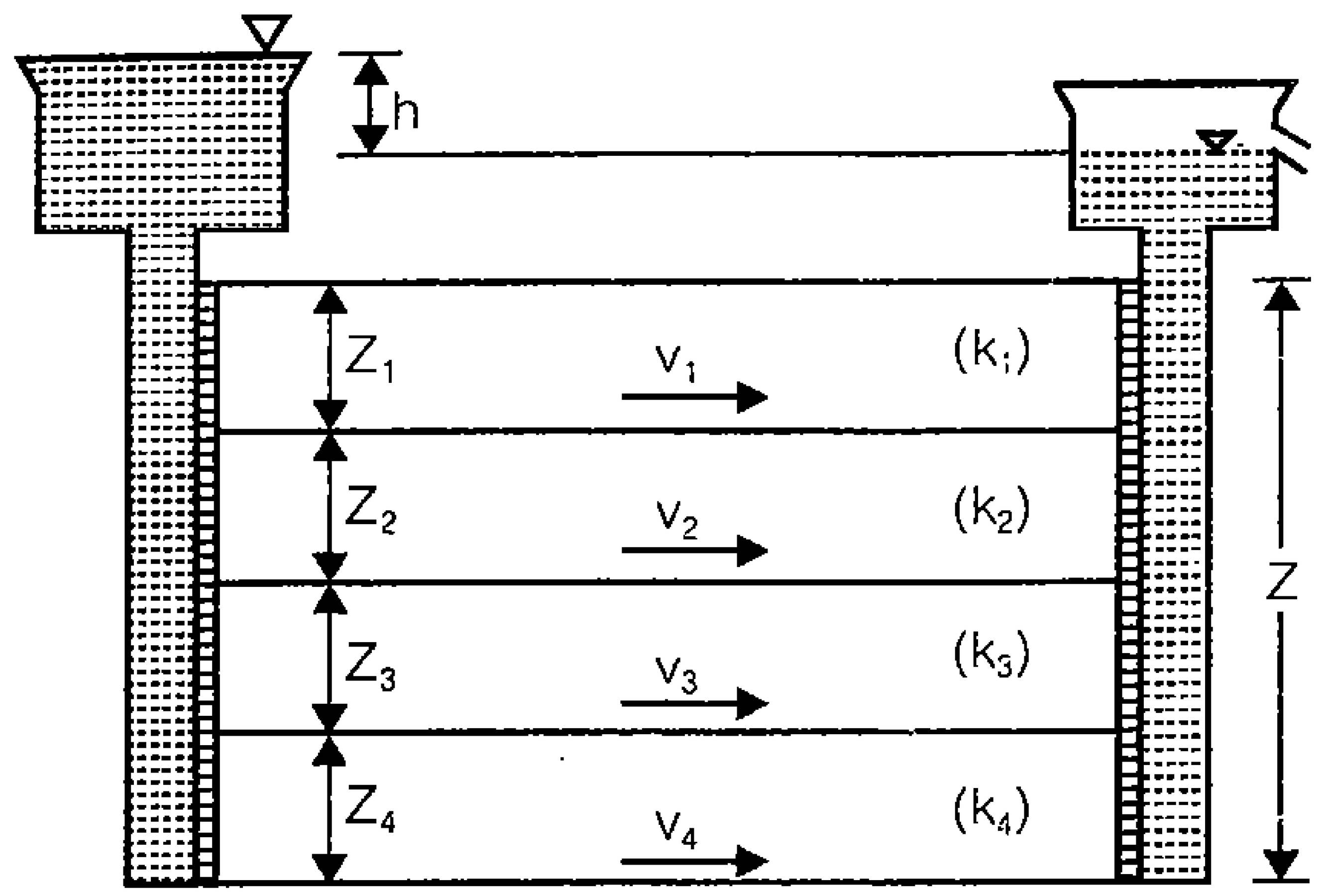
- For S = 100%, K = maximum. Also, $k_u \propto S$.

Permeability of a stratified soil

(i) Average permeability of the soil in which flow is parallel to bedding plane,

$$K_{eq} = \frac{k_1 z_1 + k_2 z_2 + \ldots + k_n z_n}{z_1 + z_2 + \ldots + z_n}$$

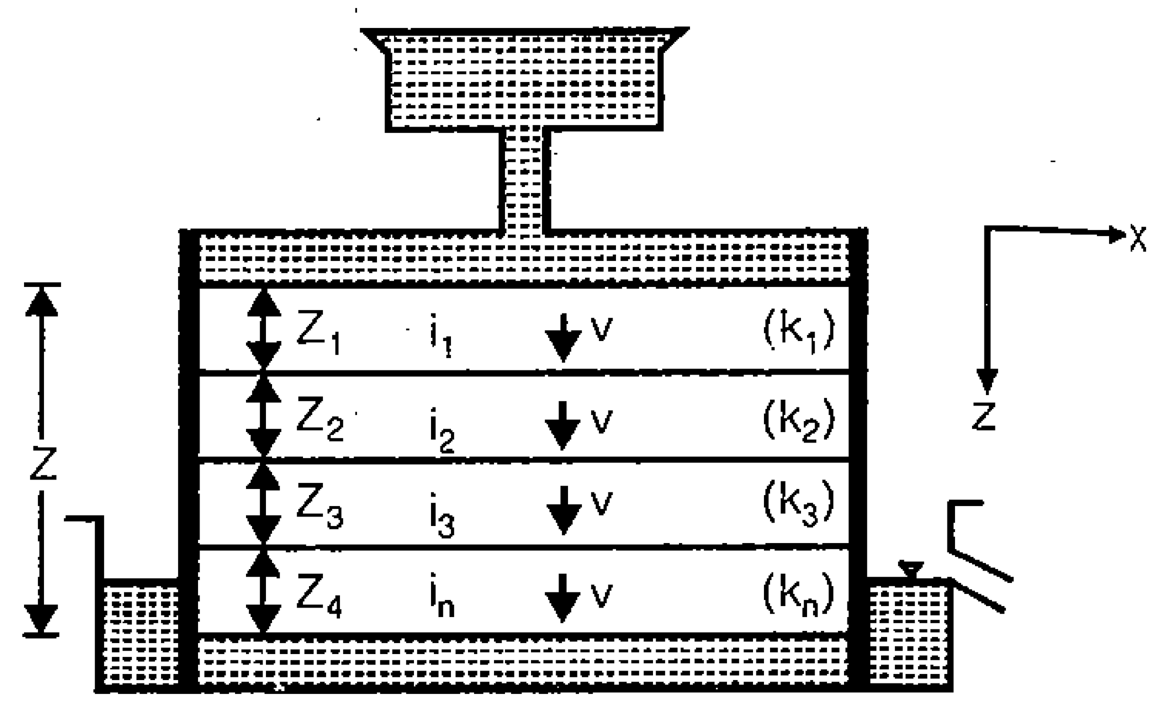
$$K_{eq} \sim k_x$$



(ii) Average permeability of soil in which flow is perpendicular to bedding plane.

$$k_{eq} = \frac{z_1 + z_2 + \ldots + z_n}{\frac{z_1}{k_1} + \frac{z_2}{k_2} + \ldots + \frac{z_n}{k_n}}$$

$$k_{eq} \sim k_z$$



(iii) For 2-D flow in x and z direction

$$k_{eq} = \sqrt{k_x \cdot k_z}$$

(iv) For 3-D flow in x, y and z direction

$$k_{eq} = (k_x \cdot k_y \cdot k_z)^{1/3}$$

Coefficient of absolute permeability (k_0)

$$k_0 = k \cdot \frac{\mu}{\gamma_w}$$

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Well Hydraulics 4

Specific yield (S_y)

The specific yield of an unconfined aquifer is the ratio of volume of water which will flow under saturated condition due to gravity effect to the total volume of aquifer (v).

$$S_y = \frac{V_{wy}}{V}$$

where, V_{wy} = Volume of water yielded under gravity effect
and V = Total volume of water.

Specific retention

The specific retention of an unconfined aquifer is the ratio of volume of water retained against gravity effect to the total volume of aquifer (v).

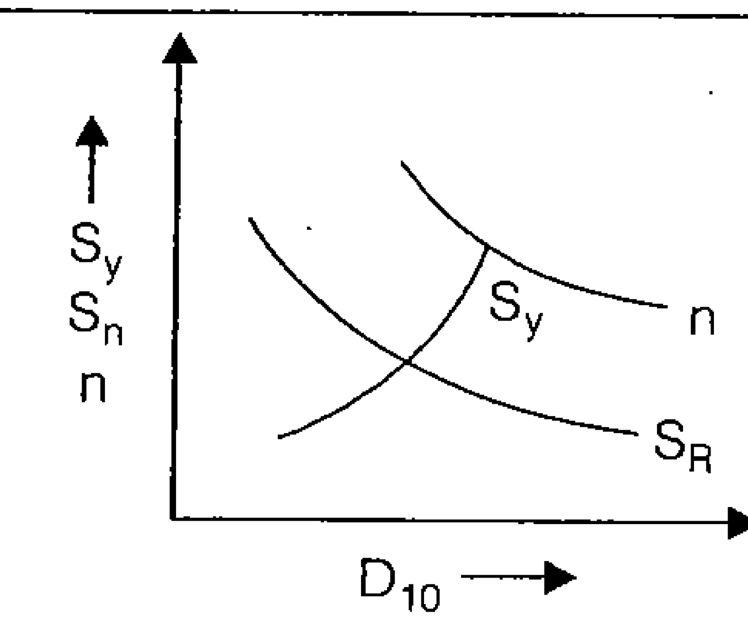
$$S_R = \frac{V_{WR}}{V}$$

where, V_{WR} = Volume of water retained under gravity effect.

Remember

$$S_y + S_R = n$$

where, n = Porosity

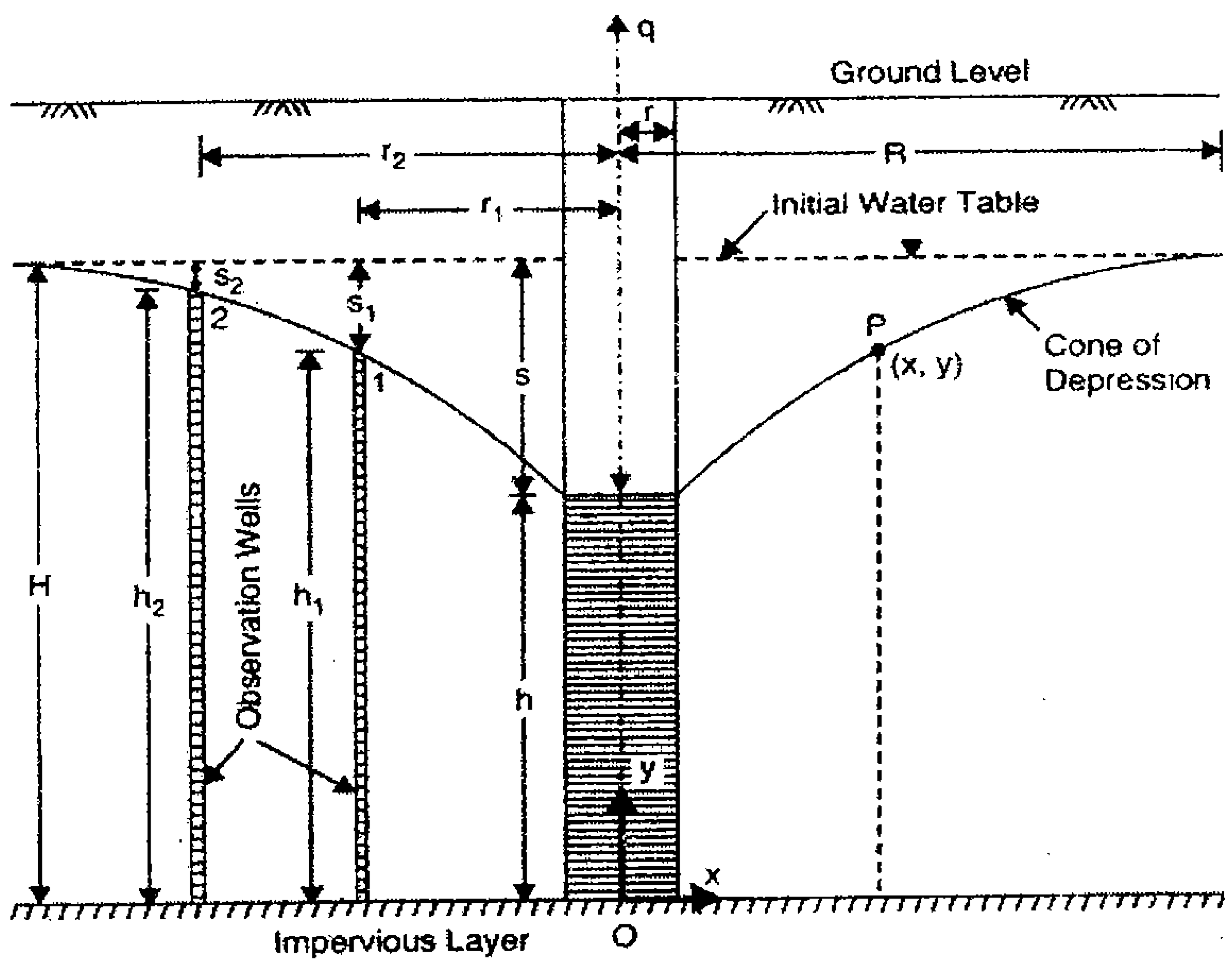


Coefficient of transmissibility

$$T = kH$$

where, H = Thickness
k = Coefficient of permeability

Unconfined Aquifer



(a) Theims Theory

$$\boxed{q = \frac{k\pi}{2.303} \frac{(h_2^2 - h_1^2)}{\log_{10}(r_2/r_1)}}$$

$$\boxed{h_1 + s_1 = h_2 + s_2}$$

where, q = Rate of flow in m^3/s
h_1 = Height of water table of 1^{st} observation well
h_2 = Height of water table of 2^{nd} observation well
s_1 = Drawdown of 1^{st} test well
s_2 = Drawdown of 2^{nd} test well.

r_1 and r_2 are radius of Ist and 2^{nd} observation wells respectively.

(b) Dupits Theory

$$\boxed{q = \frac{k\pi}{2.303} \frac{(H^2 - h^2)}{\log_{10}(R/r)}}$$

$$\boxed{R = 3000.S\sqrt{k}} \text{ and}$$

$$\boxed{S = H - h}$$

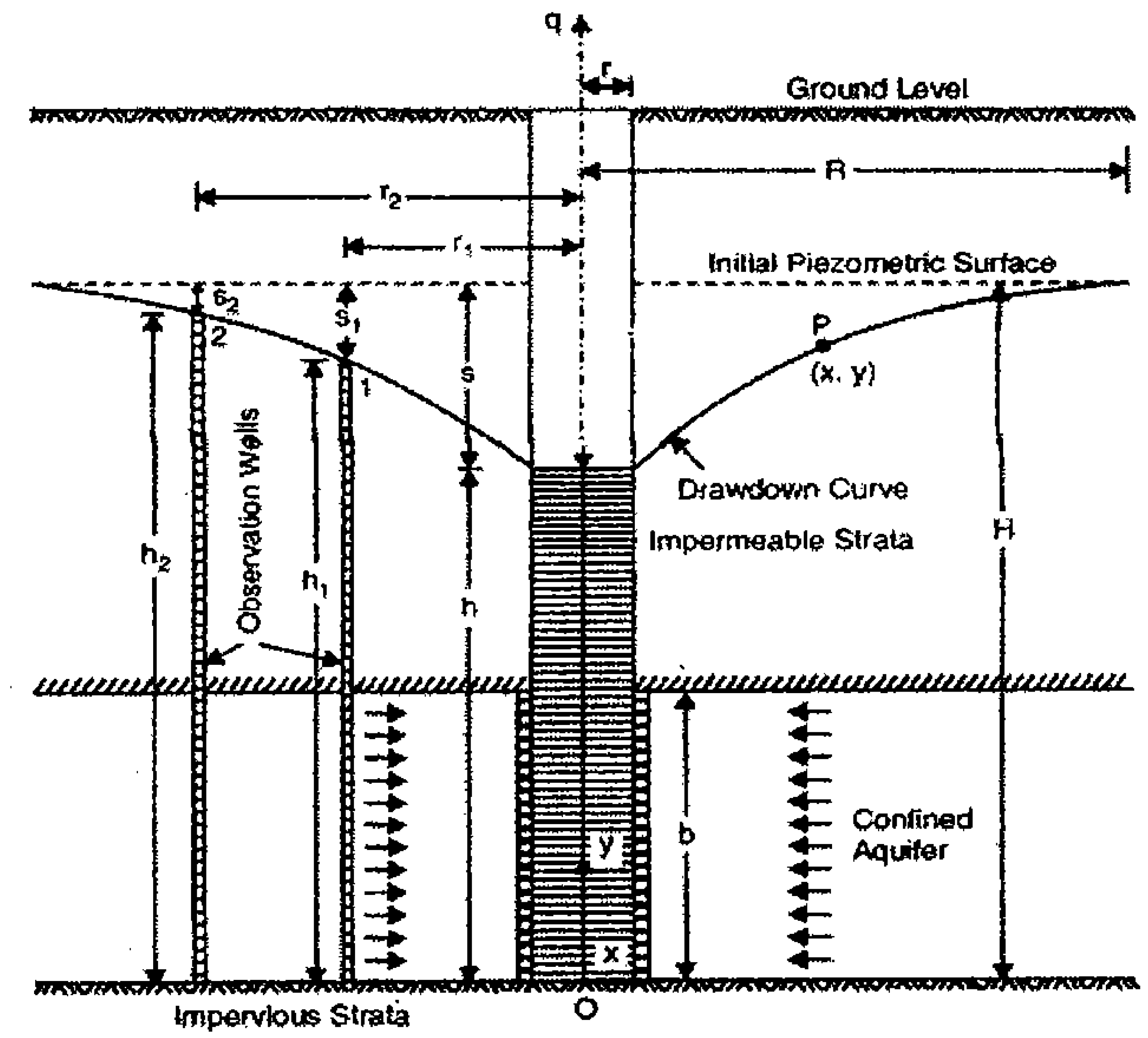
Where, S = Drawdown in the well
k = Permeability coefficient in m/s.
R = Radius of influence in 'm'

$$\boxed{150\,m \le R \le 300\,m}$$

r = Radius of test well in 'm'.

Results of dupits theory are not accurate because 'R' is based on empirical relation.

Confined Aquifer



(a) Theims theory $\boxed{q = \dfrac{2\pi bK(h_2 - h_1)}{2.303 \log_{10}\left(\dfrac{r_2}{r_1}\right)}}$ where, b = width

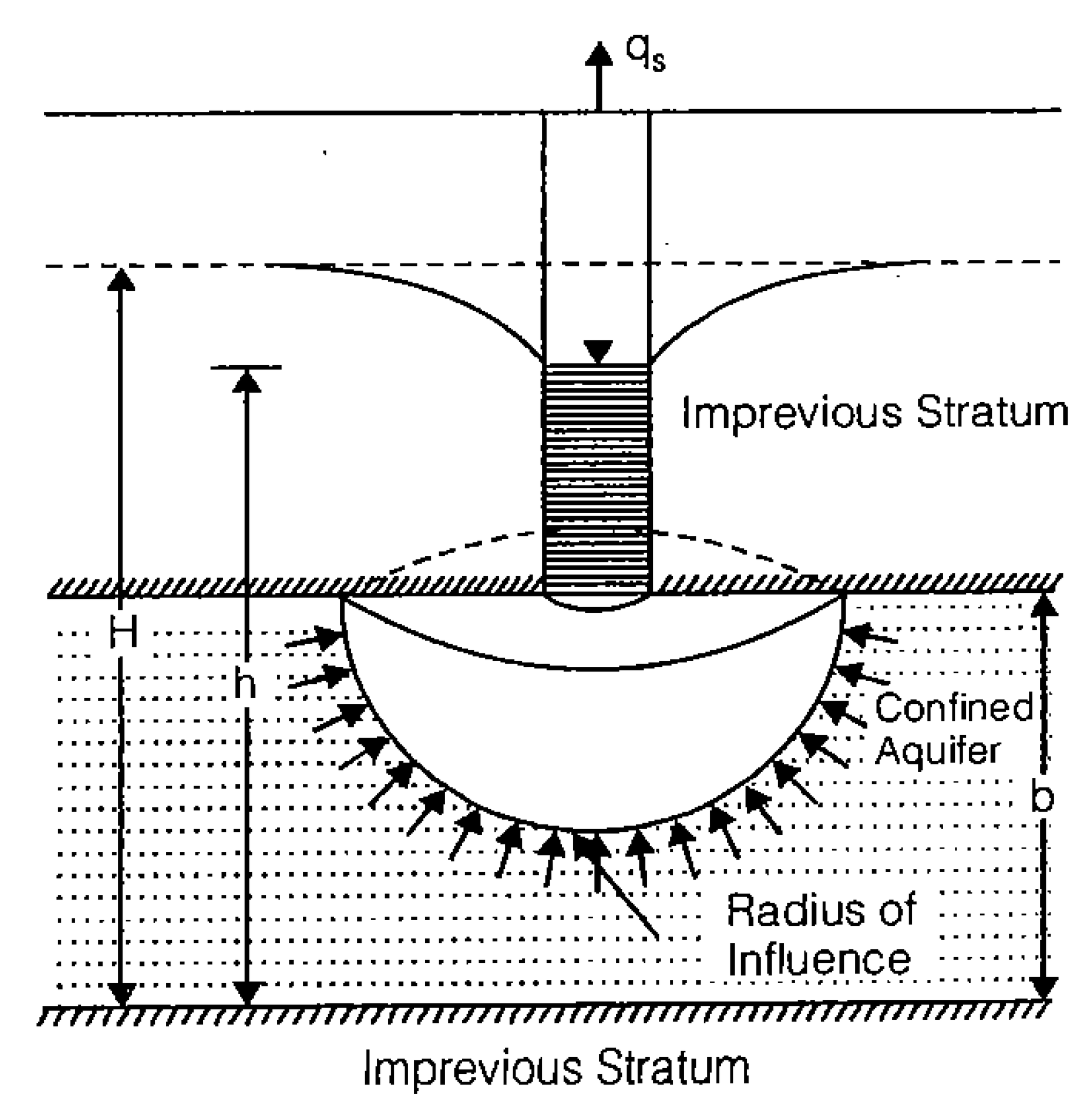
(b) Dupits theory $\boxed{q = \dfrac{2\pi bK(H - h)}{2.303 \log_{10}\left(\dfrac{R}{r}\right)}}$

Spherical flow through well

$$\boxed{q_s = K.2\pi r.s}$$

where, r = Radius of well
S = Drawdown
q_s = Rate of flow through spherical well in m^3/s

$$\boxed{q_s = \frac{1}{30}.q_{\text{radial flow}}}$$



Pumping-In-Test

(a) Open end test

$$K = \frac{q}{5.5rh}$$

where, r = Radius of pipe
h = Head of water above the base of pipe, it may include gravity head and pressure head.

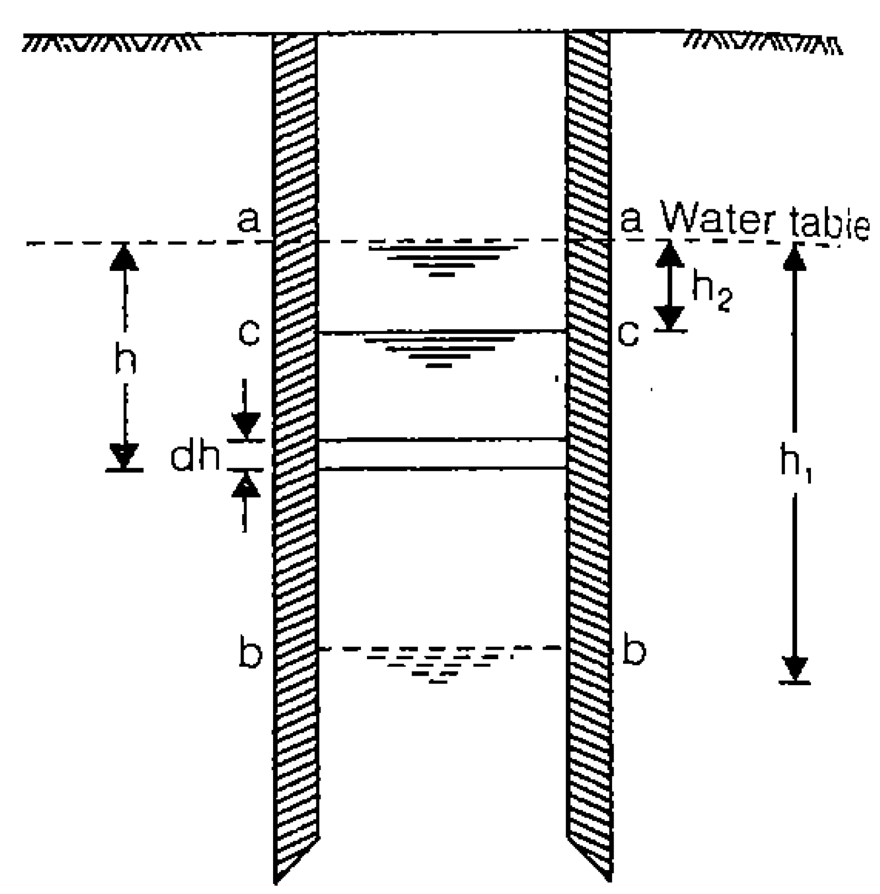
(b) Tacker test

$$K = \frac{q}{2\pi L h}\log_{10}\left(\frac{L}{r}\right) \quad \text{... when } L > 10r$$

$$k = \frac{q}{2\pi L h}\sin^{-1}\left(\frac{L}{2r}\right) \quad \text{... when } L < 10r$$

where,
L = Length of perforated section of pipe
r = Radius of pipe
h = Head of which water is added.

Open well (Recuperation test)



$$q = \frac{C}{A}.\text{Volume}$$

where, $$\frac{C}{A} = \frac{2.303}{T}\log_{10}\left(\frac{h_1}{h_2}\right)$$

Volume = A.H
A = Area of well

$\frac{C}{A}$ = Specific yield or specific capacity of an open well.

T = Time in 'sec'
h_1 = Position of water table of t = 0
h_2 = Position of water table of t = T

Values of Permeability

	SOIL	K(cm/sec)	Degree of Permeability
1.	Coarse gravel	>1	High
2.	Fine gravel-Fine sand	1 to 10^{-2}	Medium
3.	Silt-Sand admixtures, loose silts, rock flour and loess	10^{-2} to 10^{-4}	Low
4.	Dense silt, clay silt admixtures, non-homogenous clays	10^{-4} to 10^{-6}	Very low
5.	Homogenous clays	$<10^{-6}$	Impervious

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Compressibility and Consolidation

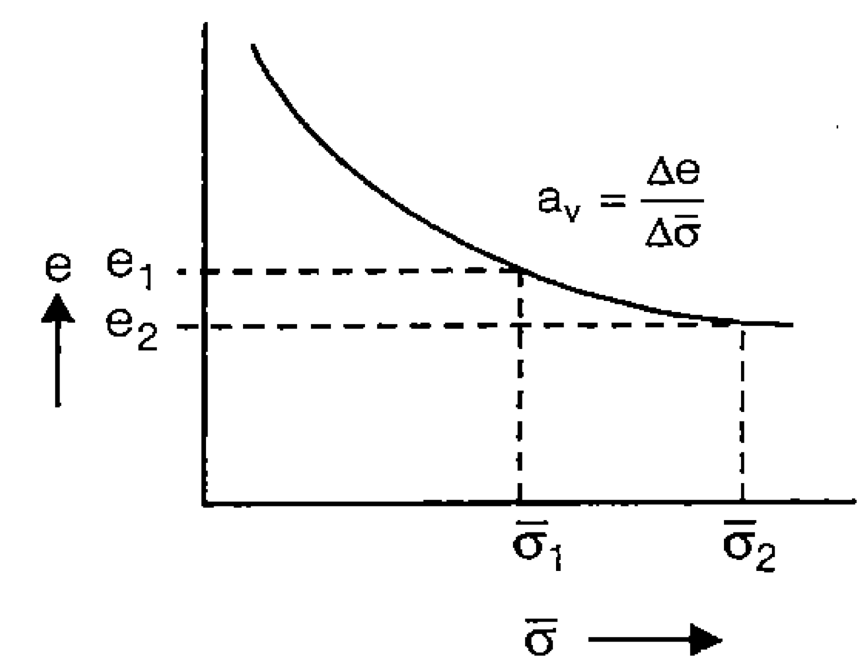
5

Coefficient of Compressibility (a_v)

$$a_v = \frac{e_1 - e_2}{\overline{\sigma_2} - \overline{\sigma_1}}$$

e_1 = Void ratio at effective stress $\overline{\sigma_1}$

e_2 = Void ratio at effective stress $\overline{\sigma_2}$



$$\frac{\Delta V}{V_0} = \frac{\Delta H}{H_0}$$

ΔV = Change in volume in m^3, or cm^3.
V_0 = Initial volume in m^3 or cm^3.
ΔH = Change in depth in 'm' or 'cm'.
H_0 = original depth in 'm' or 'cm'.

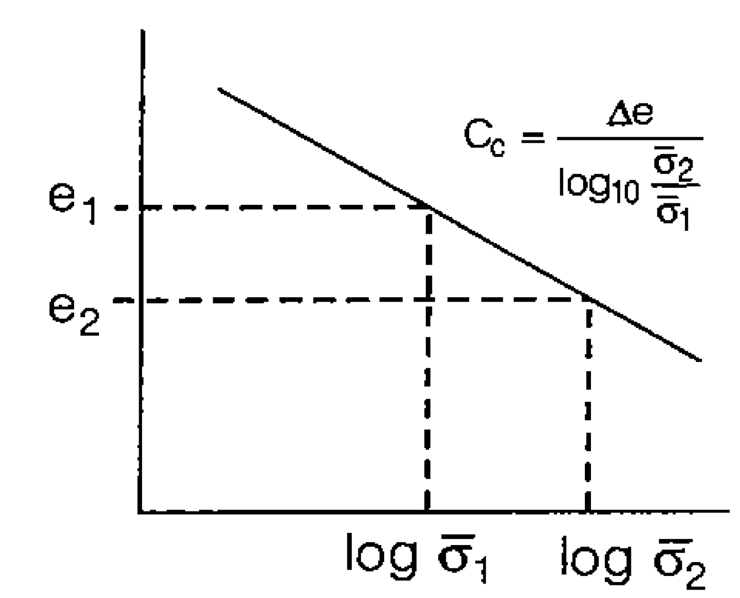
Remember: Consolidation settlement is a function of effective stress and not the function of total stress.

Coefficient of Compression (C_c)

(a) $$C_c = \frac{e_1 - e_2}{\log_{10}\overline{\sigma_2} - \log_{10}\overline{\sigma_1}}$$

↓

$$C_c = \frac{e_1 - e_2}{\log_{10}\left(\frac{\overline{\sigma_2}}{\overline{\sigma_1}}\right)}$$



(b) $C_c = 0.009(W_L - 10)$ For undisturbed soil of medium sensitivity.
W_L = % liquid limit.

(c) $C_c = 0.007(W_L - 7)$ For remolded soil of low sensitivity

(d) $C_c = 0.40(e_0 - 0.25)$ For undisturbed soil of medium sensitivity
e_0 = Initial void ratio

(e) For remoulded soil of low sensitivity. $C_c = 1.15(e_o - 0.35)$

(f) $C_c = 0.115\,w$ where, w = Water content

Remember: $C_r = \frac{1}{5}$ of C_c to $\frac{1}{10}$ of C_c where, C_r = Coef. of recompression

Over Consolidation Ratio

$$O.C.R = \frac{\text{Maximum effective stress applied in the past}}{\text{Existing effective stress}}$$

$O.C.R > 1$

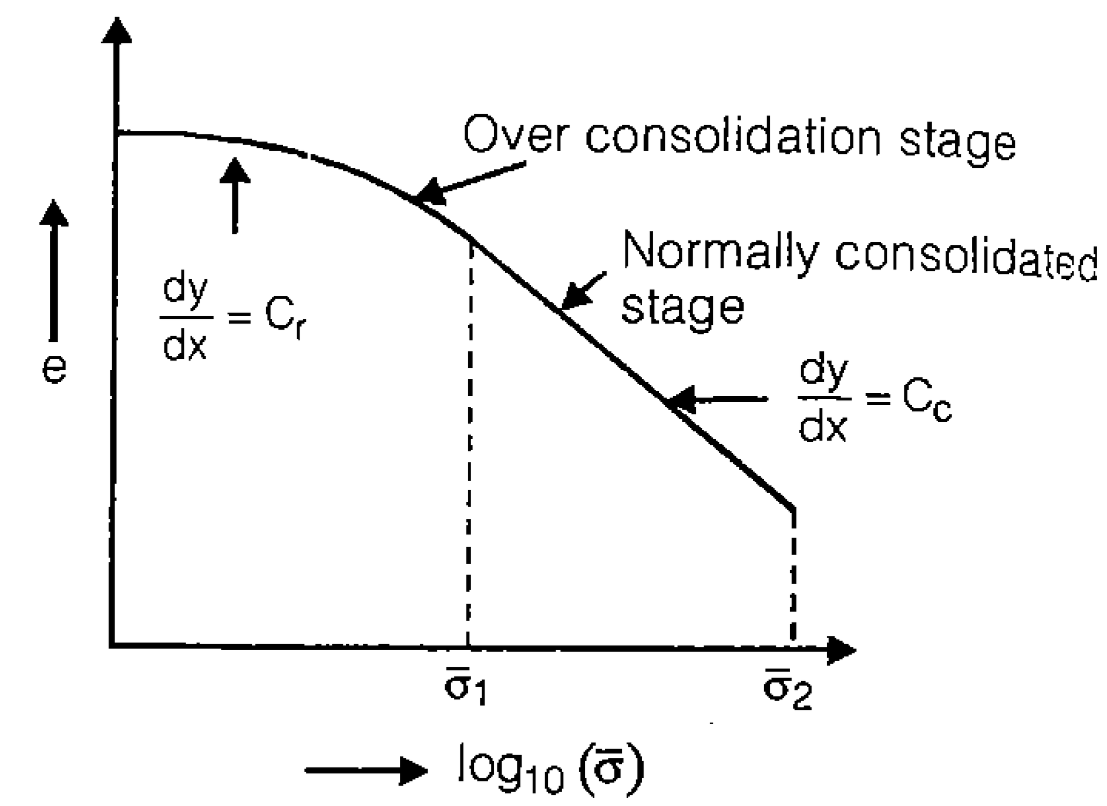
For over consolidated soil.

$O.C.R = 1$

For normally consolidated soil.

$O.C.R < 1$

For under consolidated soil.



Differential Equation of 1-D Consolidation

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}$$

where, u = Excess pore pressure,

$\frac{\partial u}{\partial t}$ = Rate of change of pore pressure

C_v = Coefficient of consolidation

$\frac{\partial u}{\partial z}$ = Rate of change of pore pressure with depth.

Coefficient of volume compressibility

$$m_v = \frac{a_v}{1 + e_0}$$

where, e_0 = Initial void ratio

m_v = Coefficient of volume compressibility

Compression modulus

$$E_c = \frac{1}{m_v}$$

where, E_c = Compression modulus.

Degree of consolidation

(i) $$\%U = \left(1 - \frac{U}{U_i}\right) \times 100$$

where,

%U = % degree of consolidation.

U = Excess pore pressure at any stage.

$U_i = \overline{\Delta\sigma}$ = Initial excess pore pressure

at t = 0, $u = u_i \Rightarrow \%u = 0\%$

at $t = \infty$, $u = 0 \Rightarrow \%u = 100\%$

(ii) $$\%u = \frac{e_0 - e}{e_0 - e_f} \times 100$$

where,

e_f = Void ratio at 100% consolidation. i.e., of $t = \infty$

e = Void ratio at time 't'

e_0 = Initial void ratio i.e., at t = 0

(iii) $$\%u = \frac{\Delta h}{\Delta H} \times 100$$

where,

ΔH = Final total settlement at the end of completion of primary consolidation i.e., at $t = \infty$

Δh = Settlement occurred at any time 't'.

Time factor

$$T_v = C_v \cdot \frac{t}{d^2}$$

where, T_v = Time factor

C_v = Coeff. of consolidation in cm^2/sec.

d = Length of drainage path

t = Time in 'sec'

$d = \frac{H_0}{2}$ For 2-way drainage

$d = H_0$ For one-way drainage.

where, H_0 = Depth of soil sample.

(i) $T_v = \frac{\pi}{4}(u)^2$ ... If $u \le 60\%$ $T_{50} = 0.196$

(ii) $T_v = -0.9332\log_{10}(1-u) - 0.0851$ ... If $u > 60\%$

Method to Find 'C_v'

(i) Square Root of Time Fitting Method

$$C_v = \frac{T_{90} \cdot d^2}{t_{90}}$$

where, T_{90} = Time factor at 90% consolidation

t_{90} = Time at 90% consolidation

d = Length of drainage path.

(ii) Logarithm of Time Fitting Method

$$C_V = \frac{T_{50} \cdot d^2}{t_{50}}$$

where, T_{50} = Time factor at 50% consolidation
t_{50} = Time of 50% consolidation.

Remember
- Square root of time fitting method is better for soil having higher secondary consolidation such as highly plastic clay.

Compression Ratio

(i) Initial Compression Ratio

$$r_i = \frac{R_i - R_0}{R_i - R_f}$$

where, R_i = Initial reading of dial gauge.
R_0 = Reading of dial gauge at 0% consolidation.
R_f = Final reading of dial gauge after secondary consolidation.

(ii) Primary Consolidation Ratio

$$r_p = \frac{R_0 - R_{100}}{R_i - R_f}$$

where, R_{100} = Reading of dial gauge at 100% primary consolidation.

(iii) Secondary Consolidation Ratio

$$r_s = \frac{R_{100} - R_f}{R_i - R_f} \qquad r_i + r_p + r_s = 1$$

Total Settlement

$$S = S_i + S_p + S_s$$

where, S_i = Initial settlement
S_p = Primary settlement
S_s = Secondary settlement

(i) Initial Settlement

$$S_i = \frac{H_0}{C_s} \cdot \log_{10} \frac{(\bar{\sigma}_0 + \overline{\Delta\sigma})}{\bar{\sigma}_0}$$ For cohesionless soil.

where, $C_s = 1.5 \frac{C_r}{\bar{\sigma}_0}$ where, C_r = Static one resistance in kN/m²
H_0 = Depth of soil sample

- $$S_i = \frac{q\sqrt{A}(1-\mu^2)}{E_s}(I_t)$$ for cohesive soil.

where; I_t = Shape factor or influence factor
A = Area.

Remember
For square footing, $A = B^2$ and $I_t = 1$

$$S_i = \frac{qB(1-\mu^2)}{E_s} \times I_t$$ for strip footing

(ii) Primary Settlement

- $$S_P = \Delta H = H_0 \frac{\Delta e}{1+e_0}$$
- $$\Delta H = H_0 m_v \overline{\Delta\sigma}$$
- $$\Delta H = \frac{C_c H_0}{1+e_0} \log_{10}\left(\frac{\bar{\sigma}_0 + \overline{\Delta\sigma}}{\bar{\sigma}_0}\right)$$
- $S_P = S_{C_1} + S_{C_2}$ $S_{C_1} << S_{C_2}$ → $S_P \sim S_{C_2}$

S_{C_1} = Settlement for over consolidated stage
S_{C_2} = Settlement for normally consolidated stage

$$S_P = \frac{C_r H_0}{1+e_0} \log_{10}\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right) + \frac{C_c H_0'}{1+e_0'} \log_{10}\left(\frac{\bar{\sigma}_2}{\bar{\sigma}_1}\right)$$

(ii) Secondary Settlement

$$S_S = \frac{C_s H_0}{1+e_{100}} \log_{10}\left(\frac{t_2}{t_1}\right)$$

where, $H_0 \sim H_{100}$
H_{100} = Thickness of soil after 100% primary consolidation.
e_{100} = Void ratio after 100% primary consolidation.
t_2 = Average time after t_1 in which secondary consolidation is calculated.

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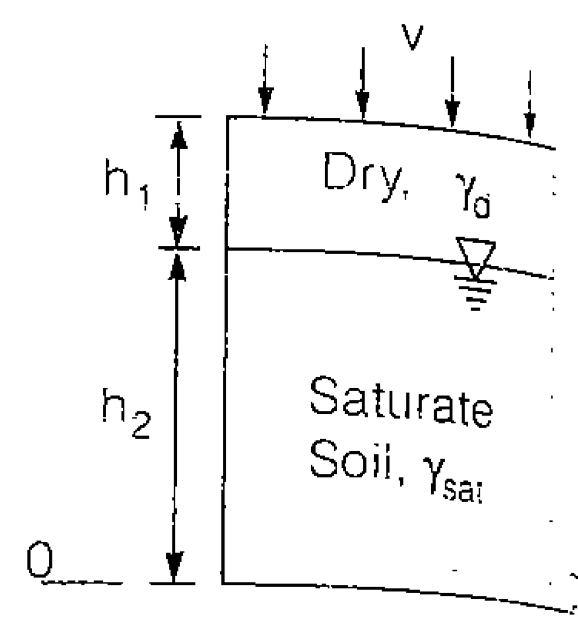
Effective Stress, Capilarity, Seepage

6

1. Effective stress concept involve physical interaction between solid and water.
2. Total stress = Complete overburden stress at 0 – 0

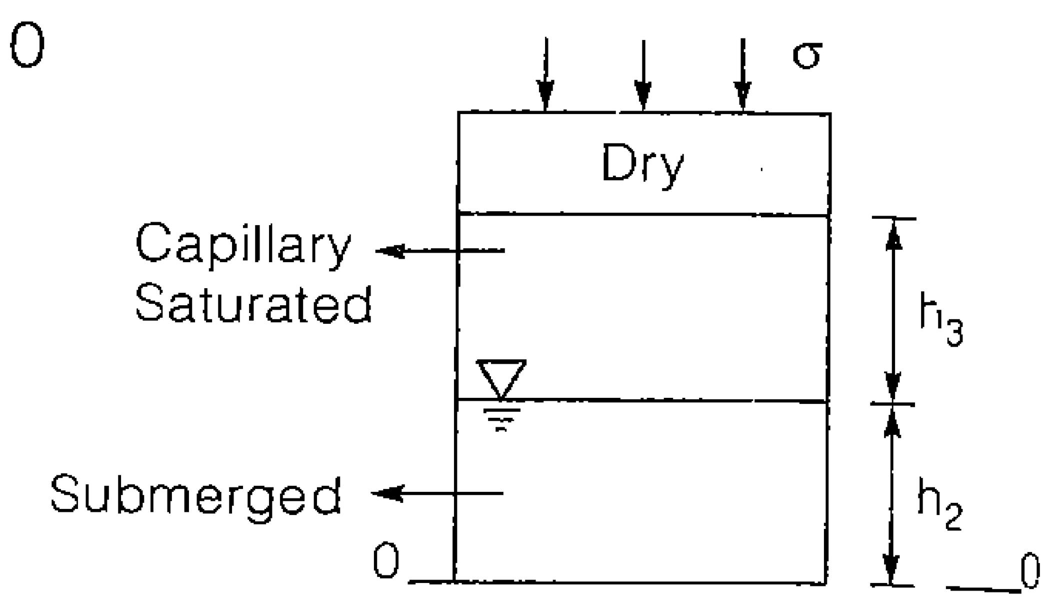


$$\text{(Physical Parameter)} = \frac{V + \gamma_d h_1 \times A + \gamma_{sat} h_2 \times A}{A}$$

3. Pore water Presence or Neutral Stress (Physical Parameter)
   Pressure applied by water occupying voids
   Static Pore water press at 0 – 0

$$u_s = \gamma_w H_2$$

$$H_3 = \frac{\sigma}{\gamma_w}$$

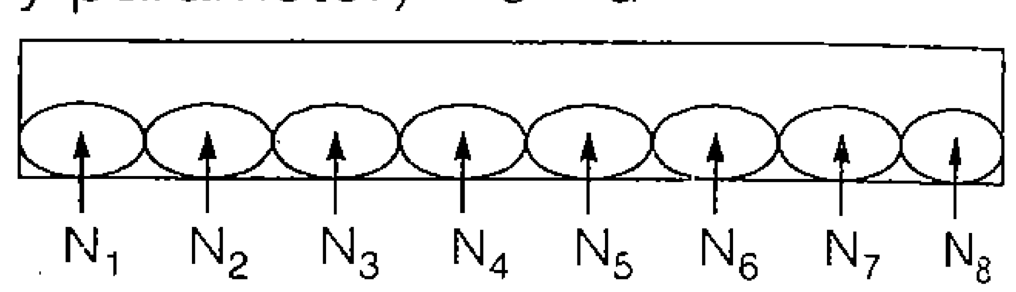


Total pore water pressure,

$$u = u_s + \sigma$$
$$= r_w H_2 + \sigma$$

4. Effective stress ($\bar{\sigma}$) = (imaginary parameter) = $\sigma - u$

$$\bar{\sigma} = \frac{\Sigma N}{A}$$



Capillary Water

Water held above the GWT due to surface tension.

1. $$h_c = \frac{4\sigma \cos\alpha}{\gamma_w d}$$

2. $$h_c(m) = \frac{0.29}{d(mm)}$$

   [Using σ = 73 N/m (super tension of pure water in glass tube)

3. For sands and silts, $d = 0.2\,D_{10}$
   D_{10} = Effective size of particle in (mm)

4. $$h_c(cm) = \frac{C}{eD_{10}}$$ C → constant having vlaue 0.1 – 0.5 cm^2

   D_{10} → Effective size in (cm)
   e → void ratio

Effects of Capillary Water :

1. Cause soil suction (apparent attractive force between soil particles) resulting in enhanced shear strength.
2. Bulking of sand capillary water results in apparent cohesion, holding particles in cluster, enclosing honey comb.

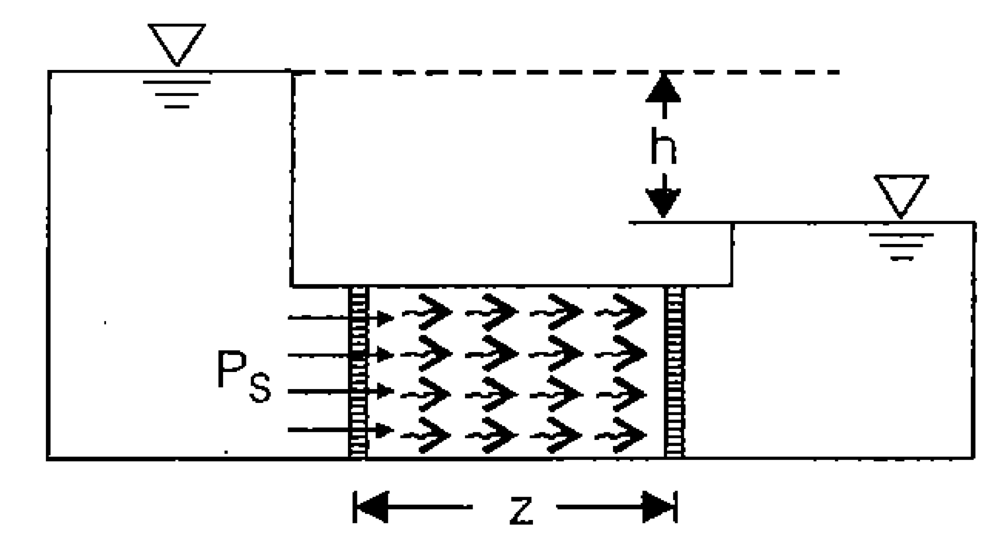
Seepage Pressure and Seepage Force

Seepage pressure is exerted by the water on the soil due to friction drag. This drag force/seepage force always acts in the direction of flow.

The seepage pressure is given by

$$P_S = h\gamma_\omega$$

where, P_s = Seepage pressure
γ_ω = 9.81 kN/m³.
Here, h = head loss
and z = length



(ii) $F_s = hA\gamma_\omega$ where, F_s = Seepage force.

(iii) $f_s = i\gamma_\omega$ where, f_s = Seepage force per unit volume.

$i = h/z$ where, i = Hydraulic gradient.

NOTE: P_s, F_s and f_s acts in the same direction i.e., in the direction of flow.

Effective Stresses

The effective stress is equal to the total stress minus the pore water pressure.

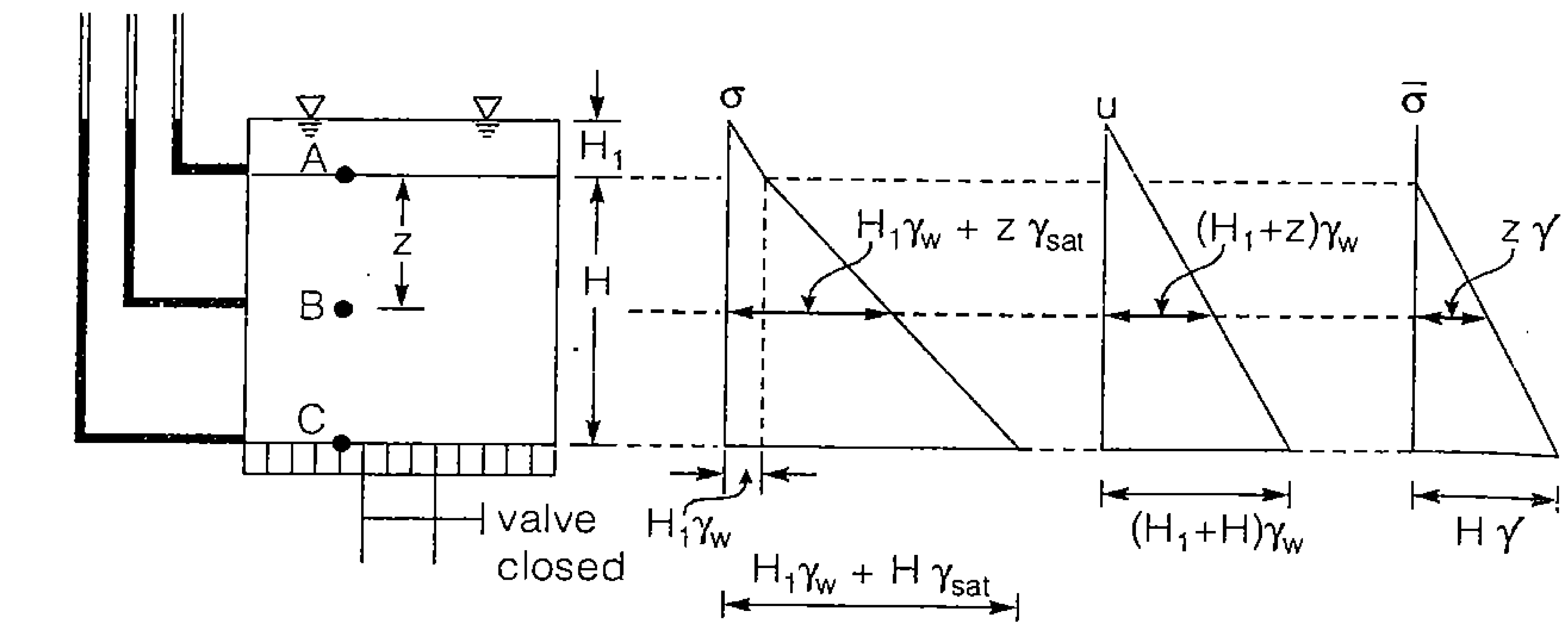
(i) Net effective vertical stress = $\bar{\sigma} \pm P_s$

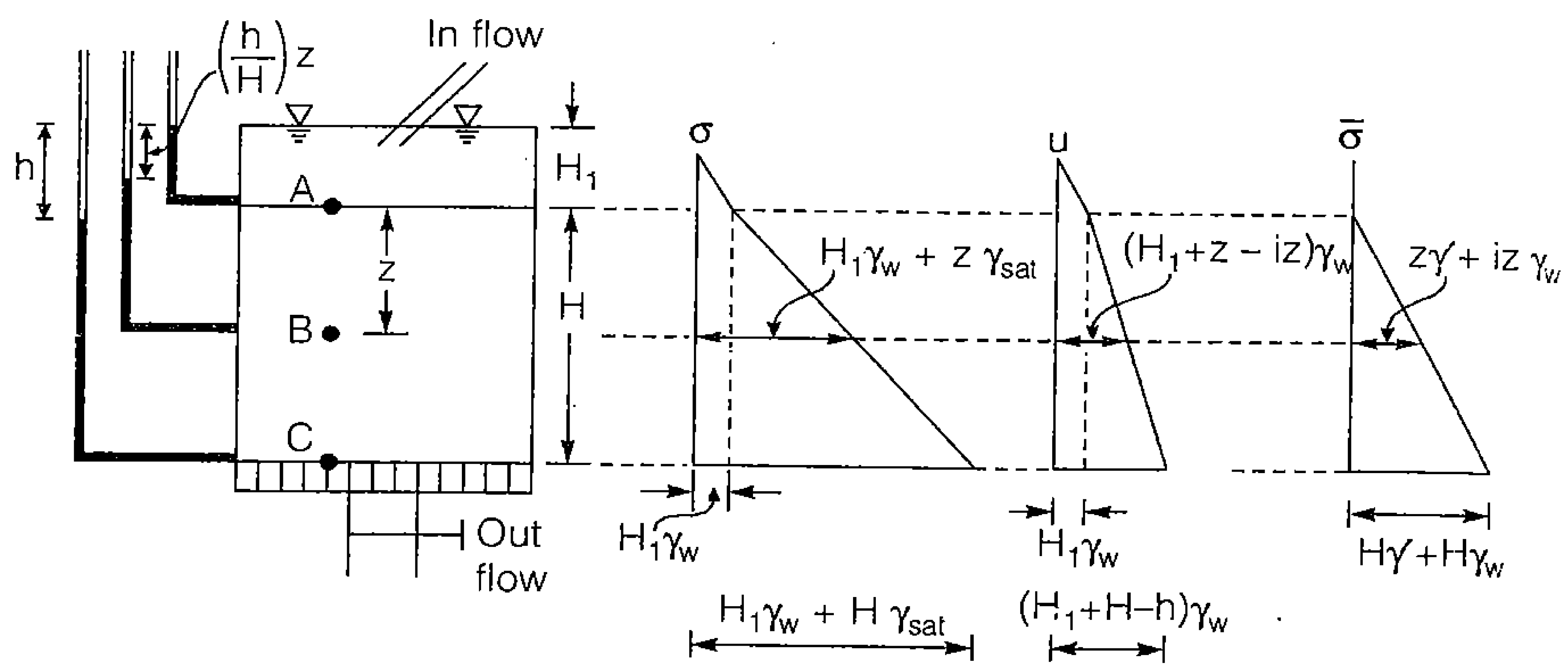
sign convn → +ve when flow is downward.
→ –ve when flow is upward.

(ii) For Vertically Upward Flow

Net effective vertical stress = $\bar{\sigma} - h\gamma_w$

where, $\boxed{\bar{\sigma} = \gamma_{sub} \cdot Z}$ and $\boxed{\gamma_{sub} = \left(\frac{G-1}{1+e}\right)\gamma_w}$





Quick Sand Condition

It is a condition but not the type of sand in which the net effective vertical stress becomes zero, when seepage occurs vertically up through the sands/cohesionless soils.

$\boxed{\text{Net effective vertical stress} = 0}$

$\boxed{i_c = \frac{G-1}{1+e}}$ where, i_c = Critical hydraulic gradient.

$2.65 \le G \le 2.70$

$0.65 \le e \le 0.70$

- To Avoid Floating Condition $\boxed{i < i_c}$ and $\boxed{F \cdot O \cdot S = \frac{i_c}{i} > 1}$

Laplace Equation of Two Dimensional Flow and Flow Net : Graphical Solution of Laplace Equation

(i) $\boxed{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0}$ where, ϕ = Potential function = kH

H = Total head

and k = Coefficient of permeability

(ii) $\boxed{\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial Y^2} = 0}$ ... 2D Laplace equation for Homogeneous soil.

where, $\phi = k_x H$ and $\phi = k_y \cdot H$ For Isotropic soil, $\boxed{k_x = k_y}$

Seepage discharge (q)

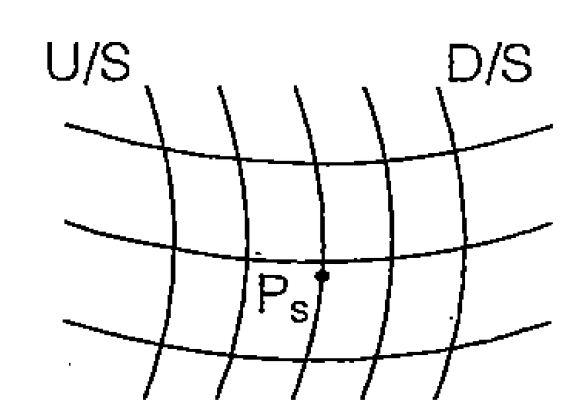
$\boxed{q = kh \cdot \frac{N_f}{N_d}}$ where, h = hydraulic head or head difference between upstream and downstream level or head loss through the soil.

- $\boxed{\text{Shape factor} = \frac{N_f}{N_d}}$
- $\boxed{N_f = N_\psi - 1}$ where, N_f = Total number of flow channels
  N_ψ = Total number of flow lines.
- $\boxed{N_d = N_\phi - 1}$ where, N_d = Total number equipotential drops.
  N_ϕ = Total number equipotential lines.
- $\boxed{\text{Hydrostatic pressure} = U = h_w\gamma_w}$ where, U = Pore pressure
  h_w = Pressure head

  $\boxed{h_w = \text{Hydrostatic head} - \text{Potential head}}$
- **Seepage Pressure**

  $\boxed{P_s = h'\gamma_w}$

  where, $\boxed{h' = h - \left(\frac{2h}{N_d}\right)}$



- Exit gradient,

$\boxed{i_e = \frac{h}{b.N_d}}$ where, size of exit flow field is b × b.

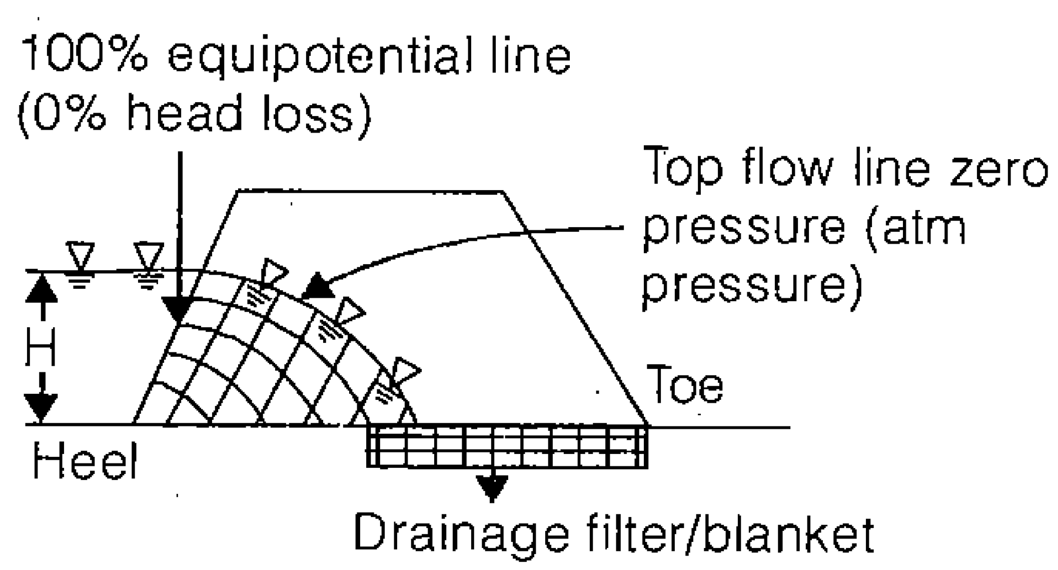
and $\Delta h = \frac{h}{N_d}$ is equipotential drop.

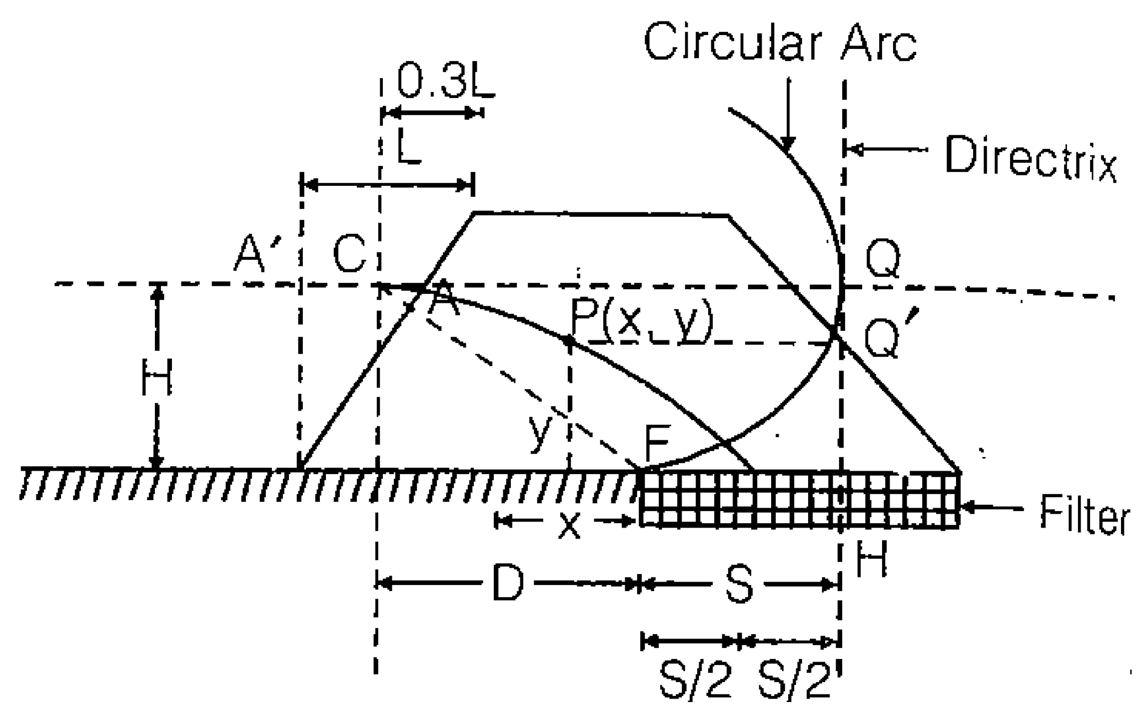
- Flow net will remain same if U/S and D/S water levels are interchanged and direction of flow is reversed provided the flow boundary is not changed.
- Flow net is changed if boundary conditions are changed it means flow net is unique for a given set of boundary conditions.

Phreatic Line

It is top flow line which follows the path of base parabola. It is a stream line. The pressure on this line is atmospheric (zero) and below this line pressure is hydrostatic.

(a) Phreatic Line with Filter





Phreatic line (Top flow line).

↓

Follows the path of base parabola

CF = Radius of circular arc = $\sqrt{D^2 + H^2}$

C = Entry point of base parabola

F = Junction of permeable and impermeable surface

S = Distance between focus and directrix

= Focal length.

FH = S

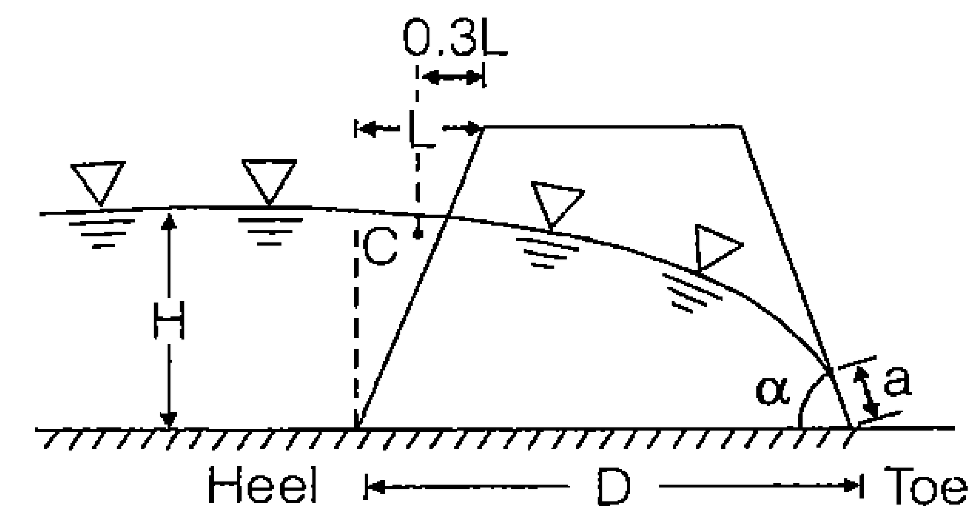
(i) $\boxed{q = ks}$ where, q = Discharge through unit length of dam.

(ii) $\boxed{S = \sqrt{D^2 + H^2} - D}$

(iii) $\boxed{K = \sqrt{k_x k_y}}$ → For 2D

$\boxed{K = (k_x k_y k_z)^{1/3}}$ → For 3D

(b) Phreatic Line without Filter



(i) For $\boxed{\alpha < 30°}$

$\boxed{q = ka\sin^2\alpha}$ and $\boxed{a = \frac{D}{\cos\alpha} - \sqrt{\frac{D^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}}}$

(ii) For $\boxed{\alpha > 30°}$

$\boxed{q = ka\sin\alpha\tan\alpha}$ and $\boxed{a = \sqrt{D^2 + H^2} - \sqrt{D^2 - H^2\cot^2\alpha}}$

- $\boxed{\frac{(D_{15})\text{Filter}}{(D_{85})\text{Soil}} < 5}$; • $\boxed{4 < \frac{(D_{15})\text{Filter}}{(D_{15})\text{Soil}} < 20}$; • $\boxed{\frac{(D_{50})\text{Filter}}{(D_{50})\text{Soil}} < 25}$

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Compaction of Soil

7

- Increasing the unit weight of soil by Reducing air voids
- Static or dynamic loads applied to soil

Factors Affecting Compaction :

1. Moisture content
2. Soil type
3. Method of compaction,
4. Amount of effort

Optimum moisture content

$$(\delta_d)_{maximum} = \frac{\delta}{1 + w_{optimum}}$$

where, $(\delta_d)_{max.}$ = Maximum dry density

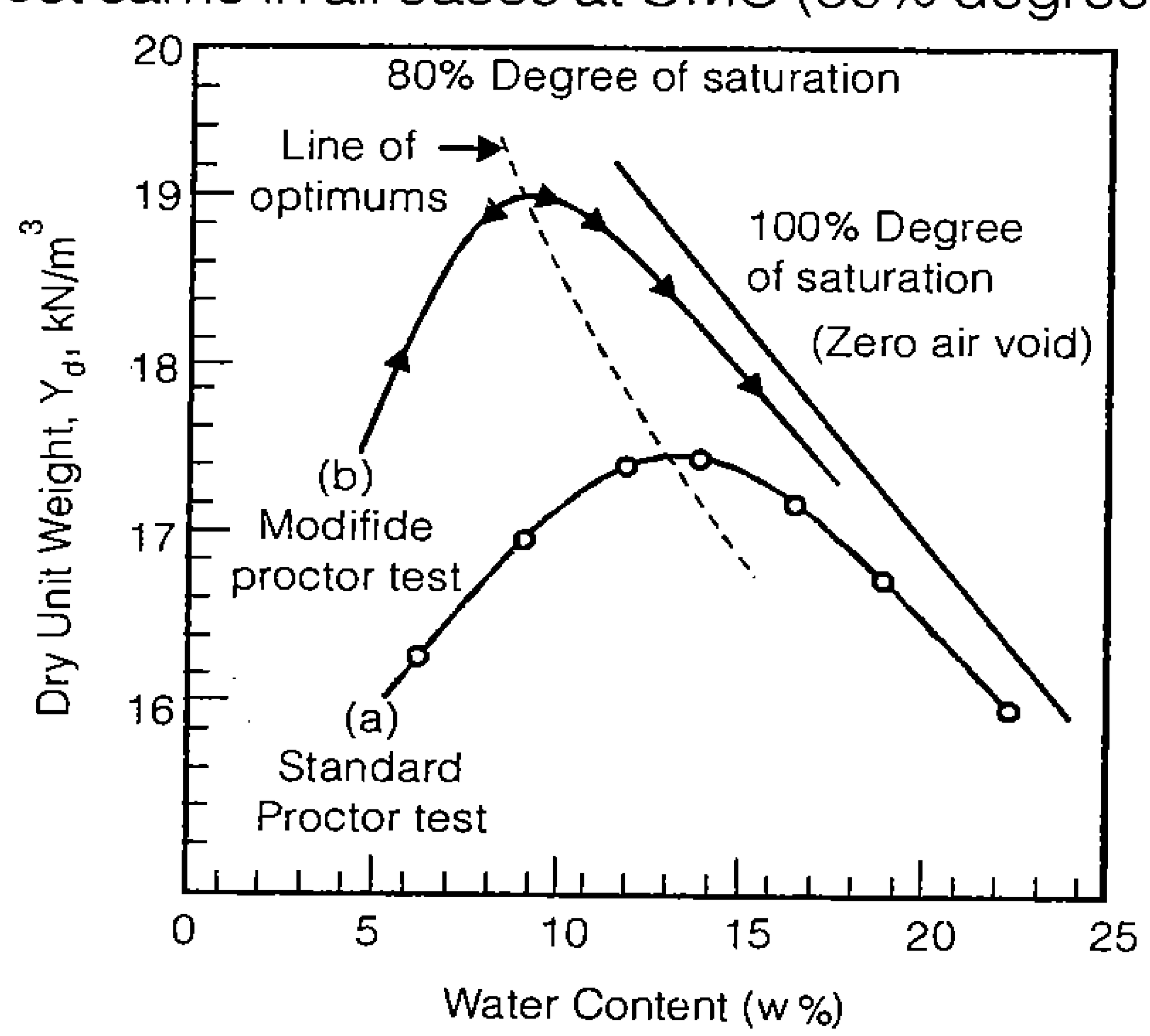
δ = Density of soil

$w_{optimum}$ = Optimum moisture content

Comparison of Standard & Modified Proctor Test

Inference

1. With large compactive effort on wet of optimum side, γ_d don't increase much.
2. S_r is almost same in all cases at OMC (80% degree of saturation line)



1. $$\gamma_d = \frac{G\gamma_w}{1 + \frac{wG}{S}}$$ for, $r_{d\,max}$, S = 1, h_a = 0 correspond to 100% saturation or zero air void line.

2. $$\gamma_d = \frac{(1 - n_a)G\gamma_w}{1 + wG}$$

Ratio of total energy given in heavy compaction test to that given in light compaction test $= \frac{4.9 \times g \times (5 \times 25) \times 450}{2.6 \times g \times (3 \times 25) \times 310} = 4.5$

Compaction Equipments

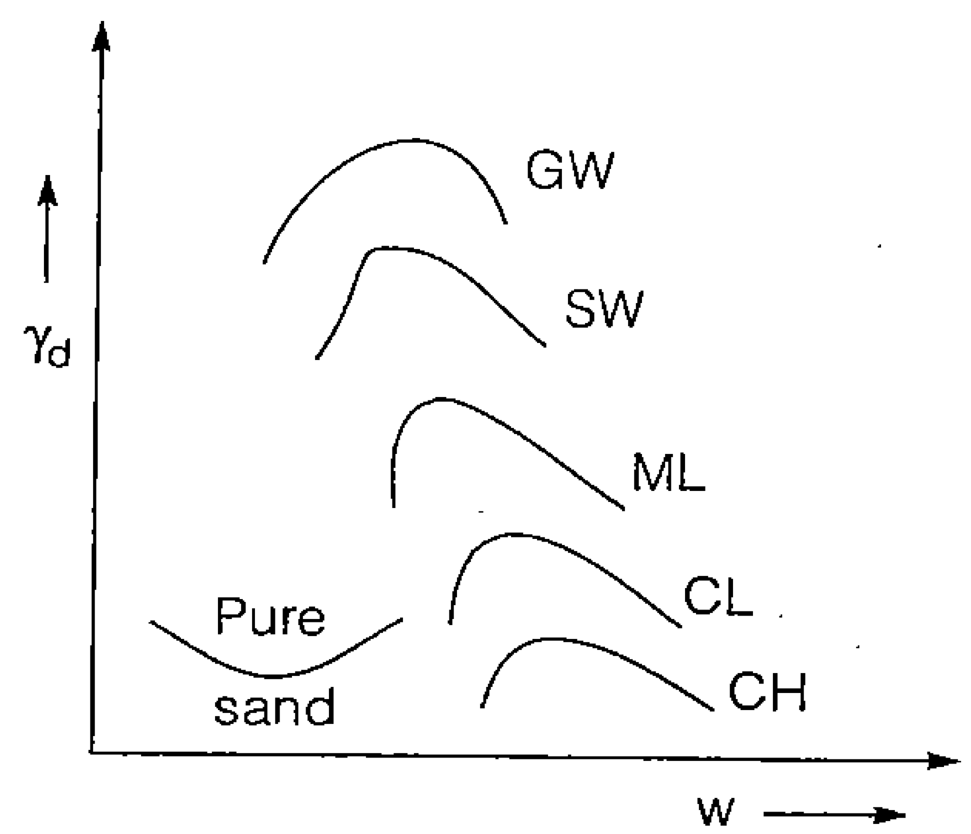
	Type of Equipment	Suitability for soil type	Nature of project
1.	Rammers or Tampers	All soils	In confined areas such as fills behind retaining walls, basement walls etc. Trench fills.
2.	Smooth wheeled rollers	Crushed rocks, gravels sands	Road construction
3.	Pneumatic tyred rollers	Sands, gravels silts, clayey soils	Base, sub-base and embankment compaction for highways, air fields etc. Earth dams.
4.	Sheep foot Rollers	Clayey soils	Core of earth dams.
5.	Vibratory Rollers	Sands	Embankments for oil storage tanks etc.

Compaction Tests

Standard proctor test (Light compaction test)	Modified proctor test (Heavy compaction test)
• Volume of mould 942cc	• Volume of mould 942 cc
• No. of layers -3	• No. of layers -5
• No. of blows per layer -25	• Nol of blows per layer -25
• Height of free fall -304.8 mm (12 inches)	• Height of free fall -457.2 mm (18 inches)
• Wt. of hammer -2.495 kg (5.5 *l*b)	• Wt. of hammer -4.54 kg (10 *l*b)

Indian standard light compaction test	Indian standard heavy compaction test
V – Volume of mould 1000 cc	• Volume of mould 1000 cc
H – Height of free fall 310 mm	• Height of free fall 450 mm
W – Wt. of hammer 2.6 kg	• Wt. of hammer 4.9 kg
n – No. of layers 3	• No. of layer 5
N – Blows per layer 25	• Blows per layer 25

Compactive energy applied per unit $\boxed{\text{Volume} = \frac{wH_nN}{V}}$



1. Coarse glained well graded - Higher γ_d
2. In Clays with higher plasticity - γ_d decrease
3. V shape due to bulking of pure sand

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Stress Distribution in The Soil

8

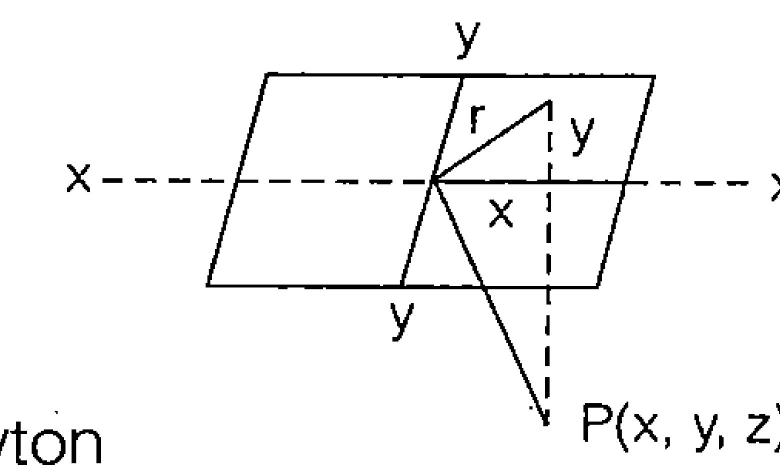
Stress in the soil may be caused by:

1. Self weight of soil
2. Applied load on soil

Boussinesq's Theory

Vertical stress at point 'P'. (σ_z)

(i) $$\boxed{\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2}}$$



where, Q = Point load in newton

(ii) $$\boxed{\sigma_z = k_B \cdot \frac{Q}{z^2}}$$

where, $k_B = \frac{3}{2\pi}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2}$ $\boxed{k_B|_{max} = \frac{3}{2\pi} = 0.4775}$

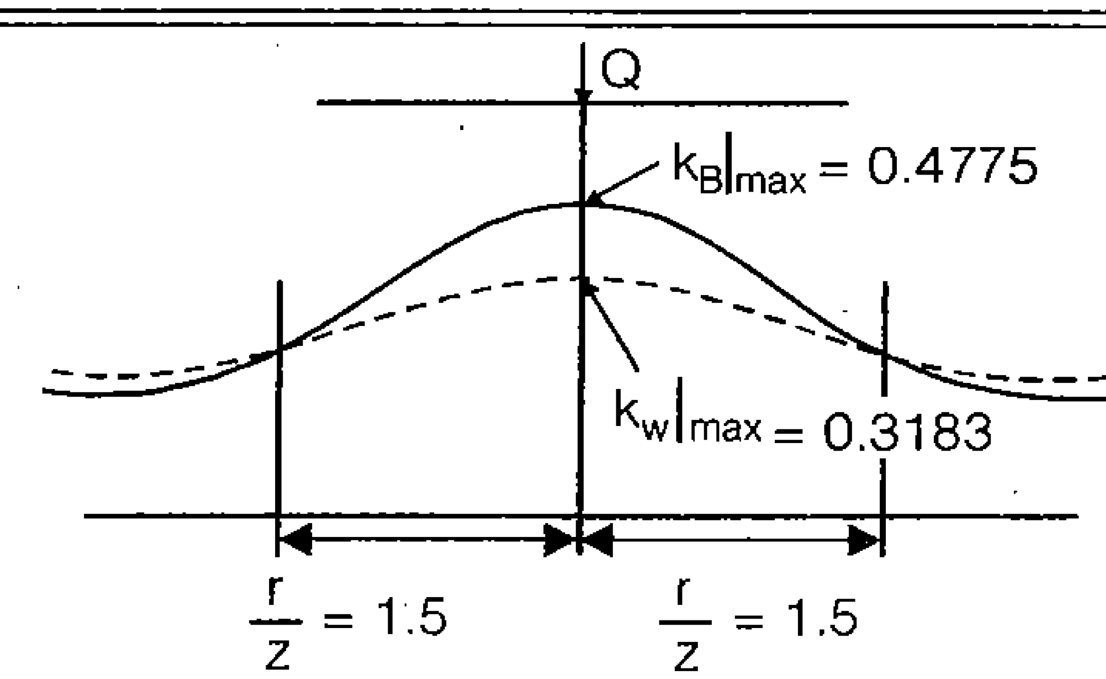
(iii) 'σ_z' below the point load at depth z, $\boxed{\sigma_z = 0.4775.\frac{Q}{z^2}}$

Westergaard's Theory

(i) $\boxed{\sigma_z = \frac{Q}{\pi z^2}\left[\frac{1}{1+\frac{2r^2}{z^2}}\right]^{3/2}}$ (ii) $\boxed{\sigma_z = k_W \cdot \frac{Q}{z^2}}$ (iii) $\boxed{k_W|_{max} = 0.3183}$

NOTE: Boussinesq's theory is applicable for Isotropic soil where as westergaards theory is applicable for non-Isotropic soil.

Remember



if $\frac{r}{z} < 1.5 \Rightarrow \boxed{k_B > k_W}$

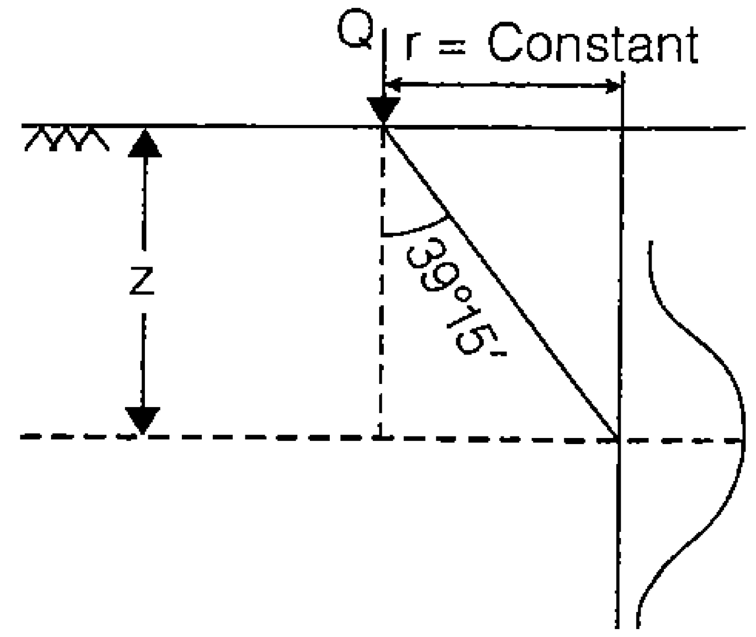
if $\frac{r}{z} > 1.5 \Rightarrow \boxed{k_W > k_B}$

if $\frac{r}{z} = 1.5 \Rightarrow \boxed{k_W = k_B}$

Boussinesq's Result

$$\sigma_z|_{max} = 0.0888\frac{Q}{r^2}$$

$$\sigma_z|_{max} = 0.1332\frac{Q^2}{z^2}$$



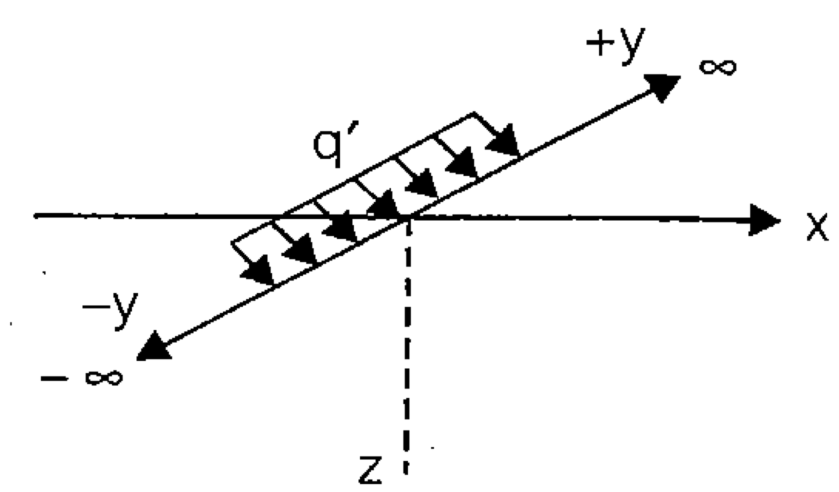
$$\sigma_z|_{maximum} = 0.0888\frac{Q}{r^2}$$

$$\sigma_z|_{maximum} = 0.1332\frac{Q}{z^2}$$

Westergaard's Results

(i) Vertical Stress due to Live Loads

$$\sigma_z = \frac{2q'}{\pi z}\left[\frac{1}{1+\frac{x^2}{z^2}}\right]^2$$



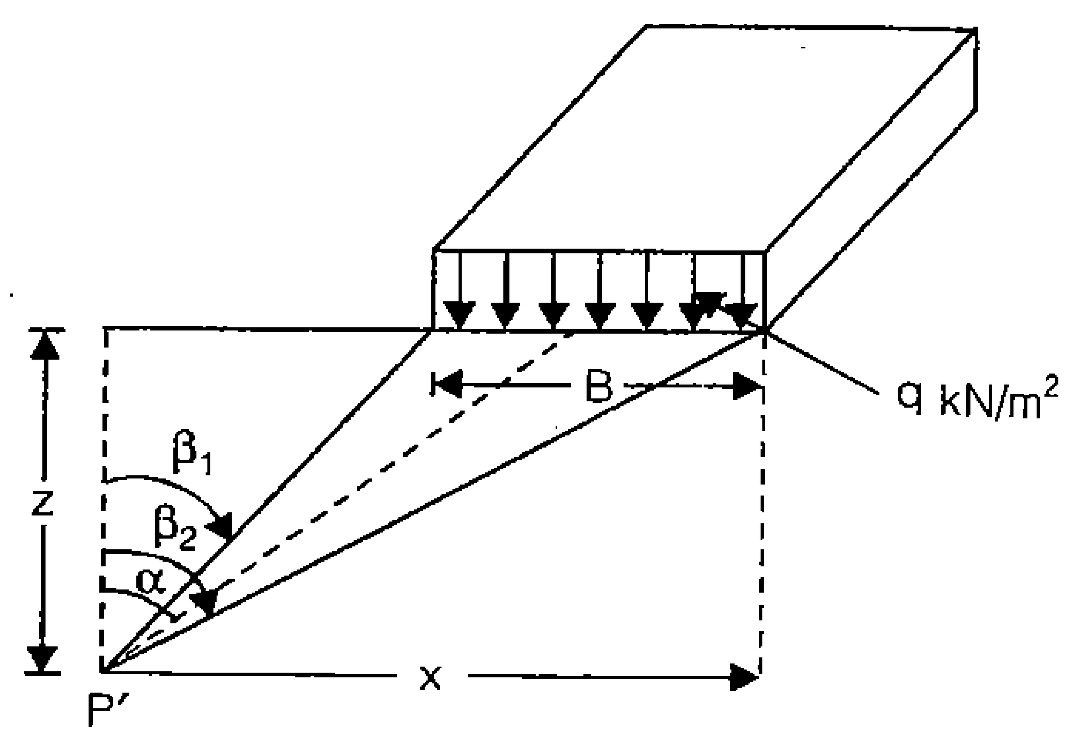
where, σ_z = Vertical stress of any point having coordinate (x, z)

Load intensity = q'/m

at x = 0 → $\sigma_z = \frac{2q'}{\pi z}$

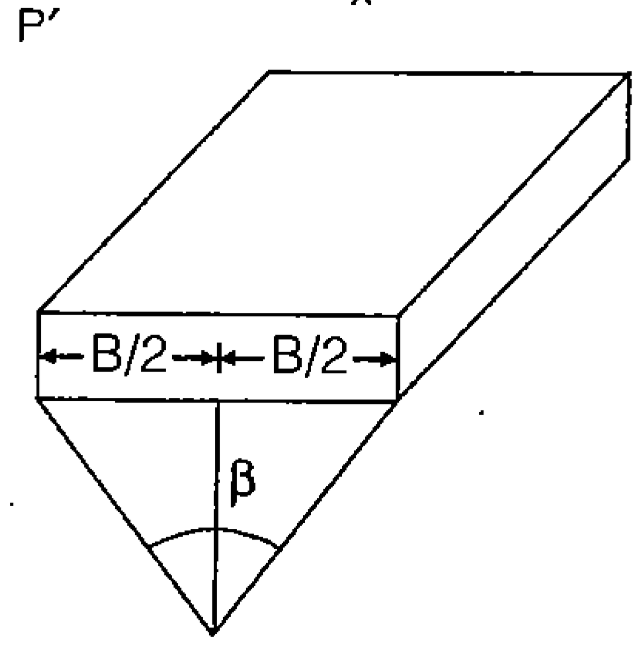
(ii) Vertical Stress due to Strip Loading

$$\sigma_z = \frac{2q}{\pi}\left(\frac{x}{B}\alpha - \frac{\sin 2\beta}{2}\right)$$



where, σ_z = Vertical stress at point 'p'

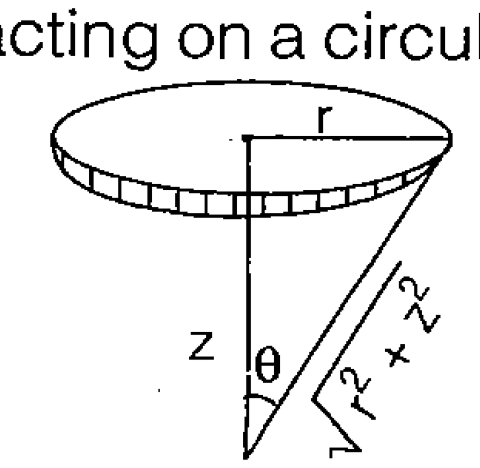
(iii) $\sigma_z = \frac{q}{\pi}[\beta + \sin\beta]$



(iv) Vertical stress below uniform load acting on a circular area.

$$\sigma_z = q(1-\cos^3\theta)$$

where, $\cos\theta = \frac{z}{\sqrt{r^2+z^2}}$



Newmark's Chart Method

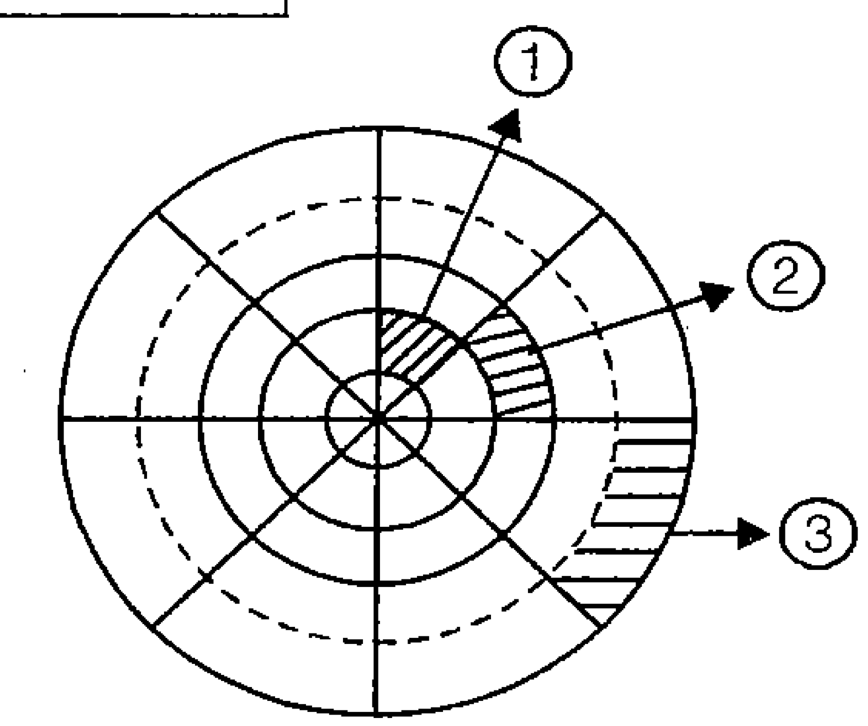
This method is applicable to semi-infinite, homogeneous, isotropic and elastic soil mass. It is not applicable for layered structure. The greatest advantage of this method is that it can be applied for uniformly distributed area of irregular shape. Newmark's chart is made of concentric circles and radial lines. Normally there are 10 concentric circles and 20 radial lines.

no. of concentric circle = 10 no. of radial lines = 20

Influence of area (1) = Influence of area (2) = Influence of area (3)

$$\text{Influence of each area} = \frac{1}{\text{Total no of sectoral area}} = 0.005$$

$$\sigma_z = 0.005qN_A$$



where, N_A = Total number of sectorial area of Newmark's chart.

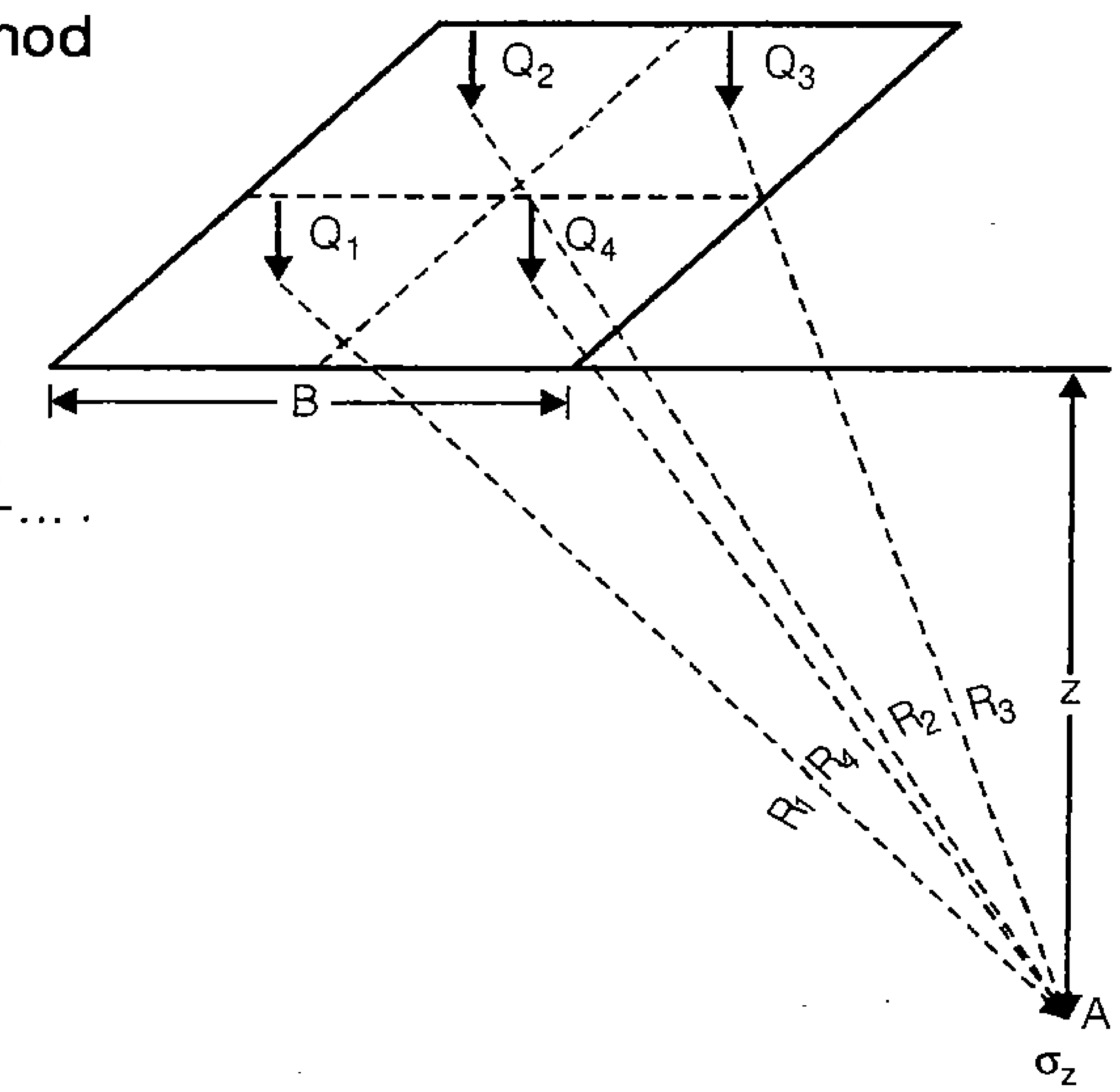
Approximate Method

(i) Equivalent Load Method

$$\sigma_z = \sigma_{z_1} + \sigma_{z_2} + \sigma_{z_3} + ...$$

where, $\sigma_{z_1} = k_{B_1}\frac{Q_1}{z^2}$

$\sigma_2 = k_{B_2}.\frac{Q_2^2}{z^2}$ ....



(ii) Trapezoidal Method

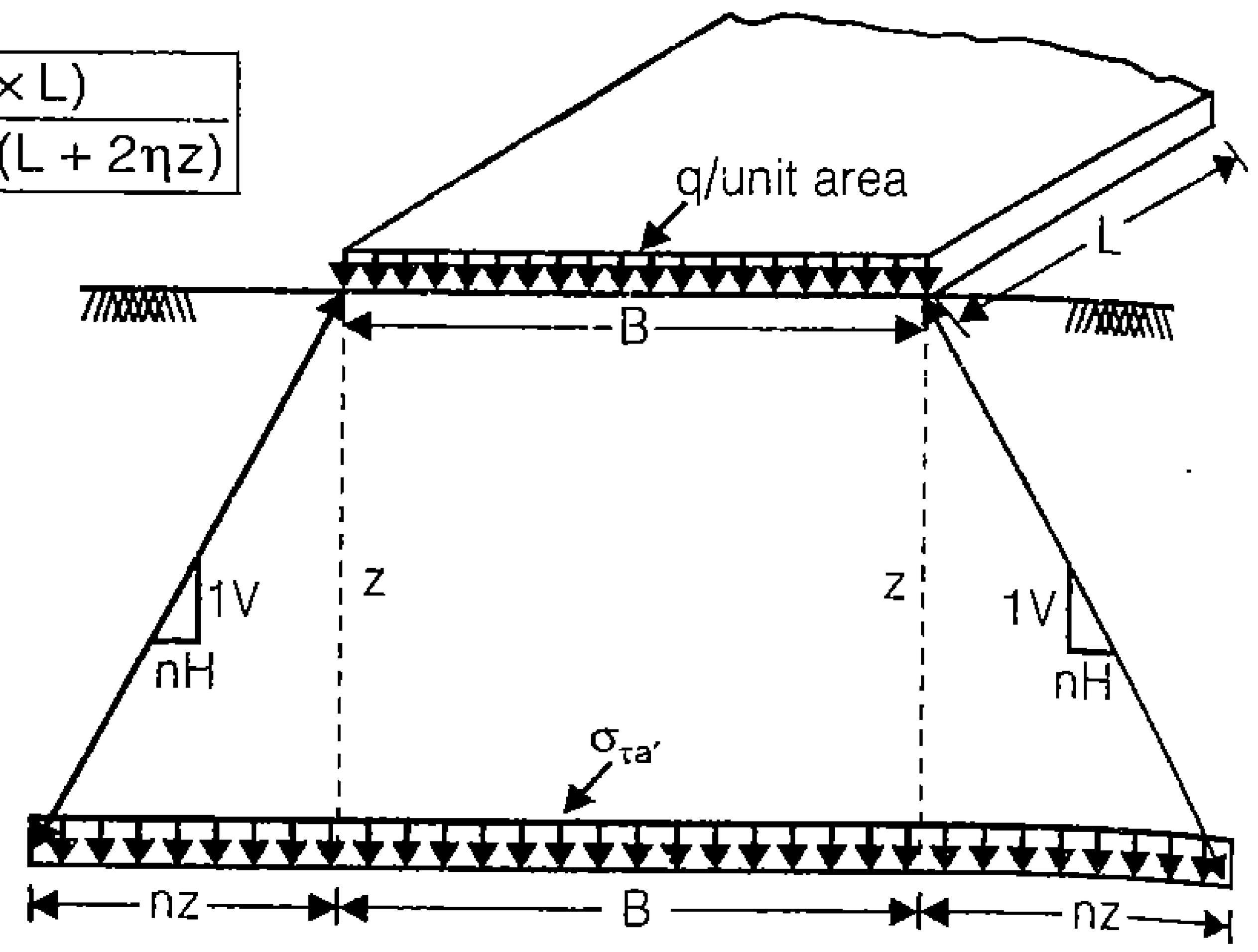
$$\sigma_z \text{ at depth 'z'} = \frac{q(B \times L)}{(B + 2\eta z)(L + 2\eta z)}$$

For 1H : 1V

$$\sigma_z = \frac{q(B \times L)}{(B + 2z)(L + 2z)}$$

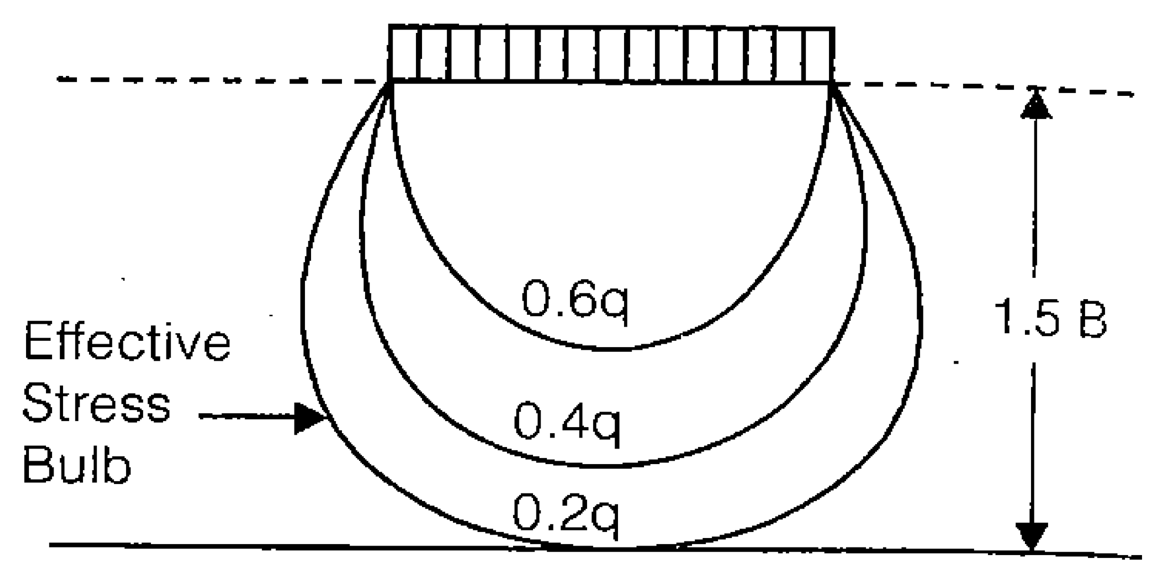
For 2H : 1V

$$\sigma_z = \frac{q(B \times L)}{(B + 4z)(L + 4z)}$$



(iii) **Stress Isobar Method** : Area bounded by 0.2q stress isobar is considered to be stressed by vertical stress on loading.

0.2q = 20% Stress Isobar



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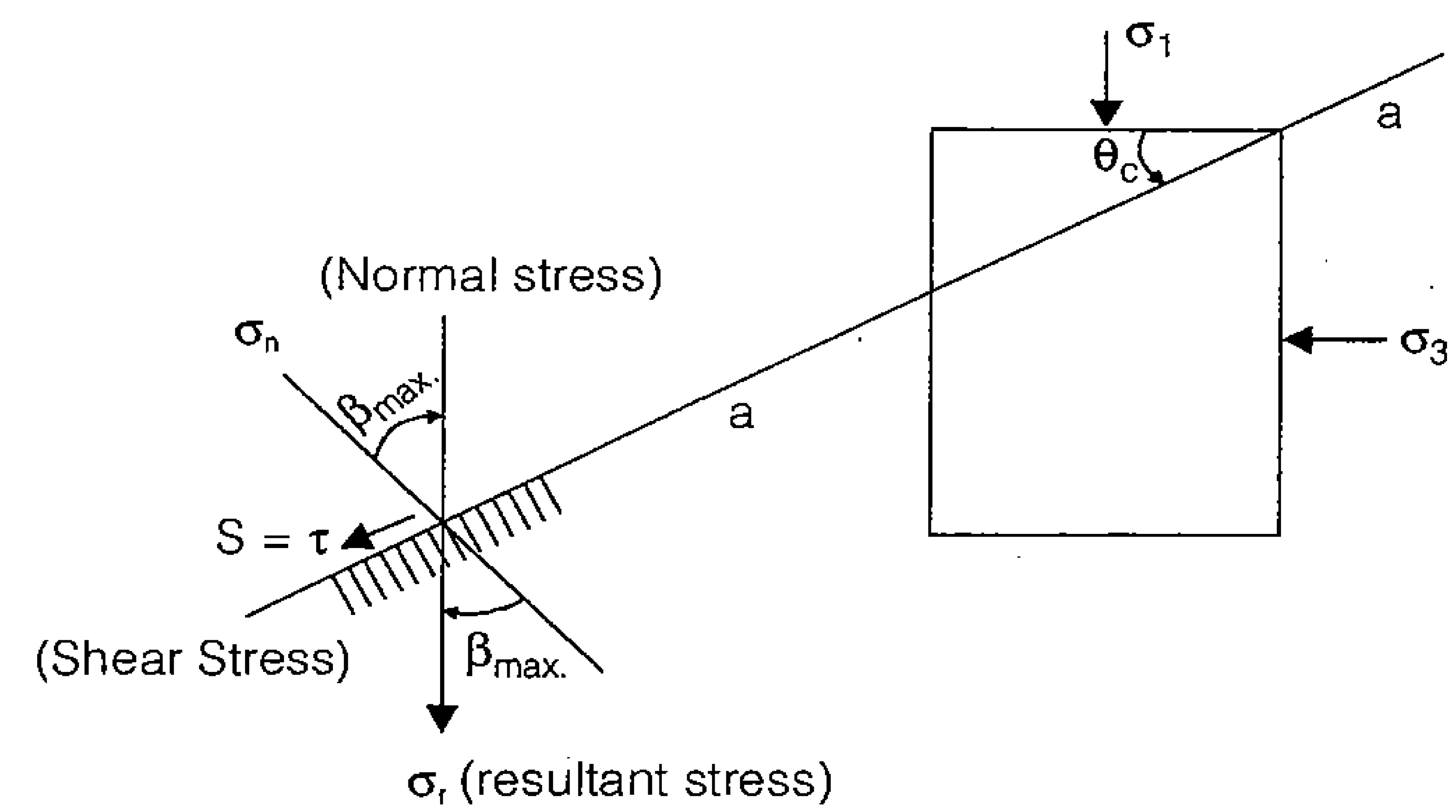
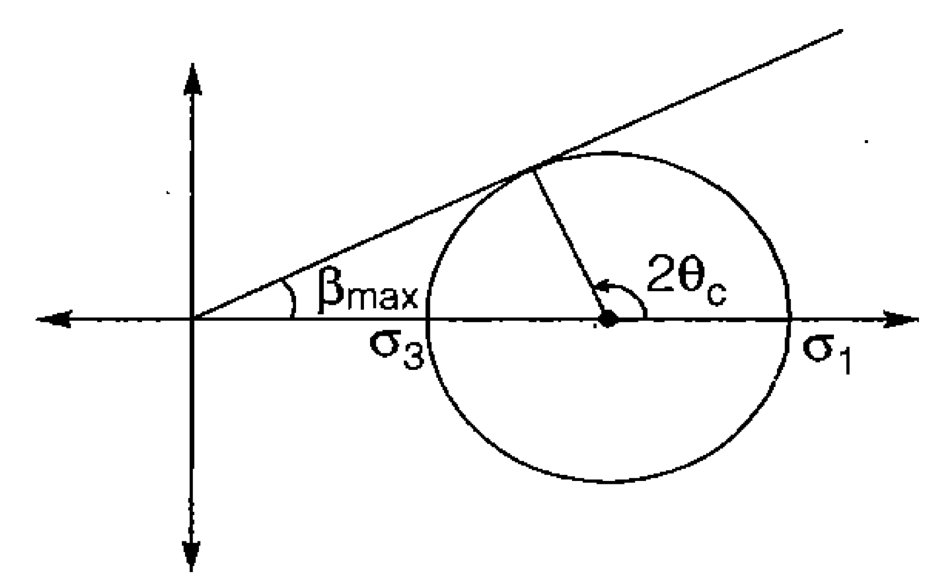
Shear Strength of Soil 9

Shear Strength

Shear strength of a soil is the capacity of the soil to resist shearing stress. It can be defined as the maximum value of shear stress that can be mobilized within a soil mass.





> **NOTE**
> Failure plane is one at which angle of obliquity is max. and max. and angle of obliquity = Angle of friction
> Failure shear stress is one at which differenc of shear stress and shear strength is minimum.

- Plane a-a is critical plane
- θ_c = Angle of critical plane (a – a)
- σ_1 and σ_3 are stresses on given planes

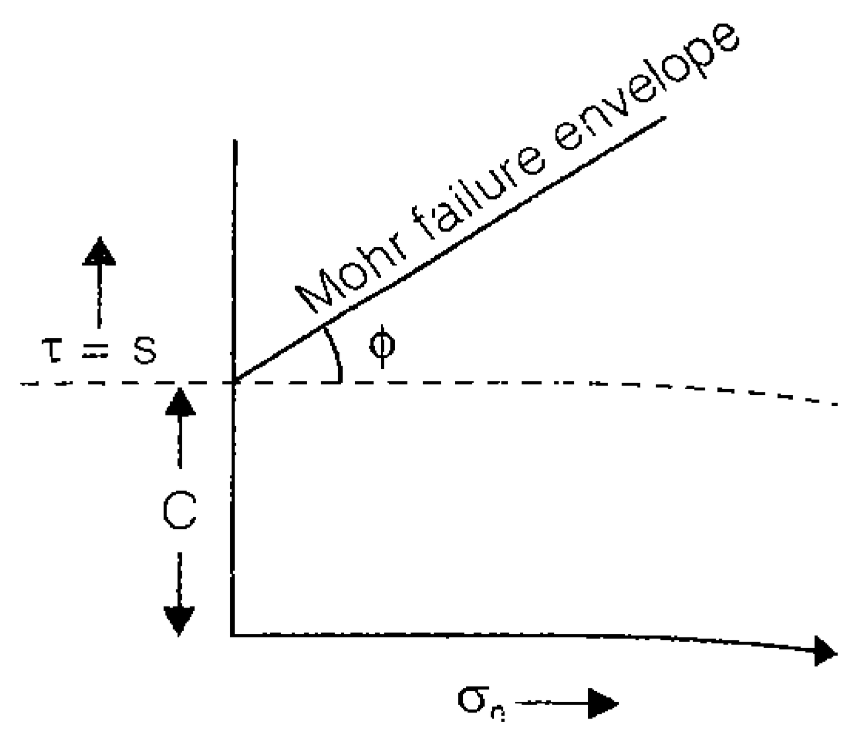
(i) $$\theta_c = \frac{\pi}{4} + \frac{\beta_{maximum}}{2}$$

where, $\beta_{max.}$ = Angle between resultant stress and normal stress on critical plane.
= Friction angle of soil = ϕ

$$\theta_c = \frac{\pi}{4} + \frac{\phi}{2}$$

↓ for clay's $\phi = 0$

$$\theta_c = \frac{\pi}{4}$$



(ii) $\tau = \sigma_n \tan\phi$, for sands or ϕ-soil.

(iii) $\tau = C + \sigma_n \tan\phi$, for C-$\phi$ soil.

NOTE: C and ϕ are not inherent properties of soil, rather depends on type of test.

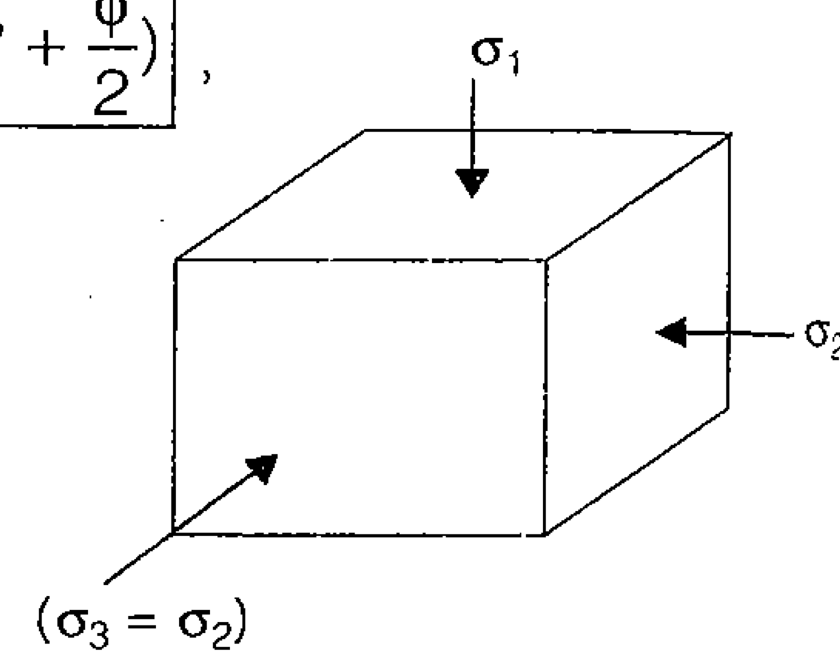
(iv) $\tau = C$, for C-soil (clays).

(v) $\sigma_1 = \sigma_3 \tan^2(45° + \frac{\phi}{2}) + 2C \tan(45° + \frac{\phi}{2})$,

for C-ϕ soil

(vi) $\sigma_1 = \sigma_3 \tan^2(45° + \frac{\phi}{2})$, for ϕ-soil.

(vii) $\sigma_1 = 2C$, for C-soil.



Mohr Coulomb's Theory

$$\tau = s = C' + \overline{\sigma_n} \tan\phi'$$

where,

C' = Effective cohesion

$\overline{\sigma_n}$ = Effective normal stress

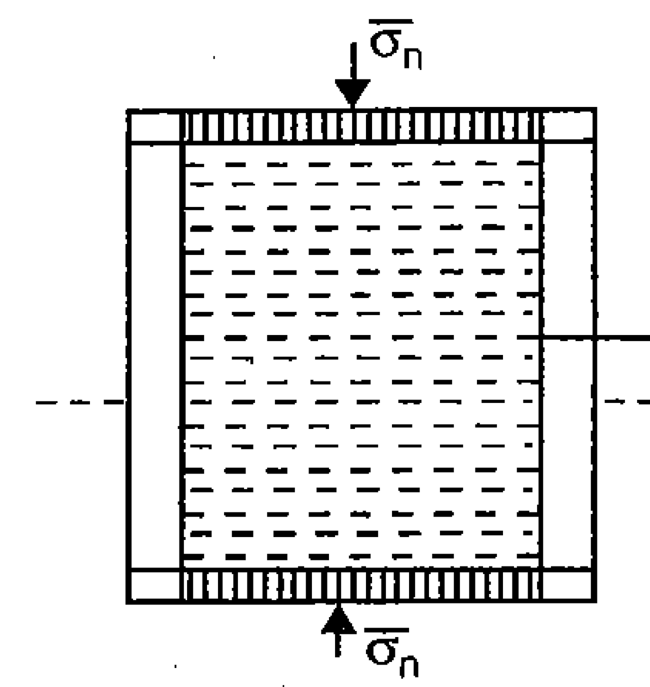
and ϕ' = Effective friction angle

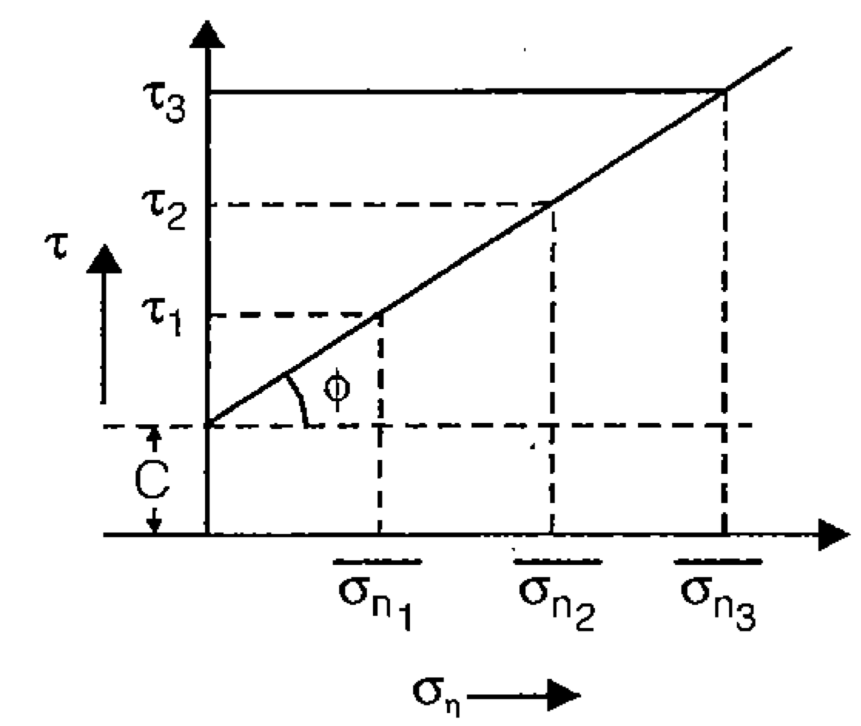
NOTE: Under dry stage total stress parameter and effective stress parameters are equal.
In drained condition effective stress analysis used.
In undrained condition, total stress analysis is done by assuming pore water pressure developed in lab is same as those in field,

Drained Condition	Effective stress analysis and post construction stability is checked.
Undrained condition with positive pole water pressure	Total stress analysis and stability should be checked immediately after construction.
Undrained condition with negative pore water pressure	Effective stress analysis and long term stability should be checked.

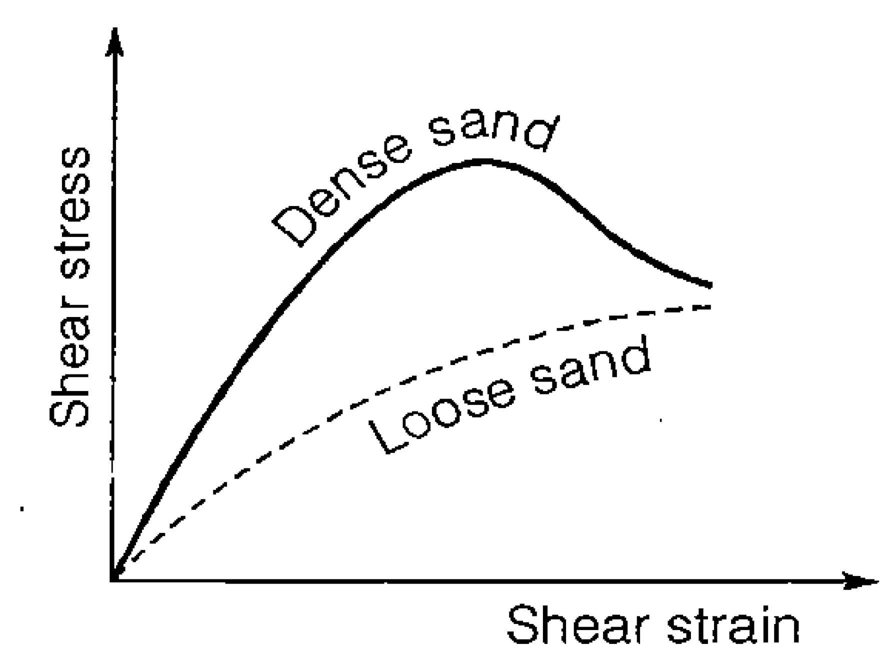
Direct Shear Test

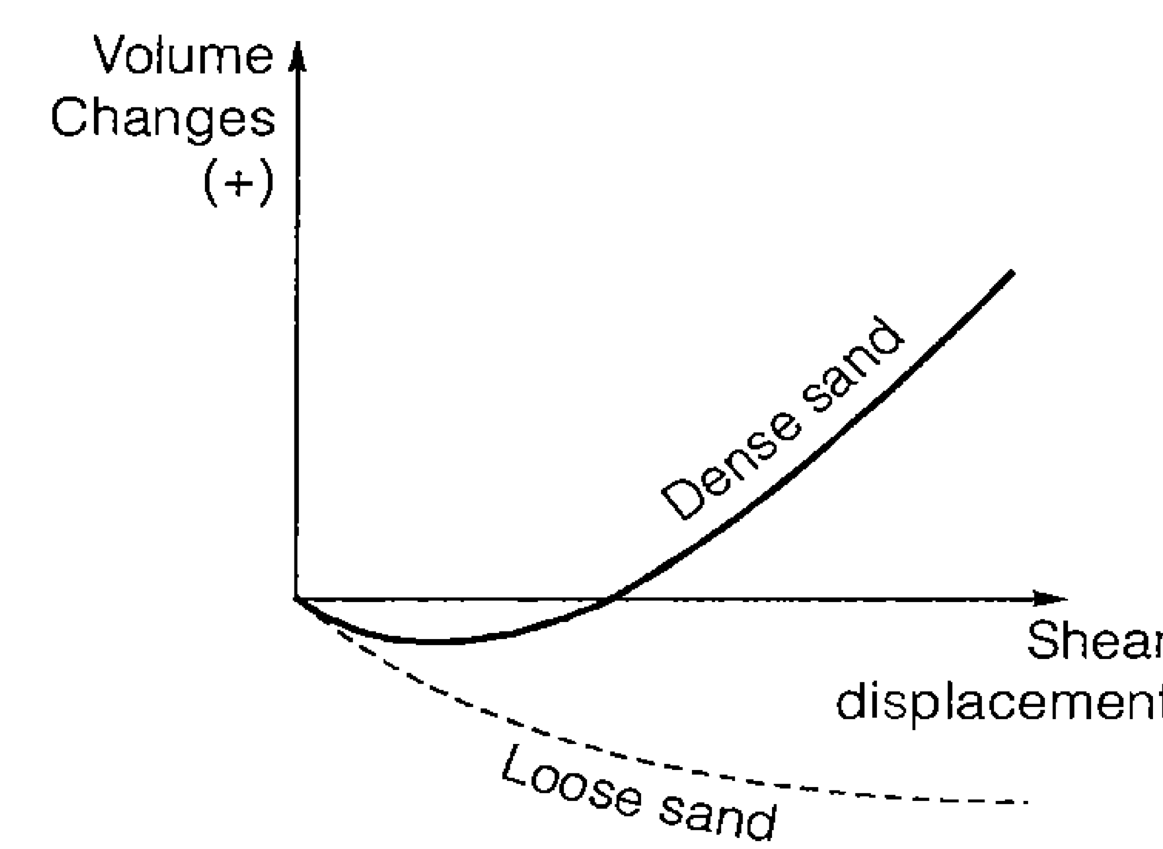
$$\tau = s = C' + \overline{\sigma_n} \tan\phi'$$





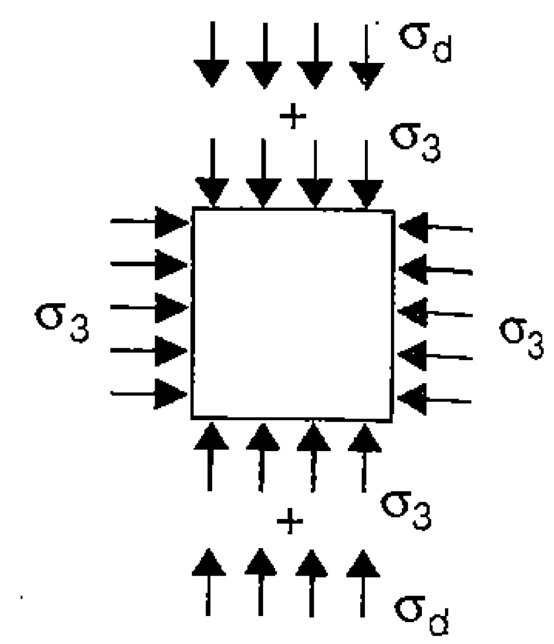
Results of Direct Shear Test





Drained conditions maintained, hence C taken as zero in results (as cohesion doesn't mobilises in drained condtion)

Triaxial Shear Test



- $\boxed{\sigma_1 = \sigma_3 + \sigma_d}$
- $\boxed{(\sigma_d)_{failure} = (\sigma_1 - \sigma_3)_{failure} = \frac{P}{A}}$
- $\boxed{\tau = S = C + \overline{\sigma_n} \tan\phi}$

where, σ_3 = Cell pressure or allround confining pressure
σ_d = Deviator stress
A = Area of failure

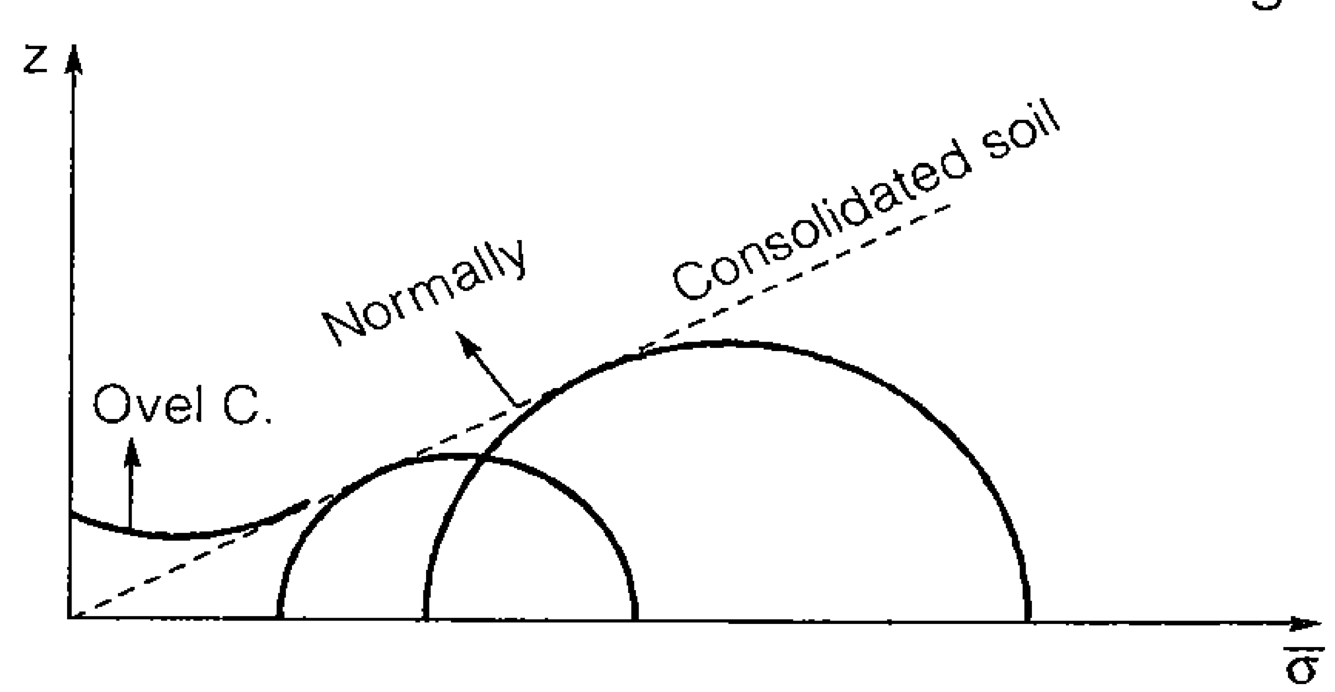
$\boxed{A = \frac{A_0(1 \pm \epsilon_v)}{(1 - \epsilon_L)}}$ where, A_0 = Area of beginning
ϵ_v = Volumetric strain

$\boxed{\epsilon_v = 0 \text{ for U – U – test}}$ where, ΔV = Volume of water escaped out

$\boxed{\epsilon_v = \frac{\Delta V}{V} \text{ for C – D test}}$ $V = \frac{\pi}{4} D^2 L$ = Initial volume
ϵ = Axial strain

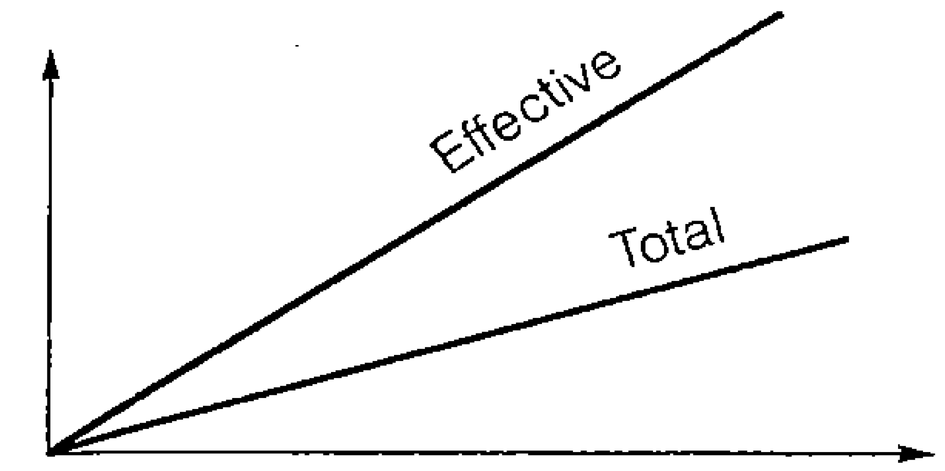
Important Points regarding Triaxial Test

1. During triaxial, either pore water line is open (to get pore pressure) or Drainage line is open (to get volume change)
2. UD (not possible in field)
3. CD — Total = effective stress : To check long term stability of embankment which has been in existence since long ago.

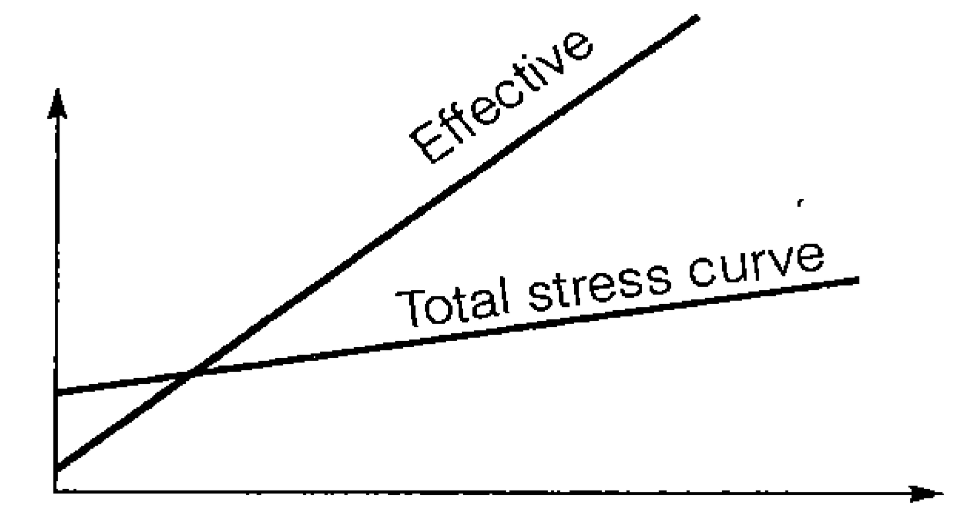


4. With $\bar{\sigma}_3 \uparrow$, more σ_1 required to canse failure hence mohr circle bigger.
   For normal consolidated soil, at $\sigma_3 = 0$, $\tau = 0$
   But for OC soil, at $\sigma_3 = 0$, τ have some value.

5. CU — Undrained strength comes higher than in site due to isotropic confining pressure of lab and anisotripic confining in site soil.



Norm alConsoliClay



O verConsoliClay

(Direct Shear) drained condition → C can't be modified
(Unconfined undrained compression test) → ϕ can't be modified

Remember

- Unconsolidated undrained test (UU test) suitable for construction of building over saturated clays.
- Consolidated undrained test (CU) suitable for stability analysis of earthern dam during sudden drawdown.
- Consolidated drained test (CD) suitable for stability analysis of retaining wall having sandy fills.

6. UU Test
   With ↑ in σ_3, effective $\bar{\sigma}_3$ doesn't change hence no decrease in void ratio or increase in strength is noted, hence for all σ_3, same incremental σ_1, will come and only one Mohr's circle is obtained.

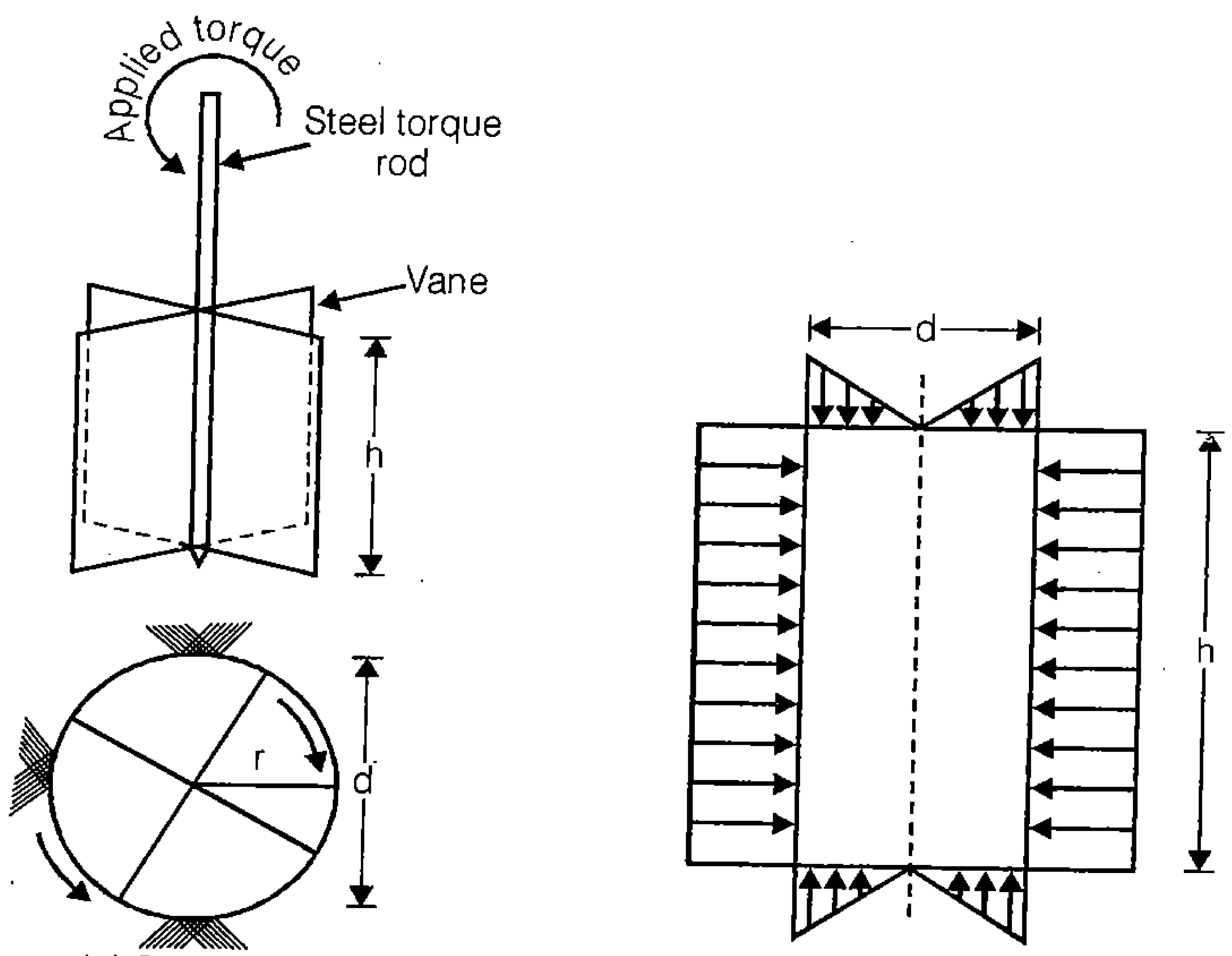
Unconfined Compression Test

- $\boxed{q_u = (\sigma_1)_f}$
  where, q_u = unconfined compressive strength.
  Here, $\boxed{\sigma_3 = 0}$
- $\boxed{(\sigma_1)_f = 2C\tan\left(45° + \frac{\phi}{2}\right)}$, for C-$\phi$ soil
- $\boxed{(\sigma_1)_f = 2C}$, for C-soil.
- $\boxed{\tau = S = C = \frac{q_u}{2}}$, for clay's or c-soil.

- For clays as sand/coarse grained soil/ can't sland in equipment with no latural pressure.
- Used to rapidly assess clay consistency in field.
- To get sensitivity values of clay.

Vane Shear Test

- It is suitable for sensitive clays.



(a) Shear vane

(b) Assumed shear stress distribution

	Lab Size	Field Size
Height of vane (H)	20 mm	10 to 20 cm
Dia of vane (D)	12 mm	5 to 10 cm
Thickness of vane (t)	0.5 to 0.1 mm	2 to 3 cm

Shear Strength

- $$S = \tau = \frac{T}{\pi D^2\left(\frac{H}{2} + \frac{D}{6}\right)}$$

When top and bottom of vanes both take part in shearing.

- $$S = \tau = \frac{T}{\pi D^2\left(\frac{H}{2} + \frac{D}{12}\right)}$$

When only bottom of vanes take part in shearing.

- $$S_t = \frac{(q_u)_{undisturbed}}{(q_u)_{remolded}}$$ Where s_f = Sensitivity

Pore Pressure Parameter

(i) $$B = \frac{\Delta U_c}{\Delta \sigma_c} = \frac{\Delta U_c}{\Delta \sigma_3}$$

where, B = Pore pressure parameter

ΔU_C = Change in pore pressure due to increase in cell pressure

$\Delta \sigma_c$ = $\Delta \sigma_3$ = Change in cell pressure.

- $0 \le B \le 1$
- $B = 0$, for dry soil.
- $B = 1$, for saturated soil.

(ii) $\bar{A} = A.B$

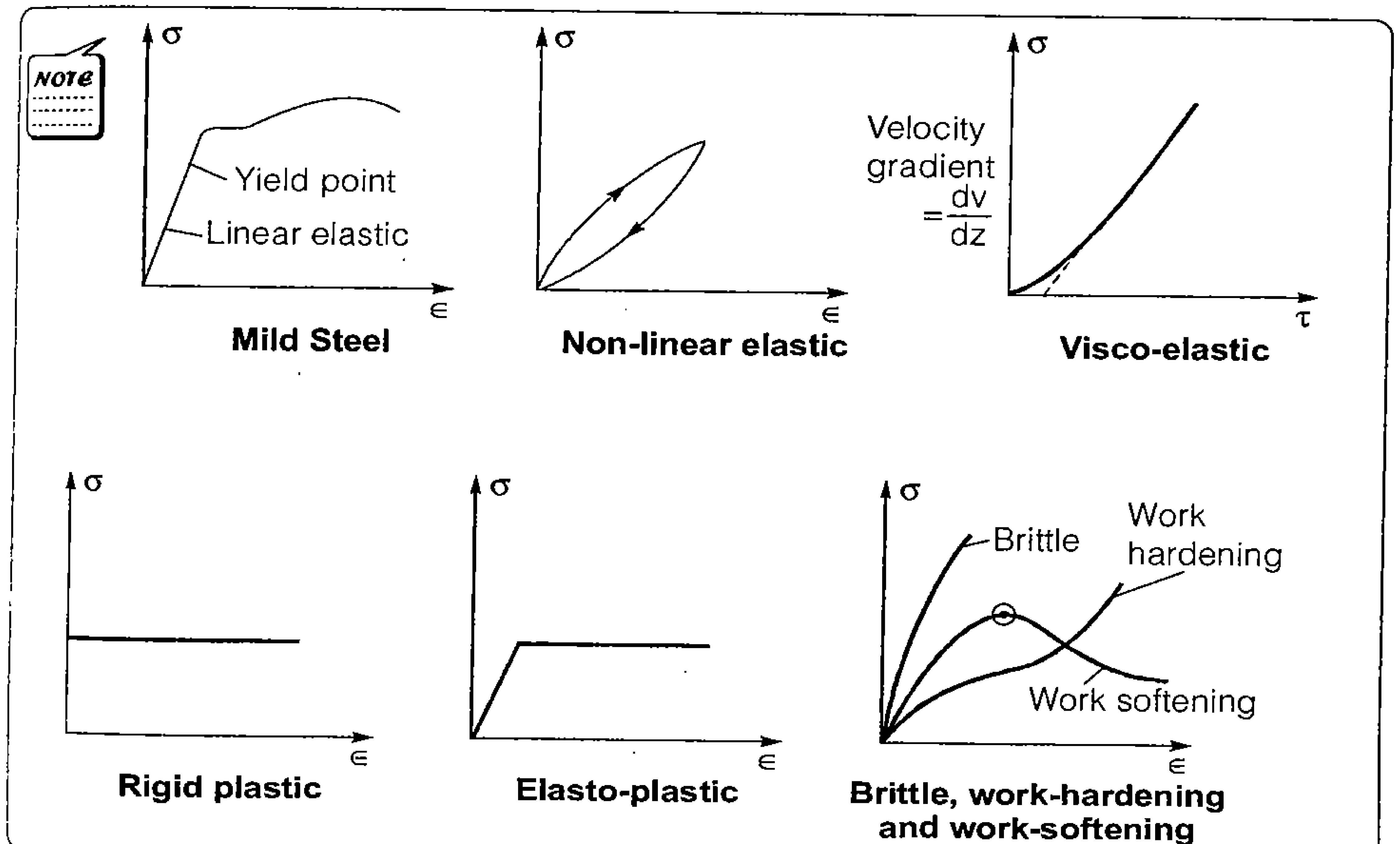
where A = Pore pressure parameter

- $$\bar{A} = \frac{\Delta U_d}{\Delta \sigma_d}$$ where, ΔU_d = Change in pore pressure due to deviator stress.

  $\Delta \sigma_d$ = Change in deviator stress

(iii) $\Delta U = \Delta U_c + \Delta U_d$ ΔU = Change in pore pressure

(iv) $\Delta U = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]$



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Retaining Wall/Earth Pressure Theories

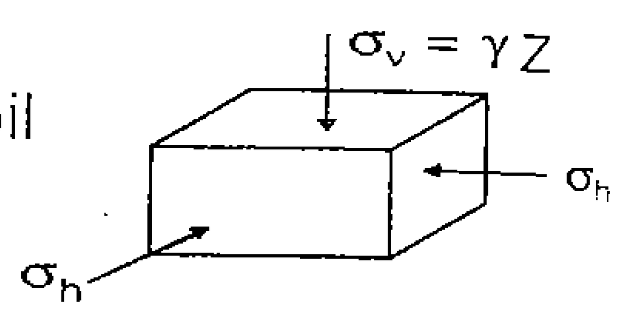
10

Contrasting Points of Active and Passive Pressures

	ACTIVE		PASSIVE
1.	Very little movement is required to mobilise the active pressure. (about 0.5% horizontal strain)	1.	Much higher movement is required to mobilise the pressure (about 2% of horizontal strain)
2.	Failure plane is inclined at 45° + ϕ/2 with horizontal	2	Failure plane is inclined at 45° − ϕ/2 with horizontal
3.	Width of sliding wedge at the top of wall is H.Cot (45° + ϕ/2)	3.	Width of sliding wedge at the top of wall is H.Cot (45° − ϕ/2)

Earth Pressure at Rest

$\boxed{\sigma_h = K_0 . \gamma . z}$, $\boxed{K_0 = \frac{\sigma_h}{\sigma_v}}$, $\boxed{K_0 = \frac{\mu}{1-\mu}}$, for C-soil



where, σ_h = Earth pressure at rest

k_0 = Coefficient of earth pressure at rest

μ = Poissons ratio of soil $\simeq$ 0.4

$\boxed{k_0 = 1 - \sin\phi}$ → for ϕ soil.

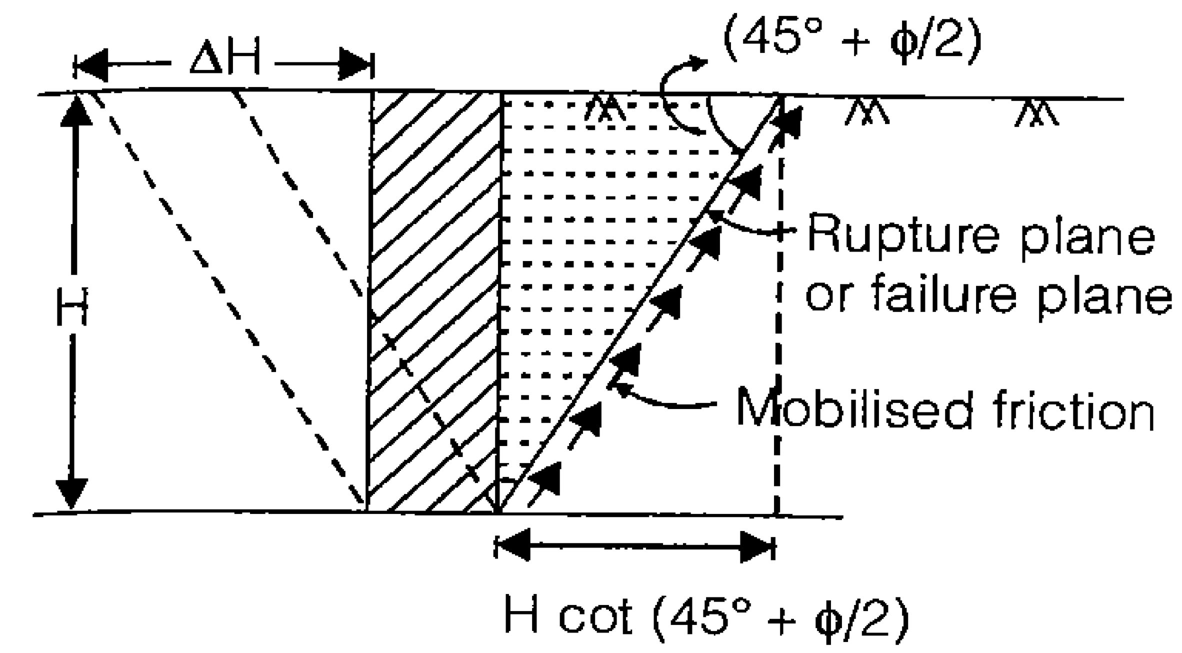
where, ϕ = Angle of internal friction.

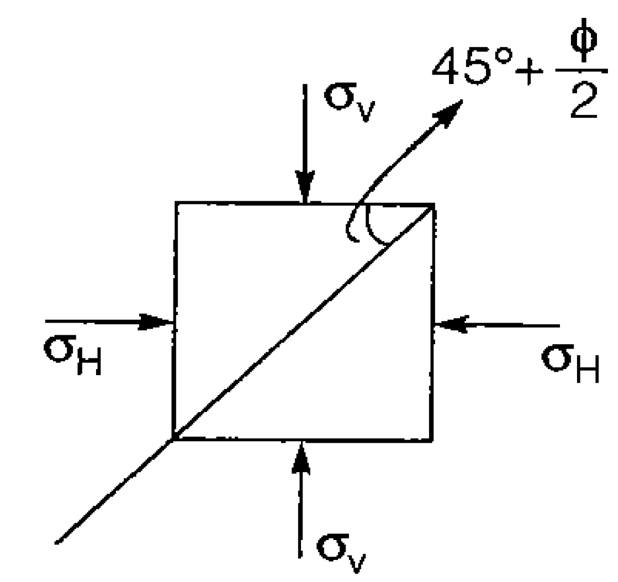
- $\boxed{(k_o)_{\text{over consolidated}} = (k_o)_{\text{normally consolidated}} \sqrt{OCR}}$

where, OCR = Over Consolidation Ratio.

Type of soil		k_o
Dense sand	–	0.4 to 0.45
Loose sand	–	0.45 to 0.5
Mechanically compacted sand	–	0.8 to 1.5
Normally consolidated clay	–	0.5 to 0.6
Over consolidated clay	–	1.0 to 4.0

Active Earth Pressure



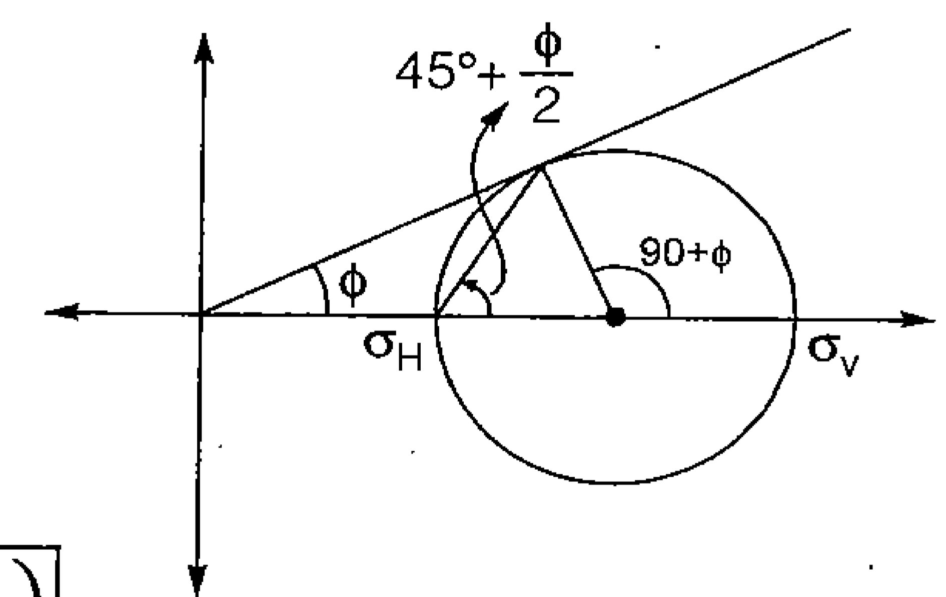


ΔH = 0.2% to 0.5% of H

- $\boxed{\text{Length of failure block} = H\cot\left(45° + \frac{\phi}{2}\right)}$
- $\boxed{\Delta H = 0.2\% \text{ of } H}$ for dense sand

  $\boxed{\Delta H = 0.5\% \text{ of } H}$ for loose sand

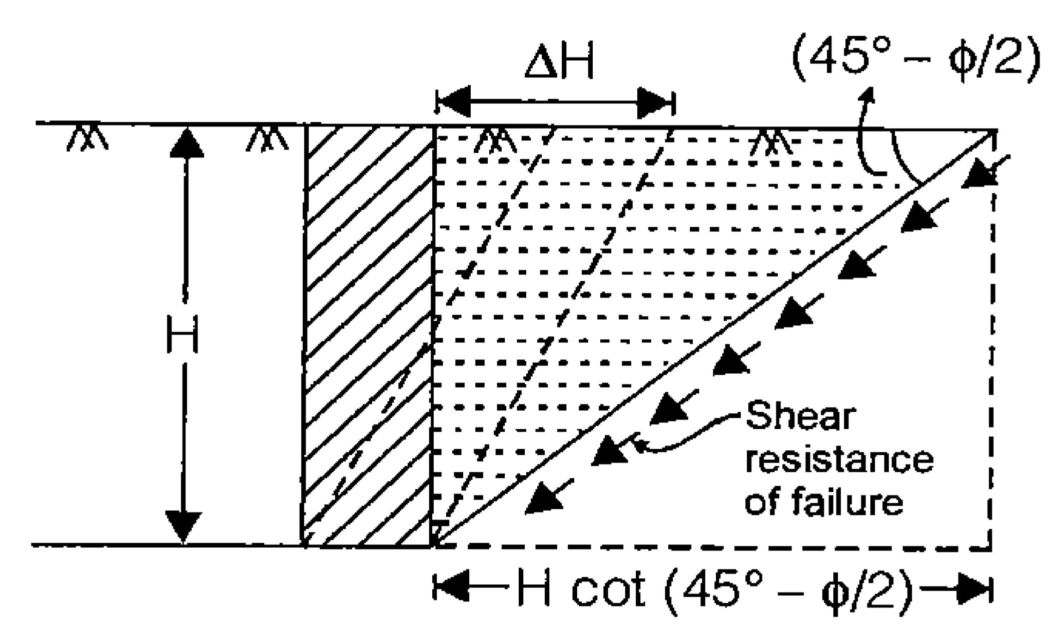
  $\boxed{\Delta H = 0.4\% \text{ of } H}$ for clay's.

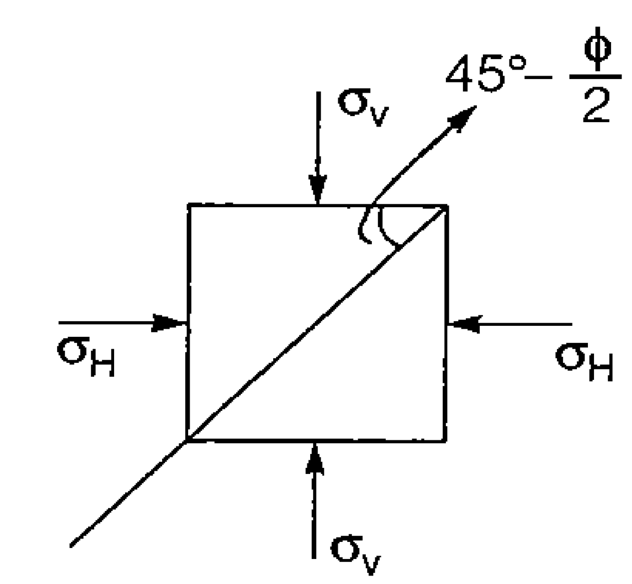


- $\boxed{k_a = \frac{1-\sin\phi}{1+\sin\phi}}$ $\boxed{k_a = \tan^2\left(45° - \frac{\phi}{2}\right)}$

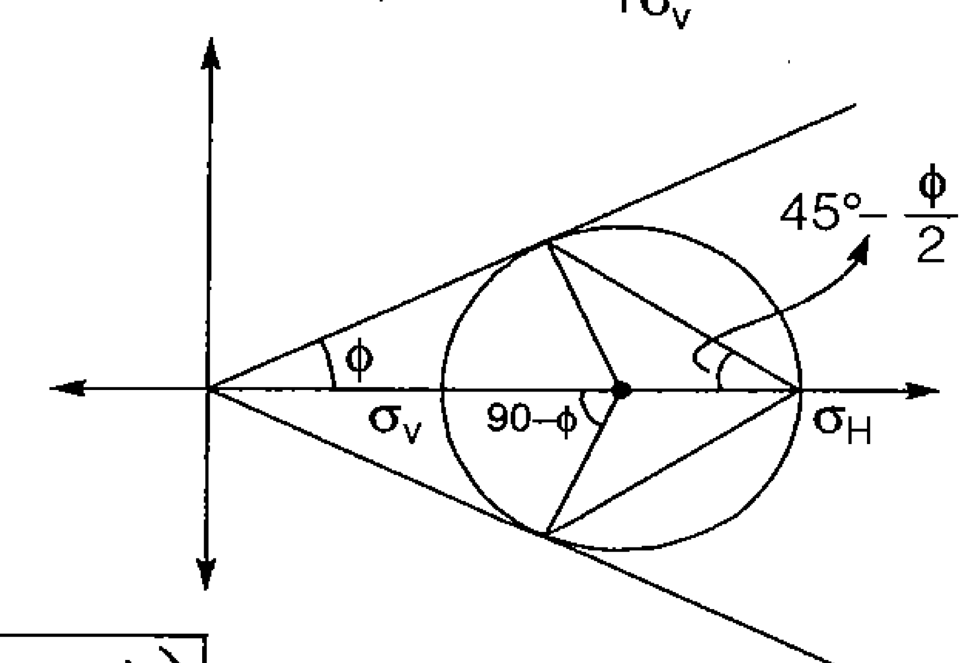
where k_a = Coefficient of active earth pressure.

Passive Earth Pressure





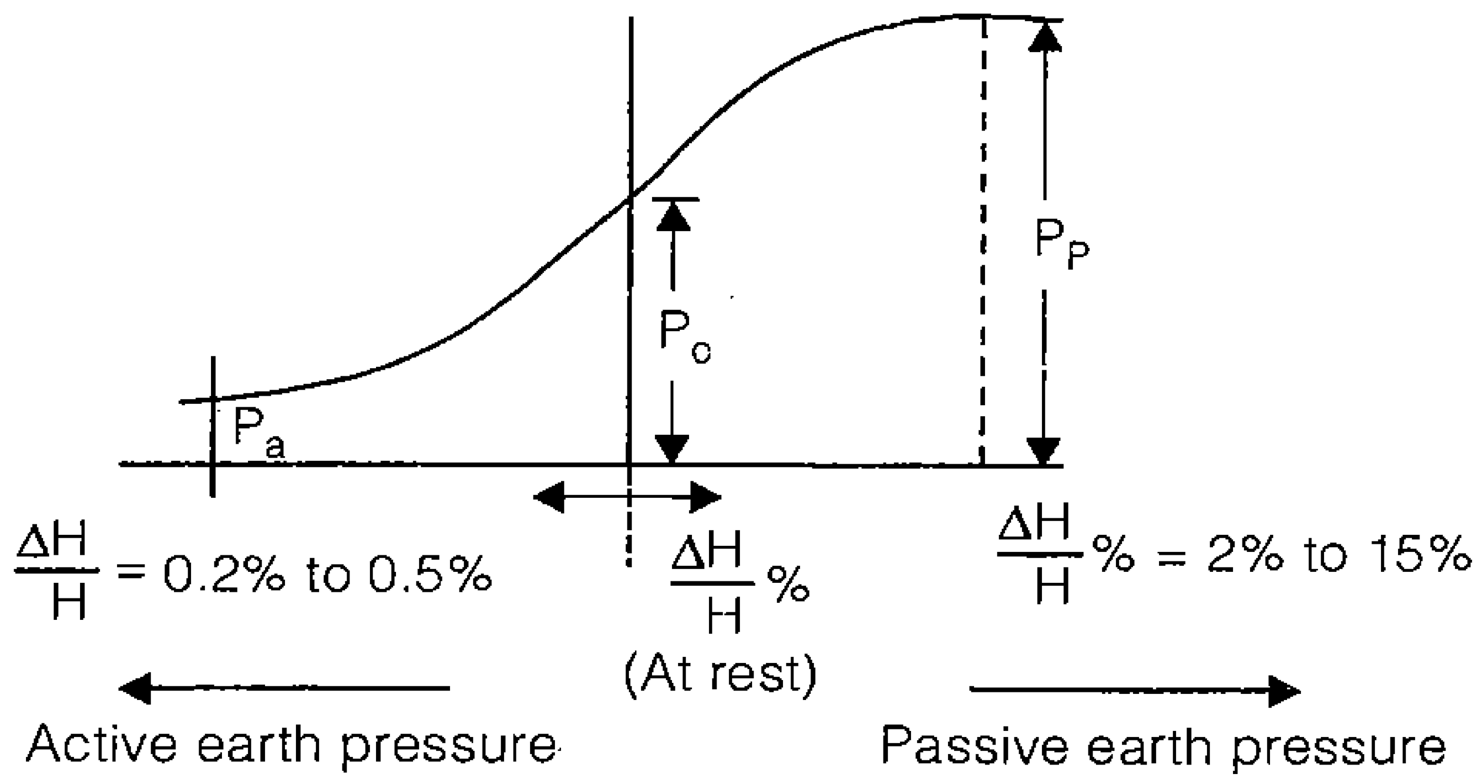
- $\boxed{\text{Length of failure block} = H\cot\left(45° - \frac{\phi}{2}\right)}$
- $\boxed{\Delta H = 2\% \text{ of } H}$ for dense sand.
- $\boxed{\Delta H = 15\% \text{ of } H}$ for loss sand.



- $\boxed{K_P = \frac{1+\sin\phi}{1-\sin\phi}}$ or $\boxed{K_P = \tan^2\left(45° + \frac{\phi}{2}\right)}$

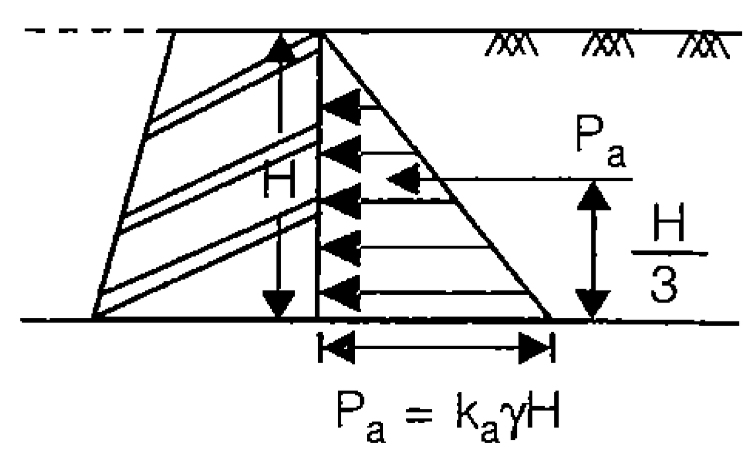
where K_p = Coefficient of passive earth pressure .

- $\boxed{K_a \cdot K_p = 1}$
- $\boxed{P_a < P_o < P_P}$ where, P_a = Active earth pressure.
  P_o = Earth pressure at rest.
  P_P = Passive earth pressure.



Active Earth Pressure by Rankine Theory

(i) $\boxed{P_a = \frac{1}{2}K_a\gamma H^2}$

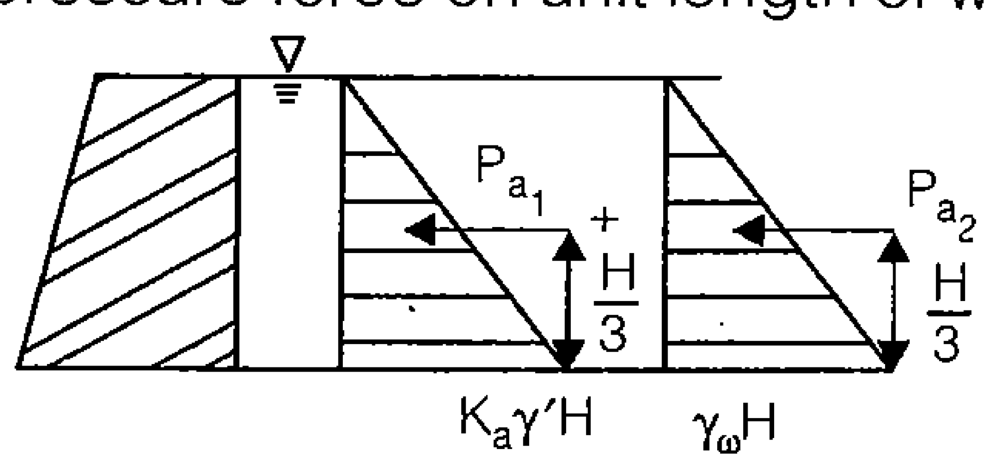


acts at $\frac{H}{3}$ from base.

where,

P_a = Active earth pressure force on unit length of wall.

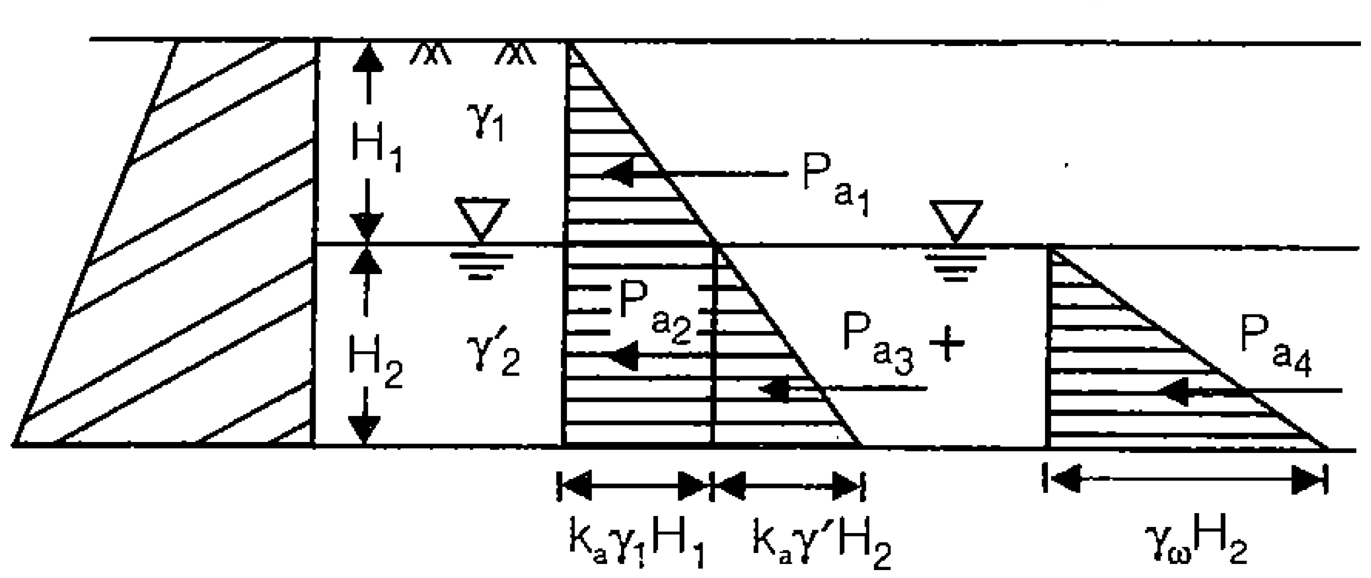
(ii) $\boxed{P_a = \frac{1}{2}k_a\gamma' H^2 + \frac{1}{2}\gamma_w H^2}$



acts at $\frac{H}{3}$ from base

where γ' = Submerged unit weight of soil.

(iii)



$\boxed{P_{a_1} = \frac{1}{2}k_a\gamma_1 H_1^2}$ --- acts of $\left(H_2 + \frac{H_1}{3}\right)$ from base = $\overline{H_1}$

$\boxed{P_{a_2} = k_a\gamma_1 H_1 H_2}$ --- acts of $\left(\frac{H_2}{2}\right)$ from base = $\overline{H_2}$

$\boxed{P_{a_3} = \frac{1}{2}k_a\gamma' H_2^2}$ --- acts at $\frac{H_2}{3}$ from base = $\overline{H_3}$

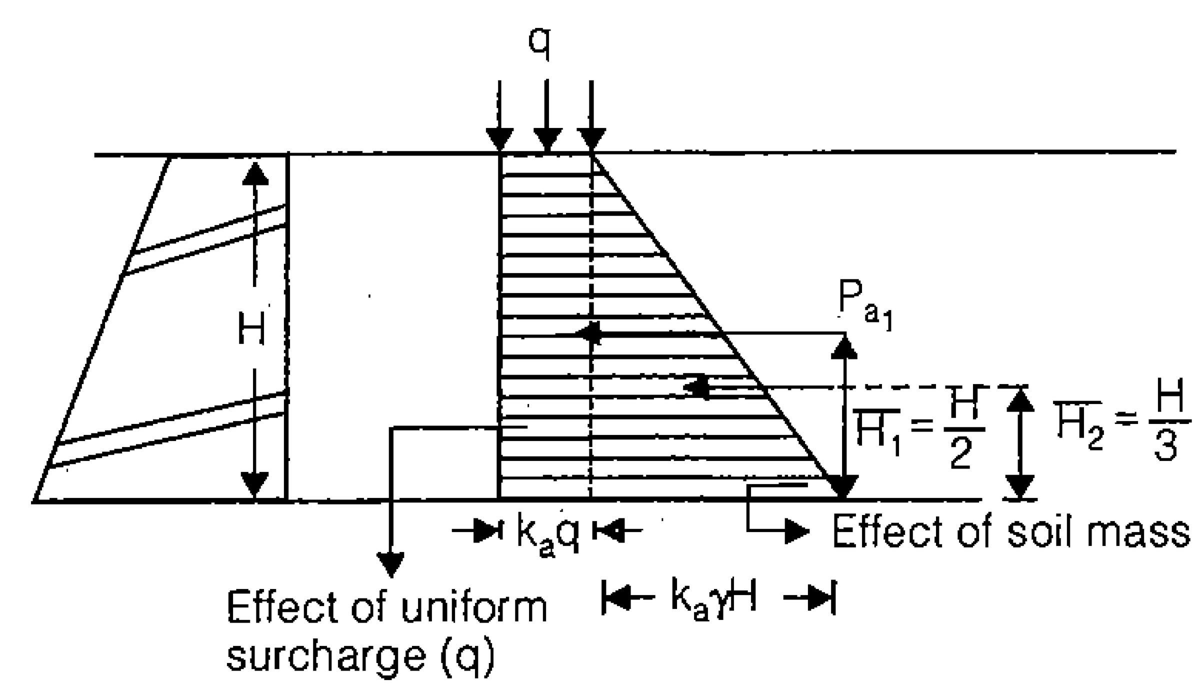
$\boxed{P_{a_4} = \frac{1}{2}\gamma_w H_2^2}$ --- acts of $\frac{H_2}{3}$ from base = $\overline{H_4}$

- **Total Earth Pressure (P_a)**

$\boxed{P_a = P_{a_1} + P_{a_2} + P_{a_3} + P_{a_4}}$ acts at $\overline{H}$ from base

where, $$\boxed{\overline{H} = \frac{P_{a_1}\overline{H_1} + P_{a_2}\overline{H_2} + P_{a_3}\overline{H_3} + P_{a_4}\overline{H_4}}{P_{a_1} + P_{a_2} + P_{a_3} + P_{a_4}}}$$

(iv)

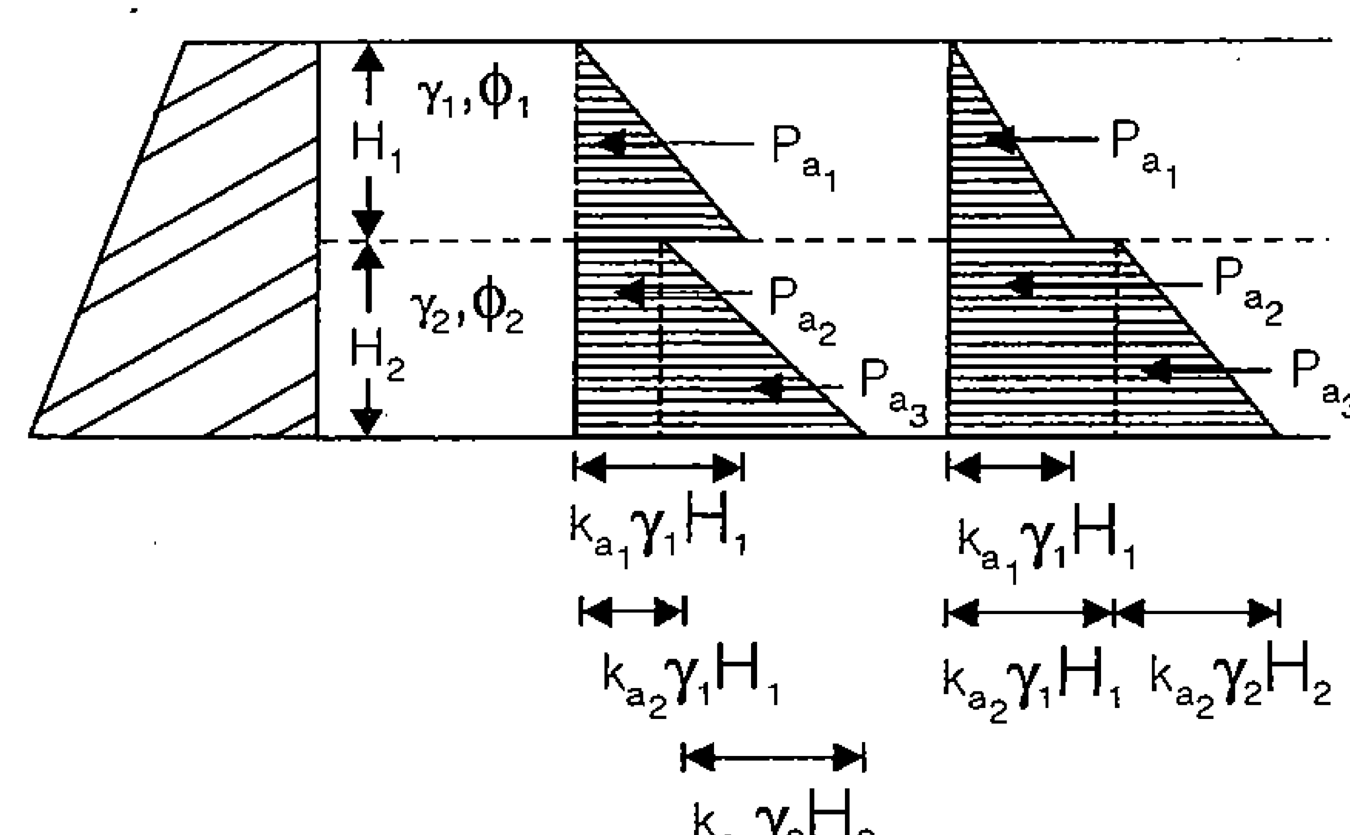


$\boxed{P_{a_1} = K_a q.H}$ acts at $\overline{H_1} = \frac{H}{2}$ from base

$\boxed{P_{a_2} = \frac{1}{2}K_a\gamma H^2}$ acts at $\overline{H_2} = \frac{H}{3}$ from base

$\boxed{P_a = P_{a_1} + P_{a_2}}$ acts at $\boxed{\overline{H} = \frac{P_{a_1}\overline{H_1} + P_{a_2}\overline{H_2}}{P_{a_1} + P_{a_2}}}$ from base.

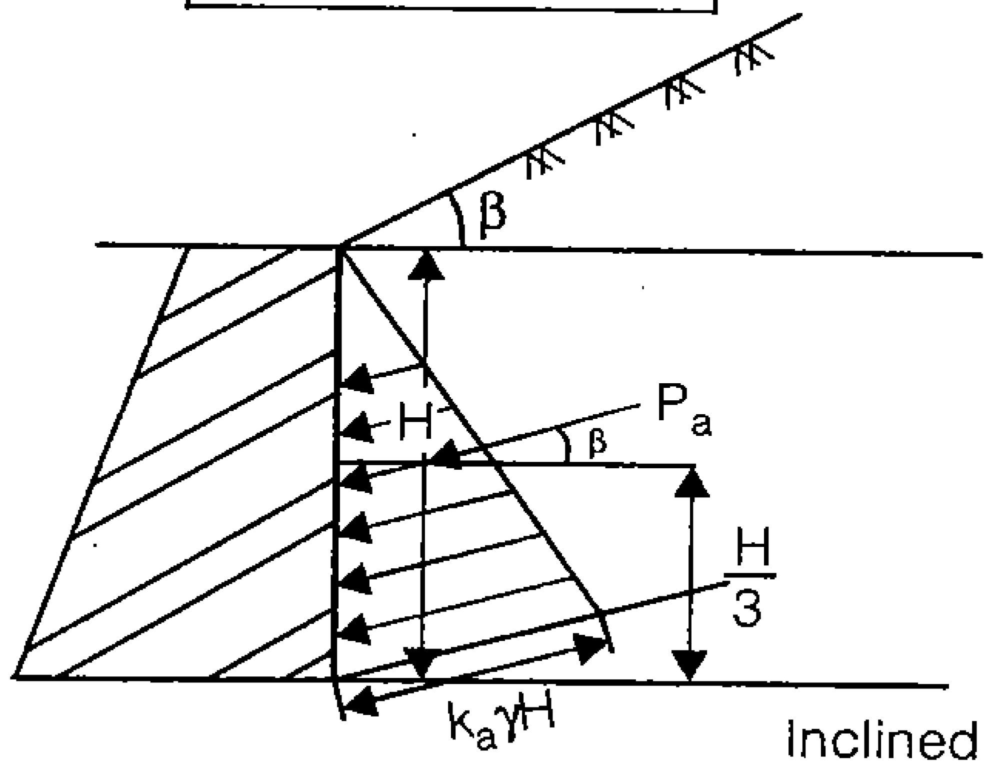
(v)



$\boxed{\phi_1 < \phi_2}$ ⇒ $\boxed{k_{a_1} > k_{a_2}}$ and $\boxed{\phi_1 > \phi_2}$ ⇒ $\boxed{k_{a_1} < k_{a_2}}$

- $P_{a_1} = \frac{1}{2}k_{a_1}\gamma_1 H_1^2$ acts at $\left(H_2 + \frac{H_1}{3}\right)$ from base.
- $P_{a_2} = k_{a_2}\gamma_1 H_1 H_2$ acts at $\left(\frac{H_2}{2}\right)$ from base.
- $P_{a_3} = \frac{1}{2}k_{a_2}\gamma_2 H_2^2$ acts at H_2 from base.

(vi)



Inclined Backfill

$$K_a / K_P = \cos\beta\left[\frac{\cos\beta \pm \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta \mp \sqrt{\cos^2\beta - \cos^2\phi}}\right]$$

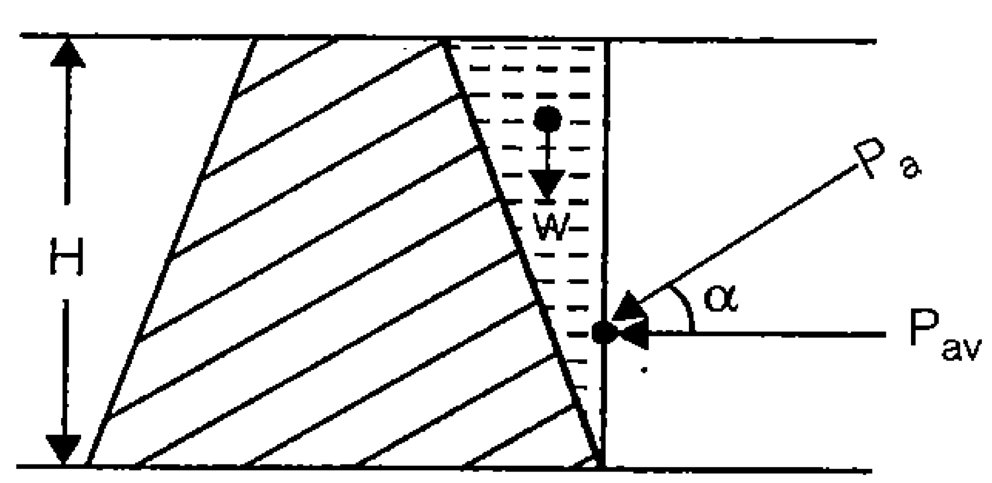
$$k_a . k_p = \cos^2\beta \qquad P_a = \frac{1}{2}k_a\gamma H^2$$

acts at H/3 from base but line of action is parallel to backfill.

$$P_P = \frac{1}{2}k_P\,\gamma H^2$$

acts at H/3 from base but line of action is parallel to backfill.

(vii)



Horizontal Backfill on Inclined back of wall

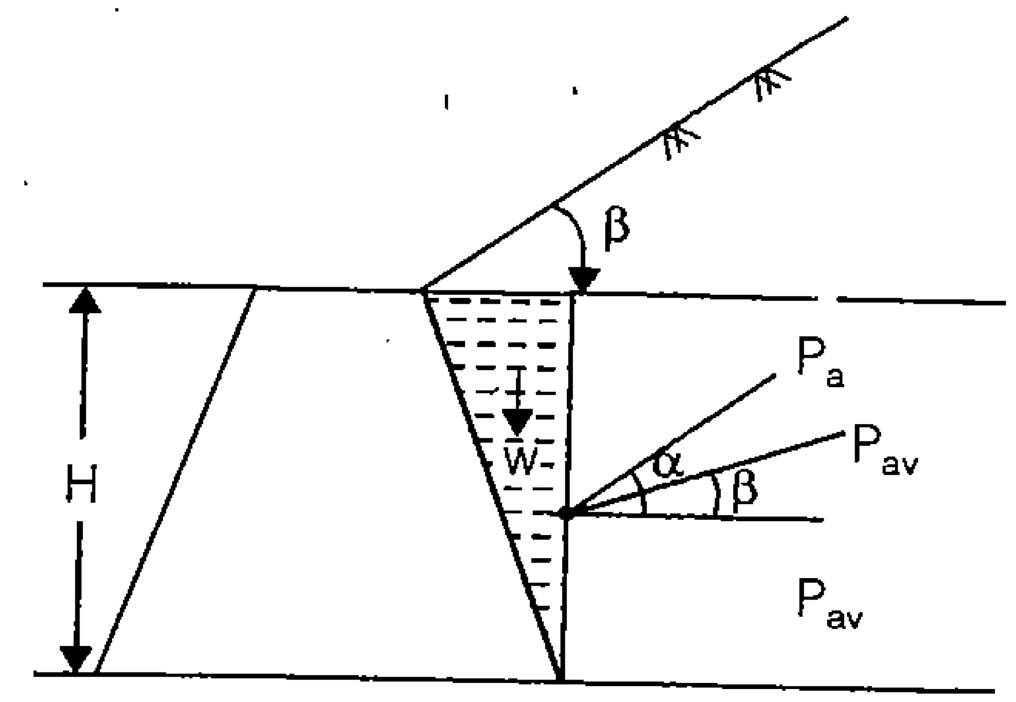
$$P_a = \sqrt{P_{av}^{\,2} + W^2}$$ where, W = Weight of soil over inclined back of wall of unit length.

$$P_{av} = \frac{1}{2}k_a\gamma H^2$$ P_{av} = Active earth pressure on imaginary wall of vertical back.

$$\tan\alpha = \frac{W}{P_{av}}$$ P_a = Resultant active earth pressure on the wall

α = Angle of inclination of resultant P_a with the horizontal.

(viii)



$$P_a = \sqrt{P_{av}^{\,2} + W^2 + 2P_{av}.W\sin\beta}$$

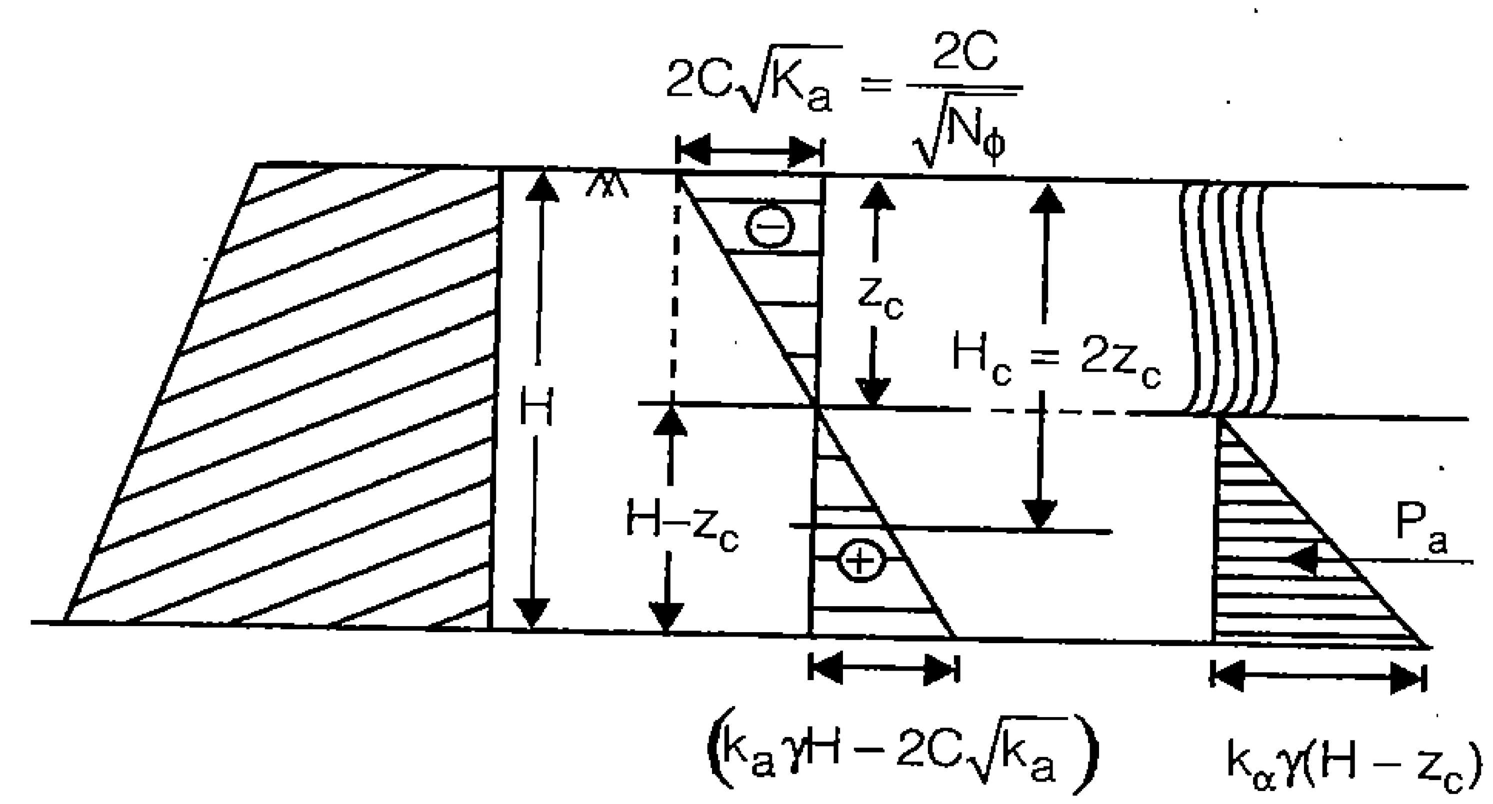
$$\tan\alpha = \frac{W + P_{av}\sin\beta}{P_{av}\cos\beta}$$

Where, P_{av} = Active earth pressure on Imaginary vertical back of wall. Acts parallel to back fill i.e., of β angle with horizontal.

W = Weight of soil block of unit length.

Active Earth Pressure for Cohesive Soil

Assuming no tension crack development



- $K_a = \tan^2\left(45° - \frac{\phi}{2}\right) = \frac{1}{\tan^2\left(45° + \frac{\phi}{2}\right)} = \frac{1}{N_\phi}$

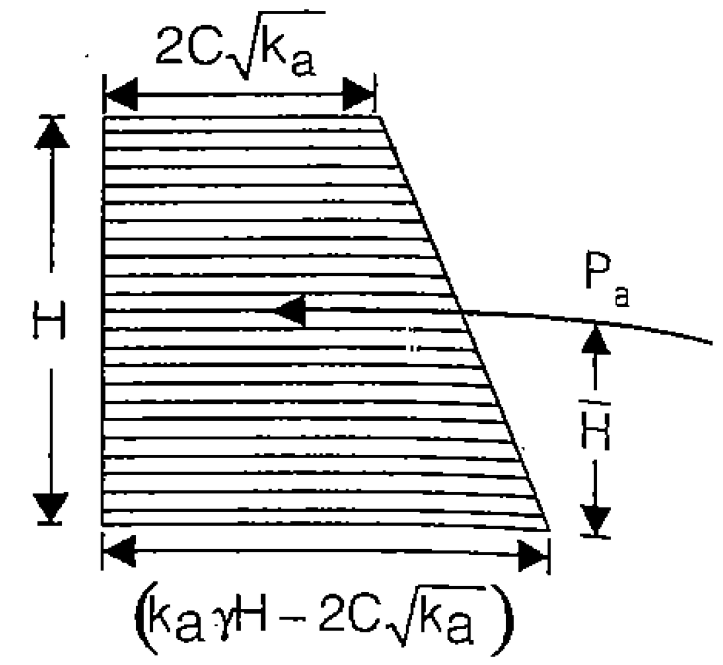
  where N_ϕ = Influence Factor.
- Active Earth Pressure of Any Depth z $P_a = k_a\gamma z - 2c\sqrt{k_a}$
- Active Earth Pressure of Surface. i.e., at z = 0 $P_a = -2c\sqrt{k_a}$
- At $z = z_c$ → $P_a = O$

  $Z_c = \frac{2c}{\gamma}\tan\left(45° + \frac{\phi}{2}\right)$ Where, z_c = depth of tension crack.
- $H_C = \frac{4C}{\gamma}\tan\left(45° + \frac{\phi}{2}\right)$ where, H_C = Critical depth = $2z_c$ = Max. possible unsupported depth.

- When Tension Cracks are not Developed

$$P_a = \frac{1}{2}k_a \gamma H^2 - 2CH\sqrt{k_a}$$

acts at centroid of trapezoidal.

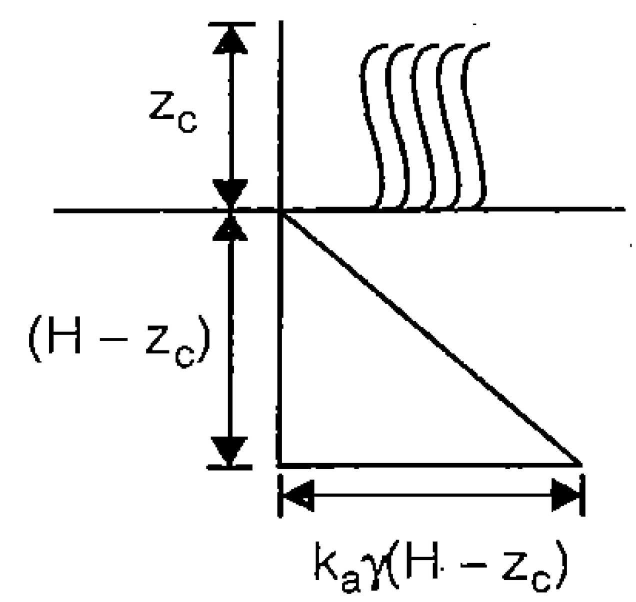


- When Tension Cracks are Developed

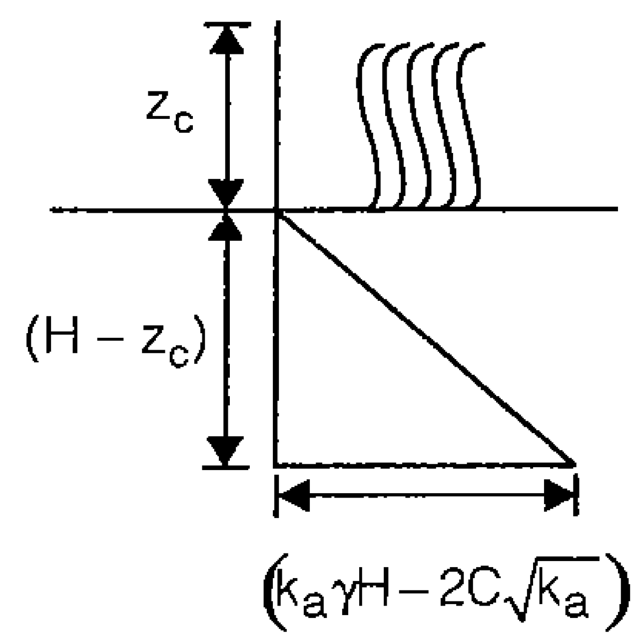
$$P_a = \frac{1}{2}(k_a\gamma H - 2C\sqrt{k_a})(H - Z_c)$$

$$P_a = \frac{1}{2}k_a\gamma H^2 - 2CH\sqrt{k_a} + \frac{2C^2}{\gamma}$$

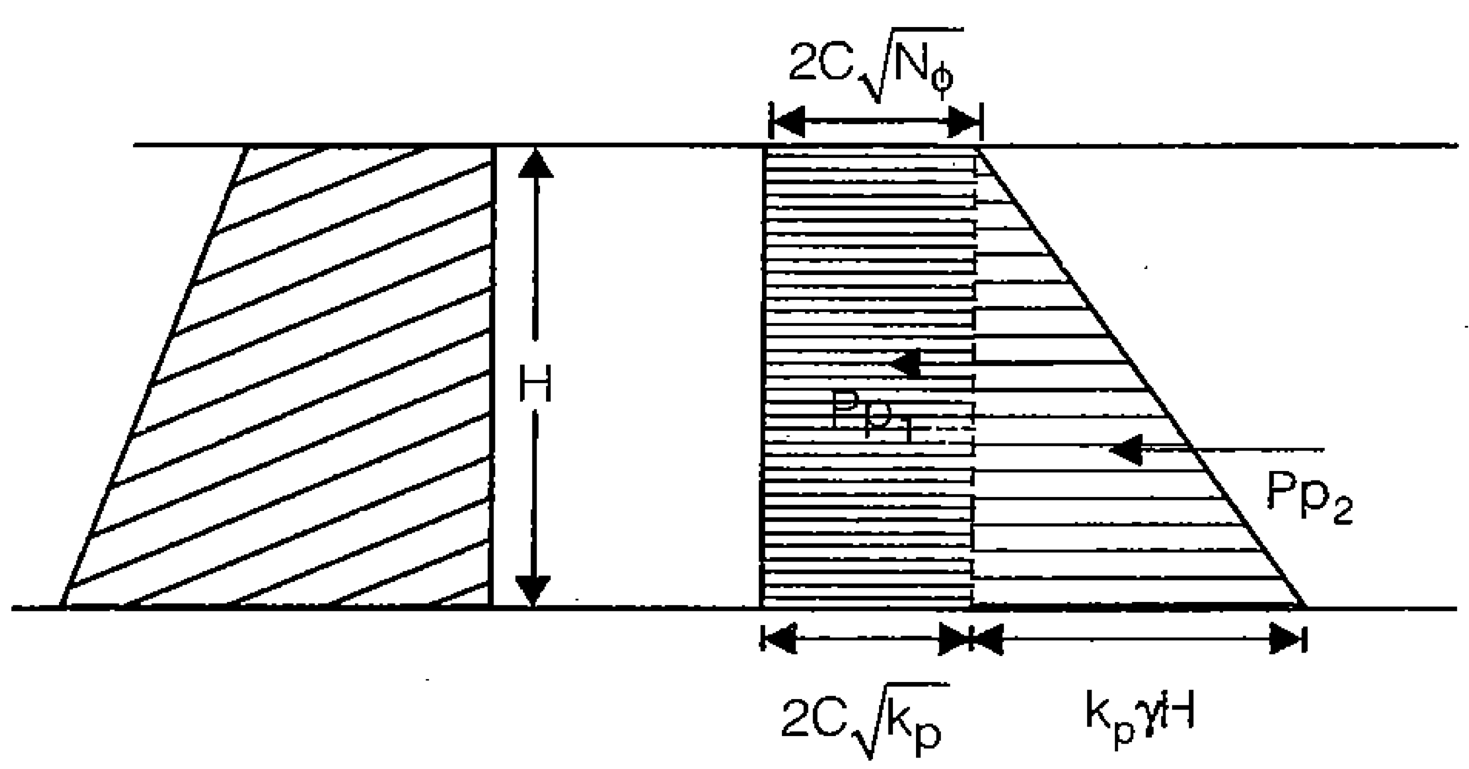
or $$P_a = \frac{1}{2}k_a\gamma(H - z_c)^2$$ acts at $\left(\frac{H - Z_c}{3}\right)$



or



Passive Earth Pressure for Cohesive Soil



- Passive Earth Pressure at any depth 'z',

$$P_p = k_p\gamma z + 2c\sqrt{k_p}$$

- Total P_p on Unit Length

$$P_p = \frac{1}{2}k_p\gamma H^2 + 2c\sqrt{k_p}H$$

Coulombs Wedge Theory

Assumption

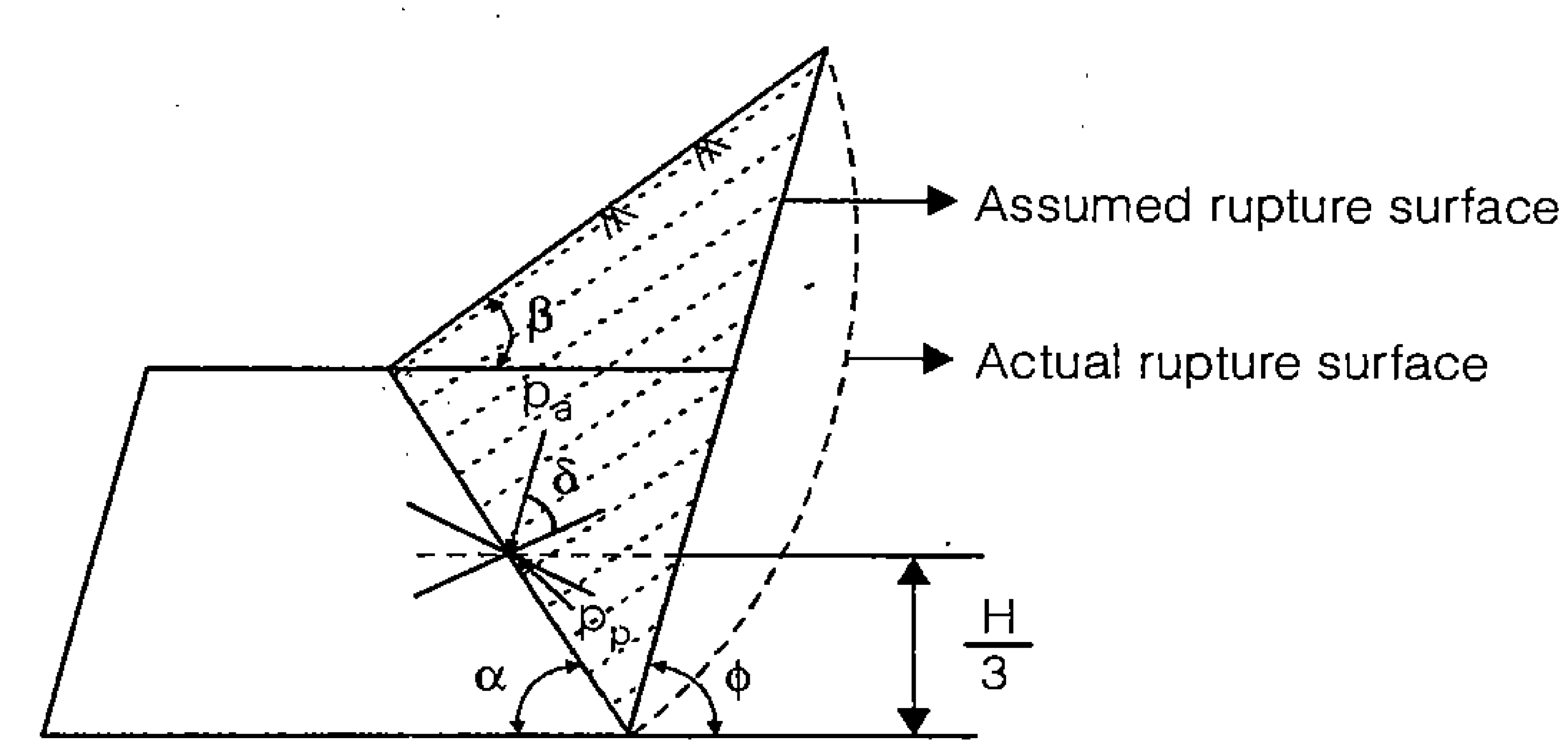
1. Backfill - Dry, Cohesionless, Isotropic
2. Back of wall can be inclined
3. Backfill can be inclined
4. Friction exist between wall + soil
5. Failure plane assumed planer
6. Sliding wedge assumed rigid body
7. Position and line of action of earth pressure in advance.

For C = 0

$$k_a = \left[\frac{\frac{\sin(\alpha+\phi)}{\sin\alpha}}{\sqrt{\sin(\alpha-\delta)} + \sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha+\beta)}}}\right]^2$$

$$k_p = \left[\frac{\frac{\sin(\alpha-\phi)}{\sin\alpha}}{\sqrt{\sin(\alpha+\delta)} + \sqrt{\frac{\sin(\phi+\delta)\sin(\phi+\beta)}{\sin(\alpha+\beta)}}}\right]^2$$

where, α = Angle of back of wall with the horizontal
β = Angle of sloping ground.
δ = Angle of roughness between soil and wall
= 0° for smooth walls



> **NOTE**
> For active earth P, earth pressure is highest value obtained for various trial wedges.
> For passive earth P, earth pressure is minimum wedges.

Special Points :

- Retaining wall are designed for active earth P.
- Ranking theory

  Overstimate → Active earth pressure

  Underestimates → Passive earth pressure

> **Remember**
> - Cohesion decreases the active earth pressure while increases the passive earth pressure.
>
> $P_a = \frac{1}{2}k_a\gamma H^2$ acts at $\frac{H}{3}$ from base of an angle δ with the normal of wall. $P_P = \frac{1}{2}k_p\gamma H^2$
>
> - For cantilever and counterfort walls rankines theory is used. For gravity and for semi-gravity retaining walls coulomb's theory is preferred.

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Stability Analysis of Slopes

11

Factor of Safety w.r.t. Shear Strength (F_s)

- $F_s = \frac{C + \bar{\sigma}\tan\phi}{\tau}$

  where, τ = Developed shear strength.

  $(C + \bar{\sigma}\tan\phi)$ = Developed or mobilized shear stress

  C = Effective cohesion

  φ = Effective friction

  $\bar{\sigma}$ = Effective normal stress

- $\tau = C_m + \bar{\sigma}\tan\phi_m$

  where, C_m = Mobilized Cohesion

  ϕ_m = Mobilized Friction Angle

$$C_m = \frac{C}{F_s} \quad \text{and} \quad \tan\phi_m = \frac{\tan\phi}{F_s}$$

Factor of Safety w.r.t. Cohesion (F_c)

$$F_c = \frac{H_c}{H} \quad \text{and} \quad F_c = \frac{C}{C_m}$$

where, H_c = Critical depth

H = Actual depth

$$H_c = \frac{4C}{\gamma}\tan\left(45° + \frac{\phi}{2}\right)$$

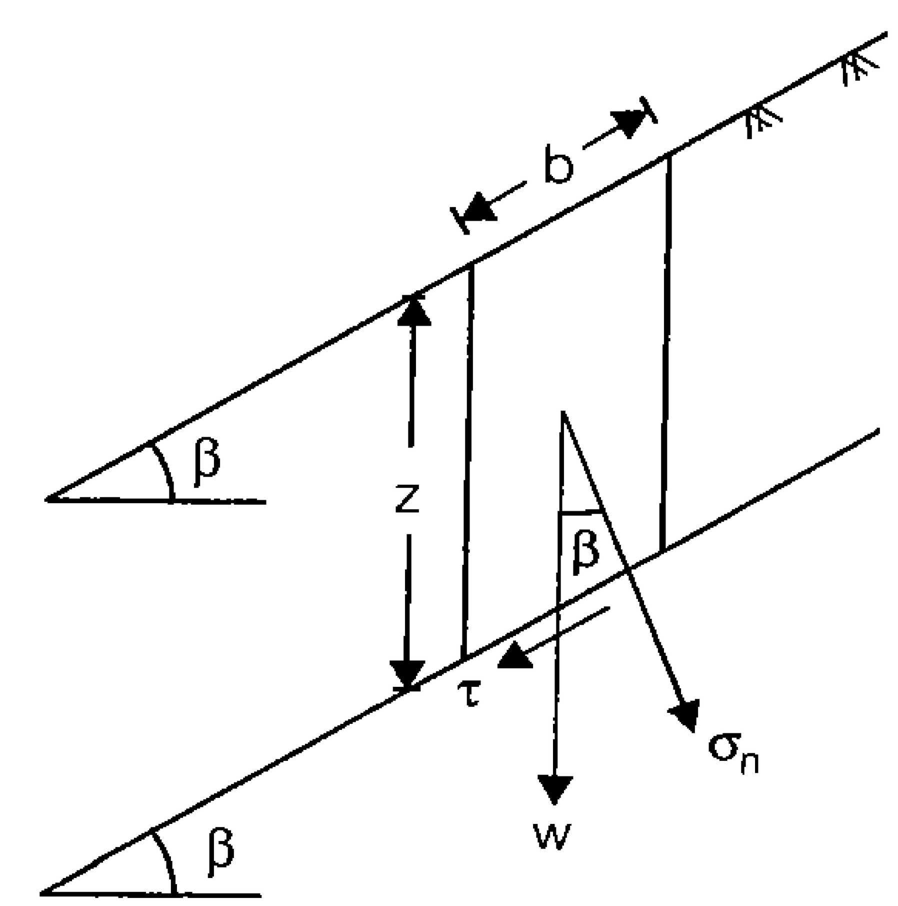
Stability Analysis of Infinite Slopes

(i) Cohesionless dry soil/dry sand

$$W = \gamma Z \cos\beta$$

$$\tau = \frac{W\sin\beta}{(b \times 1)} \Rightarrow \tau = \gamma Z \sin\beta\cos\beta$$

$$\sigma_n = \frac{W\cos\beta}{(b \times 1)} \Rightarrow \sigma_n = \gamma Z \cos^2\beta$$



where, τ = Developed shear stress or mobilized shear stress
σ_n = Normal stress.

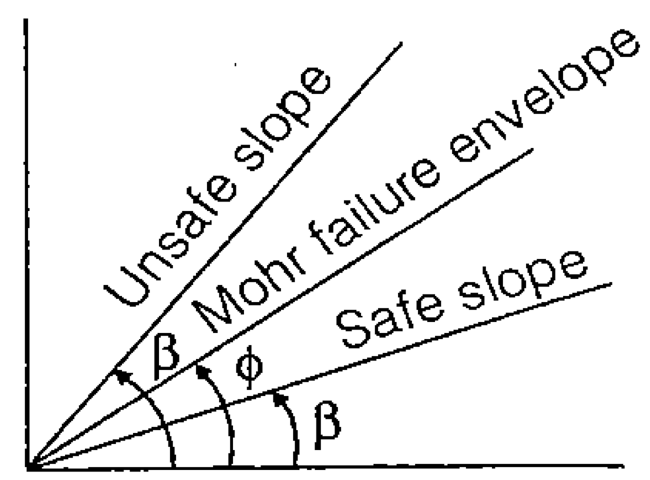
$$\boxed{F_s = \frac{\tan\phi}{\tan\beta}}$$

where, F_s = Factor of safety against sliding

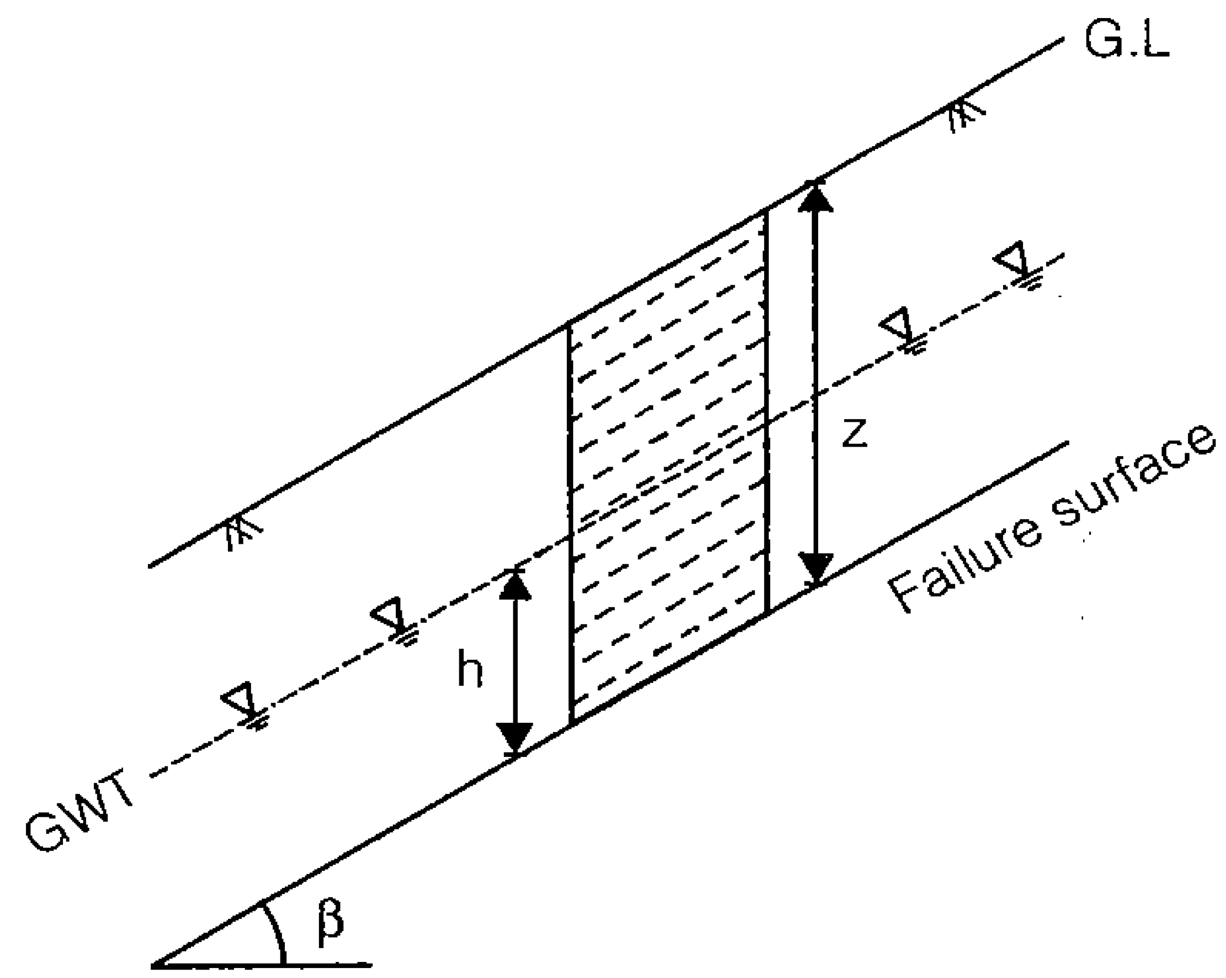
$$= \frac{S}{\tau} = \frac{C + \overline{\sigma_n}\tan\phi}{\tau}$$

- For Safety of Slopes

$$\boxed{\beta < \phi} \Rightarrow \boxed{F_s > 1}$$



(ii) **Seepage taking place and water table is parallel to the slope in Cohesionless soil**



h = Height of water table above the failure surface.

$$\boxed{F_s = \left[1 - \left(\frac{\gamma_w}{\gamma}\right)\left(\frac{h}{z}\right)\right]\frac{\tan\phi'}{\tan\beta}}$$

ϕ' is effective friction angle
γ–avg. total unit weight of soil above the slip surface upto ground level.

$$\boxed{\gamma = \frac{\gamma_1 h_1 + \gamma_2 h_2}{h_1 + h_2}}$$

(iii) **If water table is at ground level : i.e.,**

$$\boxed{h = z} \quad \boxed{F_s = \frac{\gamma'}{\gamma_{Sat}} \cdot \frac{\tan\phi}{\tan\beta}} \quad \boxed{F_s \simeq \frac{1}{2} \cdot \frac{\tan\phi}{\tan\beta}}$$

(iv) **Infinite Slope of Purely Cohesive Soil**

$$\boxed{F_s = F_c = \frac{C}{\gamma z \sin\beta \cdot \cos\beta}} \quad \boxed{F_c = \frac{H_c}{H}}$$

Here H = z = depth of slice/cut.

At Critical Stage $\boxed{F_c = 1}$

$$\boxed{S_\eta = \frac{C}{\gamma H_c} = \sin\beta.\cos\beta = \frac{C}{\gamma F_c H} = \frac{C}{\gamma F_c z}}$$

where S_η = Stability Number.

(v) **C–φ Soil in Infinite Slope** $\boxed{F_s = \frac{C}{\gamma H \sin\beta.\cos\beta} + \frac{\tan\phi}{\tan\beta}}$

(vi) **Taylor's stability no.**

$$\boxed{S_n = \frac{C}{\gamma.H_c} = \sin\beta.\cos\beta}$$ **(for cohesive soil)**

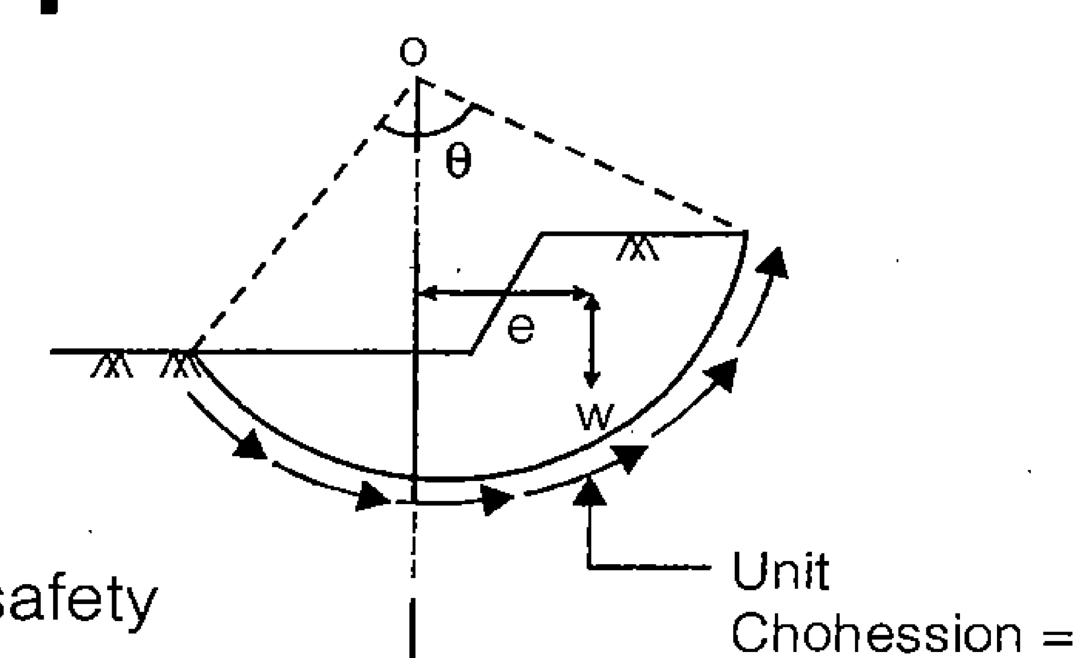
Max. theoritical value of stability no. = 0.5
Max. practical value is = 0.261

$$\boxed{S_n = [\tan\beta - \tan\phi]\cos^2\beta}$$ **(for C-φ solis)**

Stability Analysis of Finite Slopes

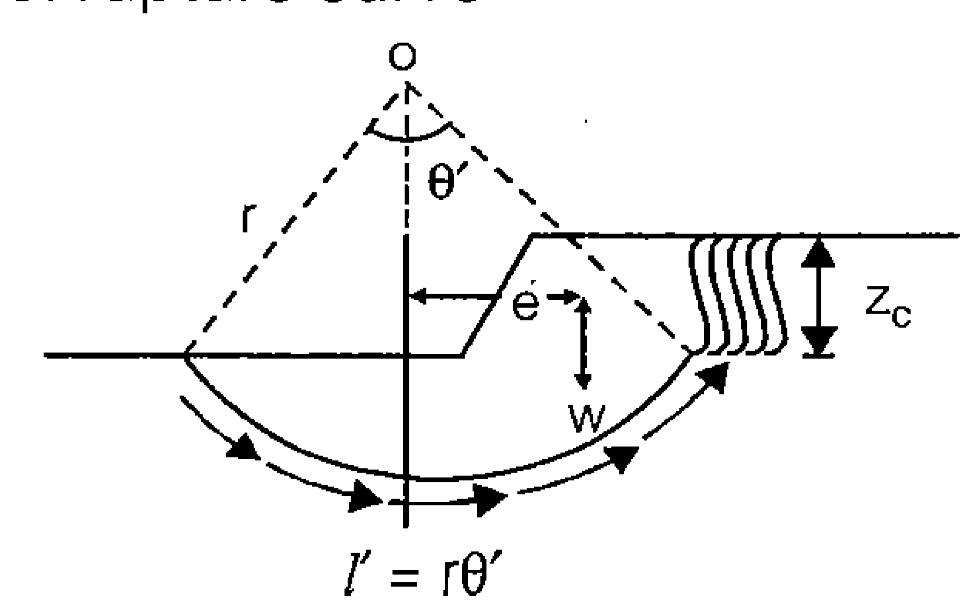
(i) **Fellinious Method**
(For Purely Cohesive Soil)

(a) $$\boxed{F = \frac{Cr^2\theta}{we}}$$



where, F = Factor of safety
r = Radius of rupture curve
l = Length of rupture curve

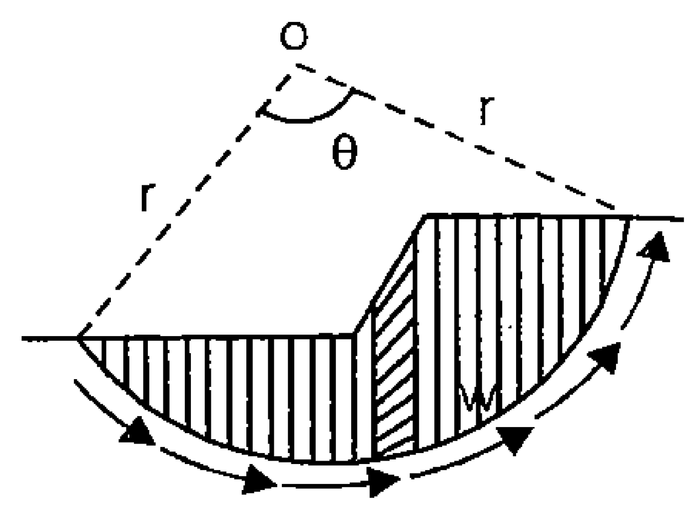
(b) $$\boxed{F = \frac{Cr^2\theta^1}{we}}$$



F = Factor of safety it tension cracks has developed.

$$z_c = \frac{2C}{\gamma}$$

(ii) Swedish Circle Method



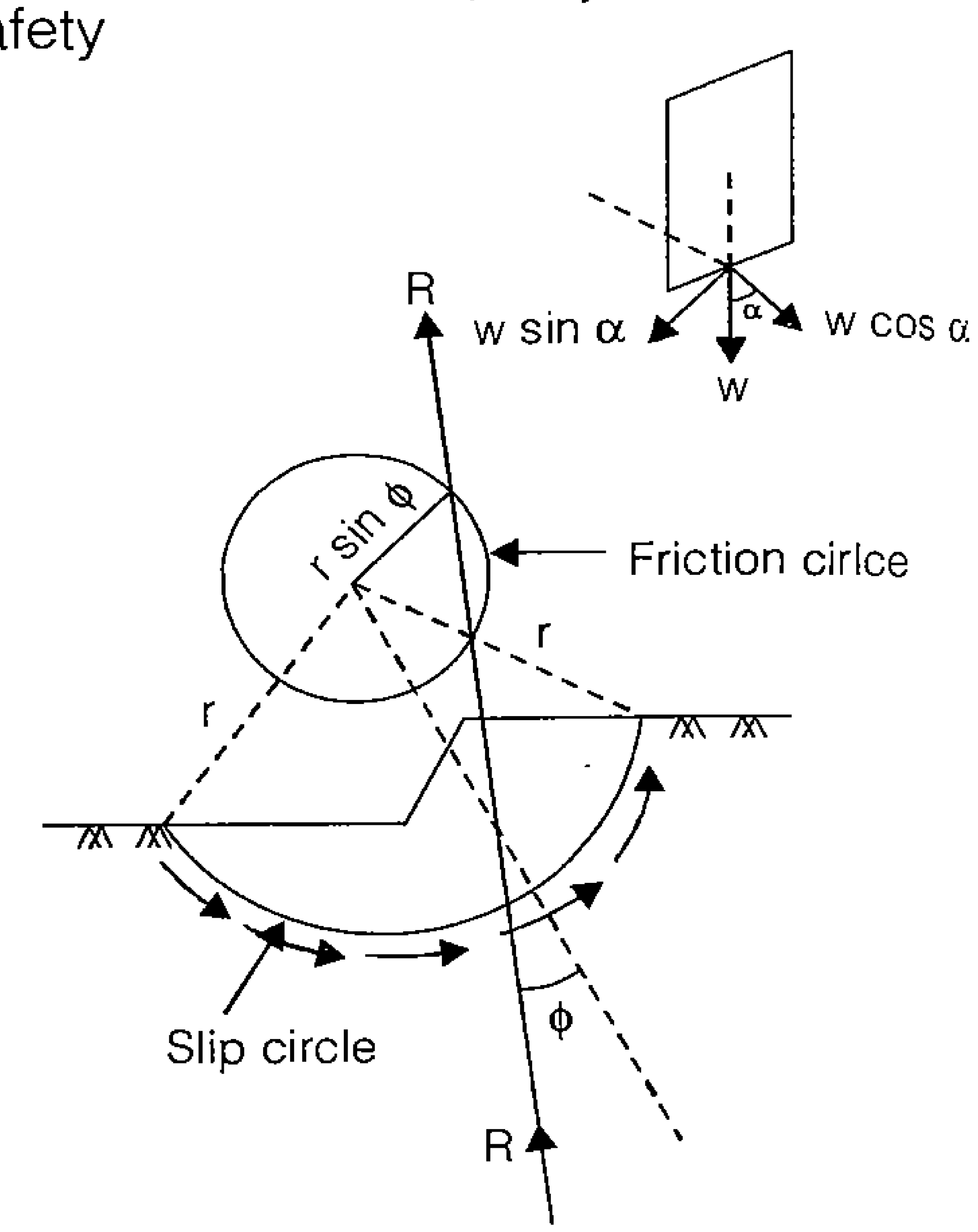
$$F = \frac{C r \theta + \Sigma w \cos\alpha \cdot \tan\phi}{\Sigma w \sin\alpha}$$

where, F = Factor of safety

(iii) Friction Circle Method

$$F_C = \frac{C}{C_m}$$

$$F_\phi = \frac{\tan\phi}{\tan\beta} = \frac{\tan\phi}{\tan\phi_m}$$



(iv) Taylor's Stability Method (C–φ soil)

$$S_\eta = \frac{C}{\gamma H_c} = \frac{C}{\gamma F_C H}$$

In case of submerged slope γ' should be used instead of γ and if slope is saturated by capillary flow they γ_{Sat} should be used in stead of γ.

$$\phi_w = \frac{\gamma'}{\gamma_{Sat}} \cdot \phi$$ where ϕ_w = weight friction angle.

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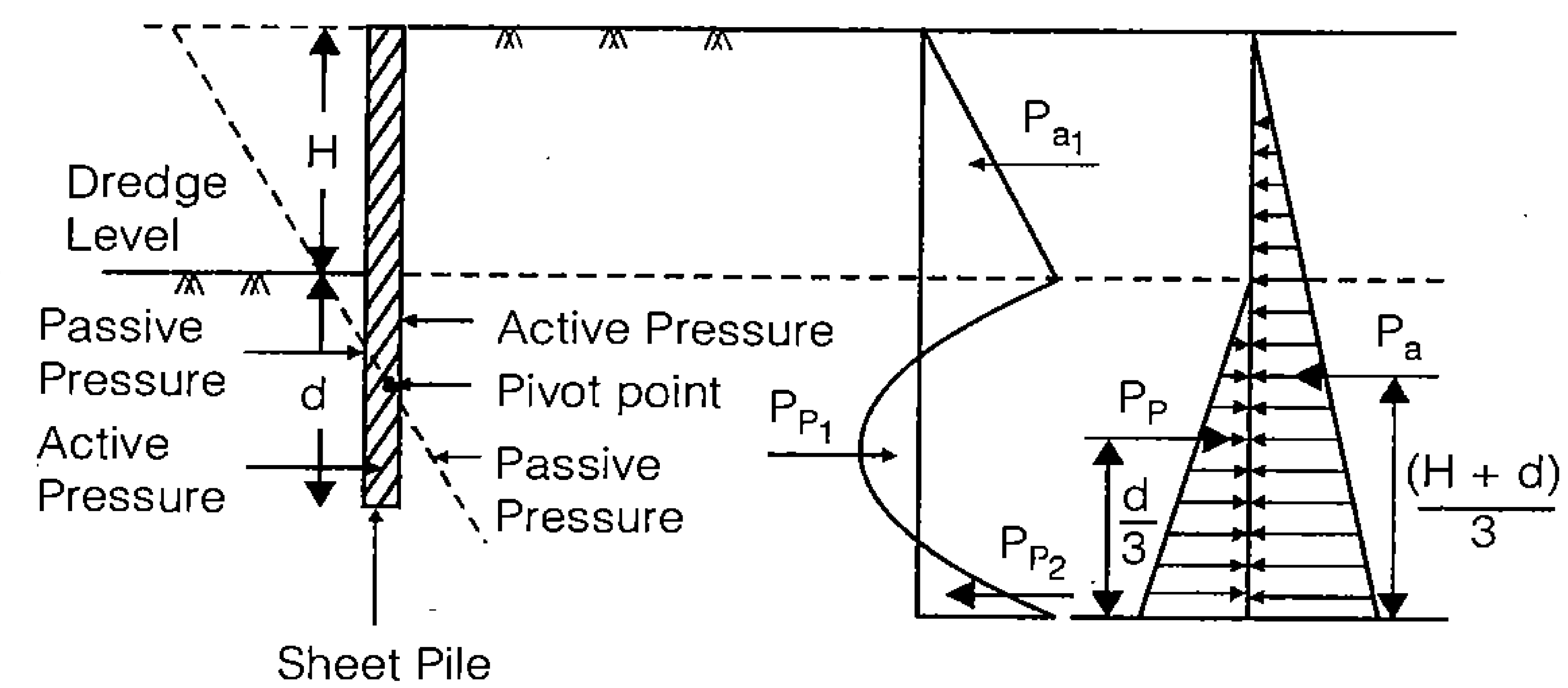
Sheet Pile Walls 12

Sheet Pile Walls Embedded in Sands



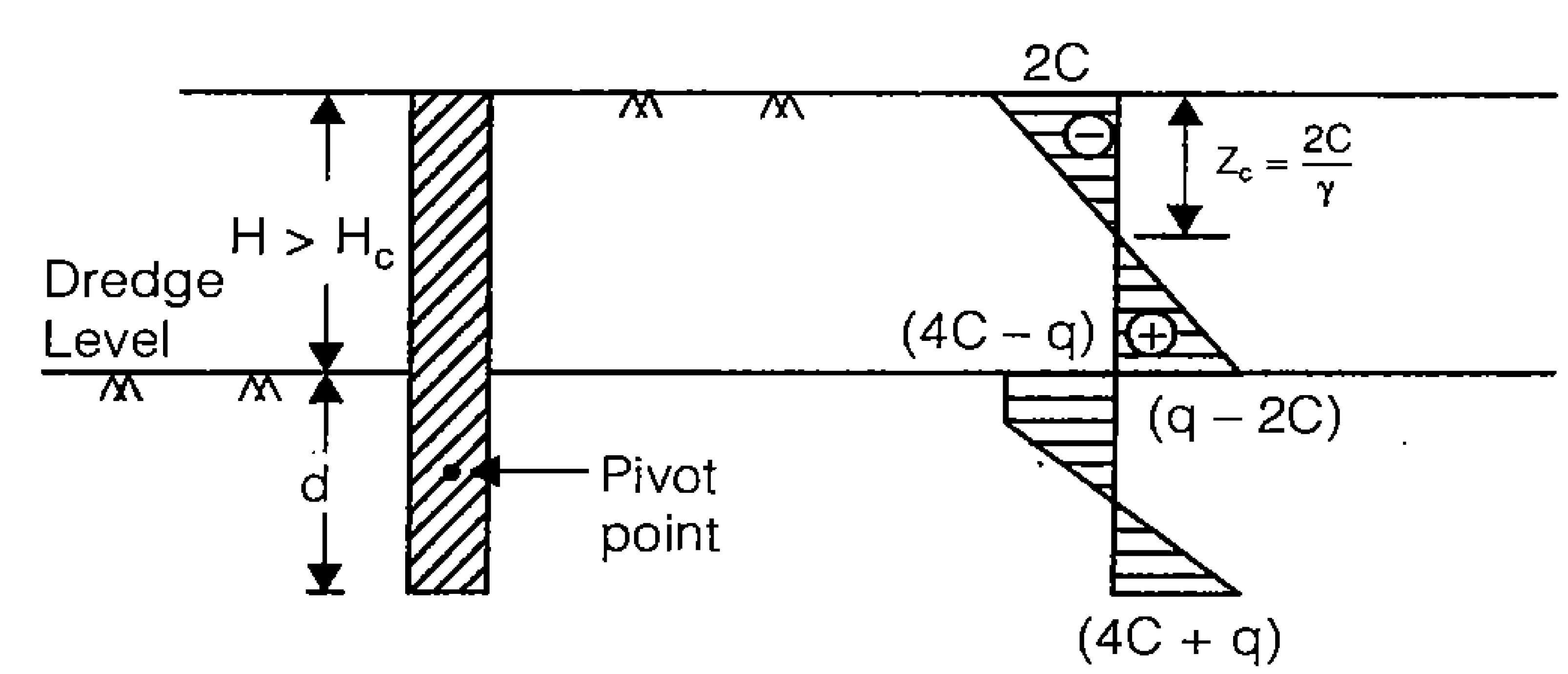
$$P_p \cdot \frac{d}{3} = P_a \frac{(H+d)}{3}$$ ... without factor of safety.

$$\frac{P_p}{Fos} \frac{d}{3} = P_a \frac{(H+d)}{3}$$ ... with factor of safety.

$$P_p = \frac{1}{2} k_p \gamma d^2$$

$$P_p = \frac{1}{2} k_a \gamma (H+d)^2$$

Sheet Pile Walls Embedded in Clays



$$q = \gamma H$$

Active earth pressure at depth H,

$$\boxed{P_a = q - 2C}$$

Passive earth pressure at depth 'H'.

$$\boxed{P_p = 2c}$$

Resultant earth pressure of depth H, is $(P_p - P_a)$

$$\boxed{P_p - P_a = 4c - q}$$

Resultant earth pressure at base i.e., at depth (H + d) is $(P_p - p_a)$

$$\boxed{P_p - P_a = (4c - q)}$$

Resultant earth pressure of base i.e., of depth (H +d) is $(P_p - P_a)$

$$\boxed{P_p - P_a = (4c + q)}$$

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Shallow Foundation & Bearing Capacity 13

Bearing Capacity

The load carrying capacity of foundation soil or rock which enables it to bear and transmit loads from a structure.

Gross Pressure Intensity

It is the total pressure at the base of the footing due to the weight of the super structure, self weight of the footing and weight of the earth fill.

Net Pressure Intensity

It is defined as excess of gross pressure to over burden pressure.

$$\boxed{q_{net} = q_g - \bar{\sigma}}$$

where, q_{net} = Net Pressure Intensity

q_g = Gross Pressure

$\bar{\sigma}$ = Effective Stress = $\gamma' D_f$.

Net ultimate bearing capacity

It is the minimum net pressure causing shear failure of soil.

$$\boxed{q_{nu} = q_u - \bar{\sigma}}$$

where, q_{nu} = Net ultimate bearing capacity

q_u = Ultimate bearing capacity

net safe bearing capacity

$$\boxed{q_{ns} = \frac{q_{nu}}{F_s}}$$

where, q_{ns} = Net safe bearing capacity

F_s = Factor of safety

Safe bearing capacity

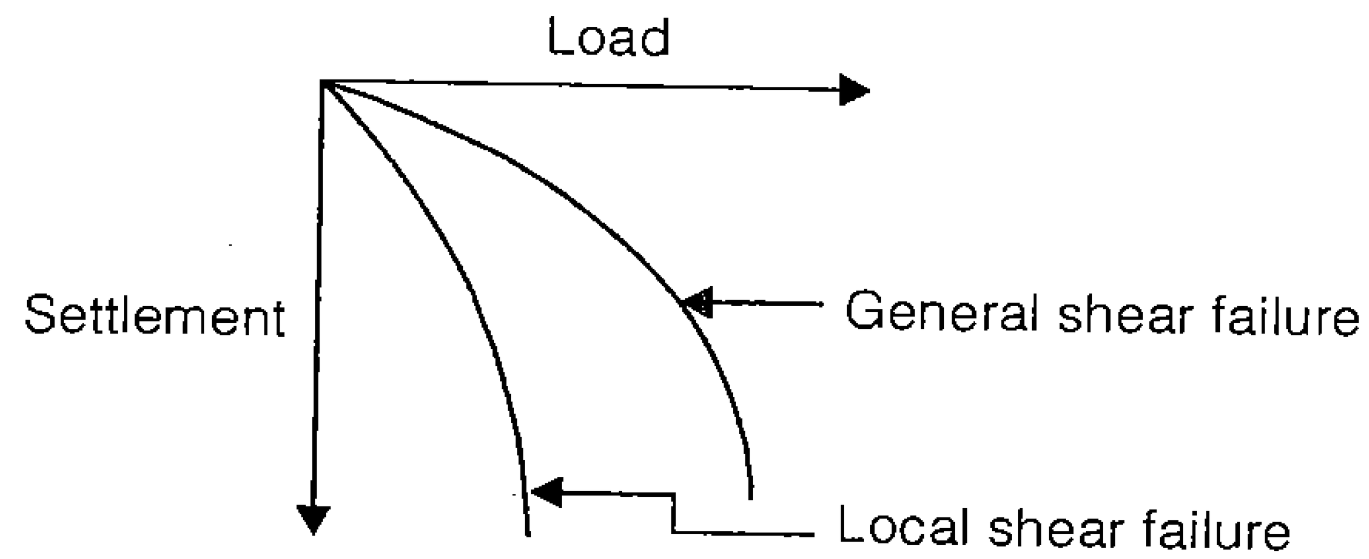
$$\boxed{q_s = q_{ns} + \bar{\sigma}}$$

where, q_s = Safe bearing capacity.

Remember

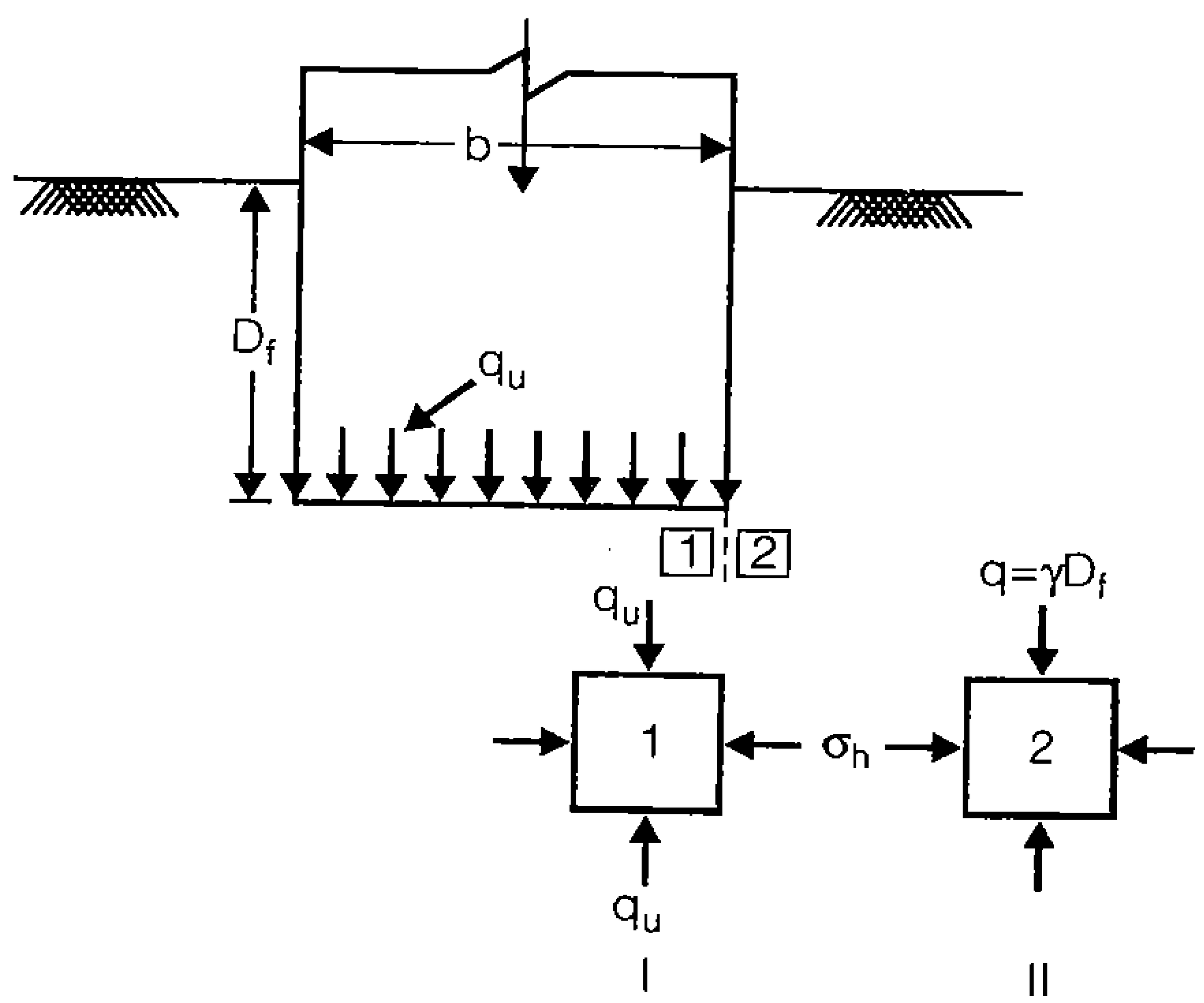
Parameter	General shear failure	Local shear failure
1. Friction angle (ϕ)	> 36°	< 28°
2. Strain of failure	≤ 5°	≥ 15°
3. S.P.T number	> 30	< 5
4. Relative density	> 17%	< 20%

5.	Void ratio	< 0.55	> 0.75
6.	Unconfined compressive strength	> 100 kN/m^2	< 80 kN/m^2



Method to determine bearing capacity

(i) Rankines Method (ϕ – soil)



Rankine's method for bearing capacity of a footing

$$q_u = \gamma D_f \tan^4\left(45° + \frac{\phi}{2}\right) \quad \text{or} \quad q_u = \gamma D_f\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2$$

(ii) Bells Theory (C – ϕ)

$q_u = CN_c + \gamma D_f N_q$ where, N_c and N_q are bearing capacity factors.

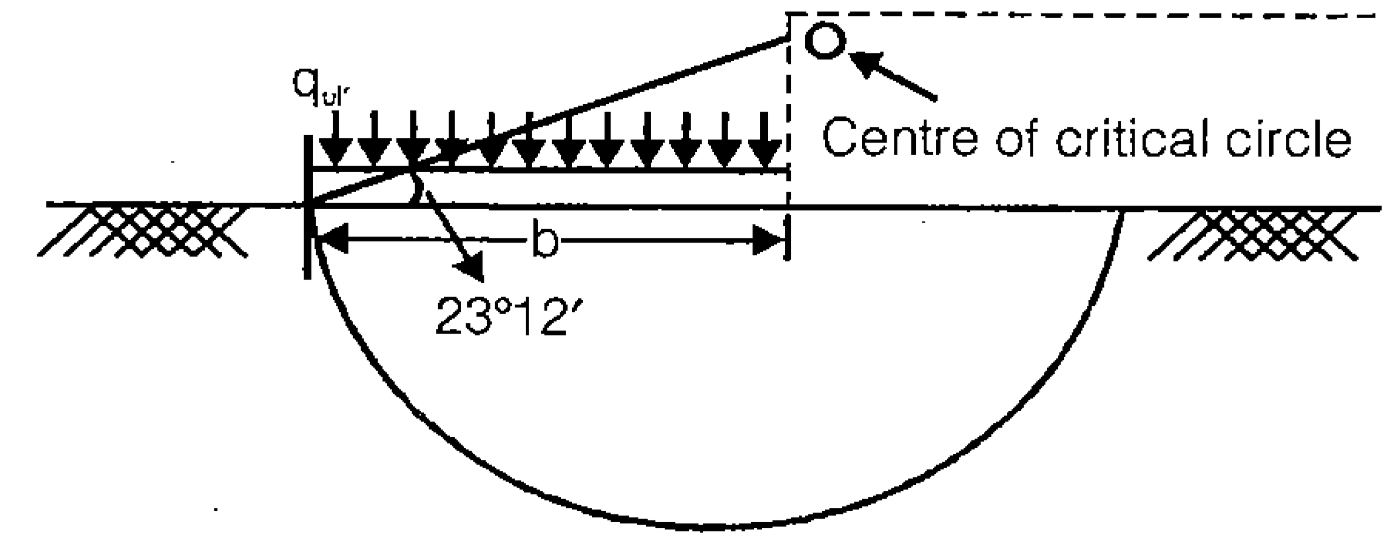
For pure clays → C = 4, q = 1

(iii) Fellinious Method : (C-soil)

- The failure is assumed to take place by slip and the consequent heaving of a mass of soil is on one side.

$$q_{ult} = \frac{W \cdot l_r + CR}{b \cdot l_o} \qquad q_{ult} = 5.5C$$

- Location of Critical circle



Location of critical circle for surface footing in Fellenius' method

(iv) Prandtl Method : (C-ϕ)

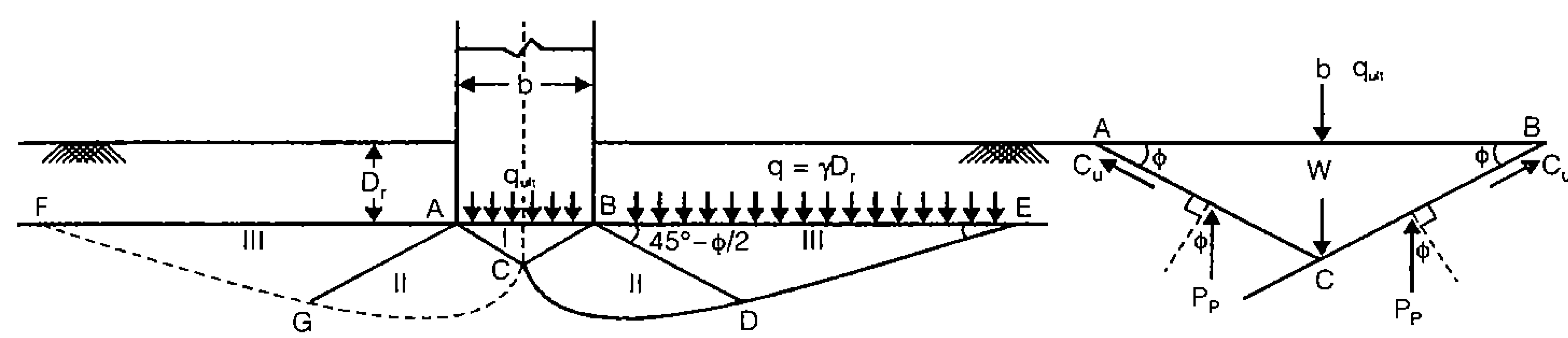
$$q_u = CN_c + \gamma D_f N_q + \frac{1}{2}\gamma B N_\gamma \rightarrow \text{For strip footing}$$

For C-soil $N_c = 5.14$, $N_q = 1$, $N_\gamma = 0$

(v) Terzaghi Method (C-ϕ)

Assumptions

S - Strip footing, S - Shallow foundation, G - General shear failure, H - Horizontal ground, R - Rough base



(a) Terzaghi system for ideal soil, rough base and surcharge (b) Forces on the elastic wedge

Terzaghi's method for bearing capacity of strip footing

Total zones = 5

For strip footing

$$q_u = CN_c + \gamma D_f N_q + \frac{1}{2}\gamma B N_\gamma$$

For square footing

$$q_u = 1.3CN_c + \gamma D_f N_q + 0.4\gamma BN_\gamma$$

For rectangular footing

$$q_u = \left(1 + 0.3\frac{B}{L}\right)CN_c + \gamma D_f N_q + \frac{1}{2}\left(1 - \frac{0.2B}{L}\right)\gamma BN_\gamma$$

For circular footing

$$q_u = 1.3CN_C + \gamma D_f N_q + 0.3\gamma DN_\gamma$$

where, D = Dia of circular footing

$CN_C \rightarrow$ Contribution due to constant component of shear strength of soil.

$\gamma D_f N_q \rightarrow$ Contribution due to surcharge above the footing

$\frac{1}{2}\gamma BN_\gamma \rightarrow$ Contribution due to bearing capacity due to self weight of soil.

Bearing capacity factors

$N_q = N_\phi \cdot e^{\pi \tan\phi}$ where N_ϕ = Influence factor

$N_\phi = \tan^2\left(45° + \frac{\phi}{2}\right)$ $N_\gamma = 1.8\tan\phi(N_q - 1)$

$N_C = \cot\phi(N_q - 1)$

For C-soil : $N_C = 5.7$, $N_q = 1$, $N_\gamma = 0$

Remember

- The surface of zone II is circular for C-soils whereas for C-ϕ soils surface is spiral (logrithm spiral).
- Terzhagi has considered general shear failure but if soil is loose and failure is local shear failure then modified values of C and ϕ should be used.

$$C' = \frac{2}{3}C, \qquad \phi' = \tan^{-1}\left(\frac{2}{3}\tan\phi\right)$$

(vi) Skemptons Method (C-soil)

This method gives net ultimate value of bearing capacity. Applicable for purely cohesive soils only.

$$q_{nu} = CN_c$$

For strip footing. $N_C = 5$ to 7.5

For circular and square footing. $N_c = 6$ to 9.0

Values of N_C

- If $\frac{D_f}{B} = 0$ i.e. of the surface.

  Then $N_C = 5$ For strip footing

  $N_C = 6.0$ For square and circular footing.

  where D_f = Depth of foundation.

- If $0 \le \frac{D_f}{B} \le 2.5$

  $N_C = 5\left[1 + 0.2\frac{D_f}{B}\right]$, for strip footing

  $N_C = 6\left[1 + 0.2\frac{D_f}{B}\right]$, for square and circular footing.

  $B = D$ in case of circular footing.

  $N_C = 5\left[1 + 0.2\frac{B}{L}\right]\left[1 + 0.2\frac{D_f}{B}\right]$ for rectangular footing.

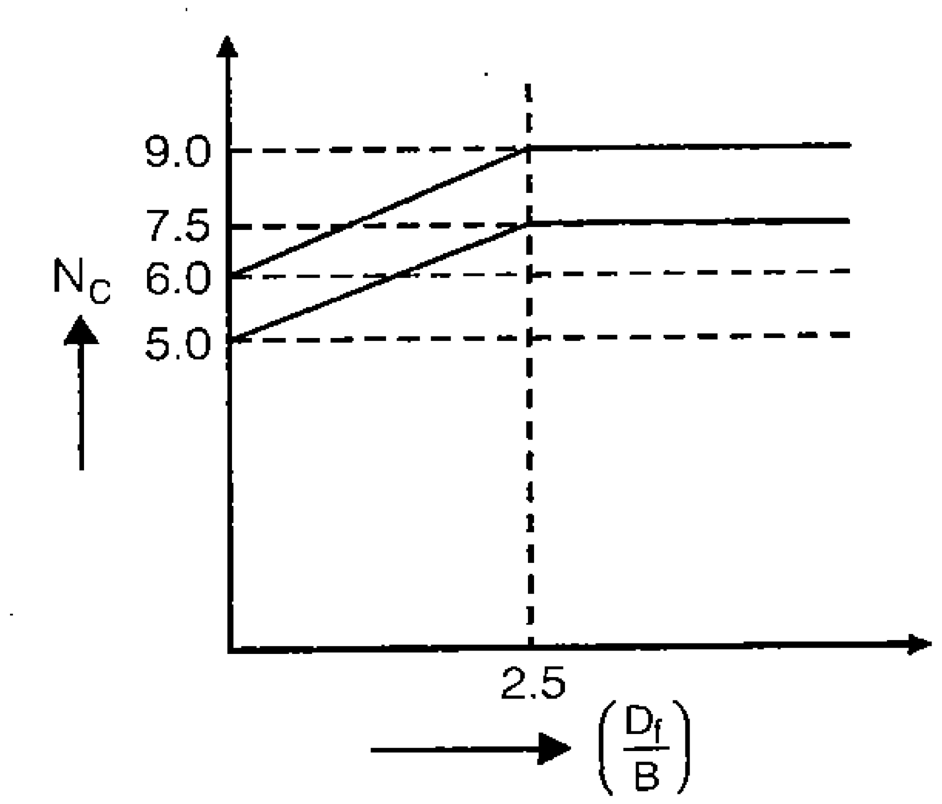
- If $\frac{D_f}{B} > 2.5$

  $N_C = 7.5$

  for strip footing

  $N_c = 9.0$

  for circular, square and rectangular footing.



(vii) Meyorhoff's Method→(C – ϕ soil)

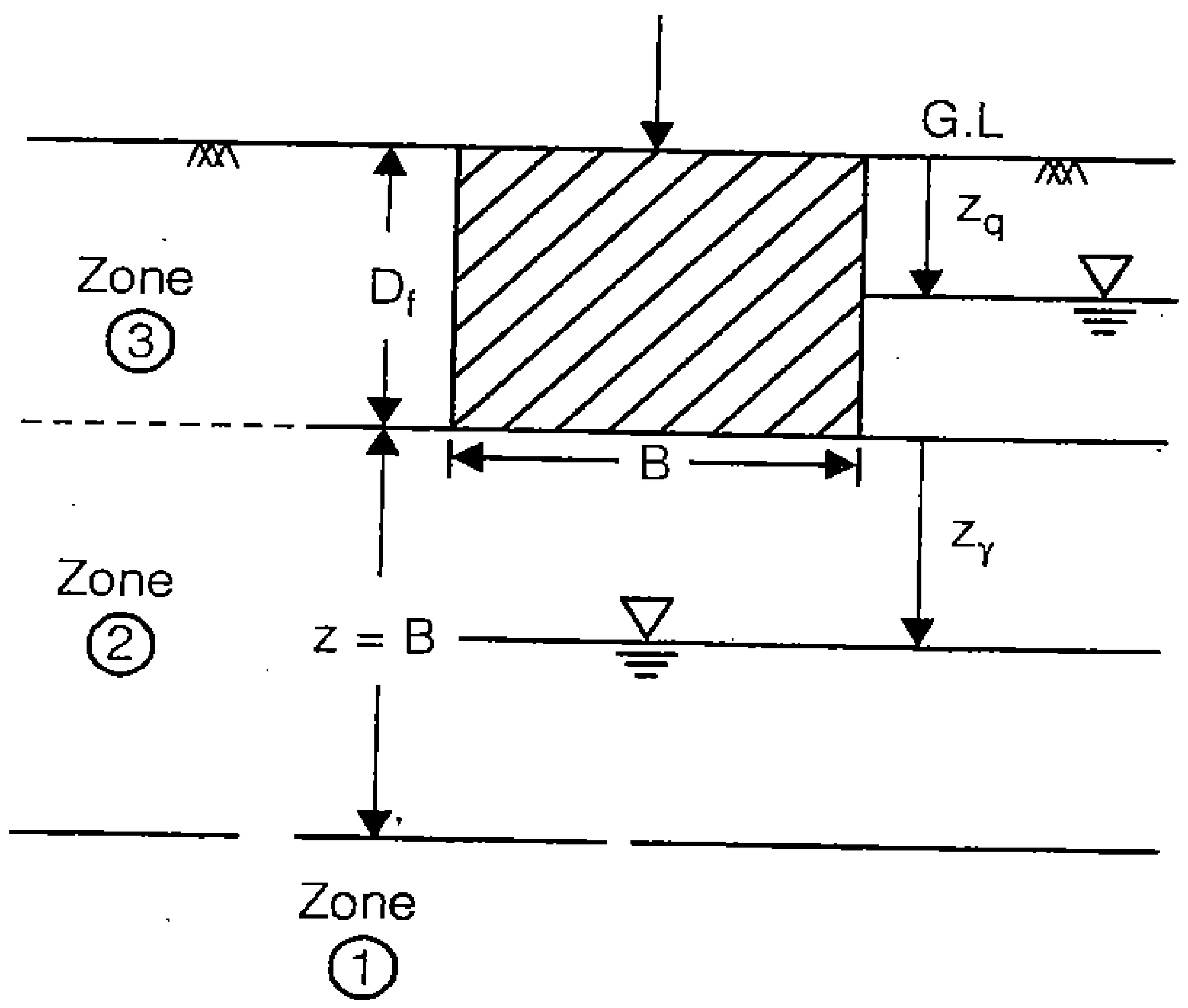
$$q_u = CN_c \cdot s_c \cdot d_c \cdot i_c + \gamma D_f N_q \cdot s_q \cdot d_q \cdot i_q + \frac{1}{2}\gamma BN_\gamma s_\gamma \cdot d_\gamma \cdot i_\gamma$$

where s, d and i are shape, depth and inclination correction factor.

(viii) IS code : $q_{nu} = CN_c\, s_c\, i_c\, d_c + \gamma D_f(N_q - 1)\, s_q\, i_a\, d_c + \frac{1}{2}B\gamma N_\gamma s_\gamma d_\gamma$

Effect of Water Table

$$q_u = CN_c + \gamma D_f N_q R_q^* + \frac{1}{2}\gamma B N_\gamma R_\gamma^*$$



where R_q* and R_γ* are water table correction factor.

$$R_q^* = \frac{1}{2}\left[1+\frac{z_q}{D_f}\right] \qquad R_\gamma^* = \frac{1}{2}\left[1+\frac{z_\gamma}{B}\right]$$

when $0 \le z_q \le D_f$ when $0 \le z_\gamma \le B$.

If $z_\gamma > B$ they $R_\gamma^* = 1$

If $z_\gamma \le 0$ they $R_\gamma^* = \frac{1}{2}$

If water table rise to G.L

$$R_q^* = \frac{1}{2} \text{ and } R_\gamma^* = \frac{1}{2}$$

Plate Load Test

1. Significant only for cohesionless.
2. Short duration test hence only result in immediate settlement.

(i) $\frac{q_{uf}}{q_{up}} = \frac{B_f}{B_p}$ ...for ϕ-soil

(ii) $q_{uf} = q_{up}$ ..for C-soil

If plate load test carried at foundation level then

$$\frac{S_f}{S_p} = \left[\frac{B_f(B_p+0.3)}{B_p(B_f+0.3)}\right]^2$$

When foundation is located at deeper depth say by amount D_2

$$S_{f\ corrected} = S_f \times \left[\frac{1}{\left(1+\frac{D_2}{B_f}\right)}\right]^{0.5}$$

(iii) $\frac{S_f}{S_p} = \left[\frac{B_f(B_p+0.3)}{B_p(B_f+0.3)}\right]^2$ ...for dense sand.

(iv) $\frac{S_f}{S_p} = \frac{B_f}{B_p}$ ... for clays

(v) $\frac{S_f}{S_p} = \left(\frac{B_f}{B_p}\right)^{n+1}$ ... for silts.

where,
q_{uf} = Ultimate bearing capacity of foundation
q_{up} = Ultimate bearing capacity of plate
S_f = Settlement of foundations
S_p = Settlement of plate
B_f = Width of foundation in m
B_p = Width of plate in m

Housels Approach

$$Q_P = mA_p + nP_p$$

$$Q_f = mA_f + nP_f$$

where, Q_p = Allowable load on plate m and n are constant
P = Perimeter
A_p = Area of plate
A_f = Area of foundation

Standard Penetration Test

Significant for Granular Soil

(i) $N_1 = N_0 \frac{350}{(\bar{\sigma}+70)}$ and $\bar{\sigma} \not> 280$

where, N_1 = Overburden pressure correction
N_0 = Observed value of S.P.T. number.
$\bar{\sigma}$ = Effective overburden pressure at the level of test in kN/m^2.

(ii) For Saturated $\bar{\sigma}$ fine sand and silt, when $N_1 > 15$

$$N_2 = \frac{1}{2}(N_1 - 15) + 15$$

where, N_2 = Dilatancy correction or water table correction.

- $N_q + N_\gamma$ related to N value using peck henson curve or (code method)
- Teng's formula relate N value with reading capacity of granular soil.

Pecks Equation

$$q_{a\,net} = 0.44\,NSC_w kN/m^2$$

$$C_w = 0.5\left(1 + \frac{D_w}{D_f + B}\right)$$

where,

D_w = depth of water table below G.L

D_f = Depth of foundation

B = Width of foundation

N = Avg. corrected S.P.T. no.

S = Permissible settlement of foundation

C_w = Water table correction factor

$q_{a\,net}$ = Net allowable bearing pressure.

Teng's Equations

$$q_{ns} = 1.4(N-3)\left(\frac{B+0.3}{2B}\right)^2 SC_wC_D\ kN/m^2$$

$$C_W = 0.5\left(1 + \frac{D_W}{B}\right)$$

$$C_D = \left(1 + \frac{D_f}{B}\right) \le 2$$

where

C_w = Water table correction factor

D_w = Depth of water table below foundation level

B = Width of foundation

C_d = Depth correction factor

S = Permissible settlement in 'mm'.

I.S. Code Method

$$q_{ns} = 1.38(N-3)\left(\frac{B+0.3}{2B}\right)^2 SC_w$$

q_{ns} = Net safe bearing pressure in kN/m²

B = Width in meter.

S = Settlement in 'mm'.

I.S. Code Formula for Raft: $q_{ns} = 0.88NSC_w$

C_w : Same as of peck henson.

Meyer-hoffs Equation

$$q_{ns} = 0.49NSC_wC_d$$

where, q_{ns} = Net safe bearing capacity in kN/m².

B < 1.2 m

$$C_d = \left(1 + \frac{D_f}{B}\right) \le 2 \qquad C_w = \frac{1}{2}\left(1 + \frac{D_w}{B}\right)$$

$$q_{ns} = 0.32N\left(\frac{B+0.3}{B}\right)^2 .S.C_d.C_w$$

B ≥ 1.2 m (where q_{ns} is in kN/m²)

Cone Penetrations Test

(i) $$C = 1.5\left[\frac{q_c}{\bar{\sigma}_0}\right]$$

where, q_c = Static cone resistance in kg/cm²

c = Compressibility coefficient

$\bar{\sigma}_0$ = Initial effective over burden pressure in kg/cm².

(ii) $$S = 2.3\frac{H_0}{C}\log_{10}\left[\frac{\bar{\sigma}_0 + \Delta\sigma}{\bar{\sigma}_0}\right]$$

where, 'S' = Settlement.

(iii) $q_{ns} = 3.6q_sR_w$ when B < 1.2 m.

where, q_{ns} = Net safe bearing pressure in kN/m².

(iv) $q_{ns} = 2.7q_c.R_w$ when B > 1.2 m

where, R_w = Water table correction factor.

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Deep Foundation

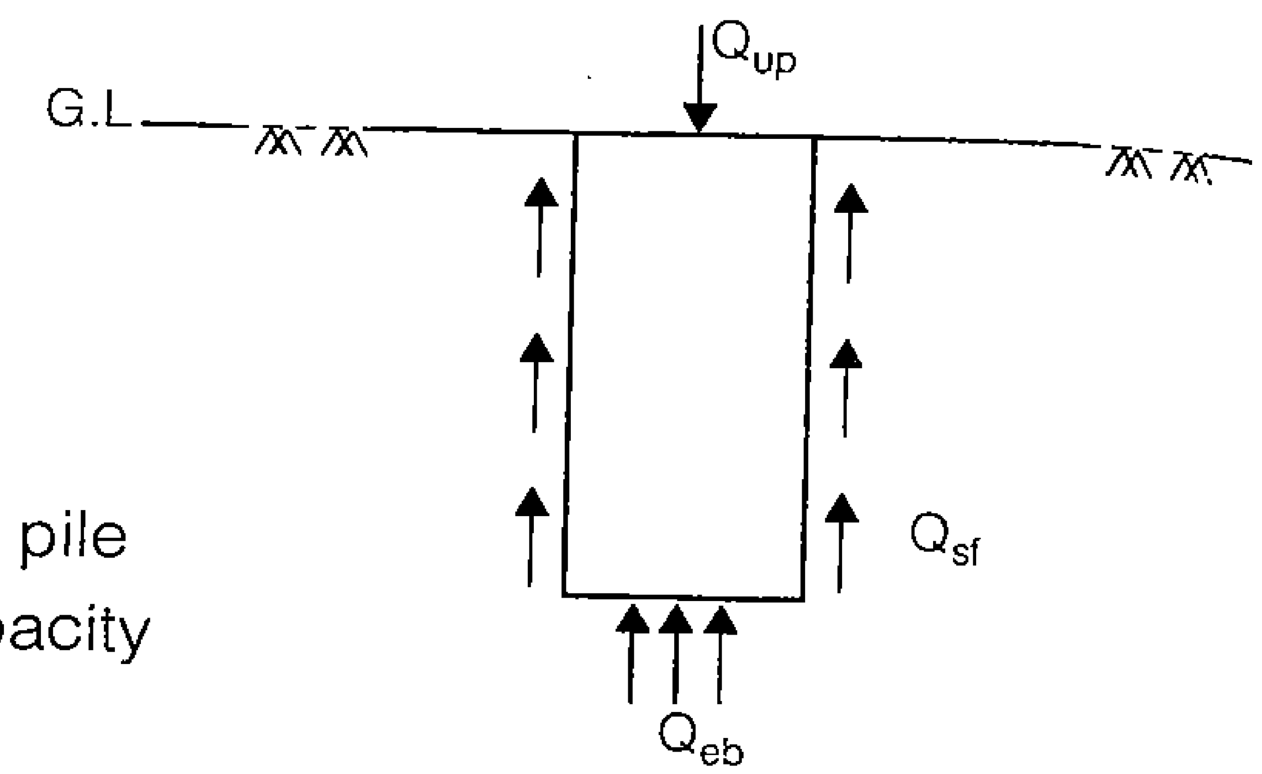
14

Bearing capacity of piles

The ultimate bearing capacity of a pile is the maximum load which it can carry without failure or excessive settlement of the ground. The bearing capacity also depends on the methods of installation

A. Analytical Method

(i) $Q_{up} = Q_{eb} + Q_{sf}$



(ii) $Q_{up} = q_b A_b + q_s A_s$

where,

Q_{up} = Ultimate load on pile
Q_{eb} = End bearing capacity
Q_{sf} = Skin friction
q_b = End bearing resistance of unit area.
q_s = Skin friction resistance of unit area
A_b = Bearing area
A_s = Surface area

(iii) $q_b \sim 9C$ where, C = Unit Cohesion at base of pile for clays

(iv) $q_s = \alpha\bar{C}$ where, α = Adhesion factor
$\alpha\bar{c} = C_a$ = Unit adhesion between pile and soil.
$\bar{c}$ = Average Cohesion over depth of pile.

(v) $Q_{safe} = \dfrac{Q_{up}}{F_s}$ where, F_s = Factor of safety.

(vi) $Q_{safe} = \dfrac{Q_{eb}}{F_1} + \dfrac{Q_{sf}}{F_2}$ $F_1 = 3$ and $F_2 = 2$
$\simeq F_1 = F_2 = 2.5$

(vii) For Pure Clays $Q_{up} = 9C.A_b + \alpha\bar{c}A_s$

B. Dynamic Approach

Dynamic methods are suitable for dense cohesionless soil only.

(i) Engineering News Record Formula

(a) $Q_{up} = \dfrac{WH}{S+C}$

(b) $Q_{ap} = \dfrac{Q_{up}}{6} = \dfrac{WH}{6(S+C)}$

where,

Q_{up} = Ultimate load on pile
Q_{ap} = Allowable load on pile
W = Weight of hammer in kg.
H = Height of fall of hammer in cm.
S = Final set (Average penetration of pile per blow of hammer for last five blows in cm)
C = Constant
= 2.5 cm → for drop hammer
= 0.25 cm → for steam hammer (single acting or double acting)

(c) For drop hammer $Q_{ap} = \dfrac{WH}{6(S+2.5)}$

(d) For Single Acting Steam Hammer $Q_{ap} = \dfrac{WH}{6(S+0.25)}$

(e) For Double Acting Steam Hammer $Q_{ap} = \dfrac{(W+ap)H}{6(S+0.25)}$

where P = Steam pressure
and a = Area of hammer on which pressure acts.

(ii) Hiley Formula (I.S. Formula)

$Q_{up} = \dfrac{\eta_h.\eta_b.WH}{S+\dfrac{C}{2}}$ $Q_{ap} = \dfrac{Q_{up}}{F_s}$ where, F_s = Factor of safety = 3
η_h = Efficiency of hammer
η_b = Efficiency of blow.

$\eta_h = 0.75$ to 0.85 for single acting steam hammer

$\eta_h = 0.75$ to 0.80 for double acting steam hammer

$\eta_h = 1$ for drop hammer.

$$\eta_b = \frac{\text{Energy of hammer after impact}}{\text{Energy of hammer just before Impact}}$$

$$\eta_b = \frac{W + e^2P}{W + P} \quad \text{when } w > e \cdot p$$

$$\eta_b = \left(\frac{W + e^2P}{W + P}\right) - \left(\frac{w - ep}{w + p}\right)^2 \quad \text{... when } w < e \cdot p$$

where, w = Weight of hammer in kg.

p = Weight of pile + pile cap

e = Coefficient of restitutions

= 0.25 for wooden pile and cast Iron hammer

= 0.4 for concrete pile and cast Iron hammer

= 0.55 for steel piles and cast Iron hammer

S = Final set or penetrations per blow

C = Total elastic compression of pile, pile cap and soil

H = Height of fall of hammer.

C. Field Method

(i) Use of Standard Penetrations Data

$$Q_{up} \doteq 400N A_b + 2\bar{N} A_s$$

where, N = Corrected S.P.T. Number

$\bar{N}$ = Average corrected S.P.T. number for entire pile length

$$Q_{ap} = \frac{Q_{up}}{F_s}$$

F_s = Factor of safety

= 4 → For driven pile

= 2.5 → for bored pile.

$q_b = 400N$ and $q_s = 2\bar{N}$

- For non Displacement Pile (H-Piles)

$q_b = 200N$ $q_s = \bar{N}$

(ii) Cone penetration test $Q_{up} = q_c A_b + \frac{\bar{q}_c}{2} A_s$

Where, q_c = static cone resistance of the base of pile in kg/cm²

$\bar{q}_c$ = average cone resistance over depth of pile in kg/cm²

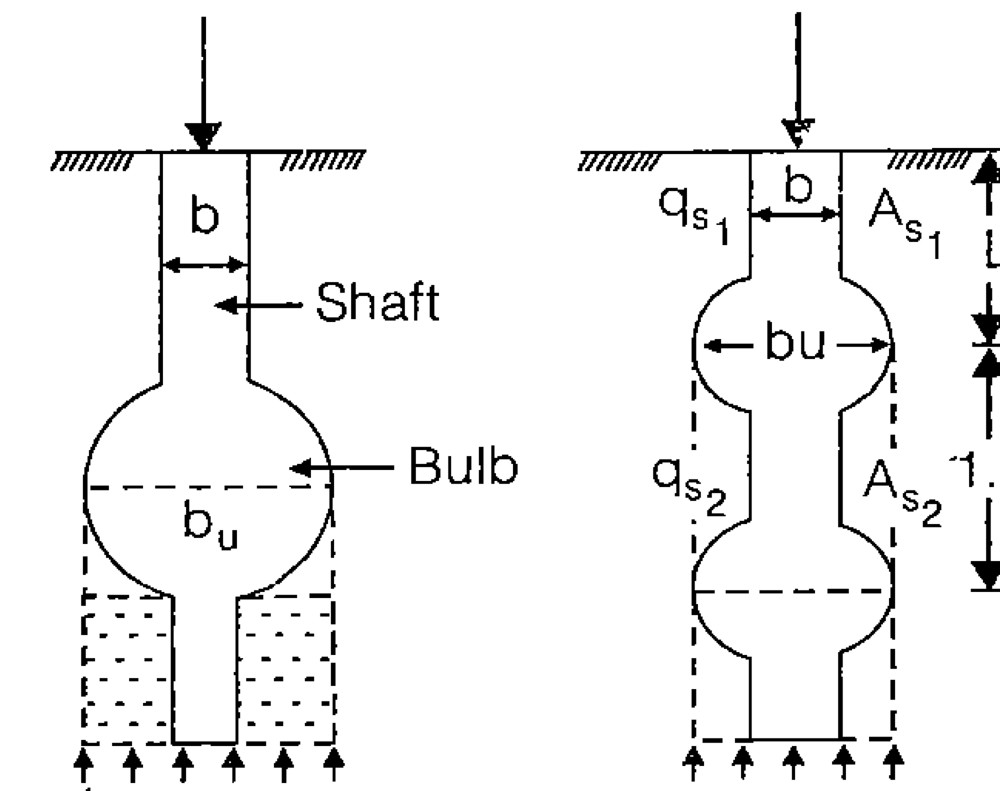
$$A_b = \frac{\pi}{4}(b_u)^2 = \text{Area of bulb (m}^2)$$

NOTE: q_c and $\bar{q}_c$ are in kg/cm², A_b and A_s are in m² and Q_{up} is in kN.

Under-Reamed Pile

An 'under-reamed' pile is one with an enlarged base or a bulb; the b[...] is called 'under-ream'.

Under-reamed piles are cast-in-situ piles, which may be installed both in sandy and in clayey soils. The ratio of bulb size to the pile shaft size may be 2 to 3; usually a value of 2.5 is used.



$\frac{bu}{b}$ = 2 to 3

Single bulb under reamed pile

Multi bulb under reamed pile

$A_{s_1} = \pi b L_1$

$q_{s_1} = \alpha C$ $\alpha < 1$.

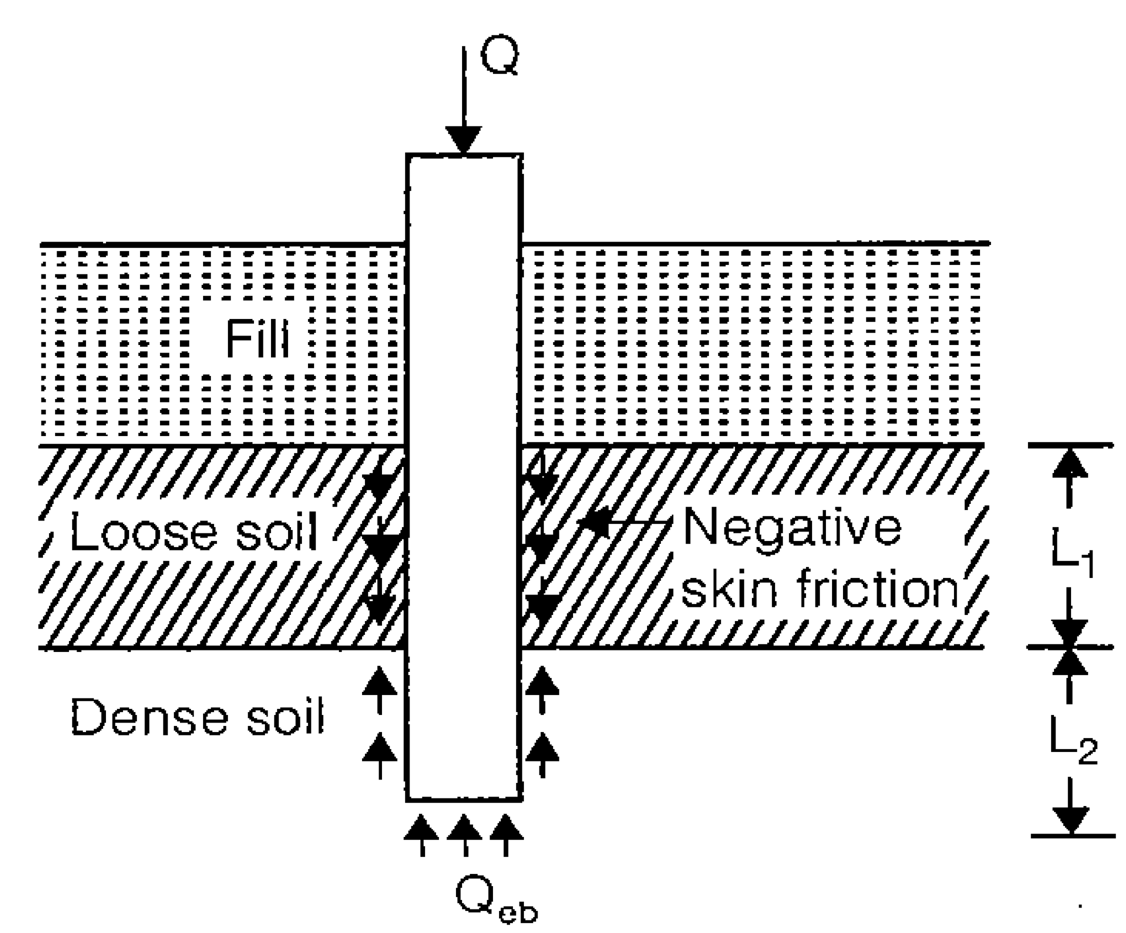
$A_{s_2} = \pi b_u L_2$

$q_{s_2} = \alpha C$ $\alpha = 1$ Where, b_u = dia of bulb, Spacing = 1.5[...]

$$Q_{up} = q_b A_b + q_{s_1} A_{s_1} + q_{s_2} A_{s_2}$$

Remember: The ratio of bearing resistance for double underreamed pile [...] that to single underreamed pile is 1.5 for sandy and clayey soi[...] including black cotton soils.

Negative Skin Friction



(i) **For Cohesive soil**

$\boxed{Q_{nf} = \text{Perimeter} \cdot L_1 \alpha C}$ for Cohesive soil.

where. Q_{nf} = Total negative skin frictions

$$F_s = \frac{Q_{up} - Q_{nf}}{\text{Applied load}}$$ where, F_s = Factor of safety.

(ii) **For cohesion less soils**

$$Q_{nf} = P \times \text{force per unit surface length of pile} = P \times \frac{1}{2} K \gamma D_n^2 \cdot \tan\delta$$

$$Q_{nf} = \frac{1}{2} P D_n^2 K \tan\delta \cdot \gamma \qquad \text{(friction force} = \mu H)$$

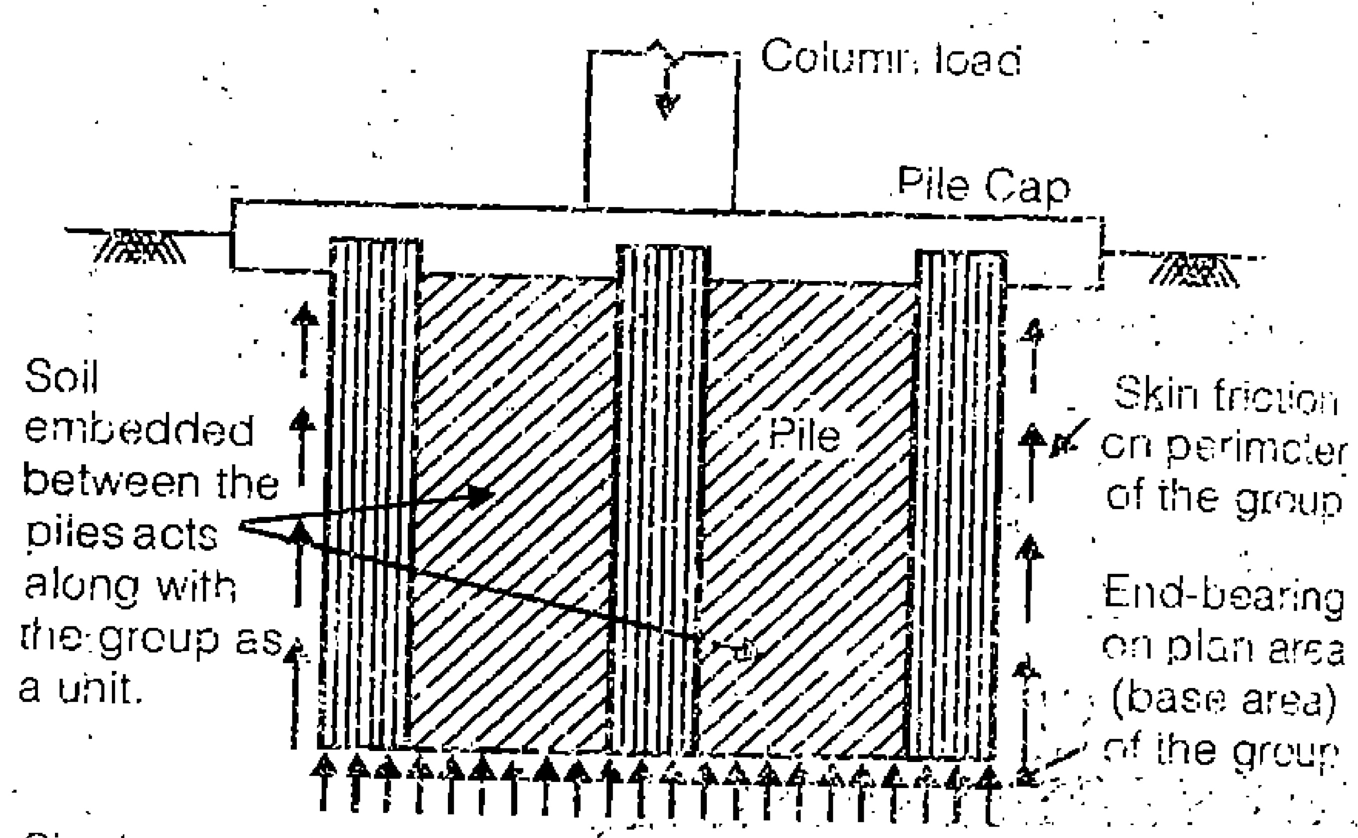
where γ = unit weight of soil.

K = Earth pressure coefficient ($K_a < K < K_p$)

δ = Angle of wall friction. ($\phi/2 < \delta < \phi$)

Group Action of Pile

The ultimate load carrying capacity of the pile group is finally chosen as the smaller of the (i) Ultimate load carrying capacity of n pile (n Q_{up}) and (ii) Ultimate load carrying capacity of the single large equivalent (block) pile (Q_{ug}). To determine design load or allowable load, apply a suitable factor of safety.



Single equivalent large pile concept for a group (block failure)

(i) **Group Efficiency (η_g)**

$$\eta_g = \frac{Q_{ug}}{n \cdot Q_{up}}$$

Q_{ug} = Ultimate load capacity of pile group

Q_{up} = Ultimate load on single pile

For sandy soil → $\boxed{\eta_g > 1}$

For clay soil → $\boxed{\eta_g < 1}$ and $\boxed{\eta_g > 1}$

Minimum number of pile for group action = 3.

$\boxed{Q_{ug} = q_b A_b + q_s A_s}$ where $\boxed{q_b = 9C}$ for clays

$\boxed{A_b = B^2}$ $\boxed{q_s = \bar{C}}$ $\boxed{A_s = 4B.L}$

- For Square Group
  Size of group,

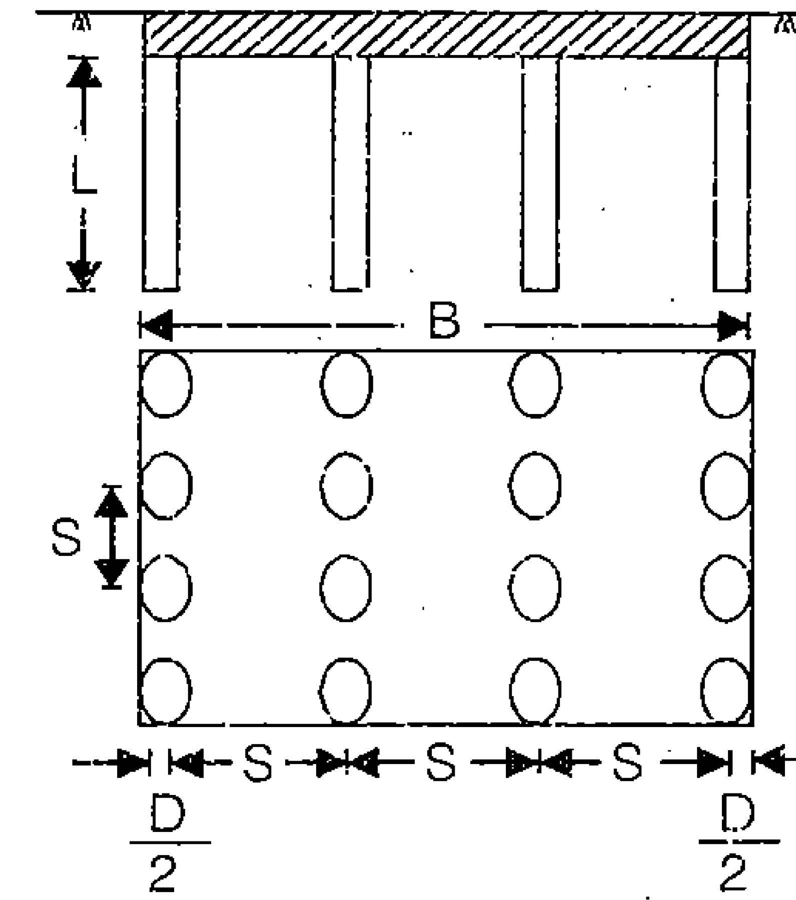
  $\boxed{B = (n-1)S + D}$

  where, η = Total number of pile

  If size of group is x.x

  They $\boxed{\eta = x^2}$

- $\boxed{Q_{ug} = \eta . Q_{up}}$



- $$Q_{ag} = \frac{Q_{ug}}{FOS}$$ where, Q_{ag} = Allowable load on pile group.

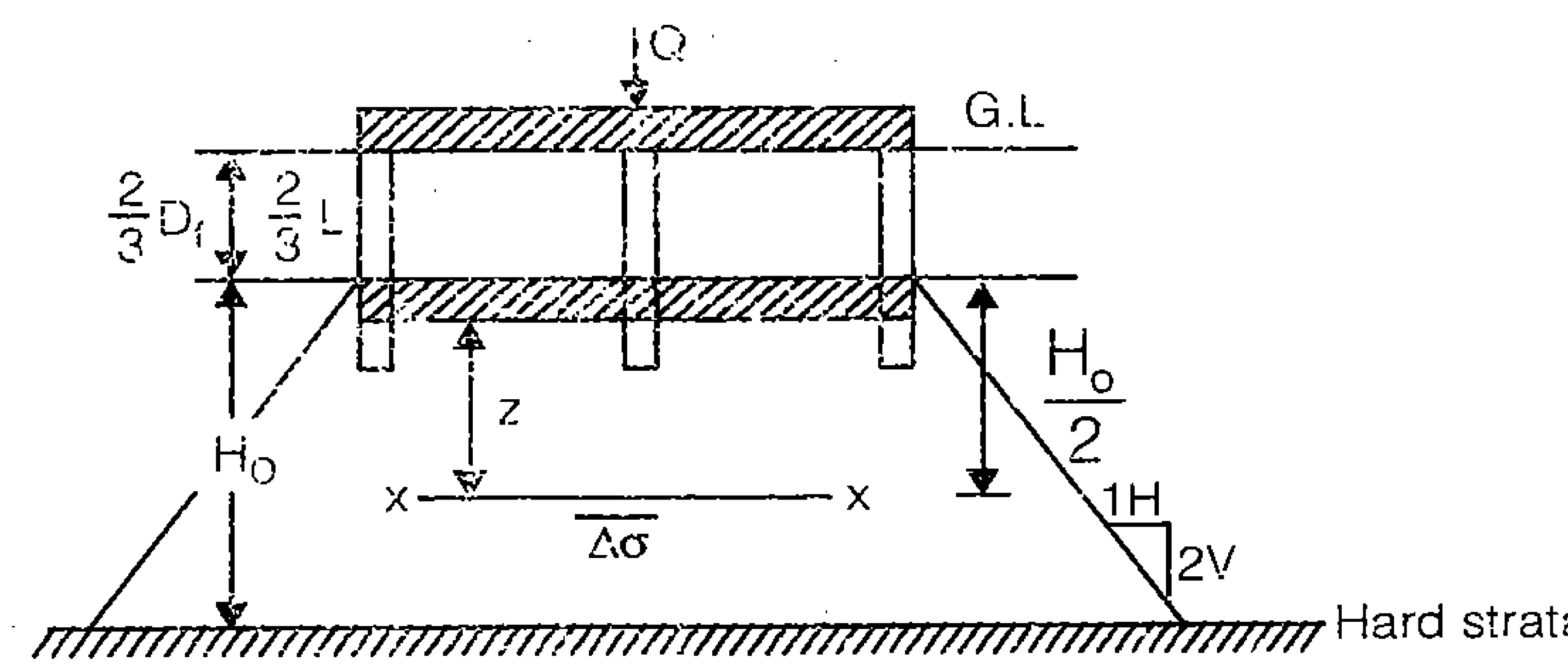
- $$S_r = \frac{S_g}{S_i}$$ where, S_r = Group settlement ratio

  S_g = Settlement of pile group

  S_i = Settlement of individual pile.

(ii) When Piles are Embended on a Uniform Clay

$$S_g = \Delta H = \frac{C_c H_0}{1 + e_0} \log_{10}\left(\frac{\overline{\sigma_0} + \overline{\Delta\sigma}}{\overline{\sigma_0}}\right) \quad \text{and} \quad \overline{\sigma_0} = \frac{Q}{(B+z)^2}$$



(iii) In Case of Sand

$$S_r = \frac{S_g}{S_i} = \left(\frac{4B + 2.7}{B + 3.6}\right)^2$$ where, B = Size of pile group in meter.

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15 Soil Stabilization

Stabilization of soils

Stabilization is the process by which the strength and stability of a soil mass is improved and increased

Test on Expansive Soil

(i) Free Swell Test

$$\%\text{Free soil} = \frac{\text{Final volume} - \text{Initial volume}}{\text{Initial volume}} \times 100$$

(ii) Differential Free Soil test

$$\%\text{DFS} = \frac{\text{Soil volume in water} - \text{Soil volume in kerosine}}{\text{Soil volume in kerosine}} \times 100$$

Degree of Expansiveness	% DFS
Low	< 20%
Medium	20 to 35%
High	35 to 50%
Very High	> 50%

Plasticity Index	Swelling Potential
0 to 15	low
10 to 35	medium
20 to 40	high
> 35	very high

NOTE: I_P, shrinkage limit, colloidal content all effect expansiveness of soil.

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16 Soil Exploration

Soil Sample and Samplers

The soil samples can be of two types: disturbed and undisturbed. A disturbed sample is that in which the natural structure of soils get partly or fully modified and destroyed, although with suitable precautions the natural water content may be preserved. Such a sample should, however, be representative of the natural soil by maintaining the original proportional of the various soil particles intact. An undisturbed sample is that in which the natural structure and properties remain preserved.

Remember: For the purpose of consistency limit, specific gravity, grain size distribution either undisturbed samples or representative samples should be used, whereas for coefficient of permeability, consolidation parameters and shear strength parameters undisturbed samples should be used.

(i) Inside Clearance

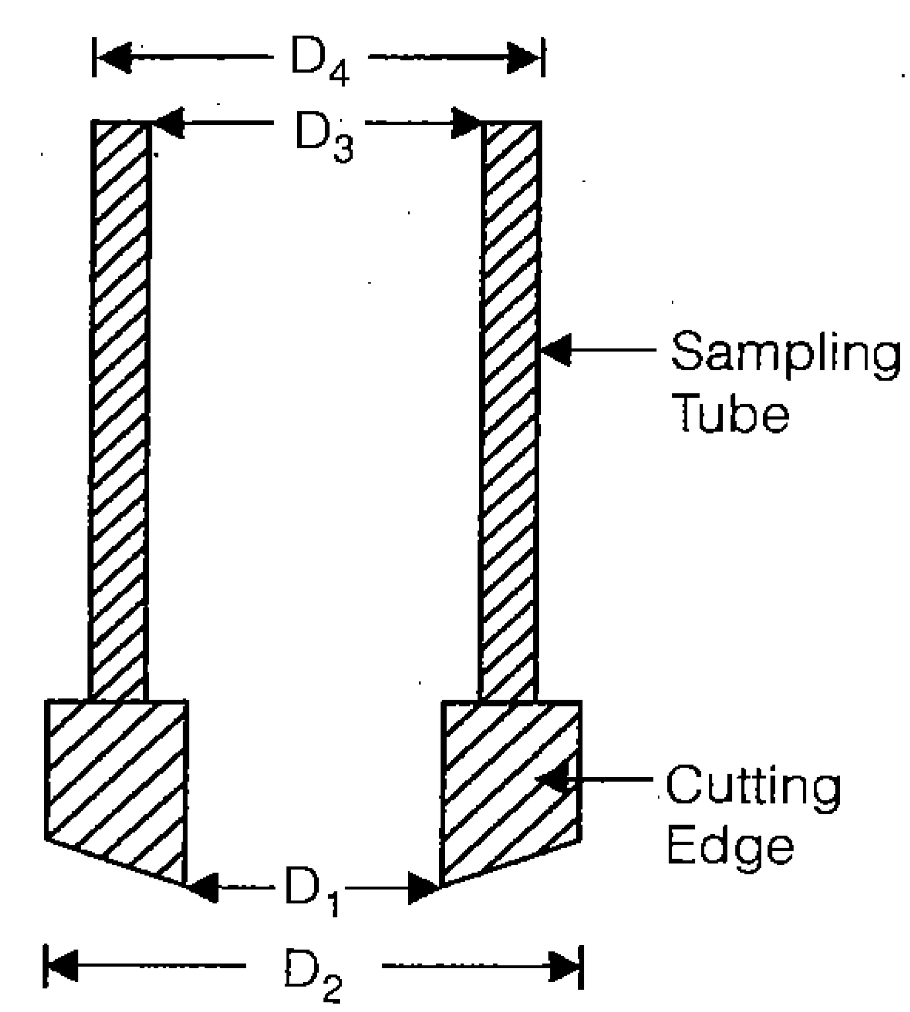
$$C_i = \frac{D_3 - D_1}{D_1} \times 100 \quad 1 < C_i < 3$$

(ii) Outside Clearance

$$C_0 = \frac{D_2 - D_4}{D_4} \times 100 \quad 0 < C_0 < 2$$

(iii) Area Ratio (A_r)

$$A_r = \frac{D_2^2 - D_1^2}{D_1^2} \times 100$$



A_r < 20% for stiff fomation, A_r < 10% for soft clays

(iv) Recovery Ratio (L_r)

$$L_r = \frac{\text{Recovered length of sample}}{\text{Penetrations length of sample}}$$

$L_r = 1 \rightarrow$ Good recovery

$L_T < 1 \rightarrow$ Compressed

$L_T > 1 \rightarrow$ Swelled

where, D_3 = Inner dia of sampling tube

D_4 = Outer dia of sampling tube

D_1 = Inner dia of cutting edge

D_2 = Outer dia of cutting edge

Seismic Method

$$D = \frac{d}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}}$$

where, d is distance between two stations

D is depth of strata

V_1 is velocity of direct wave

V_2 is velocity of refracted wave

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A Handbook on Civil Engineering

8

Fluid Mechanics & Fluid Machines

CONTENTS

Introduction

1

Ideal Fluid and Real Fluid

- **Ideal fluid:** A fluid is said to be ideal if it is assumed to be both incompressible and non-viscous. Its bulk modulus is infinite.
- **Real fluid:** Real fluid have viscosity, finite compressibility and surface tension.

> Remember
> - Ideal fluid has no surface tension.
> - Ideal fluid are imaginary and do not exist in nature.

Specific Weight, Specific Volume, Specific Gravity

- $$\text{Specific weight } (\omega) \text{ or weight density} = \frac{\text{Weight}}{\text{Volume}} = \frac{mg}{V} = \rho g$$

  Here, ρ = Density
  g = Acceleration due to gravity

  Specific weight of water = 9810 N/m^3

- $$\text{Specific Volume} = \frac{1}{\text{Density}}$$

- Specific gravity (S) or Relative density

$$\text{Specific gravity} = \frac{\text{Density of fluid}}{\text{Density of standard fluid}} = \frac{\text{Specific weight of fluid}}{\text{Specific weight of standard fluid}}$$

> Remember
> - Specific gravity for water is 1.0 at 4°C and for mercury it is 13.6
> - Specific gravity varies with temperature therefore it should be determined at specified temperature (4°C or 27°C).

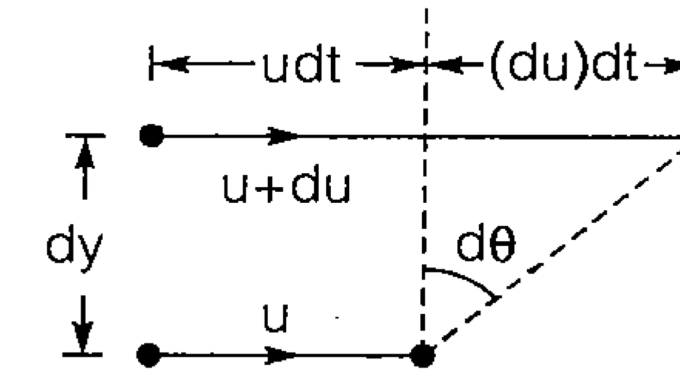
Newton's Law of Viscosity

$$\tau = \mu \cdot \frac{du}{dy} = \mu \frac{d\theta}{dt}$$

τ = shear stress
μ = coefficient of viscosity or absolute viscosity or dynamic viscosity

Here, $\frac{du}{dy}$ = Velocity gradient

$\frac{d\theta}{dt}$ = Rate of angular deformation or Rate of shear strain



$$\Rightarrow \frac{du\,dt}{dy} = d\theta$$

- For newtonian fluid, coefficient of viscosity remains constant.

Viscosity/Kinematic Viscosity

Due to viscosity a fluid offer resistance to flow

(i) **Dynamic Viscosity (μ)**
- Its SI unit is pascal-second or **N-sec/m^2**
- Its CGS unit is Poise = Dyne-sec/cm^2
- 1 poise = **0.1** N-s/m^2

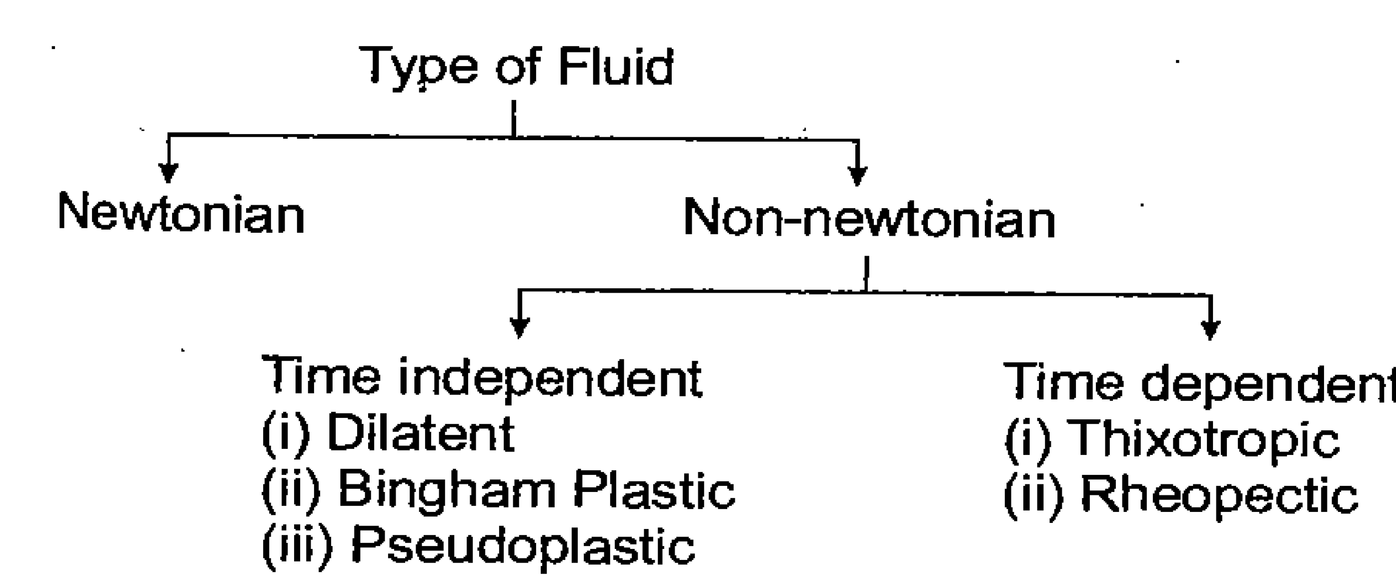
(ii) **Kinematic Viscosity**

$$\nu = \frac{\text{Dynamic viscosity } (\mu)}{\text{Mass density } (\rho)}$$

- Its SI unit is m^2/s.
- Its CGS unit is cm^2/s or stoke.
- 1 stoke = 10^{-4} m^2/s

> Remember
> - Viscosity of liquids is due to **cohesion** and for gases it is due to ***molecular momentum transfer.***
> - Viscosity of ***liquids*** decreases with temperature whereas viscosity of ***gases*** increases with increase in temperature.
> - Liquids with increasing order of viscosity are gasoline, water, crude oil, castor oil.
> - Viscosity of ***water*** at 1°C is 1 centipoise.

Type of Fluid

- 

- **Non-Newtonian Fluids:** These do not follow Newton's law of viscosity. The relation between shear stress and velocity gradient is

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

where A and B are constants depending upon type of fluid and condition of flow.

(i) For Dilatant Fluids: $n > 1$ & $B = 0$,
Ex. Butter, Quick sand.

(ii) For Bingham Plastic Fluids: $n = 1$ & $B \neq 0$
Ex. Sewage sludge, Drilling mud, tooth paste and gel.
These fluids always have certain minimum shear stress before they yield.

(iii) For Pseudoplastic Fluids: $n < 1$ & $B = 0$
Ex. Paper pulp, Rubber solution, Lipsticks, Paints, Blood, Polymetric solutions etc.

(iv) For Thixotropic Fluids: $n < 1$ & $B \neq 0$
Viscosity increases with time.
Ex. Printers ink and Enamels.

(v) For Rheopectic Fluids: $n > 1$ & $B \neq 0$
Viscosity decreases with time.
Ex. Gypsum solution in water & Bentonite solution.

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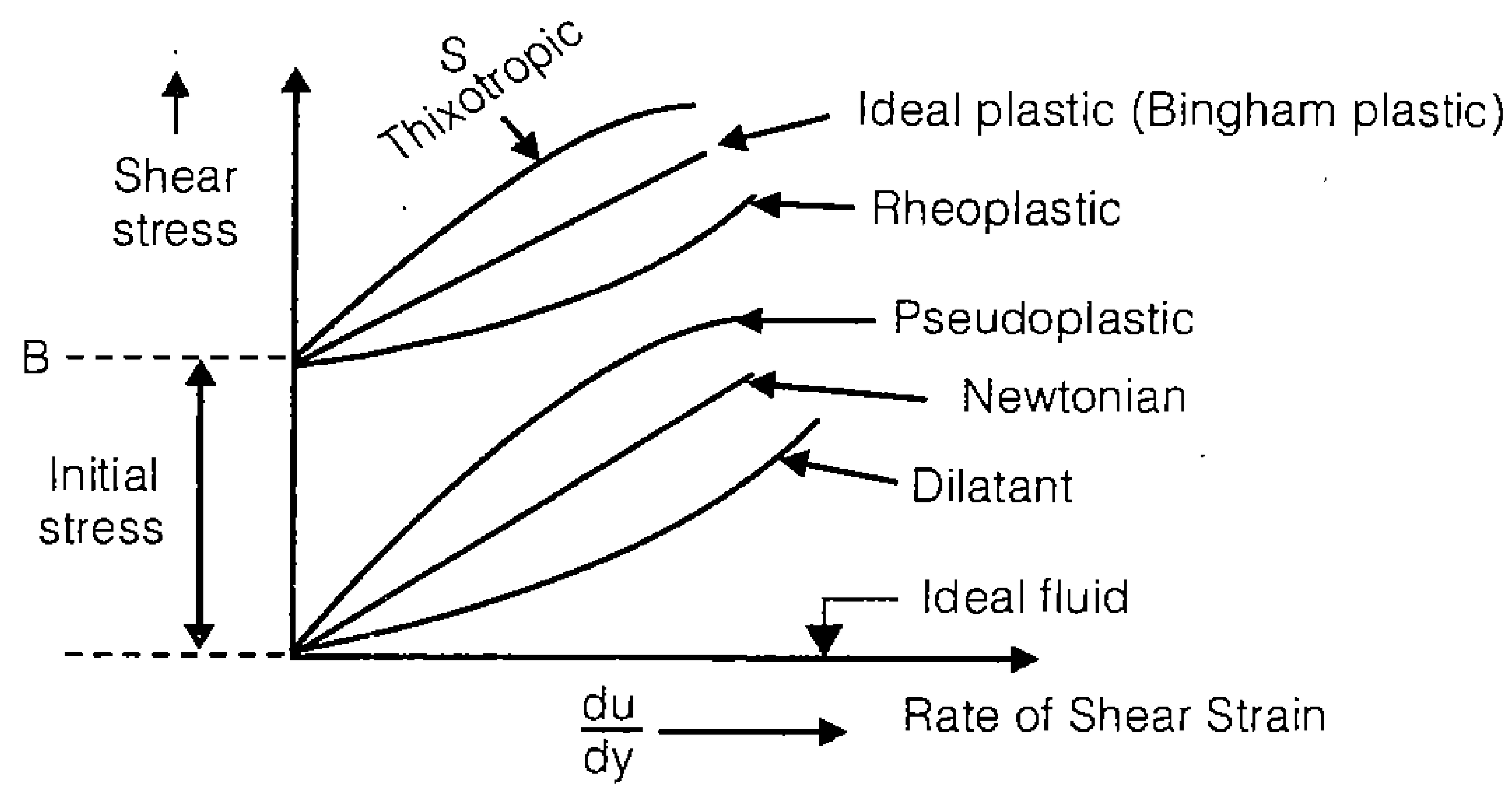

Compressibility (β), Isothermal Bulk Modulus (k_T) and Adiabatic Bulk Modulus

- **Compressibility (β):** It is inverse of bulk modulus of elasticity.

$$\beta = \frac{1}{k} = \frac{-dv}{vd\rho} \qquad \beta = \rho\cdot\left(\frac{d\rho}{dP}\right)$$

Here, k = bulk modulus of elasticity
ρ = Density
v = Volume

[∵ mass = constant]

$\rho v = c$

$$\rho dv + v d\rho = 0 \Rightarrow -\frac{dv}{v} = \frac{d\rho}{\rho}$$

- **Isothermal bulk modulus (k_T)** $k_T = P_{final} = \rho RT$
- **Adiabatic bulk modulus** $k_a = \gamma \cdot P_{final}$

$$\gamma = \frac{C_p}{C_v}$$

Here, C_p = Specific heat at constant pressure
C_v = Specific heat at constant volume

Surface Tension/Pressure Inside Drop, Bubble and Jet

Surface tension occur at the interface of liquid and a gas ***or*** at the interface of two liquid. Surface tension is inversely proportional to temperature and it also acts when fluid is at rest.

- **Pressure inside drop (Solid like sphere)** $P = \frac{4\sigma}{d}$
- **Pressure inside bubble** $P = \frac{8\sigma}{d}$
- **Pressure inside jet** $P = \frac{2\sigma}{d}$

Here, d = Diameter of drop
P = Gauge pressure

Remember
- It is a ***surface*** phenomenon
- It is force per unit length (N/m)
- For ***water-air*** interface at 20°C its value is 0.0736 N/m
- At critical point, liquid-vapour state are same thus surface tension = 0
- It is due to ***cohesion*** only

Capillary Action

- **Height of water in capillary tube**

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

Where, h = rise in capillary
σ = surface tension of water & glass
d = dia of tube
θ = angle of contact between the liquid and the material.
θ = 0° for water and glass
θ = 128° for mercury and glass

- **When a liquid surface supports another liquid of density "ρ_b", then rise in capillary is given as**

$$h = \frac{4\sigma\cos\theta}{(\rho - \rho_b)gd}$$

Remember
- Capillary action is due to adhesion and cohesion, ***both***.
- For capillary action diameter of tube should be ***less*** than 3 cm.

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Manometry

2

Pascal's Law

The intensity of pressure at any point in a stationary fluid is same in all directions.

$$\boxed{p_x = p_y = p_z}$$

Remember

- Pressure varies ***only with depth*** in stationary fluids, whereas if fluids is in motion pressure may vary in horizontal direction also.
- Fluid pressure is measured in Force/Area and it is expressed in Pascal (N/m^2) or Bar.

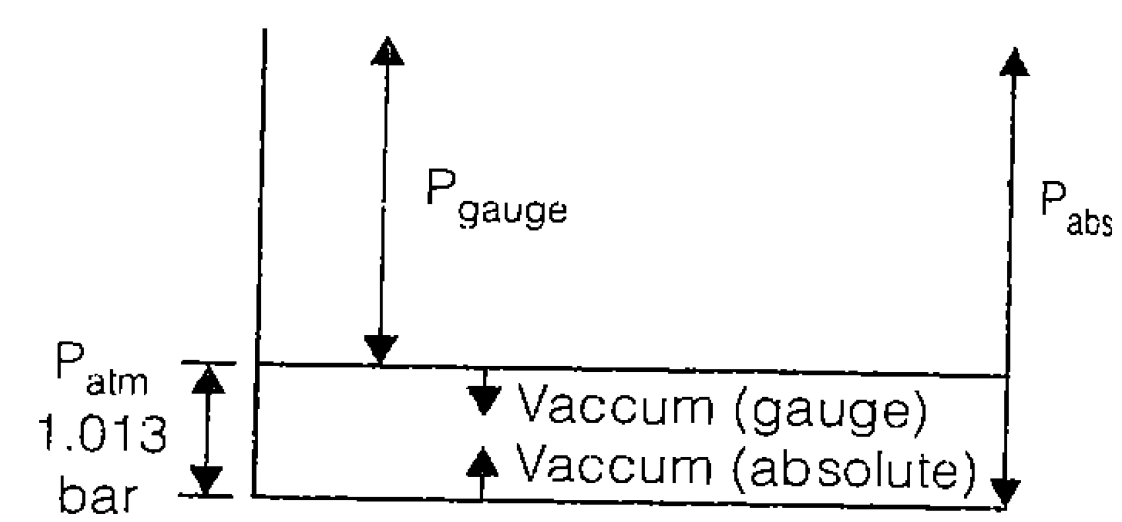
$$1 \text{ Bar} = 10^5 \text{ N/m}^2$$
$$1 \text{ MPa} = 10 \text{ Bar}$$

- Barometer shows ***atmospheric*** pressure.
- 1 kgf = 9.81 Newton.
- Pressure is a scalar quantity.

Absolute Pressure

Pressure measured with reference to absolute zero. Absolute pressure cannot be negative

Absolute pressure = gauge pressure + local atmospheric pressure



- $\boxed{P_{gauge} = \rho g h}$

Here, ρ = Density of fluid
g = Acceleration due to gravity
h = Height

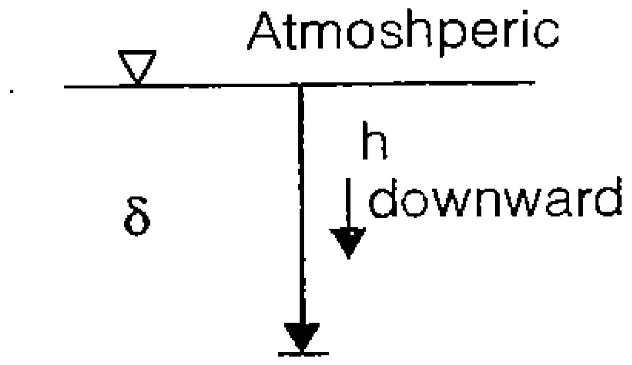
Remember

- ***Gauge*** pressure can be positive, negative or zero.
- Atmospheric pressure varies with ***altitude, temperature*** and ***local*** conditions.
- At ***mean sea level*** atmospheric pressure is 1.01×10^5 Pascal or 1 Bar or 10.3 mts. of height of water or 76 cm height of ***mercury***.

Hydrostatic Law

- For downward 'h'

$$\boxed{\frac{dP}{dh} = \omega}$$



- For upward 'h'

$$\boxed{\frac{dP}{dh} = -\omega}$$

Conversion of one Fluid Column to Another Fluid Column

$$\boxed{\rho_1 h_1 = \rho_2 h_2} \quad \boxed{s_1 h_1 = s_2 h_2}$$

Here, ρ = Density of fluid
s = Relative density

Remember

- Piezometer is suitable for ***small*** and ***positive*** pressure measurement.
- The manometric liquid should have ***high density*** and low ***vapour pressure***.

- Simple manometer/U-tube manometer can measure both ***positive*** and ***negative*** pressure.
- Aneroid/Mercury barometer used to measure ***local*** atmospheric pressure on ***absolute*** scale.
- Density of mercury = 13.6×10^3 kg/m^3
  Density of air = 1.24 kg/m^3

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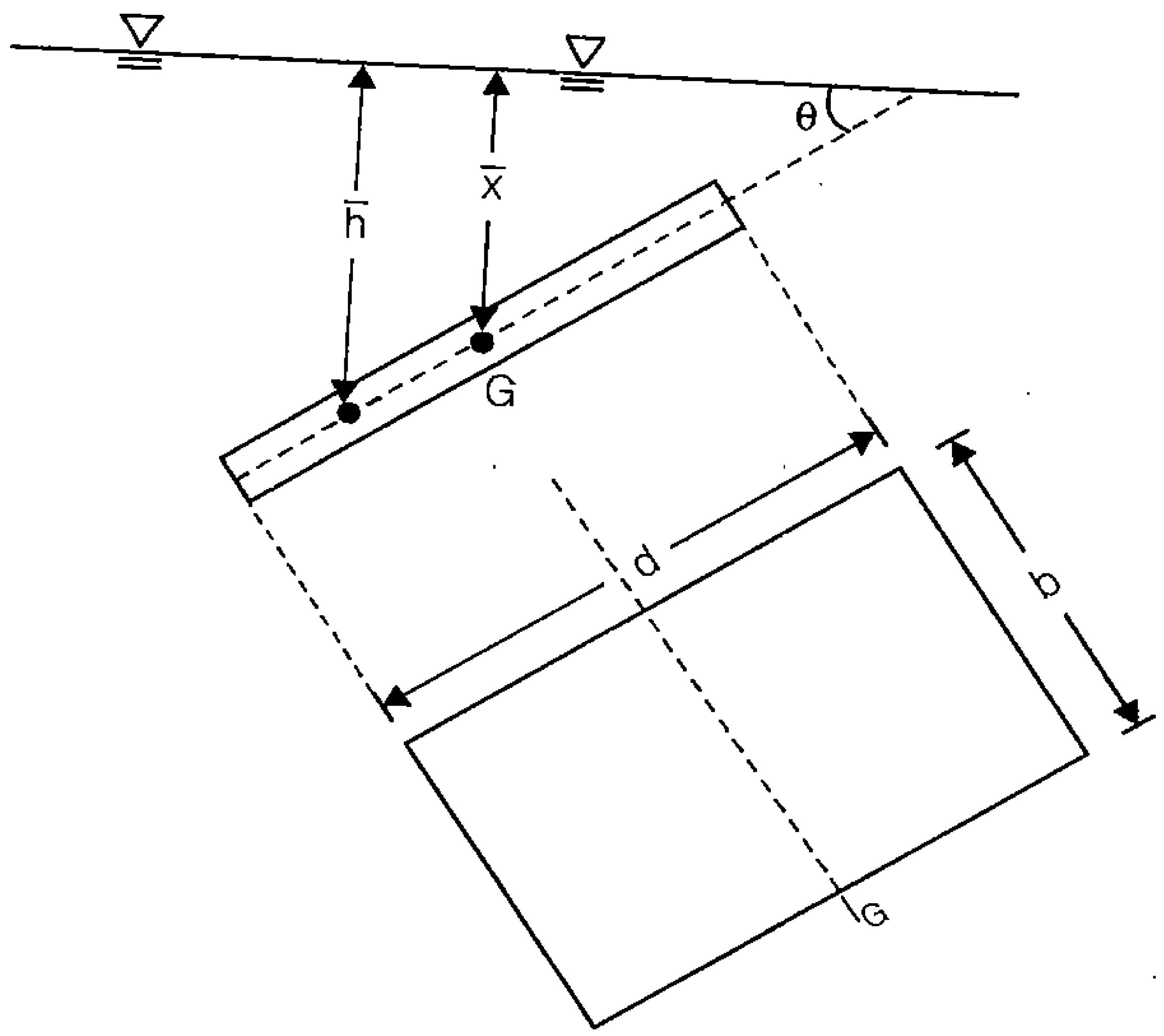
Hydrostatic Forces

3

Hydrostatic Forces on Submerged Surface

Case	Force	Center of pressure (h)
Horizontal Position	$wA\bar{x}$	$h = \bar{x}$
Vertical Position	$wA\bar{x}$	$h = \bar{x} + \frac{I_G}{A\bar{x}}$
Inclined Position	$wA\bar{x}$	$h = \bar{x} + \frac{I_G}{A\bar{x}} \sin^2\theta$



$$I_G = \frac{bd^3}{12}$$ (For rectangular plate)

$$I_G = \frac{\pi}{64}(\text{diameter})^4$$ (For circular plate)

Here, A = Area of surface touching fluid = b × d

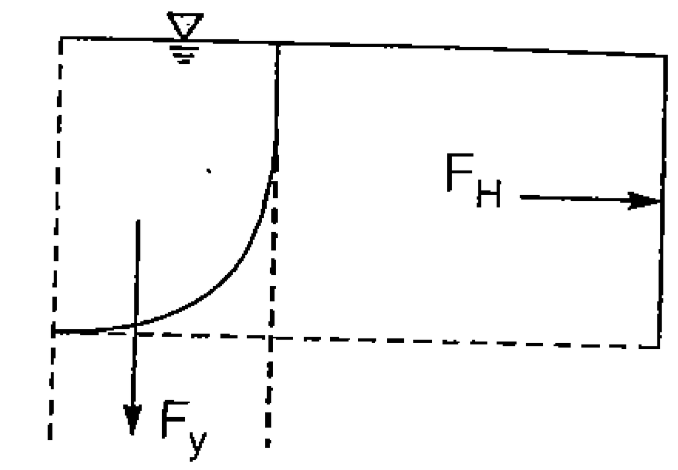
I_G = Area moment of inertia about centroidal axis and parallel to free axis.

$\bar{x}$ = Vertical distance of C.O.G. of body from free surface.

ω = Specific weight

θ = Angle at which the surface is inclined with horizontal

Hydrostatic Forces on Curved Surface



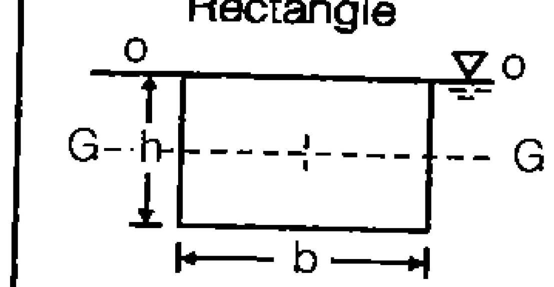
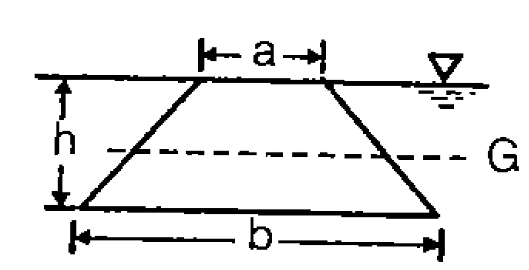
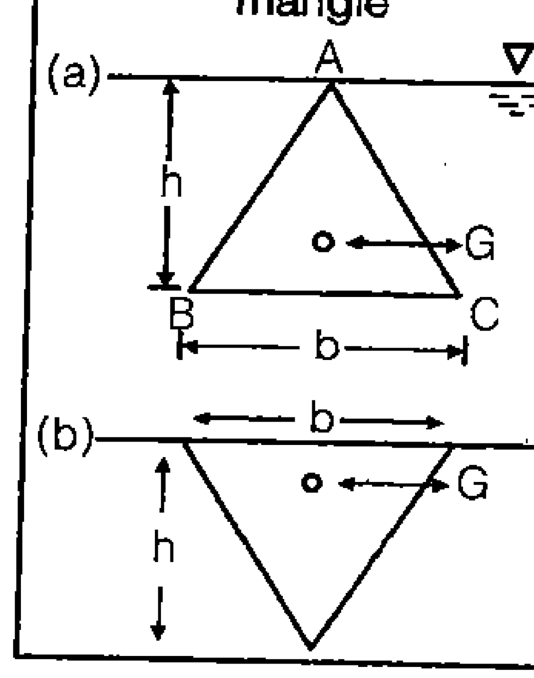
- **Horizontal Force (F_H)**
  Horizontal component of the resultant hydrostatic force 'F_x' of curved surface may be computed by projecting the surface upon a vertical plane and multiplying the projected area by the pressure at its own centre of area.
- **Vertical Force (F_V)**
  Vertical component of force 'F_y' is equal to the weight of the liquid block lying above the curved surface upto free surface.
- **Resultant Force (F)**

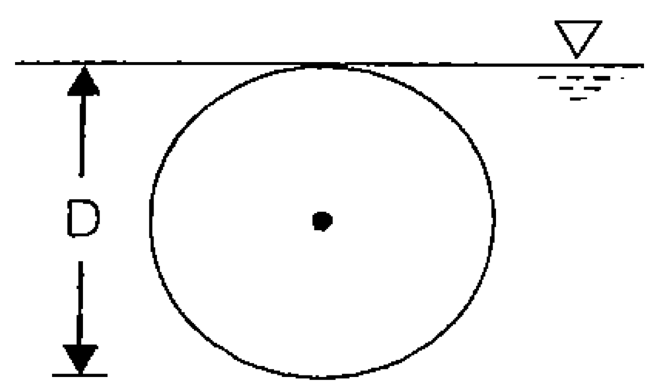
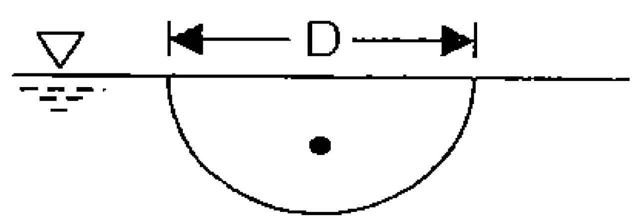
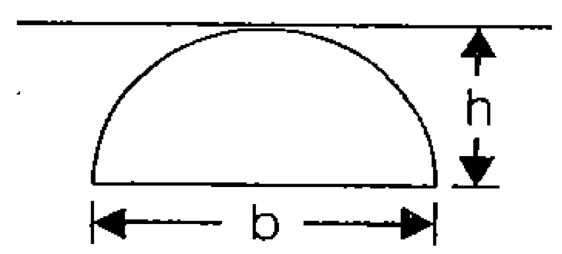
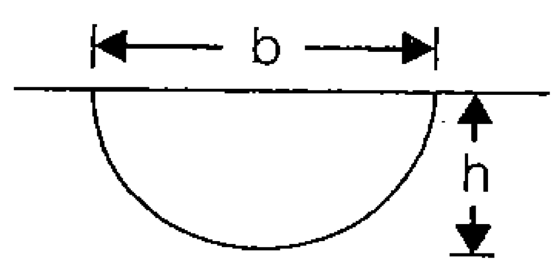
$$F = \sqrt{(F_H)^2 + (F_V)^2}$$

Angle of line of action of resultant force with the horizontal is given by

$$\tan\theta = \frac{F_y}{F_x}$$

Depth of Center of Pressure For Some Vertical Plane Surfaces From Liquid Surface

SURFACE	C.G. ($\bar{x}$)	C.P. ($\bar{h}$)
Rectangle 	$\frac{h}{2}$	$\frac{2h}{3}$
Trapezium 	$\frac{a+2b}{a+b}\cdot\frac{h}{3}$	$\frac{a+3b}{a+2b}\cdot\frac{h}{2}$
Triangle (a) 	$\frac{2h}{3}$	$\frac{3h}{4}$
(b)	$\frac{h}{3}$	$\frac{h}{2}$

Circle 	$\frac{D}{2}$	$\frac{5D}{8}$
Semi Circle 	$\frac{2D}{3\pi}$	$\frac{3\pi D}{32}$
Parabola (a) 	$\frac{3h}{5}$	$\frac{5h}{7}$
(b) 	$\frac{2h}{5}$	$\frac{4h}{7}$

Remember

In case of vertical surface, when depth of immersion ($\bar{x}$) is very large then
centre of pressure = centre of gravity

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4 Buoyancy and Floatation

Archimedes Principle

When a body is submerged either fully of partially then it is acted upon by a force of buoyancy vertically up which is equal to weight of liquid displaced by the body.

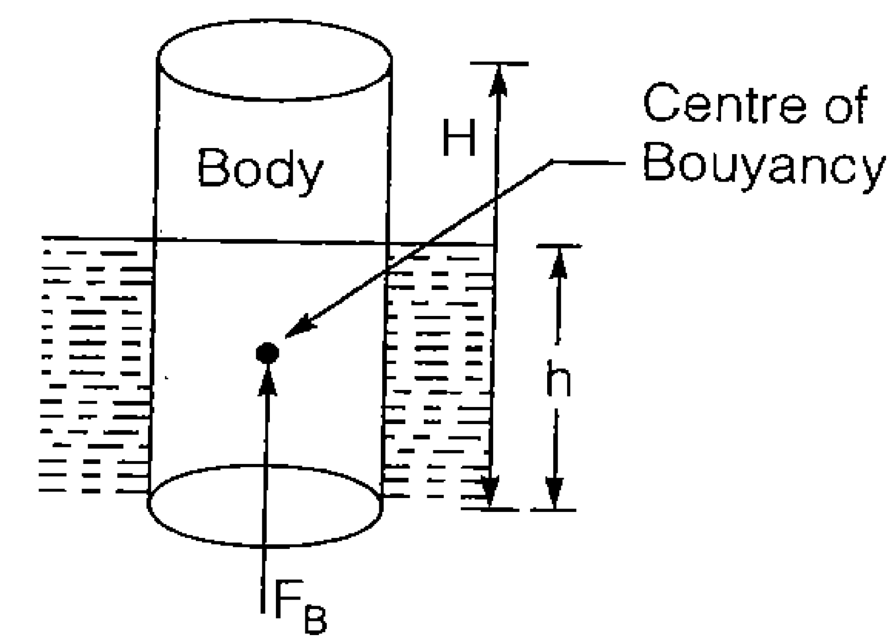
Remember

- This force of buoyancy always acts through the centroid of liquid displaced.
- Centre of Buoyancy is that point through which buoyant force act.

Principal of Flotation

$$H\rho_{Body} = h \cdot \rho_{fluid}$$

Here, H = Height of body
h = Height of body that is submerge in fluid



Condition for Equilibrium for Floating/Submerged Body

- For stable equilibrium
  - In case of floating body, metacenter should be above centre of gravity.
  - In case of submerged body, center of buoyancy should be above centre of gravity.
  - Distance between metacenter and centre of buoyancy

$$B.M = \frac{I_{min}}{V_{immersed}}$$

Here, I_{min} = Moment of inertia of top view of floating body about longitudinal axis
V = Volume of body immersed in liquid

Remember

- Metacentric height for rolling condition will be less then metacentric height for pitching condition.

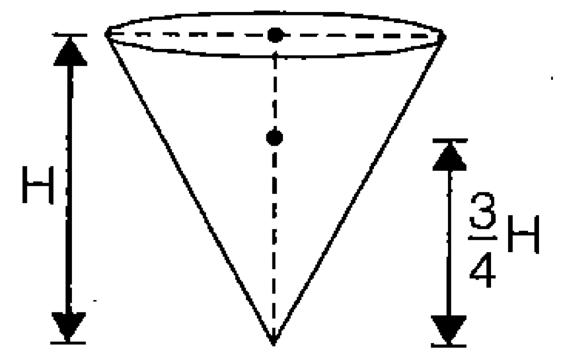
Time Period of Oscillation

If a floating body oscillates then its time period of transverse oscillation is given by

$$T = 2\pi\sqrt{\frac{K_G^2}{g.GM}}$$

Here, K_G = Least radius of gyration
GM = Metacentric height

For cone the center of gravity lies at $\frac{3}{4}H$ from the pointed end.



Metacentric height (Gm)

GM = (BM – BG)

$GM = \left(\frac{I}{V} - BG\right)$; V is volume displaced

- **For Stability:** GM > 0 ⇒ BM > BG
- **For Unstability:** GM < 0 ⇒ BM < BG
- **For Neutal :** GM = 0 ⇒ BM = BG

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Fluid Kinematics 5

Lagrangian concept focuses on single fluid particle.
Eularian concept focuses on all particles at once passing through particular section or point.

Steady and Unsteady Flow

If the fluid and flow characteristics (such as density, velocity, pressure etc.) ***at a point*** do not change with time, the flow is said to be steady flow. If the fluid and flow variables at a point may change with time, the flow will be unsteady.

For steady flow $\frac{dv}{dt} = 0$, $\frac{d\rho}{dt} = 0$ v = velocity, ρ = density

> It is applicable for ***all*** properties.

Uniform and Non-Uniform Flow

If the velocity vector at all points in the flow is same ***at any instant of time***, the flow is uniform flow. If the velocity vector varies from point to point at any instant of time, the flow will be non-uniform.

For uniform flow $\frac{dv}{ds} = 0$

> It is applicable ***only*** for velocity

Laminar and Turbulent Flow

In laminar flow, the particles moves in layers sliding smoothly over the adjacent layers while in turbulent flow particles have the random and erratic movement, intermixing in the adjacent layers.

Streakline

When a dye is injected in a liquid or smoke in a gas so as to trace the subsequent motion of fluid particles passing a fixed point, the path followed by the dye or smoke is called the streakline.

Pathline

A pathline is a curve traced by a single fluid particle during its motion.

Streamline

A streamline is an imaginary line drawn in a flow field such that a tangent drawn at any point on this line represents the direction of velocity vector at that point.

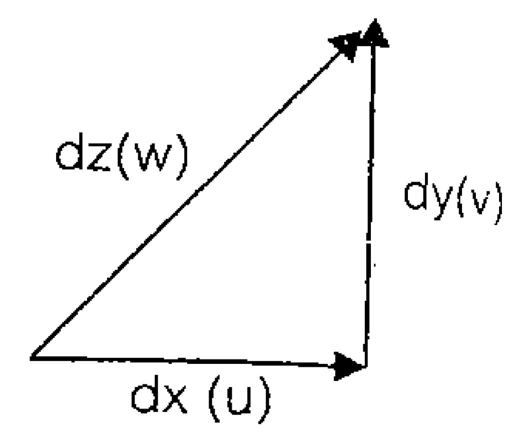
> Remember: There is no ***velocity*** component normal to stream lines.

Equation of stream line

Tangent to stream line gives velocity

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Here, u, v, w = Components of velocity in x, y, z direction



Continuity Equation (Conservation of Mass)

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Here, ρ = Density
A = Area
V = Velocity

For incompressible fluid density will be constant thus continuity equation will be

$$A_1 V_1 = A_2 V_2$$

General Continuity Equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Special Case:

If flow is steady then $\left(\frac{\partial \rho}{\partial t} = 0\right)$

Thus continuity equation will be

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

For steady, incompressible fluid density will be constant thus

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Total Acceleration of fluid

$$a_x = \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{Convective acceleration}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{Temporal or local acceleration}}$$

$$a_y = \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) + \frac{\partial v}{\partial t}$$

- Total Acceleration = Convective acceleration with respect to space + local acceleration with respect to time

Type of flow	Convective Acceleration	Temporal Acceleration
Steady & uniform	0	0
Steady & non-uniform	Exists	0
Unsteady & uniform	0	Exists
Unsteady & non-uniform	Exists	Exists

Rotational Component/Vorticity/Circulation

- Rotational component (ω)

$$\omega = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \qquad \omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

For irrotational flow $\omega_x = \omega_y = \omega_z = 0$

- **Vorticity (ξ)** $\xi = 2 \times \text{rotational component}$
- **Circulation (Γ)**
  It is line integral of tangential component of velocity around a closed curve.

$$\Gamma = \text{Vorticity} \times \text{Area of loop}$$

> Remember
> - In irrotational flow, the vorticity is zero, at all points in the flow region while for rotational flow, vorticity is non-zero.
> - Flow outside the boundary layer has irrotational characteristic while that within the boundary layer is rotational characteristic.

Velocity Potential Function (ϕ)

$$u = \frac{-\partial\phi}{\partial x}, \quad v = \frac{-\partial\phi}{\partial y}, \quad w = \frac{-\partial\phi}{\partial z}$$

Remember
- Flow always occurs in the direction of decreasing potential.
- Velocity potential function exists only for irrotational flow.
- Laplace equation is given by

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

If velocity potential function satisfies Laplace equation then it also satisfies continuity equation and hence the flow is possible.

Streamline Function (ψ)

$$v = \frac{\partial\psi}{\partial x}, \quad u = -\frac{\partial\Psi}{\partial y}$$

- If Ψ exists, then it satisfies continuity equation and flow can be rotational or irrotational.
- If stream function satisfies Laplace equation then it is case of irrotational flow.
- Discharge per *unit* length = $\psi_1 - \psi_2$

Cauchy-Reimann equation

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \quad -\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}$$

- Equipotential lines and constant stream function lines are *orthogonal*.

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Fluid Dynamics & Flow Measurements

6

Different Kind of Force Acting on Fluid Particle

- Pressure force F_P
- Gravity force F_g
- Viscous force F_v
- Turbulence F_E

If all the three force are taken into account then equation obtained is known as Navier stokes equation.

Remember
Euler's Equation
It represents momentum equation in a 2-D, ***inviscid*** steady flow.

$$\frac{dP}{\rho} + g.dz + v.dv = 0$$

- No viscous effects are considered.

Bernoulli's Equation

- **Assumptions in Bernoulli's equation:**
  - fluid is ideal
  - flow is steady
  - flow is continuous
  - fluid is incompressible
  - flow is non-viscous
  - flow is irrotational
  - applicable along a stream line

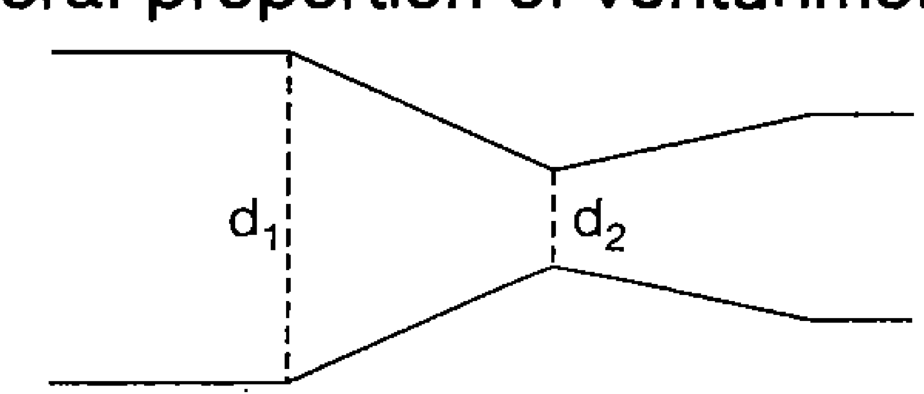
  This equation is obtained by integrating Euler's equation.

$$\frac{P}{w} + z + \frac{V^2}{2g} = C$$

Here, $\frac{P}{w} + z$ = Pressure head + gravitational head = Piezometric head

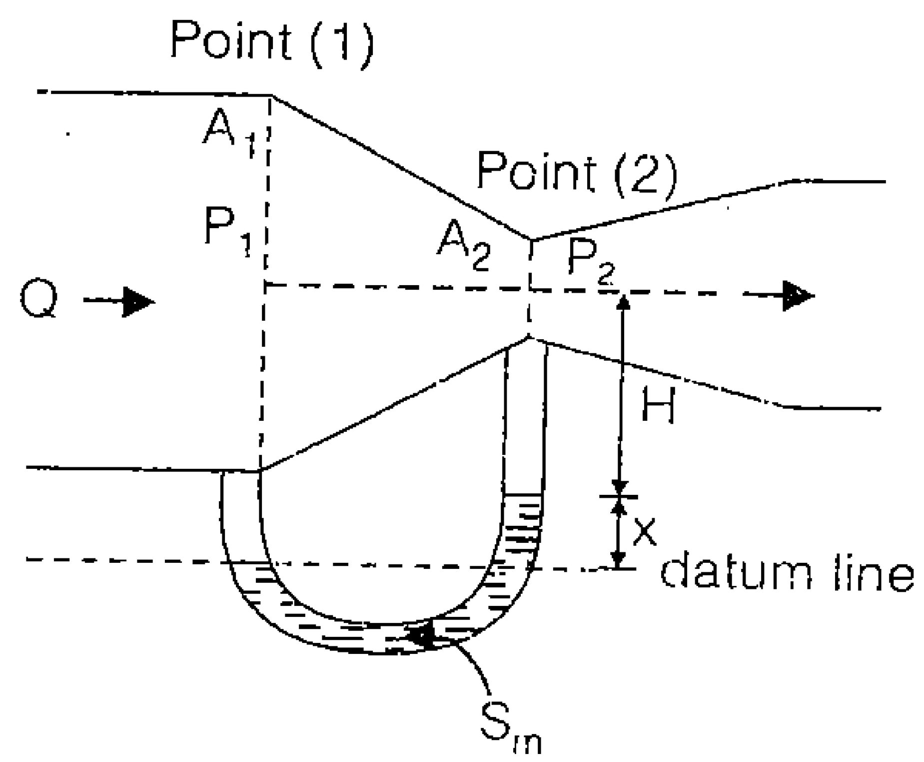
Venturimeter

- **General proportion of venturimeter**



$$d_2 = \left(\frac{1}{3} \text{ to } \frac{1}{2}\right)d_1$$

Angle of convergence = 20 – 30°
Angle of divergence = 6 – 7° and it should be not greater than 7° to avoid ***flow separation***.



- It is used for measuring discharge
- $Q_{ideal} = \dfrac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$

h = Piezometric head difference
= Pressure head difference

$= \dfrac{P_1 - P_2}{\omega} = \dfrac{V_2^2 - V_1^2}{2g}$

($\because$ gravitational head difference = 0)

$h = x\left|\dfrac{S_m}{S} - 1\right|$

Here, x = manometric deflection
s_m = Relative density of manometric fluid
s = Relative density of flowing fluid

- $Q_{actual} = C_{dv} \cdot Q_{ideal}$
  For venturimeter $C_{dv} = 0.94 - 0.98$
  Here, C_{dv} = coefficient of discharge

$C_{dv} = \sqrt{\dfrac{h - h_l}{h}}$ h_l = Head loss in convergent divergent section

Orifice Meter

- It is cheaper arrangement but has more energy loss.

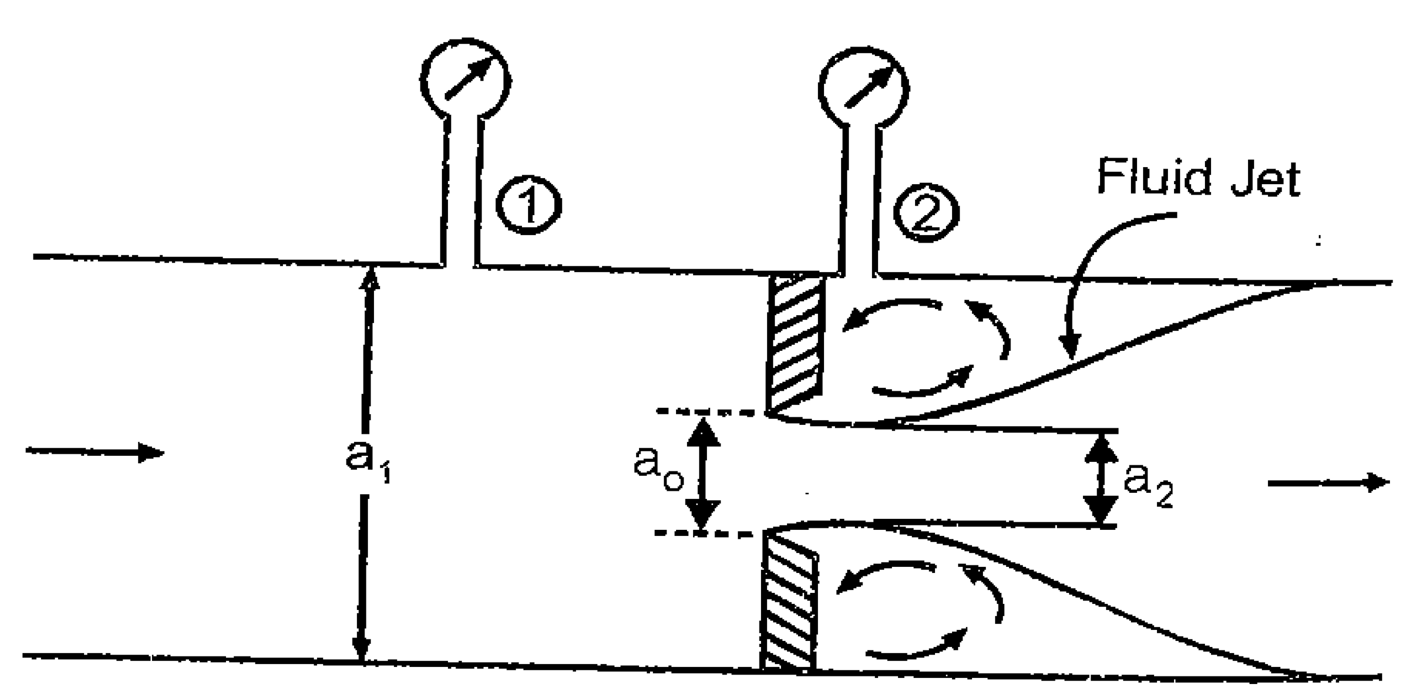
$Q = \dfrac{C_d a_1 a_o}{\sqrt{a_1^2 - a_o^2}} \sqrt{2gh}$

$a_1 = \dfrac{\pi}{4} D^2$

$a_o = \dfrac{\pi}{4} d_o^2$

d_o = dia of orifice

- It is used to measure discharge



ORIFICE METER

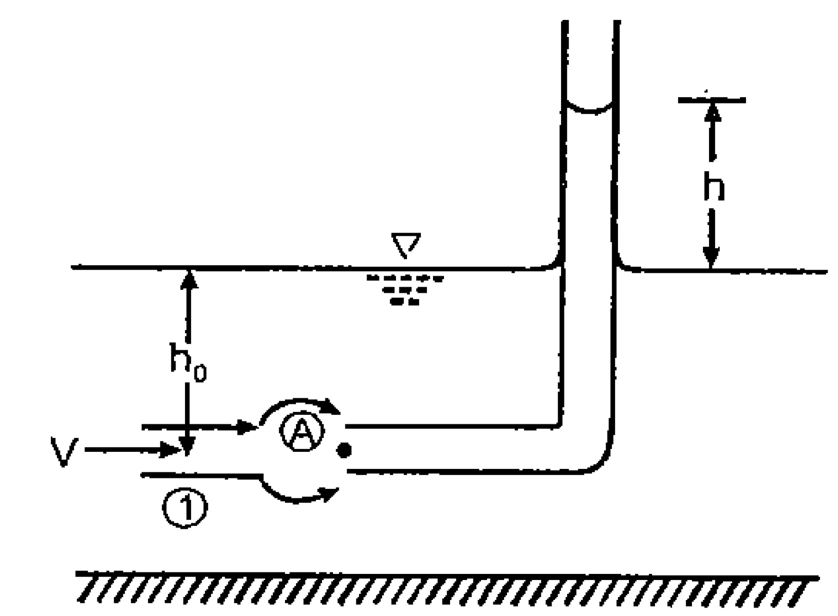
- $C_d = 0.64 - 0.76$
- For orifice

$C_c = \dfrac{C_d}{C_v}$

Here, C_c = Coefficient of contraction
C_d = Coefficient of discharge
C_v = Coefficient of velocity

Pitot Tube

It is based on principle of conversion of kinetic head into pressure head. The point at which velocity reduces to zero is called stagnation point.



Simple Pitot Tube

$V_{th} = \sqrt{2gh}$

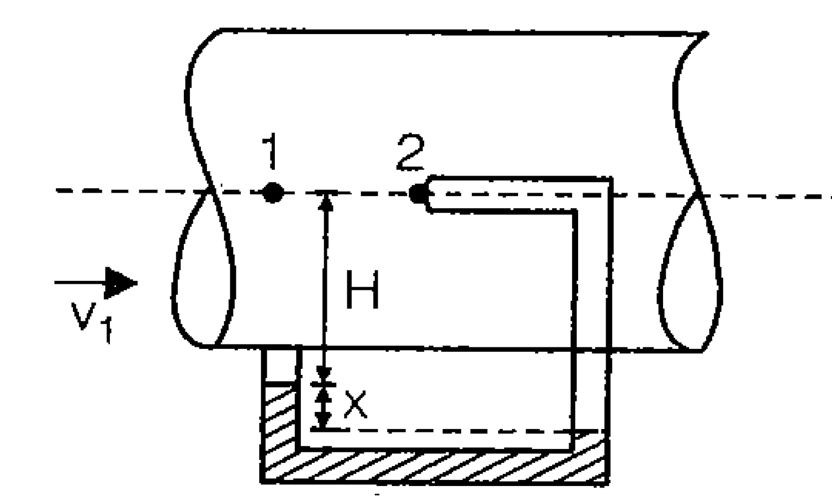
$= \sqrt{2g\left(\dfrac{p_s - p_o}{\rho g}\right)}$

$V_{ac} = C_V V_{th}$
C_V = Coefficient of velocity (0.98)
$p_s/\rho g$ = stagnation head &
$p_o/\rho g$ = static head.

Velocity head is indicated by the difference in liquid level between the Pitot tube and the piezometer. The Pitot tube measures the total head and therefore known as total head tube.

- **Application of Pitot Tube in Pipes**

$v_1 = \sqrt{2gh}$ $h = x\left(\dfrac{s_m}{s} - 1\right)$



Here, s_m = Relative density of manometric fluid
s = Relative density of flowing fluid

Hydraulic Coefficients

- Contraction coefficient $(C_c) = \dfrac{\text{Area of jet at vena contraction}}{\text{Area of orifice}}$
- Coefficient of velocity $(C_v) = \dfrac{\text{Actual velocity } (V_{ac})}{\text{Theoretical velocity } (V_{th})}$
- Coefficient of discharge $(C_d) = \dfrac{\text{Actual discharge } (Q_{ac})}{\text{Theoretical discharge } (Q_{th})}$

Devices and Their Uses

Device	Measurement
Venturimeter	rate of flow (discharge)
Flow nozzle	rate of flow
Orifice meter	rate of flow
Bend meter	rate of flow
Rotameter	rate of flow
Pitot tube	velocity
Hot wire anemometer	air & gas velocity
Current meter	velocity in open channels

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7 Viscous Flow of Incompressible Fluid

In viscous flow fluid particles move along straight parallel paths in ***layers***. It occurs at low velocity and ***viscous force*** predominates inertial force.

Reynold's Number (R_e)

$$R_e = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho v d}{\mu}$$

Here, ρ = Density
d = characteristics length
V = average velocity

Nature of Flow According to Reynolds Number For Pipe and Open Channel Flow

The limiting values of Reynold's Number corresponding to which flow is Laminar is given by:

Flow Condition	Pipe flow	Open channel flow
Laminar Flow	$Re \leq 2000$	$Re \leq 500$
Transitional Flow	$2000 < Re < 4000$	$500 < Re < 1000$
Turbulent Flow	$Re > 4000$	$Re > 1000$

Entrance Length (L_e)

The length of pipe from its entrance upto the point where flow attains fully developed velocity profile and which remains unaltered beyond that known as entrance length.

The entrance length required to establish fully developed ***laminar flow*** is given by

$$\frac{L_e}{D} = 0.07\, R_e$$

The entrance length for fully developed ***turbulent flow*** is given by

$$\frac{L_e}{D} = 50$$

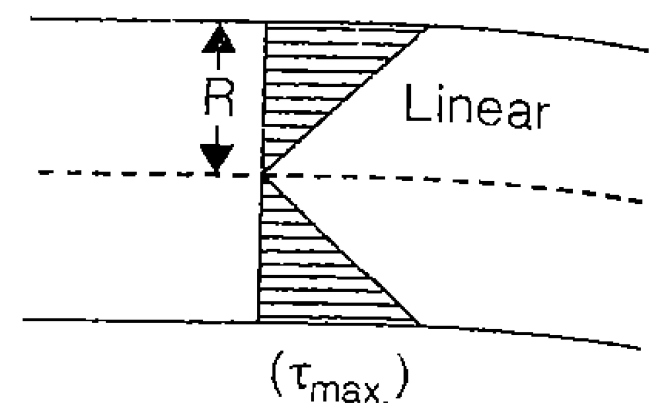
Laminar Flow Through Circular Pipe

(Hagen-Poiseulle Flow)

- Shear stress (τ) distribution

$$\tau = \left(-\frac{\partial P}{\partial X}\right)\cdot\frac{r}{2}$$

The negative sign on $\frac{\partial p}{\partial x}$ indicates decrease in pressure in the direction of flow. The pressure must decrease because pressure force is the only means available to compensate for resistance to the flow, the potential and kinetic energy remain constant.

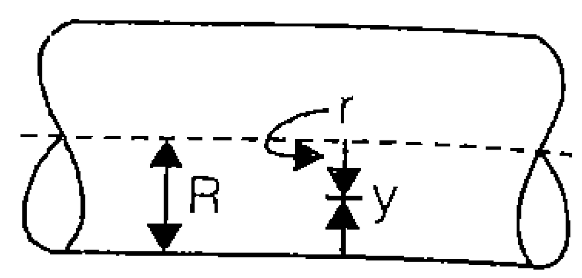


Remember

- Above equation is valid for steady and uniform flow.
- The maximum value of stress occur at r = R.
- In laminar flow shear stress is entirely due to ***viscous*** action.

- Velocity distribution

  r is the distance measured from axis

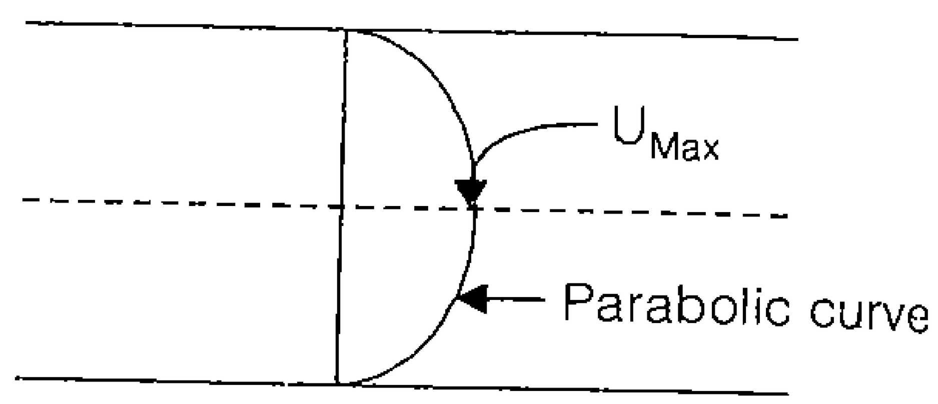


$$U = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)(R^2 - r^2)$$

At center, r = 0 thus

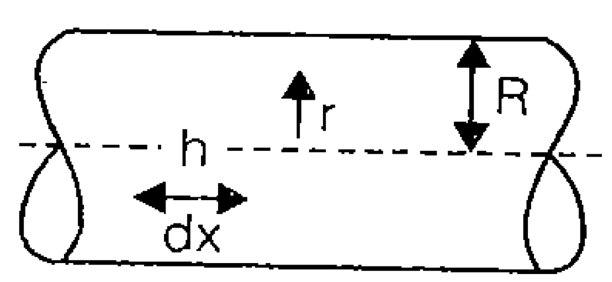
$$U_{max} = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)\cdot R^2$$

Here, U_{max} = Maximum velocity



$$U = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

- Discharge (Q) through a pipe



$Q \neq AV$, because 'V' is changing

- Heigen-Poiseulle equation, for *viscous* flow

$$Q = \frac{\pi}{128\mu}\cdot\left(\frac{-\partial P}{\partial x}\right)\cdot D^4$$

Mean velocity $= \frac{U_{max}}{2}$

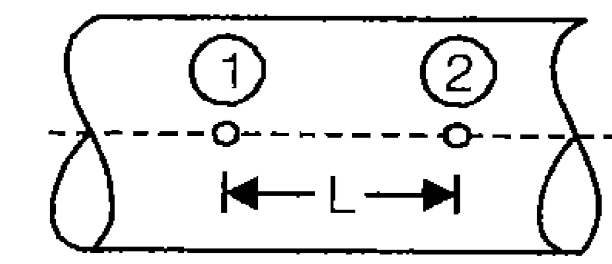
Remember

- The maximum velocity occur at axis.
- The point where local velocity is equal to mean velocity is given by

$$r = \frac{R}{\sqrt{2}} = 0.707\,R$$

- **Pressure drop in pipe**

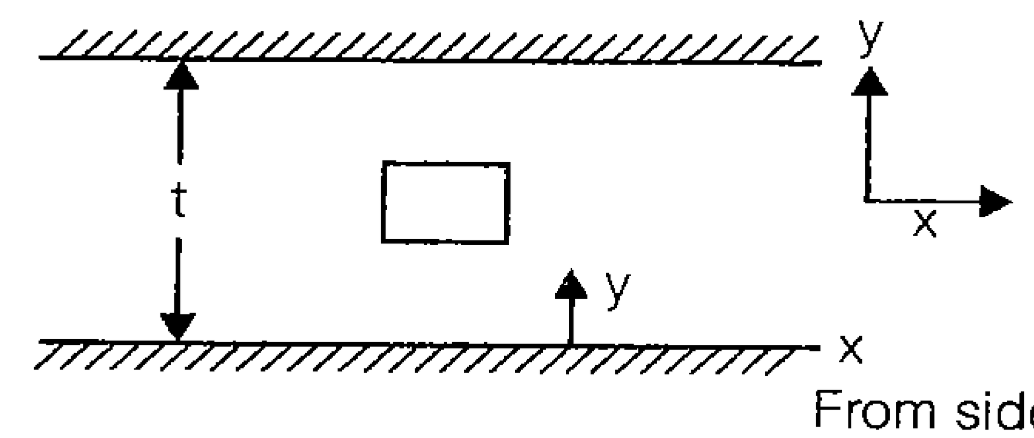
$$P_1 - P_2 = \frac{32\mu V_{avg.} L}{D^2}$$



Laminar Flow Between Two Fixed Parallel Plate

- Velocity

$$u = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)(ty - y^2)$$



Remember

- The maximum velocity occurs at $y = \frac{t}{2}$ and it is given by,

$$u_m = \frac{t^2}{8\mu}\left(-\frac{\partial p}{\partial x}\right)$$

- $V_{avg} = 0.66\,u_{max}$

- Discharge per unit width

$$Q = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial x}\right)t^3$$

- Pressure drop in given length

$$P_1 - P_2 = \frac{12\mu VL}{t^2}$$

Momentum correction factor (β)

It is defined as the ratio of momentum/sec based on ***actual*** velocity to the momentum/sec based on ***average*** velocity.

$$\beta = \frac{1}{AV^2}\int u^2 \cdot dA$$

V = Average velocity

u = Local velocity at distance r

- For laminar flow, **β** = 1.33
- For turbulent flow, **β** = 1.2

Kinetic energy correction factor (α)

It is defined as the ratio of kinetic energy/second based on ***actual*** velocity to the kinetic energy/second based on ***average*** velocity.

$$\alpha = \frac{1}{AV^3}\int u^3 \cdot dA$$

- For laminar flow, **α** = 2
- For turbulent flow, **α** = 1.33

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8 Flow Through Pipes

Friction Loss/Darcy Weisbach Equation

- Darcy Weisbach Equation

$$h_f = \frac{fLV^2}{2gD}$$

Here, L = Length of pipe
D = Dia of pipe
V = Mean velocity of flow
f = Friction factor (0.02 to 0.04 for metals)
h_f = Head loss due to friction

- For Laminar Flow $f = \frac{64}{R_e}$

Remember
- Friction factor is directly proportional to diameter of pipe.
- Head loss (h_f) is inversely proportional to fourth power of the diameter.

- Turbulent Flow $f = \frac{0.316}{Re^{1/4}}$

Remember
- Friction factor ∝ (diameter of pipe)$^{1/4}$
- Head loss (h_f) ∝ $\frac{1}{(\text{Diameter})^{19/4}}$

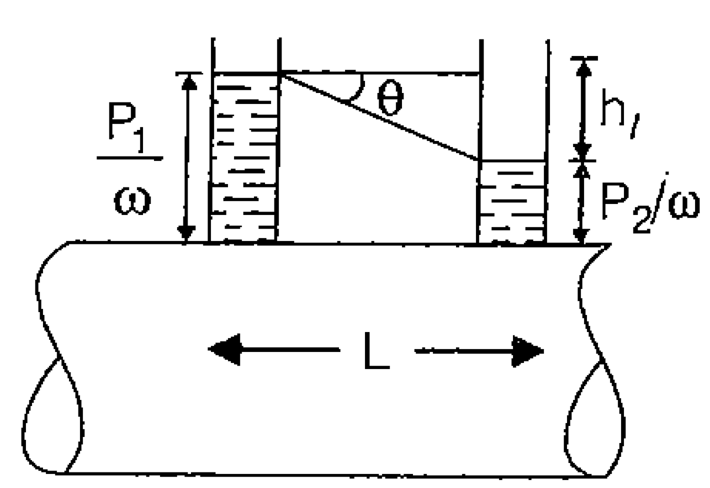
Friction loss in pipe whose end is closed and flow takes place through sides at regular interval

$$f = \frac{1}{3}\frac{fLV^2}{2gD}$$

Chezy's Formula

$$V = C\sqrt{mi}$$

C = Chezys const.
m = Hydraulic mean depth
$= \frac{\text{Area}}{\text{Wetted Perimeter}}$



$$i = \text{Hydraulic slope} = \frac{h_L}{L} = \tan\theta$$

Relation between Chezys constant and friction factor. $f = \frac{8g}{C^2}$

Series and parallel combination of pipes

- Series Combination

Dia of equivalent pipe is given by $\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \dots$

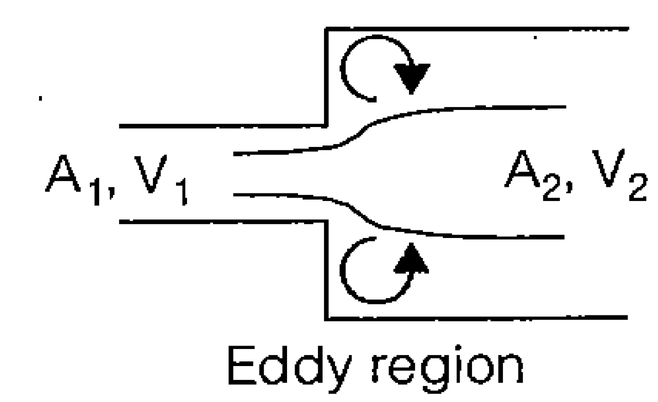
- Parallel combination of pipe: $h_{L_1} = h_{L_2}$

$$\frac{f_1L_1V_1^2}{2gD_1} = \frac{f_2L_2V_2^2}{2gD_2}$$

Different Type of Minor Losses in Pipe

- Losses due to sudden expansion

$$h_{le} = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

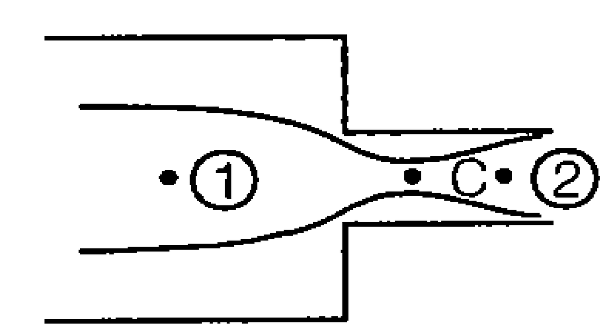


Remember For sudden expansion, the optimum ratio of diameters of the pipe so that the pressure loss is minimum is given by,

$$\frac{d_1}{d_2} = \frac{1}{\sqrt{2}}$$

- Losses due to sudden contraction

$$h_{le} = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$



Coefficiency of contraction $(C_c) = \frac{V_2}{V_c}$

If C_c is not given $h_{lc} = \frac{0.5V_2^2}{2g}$

- Losses at exit of pipe $h_l = \frac{V^2}{2g}$
- Losses at entrance to pipe $h_l = \frac{0.5\,V^2}{2g}$

Here, V = ***Mean*** velocity of flow in pipe

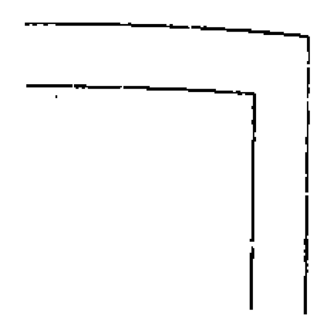
- Loss due to gradual expansion

$$h_L = K_L \frac{(V_1 - V_2)^2}{2g} = K_L \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

K_L depends upon angle of expansion.

- Losses due to bends

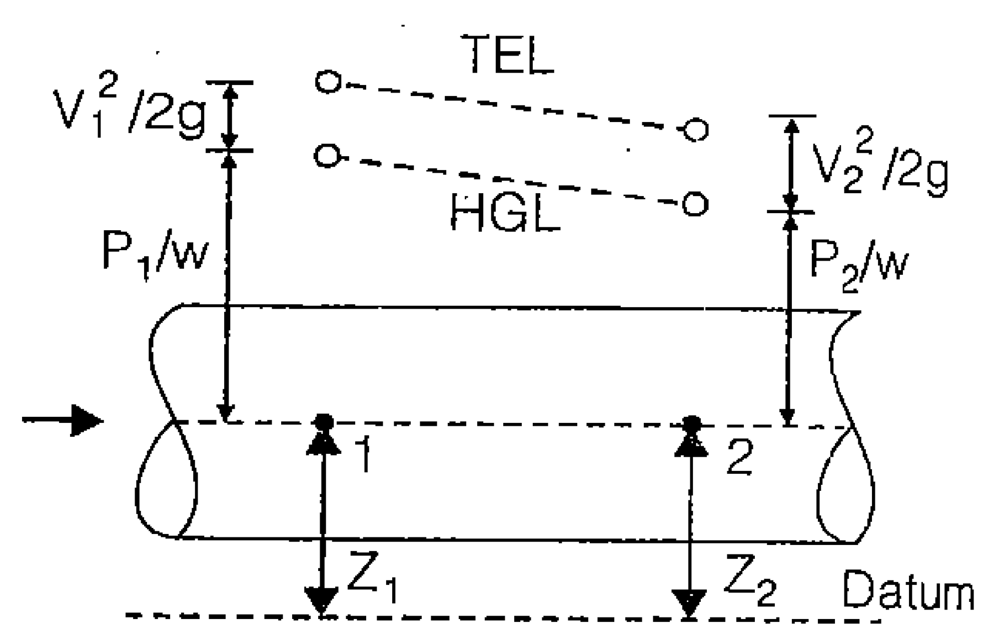
$h_l = \frac{KV^2}{2g}$ $\quad K \simeq 1.2$ for 90° bend; $K \simeq 0.4$ for 45° bend



Hydraulic Gradient Line (HGL) and Total Energy Line (TEL)

HGL: It joins Piezometric head $\left(\frac{P}{w} + z\right)$ at various points.

TEL: It joins total energy head at various points $\left(\frac{P}{w} + z + \frac{V^2}{2g}\right)$

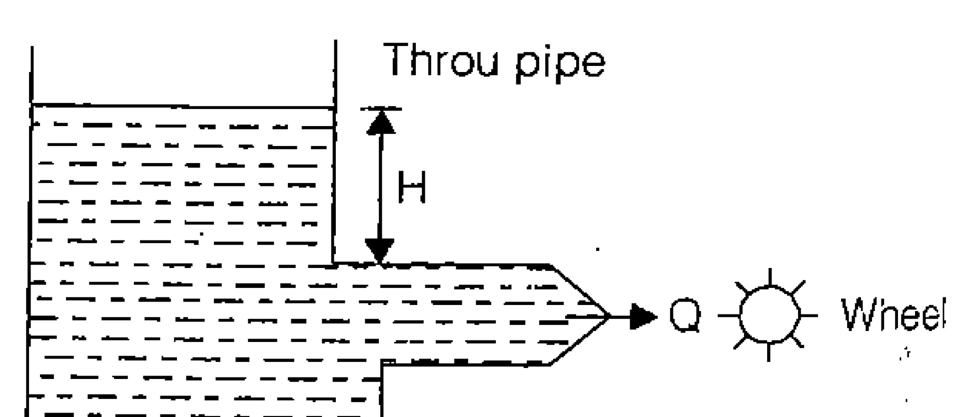


HGL is always parallel and lower than TEL.

Power Transmission Through Pipe (P)

- $P_{ideal} = \rho g Q H$
- $P_{actual} = \rho g Q(H - H_l)$

  Here, h_l=Head loss
- Efficiency $(\eta) = \frac{P_{actual}}{P_{ideal}} = \frac{H - h_l}{H}$

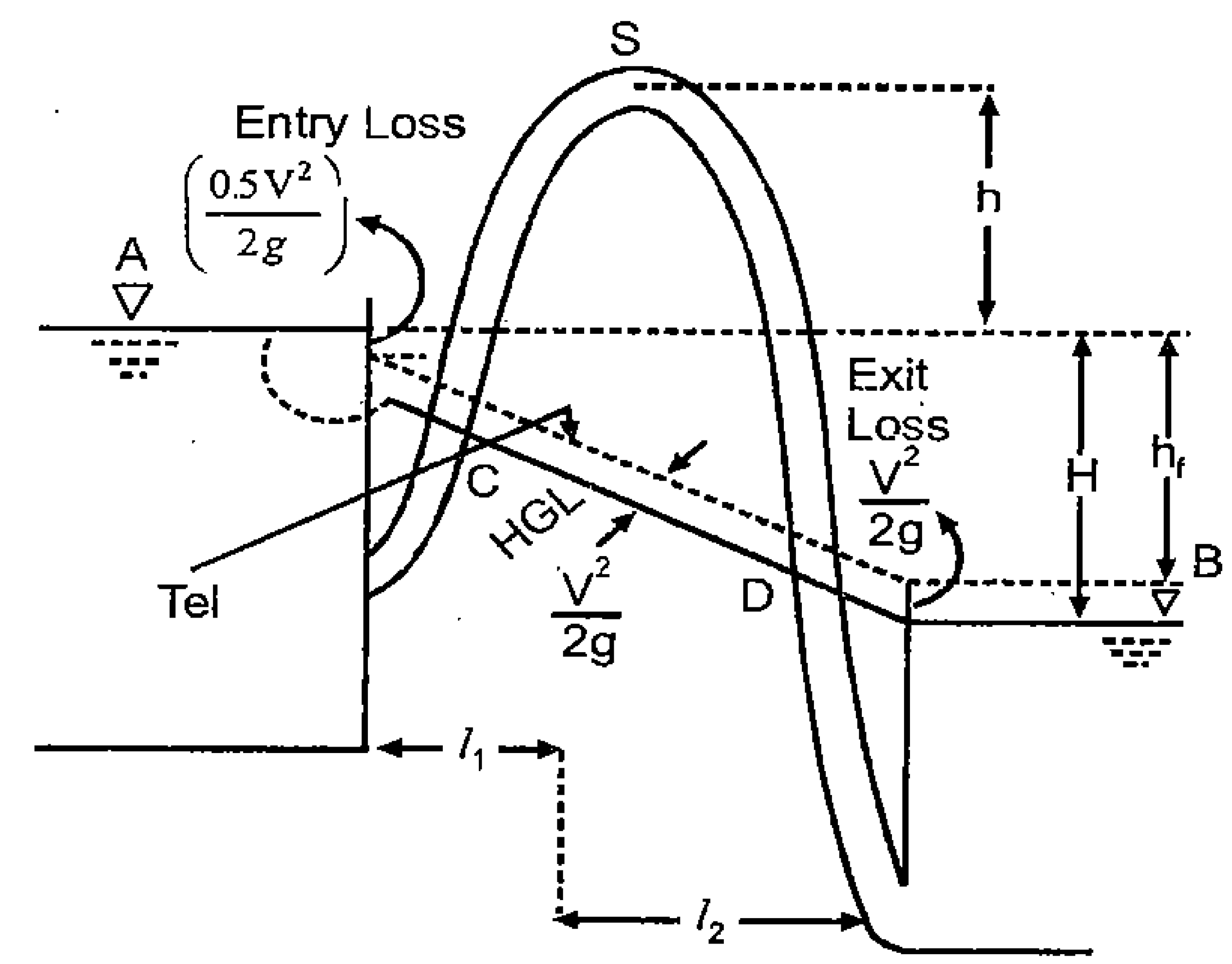


- Power delivered by a given pipeline is ***maximum*** when the flow is such that ***one third*** of static head is consumed in pipe friction. Thus efficiency is limited to only 66.66% (For η_{max}, $h_l = H/3$).

Siphon Action of Pipe

A pipe which raises above its hydraulic grade line has negative pressure and is known as a siphon.

- Formation of air lock at highest portion of pipe.
- Summit pressure (p_s) should not be less than vapour pressure of liquid to avoid air lock formation at summit. (For water it is 2.5 m at 20°C) (absolute)
- Pressure at C and D = atmospheric pressure
- Pressure below C and D = Positive



- Pressure above C and D = Negative (Suction pressure).
- It total head loss between A and B is H then,

  H = entry loss + friction loss + exit loss

$$H = 0.5\frac{V^2}{2g} + \frac{fLV^2}{2gD} + \frac{V^2}{2g} \quad ..(i)$$

- If *h* is height of summit above reservoir A, then by applying Bernoulli's equation between A and S,

$$\frac{p_A}{w} = \left(\frac{p_s}{w} + h\right) + \frac{V^2}{2g} + 0.5\frac{V^2}{2g} + \frac{fl_1V^2}{2gD} \quad ...(ii)$$

from (i) and (ii) p_s and h can be calculated.

Water Hammer in Pipe

- There are three cases of water hammer
  (i) Gradual closure of valve
  (ii) Sudden closure of valve and pipe is rigid
  (iii) Sudden closure of valve and pipe is elastic

- The valve closure is gradual if $t > \frac{2L}{C}$
- The valve closure is sudden if

$$t < \frac{2L}{C}$$

where, L = Length of pipe
t = time in sec.
C = velocity of pressure wave

- **Gradual Closure**

In this case the pressure head is given by

$$\frac{p}{W} = \frac{LV}{gt} \quad ...(i)$$

where, L = Length of pipe
V = Velocity of flow
t = time in second to close the valve.

- **Sudden Closure With Rigid Pipe**

The pressure head in this case is given by

$$\frac{p}{W} = \frac{VC}{g} \text{(Allievi formula)} \quad ...(ii)$$

where, $C = \sqrt{\frac{K}{\rho}}$ = Velocity of pressure wave
K = Bulk modulus of water
ρ = density of water.

- **Sudden Closure With Elastic Pipe**

The pressure head is given by:

$$\frac{p}{W} = \frac{V}{\sqrt{\rho g^2\left(\frac{1}{K} + \frac{D}{tE}\right)}} \quad ...(iii)$$

where, D = Diameter of pipe
t = Thickness of pipe
E = Modulus of elasticity of pipe material

(In deriving equation (iii), the Poisson's ratio of pipe material is assumed as 0.25.)

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Vortex Motion 9

A *whirling* mass of fluid is called vortex flow.

Free Vortex Flow

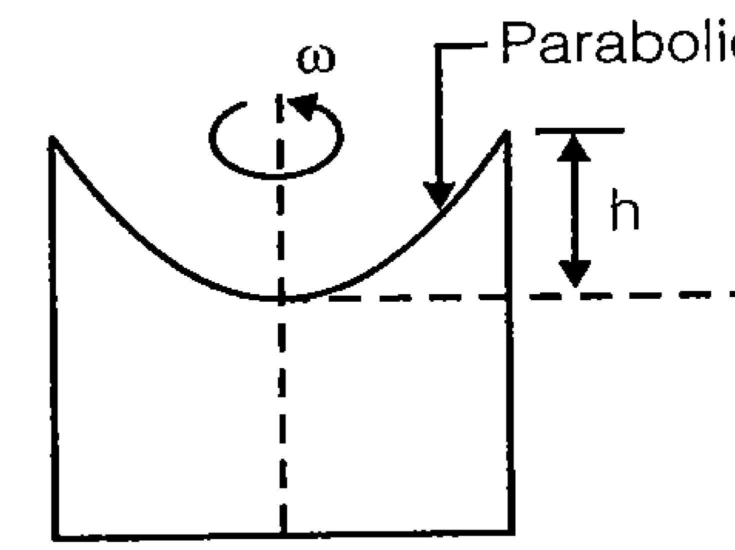
In this flow fluid mass rotates due to conservation of angular momentum. The velocity profile is inversely proportional with the radius.

$$v\,r = \text{constant}$$

The point at the centre of rotation is called singular point, where velocity approaches to infinite. ***Example:*** Whirling mass of liquid in wash basin, whirlpool in rivers etc.

> Remember
> No ***external*** torque or energy is required.
> In free vortex flow ***Bernoulli's equation*** can be applied.

Forced Vortex Flow



When a fluid is rotated about a vertical axis at constant speed, such that every particle has the same angular velocity, motion is known as the forced vortex.

$$V = r\omega \qquad h = \frac{\omega^2 r^2}{2g}$$

where h is height of paraboloid, and r is radius of cylinder.

$$\text{Volume of paraboloid} = \frac{1}{2}\pi r^2 h$$ = ½ of volume of circumscribing cylinder

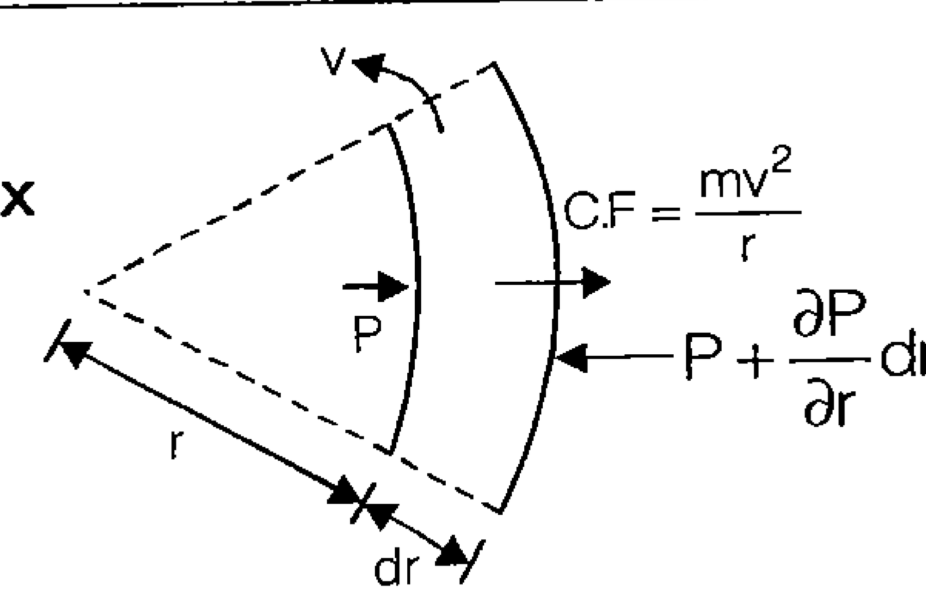
> Remember
> - The surface profile of forced vortex flow is ***parabolic***.
> - Forced vortex requires const. supply of ***external*** energy/torque. Example of forced vortex flow are rotating cylinder & flow inside centrifugal pump.

Variation of Pressure

- **Valid for Free as Well as Forced Vortex**

$$dp = \frac{\rho v^2}{r} \cdot dr - \rho g.\, dz$$

(z in upward direction)



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Boundary Layer Theory

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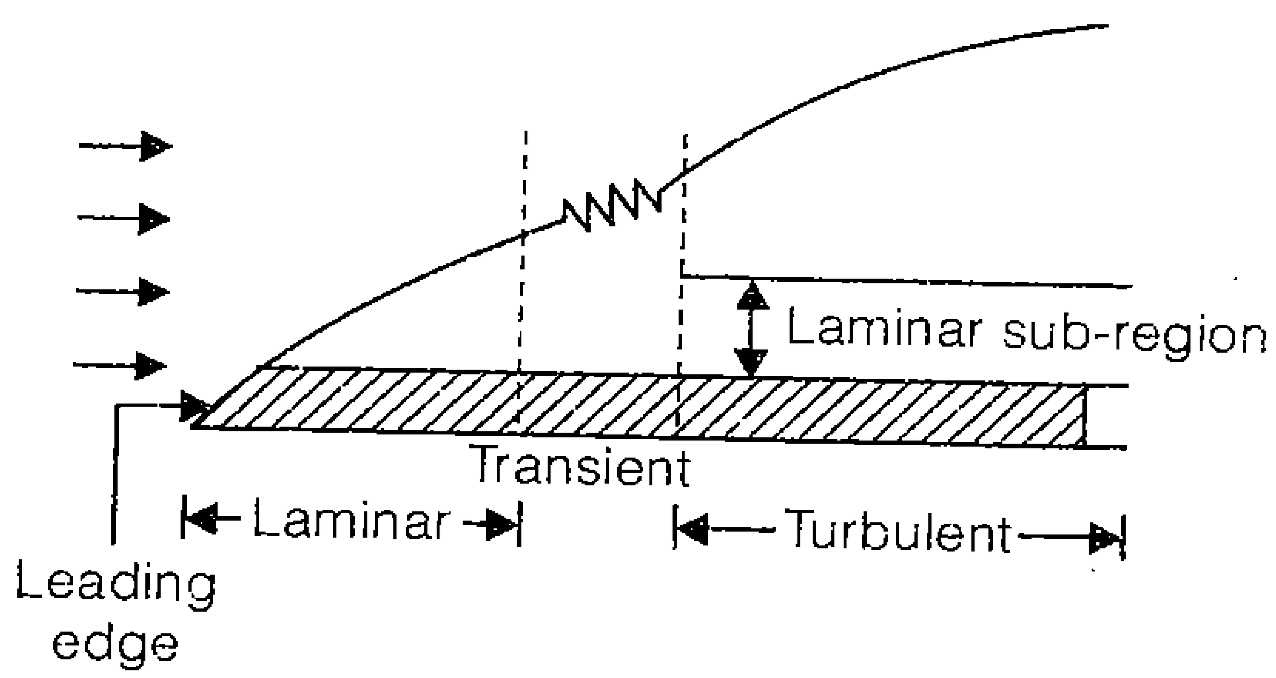
Boundary layer is a region in the immediate vicinity of the boundary surface in which the velocity of flowing ***fluid increases gradually*** from zero at the boundary surface to the velocity of the main stream.

Development of Boundary Layer Region



Various Terms Associated With Boundary Layer Theory

- **Boundary layer thickness (δ):** It is defined as the distance from the boundary surface in which the velocity reaches the 99% of the velocity of the main stream.

$$y = \delta$$

for $$v = 0.99\ V_0$$

- **Displacement Thickness (δ*):** It is the distance measured normal to the boundary, by which the solid boundary should be displaced in order to compensate for the reduction in mass flow rate due to boundary layer growth.

$$\delta^* = \int_0^\delta \left(1 - \frac{v}{V_0}\right) dy$$

The quantity $(V_0 - v)$ known as the velocity defect.

- **Momentum Thickness (θ)**

$$\theta = \int_0^\delta \frac{v}{V_0}\left(1 - \frac{v}{V_0}\right) dy$$

- **Energy Thickness (δ_e)**

$$\delta_e = \int_0^\delta \frac{v}{V_0}\left(1 - \frac{v^2}{V_0^2}\right) dy$$

Remember

- For $\frac{v}{V_0} = \frac{y}{\delta}$,

$$\delta^* = \frac{\delta}{2},\ \theta = \frac{\delta}{6},\ \delta_e = \frac{\delta}{4} \text{ thus } \delta^* > \delta_e > \theta$$

- The ratio of displacement thickness to momentum thickness is called the ***shape factor*** (H). Its value should be always greater than 1.

General velocity profile for laminar and turbulent flow

- For laminar flow $$\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

- For turbulent flow $$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/2}$$

Reynold number for different types of flow over flat plate

$$R_e = \frac{\rho V x}{\mu}$$

Here, x = Distance from where solid surface starts

$R_e < 5 \times 10^5$ – laminar flow

$R_e > 5 \times 10^5$ – turbulent flow

Blassius Experiment Results/When Velocity Profile is Not Given

Laminar	Turbulent
1. $\frac{\delta}{x} = \frac{5}{\sqrt{R_{ex}}}$	1. $\frac{\delta}{x} = \frac{0.376}{(R_{ex})^{1/5}}$
2. $C_{fx} = \frac{0.664}{\sqrt{R_{ex}}}$	2. $C_{fx} = \frac{0.059}{(R_{ex})^{1/5}}$
3. $C_d = \frac{1.328}{\sqrt{R_{el}}}$	3. $C_d = \frac{0.074}{(R_{el})^{1/5}}$

For laminar flow $\delta \propto \sqrt{x}$ $\tau \propto \frac{1}{\sqrt{x}}$

Here, δ = Boundary layer thickness

τ_o = Shear stress at solid surface

x = Distance from where solid surface starts

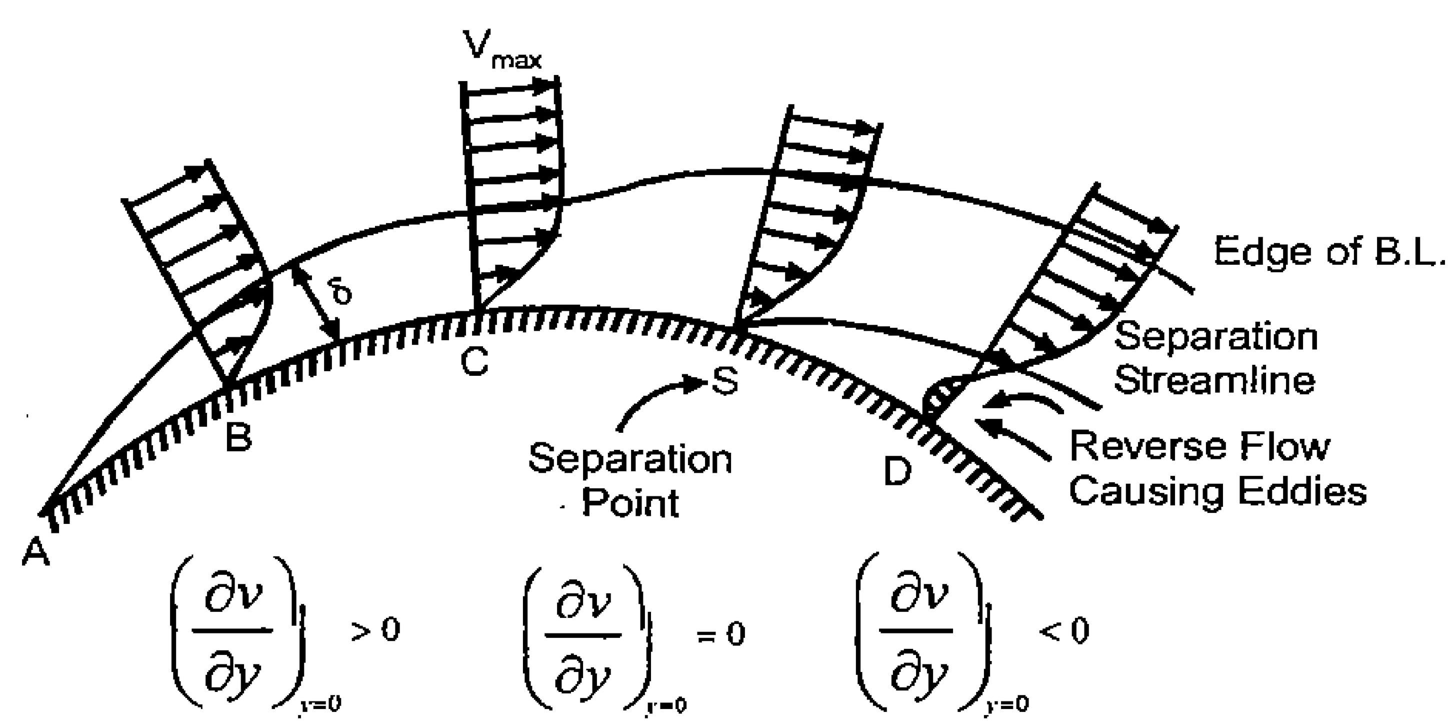
Hardness

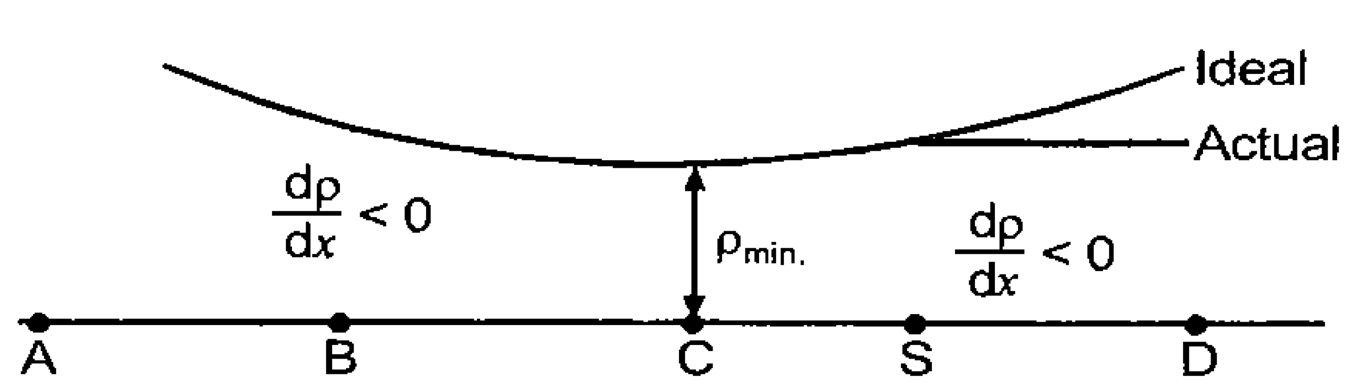
Local skin friction coefficient $\boxed{C_{fx} = \dfrac{\tau_0}{\frac{1}{2}\rho u_\infty^2}}$

Average drag coefficient $\boxed{C_d = \dfrac{F_d}{\frac{1}{2}\rho A U_\infty^2}}$

Condition For Boundary Layer Separation

It is caused by ***adverse pressure*** gradient $\left(\dfrac{dp}{dx} > 0\right)$





- Location of Separation point

The separation point S is determined from the condition $\left(\dfrac{\partial v}{\partial y}\right)_{y=0} = 0.$

For a given velocity it can be determined whether the B.L. has separated or on the verge of separation or will not separate from the following conditions:

- If $\left(\dfrac{\partial v}{\partial y}\right)_{y=0} < 0$: Flow has separated.
- If $\left(\dfrac{\partial v}{\partial y}\right)_{y=0} = 0$: Flow is on verge of separation.
- If $\left(\dfrac{\partial v}{\partial y}\right)_{y=0} > 0$: Flow is attached with the surface

Methods of Preventing Separation

- Rotating the boundary in the direction of flow.
- Suction of the slow moving fluid by a suction slot.
- Supplying additional energy from a blower.
- Providing a bypass in the slotted wing.
- Providing guide blades in a bend.
- Injecting fluid into boundary layer.
- Streamlining of body shapes.

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11 Turbulent Flow

Shear Stress in Turbulent Flow

$$\tau = \mu \frac{d\bar{v}}{dy} + \eta \frac{d\bar{v}}{dy}$$

where, μ = dynamic coefficient of viscosity (*fluid* characteristic)
η = eddy viscosity coefficient (*flow* characteristic)

- Eddy viscosity comes in picture due to the turbulence effect.

Hydro Dynamically Smooth And Rough Pipes

- If the ***average height of irregularities*** (k) is much less the thickness of ***laminar*** sublayer (δ'), then the boundary is called hydrodynamically smooth.
- If the average height of irregularities (k) is much greater than the thickness of laminar sublayer (δ'), then the boundary is called hydrodynamically rough.
- On the basis of NIKURADSE'S EXPERIMENT the boundary is classified as:

Hydrodynamically smooth: $\frac{k}{\delta'} < 0.25$

Boundary in transition: $0.25 < \frac{k}{\delta'} < 6.0$

Hydrodynamically Rough: $\frac{k}{\delta'} > 6.0$

- $\frac{R}{k}$ is known as specific roughness. where 'k' is average height of roughness and 'R' is radius of the pipe.

Velocity Distribution For Turbulent Flow In Pipes

- Prandtl's universal velocity distribution equation

$$v = v_{max} + 2.5V^* \log_e\left(\frac{y}{R}\right)$$

where $V^* = \sqrt{\frac{\tau_0}{\rho}}$ = shear or friction velocity.
y = distance from pipe wall
ρ = Density of fluid

- The above equation is valid for both smooth and rough pipe boundaries.
- Shear velocity is still an ambigous quantity.

- Karman - Prandtl Velocity distribution equation

(i) Hydro Dynamically Smooth pipe

$$\frac{v}{V^*} = 5.75 \log_{10}\left(\frac{V^* y}{\nu}\right) + 5.5$$

(ii) Hydro Dynamically Rough pipe

$$\frac{v}{V^*} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5$$

where
V^* = shear velocity
y = distance from pipe wall
k = average height of roughness
ν = kinematic viscosity.
v = Average velocity

- Velocity distribution in terms of mean velocity

$$\frac{v - V^*}{V^*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

The above equation is for ***both rough and smooth*** pipes.

Friction Factor

- Friction factor 'f' for laminar flow

$$f = \frac{64}{Re}$$ where Re = Reynolds number

- Friction factor (f) for turbulent flow in smooth pipes

$$f = \frac{0.316}{(Re)^{1/4}} \quad \left(4 \times 10^3 < Re < 10^5\right)$$

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10}\left(Re\sqrt{f}\right) - 0.8 \quad \left(5 \times 10^4 < Re < 4 \times 10^7\right)$$

- Friction factor (f) for turbulent flow in Rough pipes

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10}\left(\frac{R}{k}\right) + 1.74$$

This eq. shows that for rough pipes friction factor depends only on $\frac{R}{K}$ (Relative smoothness) and not on Reynolds number (Re)

- Friction factor for commercial pipes

$$\frac{1}{\sqrt{f}} - 2.0\log_{10}\left(\frac{R}{k}\right) = 1.74 - 2.0\log_{10}(1 + 18.7)\frac{R/k}{Re\sqrt{f}}$$

In this equation, K = equivalent sand grain roughness.

Remember: There is no specific relationship between f and R_e for transition flow in pipes.

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Dimensional Analysis 12

- Velocity potential = $[L^2 T^{-1}]$
  Stream function = $[L^2 T^{-1}]$
  Acceleration = $[LT^{-2}]$
  Vorticity = $[T^{-1}]$
- Total no. of variables influencing the problem is equal to the no. of independent variables plus one, one being the no. of dependent variable.
- **Buckingham π theorem** states that if all the n-variable are described by m fundamental dimensions, they may be grouped into (n - m) dimensionless π terms.
- Selection of 3 repeating variables from the geometry of flow, fluid properties and fluid motion.
- **Geometric similarity - similarity of shape**
  Kinematic similarity - similarity of motion
  Dynamic similarity - similarity of forces

Number	Equation	Significance
Reynolds No.	$\frac{F_i}{F_v} = \frac{\rho VL}{\mu}$	Flow in closed conduit pipe
Froude No.	$\sqrt{\frac{F_i}{F_g}} = \frac{V}{\sqrt{gL}}$	where a free surface is present, structure eg. weirs spillway, channels, etc. where *gravity* force is predominant.
Eulers No.	$\sqrt{\frac{F_i}{F_p}} = \frac{V}{\sqrt{\frac{p}{\rho}}}$	In cavitation studies.
Mach No.	$\sqrt{\frac{F_i}{F_e}} = \frac{V}{C}$	where fluid compressibility is important.
Weber No.	$\sqrt{\frac{F_i}{F_\sigma}} = \frac{V}{\sqrt{\sigma/\rho L}}$	In capillary studies.

Here, F_i = Inertia force F_v = Viscous force
F_p = Pressure force F_e = Elastic force
F_σ = Surface tension force

Reynolds Model Law

$$(Re)_m = (Re)_p \qquad \frac{\rho_r V_r L_r}{\mu_r} = 1$$

- Applications of Reynold's Model Law
  - Flow through small sized pipes.
  - Low velocity motion around automobiles and aeroplane.
  - Submarines completely under water.
  - Flow through low speed turbo machines.

Froude's Model Law

$$(F_r)_m = (F_r)_p \qquad \frac{V_r}{\sqrt{L_r g_r}} = 1$$

- Applications of Froude's Model Law
  - Open channels
  - Notches & weirs
  - Spill ways & dams
  - Liquid jets from orifice
  - Ship partially submerged in rough & turbulent sea

Distorted model law

Vertical dimension simulates fronde law, while other two dimensions suit available space.

(i) Velocity scale ratio: $V_r = \sqrt{(L_r)_V}$

(ii) Area scale ratio: $A_r = (L_r)_H (L_r)_V$

(iii) Discharge scale ratio: $Q_r = (L_r)_H (L_r)_V^{3/2}$

(vi) Manning's Roughness Ratio : $n_r = \frac{R_r^{2/3}}{L_r^{1/2}}$

R = Hydraulic radius

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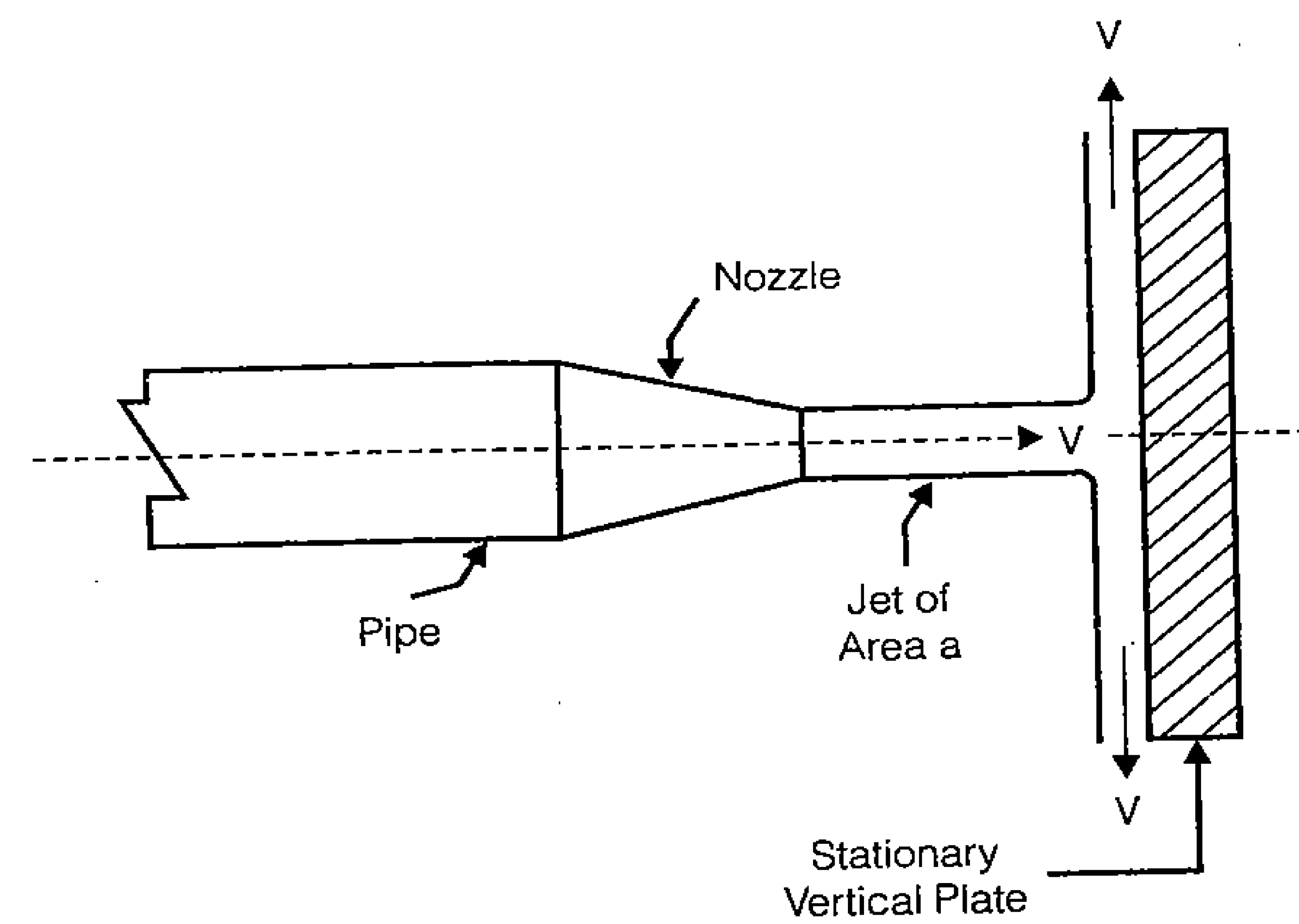
13 Impact of Jets

Impact of jet means the force exerted by a jet on a plate which may be stationary or moving. This force is obtained from ***impulse-momentum*** equation.

Force Exterted by Jet on a Stationary Plate

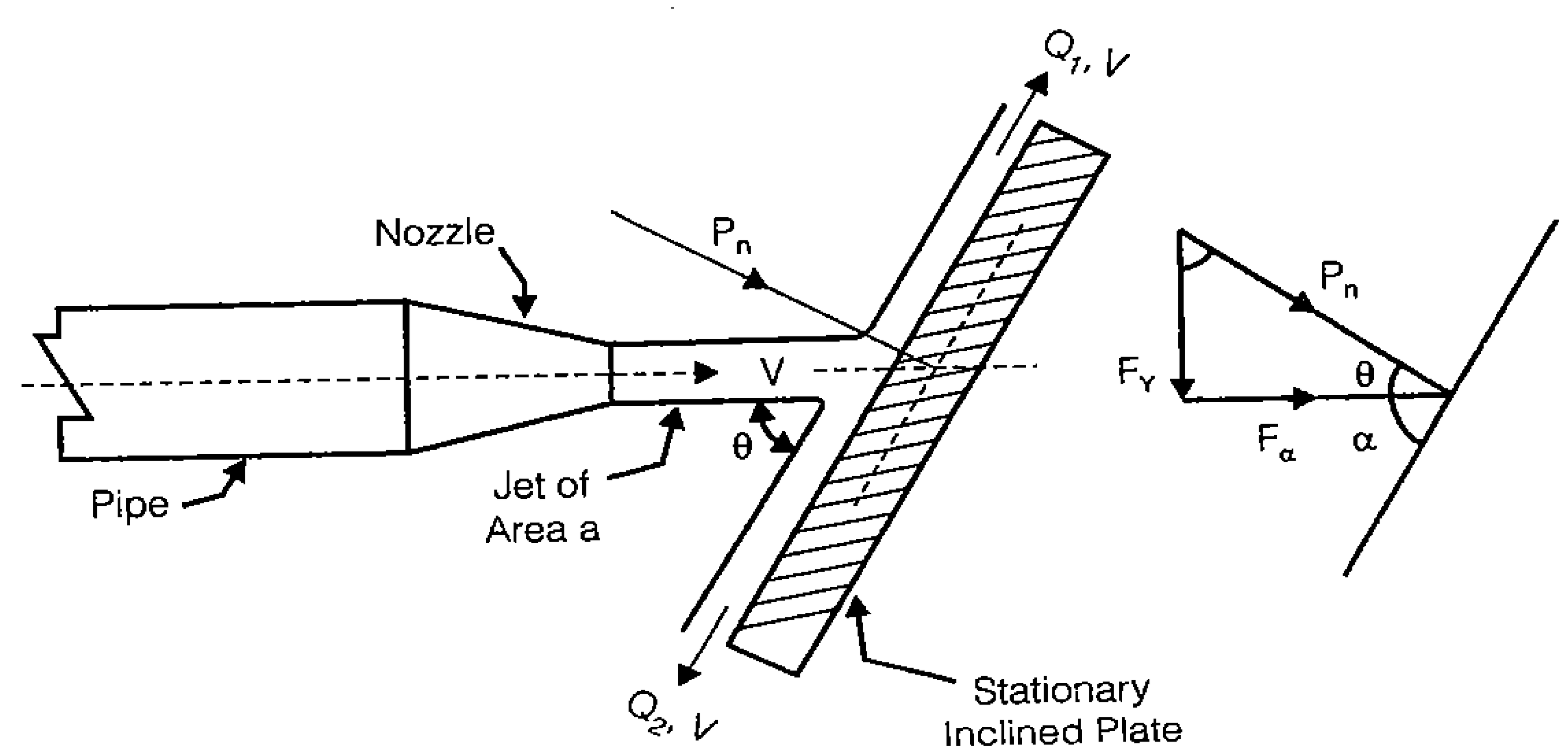
- **Plate is vertical to the jet**
  Jet Strikes normal to the flat stationary plate.



Force exerted by the jet normal to the plate

$P_n = \rho a V^2$
a = area of jet
V = velocity of jet

- **Plate is inclined to the jet**
  Jet strikes on an inclined stationary plate.



Force exerted by the jet normal to the plate

$$P_n = \rho\, a\, V^2 . \sin\theta \qquad Q_1 = \frac{Q}{2}(1+\cos\theta)$$

$$Q_2 = \frac{Q}{2}(1-\cos\theta) \qquad \frac{Q_1}{Q_2} = \frac{1+\cos\theta}{1-\cos\theta}$$

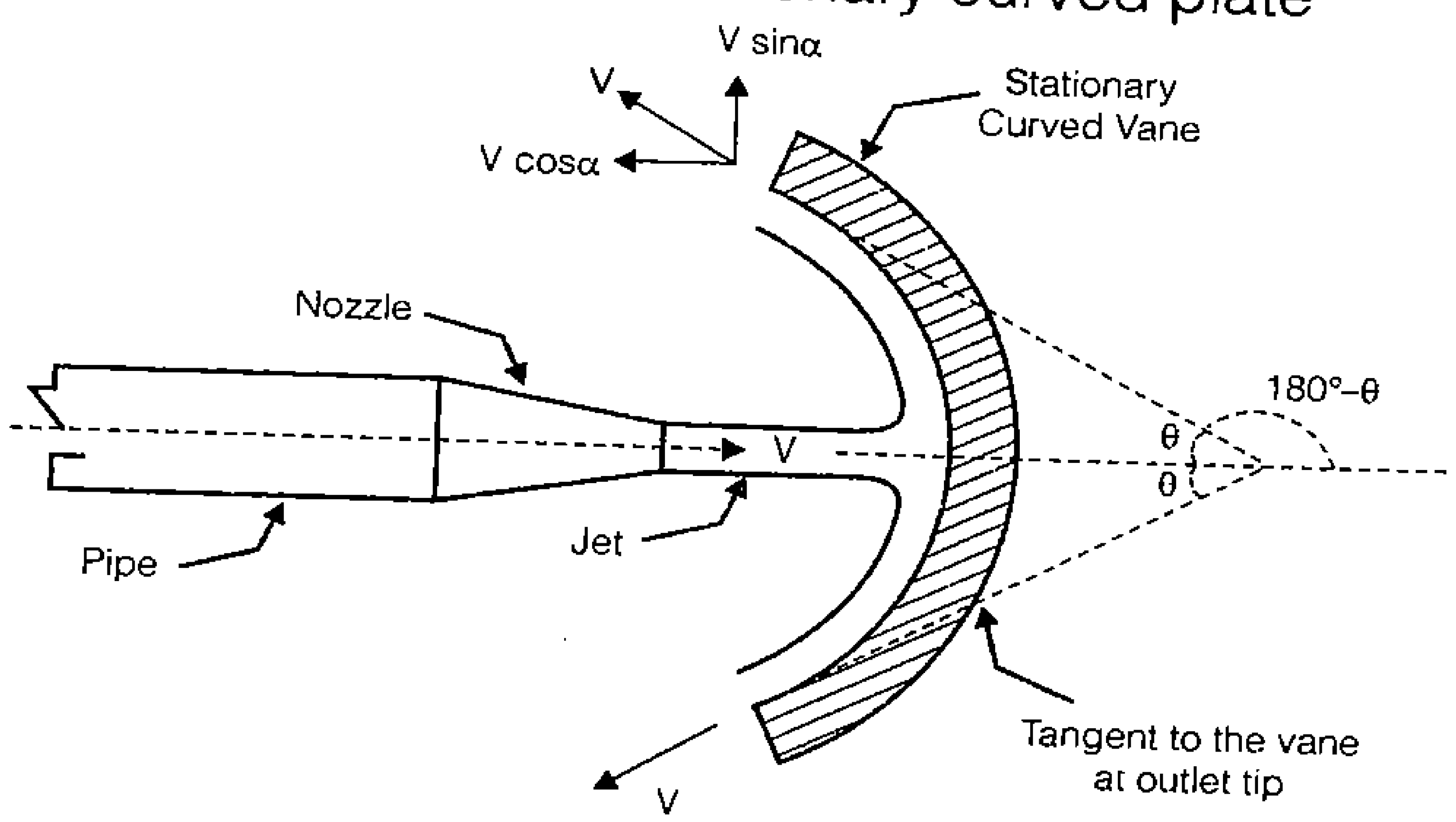
$$P_x = \rho a V^2 \sin^2\theta$$

$$P_y = \rho a V^2 \sin\theta \cos\theta$$

> **NOTE** Whenever jet strikes a flat plate, its resultant always act normal to the plate.

- **Plate is curved**

  Jet striking on a symmetrical stationary curved plate



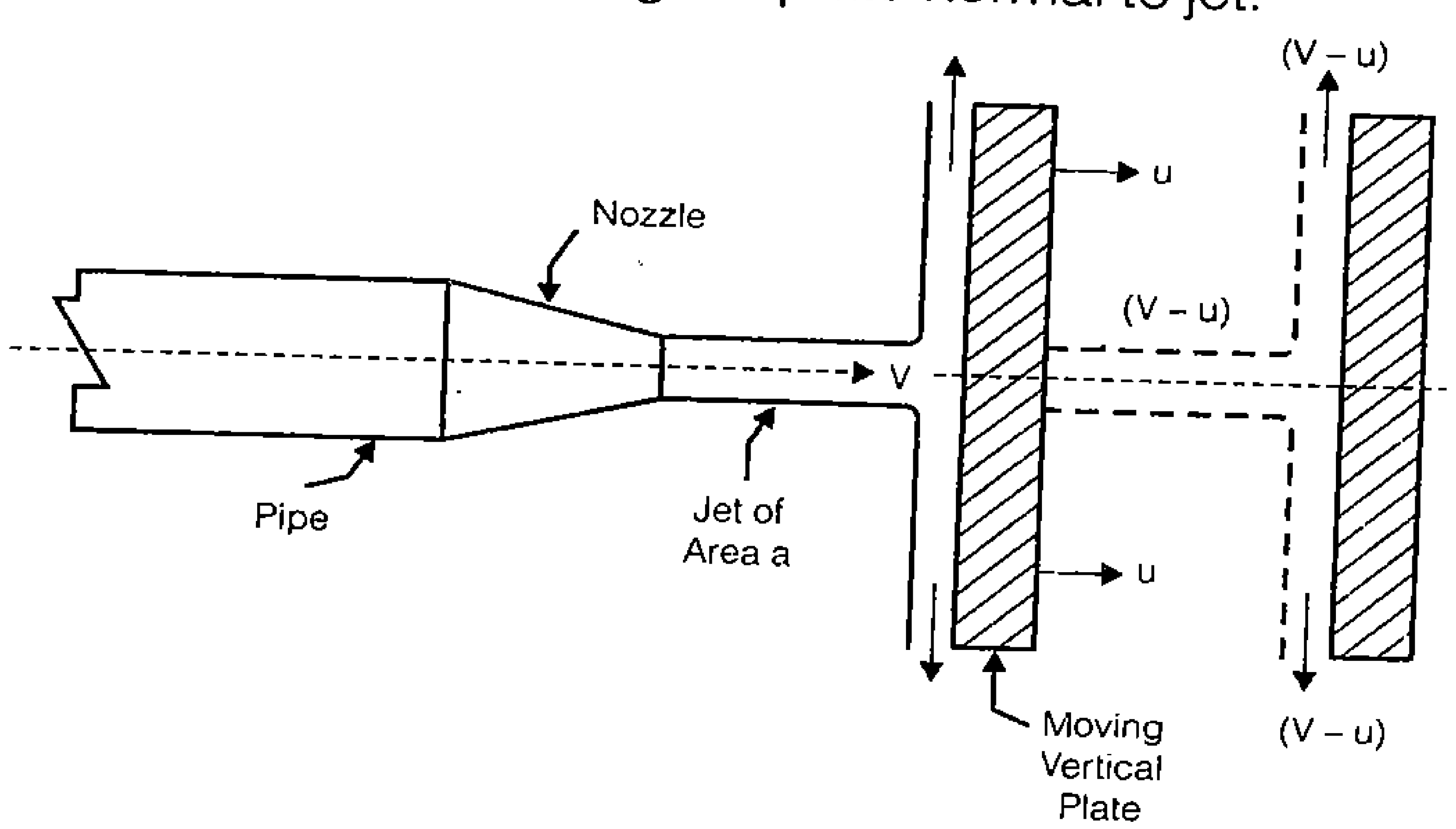
$$P_n = \rho a V^2 (1+\cos\theta)$$

- Force exerted by a jet in its direction of flow on a curved vane is ***always greater*** than that exerted on a flat plate.
- Angle of deflection = $(180 - \theta^\circ)$

Force Exerted by Jet On a Moving Plate

- **Plate is vertical to the jet**

  Force exerted by jet on moving flat plate normal to jet.



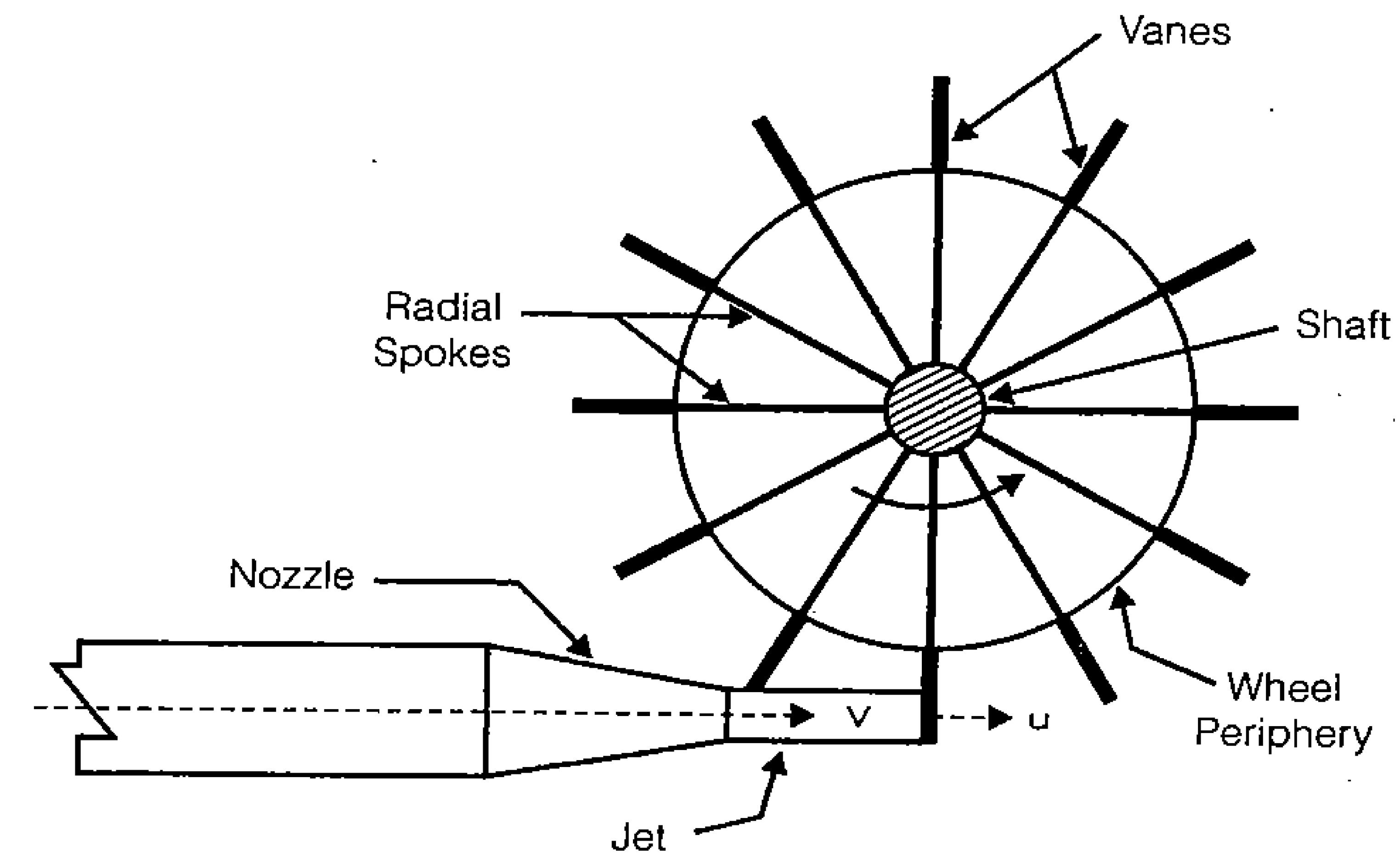
u = plate velocity

$$P_n = \rho a (V-u)^2$$

Work done per second (W) = $P_n \times u = \rho a [V-u]^2 \times u$

- **Plate mounted on the periphery of wheel**

  Jet strikes on series of flat plat mounted on the periphery of wheel.



$$P_n = \rho a V (V-u)$$

Work done by the jet = $P_n \times u$

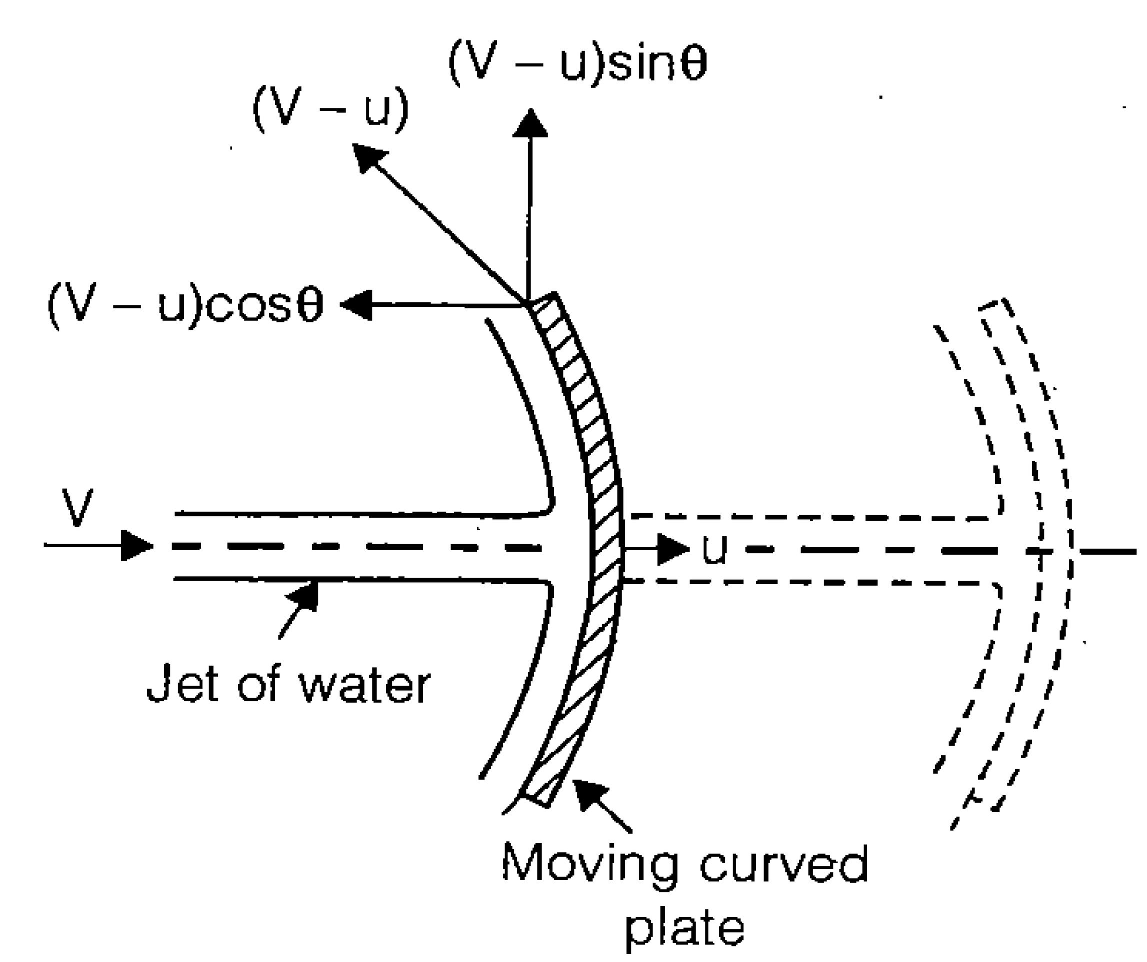
$$W = \rho a V (V-u) u$$

Efficiency of the work done of wheel $\eta = \dfrac{2u(V-u)}{V^2}$

When peripheral velocity will be half of the velocity of jet i.e.

$u = \dfrac{V}{2}$ then efficiency will be maximum i.e. **$\eta_{max} = 50\%$**

- **Curve plate when the plate is moving in the direction of jet**



Force exterted by the jet of water on the curved plate in the direction of the jet.

$$P = \rho a(V-u)^2(1+\cos\theta)$$

Work done by the jet on the plate per second

$$W = \rho a(V-u)^2 \times u[1+\cos\theta]$$

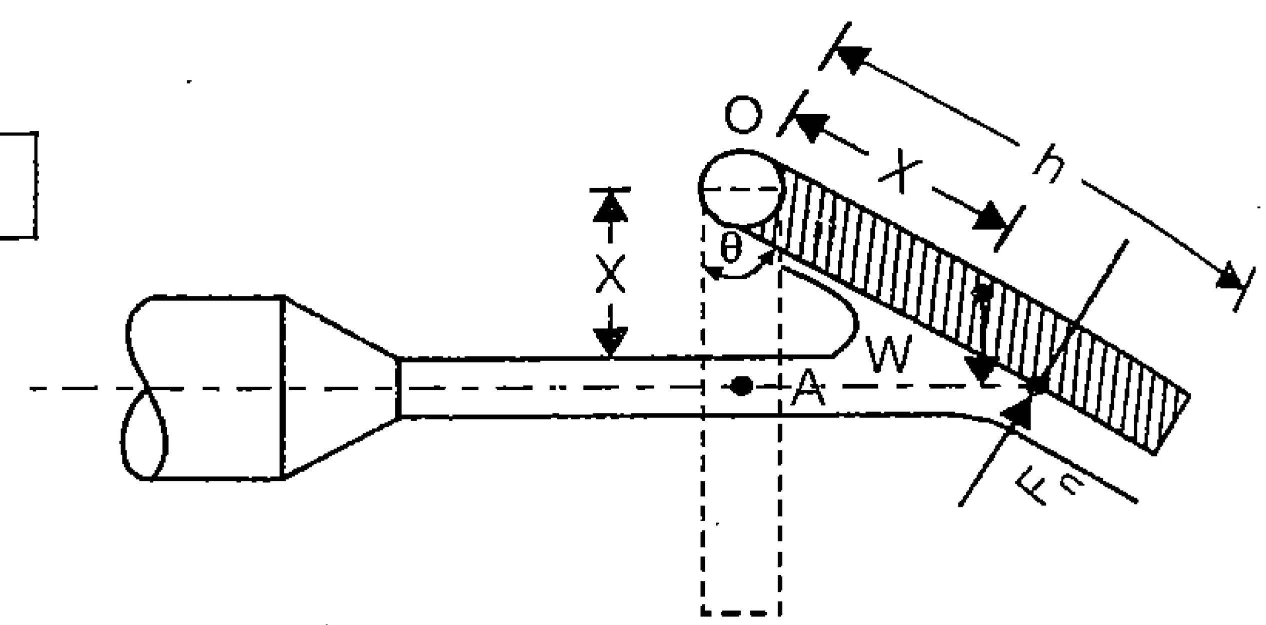
Force Exerted By a Jet on a Hinged Plate

- Force due to jet of water, normal to the plate

$$P_n = \rho a V^2 \sin(90° - \theta)$$

- For equilibrium of plate

$$\sin\theta = \frac{\rho a V^2}{W}$$



Concept of Tangential Flow Runners

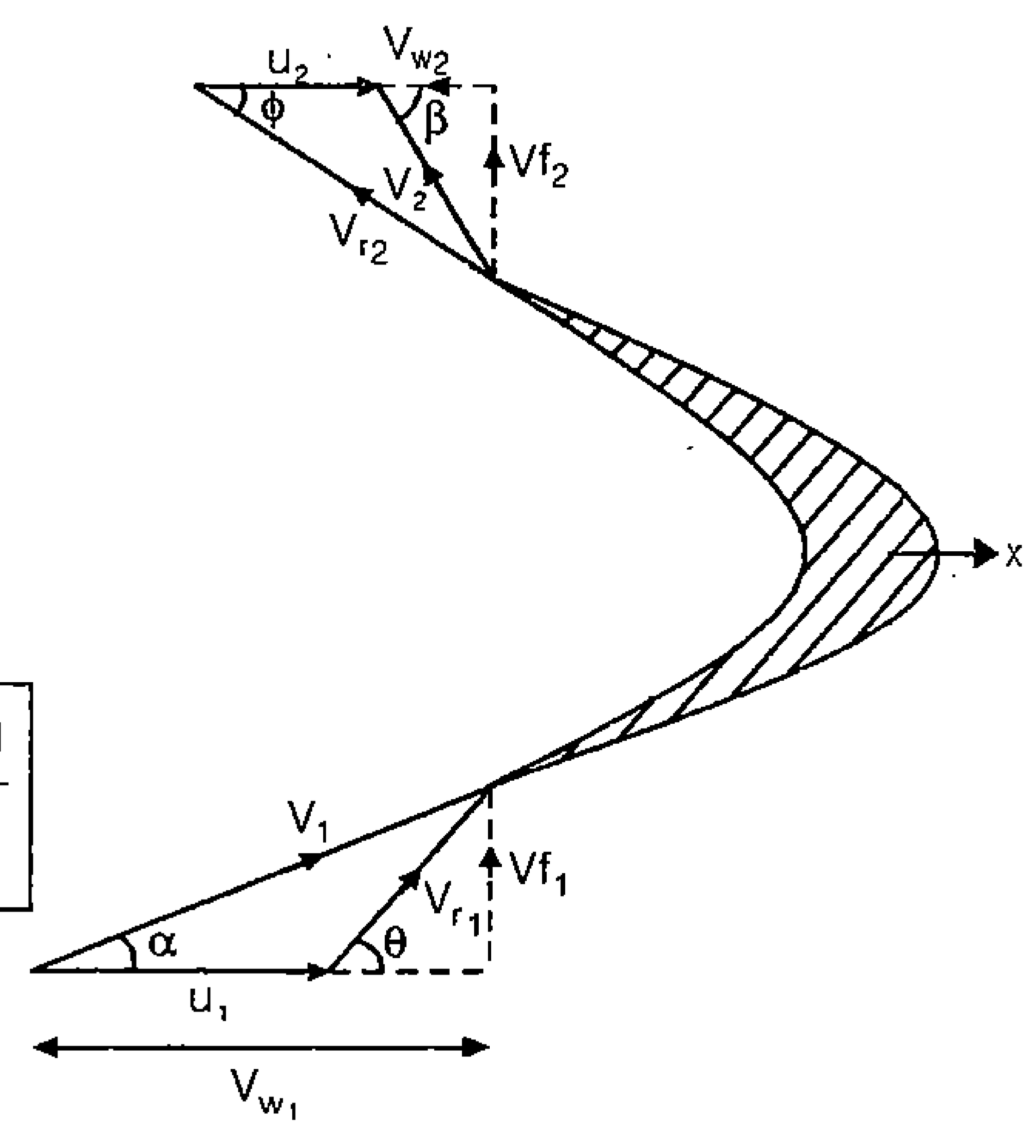
$$u_1 = u_2 = \frac{\pi DN}{60} = u$$

$$P = \dot{m}\left(V_{w_1} + V_{w_2}\right)u$$

$$\eta = \frac{\text{Power obtained}}{\text{Power provided}}$$

$$\eta_{\text{system}} = \frac{P}{\frac{1}{2}\dot{m}_{\text{nozzle}}V_1^2} = \frac{\dot{m}\left(V_{w_1} + V_{w_2}\right)u}{\frac{1}{2}(\rho a v_1)V_1^2}$$

$(\dot{m} = \rho a v r_1)$

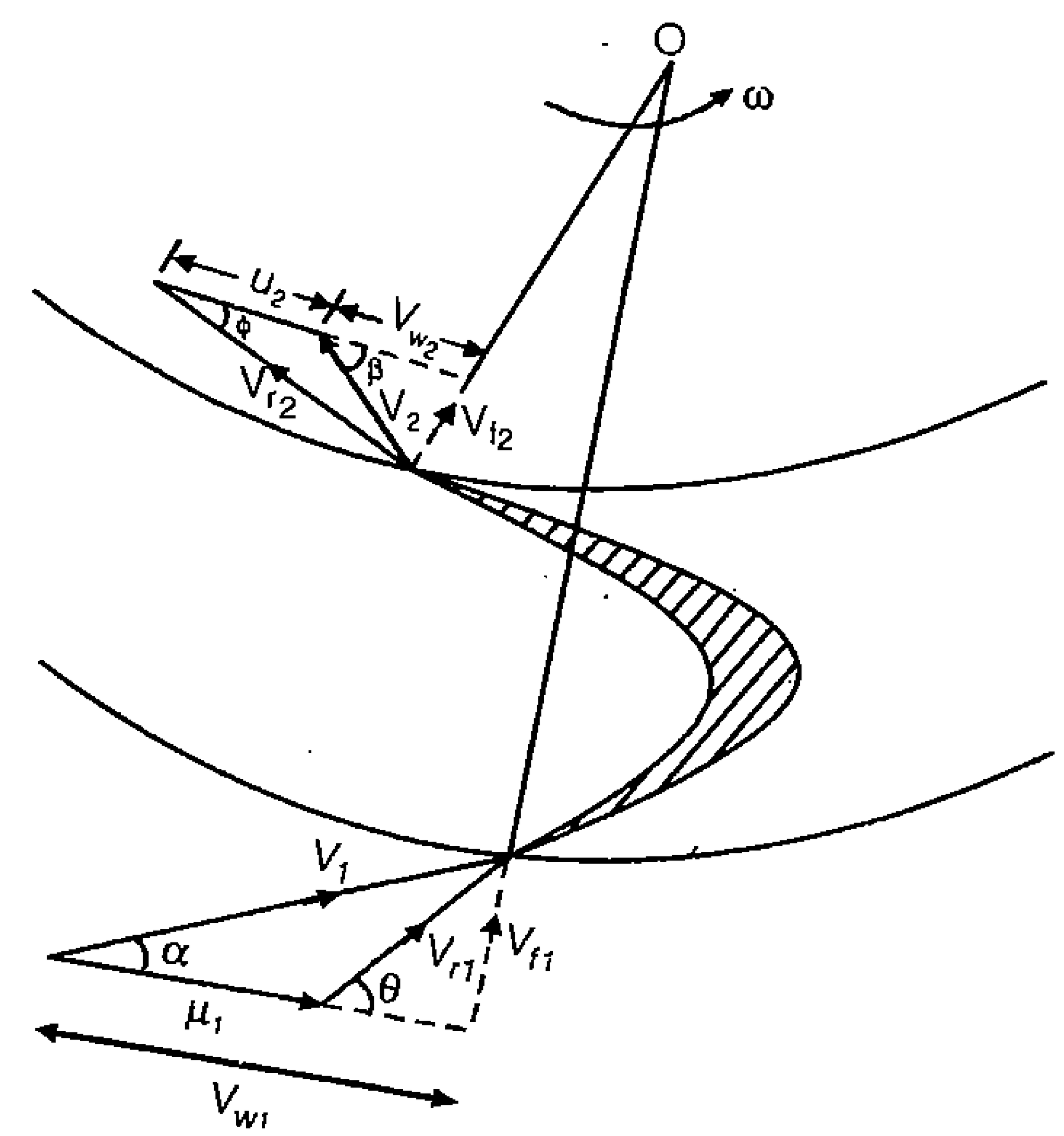


Concept of radial flow runners

Flow of water can be inward or outward.

Inward flow radial runner $R_1 > R_2$

$$\left.\begin{aligned} u_1 &= R_1\omega \\ u_2 &= R_2\omega \end{aligned}\right\} u_1 > u_2$$



$$\dot{w} = m\left(V_{w_1}u_1 + V_{w_2}u_2\right)$$

$$\eta_B = \frac{\dot{m}(V_{w_1}u_1 + V_{w_2}u_2)}{\frac{1}{2}\dot{m}_{\text{nozzle}}V_1^2}$$

$$\dot{m}_{\text{nozzle}} = \rho a V_1^2, \dot{m} = \rho a V_{r_1}$$

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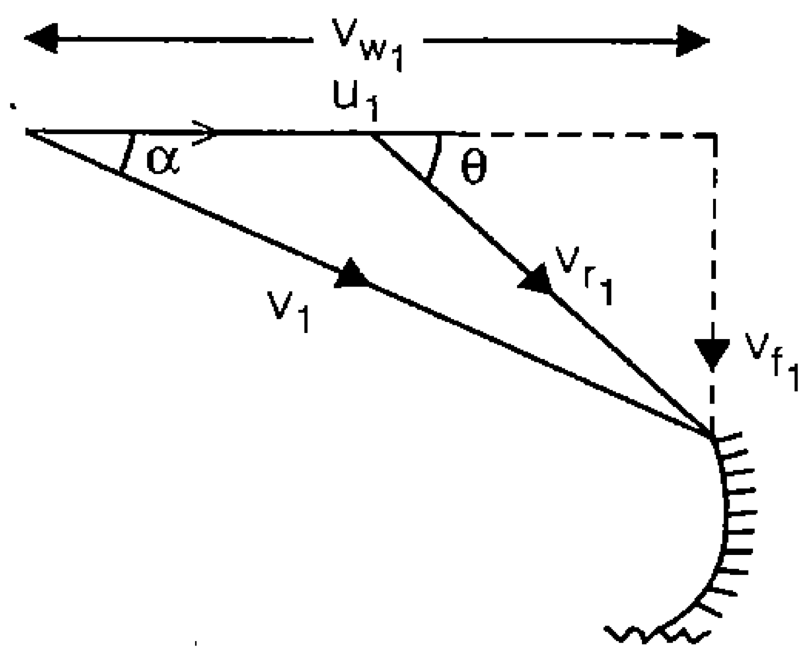
Turbines

14

Velocity Diagram

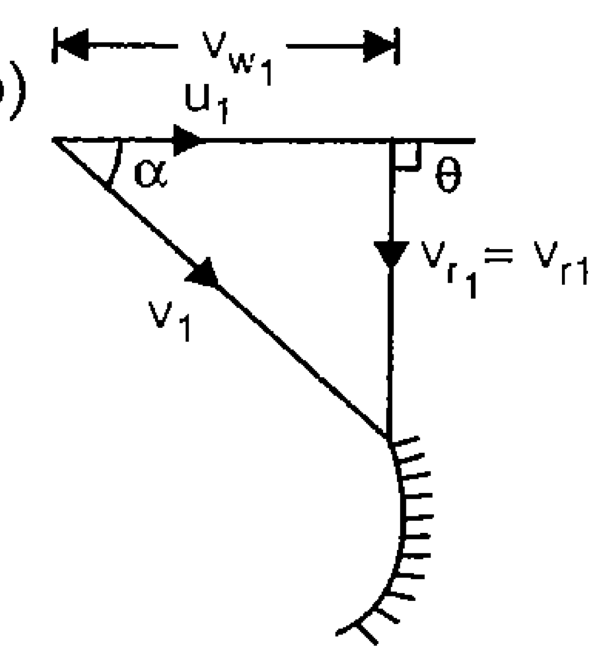
1. At Inlet

(a).



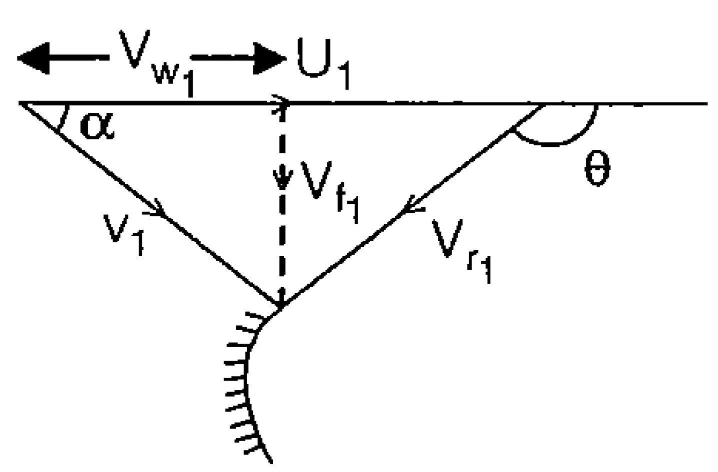
- $\theta < 90°$
- Forward vanes
- Common case of
- Francis Turbine
- $\boxed{\tan\theta = \dfrac{V_{f_1}}{V_{w_1} - U_1}}$

(b)



- $\theta = 90°$
- Radial vanes
- $U_1 = V_{w_1}$
- $V_{F_1} = V_{r_1}$
- $\boxed{\tan\theta = \dfrac{V_{f_1}}{V_{w_1}}}$

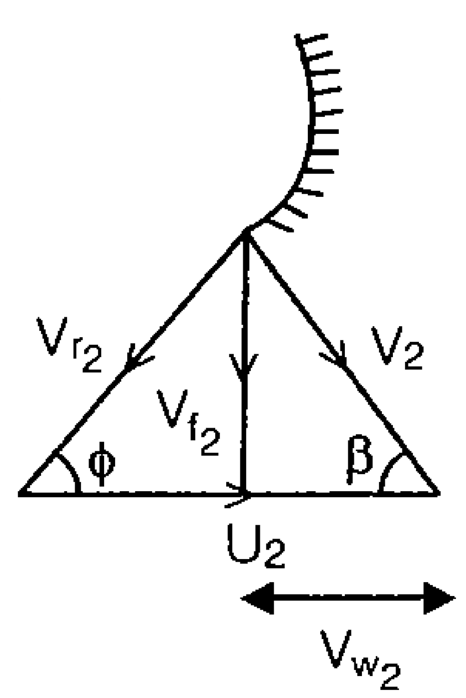
(c)



- $\theta > 90$
- Backward vanes
- $\tan(180° - \theta) = \dfrac{V_{f_1}}{U_1 - V_{W_1}}$

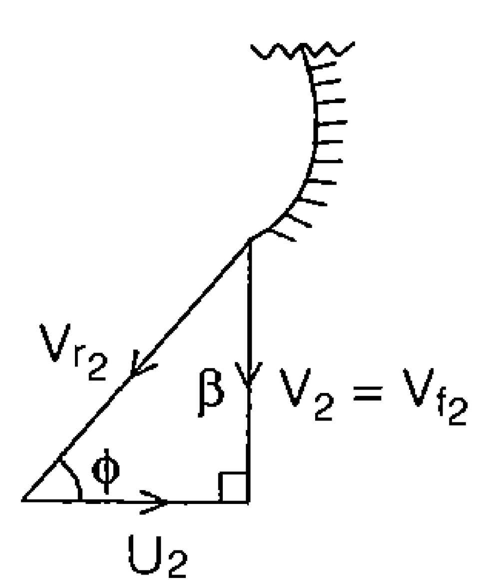
2. At Outlet

(a)



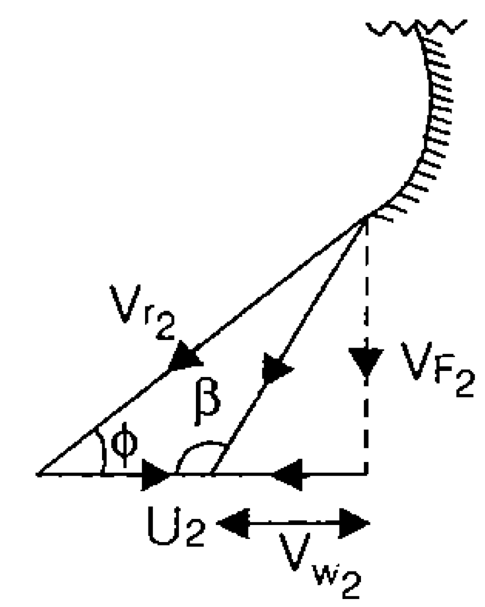
- $\beta < 90°$
- V_{W_2} is in the direction of U_2.

(b)



- $\beta = 90°$
- Common case of Francies turbine
- Use fuel for maximum efficiency.

(c)



- $\beta > 90°$
- Common case of Pelton wheel turbine

Here, α = guide vane angle between V_1 and U_1 at Inlet.
β = guide vane angle between V_2 and U_2 at outlet.
θ = Runner vane angle between V_{r_1} and U_1 at Inlet.
ϕ = Runner vane angle between V_{r_2} and U_2 at outlet.
V_1 = Absolute velocity of water at Inlet.
V_2 = Absolute velocity of water of outlet.

u_1 and u_2 are relative velocities at inlet and outlet respectively.
V_1 and V_2 are tangential also called circumferential velocity at inlet and outlet respectively.

Francis Turbine

1. Powers

(i) $\boxed{H.P = wQH}$ in kW

$= \dfrac{\rho QH}{75}$ in HP

H.P = Hydraulic power, also called water power
w = Specific weight = ρg

It is also denoted by 'γ'

where γ = unit wt or specific wt
1 H.P = 746 walt (British H.P)
1 H.P = 736 walt (Metric H.P)
ρ = density, Q = Discharge,
H = Head = $H_g - H_F$
H_g = Gross Head, H_F = Fricloss.

(ii) $\boxed{R.P = \rho Q[V_{w_1}U_1 - V_{w_2}U_2]}$

$= \dfrac{wQ}{g}[V_{W_1}U_1 - V_{W_2}U_2]$ R.P = Runner power

For maximum efficiency $V_{W_2} = 0$.

(a) For maximum efficiency

$$\boxed{R.P = \frac{wQ}{g}(V_{w_1 u_1})}$$

V_{w_1} = Whirl velocity at Inlet.
V_{w_2} = Whirl velocity at outlet.

(iii) $\boxed{S.P = \tau\omega}$ S.P = R.P – Mechanical friction losses

Where, τ = Torque produced by shaft
ω = Angular velocity of shaft
S.P = Shaft power.

> **NOTE** In Francies Turbine H.P > R.P > S.P

2. Efficiency

(i) $\boxed{\eta_h = \frac{R \cdot P}{H \cdot P}}$ $\Rightarrow \eta_h = \frac{(V_{w_1 u_1} - V_{w_2 u_2})}{gH}$

η_h = Hydraulic efficiency

(ii) $\boxed{\eta_m = \frac{S \cdot P}{R \cdot P}}$ $= 1 - \frac{\text{loss}}{R \cdot P}$

η_m = Mechanical efficiency.

(iii) $\boxed{\eta_0 = \frac{S \cdot P}{H \cdot P}}$ $\eta_o = \eta_m \cdot \eta_h$.

η_o = Overall efficiency.

(iv) $\boxed{\eta_{vol} = \frac{Q - \Delta Q}{Q}}$ η_{vol} = Volumetric efficiency
Q = Discharge (m^3/s)
and ΔQ = Charge in discharge (m^3/s).

(v) $\boxed{\eta_b = \frac{v_1^2 - v_2^2}{v_1^2}}$ η_b = Blade efficiency.

V_1 and V_2 are absolute velocity of water at inlet and outlet respectively.

(vi) $\boxed{\eta_o = \eta_h \cdot \eta_m \cdot \eta_v \cdot \eta_b}$ η_o = Overall efficiency.

3. Design Parameter of Francis Turbine

(i) $\boxed{\frac{D_1}{D_2} \sim 2}$ D_1 and D_2 are dia of inlet and outlet respectively.

(ii) For maximum efficiency, $V_{w_2} = 0$.

(iii) $\boxed{\phi = \frac{u_1}{\sqrt{2gH}} \simeq 0.6 + 0.9}$

ϕ = Speed ratio
U_1 = Tangential also called circumferential velocity of inlet.
H = Head = $H_g - H_f$
$\sqrt{2gH}$ = Spout velocity.

(iv) $\boxed{\psi = \frac{V_{f_1}}{\sqrt{2gH}} \simeq 0.15 \text{ to } 0.30}$

ψ = Flow ratio
V_{F_1} = Flow velocity of inlet.

(v) $\boxed{\eta = \frac{B_1}{D_1} \simeq 0.1 \text{ to } 0.45}$ η = width ratio

(vi) $\boxed{A_{f_1} = (1-k)\pi D_1 B_1}$ $\boxed{A_{f_2} = (1-k)\pi D_2 B_2}$

A_{f_1} and A_{f_2} are area of flow at inlet and outlet respectively.
K = Vane thickness coefficiency $\simeq$ 5%
D_1 and D_2 are diameter at inlet and outlet respectively.
B_1 and B_2 are width of plate at inlet and outlet respectively.

(vii) $\boxed{R = \frac{\frac{P_1}{\gamma} - \frac{P_2}{\gamma}}{\left(\frac{V_{w_1} u_1 - V_{w_2} u_2}{g}\right)}}$ $R = \frac{\frac{P_1}{\gamma} - \frac{P_2}{\gamma}}{\left(\frac{V_{w_1} u_1}{g}\right)}$ → For maximum efficiency

Here, R = Degree of reaction
γ = Unit weight or specific wt = rg
ρ = Density

Also, $R = 1 - \left(\frac{v_1^2 - v_2^2}{2v_{w_1} \cdot u_1}\right)$

(viii) $\boxed{H = \frac{V_2^2}{2g} + \frac{V_{w_1} U_1}{g}}$ (Master Formula) where, H = Head

Assumption, $V_{w_2} = 0, \frac{P_2}{\rho g} = 0,$

4. Model Relationship for Turbine

Dimensional Parameter	*Diamensionless Parameter*
1. $N_S = \frac{N\sqrt{P}}{(H)^{5/4}}$	1. $N_S = \frac{\omega\sqrt{\frac{P}{\rho}}}{(gH)^{5/4}}$

2. $C_H = \dfrac{H}{N^2D^2}$ 2. $C_H = \dfrac{gH}{\omega^2D^2}$

3. $C_Q = \dfrac{Q}{ND^3}$ 3. $C_Q = \dfrac{Q}{\omega D^3}$

4. $C_P = \dfrac{P}{N^3D^5}$ 4. $C_P = \dfrac{P}{\rho\omega^3D^5}$

Here, N_S = Specific speed, C_H = Head coefficient

C_Q = Discharge coefficient, C_P = Power coefficient

$g \rightarrow$ Acln. due to gravity = 9.81 m/s^2.

$w \rightarrow$ Angular speed = $\dfrac{2\pi N}{60}$, $N \rightarrow$ No. of revolution/minute

$H \rightarrow$ Head (m), $P \rightarrow$ Pressure (N/m^2)

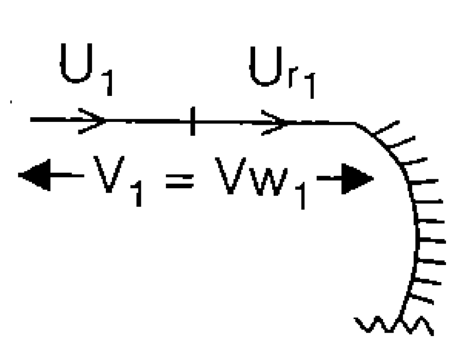
$\rho \rightarrow$ Density (kg/m^3), $D \rightarrow$ Diameter (m)

$Q \rightarrow$ Discharge (m^3/sec)

Pelton Wheel Turbine

1. Velocity Triangle

(i) At Inlet



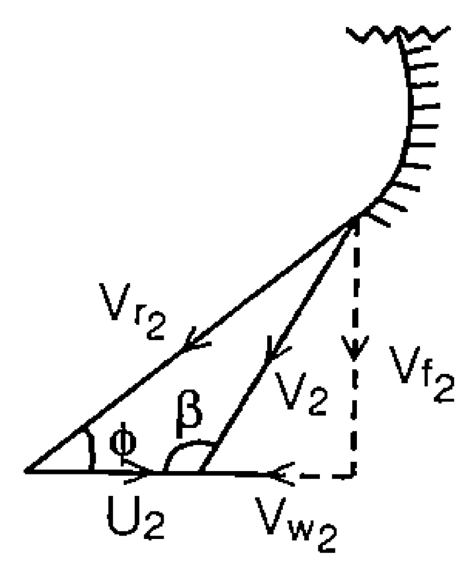
$V_1 = V_{w_1}$

$V_{F_1} = 0$

Here, $\alpha = 0°$

$\theta = 0°$

(ii) At outlet



- $\beta > 90°$
- $V_{r_2} = kV_{r_1}$
- k = Friction factor
- k = 1

For frictionless vane

- $|V_{r_2}\cos\phi| = |U_2| + |V_{w_2}|$

2. Power

(i) $H.P = WQH$ ... kW

$= \dfrac{\rho QH}{75}$ ... H.P

H.P = Hydraulic power.

(ii) Jet power or k.E/sec of Jet $= \dfrac{1}{2}\rho Q v_1^2 = \dfrac{1}{2}\cdot\dfrac{wQ}{g}\cdot v_1^2$

(iii) $R.P = \dfrac{wQ}{g}[V_{w_1} + V_{w_2}]u$

R.P = Runner power

(iv) S.P = R.P – Mechanical friction losses.

> NOTE: H.P > K.E/sec. of Jet > R.P > S.P

3. Efficiencies

(i) $\eta_{nozzle} = \dfrac{k\cdot E/\sec \text{ of jet}}{H\cdot P \text{ available of base of nozzle}}$

$= \dfrac{V_1^2}{2gH} = C_V^2$ $\left[\because v_1 = C_v\sqrt{2gH}\right]$

where, η_{nozzle} = nozzle efficiency.

(ii) $\eta_h = \dfrac{R\cdot P}{k\cdot E/\sec \text{ of Jet}}$

$\eta_h = \dfrac{2[V_{w_1} + V_{w_2}]u}{v_1^2} = \dfrac{2(v_1 - u)(1 + k\cos\phi)u}{v_1^2}$

$\eta_h\big|_{max} = \dfrac{(1 + k\cos\phi)}{2}$ where, η_h = Hydraulic efficiency.

$\eta_h\big|_{max}$ = Maximum hydraulic efficiency.

(iii) $\eta_m = \dfrac{S\cdot P}{H\cdot P}$ where, η_m = Mechanical efficiency.

(iv) $\eta_o = \dfrac{S\cdot P}{H\cdot P} = \eta_{nozzle}\cdot\eta_h\cdot\eta_{mechanical}$ where, η_o = overall efficiency

4. Design Criteria

(i) $\phi = \dfrac{u_1}{\sqrt{2gH}}$ 0.45 to 0.47 where, ϕ = Speed ratio.

(ii) $\psi = \dfrac{V_{f_1}}{\sqrt{2gH}}$ = zero where, ψ = Flow ratio

(iii) $m = \frac{D}{d} = 11 \text{ to } 15$ where, m = Jet ratio
D = Dia of pitch circle
d = Dia of jet.

(iv) $n = \left(15 + \frac{m}{2}\right)$ this is Tygon formula.

= 18 to 25 where, η = Number of vanes

(v) $\text{No. of jet} = \frac{\text{Total discharge through penstock}}{\text{Discharge through each jet}} \not> 6$

Keplan and propeller turbine (Axial Flow reaction turbine)

1. $Q = A_{F_1}.V_{F_1} = A_{F_2}.V_{F_2}$ where, Q = Discharge

$$A_{F_1} = \frac{\pi}{4}(D_0^2 - D_b^2)$$

2. $U_1 = U_2 = \frac{\pi DN}{60}$ where, D = D_o at expressed edge
D = D_b at inner edge

and $D = \frac{D_o + D_b}{2}$ at mid point

D_o = Outer dia of runner

D_b = Dia of hub or boss.

NOTE: The analysis of velocity triangle, powers. Are Similar to that of francies turbines. In this case generally $\theta > 90°$ and $V_{w_2} = 0$.

Master formular (Beronoulies principle) can be applied.

3. **Design Parameter**

(i) $\phi = \frac{U_1}{\sqrt{2gH}}$ is of the order of 2.0

(ii) $\psi = \frac{V_{f_1}}{\sqrt{2gH}}$ = 0.5 to 0.7

(iii) Number of vanes on the runner are generally 3 to 8.

Impulse turbine (Pelton): High head and low discharge.
Francis turbine: Medium head and medium discharge.
Kaplan and Propeler turbine: Low head and high discharge.

Turbine	Specific speed, N_s,(MKS)
1. Pelton wheel turbine (single jet)	10 - 35
2. Pelton wheel turbine (multiple jet)	35 - 60
3. Francis turbine	60 - 300
4. Kaplan turbine	> 300

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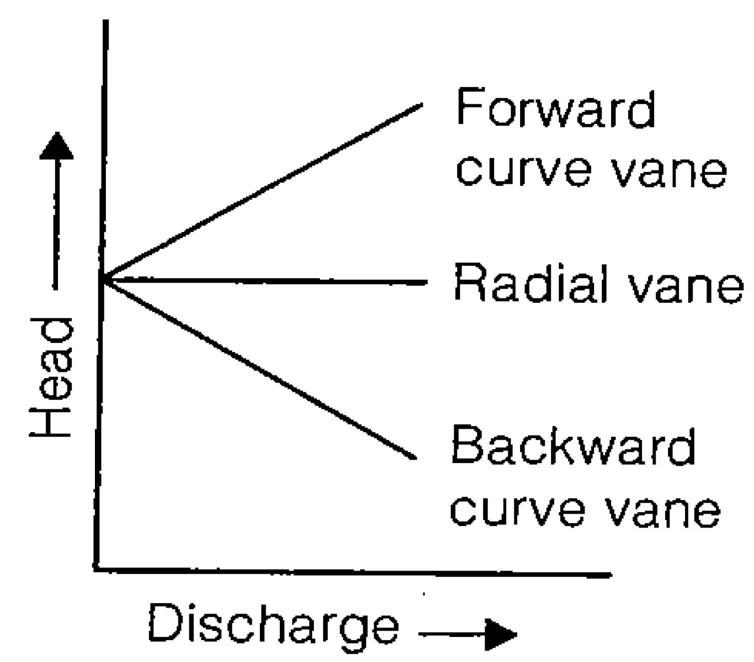
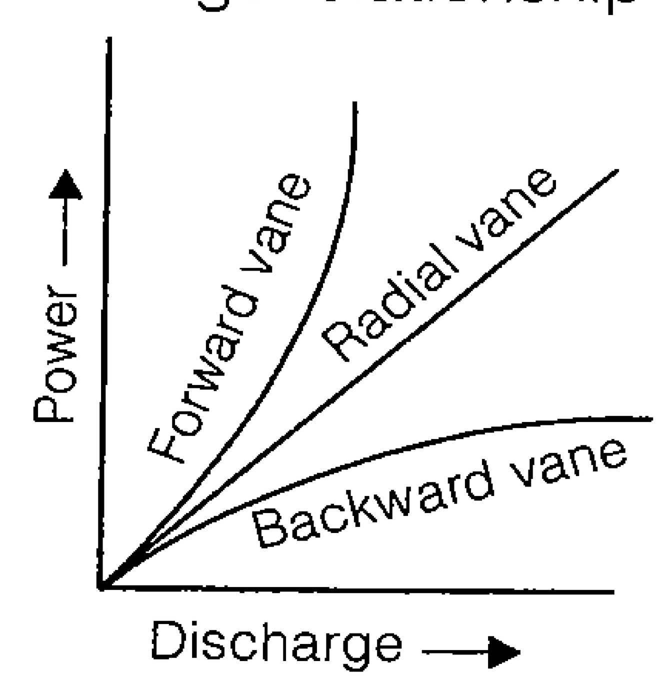
15 Hydraulic Pumps

Centrifugal pump

- Centrifugal pump is reverse of inward flow reaction turbine. It works on principle of forced vortex motion. It has high discharging capacity and can be used for lifting highly viscous liquids e.g. sewage water, chemicals etc.
- Priming is an operation in which liquid is completely filled in the chamber of pump so that air or gas or vapour from the portion of pump is driven out & no air pocket is left.
- In volute pump cross sectional area results in developing a uniform velocity throughout the casing & free vortex is formed.
- Centrifugal pump has high output and high efficiency.
- Head Vs discharge and Power Vs discharge relationship





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Types of Pump	Range of Head
Low head pump	upto 15m head
Medium head pump	15 m to 40 m
High head pump	above 40 m

-

Pump	Specific Speed
Radial Flow	10 to 80
Mixed Flow	80 to 160
Axial Flow	160 to 450

- The specific speed of a centrifugal pump may be defined as the speed in revolution per minute of a geometrically similar pump of such a size that under corresponding conditions it would deliver 1 liter of liquid per second against of a head of 1m.

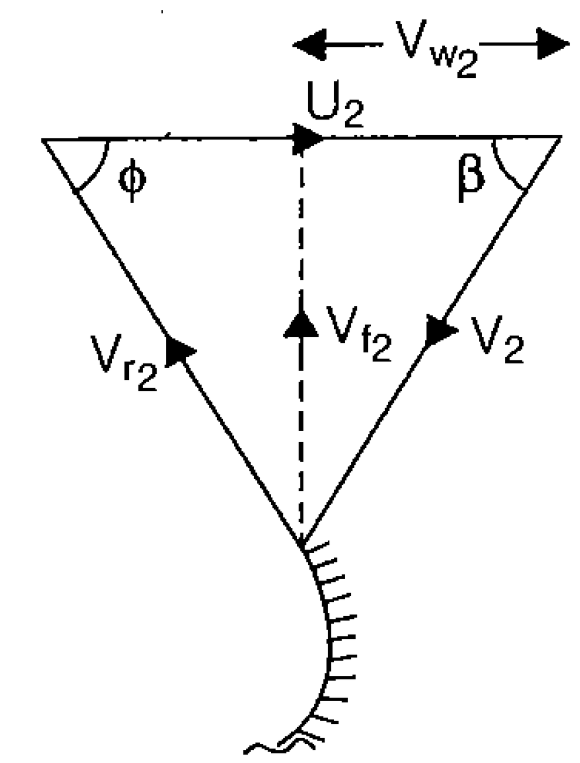
For multi stage $(H_m) = \dfrac{\text{Total head}}{\text{No. of stage}}$

Specific speed $(N_s) = \dfrac{N\sqrt{Q}}{(H_m)^{3/4}}$

- For optimum efficiency impeller should be designed such that whirl velocity at inlet is zero. It means discharge should enter in the pump radially $\left(V_{w_1} = 0\right)$.
- **Velocity Triangle**

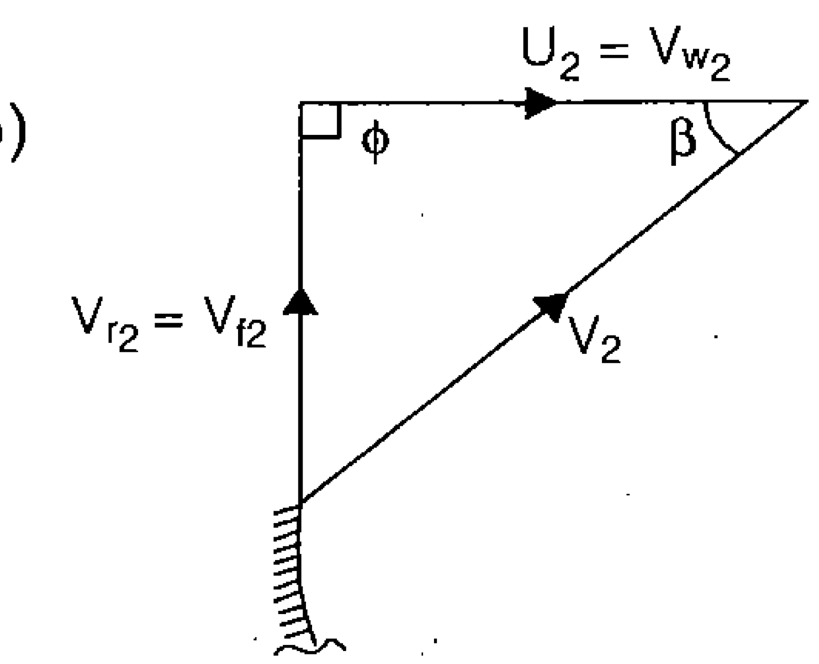
(i) At Outlet

(a)



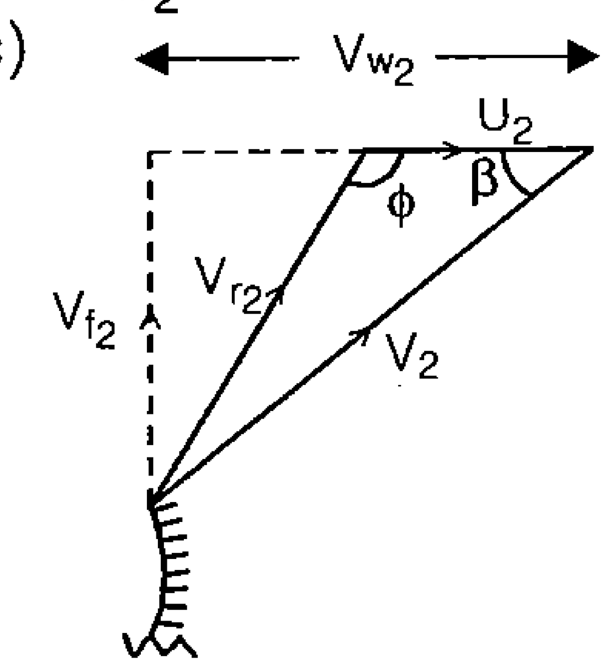
- $\phi < 90°$
- Backward vanes
- Common case of centrifugal pump.
- More efficient
- $\tan\phi = \dfrac{V_{f_2}}{U_2 - V_{W_2}}$
- $V_{F_2} = V_2 \sin\beta$.
- $V_{W_2} = V_2 \cos\beta$.

(b)



- $\phi = 90°$
- Radial vanes
- $V_{r_2} = V_{F_2}$
- $U_2 = V_{w_2}$

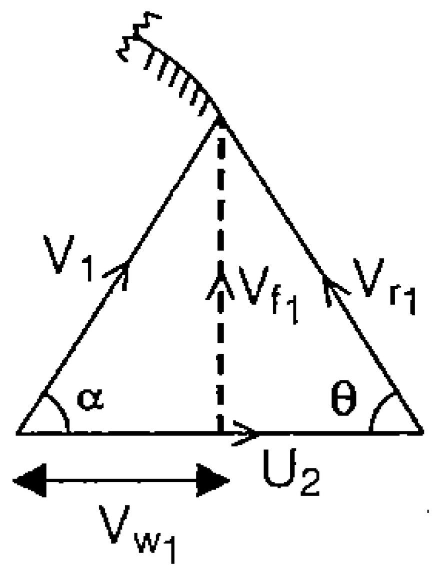
(c)



- $\phi < 90°$
- Forward vanes
- $\tan(180° - \phi) = \dfrac{V_{f_2}}{(V_{w_2} - U_2)}$
- $V_{f_2} = V_2 \sin\beta$
- $V_{w_2} = V_2 \cos\beta$

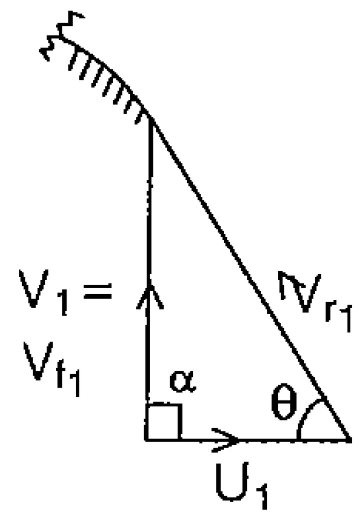
(ii) At Inlet

(a)



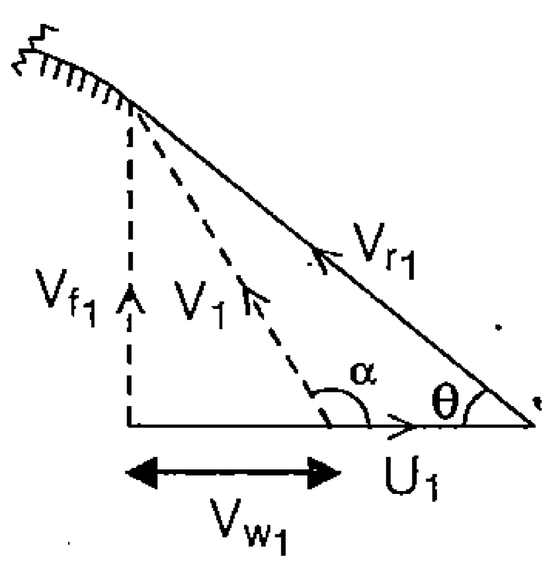
- $\alpha < 90°$
- V_{W_1} is in the direction of U_1
- $\tan\theta = \dfrac{V_{f_1}}{U_1 - V_{W_1}}$
- $V_{w_1} = V_1 \cos\alpha$
- $V_{F_1} = V_1 \sin\alpha$

(b)



- $\alpha = 90°$
- $V_{w_1} = 0$
- $\tan\theta = \dfrac{V_{f_1}}{U_1} = \dfrac{V_1}{U_1}$
- Common case of centrifugal pump.

(c)



- $\alpha > 90°$
- V_{w_1} is in the opposite direction of U_1
- $\tan\theta = \dfrac{V_{f_1}}{|U_1| + |V_{w_1}|}$

Here, θ = Impeller vane angle of inlet (Angle between V_{r_1} and U_1)
ϕ = Impeller vane angle of outlet (Angle between V_{r_2} and U_2)
α = Angle between V_1 and U_1 at Inlet (Blade Angle)
β = Angle between V_2 and U_2 of outlet (Blade Angle)
V_1 = Absolute velocity of water at inlet
V_2 = Absolute velocity of water at Inlet.

V_{r1} and V_{r2} are relative velocity of water at Inlet and outlet respectively.
U_1 and U_2 are tangential also called circumferential velocity at inlet and outlet respectively.

- **Design parameters of pump**

1. (a) $A_{f_1} = \pi D_1 B_1$ ... neglecting vane thickness
$= (1-k)\pi D_1 B_1$

Here, A_{f_1} = Area of flow at inlet
k = Vane thickness coefficient $\simeq$ 5%

(b) $A_{f_2} = \pi D_2 B_2$ ... neglecting vane thickness
$= (1-k)\pi D_2 B_2$

Here, A_{f_2} = Area of flow of outlet
k = Vane thickness coefficient
D_1 = Dia of Impeller of Inlet
D_2 = Dia of impeller of outlet
B_1 = Width of impeller of inlet
B_2 = Width of impeller of outlet.

2. $Q = A_{f_1}V_{f_1} = A_{f_2}V_{f_2}$ Here, Q = Discharge

V_{f_1} and V_{f_2} is velocity of flow at inlet and outlet respectively.

3. Number of vanes may be 6 to 12.

4. $\phi = \dfrac{U_2}{\sqrt{2gHm}}$ Here, ϕ = Speed ratio
g = 9.81 m/s²
H_m = Manometric Head.

5. $\psi = \dfrac{V_{f2}}{\sqrt{2gHm}}$ Here, ψ = Flow ratio

6. $\dfrac{D_2}{D_1} \simeq 0$

7. $\omega_{win} = \sqrt{\dfrac{8gHm}{D_2^2 - D_1^2}}$ ω = Angular speed in rad/sec.
$= \dfrac{2\pi N}{60}$ 'N' is in revolution/min (r.p.m)

8. If $D_2 = 2D_1$ than $D_2 = \dfrac{10.23}{\omega}\sqrt{H_m}$

- **Powers In Pump**

(i) $S.P = T\omega$ Here, S.P = Shaft power

$\omega = \dfrac{2\pi N}{60}$ rad/sec.
ω = Angular speed of shaft
T = Torque produced in the shaft.
N in r.p.m (revolution per minute)

(ii) (a) $I.P = \dfrac{wQ}{g}[V_{w_2}U_2 - V_{w_1}U_1]$

(b) $I.P|_{max} = \dfrac{wQ}{g}[V_{w_2}U_2]$ [Why $V_{w_1} = 0$]

(c) $\boxed{I.P = S.P - \text{Mechanical Frictional losses}}$

Here, I.P = Impeller power, θ = Discharge, $\omega = \rho g$
= γ = unit weight or specific weights
= weight per unit volume (kN/m^3)

(iii) $\boxed{M.P = wQH_m}$ ... kW

$$= \frac{\rho Q H_m}{75} \text{ ... H.P}$$

Here, M.P = Manometric power,
$\omega = \rho g = \gamma$ in kN/m^3 = Specific weight
H_m = Manometric head, ρ = Density in kg/m^3.

- S.P > I.P > M.P
- Manometric Head (H_m)

(i) $H_m = h_s + h_d + h_{fs} + h_{fd} = H_s + H_d$
h_s = Suction head, h_d = delivery head
h_{fs} = Friction head loss in suction pipe
h_{fd} = Friction head loss in delivery pipe
Hs = Static head = $h_s + h_d$.
H_F = Total friction loss = $h_{fs} + h_{fd}$.

(ii) $\boxed{H_m = \frac{P_d}{\rho g} - \frac{P_s}{\rho g} \text{ or } \frac{P_d}{\gamma} - \frac{P_s}{\gamma}}$ P_d = Pressure in delivery pipe
P_s = Pressure in suction pipe.

(iii) $\boxed{H_m = \frac{V_{w_2} \cdot U_2}{g}}$

- Efficiencies of the Pump

(i) $\boxed{\eta_{mech.} = \frac{I \cdot P}{S \cdot P}}$ where, η_{mech} = Mechanical efficiency

(ii) $\boxed{\eta_{man} = \frac{M \cdot P}{I \cdot P}}$ where, η_{man} = Manometric efficiency

$$= \frac{gH_m}{(V_{w_2}U_2 - V_{w_1}U_1)}$$

(iii) $\boxed{\eta_o = \frac{M \cdot P}{S \cdot P}}$ where, η_o = Overall efficiency
$\eta_o = \eta_{mech} \cdot \eta_{vol} \cdot \eta_{max}$
$\eta_o = \eta_{mech} \cdot \eta_{man}$[where η_{vol} is neglected]

(iv) $\boxed{\eta_{vol} = \frac{Q - \Delta Q}{Q}}$ where, η_{vol} = Volumetric efficiency.

- **Model Relationship for Pumps**

Dimensionless Parameter	Dimensional Parameter
(i) $C_H = \frac{gH}{\omega^2 D^2}$	(i) $C_H = \frac{H}{N^2 D^2}$
(ii) $C_Q = \frac{Q}{\omega D^3}$	(ii) $C_Q = \frac{Q}{N D^3}$
(iii) $C_P = \frac{P}{\rho \omega^3 D^5}$	(iii) $C_P = \frac{P}{N^3 D^5}$
(iv) $N_S = \frac{\omega\sqrt{Q}}{(gH)^{3/4}}$	(iv) $N_S = \frac{N\sqrt{Q}}{(H)^{3/4}}$

Where, C_H = Head coefficient, C_Q = Discharge coefficient
C_P = Power coefficient, N_S = Specific speed of pump
P = Power

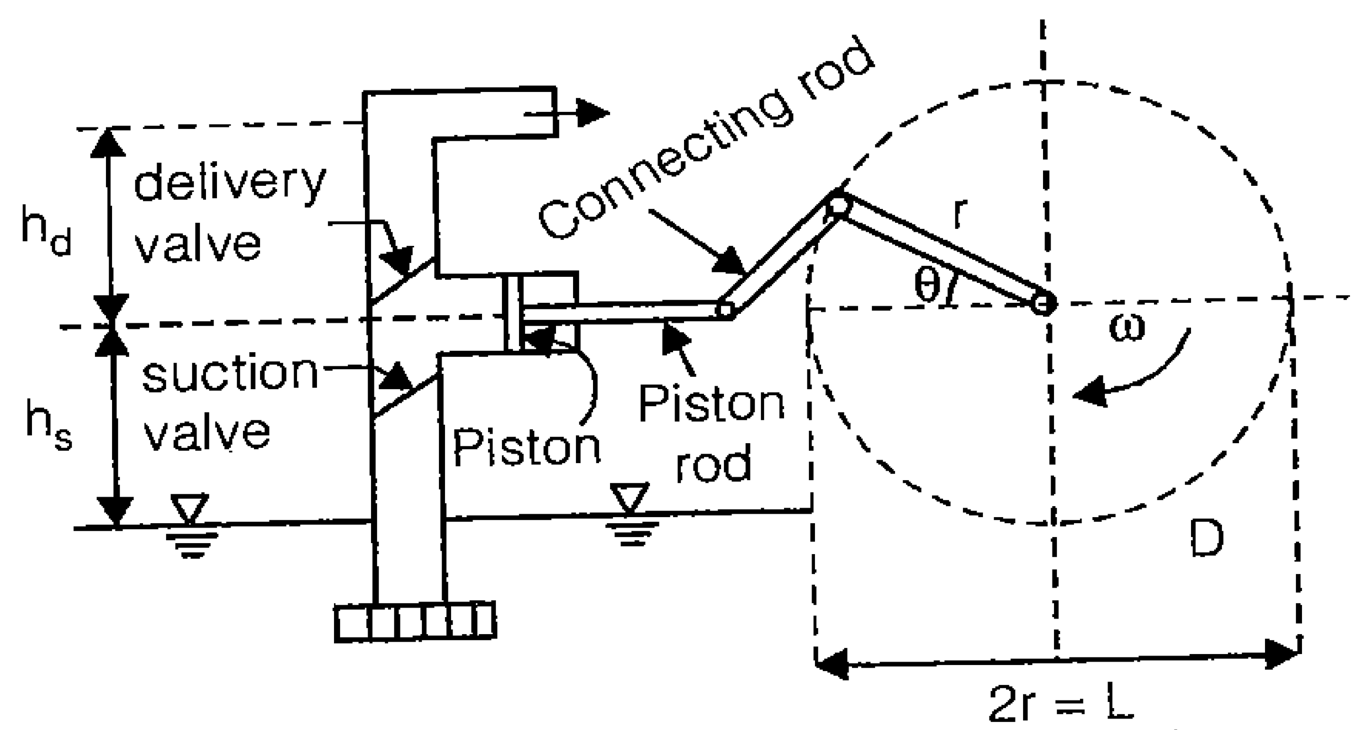
- Overall efficiency of pump(η_o) = $\frac{C_Q \cdot C_H}{C_P}$
- Net Positive Suction Head (NPSH)
- NPSH = $\left(\frac{P_{atm}}{\rho g} - h_s - h_{fs}\right) - \frac{P_v}{\rho g}$

where, $\frac{P_{atm}}{\rho g}$ = Atmospheric pressure head $\simeq$ 10.3 m at mean sea level.

$\frac{P_v}{\rho g}$ = Vapour pressure head $\simeq$ 2.5 m at 20 °C for water and 10.3 m at 100 °C for water

- Thomas Cavitation Number (σ) = $\frac{NPSH}{H}$
- Critical Thomas number (σ_c) = $1.042(H)^{4/3}$
- For no cavitation $\sigma \geq \sigma_c$ or NPSH $\geq \sigma_c H$.

RECIPROCATING PUMP



Volume of water discharged per second,

$$Q = \frac{ALN}{60} m^3/sec$$

A = Area of cylinder (in m^2)

L = Length of cylinder (in m)

N = Crank speed (in rpm)

- If the head against which water is to be lifted is

$H_s = (h_s + h_d)$

h_s = suction head

h_d = delivery head

- Work done per second = $\gamma Q(h_s + h_d)$
- Reciprocating pumps are used to lift water against high head at low discharge.
- To increase discharge and to maintain it more uniform, double acting reciprocating pumps are used.

$$Q \approx \frac{2ALN}{60}$$, thus power also gets doubled.

- Slip in Percentage is given by

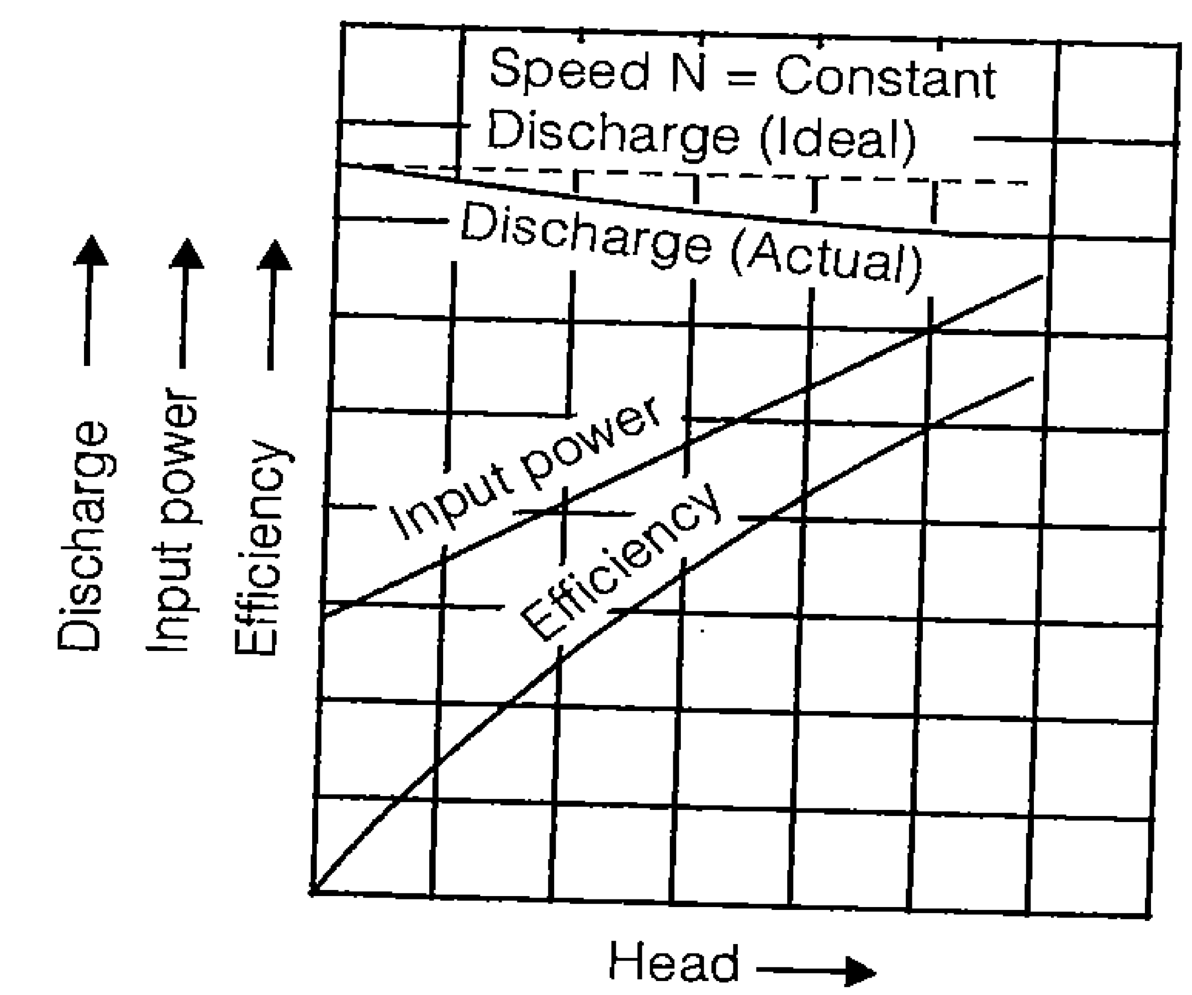
$$\% \text{ slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100 = (1 - C_d) \times 100$$

where C_d = coefficient of discharge

Slip is negative when (i) delivery pipe is small and suction pipe is long (ii) Pump is running at very high speed.

- **Indicator diagram** is a graph between the pressure head in the cylinder and the distance travelled by the piston flow inner dead centre for one complete revolution of the crank, work done by pump is proportional to area of indicator diagram.

- **Air Vessel** is used to obtain continuous supply of water at uniform rate, to save a considerable amount of work and to run the pump at a high speed without separation.
- Percentage of work saved is 84.8% when single acting pump with air vessel is used while this saving is only 39.2% when air vessel is used in double acting pump.
- Advantage of multicylinder pumps are that the pump even without air vessels deliver liquid more uniformly as compared to single cylinder pump.
- Operating Characteristic Curve of Reciprocating Pump is given below:



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Open Channel Flow

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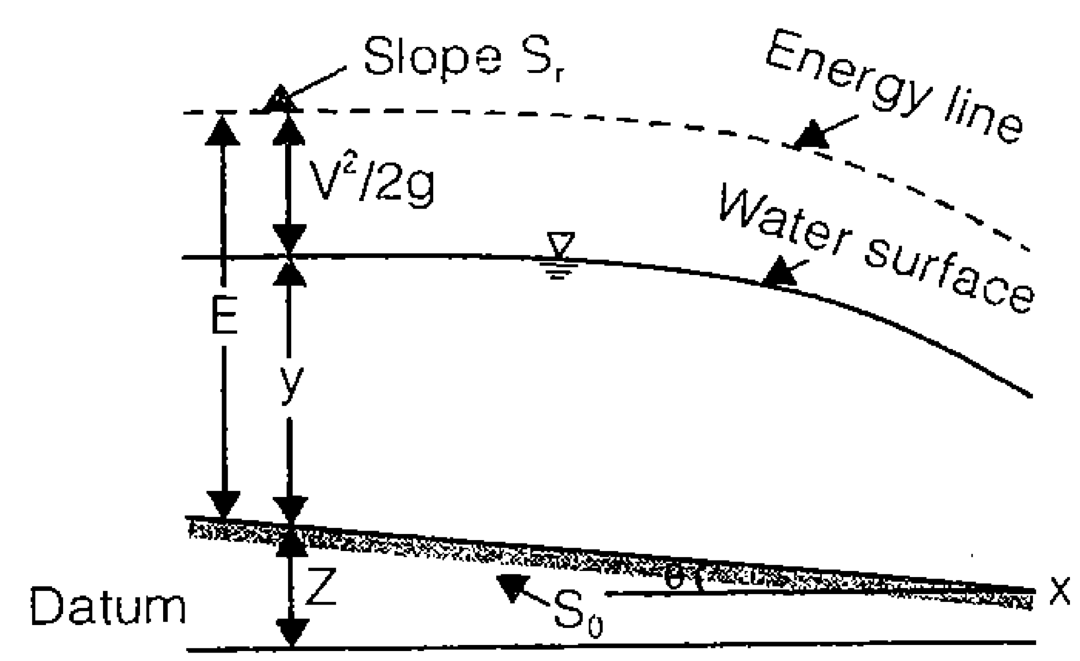
Gradually Varied Flow

In gradually varied flow depth of flow changes over large length. Specific energy is constant in G.V.F.

- **Assumptions of G.V.F**
  1. Chezys formula and manings formula are used to determine energy slope.
  2. Energy correction factor (α) = 1.
  3. Pressure distribution is hydrostatic.
  4. Flow is steady i.e., discharge is constant.
  5. Channel is prismatic.
  6. The bottom slope of the channel is very small.
  7. Roughness coefficient of the channel is independent of depth.
- **Specific Energy (E)**



(a) $$E = z + y + \frac{\alpha v^2}{2g}$$

[α = 1 for G.V.F]

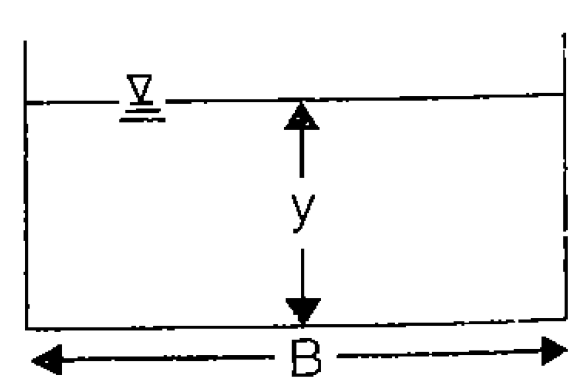
(b) $$E = y + \frac{v^2}{2g}$$

If channel bottom is datum.

where, y = Depth of flow
v = Velocity of flow (m/s)
α = Energy correction factor = 1 for G.V.F.

- **G.V.F. For Rectangular Channel**

$$E = y + \frac{q^2}{2g\,y^2} = E_P + E_K$$

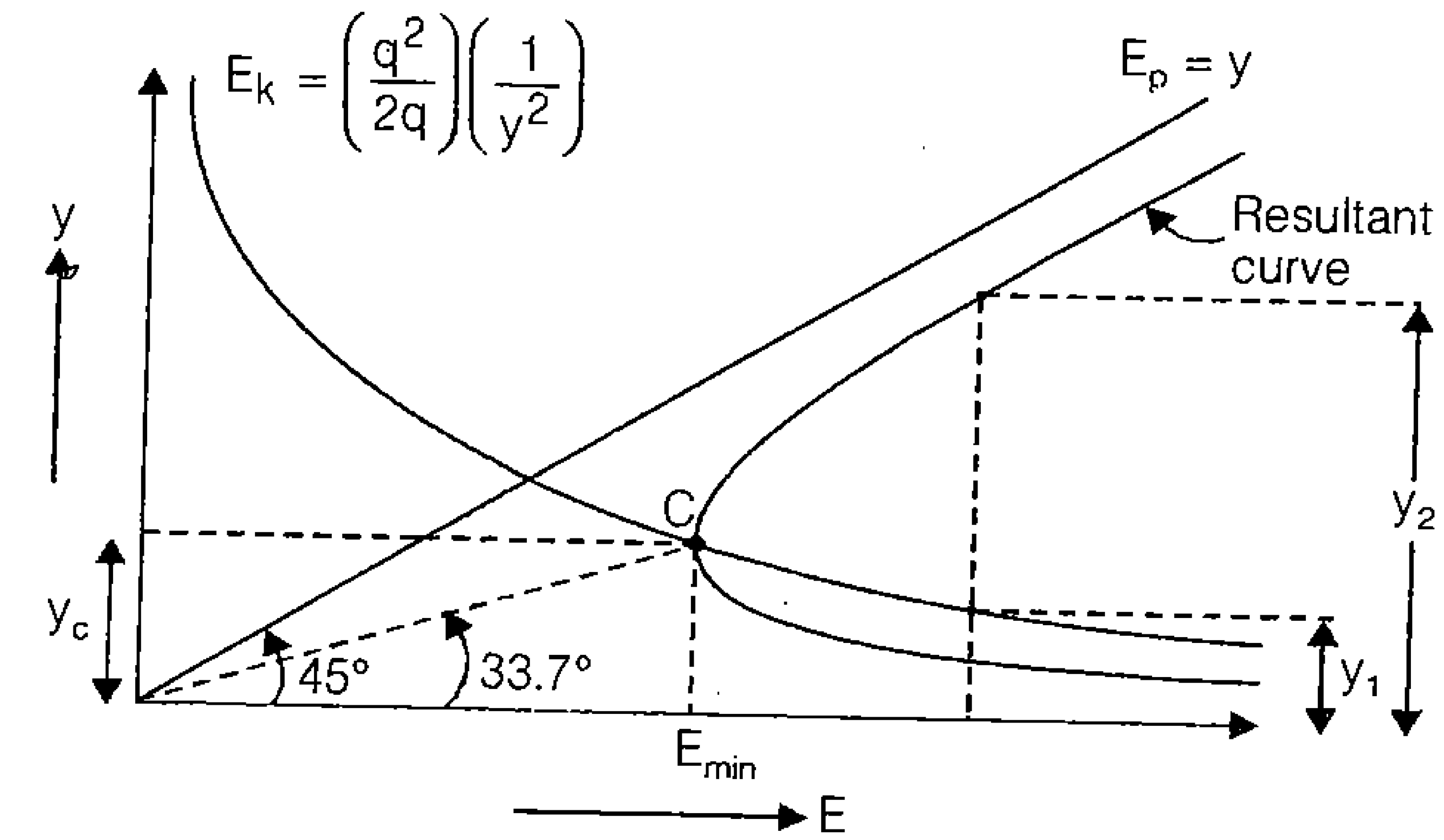


A = By

Where, A = Area of channel (m^2)
B = Width and y = depth

$$q = \frac{Q}{B} = \text{Discharge per unit width.}$$

- **Specific Energy Curve for Rectangular Section**



(a) At minimum specific energy only one depth of known as critical depth of flow (y_c).

(b) If specific energy is other than E_{min} then there will be two depth of flow known as alternate depth or sequent depth (y_1 and y_2).

- Critical Depth (y_c) = $\left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ → for rectangular channel.

  At critical depth = $\frac{dE}{dy} = 0$

- **Minimum Specific Energy**

  For, Rectangular channel → $E_{min} = \frac{3}{2} y_C$

  i.e., kinetic head is half of potential head

  Parabolic channel → $E_{min} = \frac{4}{3} y_C$

  i.e., kinetic head is $\frac{1}{3}$ of potential head

  Triangular channel → $E_{min} = \frac{5}{4} y_C$

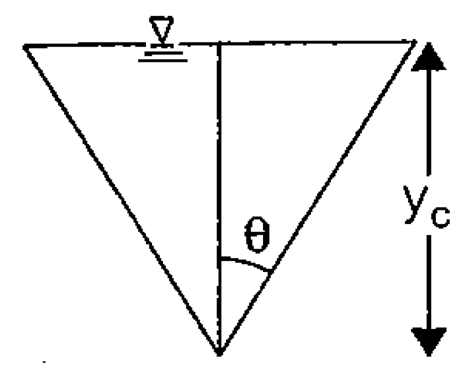
  i.e., kinetic head is $\frac{1}{4}$ of potential head.

- **Condition for critical flow for any shape of channel**

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

- **Critical Depth for Triangular Channel**

$$y_c = \left[\frac{2Q^2}{g\tan^2\theta}\right]^{1/5}$$



- **Dynamic Equations for G.V.F**

(i) $\frac{dy}{dx} = \frac{S_O - S_F}{1 - \frac{q^2}{gy^3}}$ ⟶ For rectangular channel

(ii) $\frac{dy}{dx} = \frac{S_O - S_F}{1 - \frac{V^2}{gy}}$ ⟶ For rectangular channel

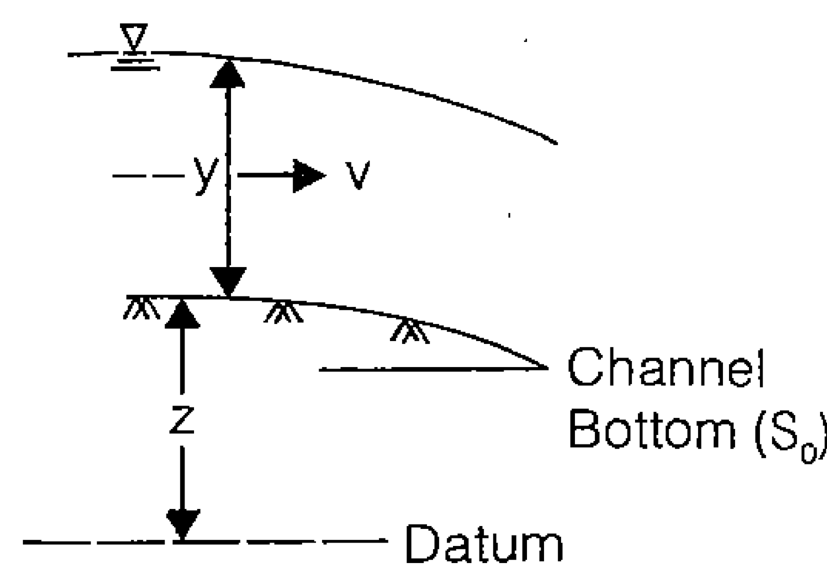
(iii) $\frac{dy}{dx} = \frac{S_O - S_F}{1 - F_r^2}$ ⟶ For rectangular channel

where, $\frac{dy}{dx}$ = Change in depth of flow in the direction of flow.

S_O = Channel bottom slope

S_F = Energy slope

F_r = Froud number = $\frac{V}{\sqrt{gy}}$



(iv) $\frac{dy}{dx} = S_O\left[\frac{1-\left(\frac{y_n}{y}\right)^3}{1-\left(\frac{y_c}{y}\right)^3}\right]$ ⟶ Chezy's result

(v) $\frac{dy}{dx} = S_O\left[\frac{1-\left(\frac{y_n}{y}\right)^3}{1-\left(\frac{y_c}{y}\right)^3}\right]$ ⟶ Mannings result

where, y_n = Normal depth of flow. (Depth at which at flow is uniform).

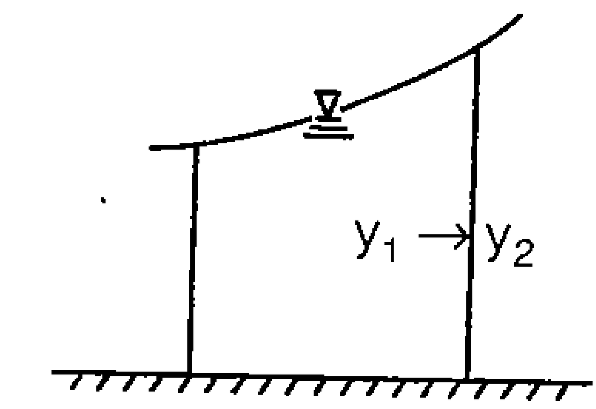
and y_c = Critical depth of flow (Depth at which specific energy is minimum).

- **Dynamic equation for G.V.F for any shape of channel**

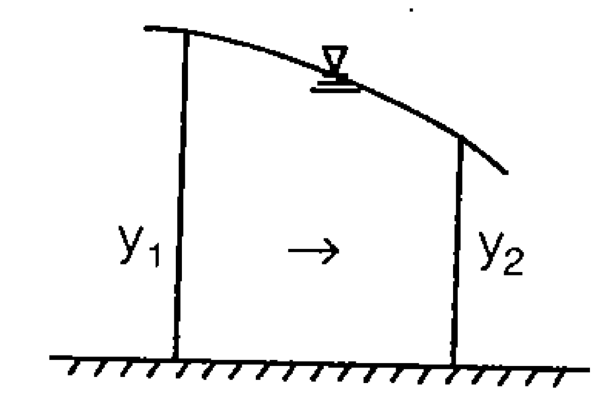
$$\frac{dy}{dx} = \frac{S_O - S_f}{1 - \frac{Q^2T}{gA^3}}$$

- **Water Surface Profile**

If $\frac{dy}{dx} > 0$ ⟶ Rising curve or backwater curve.



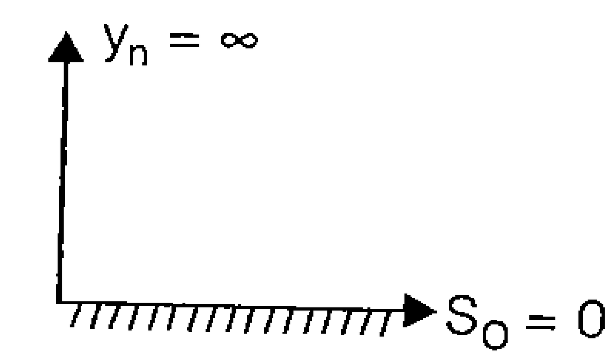
It $\frac{dy}{dx} < 0$ ⟶ drawdown curve.



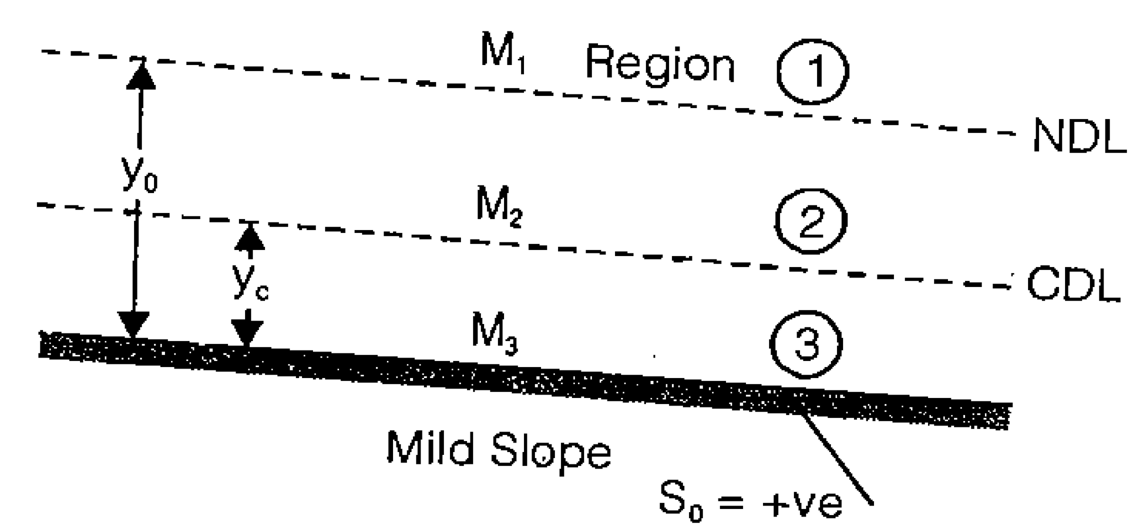
If $\frac{dy}{dx} = 0$ ⟶ Water surface have no profile.

- **Type of Channel Slope**

(i) Horizontal slope ⟶ (S_0 = O) ⟶

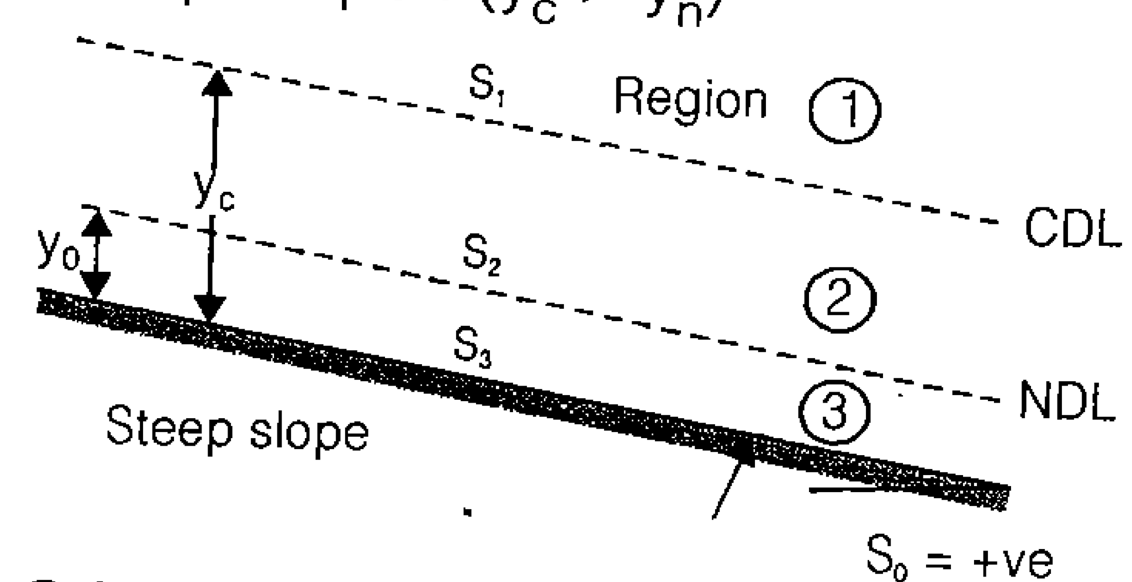


(ii) Mild Slope : ($y_n > y_c$) ⟶

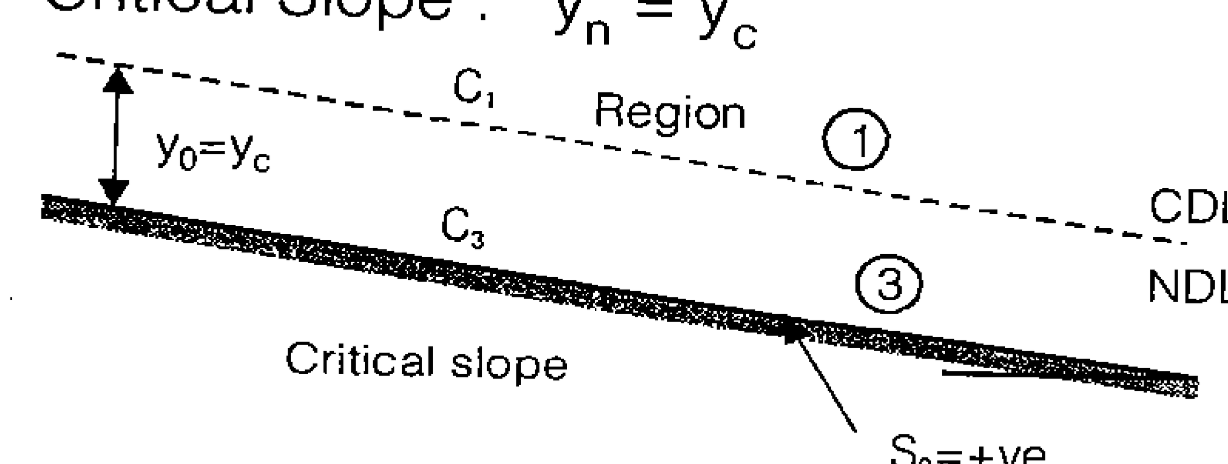


where, NDL = Normal depth line, CDL = Critical depth line.

(iii) Steep Slope : ($y_c > y_n$)

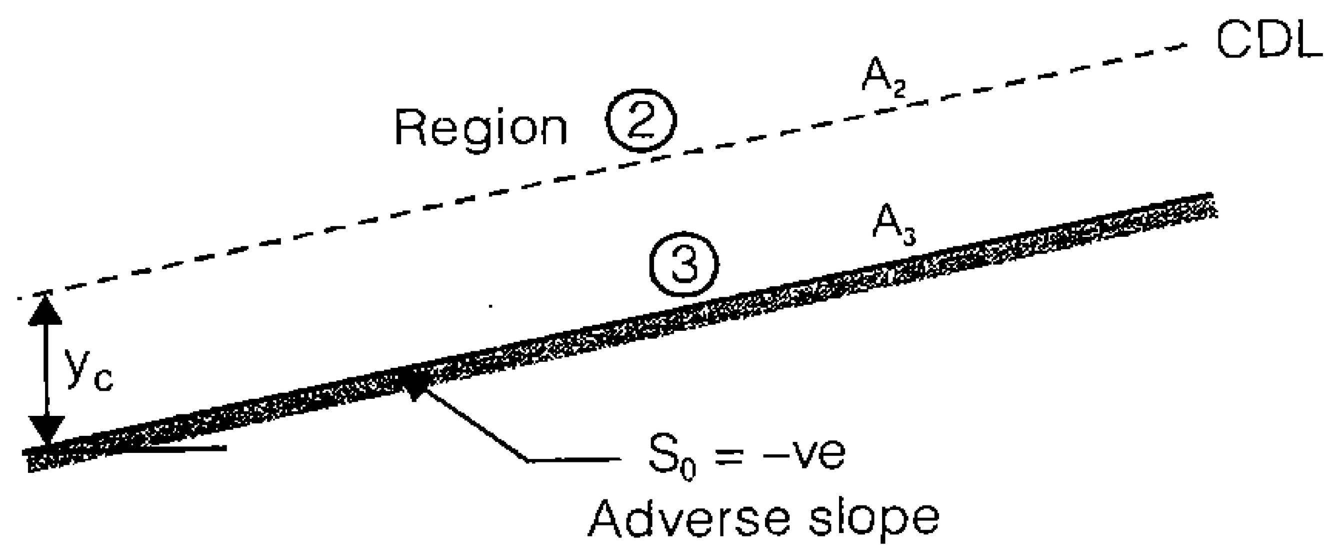


(iv) Critical Slope : $y_n = y_c$



C_2 will not exist.

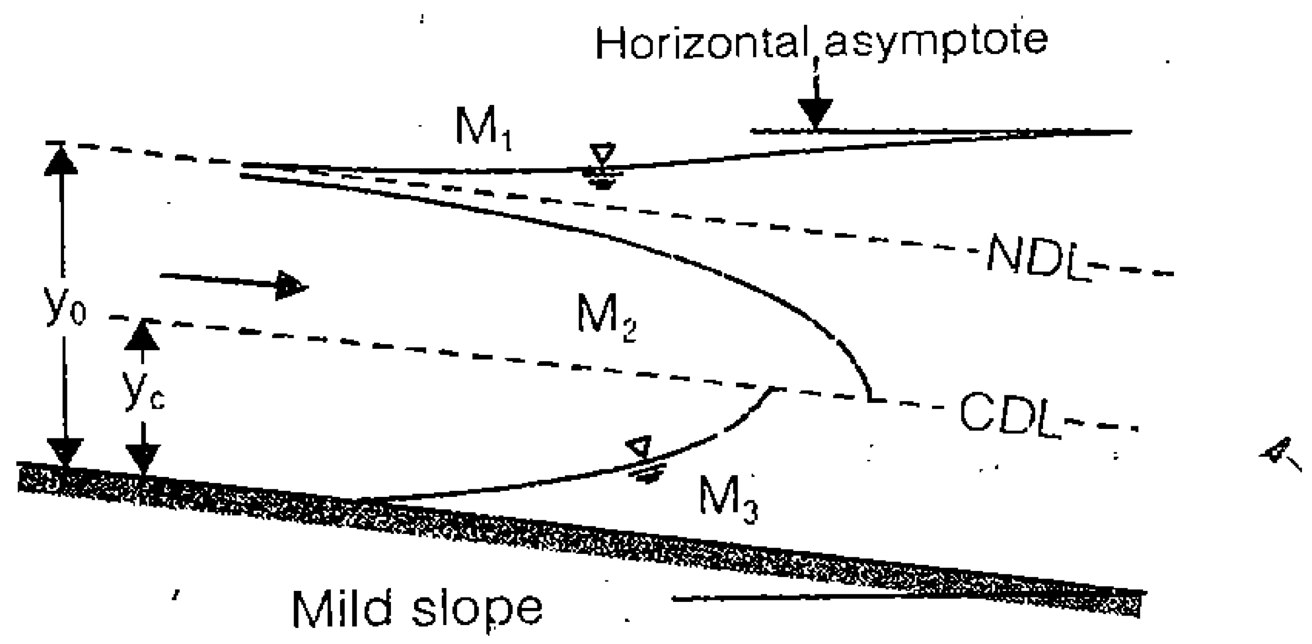
(v) Adverse Slope $y_n < 0$



'A_1' will not exist.

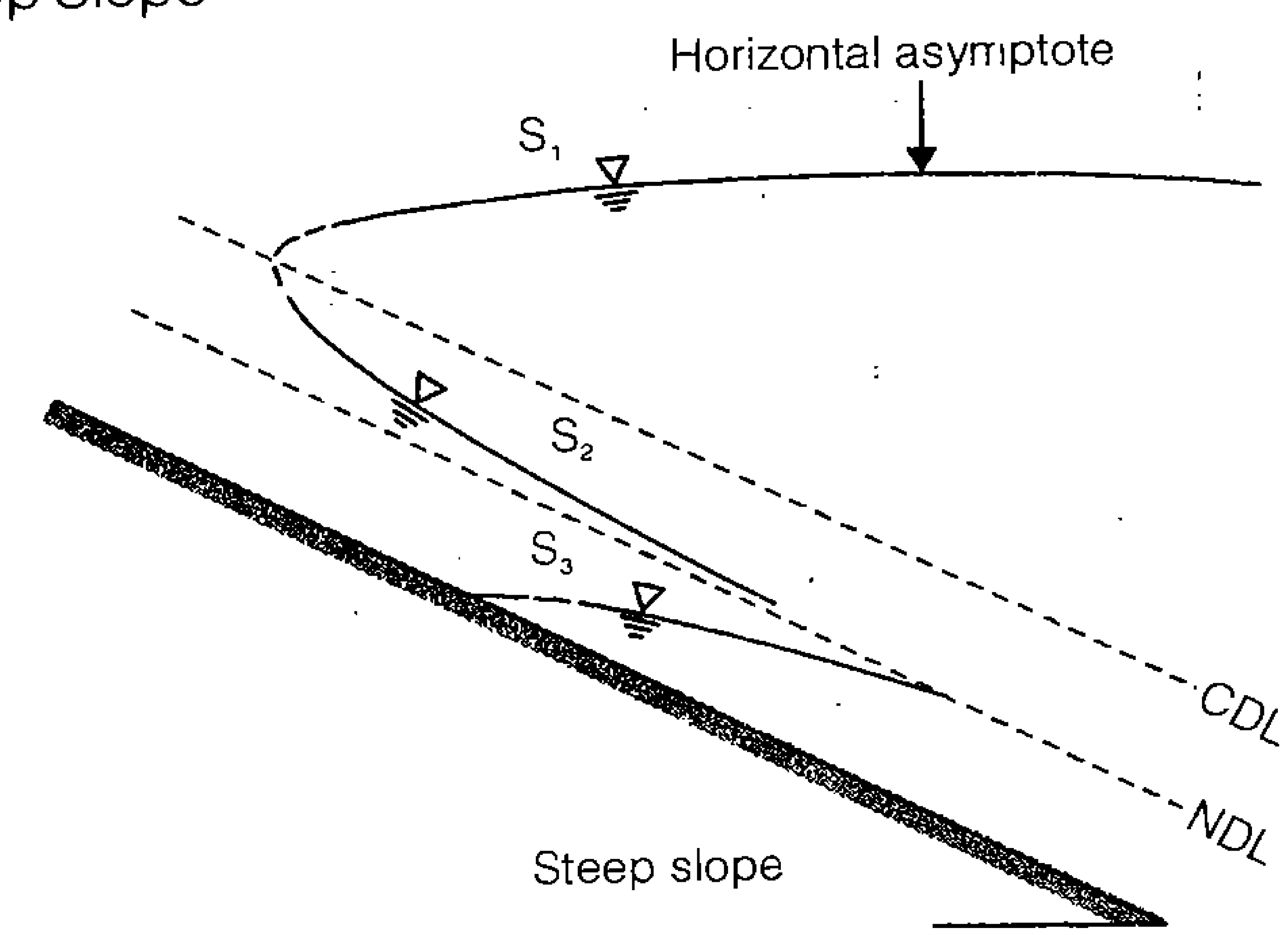
- **Type of Surface Profile Curve**

(i) Mild Slope

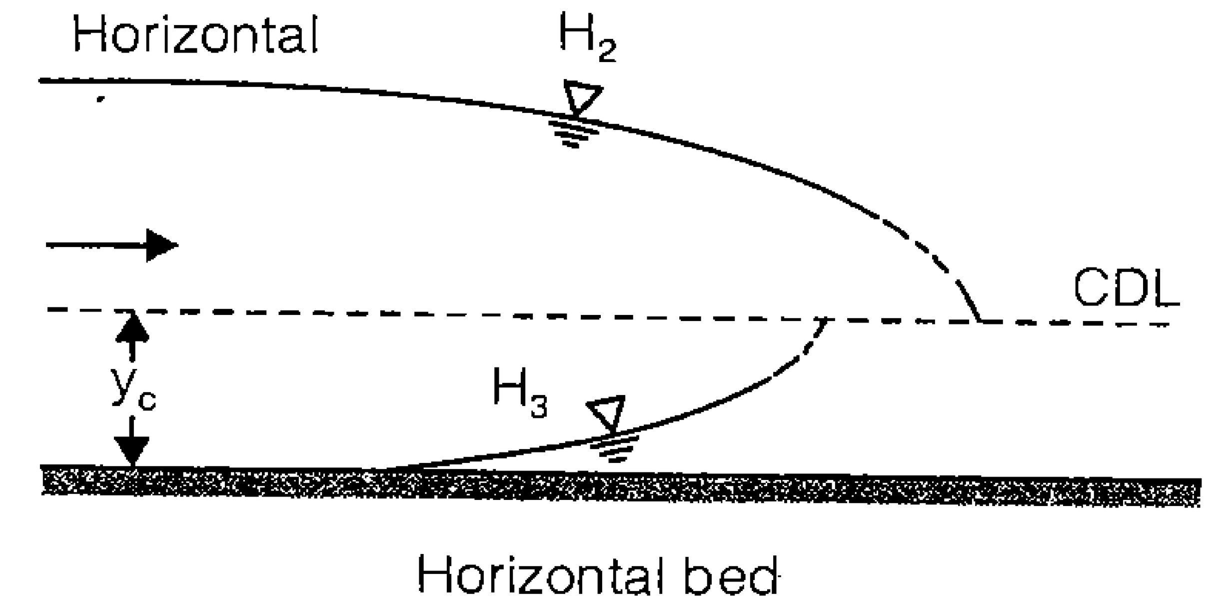


> **NOTE**
> - Water surface profile meets asymptotically to the normal depth line whereas it meets normally to the C.D.L and channel bottom slope.
> - Zone (I) and zone (III) curve is always rising or back water curve whereas zone (II) curve is drawdown curve.

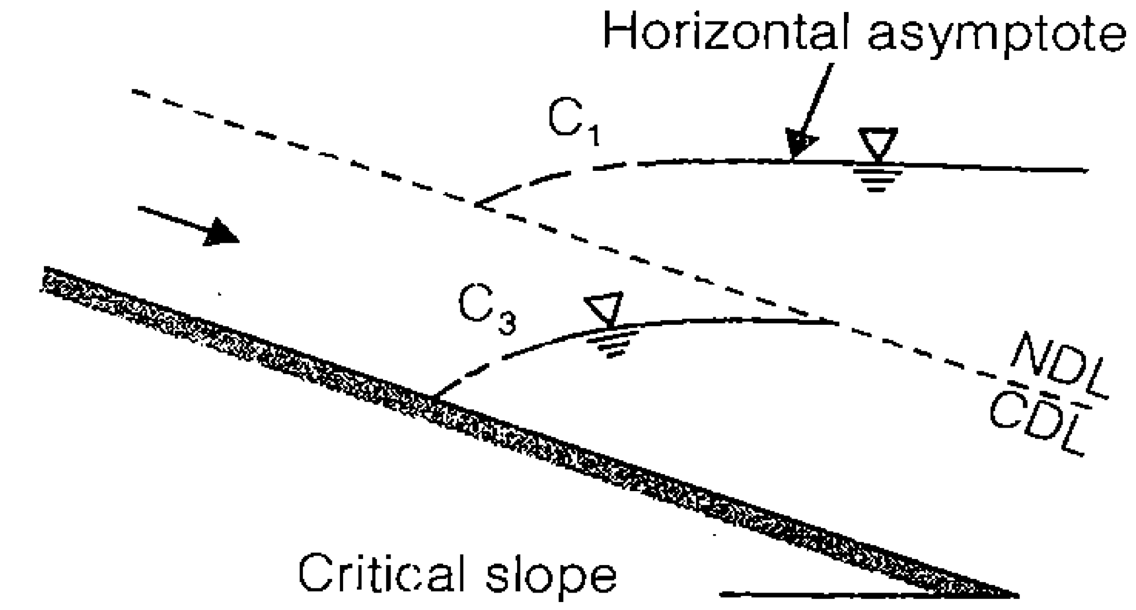
(ii) Steep Slope



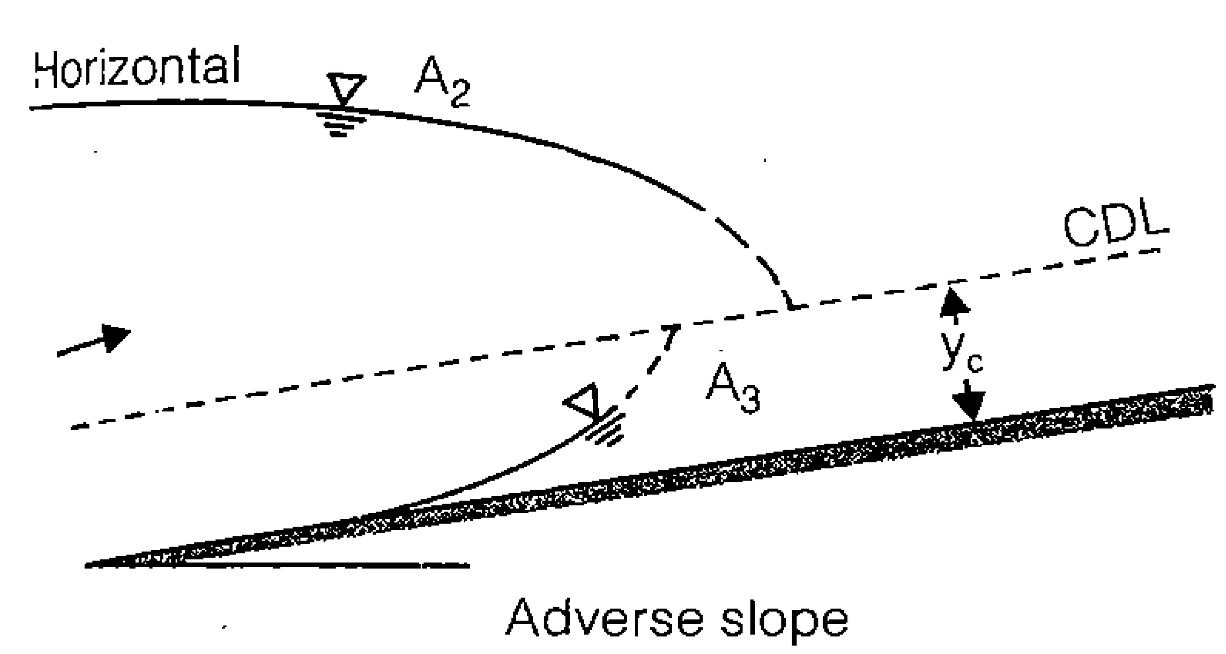
(iii) Horizontal

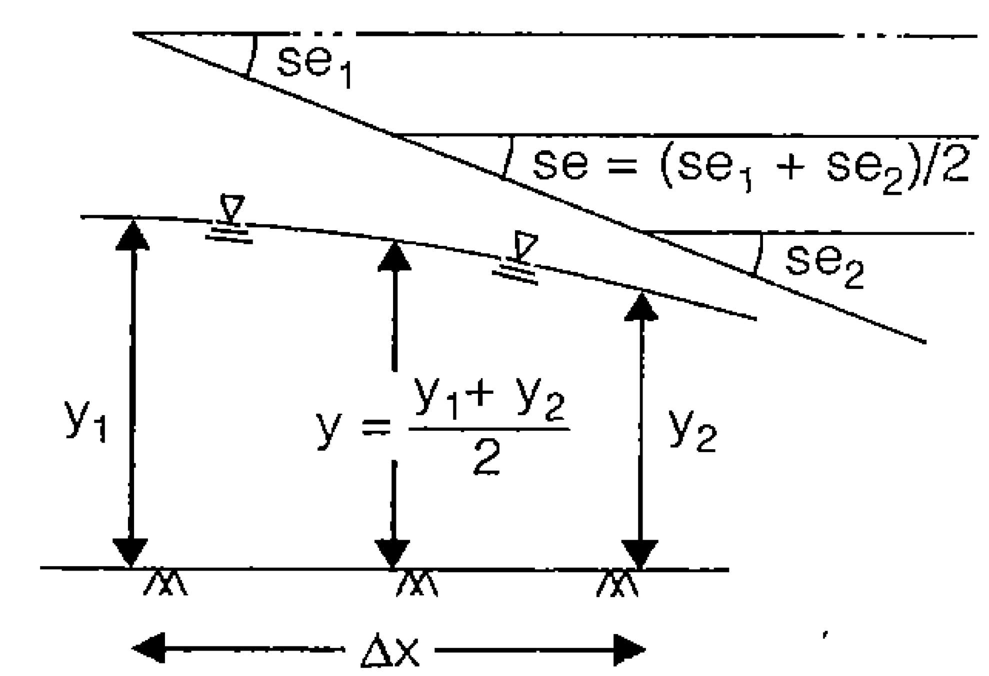


(iv) Critical Slope



(v) Adverse Slope





- **Length of Curve Surface Profile between Two Sections**

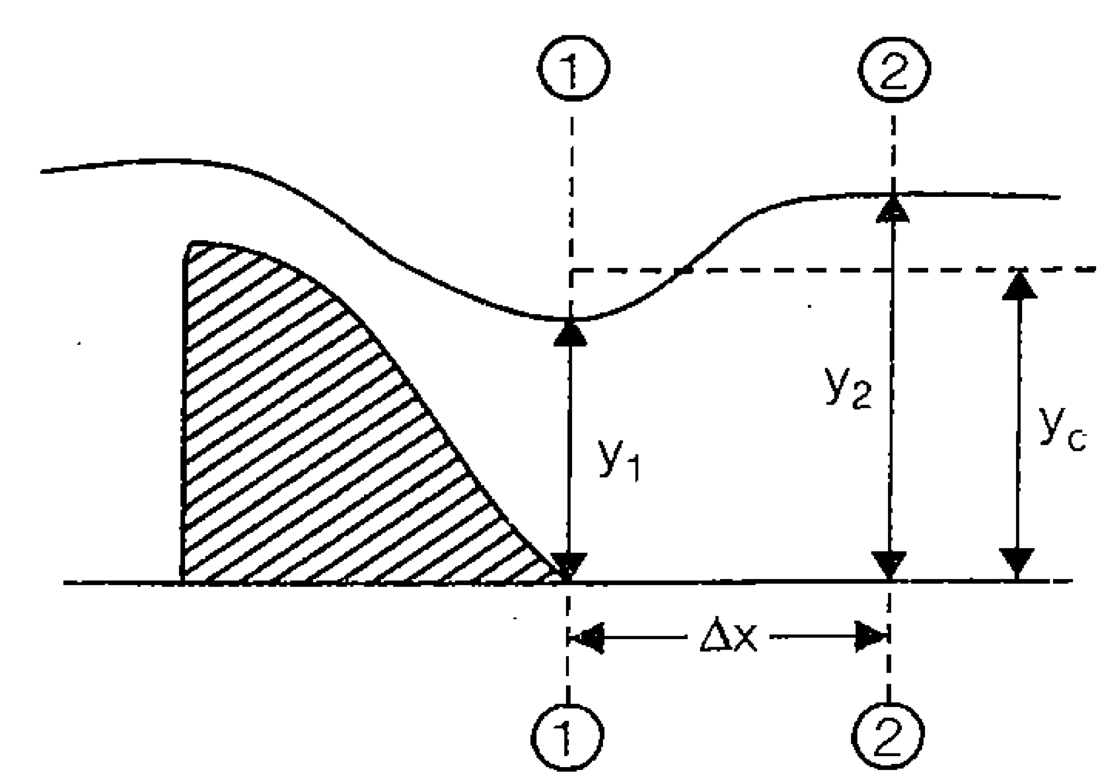
$$\Delta x = \frac{E_2 - E_1}{S_0 - S_e}$$

where, Δx = Length of curve surface profile

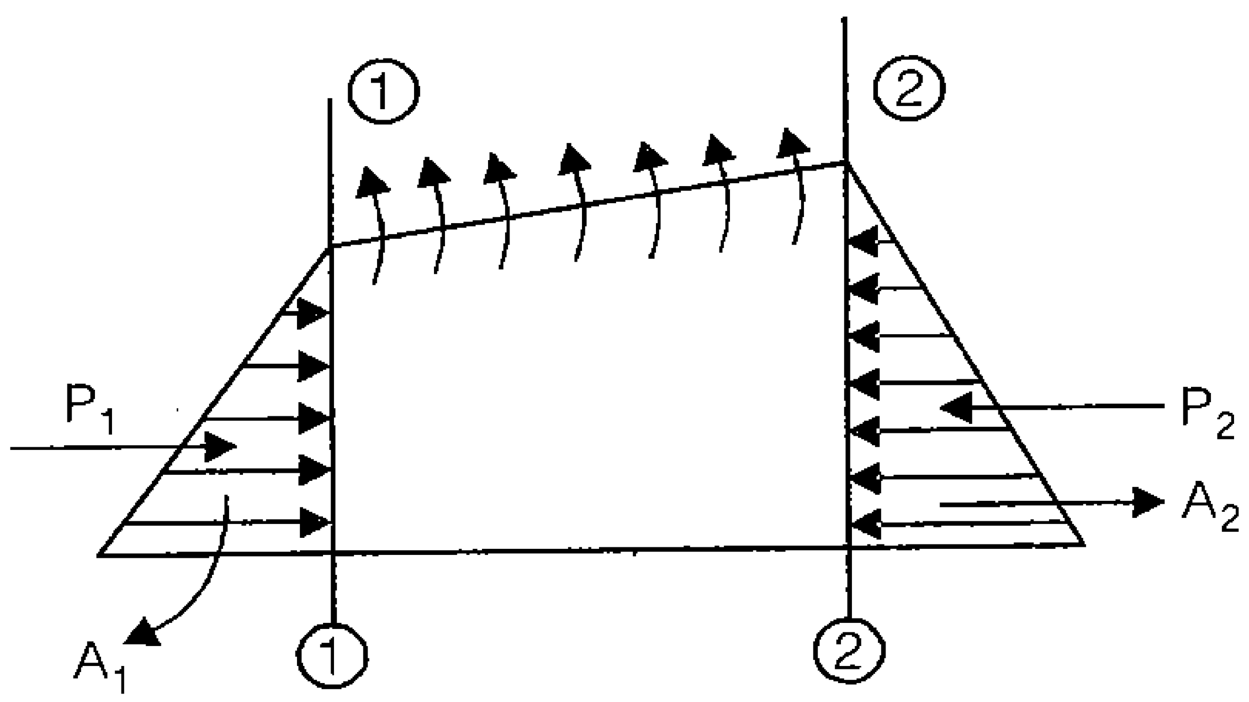
S_e = Energy slope

S_o = Bed slope

E_1 and E_2 are specific energy of section (I) and (II) respectively.



- **Rapidly Varied Flow**
  - Depth of flow changes suddenly over small length.
  - Flow changes from super critical stage to subcritical stage.
  - Δx is very small and $y_1 < y_c \le y_2$.
  - The change in depth from y_1 to y_2 is called hydraulic jump and y_1 and y_2 is called conjugate depth and sequent depth.
  - Specific force is constant. $\boxed{F = \frac{Q^2}{Ag} + A\bar{z} = \text{const.}}$



$$\boxed{\frac{Q^2}{A_1 g} + A_1\overline{z_1} = \frac{Q^2}{A_2 g} + A_2\overline{z_2}}$$

Where, $\overline{z_1}$ = Depth of centroid of area A_1 from free surface.

$\overline{z_2}$ = Depth of centroid of area A_2 from free surface.

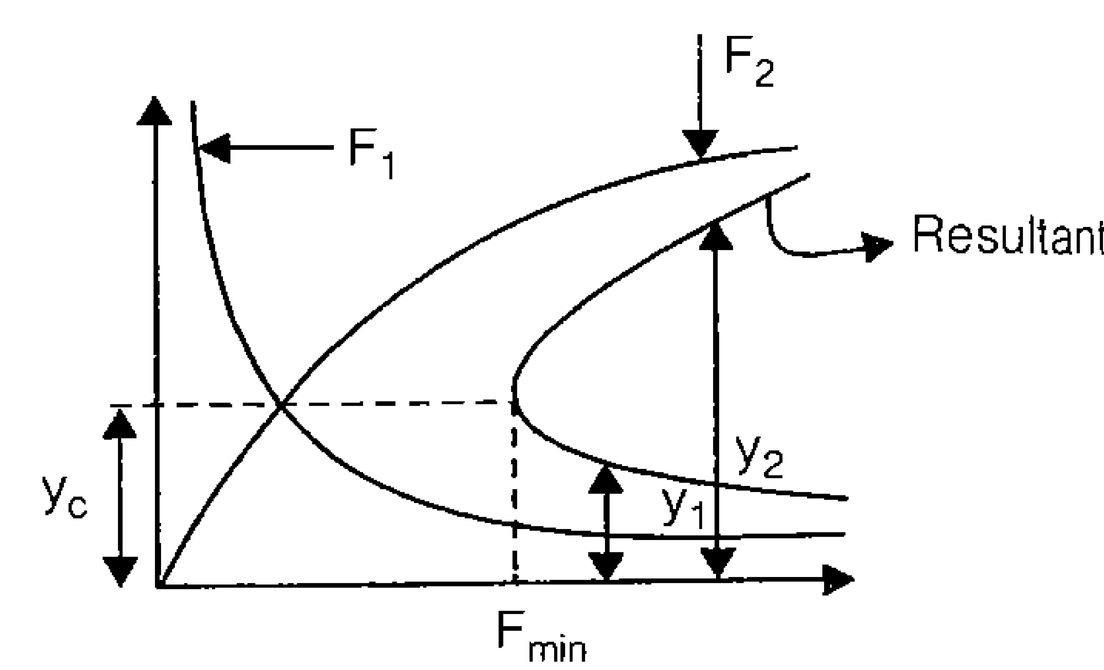
- **Specific Energy Curve for Rectangular Channel?**

$\because \quad \boxed{F = \frac{Q^2}{Ag} + A\bar{z}}$

$$= \frac{q^2 B}{g}\cdot\frac{1}{y} + \frac{B}{2}y^2 = F_1 + F_2$$

F_1 is rectangular hyperbola.

F_2 is parabola.



- **Critical Depth**

$$\boxed{y_c^3 = \left(\frac{q^2}{g}\right)} \quad \text{... for rectangular channel}$$

- Condition for critical depth for any shape of channel $\boxed{\frac{Q^2}{g} = \frac{A^3}{T}}$
- Some Relations

(i) $y_c^3 = \dfrac{y_1 y_2 (y_1 + y_2)}{2}$

(ii) $\dfrac{y_2}{y_1} = \dfrac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$ where $Fr_1 > 1$

(iii) $\dfrac{y_1}{y_2} = \dfrac{1}{2}\left[-1 + \sqrt{1 + 8Fr_2^2}\right]$ where $Fr_2 > 1$

where, Fr_1 = Froud number at (i) = $\dfrac{v_1}{\sqrt{gy_1}}$

and Fr_2 = Froud number at of (ii) = $\dfrac{v_2}{\sqrt{gy_2}}$

- **Power Loss of Hydraulic Jump**

$$\boxed{\text{Power Loss} = WQ(\Delta E)}$$

where $W = \gamma = \rho g$ = unit weight, Q = Discharge

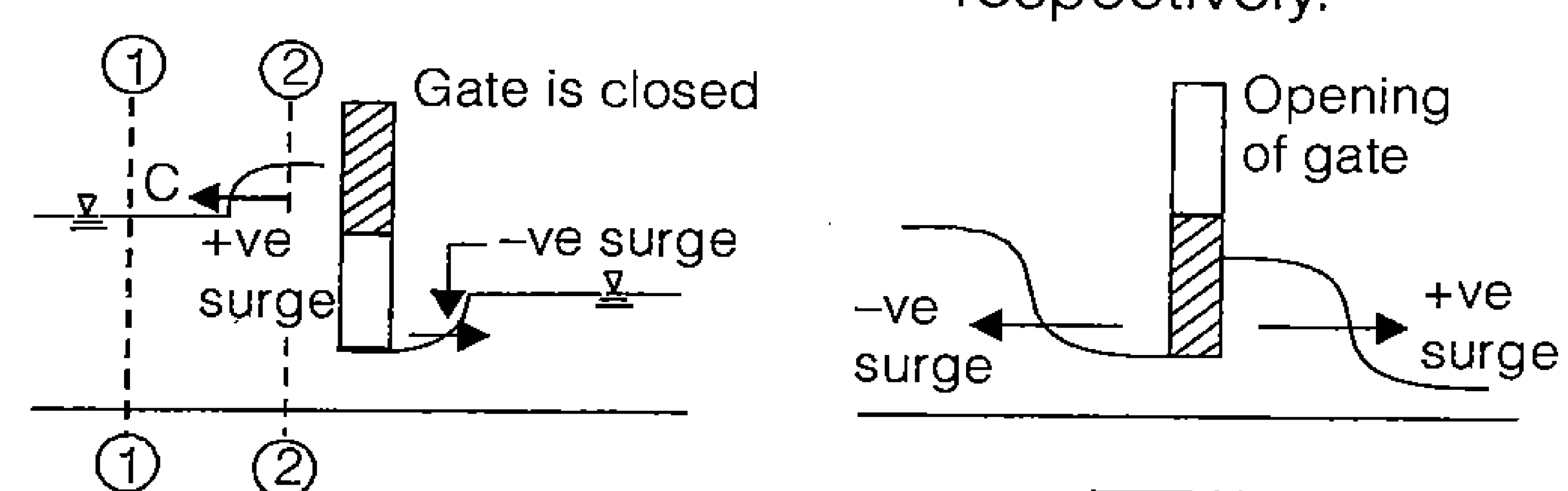
$$\Delta E = E_1 - E_2 = \frac{(y_2 - y_1)^3}{4y_1y_2} \text{ and } \Delta E = \frac{(v_1 - v_2)^3}{2g(v_1 + v_2)}$$

  - Height of jump (Hj) = $y_2 - y_1$
  - Length of jump = $L_j = \Delta x = 6.9(y_2 - y_1)$

- **Surge or Standing Wave**

Where the flow in a channel is obstructive surge wave is created.
Example of surge is closure of gate and opening of gate.
A +ve surge is that which causes increase in depth in the direction of flow.
A –ve surge is that which causes decrease in depth in the direction of flow.

  - $\boxed{C = \frac{v_1 y_1 - v_2 y_2}{y_2 - y_1}}$ where, C = Celerity of wave (velocity of wave) v_1 and v_2 is velocity of flow of section (1)-(1) and (2)-(2) respectively.



  - Absolute velocity of flow of section (1) is $\boxed{V_1 + C = \sqrt{\frac{gy_2}{2y_1}(y_1 + y_2)}}$
  - Absolute velocity of flow at section (2) is $C - V_1$.
  - When $y_1 \simeq y_2 \simeq y$

they $C = \sqrt{gy} - V_1 \quad \Rightarrow C = \sqrt{gy}(1 - Fr_1)$

  - It Fr > 1 than C < O $\Rightarrow$ Super critical flow
$\Rightarrow$ Surge moves in downstream

Fr < 1 than C > O $\Rightarrow$ Subcritical Flow.
$\Rightarrow$ Surge moves in upstream.

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A Handbook on
Civil Engineering

9

Enviromental Engineering

CONTENTS

1 Water Demand

Fire Demand

Rate of fire demand is sometimes treated as a function of population and is worked out on the basis of empirical formulas:

(i) **As per GO Fire Demand** = $\boxed{\sqrt{100\times \text{Population in thousands}}}$ Kl

(ii) **Kuichling's Formula**

$\boxed{Q = 3182\sqrt{P}}$ where, Q = Amount of water required in litres/minute.
P = Population in thousand.

(iii) **Freeman Formula**

$$\boxed{Q = 1136\left[\frac{P}{10}+10\right]}$$

(iv) **National Board of Fire Under Writers Formula**

(a) For a central congested high valued city

(i) Where population < 200000

$$\boxed{Q = 4637\sqrt{P}\left[1-0.01\sqrt{P}\right]}$$

(ii) Where population > 200000

Q = 54600 lit/minute for first fire

and Q = 9100 to 36,400 lit/minute for a second fire.

(b) For a residential city.

(i) Small or low building,
Q = 2,200 lit/minutes.

(ii) Larger or higher buildings,
Q = 4500 lit/minute.

(iii) High value residences, apartments, tenements
Q = 7650 to 13,500 lit/minute.

(iv) Three storeyed buildings in density built up sections,
Q = 27000 lit/minute.

(iv) **Buston's Formula**

$$\boxed{Q = 5663\sqrt{P}}$$

The probability of occurrence of a fire, which, inturn, depends upon the type of the city served, has been taken into consideration in developing a above formula on the basis of actual water consumption in fire fighting for Jabalpur city of India. The formula is given as

$$Q = \frac{4360R^{0.275}}{(t+12)^{0.757}}$$

where, R = Recurrence interval of fire i.e., period of occurrence of fire in years, which will be different for residential, commercial and industrial cities.

$(R)_{minimum}$ = 1 year

t = Duration of fire in minutes,

$t_{minimum}$ = 30 minute.

Per Capita Demand (q)

$$q = \frac{\text{Total yearly water requirement of the city in litre}}{365 \times \text{Design population}}$$

Assessment of Normal Variation

(i) Maximum daily demand = 1.8 × Avg daily demand

(ii) Maximum hourly demand = 1.5 × Avg. demand hourly of max. day

(iii) Maximum hourly demand or peak demand = 2.7 × Avg Hourly demand

(iv) $\frac{\text{Maximum daily demand}}{\text{Avg daily demand}} = 180\%$

(v) $\frac{\text{Maximum weekly demand}}{\text{Avg weekly demand}} = 148\%$

(vi) $\frac{\text{Maximum monthly demand}}{\text{Avg monthly demand}} = 128\%$

Population Forecasting Methods

(i) Arithmetic increase method

$$P_n = P_0 + n\bar{x}$$

where, P_n = Prospective or forecasted population after n decades from the present (i.e., last known census)

P_O = Population at present (i.e., last known census)

n = Number of decades between now & future.

$\bar{x}$ = Average (arithmetic mean) of population increases in the known decades.

(ii) Geometric Increase Method

$$P_n = P_o\left(1+\frac{r}{100}\right)^n$$

where, P_O = Initial population.

P_n = Future population after 'n' decades.

r = Assumed growth rate (%).

$$r = \sqrt[t]{\frac{P_2}{P_1}} - 1$$

where, P_2 = Final known population

P_1 = Initial known population

t = Number of decades (period) between P_1 and P_2.

$$r = \sqrt[t]{r_1 r_2 \ldots r_t}$$

(iii) Incremental Increases Method

$$P_n = P_o + n\bar{x} + \frac{n(n+1)}{2}\bar{y}$$

where, $\bar{x}$ = Average increase of population of known decades

$\bar{y}$ = Average of incremental increases of the known decades.

(iv) Decreasing rate of growth method

Since the rate of increase in population goes on reducing, as the cities reach towards saturation, a method which makes use of the decrease in the percentage increase, in many a times used, and gives quite rational results. In this method, the average decrease in the percentage increase is worked out, and is then subtraced from the latest percentage increase for each successive decade. This method is however, applicable only in cases, where the rate of growth shows a downward trend.

(v) Logistic Curve Method

(a) $$\log_e\left(\frac{P_s - P}{P}\right) - \log_e\left(\frac{P_s - P_o}{P_o}\right) = -k P_s t$$

where,

P_o = Population of the start point.

P_s = Saturation population

P = Population at any time t from the origin.

k = Constant.

(b) $P = \frac{P_s}{1 + m\log_e^{-1}(nt)}$

(c) $P_s = \frac{2P_0P_1P_2 - P_1^2(P_0 + P_2)}{P_0P_2 - P_1^2}$

(d) $m = \frac{P_s - P_o}{P_o}$

(e) $n = \left(\frac{1}{t_1}\right)\log_e\left[\frac{P_o(P_s - P_1)}{P_1(P_s - P_o)}\right]$

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Conduits for Transporting Water 2

Head Loss

- Darcy-weisbach Formula

$H_L = \frac{fLV^2}{2gD}$ where, H_L = Head loss in meters.

L = Length of pipe in metres.

d = Diameter of the pipe in metres

V = Mean velocity of flow through the pipe in m/sec.

g = Acceleration due to gravity (m/s².)

f = Friction factor.

= 0.02 for new smooth pipes

= 0.075 for old rough pipes.

$f = 0.04\left[1 + \frac{1}{35d}\right]$ → for old pipes.

$f = 0.02\left[1 + \frac{1}{35d}\right]$ → for new pipes.

$f = a + \frac{b}{R_e^m}$ where a, b and m are constants depending upon R_e and δ.

Re = Reynolds number

and δ = Relative roughness.

- For laminar flow ($R_e \le 2000$)

$f = \frac{64}{R_e}$ and $R_e = \frac{VD}{\nu}$

V = Velocity of water (m/s)

ν = Kinematic viscosity (m²/s)

D = Dia of pipe (m)

- For turbulent flow ($R_e \ge 4000$)

(i) $\frac{1}{\sqrt{f}} = 2\log_{10} R_e\sqrt{f} - 0.8$ → for smooth pipe

(ii) $\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{k}\right) + 1.74$ → for rough pipe

where, R = Radius of pipe = $\frac{D}{2}$

Hazen-William's Formula

$$V = 0.85 C_H \cdot R^{0.63} \cdot S^{0.54}$$

where, C_H = Coefficient of hydraulic capacity

R = Hydraulic mean depth of pipes in metres

$= \frac{d}{4}$ for circular pipes flowing full

S = Slope of the energy lines.

V = Flow velocity through the pipe in m/sec.

Pipe material	Value of C_H Depending upon the smoothness of the pipe material
Concrete (regardless of age)	130
Cast Iron	
New	130
5 years old	120
20 years old	100
Welded steel (New)	120
Riveted steel (New)	110
Vitrified clay	110
Brick Sewers	100
Asbestos-Cement	140

Modified Hazen-William's Formula

$$V = \frac{3.83 C_R [d^{0.6575} (g.s)^{0.5525}]}{\nu^{0.105}}$$

where, C_R = Dimensionless coefficient of roughness.

d = Pipe diameter.

g = Acceleration due to gravity.

$= 9.81\ m/s^2$.

s = Friction slope $= \frac{H_L}{L}$.

ν = Viscosity of liquid.

$$V = 143.534 C_R \cdot R^{0.6575} \cdot S^{0.5525}$$

where, R = Hydraulic radius or hydraulic mean depth $= \frac{A}{P}$

MANNINGS FORMULA

$$H_L = \frac{n^2 \cdot V^2 \cdot L}{(R)^{4/3}}$$

where, η = Mannings rugosity coefficient.

L = Length of pipe in metres.

V = Flow velocity (m/s).

$R = \frac{A}{P} = \frac{d}{H}$ for circular pipe.

Forces Acting on Pressure Conduits

(i) Hoop tension or circumferential tension (σ)

$$\sigma = \frac{PD}{2t}$$

where, P = Total internal pressure including the full static water pressure and water hammer pressure

d = Diameter of the pipe in metres

t = Thickness of the pipe in metres.

(ii) Water Hammer Pressure

$$P_{h(maximum)} = \delta_w U_P \cdot V$$

where,

$P_{h(maximum)}$ = Maximum water hammer pressure developed in N/m^2.

U_P = Velocity of the pressure wave generated.

δ_w = Density of water $\left(\gamma_w / g\right)$.

γ_w = Unit wt. of water = 9.81 kN/m^3.

V = Velocity of water in the pipe.

$$U_P = \sqrt{\frac{E_w}{\delta_w} \cdot \frac{1}{\sqrt{1 + \frac{E_w}{E_P} \cdot \frac{d}{t}}}}$$

where, E_w = Modulus of elasticity of water or Bulk modulus of compression of water

E_P = Modulus of elasticity of pipe material.

d = Diameter of pipe

t = Thickness of the pipe shell

δ_w = Density of water.

- $$P_{h(maximum)} = \frac{1433V}{\sqrt{1 + \frac{k \cdot d}{t}}} kN/m^2$$ where, $k = \frac{E_W}{E_P}$
- $$P_h = (P_h)_{maximum} \cdot \left(\frac{T_C}{T}\right)$$ where, T_C = Critical time, T = Actual time of closure.
- $$T_C = \frac{2\delta}{U_P}$$ where, s = Distance of the value from the reservoir.

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Development of Ground Water

3

Darcy Law's

(i) $\boxed{Q = kiA}$ (For Laminar flow)

where, Q = Discharge

k = Coefficient of permeability

i = Hydraulic gradient = $\frac{\Delta h}{l}$

A = Area of flow.

(ii) $\boxed{\frac{Q}{A} = V = ki}$ where, V = Discharge velocity

(iii) $\boxed{V_S = \frac{V}{n}}$ where, V_S = Seepage velocity

n = Porosity.

(iv) $\boxed{V_S = \frac{k' i D_{10}^2}{Y}}$ where, k' = Constant having value 400.

i = Hydraulic gradient

D_{10} = Effective size of soil particle

Y = Dynamic viscosity.

(v) $\boxed{k = \frac{C \cdot d^2 \cdot g}{\nu}}$ where, C = Shape factor (which is a function of porosity, packing and grain size distribution).

d = Average size of particle.

ν = Kinematic viscosity.

Specific yield

$\boxed{S_y = \frac{V_{wy}}{V}}$ where, S_y = Specific yield.

V_{wy} = Volume of water yielded under gravity effect.

V = Total volume of water drained.

Specific retention

$\boxed{S_R = \frac{V_{WR}}{V}}$ where, S_R = Specific retention.

V_{WR} = Volume of water retain under gravity effect.

V = Total volume of water.

$\boxed{S_y + S_R = n}$ where, n = Porosity.

Slot Opening

Slot size = $D_{10} \pm 8\%$ of D_{10} of gravel pack material.

Slot size = D_{60} of aquifer design on the basis of finest aquifer.

Pack Aquifer Ratio (P.A)

$$\boxed{\text{Pack Aquifer Ratio (P.A)} = \frac{D_{50} \text{ of gravel}}{D_{50} \text{ of aquifer}}}$$

$$\boxed{9 < PA < 12.5} \text{ if } \left(C_u = \frac{D_{60}}{D_{10}}\right) \le 2.$$

$$\boxed{12 < PA < 15.5} \text{ if } (C_u > 2)$$

Well Losses

- Jacob-equilibrium formula for confined aquifer,

$$\boxed{s = \frac{Q}{4\pi T}\left\{\log_e \frac{4Tt}{r^2 A} - 0.5772\right\}}$$

where, s = Drawdown in observation well after time t.

r = Radial distance of observation well from main pump well.

T = Coefficient of transmissibility = k.d

A = Coefficient of storage.

- $\boxed{s_2 - s_1 = \frac{2.303Q}{4\pi T} \log_{10} \frac{t_2}{t_1}}$

s_2 = Drawdown of observation well at time t_2.

s_1 = Drawdown of observation well at time t_1.

- $\boxed{\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2}}$ where, r_1 and r_2 is the distance of drawdown in time t_1 and t_2 respectively.

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Quality Control of Water Supplies

4

Total Solid and Suspended Solid

$$\boxed{\text{Total solid} - \text{Suspended solid} = \text{Dissolved solid}}$$

Acceptable limit of total solid = 500 mg/lit

Threshold odour number

$$\boxed{\text{T.O.N} = \frac{\text{Final volume at which odour is hardly detectable}}{\text{Sample volume}}}$$

where, TON = Threshold odour number

$$\boxed{1 \le \text{TON} \le 3}$$

Remember: Dissolved solid in mg / lit = $\frac{\text{Micro mho}}{\text{cm}}$ at 25°C × 0.65

Constituents of Alkalinity

Major sources → CO_3^{2-}, HCO_3^-, OH^-

Minor sources → $HSiO_3^-, H_2BO_3^-, HPO_4^{2-}, HS^-$

(a) $\boxed{\text{Equivalent weight} = \frac{\text{Molecular weight}}{\text{valency}}}$

$$\text{Equivalent weight of } CaCO_3 = \frac{40 + 12 + 3 \times 16}{2} = 50$$

$$\boxed{\text{gram equivalent or number of equivalent} = \frac{\text{weight}}{\text{equivalent weight}}}$$

(b)

Ca^{2+}	+	$2Cl^-$	⟶	$CaCl_2$
1 mole		2 mole		1 mole
1 equivalent		1 equivalent		1 equivalent

∴ equivalent of Ca^{2+}

= equivalent of Cl^-

= equivalent of $CaCl_2$

(c) $\boxed{\text{Alkalinity of water} = (x + y + z)\text{mg/lit of } CaCO_3}$

where, CO_3^{2-} ⟶ x mg/lit of $CaCO_3$

HCO_3^- ⟶ y mg/lit of $CaCO_3$

OH^- ⟶ z mg/lit of $CaCO_3$

pH Value of water

- $\boxed{pH = -\log_{10}[H^+]}$
- $\boxed{pOH = -\log_{10}[OH^-]}$
- $\boxed{pH + pOH = 14}$
- $\boxed{[H^+][OH^-] = 10^{-14}}$
- $\boxed{[H^+] > 10^{-7}}$ for strong acid.
- $\boxed{[H^+] < 10^{-7}}$ for strong base
- $\boxed{[H^+] = [OH^-] = 10^{-7}}$ for neutral solution

Hardness of Water

$$\boxed{\text{Total Hardness} = \left[\left\{\frac{[Ca^{2+}]}{\text{Equivalent weight of } Ca^{2+}}\right\} \times \text{Equivalent weight of } CaCO_3\right] + \left[\left\{\frac{[Mg^{2+}]}{\text{Equivalent weight of } Mg^{2+}}\right\} \times \text{Equivalent weight of } CaCO_3\right]}$$

where, $[Ca^{2+}]$ is concentration of Ca^{2+} in mg/lit.

$[Mg^{2+}]$ is concentration of Mg^{2+} in mg/lit.

(a) $\boxed{\text{Total hardness} = \left[\frac{[Ca^{2+}]}{20} \times 50 + \frac{[Mg^{2+}]}{12} \times 50\right] \text{mg/lit as } CaCO_3}$

where, atomic weight of Ca = 40

atomic weight of Mg = 24

atomic weight of O = 16

atomic weight C = 12

Valency of Ca = 2

Valency of Mg = 2

$$\text{Equivalent weight of } Ca^{2+} = \frac{\text{atomic weight}}{\text{valency}} = \frac{40}{2} = 20$$

$$\text{Equivalent weight of } Mg^{2+} = \frac{\text{atomic weight}}{\text{valency}} = \frac{24}{2} = 12$$

$$\text{Equivalent weight of } CaCO_3 = \frac{\text{Moleculas weight}}{\text{Valency}}$$

$$= \frac{40+17+(3\times16)}{2} = 50$$

As $CaCO_3 \rightarrow Ca^{2+} + CO_3^{2-}$

$\therefore$ Valency or n-factor = 2.

(b) $\boxed{\text{Carbonate Hardness} = \text{Minimum of} \begin{cases} \text{Total hardness} \\ \text{Alkalinity} \end{cases}}$

(c) $\boxed{\text{Non carbonate hardness} = \text{Total hardness} - \text{Alkalinity}}$

Note: –ve value is taken as zero.

(d) Hardness

$0 - 55 \xrightarrow{\text{mg/lit as } CaCO_3}$ Soft

$56 - 100 \xrightarrow{\text{mg/lit as } CaCO_3}$ Slightly hard

$101 - 200 \xrightarrow{\text{mg/lit as } CaCO_3}$ Moderately hard

$201 - 500 \xrightarrow{\text{mg/lit as } CaCO_3}$ Very hard

House hold supplies limits 74 – 115 mg/l

(e) Acceptable limit of hardness = 200 mg/lit as $CaCO_3$ and cause of rejection is 600 mg/lit as $CaCO_3$.

Biochemical Oxygen Demand (BOD)

$$\boxed{(BOD)_5 = (DO_i - DO_f) \times \text{Dilution Ratio}}$$

where, BOD_5 = BOD of 5 days

$DO_i - DO_f$ = Loss of oxygen in mg/lit.

Do_i = Initial dissolved oxygen concentration in the diluted sample.

Do_f = Final dissolved oxygen concentration in the diluted sample.

(After 5 day of incubation of 20° C)

Remember

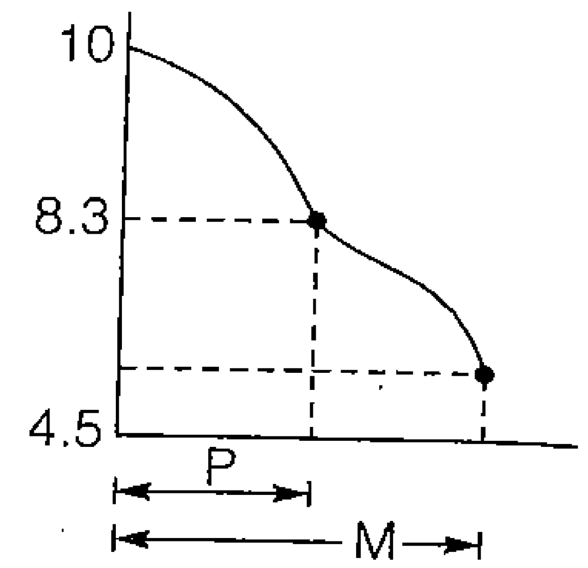
- Aerobic bacteria—i.e., those which require oxygen for their survival.
- Anaerobic bacteria—i.e., those which flourish and thrive in the absence of free oxygen

Facultative bacteria—i.e., those which can survive with or without free oxygen

- Most bacteria are harmless, and under certain conditions beneficial to human begins, animals and crops. Such bacteria or micro-organisms are called non-pathogenic bacteria or non-pathogens. However, certain other bacteria are the deadly foes of man and animals and may enter their tissues, causing serious water borne diseases, such as cholera, typhoid, infectious hepatitis, etc. Such harmful bacteria or organisms are known as pathogenic bacteria or pathogens.

Water Quality Parameters	Instrument Used	Additional Information
1. Turbidity	1. Turbidity Rod (Absorption Principle) Field Method	
	2. Jackson Tubidmeter method (Absorption Principle) Haboratory method	(i) Only for sample with turbidity > 25 units (ii) Generally for water in natural water bodies and not for water supply.
	3. Baylis Turbidmeter (Absorption) (colour matching technique)	(i) Can measure turbidity < 1 unit
	4. Nephlometer	(i) Can measure turbidity < 1 unit
2. Colour	Tintometer (using nessler tubes)	Std. units TCU Pt in (chloroplatinate form) in 1 *l* of water
3. Taste and Odour	Osmoscope	Std. units TON

- Turbidity > 5 units - detectable by naked eye.
- In Nephlometer, turbidity is expressed in FTU.

Chemical W.Q.P.	Measurement	
1. Alkalinity	Titratant : $0.02\ N\ H_2SO_4$ Indicates : Phenolphthaline (Basic Indicator) Methyl Orange (acidic indicator)	$OH^- + H^+ \rightarrow H_2O$ $CO_3^{2-} + H^+ \rightarrow HCO_3^-$  (i) $P < \frac{M}{2}$ (Most common observation)
2. PH	Instrument : Acquascope Potentiometer (with Calomel Rods)	
3. Hardness	Titration Titiant : 0.01 M EDTA Indicator : EBT	British Degree Of Hardness = 14.25 mg/l (as $CaCO_3$) French D.O.H. = 10 mg/l (as $CaCO_3$)
4. Dissolved solids	i. Graviometric Technique ii. Di-ionic tester	Electrical conductivity calliberated against total dissolved solids
5. Chloride Content	Gitrant : $AgNO_3$ solution Indicator : $K_2C_rO_4$	
6. Nitrogen content	i. Free $NH_3 \rightarrow$ Simple boiling and distribution ii. Organic : Addition of $KMnO_4$ to already boiled water sample iii. Nitrite : Colour making (Sulphonic acid and Napthamine) iv. Nitrate : Phenol disulphonic + KOH acid	INDICATE Recent Pollution N_2 before decomposition

7. Fluoride content	Colour Matching Colour indicued by Zirconium	If < 1 ppm - Dental cavities If > 1.5 ppm - Fluorosis
8. Oxygen	Winkler's Method	If nitrite ion present. Modified Winkler's adopted

Biological WQP

Testing of Colliforms is done

Tests/Technique	Remarks
1. MPN Test	i. Multiple tube Feimentation test ii. Nutrient used : Lactose broth iii. More is the dilution of sample lesser is the possibility of getting +ve test.
2. Membrane Filter Technique	i. Nutrient : M-Endo Medium ii. Colliform colonies are counted
3. Colliform Index Test	Reciprocal of smallest quantity of sample giving +ve B-coli test

Water Quantity Parameter	Permissible Units	Cause for Rejection Value
1. Suspended solids	500 mg/*l*	2000 mg/*l*
2. Turbidity	1 NTU	10 NTU
3. Colour	5 TCU	25 TCU
4. Taste and Odour	1 TON	3 TON
5. Alkalinity	200 mg/*l*	600 mg/*l*
6. pH	7 - 8.5	< 6.5
7. Hardness	200 mg/*l*	600 mg/L
8. Chloride content	200 mg/*l*	1000 mg/L

	Water Quantity Parameter	Permissible Units	Cause for Rejection Value
9.	Fre NH_3	0.15 mg/l	0.15 mg/l
	Organic NH_3	0.3 mg/l	0.3 mg/l
	Nitrite	0	0
	Nitrate	45 mg/l	
10.	Fluoride	1 mg/l	1.5 mg/l
11.	Fe	0.1 mg/l	1 mg/l
12.	Mn	0.05 mg/l	0.5 mg/l
13.	Cu	0.05 mg/l	1.5 mg/l

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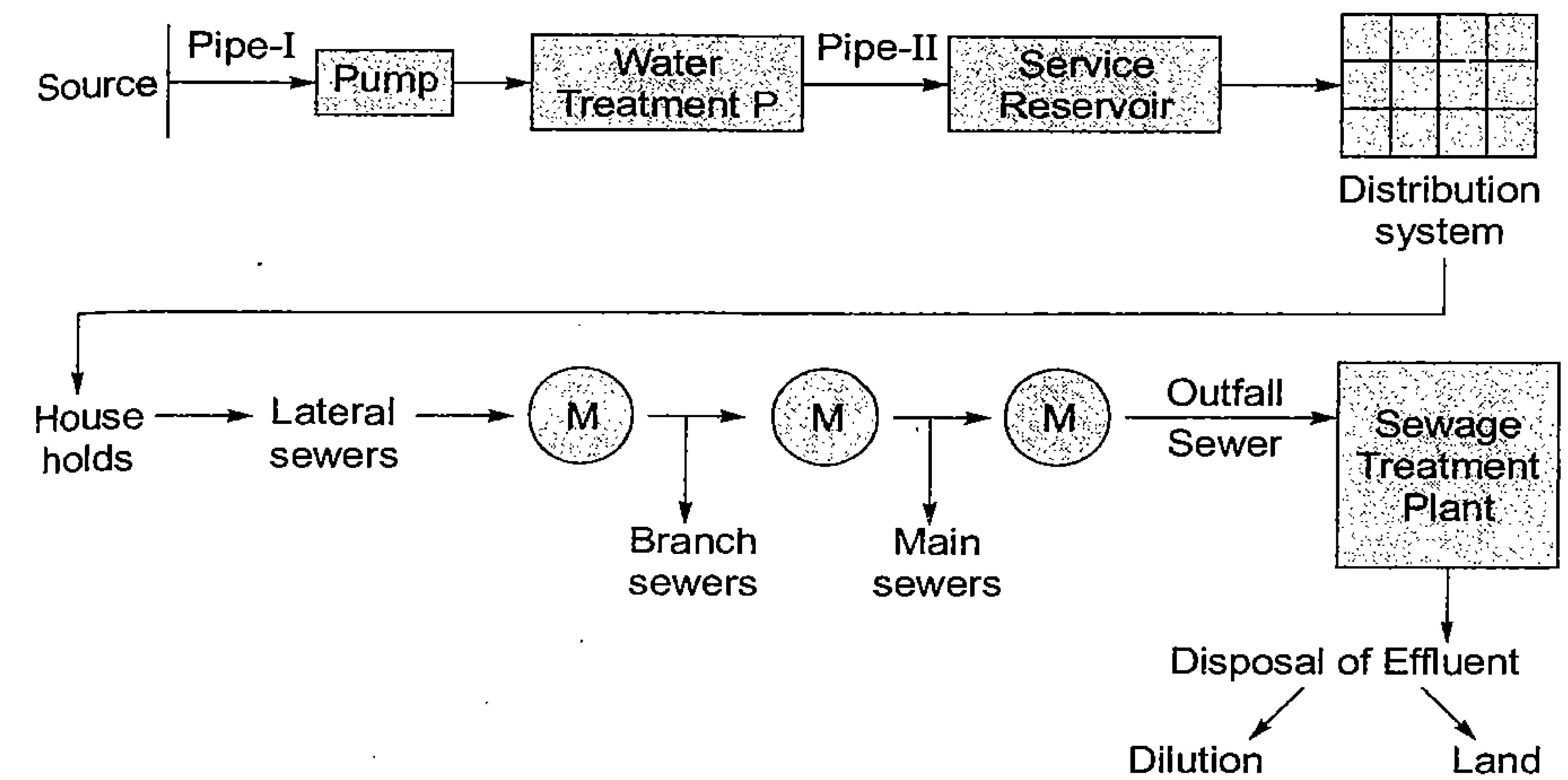
5 Water Treatment



Theory of Sedimentation

- Stokes Law

(a) $$V_S = \frac{g}{18}(G-1)\frac{d^2}{\nu}$$ for d < 0.1 mm.

where, V_S = Velocity of settlement of particle in m/s.
d = Diameter of the particle in meter.
G = SP gravity of the particle

$$= \frac{\gamma_s}{\gamma_w} \text{ or } \frac{\delta_s}{\delta_w}$$

ν = Kinematic viscosity of water in m^2/sec.

(b) $$V_S = \left[\frac{\frac{4}{3}gd(G-1)}{C_D}\right]^{1/2}$$

$C_D = 0.4$ → For ($R_e > 10^4$) $C_D = \frac{24}{R_e}$ → For ($R_e < 0.5$)

$$C_D = \frac{24}{R_e} + \frac{3}{\sqrt{R_e}} + 0.34 \quad \text{For } 0.5 \le R_e \le 104$$

(c) $$V_S = 418(G-1)d^2\left(\frac{3T+70}{100}\right) \quad \text{for } d < 0.1 \text{ mm}$$

where, T = Temperature of water in °C

V_S is in mm/sec.

d is in mm.

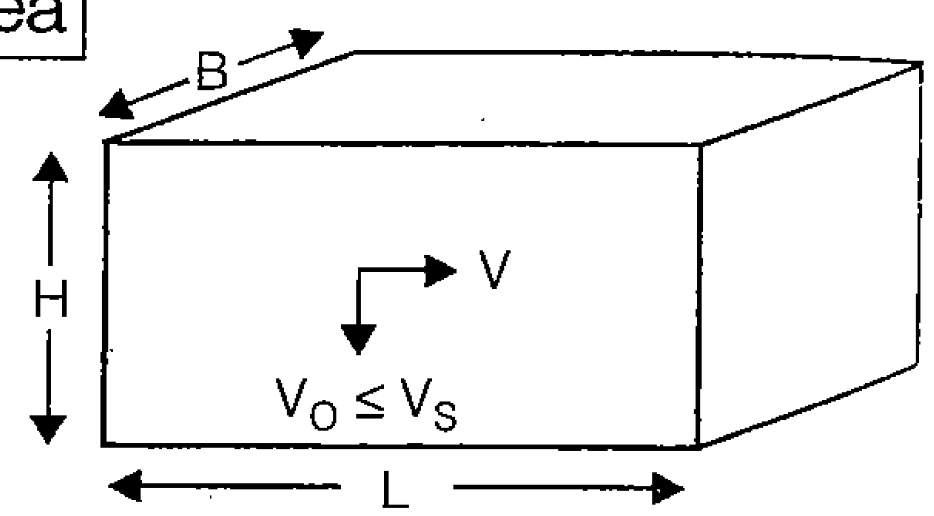
(d) $$V_S = 1.8\sqrt{gd(G-1)} \quad \text{For } d > 0.1 \text{ mm.}$$

(e) $$V_S = 418(G-1)d\left(\frac{3T+70}{100}\right) \quad \text{For } 0.1 < d < 1 \text{ mm.}$$

Sedimentation Tank : Surface Over flow Rate

(a) $$\text{Over flow rate, } V_O = \frac{\text{Discharge}}{\text{Surface area}}$$

$$V_O = \frac{Q}{BL}$$



V_O = 12000 to 18000 lit/m²/day for plain sedimentation.

V_O = 24000 to 30,000 lit/m²/day for sedimentation with coagulation.

(b) Velocity of flow, $$V_f = \frac{Q}{BH}$$

(c) Time of horizontal flow, or Detention time $$T = \frac{L}{V_f} = \frac{L}{Q/BH} = \frac{LBH}{Q}$$

(d) Time of falling through height 'H' $$T = \frac{H}{V_S} = \frac{LBH}{Q}$$

(e) Detention time, $$t_d = \frac{L}{V_f} = \frac{H}{V_S}$$

4 to 8 hr → For plain sedimentation

2 to 4 hr → For sedimentation with coagulation

(f) η, Efficiency

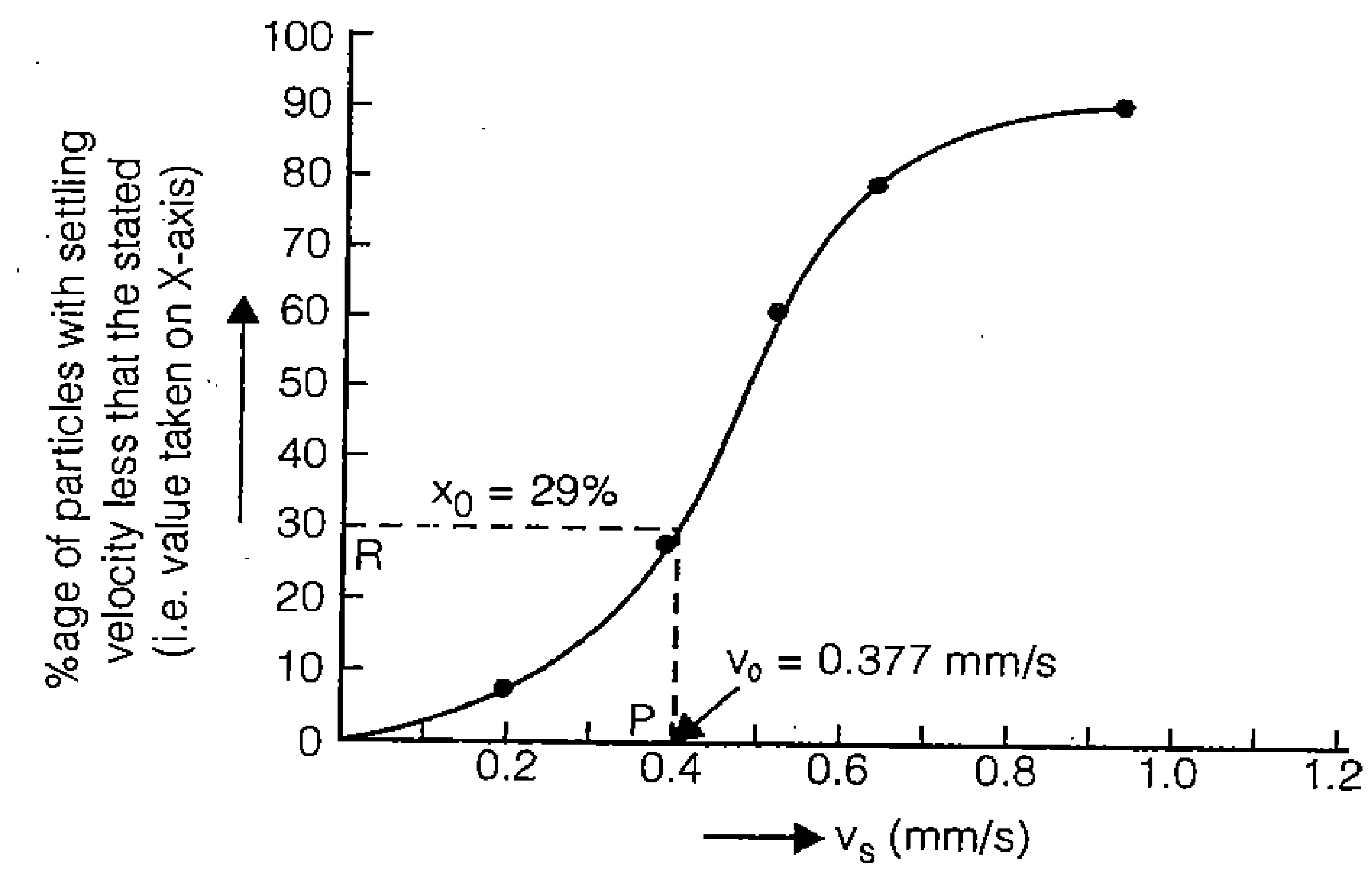
$$P_\rho = \frac{V_S}{V_O} \times 100$$

where, P_e = % of lighter particles (with settling velocity (V_S) less than V_O) which shall be removed in an ideal settling basin.

(g) % of particle removed

$$= (100 - x_0) + \int_{x=0}^{x=x_0} \left(\frac{V_S}{V_O} \times 100\right) \cdot dx$$

where, x_0 corresponds to V_0



(h) Detention time 't'

$$t = \frac{BLH}{Q} \quad \text{for rectangular tank.}$$

$$t = \frac{d^2(0.011d + 0.785H)}{Q} \quad \text{for circular tank}$$

where, d = Dia of the tank

H = Vertical depth of wall or side water depth

(i) $$\text{Displacement efficiency} = \frac{\text{Flowing through period}}{\text{Detention period}}$$

(j) Scour velocity, V_d $$V_d = \sqrt{\frac{8\beta}{f'} g(G-1)d}$$

where, β = 0.04 for unigranular sand and 0.06 or more for non-uniform (interlocking) sticky material.

f′ = Darcy weisback friction factor.

= 0.025 to 0.03 for settling tanks.

Chemicals used for Coagulation

- Alum $(Al_2(SO_4)_3.18H_2O)$

$Al_2(SO_4)_3.18H_2O + 3Ca(HCO_3)_2 \longrightarrow 3CaSO_4 + 2Al(OH)_3\downarrow + 6CO_2\uparrow$

666 gm of Alum racts with 48 gm of $Ca(HCO_3)_2$ to give 146 gm of $Al(OH)_3$

⇒ 1 gm of Alum reacts with 0.73 gm of $Ca(HCO_3)_2$ alk. or 0.45 gm of $CaCO_3$ alk.
to give 0.234 gm of $Al(OH)_3$ ppt.

$Al_2(SO_4)_3.18H_2O + 3Ca(OH)_2 \longrightarrow 3CaSO_4 + Al(OH)_3\downarrow + 18H_2O$

$Al_2(SO_4)_3.18H_2O + 3Na_2CO_3 \longrightarrow 3Na_2SO_4 + 2Al(OH)_3\downarrow + 3CO_2\uparrow + 15H_2O$

- Copperas $(FeSO_4.7H_2O)$

$FeSO_4.7H_2O + Ca(OH)_2 \longrightarrow CaSO_4 + Fe(OH)_2 + 7H_2O$

Copperas Hydrated line Ferrous hydroxide

$FeSO_4.7H_2O + Ca(HCO_3)_2 \longrightarrow Fe(HCO_3)_2 + CaCO_3 + 7H_2O$

$Fe(HCO_3)_2 + 2Ca(OH)_2 \longrightarrow Fe(OH)_2 + 2CaCO_3 + 2H_2O$

$Fe(OH)_2 + O_2 + 2H_2O \longrightarrow 4Fe(OH)_3\downarrow$

Ferric Hydroxide

- Chlorinated Copperas : $(Fe_2(SO_4)_3$ and $FeCl_3)$

$6(FeSO_4.7H_2O) + 3Cl_2 \longrightarrow 2Fe_2(SO_4)_3 + 2FeCl_3 + 42H_2O$

Ferric sulphate

$Fe_2(SO_4)_3 + 3Ca(OH)_2 \longrightarrow 3CaSO_4 + 2Fe(OH)_3\downarrow$

Ferric Sulphate Hydrated line Ferric hydroxide ppt

$2FeCl_3 + 3Ca(OH)_2 \longrightarrow 3CaCl_2 + 2Fe(OH)_3\downarrow$

Ferric Chloride Hydrated line Ferric hydroxide ppt

- Sodium Aluminate $(Na_2Al_2O_4)$

$Na_2Al_2O_4 + Ca(HCO_3)_2 \longrightarrow CaAl_2O_4\downarrow + Na_2CO_3 + CO_2\uparrow + H_2O$

$Na_2Al_2O_4 + CaCl_2 \longrightarrow CaAl_2O_4\downarrow + 2NaCl$

$Na_2Al_2O_4 + CaSO_4 \longrightarrow CaAl_2O_4\downarrow + Na_2SO_4$

Mixing Basin

$$G' = \left[\frac{P}{\mu V}\right]^{1/2}$$

where,
G′ = Temporal mean Velocity gradient (per second).
P = Power dissipated in watts i.e., N-m/s.

V = Volume of raw water to which P is applied in m^3.
μ = Dynamic viscosity (N-s/m^2).

Flocculation

- Velocity gradient, $G' = \left[\frac{P}{\mu V}\right]^{1/2}$.
- $20\ sec^{-1} < G' < 75\ sec^{-1}$.
- Detention time, t_d is 10 to 30 minute.
- $G' \cdot t_d$ (Conjuction opportunity) $= \frac{\text{Power induced rate of flow}}{\text{Displacement induced rate of flow}}$
- Number of particle collision $\alpha\ G't_d$.
- $G'td = 2 \times 10^4$ to 6×10^4 for Alum.
  $= 1 \times 10^5$ to 1.5×10^5 for Iron salt.
- $\frac{G' \text{of influent end}}{G' \text{of effluent end}} = 2$

Filtration

A. Slow Sand Filter

- Depth of filter is 2.5 to 3.5 m.
- Plan area of filter is 100 to 200 m^2.
- $0.2 \le D_{10}$ of sand ≤ 0.3 mm.
- $\frac{D_{60}}{D_{10}} = 5$.
- Design period = 10 years.
- Depth of sand is 90 to 110 cm.
- Frequency of cleaning is 1 to 3 months
- Rate of filtration = 2400 to 4800 lit/m^2/day or 100 to 200 lit/m^2/hr.
- Efficiency of bacteria removal = 98 to 99%.
- It can not be used if turbidity > 50 ppm.
- It is designed for maximum daily demand.
- $\frac{\text{Discharge}}{\text{Rate of filteration}} = \text{Plan area}$.

B. Rapid Sand Filter

- $N = 1.22\sqrt{Q}$ where, N = Number of unit required
  Q = Plant capacity in million lit/day (MLD)

- $\frac{D_{60}}{D_{10}} = 1.2$ to 1.8
- Sand layer depth is 60 to 90 cm.
- D_{10} of sand is 0.35 to 0.55 mm.
- Depth of tank = 2.5 m to 3.5 m.
- Area = 10 to 80 m^2 each unit.
- Rate of washing is 15 to 90 cm rise/minute.
- Rate of filtration 3000 to 6000 lit/m^2/hour (slow sand filter × 30)
- Cross-sectional area of manfold =2 × cross-sectional area of lateral.
- Cross-sectional area of each lateral = 2 to 4 times cross-sectional area of perforations in it.
- Total cross-sectional area of perforation = 0.2% of total area of 1 filter bed
- $\frac{\text{Length of each lateral}}{\text{Dia of lateral}} \ngtr 60$
- 4-5% of filtered water is used as back wash.
- 30 min. used for back wash.

> **Remember:** Economical dia of rising main is given by Lea $\boxed{D = 1.22\sqrt{Q}}$
> Q is in m^3/sec, D is in meter.

Hydraulics of Sand Gravity Filters

$$\boxed{h_L = \frac{1.067V^2}{\phi \cdot g \cdot n^4} \in \frac{C_D \cdot f}{d}}$$

where,
h_L = Frictional head loss through the filter in meter.
V = Approach velocity or filtration velocity in m/s.
D = Depth of filter in meter
ϕ = Shape factor (for non spherical particle)
d = Diameter of sand particles in meter.
g = Accelerations due to gravity in m/s^2.
n = Porosity
C_D = Newton's dray coefficient.
f = Mass friction of sand particle of dia d.

- **Rose Equation,** $\boxed{h_L = \left|\frac{1.067V^2 D}{\phi g n^4} \cdot \frac{C_D}{d}\right]}$

Hydraulic head loss and expansion of the filter during backwash of RSF

- $\boxed{H_{L_e}\gamma_w = D\gamma_{sub}}$ where, H_{Le} = Head loss through the filter bed required to initiate expansion in meter.
  γ_w = Unit weight of mater in kN/m^3.
  D = Depth of filter bed in meter.
  γ_{sub} = Submerged unit weight of sand in bed of depth 'D' in kN/m^3.
- $\boxed{H_{L_e} = D(1-n)(G-1)}$
- $\boxed{H_{L_e} = D_e(1-n_e)(G-1)}$

  where, D_e = Depth of expanded/fluidized bed in meter.
  n_e = The porosity of the expanded fluidized bed.
- $\boxed{D_e = \frac{(1-n)D}{(1-n_e)}}$
- $\boxed{D_e = (1-n)D.\Sigma\frac{f}{1-n_e}}$

  where, f =mass fraction of sand of various sizes in the sand (as per sieve analysis)
- $\boxed{n_e = \left(\frac{V_b}{V_s}\right)^{0.22}}$ where, n_e = Porosity of expanded bed
  V_b = Backwash velocity in m/s
  V_s = Settling velocity in m/s.
- $\boxed{V_s = \left[\frac{4}{3}\frac{gd(G-1)}{C_D}\right]^{1/2}}$

> **Remember**
> - **Pressure Filters:** Pressure filters are just like small rapid gravity filters placed in closed vessels, and through which water to be treated is passed under pressure.
> - Rate of Filtration—6,000 to 15,000 litre/hour/m^2 (Rapid Sand Filter × 2)
> - The pressure filter are less efficient than the rapid gravity filters, in removing bacteria and turbidities.

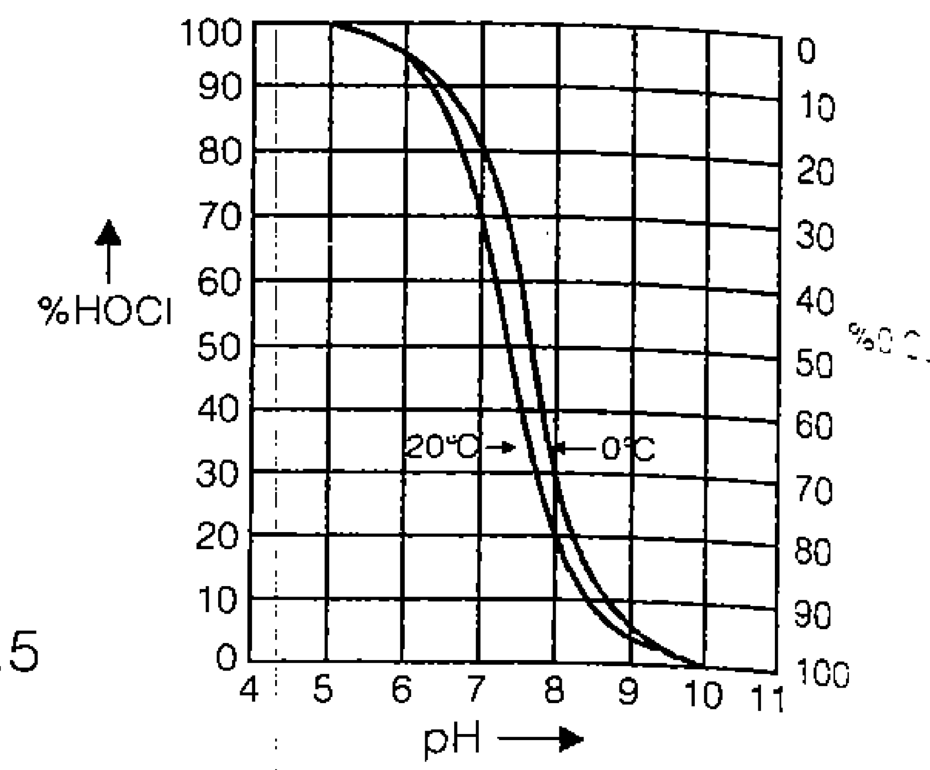
Disinfection or Sterilization

(i) Treatment with Ozone

$$\underset{\substack{\text{Oxygen}\\\text{Molecule}\\\text{3 Mole}}}{3O_2} \xrightarrow[\text{Arc voltage}]{\text{Under high electric}} \underset{\substack{\text{Ozone}\\\text{Molecule}\\\text{2 Mole}}}{2O_3}$$

$$\underset{\text{Ozone}}{O_3} \longrightarrow \underset{\text{Oxygen}}{O_2} + \underset{\text{Nascent oxygen}}{[O]}$$

(ii) Disinfecting Action of Chlorine



$Cl_2 + H_2O \xrightarrow{PH > 5} HOCl + HCl$

Hypochlorous acid.

$HOCl \underset{PH<7}{\overset{PH>8}{\rightleftharpoons}} H^+ + OCl^-$

Hydrogen ion Hypochlorite ion

$NH_3 + HOCl \rightarrow NH_2Cl + H_2O$ PH > 7.5

Monochloro Amine

$NH_2Cl + HOCl \rightarrow NHCl_2 + H_2O$ PH→5 to 6.5

Di-chloroamine

$NHCl_2 + HOCl \rightarrow NCl_3 + H_2O$ PH < 4.4

Nitrogen Trichloroamine

(iii) Doses of Chlorine

Type of virus to be killed	Quantity of free chlorine required in mg/l with about 30 minutes contact period for water of pH lower than 7 or so
Poliomyelitis virus	0.1
Hepatitis virus	0.4
Cysts of E.histolytica, i.e. the organism causing ameobic dysentery	3.0 or even lower
Tuberculosis organisms	3.0
Coxsaickie Virus	Very huge dose varying from 21 to 138 mg/l.

(iv) Forms in which chlorine is applied

(a) Free chlorine
(b) Hypochlorites & Bleaching Powder
(c) Chloramines
(d) Chlorine dioxide (ClO_2)

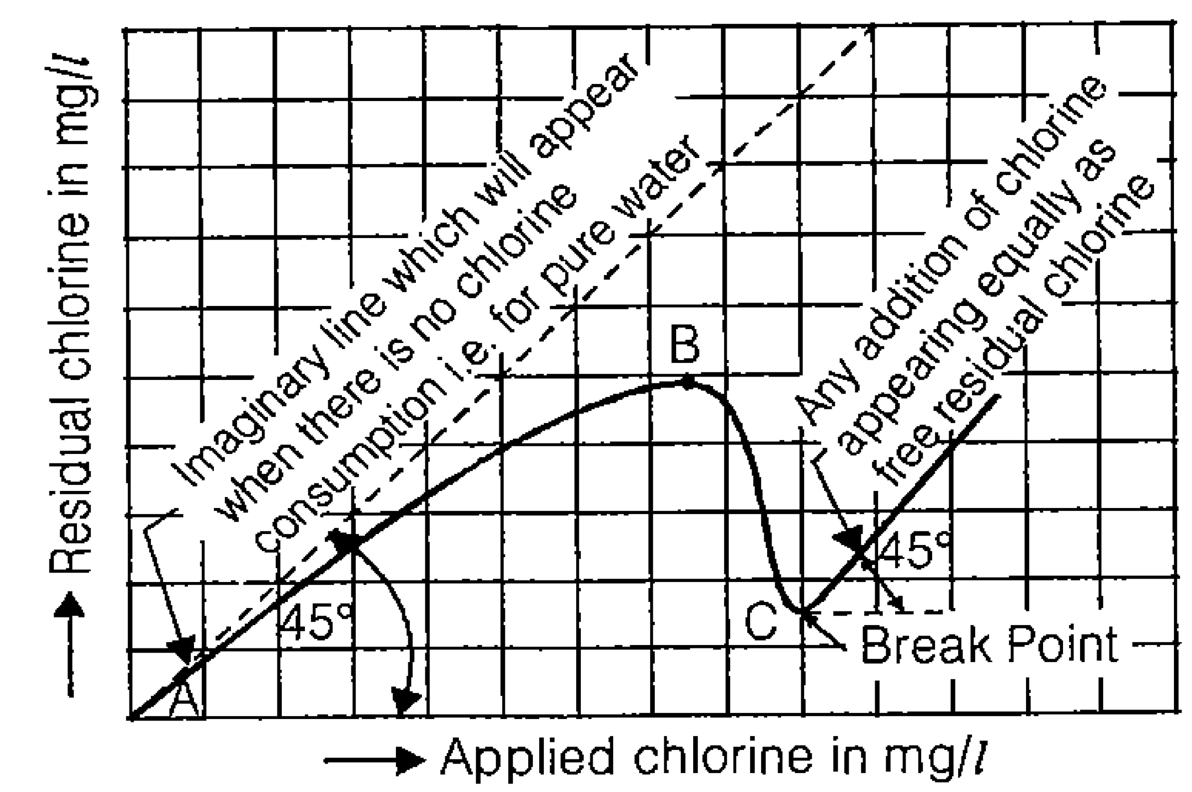
$Ca(OCl)_2 \xrightarrow{H_2O} Ca^{2+} + 2OCl^-$

Calcium Hypochlorite Calcium ion Hypochlorite ion

$OCl^- + H^+ \underset{PH>8}{\overset{PH<7}{\rightleftharpoons}} HOCl$ (Hypochlorous acid)

Type of Chlorination

(i) Plain chlorination
(ii) Pre-chlorination
(iii) Post-chlorination
(iv) Double chlorination
(v) Break point chlorination
(vi) Super chlorination
(viii) Dechlorination



Test of Chlorine Residual

1. Orthotoulidine test : colour - mathcing method
2. Arsenide orthotoulidine test - when mineral present in water sample, Also a colour matching method.
3. DPD and chlorotex test (Di-ethyl phenyline diamine): colourmaking method.
4. Starch iodide Test

$$\begin{bmatrix}\text{Quantity of chlorine in}\\ \text{mg/lit in the original}\\ \text{sample of water}\end{bmatrix} = 0.355 \begin{bmatrix}\text{Number of ml of}\\ \text{thiosulphate}\\ \text{required to remove}\\ \text{the blue colour}\end{bmatrix}$$

Water Softening

- **Methods of Removing Temporary Hardness**

(i) Boiling

$Ca(HCO_3)_2 + \text{Heat} \longrightarrow CaCO_3\downarrow + CO_2\uparrow + H_2O$

Calcium bi Carbonate; Calcium Carbonate

(ii) Addition of Lime

$MgCO_3 + Ca(OH)_2 \longrightarrow Mg(OH_2)\downarrow + CaCO_3\downarrow$

Magnesium carbonate; Hydrated lime; Magnesium Hydroxide; Calcium Carbonate

$Mg(HCO_3)_2 + Ca(OH)_2 \longrightarrow Ca(HCO_3)_2 + Mg(OH)_2\downarrow$

$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + 2H_2O$

- **Method of Removing Permanent Hardness**

(i) Lime-Soda Process

(a) $Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + 2H_2O$

(b) (i) $Mg(HCO_3)_2 + Ca(OH)_2 \longrightarrow Ca(HCO_3)_2 + Mg(OH)_2\downarrow$

(ii) $MgCO_3 + Ca(OH)_2 \longrightarrow Mg(OH)_2 + CaCO_3\downarrow$

(c) $MgCl_2 + Ca(OH)_2 \longrightarrow Mg(OH)_2\downarrow + CaCl_2$

(d) $MgSO_4 + Ca(OH)_2 \longrightarrow Mg(OH)_2\downarrow + CaSO_4$

(e) $CO_2 + Ca(OH)_2 \longrightarrow CaCO_3\downarrow + H_2O$

(f) $CaCl_2 + Na_2CO_3 \longrightarrow CaCO_3\downarrow + 2NaCl$

(g) $CaSO_4 + Na_2CO_3 \longrightarrow CaCO_3\downarrow + Na_2SO_4$

- $\boxed{\text{Dry sludge produced in mg/lit} = [C_{aR} + 0.58M_{gR} + L_{iA}]}$

where, C_{aR} = Calcium hardness removed in mg/lit (expressed as $CaCO_3$)

M_{gR} = Magnesium hardness removed in mg/lit (expressed as $CaCO_3$)

L_{iA} = Lime added in mg/lit (expressed as $CaCO_3$)

- **Zeolite or Base Exchange or Cation-Exchange Process for Removing Hardness**

$$Na_2z + \begin{matrix} Ca \\ Mg \end{matrix}\left\{\begin{matrix} (HCO_3)_2 \\ SO_4 \\ Cl_2 \end{matrix}\right. \longrightarrow Na_2\left\{\begin{matrix} (HCO_3)_2 \\ SO_4 \\ Cl_2 \end{matrix}\right. + \begin{matrix} Ca \\ Mg \end{matrix}\Big\{ z$$

Sodium zeolite or active zeolite; Calcium or Magnesium Salt; Sodium slats which don't cause hardness; Calcium or Magnesium zeolite

$$\left.\begin{matrix} Ca \\ Mg \end{matrix}\right\} z + 2NaCl \longrightarrow Na_2z + \left.\begin{matrix} Ca \\ Mg \end{matrix}\right\} Cl_2$$

Used zeolite; Sodium chloride solution; Regenerated zeolite

- **Demineralization Process for Removing Hardness**

$Ca(HCO_3)_2 + H_2R \longrightarrow CaR + 2H_2O + 2CO_2\uparrow$

(H_2R: Fresh cation exchange resin; CaR: Exhausted rasin)

$CaCl_2 + H_2R \longrightarrow CaR + 2HCl$

$MgSO_4 + H_2R \longrightarrow MgR + H_2SO_4$

$2NaCl + H_2R \longrightarrow Na_2R + 2HCl$

$R\text{-}OH + HCl \longrightarrow RCl + H_2O$

(R-OH: Fresh anion exchange resin; RCl: Exhausted resin; H_2O: Water)

$2R\text{-}OH + H_2SO_4 \longrightarrow R_2SO_4 + 2H_2O$

(2R-OH: Fresh anion exchange resin; R_2SO_4: Exhausted resin; $2H_2O$: Water)

NOTE: Hardness for drinking water = 75 – 115 mg/lit. is recommended.

Drinking water specification: IS : 10500, 1992 (Reaffirmed 1993)

Tolerance Limit

S. No.	Parameter	IS: 10500 Requirement (desirable limit)	Undesirable effect outside the desirable limit	IS: 10500 Permissible limit in the absence of alternate source
	Essential Characteristics			
1.	pH	6.5-8.5	Beyond this range the water will effect the mucous membrane and/or water supply system	No relaxation
2.	Colour (hazen units), Maximum	6.5-8.5	Above 5, consumer acceptance decreases	25
3.	Odour	Unobjectionable	----	---
4.	Taste	Agreeable	----	---
5.	Turbidity, NTU, Max	5	Above 5, consumer acceptance decreases	10
	Following Results are expressed in mg/l:			
6.	Total hardness as $CaCO_3$ Max	300	Encrustation in water supply structure and adverse effects on domestic use	600
7.	Iron as Fe, Max	0.30	Beyond this limit taste/appearance are affected, has adverse effect on domestic uses and water supply structures, and promotes iron bacteria.	1.0
8.	Chlorides as Cl, Max	250	Beyond this limit tast, corrosion and palatability are effected	1000
9.	Residual, Free Chlorine, Min	0.20	---	---

Desirable Characteristics				
10.	Dissolved solids, Max	500	Beyond this palatability decreases and may cause gastro intentional irritation	2000
11.	Calcium as Ca, Max	75	Encrustation in water supply structure and adverse effects on domestic use	200
12.	Magnesium as Mg. Max	30	---	100
13.	Nitrates as NO_3	45	Beyond this methane-moglobinemia takes place	100
14.	Fluoride, Max	1.0	Fluoride may be kept as low as possible. High fluoride may cause fluorosis	1.5
15.	Alkalinity, Max	200	Beyond this limit taste becomes unpleasant.	600

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6 Design of Sewer

Hydraulic Formulas for Determining Flow Velocities in Sewers and Drains

(i) Chezy's Formula

$$\boxed{V = C\sqrt{RS}}$$

where, C = Chezy's constant
S = Hydraulic gradient
R = Hydraulic mean depth or Hydraulic Radius

$$= \frac{\text{Area}}{\text{Wetted perimeter}}$$

$$\boxed{R = \frac{D}{4}}$$ for circular sewer of dia D (in Running full)

Discharge, $\boxed{Q = AV}$ where, A = Flow area of cross-section.

(a) Kutters Formula

$$\boxed{C = \frac{23 + \frac{0.00155}{S} + \frac{1}{n}}{1 + \left(\frac{23 + 0.00155}{S}\right)\frac{n}{\sqrt{R}}}}$$

where, n = Rugosity coefficient
S = Bed slope

(b) Bazin formula

$$\boxed{C = \frac{157.6}{1.81 + \frac{k}{\sqrt{R}}}}$$

where, k = Bazin constant.

(ii) Manning's Formula

$$\boxed{V = \frac{1}{n} \cdot R^{2/3} \cdot S^{1/2}}$$

(iii) Crimp and Burge's Formula

$$\boxed{V = 83.5 R^{2/3} \cdot S^{1/2}}$$

This formula is close to Manning's formula for all those sewer materials for which,

$$\boxed{n = 0.012}$$

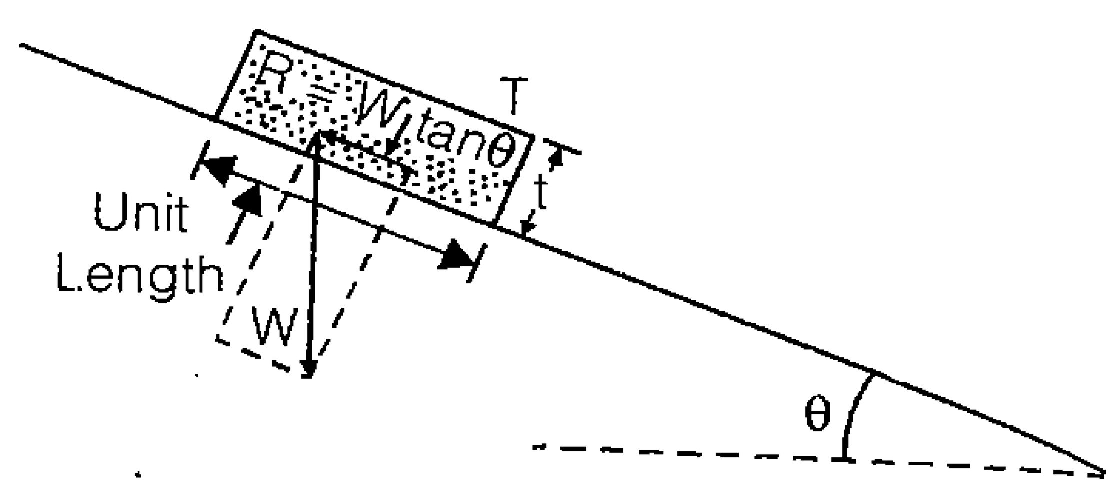
(iv) William Hazen's Formula

$$\boxed{V = 0.85C_H \cdot R^{0.63} \cdot S^{0.54}}$$

S. No.	Type of pipe material	Value of C_H for	
		New pipes	Design purposes
1.	Concrete and R.C.C. pipes	140	110
2.	Cast iron pipes	130	100
3.	Galvanised iron pipes	120	100
4.	Steel pipes with welded joints	140	100
5.	Steel pipes with rivetted joints	110	95
6.	Steel pipes with welded joints lined with cement for bituminous enamel	140	110
7.	Asbestos cement pipes	150	120
8.	Plastic pipes	150	120

SHIELDS EXPRESSION FOR SELF CLEANSING VELOCITY

$$\boxed{V = \sqrt{\frac{8g}{f}(1-n)\sin\theta\,(G-1)d'}}$$



where, f = Friction Factor
n = Porosity
$\sin\theta \sim \tan\theta$ = Slope
$\Rightarrow \theta = \tan^{-1}$ (Slope)
G = Specific gravity of solid
d = Avg size of solid.

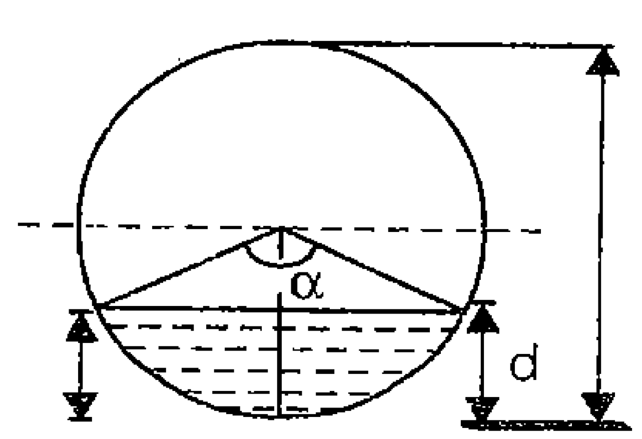
$$\boxed{V = \sqrt{\frac{8k}{f}(G-1)gd'}}$$ where, $k = (1-n)\sin\theta$

Hydraulic Characteristics of Circular Sewer Free Board Provision

(i) Depth of partial flow

$$\boxed{d = \left[\frac{D}{2} - \frac{D}{2}\cos\frac{\alpha}{2}\right]}$$

Partially filled circular sewer section



Full section of Dia D

where, α = Central angle in degree
d = Depth of partial flow
D = Dia of full section

(ii) **Proportionate depth, (d/D)** $\boxed{\frac{d}{D} = \frac{1}{2}\left(1-\cos\frac{\alpha}{2}\right)}$

(iii) **Proportionate area (a/A)** $\boxed{\frac{a}{A} = \left(\frac{\alpha}{360^\circ} - \frac{\sin\alpha}{2\pi}\right)}$

where, $\boxed{a = \frac{\pi D^2}{4}\cdot\frac{\alpha}{360^\circ} - \frac{D}{2}\cos\frac{\alpha}{2}\cdot\frac{D}{2}\sin\frac{\alpha}{2}}$ $\boxed{A = \frac{\pi D^2}{4}}$

(iv) **Proportionate perimeter** $\boxed{\frac{p}{P} = \frac{\alpha}{360^\circ}}$

where, $\boxed{p = \pi D\frac{\alpha}{360^\circ}}$ $\boxed{P = \pi D}$

(v) **Proportionate Hydraulic Mean Depth** $\boxed{\frac{r}{R} = \left[1 - \frac{360^\circ\sin\alpha}{2\pi\alpha}\right]}$

(vi) **Proportionate Velocity** $\boxed{\frac{v}{V} = \frac{N}{n}\cdot\frac{r^{2/3}}{R^{2/3}}}$

(a) If n = N $\boxed{\frac{v}{V} = \left[1 - \frac{360^\circ\sin\alpha}{2\pi\alpha}\right]^{2/3}}$

(vii) **Proportionate Discharge** $\boxed{\frac{q}{Q} = \left[\frac{\alpha}{360^\circ} - \frac{\sin\alpha}{2\pi}\right]\left[1 - \frac{360^\circ\sin\alpha}{2\pi\alpha}\right]^{2/3}}$

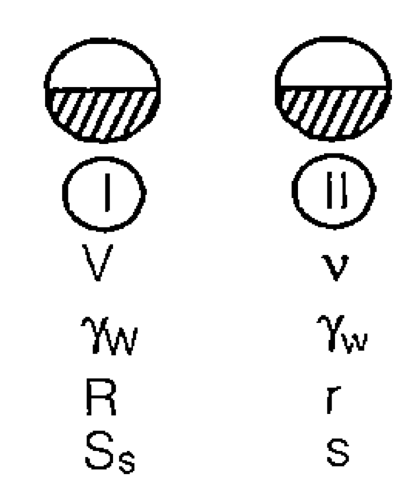
(viii) For two sewers whose degree of cleansing is same.

i.e. $\boxed{\gamma_w rs = \gamma_w RS_s}$

(a) $\frac{S_S}{S} = \frac{R}{r}$

(b) $\frac{V_S}{V} = \frac{N}{n}\cdot\left(\frac{r}{R}\right)^{1/6}$

(c) $\frac{q_S}{Q} = \frac{N}{n}\cdot\left(\frac{a}{A}\right)\left(\frac{r}{R}\right)^{1/6}$



- $\boxed{\phi_{sewer} < 0.4\,m}$ design to run $\frac{1}{2}$ full of Q_{design}.
- $\boxed{0.4\,m < \phi_{sewer} < 0.9\,m}$ design to run $\frac{2}{3}$ full of Q design.
- $\boxed{\phi_{sewer} > 0.9}$ design to run $\frac{3}{4}$ full of design.

where, ϕ = Dia of sewer.

Q_{design} = Design discharge.

Sewer is Designed for Maximum Hourly Discharge

(i) **Maximum hourly discharge**

= 1.5 (Maximum daily discharge)

(ii) **Maximum daily discharge**

= 2.0 (Avg daily discharge)

(iii) **Maximum hourly discharge**

= 3.0 (Avg daily discharge)

NOTE: Sewer is generally design to run $\frac{1}{2}$ to $\frac{3}{4}$th full condition under maximum hourly discharge.

- Minimum discharge = Minimum hourly discharge

$$= \frac{1}{3} \cdot \text{Avg daily discharge}$$

- **Estimation of Peak Drainage Discharge**

(i) $$k_{equivalent} = \frac{k_1A_1 + k_2A_2 + \ldots + k_nA_n}{A_1 + A_2 + \ldots + A_n}$$

where, k = Runoff/rainfall

Suffix 1, 2, ... n used for k is runoff of 1, 2, ... n.

A = Area.

(ii) Time of concentration = Time of inlet (i.e., over land flow) + Channel flow time

(iii) $$p_c = p_o\left(\frac{2}{1+T_c}\right)$$

where, p_c = Maximum rainfall intensity of a particular frequency having duration equal to time of concentration.

p_o = (Point rainfall intensity of same frequency as P_c) × Area dispersion factor. (cm/hr)

T_c = Time of concentration in hr.

(iv) $$Q_P = \frac{1}{36} \cdot k \cdot P_C \cdot A$$

where, Q_P = Peak discharge in m^3/sec.

A = Catchment area in **(Hectare)**

p_c = Critical rainfall in (cm/hour)

Type of Sewer	Ratio to maximum flow to average flow
Trunk mains above 1.25 m in dia.	1.5
Mains up to 1 m in dia.	2.0
Branches up to 0.5 m in dia.	3.0
Laterals and small sewers upto 0.25 m in dia.	4.0

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Quality & Characteristics of Sewage

7

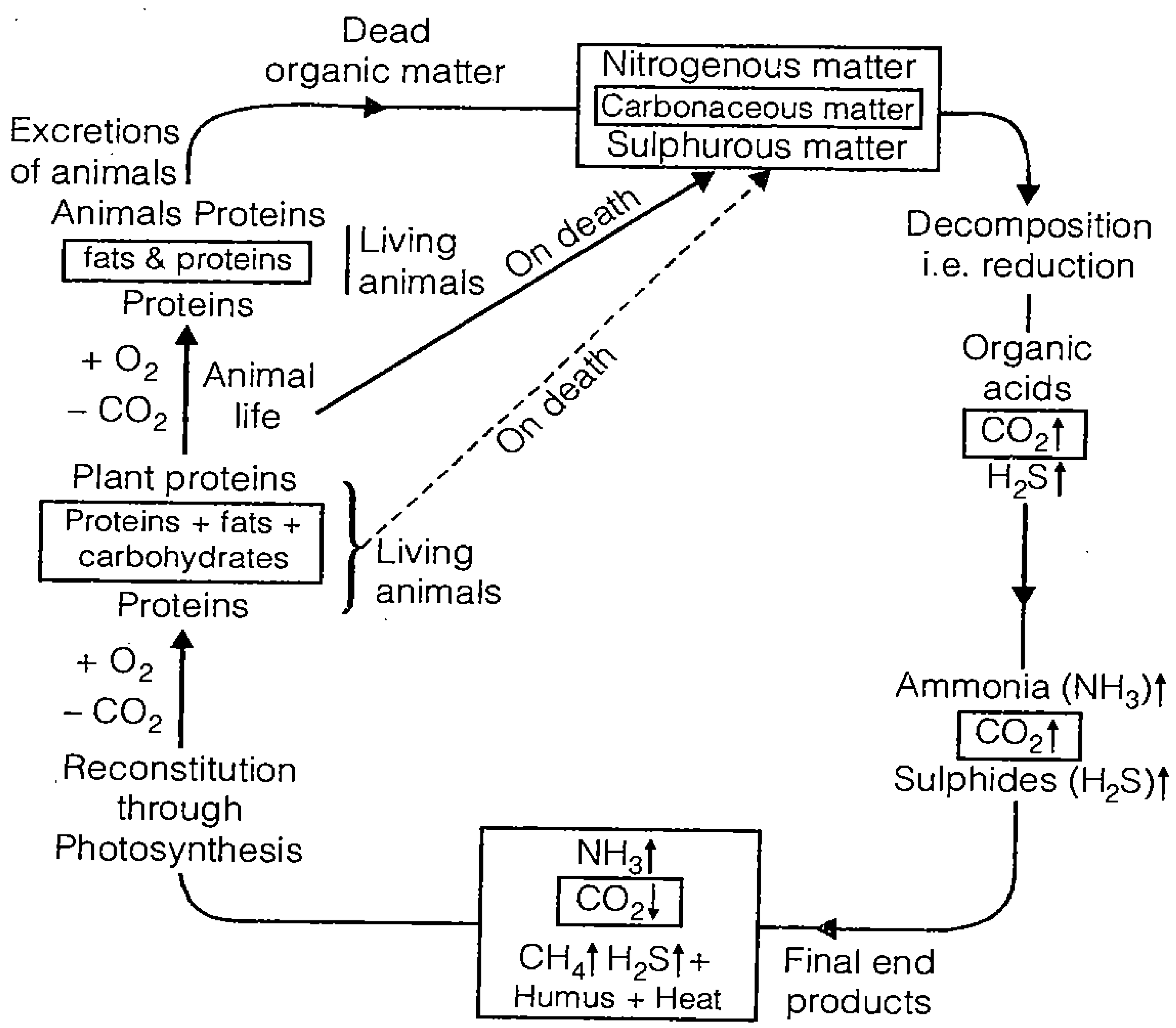
Aerobic Decomposition

(i) Nitrogenous organic matter $\xrightarrow[\text{Aerobic}]{\text{Oxidation by}}$ $NO_3^- + NH_3\uparrow$ + Energy
Nitrate

(ii) Carbonaceous organic matter $\xrightarrow[\text{Aerobic}]{\text{Oxidation by}}$ $CO_2 + H_2O$ + Energy
Carbondioxide

(ii) Sulphurous organic matter $\xrightarrow[\text{Aerobic}]{\text{Oxidation by}}$ SO_4^{2-} +Energy
Sulphate

- Nitrogen Cycle under Aerobic Decomposition



Anaerobic Decomposition

(i) Nitrogeneous organic matter $\xrightarrow[\text{Anaerobic}]{\text{Reduction by}}$ $N_2\uparrow + NH_3$ + Organic acids + Heat energy

(ii) Carbonaceous organic matter $\xrightarrow[\text{Anaerobic}]{\text{Reduction by}}$ $CO_2\uparrow$ + Heat energy.

(iii) Sulphurous organic matter $\xrightarrow[\text{Anaerobic}]{\text{Reduction by}}$ $H_2S\uparrow$ + Heat energy.

(iv) Organic acids $\xrightarrow[\text{Anaerobic bacteria}]{\text{Methane forming}}$ $CH_4\uparrow + CO_2\uparrow$ + Heat energy

Threshold Odour Number (TON)

$$\boxed{TON = \frac{V_S + V_D}{V_S}}$$

V_S = Volume of the sewage
V_D = Volume of distilled water or odourless water.

Total Solids, Suspended Solids and Settleable Solids

(i) $\boxed{S_3 = S_1 - S_2}$ where, S_3 = Dissolved solids plus colloidals or filterable solids in mg/lit
S_2 = Non-filterable solids in mg/lit
S_1 = Total amount of solids in mg/lit

(ii) $\boxed{S_2 - S_4 = S_5}$ S_4 = Volatile suspended solids, in (mg/lit.)
S_5 = Fixed solids

Type of solid	Strength of Sewage		
	Weak	Weak	Strong
Total solids	400	800	1200
Suspended solids			
Total	100	200	350
Volatile	75	135	210
Settleable solids	2.5	5	7
Ether soluble matter such as fats, oils and grease.	6	14	20

Total Solids
50% → Dissolved
25% → Suspended
25% → Settlable

Chemical Oxygen Demand

(i) Biodegradable + non Biodegrable O.M.

(ii) $K_2Cr_2O_7 + H_2SO_4$ added and O_2 used is measured.

Theoretical Oxygen Demand

$$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

$$\underset{\text{Benzene}}{C_6H_6} + 7.5O_2 \longrightarrow 6CO_2 + 3H_2O$$

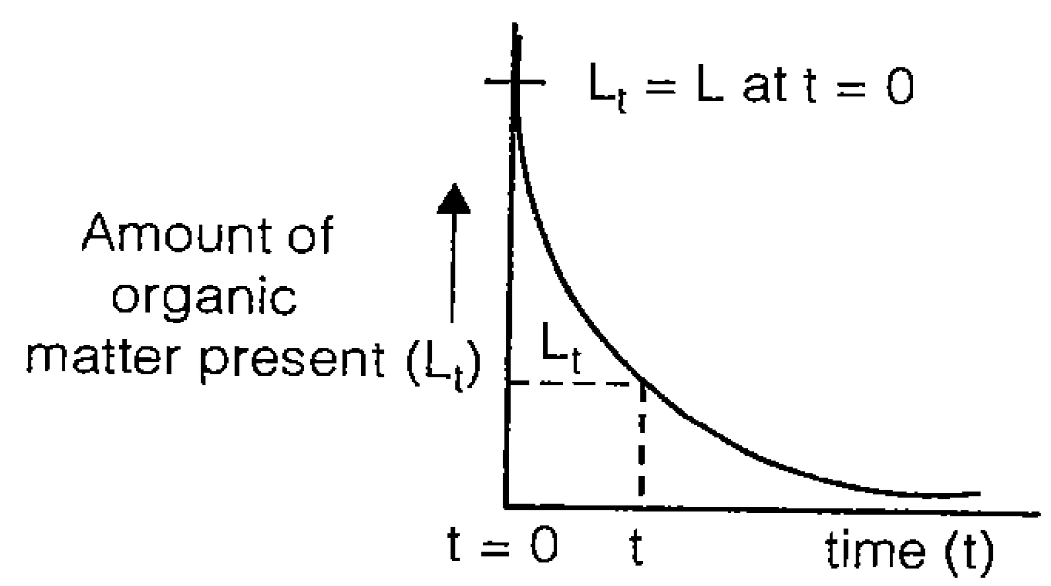
$$\underset{\text{Glucose}}{C_6H_{12}O_6} + 6O_2 \longrightarrow 6CO_2 + 6H_2O$$

Biochemical Oxygen Demand

- $\boxed{BOD = (DO_i - DO_f)\text{ Dilution Factor}}$

where, BOD = Biochemical oxygen demand in ppm or mg/lit.
DO_i = Initial dissolved oxygen in mg/lit.
DO_f = Final dissolved oxygen in mg/lit.

- $\boxed{\text{Dilution Factor} = \dfrac{\text{Volume of the diluted sample}}{\text{Volume of the undiluted sewage sample}}}$

- 

Ist Stage BOD Curve

(i) $\boxed{\dfrac{dL_t}{d_t} = -kL_t}$ where, k = Rate constant signifying the rate of oxidation of organic matter and it depends upon the nature of organic matter and temperature. Its unit is per day.

L_t = O_2 equivalent of organic matter present after t days.

(ii) $\boxed{L_t = L\cdot(10)^{-k_Dt}}$ where, k_D = Deoxygenation constant.
L = Organic matter present at t = 0

(iii) $\boxed{k_D = 0.434K}$

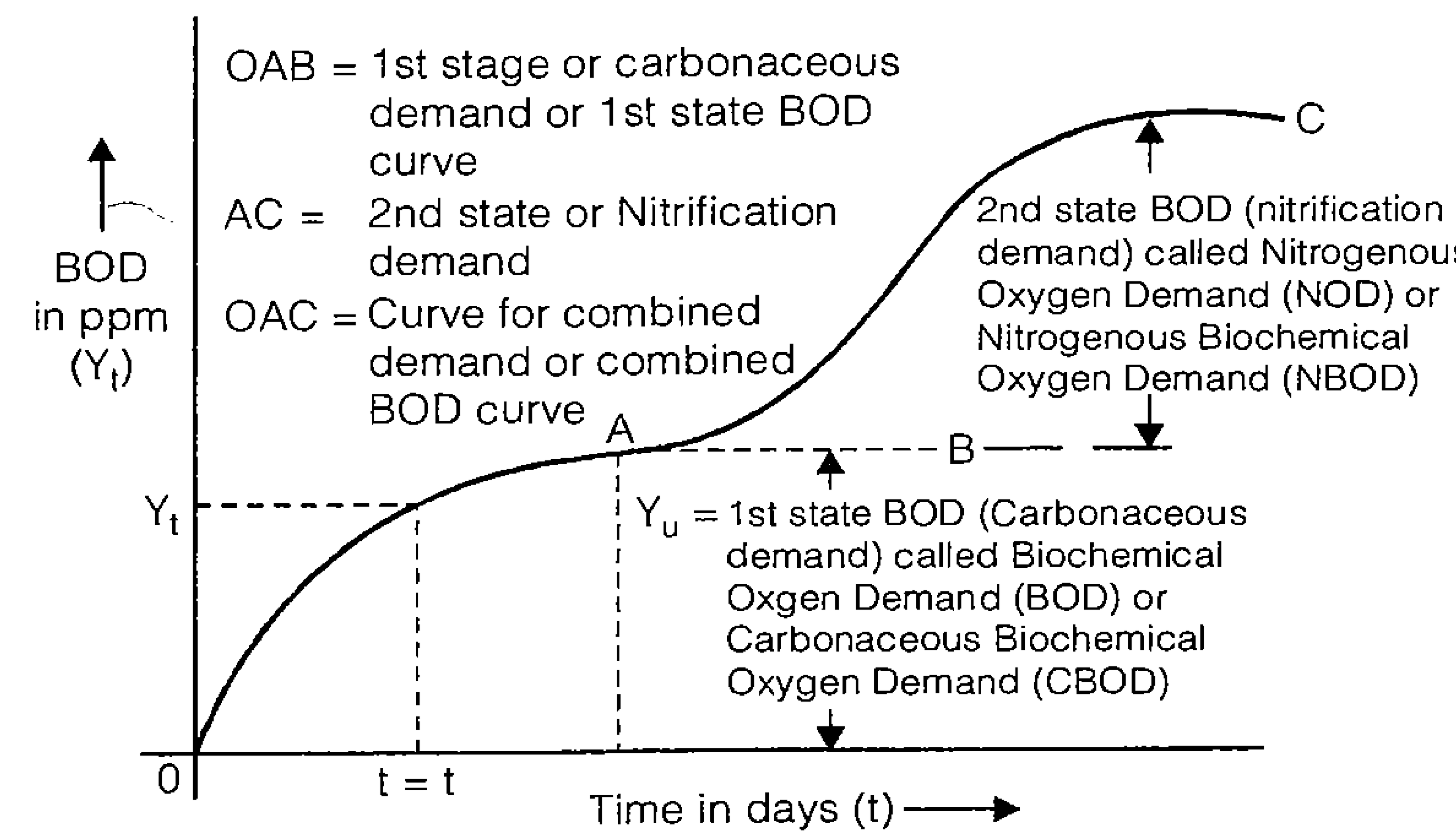
(IV) $\boxed{Y_t = L - L_t}$ where, Y_t = Total amount of organic matter oxidized in t days i.e. BOD.

(v) $\boxed{Y_t = L\left[1-(10)^{-k_Dt}\right]}$

(vi) $\boxed{Y_u = L}$ where, Y_u = Ultimate B.O.D of t = ∞ days.

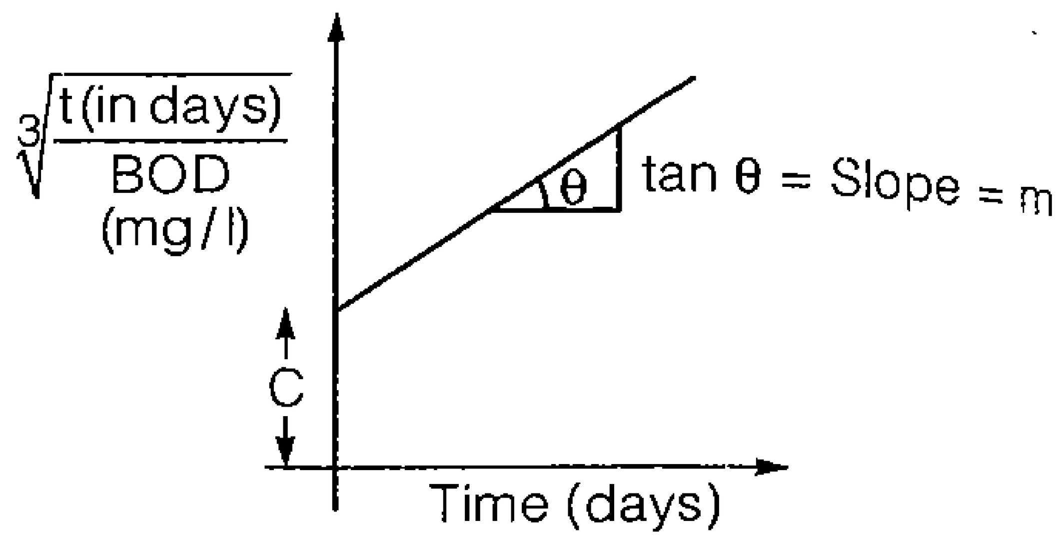
(vii) $\boxed{k_{D(T°C)} = k_{D(20°C)}\left[1.047\right]^{T-20\,°C}}$

Water type	K_D value per day
Tap waters	< 0.05
Surface waters	0.05 – 0.1
Municipal waste waters	0.1 – 0.15
Treated sewage effluents	0.05 – 0.1



Parameter/Characteristic	Strength of Sewage		
	Weak	Medium	Strong
Total suspended solids (SS)	100	200	350
Volatile suspended solids	75	135	210
BOD	100	200	400
COD	175	300	600
TOC*	100	200	400
Ammonia-N	5	10	20
Organic-N	8	20	40
PO_4-P	7	10	20

(viii) Laboratory Estimations of k_D and L values (Thomas Method)



$$k_D = 2.61\frac{m}{C}$$

where, m = Slope of the line
C = Intercept of the line on y-axis.

$$L = Y_u = \frac{1}{2.3k_D C^3}$$

Relative Stability (s)

$$S = 100\left[1-(0.794)^{t_{20}}\right] \qquad S = 100\left[1-(0.630)^{t_{37}}\right]$$

where, t_{20} = time in days at 20 °C.
t_{37} = time in days at 37 °C.

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8 Disposing of The Sewage Effluents

Standards of Dilutation for Discharge of Wastewaters into Rivers

- Standards of Dilutation based on Royal Commission Report

Dilution factor	Standards of purification required
Above 500	No treatment is required. Raw sewage can be directly discharged into the volume of dilution water.
Between 300 to 500	Primary treatment such as plain sedimentation should be given to sewage, and the effluents should not contain suspended solids more than 150 ppm.
Between 150 to 300	Treatments such as sedimentation, screening and essentially chemical precipitation are required. The sewage effluent should not contain suspended solids more than 60 ppm.
Less than 150	Complete thorough treatment should be given to sewage. The sewage effluent should not contain suspended solids more than 30 ppm., and its 5 days BOD at 18.3° C should not exceed 20 ppm.

- BIS Standards for Discharge of Sewage and Industrial Effluents in Surface Water Sources and Public Sewers

Characteristic of the Effluent	Tolerance limit for sewage effluents discharged into surface water sources, as per IS 4764-1973	Tolerance limit for Industrial effluents discharged into	
		Inland surface waters, as per IS 2490-1974	Public sewers as per IS 3306-1974
(1)	(2)	(3)	(4)
BOD_5	20 mg/*l*	30 mg/*l*	500 mg/*l*
COD	—	250 mg/*l*	—
pH value	—	5.5 to 9.0	5.5 to 9.0
Total suspended solids (TSS)	30 mg/*l*	100 mg/*l*	600 mg/*l*
Temperature	—	40°C	45°C
Oil and grease	—	10 mg/*l*	100 mg/*l*
Phenolic compounds (as Phenol)	—	1 mg/*l*	2 mg/*l*
Cyanides (as CN)	—	0.2 mg/*l*	—
Sulphides (as S)	—	2 mg/*l*	—
Fluorides (as F)	—	2 mg/*l*	—
Total residual chlorine	—	1 mg/*l*	—
Insecticides	—	Zero	—

- **General Standards for Discharge of Environment Pollutants from Effluents into Surface Water Sources, Public Sewers, and Marine Coasts Under Environment (Protection) Rules, 1986**

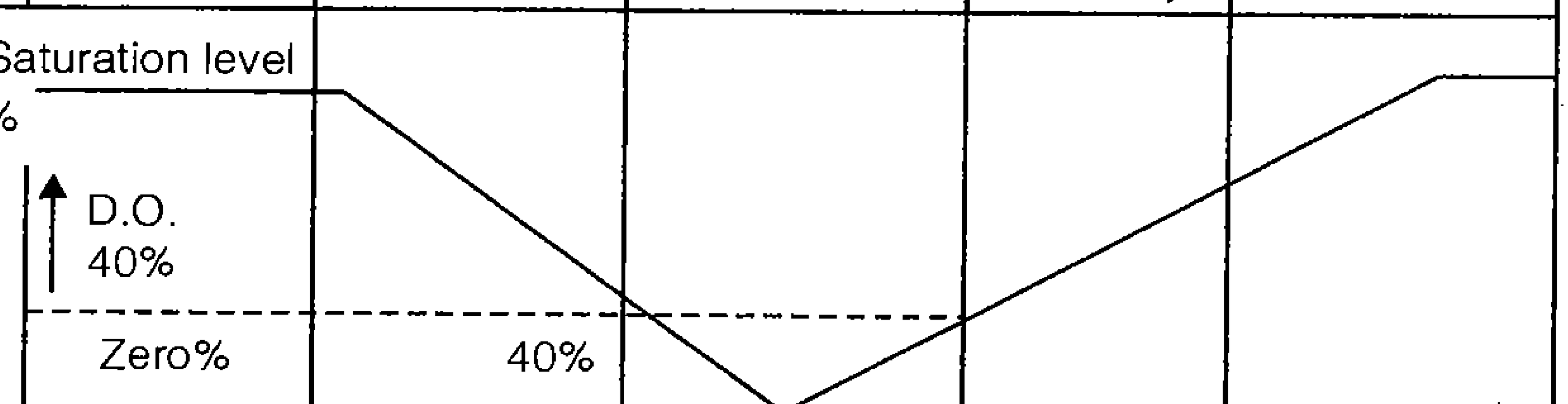
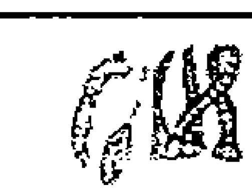
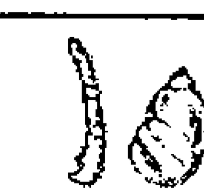
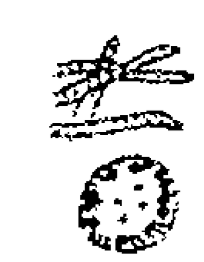
Characteristic of the effluent i.e. Name of pollutant in the effluent	Standard Prescribed under Environment (Protection Rules, 1986 of GOI for		
	Inland surface waters	Public sewers	Marine coasts i.e. seas and oceans
(1)	(2)	(3)	(4)
Colour and odour	All efforts should be made to remove colours and unpleasant odours, as for as possible.	All efforts should be made to remove colour and unpleasant odours, as for as possible.	All efforts should be made to remove colour and unpleasant odours, as for as possible.
Total suspended solids (TSS)	100 mg/l	600 mg/l	(i) 100 mg/l for process waste water. (ii) For cooling water effluent, 10% above total suspended mater of influent.
Particle size of suspended solids	Shall pass 850 micron sieve	—	(a) Floatable solids: max 3 mm (b) Settleable solids: max 850 micron.
BOD5 at 20°C	30 mg/l	350 mg/l	100 mg/l
COD	250 mg/l		250 mg/l
pH value	5.5 – 9.0	5.5 – 9.0	5.5 – 9.00
Temperature	Shall not exceed 5°C above the temperature of receiving water.	—	Shall not exceed 5°C above the temperature of receiving water.
Oil and grease	10 mg/l	20 mg/l	20 mg/l
Total residual chlorine	1.0 mg/l	—	1.0 mg/l
Ammonical Nitrogen (as N)	50 mg/l	50 mg/l	50 mg/l
Total Kjeldahl nitrogen (as N)	100 mg/l	—	100 mg/l
Free Ammonia (as NH_3)	5.0 mg/l	—	5.0 mg/l
Phenolic compounds (as C_6H_5OH)	1.0 mg/l	5.0 mg/l	5.0 mg/l
Sulphide (as S)	2.0 mg/l	—	5.0 mg/l

Dilution and Dispersion

$$C_{mix} = \frac{C_S \cdot Q_S + C_R Q_R}{Q_S + Q_R}$$

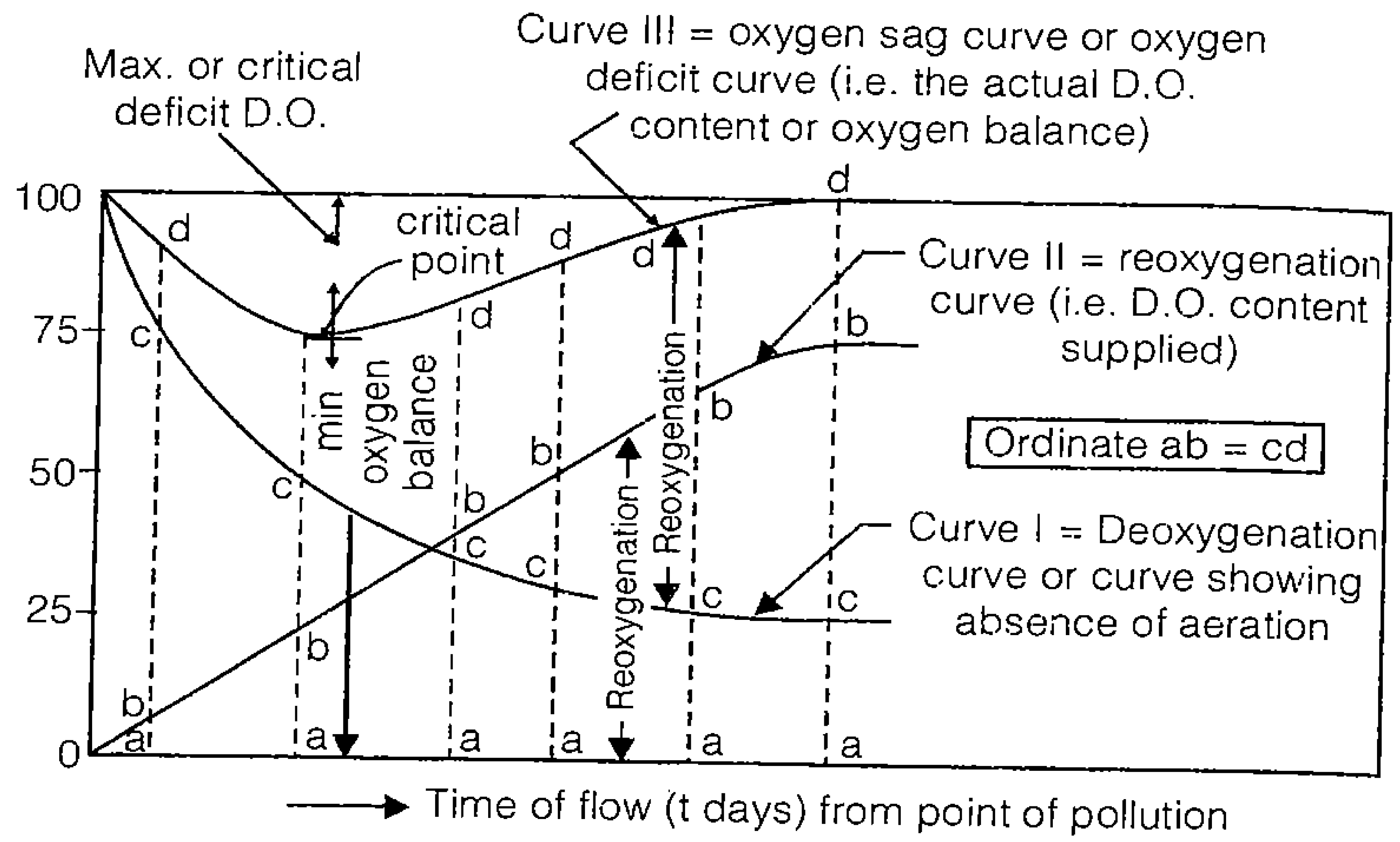
where, C_S = Concentration of sewage in mg/lit.
Q_S = Flow rate of sewage in m^3/sec or lit/sec.
C_R = Concentration of river in mg/lit.
Q_R = Flow rate (discharge) in m^3/sec or lit/sec.
C_{mix} = Concentration of mixture.

Zone of Pollution in River Stream

	← Zones of pullution →				
	Clear water	Zone of degradation	Zone of active decomposition	Zone of recovery	Zone of clearer water
Dissolved oxygen sag curve					
Physical Indices	Clear water, no bottom sludge, no colour	Floating solids, bottom sludge, present, colour getting turbid	Darker and greyish colour, evolution of gases like CH_4CO_2 H_2S etc. lot of sludge coming to the surface forming an ugly scum layer at top	Turbid with bottom sludge	Clear water with no bottom sludge
Fish presence	Ordinary fish like game, pan, food and forage etc. present	Tolerant fishes like carp, buffalo, gary, etc. present	No fish present	Tolerant fish like carp, buffalo, etc. are present	Ordinary fish like game, pan, food and forage etc. present
Bottom Animals					
Algae & Protozoa etc. called plankton					

- $\boxed{\text{Oxygen deficit} = \text{Saturation D}\cdot\text{O} - \text{Actual}\cdot\text{D}\cdot\text{O}}$

Saturation D.O at 20 °C ⟶ 9.2 mg/lit.
Saturation D.O at 30 °C ⟶ 7.6 mg/lit.
Saturation D.O at 0 °C ⟶ 14.6 mg/lit.



ab = cd

- $\boxed{\text{TOD} > \text{COD} > \text{BOD}}$ where, TOD = Theoretical oxygen demand
- $\boxed{\text{COD} > (\text{BOD})_U > \text{BOD}_5}$ BOD = Biological oxygen demand

COD = Chemical oxygen demand

$(\text{BOD})_U$ = Ultimate BOD (Y_u).

- $\left[\begin{array}{l}\text{Standard BOD}\\\text{(5 days)}\\\text{of industrial}\\\text{sewage}\end{array}\right] = \left[\begin{array}{l}\text{Standard BOD}\\\text{(5 days) of domestic}\\\text{sewage per person}\\\text{per day}\end{array}\right] \times \text{Population equivalent}$

Stretcher-PHELPS EQUATION

- $$D_t = \frac{k_D L}{k_R - k_D}\left[(10)^{-k_D t} - (10)^{-k_R t}\right] + \left[D_o \cdot (10)^{-k_R t}\right]$$
- $$k_{D(T°C)} = k_{D(20°C)}[1.047]^{(T-20°C)}$$
- $$k_{R(T°C)} = k_{R(20°C)}[1.016]^{(T-20°C)}$$

- $$f = \frac{k_R}{k_D}$$
- $$t_C = \frac{1}{k_D(f-1)}\log_{10}\left[\left\{1-(f-1)\frac{D_O}{L}\right\}f\right]$$
- $$D_C = \frac{L}{f}\cdot[10]^{-k_D \cdot t_C}$$

where, D_t = D.O deficit in mg/lit after t days.

L = Ultimate first stage BOD of the mix at a point of waste discharge in mg/lit.

D_O = Initial oxygen deficit of the mix at the mixing point in mg/lit.

k_R = Reoxygenation constant

k_D = Deoxygenation constant

f = Self purification constant

t_c = Critical time at which minimum dissolved oxygen occurs i.e.

$\boxed{\frac{dD_t}{dt} = 0}$ D_C = Critical maximum oxygen deficit.

- $$\left(\frac{L}{D_C \cdot f}\right)^{f-1} = f\left[1-(f-1)\frac{D_O}{L}\right]$$

Constituent Pollutant contained in the Wastewater Effluent	Tolerance limit
(2)	(3)
BOD_5	100 mg/l
COD	250 mg/l
pH value	5.5 to 9.0
Total suspended solids	100 mg/l
Oil and grease	20 mg/l
Fluorides (as F)	15 mg/l
Ammoniacal Nitrogen (as N)	50 mg/l

Type of soil	Doses of sewage in cubic metres per hectare per day	
	Raw sewage	Settled sewage
Sandy	120 – 150	220 – 280
Sandy loam	90 – 100	170 – 220
Loam	60 – 80	110 – 170
Clayey loam	40 – 50	60 – 110
Clayey	30 – 45	30 – 60

Characteristics of effluent i.e. pollutant of waste water	Standards under Environment (Protection) Rules, 1986
(1)	(2)
Colour and odour	All efforts should be made to remove colour and unpleasant odour as far as practicable
BOD_5 at 20°C	100 mg/*l**
Suspended Solids (SS)	200 mg/*l*
pH value	5.5 to 9.0
Oil and grease	10 mg/*l*
Arsenic (As)	0.2 mg/*l*
Cyanide (as CN)	0.2 mg/*l*
Radioactive materials	
(a) Alpha emitters	10^{-8} μC/m*l*
(b) Beta emitteres	10^{-7} μC/m*l*

■■■

9 Treatment of Sewage

Settling Velocity

(i) $V_S = \frac{g}{18}(G-1)\frac{d^2}{\nu}$ for d < 0.1 mm

where, V_S = Velocity of settlement of particle or settling velocity in m/sec.

d = Diameter of the particle in meter.

G = Specific gravity of the particle.

ν = Kinematic viscosity of water in m²/sec.

$\nu = \frac{Y}{\delta}$

Y = Dynamic viscosity

δ = Density

(ii) $V_S = \sqrt{\frac{\frac{4}{3}g\cdot(G-1)d}{C_D}}$ where, C_D = Coefficient of drag $= \frac{24}{R_e}$

→ For laminar flow

$$R_e = \text{Reynolds number} = \frac{\delta v_s d}{Y} = \frac{V_S d}{\nu}$$

$C_D = \frac{24}{R_e} + \frac{3}{(R_e)^{0.5}} + 0.34$ $C_D = \frac{18.5}{R_e^{0.6}}$ ⟶ for transition flow.

$C_D = 0.34 + 0.4$ ⟶ for turbulent flow.

(iii) $V_S = \left[\frac{g(G-1)}{13.88}\frac{d^{1.6}}{\nu^{0.6}}\right]^{0.714}$ 0.1 mm < d < 1.0 mm.

(iv) Newtons Equation for Turbulent Settling

$V_S = 1.8\sqrt{gd(G-1)}$ for d > 1 mm

(v) Modified Hazen's Equation for Transition Zone

(a) $V_S = 60 \cdot 6d(G-1) \cdot \left(\frac{3T+70}{100}\right)$ For $0.1 < d < 1$ mm

Where T = Temperature in °C.

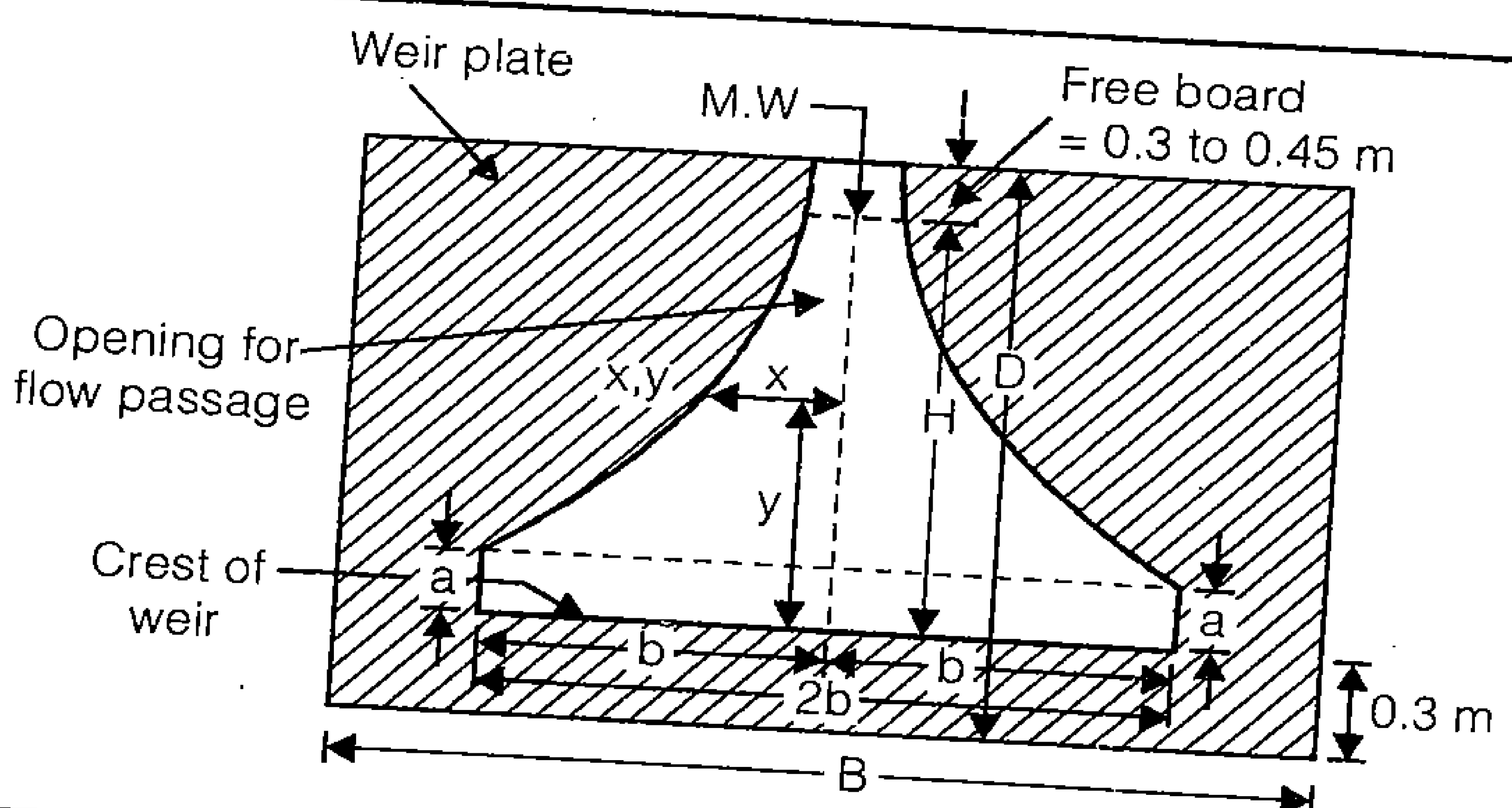
(b) Putting G = 2.65 for Inorganic Solids $V_S = d(3T+70)$

(c) Putting G = 1.2 for Organic Solids $V_S = 0.12d(3T+70)$

Critical Scour Velocity in Constant Velocity Horizontal Flow Grit Chamber (V_H)

$$V_H = 3 \text{ to } 4.5\sqrt{gd(G-1)}$$

PROPORTIONAL FLOW WEIR



$x = \frac{2BV_h}{C_d \cdot \sqrt{2g} \cdot \pi\sqrt{y}}$

$b = 1.467\,B\,V_h$

where, B = Width of channel.

V_h = Horizontal flow velocity.

C_d = Coefficient of discharge.

x and y are coordinates on weir profile.

Parabolically or V-Shaped Grit Chamber Provided with a Parshall Flume

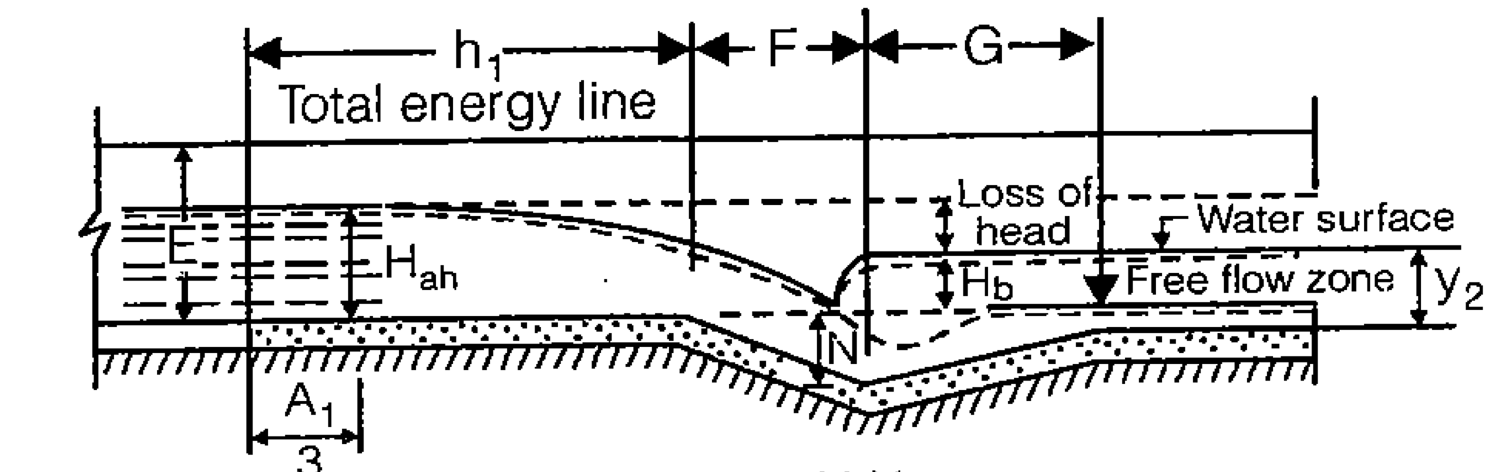
(i) Parshall Flume $Q = 2.264W(H_a)^{3/2}$

where, W = Width of the throat in meter.

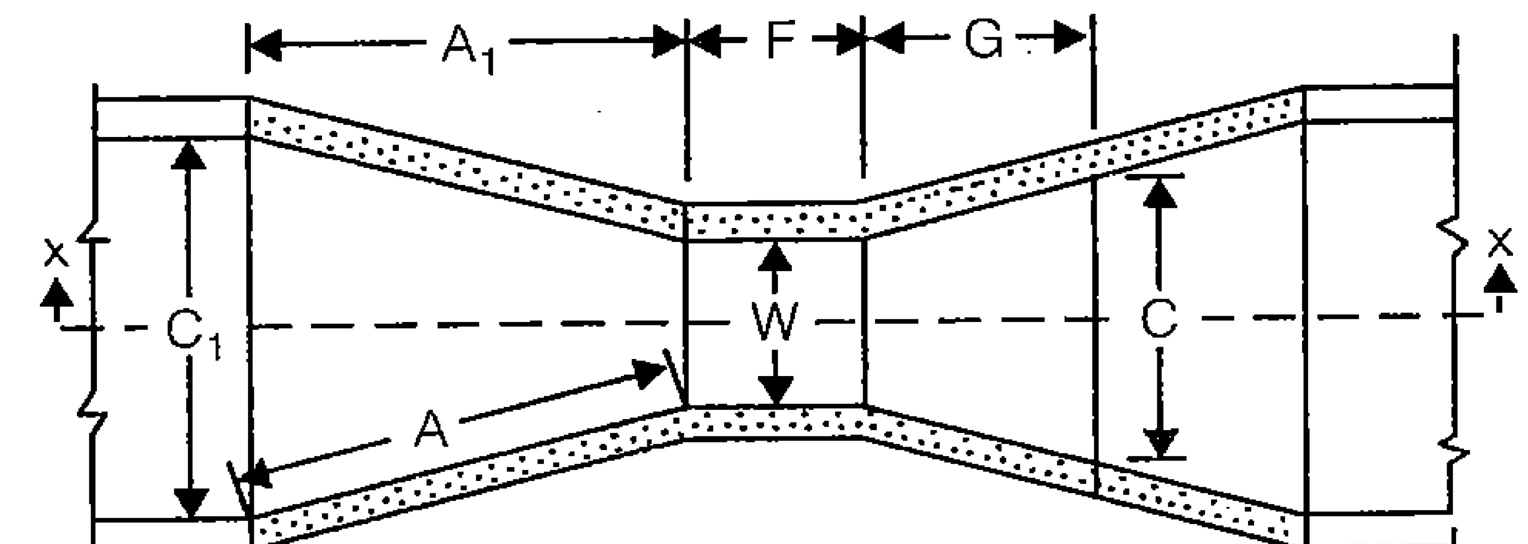
Q = Flow in (m^3/sec) through Parshall flume.

H_a = Depth of flow in upstream leg of flume of one third

portion in meter.



Section X-X



The Parshall flume

(ii) Parabolic Grit Channel

$Q = C_1 \cdot \int_{y=0}^{y=y} x\,dy$ where $V_h = \frac{Q}{\int_0^y x\,dy} = C_1$

$Q = C_2 y^n$ where, n = Discharge coefficients of the control section.

$Y^{n-1} = \frac{C_1}{C_2} \cdot x$

= 1.5 for parshall flume.

= 1 for proportional flow weir.

- **Aerated Grit Channels** Detention period = 3 minutes
- **Detritus Tank** Detention period = 3 to 4 minutes

Skimming Tank

(i) Detentions Period = 3 to 5 minutes.

(ii) Amount of compressed air required = 300 to 6000 m^3 per million litre of sewage.

(iii) Surface Area,

$A = 0.00622\frac{q}{V_r}$

where, q = Rate of flow of sewage in m^3/day.

V_r = Min. rising velocity of greasy material to be removed in m/min

= 0.25 m/min mostly.

- **Vacuators**

  Vacuum Pressure = 0 to 25 cm of Hg

  For 10 to 15 minutes.

Sedimentation Tank

(i) Overflow rate = 40000 to 50000 lit/m² day for plain sedimentation.

= 50000 to 60000 lit/m²/day for sedimentation with coagulation.

= 25000 to 35000 lit/m²/day for secondary sedimentation tank

(ii) Depth ~ 2.4 to 3.6 m.

(iii) Detention time = 1 to 2 hour.

(iv) Width = 6.0 m

(v) Length = 4 to 5 times width.

(vi) Velocity of flow $V_f = 0.3$ m/min.

(vii) $$V = \frac{Q}{BH}$$

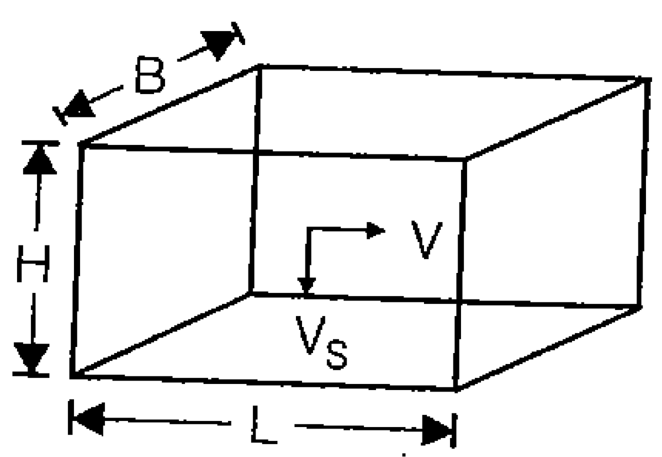
where, V = Flow velocity

B = Width of the Basin

H = Depth of sewage in the tank.

(viii) $$\frac{V}{V_S} = \frac{L}{H}$$

(ix) $$V_S = \frac{Q}{BL}$$



- **Detention Time**

(a) $$t = \frac{BLH}{Q}$$ For rectangular Tank

(b) $$t = \frac{d^2(0.011d + 0.785H)}{Q}$$ for circular tank

where d = Dia of the tank

H = Vertical depth of wall or side water depth

- **Displacement Efficiency (η)**

$$\eta = \frac{\text{Flowing through period}}{\text{Detention period}}$$

Trickling Filter

(a) Conventional Trickling Filter or Low Rate Trickling Filter

$$\eta(\%) = \frac{100}{1 + 0.0044\sqrt{u}}$$

where, η = Efficiency of the filter and its secondary clarifier, in terms of % of applied BOD

u = Organic loading in kg/ha-m/day applied to the filter (called unit organic loading)

(b) High Rate Trickling Filter

(i) $$F = \frac{\left(1 + \frac{R}{I}\right)}{\left(1 + 0.1\frac{R}{I}\right)^2}$$

where, F = Recirculation factor

$\frac{R}{I}$ = Recirculation ratio

(ii) $$\eta(\%) = \frac{100}{1 + 0.0044\sqrt{\frac{Y}{VF}}}$$

where, Y = Total organic loading in kg/day applied to the filter i.e. the total BOD in kg.

$\frac{Y}{VF}$ = Unit organic loading in kg/Ha-m/day

V = Filter volume in Ha-m.

η% = % efficiency of single stage high rate trickling filter.

(iii) $$\eta'(\%) = \frac{100}{1 + \frac{0.0044}{1-\eta}\sqrt{\frac{Y'}{V'F'}}}$$

where,

η′ = Final efficiency in the two stage filter.

Y′ = Total BOD in effluent from first stage in kg/day.

F′ = Recirculation factor for second stage filter

V′ = Volume in second stage filter in ha-m.

Characteristics (2)	Conventional or Standrad rate filters (3)	High rate filters (4)
Depth of filter media	Varies between 1.6 to 2.4 m.	Varies between 1.2 to 1.8 m.
Size of filter media	25 to 75 mm.	25 to 60 mm.
Land required	More land area is required, as the filter loading is less.	Less land area is required as the filter loading is more.

Cost of operation	It is more for treating equal quantity of sewage.	It is less for treating equal quantity of sewage.
Method of operation	Continuous application, less flexible requiring less skilled supervision.	Continuous application, more flexible, and more skillful operation is required.
Type of effluent produced	The effulent is highly nitrified and stabilized, with BOD in effluent ≤ 20 ppm or so.	The effluent is nitrified up to nitrite stage only and is thus less stable, and hence it is slightly inferior quality. BOD in effluent ≥ 30 ppm. or so.
Doing interval	It generally varies between 3 to 10 minutes. The sewage is generally not applied continuously but is applied at intervals.	It is not more than 15 seconds, and the sewage is thus applied continuously.
Filter loading values (i) Hydraulic loading	Varies between 20 to 44 M.L. per day	Varies between 11 to 330 M.L. per hectare day.
(ii) Organic loading	Varies between 900 to 2200 kg of BOD_5 per hectare-metre of filter media per day.	Varies between 6000 to 18,000 kg of BOD_5 per hectare metre of filter media per day.
Recirculation system	Not provided generally.	Always provided for increasing hydraulic loading.
Quality of secondary sludge produced	Black, highly oxidised with slight particles.	Brown, not fully oxidised with fine particles.

Dunbar Filter

Surface loading = 2500 Ml/m²/day.

BOD removed = 85%

Sludge and its Moisture Content

$$V = V_1\left[\frac{100 - P_1}{100 - P}\right]$$

V_1 = Volume of sludge at moisture content P_1%

V = Volume of sludge at moisture content P%

Sludge Digestion Tank

(i) When the change during digestion is linear.

(a) $$V = \left(\frac{V_1 + V_2}{2}\right)t$$

where, V = Volume of digestion in m³.

V_1 = Raw sludge added per day (m³/day)

V_2 = Equivalent digested sludge produced per day on completion of digestion, m³/day.

t = Digestion period in day.

(b) $$V = \left(\frac{V_1 + V_2}{2}\right)t + V_2T$$ with monsoon storage

where, T = Number of days for which digested sludge (V_2) is stored (monsoon storage)

(ii) When the change during digestion is parabolic

(a) $$V = \left[V_1 - \frac{2}{3}(V_1 - V_2)\right]t$$ without monsoon storage

(b) $$V = \left[V_1 - \frac{2}{3}(V_1 - V_2)\right]t + V_2T$$ without monsoon storage

Type of raw sludge to be digested	kg. of volatile solids present per cu. m. of sludge per month	Per capita capacity in cum/capita
Primary sludge	8	0.021
Mixture of primary sludge and secondary sludge from trickling filters (humus tanks)	7.36	0.036
Mixture of primary sludge and secondary activated sludge	5.76	0.061
Chemically coagulated sludge	—	

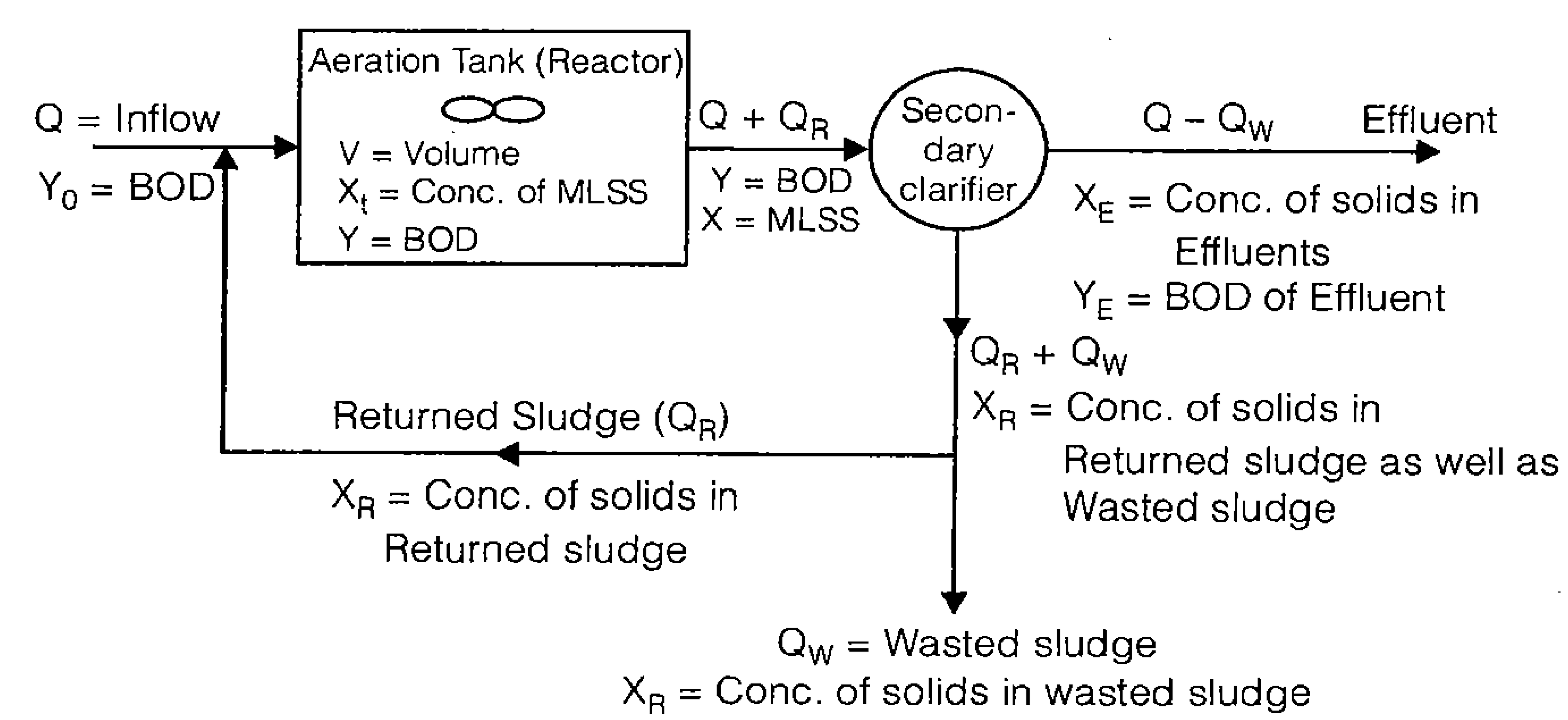
Destruction and Removal Efficiency (DRE)

$$DRE = \frac{W_{in} - W_{out}}{W_{in}} \times 100$$

where, W_{in} = Mass fill rate of one POHC (Principal organic Hazardous constituent) in the waste stream.

W_{out} = Mass emissionrate of the same POHC present in the exhaust emission prior to release to the atmosphere.

Aeration Tank



(i) Detention period, $t = \frac{V}{Q}$

where V = Volume of the tank in m^3.

Q = Quantity of waste water flow into the aeration tank excluding the quantity of recycled sludge (m^3/day)

(ii) Volumetric BOD Loading or Organic Loading, (U) $u = \frac{QY_O}{V}$

where, QY_O = Mass of BOD applied per day to the aeration tank through influent sewage in gm.

V = Volume of aeration tank in m^3.

Q = Sewage flow into the aeration tank in m^3.

Y_O = BOD_5 in mg/lit (or gm/m^3) of the influent sewage.

(iii) $\frac{F}{M} = \frac{QY_O}{VX_t}$ where, $\frac{F}{M}$ = Food (F) to Microorganism (M) ratio

QY_O = Daily BOD applied to the aeration system in gm.

Y_O = 5 day BOD of the influent sewage in mg/lit.

Q = Flow of influent sewage in m^3/day.

X_t = MLSS (Mixed liquor suspended solids) in mg/lit.

V = Volume of the Aeration Tank (lit).

M = X_tV = Total microbial mass in the system in gm.

(iv) Sludge Age (θ_C)

(a) $\theta_C = \frac{\text{Mass of suspended solids (MLSS) in the system (M)}}{\text{Mass of solids leaving the system per day}}$

(b) $\theta_C = \frac{VX_T}{Q_wX_R + (Q - Q_w)X_E}$

where,

X_T = Concentration of solids in the influent of the Aeration Tank called the MLSS i.e. mixed liquor suspended solids in mg/lit.

V = Volume of Aerator

Q_W = Volume of wasted sludge per day

X_R = Concentration of solids in the returned sludge or in the wasted sludge (both being equal) in mg/lit.

Q = Sewage inflow per day.

X_E = Concentration of solids in the effluent in mg/lit.

(v) Sludge Volume Index (S.V.I)

$SVI = \frac{V_{ob}}{X_{ob}} \times 1000$ where, X_{ob} = Concentration of suspended solids in the mixed liquor in mg/lit.

V_{ob} = Settled sludge volume in ml/lit.

S.V.I. = Sludge volume index in ml/gm.

(vi) Sludge Recycle and Rate of Return Sludge

$Q_R.X_R = (Q + Q_R) \times t$

$\frac{Q_R}{Q} = \frac{X_t}{X_R - X_t}$ where, Q_R= Sludge recirculation rate in m^3/day.

X_t = MLSS in the aeration tank in mg/lit.

X_R = MLSS in the returned or wasted sludge in mg/lit.

$X_R = \frac{10^6}{S.V.I}$ S.V.I = Sludge volume index in ml/gm.

- **Specific substrate utilization rate**

$U = \frac{Q(Y_0 - Y_E)}{V.X_t}$ $\frac{1}{\theta_C} = \alpha_y U - k_e$

α_y = 1 for MLSS and 0.6 for MLVSS, k_e = 0.06

- **Oxygen Requirement of the Aeration Tank**

$$O_{2(required)} = \left[\frac{Q(Y_O - Y_E)}{f} - 1.42Q_W \cdot X_R\right] gm/day$$

where, $f = \frac{BOD_5}{BOD_u} \simeq \frac{5\,day\,BOD}{Ultimate\,BOD} = 0.68$

- **Oxygen Transfer Capacity (N)**

$$N = \frac{N_S \cdot (D_S - D_L) \cdot (1.024)^{T-20°C} \cdot \alpha}{9.17}$$

where, N = Oxygen transferred under field conditions in kg O_2/k.wh (Or MJ)

N_S = Oxygen transfer capacity under standard conditions in kg O_2/kwh (or MJ)

D_S = Dissolved oxygen-saturation value for sewage at operating temperature.

D_L = Operation D·O level in Areation tank usually 1 to 2 mg/lit.

T = Temperature in °C

α = Correction factor for oxygen transfer for sewage usually 0.8 to 0.85.

Oxidation Ponds

- Depth → 1.0 to 1.8 m.
- Detention period → 2 to 6 weeks.
- Organic loading → 150 to 300 kg/ha/day.
  Under hot condition → 60 to 90 kg/ha/day.
  Under cold conditions.
- Length to width ratio = 2
- Sludge Accumulation = 2 to 5 cm/year
- Minimum depth to be kept = 0.3 m.

For Inlet Pipe Design

Assume v = 0.9 m/s

Assume flow for 8 hrs.

For Outlet Pipe Design

Dia of outlet = 1.5 dia of inlet pipe

Septic Tank

- Detention time = 12 to 36 hr.
- Sludge accumulation rate = 30 lit/cap/year.
- Sewage flow = 90 to 150 lit/capita/day.
- Cleaning period = 6 to 12 months
- Length to width ratio = 2 to 3 m.
- Depth = 1.2 to 1.8 m
- Width ≮ 0.9 m.
- Free board = 0.3 m.
- Volume of septic tank = (Sewage flow × Detention time) + (Sludge accumulationrate) × Cleaning rate

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10 Air and Sound Pollution

Primary Pollutants

(i) Sulphur dioxide (especially $SO_2\uparrow$)

(ii) $CO\uparrow$

(iii) Nitrogen oxides NO and NO_2

(iv) Lead (Pb)

(v) Hydrocarbons

(vi) Allergic agents like pollens and spores

(vii) Radio active substances etc.

Certain less important primary pollutants are H_2S, H_2F and other fluorides, methyl and Ethyl mercaptans. etc.

Secondary Pollutants

(i) H_2SO_4 (more toxic then $SO_2\uparrow$)

(ii) Ozone (O_3)

(iii) Formaldehydes

(iv) Peroxyacetyl-nitrate (PAN)etc.,

Wind speed

Wind speed is generally measured by Anemometer at a height say Z_0, knowing the wind speed say U_0, At a any height Z.

We can workout velocity U as

$$U = U_0\left(\frac{Z}{Z_0}\right)^k$$

where, $k = \text{const.} \approx \frac{1}{9}$ for large lapse rate

$= \frac{1}{3}$ for marked inversions

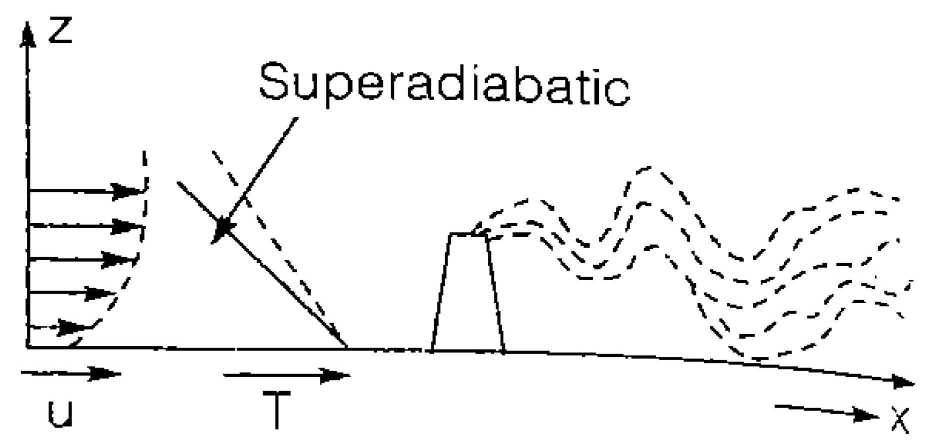
$\approx \frac{1}{7}$ (average normal value)

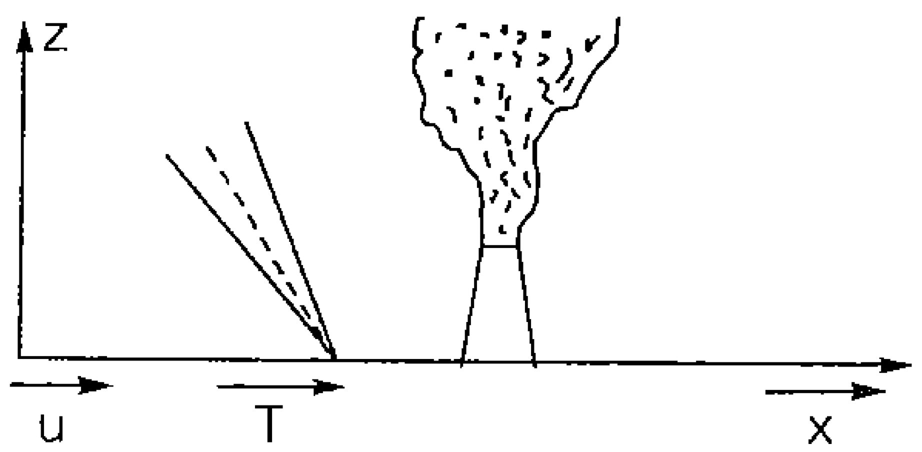
Plume

The emitted gases being known as a *plume* and their source of origin as *stack*.

(i) Looping plume

- Occurs in super adiabatic environment. Which produces highly unstable environment because of rapid mixing.
- Higher stacks are needed.



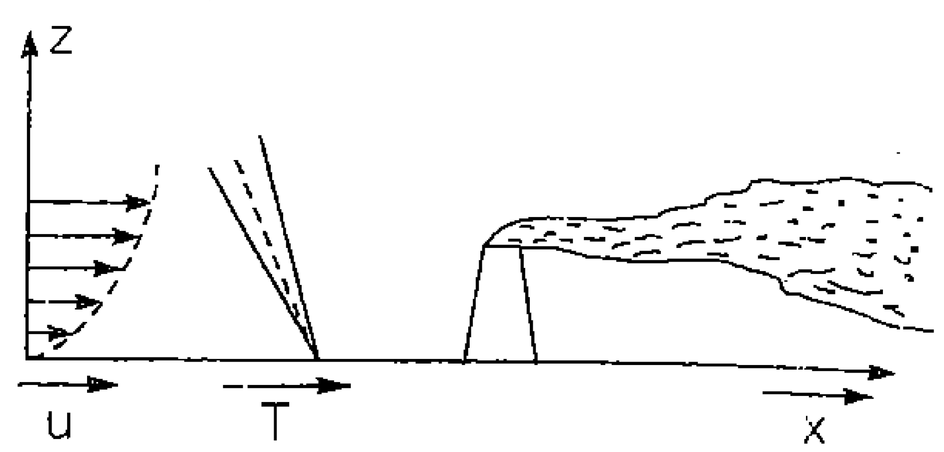


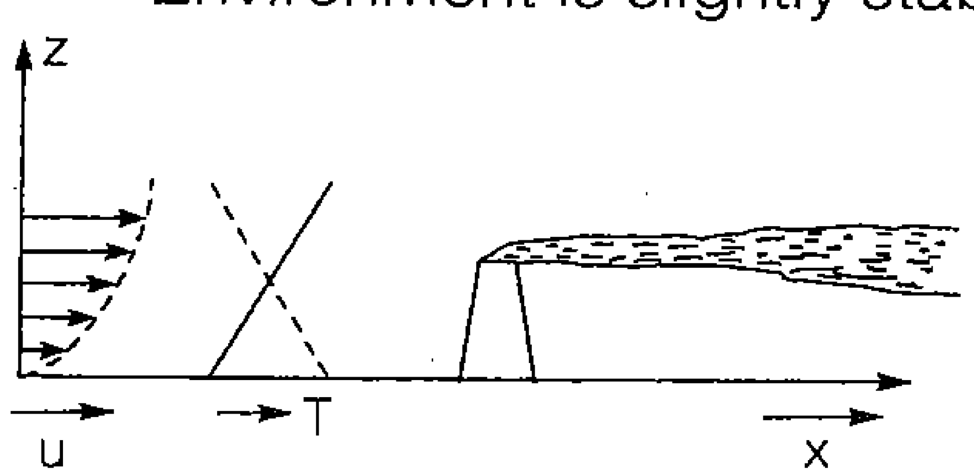
(ii) Neutral plume

- Upward vertical rise
- when ELR = ALR

(iii) Coning plume

- When wind velocity > 32 km/hr and when clouds are present.
- Also occurs under sub-adiabatic condition. (ELR < ALR)
- Environment is slightly stable.

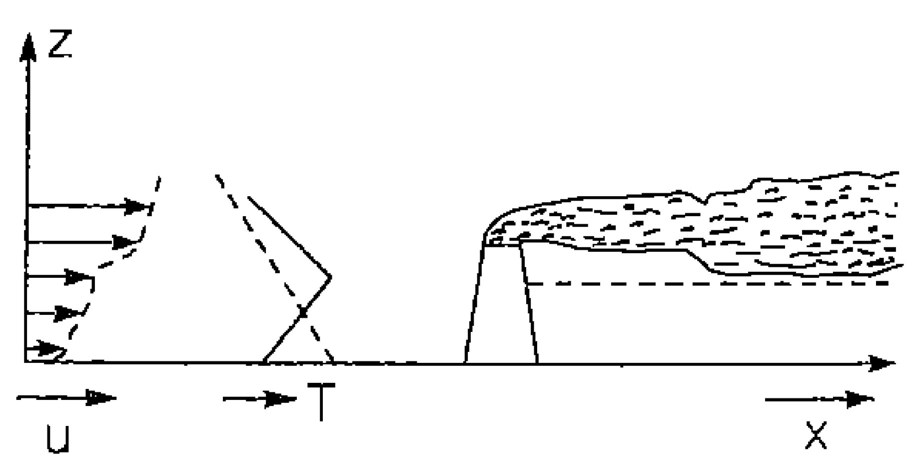




(iv) Fanning plume

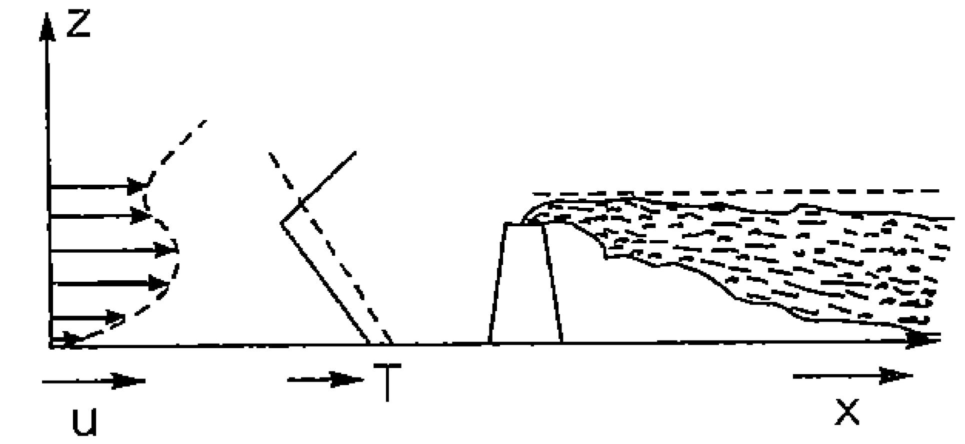
- Under extreme inversion conditions
- Emission will spread only horizontally
- High rising stacks are needed.

(v) Lofting plume ()



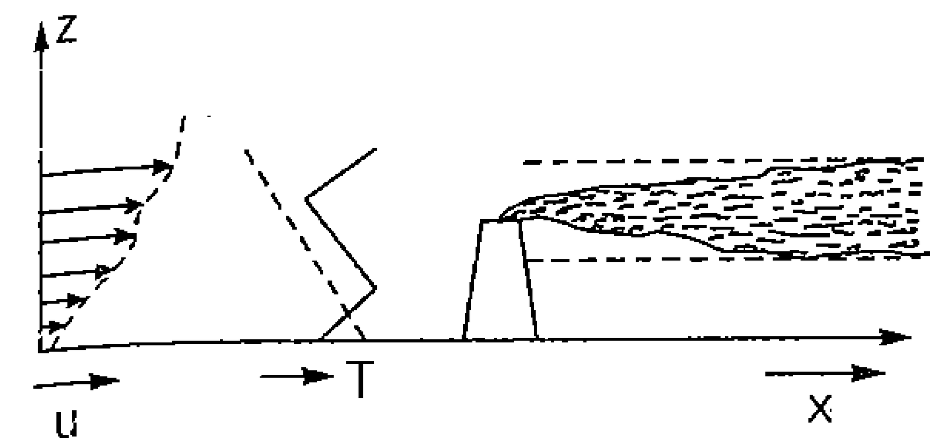
- When there exists a strong super adiabatic L.R. above surface inversion.
- Such plume has minimum downward mixing as its downward motion is prevented by inversion but upward mixing will be rapid and turbulent.

(vi) Fumigating plume ()



- When inversion layer occurs at a short distance above the top of the stack and super adiabatic conditions prevail below the stack.
- Pollutants can not escape above the top of the stack because of inversion layer.

(vii) Trapping plume



- When inversion layer exists above the emission source as well as below the source. Naturally the emitted plume will neither go up nor down.

- **Various mechanical devices used for removal of particulate matters**

Name of device	Min. particle size (μm)	Efficiency of Removal
1. Gravitational settling chamber	> 50	< 50%
2. Centrifugal collectors including cyclone collectors or separators and dynamic precipitator	5-25	30-90%
3. Wet scrubbers of collectors including spray towers	> 10	< 80%
4. Cyclonic scrubbers	> 2.5	< 80%
5. Venturi scrubbers (Also used for gaseous pollutants)	> 0.5	< 99%
6. Electrostatic precipitator	> 1	95-99%
7. Fabric filters	< 1	> 99%

Controlling air pollution from Auto mobiles

Automobiles normally emit

CO — 0.5 to 6.4%
HC — 300 to 1000 ppm
NO_x — 500 to 3000 ppm

depending upon type and condition of engine also.

Vanadium pentoxide: used for removing SO_2

Platinum metal: for removing NO_x

Activated alumina: for removing Hydrocarbons (impregnated with metallic compounds)

Pd^{++} & Cu^{++} for converting CO to CO_2

Characteristics of Sound and its measurement

$$\boxed{P = \frac{1}{f}}$$

where, P = period

f = frequency

c = $f\lambda (V = n\lambda)$

λ = wave length, V = velocity of sound

n = frequency, $prms = \sqrt{p^2(t)} = \sqrt{\frac{1}{T}\int_0^T p^2(t)\,dt}$

p(t) = pressure at any time 't', prms = sound pressure

Sound pressure = Atmospheric pressure — barometric pressure.

Power of Sound

The rate of doing work by a travelling sound wave in the direction of propagation of wave.

or

The energy transmitted by a sound wave in the direction of its propagation is thus defined as its power.

(i) Intensity of sound

'Sound power average' over the time per unit area normal to the direction of propagation of wave.

$$\boxed{I = \frac{W}{a}}$$ a = unit area ⊥ to the direction of wave motion.

$$\boxed{I = \frac{p^2rms}{\rho \times v}}$$ P = density of medium, v = vel. in m/sec.

$$\boxed{v = 20.05\sqrt{T}}$$ T = temperature in K

(ii) Level of Noise

- Sound pressure which can be heard by human ≈ 20 μΠα.
- Loudest sound produced by rocket ≈ 200 Pa.
- Level (Measurement

$$\boxed{L = \log_{10}\left(\frac{Q}{Q_0}\right) \text{bels}}$$

$$= 10\text{Log}_{10}\left(\frac{Q}{Q_0}\right) \text{ decibel (dB)}$$

where, Q = measured quantity, Q_o = Reference standard.

≈ 20 μΠα (if pressure)

$$\boxed{L_p = 10\log_{10}\left(\frac{prms}{20\mu Pa}\right)^2}$$

$$= 20\log_{10}\left(\frac{prms}{20\mu Pa}\right)$$

If Reference is power (≈ 10^{-12} watts) $\boxed{L_w = 10\log_{10}\left(\frac{W}{10^{-12}}\right)}$

If Reference Q_0 is sound Intensity (≈ 10^{-12} W/m²)

$$\boxed{L_I = 10\log_{10}\left(\frac{I}{10^{-12}}\right)}$$

(iii) Averaging sound pressure levels

The average value of the various records at a place is given by

$$\boxed{\bar{L}_p = 20\log\frac{1}{N}\sum_{n=1}^{N}(10)^{Ln/20}}$$

$\bar{L}_p$ = average sound pressured level in dB(Re ≈ 20 μΠα)

N = No. of measurement reading

L_n = nth sound pressure level in dB (Re -20 μPa) say for e.g.. average of 4 measurement 40, 50, 62 and 72 dB (Re = 20 μPa)

$$\sum_{n=1}^{4}(10)^{Ln/20} = 10^{40/20} + 10^{50/20} + 10^{62/20} + 10^{72+20} = 5656.22$$

$$\boxed{\bar{L}_p = 20\log_{10}\frac{1}{4}(5656.22)} \quad \boxed{\bar{L}_p = 63\text{dB}}$$

Noise Rating systems

A noise may consist of different types of sounds with different pressure levels. Operating for different time intervals, the frequency may also vary. Hence to find combined effect there are two concepts L_n and Leq concepts.

(i) The L_n Concept

The value of L_n will represent the sound pressure level that will exceed for N% of the gauging time. e.g. 70 dB value of L_{60} means that the sound level will exceed 70 dB for 60% of the measuring time.

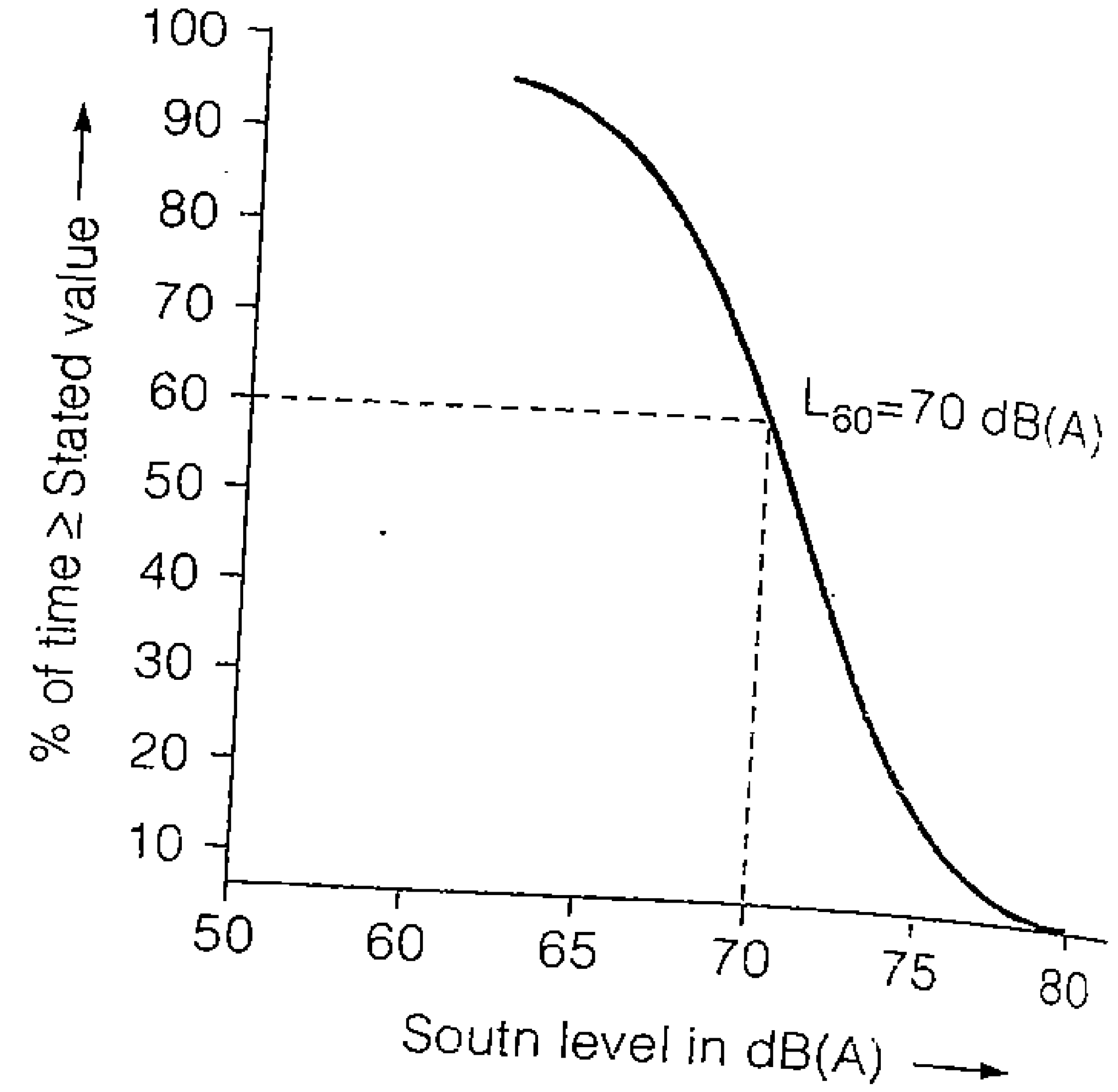
Graph between L_n and N (N = ...%)

(ii) L_{eq} Concept

(Equivalent NOISE level)

Leq is that statistical value of sound pressure level that can be equated to any fluctuating noise level.

Thus, Leq is defined as the const. noise level which over a given time expands the same amount of energy as is expanded by the fluctuating levels over the same time, this value is expressed by the equation



$$L_{eq} = 10\log\sum_{i=1}^{n}(10)^{Li/10}\times t_i$$

where, n = total no of sound samples

L_i = noise level of the i^{th} sample

t_i = time duration of i^{th} sample expressed as % total time.

L_{eq} value for fluctuating noise level of 95 minutes indicated earlier (i.e. the one with 80 dB lasting for 10 minutes, followed by 60 dB for 80 minutes followed by 100 dB for 5 min. can be worked out as below.

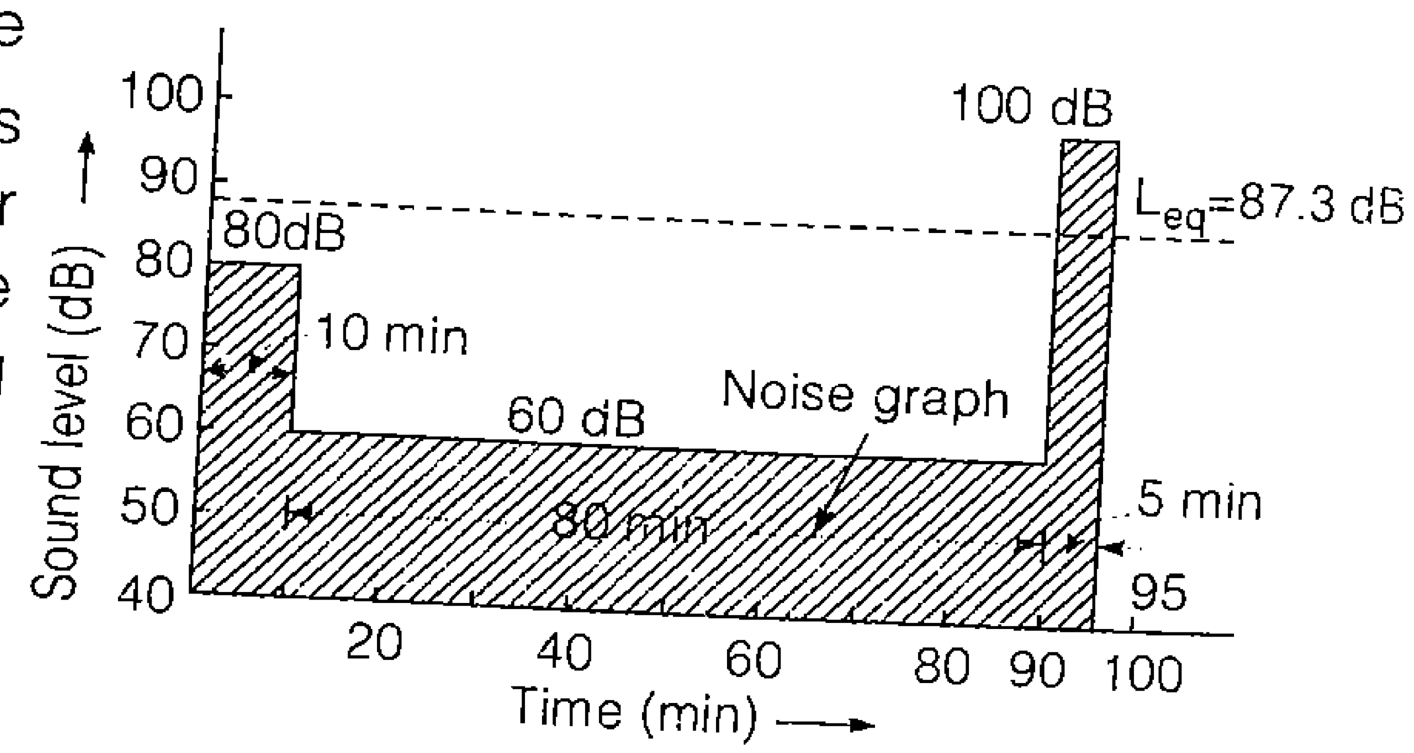
$$\sum_{1}^{3}(10)^{Li/10}\times t_i = \left[(10)^{80/10}\times\frac{10}{95}+(10)^{60/10}\times\frac{80}{95}+(10)^{100/10}\times\frac{5}{95}\right]$$

$= 537.69\times 10^6$

$L_{eq} = 10\log_{10}(537.69\times 10^6)$

$$L_{eq} = 87.3 \text{ dB}$$

- The equivalent noise level (L_{eq}) constitutes an important parameter for evaluating the impact of fluctuating noise of all kinds.



- The duration in hours, over which L_{eq} is worked out for a given site is further mentioned in bracket, such as L_{eq} (8) which means that L_{eq} is based on 8 hr., measurement, when, however no such time is mentioned, then L_{eq} is always corresponds to one hr. measurement.

(iii) Noise levels of different sources

1. Air traffic = 90 – 110 dB
2. Rail traffic = 90 – 110 dB (about 30 m away)
3. Heavy Road traffic = 80 – 90 dB
4. Medium Road traffic = 70 – 80 dB
5. Light Road traffic = 60 – 70 dB

(iv) Noise reduction

$$\text{Noise reduction (dB)} = 10\log_{10}\left(\frac{20H^2}{\lambda R}\right)$$

R = Distance between source and wall.

H = Height of barriers wall

λ = Wave length of sound

D = Distance between barrier and the receiving point.

CPCB standards of noise levels (d_B) ⇒ $d_{B(A)}$

Rural	Suburban	Residential (urban)	Urban (Residential & business)	City	Industrial
25-35	30-40	35-45	40-50	45-50	50-60

Ambient air quality standards $d_{B(A)}$, L_{eq}.

Area	Day time	Night time
Industrial area	75	70
Commercial area	65	55
Residential area	55	45
Silence zone	50	40

■■■

A Handbook on
Civil Engineering

10

Highway Engineering

CONTENTS

1 Highway Development and Planning

Nagpur Road Plan or First 20-year Road Plan

(i) $$(NH + SH + MDR)_{in\ km} = \left[\frac{A}{8} + \frac{B}{32} + 1.6N + 8T\right] + D - R$$

where,

NH + SH + MDR = Total length of first category or metalled roads for national & state highways & major district roads in km.

A = Agricultural area in km^2

B = Non-agricultural area in km^2

N = Number of towns & villages with population range 2001-5000

T = Number of towns & villages with population over 5000.

D = Development allowance of 15% of road length calculated to be provided for agricultural & industrial development during the next 20 years.

R = Existing length of railway track in km.

(ii) $$(ODR + VR)_{in\ km} = [0.32V + 0.8Q + 1.6P + 3.2S + D]$$

where,

ODR + VR = Total length of second category roads for other district roads & village roads in km.

V = Number of villages with population range 500 or less.

Q = Number of villages with population range 501-1000.

P = Number of villages with population range 1001-2000.

S = Number of villages with population range 2001-5000.

D = Development allowance of 15% for next 20 years.

Second Twenty Year Road Plan (1961-81) (Bombay Road Plan)

(i) $$NH = \left[\frac{A}{64} + \frac{B}{80} + \frac{C}{96}\right] + [32K + 8M] + D$$

where, NH = Length of national highways in km.

(ii)
$$(NH + SH)_{in\,km} = \left[\frac{A}{20} + \frac{B}{24} + \frac{C}{32}\right] + [48K + 24M + 11.2N + 1.6P] + D$$

where, $(NH + SH)_{in\,km}$ = Length of national & state highway in km.

(iii)
$$(NH + SH + MDR)_{in\,km} = \left[\frac{A}{8} + \frac{B}{16} + \frac{C}{24}\right] + [48K + 24M + 11.2N + 9.6P + 6.4Q + 2.4R] + D$$

where, $(NH + SH + MDR)_{in\,km}$ = Length of national & state highway & major district road.

(iv)
$$(NH + SH + MDR + ODR)_{in\,km} = \left[\frac{3A}{16} + \frac{3B}{32} + \frac{C}{16}\right] + [48K + 24M + 11.2N + 9.6P + 12.8Q + 4R + 0.8S + 0.32T] + D$$

where, $(NH + SH + MDR + ODR)_{in\,km}$ = Length of national & state highway & major district road & other district road.

(v)
$$(NH + SH + MDR + ODR + \text{Village road})_{in\,km} = \left[\frac{A}{4} + \frac{B}{8} + \frac{C}{12}\right] + [48K + 24M + 11.2N + 9.6P + 12.8Q + 5.9R + 1.6S + 0.64T + 0.2V] + D$$

where,
A = Developed & agricultural areas in km^2
B = Semi-developed area in km^2
C = Undeveloped area in km^2
K = Number of towns with population over 1,00,000
M = Number of towns with population range 1,00,000 - 50,000
N = Number of towns with population range 50,000 – 20,000
P = Number of towns with population range 20,000 – 10,000
Q = Number of towns with population range 10,000 – 5,000
R = Number of towns with population range 5,000 – 2,000
S = Number of towns with population range 2,000 – 1,000
T = Number of towns with population range 1,000 – 500
V = Number of towns with population range below 500.
D = Development allowance of 5% of road length calculated for further development & other unforeseen factors.

Highway Development and Planning

Some Important Years

1.	Jayakar committee	Formed in Nov. 1927 Recommendation Feb. 1928
2.	Central road fund	1929
3.	Indian road congress	1934
4.	Motor vehicle act	1939
5.	First 20 years road plan (Nagpur road plan)	1943-1963 (but it finished in 1961)
6.	CRRI (Central road research institute)	1950
7.	Second 20 year road plan (Bombay road plan)	1961-1981
8.	Third 20 year road plan (Lucknow road plan)	1981-2001
9.	National highway act	1956

Important Recommendations

Jayakar Committee

1. Road development should be considered as a matter of national interest.
2. Tax on petrol should be levied to collect fund for road development work.

   Result - Central road fund was formed in 1928.
3. A semi official technical body should be formed to act as advisory body on various aspect of roads.

   Result - IRC was formed in 1934.
4. A research organisation should be instituted to carryout research and development work.

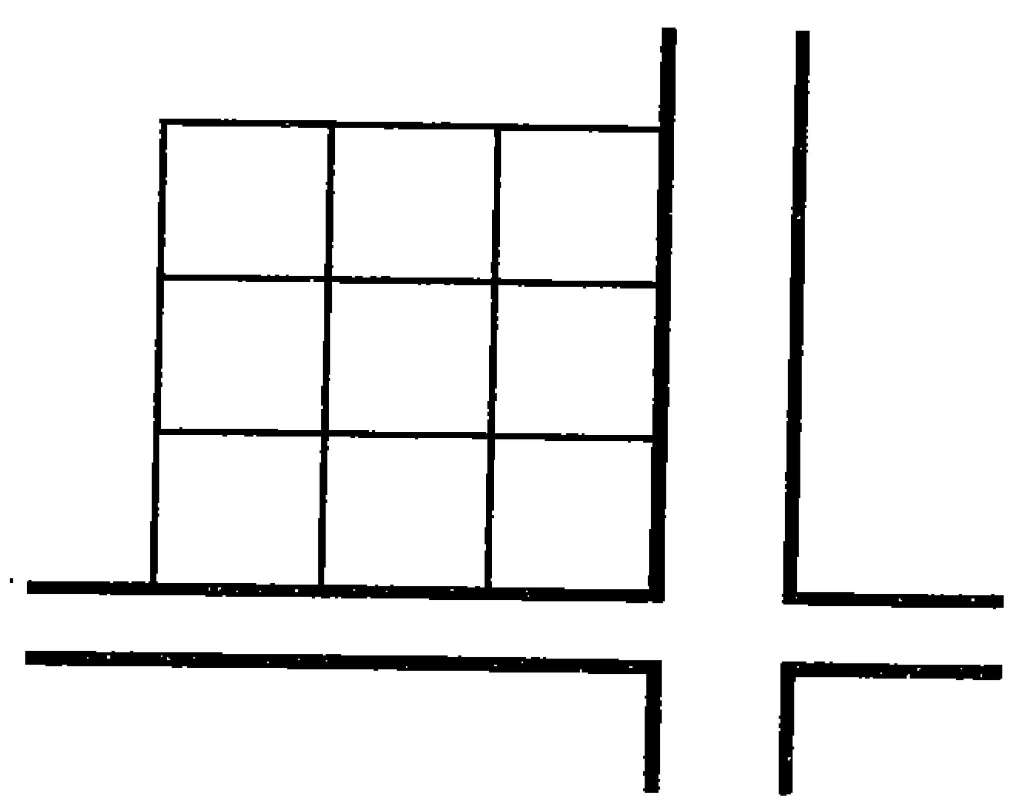
   Result - CRRI was formed in 1950.

Three Road Development Plan

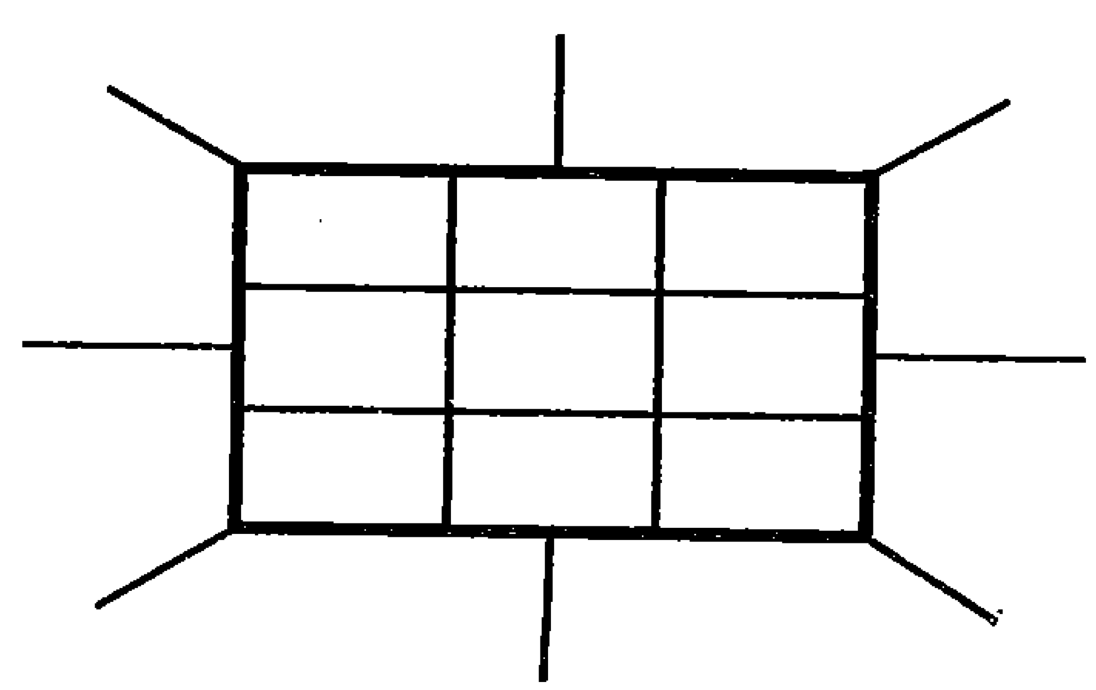
		Ist 20 year road plan	IInd 20 year road plan	IIIrd 20 year road plan
1.	Venue	Nagpur	Bombay	Lucknow
2.	Year	1943	1961	1921
3.	Target at end	16 km/100 km² area	32 km/100 km² area	82 km/100 km² area
4.	Total road length target	5.29 lakhs km	10.57 lakhs km	27.02 lakhs km
5.	Outlay	448 crore	5200 crore	—
6.	Other points	Roads are divided into 5 categories (i) NH (ii) SH (iii) MDR (iv) ODR (v) VR	Expressway was added	Roads are divided into three major categories (i) Primary → Express ways, NH (ii) Secondary → SH, MDR (iii) ODR & VR

Different Road Patterns

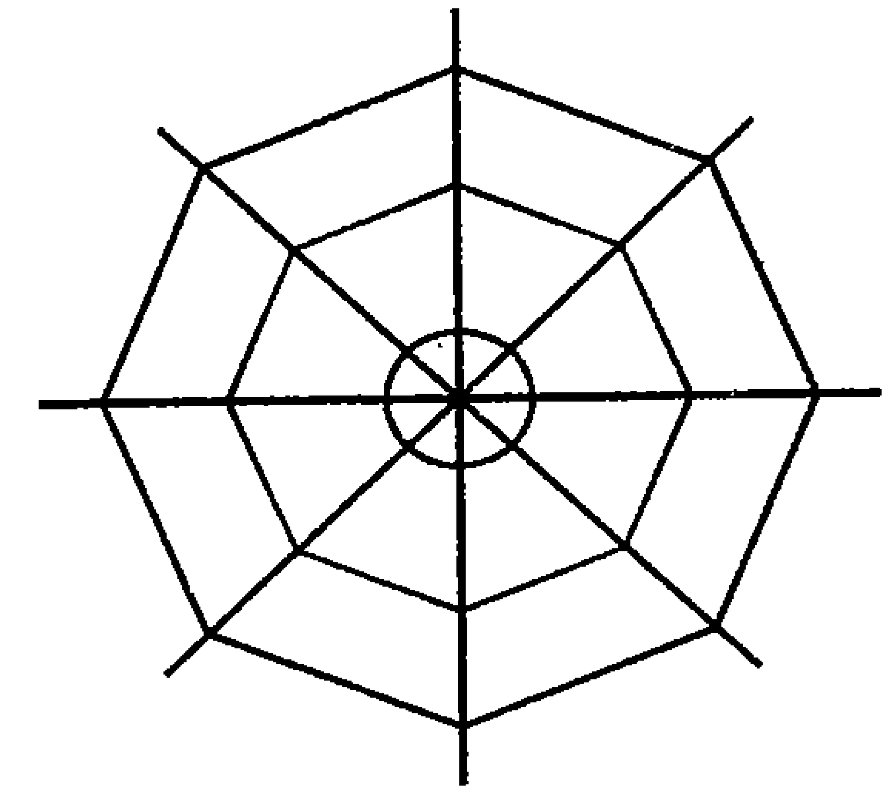
1. Rectangular and block pattern



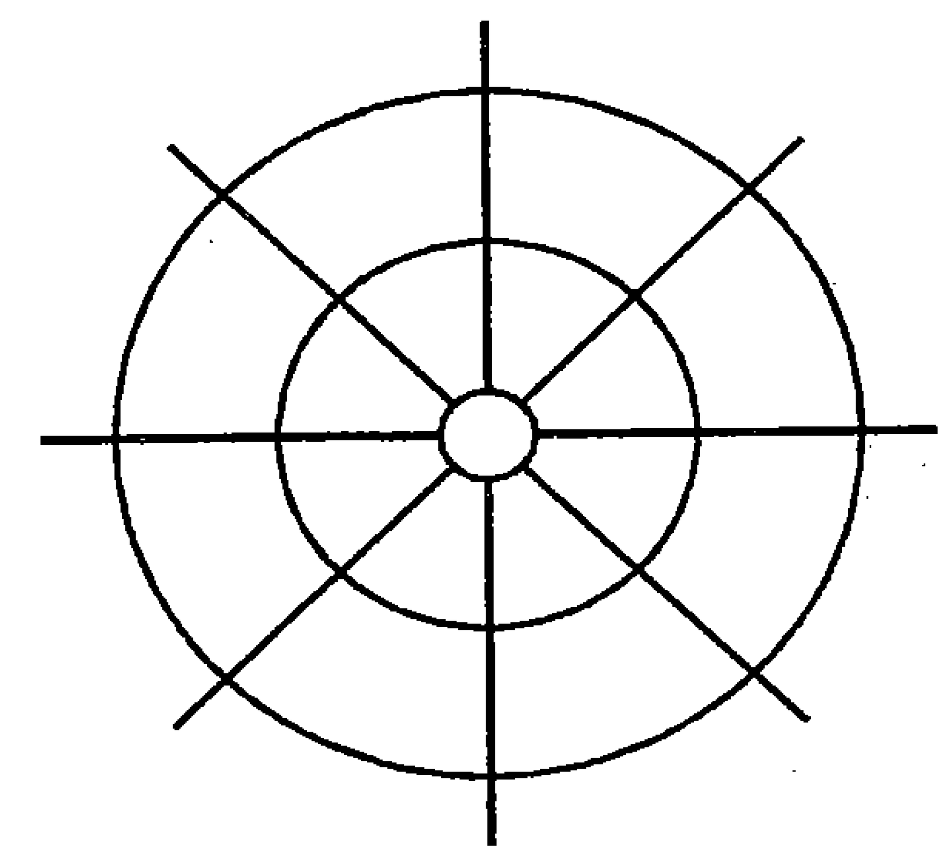
2. Star and block pattern



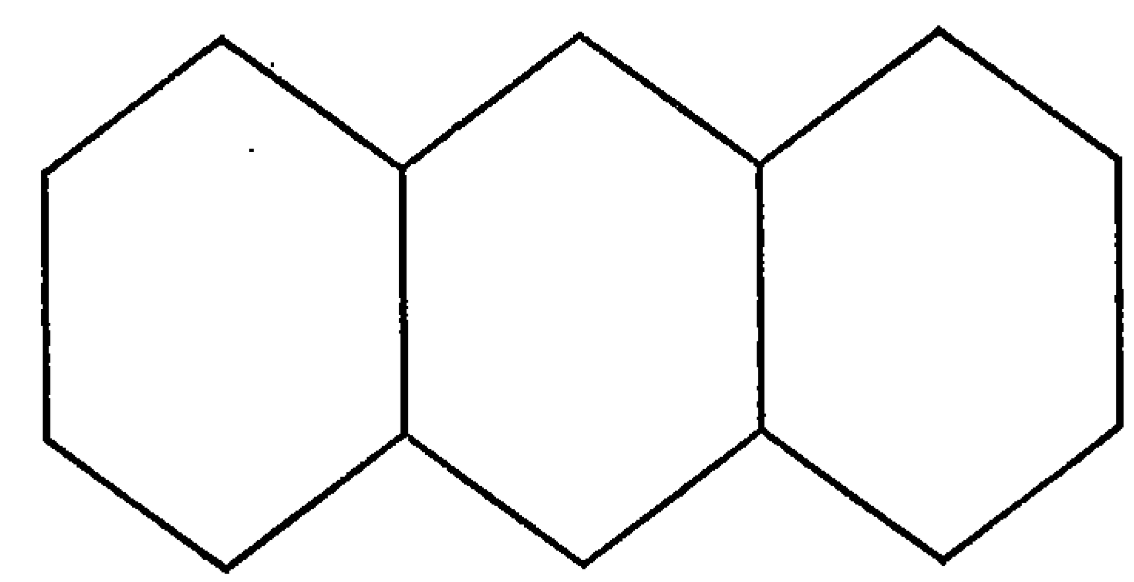
3. Star and grid pattern (adopted in India)



4. Star and circular pattern



5. Hexagonal road pattern



Highway Geometric Design

2

Stopping Sight Distance (SSD)

The minimum sight distance available in a highway at any spot should be of sufficient length to stop a vehicle traveling as design speed, safely without collision with any other obstruction. The absolute minimum sight distance is therefore equal to the stopping sight distance, which is also some times called non-passing sight distance.

The stopping distance of a vehicle is the sum of :

1. The distance travelled by the vehicle during the total reaction time known as lag distance and
2. The distance travelled by the vehicle after the application of the brakes, to a dead stop position which is known as the braking distance.

(i) $\boxed{\text{lag distance} = 0.278Vt}$ where, V = Speed in km/hr.
t = Reaction time in sec

(ii) $\boxed{\text{Breaking distance} = \dfrac{V^2}{254(f \pm S\%)}}$ where,
f = Coefficient of friction
= 0.40 for v = 20 to 30 km/hr
= 0.35 for v = 100 km/hr

(iii) SSD = lag distance + breaking distance

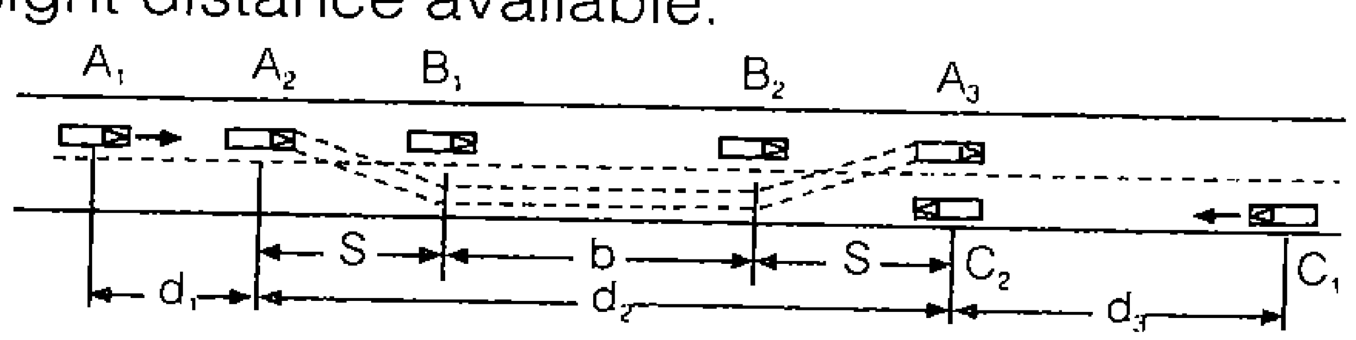
$$\boxed{SSD = 0.278Vt + \frac{V^2}{254(f \pm S\%)}}$$

where, S% is gradient + ve sign for ascending gradient & –ve sign for descending gradient

SSD = Stopping sight distance in 'm'.

Overtaking Sight Distance (OSD)

The minimum distance open to the vision of the driver of a vehicle intending to overtake slow vehicle ahead with safety against the traffic of opposite direction is known as the minimum overtaking sight distance (OSD) or the safe passing sight distance available.



Overtaking Manoeuvre

$$\boxed{OSD = d_1 + d_2 + d_3}$$

where, O.S.D = Overtaking sight distance in 'm'

d = Distance travelled by overtaking vehicle A during the reaction time t sec of the driver from position A_1 to A_2.

$$\boxed{d_1 = 0.278V_b t}$$

d_2 = Distance travelled by the vehicle A from A_2 to A_3 during the actual overtaking operation in time T sec.

$$\boxed{d_2 = b + 2s} \text{ and } \boxed{d_2 = 0.278V_b T + \frac{1}{2}aT^2}$$

where S = Minimum spacing between two vehicle.

$\boxed{s = 0.2V_b + 6}$ here V_b is in km/hr.

$\boxed{T = \sqrt{\dfrac{4s}{a}}}$ where, a = acceleration in m/s^2.

$\boxed{T = \sqrt{\dfrac{14.4s}{a}}}$ where a is in km/hr/sec.

$\boxed{d_3 = 0.278V_C T}$

where, d_3 = Distance travelled by on coming vehicle C from C_1 to C_2 during the overtaking operation of A i.e., T sec.

V_C = V = Speed of overtaking vehicle or design speed (km/hr)

if V_b is not given then

$$\boxed{\begin{array}{c} V_b = (V - 16)\,km/hr \\ V_b = (v - 4.5)\,m/s \end{array}}$$ v = design speed in m/s.

Overtaking Zone

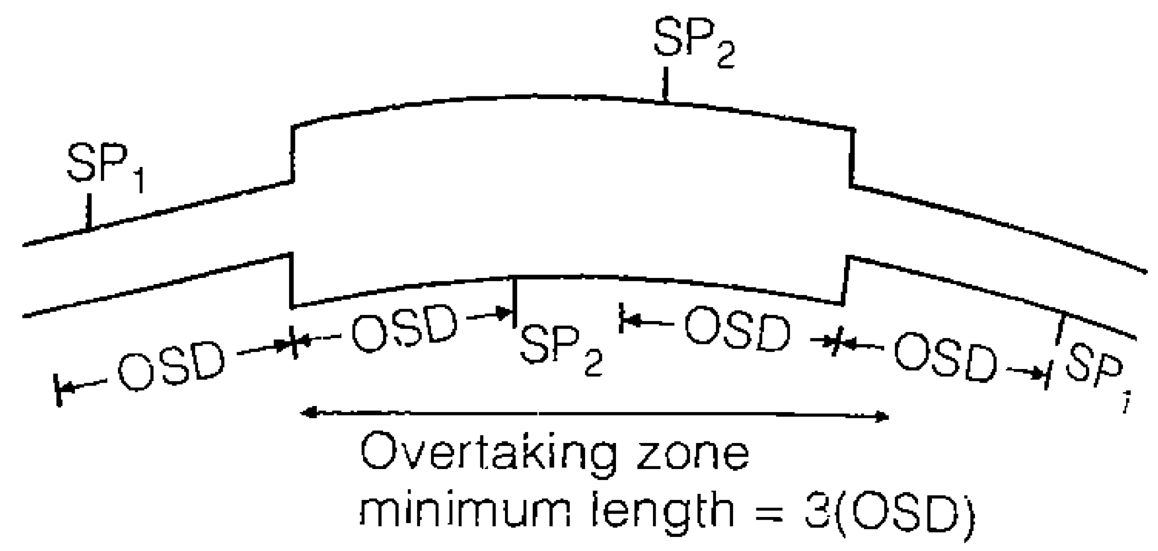
It is desirable to construct highways in such a way that the length of road visible ahead at every point is sufficient for safe overtaking. This is seldom practicable and there may be stretches where the safe overtaking distance can not be provided. In such zones where overtaking or passing is not safe or is not possible, sign posts should be installed indicating "No passing" or "Overtaking Prohibited" before such restricted zones starts. But the overtaking opportunity for vehicles moving at design speed should be given at frequency intervals. These zones which are meant for overtaking are called overtaking zones.

$\boxed{O \cdot S \cdot D = d_1 + d_2}$

For one way traffic

$\boxed{OSD = d_1 + d_2 + d_3}$

For two way traffic



$\boxed{\text{Minimum length of overtaking zone} = 3 \cdot (OSD)}$

$\boxed{\text{Desirable overtaking zone} = 5(OSD)}$

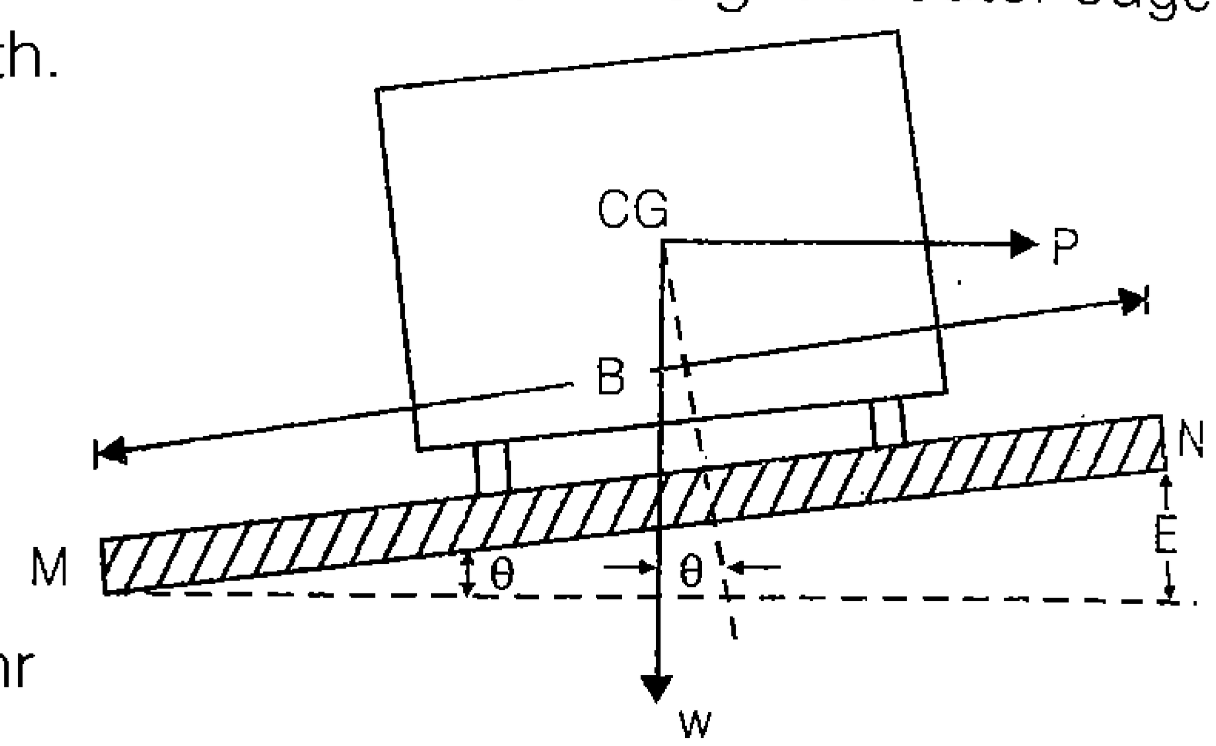
Super Elevation (e)

In order to counteract the effect of centrifugal force and to reduce the tendency of the vehicle to overturn or skid, the outer edge of the pavement is raised with respect to the inner edge, thus providing a transverse slope throughout the length of the horizontal curve. This transverse inclination to the pavement surface is known as superelevation or cant or banking.

The superelevation 'e' is expressed as the ratio of the height of outer edge with respect to the horizontal width.

(a) $\boxed{e = \frac{NL}{ML} = \tan\theta}$

(b) $\boxed{e + f = \frac{V^2}{127R}}$



where V = Speed in km/hr
R = Radius in 'm'
f = Design value of lateral friction = 0.15
e = Rate of super elevation ~ tan θ

- Maximum Super Elevation (e_{max})

$$\boxed{e_{max} = \begin{cases} 0.07 \rightarrow \text{For plain \& Rolling Terrain} \\ 0.10 \rightarrow \text{For Hilly Area} \\ 0.04 \rightarrow \text{For Urban roads with frequent intersections} \end{cases}}$$

Ruling Minimum Radius of the Curve (R_{ruling})

$$\boxed{R_{ruling} = \frac{V^2}{127(e+f)}}$$

where, V = Ruling design speed in km/hr
e = Rate of super elevation
f = Coefficient of friction ~ 0.15

Extra Widening (E_W)

The extra widening of pavement on horizontal curves is divided into two parts (i) Mechanical and (ii) Psychological widening.

$$\boxed{E_W = W_m + W_P} \rightarrow \boxed{E_W = \frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}}}$$

Mechanical Widening (W_m): The widening required to account for the off-tracking due to the rigidity of wheel base is called mechanical widening. (W_m).

$$\boxed{W_m = \frac{n l^2}{2R}}$$

Psychological widening (W_p): Extra width of pavement is also provided for psychological reasons such as, to provide for greater maneuverability of steering at higher speeds, to allow for the extra space requirements for the overhangs of vehicles and to provide greater clearance for crossing and overtaking vehicles on the curves. Psychological widening is therefore important in pavements with more than one lane.

$$\boxed{W_P = \frac{V}{9.5\sqrt{R}}}$$

where, n = number of traffic lanes
l = length of wheel base (m)
R = radius of the curve (m)
V = velocity (kmph)

TRANSITION CURVE

The Indian Roads Congress recommends the use of the spiral as transition curve in the horizontal alignment of highways due to the following reasons:

(i) The spiral curve satisfies the requirements of an ideal transition.

(ii) The geometric property of spiral is such that the calculations and setting out the curve in the field is simple and easy.

- Length of Transition Curve (L)

(i) According to rate of change of centrifugal acceleration

$$L = \frac{0.0215V^3}{CR}$$

where, V = Speed of vehicle in (km/hr)

C = Allowable rate of change of centrifugal acceleration in m/sec³

$$C = \frac{80}{75 + V}$$

R = Radius of curve in 'm'.

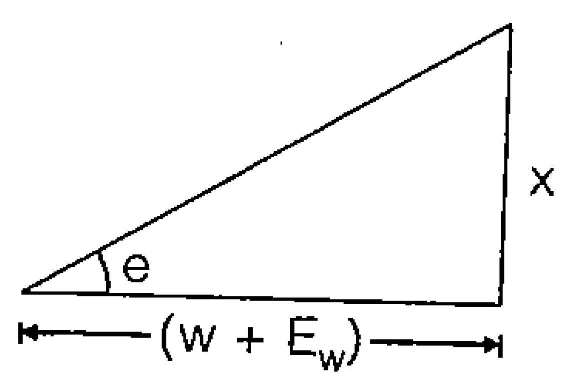
L = Length of transition curve in 'm'.

(ii) According to rate of change of super elevation

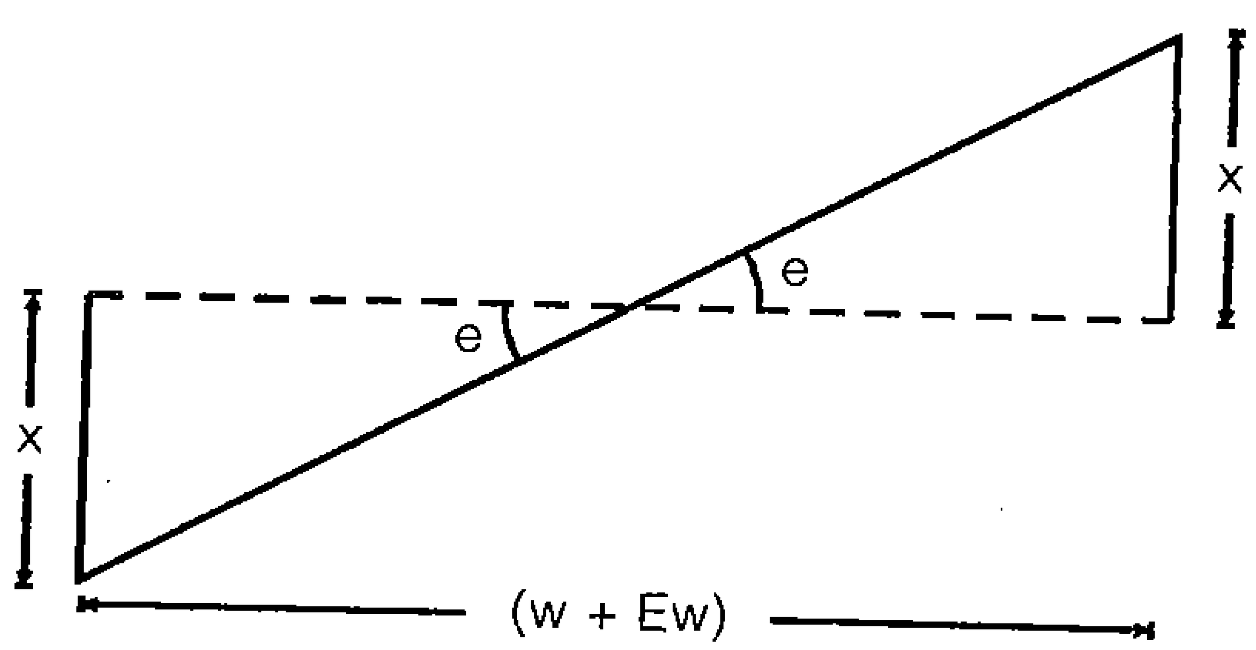
$$L = \begin{cases} 150x \rightarrow \text{For plain \& Rolling Terrain} \\ 100\,x \rightarrow \text{For Built up Area} \\ 60\,x \rightarrow \text{For Hilly Area} \end{cases}$$

where, x = Raise of outer line of road.

$$x = (w + E_w)e$$ it pavement is rotated about inner side.



$$x = (w + E_w)\frac{e}{2}$$ it pavement is rotated about centre line.



(iii) According to empirical formula

$$L = \frac{2.7V^2}{R}$$ For Plain & Rolling Terrain

$$L = \frac{V^2}{R}$$ For Hilly Area.

Set Back Distance (m)

The clearance distance or set back distance required from the centre line of a horizontal curve to an obstruction on the inner side of the curve to provide adequate sight distance depends upon the following factors:

(i) Required sight distance (SSD)

(ii) Radius of horizontal curve, (R)

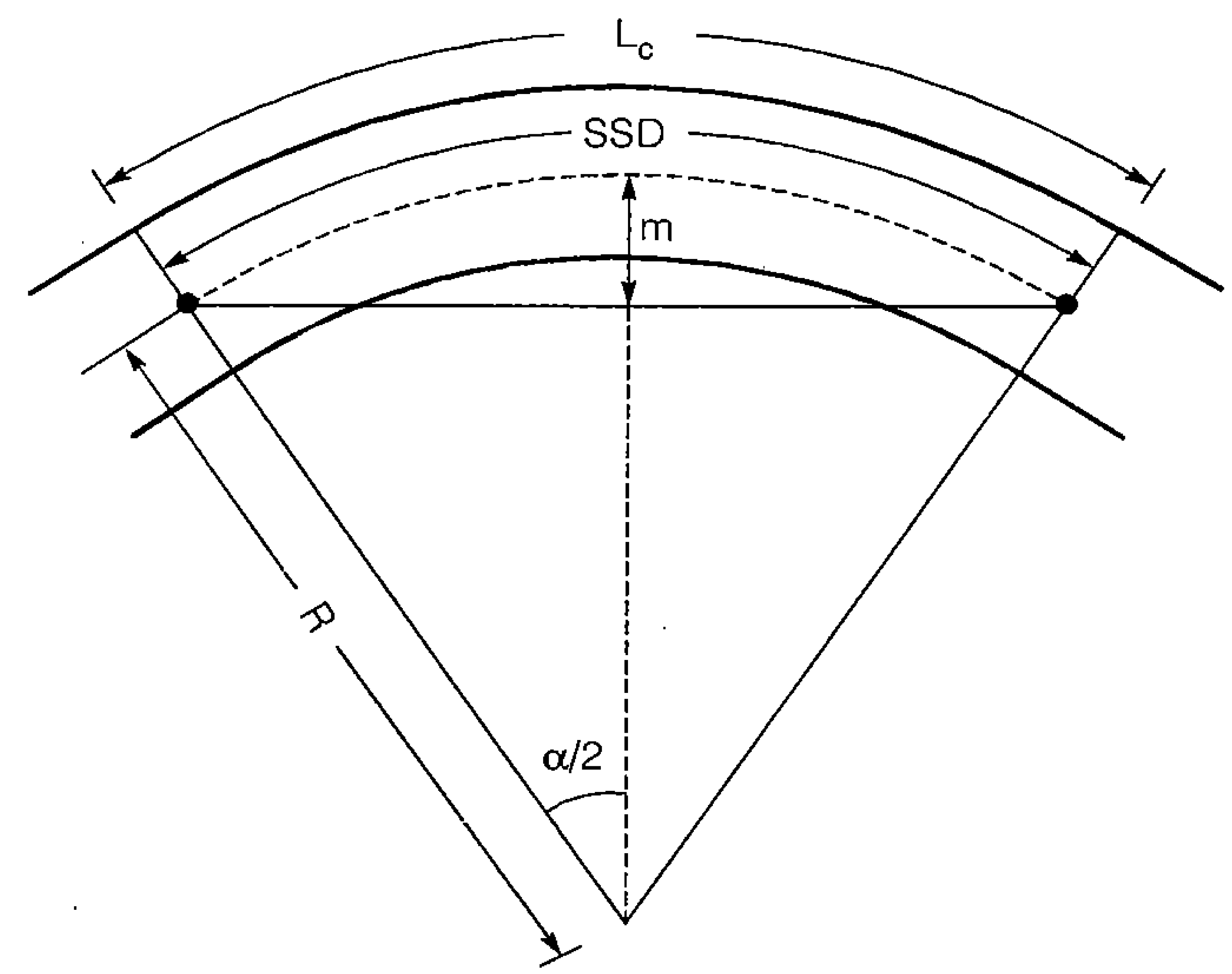
(iii) Length of the curve (L_c)

1. For single lane road

(a) When $L_C >$ **SSD**

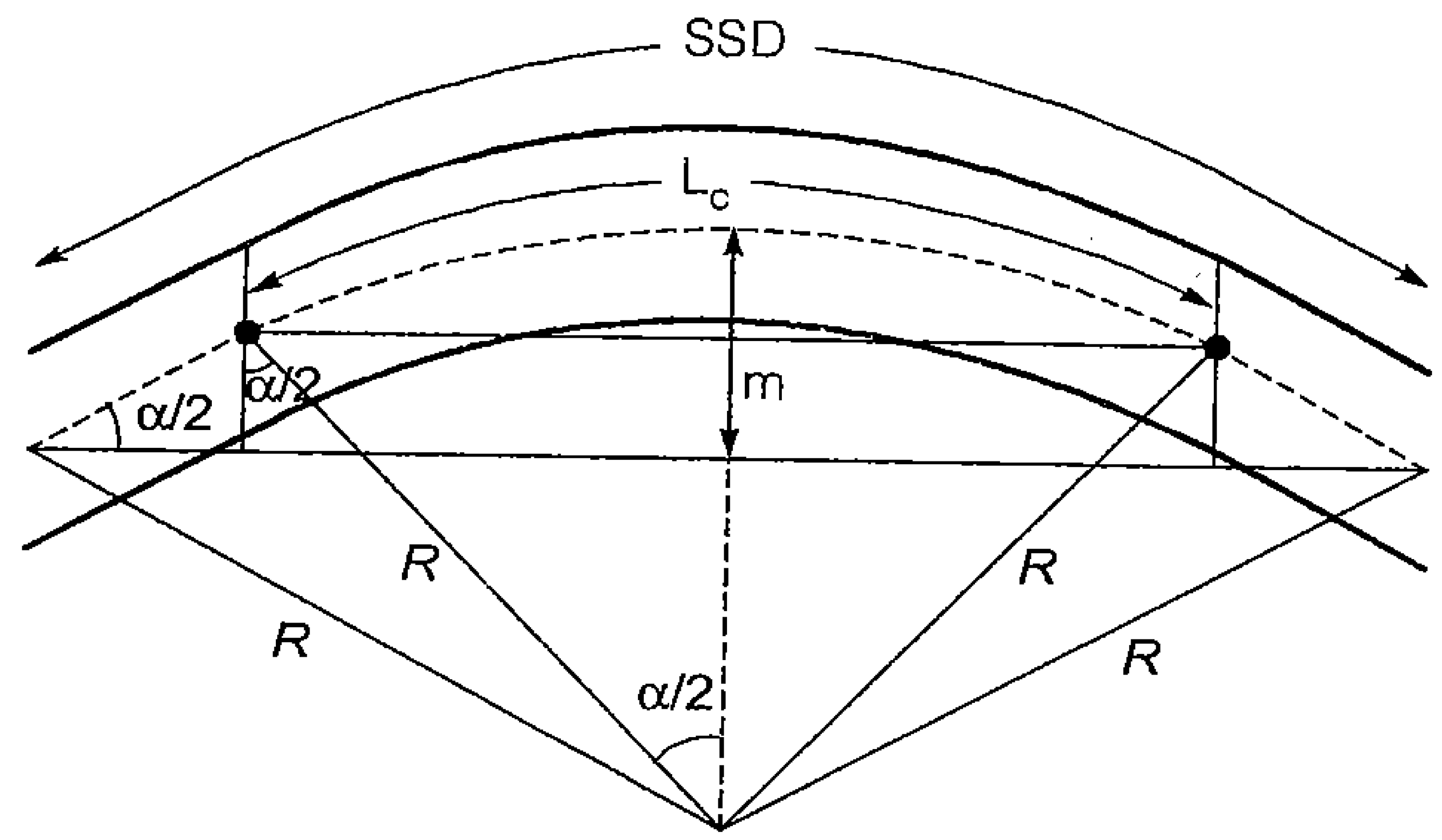
$$m = R - R\cos\frac{\alpha}{2} \quad \& \quad \frac{\alpha}{2} = \frac{180s}{2\pi R}$$

where, L_C = Length of curve & s = SSD



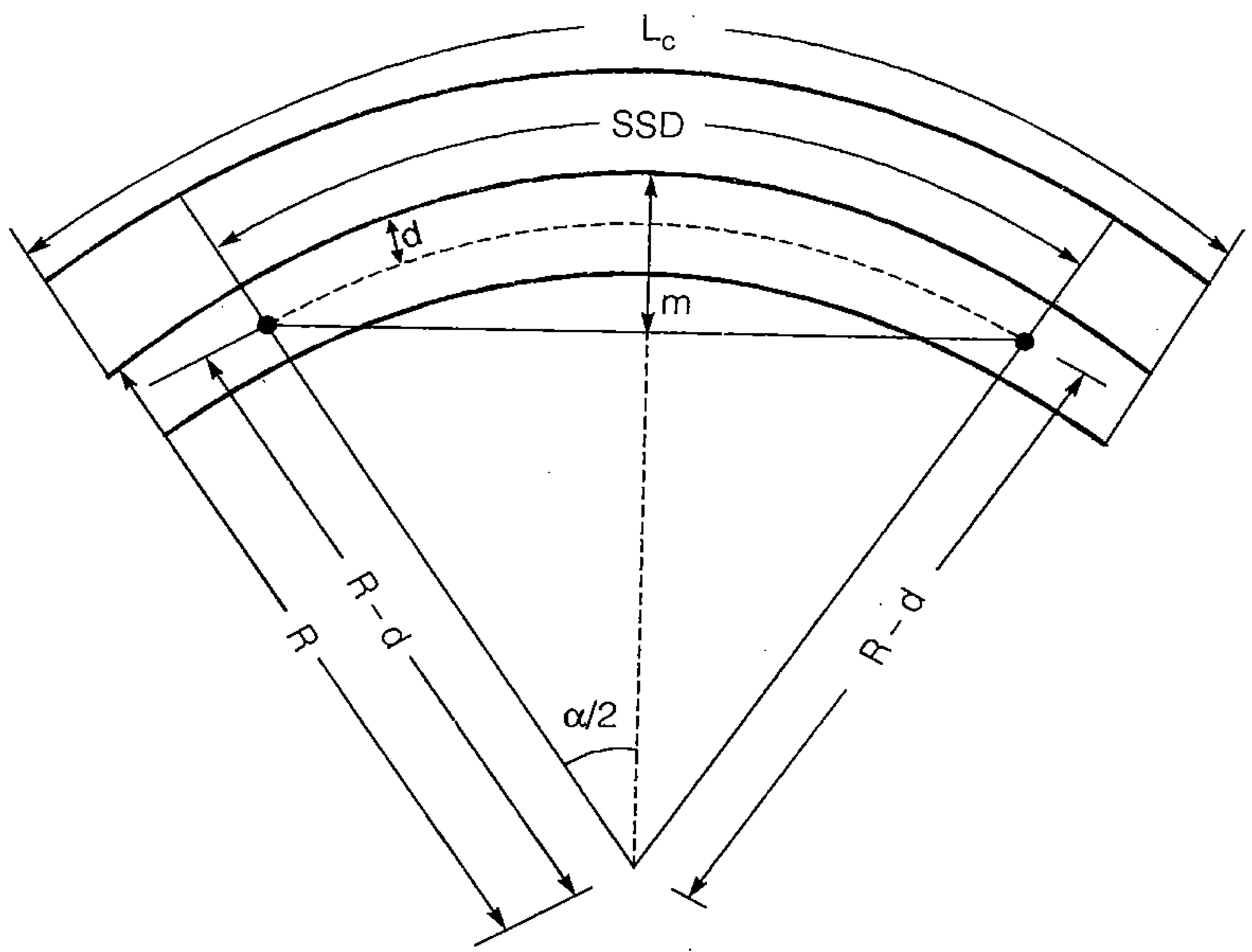
(b) When $L_C <$ **SSD**

$$m = R\left(1 - \cos\frac{\alpha}{2}\right) + \frac{S - L_C}{2}\sin\frac{\alpha}{2} \quad \& \quad \frac{\alpha}{2} = \frac{180L_C}{2\pi R}$$



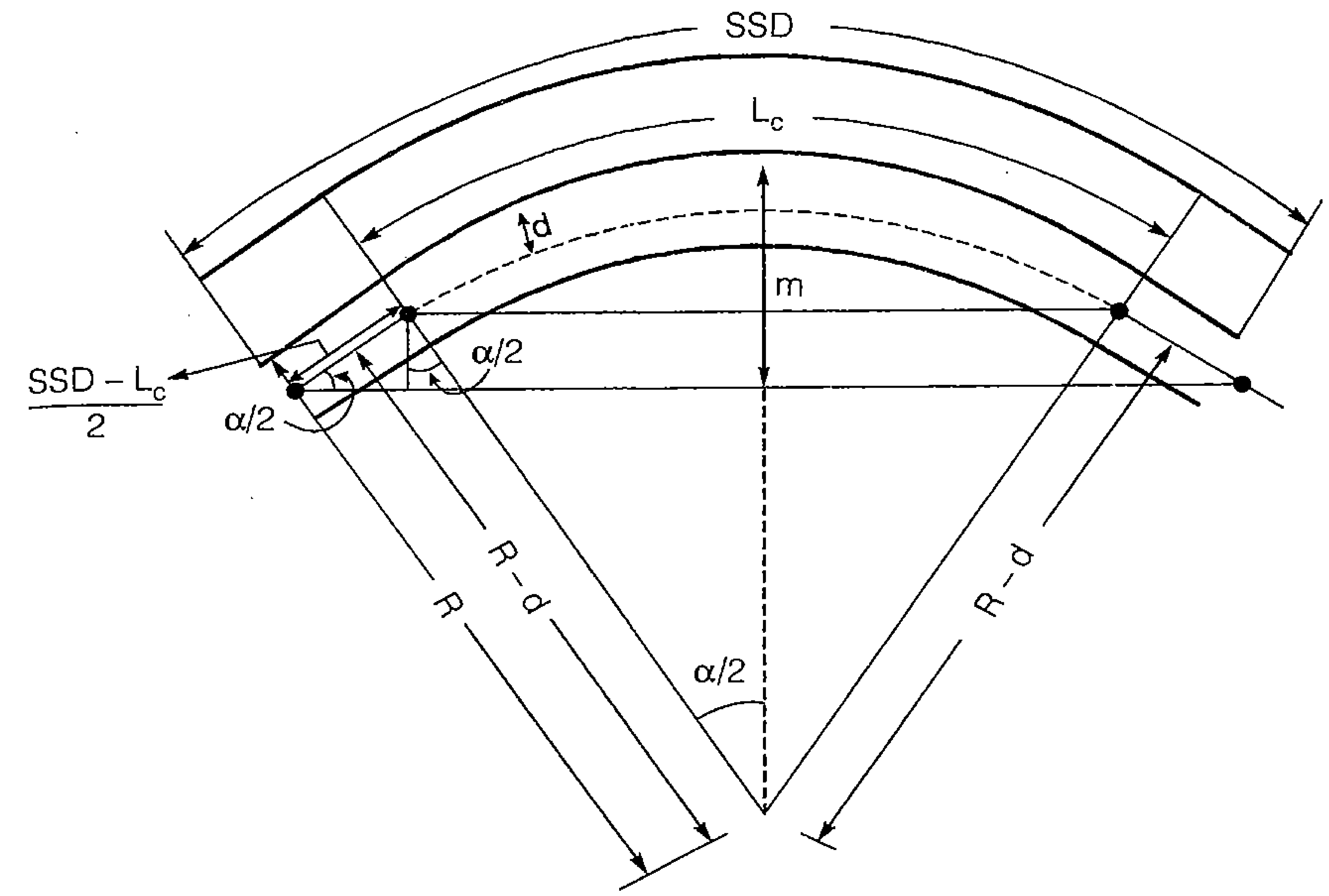
2. For two lane road
   (a) When L_C > SSD

$$m = R - (R - d)\cos\frac{\alpha}{2} \quad \& \quad \frac{\alpha}{2} = \frac{180s}{2\pi(R-d)}$$



   (b) When L_C < SSD

$$m = R - (R - d)\cos\frac{\alpha}{2} + \left(\frac{S - L_C}{2}\right)\sin\frac{\alpha}{2} \quad \& \quad \frac{\alpha}{2} = \frac{180 L_C}{2\pi(R-d)}$$



- **Grade Compensation**

$$\text{Grade compensation} = \frac{30 + R}{R}\% \quad \text{and}$$

$$\text{Maximum value of grade compensation} = \frac{75}{R}\%$$

where, R = Radius of curve in meter.

Vertical Curve

Due to changes in grade in the vertical alignment of highway, it is necessary to introduce vertical curve at the intersections of different grades to smoothen out the vertical profile and thus ease off the changes in gradients for the fast moving vehicles.

The vertical curves used in highway may be classified into two categories:

(i) Summit curves or crest curves with convexity upwards

(ii) Valley or sag curves with concavity upwards.

- **Summit Curves (Crest Curve with Convexity Upward):** Summit curves with convexity upwards are formed in any one of the case illustrated in fig. The deviation angles between the two interacting gradients is equal to the algebraic difference between them. Of all the cases, the deviation angle will be maximum when an ascending gradient meets with a descending gradient i.e., $N = n_1 - (-n_2) = (n_1 + n_2)$

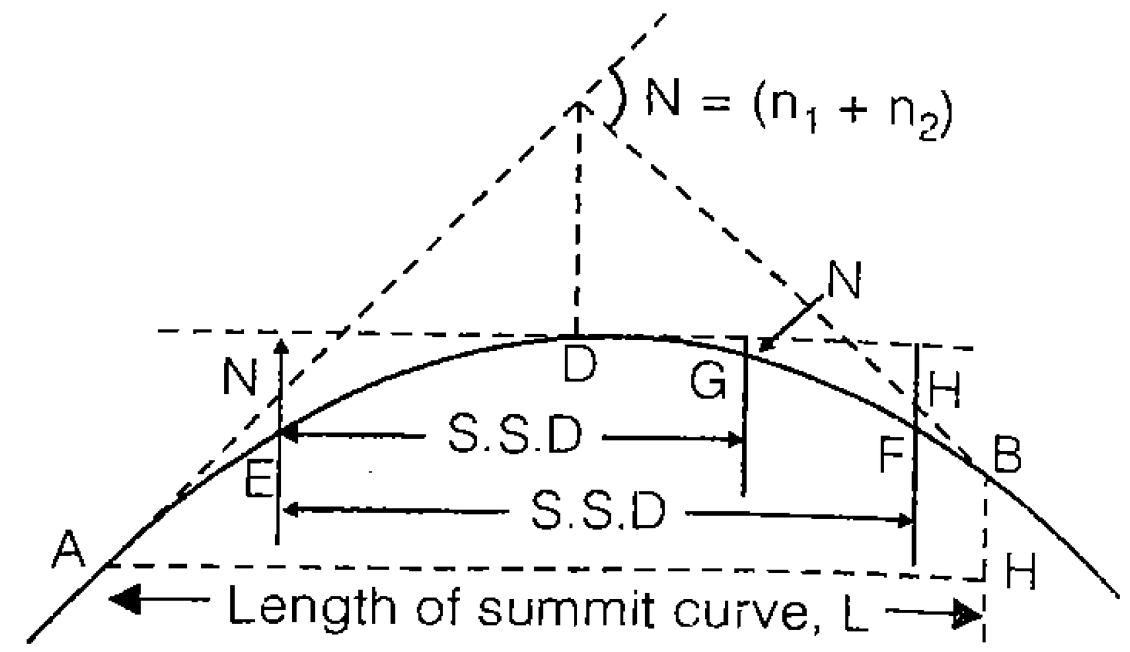
(i) Length of summit curve for SSD

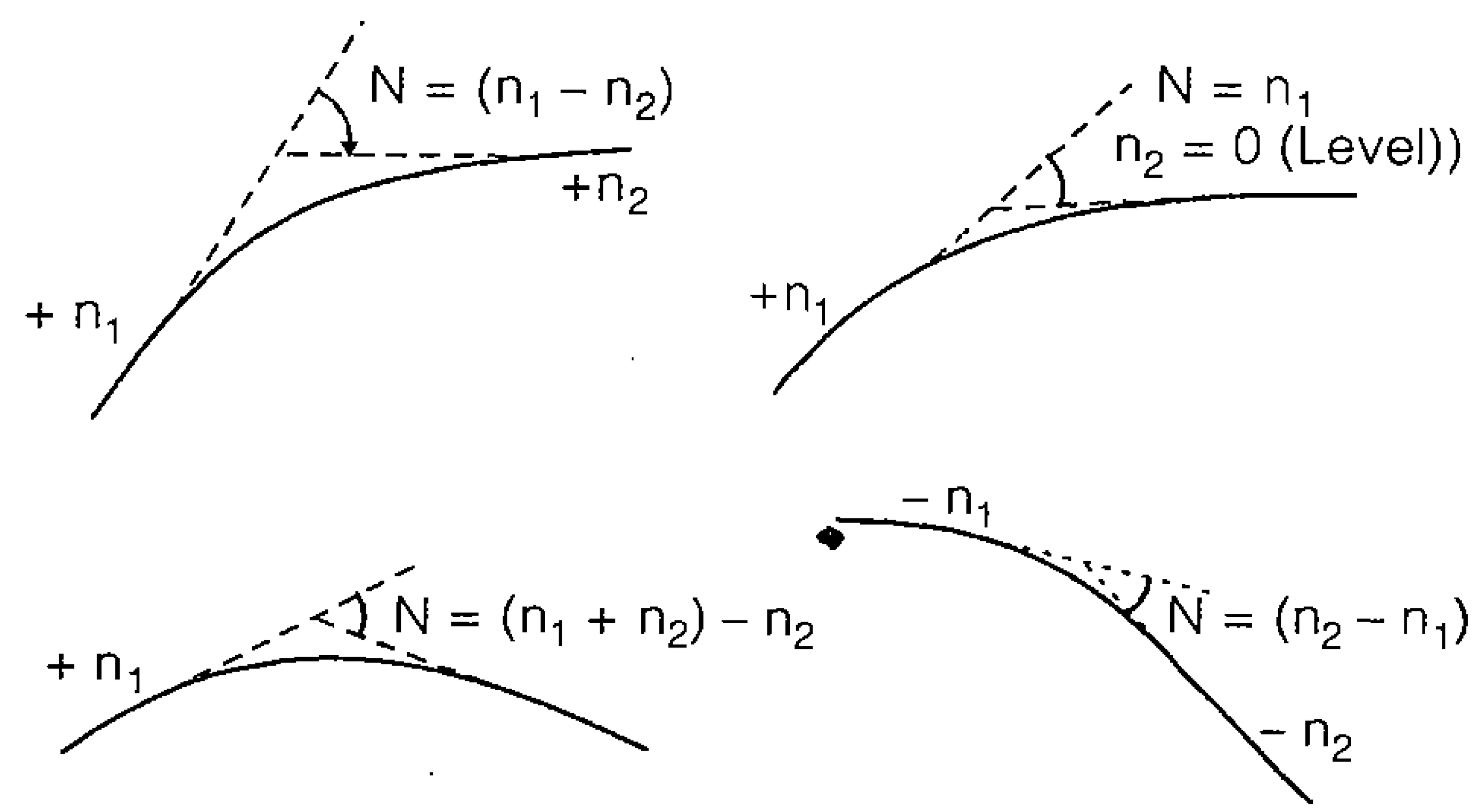
(a) When L > **SSD**

$$\boxed{L = \frac{NS^2}{\left(\sqrt{2H}+\sqrt{2h}\right)^2}}$$

$$\downarrow$$

$$\boxed{L = \frac{NS^2}{4\cdot 4}}$$





where,
L = Length of summit curve in meter
S = SSD (m)
N = Deviation angle
= Algebraic difference of grade
H = Height of eye level of driver above road way surface = 1.2 m
h = Height of object above the pavement surface = 0.15 m

(b) When L < **SSD**

$$\boxed{L = 2S - \frac{(\sqrt{2H}+\sqrt{2h})^2}{N}} \rightarrow \boxed{L = 2S - \frac{4\cdot 4}{N}}$$

(ii) Length of summit curve for safe overtaking sight distance (OSD) or intermediate sight distance (ISD)

(a) When L > **OSD**

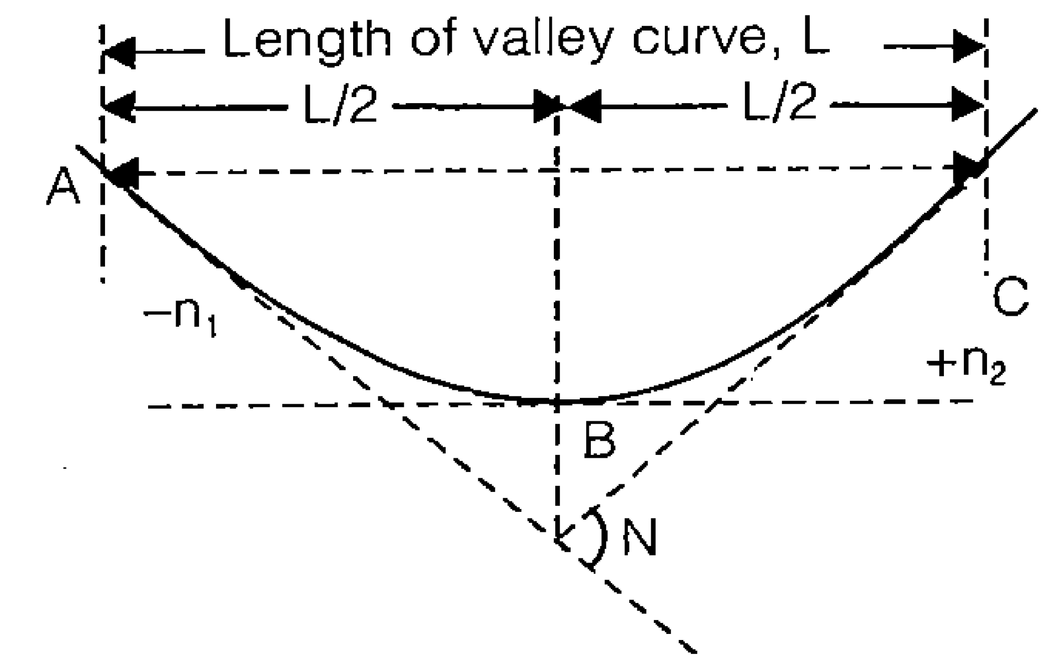
$$\boxed{L = \frac{NS_0^2}{\left(\sqrt{2H}+\sqrt{2h}\right)^2}} \rightarrow \boxed{L = \frac{NS_0^2}{9.6}}$$

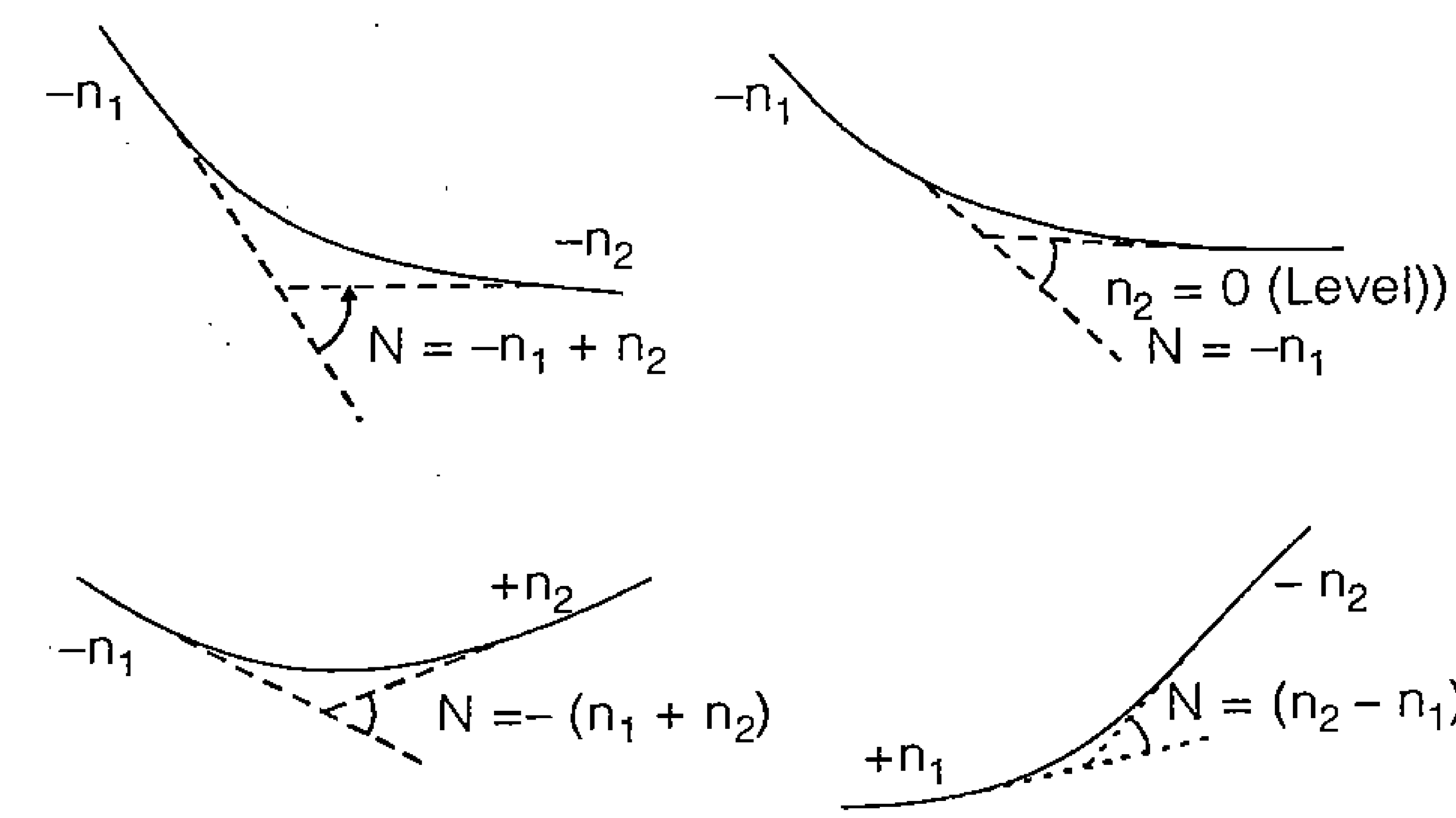
where, S_0 = Overtaking or Intermediate sight distance

(b) Whey L < **OSD**

$$\boxed{L = 2S - \frac{(\sqrt{2H}+\sqrt{2h})^2}{N}} \rightarrow \boxed{L = 2S - \frac{9.6}{N}}$$

- **Valley Curves (Sag Curve with Concavity Upward):** Valley curves or sag curves are formed in any one of the cases illustrated in fig. In all the cases the maximum possible deviation angle is obtained when a descending gradient meets with an ascending gradient.





(i) Length of valley curve as per comfort condition (transition curves are provided back to back).

$$\boxed{L = 2\left[\frac{NV^3}{C}\right]^{1/2}} \quad \text{if } C = 0.6\ \text{m/s}^3 \text{ than } \boxed{L = 0.38(NV^3)^{1/2}}$$

(ii) Length of valley curve for head light sight distance (parabolic curve is provided).

(a) When L > SSD

$$\boxed{L = \frac{NS^2}{2h_1 + 2s\tan\alpha}} \rightarrow \boxed{L = \frac{NS^2}{1.50 + 0.035s}}$$

where,

L = Total length of valley curve
S = SSD (m)
N = Deviation angle
α = Beam angle $\simeq 1°$
h_1 = Avg. height of head light = 0.75 m

(b) When **L < SSD**

$$L = 2S - \frac{(2h_1 + 2s\tan\alpha)}{N} \rightarrow L = 2S - \frac{(1.50 + 0.035S)}{N}$$

■■■

Traffic Engineering 3

Traffic engineering is that phase of engineering which deals with planning and geometric design of streets, highways, abutting lands, and with traffic operation thereon, as their use is related to the safe, convient and economic transportation of persons and goods.

Theoretical maximum capacity, (C)

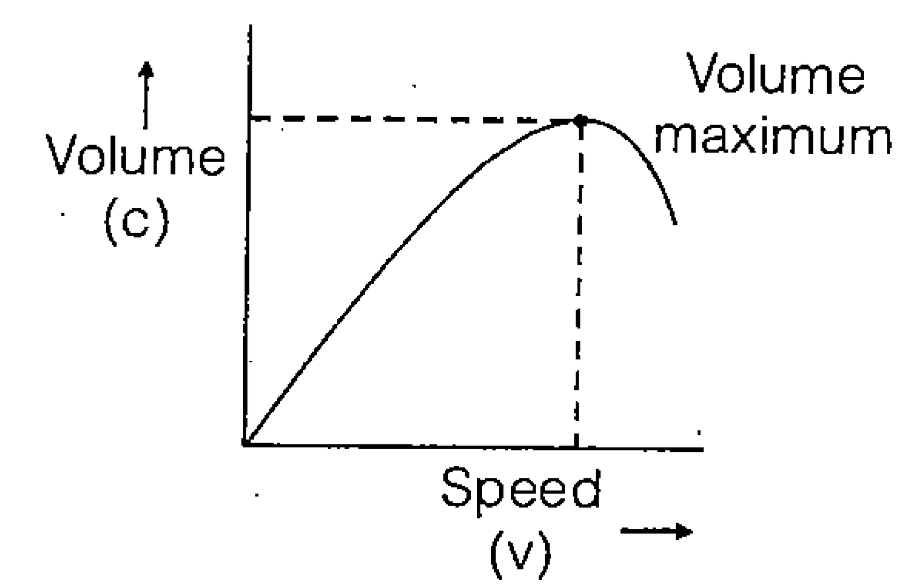
$$C = \frac{1000V}{S}$$ where, S = Minimum clear distance between two vehicles (m).

$$S = 0.2V + 6$$ where, v = Speed of vehicle in km/hr.

$$C = \frac{3600}{H_t}$$ C = Theoretical maximum capacity in vehicle/hour
H_t = Time headway in 'Sec'.

$$C = V \cdot \delta$$

where, C = Traffic capacity or traffic volume in vehicle/hour
δ = Traffic density in vehicle/km
V = Traffic velocity in km/hr.



Passenger Car Unit

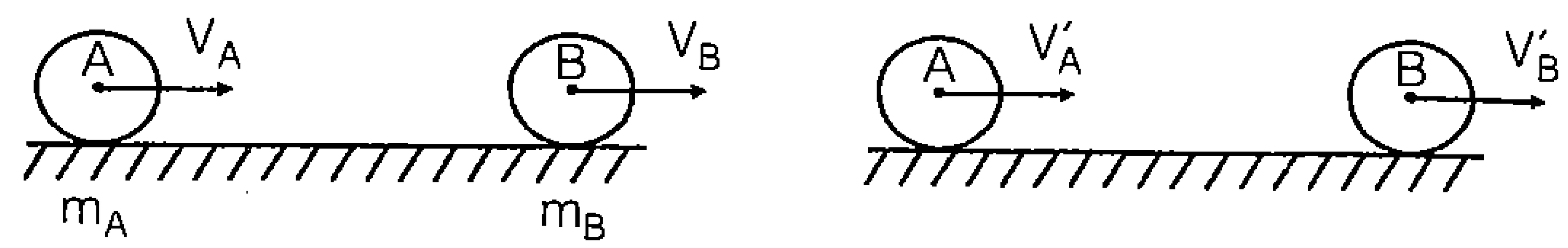
The PCU may be considered as a measure of the relative space requirement of a vehicle class compared to that of passenger car under a specified set of roadway, traffic and other conditions.

$$PCU = \frac{\text{Capacity of roadway with passenger car only}}{\text{Capacity of roadway with a particular class vehicle only}}$$

Accident Studies

The problem of accident is very acute in highway transportation due to complex flow patterns of vehicular traffic presence of mixed traffic and pedestrains. Traffic accidents may involve property damages, personal injuries or even casualties.

(i) $$e = \frac{V'_B - V'_A}{V_A - V_B}$$ where, e = Coefficient of restitution
$(V'_B - V'_A)$ = Velocity of separation
$(V_A - V_B)$ = Velocity of approach.



Before Collision ($V_A > V_B$)

After Collision ($V_B' > V_A'$)

V_A = Velocity of vehicle 'A' of mass m_A before collision.
V_B = Velocity of vehicle 'B' of mass m_B before collision.
V_A' = Velocity of vehicle A after collision
V_B' = Velocity of vehicle B after collision.

- $\boxed{e = 1}$ for perfectly elastic collision.
- $\boxed{e = 0}$ for perfectly inelastic collision or plastic collapse.
  i.e., both vehicle move with same velocity after collision.
- **Momentum Equation**

$$\boxed{m_A v_A + m_B v_B = m_A v_A' + m_B v_B'}$$

- **Types of Collision**
  (i) Collision of moving vehicle with parked vehicle (assumption : collision is perfectly plastic)

  (a) $\boxed{v_3 = \sqrt{2gfS_2}}$

  (b) $\boxed{v_2 = \dfrac{(m_A + m_B)}{m_A} \cdot v_3}$

  (c) $\boxed{v_1 = \sqrt{v_2^2 + 2gfS_1}}$

  where, v_1 = Initial velocity of moving vehicle in km/hr.
  v_2 = Velocity of moving vehicle after travelling distance 's_1' (in meter)
  v_3 = Common velocity of moving & parked vehicle at the time of collision.
  s_2 = Distance travelled by both vehicle till both vehicle comes in rest finally.

- **Two Vehicle Approaching from Right Angle Collide at an Intersection**

  (i) $\boxed{V_{A_3} = \sqrt{2gfS_{A_2}}}$

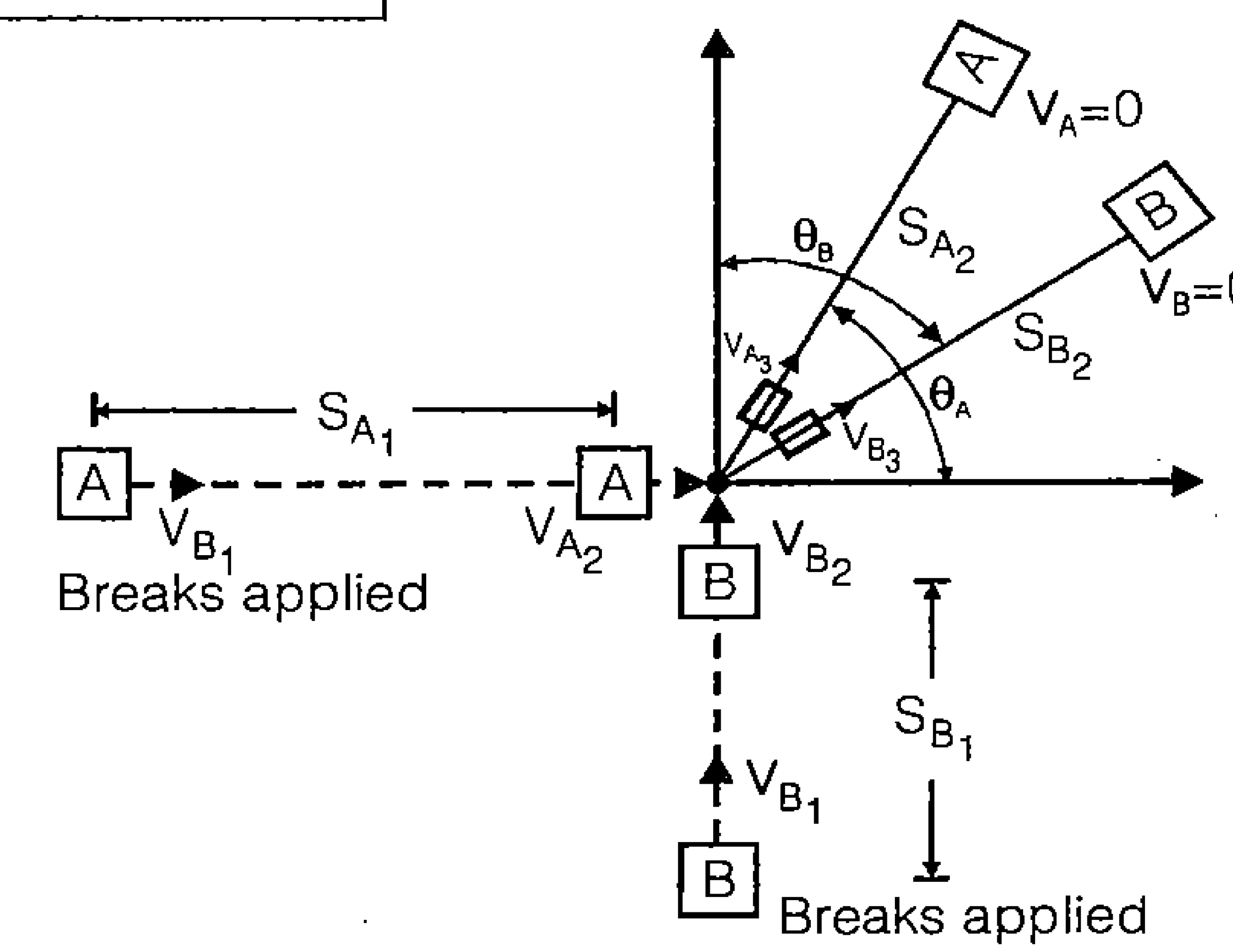
  (ii) $\boxed{V_{B_3} = \sqrt{2gfS_{B_2}}}$

(iii) $\boxed{V_{A_2} = V_{A_3}\cos\theta_A + \dfrac{m_B}{m_A} \cdot V_{B_3}\sin\theta_B}$

(iv) $\boxed{V_{B_2} = \dfrac{m_A}{m_B} \cdot V_{A_3}\sin\theta_A + V_{B_3}\cos\theta_B}$

(v) $\boxed{V_{A_1} = \sqrt{V_A^2 + 2gfS_{A_1}}}$

(vi) $\boxed{V_{B_1} = \sqrt{V_{B_2}^2 + 2gfS_{B_1}}}$



where,
S_{A_1} & S_{B_1} are skid distance just before collision.
S_{A_2} & S_{B_2} are skid distance after collision.
V_{A_3} & V_{B_3} are speed of vehicle A & B respectively after the collision.
V_{A_2} & V_{B_2} are speed of vehicle A & B respectively after skidding a distance S_{A_1} & S_{B_1}.
V_{A_1} & V_{B_1} are speed of vehicle A & B respectively before skidding.

Design of Signal

(i) Trial Cycle Method

$$\boxed{X_A = \frac{\eta_A}{15 \times 60} \times T} \quad \boxed{X_A = \frac{\eta_B}{15 \times 60} \times T}$$

where, X_A = Number of vehicle accumulated in one cycle time on Road A.
X_B = Number of vehicle accumulated in one cycle time on Road B.
T = Total cycle time in 'sec' (assumed)

$\boxed{G_A = 2.5X_A}$ η_A = Traffic count on road A in 15 minutes.

$\boxed{G_B = 2.5X_B}$ η_B = Traffic count on road B in 15 minutes.

$$\boxed{T' = (G_A + A_A) + (G_B + A_B)}$$

where, T' = Total cycle time (Actual)
A_A = Amber time on road A
A_B = Amber time on road B
G_A & G_B are green time on road A & B respectively.

if $\boxed{T' = T}$ **then O.K.** otherwise repeat the process.

(ii) Approximate Method

$$\boxed{R_A = G_{AP} = \left(7 + \frac{w_A}{1.2}\right)} \quad \boxed{R_B = G_{BP} = \left(7 + \frac{w_B}{1.2}\right)}$$

whey, R_A = Red time on road A
R_B = Red time on road B
G_{AP} = Green time on road A for Pedestrians
G_{BP} = Green time on road B for Pedestrians
w_A = Width of road A
w_B = Width of road B
1.2 m/s = Speed of pedestrians

$$\boxed{G_A = R_B - A_A} \quad \boxed{G_B = R_A - A_B}$$

where, G_A = Green time on road A
G_B = Green time on road B
A_A & A_B are Amber time on road A & B respectively.

(iii) Websters Method

$$\boxed{C_O = \frac{1.5L + 5}{1 - Y}}$$

where, C_O = Optical cycle time
L = Total lost time

$$\boxed{L = 2n + R}$$

where, n = number of phase
R = All red time

$$\boxed{y = y_1 + y_2} \rightarrow \boxed{y_2 = \frac{q_B}{s_B}}$$

$$\downarrow$$

$$\boxed{y_1 = \frac{q_A}{s_A}}$$

where, q_A = Normal flow on road A
q_B = Normal flow on road B
s_A = Saturation flow on road A
s_B = Saturation flow on road B

$$\boxed{G_A = \frac{y_1}{y}(C_O - L)} \quad \boxed{G_B = \frac{y_2}{y}(C_O - L)}$$

where, G_A & G_B are green time on road A & B respectively.

Annual Average Daily Traffic (AADT or ADT)

$$\boxed{AADT = \frac{\text{Total yearly traffic}}{365}}$$

Space Mean Speed (V_s)

$$\boxed{V_s = \frac{3.6 \cdot d \cdot n}{\sum_{i=1}^{n} t_i}}$$

where, V_s = Space mean speed in km/hr.
d = Length of road in meter
n = Number of individual vehicle observations
t_i = Observed travel time (sec) for i^{th} vehicle of travel distance 'd' meter.

Time mean speed (v_t)

$$\boxed{V_t = \frac{\sum_{i=1}^{n} V_i}{n}}$$

where, V_t = Time mean speed (km/hr)
V_i = Observed instantaneous speed of i^{th} vehicles (km/hr)
n = number of vehicles observed.

Speed & delay study by floating car method

Average journey time (t) in minute

$$\boxed{\bar{t} = t_w - \frac{n_y}{q}}, \quad \boxed{q = \frac{n_a + n_y}{t_a + t_w}}$$

where,
q = Flow of vehicles (volume per minutes) in one direction of the stream.
n_a = Average number of vehicles counted in the direction of the stream when the test vehicle travel in the opposite direction.
n_y = The average number of vehicles overtaking the test vehicle minus the number of vehicle overtaken when the test is in the direction of 'q'.
t_w = Average journey time when the test vehicle is travelling with the stream q.
t_a = Average journey time, in minute when the test vehicle is running against the stream 'q'.

Relationship between speed, travel time, volume, density & capacity

Travel time per unit length of road,

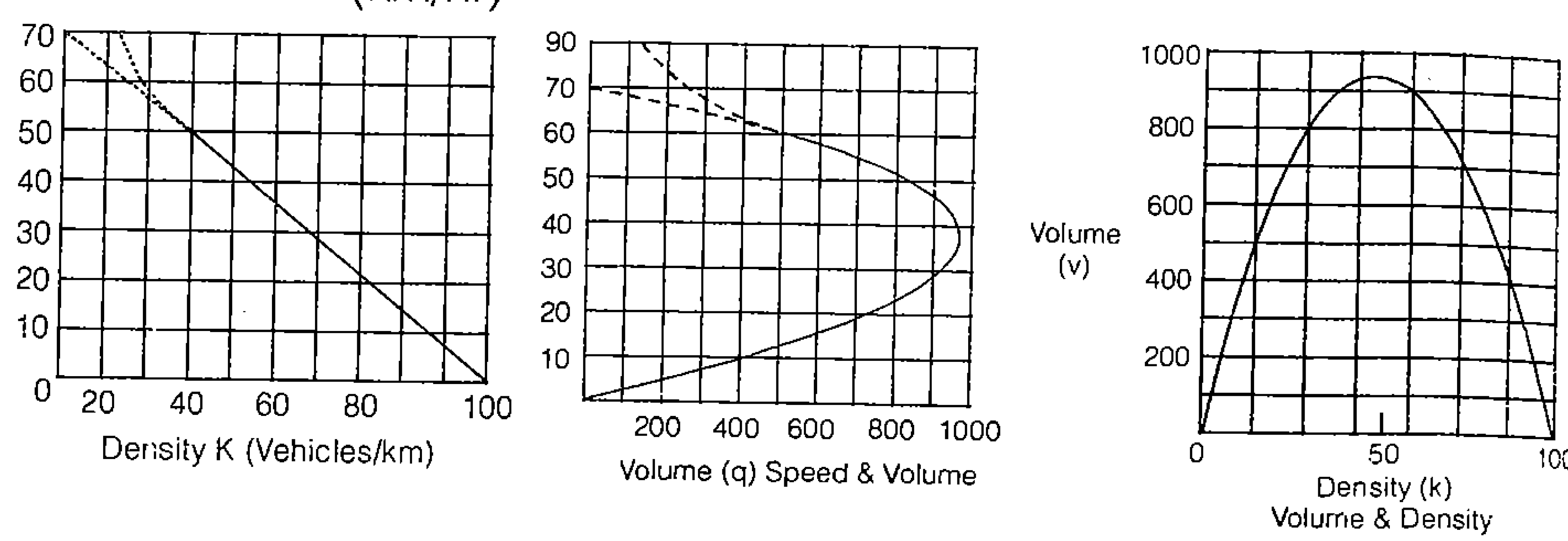
$$T_{(sec/km)} = \frac{3600}{V}$$ where, V = Speed in km/hr

$$q = kV_s$$

where, q = Average volume of vehicle passing a point during a specified period of time (vehicle per hour).

k = Average density or number of vehicle occupying a unit length of roadway at a given instant (vehicles/km).

V_s = Space-mean speed of vehicles in a unit roadway length (km/hr)



Capacity flow or maximum flow, (q_{max})

$$q_{max} = \frac{V_{SF} \cdot k_j}{4}$$

where, V_{SF} = Free mean speed i.e., maximum speed at zero density.

k_j = Jam density i.e., maximum density at zero speed.

$$k_j = \frac{1000}{S}$$ where, s = Spacing between vehicles.

Rotary Intersection

A rotary intersection or traffic rotary is an enlarged road intersection where all converging vehicles are forced to move round a large central island in one direction (clockwise direction) before they can weave out of traffic flow into their respective directions radiating from the central island.

(i) $V = 40\ km/hr$ → For Rural Areas.

$V = 30\ km/hr$ → For Urban Areas.

where, V = Design speed

(ii) Radius of rotary, (R)

$$R = \frac{V^2}{127f}$$

where, V = Design speed of vehicle (km/hr)

f = Coefficient of friction may be taken as 0.43 & 0.47 for the speed of 40 & 30 km/hr respectively after allowing a factor of safety of 1.5.

$$(R_{min})_{Central\ Island} = 1.33\,(R)_{Entry\ Curve}$$

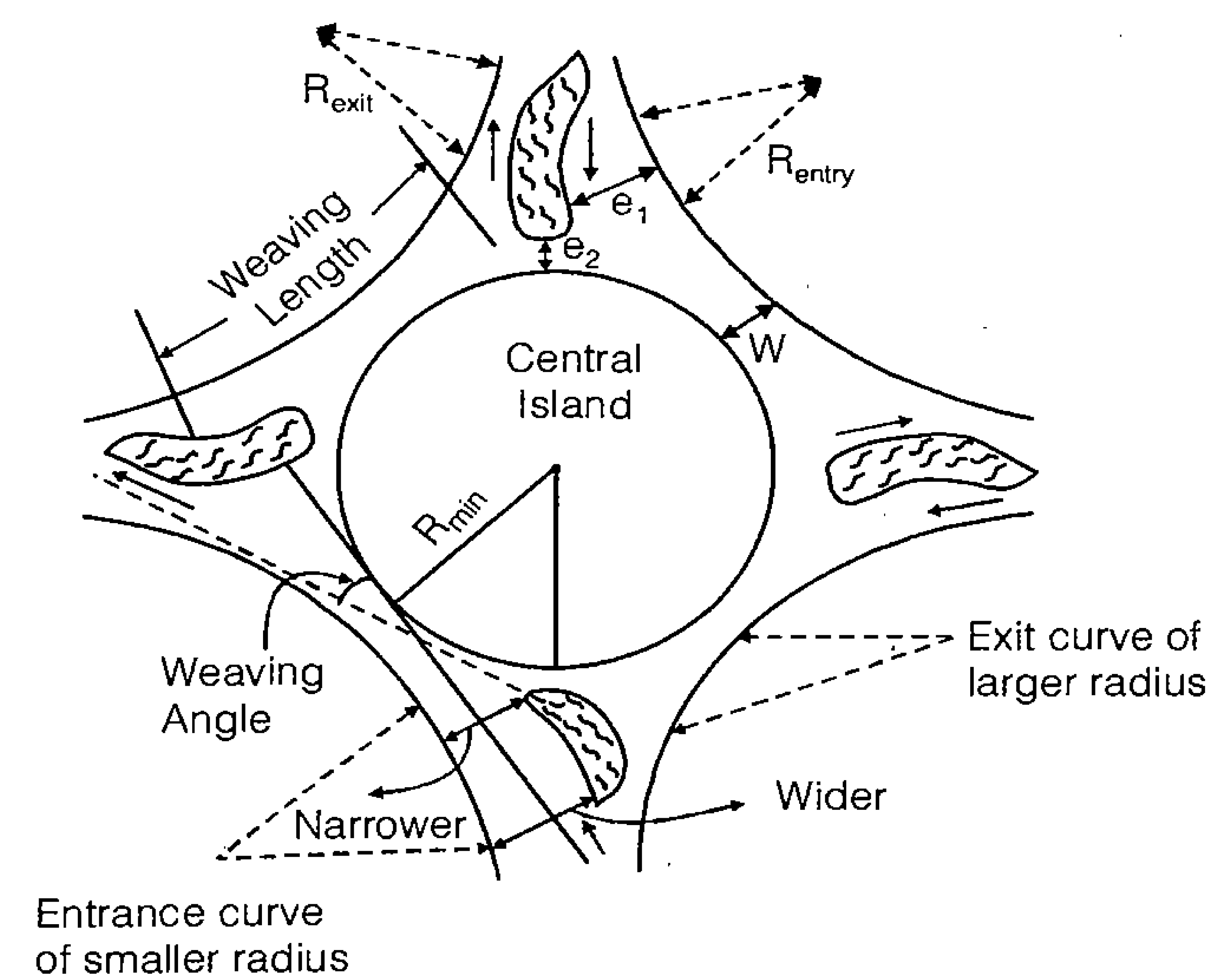
$(R_{min})_{Central\ Island}$ = Minimum radius of central Island.

$(R)_{Entry\ Curve}$ = Radius of entry curve.

$20\ m \le (R)_{entry\ curve} \le 35\ m$ for v = 40 km/hr.

$15\ m \le (R)_{Entry\ Curve} \le 25\ m$ for v = 30 km/hr.

(iii) $$w = \left[\left(\frac{e_1 + e_2}{2}\right) + 3.5\right] \text{metre}$$



where, w = Width of weaving section.

e_1 = Entry width,

e_2 = Width of non weaving section.

(iv) Capacity of Rotary (Q_P)

$$Q_P = \frac{280w\left(1+\frac{e}{w}\right)\left(1-\frac{p}{3}\right)}{\left(1+\frac{w}{L}\right)}$$

where, Q_P = Practical capacity of weaving section of a rotary in PCU per hour.

w = Width of weaving section (6 to 18 m).

e = Average width of entry 'e_1' & width of non-weaving section e_2 for the range e/w = 0.4 to 1.0 ~ $\frac{e_1+e_2}{2}$.

L = Length of weaving section between the ends of channelizing islands in meter for the range of $\frac{w}{L}$ = 0.12 to 0.4.

p = Proportion of weaving traffic or weaving ratio.

(v) $p = \frac{b+c}{a+b+c+d}$ $0.4 \le p \le 0.90$

where, a = Left turning traffic moving along left extreme lane.

d = right turning moving along right extreme lane.

b = Crossing/weaving traffic turning towards right while entering the rotary.

c = Crossing/weaving traffic turning towards left while leaving the rotary.

Parking Facilities

- Number of spaces (N)

(i) $N = \frac{L}{6.6}$ for parallel parking with equal spacing facing the same direction.

(ii) $N = \frac{L}{6.75}$ for parallel parking when two cars placed closely.

(iii) $N = \frac{L-0.85}{5.1}$ for 30° angle parking

(iv) $N = \frac{L-2.0}{3.6}$ for 45° angle parking

(v) $N = \frac{L-2}{2.9}$ for 60° angle parking.

(vi) $N = \frac{L}{2.5}$ for 90° angle parking.

- Out of various angles used in angle parking, 45 degree angle is considered the best from all considerations discussed above.

Highway Lighting

Spacing between lighting units =

$$\frac{\text{Lamp lumen} \times \text{Coefficient utilization} \times \text{Maintenance factor}}{\text{Average lux} \times \text{Width of road}}$$

Trip Distribution

$$T_{ij} = \frac{G_i A_j F_{ij}}{\sum_{j=l}^{n} A_j F_{ij}}$$

where, T_{ij} = Number of trips from zone i to zone j.

G_i = Trips generated in zone i.

A_j = Trips attracted to zone j.

F_{ij} = Empirically derived 'Friction Factor' calculated on area wise basis.

n = Number of zones in the urban area.

■■■

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n = Number of zones in the urban area.

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Highway Materials

4

Group Index of Soils (G.I)

In order to classify the fine grained soils within one group and for judging their suitability as subgrade material, an indexing system has been introduced in HRB classification which is termed as group index. Soils are thus assigned arbitrary numerical numbers known as group index (GI). Group index is function of percentage and is given by the equation

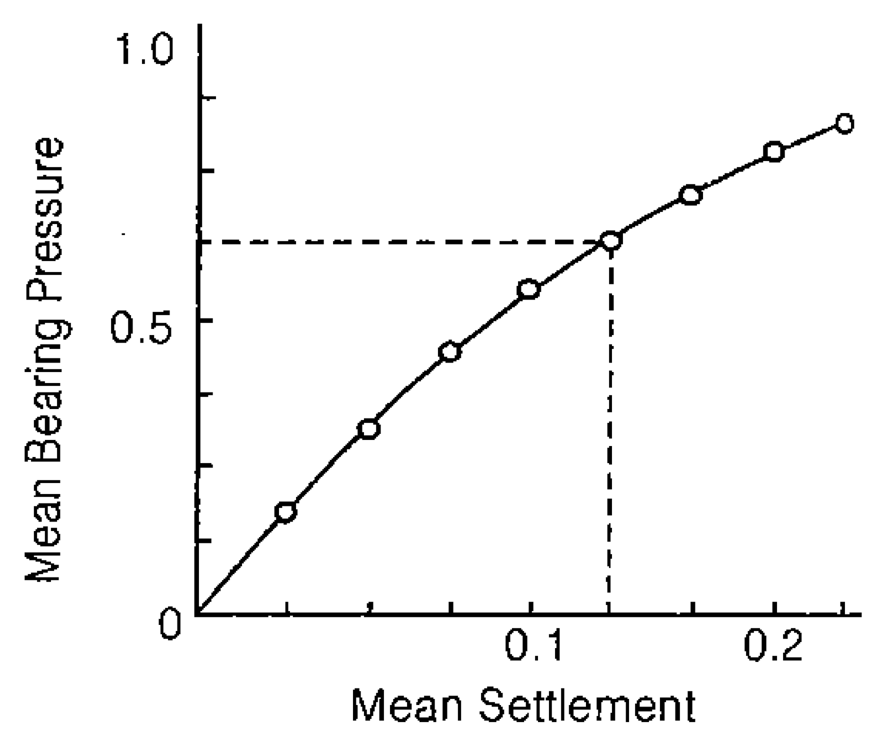
$$\boxed{G.I = 0.2\,a + 0.005\,ac + 0.01\,bd}$$

where, $a = (P - 35) \ngtr 40$
$b = (P - 15) \ngtr 40$
$c = (W_L - 40) \ngtr 20$
$d = (I_P - 10) \ngtr 20$

where, W_L = Liquid limit, I_P = Plasticity Index.
P = Percentage fines passing from 0.074 mm sieve.

$\boxed{0 \le G.I \le 20}$ Lower the group index → best quality.

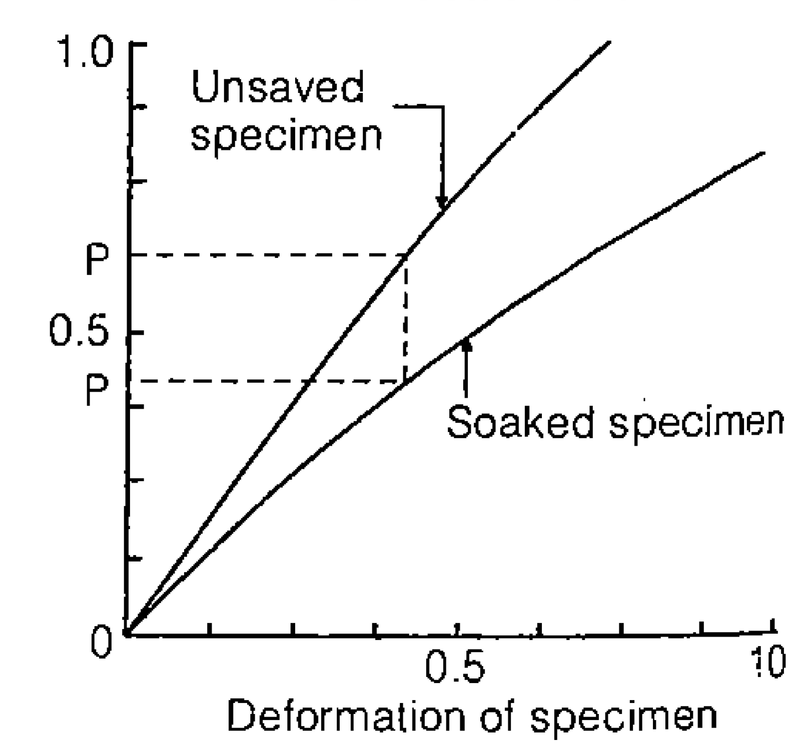
Plate Bearing Test



(i) $$\boxed{k = \frac{P}{0.125}\,kg/cm^3}$$

Here,
k = Modulus of subgrade reaction
P = Pressure corresponding to settlement of 0.125 cm.

(ii) $$\boxed{k_s = k \cdot \frac{P_s}{P}}$$



where,
k_s = Modulus of subgrade reaction for soaked condition.
P_s = Pressure required in the soaked condition to produce same deformation as deformation Produce by pressure 'P' in consolidated condition.
k = Modulus of subgrade reaction for consolidated stage.

(iii) $$\boxed{\Delta = \frac{1.18\,Pa}{E}}$$

$$\boxed{k = \frac{P}{\Delta} = \frac{E}{1.18\,a}}$$

$$\boxed{k \cdot a = \text{constant}}$$

$$\boxed{k = k_1 \cdot \frac{a_1}{a}}$$

where, Δ = Deformation in 'cm'.
a = Radius of rigid plate in 'cm'.
E = Modulus of elasticity of soil subgrade in kg/cm^2.
a_1 = Radius of smaller plate (other plate)
k_1 = Modulus of subgrade reaction of other plate of radius 'a_1' cm.

California Bearing Ratio (CBR) Test

$$CBR\% = \frac{\left[\begin{array}{c}\text{Load (or pressure) sustained by the}\\ \text{specimen at 2.5 or 5.0 mm penetration}\end{array}\right]}{\left[\begin{array}{c}\text{Load (or pressure) sustained by standard}\\ \text{aggregates at the corresponding penetration level}\end{array}\right]} \times 100$$

Test for Road Aggregate

	Machine	Base Course	Surface Course
(i) Aggregate Abrasion value	Los Angeles Abrason test	$\ngtr$ 50%	$\ngtr$ 30%
(ii) Aggregate crushing value	UTH	$\ngtr$ 45%	$\ngtr$ 30%
(iii) Aggregate Impact value	Impact Testing Machine	$\ngtr$ 40% (for Bitcemine machine) $\ngtr$ 35% (for WBM)	$\ngtr$ 30%

Indicator	Respective Test Results
1. Flakiness Index	$\ngtr$ 15%
2. Elongation	$\ngtr$ 15%
3. Angularity Number	0 - 11
4. Soundness index	$\ngtr$ 18% ($MgSO_4$), $\ngtr$ 12% (Na_2SO_4)
5. Water absorption value	$\ngtr$ 0.6%
6. Stipping value	$\ngtr$ 25%

(i) Aggregate crushing test

$$\text{Aggregate crushing value} = \frac{100w_2}{w_1}\%$$

where, w_1 = Weight of the test sample in 'gm'

w_2 = weight of the crushed material in 'gm' passed through 2.36 mm sieve.

(ii) Shape Tests

$$\text{Angularity number} = 67 - \frac{100w}{C \cdot Ga}$$

Where, Ga = specific gravity of aggregate

w = mass of mould containing aggregate

C = mass of mould containing water

(iii) Abrasion Test

(a) Los Angeles Abrasion Test

$$\text{Coefficient of hardness} = 20 - \frac{\text{Loss in weight in 'gm'}}{3}$$

Bituminous Material

1. Product of fracitonal distillation of Petroleum: Gasoline, Naptha, Kerosene, Lubricating oil and Residue - Petroleum Bitumen.
2. Cutback Bitumen : Reduced Viscosity Bitumen

	Types	Volatile Diluent Used
i.	RC - N	Gasoline/Naptha
ii.	MC - N	Kerosene
iii.	SC - N	High boiling point gas

N - Numeral [0, 1, 2, 3, 4, 5]

Show progressive thickening from 0 to 5

3. Specific Gravity :

Bituminous → 0.97 - 1.02

Tar → 1.1 - 1.5

Bituminous Mixes

(i) Determination of Specific Gravity

$$G_a = \frac{100}{\frac{W_1}{G_1} + \frac{W_2}{G_2} + \frac{W_3}{W_3} + \frac{W_4}{G_4}}$$

where, G_a = Average specific gravity of blended aggregate mix.

W_1, W_2, W_3, W_4 are % by weight of aggregate 1, 2, 3 & 4 respectively.

G_1, G_2, G_3 & G_4 are specific gravities of the aggregate 1, 2, 3 & 4 respectively.

(ii) Specific Gravity of Compacted Specimen

(a)

$$G_t = \frac{100}{\frac{(100 - W_b)}{G_a} + \frac{W_b}{G_b}}$$

where, G_t = Theoretical maximum specific gravity of the mix.

W_b = % by weight of bitumen.

G_b = Specific gravity of bitumen.

G_a = Average specific gravity of aggregates.

(b) Theoretical density γ_t, percent solids by volume

$$\gamma_t = \frac{100G}{G_t}$$

where, G = Actual specific gravity of test specimen

G_t = Theoretical maximum specific gravity.

(c) Voids in the Mineral Aggregate (VMA)

$$VMA = (V_v + V_b) = 100 - \frac{G}{W_a}$$

where, V_b = % of bitumen

W_a = Aggregate content percent by weight

V_v = % air voids in the specimen.

$$V_v = 100 - \gamma_t = \frac{100(G_t - G)}{G_t}$$

(d) % Voids Filled with Bitumen (VFB)

$$VFB = \frac{100V_b}{VMA}$$

Marshall Method Bituminous Mix Design

- Percentage Air Voids = $V_v = \frac{G_t - G_m}{G_m} \times 100$

where, G_m = Bulk density or mass density of the specimen

G_t = Theoretical specific gravity of mixture

$$G_t = \frac{100}{\frac{W_1}{G_1} + \frac{W_2}{G_2} + \frac{W_3}{G_3} + \frac{W_4}{G_4}}$$

where, W_1 = Percent by weight of coarse aggregate in total mix
W_2 = Percent by weight of fine aggregate in total mix
W_3 = Percent by weight of filler in total mix
W_4 = Percent by weight of bitumen in total mix
G_1 = Apparent specific gravity of coarse aggregate
G_2 = Apparent specific gravity of fine aggregate
G_3 = Apparent specific gravity of filter
G_4 = Specific gravity of bitumen

Percent Voids in Mineral Aggregate (VMA)

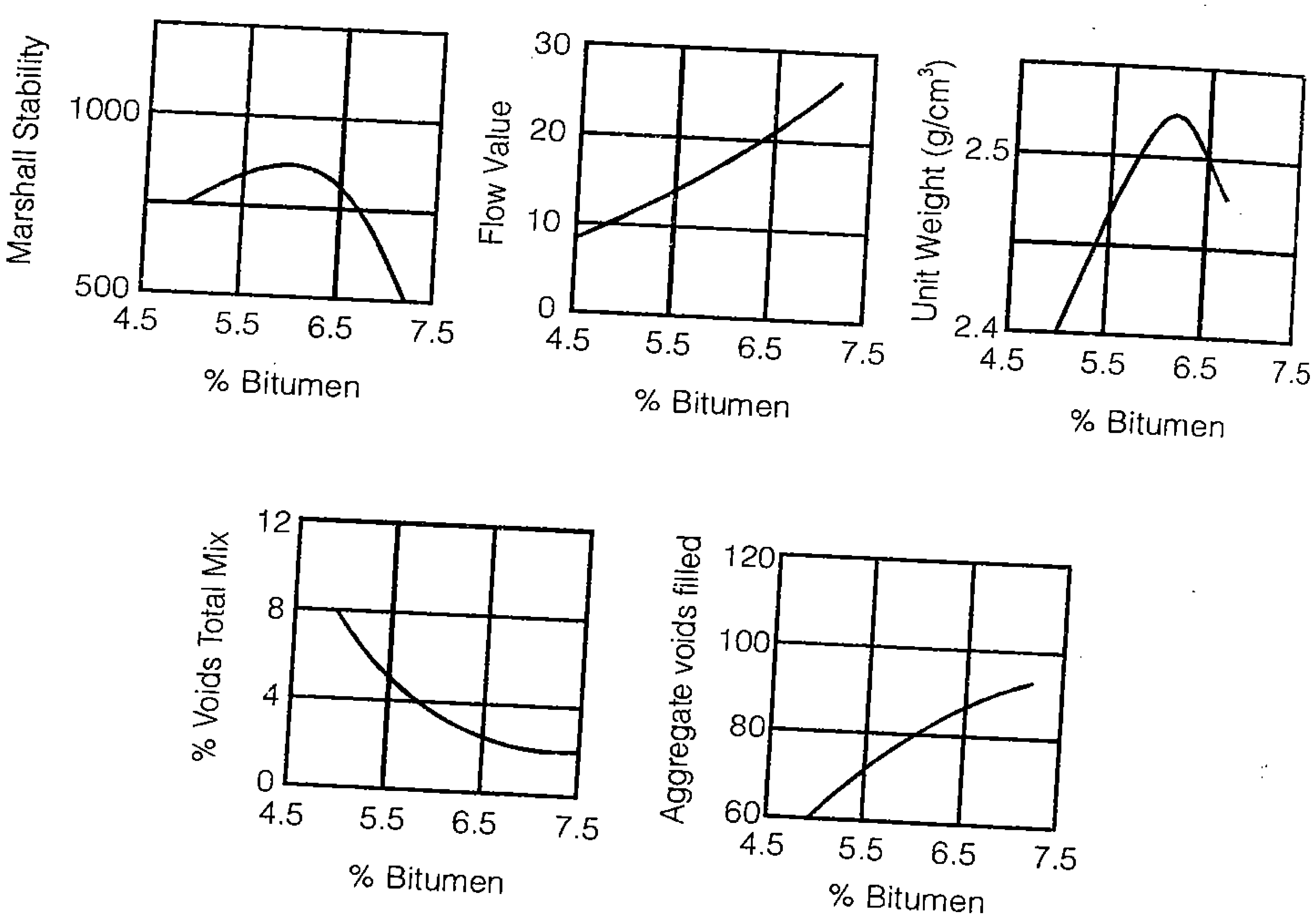
$$\boxed{VMA = V_v + V_b}$$

Here, V_v = Volume of air voids, %

$$V_b = \text{Volume of bitumen, \%} = G_m \cdot \frac{W_4}{G_4}$$

Percent Voids Filled with Bitumen (VFB)

$$\boxed{VFB = \frac{100V_b}{VMA}}$$



HveeM Method of Bituminous mix Design Stabilimeter Value, (s)

$$\boxed{s = \frac{22.2}{\frac{P_h - D_2}{P_v - P_h} + 0.22}}$$

where, P_v = Vertical pressure at 28 kg/cm^2 or at a total load of 2268 kg.
P_h = Horizontal pressure corresponding to P_v = 28 kg/cm^2.
D_2 = Displacement on specimen represented as number of turns of pump handle to raise P_h from 0.35 to 7 kg/cm^2.

Cohesiometer Value, (c)

$$\boxed{c = \frac{L}{w(0.2H + 0.0176H^2)}}$$

where, L = Weight of shots in gm.
w = Diameter or width of specimen in cm
H = Height of specimen in cm.

Stabilimeter Resistance R-value

$$\boxed{R = 100 - \frac{100}{\frac{2.5}{D_2}\left(\frac{P_v}{P_h} - 1\right) + 1}}$$

where, P_v = Vertical pressure applied (11.2 kg/cm^2)
P_h = Horizontal pressure transmitted at P_v = 11.2 kg/cm^2.
D_2 = Displacement of stabilimeter fluid necessary to increase the horizontal pressure from 0.35 to 7 kg/cm^2, measured in number of revolutions of the calibrated pump handle.

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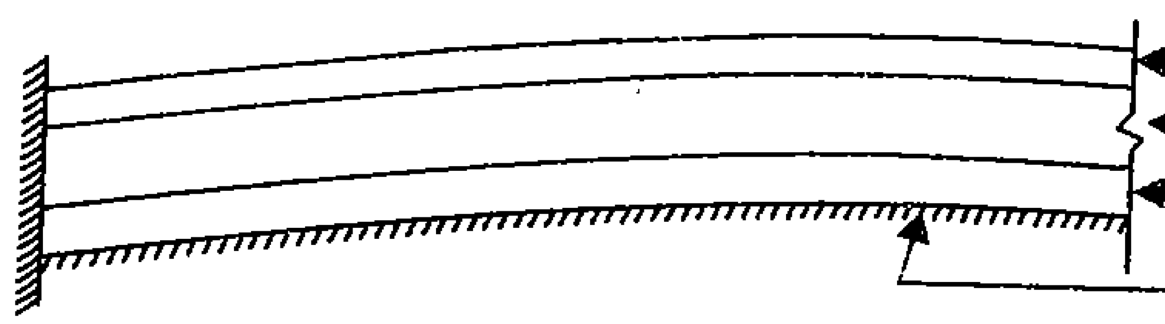
Pavement Design

5

Flexible Pavements

Flexible pavements are those, which on the whole have low or negligible flexural strength and are rather flexible in their structural action under the loads.

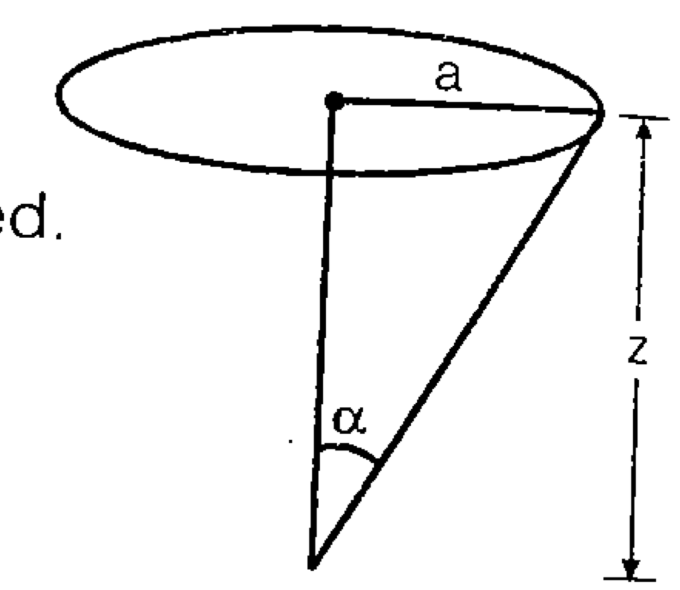
A typical flexible pavement consists of four components : 1. soil subgrade 2. sub-base course 3. base course 4. surface course.



(i) Stress Under Road Surface as per Boussineq's Equation,

$$\sigma_z = q\left[1-\cos^3\alpha\right], \text{ where } \cos\alpha = \frac{z}{\sqrt{a^2+z^2}}$$

where, σ_z = vertical stress at depth z.
q = surface pressure.
z = depth at which σ_z is computed.
a = radius of loaded area.



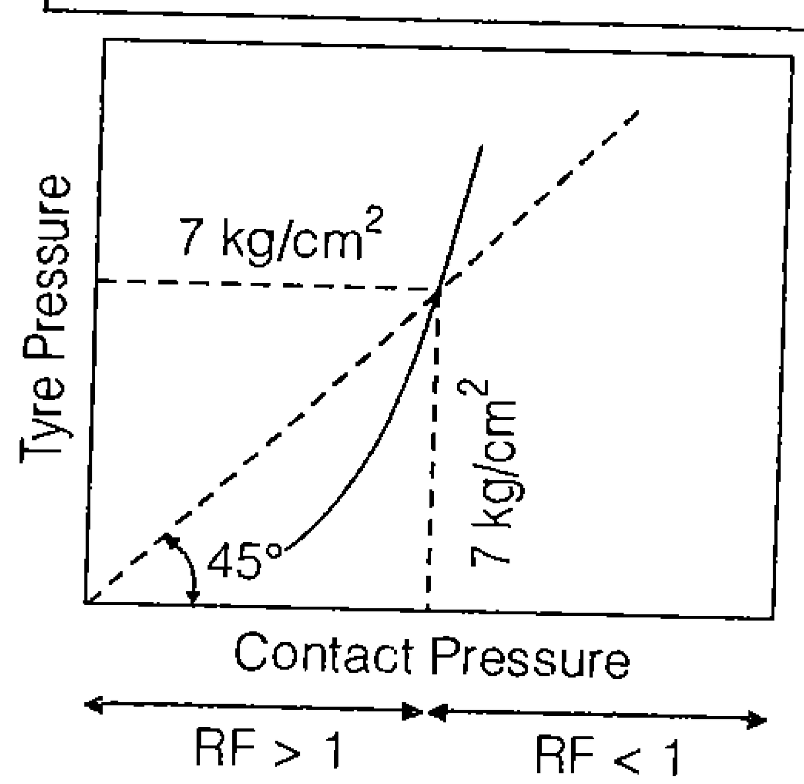
(ii) As Per IRC

Maximum legal axle load = 8170 kg

Equivalent single wheel load = 4085 kg.

(iii) $$\text{Contact pressure} = \frac{\text{Load on wheel}}{\text{Contact area or area of imprint}}$$

(iv) $$\text{Rigidity factor (R}\cdot\text{F)} = \frac{\text{Contact pressure}}{\text{Tyre pressure}}$$

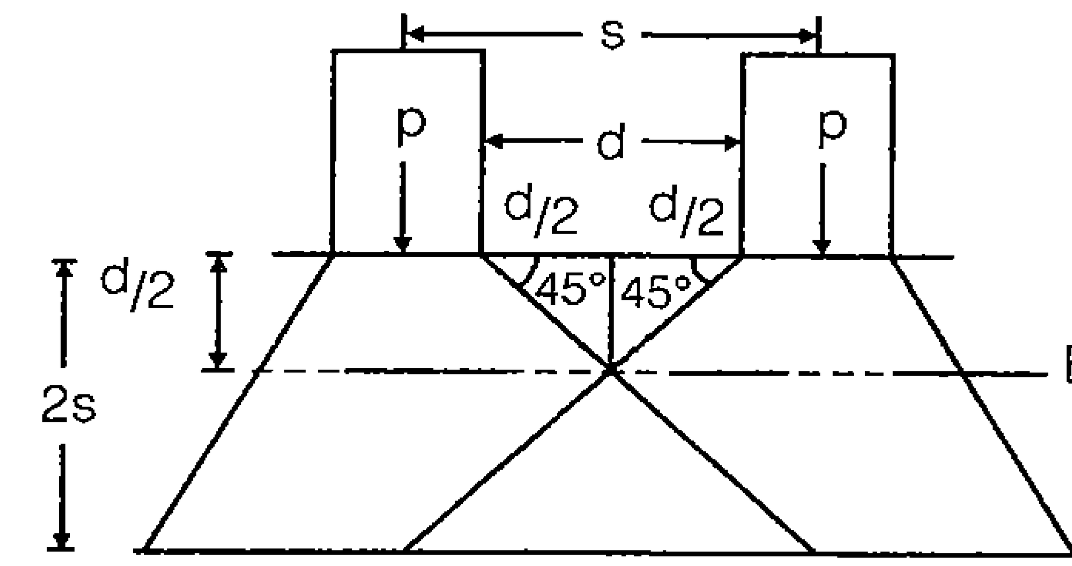


Remember

- Tyre pressure is important for upper layers.
- Contact pressure is important for deeper layers.

(v) Equivalent Single Wheel Load (ESWL)

$$\text{(ESWL) at } \frac{d}{2} \text{ depth} = P \qquad \text{(ESWL) at 2s depth} = 2P$$



Methods of Flexible Pavement Design

(i) Group Index Method

$$G\cdot I = 0.2a + 0.005ac + 0.01bd$$

(ii) C.B.R Method

(a) $$C\cdot B\cdot R \text{ values} = \frac{\text{Load on soil sample}}{\text{Standard load}} \times 100$$

Penetration	Standard load
2.5 mm	1370 kg
5.0 mm	2055 kg

(b) Thickness of Pavement, (T)

$$T = \sqrt{P}\left[\frac{1.75}{CBR} - \frac{1}{p\pi}\right]^{1/2} \quad \text{or} \quad T = \left[\frac{1.75P}{CBR} - \frac{A}{\pi}\right]^{1/2}$$

where, P = Wheel load in kg.
CBR = California bearing ratio in percent
p = Tyre pressure in kg/cm²
A = Area of contact in cm².

$$A = \pi a^2$$

a = Radius of contact area.

(c) Number of heavy vehicle per day for design (A),

$$A = P[1+r]^{(n+10)}$$

(i) Modulus of subgrade reaction (k),

$$k = \frac{P}{\Delta}$$

where, k = Modulus of subgrade reaction (kg/cm²/cm)
P = Pressure required for 'Δ' deflection (kg/cm²)
Δ = Deflection (cm).

For 75 cm dia plate, Δ = 0.125 cm.

(ii) Radius of Relative Stiffness (l)

$$l = \left\{\frac{Eh^3}{12k(1-\mu^2)}\right\}^{1/4}$$

where, l = Radius of relative stiffness, cm
E = Modulus of elasticity of cement concrete (kg/cm²)
μ = Poisson's ratio for concrete = 0.15
h = Slab thickness (cm)
k = Subgrade modulus or modulus of subgrade reaction (kg/cm³)

(iii) Equivalent Radius of Resisting Section (b)

(a) $$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

when a < 1.724 h

(b) $b = a$ when a > 1.724 h

where, a = Radius of contact area (cm)
h = Slab thickness (cm)

(iv) Glodbeck's Formula for Stress due to Corner Load

$$S_C = \frac{3P}{h^2}$$

where, S_C = Stress due to corner load (kg/cm²)
P = Corner load assumed as a concentrated point load, (kg)
h = Thickness of slab (cm).

(v) Westergards Stress Equation

(a) Stress at Interior Loading (s_i)

$$s_i = \frac{0.316P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 1.069\right]$$

(b) Stress at Edge Loading (s_e)

$$s_e = \frac{0.572P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 0.359\right]$$

(c) Stress at Corner Loading (s_c)

$$s_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

where, h = Slab thickness (cm)
P = Wheel load (kg)
a = Radius of contact area (cm)
l = Radius of relative stiffness (cm)
b = Radius at resisting section (cm).

(vi) Warping Stresses

(a) Stress at Interior Region (S_{t_i})

$$S_{t_i} = \frac{E\alpha T}{2}\left[\frac{C_x + \mu C_y}{1-\mu^2}\right]$$

where, S_{t_i} = Warping stress at interior region (kg/cm²)
E = Modulus of elasticity of concrete, (kg/cm²)
α = Coefficient of thermal expansion(/°c)
C_x = Coefficient based on $\left(\frac{L_x}{l}\right)$ in desired direction.
C_y = Coefficient based on $\left(\frac{L_y}{l}\right)$ in right angle to the above direction.
μ = Poissons's ratio ~ 0.15.

L_x/l or L_y/l	C_x or C_y
4	0.6
8	1.1
12	1.02

L_x & L_y are the dimensions of the slab considering along x & y directions along the length & width of slab.

(b) Stress at Edge Region (s_{t_e})

$$S_{t_e} = \text{maximum}\begin{cases}\frac{E\alpha T}{2}\cdot C_x \\ \frac{E\alpha T}{2}\cdot C_y\end{cases}$$

(c) Stress at Corner Region (S_{t_c})

$$S_{t_c} = \frac{E\alpha T}{3(1-\mu)}\sqrt{\frac{a}{l}}$$

where, a = Radius of contact area

l = Radius of relative stiffness

(vii) Frictional Stress (s_f) $S_f = \frac{WLf}{2 \times 10^4}$

where, S_F = Frictional stress (kg/cm²)

W = Unit weight of concrete, (kg/m³)

f = Friction constant or coefficient of subgrade restraint

L = Slab length (m)

B = Slab width (m)

(viii) Combination of Stresses

A. Critical Combination During Summer

(a) Stress for edge/interior regions at Bottom = (+ load stress) + (warping stress of day time) – Frictional stress

(b) Stress for corner region at top = (+ load stress + warping stress at night)

B. Critical Combination During Winter

(a) Stress for edge/interior at bottom = (+ load stress + warping stress at day time + frictional stress)

(b) Stress for Corner at Top = (load stress + warping stress at night)

Design of Joints in Cement Concrete Pavements

(i) Spacing of expansion joints, (L_e)

$$L_e = \frac{\delta'}{100\,\alpha\,(T_2 - T_1)}$$

where, δ' = Maximum expansion in slab (cm)

L_e = Spacing of expansion joint (m)

α = Coefficient of thermal expansion of concrete (/°c)

(ii) Spacing of contraction joint, (L_c)

(a) When reinforcement is not provided

$$L_C = \frac{(2 \times 10^4)s_c}{w.f}$$

where, L_c = Spacing of contraction joint (m)

s_c = Allowable stress in tension in cement concrete.

f = Coefficient friction ~ 1.5

w = Unit weight of cement concrete (kg/m³).

(b) When reinforcement is provided

$$L_c = \frac{200 S_s A_s}{bhwf}$$

where, S_s = Allowable tensile stress in steel (kg/cm²) $\simeq$ 1400 kg/cm².

A_s = Total area of steel in cm².

(iii) Longitudinal Joints

(a) $$A_s = \frac{bfhw}{100\,S_s}$$

where, As = Area of steel required per meter length of joint (cm²)

b = Distance between the joint & nearest free edge (m)

h = Thickness of the pavement (cm)

f = Coeff. of friction $\simeq$ 1.5

w = Unit wt. of concrete (kg/cm³)

S_s = Allowable working stress in tension for steel (kg/cm²)

(b) $$L_t = \frac{d}{2} \cdot \frac{S_s}{S_b}$$

where, L_t = Length of tie bar

S_s = Allowable stress in tension (kg/cm²) $\simeq$ 1400

S_b = Allowable bond stress in concrete (kg/cm²)

= 24.6 kg/cm² for deformed bars

= 17.5 kg/cm² for plain tie bars

d = diameter of tie bar (cm).

IRC recommendations for design of cement concrete pavements

$$A_d = P'[1 + r]^{(n+20)}$$

where, A_d = Number of commercial vehicles per day (laden weight > 3 tonnes)

P′ = Number of commercial vehicles per day at last count.

r = Annual rate of increase in traffic intensity.

n = Number of years between the last traffic count & the commissioning of new cement concrete pavement.

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Highway Maintenance

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Flexible Overlay Over Flexible Pavement by Conventional Design Method

$$h_0 = h_d - h_e$$

where, h_o = Overlay thickness required (cm)
h_d = Total design thickness required (cm)
h_e = Total thickness of the existing pavement (cm).

Analysis of Data

$$\bar{D} = \frac{\Sigma D}{n} \text{mm}$$

where, $\bar{D}$ = Mean value of deflections at 'n' points.
$\Sigma D = D_1 + D_2 + \ldots$
where, $D_1, D_2 \ldots$ are rebound deflection values.

Standard Deviation of Deflection (σ)

$$\sigma = \sqrt{\frac{\Sigma(\bar{D} - D)^2}{(n-1)}}$$

Characteristics Deflection (D_C),

$$D_C = D + t \cdot \sigma$$

Overlay Thickness Design

$$h_0 = \frac{R}{0.434} \log_{10} \frac{D_c}{D_a}$$ Ruiz's Equation

where, h_o = Thickness of bituminous overlay in cm.
R = Deflection reduction factor depending on the overlay material.

As Per I.R.C $$h_o = 550 \log_{10} \frac{D_c}{D_a}$$

where, h_o = Thickness of granular or WBM overlay in mm.
D_c = $(\bar{D} + 6)$, after applying the corrections for pavement temperature & subgrade moisture.
D_a = 1.00, 1.25 & 1.50 mm if the projected design traffic A is 1500 to 4500, 450 to 1500 & 150 to 450 respectively.

Rigid Overlay Over Rigid Pavement

Overlay thickness, (h_o) $$h_o = \left(h_d^n - xh_e^b\right)^n$$

Here, h_o = Rigid overlay thickness
h_d = Design thickness
h_e = Existing pavement thickness values of a, b, x & n depend upon the pavement & the method of overlay construction.

Flexible Overlay over Rigid Pavement

$$h_f = 2.5(Fh_d - h_e)$$

where, h_f = Flexible overlay thickness
h_e = Existing rigid pavement thickness
h_d = Design thickness of rigid pavement
f = Factor which depend upon modulus of existing pavement.

$$h_b = 1.66(Fh_d - h_e)$$

where, h_b = Thickness of bituminous overlay.

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A Handbook on
Civil Engineering

11

Surveying

CONTENTS

1 Fundamental Concepts of Surveying

Introduction

Surveying is the art of determining the relative positions of points on, above or beneath the surface of earth by means of direct or indirect measurements of distance, directions & elevation.

Plane Surveying

Plane surveying is that type of surveying in which the mean surface of the earth is considered as a plane and the spheroidal shape is neglected.

Geodetic Surveying

It is that type of surveying in which the shape of the earth is taken into account.

Scales

Scale is the fixed ratio that every distance on the plan bears with corresponding distance on the ground.

Representative Fraction (RF)

$$RF = \frac{\text{Map distance}}{\text{Ground distance}}$$

(a) **Plain scale:** Plain scale is one on which it is possible to measure two dimensions only such as units and lengths and diameters, miles and furlongs etc.

(b) **Diagonal scale:** On a diagonal scale it is possible to measure three dimensions such as meters, decimeters and centimeters; units, tenth and hundreds; yards, feet and inches.

(c) **The Vernier:**

(i) **Direct vernier:** It is so constructed that (n – 1) divisions of the main scale is equal to n division of the vernier.
In direct vernier, vernier scale moves in same direction of main scale.

$$\text{Least count} = \frac{s}{n}$$

where s = value of one smallest division of main scale
n = number of division on the vernier
v = value of one smallest division of vernier

also $\boxed{nv = (n-1)s}$

(ii) **Retrograde vernier:** It is so constructed that (n + 1) division of main scale is equal to n division of vernier. In retrograde vernier, vernier scale moves in opposite direction of main scale.

$\boxed{\text{Least count} = \frac{s}{n}}$ also $\boxed{nv = (n+1)s}$

(d) Shrunk scale

$\boxed{\text{Shrunk scale} = \text{original scale} \times \text{shrinkage factor}}$

$$\text{Shrinkage factor} = \frac{\text{Shrunk length}}{\text{Actual length}}$$

Type or purpose of syrvey	Scale	R.F.
(a) Topographic survey		
1. Building sites	1 cm = 10 m or less	$\frac{1}{1000}$ or less
2. Town planning schemes, reservoirs etc.	1 cm = 50 m to 100 m	$\frac{1}{5000}$ to $\frac{1}{10000}$
3. Location surveys	1 cm = 50 m to 200 m	$\frac{1}{5000}$ to $\frac{1}{20000}$
4. Small scale topographic maps	1 cm = 0.25 km to 2.5 km	$\frac{1}{25000}$ to $\frac{1}{250000}$
(b) Cadastral maps	1 cm = 5 m to 0.5 km	$\frac{1}{500}$ to $\frac{1}{5000}$
(c) Geographical maps	1 cm = 5 km to 160 km	$\frac{1}{500000}$ to $\frac{1}{16000000}$
(d) Longitudinal sections		
1. Horizontal scale	1 cm = 10 m to 200 m	$\frac{1}{1000}$ to $\frac{1}{20000}$
2. Vertical scale	1 cm = 1m to 2m	$\frac{1}{100}$ to $\frac{1}{200}$
(e) Cross-secions		
(Both horizontal and vertical scales equal)	1 cm = 1 m to 2 m	$\frac{1}{100}$ to $\frac{1}{200}$

Error Due to Use of Wrong Scale

(a) $$\text{Correct length} = \frac{\text{RF of wrong scale}}{\text{RF of correct scale}} \times \text{Measured length}$$

(b) $$\text{Correct Area} = \left(\frac{\text{RF of wrong scale}}{\text{RF of correct scale}}\right)^2 \times \text{Calculated Area}$$

Error Due to Incorrect Length of Chain or Tape

(a) True length of the line, (l)

$$\boxed{l = \left(\frac{L'}{L}\right) \times l'}$$

Here, L = Designated length of tape/chain
L′ = Actual but wrong length of the chain or tape
l' = Wrong measured length of the line
l = Actual true length of the line

Case (a): In case of Area

$$\boxed{A = \left(\frac{L'}{L}\right)^2 \cdot A'}$$

where, A = True area
A′ = Wrong measured area

Case (b): In case of volume

$$\boxed{V = \left(\frac{L'}{L}\right)^3 \cdot V'}$$

where, V = True volume
V′ = Wrong measured volume

Most Probable Value

$$\boxed{E_s = \pm 0.6745\sqrt{\frac{\Sigma V^2}{n-1}}} \quad \boxed{E_m = \pm 0.6745\sqrt{\frac{\Sigma V^2}{n(n-1)}} = \frac{E_s}{\sqrt{n}}}$$

where E_s = Probable error of single observation
V = Difference between any single observation and the mean of the series
E_m = Probable error of the mean
n = Number of observation in the series

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Linear Measurements

2

Tape Corrections

(a) Correction due to standardization or (correction due to absolute length)

$$C_a = \left(\frac{C}{l}\right)L$$

where, C_a = Correction for absolute length
L = Measured length of the line
l = Designated length of the tape
C = Correction per tape length

(b) Correction due to temperature

$$C_T = L\alpha\left(T_m - T_0\right)$$

where, T_m = Temp. at the time of measurement
T_0 = Temp. at the time of standardization of the tape
α = Coeff. of thermal expansion
L = Measured length

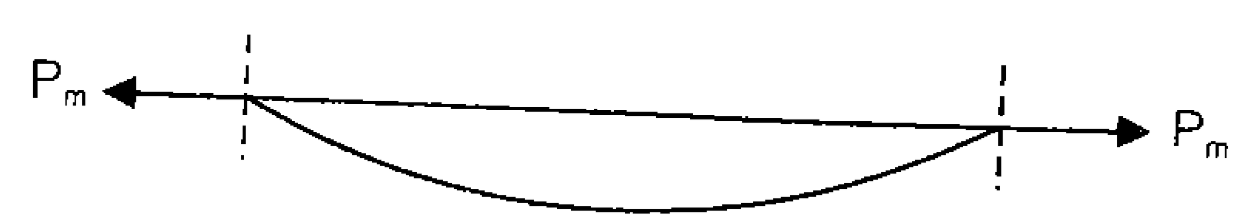
Remember

(i) If $T_m > T_0$ — Length of tape → Increases
⇒ Error → Negative
⇒ Correction → Positive

(ii) If $T_m < T_0$ — Length of tape → decreases
⇒ Error → Positive
⇒ Correction → Negative

(c) Correction for pull or tension

$$C_p = \frac{(P_m - P_0)L}{AE}$$



where, C_p = Correction for pull
P_m = Pull applied at the time of measurement
P_0 = Pull applied at the time of standardization
L = Measured length
A = Cross-sectional area
E = Young's modulus of tape

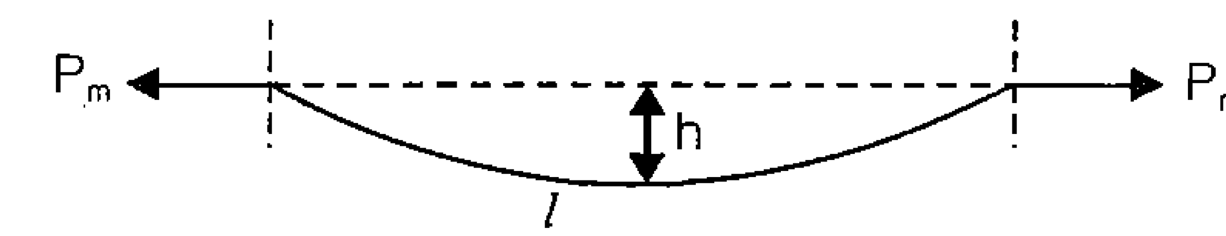
Remember

If $P_m > P_0$ — Length of tape → Increases
⇒ Error → Negative
⇒ Correction → Positive

If $P_m < P_0$ — Length of tape → decreases
⇒ Error → Positive
⇒ Correction → Negative

(d) Sag Correction

$$C_{sag} = \frac{W^2 l}{24P_m^2}$$ (Subtractive)



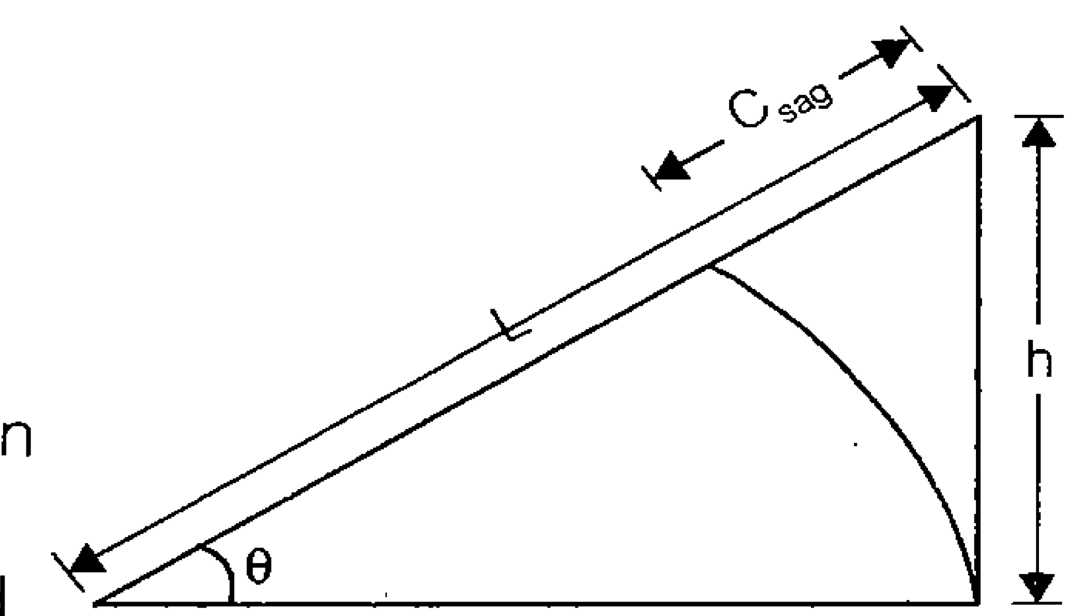
where, P_m = Pull applied, W = Total weight of tape = wl
w = Weight per meter length, l = Length of tape

- **Case: Normal Tension**
At a particular value of pull (where $P_m > P_0$) pull correction and sag correction neutralize each other. This value is called normal tension.

$$\frac{(P_m - P_0)l}{AE} = \frac{W^2 l}{24P_m^2} \rightarrow P_m^2\left(P_m - P_0\right) = \frac{W^2 AE}{24}$$

(e) Correction due to slope

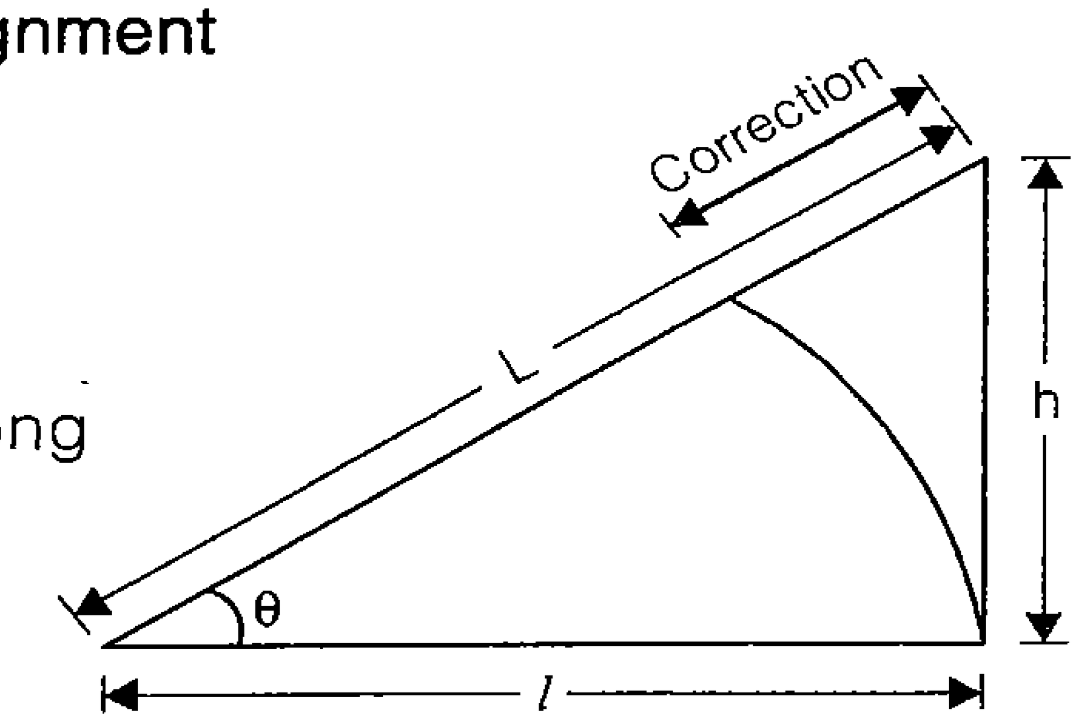
$$C_s = \frac{h^2}{2L}$$ (Subtractive)



where,
h = difference in elevation between the ends
L = Inclined length measured
l = Horizontal length
C_s = Correction due to sag

(f) Correction due to wrong alignment

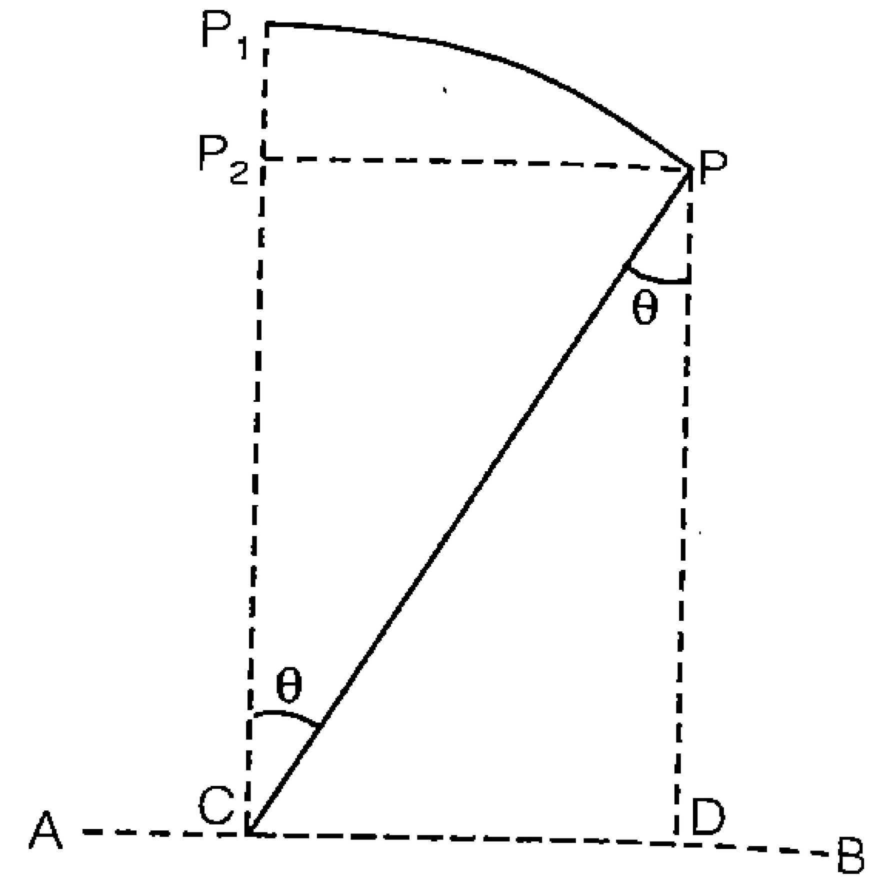
$$C_{al} = \frac{h^2}{2L}$$ (Subtractive)



where,
L = Length measured along wrong alignment
l = Correct length
h = Error in alignment

Limiting Length of Offset

(a) Effect of error in laying out direction only

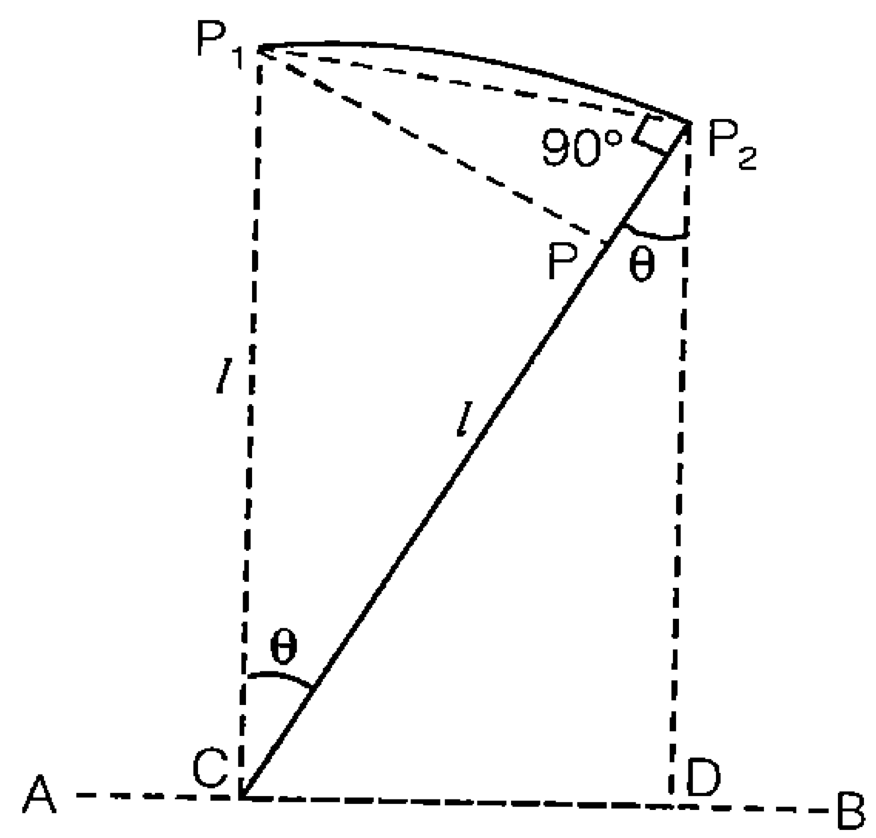


$$l = \frac{S}{40 \sin\theta}$$

where, l = Limiting length of offset
S = Scale (1 cm = S meter)

Remember: Maximum allowed length of error on drawing = 0.025 cm.

(b) Combined error in length and direction



$$l = \frac{1}{\sin\theta}\sqrt{\left(\frac{S}{40}\right)^2 - x^2}$$

where,
x = error in length measurement.
S = Scale (1 cm = S meter)

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3 Compass Surveying and Theodolites

Units of Angle Measurement Systems

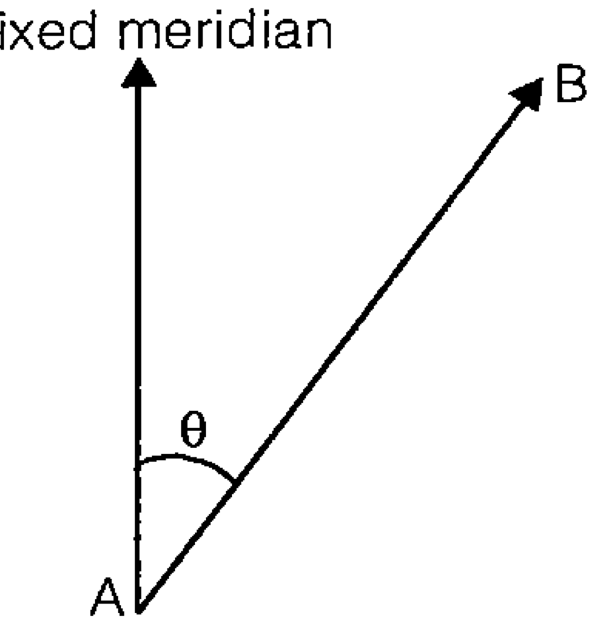
(i) **Sexagesimal system**
1 circumference = 360° degree
1 degree = 60 minutes
1 minute = 60 seconds

(ii) **Centesimal system**
1 circumference = 400 grades
1 grade = 100 centigrades
1 centigrade = 100 centi-centigrades

(iii) **Hour system**
1 circumference = 24 hour
1 hour = 60 minute
1 minute = 60 seconds

Important Terms

(i) **Bearing :** Direction of a line w.r.t a fixed meridian is called bearing.

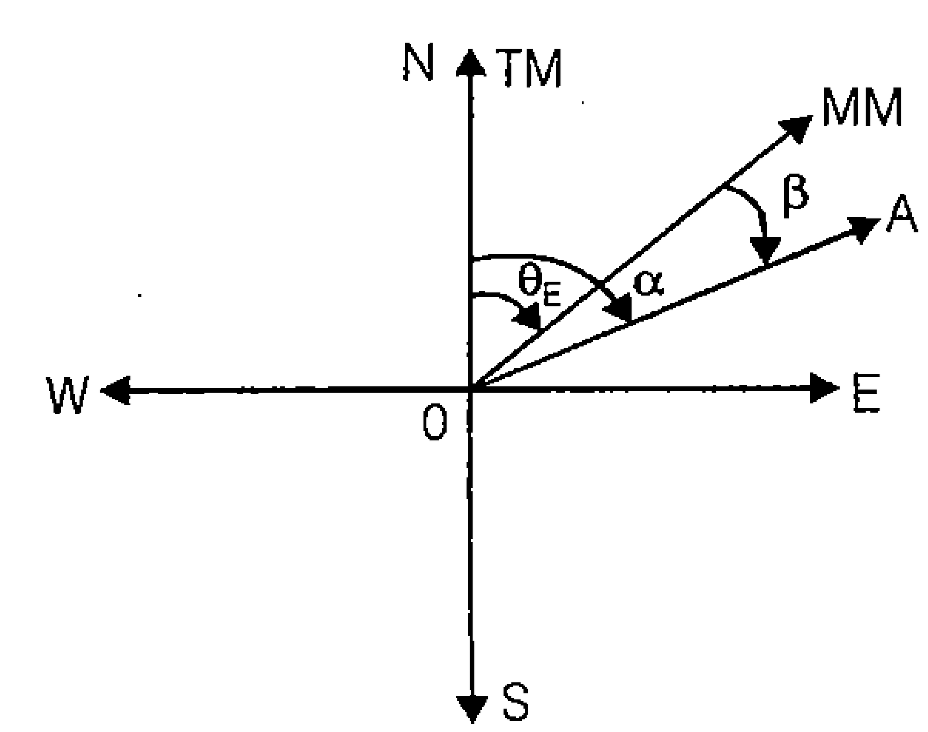


(ii) **True meridian and true bearing:** True meridian is a line joining true north pole, true south pole and point of reference.

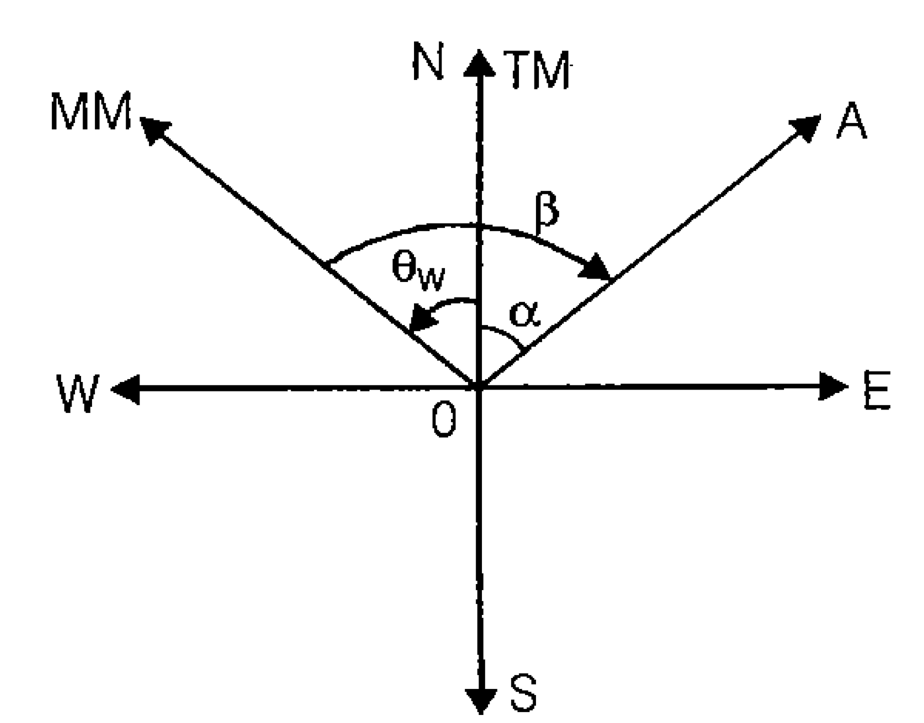
Angle measured for any line w.r.t true meridian is called true bearing.

(iii) **Magnetic meridian and magnetic bearing:** Line joining magnetic north pole, magnetic south pole and point of reference is called magnetic meridian.

Bearing taken w.r.t magnetic meridian is called magnetic bearing.



Eastern Declination



Western Declination

(iv) **Magnetic declination:** At any place horizontal angle between true meridian and magnetic meridian is called magnetic declination

For Eastern declination

$\boxed{\alpha = \beta + \theta_E}$ or $\boxed{T.B = M.B + \theta_E}$

Here, α = true bearing or T.B

β = magnetic bearing or M.B

θ_E = eastern declination

For western declination

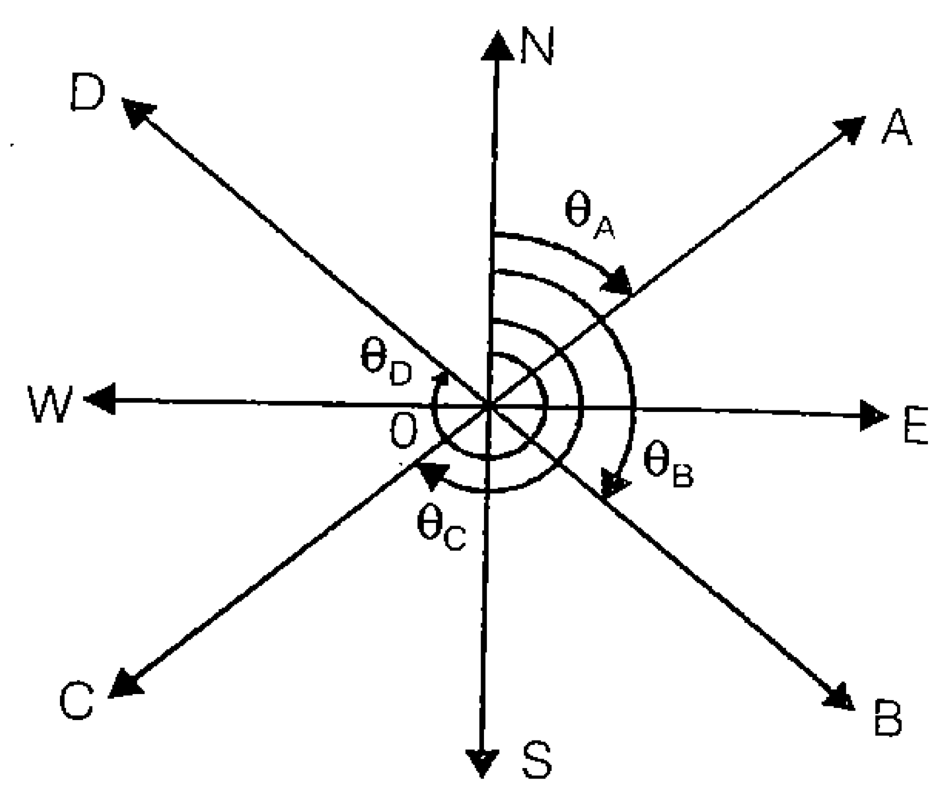
$\boxed{\alpha = \beta - \theta_W}$ or $\boxed{T.B = M.B - \theta_W}$

Here, θ_W = western declination.

Designation of Bearing

(i) **WCB (whole circle bearing system):**

All bearings are taken wrt north direction, going in clockwise.



Bearing of line OA = θ_A

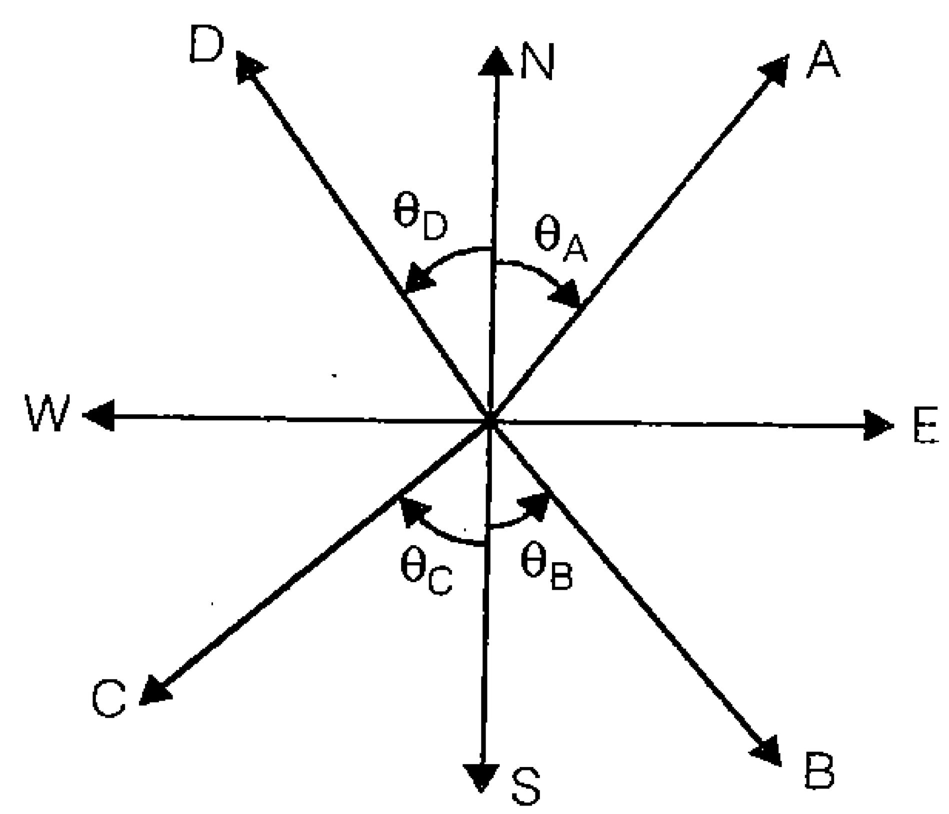
Bearing of line OB = θ_B

Bearing of line OC = θ_C

Bearing of line OD = θ_D

- These bearing are observed by Prismatic compass.

(ii) **QSB (quadrantal system of bearing)**



- In this system, the bearing of a line is measured eastward or westward from north or south pole whichever is nearer.
- These bearings are observed by surveyor compass.
- It is also called reduced bearing system.

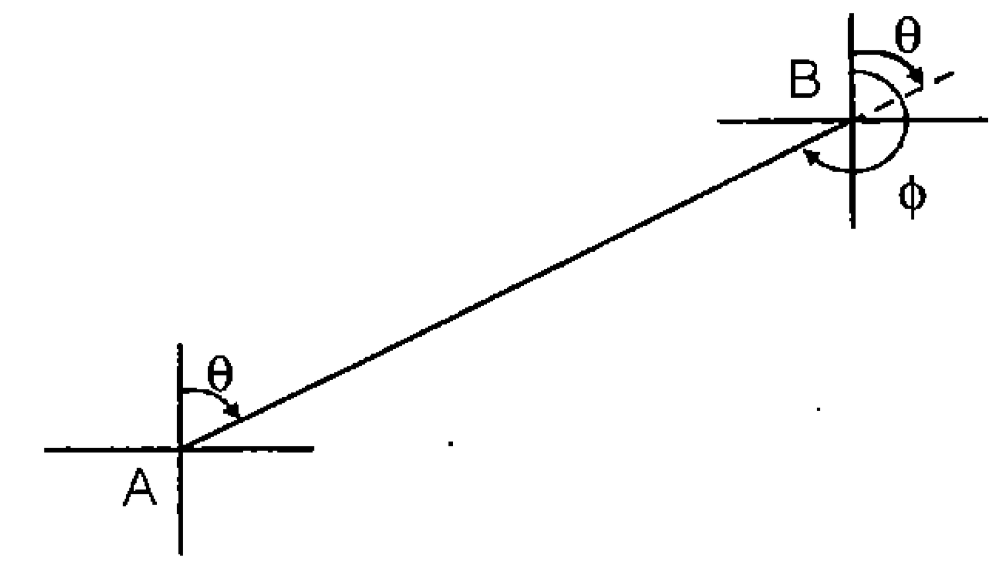
Baring of line 'OA' is $N\theta_AE$

Baring of line 'OB' is $S\theta_BE$

Baring of line 'OC' is $S\theta_CW$

Baring of line 'OD' is $N\theta_DW$

- **Fore bearing and back bearings**



$\boxed{B.B = F.B \pm 180°}$

Where,

B.B = Backbearing

FB = Fore bearing

θ = F.B. of line AB

ϕ = F.B of line BA or B.B of line AB

+ve sign is used when F.B < 180°

–ve sign is used when F.B. > 180°

- **Local attraction**

Direction of magnetic needle can be diverted due to presence of some magnetic objects near the instrument set of a station. This will cause local attraction due to which same error will be there in all measurements taken from that station.

Remember: If the difference between fore bearing and back bearing is 180° then the adjoining stations are free from local attraction.

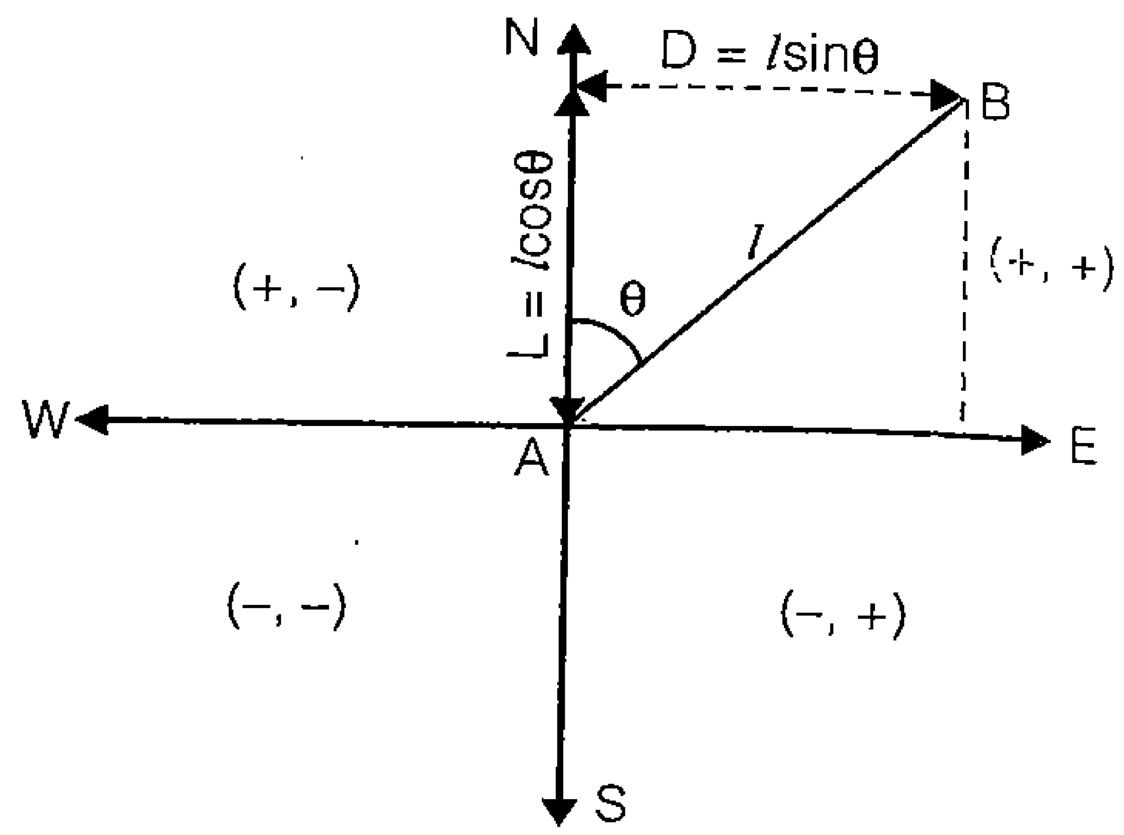
■■■

Traverse Surveying 4

Latitude and Departure

The latitude and departure of the line AB of length l and reduced bearing θ are given by

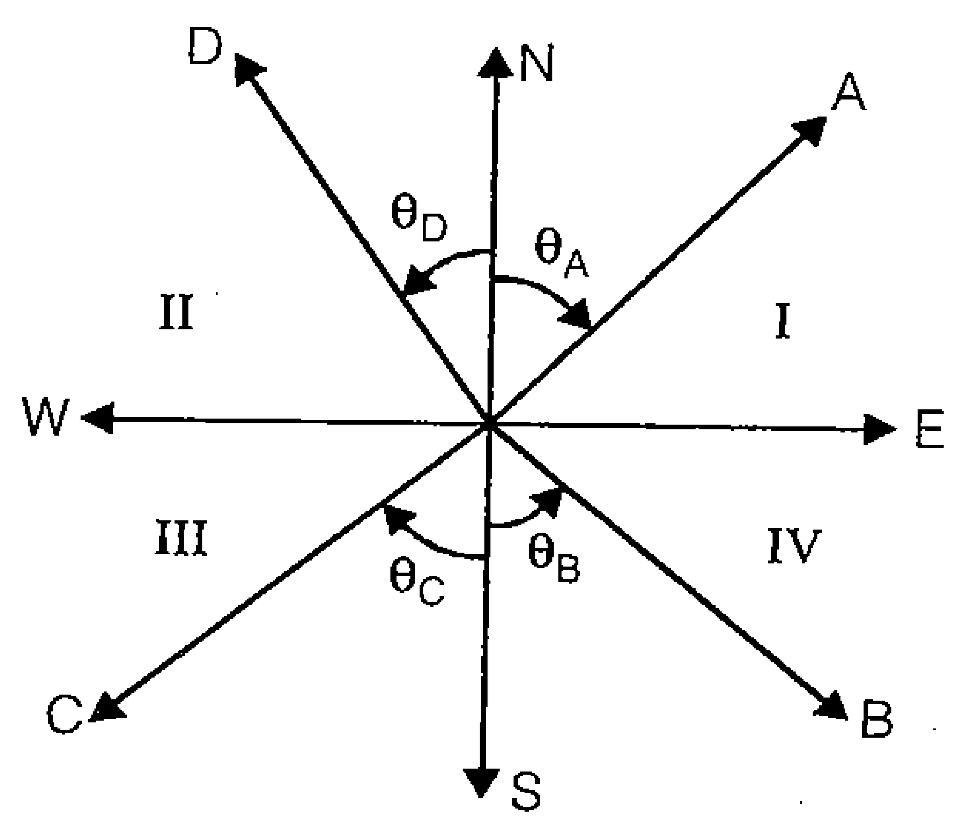
$$L = +l\cos\theta$$
$$D = +l\sin\theta$$



Latitude : Projection of a line on N-S direction is called latitude.

Departure : Projection of a line on E – W direction is called departure.

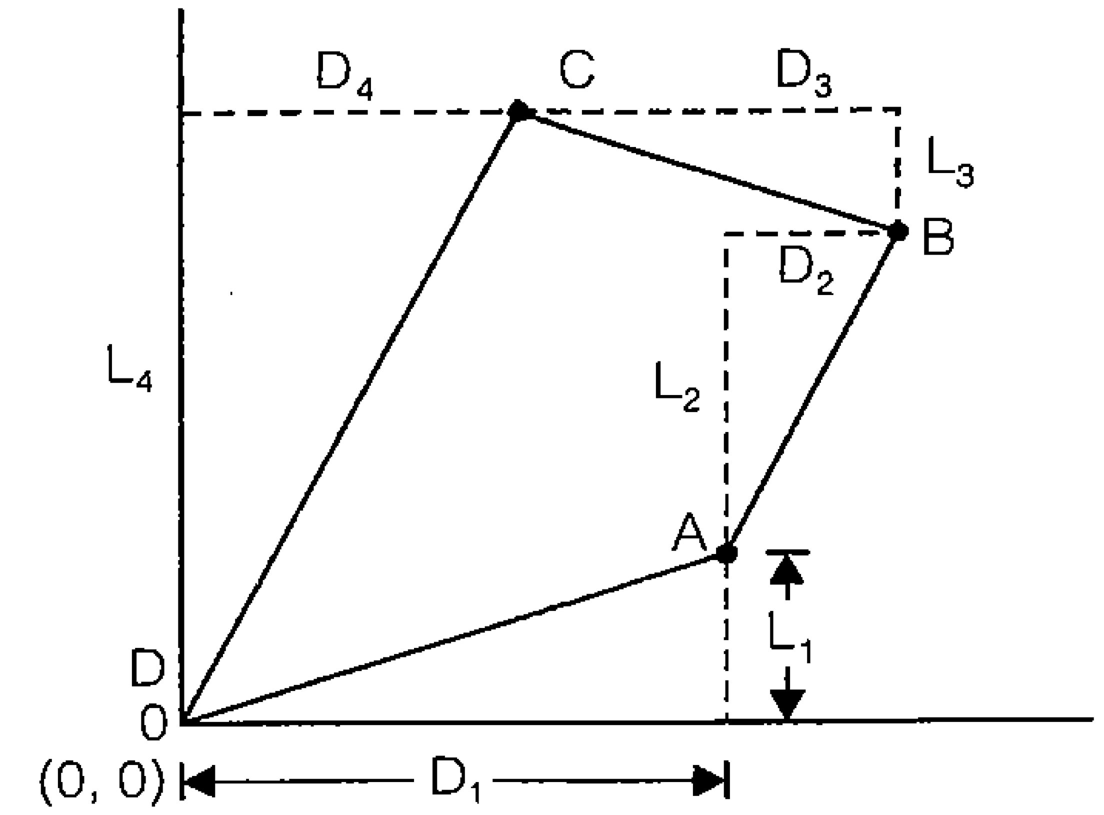
- Latitude and departure in various quadrants



Line	Reduced Bearing	Quadrants	Latitude	Departure
OA	$N\theta_A E$	I	$+l_1 \cos\theta_A$	$+l_1 \sin\theta_A$
OB	$S\theta_B E$	IV	$-l_2 \cos\theta_B$	$+l_2 \sin\theta_B$
OC	$S\theta_C W$	III	$-l_3 \cos\theta_C$	$-l_3 \sin\theta_C$
OD	$N\theta_D W$	II	$+l_4 \cos\theta_D$	$-l_4 \sin\theta_D$

Here, l_1, l_2, l_3, and l_4 are length of line OA, OB, OC and OD respectively.

Independent Coordinate



Remember

Coordinate of different point with respect to single origin is called independent coordinate.

$A \equiv (L_1, D_1)$

$B \equiv [(L_1 + L_2), (D_1 + D_2)]$

$C \equiv [(L_1 + L_2 + L_3), (D_1 + D_2 - D_3)]$

$D \equiv [(L_1 + L_2 + L_3 - L_4), (D_1 + D_2 - D_3 - D_4)] \sim (0, 0)$

For a closed Traverse

$$\Sigma L = 0 \quad \text{and} \quad \Sigma D = 0$$

where, ΣL = Sum of all latitude

ΣD = Sum of all departure

Closing Error

If sum of latitude,

$\Sigma L \neq 0$

and sum of departure

$\Sigma D \neq 0$

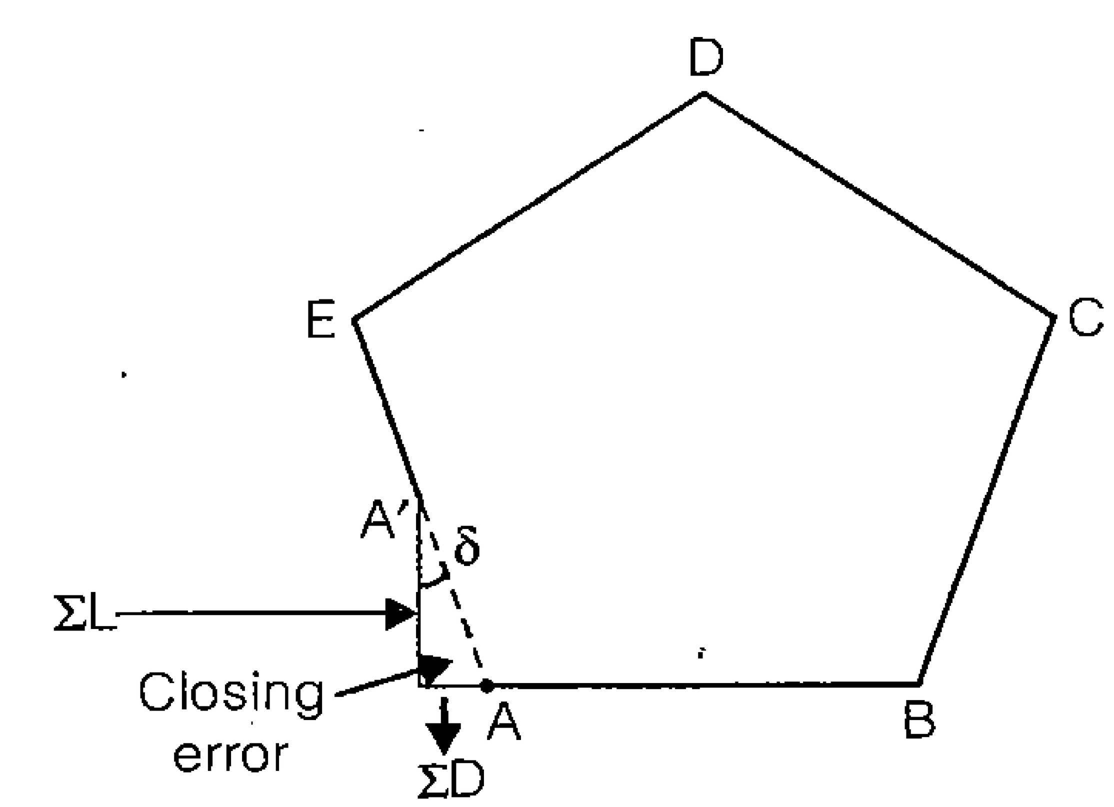
then there is a closing error

Closing error,

$$e = AA' = \sqrt{(\Sigma L)^2 + (\Sigma D)^2}$$

direction of closing error (δ)

$$\delta = \tan^{-1}\left(\frac{\Sigma D}{\Sigma L}\right)$$

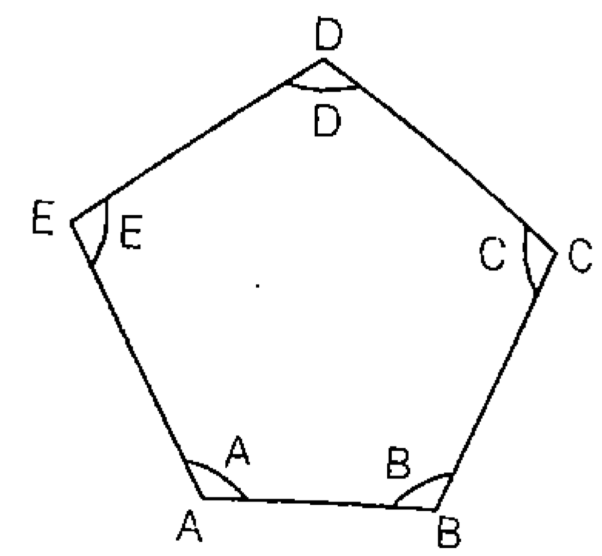


Remember

The sign of ΣD and ΣL will thus define the quadrant in which the closing error lies.

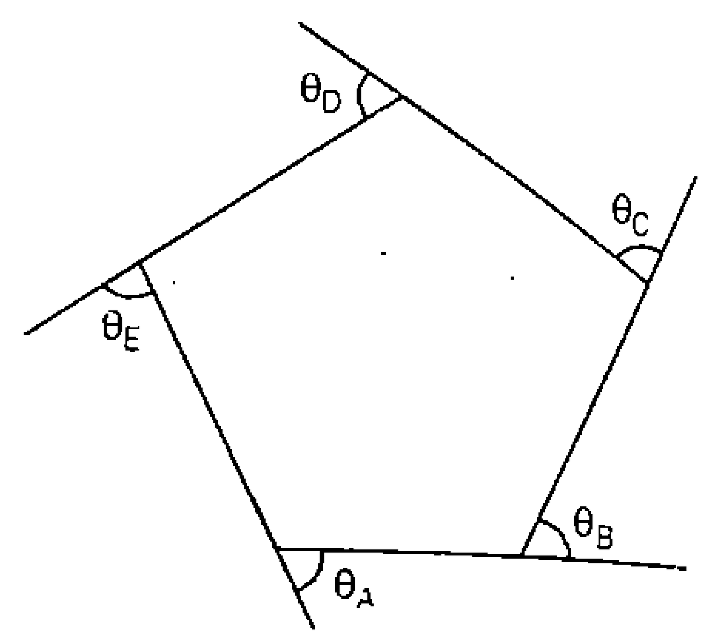
Adjustment of Closing Error

- Sum of all internal angles of a closed traverse $= (2n - 4) \times 90°$
  where n = no. of sides



- Sum of all deflection angles = 360°

  i.e. $\boxed{\theta_A + \theta_B + \theta_C + \theta_D + \theta_E = 360°}$



- Sum of latitude, $\boxed{\Sigma L = 0}$

  Sum of departure, $\boxed{\Sigma D = 0}$

Balancing the Traverse

(a) Bowditch method

Error in linear measurement $\propto \sqrt{l}$
where l = length of a line

Error in angular measurement $\propto \frac{1}{\sqrt{l}}$

correction to a particular line

$$\boxed{C_L = \left(\frac{l}{\Sigma l}\right) \times \Sigma L}$$

$$\boxed{C_D = \left(\frac{l}{\Sigma l}\right) \times \Sigma D}$$

Here, l = length of a line,
C_L = correction in latitude of a line
ΣL = total error in latitude,
C_D = correction in departure of a line
Σ_D = total error in departure
Σl = sum of length of all lines

> Remember: This method is mostly used to balance a traverse where linear and angular measurements have been taken with equal precision.

(b) Transit method

Correction in latitude of all line, $\boxed{C_L = \left(\frac{L}{L_T}\right) \times \Sigma L}$

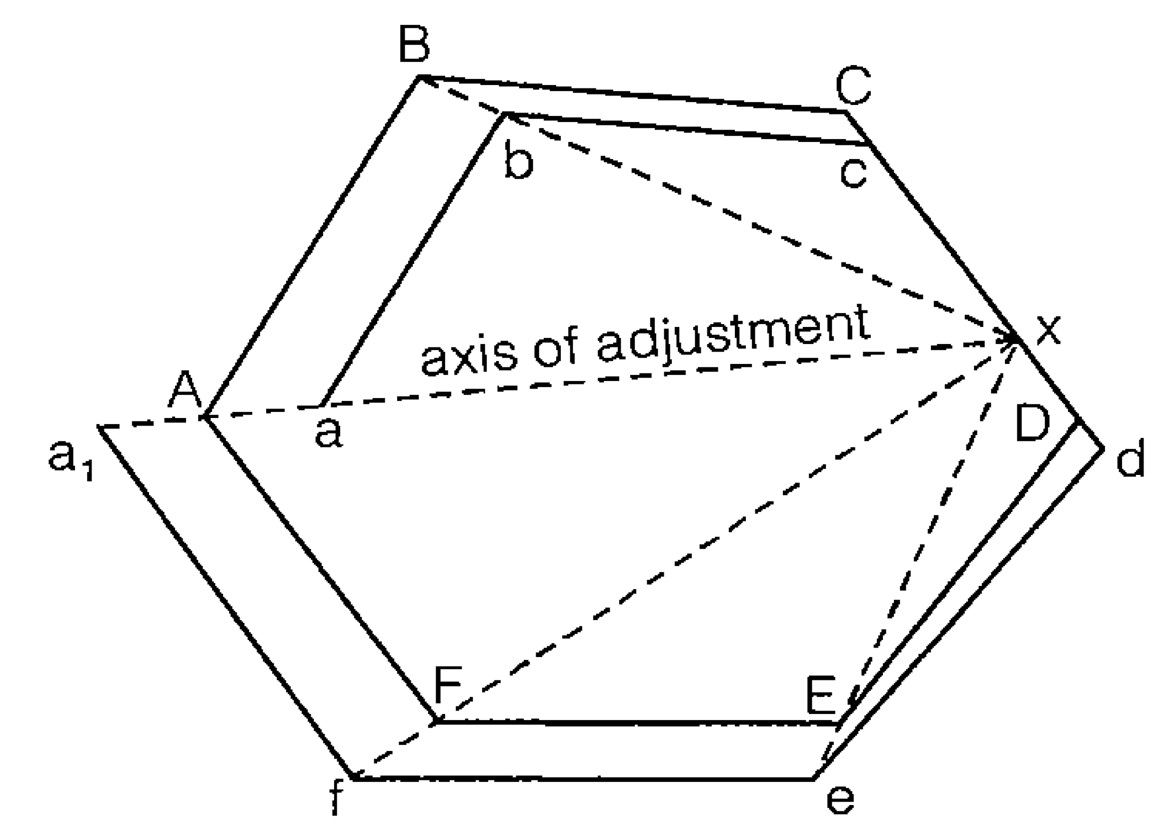
Correction in departure of a line, $\boxed{C_D = \left(\frac{D}{D_T}\right) \times \Sigma D}$

Here, ΣL = Total error in latitude
ΣD = total error in departure
L = latitude of a line
D = departure of a line
L_T = sum of all latitude without considering sign
D_T = Sum of all departure without considering sign.

> Remember: This method is suitable where angular measurements are more precise than linear measurements.

(c) Axis method

$$\boxed{\text{Correction to any length} = \text{that length} \times \frac{\frac{1}{2}\text{Closing error}}{\text{Length of axis}}}$$



i.e. $\boxed{\text{Correction of } a_{1f}.\frac{\frac{1}{2}.a_a a}{aX}}$

> Remember: This method is used where angles are measured very accurately, so correction are done in length of line only, bearing of lines are not changed.

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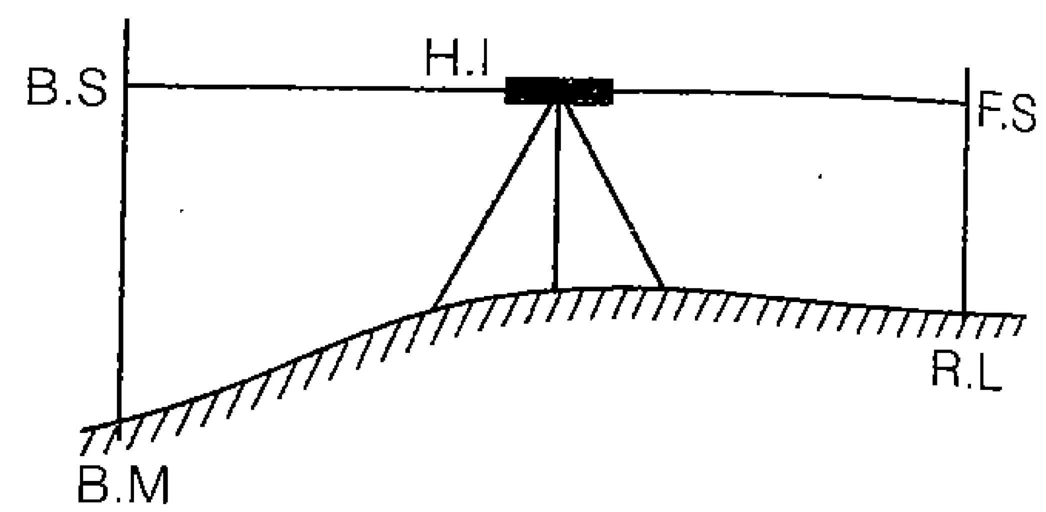
5

Levelling

Definitions

(i) **Reduced level:** The elevation of a point with respect to either Mean Sea Level (MSL) or with respect to a fixed point of known height is called reduced level.

(ii) **Bench mark:** Bench mark is relatively permanent point of reference whose elevation with respect to some assumed datum is known. It is used either as a starting point for levelling or as a point upon which to close as a check.



(iii) **Back sight:** After setting up the instrument I[st] reading taken is called back sight. It is also known as plus sight.

(iv) **Fore sight:** Last reading taken from an instrument station is called fore sight. It is also known as minus sight.

(v) **Intermediate sight:** All readings ohter than back sight and fore sight are intermediate sight.

(vi) **Height of instrument:** It is the Reduced Level (RL) of line of sight of the instrument set up at different stations.

$$\boxed{H.I = R.L + B.S}$$

$$\boxed{R.L = H.I - F.S}$$

Arithmetic Check

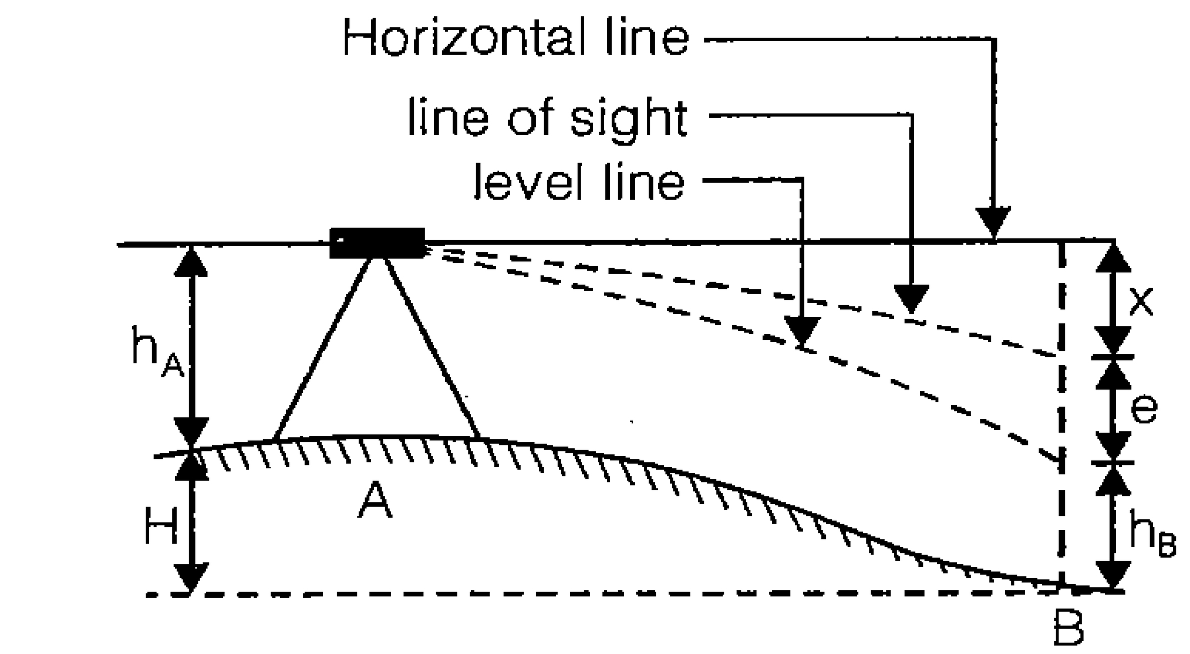
(i) For rise and fall method

$$\boxed{\Sigma B.S - \Sigma F.S = \Sigma Rise - \Sigma Fall = Last\ R.L - First\ R.L}$$

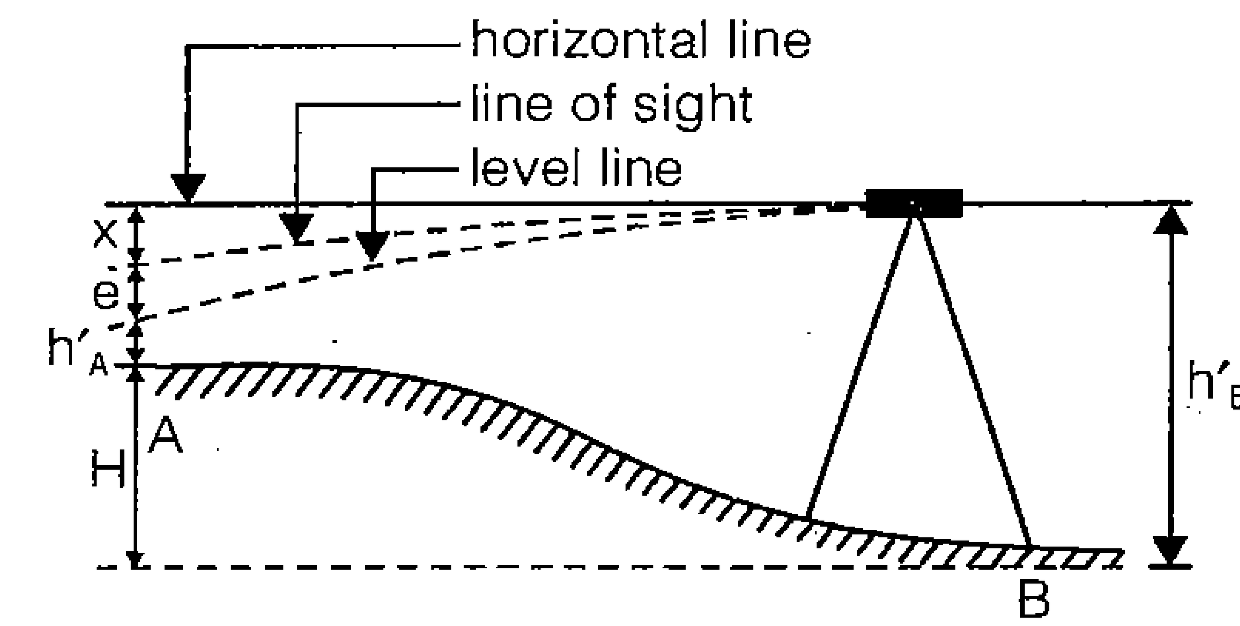
(ii) Height of instrument method

$$\boxed{\Sigma B.S - \Sigma F.S = Last\ R.L - First\ R.L}$$

Reciprocal Levelling



Here, x = error due to inclined line of sight, and
e = error due to curvature and refraction



When instrument is set up at A

Reading on staff at A = h_A

Reading on staff at B = h_B

When instrument is set up at B

Reading on staff at A = h'_A

Reading on staff at B = h'_B

$\boxed{h_A - h_B = h'_A - h'_B}$ If instrument is correct.

$$\boxed{H = \frac{(h_B - h_A) + (h'_B - h'_A)}{2}}$$

Here 'H' is the true difference of R.L between A and B.

True Readings

Instrument is at	Reading of A	Reading of B
A	h_A	$h_A + H$
B	$h'_B - H$	h'_B

Curvature Correction (C_C)

$$C_C = -\frac{d^2}{2R}$$

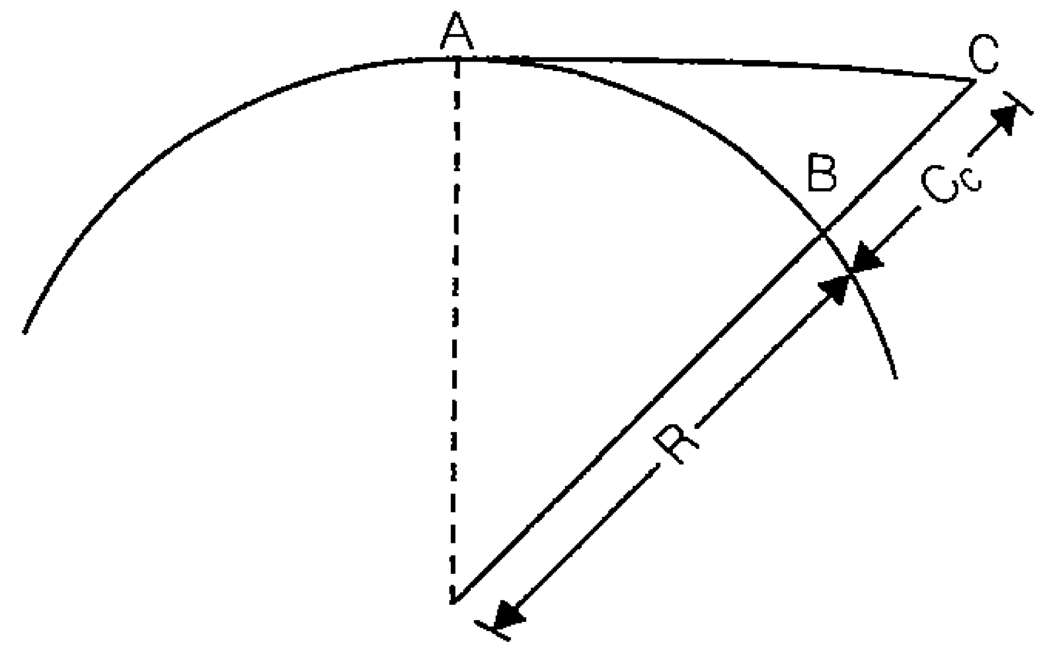
Here, d = horizontal distance between A and B

R = radius of earth

If R = 6370 km

than $C_C = -0.07849\,d^2$

here 'C_C' is in meter and 'd' is in kilometer



Refraction Correction (C_R)

$$C_R = +\frac{1}{7}C_C$$

$$C_R = \frac{1}{7}\times\frac{d^2}{2R}$$

↓ If R is 6370 km

$$C_R = 0.01121\,d^2 \text{ meter}$$

Here d is in kilometer.

Combined Correction Due to Curvature and Refraction (C)

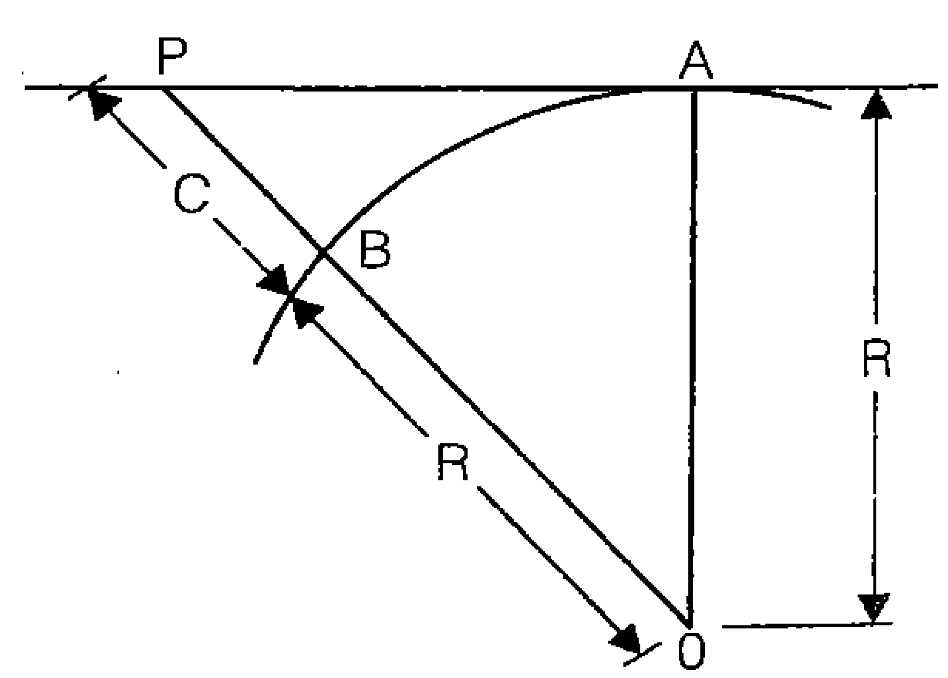
$$C = -\frac{6}{7}\times\frac{d^2}{2R}$$ If R = 6370 km

$$C = -0.06728\,d^2 \text{ meter}$$ Here d is in kilometer.

Distance of Visible Horizon

$$d = 3.8553\sqrt{c}\text{ km}$$

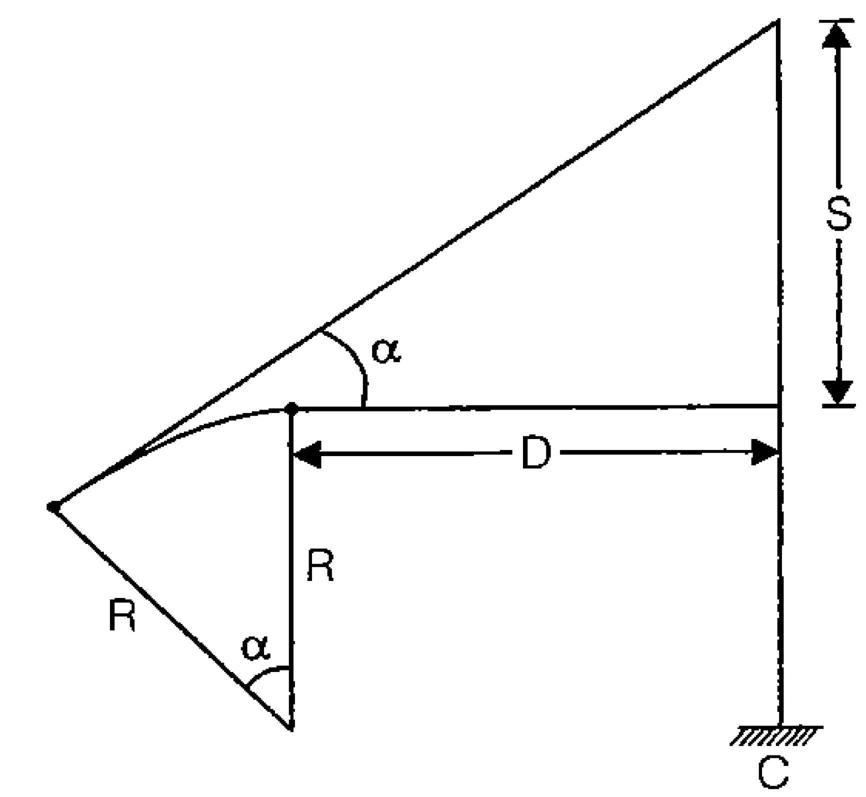
Here 'C' being in meters.
(taking both curvature and refraction into accounts)



Sensitiveness of Bubble Tube

Sensitiveness of the bubble tube is defined as the angular value of one division of the bubble tube.

α' = sensitivity of the bubble tube
= angular value of one division



$$\alpha' = \frac{S}{nD}\times 206265 \text{ seconds}$$

or $$\alpha' = \frac{S}{nD\sin 1''}\text{ seconds}$$

Here, S = difference between two staff readings.
n = no. of divisions of bubble

also, $\alpha' = \frac{l}{R}$ radian or $\alpha' = \frac{l}{R\sin 1''}$ seconds

where, l = length of one division
R = radius of curvature of bubble tube.

Remember: 1 radian = 206265 seconds = $\frac{1}{\sin 1''}$

Contouring

Contours: Contour is an imaginary line joining points of equal elevation on earth surface.

Contour interval: Vertical distance between two contour is called contour internal.

Some suitable value of contour intervals

Scale of map	Type of ground	Contour internal (metres)
Large	Flat	0.2 to 0.5
(1 cm = 10 m or less)	Rolling	0.5 to 1
	Hilly	1, 1.5 or 2
Intermediate	Flat	0.5, 1 or 1.5
(1 cm = 10 m to 100 m)	Rolling	1, 1.5 or 2
	Hilly	2, 2.5 or 3
Small	Flat	1, 2 or 3
(1 cm = 100 m or more)	Rollng	2 to 5
	Hilly	5 to 10
	Mountaineous	10, 25 or 50

Contour internal for various purposes are suggested as:

Purpose of survey	Scale	Interval (metres)
1. Building sites	1 cm = 10 m or less	0.2 to 0.5
2. Town planning schemes, reservoirs etc.	1 cm = 50 m to 100 m	0.5 to 2
3. Location surveys	1 cm = 50 m to 200 m	2 to 3

$$\text{Contour interval} = \frac{25}{\text{No. of cm per km}} \text{(metres)}$$

$$\text{Contour interval} = \frac{50}{\text{No. of inches per mile}} \text{(feet)}$$

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6 Calculation of Area and Volume

General Methods of Computing Area

(a) **By computations based directly on field measurements**

By dividing the area into a number of triangles.

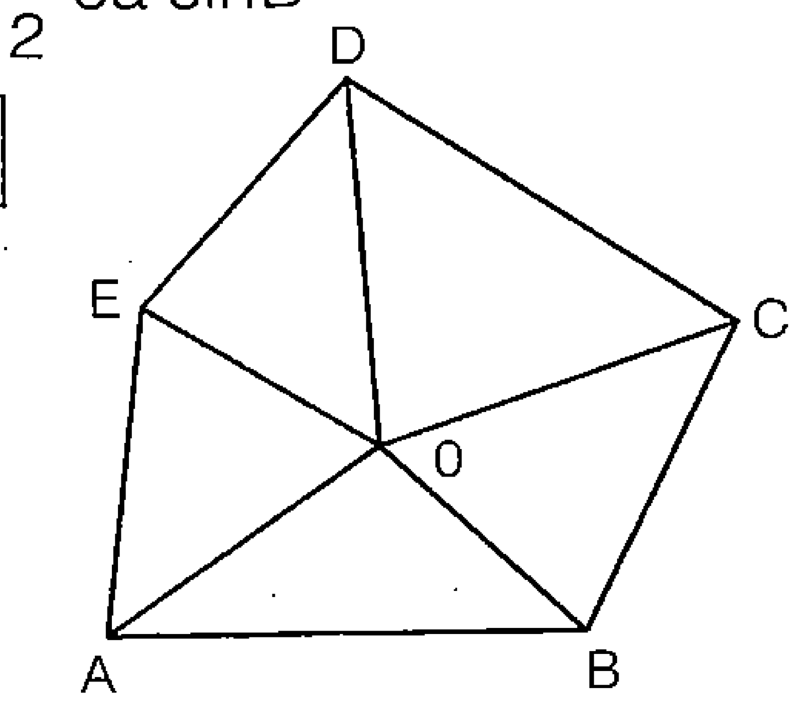
Area of $\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ca \sin B$

$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

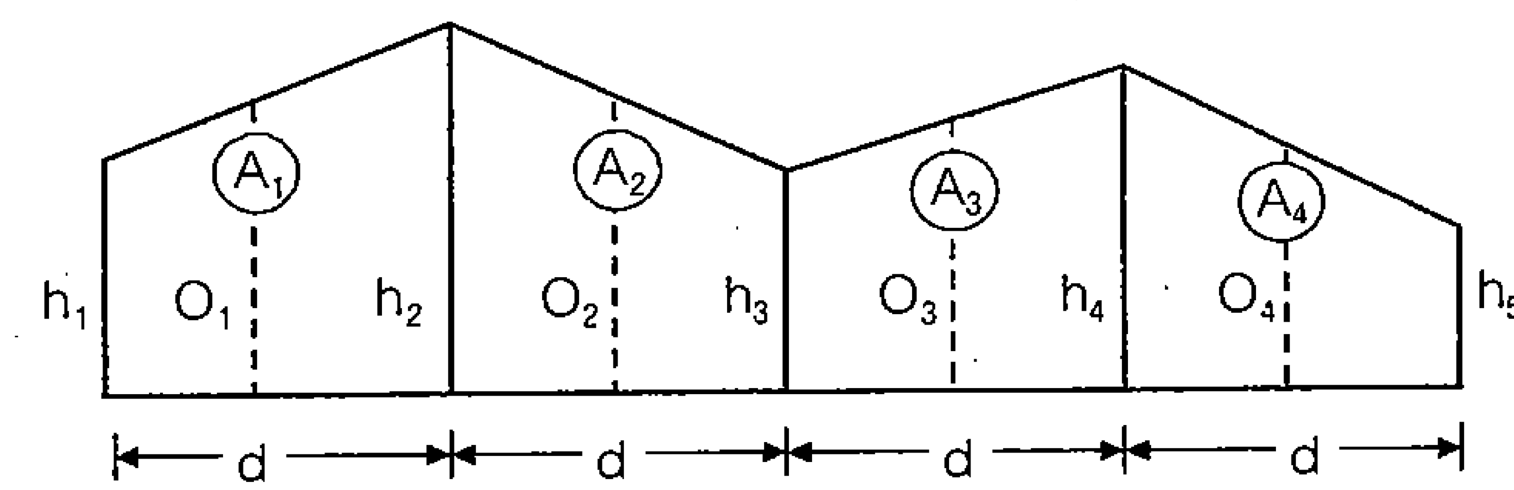
where s = semi perimeter

$$s = \frac{a+b+c}{2}$$

where a, b and c are length of sides.



(b) **Using offsets taken from a staight line**



(i) By mid ordinate method

In this case mid ordinates are measured.

$A = d(O_1 + O_2 + O_3 + ... O_n)$

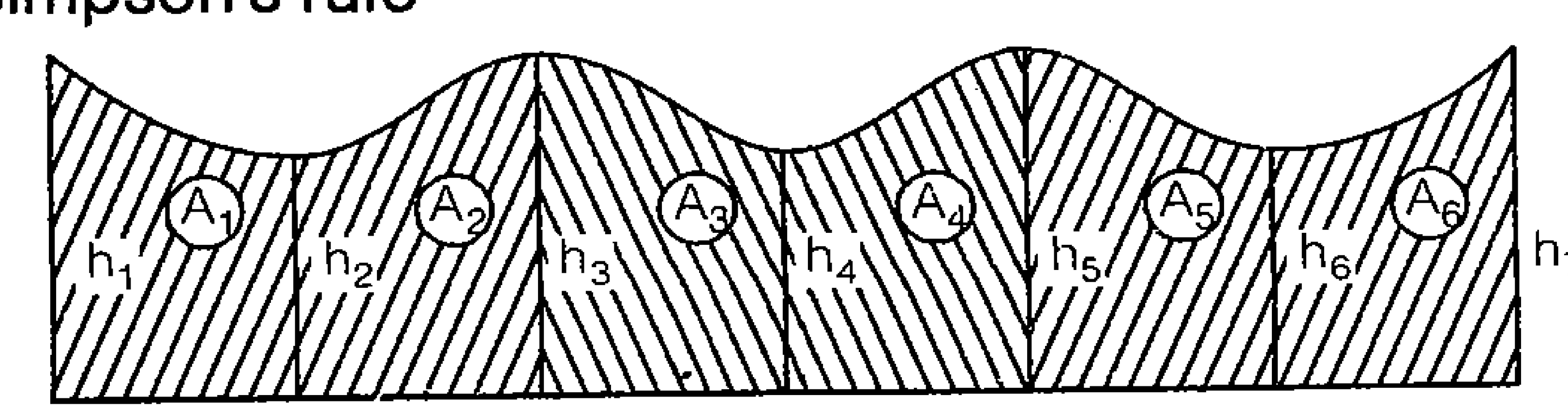
(ii) By Average ordinate method

$$A = (n-1)d\left[\frac{h_1 + h_2 + ..h_n}{n}\right]$$

(iii) Trapazoidal rule (End area Method)

$$A = d\left[\frac{h_1 + h_n}{2} + h_2 + h_3 + ..h_{n-1}\right]$$

(c) **Simpson's rule**



$$A = \frac{d}{3}\left[(h_1 + h_n) + 4(h_2 + h_4 + h_6 + ...) + 2(h_3 + h_5 + h_7 + ...)\right]$$

In this method odd no of offsets are needed. Area should be in pairs.

Note: Simpson's three point formula $A = \frac{d}{3}[h_1 + 4h_2 + h_3]$

- This rule is based on the assumptions that the figures are trapezoids.
- The rule is more accurate than previous two rules.
- Simpson's one third rule may be stated as: The area is equal to the sum of the two end ordinates plus four times the sum of even intermediate ordinates plus twice the sum of the odd intermediate ordinates, the whole multiplied by one-third the common interval between them.
- It should be clar that this rule is applicable only when number of divisions of the area is even i.e., the total number of ordinates is odd.

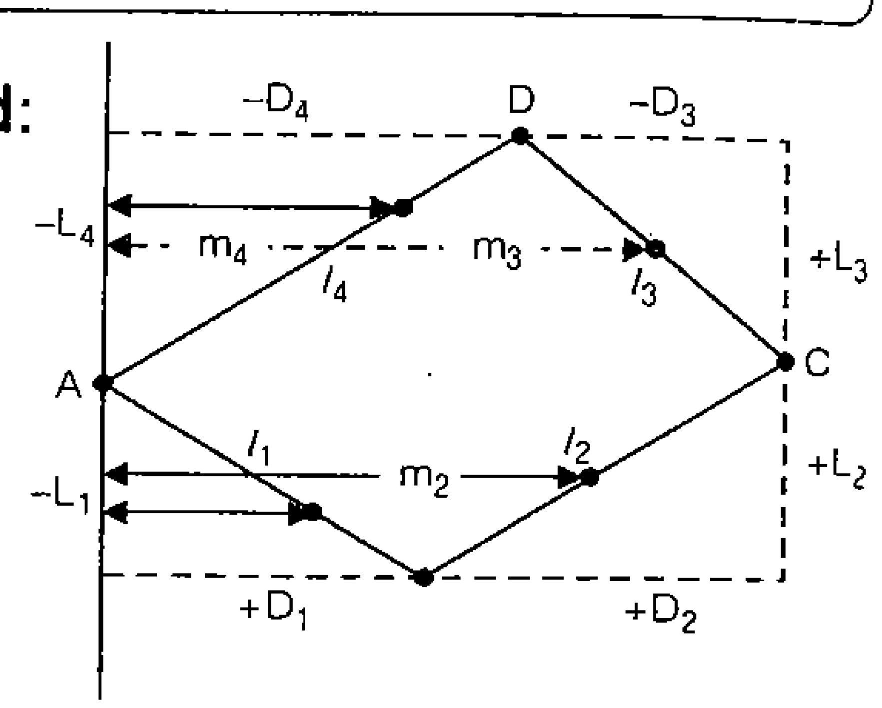
(c) Area by Meridian distance method:

Traverse,

Area of the closed

$$\Delta = \Sigma L \cdot m$$

Or,



$$\Delta = (-L_1 \times m_1) + (L_2 \times m_2) + (L_3 \times m_3) + (-L_4 \times m_4)$$

where 'm' is meridian distance,

$$m_1 = \frac{D_1}{2}$$

$$m_2 = \frac{D_1 + D_1 + D_2}{2} = m_1 + \frac{D_1}{2} + \frac{D_2}{2}$$

$$m_3 = m_2 + \frac{D_2}{2} - \frac{D_3}{2}$$

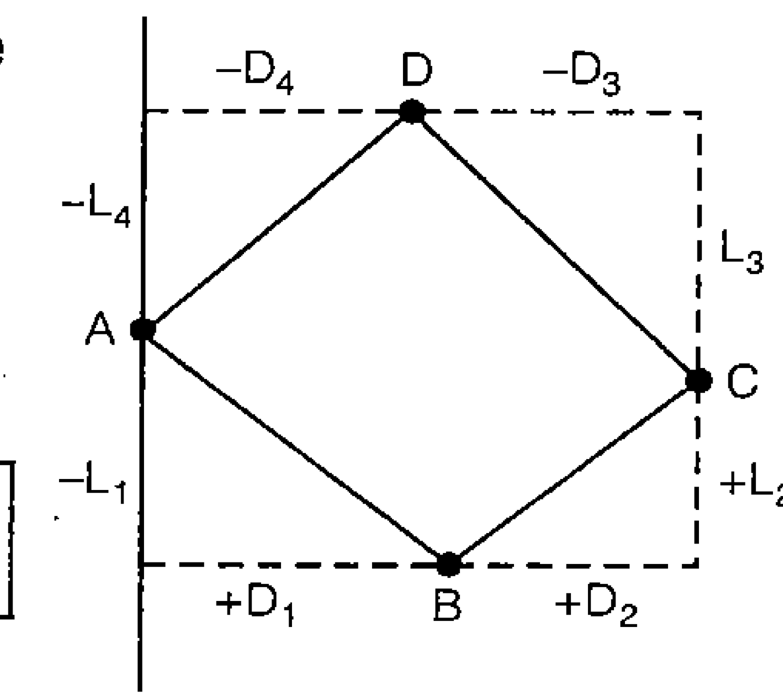
$$m_4 = m_3 - \frac{D_3}{2} - \frac{D_4}{2} = \frac{D_4}{2}$$

MD of any line = MD of previous line + half of departure of previous line + half of departure of this line

- The distance of mid point of a line w.r.t. a fixed meridian is called meridian distance.
- Here, L_1, L_2, L_3 and L_4 are latitudes line AB, BC, CD and DA. D_1, D_2, D_3 and D_4 are departure.

(d) Area by Double Meridian distance method

Total area, $\Delta = \frac{1}{2}\Sigma ML$



$$\Delta = \frac{1}{2}[(-L_1 M_1) + (L_2 M_2) + (L_3 M_3) + (-L_4 M_4)]$$

D.M.D. of AB,

$$M_1 = 0 + D_1 = D_1$$

D.M.D. of BC,

$$M_2 = D_1 + (D_1 + D_2)$$

D.M.D. of CD

$$M_3 = D_1 + D_2 + (D_1 + D_2 - D_3)$$
$$= M_2 + D_2 + (-D_3)$$

D.M.D. of DA

$$M_4 = M_3 + (-D_3) + (-D_4)$$

DMD of any line = DMD of previous line + departure of previous line + departure of this line

Volume Measurement

(a) Trapezoidal formulae:

Volume (V) of earthwork between a number of sections having areas A_1, A_2......A_n spaced at a constant distance d.

$$V = d\left[\frac{A_1 + A_n}{2} + A_2 + A_3 + ... + A_{n-1}\right]$$

(b) Simpson's formulae:

Volume (V) of the earthwork between a number of sections having Area A_1, A_2,..... A_n spaced at a constant distance d apart is

$$V = \frac{d}{3}\left[(A_1 + A_n) + 4(A_2 + A_4 + ... + A_{n-1}) + 2(A_3 + A_5 + ... + A_{n-2})\right]$$

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Tacheometric, Curve & Hydrographic Surveying

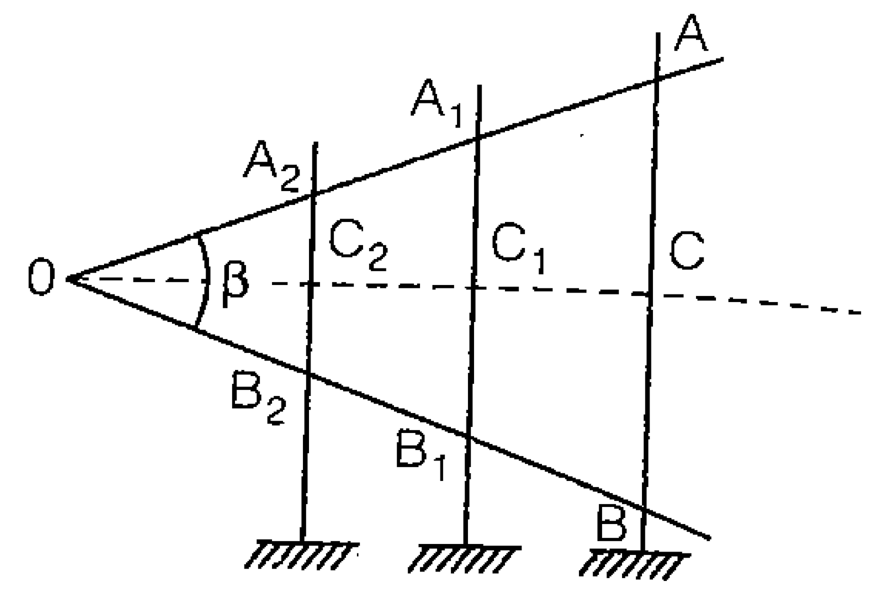
7

Stadia Method

(i) **Distance-elevation formulae for horizontal sights**

Horizontal distance between the axis and the staff is

$$\boxed{D = ks + c}$$

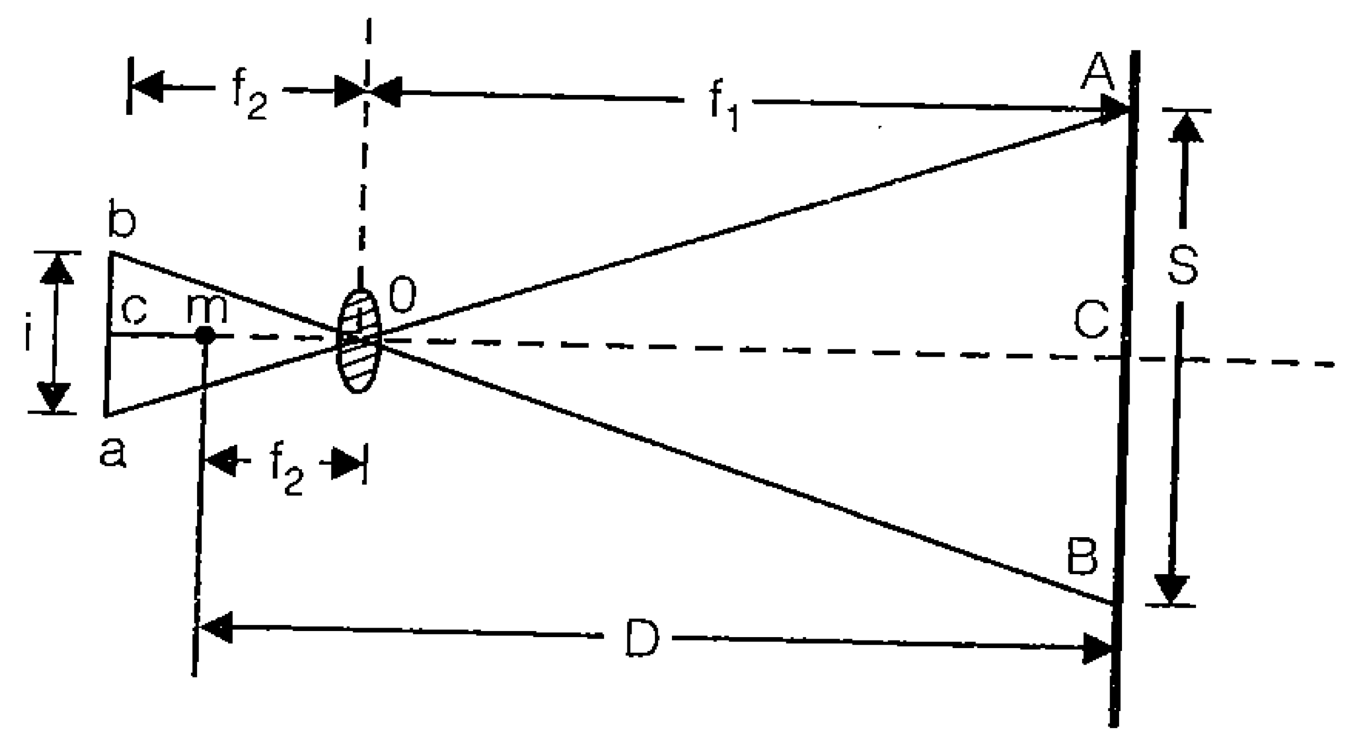


where $k = \frac{f}{i}$ = Multiplying constant

$c = (f + d)$ = additive constant

f = focal length of objectives

$$\boxed{f = \frac{f_1 f_2}{f_1 + f_2}}$$



where f_1 = Horizontal distance of staff from the optical centre of the objective

f_2 = Horizontal distance of the cross-wires from 0

d = Distance of the vertical axis of the instrument from 0

$ab = i$ = Interval between the stadia hairs of the diaphragm

$AB = s$ = Staff intercept

A, C and B = The points cut by the three lines of sight corresponding to the three wires.

'0' is the optical centre of objective of an external focusing telescope.

D = Horizontal distance of the staff from the vertical axis of the instrument.

M = Centre of the instrument, corresponding to the vertical axis.

> Remember: Stadia method is based on the principle that the ratio of the perpendicular to the base is constant in similar isosceles triangles.

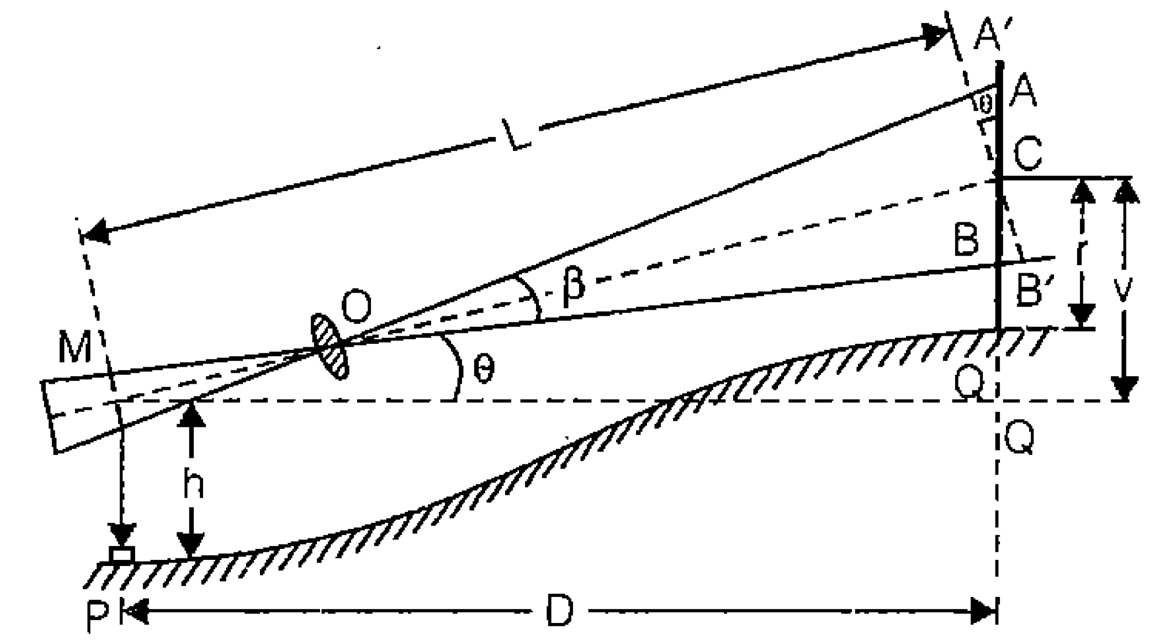
(ii) **Distance and elevation formulae for staff vertical inclined sight**

Horizontal distance,

$$\boxed{D = ks\cos^2\theta + c\cos\theta}$$

Vertical intercept

$$\boxed{V = k \cdot s\frac{\sin 2\theta}{2} + c\sin\theta}$$



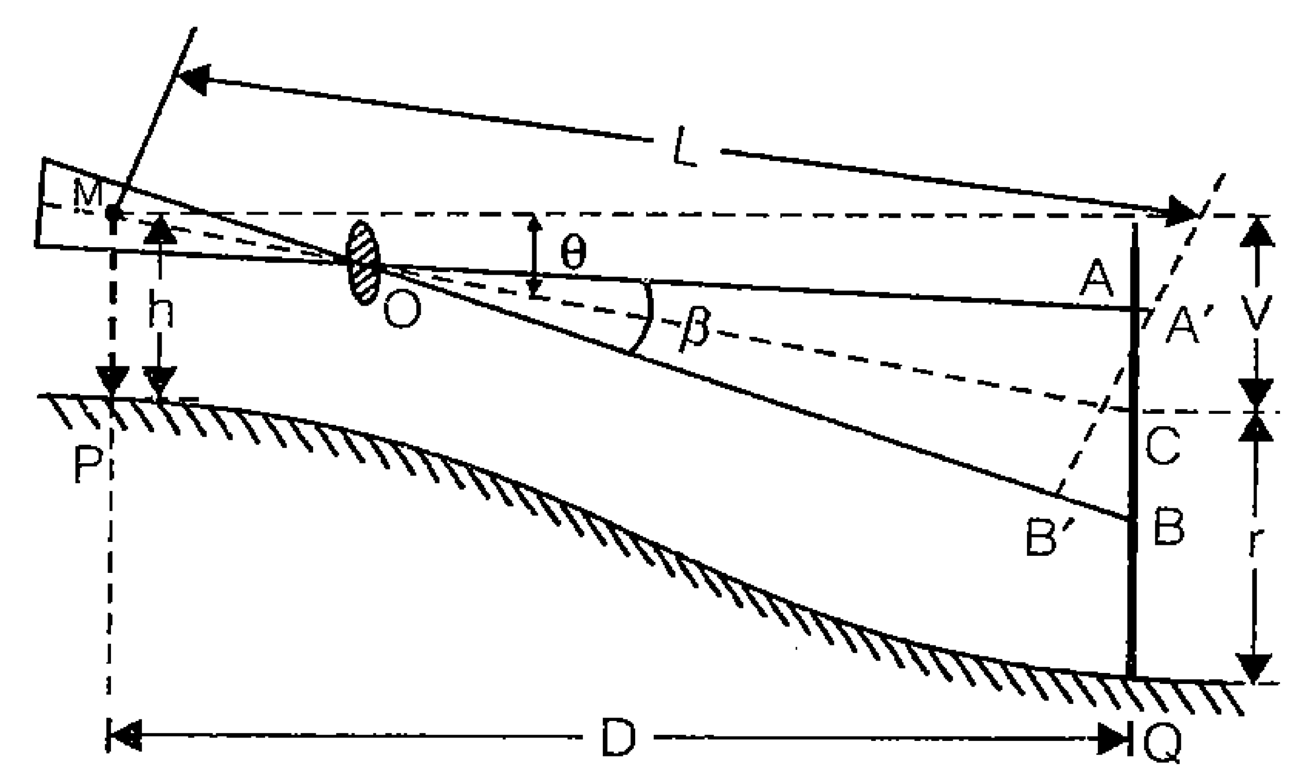
Here, θ = Inclination of line of sight from the horizontal

Elevation of the staff station for angle of elevation.

$$\boxed{\text{Elev. of staff station} = \text{Elev. of instrument station} + h + V - r}$$

Elevation of staff station for angle of depression

$$\boxed{\text{Elev.of } Q = \text{Elev. of } P + h - V - r}$$

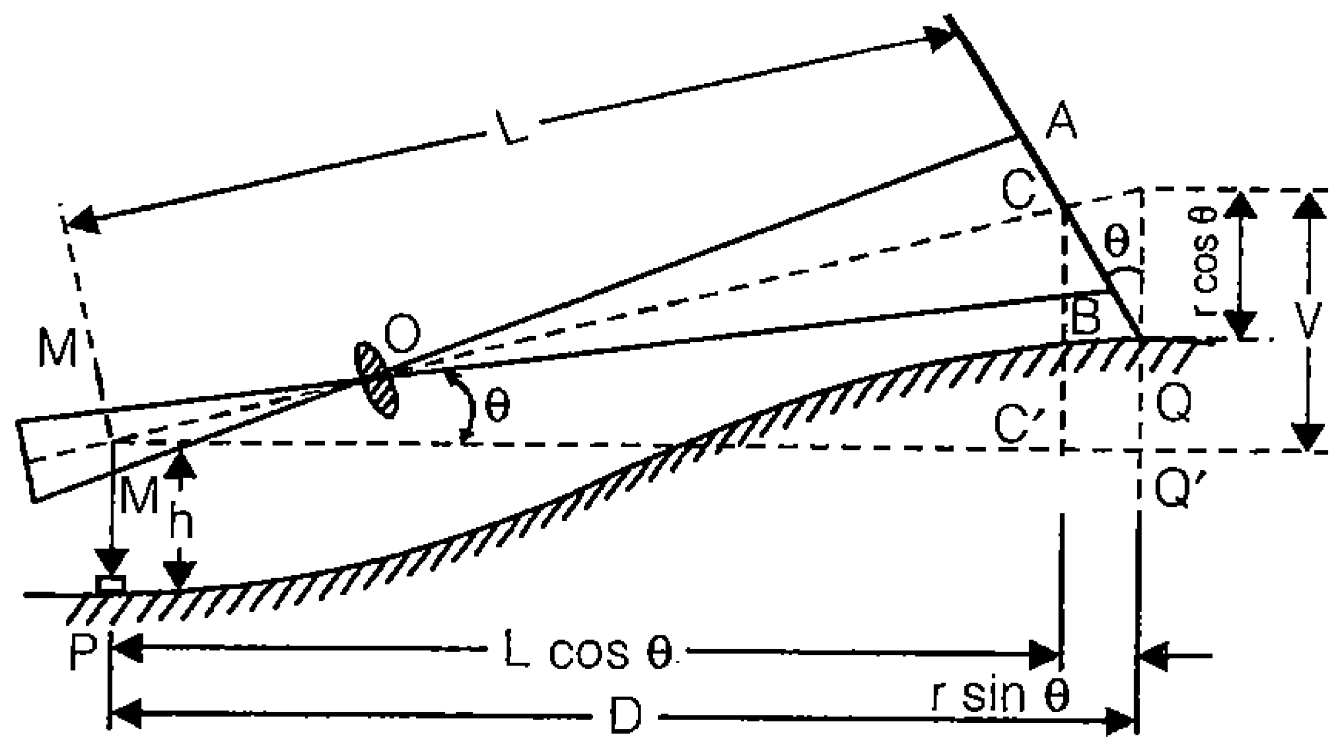


(iii) **Distance of Elevation formulae for staff normal**

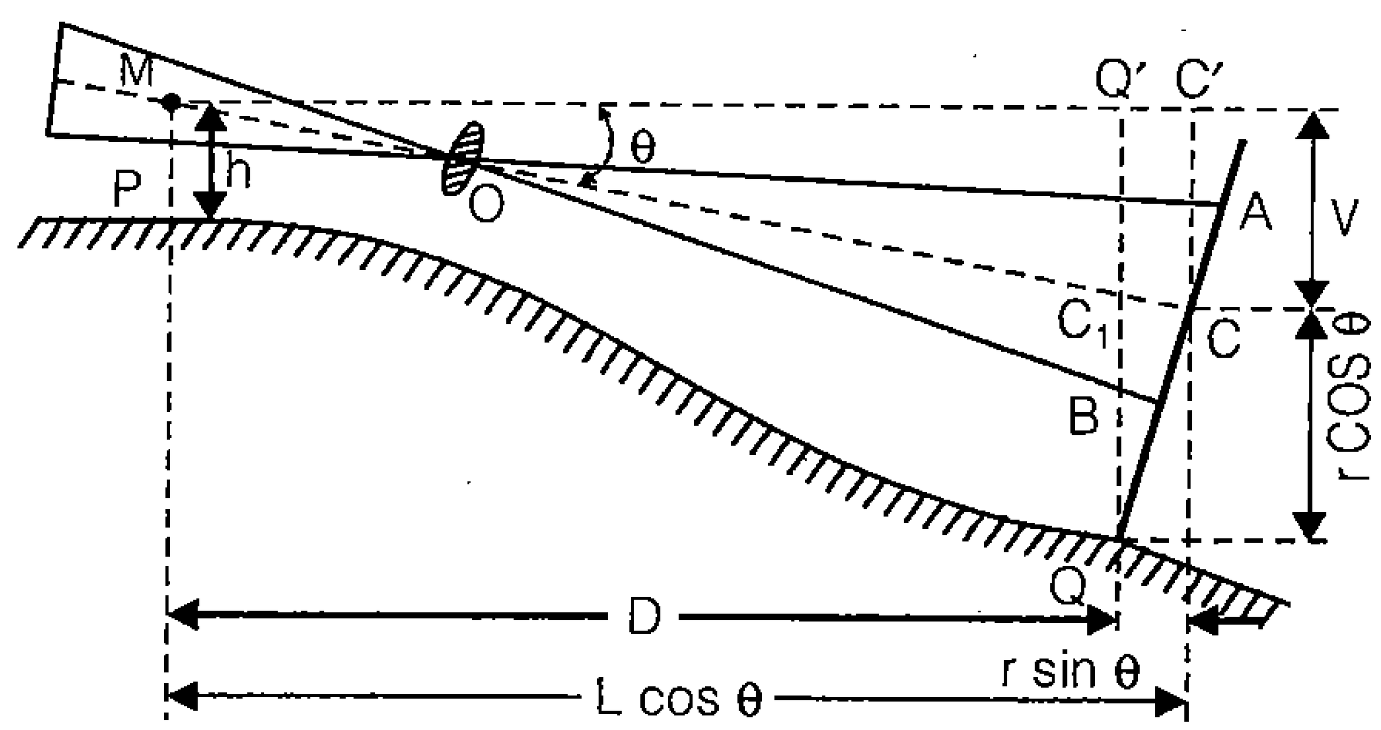
Case-(a): When line of sight at an angle of elevation θ.

$$\boxed{D = (ks + c)\cos\theta + r\sin\theta} \quad \boxed{V = (ks + c)\sin\theta}$$

$$\boxed{\text{Elev. of } Q = \text{Elev. of } P + h + V - r\cos\theta}$$



Case-(b): When line of sight at an angle of depression θ.



$$\boxed{D = (ks + c)\cos\theta - r\sin\theta}$$

$$\boxed{V = (ks + c)\sin\theta}$$

$$\boxed{\text{Elev. of } Q = \text{Elev. of } P + h - V - r\cos\theta}$$

Curve

- Length of curve,

$$l = \frac{\pi R \Delta}{180}$$

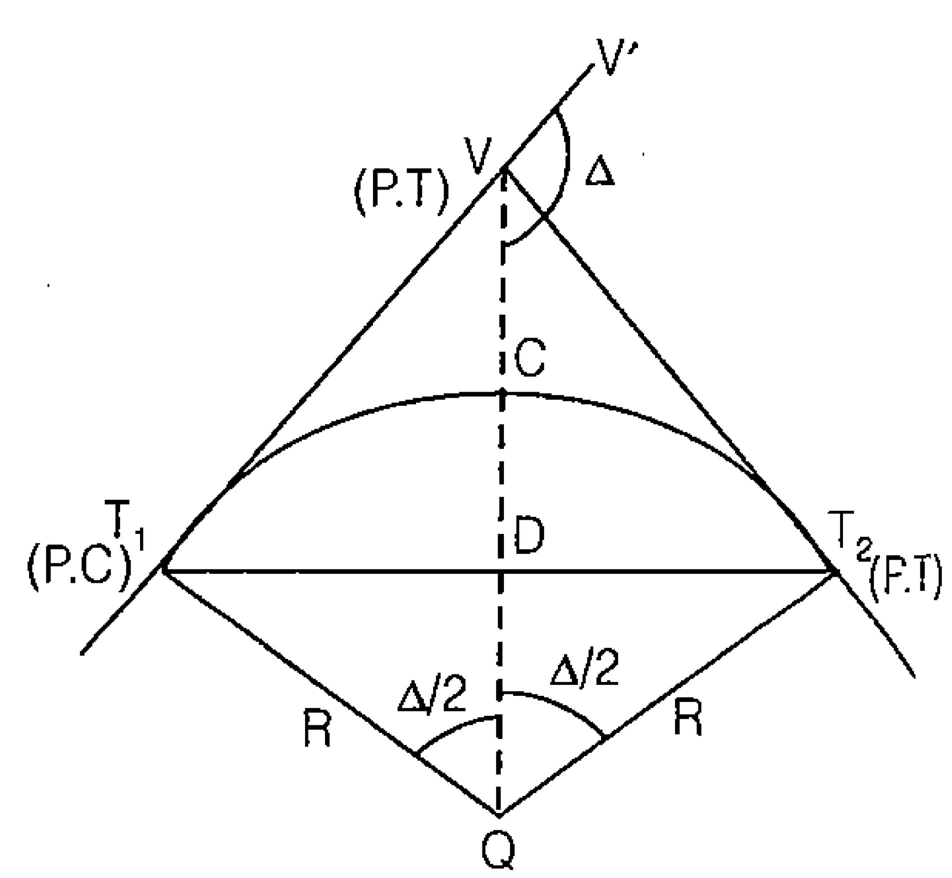
where, Δ = the angel subtended at the centre in degree

- Tangent length,

$$T = R\tan\frac{\Delta}{2}$$

- Length of chord $= 2R\sin\frac{\Delta}{2}$

- Mid ordinate $M = R\left(1 - \cos\frac{\Delta}{2}\right)$



Hydrographic Surveying

- Hydrographic surveying is that branch of surveying which deals with the measurement of bodies of water. It is the art of delineating the submarine levels, contours and features of seas, gulfs, rivers and lakes.
- **Soundings:** The measurement of depth below the water surface is called sounding.
- Equipment needed for sounding are:
  - **(i) Sounding boat:** A rowboat for sounding should be sufficiently roomy and stable.
  - **(ii) Sounding rods or poles:** A sounding rod is a pole of a sound straight-grained well seasoned tough timber usually 5 to 8 cm in diameter and 5 to 8 meters long.
  - **(iii) Lead lines:** A lead line or a sounding line is usually a length of a such cord or tiller rope of Indian hemp or braided flax or a brass chain with a sounding lead attached to the end. Lead lines, areusually used for depths over about 6 meters.

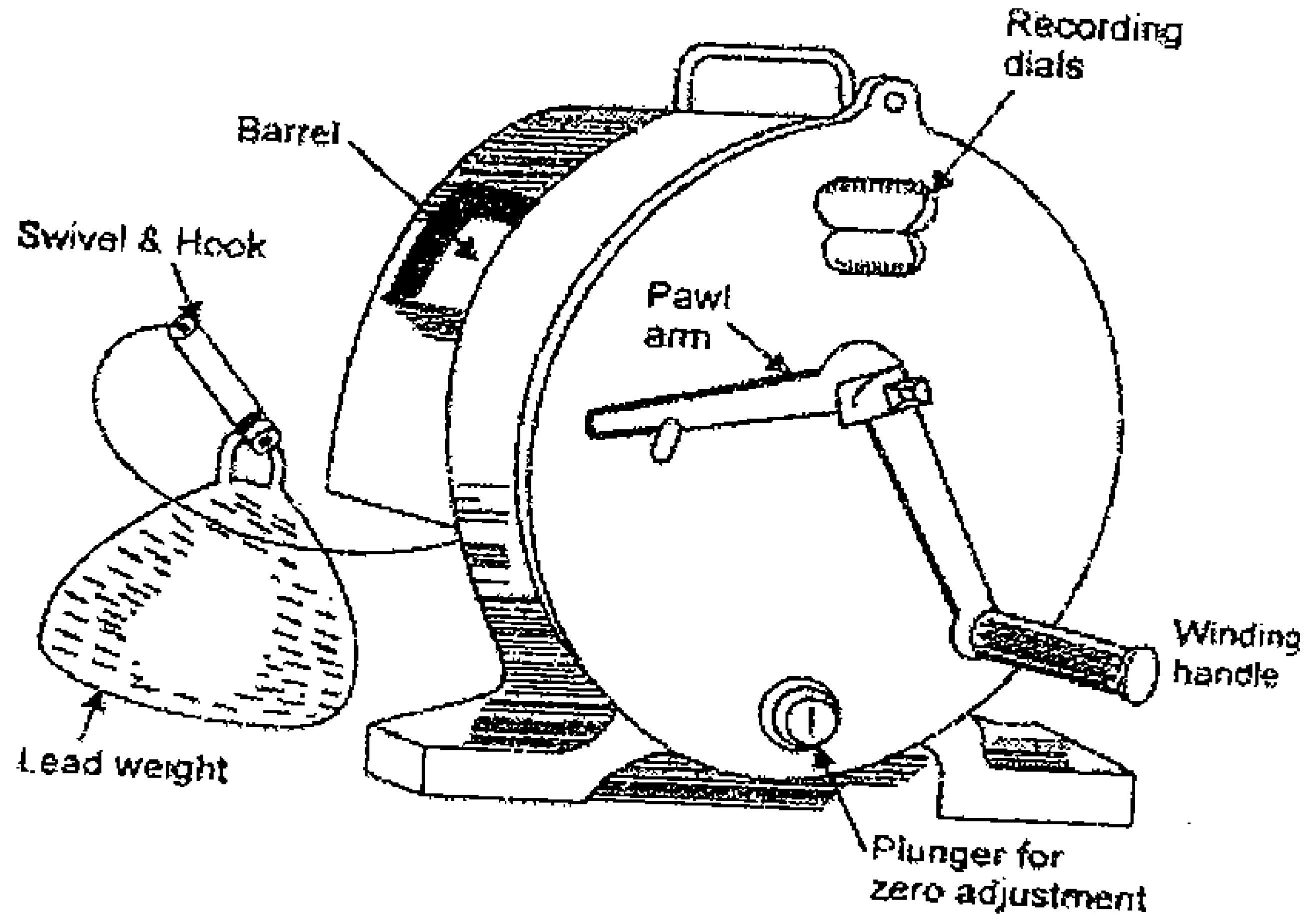
Sounding lead is a weight attached to the line. The weight is conical in shape and varies from 4 to 12 kg depending upon the depth of water and the strength of the current.

Leadline are marked in feet as follows:

Feet	Marks
2, 12, 22 etc.	Red bunting
4, 14, 24 etc.	White bunting
6, 16, 26 etc.	Blue bunting
8, 18, 28 etc.	Yellow bunting
10, 60, 110 etc.	One strip of leather
20, 70, 120 etc.	Two strip of leather
30, 80, 130 etc.	Leather with two holes
40, 90, 140 etc.	Leather with one holes
50	Star-shaped leather
100	Star shaped leather with one hole

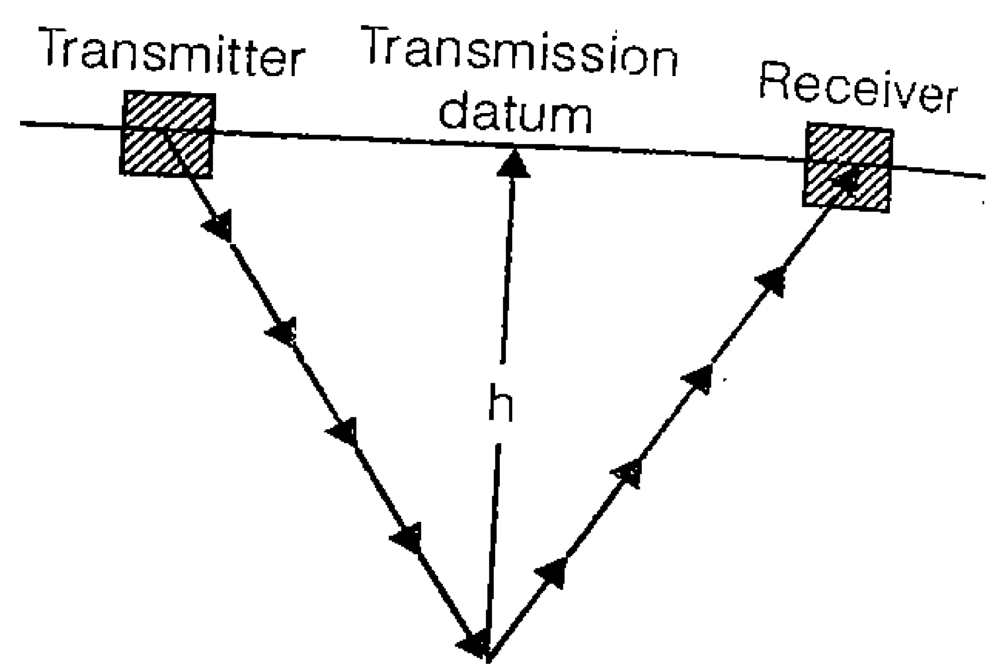
The intermediate odd feet (1, 3, 5, 7, 9 etc.) are marked by white seizings.

- **(iv) Sounding machine:** The sounding machine is mounted in a sounding boat and can be used up to a maximum depth of 100 ft.



(v) **Fathometer:** Echo-sounding

A fathometer is used for ocean sounding where the depth of water is too much and to make a continuous and accurate record of the depth of water below the boat or ship at which it is installed.



depth, (h) $h = \frac{Vt}{2}$

where, V = speed of sound in water

t = time interval between transmitter and receiver

■■■

8 The Tides

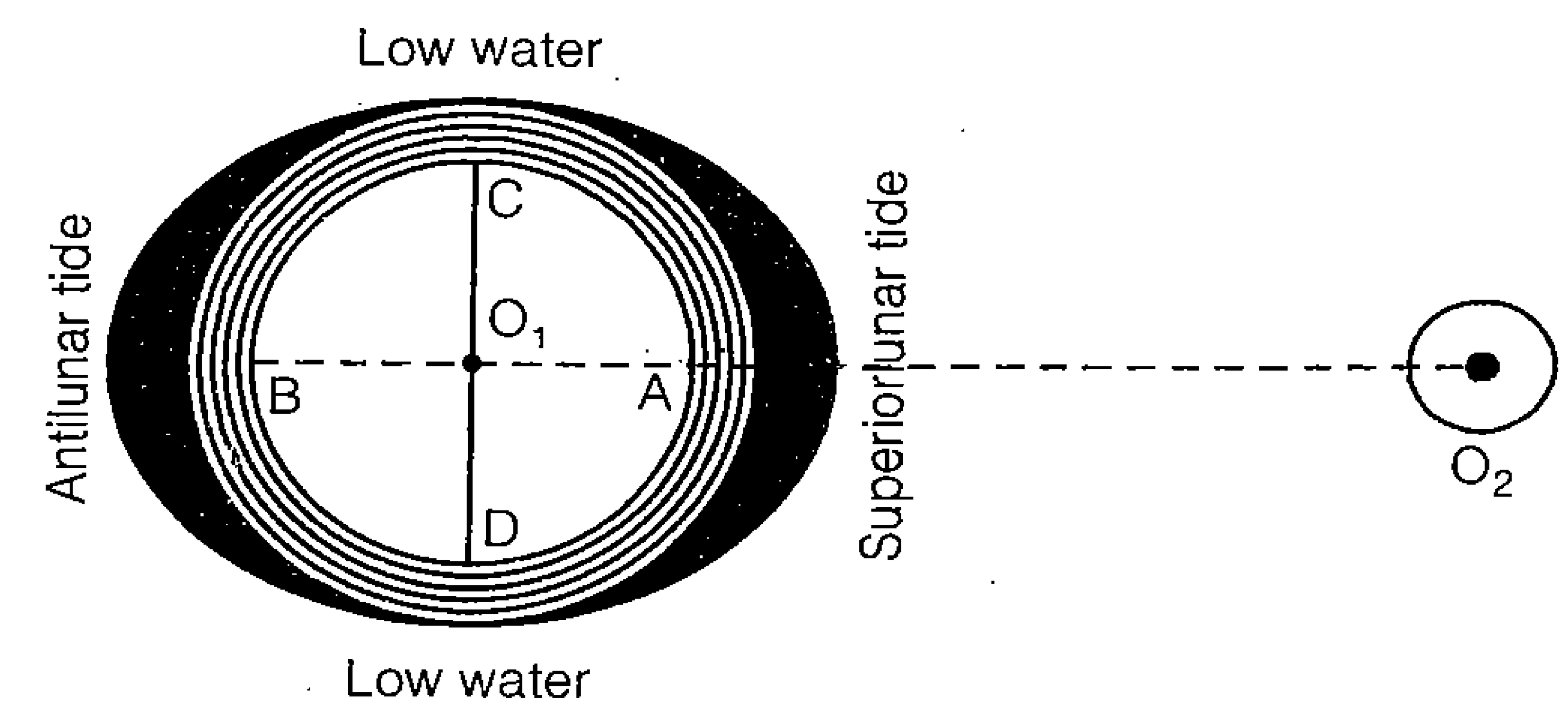
Definition

All celestial bodies exert a gravitational force on each other. These forces of attraction between earth and other celestial bodies (mainly moon and sun) cause periodical variations in the level of a water surface commonly known as tides.

Assumptions Made in the Equilibrium Theory of Tides

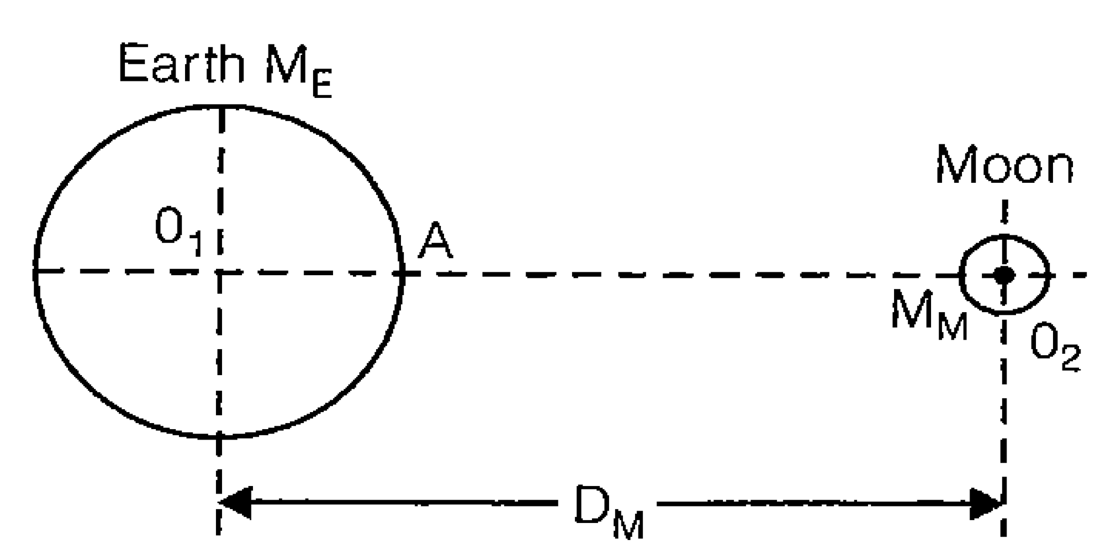
- The earth is covered all round by an ocean of uniform depth.
- The ocean is capable of assuming instantaneously the equilibrium figure required by the tide producing forces.

Lunar tide:



- There are two lunar tides at A and B, two low water positions of C and D.
- The tide of A is called the superior lunar tide or tide of moon's upper transit, while tide at B is called inferior or anti-linear tide.

Solar tide:



- Tide producing force F_M of the Moon on unit mass at A is given by

$$F_M = kM_M\left(\frac{2R}{D_M^3}\right)$$

- Tide producing force F_S of the Sun on unit mass at A is given by

$$F_S = kM_S\left(\frac{2R}{D_S^3}\right)$$

Here, M_S = Mass of Sun
M_M = Mass of Moon
D_M = Mean distance from the centre of earth to the centre of the Moon.
D_S = Mean distance from the centre of earth to the centre of the Sun.
R = Radius of earth.
K = Constant of gravitation

- $\frac{F_S}{F_M} = 0.458$ i.e. Solar tide = 0.458 · Lunar tide
- $M_M = \frac{1}{18} M_E$ and $M_S = 331000\ M_E$

where M_E = Mass of earth

Remember

The age of tide varies for different places, upto a maximum of 3 days, and is reckoned to the nearest 1/4 day. The age of tide is one of the non-harmonic constants.
- Lunitidal interval is the time interval that elapses between the moon's transit and the occurrence of the next high water.
- The time of transit at the given place can be derived by adding 2 m for every hour of west longitude and subtracting 2 m for every hour of east longitude of the place to the time of transit of Greenwich.

Vulgar Establishment

- **Definition:** Vulgar establishment is defined as the value of lunitidal interval on the day of full moon or change of moon.

Mean establishment = Vulgar establishment – lagging correction

Lagging correction = 26 M

Height of tide: (H)

$$H = h + \frac{1}{2}\cdot r\cos\theta$$

where, h = height of mean tide level above datum
r = range of tide

$$\theta = \frac{\text{Interval from high water}}{\text{Interval between high and low water}} \times 180°$$

Prediction of tide with Harmonic constants:

$$V = fH\cos(E - g)$$

where, V = value of constituent at zero hour on the day
H = mean amplitude (half range) of the constituent at the port
f = factor, the value of which is very near to unity and which varies slowly from year to year.
E = angle (same for all ports)
g = constant, special to the port and the constituent

$$E(\text{at zero hour}) = m + d$$

where, m = value of E at zero hour of the first day of each month
d = increment in E from zero hour of the first day of the month to the zero hour of the day.

Symbol for constituent	Description or name	Period
M_2	Lunar semi-diurnal	$\frac{1}{2}$ Lunar day
S_2	Solar semi-diurnal	$\frac{1}{2}$ Solar day
N_2	Larger elliptic-semi-diurnal	—
K_2	Luni-solar diurnal	$\frac{1}{2}$ Sideral day
K_1	Luni-solar diurnal	Sideral day
O_1	Larger diurnal (declinational)	—
P_1	Solar diurnal (declinational)	—
M_4	First overtake of semi-diurnal	$\frac{1}{4}$ Lunar day
MS_4	Compound luni-solar $\frac{1}{2}$ diurnal	—

■■■

Triangulation

9

Definition

The horizontal control in Geodetic survey is established either by triangulation or by precise traverse. In triangulation, the system consists of a number of inter-connected triangles in which the length of only one line is called the base line and the angles of the triangle are measured very precisely.

		First order or 1° Triangulation	Second order or 2° Triangulation	Third order or 3° Triangulation
1.	Average triangle closure →	< 1 seconds	3 seconds	6 seconds
2.	Maximum triangle closure →	≯ 3 seconds	8 seconds	12 seconds
3.	Length of base line →	5 to 15 kilometers	1.5 to 5 km	0.5 to 3 km
4.	Length of the sides of triangles →	30 to 150 kilometers	8 to 65 km	1.5 to 10 km
5.	Actual error of base →	1 in 300,000	1 in 150,000	1 in 750000
6.	Probable error of base →	1 in 1000000	1 in 500,000	1 in 250,000
7.	Discrepancy between two → measures of a section	10 mm $\sqrt{\text{kilometers}}$	20 mm$\sqrt{\text{km}}$	25 mm$\sqrt{\text{km}}$
8.	Probable error of computed → distance	1 in 60000 to 1 in 250000	1 in 20000 to 1 in 50,000	1 in 5000 to 1 in 20,000
9.	Probable error in astronomic → azimuth	0.5 seconds	2.0 seconds	5 seconds

Remember

- When the shape of the triangle is such that any error in the measurement of angle has minimum effect upon the lengths of the calculated side, than such a triangle is called well conditioned triangle.
- The best shape of well condition triangle is Isosceles with base angle equal to 56°14′.
- The triangle having angle <30° and angle > 120° should be avoided.

- **Criterion of strength of figure**

The strength of figure is a factor to be considered in establishing a triangulation system for which the computation can be maintained within a desired degree of precision.

The square of a probable error (L^2) that would occur in the sixth place of the logarithm of any side,

$$L^2 = \frac{4}{3} d^2 R$$

where, $R = \frac{D-C}{D}\Sigma\left[\delta_A^2 + \delta_A\,\delta_B + \delta_B^2\right]$

d = Probable error of an observed direction in seconds

D = Number of directions observed (forward and/or backward)

δ_A = Difference per second in the sixth place of a logarithms of the sine of the distance angle A of each triangle

δ_B = Same as δ_A but for the distance angle B

C = Number of angles and side conditions

C = (n′ – s′ + 1) + (n – 2s + 3)

n = Total number of lines

n′ = Number of lines observed in both directions

s = Total number of stations

s′ = Number of occupied stations

(n′ – s′ + 1)= Number of angle conditions

(n′ – 2δ + 3)= Number of side conditions

Signals and Towers

- **A signal is a device erected to define the exact position of an observed station.**

A. **Non Luminous Signals:** Diameter of signal in cms =1.3 D to 1.9 D
Height of signal in cms =13.3 D
where, D = distance in kms (Length of sight) for non luminous signals

B. **Luminous or Sun Signals:** Used when length of sight distance > 30 Kms.
Phase of Signals: It is the error of bisection which arises, when the signal is partly in light and partly in shade.

- Correction

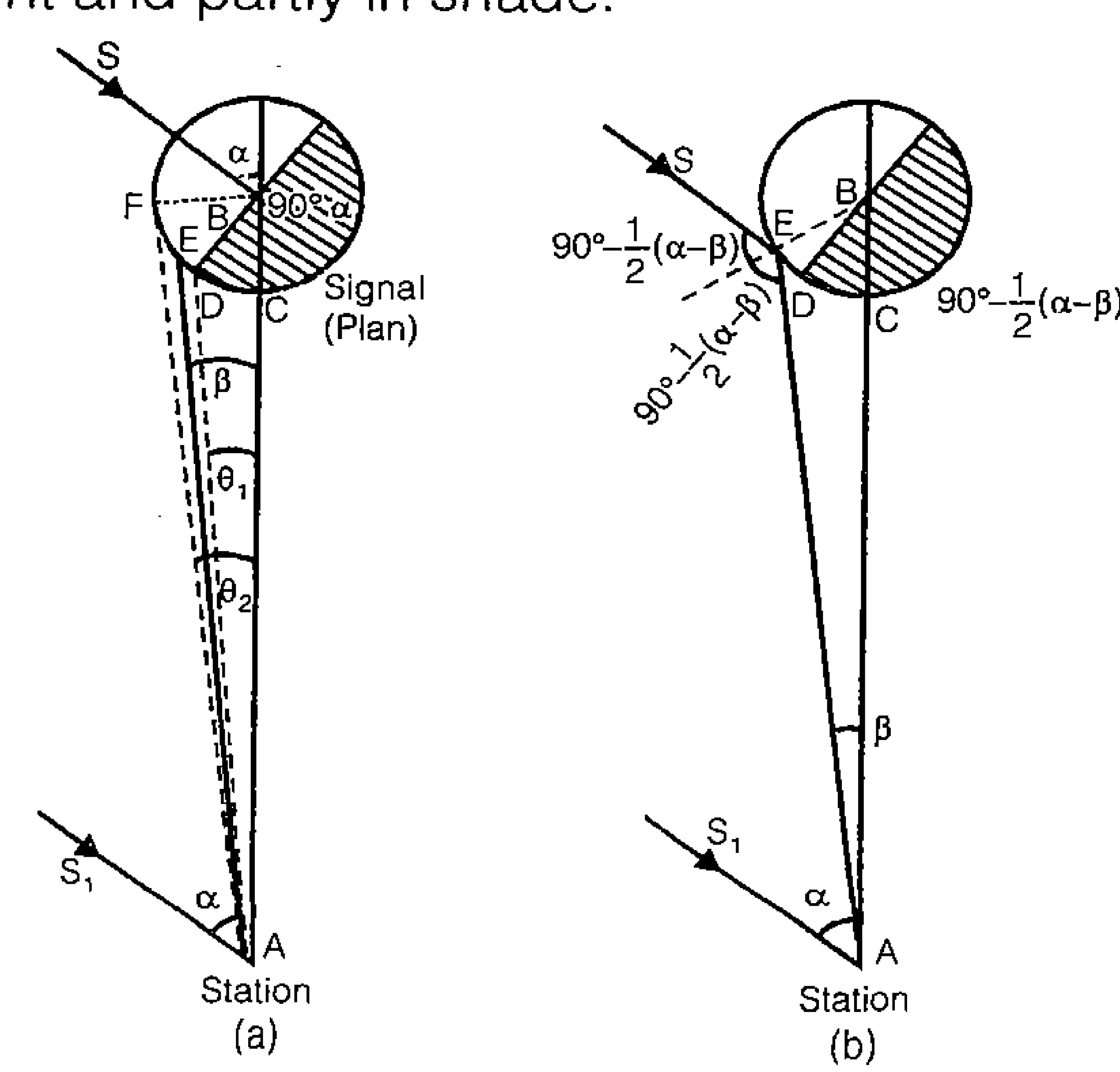
(i) When observation is made on the bright portion.
Phase correction,

$$\beta = \frac{r\cos^2\frac{\alpha}{2}}{D}\ \text{radians}$$

α = Angle which the direction of sun makes with line of sight.

r = radius of the signal.

D = Distance of sight.



(ii) When the Observation is made on the bright line:

$$\beta = \frac{r\cos\frac{\alpha}{2}}{D} \text{ radians}$$

Routine of Triangulation Survey

The routine of triangulation survey generally consists of the following operations:

1. Reconnaissance
2. Erection of singles and towers
3. Measurement of base lines
4. Measurement of horizontal angles
5. Astronomical observations at Laplace stations, and
6. Computations

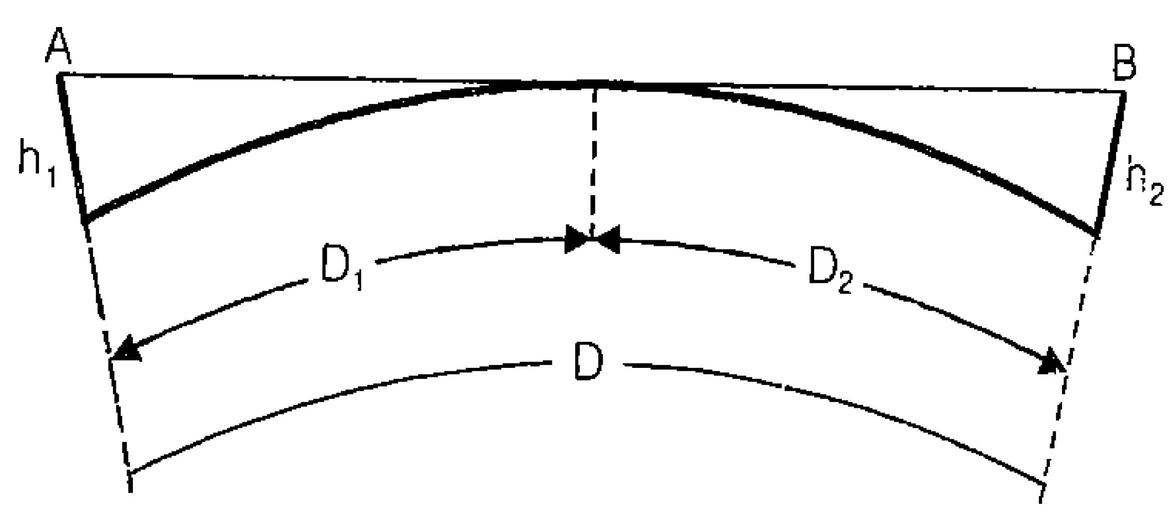
Intervisibility and Height of Stations

(a) **The distance between the stations:** If here is no obstruction due to intervening ground, the distance of the visible horizon from a station of known elevation above datum is given by

$$\boxed{h = \frac{D^2}{2R}(1-2m)}$$

where, h = height of the station above datum
D = distance to the visible horizon
R = mean radius of the earth

(b) **Relative elevation of stations:** If there is no obstruction due to intervening ground, the formula $h = \frac{D^2}{2R}(1-2m)$ may be used to get the necessary elevation of a station at distance, so that it may be visible from another station of known elevation



Let, h_1 = known elevation of station A above datum
h_2 = required elevation of B above datum
D_1 = distance from A to the point of tangency
D_2 = distance from B to the point of tangency
D = the known distance between A and B

then, $h_1 = 0.06728\, D_1^2$

$$D_1 = \sqrt{\frac{h_1}{0.0728}} = 3.8553\sqrt{h_1}$$

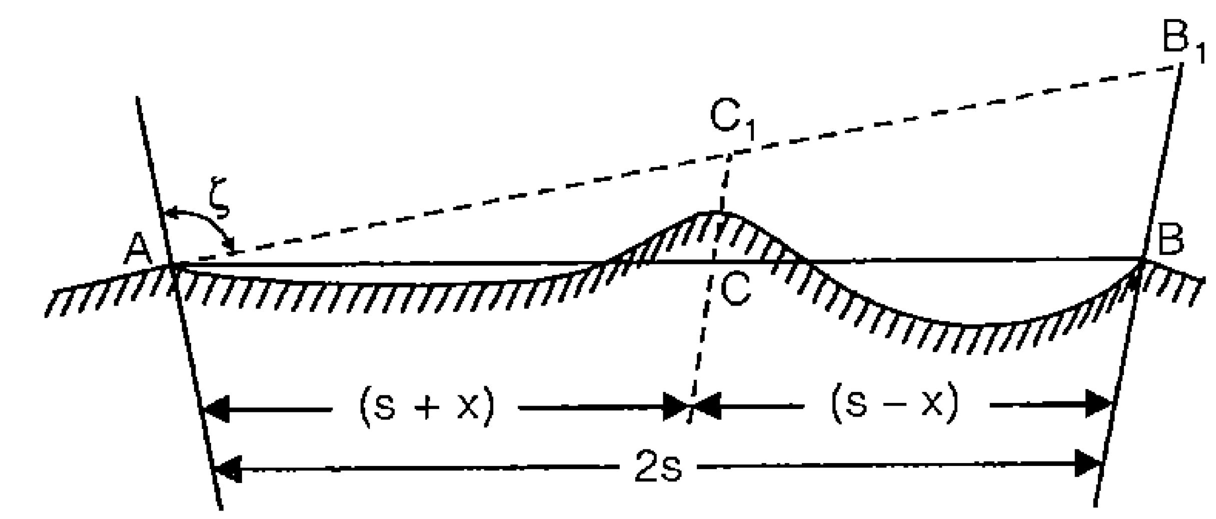
where D_1 is in km and h_1 is in meters, $D_2 = D - D_1$

$$h_2 = 0.06728\, D_2^2 \text{ meters}$$

(c) **Profile of the intervening ground:** In the reconnaissance, the elevations and positions of peaks in the intervening ground between the proposed stations should be determined. A comparison of their elevations should be made to the elevation of the proposed line of sight to ascertain whether the line of sight is clear off the obstruction or not. The problem can be solved by using the principles discussed in the factors (1) and (2) above, or by a solution suggested by Captain G.T. McCaw. The former method will be clear from the worked out examples.

Captain GT McCaw's Method

Let, h_1 = height of station A above datum
h_2 = height of station B above datum
h = height of line of sight at the obstruction C



2s = distance between the two stations A and B
(s + x) = distance of obstruction C from A
(s − x) = distance of obstruction C from B
ζ = zenith distance from A to B

The height h of the line of sight at the obstruction is given by

$$\boxed{h = \frac{1}{2}(h_2 + h_1) + \frac{1}{2}(h_2 - h_1)\frac{x}{s} - (s^2 - x^2)\operatorname{cosec}^2\zeta\left(\frac{1-2m}{2R}\right)}$$

$$\boxed{\operatorname{cosec}^2\zeta = 1 + \frac{(h_2 - h_1)^2}{4s^2}}$$. The expression $\boxed{\frac{1-2m}{2R} = 0.574}$.

If x, s and R are substituted in miles, and h_1, h_2 and h are in feet.

and $\boxed{\frac{1-2m}{2R} = 0.06728}$

If x, s and R are in km and h_1, h_2 and h are in meters.

■■■

Field Astronomy

10

Relationship between Coordinates

If Z = Zenith

δ = Declination (-ve-south + ve North equation)

θ = Latitude of the observer.

Then $\theta = \delta + Z$

$\theta = \alpha - p$.....(If the star is north of zenith but above the pole)

$\theta = \alpha + p$......(If the star is north of zenith but below the pole)

α = meridian altitude of star

P = Polar distance

- Latitude of Pole = Latitude of observer (always)
- Hour Angle of Equinox = Hour Angle of star + R.A. of star where, R.A.= Right Ascension
- 1 Tropical year = 365.24422 mean solar days
- 1 SIDEREAL days = 366.2422 sidereal days.

Remember

- 1 Solar day = $1 + \frac{1}{365.2422}$ Sidereal days

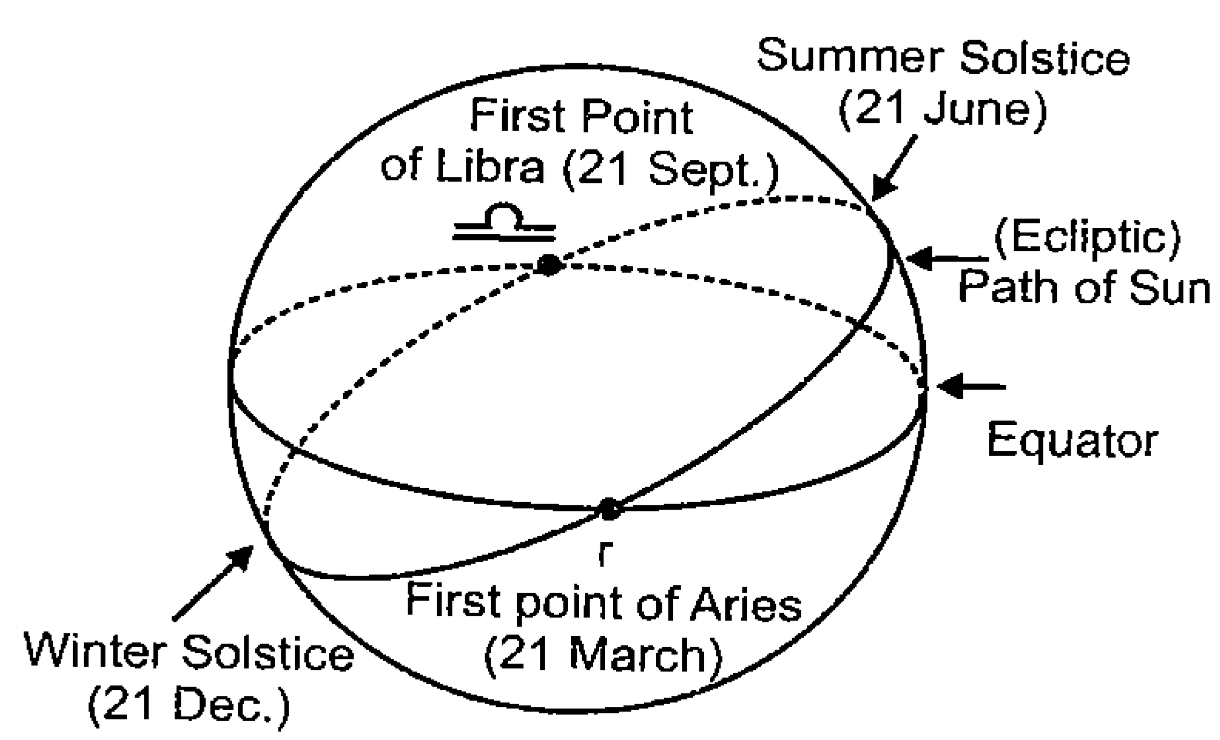
1 Solar days = 24h 3m 56.56s Sidereal time.

To convert the mean solar time to the sidereal time, we will have to add a correction of 9.8565 second per hour of mean time this correction is called the acceleration.

(i) **First point of Aries and Libra:** First point of Aries is the point where sun crosses the equator from south to north on 21st March. On this day and nights are of equal direction.

First point of Libra is point where sun crosses the equator from north to south on 21/22 Sept.

- 1st point of Aries – also known as – vernal equinox.
- 1st point of Libra - also Autumnal equinox.



- Points on the ecliptic at which north and south declination is max. are known as - Solstices.

  21st June → Summer solstices

  21/22 December → Winter solstice

  r → first point of Aries (Vernal Equinox)

  Ω → first point of Libra (Autumnal Equinox)

(ii) **Latitude (θ) :** Angular distance of any plane north or south of the Equator. Angle between zenith and celestial equator is called latitude.

(iii) **Co-Latitude :** Measured angle between pole to zenith point for any place is called colatitude.

colatitude (c) = 90 – latitude = $90 - \theta$.

(iv) **Longitude:** Angle between meridian of a place from a fixed (prime) meridian is called longitude universally adopted prime meridian is Greenwich angle is measured 0° to 180° East or west of prime meridian.

(v) **Altitude (α):** Altitude of any celestial body or star is angular distance from horizon, measures on the vertical circle passing through the body.

(vi) **Co-Altitude (z):** Angular distance of body from zenith also called zenith distance.

$z = 90 - \alpha = 90$ – Altitude.

(vii) **Azimuth (A):** Angle between vertical circle passing through the body from observer's meridian (z-p line).

(viii) **Declination (δ):** Angular distance of a body from the plane of celestial equator, measured along declination circle, declination circle is great circle passing through body and celestial pole. Varies from 0° to 90° (Nors)

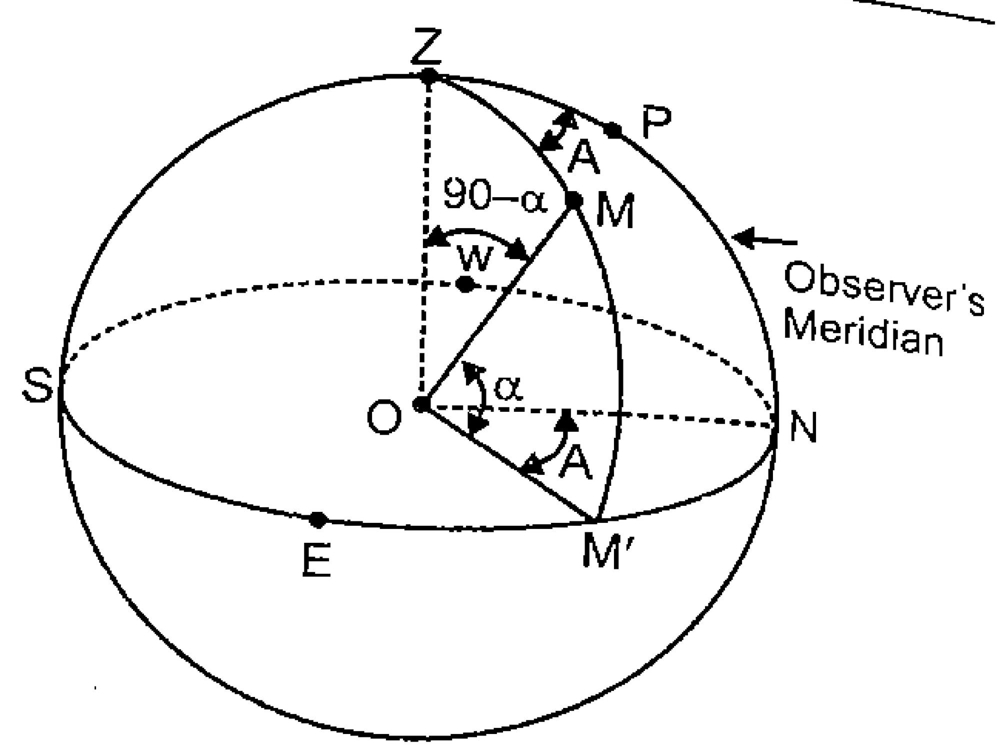
(ix) **Co-declination (ρ) or Polar distance:** Angular distance of heavenly body from Pole.

$\rho = 90 - \delta = 90°$ – declination.

(x) **Hour Angle (H):** Angle between observer's meridian and declination circle passing through the body. But measured from south in westward direction.

(xi) **Right Ascension (R.A.):** Angular distance measured eastward from first point of Aries is called right ascension. It is angle between hour circle passing from body to hour circle passing from first point of Aries, measured in east direction.

- **Coordinate System**
  - (i) **Horizon System (Altitude and Azimuth system):** In this system, zenith is the reference point, and plane of reference is horizon.

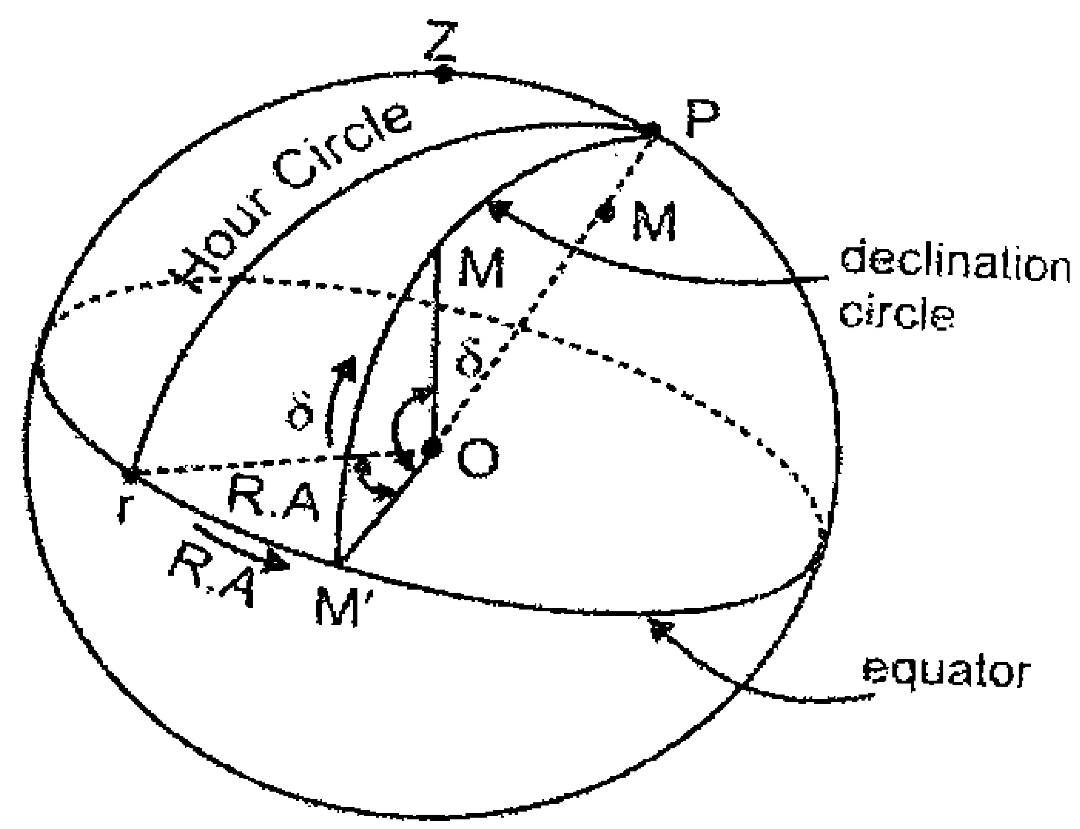


- **Angle**
  - (i) **Azimuth:** Horizontal angle between two great circle (1) observer's meridian and great circle passing from the point M. Angle NOM′ = A is called Azimuth.
  - (ii) **Altitude:** Angle above or below the horizon (LM′ OM = α) is called altitude.
    - Azimuth is the horizontal angle, whereas altitude is the vertical angle.
    - This system of measurement is dependent on the position of the observer.
    - Zenith distance (ZM) or ZOM, is the angular distance of the object form zenith.

    Zenith distance = ZM = 90 − α

  - (iii) **Declination:** Angle above or below the equator wrt to pole is called declination.

    (δ) = ∠ MOM.

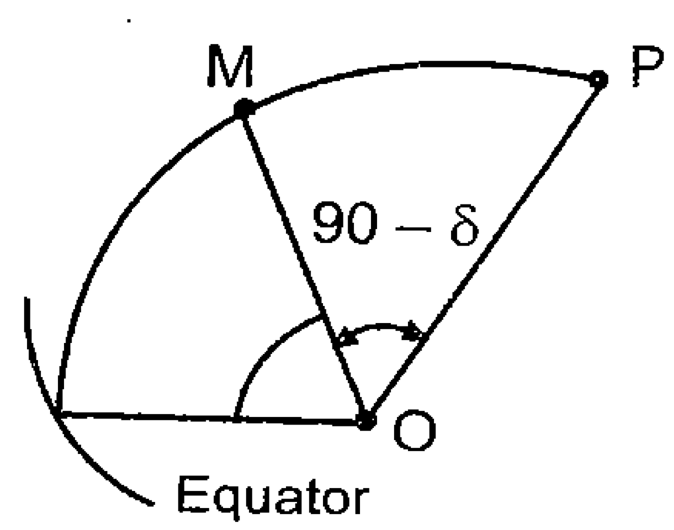


  - (iv) **Right Ascension:** Right ascension is the angle measured along the equator w.r.t. first point of Aries (r) going towards east

    R. A. = Angle γOM′.

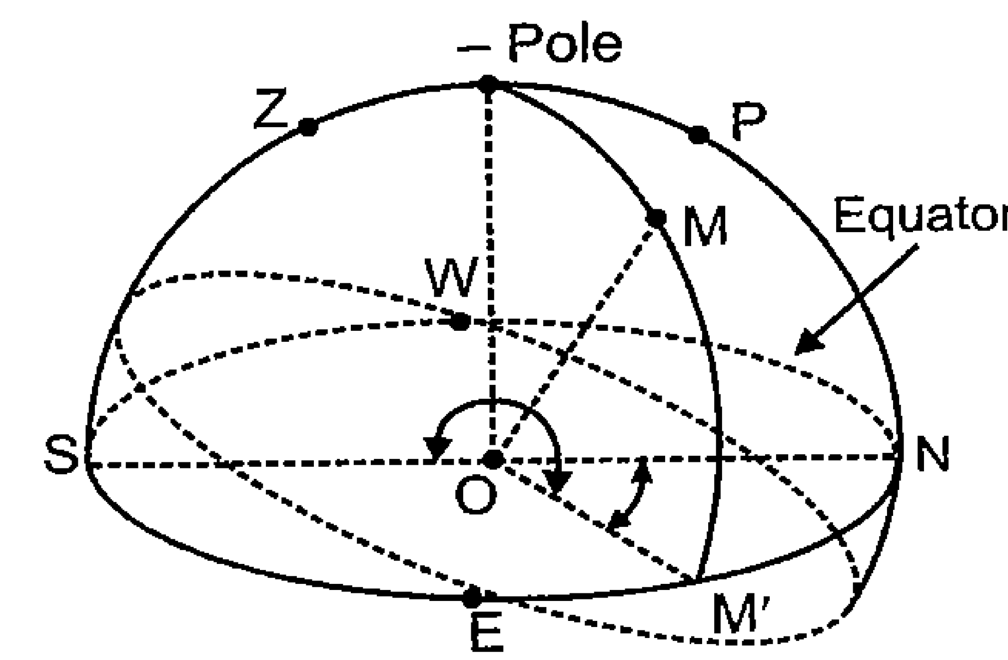
  - (v) **Co-declination:** Angular distance from pole is called co-declination.

    angle POM is called co-declination = 90 − δ.
    - This system is independent of position of observer.

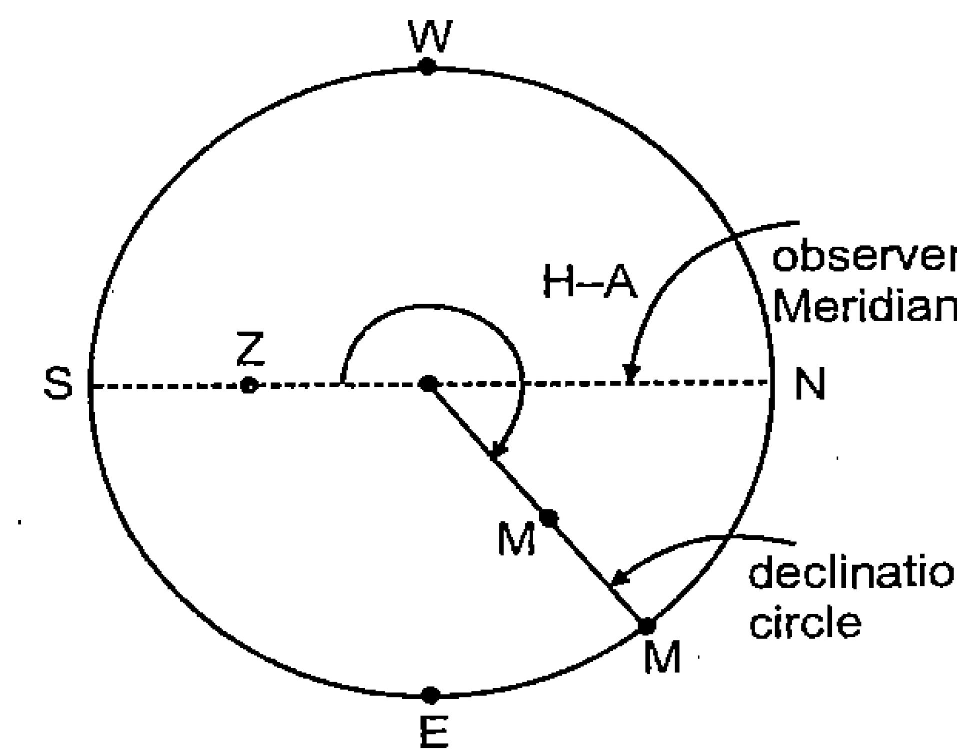


  - (vi) **The Dependent Equatorial System**

    Two reference planes are
    1. Declination circle
    2. Equator.



    Two Angles:
    1. **Declination:** Same as above
    2. **Hour angle:** Angle measured between two great circles, 1. observer's meridian and 2. great circle passing from the point and pole is called hour angle. Hour angle is measured from south going towards west upto the declination circle angle SOM is Hour angle.



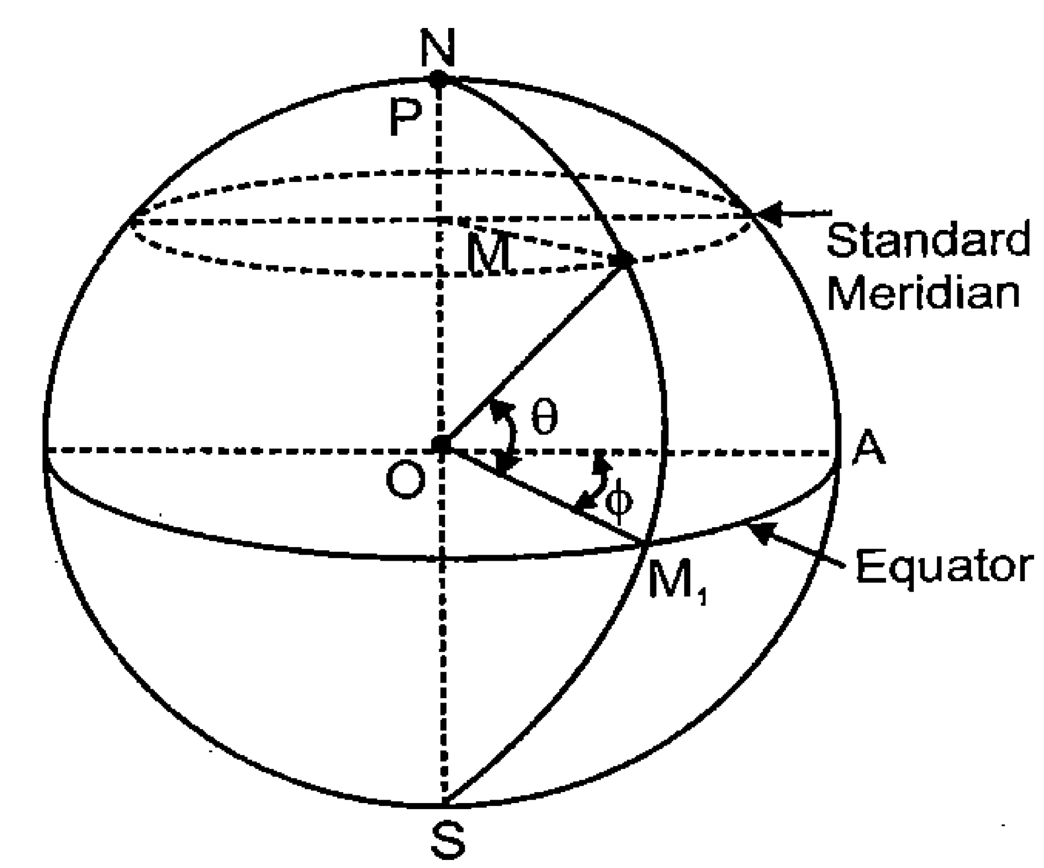
Terrestrial Latitude and Longitude

This system is used for locating position of any point on earth surface.

Axis of the Earth: Axis joining north and south pole of the earth.

Meridian: Any great circle whose plane passes through axis of earth is called Meridian or Terrestrial Meridian.

Equator: Great circle perpendicular to axis of earth is called equator.



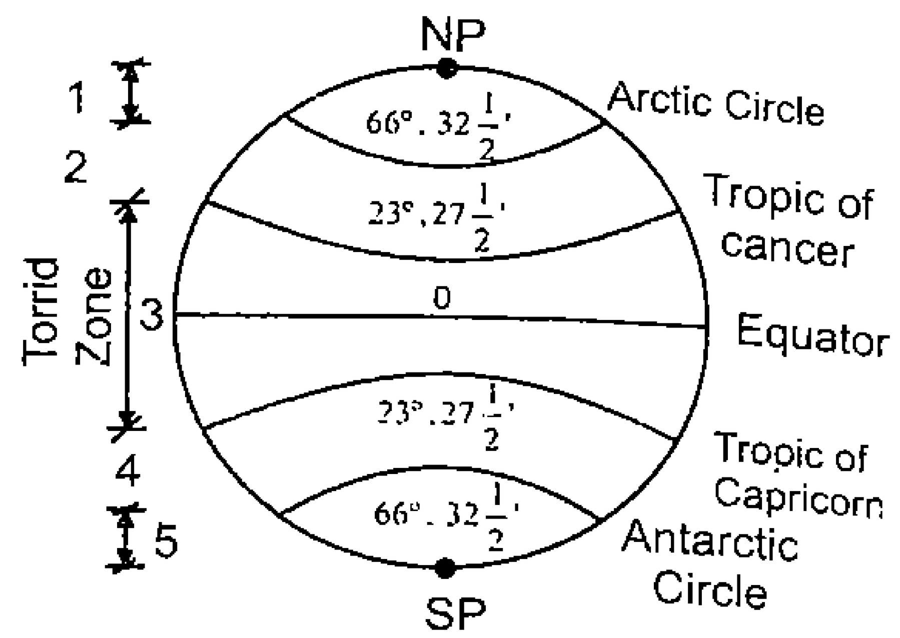
Latitude(θ): Vertical angle above or below equator is called latitude (angle MOM_1).

Longitude(ϕ): Horizontal angle between great circle (Meridian) passing through place and standard meridian is called longitude (ϕ). Angle AOM_1 is longitude. For Earth, prime meridian or standard meridian is meridian passing through Greenwich. All points on a meridian have same longitude.

Parallel of latitude: Parallel of latitude through a point is small circle passing through that point and parallel to equator. All the points on a parallel of latitude hour have same latitude.

Zones of the Earth:

1. North frigid zone
2. North Temperate zone
3. Torrid zone
4. South temperate zone
5. South frigid zone



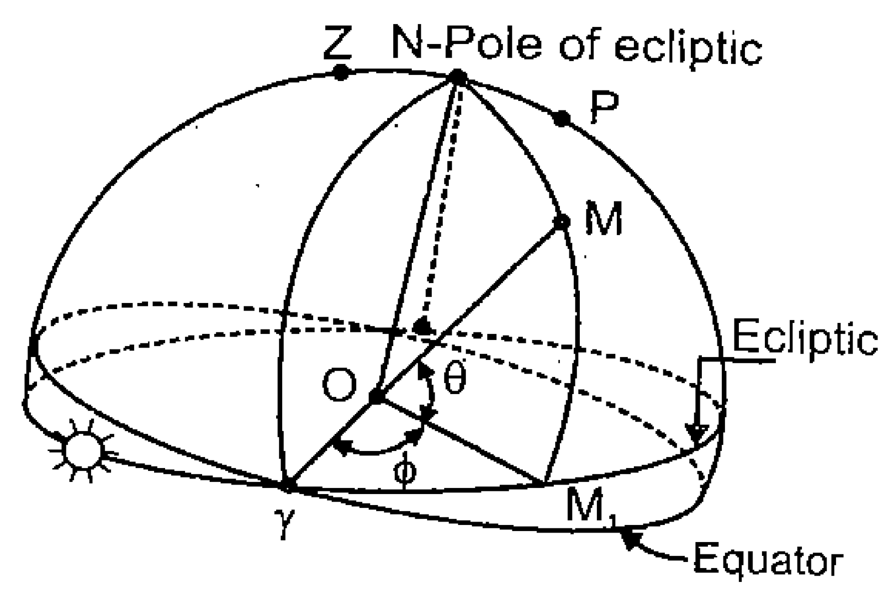
Nautical Mile

Nautical Mile = distance on arc of great circle by 1 minute angle at centre of earth.

$$= \frac{2\pi R}{360} \times \frac{1}{60} = \frac{2\pi \times 6370}{360 \times 60} = 1.852\,\text{km}$$

Celestial Latitude and Longitude System: Primary plane of reference are:

1. Plane of ecliptic - Horizontal plane.
2. Great circle passing through first point of Aries and perpendicular to plane of ecliptic.
   - Vertical plane



Coordinates are :

(i) Celestial latitude

(ii) Celestial longitude

Celestial latitude: Angle $MOM_1 = \theta$ is latitude. It is the vertical angle measured above or below arc of ecliptic.

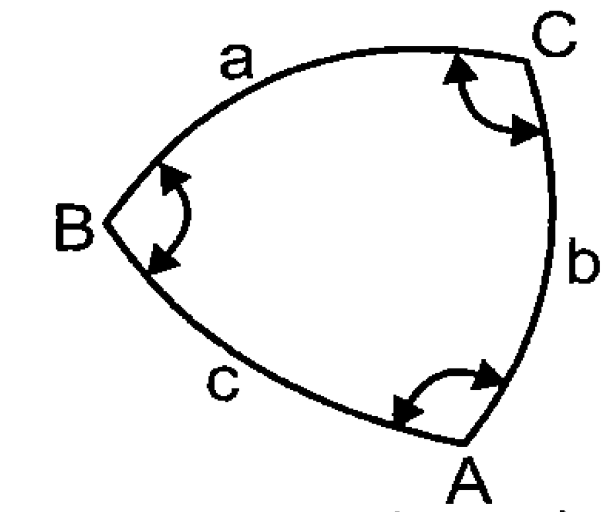
Celestial longitude: Horizontal angle measured from first point of Aries to the east. This angle may be between 0° to 360° angle γOM_1 is the longitude.

North Pole of Ecliptic: Point on great circle that is perpendicular to ecliptic and passes through first point of Aries. It is the point where all great circles perpendicular to ecliptic meets in north of ecliptic.

As per this system latitude of sun is always zero. So this system is very useful to fix the position of sun. This system is also independent of place of observation.

Spherical Triangle

- a, b, and c are the sides of spherical triangle.



- a is the angle formed at centre of sphere by arc BC.
- A is the angle between two great circle passing from AB and AC.

- **Properties:** The following are the properties:
  1. Any angle is less than 2 × 90°(π)
  2. Sum of three angles is more than two right angles and less than six right angles.

$$2\times\frac{\pi}{2} < (A+B+C) \le 6\times\frac{\pi}{2}$$

$$\pi < (A+B+C) \le 3\pi$$

  3. Sum of any two sides > third side.
  4. If some of any two sides is equal to two right angle (π), Sum of angles opposite them is also equal to 2 × 90° or π
     If $a + b = 2 \times 90° = \pi$
     then $A + B = 2 \times 90° = \pi$
  5. The smaller angle is opposite the smaller side.

- **Formulae**

1. $\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$

2. $\cos A = \frac{\cos a - \cos b.\cos c}{\sin b.\sin c}$

   $\cos a = \cos b.\cos c + \sin b.\sin c.\cos A$

3. $\cos a = \frac{\cos A + \cos B.\cos C}{\sin B.\sin C}$

   $\cos A = -\cos B.\cos C + \sin B.\sin C.\cos a$

4. $\sin\frac{A}{2} = \sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin b.\sin c}}$

   $\cos\frac{A}{2} = \sqrt{\frac{\sin S.\sin(S-a)}{\sin b.\sin C}}$

   $\tan\frac{A}{2} = \sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin S.\sin(S-a)}}$

   $S = \frac{a+b+c}{2}$

5. $\sin\frac{a}{2} = \sqrt{\frac{-\cos S\cos(S-A)}{\sin B.\sin C}}$

   $\cos\frac{a}{2} = \sqrt{\frac{\cos(S-B)(S-C)}{\sin B.\sin C}}$

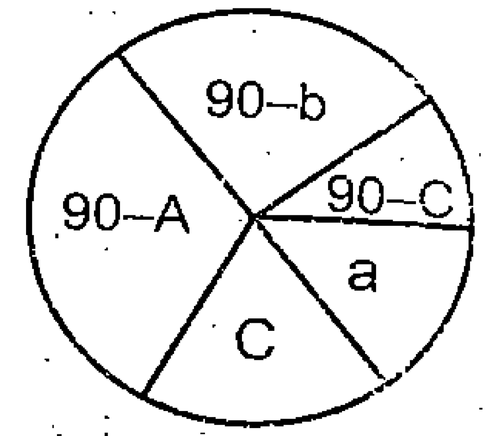
   $\tan\frac{a}{2} = \sqrt{\frac{-\cos S.\cos(S-A)}{\cos(S-B)\cos(S-C)}}$

   $S = \frac{A+B+C}{2}$

6. $\tan\frac{1}{2}(a+b) = \dfrac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\frac{1}{2}C$

$$\tan\frac{1}{2}(a-b) = \frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\cot\frac{1}{2}C$$

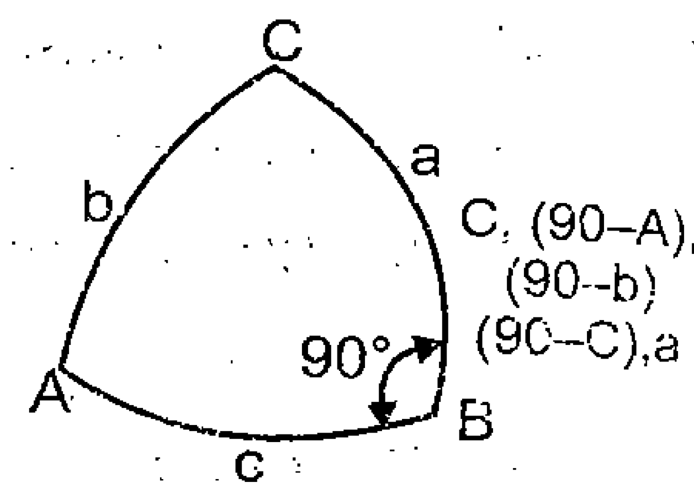
Napier's Rule (Applicable for right angled spherical triangle):



sin (middle) = tan (adjacent$_1$) × tan (adjacent$_2$)

sin (middle) = cos (app.$_1$) × cos (app.$_2$)

1. sin c = tan (90 – A). tan a
   = cot A. tan a
2. sin c = cos (90 – b). cos (90 – c)
   = sec. b sec.C



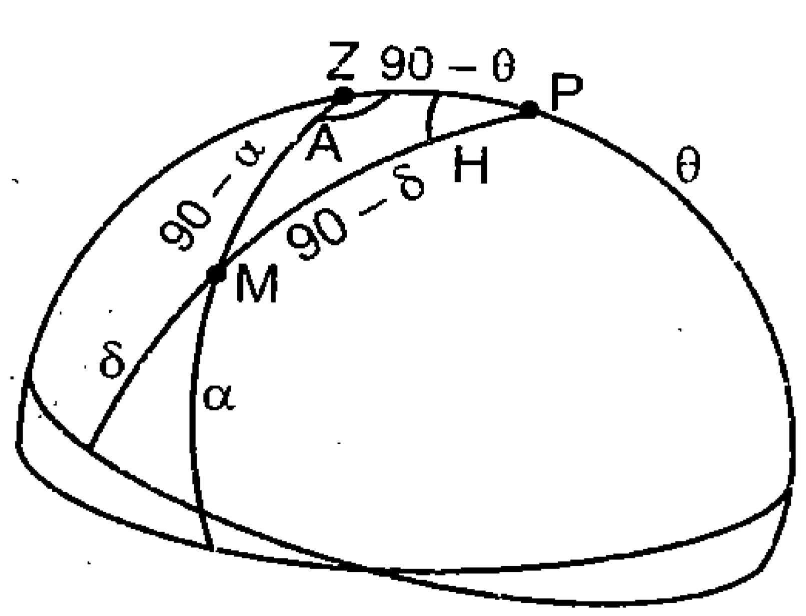
- **Spherical excesses**

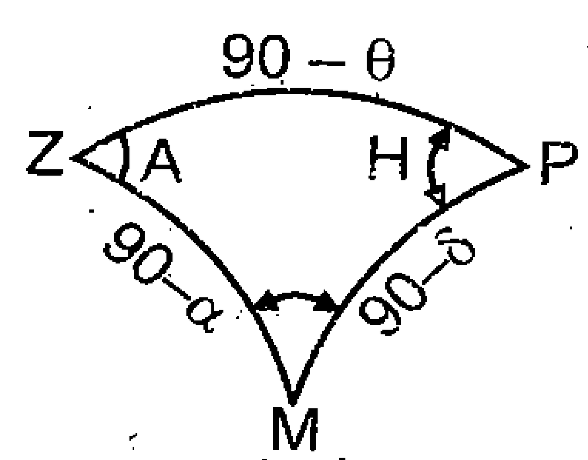
  $E = (A + B + C) - 180$

  area of spherical triangle

  $$\Delta\text{area} = \frac{\pi R^2 \times E}{180}$$

- **Astronomical Triangle**

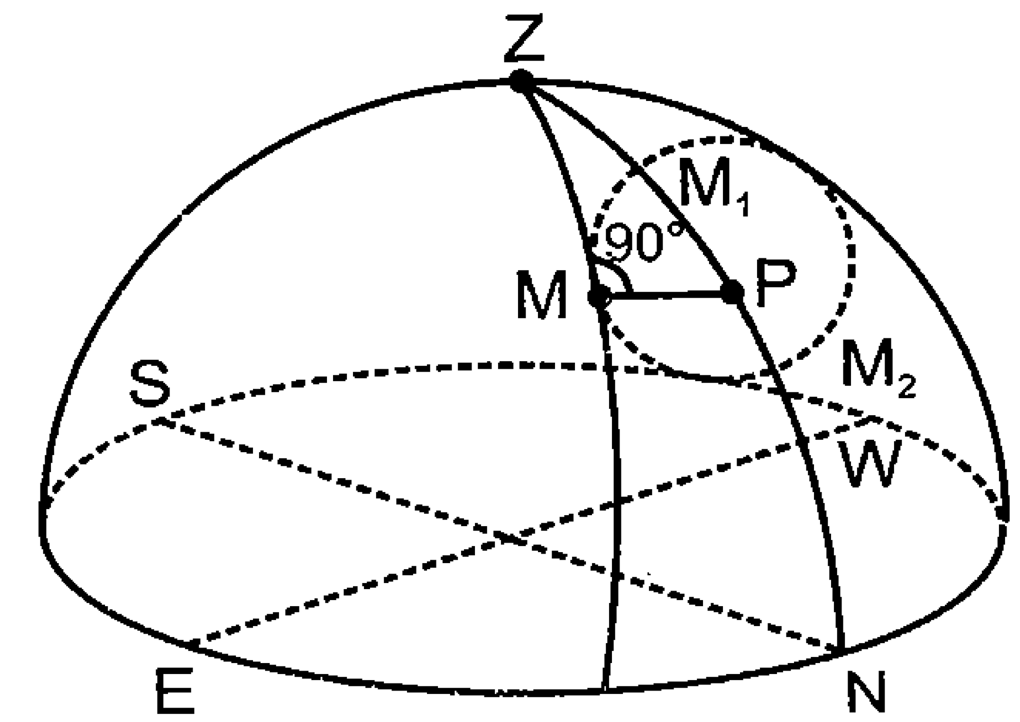


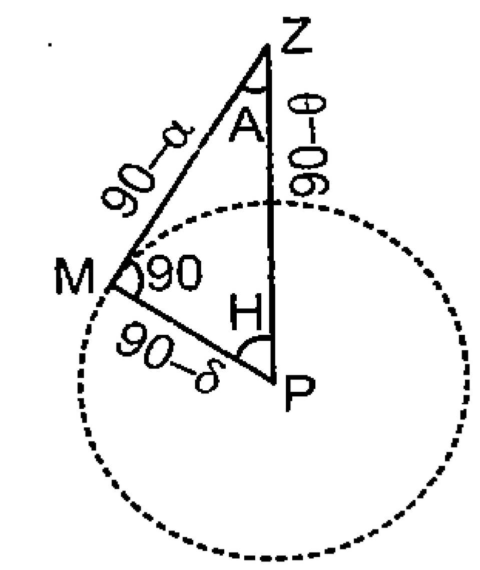


Different Position of Star w.r.t to Observer's Meridian

- **Star at Elongation**

  A star is said to be at Elongation, where it is at greatest distance from standard meridian (Z–P line). Azimuth of the star is maximum in this position.





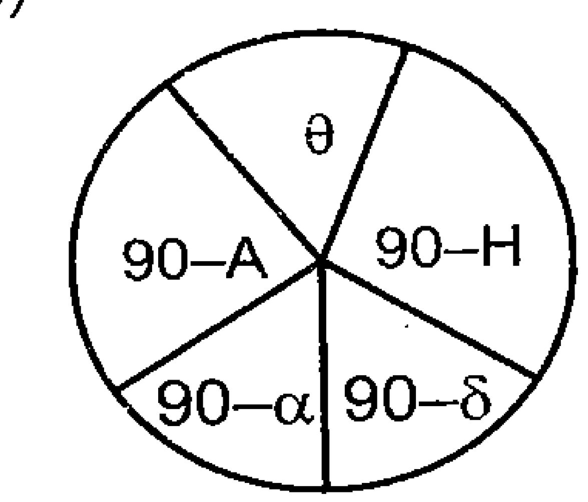
When distance is maximum to the east of observer's meridian, it is called star at eastern elongation.

And when distance is maximum to west of observer's meridian, it is called star at western elongation.

Applying Napier's Rule for Astrotriangle ZMP.

Five parts of Napier's circle will

$(90 - \alpha)$, $(90 - A)$, $90 - (90 - \theta) = \theta$ $(90 - H)$, $(90 - \delta)$



(i) Hour Angle (H)

$\sin(90 - H) = \tan(90 - \delta).\tan\theta$

$\cos H = \cot\delta.\tan\theta$

(ii) Altitude (α)

$\sin\theta = \cos(90 - \alpha).\cos(90 - \delta) = \sin\alpha.\sin\delta$

$$\text{so } \sin\alpha = \frac{\sin\theta}{\sin\delta} = \sin\theta \times \text{cosec}\delta$$

(iii) Azimuth (A)

$$\left.\begin{aligned}\sin(90-\delta) &= \cos(90-A)\cos\theta \\ \cos\delta &= \sin A.\cos\theta\end{aligned}\right\} \Rightarrow \sin A = \frac{\cos\delta}{\cos\theta}$$

- **Star at Culmination**

  Path of a star crosses the observer's meridian twice, in one revolution around the pole. A star is said to be at culmination, where it crosses the observer's meridian in above figure. M_1 is position of upper culmination and M_2 is position of lower culmination.

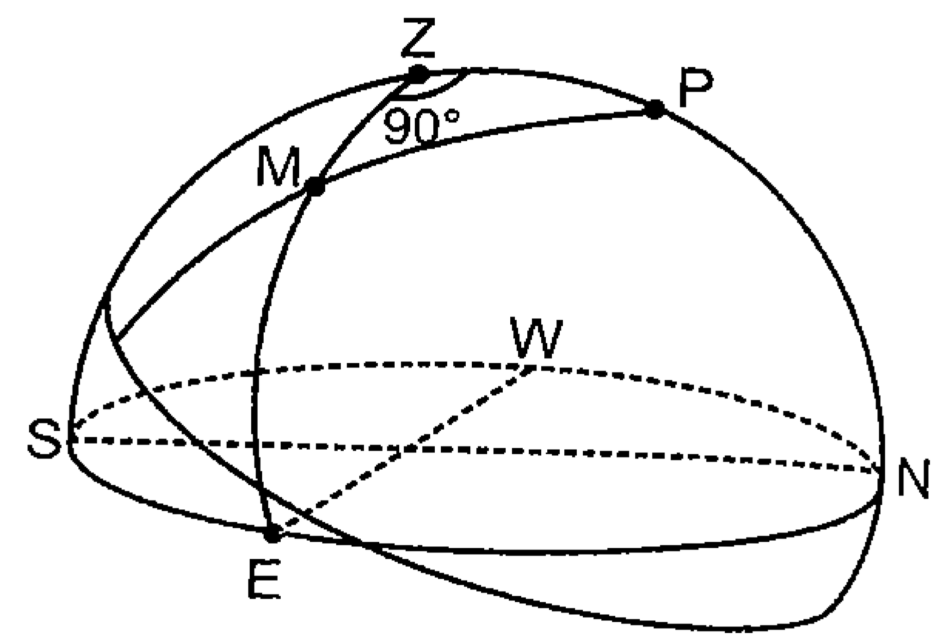
- **Star at Prime Vertical**

  A star is said to be at prime vertical when it crosses the prime vertical. At this position Azimuth of star i.e. angle at zenith is equal to 90°.

Five parts of Napier's circle will be

$(90-\theta)$, $(90-H)$, δ, $(90-M)$, $(90-\alpha)$

Given are – δ and θ



(i) Calculation of Hour angle (H)

$\sin(90-H) = \tan\delta . \tan(90-\theta)$

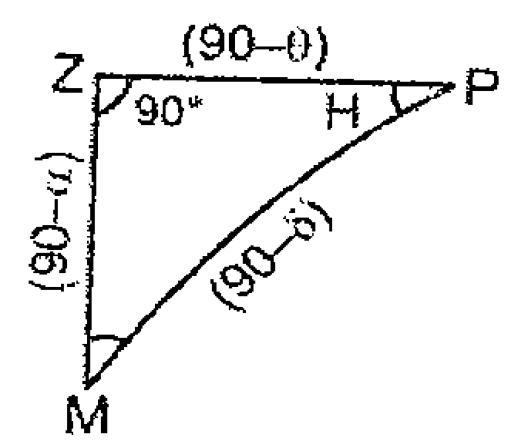
$\cos H = \tan\delta . \cot\theta$

(ii) Altitude (α)

$\sin\delta = \cos(90-\theta) . \cos(90-\alpha)$

$\sin\delta = \sin\theta . \sin\alpha$

$\sin\alpha = \sin\theta . \text{cosec}\,\delta$.



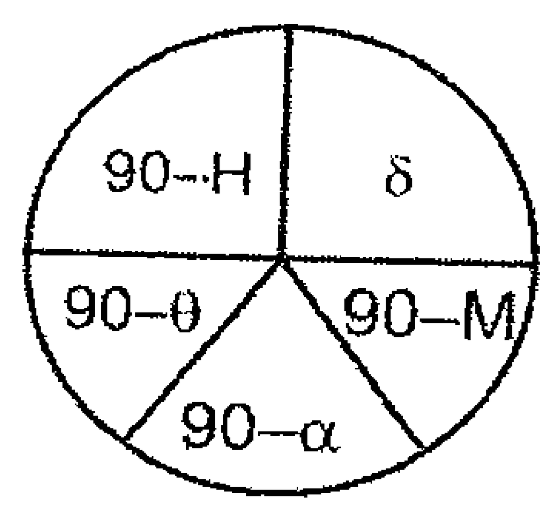
- **Star at Horizon**

When star is at Horizon, it's altitude will be equal to zero.

$\alpha = 0$ so $ZM = 90°$

$PM = 90-\delta$, $ZP = 90-\theta$

$\angle P = H$ $\angle Z = A$.

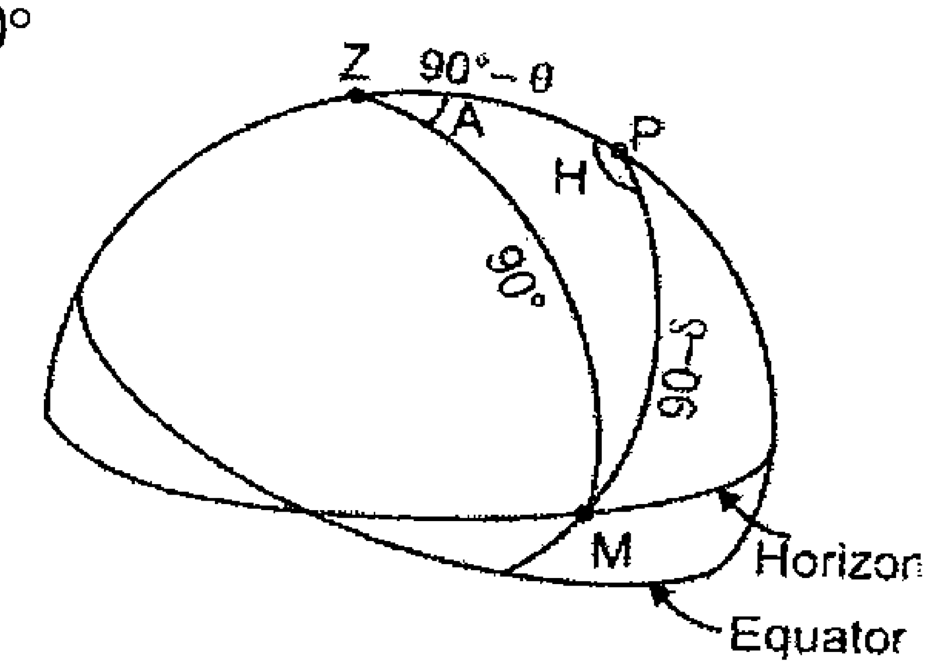


Azimuth.

$$\cos A = \frac{\cos(90-\delta)-\cos(90-\theta).\cos 90°}{\sin(90-\theta).\sin 90°}$$

$$= \frac{\cos(90-\delta)-0}{\cos\theta.1}$$

$$= \frac{\sin\delta}{\cos\theta} = \sin\delta.\sec\theta$$



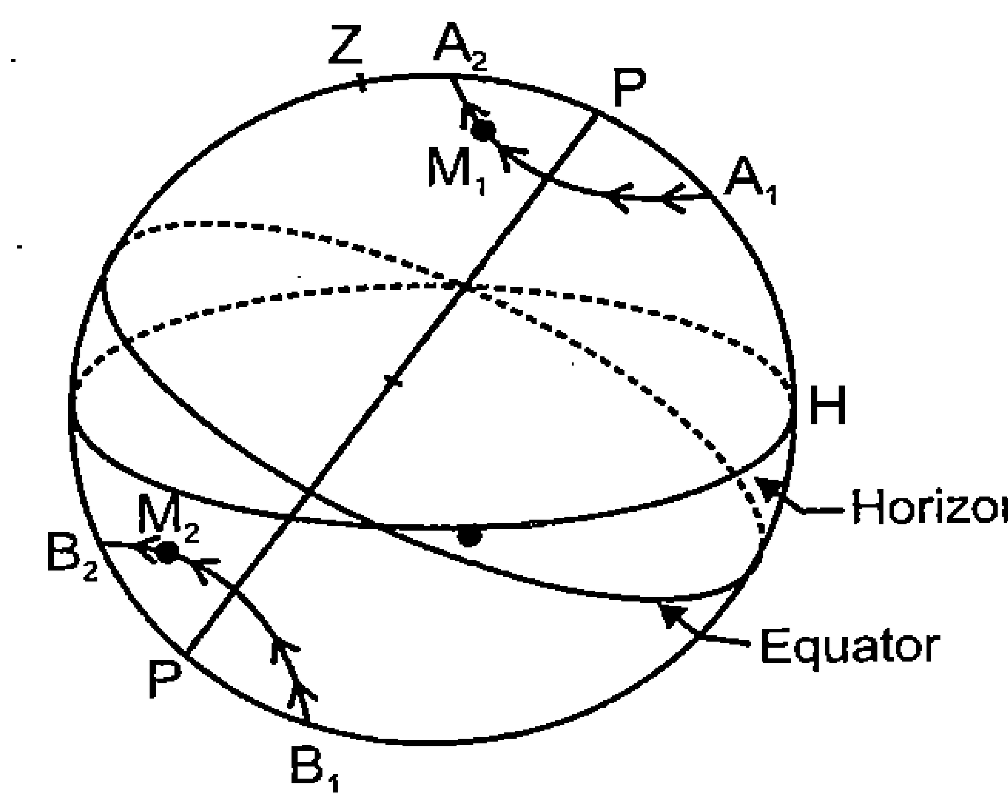
Hour angle

$$\cos H = \frac{\cos 90-\cos(90-\theta).\cos(90-\delta)}{\sin(90-\theta)\sin(90-\delta)}$$

$$= -\tan\theta . \tan\delta$$

- **Circumpolar star**

The stars which remain always above the horizon (or below the horizon) and also do not set. Any time (when above the horizon) or do not rise any time (when below the horizon), are called circumpolar star.



Arrow position, H_1 is a circumpolar star above the horizon, and M_2 is circumpolar star below the horizon.

For circumpolar star, distance of star from pole PA_1 should be less than distance of pole from horizon.

$PA_1 < PH$ and or $(90-\delta) < \theta$ so $\delta > 90-\theta$.

So declination of a circumpolar star is always greater than the colatitude of the place of observation.

- **Relation Between various coordinate**

1. Latitude of place (θ) and altitude of Pole (α_P)

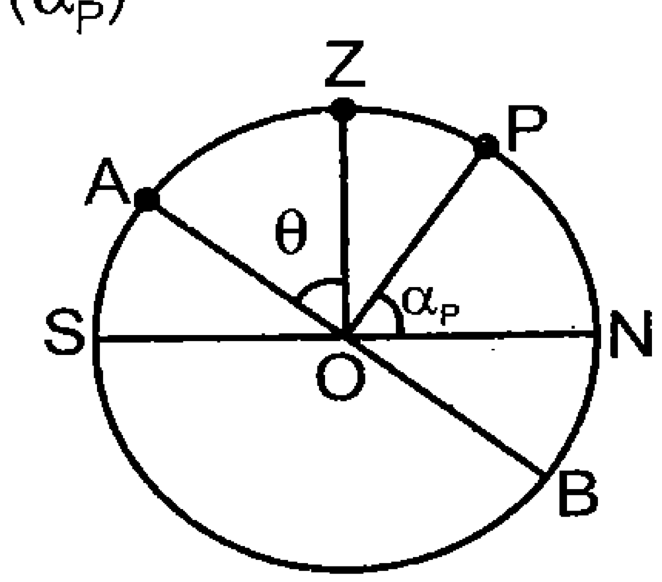
   $\angle ZOA = \theta$ = latitude of place

   $\angle PON = \alpha_P$ = Altitude of pole

   As $\angle ZOP = \angle NOB$. (from geometry)

   So, $\angle ZOA = \angle PON$

   $\theta = \alpha_P$



2. Latitude of place (θ), declination (δ) and Altitude (α) of celestial Body:

   For M_1, $PM_1 = 90-\delta = p$

   $M_1N = \alpha$

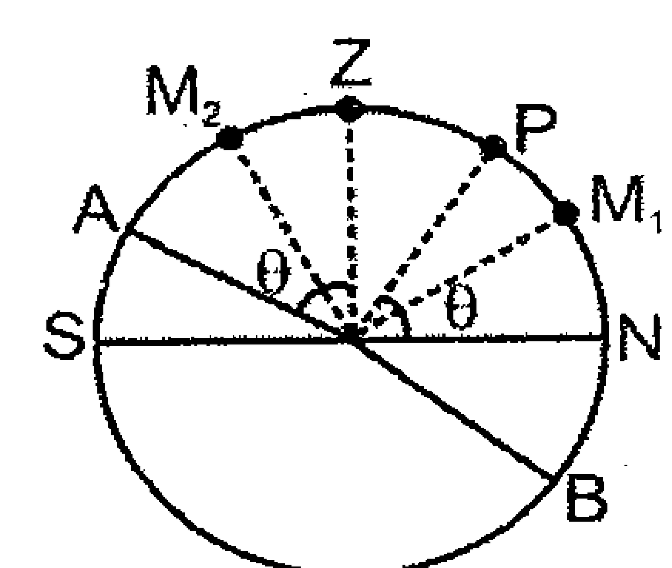
   $ZM_1 = Z = (90-\alpha)$

   $M_1 B = \delta$

   $M_1B = M_1N + NB$

   $\delta = a + (90-\delta)$ or $\theta = \alpha + (90-d) = \alpha + p$

   $\theta = \alpha + p$



For star M_2

$M_2A = \delta . M_2Z = Z$ $ZA = \theta$

$ZA = ZM_2 + M_2A$

$\Rightarrow$ $\theta = \delta + Z$ This equation covers all cases.

Time

Interval which lapses between any two instants, is termed as time. Following type of time measurements are generally used by astronomers.

(i) **Sidereal Time:** Hour angle of first point of Aries (γ) measured west ward at any instant is called sidereal time of that instant.

Interval of time between two successive upper point transit of first point of Aries is called sidereal day.

Local Sidereal Time (LST): The interval of time which elapses since the upper transit of first point of Aries over observer's meridian is known as local sidereal time of the place.

LST = RA of a star + HA of star

= Right Ascension (RA) of the observer's meridian.

(ii) **Apparent Solar Time :** Measurement of time based on daily apparent motion of the sun round the earth, is known as apparent solar time.

Interval of time between two successive lower transit (culmination) of centre of sun over meridian of the place is called apparent solar day.

(iii) **Mean Solar Time:** As the rate of movement of sun along the ecliptic is not uniform, length of apparent solar day, throughout the year is also not uniform.

To overcome this difficulty of recording the variation of apparent solar time by a clock, a fictitious sun is assumed to move at uniform rate along the equator so that to have a solar day of uniform duration. Motion of this mean sun is the average of that of the true sun in right ascension.

Interval of time between two successive lower transit of mean sun is called mean solar day

Local Mean Noon: The instant when the mean sun crosses the local meridian at its upper transit is known as local mean noon.

Local Mean time: (LMT): Hour angle of the mean sun recorded westward from 0 to 24 hours, is known as local mean time. The mean solar day begins at mid night and completes at next mid-night.

(iv **Standard Time:** As local mean time of any place is taken from lower transit of mean sun at the meridian of that place. So local mean time of different meridian will also be different for a country having difference of meridian of different places, to avoid confusion, a standard time is taken as per a central meridian of the country called standard meridian.

Standard meridian of a country is generally selected such that it lies at an exact number of hours from Greenwich. But Indian Standard Meridian is at $5^{1/2}$ hours (80° 30' longitude) east of Greenwich. All watches in a country shows the same standard time irrespective of the place.

Standard time (ST) = LMT ± difference of longitude converted to time.

- **Equation of Time**

Difference between apparent solar time and mean solar time at any instant is known as the equation of time.

Equation of time = Apparent solar time – Mean solar time.

- **Conversion of Time**

Longitude	360°	15°	1°C	15′	1′	15″
Time	24 hours	1 hour	4 minute	1 minute	4 second	1 second

- **Conversions**

1. Conversion of local time to standard time

$$LMT = MT \pm \text{diff. in longitude} \left(\frac{E}{W}\right)$$

2. Conversion of local time to Greenwich time

$$LMT = GMT \pm \text{longitude of place} \left(\frac{E}{W}\right)$$

3. Local Apparent time = Local mean time + equation of time.
4. Mean solar time
   =sidereal time – retardation
   =sidereal time – 9.8296 seconds per hour of given sidereal time
5. Sidereal time
   = Mean solar time + Acceleration
   = Mean solar time + 9.8565 sec per hour of given mean solar time.
6. LST at LMM = GST at GMM

$$\pm 9.8565 \text{ sec. per hour of longitude} \left(\frac{W}{E}\right)$$

LST at LMN = GST at GMN

$$\pm 9.8565 \text{ sec. per hour of longitude} \left(\frac{W}{E}\right)$$

7. LST at LMT = LST at LMM + SI from LMM
   SI (sidereal time interval) = LST at LMT – LST at LMM

■■■

Photogrammetric Surveying

11

Scale a Vertical Photograph

Scale $$S = \frac{\text{Map distance}}{\text{Ground distance}} = \frac{f}{H-h}$$

H = height of exposure station (or the air plane) above the mean sea level.
h = Height of ground above MSL
f = Focal length of camera

- If A and B are two points on ground having elevations h_a and h_b above MSL, then Average scale of line joining A and B is given by.

$$S = \frac{f}{H - \left(\frac{h_a + h_b}{2}\right)} \quad \text{where, } \frac{h_a + h_b}{2} = h_{av}.$$

- **Datum scale**

$$S = \frac{f}{H}$$

- **Scale of a photograph**

$$S_h = \frac{l}{L}$$

$$\frac{\text{Photo scale}}{\text{Map scale}} = \frac{\text{Photo distance}}{\text{Map distance}}$$

where, l = distance in Photograph
L = distance in ground

- **Computation of length of the line between points of different elevations from measurement on a vertical photograph.**
  (i) If A and B be two ground point having elevations h_a and h_b above MSL and coordinates (X_a, Y_a) and (X_b, Y_b)
  (ii) Let a and b be the position of corresponding points in photograph and (x_a, y_a) and (x_b, y_b) be the corresponding coordinates.

then $$\frac{x_a}{X_a} = \frac{y_a}{Y_a} = \frac{f}{H-h_a}$$

$$\frac{x_b}{X_b} = \frac{y_b}{Y_b} = \frac{f}{H-h_b}$$

where, $$X_a = \frac{H-h_a}{f}.x_a$$

$$X_b = \frac{H-h_b}{f}.x_b$$

$$Y_a = \frac{H-h_a}{f}.y_a$$

$$Y_b = \frac{H-h_b}{f}.y_b$$

The length between AB is given by

$$L = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$$

- **Relief displacement on a Vertical Photograph**
  When the ground is not horizontal the scale of the photograph varies from point to point. The ground relief is shown in perspective on the photograph. Every point on the Photograph is therefore, displaced from true orthography position. This displacement is called relief displacement.

- **Relief displacement**

$$d = \frac{Rfh}{H(H-h)}$$

$$d = \frac{r \times h}{H} = \frac{r_o \times h}{H-h}$$

(i) The relief displacement increases as the distance from the principal point increases.

(ii) $d \alpha \frac{1}{H}$

Scale of a tilted photograph

$$S_h = \frac{f \sec t - mn \sin t}{H-h}$$

$$S_h = \frac{f \sec t - y^1 \sin t}{H-h}$$

where, y′ = –x sinq + y¢ cosq + f tant
θ = 180 – s
s = Swing
t = tilt
f = Focal length
H = Flying height above datum
h = high of ground above datum.

It can be seen that the tilt and relief displacements tend to cancel in the upper part of the photograph while they are cumulative to the lower part.

Overlap in the Photographs

Longitudinal overlap = 55 to 65%

Lateral Overlap = 15 to 35%

for maximum rectangular area, to be covered by one photograph, the rectangle should have the dimension in the flight to be one-half the dimension normal to the direction of flight.

W = 2B W = 1.22H W= width of ground % overlap ≈ 60% in longitudinal direction.

Number of Photographs to Cover a Given Area

$$\boxed{N = \frac{A}{a}}$$

A = Total area to be photographed

a = net ground area covered by each photography

N = number of photographs required.

$a = L \times W$

$L = (1 - P_1)s.l.$

$W = (1 - P_w)s.l.$

$a = l.ws^2(1 - P_1)(1 - P_w)$

Where, l = length of photograph in direction of flight

W = width of photograph.

P_l = %lap in longitudinal direction

P_w = lap in longitudinal direction

$$S = \text{Scale of Photograph} = \frac{H}{f}$$

If instead of total area A, the rectangular dimensions $L_1 \times L_2$ (Parallel and Transverse to flight) are given then, the number of photograph required are given as follows.

Let L_1 = Dimension of area parallel to the direction flight

L_2 = Dimension of area Transverse the direction of flight

N_1 = Number of Photographs in each strip

N_2 = Number of strips required.

N = Total number of photographs to cover the whole area.

$N = N_1 \times N_2$

$$N_1 = \frac{L_1}{(1-P_l)s \times l} + 1$$

$$N_2 = \frac{L_2}{(1-P_w)s.l} + l$$

- Interval between exposures

$$\boxed{T = \frac{3600 \times L}{V}}$$

V = ground speed of airplane KMPH.

L = ground distance covered by each photograph in the direction of flight = $(1 - P_l)$ s.l.......in Km

Photogrammetry

(i) **Terrestrial photogrammetry:** Photographs are taken from a fixed position on or near the ground.

(ii) **Aerial photogrammetry:** Photographs are taken from a camera mounted in an aircraft flying over the area.

Phototheodolite: It is a combination of "theodolite and a terrestrial camera. Important parts are:

(i) Camera Box of a fixed focus type.

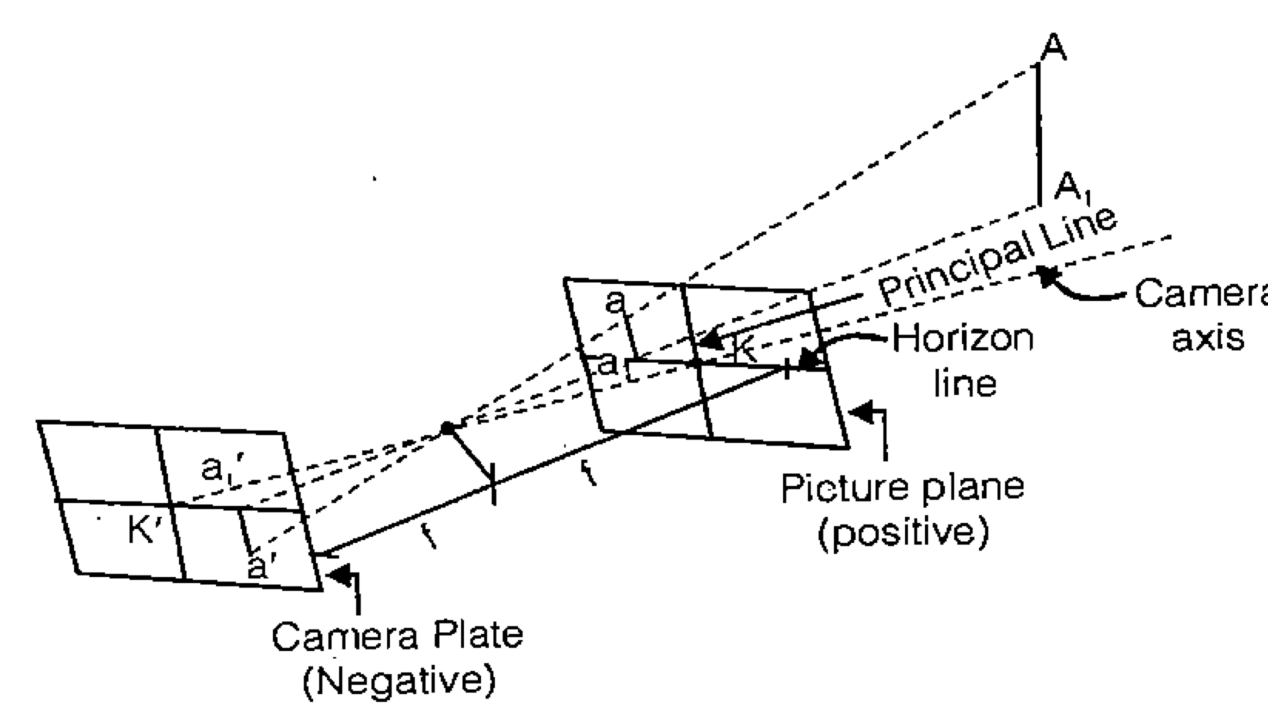
(ii) Hollow rectangular frame consist of two cross hair.

(iii) Photographic plate

(iv) Theodolite

- Important Definitions

(i) **Camera Axis:** Line passing through centre of camera lens perpendicular both to camera plate (Negative) and picture plane (photograph).



(ii) **Picture Plane:** Positive plane, perpendicular to camera axis.

(iii) **Principal point:** K or K′ point on intersection of camera axis with either picture plane or the camera plate.

(iv) **Focal length (f):** Perpendicular distance from centre of camera lens to either to picture plane or camera plate. It satisfy the relation.

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{u+v}{u.v} \Rightarrow f = \frac{uv}{u+v}}$$

(v) **Nodal point:** Nodal point is either of two points on the optical axis of a lens so located that when all object distances are measured from one point, and all image distances are measured from other. They satisfy the simple lens relation.

$$\boxed{\frac{1}{f}=\frac{1}{u}+\frac{1}{v}}$$

(vi) **Principal plane:** It is a plane which contain principal line and optical axis.

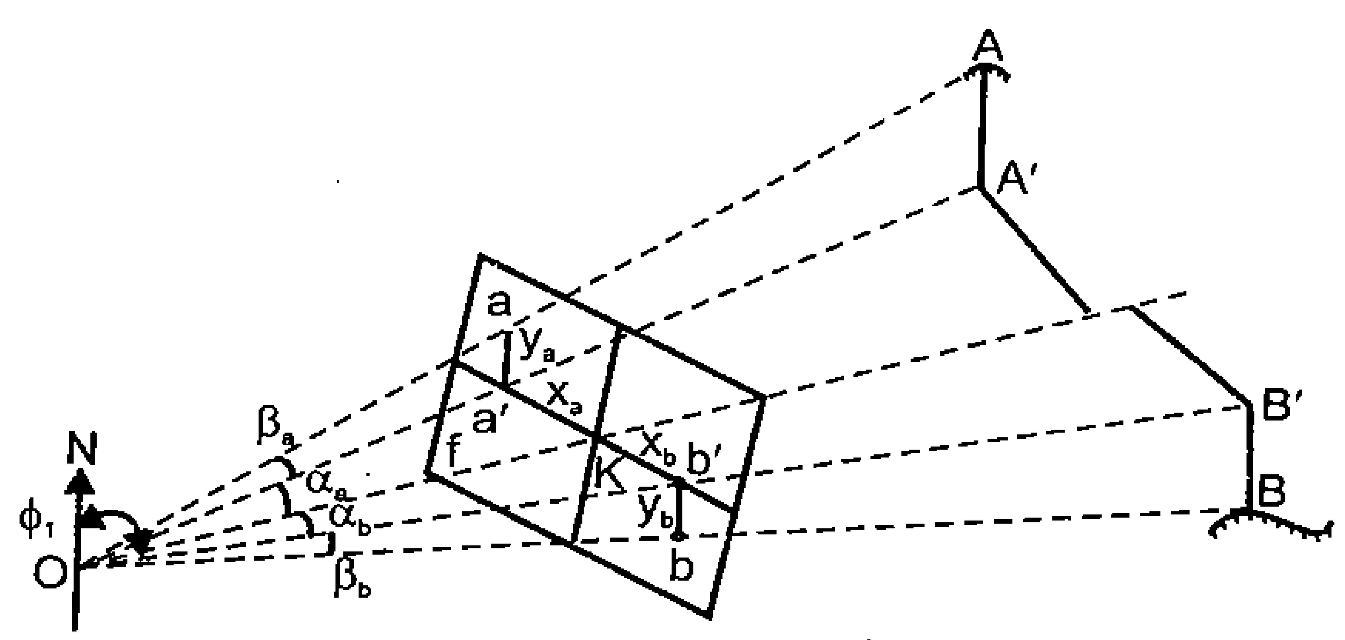
(vii) **Oblique photograph:** Photograph taken from air with axis of camera tilted from vertical are called oblique photograph, these are of two type

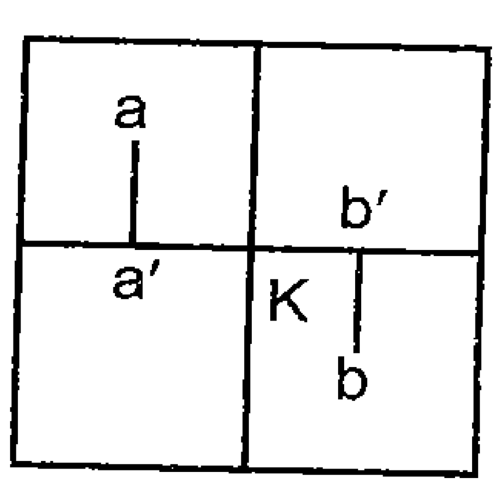
(a) **Low Oblique photograph:** An oblique photograph that does not show the horizon is called low oblique photograph.

(b) **High Oblique photograph:** If tilt is more upto such that horizon is shown in the photograph, it is called high oblique photograph.

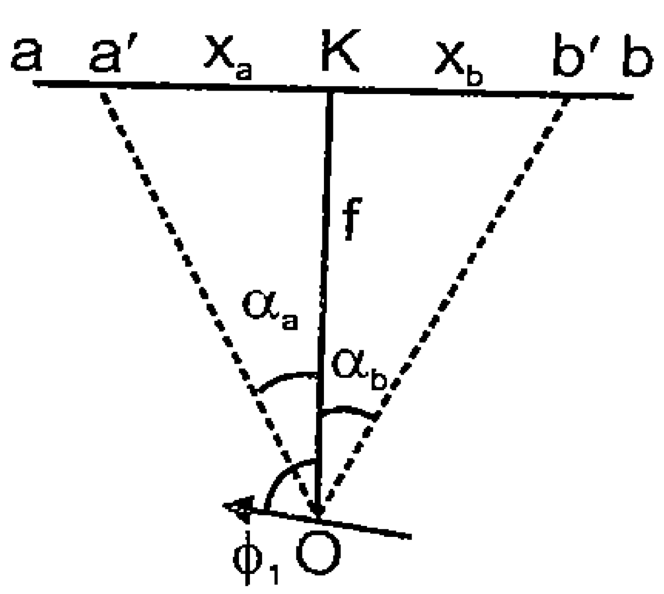
(viii) **Convergent photograph:** Low oblique photographs which are taken with two cameras exposed simultaneously at successive exposure stations, with their axes tilted at a fixed inclination from vertical, so that forward exposure of first station from a stereo pair with backward exposure of next station, these photographs are called 'Convergent Photographs'.

Horizontal and Vertical angles from terrestrial Photograph

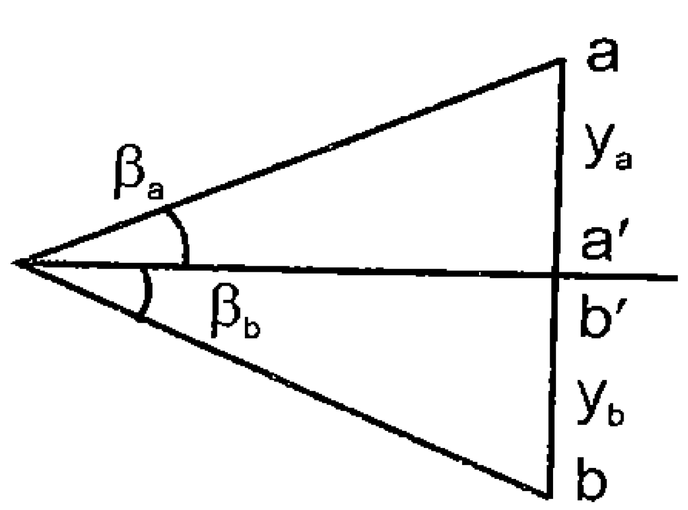




Photograph



Plan



Elevation

$Oa' = f \sec \alpha_a$

$Ob' = f \sec \alpha_b$

$x_a = f \tan \alpha_a y_a = Oa' \tan\beta_a = f \sec \alpha_a . \tan\beta_a$

$x_b = f \tan \alpha_b y_b = Ob' \tan\beta_b = f \sec \alpha_b . \tan\beta_b$

$$\boxed{\begin{aligned}\tan\alpha_a &= \frac{x_a}{f}\\ \tan\alpha_b &= \frac{x_b}{f}\end{aligned}} \qquad \boxed{\begin{aligned}\tan\beta_a &= \frac{y_a}{f\sec\alpha_a}\\ \tan\beta_b &= \frac{y_b}{f\sec\alpha_b}\end{aligned}}$$

Angle ϕ_1 is magnetic bearing of camera axis (or principal vertical plane.)

Azimuth of line Ok = ϕ_1

Azimuth of line OA = $\phi_1 - \alpha_a$ (OA is left to OK)

Azimuth of line OB = $\phi_1 + \alpha_b$ (OA is right to OK)

So, $\boxed{\text{Azimuth of a line = Camera azimuth} + \alpha}$

Elevation of a point by photographic measurement

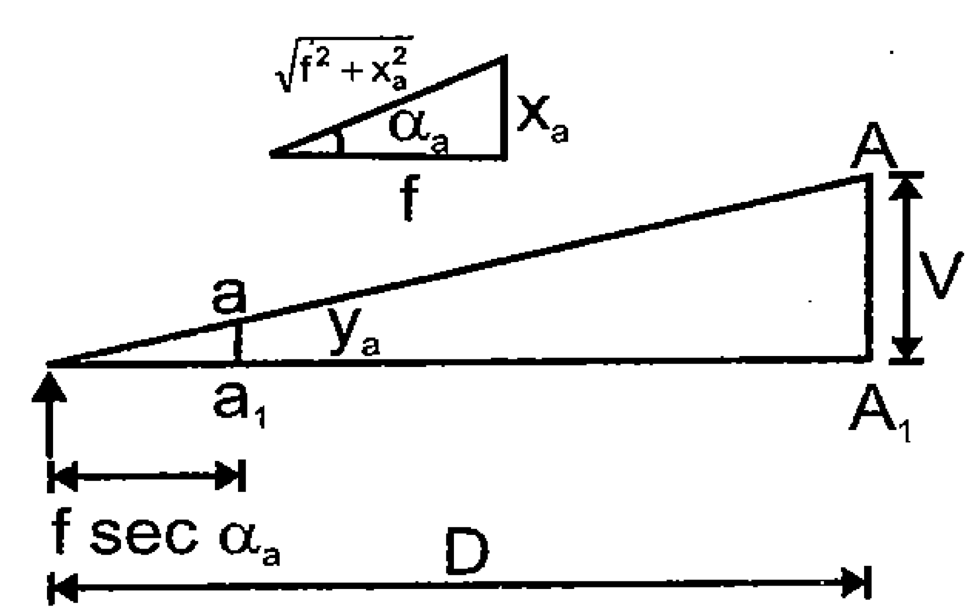
Consider Point A

$$\tan\alpha_a = \frac{x_a}{f}$$

$$\tan\beta_a = \frac{y_a}{Oa_1} = \frac{y_a}{f\sec\alpha_a} = \frac{y_a}{f}\cos\alpha_a.$$

If V = Elevation of point A above Horizontal plane through camera axis.

From Similar triangle



$$\frac{y_a}{f\sec\alpha_a} = \frac{V}{D}$$

So, $$V = \frac{y_a.D}{f\sec\alpha_a} = \frac{y_a.D}{\sqrt{f^2+x_a^2}} = \frac{y.D}{\sqrt{f^2+x^2}}$$

So, $$\boxed{V = \frac{yD}{f}\cos\alpha = \frac{y.D}{\sqrt{f^2+x^2}}}$$

Elevation of point A.

$h = H_C + V + C$

Where, H_C = Elevation of camera

V = Elevation of point A

C = Correction for curvature and refraction.

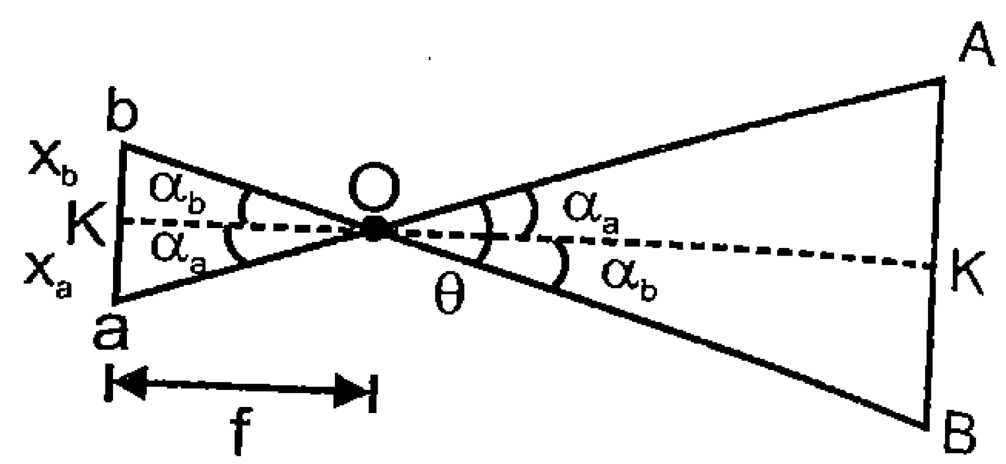
$$\boxed{h = H_C + V + C}$$

- **Determination of focal length of the lens**

  Take two points A and B. Measure angle θ very accurately from a theodolite

$$\angle AOB = \theta$$

$$ak = x_a,\ bk = x_b$$



$$\tan\alpha_a = \frac{x_a}{f} \quad \tan\alpha_b = \frac{x_b}{f}$$

$$\tan\theta = \tan(\alpha_a + \alpha_b)$$

$$= \frac{\tan\alpha_a + \tan\alpha_b}{1 - \tan\alpha_a . \tan\alpha_b}$$

$$= \frac{\frac{x_a}{f} + \frac{x_b}{f}}{1 - \frac{x_a}{f}.\frac{x_b}{f}} = \frac{(x_a + x_b)f}{f^2 - x_a.x_b}$$

$$f^2\tan\theta = x_a.x_b \tan\theta - f(x_a + x_b) = 0$$

or $$f^2 - f\left(\frac{x_a + x_b}{\tan\theta}\right) - x_a x_b = 0$$

Quadratic equation in f.

Arial Photogrammetry

Arial photograph are taken from a fast speed arial camera which have very high speed and efficient shutter, using high speed emulsion for the film.

Important Definitions

(i) **Vertical photograph:** when photograph is taken keeping camera axis vertical, coinciding with the direction of gravity, it is called a vertical photograph.

(ii) **Tilted Photograph:** camera axis inclined at an angle from vertical.

(iii) **Exposure station:** Point in space, occupied by camera lens at the time of exposure.

(iv) **Flying height:** Elevation exposure station above sea level.

(v) **Flight line:** Line drawn on the map to represent the track of the aircraft.

(vi) **Focal length:** distance from front Nodal point of lens to plane of photograph (OK)

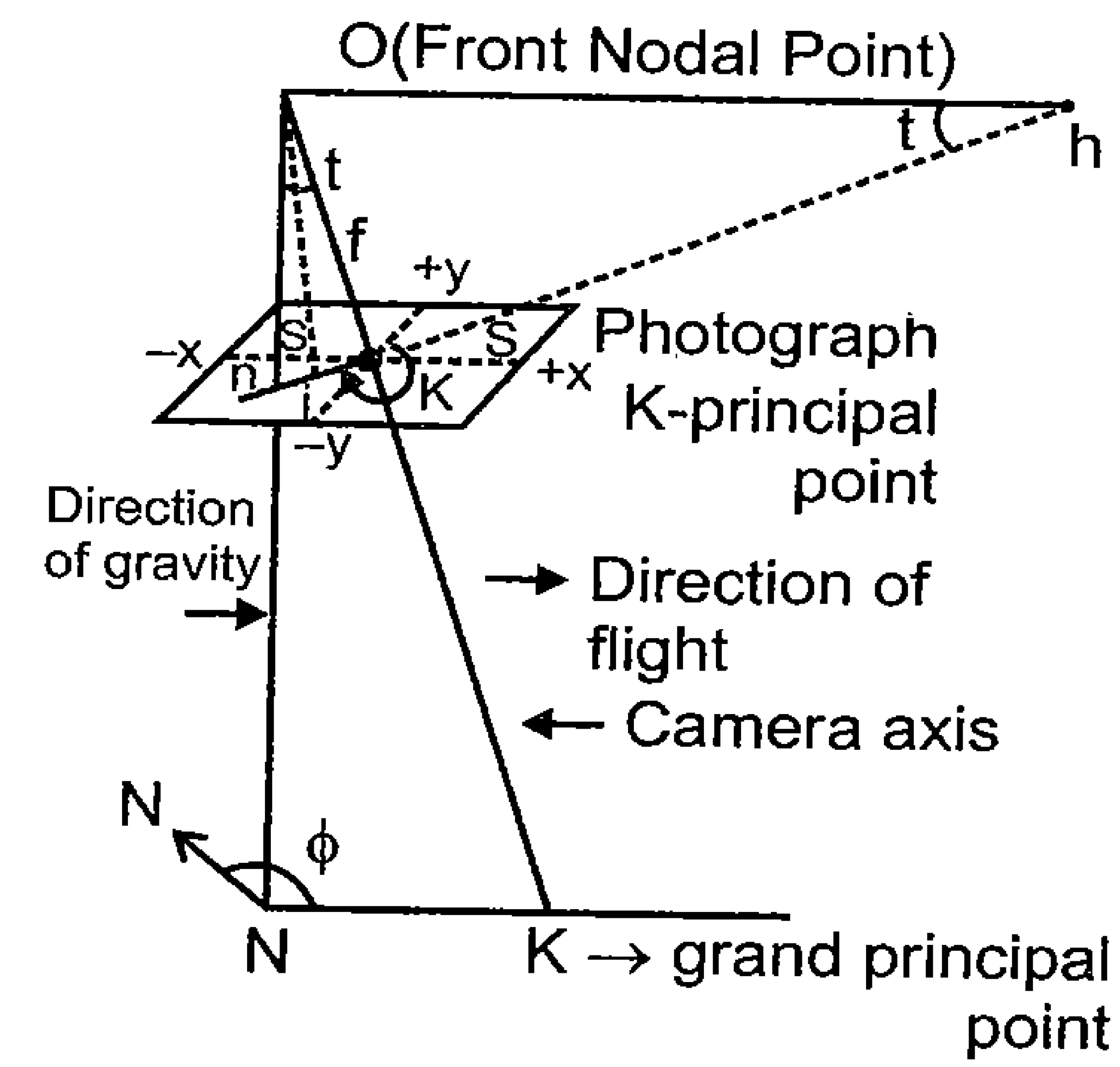
(vii) **Principal Point:** Point where perpendicular dropped from the front nodal point strikes the photograph (K).

(viii) **Nadir Point:** Nadir point is point where plumb line intersects photograph.

(ix) **Ground Nadir Point:** Point on the ground vertically beneath the exposure station (Point N).

(x) **Tilt:** Vertical angle defined by the intersection at the exposure station.

$\angle KON = t =$ tilt



(xi) **Principal Plane:** Plane defined by lens (O) ground Nadir Point (N) Principal point produced on ground (K).

(xii) **Principal Line:** Intersection of principal plane with plane of photograph (line NK)

(xiii) **Isocentre:** Isocentre is the point in which the bisector of angle of tilt meets the photograph. Oi is the bisector and i is isocentre - distance $ki = f\tan^{t/2}$.

(xiv) **Swing:** Angle measured in plane of photograph from + y axis clockwise to Nadir point.

(xv) **Azimuth of principal plane:** Clockwise horizontal angle measured about the ground nadir point from the ground survey north meridian to the principal plane of the photograph.

(xvi) **Horizon point (h):** Intersection of principal line with the horizontal line through the perspective centre. Such as point h in figure is horizon point.

(xvii) **Axis of Tilt:** Axis of tilt is a line in photograph plane perpendicular to principal line at the isocentre such as i_1, $i\,i_2$ in figure. The plane of photograph is tilted about axis.

Relation between principal point, plumb point and isocentre

(i) NK = distance of nadir point from principal point.

$$\boxed{\frac{NK}{KO} = \tan t} \Rightarrow \boxed{NK = KO\tan t = f\tan t}$$

(ii) Ki = distance of the isocentre from principal point.

$$\boxed{\frac{Ki}{KO} = \tan\frac{t}{2}} \Rightarrow \boxed{Ki = f\tan\frac{t}{2}}$$

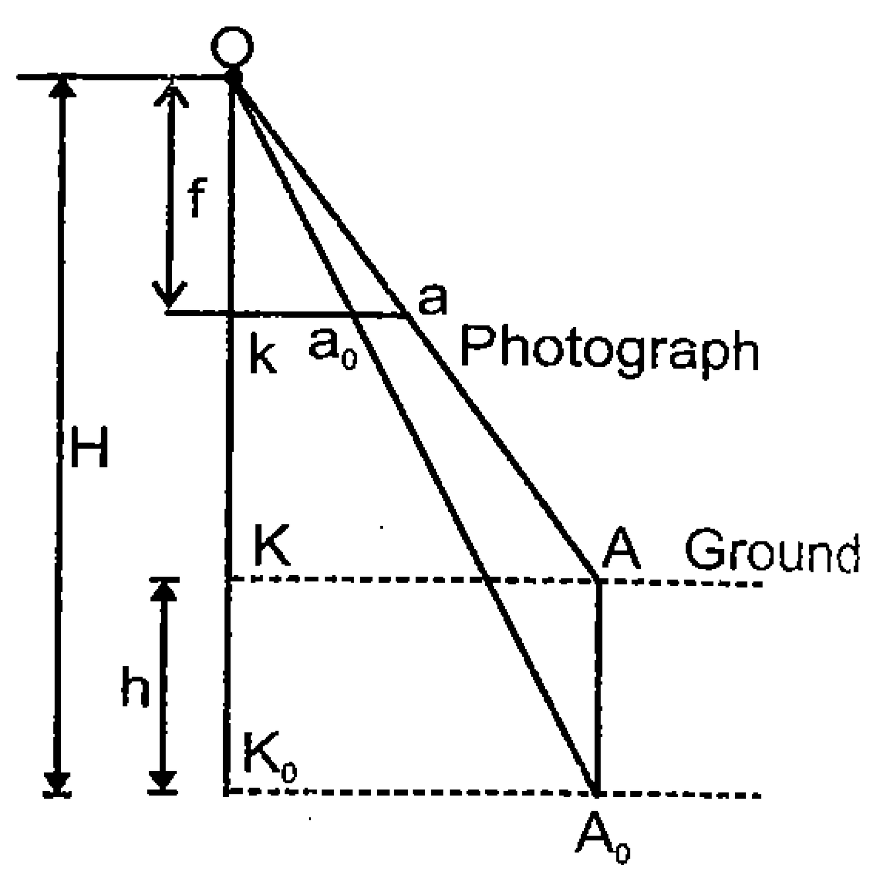
(iii) Kh = distance of principal point to horizon point.

$$\boxed{Kh = KO.\cot t} \boxed{Kh = f\cot t}$$

Scale of a Vertical Photograph

Scale varies for points of different elevation, scale will be constant only when elevation of all points are same.

If all the points are on the same elevation. Then



Scale $\boxed{S = \frac{\text{Map distance}}{\text{Ground distance}} = \frac{ka}{KA}}$

or $S = \frac{ka}{KA} = \frac{Ok}{OK} = \frac{f}{H-h}$

$\Rightarrow \boxed{S = \frac{f}{H-h}}$

Case 2: If A and B are two points having elevation h_a and h_b respectively above mean sea level.

Scale of photograph at elevation h_a.

$$\boxed{\frac{ka}{K_a.A} = \frac{ok}{OK_a} = \frac{f}{H-h_a}}$$

Scale of photograph at elevation h_b.

$$\boxed{\frac{kb}{BK_b} = \frac{ok}{OK_b} = \frac{f}{H-h_b}}$$

So in general scale of photograph for different elevations (h).

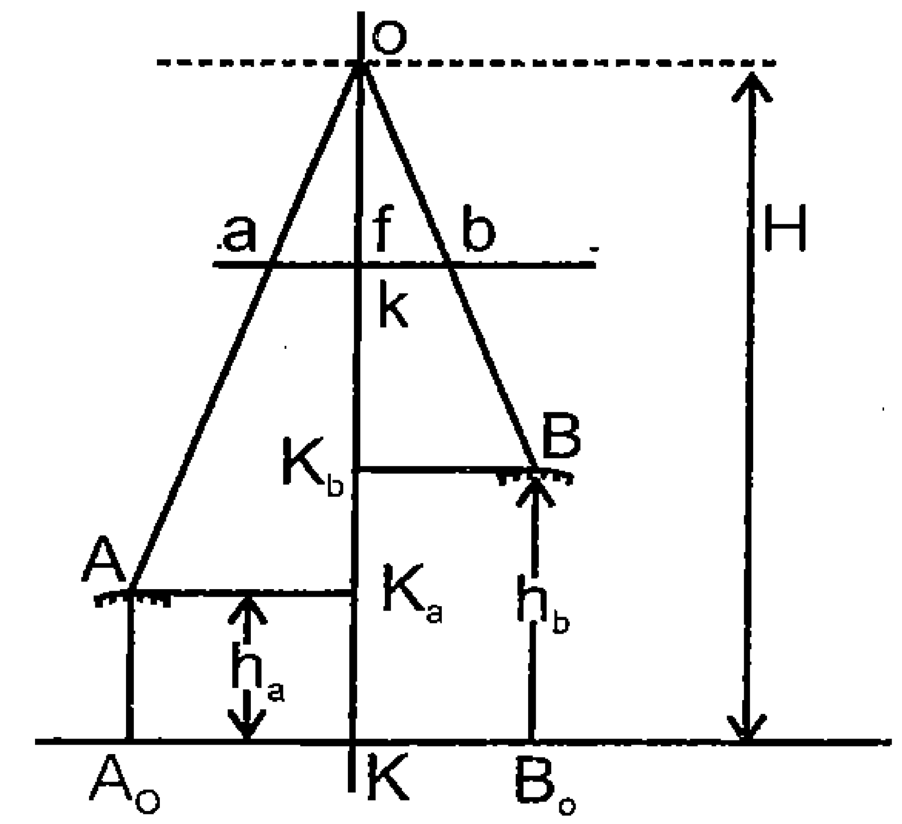
$$\boxed{S_h = \frac{f}{H-h}}$$

This can be represented by representative fraction (R_n) also.

$$\boxed{R_h = \frac{1}{\left(\frac{H-h}{f}\right)}}$$

$$\uparrow$$

$$\boxed{R_h = \frac{1}{SH} = \frac{\frac{1}{f}}{H-h} = \frac{1}{\left(\frac{H-h}{f}\right)}}$$



Different Scales

(i) **Datum scale:** if all points are projected at mean sea level.

Datum scale $S_d = \frac{ka}{KA_O} = \frac{ok}{OK} = \frac{f}{H}$

$\Rightarrow \boxed{S_d = \frac{f}{H}}$

(ii) **Average scale:** If all points are projected on a plane representing the average elevation.

$$\boxed{S_{av} = \frac{f}{H-h_{av}}}$$

Computation of length of line between points of different elevation

Coordinate of point A and B on ground in plan.

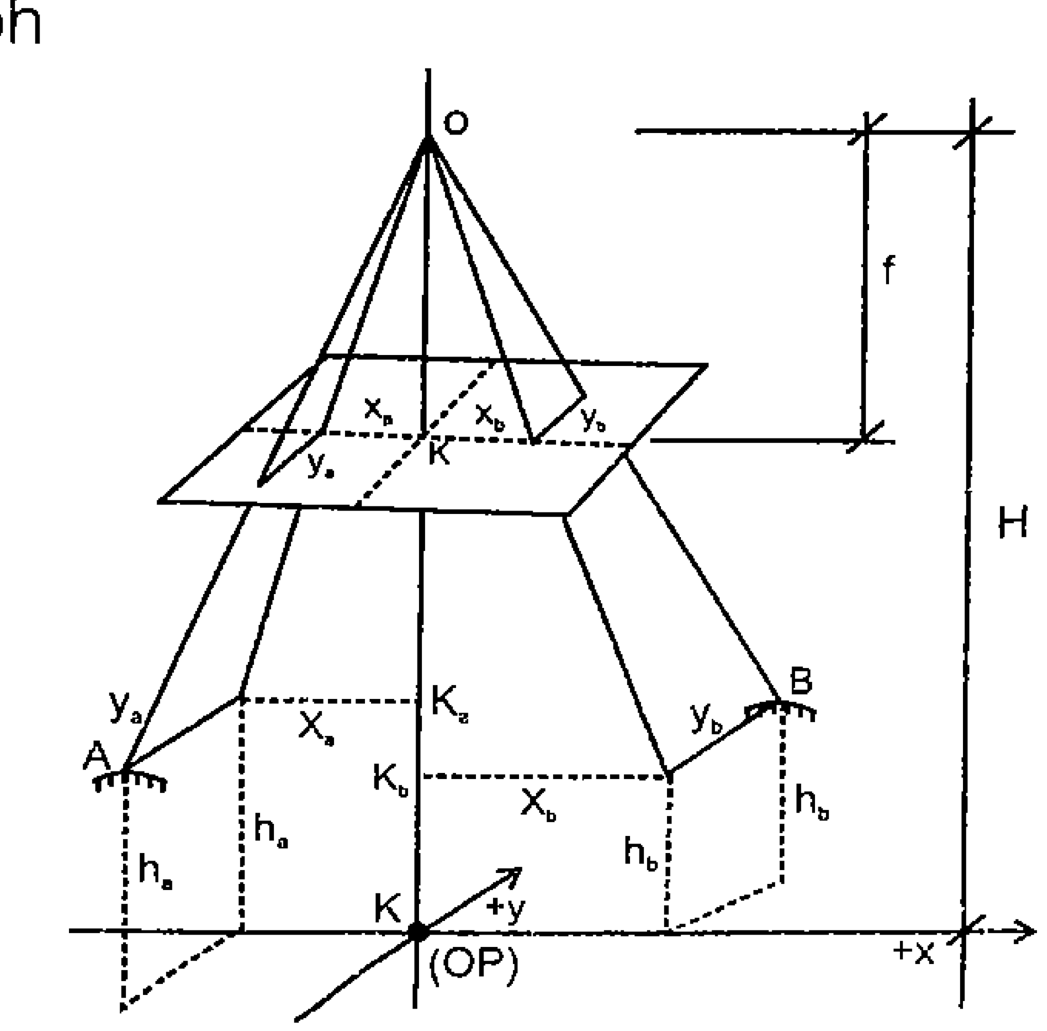
$A - X_a, Y_a$

$B - X_b, Y_b$

Corresponding points on photograph

$a - x_a, y_a$

$b - x_b, y_b$



For point (A) from similar triangles.

$$\frac{OK}{OK_a} = \frac{x_a}{X_a} = \frac{y_a}{Y_a} = \frac{f}{H-h_a}$$

For Point B.

$$\frac{OK}{OK_b} = \frac{x_b}{X_b} = \frac{y_b}{Y_b} = \frac{f}{H-h_b}$$

So, $X_a = \frac{H-h_a}{f}.x_a$ $Y_a = \frac{H-h_a}{f}.y_a$

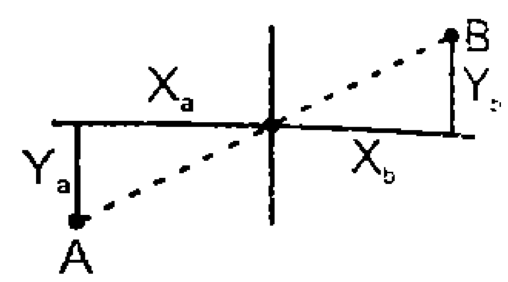
$X_b = \frac{H-h_b}{f}.x_b$ $Y_b = \frac{H-h_b}{f}.y_b$

So, In general coordinate X and Y of any point.

$X = \frac{H-h}{f}.x$ $Y = \frac{H-h}{f}.y$

Length between two points A and B is given by

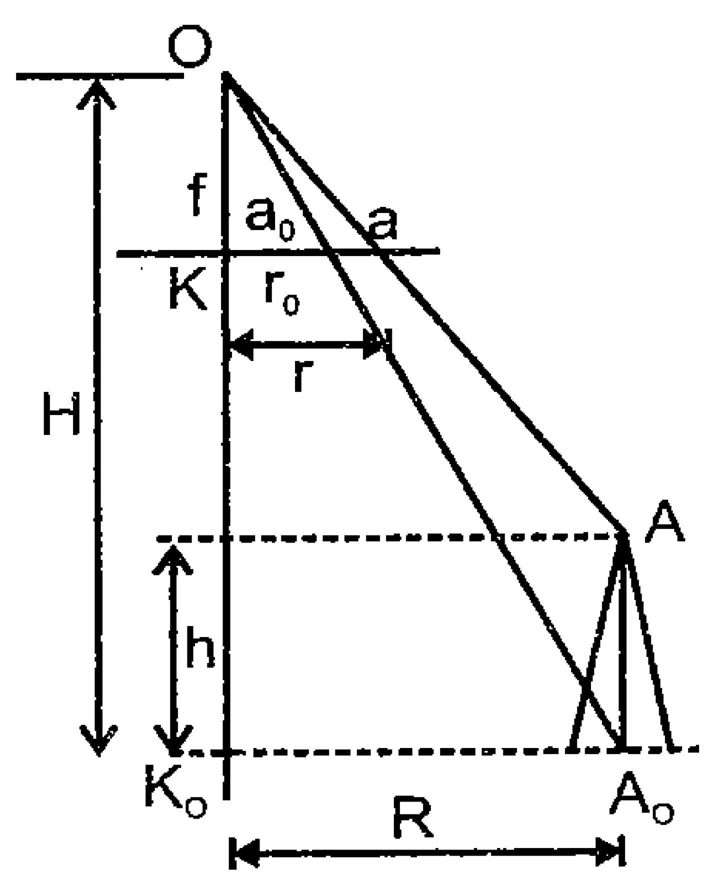
$$L = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$$



X_a, X_b, Y_a, Y_b → should be given with proper sign.

Relief Displacement

Due to different elevation of different points, every point on photograph is displaced from their original position. This displacement is called relief displacement.



$r = K_a$

$r_o = K_{a_o}$

$R = K_o A_o$

aa_o is called relief displacement.

$aa_o = r - r_o$

from similar triangle.

$$\frac{r}{R} = \frac{f}{H-h} \Rightarrow r = \frac{fR}{H-h} \quad \text{(i)}$$

$$\frac{r_o}{R} = \frac{r}{H} \Rightarrow r_o = \frac{fR}{H} \quad \text{(ii)}$$

So relief displacement

$$d = r - r_o = \frac{fR}{H-h} - \frac{fR}{H} = \frac{fR}{H(H-h)}(H-H+h)$$

$$\Rightarrow \quad d = \frac{f.R.h}{H.(H-h)} \quad \text{(iii)}$$

But $R = \left(\frac{H-h}{f}\right) r$

So, $d = \frac{fh}{H(H-h)} \times \frac{(H-h)}{f}.r$

So, $d = \frac{rh}{H}$ (iv)

from $\frac{\text{(i)}}{\text{(ii)}}$ $\frac{r}{r_O} = \frac{H}{H-h}$

$\Rightarrow$ $\frac{r}{H} = \frac{r_O}{H-h}$

So relief displacement

$$d = \frac{rh}{H} = \frac{r_O h}{H-h} \quad \text{(v)}$$

If relief displacement is known then height of an object

$$h = \frac{dH}{r}$$

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A Handbook on
Civil Engineering

12

Irrigation Engineering

CONTENTS

Water Requirements of Crops

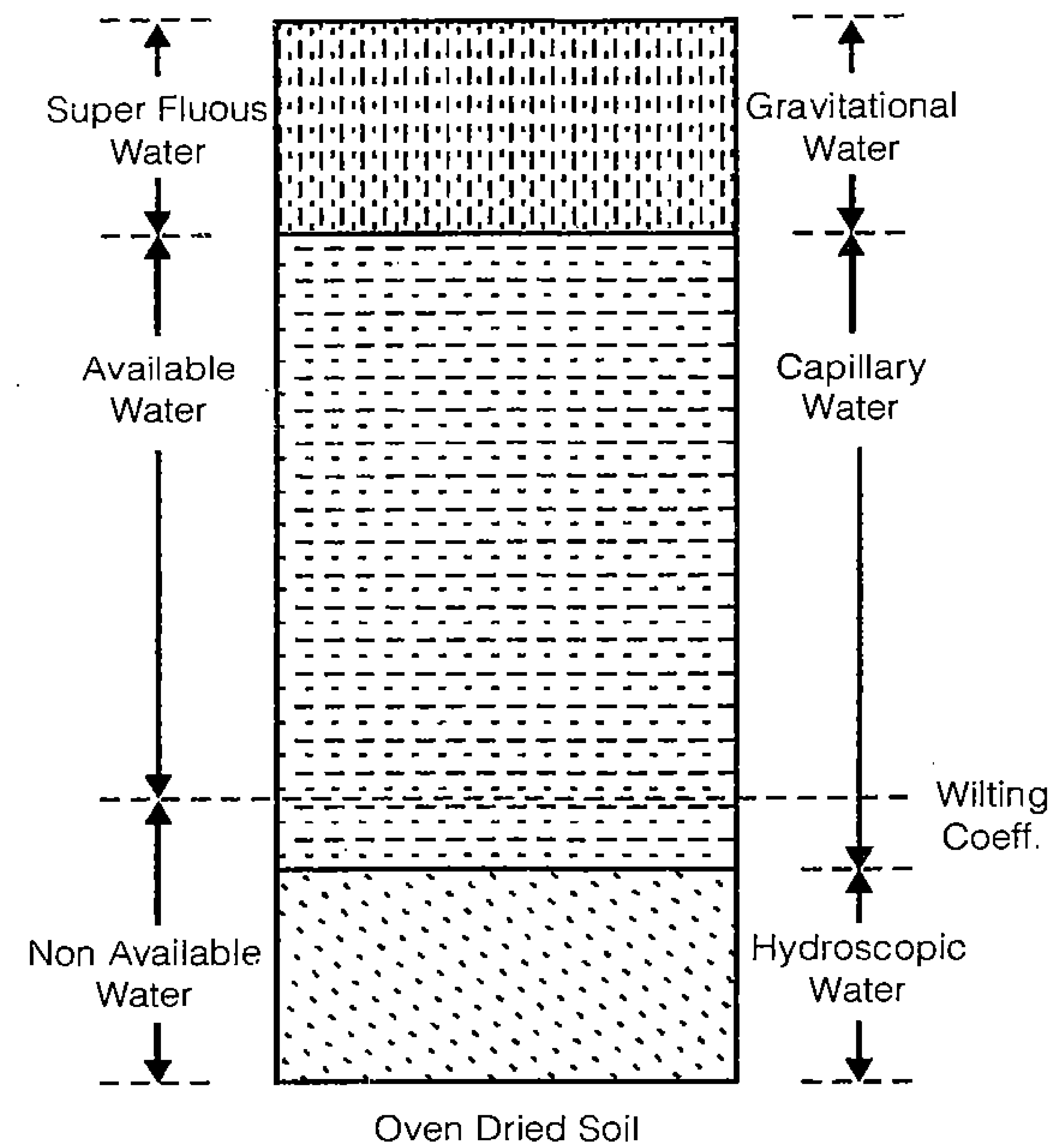
1

Soil Moisture

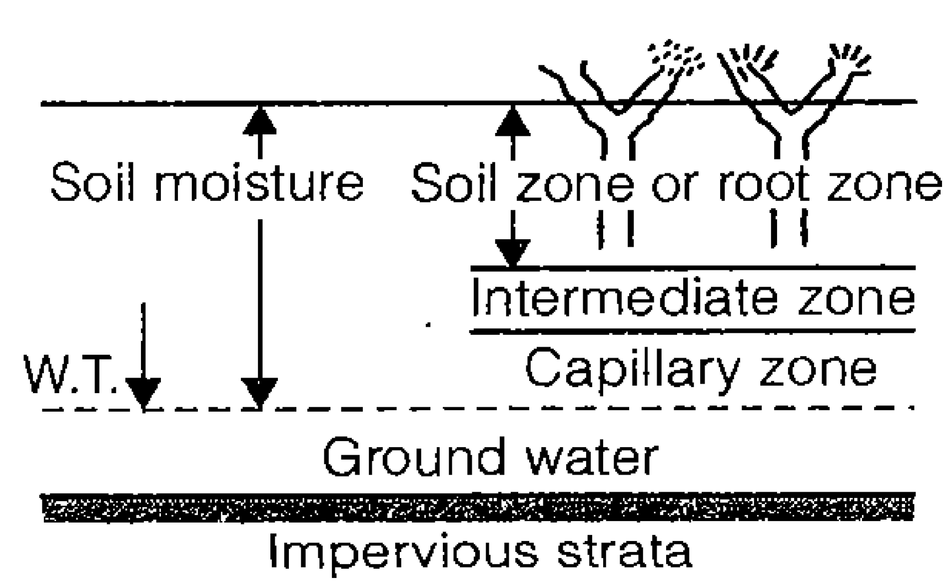
Classes and Availability of Soil Water

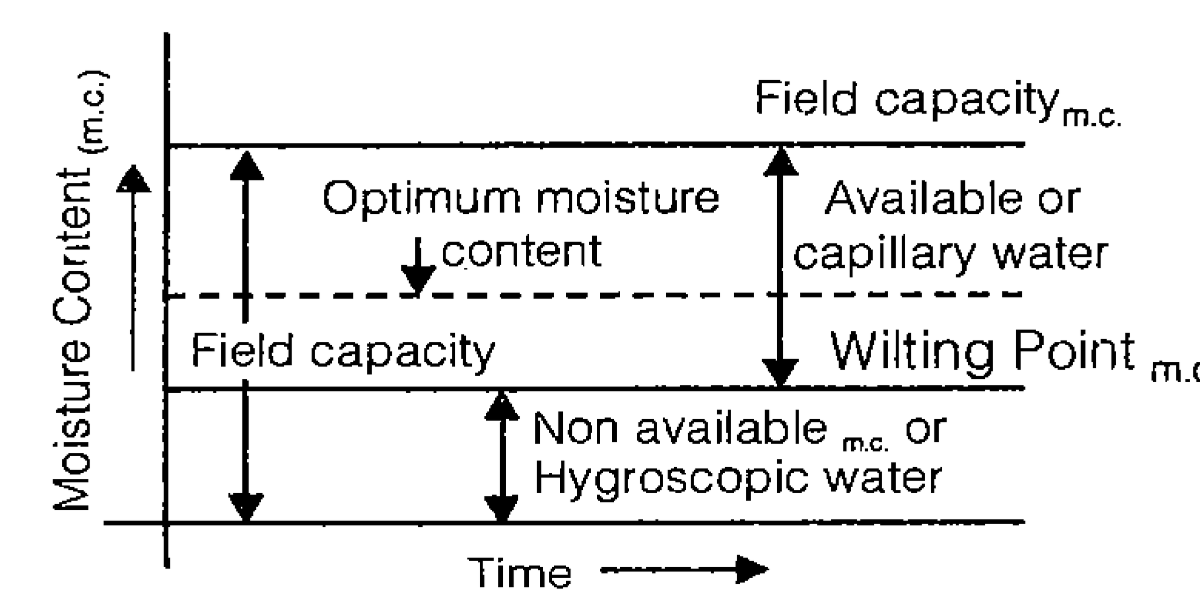
Water present in the soil may be classified under three heads:

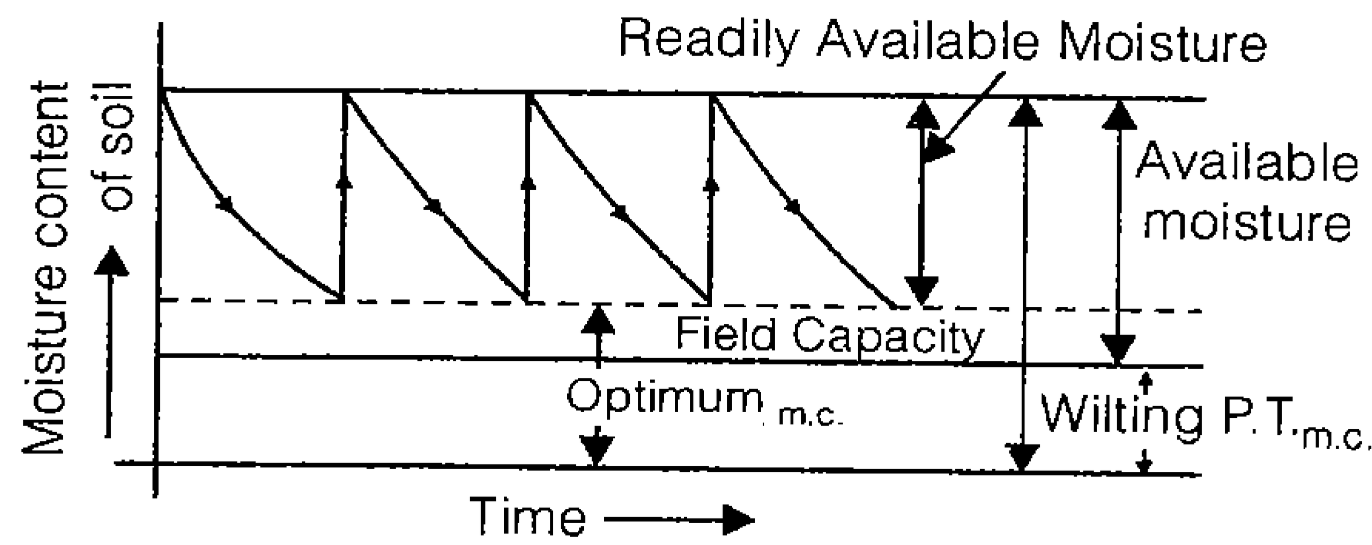
1. Hygroscopic water 2. Capillary water 3. Gravitational water



(i) Available moisture for the plant $= F_C - \phi$







(ii) Readily available moisture for the plant = $F_C - M_O$

Here, F_C = Field capacity

ϕ = Wilting point or wilting coefficients below which plant can't survive.

M_O = Readily available moisture content.

(iii) $$\text{Frequency of Irrigation} = \frac{\text{Available / Readily available moisture depth}}{\text{Consumptive use rate}}$$

(iv) $$F_C = \frac{\text{Weight of water stored in soil of unit area}}{\text{Weight of same soil of unit area}}$$

where, Weight of water stored in soil of unit area $= \gamma_w \cdot d_w \cdot 1$

Weight of same soil of unit area = $\gamma \cdot d \cdot l$

d_w = depth of water stored in root zone.

(v) $$d_w = \frac{\gamma.d}{\gamma_w} \cdot F_c$$ $\gamma \rightarrow$ dry unit wt. of soil

(vi) Available moisture depth to plant

$$d_w' = \frac{\gamma.d}{\gamma_w}(F_c - \phi)$$

(vii) Readily available moisture depth to plant

$$d_w' = \frac{\gamma.d}{\gamma_w}(F_c - m_o)$$

(viii) $F_c = n/G$ where, G = Specific gravity and n = Porosity

Duty and Delta

Duty: The duty of water is the relationship between the volume of water and the area of the crop it matures.

It is defined as the area irrigated per cumec of discharge running for base period B. The duty is generally represented by D.

Delta: It is the total depth of water required by a crop during the entire base period and is represented by the symbol Δ.

Relation between Duty and Delta

$$\Delta = \frac{8.64B}{D}$$

where, Δ = Delta in meter
D = Duty in Ha/cumec
B = Base period in days

also $$\Delta = \frac{2B}{D}$$

where, Δ = Delta in feet
B = Base period in days
D = Duty in acre/cusec

Consumptive use determination by use of Equations

The following are some of the commonly used methods.

1. Penman-Method
2. Blaney-Criddle method
3. Hargreaves class A pan evaporation Method.

Blaney-Criddle Method: Blaney-Criddle equation expresses the consumptive use in terms of temperature and day time hours. If C_u is monthly consumptive use, its value is given by $C_u = k.f$ (inches)

where, k = crop factor to be determined for each crop; its value depends upon certain environmental conditions

f = monthly consumptive use factor = t × (p/100)

t = mean temperature in °F.

p = percentage of day time hours of the year, occurring during the period.

If Expressed in metric units, the above formula becomes:

$$C_u = k.\frac{p}{40}[1.8 \cdot t + 32] = k.f$$

Where, t = temperature in °C

C_u = monthly consumptive use in cm

Irrigation Efficiencies

(i) Water Conveyance Efficiency, (η_c)

$$\eta_c = \frac{w_f}{w_r} \times 100$$

where, w_f = Water delivered to the field.
w_r = Water delivered from the reservoir.

(ii) Water Application Efficiency (η_a)

$$\eta_a = \frac{w_s}{w_f} \times 100$$

where, w_s = Water stored in the root zone.

w_f = Water delivered to the filed.

(iii) Water use Efficiency (η_u)

$$\eta_u = \frac{w_u}{w_f} \times 100$$

Where, w_u = Water use consumptively

w_f = Water delivered to the field.

(iv) Water Storage Efficiency (η_s)

$$\eta_s = \frac{w_{s'}}{w_\eta} \times 100$$

where, $w_{s'}$ = Actual water stored in the root zone.

w_η = Water needed to store to bring the water content up to field capacity.

$$w_\eta = F_c - \text{Available moisture}$$

(v) Water Distribution Efficiency, (η_d)

$$\eta_d = \left(1 - \frac{y}{d}\right) \times 100$$

where, y = Average numerical deviation in the depth of water stored from the average depth of irrigation stored.

d = Average depth during irrigation.

(vi) Consumptive use Efficiency (η_{cu})

$$\eta_{cu} = \frac{w_{cu}}{w_d} \times 100$$

where, w_{Cu} or C_u = Water used by plant consumptively.

w_d = Net amount of water depleted from root zone.

Irrigation Requirements of Crops

(i) Consumptive Irrigation Requirement, (CIR)

$$CIR = C_u - P_{eff}$$

where, C_u = Total consumptive use requirement.

P_{eff} = Effective rainfall.

(ii) Net Irrigation Requirement (NIR)

$$NIR = CIR + \text{Leaching requirement}$$

(iii) Field Irrigation Requirement (FIR)

$$FIR = \frac{NIR}{\eta_a}$$

(iv) Gross Irrigation Requirement (GIR)

$$GIR = \frac{FIR}{\eta_c}$$

■■■

2 Design of Stable Channels

Kennedy's Theory

- Design Steps

(i) $$v_0 = 0.55\, m y^{0.64}$$

where, v_o = Critical velocity

y = Trial depth

m = Critical velocity ratio

(a) $$m = \frac{v}{v_0}$$

where, v = Actual mean velocity.

m = 1 → For standard particles of upper Baridoab.

m = 1 to 1.2 → For Coarser Sediments.

m = 0.7 to 1.0 → For finner particles.

(b) y = 1 m → when $0 \le Q \le 20$ m³/sec.

y = 2 m → when $20 \le Q \le 40$ m³/sec.

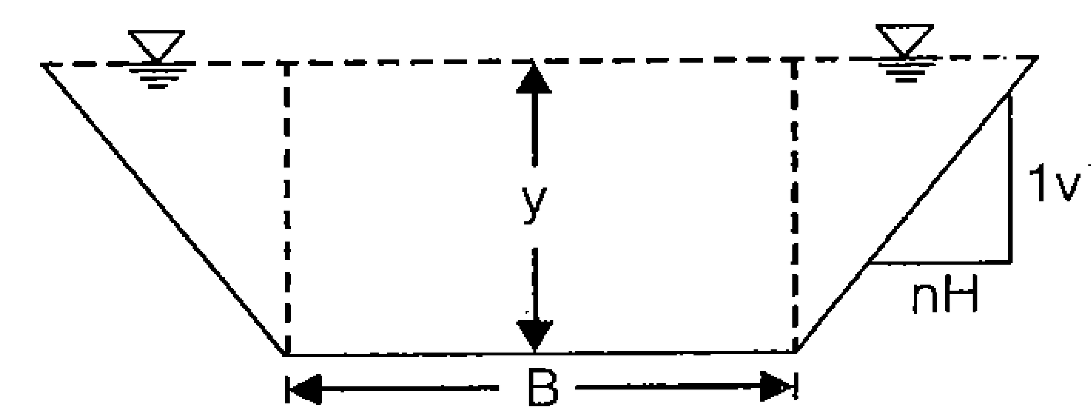
y = 2.5 m → when $40 \le Q \le 80$ m³/sec.

y = 3.0 m → when $80 \le Q \le 100$ m³/sec.

(ii) $$A = \frac{Q}{V_o}$$

$$A = (B + ny)y$$

→ Get 'B'



where, A = Area, (m²)

Q = Discharge (m³/sec)

V_o = Critical velocity.

(iii) $$P = B + 2y\sqrt{n^2 + 1}$$

(iv) $$R = \frac{A}{P}$$

where, R = Hydraulic mean depth.

(v) $$V = C\sqrt{RS}$$

(a) $$C = \frac{23 + \frac{0.00155}{s} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{s}\right)\frac{N}{\sqrt{R}}}$$

(b) $$\frac{1}{3500} \le S \le \frac{1}{5000}$$

(vi) If $V \simeq V_0$ then O.K.

Lacey's Theory

- Design Steps:

(i) $V = \left[\frac{Qf^2}{140}\right]^{1/6}$ where, V = Velocity in m/s
Q = Discharge in cumec (m^3/sec)
f = Silt factor

(a) $f = 1.76\sqrt{d_{mm}}$ where d_{mm} = dia in 'mm'.

(ii) $R = \frac{5}{2}\cdot\frac{V^2}{f}$

(iii) $P = 4.75\sqrt{Q}$ and $P = B + 2y\sqrt{n^2+1}$ → Get equation (i) where, P = Perimeter

(iv) $A = \frac{Q}{V}$ and $A = (B + ny)y$ → Get equation (ii) where, A = area.

Solve equation (i) and (ii) get B and y.

(v) $S = \frac{f^{5/3}}{3340\cdot Q^{1/6}}$ where S = bed slope.

(vi) $\text{Lacey regime scour depth} = 1.35\left(\frac{q^2}{f}\right)^{1/3}$

Lindley's Theory

$V = 0.567\cdot y^{0.57}$, $V = 0.274\cdot B^{0.355}$ and $B = 7.76y^{1.61}$

Lining of Canals

(a) Annual Benefits

$$\text{Total annual benefits} = mR_1 + PR_2$$

where R_1 = Irrigation water sold to the cultivator at a rate ₹ R_1/cumec.
m = Cumec of water is saved by lining the canal annually.
R_2 = Rate of maintenance cost in rupees per year.
P = % (fraction) of saving achieved in maintenance cost by lining the canal.
≃ 0.4.

(b) Annual Costs

$$\text{Total annual cost of lining} = \frac{C}{y} + \frac{C}{2}\left(\frac{r}{100}\right)$$

where, C = Capital expenditure required on lining
y = Life of lining
$\frac{C}{y}$ = Annual depreciation charge in rupees
r = Rate of interest (%)
$\frac{C}{2}\left(\frac{r}{100}\right)$ = Annual Interest

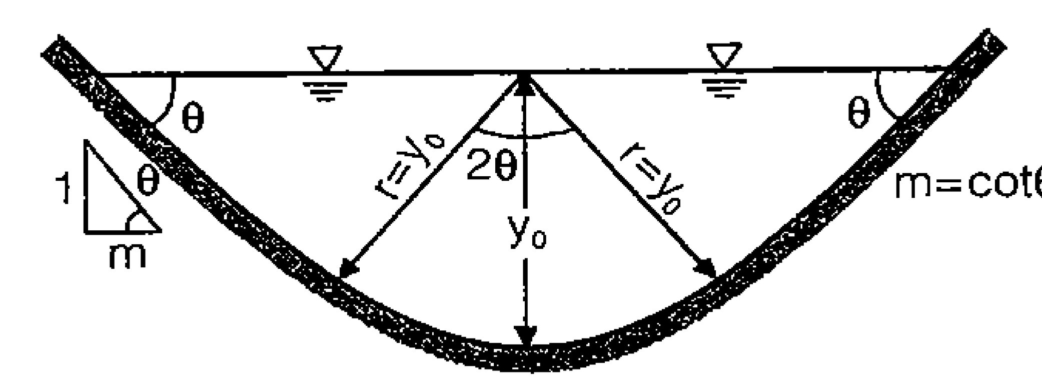
(c) $$\text{Benifit cost ratio} = \frac{mR_1 + PR_2}{\frac{C}{y} + \frac{C}{2}\left(\frac{r}{100}\right)}$$

Design of Lined Canals

A. Triangular Section

- Used when

$Q \le 150$ cumec



(i) $A = y^2(\theta + \cot\theta)$

(ii) $P = 2y(\theta + \cot\theta)$

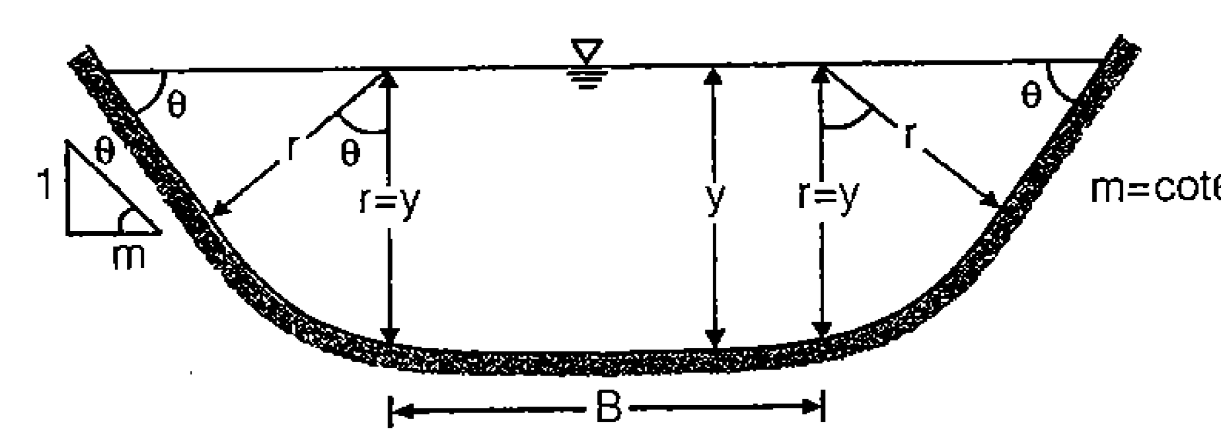
(iii) $R = \frac{A}{P} = \frac{y}{2}$ where, A = Area (m^2)
y = Central depth = Radius of circle
θ = Angle
R = Hydraulic mean depth.

B. Trapezoidal Section

$A = By + y^2(\theta + \cot\theta)$

$P = B + 2y(\theta + \cot\theta)$

$R = \frac{A}{P}$



Sediment Transport Mechanics

(i) Du-Bois Formula

$$\boxed{q_b = k_b \cdot \tau_0(\tau_0 - \tau_c)}$$

where, q_b = Bed load (volume) transported in m^3 per second per unit width of channel.

τ_o = Average shear stress on the channel boundary. (N/m^2)

τ_c = Minimum shear stress required to move the grain called shear stress. (N/m^2)

k_b = A constant depending upon the grain size & given as.

$$\boxed{k_b = \frac{0.178}{(d)^{3/4}}}$$ where d is effective grain dia in mm

(ii) Shield's Formula

$$\boxed{\frac{q_b}{q} = \frac{\gamma \cdot d(S_s - 1)}{10(\tau_0 - \tau_c)}}$$ where, $\frac{q_b}{q}$ = Load carrying capacity.

q_b = Bed load transported in m^3/sec.

S_s = Specific gravity of the bed grain.

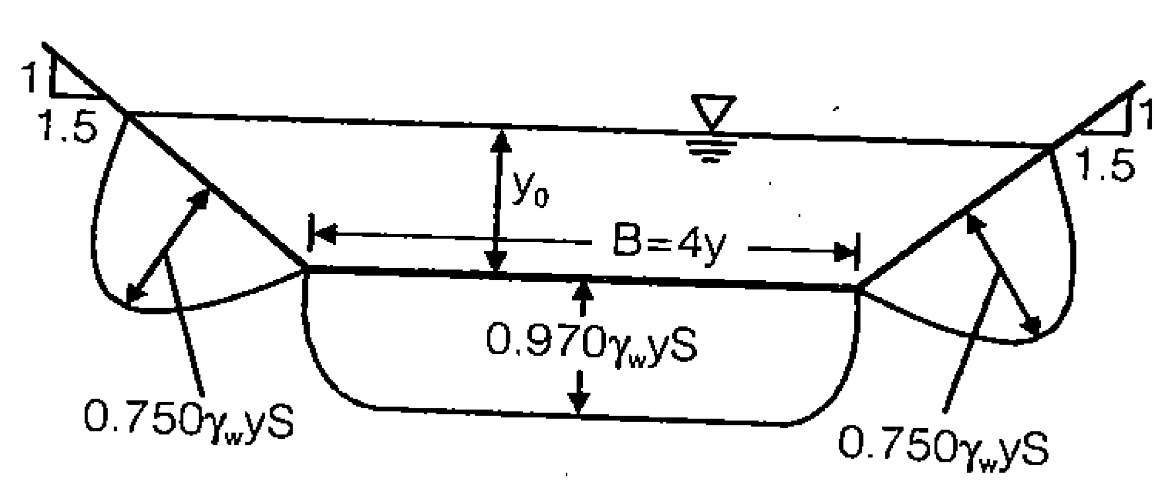
q = Discharge per unit width in m^2/s

γ = Unit weight of fluid in kN/m^3.

d = Dia of bed grain in meter.

Design of Unlined Canal

(i) $\boxed{\tau_0 = \gamma_w RS}$



where, S = Channel longitudinal slope

R = Hydraulic mean depth

τ_0 = Tractive stress (shear stress) at the channel bottom when water flows through the channel.

(ii) $\boxed{\tau_0' = 0.75\gamma_w RS}$

where, τ_0' = Average tractive stress at the channel side.

γ_w = Unit weight of water in kN/m^3.

(iii) **For No Sediment Movement**

$\boxed{\tau_0 \le \tau_c}$ where, τ_c = Critical tractive stress.

(iv) **At Channel Bottom**

$$\boxed{\tau_c = 0.056\gamma_w \cdot d(S_s - 1)}$$

where, γ_w = Unit weight of water (kN/m^3)

d = Dia of sediments.

S_s = Specific gravity of sediments $\simeq$ 2.65

$$\boxed{\tau_c = \frac{\gamma_w \cdot d}{11}}$$ at G = 2.67.

(v) **At Channel Side**

$$\boxed{\tau_c' = \tau_c\sqrt{1 - \frac{\sin^2\theta}{\sin^2\phi}}}$$ where, θ = Sides slope angle

ϕ = Angle of repose of soil.

(vi) **For No Scouring**

$\boxed{\tau_0 \le \tau_c}$ → For no scouring from channel bottom.

→ $\boxed{d \ge 11RS}$

$\boxed{\tau_0' \le \tau_c'}$ → For no scouring from channel sides.

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Design & Construction of Gravity Dams

3

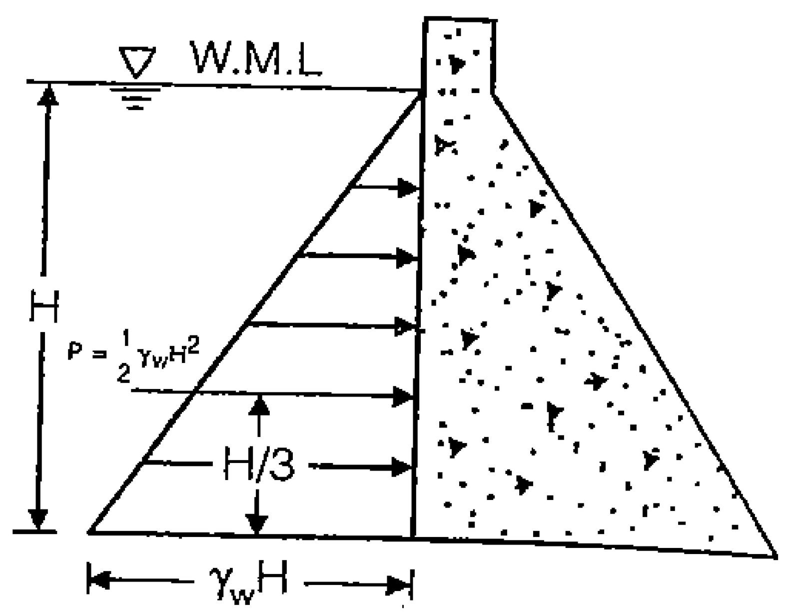
Forces Acting on Gravity Dam

(i) Water Pressure

$$P = \frac{1}{2}\gamma_w H^2$$

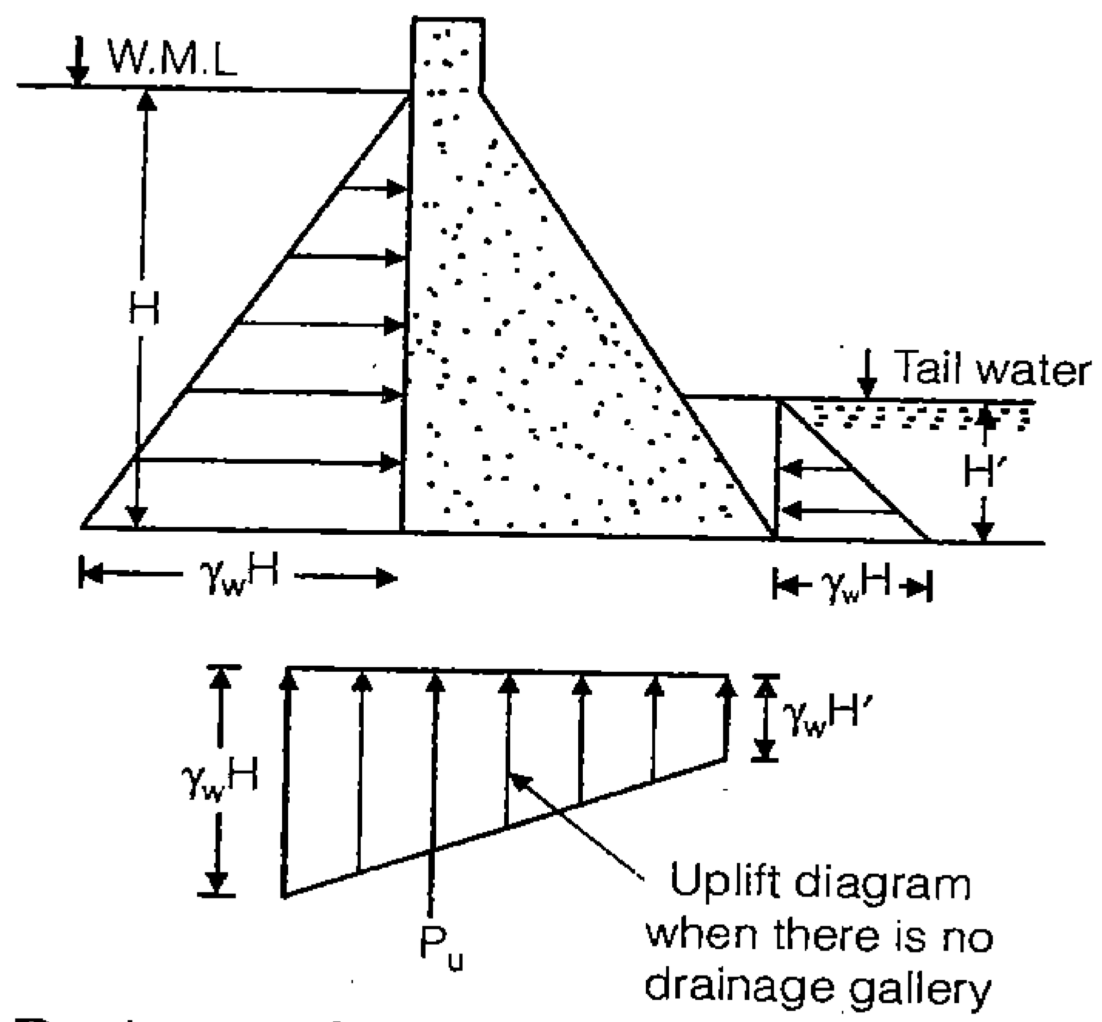
Acting at $\frac{H}{3}$ from base

where γ_w = Unit weight of water.

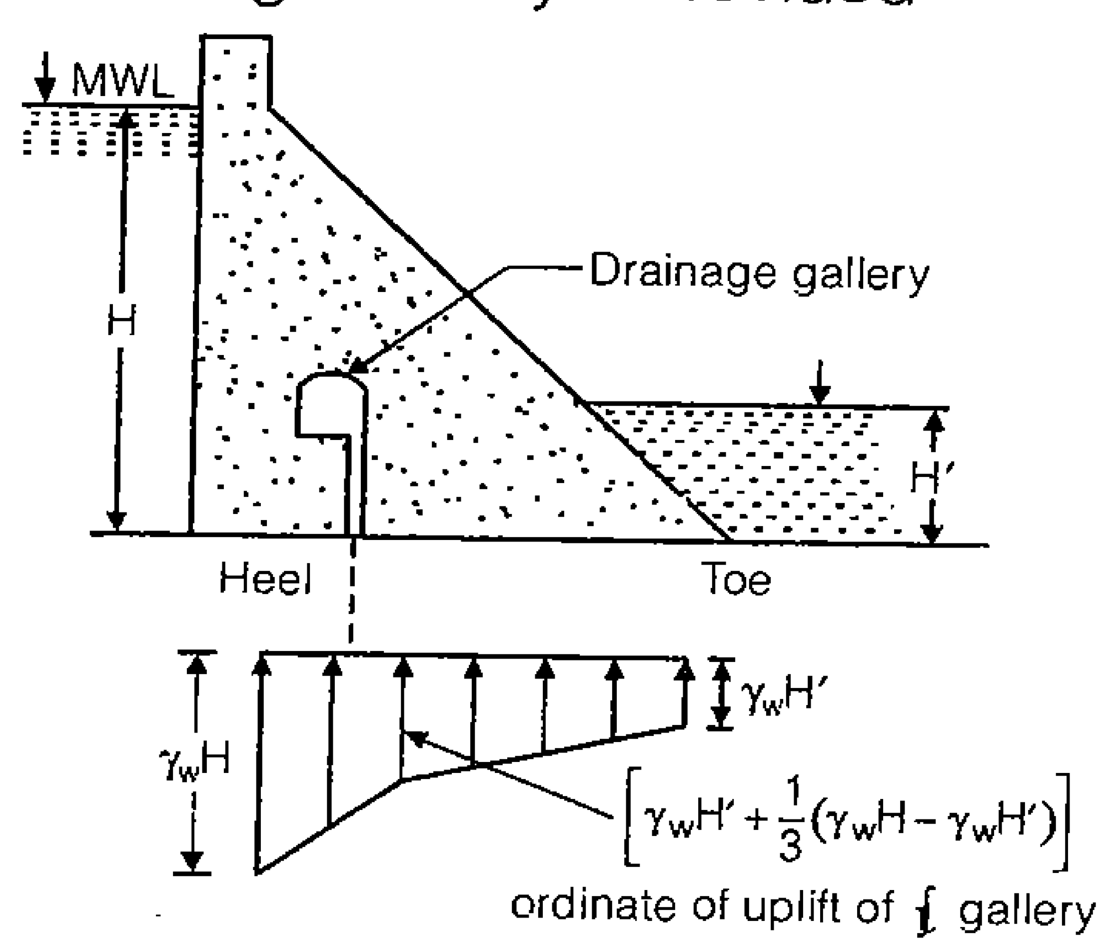


(ii) Uplift Pressure

(a) When Drainage Gallery is not Provided



(b) When Drainage Gallery is Provided



(iii) Earthquake Force

$$\alpha_H = 0.1g \text{ to } 0.2g$$

$$\alpha_v = 0.75\alpha_H$$

$$\alpha = \beta I \alpha_0$$

where, α_H = Horizontal acceleration
α_v = Vertical acceleration
α = Seismic coefficient
β = Soil foundation system factor
I = Importance factor
α_0 = Basic seismic coefficient which depends upon seismic zone of country.

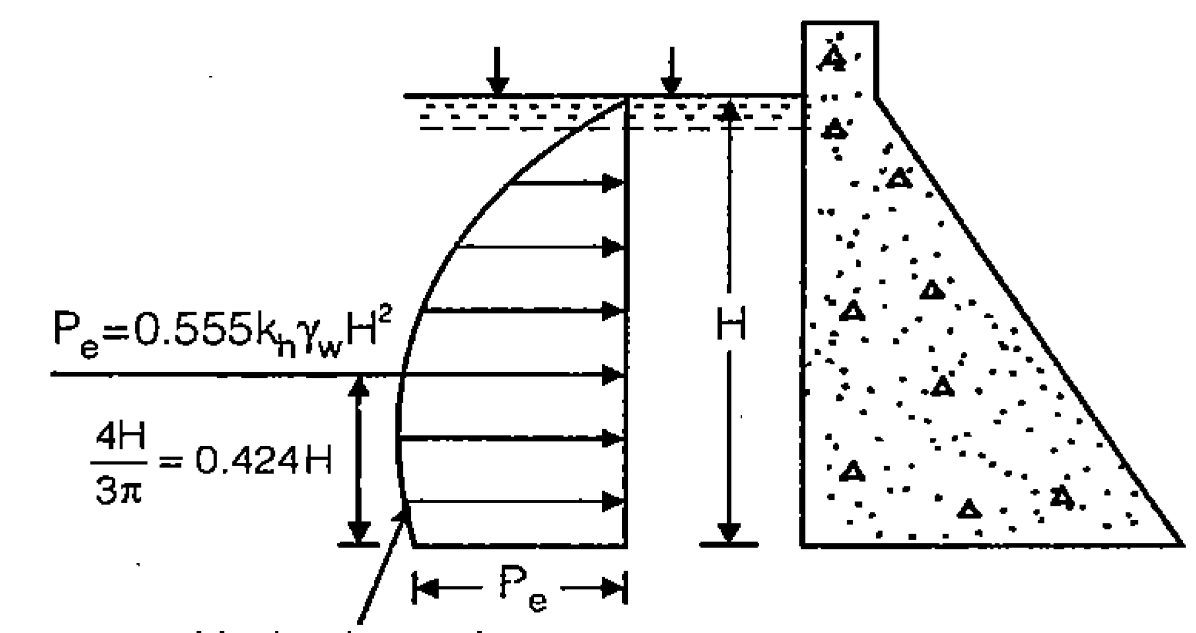
$$F_g = \frac{w}{g}(g \pm \alpha_v)$$

where, F_g = Body force
g = Acceleration due to gravity, +ve for upward & –ve for downward.

(a) Hydrodynamic force

(i) $$F_H = \left(\frac{w}{g}\right)\alpha_H$$ F_H = Horizontal Inertia force.

(ii) It effect of water pressure due to earthquake distribution of pressure is parabolic.



Hydrodynamic pressure distribution

$$P_e = 0.555\,\alpha\,\gamma_w H^2$$

at 0.424 H from base.

α = 0.1g to 0.2g

(iv) Silt Pressure

$$F_{silt} = \frac{1}{2}k_a\,\gamma_{sub}\cdot h_s^2$$

where, h_s = Height of silt from the base.

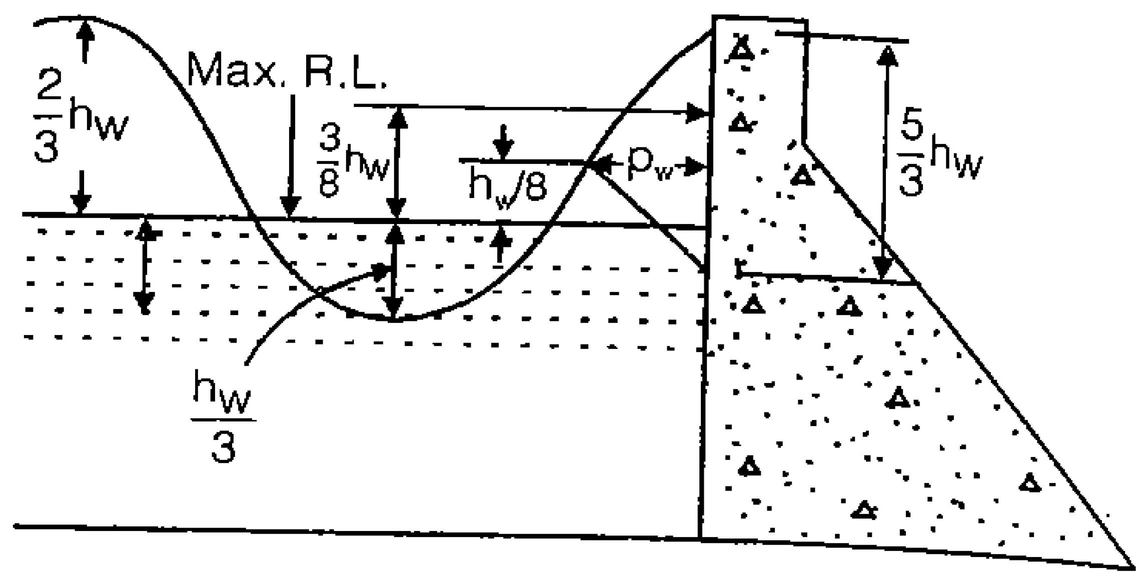
k_a = Coefficient of active earth pressure $= \frac{1-\sin\phi}{1+\sin\phi}$

γ_{sub} = Submerged unit weight.

According to U.S.B.R

$$F_{silt} = \frac{360 h_s^2}{2}(\text{kg f})$$

(v) **Wave Pressure**



$$\boxed{P_w = 2\cdot 4\gamma_w h_w}$$

Acts at $\frac{hw}{8}$ from still water level.

where, P_w = Resultant wave pressure.

$\boxed{F_w = 2\gamma_w h_w^2}$ acts at $\frac{3h}{8}$ from still water surface.

where F_w = Total wave force.

$\boxed{h_w = 0.032\sqrt{VF} + 0.763 - 0.271(F)^{3/4}}$ if $F < 32$ km.

$\boxed{h_w = 0.032\sqrt{VF}}$ when $F > 32$ km.

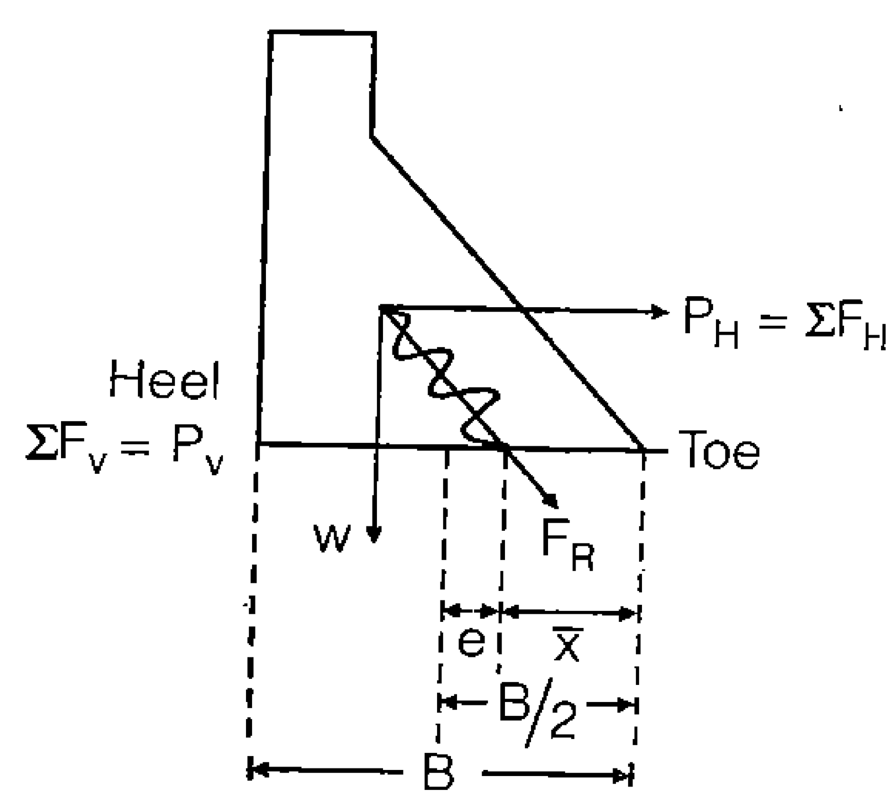
where, F = Length of reservoir in km
h_w = Height of wave in meter
V = Wind velocity in km/hr.

(vi) **Self Weight of Dam**

$\boxed{W = \gamma_c V}$ where, γ_c = Unit weight of concrete
V = Volume of dam body per unit length

Modes of Failure & Criteria for Structural Stability of Gravity Dams

(i) **Failure by Overturning About Toe**



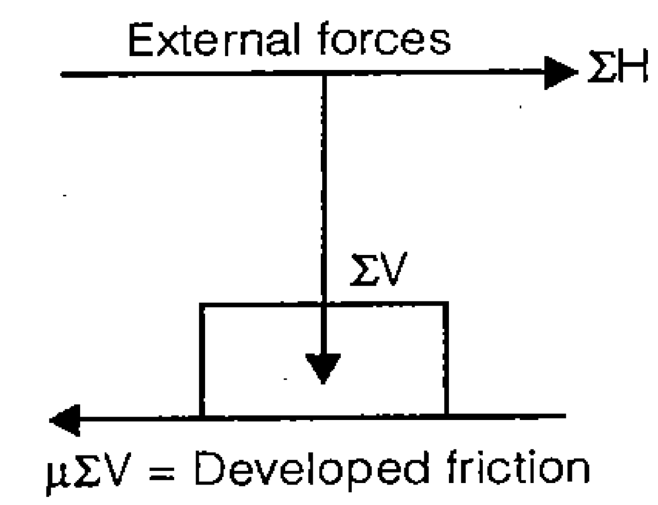
$$\boxed{F_s = \frac{M_R}{M_o} > 1.5}$$

where, F_s = Factor of safety
M_R = Restoring moment about toe (due to ΣF_V)
M_o = Overturning moment about toe (due to ΣF_H)

$\boxed{F_R = \sqrt{F_H^2 + F_V^2}}$ where, F_R = Resultant force
e = Eccentricity

$\boxed{e = \frac{B}{2} - \bar{x}}$ $\bar{x}$ = Distance of $\vec{F_R}$ from toe.

(ii) **Failure due to Sliding**



$\boxed{F_S = \frac{\mu\Sigma F_V}{\Sigma F_H}}$ $\frac{\Sigma F_H}{\Sigma F_V}$ = Sliding factor

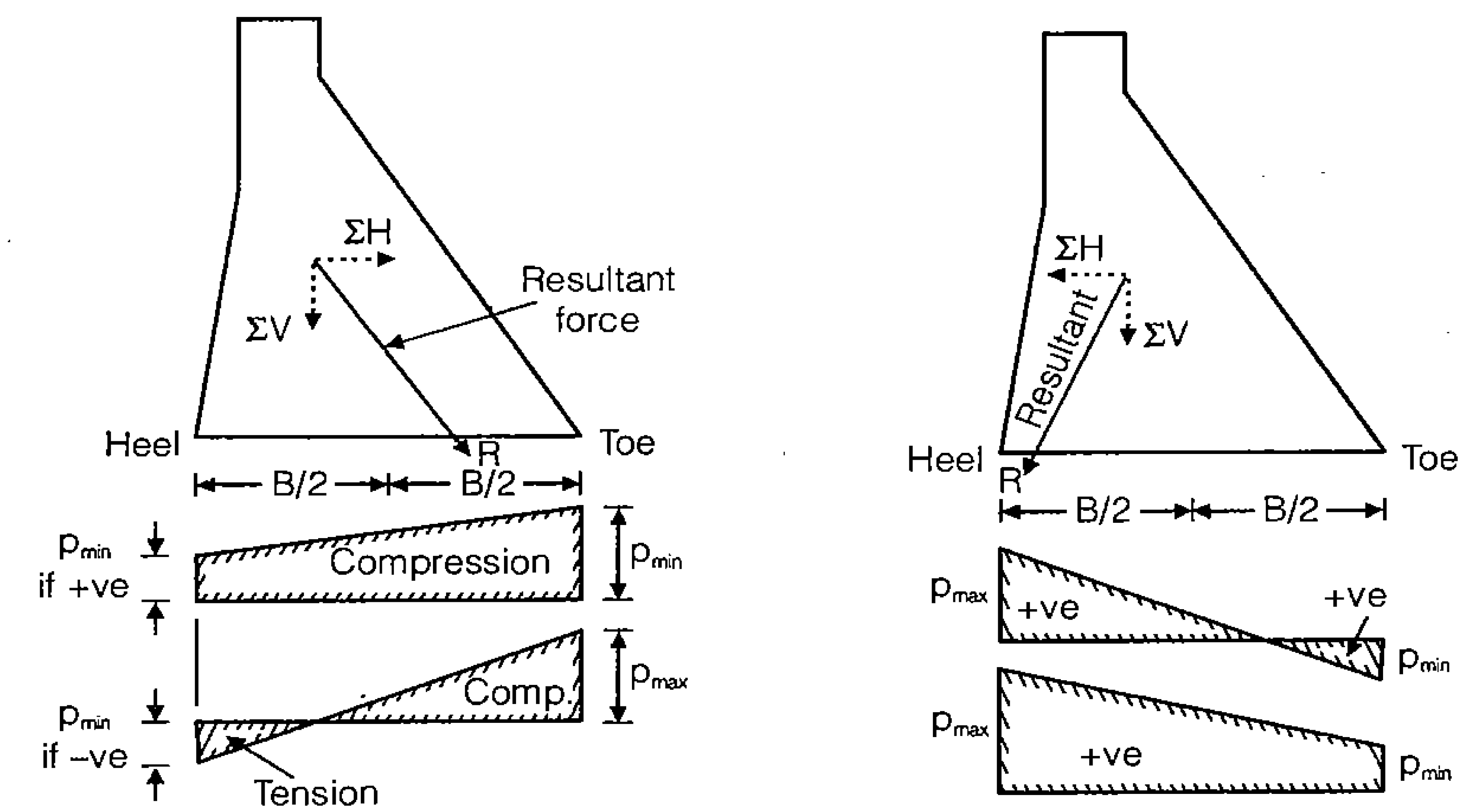
F_S = Factor of safety due to sliding.

$$\boxed{F_S = \frac{\mu}{\text{sliding factor}} > 1}$$

Case : If shear strength is also accounted then factor of safety is called shear frictional factor (S.F.F)

$\boxed{S\cdot F\cdot F = \frac{\mu\,\Sigma F_v + q(B\times 1)}{\Sigma F_H}}$ $\boxed{S\cdot F\cdot F > 3}$ where, B = Width in meter.

(iii) **Failure due to Compression or Crushing**

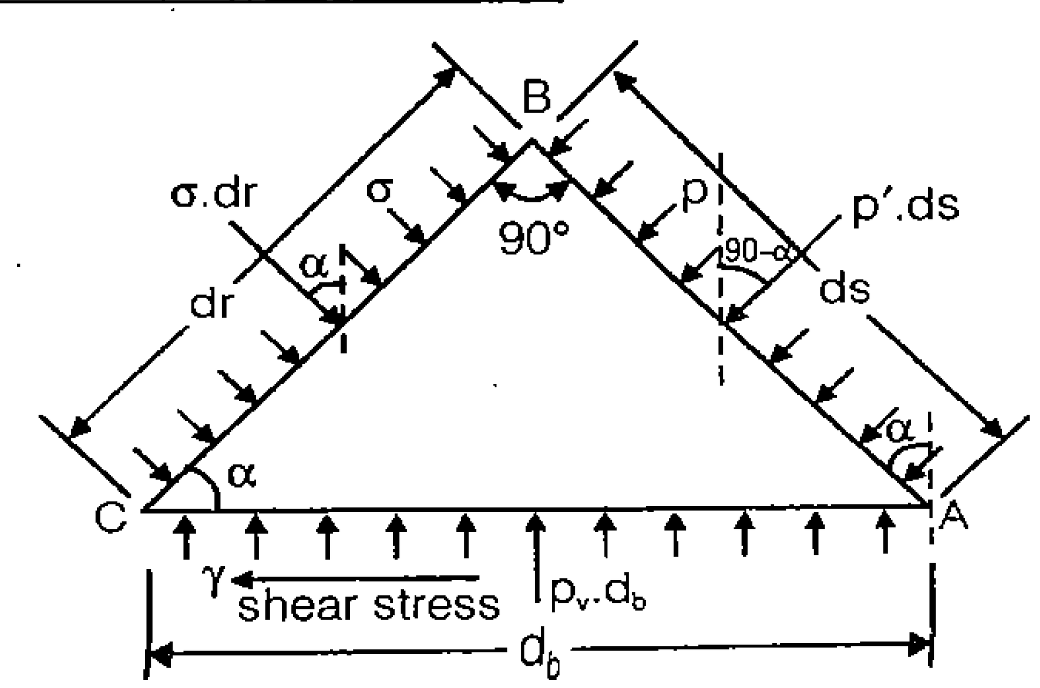


When toe failure occurs. **When heel failure occurs.**

$\boxed{\sigma_{max} = \frac{\Sigma F_V}{(B\times 1)}\left[1+\frac{6e}{B}\right]}$ $\boxed{\sigma_{min} = \frac{\Sigma F_V}{(B\times 1)}\left[1-\frac{6e}{B}\right]}$

$\boxed{\sigma_{max} \le F_C}$ for no failure

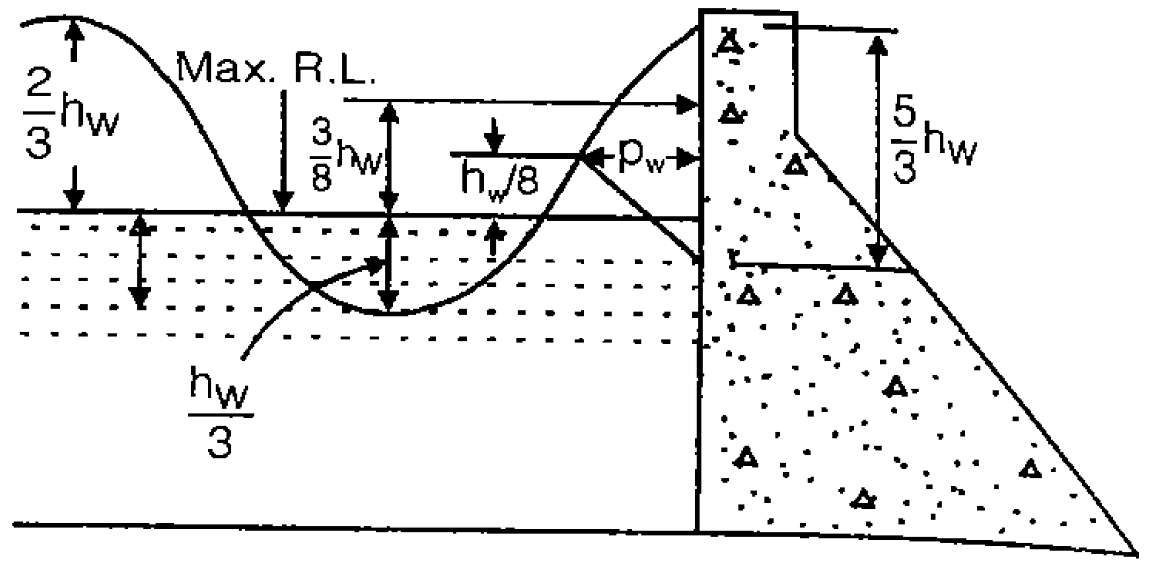
where F_C = Crushing strength.



Case : (I) When shear stress also acts on horizontal plane.

$$\boxed{\sigma_1 = \sigma_v \sec^2\alpha - \sigma_2 \tan^2\alpha}$$

(v) **Wave Pressure**



$$\boxed{P_w = 2\cdot 4\gamma_w h_w}$$

Acts at $\frac{hw}{8}$ from still water level.

where, P_w = Resultant wave pressure.

$\boxed{F_w = 2\gamma_w h_w^2}$ acts at $\frac{3h}{8}$ from still water surface.

where F_w = Total wave force.

$\boxed{h_w = 0.032\sqrt{VF} + 0.763 - 0.271(F)^{3/4}}$ if F < 32 km.

$\boxed{h_w = 0.032\sqrt{VF}}$ when F > 32 km.

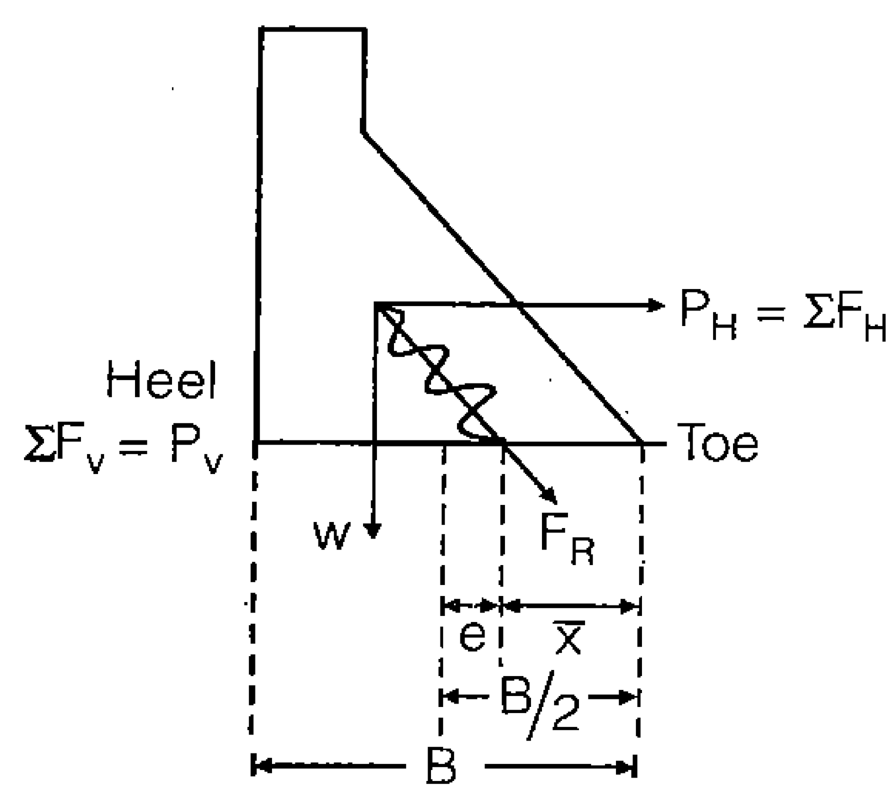
where, F = Length of reservoir in km
h_w = Height of wave in meter
V = Wind velocity in km/hr.

(vi) **Self Weight of Dam**

$\boxed{W = \gamma_c V}$ where, γ_c = Unit weight of concrete
V = Volume of dam body per unit length

Modes of Failure & Criteria for Structural Stability of Gravity Dams

(i) **Failure by Overturning About Toe**



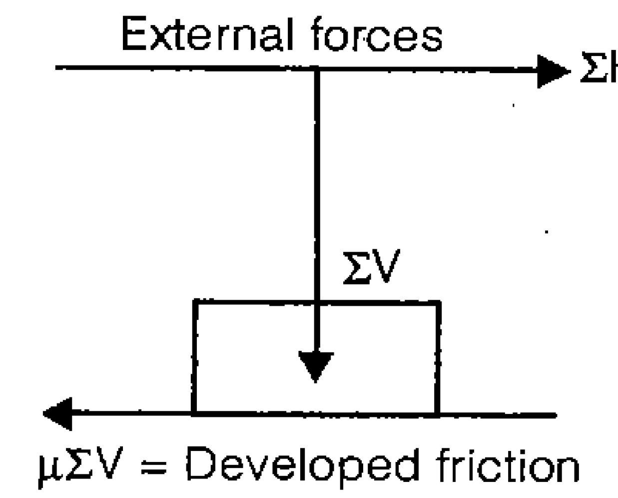
$$\boxed{F_s = \frac{M_R}{M_o} > 1.5}$$

where, F_s = Factor of safety
M_R = Restoring moment about toe (due to ΣF_V)
M_o = Overturning moment about toe (due to ΣF_H)

$\boxed{F_R = \sqrt{F_H^2 + F_V^2}}$ where, F_R = Resultant force
e = Eccentricity

$\boxed{e = \frac{B}{2} - \bar{x}}$ $\bar{x}$ = Distance of $\vec{F_R}$ from toe.

(ii) **Failure due to Sliding**



$\boxed{F_S = \frac{\mu \Sigma F_V}{\Sigma F_H}}$ $\frac{\Sigma F_H}{\Sigma F_V}$ = Sliding factor

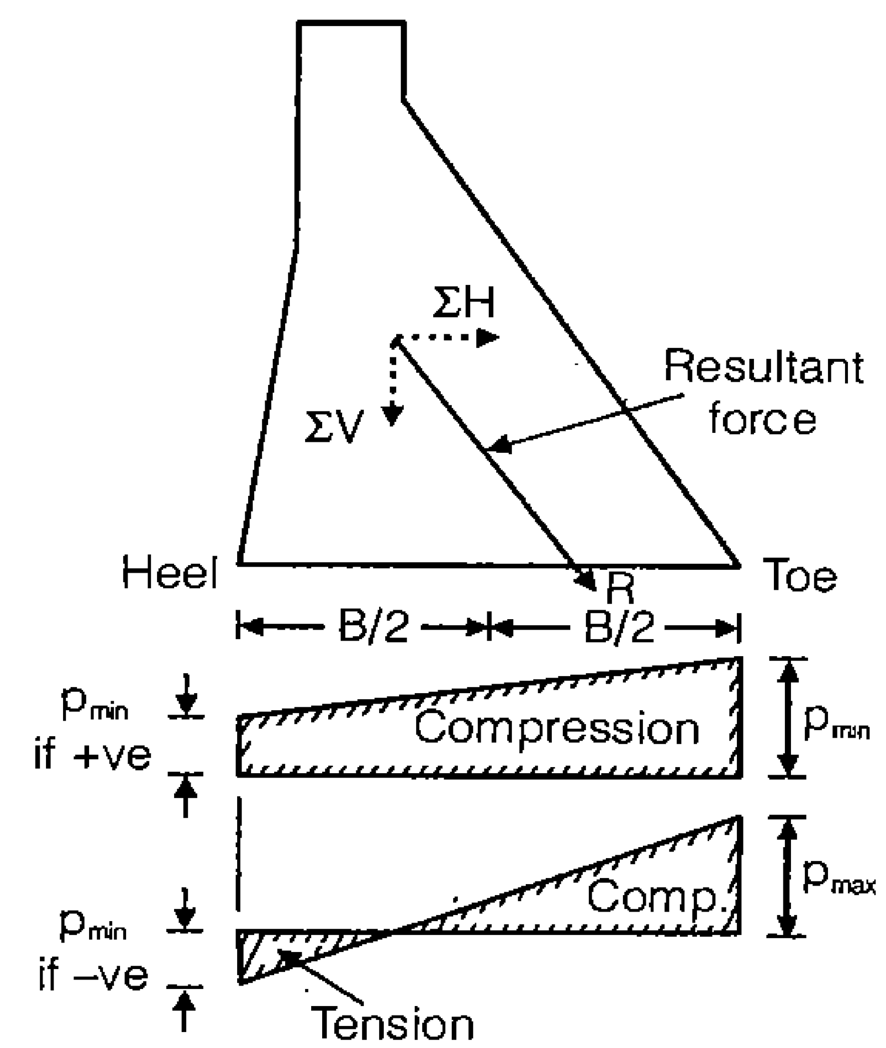
F_S = Factor of safety due to sliding.

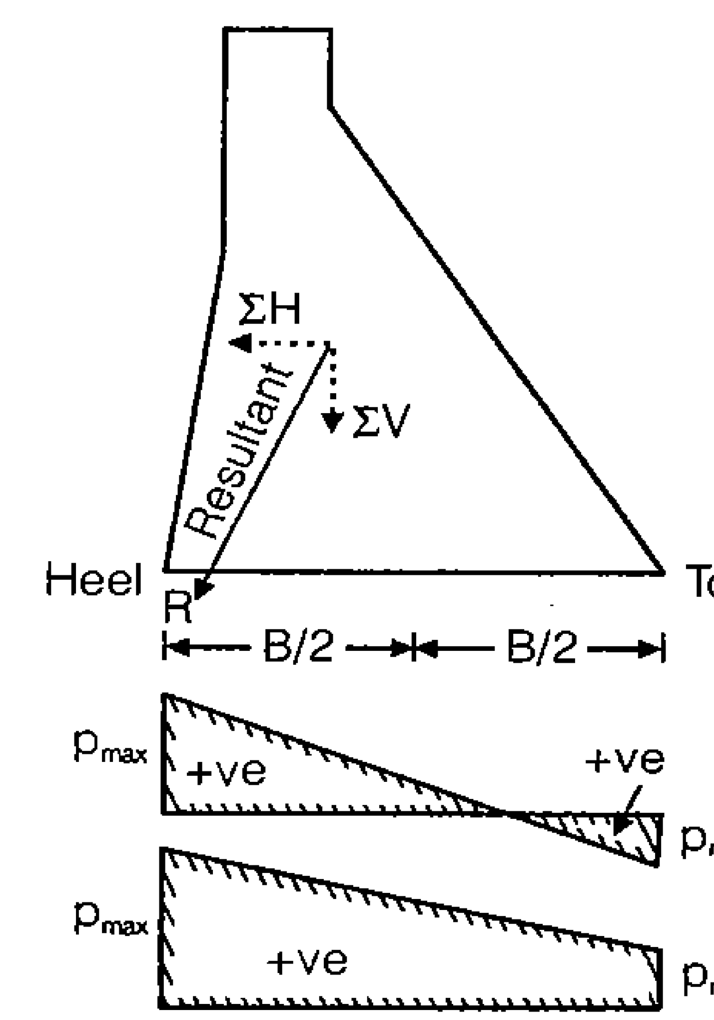
$$\boxed{F_S = \frac{\mu}{\text{sliding factor}} > 1}$$

Case : If shear strength is also accounted then factor of safety is called shear frictional factor (S.F.F)

$\boxed{S\cdot F\cdot F = \frac{\mu\,\Sigma F_v + q(B\times 1)}{\Sigma F_H}}$ $\boxed{S\cdot F\cdot F > 3}$ where, B = Width in meter.

(iii) **Failure due to Compression or Crushing**





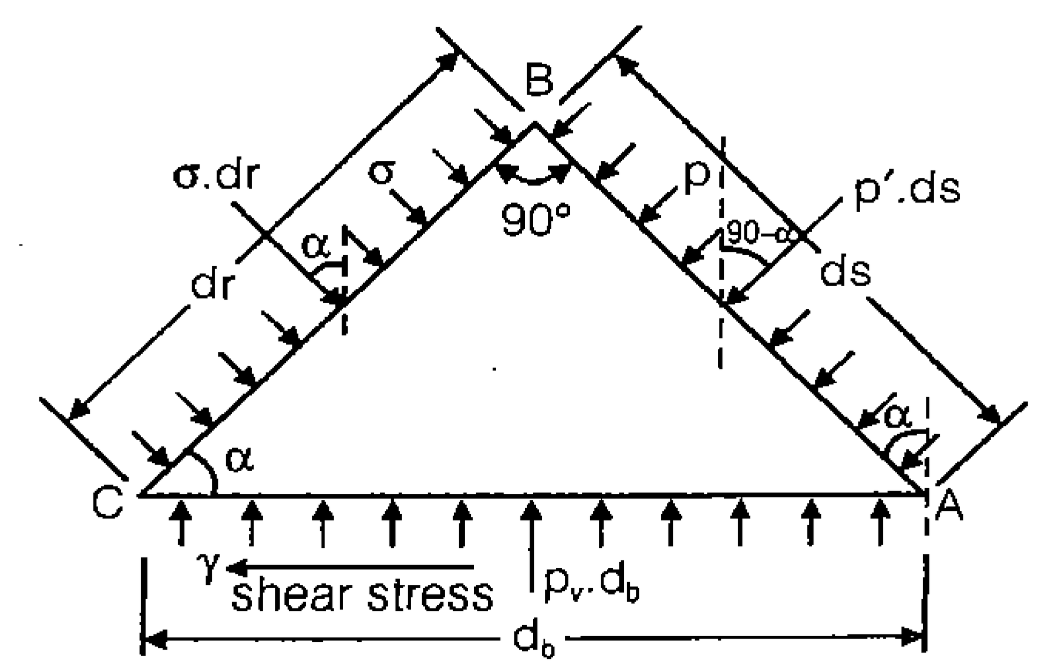
When toe failure occurs. **When heel failure occurs.**

$\boxed{\sigma_{max} = \frac{\Sigma F_V}{(B\times 1)}\left[1+\frac{6e}{B}\right]}$ $\boxed{\sigma_{min} = \frac{\Sigma F_V}{(B\times 1)}\left[1-\frac{6e}{B}\right]}$

$\boxed{\sigma_{max} \le F_C}$ for no failure

where F_C = Crushing strength.

Case : (I) When shear stress also acts on horizontal plane.



$$\boxed{\sigma_1 = \sigma_v \sec^2\alpha - \sigma_2 \tan^2\alpha}$$

$$\sigma_v = \sigma_{max} = \frac{\Sigma F_V}{(B \times 1)}\left[1 + \frac{6e}{B}\right]$$ where, α = Angle of d/s surface with vertical

For no Failure,

$$\sigma_1 \le F_C$$ where, F_c = Crushing strength of concrete.
τ = Magnitude of shear stress on horizontal plane near the toe.

$$\tau = (\sigma_v - \sigma_2)\tan\alpha$$

Case (2): When earthquake force considered then

$$\sigma_1 = \sigma_v \sec^2\alpha - (\sigma_2 - p_e)\tan^2\alpha$$

$$\tau = [\sigma_v - (\sigma_2 - p_e)]\tan\alpha$$ where, P_e = Earthquake Pressure.

(iv) **Failure due to Tension**

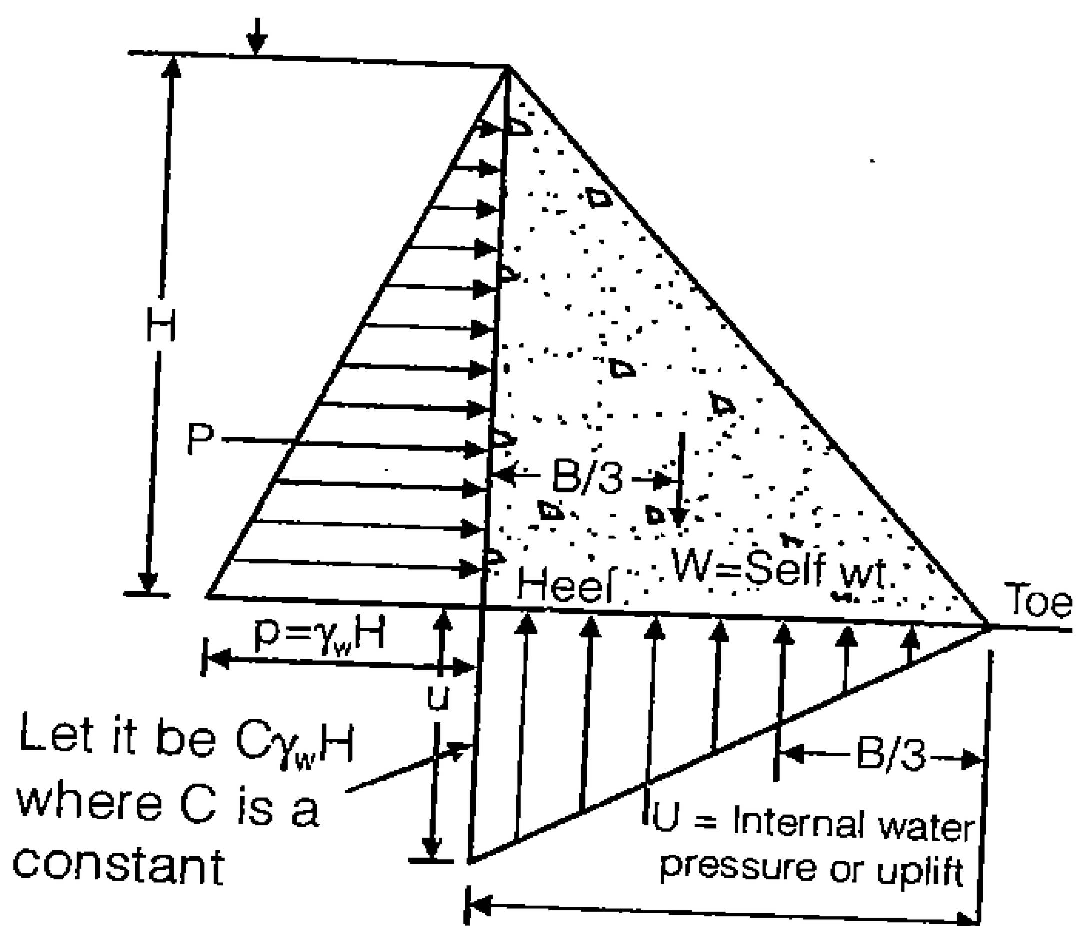
$$\sigma_{min} \ge 0 \rightarrow e \le \frac{B}{\sigma}$$

Elementary Profile of Gravity Dam

$$\Sigma F_H = P_H = \frac{1}{2}\gamma_w H^2$$

$$P_u = \frac{1}{2}C\gamma_w H \cdot B$$

where P_u = Uplift pressure i.e., Force of buoyancy.
C = Uplift pressure coefficient.



$$w = \frac{1}{2}G\gamma_w \cdot BH$$

where w = Weight of dam body for unit length

$$B = \frac{H}{\sqrt{G - C}}$$ where, B = Minimum base width required for no tension criteria.

$$B' = \frac{H}{\mu(G - C)}$$ B′ = Minimum base width for no sliding criteria.
G = Sp. gravity ofconcrete, i.e., that of the material of the dam

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Water Logging 4

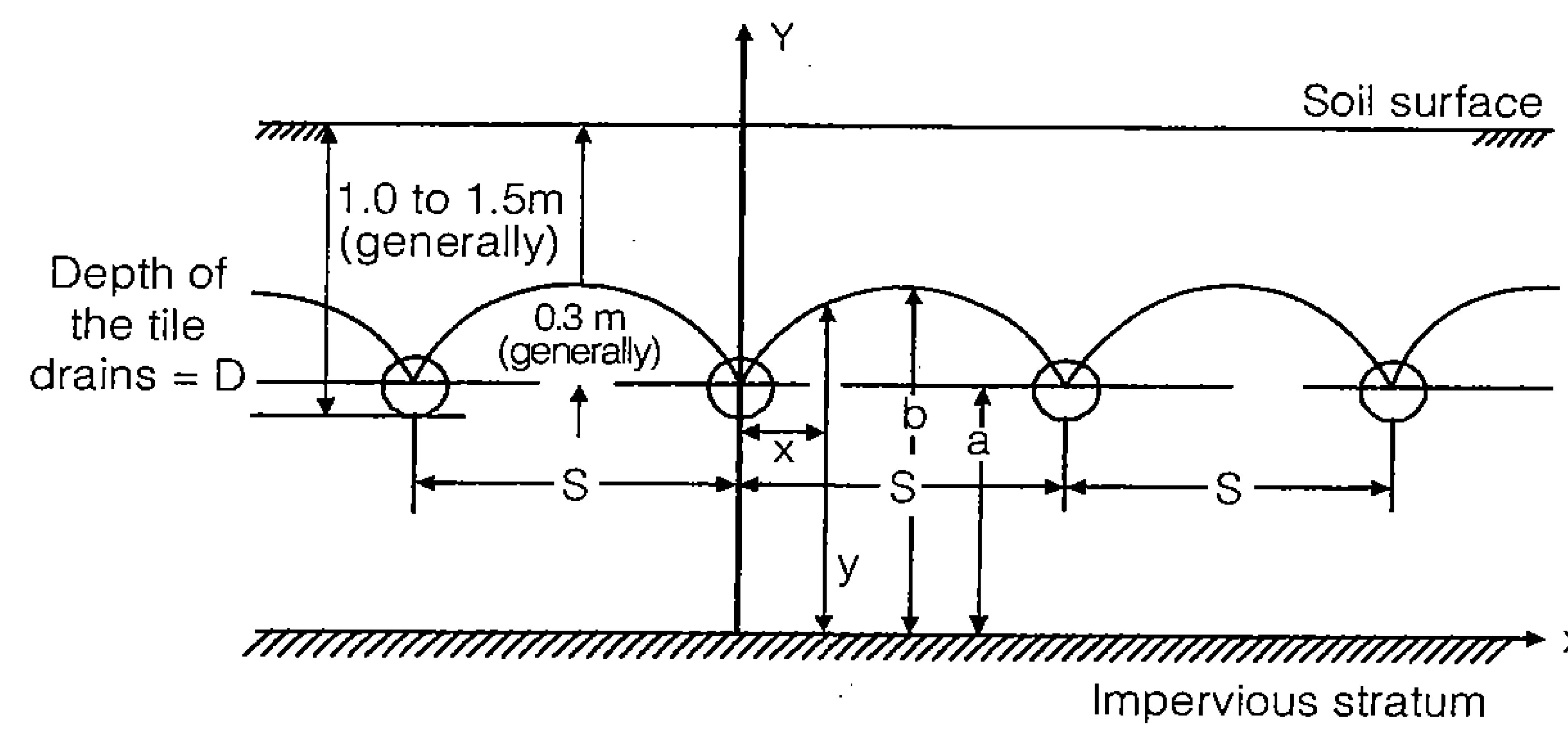
Leaching Requirement, (L.R)

$$L \cdot R = \frac{D_d}{D_a} = \frac{C_i}{C_d} = \frac{(E \cdot C)_i}{(E \cdot C)_d}$$

where, D_d = Depth of water drained out per unit area
D_a = Depth of water applied per unit area
C_i = Salt content of irrigation water
C_d = Salt content of drained water
$(E.C)_i$ = Electrical conductivity of irrigation water
$(E.C)_d$ = Electrical conductivity of drained water.

Depth & Spacing of Tile Drains

$$S = \frac{4k}{q}(b^2 - a^2)$$



where, S = Spacing of tile drains in m
k = Coefficient of permeability in m/s
q = Total discharge per unit length of tile drain $m^3/s/m$
b = height of water table above the impervious layer
a = depth of imprevious stratum below centre of the drain.

$$q = \frac{\left(\frac{p}{100}\right)}{24 \times 3600} \cdot (S \times l)$$ where, P = Annual rainfall in meter at a place.

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River Training & Diversion Geadworks

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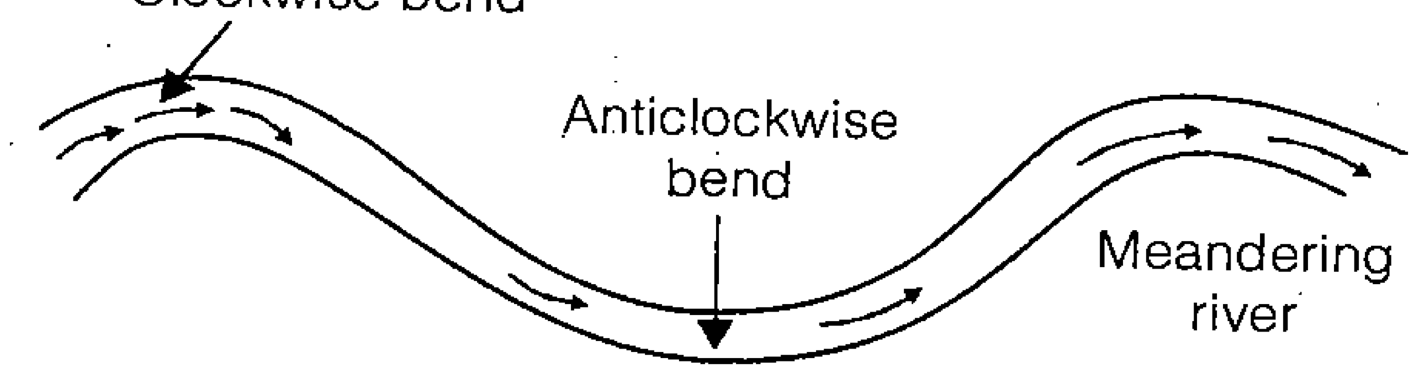
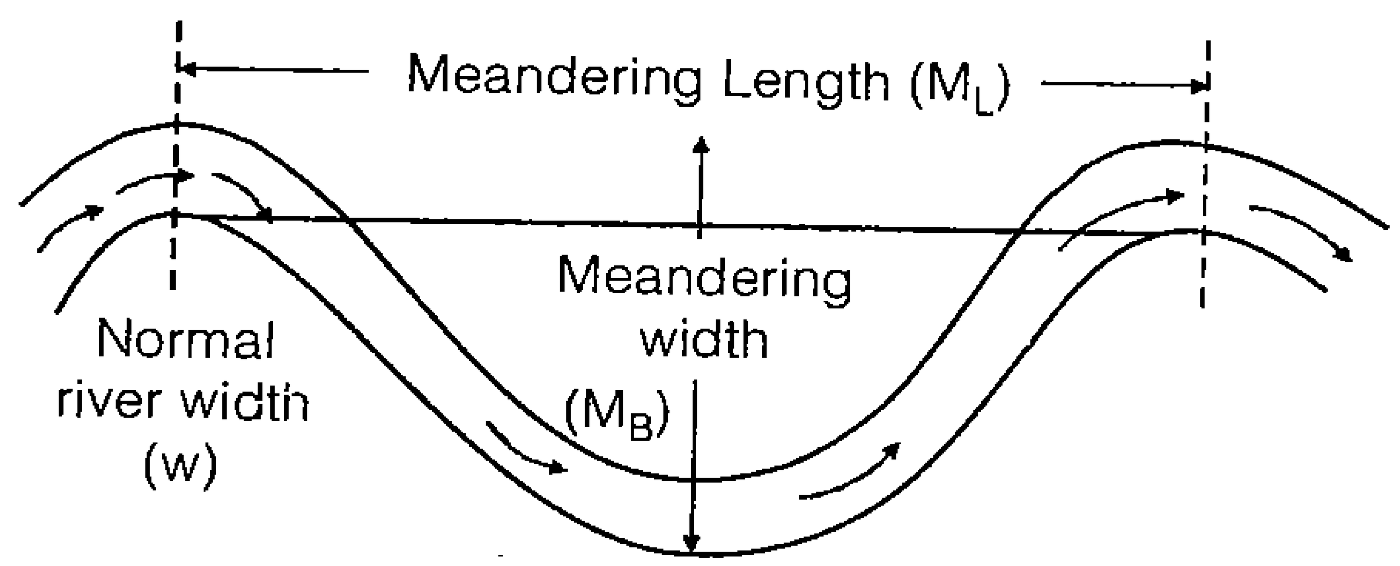
Meanders

If a river deviates from its axial path and a curvature of reverse order is developed with short straight reaches, the river is stated to be a meandering river.



Meander Parameter



(i) **Meander Ratio, (MR)**

$$\boxed{MR = \frac{M_B}{M_L}}$$

where, M_B = Meander Belt
M_L = Meander Length.

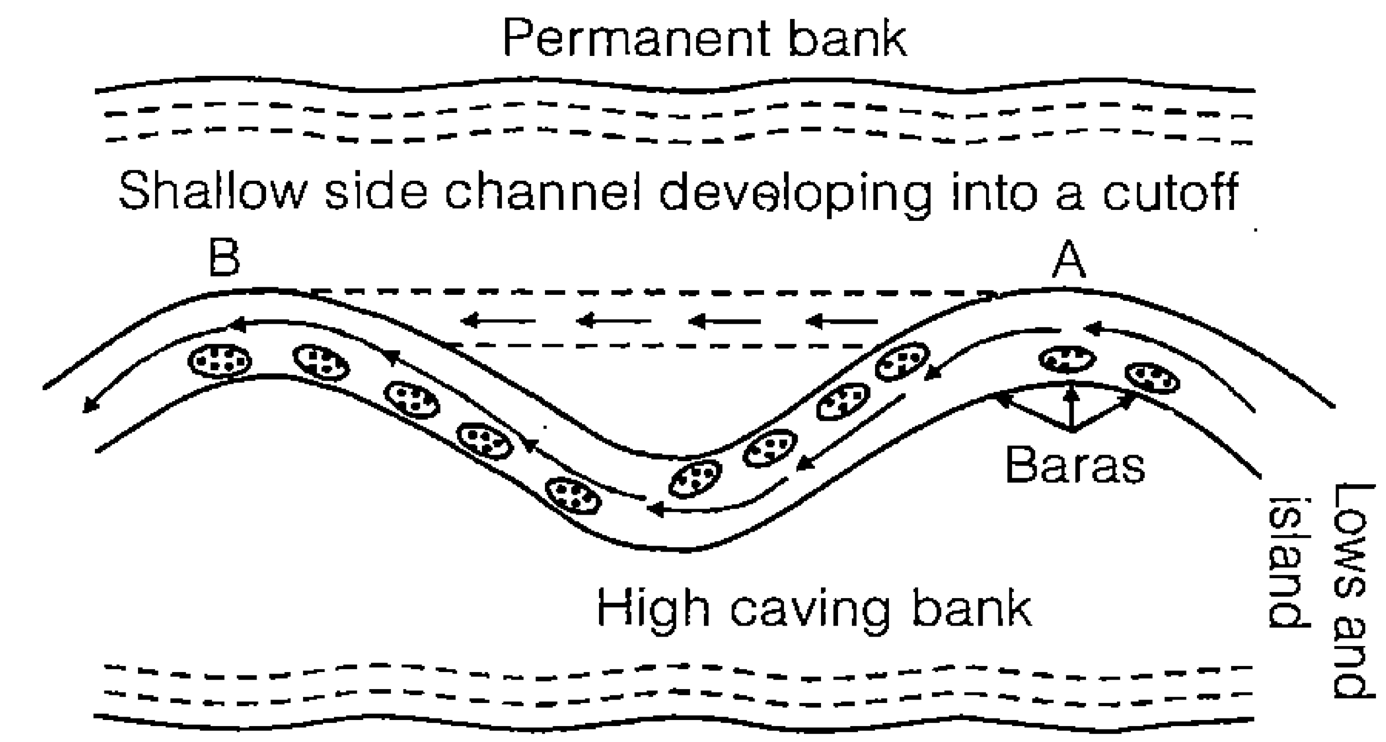
(ii) **Dominant Discharge**

$$\boxed{Q_{dominant} = \frac{1}{2} \text{ or } \frac{2}{3} \text{ of } Q_{max} = \frac{9}{16} \cdot Q_{max} \text{ (generally)}}$$

(iii) **Meander Length for Rivers in Flood Plains**

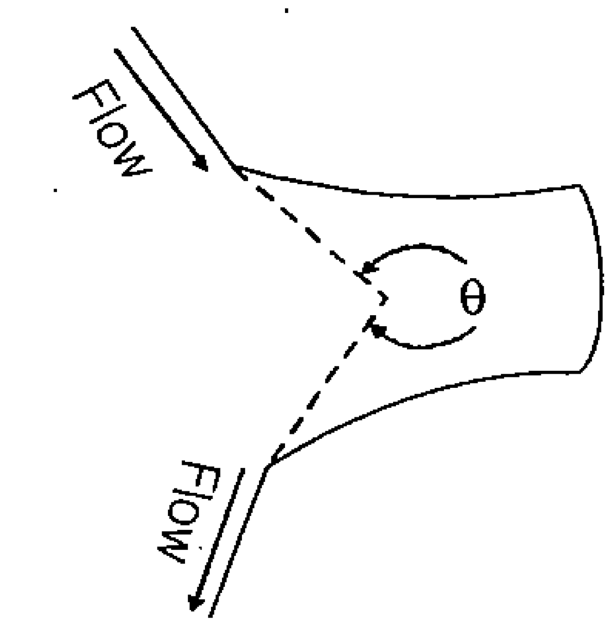
$$\boxed{M_L = 65.8\sqrt{Q_{dominant}}}$$

(iv) $\boxed{\text{Cut} - \text{off ratio} = \frac{ACB}{AB}}$ $\boxed{1.7 \le C.O.R \le 3.0}$



(v) **Angle of Swing**

$$\boxed{\theta = 180° + 2\left[\text{vers}^{-1}\left(\frac{\text{Chord}}{2 \times \text{Radius}}\right)\right]}$$



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A Handbook on
Civil Engineering

13

Engineering Hydrology

CONTENTS

Precipitation & General Aspects of Hydrology 1

Index of Wetness

- $$\text{Index of wetness} = \frac{\text{rainfall in a given year at a given place}}{\text{average annual rainfall of that place}} \times 100$$
- % Rain deficiency = 100 – % Index of wetness

Aridity Index

$$A.I = \frac{PET - AET}{PET} \times 100$$

where, A.I. = Aridity Index
PET = Potential Evapo-transpiration
AET = Actual Evapotranspiration

(a) $AI \le 0 \rightarrow$ Non arid
(b) $1 \le A.I \le 25 \rightarrow$ Mild Arid
(c) $26 \le A.I \le 50 \rightarrow$ Moderate Arid
(d) $A.I > 50 \rightarrow$ Severe Arid.

Remember: In this AI calculation, AET is calculated according to Thornthwaite's water balance technique.

Optimum Number of Rain Gauge : (N)

$$N = \left(\frac{C_v}{\epsilon}\right)^2 \qquad C_v = \frac{\sigma_{n-1}}{\bar{x}} \times 100 \qquad \sigma_{n-1} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{(n-1)}} \qquad \bar{x} = \frac{\Sigma x}{n}$$

where, C_v = Coefficient of variation, ϵ = Allowable % Error,
σ = Standard deviation of the data, n = Number of stations
$\bar{x}$ = Mean of rainfall value

Estimation of Missing Rainfall Data

(a) $$P_x = \frac{P_1 + P_2 + \ldots + P_n}{(n)}$$ If $N_1, N_2 \ldots N_n < 10\%$ of N_x

where, $N_1, N_2, \ldots N_x \ldots N_n$ are normal annual precipitation of 1, 2, ... x ... n respectively.
$P_1, P_2 \ldots P_n$ are rainfall at station 1, 2, ... n respectively.
and P_x is the rainfall of station x.
Case : A minimum number of three stations closed to station 'x'

$$P_x = \frac{P_1 + P_2 + P_3}{3}$$

(b) $$P_x = \frac{N_x}{n}\left[\frac{P_1}{N_1} + \frac{P_2}{N_2} + \ldots + \frac{P_n}{N_n}\right]$$ If any of $N_1, N_2, N_3 \ldots N_n > 10\%$ of N_x.

Mean Rainfall Data

To convert the point rainfall values at various stations into an average value over a catchment the following three methods are in use

(i) **Arithmetic Avg Method:** When the rainfall measured at various stations in a catchment show little variation, the average precipitation over the catchment area is taken as the arithmetic mean of the station values.

$$P_{avg} = \frac{P_1 + P_2 + \ldots + P_n}{n}$$

where, $P_1, P_2 \ldots P_n$ are rainfall values of station 1, 2, ... n respectively.

In practice this method is used very rarely.

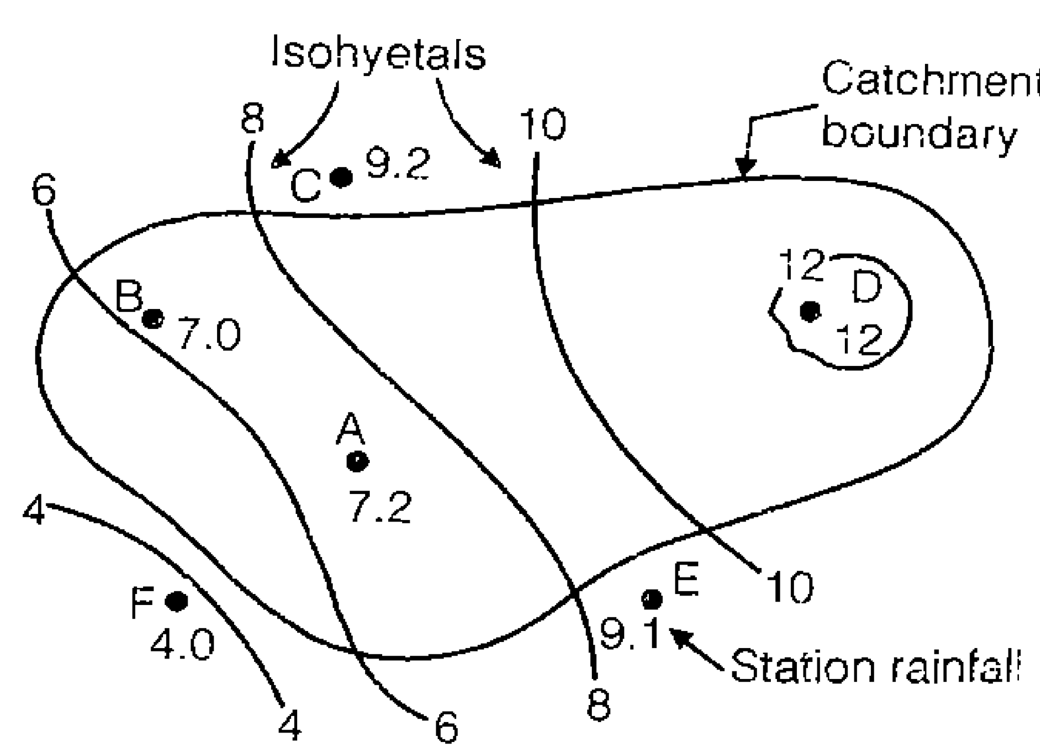
(ii) **Thiessen Polygon Method:** In this method the rainfall recorded at each station is given a weightage on thebasis of an area closest to the station.

$$P_{avg} = \frac{P_1A_1 + P_2A_2 + \ldots + P_nA_n}{A_1 + A_2 + \ldots + A_n}$$

where, $P_1, P_2, \ldots P_n$ are the rainfall data of areas $A_1, A_2 \ldots A_n$.

The Thiessen-polygon method of calculating the average percipitation over an area is superior to the arithmetic average method.

(iii) **Isohyetal Method:** An isohyet is a line joining points of equal rainfall magnitude. The recorded values for which areal average P is to be determined are then marked on the plot at appropriate stations. Neighbouring stations outside the catchment are also considered.



$$P_{avg} = \frac{A_1\frac{(P_1 + P_2)}{2} + A_2\frac{(P_2 + P_3)}{2} + \ldots A_{n-1}\frac{(P_{n-1} + P_n)}{2}}{A_1 + A_2 + \ldots + A_{n-1}}$$

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2 Evaporation and Its Measurement

Evaporation is a cooling process in which the latent heat of evaporation of about 585 cal/gm is provided by the water body. In this process liquid changes into gaseous phase at free surface, below the boiling point through the transfer of heat energy.

Dalton's Law

The rate of evaporation is proportional to the difference between the saturation vapour pressure at the water temperature, e_s and the actual vapour pressure in the air, e_a. Thus

$$E = K(e_s - e_a)$$

where, E = Rate of evaporation (mm/day)

e_s = Saturation vapour pressure of air (mm)

e_a = Actual vapour pressure of air (mm)

$$e_s - e_a = \text{Saturation defficiency}$$

Measurement of Evaporation

(i) ISI Standard Pan

$$\text{Lake evaporation} = C_p \times \text{Pan evaporation}$$

where, C_p = Pan coefficient

= 0.8 for ISI pan

= 0.7 for class-A pan

(ii) Empirical Evaporation Equations (Meyer's Formula)

$$E = k_m(e_s - e_a)\left[1 + \frac{V_9}{16}\right]$$

where k_m = Coefficients which accounts for size of water body.

= 0.36 (for large deep water)

$\simeq$ 0.50 (for small and shallow waters)

e_s = Saturation vapour pressure of air in mm of Hg.

e_a = Actual vapour pressure of overlying air in mm at Hg at specified height of 8 m.

V_9 = Monthly mean wind velocity in km/hr at about 9 m above the ground level.

1/7th Power Law

- $$\frac{V_1}{V_2} = \left(\frac{H_1}{H_2}\right)^{1/7}$$ where V_1 is the wind velocity at height H_1 and V_2 is the wind velocity at height H_2.

Water Budget Method

This is simplest method but it is least reliable. It is used for rough calculation. It is based on mass conservation principle.

$$P + V_{is} + V_{ig} = V_{os} + V_{og} + E + \Delta S + T_L$$

where, P = Daily precipitation on the water surface.
V_{is} = Daily surface inflow into lake.
V_{os} = Daily surface outflow from the lake.
V_{ig} = Daily underground inflow into the lake
V_{og} = Daily underground outflow from the lake
E = Daily Evaporation
ΔS = Change in storage of lake
= +ve if increase in storage
= –ve if decrease in storage
T_L = Daily transpiration loss from the plants on the lake.

Energy Budget Method

The energy budget method is an application of the law of conservation of energy. The energy available for evaporation is determined by considering the incoming energy, outgoing energy and energy stored in the water body over a known time interval.

$$E = \frac{H_n - H_g - H_s - H_i}{\delta \cdot L(1+\beta)}$$ where, H_n = Net heat energy received by the water surface

$$H_n = H_c(1-r) - H_b$$

$H_c(1-r)$ = Incoming solar radiation into a surface of reflection coefficient, r
H_b = Back radiation from water body
H_g = Heat flux into the ground
H_s = Heat stored in water body
H_i = Net heat conducted out the system by water flow (advected energy)
β = Bowen's ratio.
δ = Density of water
L = Latent heat of evaporation.

Evapo-Transpiration

While transpiration takes place, the land area in which plants stand also lose moisture by the evaporation of water from soil and water bodies. In hydrology and irrigation practive, it is found that evaporation and transpiration processes can be considered advantageously under one head as evapo-transpiration.

The real evapo-transpiration occurring in a specific situation is called actual evapo-transpiration (AET).

- **Penman's Method**

  Penman's equation is based on sound theoretical reasoning and is obtained by a combination of the energy balance and mass transfer approach.

$$PET = \frac{AH_n + E_a\gamma}{A + \gamma}$$

where, PET = Daily evaporation in mm/day.
A = Slope of the saturation vapour pressure v/s Temperature curve at the mean air temperature in mm of Hg per °C.
H_n = Net radiation in mm of **evaporable** water per day
E_a = Parameter including wind velocity and saturation deficit.
γ = Psychometric constant
= 0.49 mm of Hg/°C

It is based on mass transfer and energy balance.

Transpiration Loss (T)

$$T = (w_1 + w) - w_2$$

where, w_1 = Initial weight of the instrument
w = Total weight of water added for full growth of plant.
w_2 = Final weight of instrument including plant and water
T = Transpiration loss.

StreamFlow Measurement

Streamflow representing the runoff phase of the hydrologic cycle is the most important basic data for hydrologic studies.

Streamflow measurement techniques can be broadly classified into two categories as:

1. Direct determination and
2. Indirect determination.

Under each category there are a host of methods, the important ones are listed below:

1. Direct determination of stream discharge
   (a) Area velocity methods
   (b) Dilution techniques
   (c) Electromagnetic method and
   (d) Ultrasonic method
2. Indirect determination of streamflow
   (a) Hydraulic structures, such as weirs, flumes and gated structures, and
   (b) Slope-area method

Determination of Velocity

(i) **Float Method:** Float are generally used to determine approximate velocity of the surface. These are floating devices which are passed with the water along the flow of stream.

$$V_s = \frac{L}{t}$$

Here, V_s = Surface velocity

L = Distance travelled by the float in time 't'.

(ii) **Current Meters Method:** These consist of rotating elements which rotate due to reactions of stream currents. The number of revolution per sec are counted. This can be used to measure point velocity of any depth.

$$V = aN_s + b$$

where, V = Point velocity

N_s = Number of revolution per sec. a and b are current meters constant.

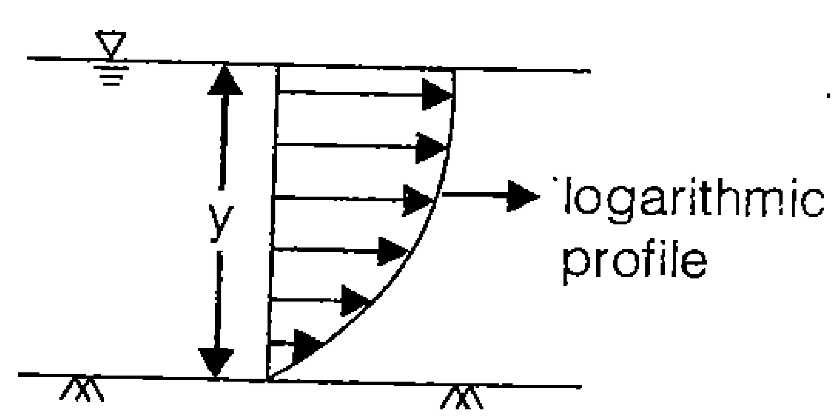
Velocity Distribution

(i) $\bar{V} = (0.85 \text{ to } 0.95)V_s$

for turbulent flow

where, $\bar{V}$ = Mean velocity

V_s = Surface velocity



(ii) **For Shallow Streams**

$\bar{V} = V_{0.6y}$ where $\bar{V}_{0.6y}$ = Point velocity at 0.6y from surface

(iii) **For Deep Streams** $\bar{V} = \dfrac{V_{0.2y} + V_{0.8y}}{2}$

- Sounding Weight

$$W = 50\bar{V}y$$

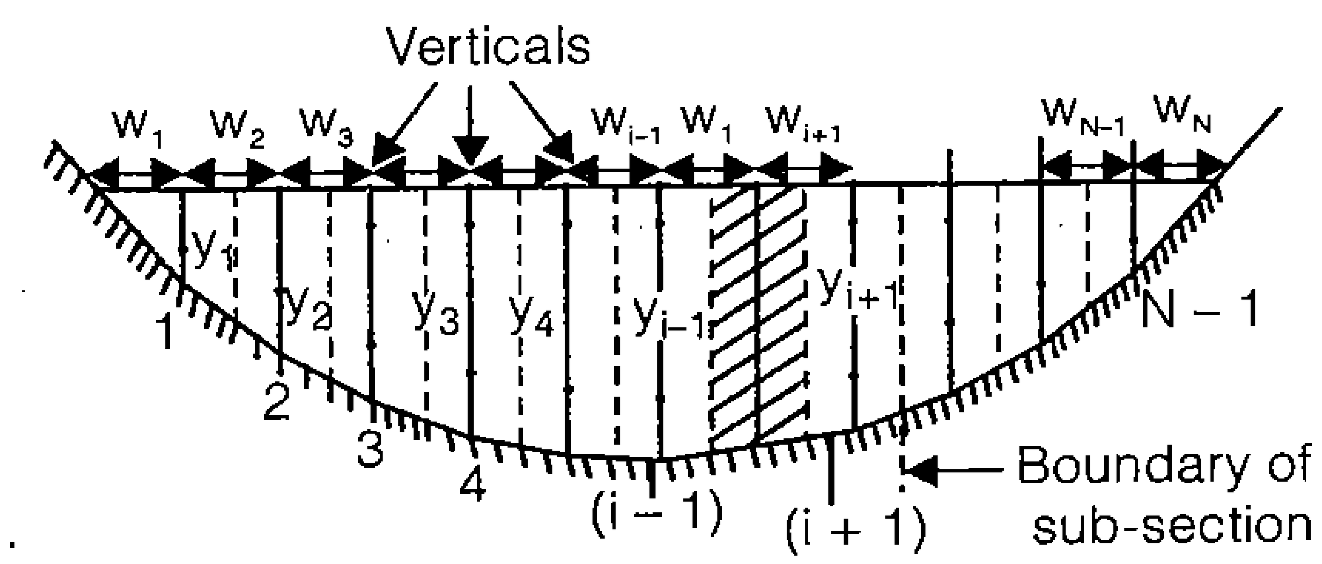
W = Weight in newton

$\bar{V}$ = Average stream velocity

y = Depth of flow in meters.

Stream Flow (discharge Measurement)

(i) **Area Velocity Method:** This method of discharge measurement consists essentially of measuring the area of cross-section of the river at a selected section called the gauging site and measuring the velocity of flow through the cross-sectional area. The gauging site must be selected with care to assure that the stage-discharge curve is reasonably constant over a long period of about a few years.



Total discharge,

$$Q = q_1 + q_2 + \ldots + q_{n-1}$$

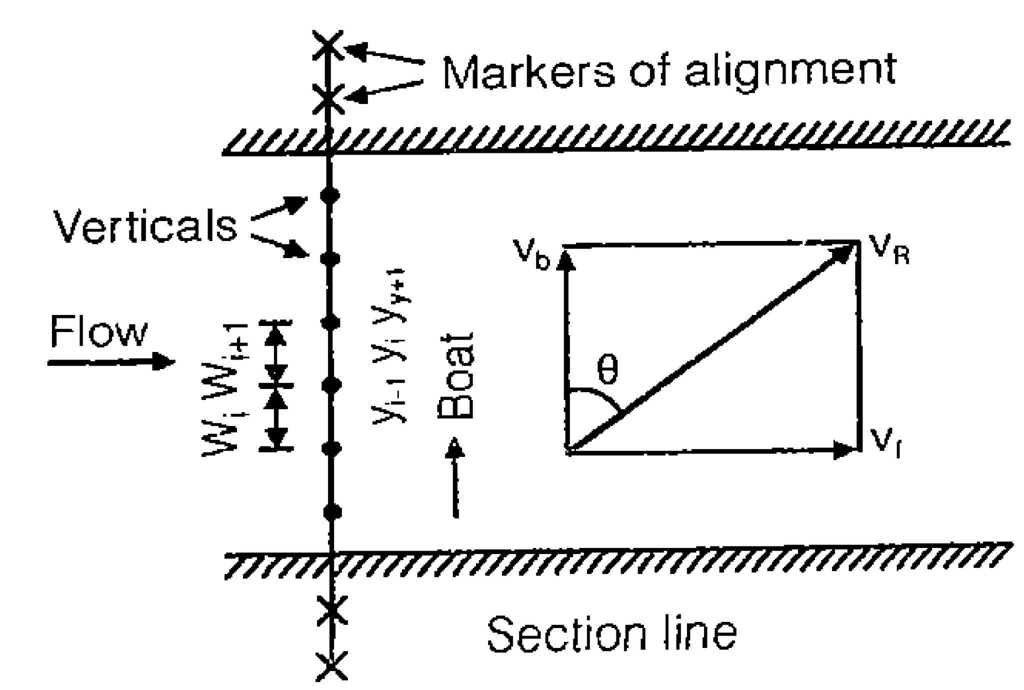
where, $q_1 = \overline{W_1}\, y_1 \overline{V_1}$

$q_2 = \overline{W_2}\, y_2 \overline{V_2}$ etc.

$$\overline{W_i} = \frac{W_i + W_{i+1}}{2} \quad \text{for } 2 \le i \le (n-1)$$

$$\overline{W_1} = \frac{(w_1 + w_2/2)^2}{2w_1} \qquad \overline{W_n} = \frac{(w_n + w_{n-1}/2)^2}{2w_n}$$

(ii) **Moving Boat Method:** In this method a special propeller type current meter which is free to move about a vertical axis is towed in a boat at a velocity V_B at right angles to the stream flow. If the flow velocity is V_F the meter will align itself in the direction of the resultant velocity V_R making an angle θ with the direction of the boat. Further, the meter will register the velocity V_R. If V_B is normal to V_F.



$$V_B = V_R \cos\theta$$

$$V_F = V_R \sin\theta$$

$$q_i = \left[\frac{y_{i-1}+y_i}{2} \times w_i\right]\overline{v_i}$$

where, q_i = Discharge through the i^{th} segment.

$$q_i = \left(\frac{y_{i-1}+y_i}{2}\right)V_R^2 \sin\theta.\cos\theta.t_i$$

t_i = Time required to pass the boat through i^{th} segment.

(iii) Dilution Method: The dilution method of flow measurement, also known as the chemical method depends upon the continuity principle applied to a tracer which is allowed to mix completely with the flow.

$$Q_0 = \frac{Q_1(C_1 - C_2)}{C_2 - C_0}$$

This technique in which Q_0 is estimated by knowing C_1, C_2, C_0 and Q_1 is known as constant rate injection method or plateau gauging.

where,

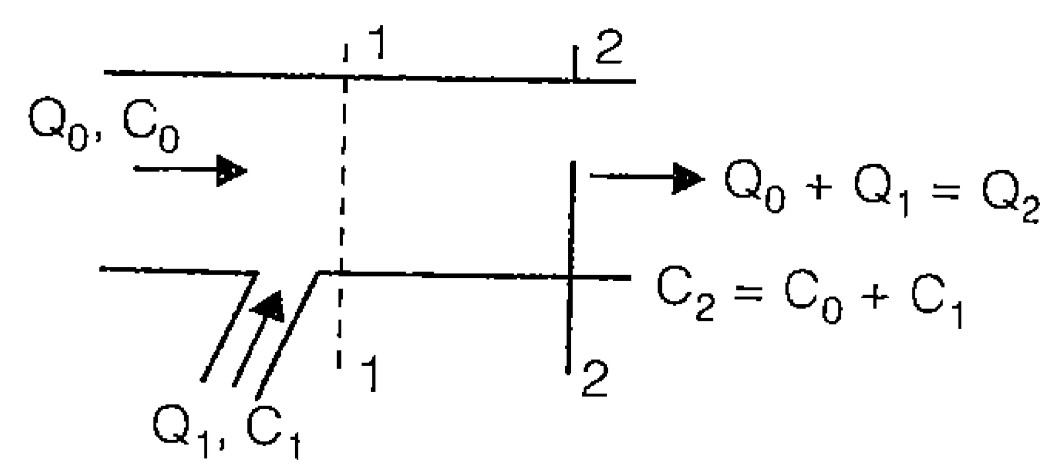
Q_0 = Discharge of stream

C_0 = Tracer intensity initially

C_1 = Tracer intensity at (1)

C_2 = Tracer intensity at (2).

Q_1 and Q_2 are discharge at (1) and (2) respectively.



(iv) Slope Area Method: It is a very versatile indirect method of discharge estimation and requires (i) the selection of a reach in which cross-sectional properties including bed elevations are known at its ends, (ii) the value of Manning's n and (iii) water-surface elevations at the two end sections.

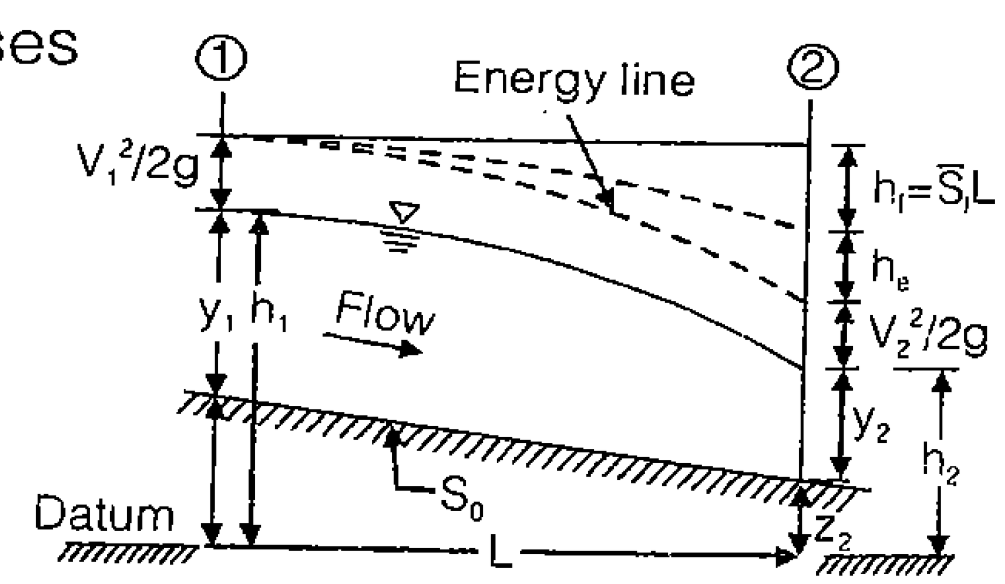
(a) $$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + H_f + H_e$$

where H_f = Frictional losses

H_e = Eddy losses

(b) $$H_e = k_e\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right]$$

k_e = Eddy loss coefficients.



(c) $$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + H_f + H_e$$

(d) $$H_f = (h_1 - h_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - k_e\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

(e) $$Q = k\sqrt{\frac{H_F}{L}}$$ where, Q = Stream flow (m^3/sec)

(f) $$k = (k_1 \cdot k_2 \ldots k_n)^{1/n}$$ where, k = Conveyance

(g) $$k = \frac{1}{n} \cdot A \cdot R^{2/3}$$ where, A = Area (m^2)

R = Hydraulic mean depth

$= \dfrac{\text{Area}}{\text{Wetted Perimeter}}$

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Infiltration, Runoff and Hydrographs

3

Infiltration

Infiltration is the flow of water into the ground through the soil surface.

- **Horton's Equation:** Horton expressed the decay of infiltration capacity with time as an exponential decay given by

$$\boxed{f_{ct} = f_{cf} + (f_{co} - f_{cf})e^{-k_h \cdot t_d}}$$

where

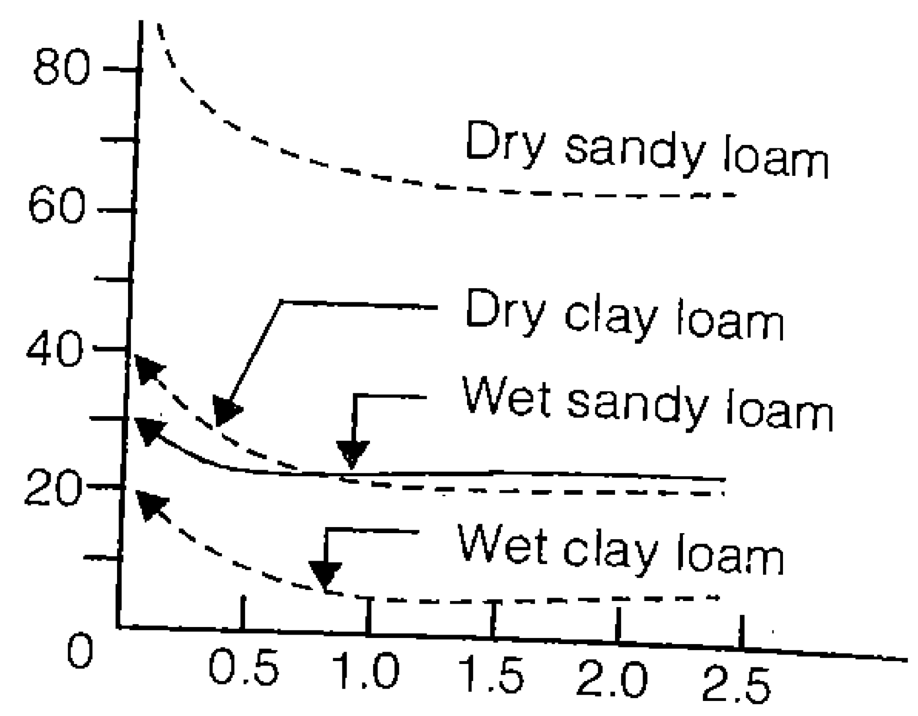
f_{ct} = infiltration capacity at any time t from start of the rainfall

f_{co} = Initial infiltration capacity at t = 0

f_{cf} = Final steady state value

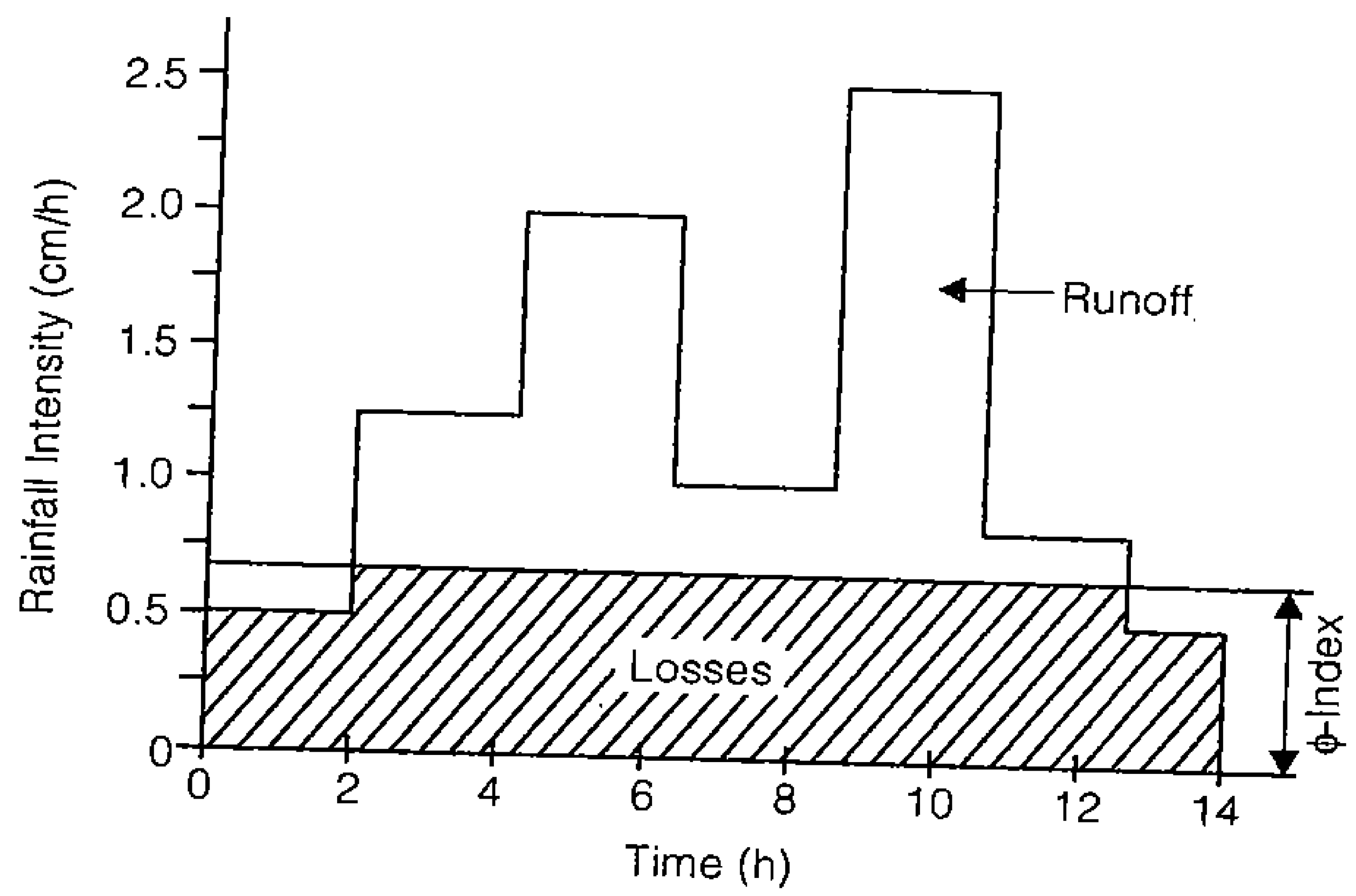
t_d = Duration of rainfall

k_h = Constant depending on soil.



Infiltration Indices

In hydrological calculations involving floods it is found convenient to use a constant value of infiltration rate for the duration of the storm. The defined average infiltration rate is called *infiltration index* and two types of indices are in common use.



(i) **W-Index:** In an attempt to refine the ϕ-index the initial losses are separated from the total abstractions and an average value of infiltration rate, called W-index, is defined as

$$\boxed{W - Index = \frac{P - R - I_a}{t_e}}$$

where, P = Total storm precipitation (cm)

R = Total storm runoff (cm)

I_a = Initial losses (cm)

t_e = Duration of rainfall excess

W–Index = Avg. rate of infiltration (cm/hr)

(ii) **ϕ–Index:** The ϕ-index is the average rainfall above which the rainfall volume is equal to the runoff volume. The ϕ-index is derived from the rainfall hyetograph with the edge of the resulting run-off volume.

$$\boxed{\phi\text{-Index} = \frac{I - R}{24}}$$

where, R = Runoff in cm from a 24-h rainfall of intensity I cm/day

Runoff

Runoff means the draining or flowing off of precipipation from a catchment area through a surface channel. It thus represents the output from the catchment in a given unit of time.

Direct Runoff: It is that part of the runoff which enters the stream immediately after the rainfall. It includes surface runoff, prompt interflow and rainfall on the surface of the stream. In the case of snow-melt, the resulting flow entering the stream is also a direct runoff. Sometimes terms such as *direct storm runoff* the *storm runoff* are used to designate direct runoff.

Base Flow: The delayed flow that reaches a stream essentially as groundwater flow is called base flow.

(i) $\boxed{\text{Direct runoff = Surface runoff + Prompt Interflow}}$

(ii) $\boxed{\text{Direct runoff = Total runoff − Base flow}}$

(iii) $\boxed{\text{Form Factor} = \frac{A}{l^2}}$ where, A = Area of the catchment

l = Axial length of basin.

(iv) $\boxed{\text{Compactness coefficient} = \frac{P}{2\pi r_e}}$

$\boxed{r_e = \sqrt{\frac{A}{\pi}}}$ r_e = Radius of equivalent circle whose Area is equal to area of catchment (A)

(v) Elevation of the water shed, (z)

$$\boxed{z = \frac{A_1z_1 + A_2z_2 + \ldots + A_nz_n}{A_1 + A_2 + \ldots A_n}}$$

where, A_1, A_2 ... . Area between successive contours.

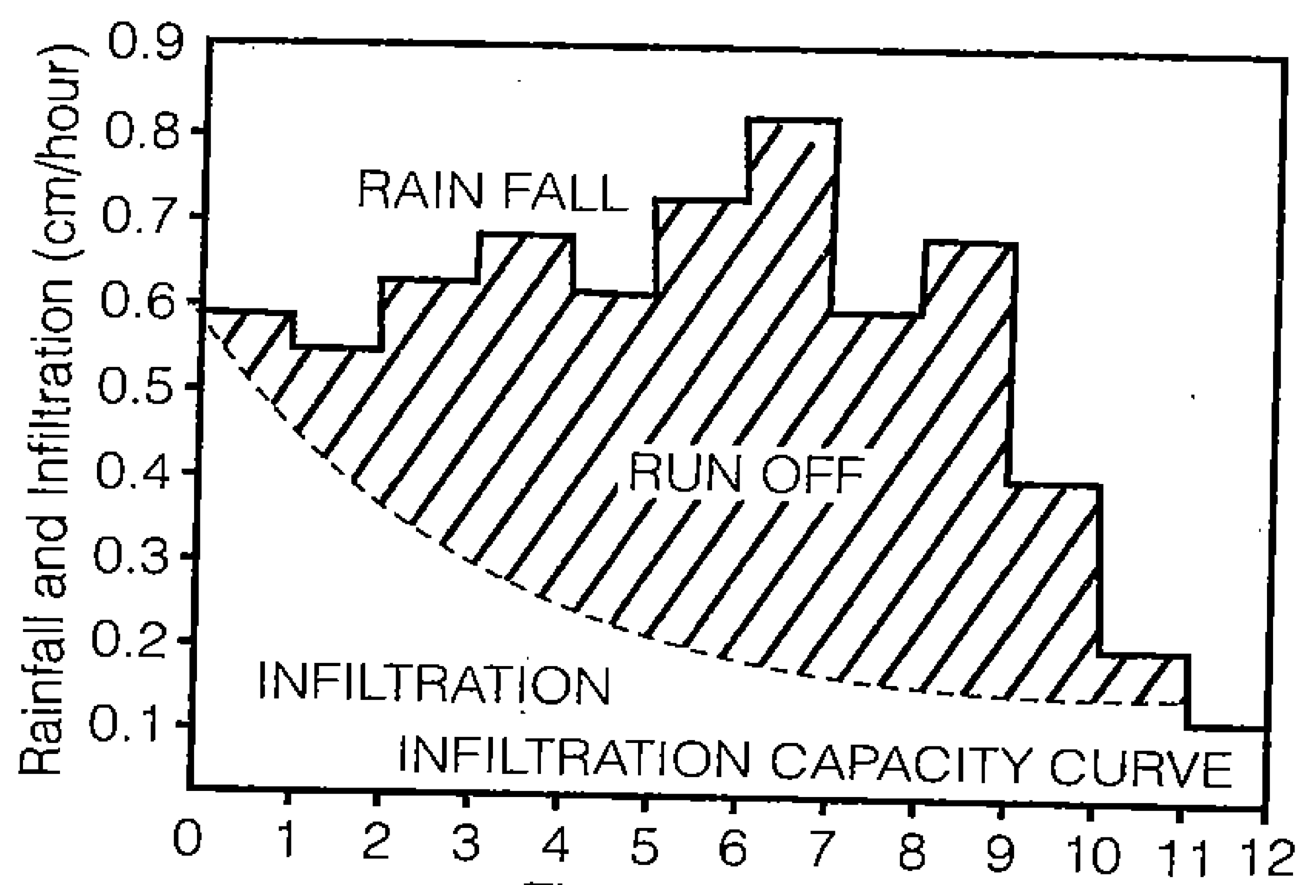
z_1, z_2 ... mean elevation between two successive contrours.

Method to Compute Runoff

(i) By Runoff Coefficient

$Q = KP$ where, P = Precipitation
K = Runoff coefficient
Q = Runoff

(ii) By Infiltration Capacity Curve



Infiltrations capacity curve and run-off

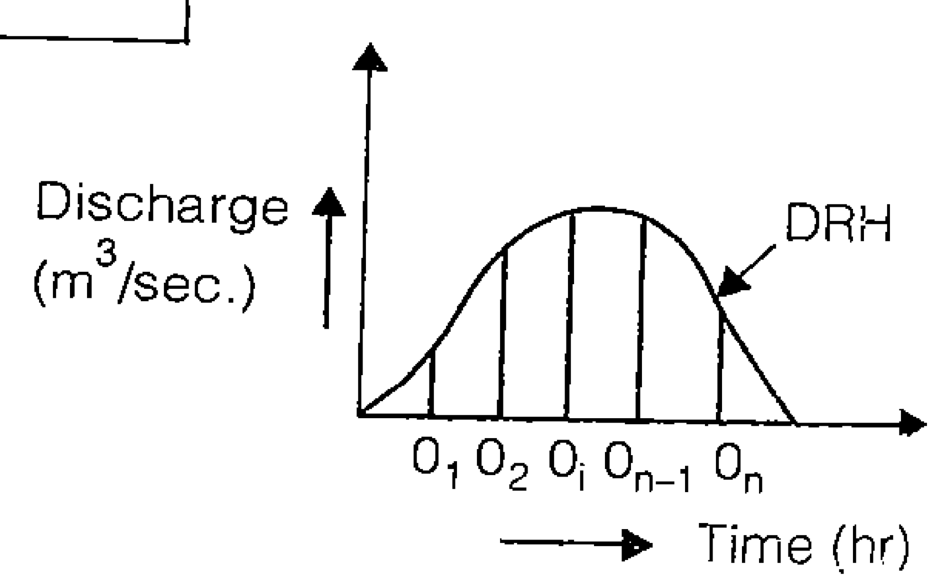
(iii) By Rational Formula

$$Q_P = \frac{1}{36} \cdot kP_C A$$

where, k = Runoff coefficient
P_C = Critical design rainfall intensity in cm/hr
A = Area of catchment in hectare
Q_P = Peak discharge in m³/sec.

(iv)
$$\text{Direct runoff depth} = \frac{0.36 \sum_{i=1}^{n} (O_i) t}{A} \text{ cm}$$

where,
A = Area in km²
t = Time in hour
O_i = Ordinate of i^{th} element i.e., discharge in m³/sec.



Hydrograph

A plot of the discharge in a stream plotted against time chronologically is called a hydrograph.

Unit Hydrograph

This method was first suggested by Sherman in 1932 and has undergone many refinements since then.

A unit hydrograph is defined as the hydrograph of direct runoff resulting from one unit depth (1 cm) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D hours).

Time Invariance: The first basic assumption is that the direct-runoff response to a given effective rainfall in a catchment is time invariant. This implies that the DRH for a given ER in a catchment is always the same irrespective of when it occurs.

Linear Response: The direct-runoff response to the rainfall excess is assumed to be linear. This is the most important assumption of the unit-hydrograph theory. Linear response means that if an input $x_1(t)$ cause an output $y_1(t)$ and an input $x_2(t)$ causes an output $y_2(t)$, then an input $x_1(t) + x_2(t)$ gives an output $y_1(t) + y_2(t)$. Consequently, if $x_2(t) = rx_1(t)$, then $y_2(t) = r_1 y_1(t)$. Thus, if the rainfall excess in a duration D is r times the unit depth, the resulting DRH will have ordinates bearing ratio r to those of the corresponding D-h unit hydrograph.

(i) $t'_B = t_B + (n-1)D$

where, t'_B = Base period of T hr U.H
t_B = Base period of D hr U.H

also, T > D

$T = n \cdot D$ where 'n' is an integer.

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Floods, Flood Routing and Flood Control

4

A flood is an unusually high stage in a river, normally the level at which the river overflows its banks and inundates the adjoining area.

The design of bridges, culvert waterways and spilways for dams and estimation of scour at a hydraulic structure are some examples wherein flood-peak values are required.

To estimate the magnitude of a flood peak the following alternative methods are available:

1. Rational method
2. Empirical method
3. Unit-hydrograph technique
4. Flood-frequency studies

Rational Method

If $t_P \geq t_C$

$$Q_P = \frac{1}{36} \cdot k \cdot P_C \cdot A$$

where, Q_p = Peak discharge in m³/sec.
P_C = Critical design rainfall in cm/hr
A = Area of catchment in hectares
K = Coefficient of runoff.
t_D = Duration of rainfall
t_C = Time of concentration

Empirical Formulae

(a) Dickens Formula (1865)

$$Q_P = C_D \cdot A^{3/4}$$

where, Q_P = Flood peak discharge in m³/sec.
A = Catchment area in km².
C_D = Dickens constant, $6 \leq C_D \leq 30$.

(b) Ryve's Formula (1884)

$$Q_P = C_R \cdot A^{2/3}$$

where,
C_R = Ryve's constant
= 8.8 for constant area within 80 km from the cost.
= 8.5 if distance of area is 80 km to 160 km from the cost.
= 10.2 if area is Hilley and away from the cost.

(c) Inglis Formula (1930)

$$Q_P = \frac{124A}{\sqrt{A+10.4}} \simeq 123\sqrt{A}$$

where, A = Catchment area in km².
Q_P = Peak discharge in m³/sec.

Flood Frequency Studies

(i) Recurrence Interval or Return Period:

$$T = \frac{1}{P}$$ where, P = Probability of occurrence

(ii) Probability of non-occurrence: $q = 1 - P$

(iii) Probability of an event occurring r times in 'n' successive years:

$$P_{r,n} = {}^nC_r \cdot P^r \cdot q^{n-r}$$ where, $${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

(iv) Reliability : (Probability of non Occurrence/Assurance) = q^n.

(v) $\text{Risk} = 1 - q^n \rightarrow \text{Risk} = 1 - (1-p)^n$

(vi) $$\text{Safety Factor} = \frac{\text{Design value of hydrological parameter adopted}}{\text{Estimated value of hydrological parameter}}$$

(vii) Safety Margin = Design value of hydrological parameter − Estimated value of hydrological parameter

Gumbel's Method

The extreme values distribution was introduced by Gumbel (1941) and is commonly known as Gumbel's distribution. It is one of the most widely used probability distribution functions for extreme values in hydrologic and meteorologic studies for prediction of flood peaks, maximum rainfalls, maximum wind speed.

Gumbel defined a flood as the largest of the 365 daily flows and the annual series of flood flows constitute a series of largest values of flows.

Based on probability distribution,

$$P_{(x \geq x_0)} = 1 - e^{-e^{-y}}$$

(i) $$X_T = \bar{X} + K \cdot \sigma$$ where, x_T = Peak value of hydrological data
k = frequency factor

(ii) $$k = \frac{y_T - \bar{y}_n}{S_n}$$ y_T = Reduced variate

$$y_T = -\log_e \log_e \left(\frac{T}{T-1}\right)$$

T = Recurrence interval in year
$\bar{y}_n$ = Reduced mean = 0.577
S_n = Reduced standard deviation.
S_n = 1.2825 for $N \rightarrow \infty$

(iii) $\sigma = \sqrt{\left(\frac{N}{N-1}\right)\left[\overline{X^2} - \overline{X}^2\right]}$

$$\overline{X} = \frac{\Sigma x}{N}$$

and $\overline{X^2} = \frac{\Sigma x^2}{N}$.

Confidence Limit

Since the value of the variate for a given return period, x_T determined by Gumbel's method can have errors due to the limited sample data used, an estimate of the confidence limits of the estimate is desirable. The confidence interval indicates the limits about the calculated value between which the true value can be said to lie with a specific probability based on sampling errors only.

For a confidence probability c, the confidence interval of the variate x_T is bounded by values x_1 and x_2 given by

$$X_2/X_1 = X_T \pm f(c) \cdot S_e$$

where, f(c) is a function of confidence probability 'C'.

C(in%)	50	68	80	90	95	99
f(C)	0.674	1.00	1.282	1.645	1.96	2.58

S_e = Probable error

$$S_e = \frac{b\sigma}{\sqrt{N}}$$

where, N = Sample size

b = Factor

$$b = \sqrt{1 + 1 \cdot 3k + 1 \cdot 1k^2}$$

σ = Standard deviation

$$k = \frac{y_T - \overline{y_n}}{S_n}$$

k = Frequency factor.

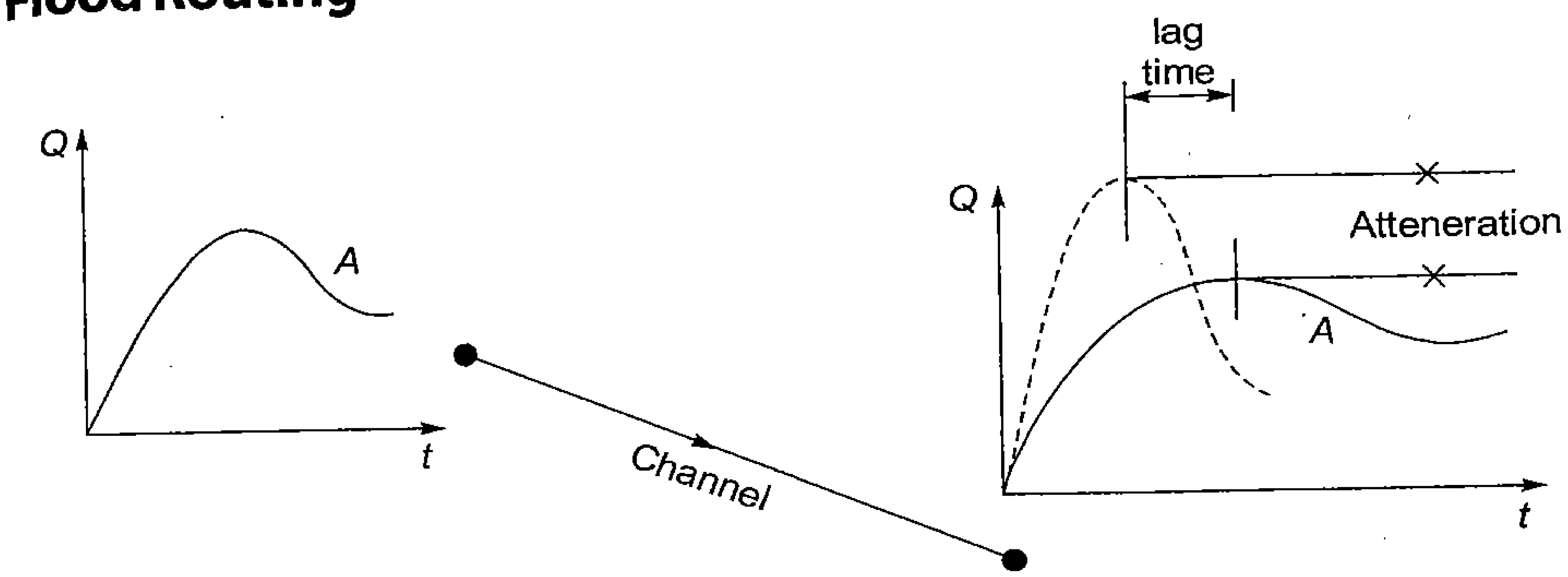
Flood Routing

Flood routing is the technique of determining the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections. The hydrologic analysis of problems such as flood forecasting, flood protection, reservoir design and spillway design invariably includes flood routing.

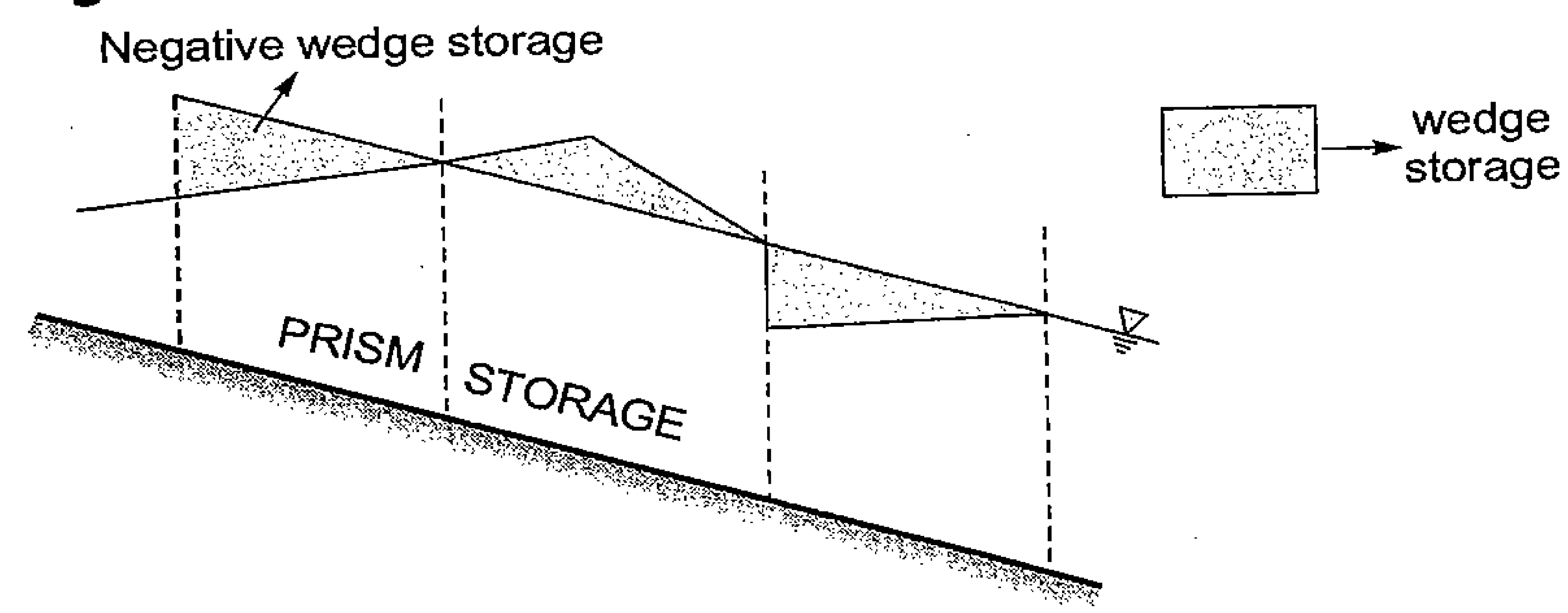
Prism Storage: It is the volume that would exist if the uniform flow occurred at the downstream depth, i.e., the volume formed by an imaginary plane parallel to the channel bottom drawn at the outflow section water surface.

Wedge Storage: It is the wedge like volume formed between the actual water surface profile and the top surface of the prism storage.

Flood Routing



Muskingum Method



$$S = S_P + S_W$$

where, S = Total storage in the channel.

S_P = Prism storage

= f(Q) = Function of outflow discharge.

S_W = Wedge storage

= f(I) = Function of Inflow discharge.

$$S = f(Q) + f(I) \qquad S = k\left[XI^m + (1-X)Q^m\right]$$

where, X = Weighting factor

m = Constant = 0.6 for rectangular channels

= 1.0 for natural channels

k = Storage time constant

Methods of Channel Routing

- **Muskingum Method : Hydrologic Channel Routing**

(i) $\Delta S = (\overline{I} - \overline{Q})\Delta t$ where, $\Delta S \rightarrow$ Change in storage in time Δt

$\Delta t \rightarrow$ Time interval at which observations are taken. (Routing Interval)

$\overline{I} \rightarrow$ Avg. in flow rate over the period Δt

$\overline{Q} \rightarrow$ Average outflow rate over time period Δt.

- $\Delta S = \left[\left(\frac{I_1 + I_2}{2}\right) - \left(\frac{Q_1 + Q_2}{2}\right)\right]\Delta t$
- $S_1 = k[XI_1 + (1-x)Q_1]$
- $S_2 = k[XI_2 + (1-x)Q_2]$
- $(S_2 - S_1) = k[x(I_2 - I_1) + (1-x)(Q_2 - Q_1)]$
- $Q_1 = I_1$
- $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$
- $Q_n = C_0 I_n + C_1 I_{n-1} + C_2 Q_{n-1}$

where C_0, C_1 and C_2, are Muskingum constant

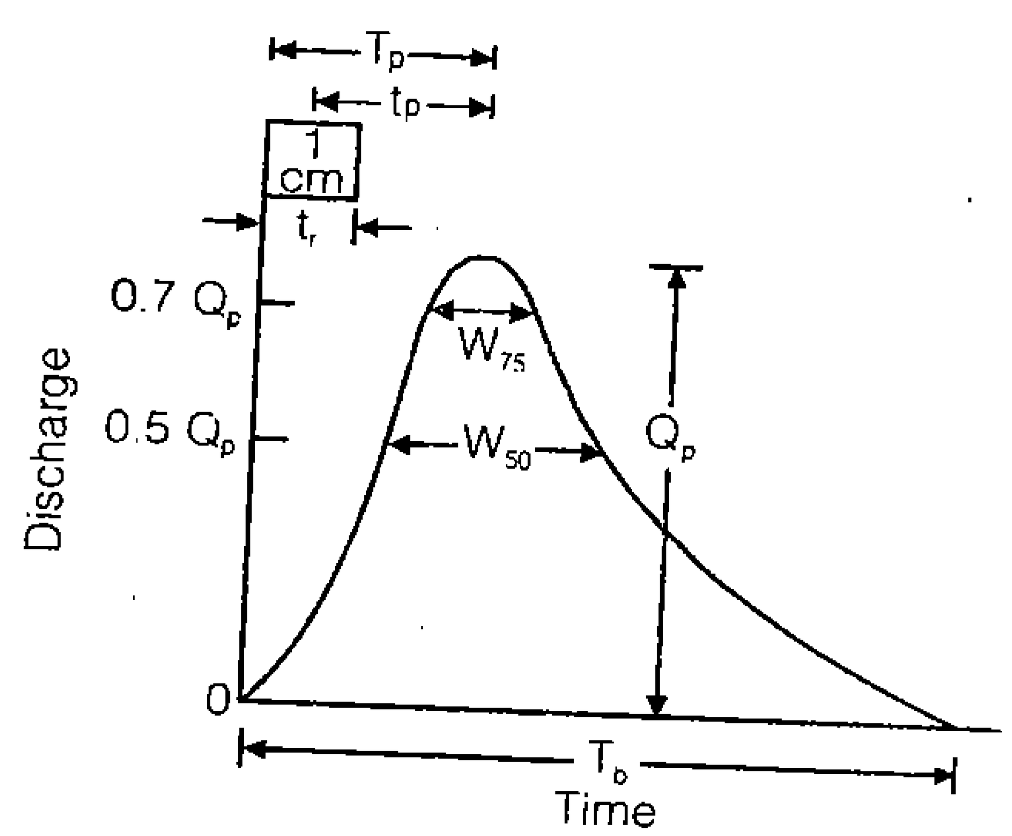
$$C_0 = \frac{-kx + 0.5\Delta t}{k(1-x) + 0.5\Delta t} \qquad C_1 = \frac{kx + 0.5\Delta t}{k(1-x) + 0.5\Delta t}$$

$$C_2 = \frac{k(1-x) - 0.5\Delta t}{k(1-x) + 0.5\Delta t} \qquad C_0 + C_1 + C_2 = 1$$

- For best result, $2kx < \Delta t < k$

Synthetic Hydrograph

Synder's Method: Synder (1938), based on a study of a large number of catchments in the appalachian Highlands of eastern United States developed a set of empirical equations for synthetic unit hydrographs in those areas. These equations are in use in the USA, and with some modifications in many other countries, and consititute what is known as Synder's synthetic unit hydrograph.



(i) $t_P = C_t[L \cdot L_{Ca}]^{0.3}$

where, t_p = Time interval between mid point of unit rainfall excess and peak of unit hydrograph in hour

L = Length of main stream

L_{Ca} = The distance along the main stream from the basin outlet to a point on the stream which is nearest to the centroid of basis (in km)

C_t = Regional constant $0.3 < C_t < 0.6$

(ii) $t_P = C_t\left[\frac{L \cdot L_{Ca}}{\sqrt{S}}\right]^n$ S= Basin slope. n= Constant = 0.38.

(iii) $t_r = \frac{t_P}{5.5}$ t_r = Standard duration of U.H in hour.

(iv) $Q_{PS} = \frac{2.78 C_P A}{t_P}$

where, C_P = Regional constant = 0.3 to 0.92.

A = Area of catchment in km^2.

Q_{PS} = Peak discharge in m^3/s.

(v) $t'_P = \frac{21}{22} t_P + \frac{t_R}{4}$ where, t_R = non standard rainfall duration. t'_p = Basin lag for non standard U.H.

(vi) $Q_P = \frac{2.78 C_P A}{t'_P}$

(vii) $t_B = (72 + 3t'_P)$ hour, for large catchment.

where, t_B = Base time of synthetic U.H

$t_B = 5\left[t'_P + \frac{t_R}{2}\right]$ hour, for small catchment.

(viii) $W_{50} = \frac{5.87}{(q)^{1.08}}$ W_{50} = Width of U.H in hour at 50% peak discharge.

(ix) $W_{75} = \frac{W_{50}}{1.75}$ W_{75} = Width of U.H in hours at 75% peak discharge.

(x) $q = \frac{Q_P}{A}$ where, Q_P = Peak discharge in m^3/sec. A = Area in km^2.

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A Handbook on

Civil Engineering

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Railway Engineering

CONTENTS

1 Rail Joints and Welding of Rails

Long Welded Rail

A LWR, is the rail whose central part does not undergo any longitudinal movement under temperature variations.

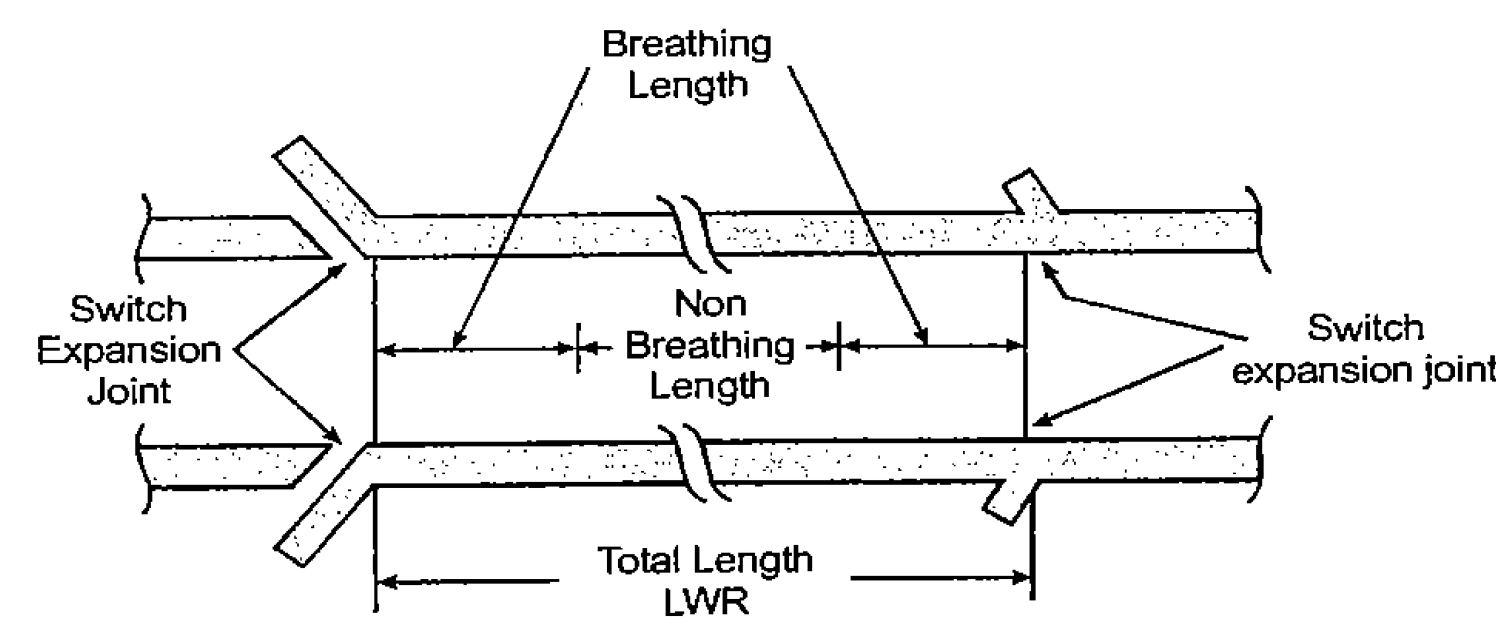
Normally, any length greater than 200 m on BG, and 300 m on MG will function as LWR.

Length of rail required in one direction $\boxed{L = (n-1)\,s}$

Total minimum length of LWR so that central portion does not move = 2L

Minimum no. of sleepers required to prevent P_f force

$$\boxed{n = \frac{F}{P_f}} \text{ where } \boxed{F = \alpha T E_S \cdot A_S}$$



where,
- F = Force in a fully developed rail neglecting the creep effects.
- A_S = Cross-sectional area of rail section.
- E_S = Modulus of elasticity of rail steel. ~2150 tonnes/sq. cm.
- α = Coefficient of linear expansion for rail steel ~0.00001152/°C.
- T = Change in temperature in °C.
- P_f = Force resisted by one fixtures.
- S = Spacing of fixtures.
- n = minimum no. of sleepers required to prevent F force

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Ballast, Formation and Sleepers

2

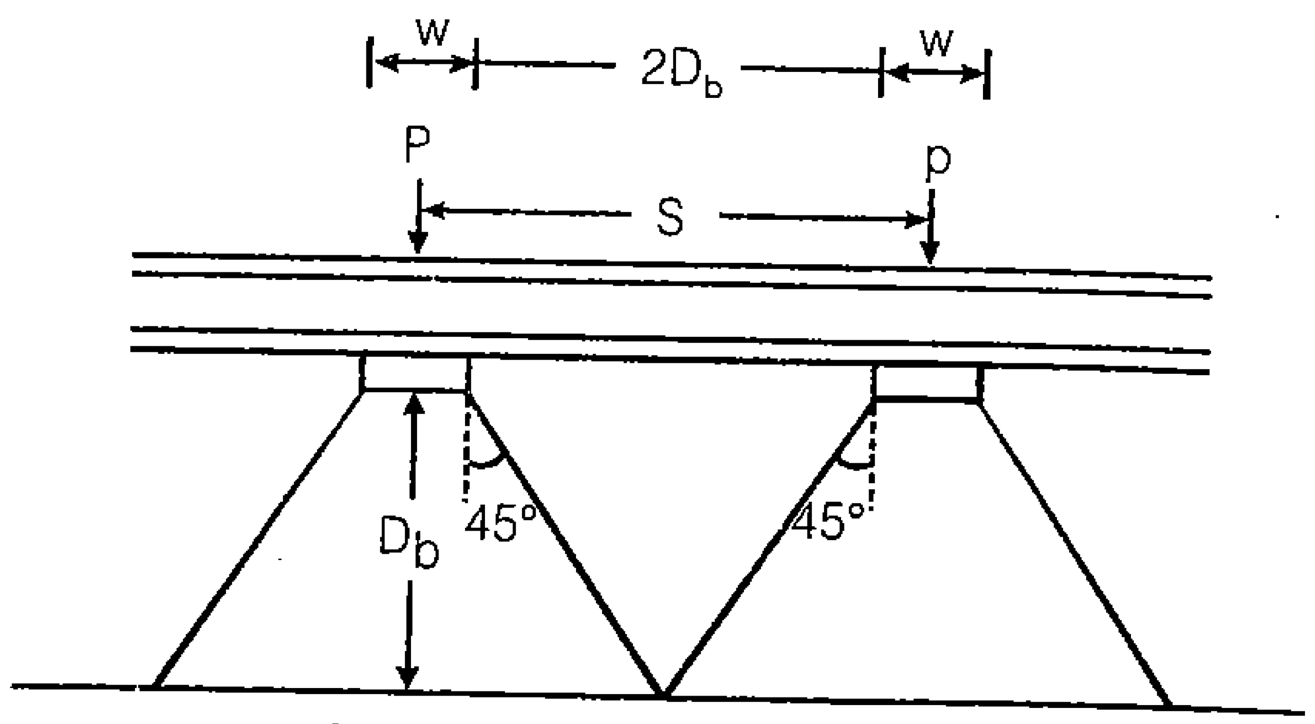
Ballast and Ballast Cushion

Ballast: It is high quality crushed stone with desired specifications placed immediately beneath the sleeper.

Ballast Cushion: The depth of ballast below the bottom of the sleeper, normally measured under the rail seat.

Minimum depth of ballast Cushion

$$D_b = \frac{S - W}{2}$$



where,

D_b = Min. depth of ballast cushion.

S = Centre to centre distance between two sleepers or sleeper spacing.

W = Width of sleeper.

Composite sleeper index

It is the hardness index of a timber to determine the suitability of a particularly timber to use as a sleeper.

$$C \cdot S \cdot I = \frac{S + 10H}{20}$$

where,

CSI = Composite sleeper index.

S = Strength index of timber at 12% moisture content.

H = Hardness index of timber at 12% moisture content.

	CSI Values
Track sleepers →	783
Crossing sleepers →	1352
Bridge sleepers →	1455

Sleeper Density

No. of sleepers to be used for one rail length.

Denoted by (n + x)

vary from (n + 3) to (n + 6)

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Geometric Design of The Track

3

Different gauges

Gauge	Distance between rails
Broad gauge	1.676 m
Meter gauge	1.0 m
Narrow gauge	0.762 m
Light gauge (feather track)	0.610 m
Standard gauge (used in delhi metro)	1.435 m

Safe Speed on Curves Based on Martins Formula

(a) For Transition Curve

(i) For B.G & M.G. $V = 4.35\sqrt{R - 67}$ where, V is in kmph.

(ii) For N.G $V = 3.65\sqrt{R - 6}$ for V is km/hr.

(b) For Non-Transition Curve $V = 0.80 \times \text{speed calculated in (a)}$

(c) For High Speed Trains $V = 4.58\sqrt{R}$

Safe Speed Based on Super Elevation

(a) For Transition curves

(i) For B.G $v = 0.27\sqrt{(c_a + c_d)R}$

(ii) For M.G $v = 0.347\sqrt{(c_a + c_d)R}$

The above two formula based on the assumption that G = 1750 mm for B.G

G = 1057 mm for N.G

and $e = \frac{Gv^2}{127R}$ where, e = super elevation.

(iii) For N.G $\boxed{v = 3.65\sqrt{R-6}}$

where, v = Speed in km/hr
R = Radius of curve in 'mm'
c_a = Actual cant in 'mm'
c_d = Cant deficiency in 'mm'.

Speed from the Length of Transition Curve

(a) For speed upto 100 km/hr.

$$\boxed{V_{max.} = \frac{134L}{c_a} \text{ or } \frac{134L}{c_d}}$$ (min. of two is adopted)

where, L = Length of transition curve based on rate of change of cant as 38 mm/sec. for speed upto 100 km/hr & 55 mm/sec for speed upto 100 km/hr & 55 mm/sec for high speeds.
c_a = Actual cant in 'mm'.
c_d = Cant deficiency in 'mm'.

(b) For high speed trains (speed > 100 km/hr)

Either, $$\boxed{V_{max} = \frac{198\ L}{c_a} \text{ or } \frac{198L}{c_d}}$$

Minimum of the two is adopted.

Radius & Degree of Curve

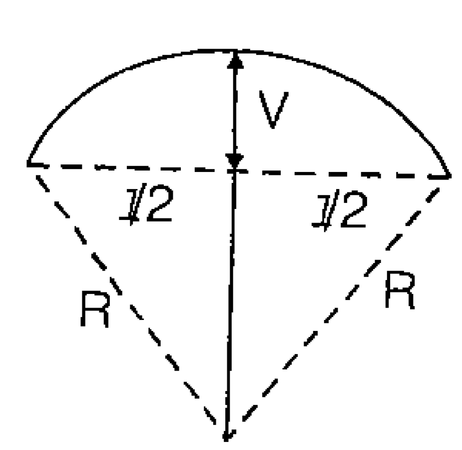
$$\boxed{D = \frac{1720}{R}}$$ If one chain length = 30 m.

$$\boxed{D = \frac{1150}{R}}$$ If one chain length = 20 m
where, R = Radius

$$\boxed{D = \text{Degree of curve} = \begin{cases} 10° \rightarrow \text{B.G} \\ 16° \rightarrow \text{M.G} \\ 40° \rightarrow \text{N.G} \end{cases}}$$

Versine of Curve (v)

$$\boxed{V = \frac{L^2}{8R}}$$



Grade Compensation

For B.G → 0.04% per degree of curve
M.G → 0.03% per degree of curve
M.G → 0.02% per degree of curve

Super Elevation (cant) (e)

$$\boxed{e = \frac{GV^2}{127R}}$$

where, e = Super elevation in meter
V = Speed in km/hr., R = Radius in meter
G = Gauge distance between the centre of rails.

Equilibrium Cant (e')

$$\boxed{e' = \frac{GV_{av}^2}{127R}}$$

where, V_{av} = Average speed or equilibrium speed.

Equilibrium Speed or Average Speed (V_{av})

(a) When maximum Sanctioned Speed > 50 km/hr.

$$\boxed{V_{av} = \text{minimum}\begin{cases} \frac{3}{4} \times V_{max} \\ \text{Safe speed by} \\ \text{martins formula} \end{cases}}$$

(b) When Sanctioned speed < 50 km/hr

$$\boxed{V_{av} = \text{minimum}\begin{cases} V_{max} \\ \text{Safe speed by} \\ \text{martins formula} \end{cases}}$$

(c) Weighted Average Method

$$\boxed{V_{av} = \frac{n_1V_1 + n_2V_2 + ...}{n_1 + n_2 + ...}}$$

where, n_1, n_2, n_3... etc. are number of trains running at speeds v_1, v_2, v_3 ... etc.

Maximum Value of Cant (e_{max})

	B.G. Track		M.G.	N.G.
	<120 kmph	>120 kmph		
e_{max} (actual)	16.5 cm	18.5 cm	10.0 cm	7.6 cm

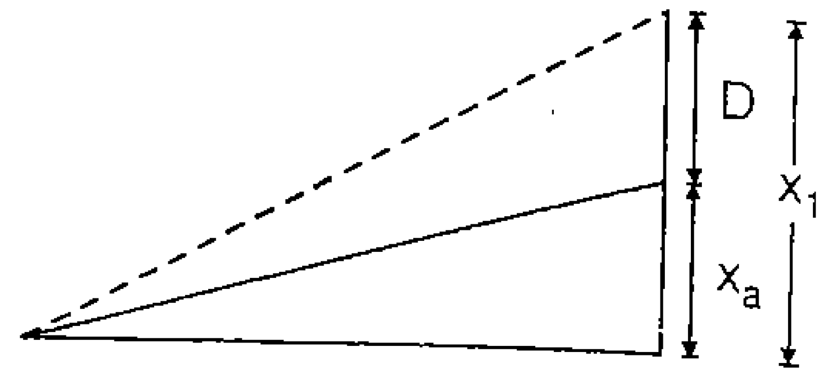
Cant Deficiency (D)

$$\text{Cant deficiency} = x_1 - x_A$$

where,

x_A = Actual cant provided as per average speed

x_1 = Cant required for a higher speed train.

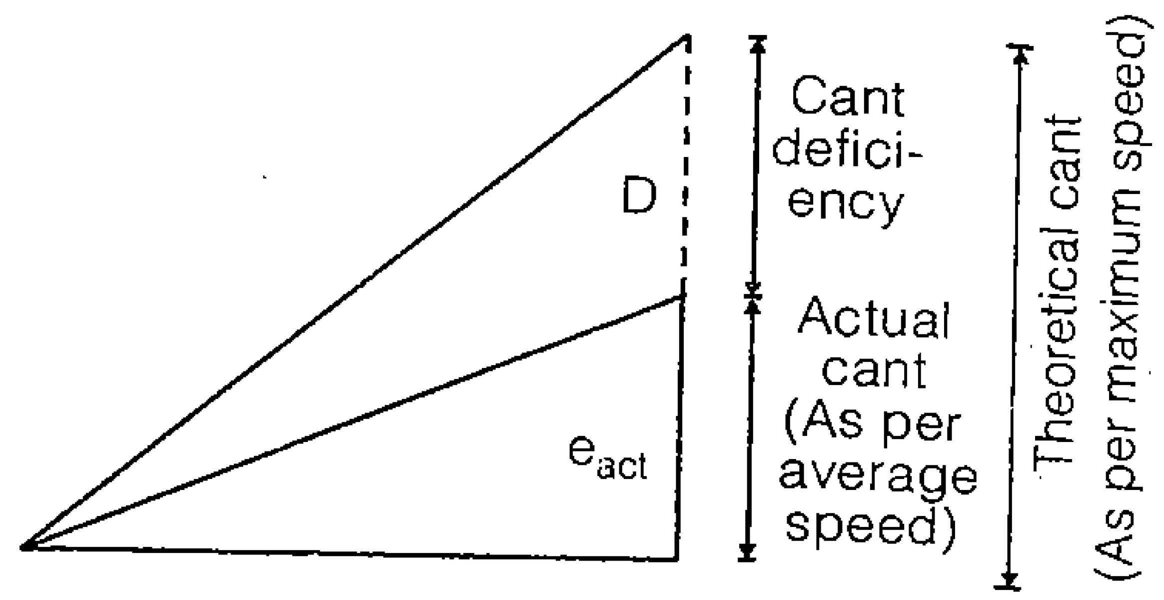


$$e_{th} = e_{act} + D$$

where, e_{th} = Theoretical cant

e_{act} = Actual cant

D = Cant deficiency.

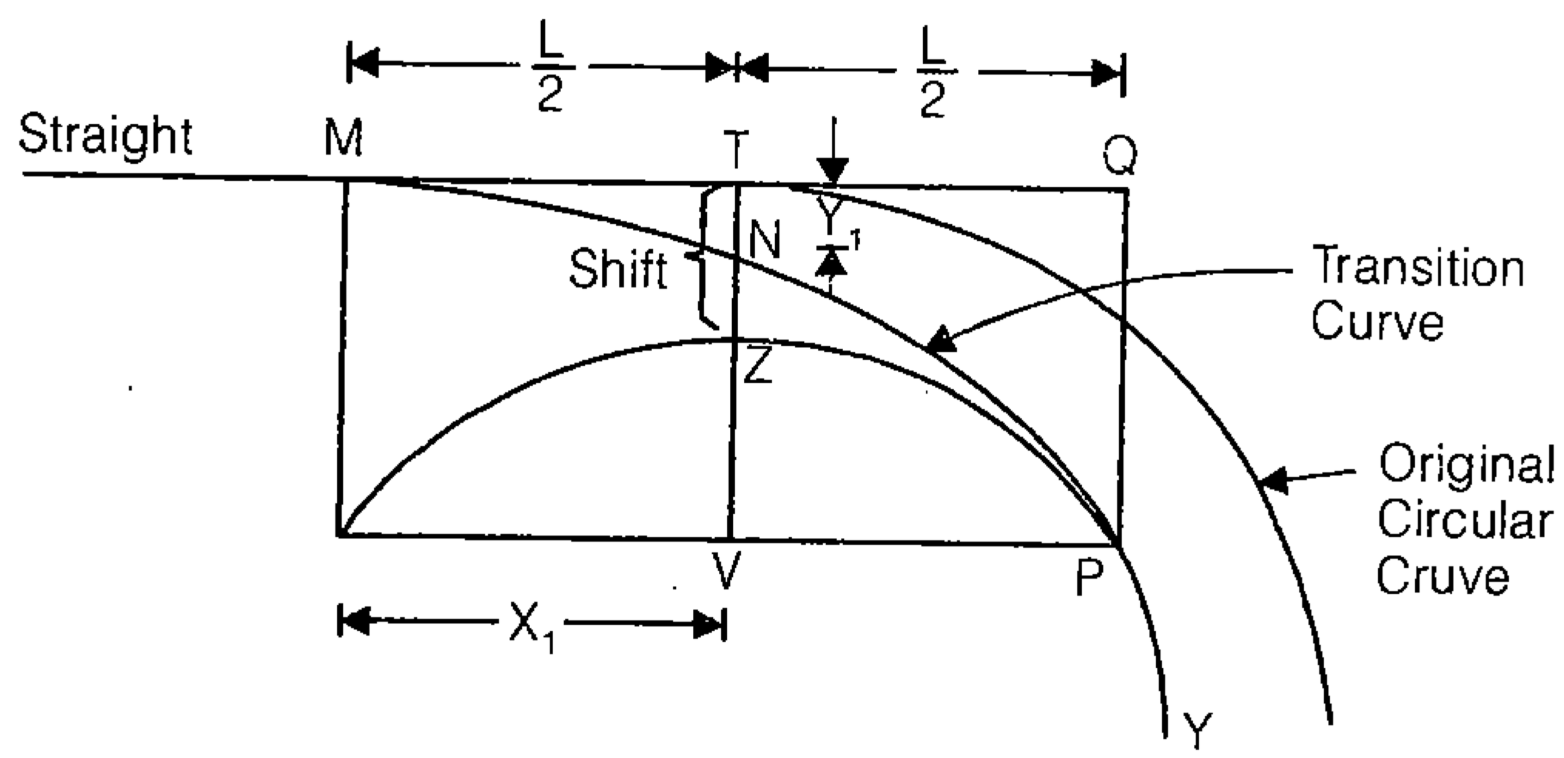


	B.G. Track		M.G.	N.G.
	<100 kmph	>100 kmph		
D_{max}	7.60 cm	10.0 cm	5.10 cm	3.80 cm

Transition Curve (Cubic Parabola)

Equation of transition curve;

$$y = \frac{X^3}{6RL}$$



(a) Shift (S)

$$S = \frac{L^2}{24R}$$

where, S = Shift in 'm'

L = Length of transition curve in 'm'

R = Radius of circular curve in 'm'.

(b) Length of Transition Curve: According to Indian Railway.

$$L = \max.\begin{cases} 0.073\, C_a \cdot V_{max} \\ 0.073\, C_d \cdot V_{max} \\ 7.20\, C_a. \end{cases}$$

where,

L = Length of transition curve in 'm'.

V_{max} = Maximum permissible speed in km/hr.

C_a = Actual cant on curve in 'cm'.

C_d = Cant deficiency in 'cm'.

Another Approach

L = Maximum of (I), (II), (III) and (IV).

where,
(i) As per railway code, $L = 4.4\sqrt{R}$ where L & R in 'm'.
(ii) At the rate of change of super elevation of 1 in 360.
(iii) Rate of change of cant deficiency, say 2.5 cm is not exceeded.
(iv) Based on rate of change of radial acceleration with radial acceleration of 0.3048 m/s^2.

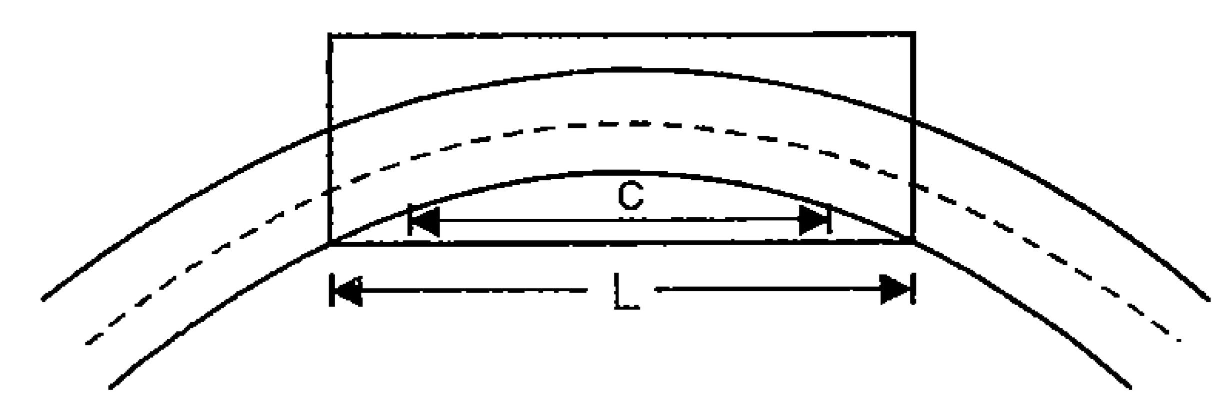
$$L = \frac{3.28V^3}{R}$$ where V is in m/s.

Extra Lateral Clearance on Curves

(a) Over throw or extra clearance needed of centre

$$= \frac{C^2}{8R}$$

(b) End throw or extra clearance needed at end



$$= \frac{L^2}{8R} - \frac{C^2}{8R}$$

where,

L = End to end length of bogie

C = Centre to centre distance of two bogie.

R = Radius of curve.

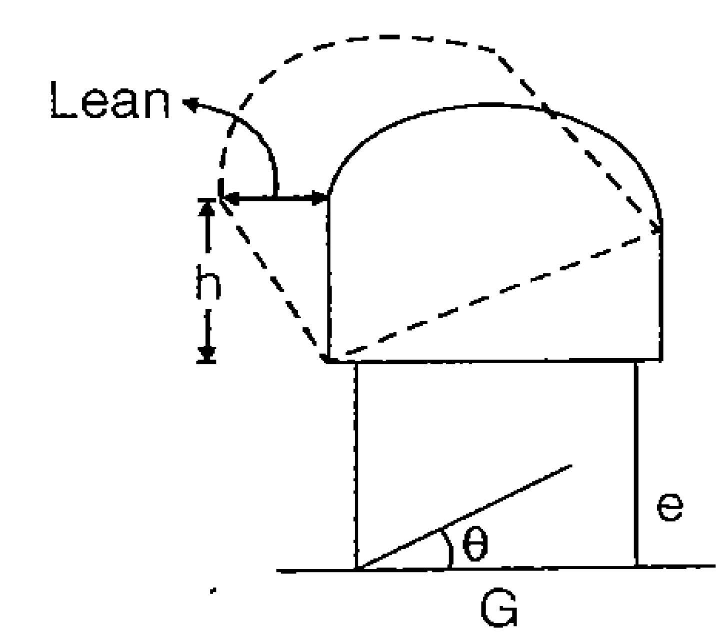
(c) Lean (L)

$$L = \frac{h \cdot e}{G}$$

where, h = Height of vehicle

e = Super elevation

G = Gauge.



(d) Total Extra Lateral Clearance Needed Outside the Curve

$$E_1 = \text{end throw} = \frac{L^2 - C^2}{8R}$$

(e) Total Extra Lateral Clearance Inside the Curve

$$E_2 = \text{Overthrow} + \text{Lean} + \text{Sway} \qquad E_2 = \frac{c^2}{8R} + \frac{he}{G} + \frac{1}{4} \cdot \frac{he}{G}$$

where, R = Radius of curve in 'mm'.
L = End to end length of bugie = 21340 mm for B.G
= 19510 mm for M.G
h = Height of bougie = 4025 mm for B.G
= 3355 mm for M.G
c = Bogie centres distance = 1475 mm for B.G
= 13715 mm for M.G
e = Super elevation in mm
G = 1.676 m for B.G = 1.0 m for M.G

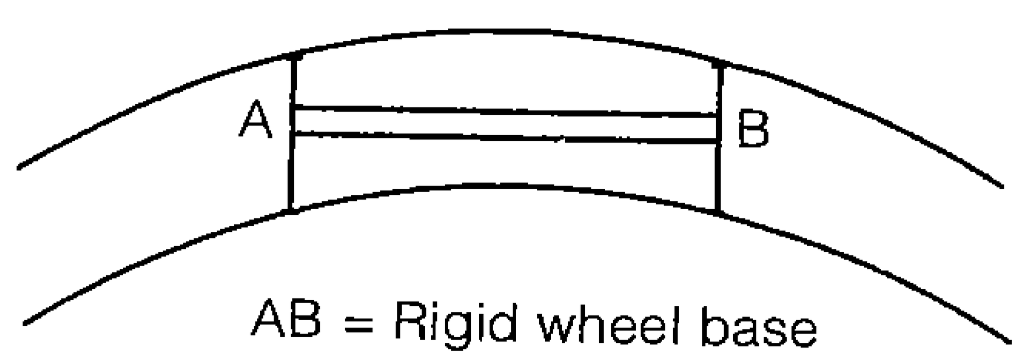
Extra Clearance on Platforms

(a) For platforms situated inside of curve
$= E_2 - 41$ mm.

(b) For platforms situated outside the curve
$= E_1 - 25$ mm.

Gauge Widening on Curves

$$w_e = \frac{13(B+L)^2}{R}$$



where,
B = Rigid wheel base in meters.
= 6 m for B.G.
= 4.88 m for M.G
R = Radius of curve in m.
L = Lap of flange in 'm'. = $0.02\sqrt{h^2 + Dh}$
h = Depth of wheel flange below rails in cm.
D = Diameter of wheel in cm.
w_e = Gauge widening in cm.

■■■

4 Points and Crossing

Heel Divergence or Heel Clearance

For B.G → 13.7 cm to 13.3 cm
For M.G → 12.1 cm to 11.7 cm.
For N.G → 9.8 cm

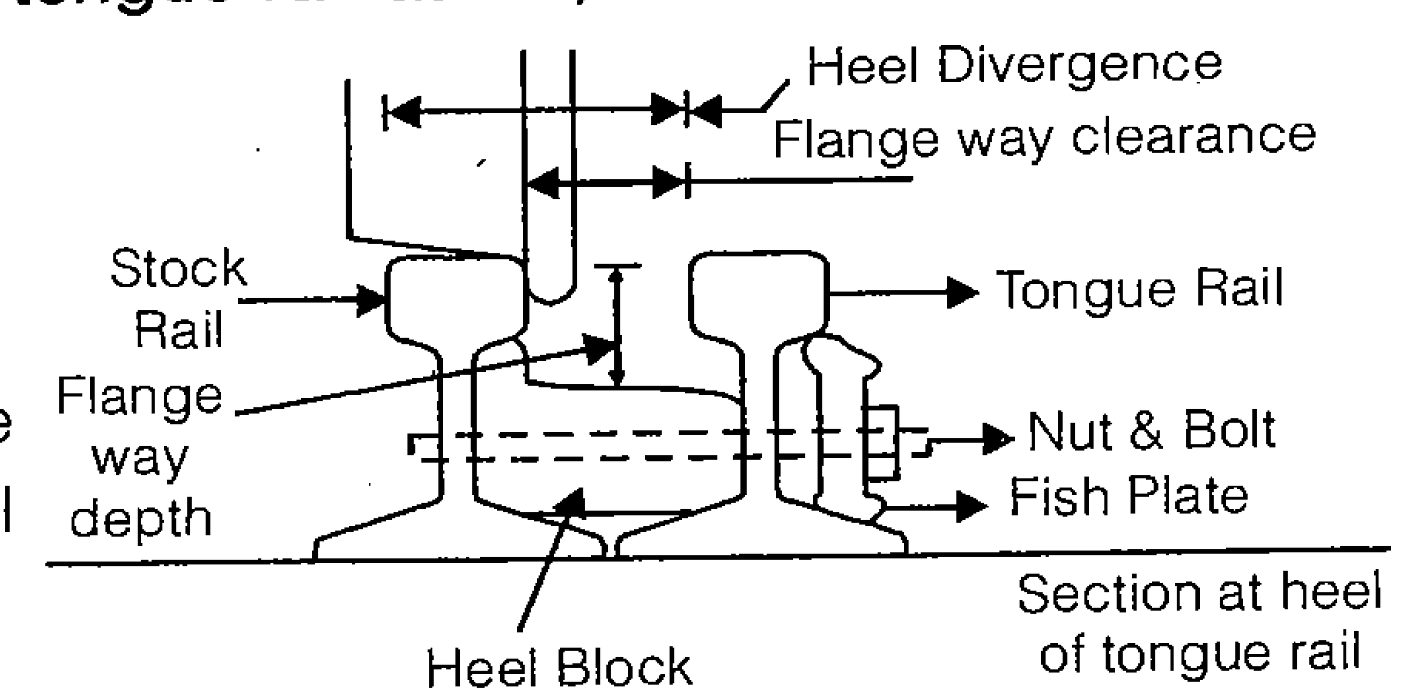
Switch Angle (α)

$$\alpha = \frac{\text{Heel divergence}}{\text{Length of tongue rail}}$$

(a) When thickness of tongue rail at toe, t = 0

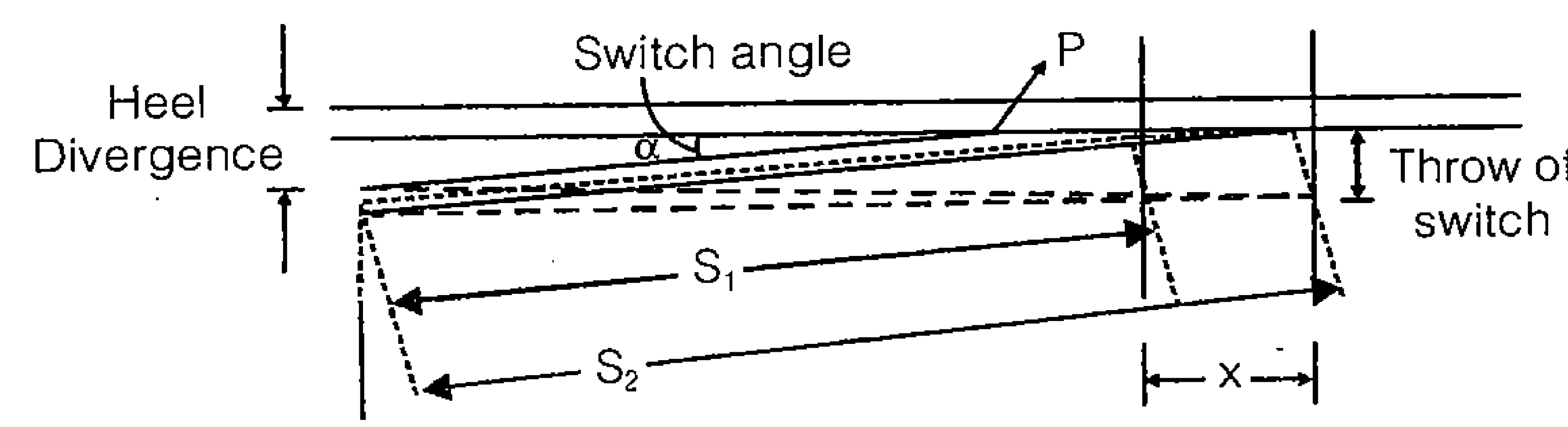
$$\alpha = \sin^{-1}\left(\frac{d}{s_2}\right)$$

where,
d = Heel divergence
s_2 = Theoretical length of tongue rail



(b) When thickness of tongue rail at toe = t

$$\alpha = \frac{\sin^{-1}(d-t)}{s_1}$$ where, s_1 = Actual length of tongue rail.



Flange Way Clearance

For 1 in 12 crossing: Flange way clearance = 6.3 cm.
For 1 in 8½ crossing: Flange way clearance = 6.6 cm.

Minimum Length of Tongue Rail (S)

$$S = R\tan\frac{\alpha}{2}$$

where, S = Theoretical length of tongue rail
R = Radius of curve at turnout.

Distance between TNC & ANC

$$d_{ta} = N \cdot t$$

where, d_{ta} = Distance between TNC & ANC
N = Number of crossing
t = Thickness of nose of crossing.

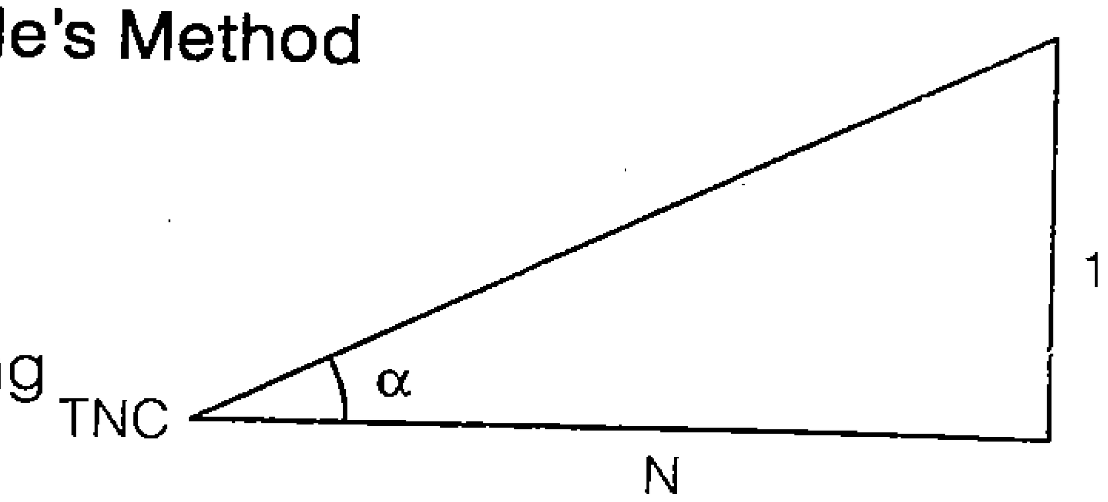
Number & Angle of Crossing

$$N = \frac{\text{Spread at the leg of crossing}}{\text{Length of crossing from T}\cdot\text{NC}}$$

(a) Right Angle Method, or Cole's Method

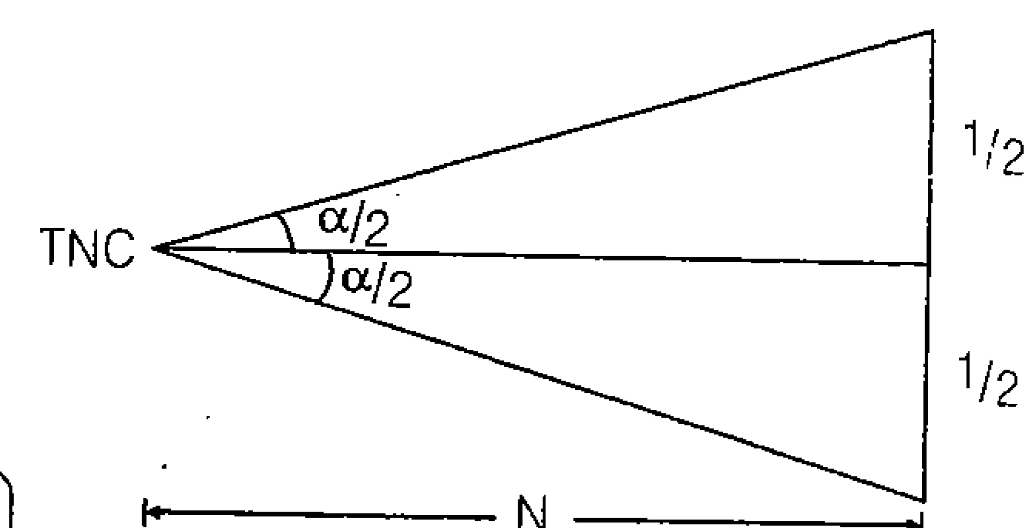
$$N = \cot\alpha$$

where, α = Angle of crossing
and N = Number of crossing



(b) Centre Line Method

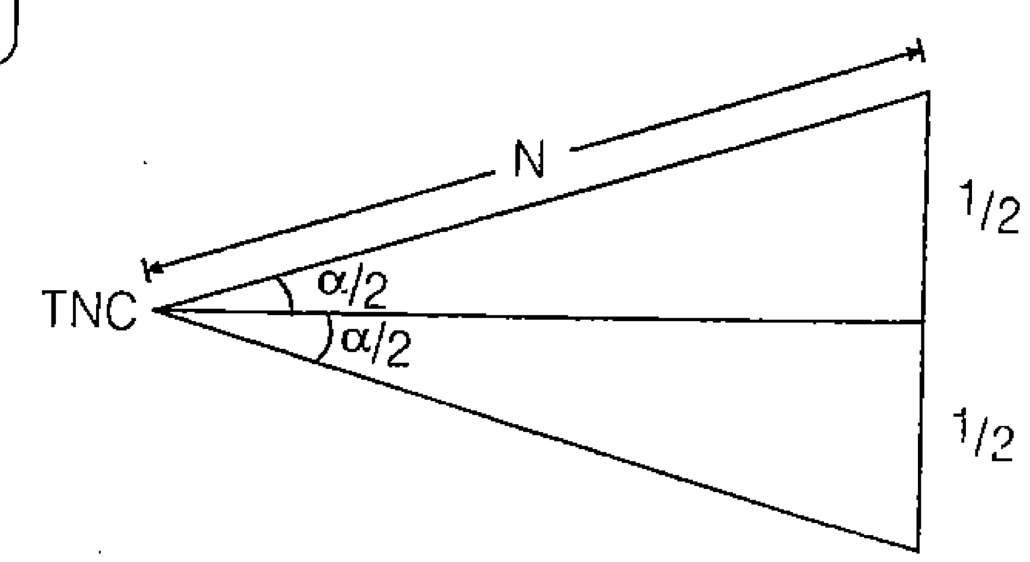
$$N = \frac{1}{2}\cot\left(\frac{\alpha}{2}\right)$$



For 1 in 12 crossing, N = 12

(c) Isosceles Triangle Method

$$N = \frac{1}{2}\text{cosec}\left(\frac{\alpha}{2}\right)$$

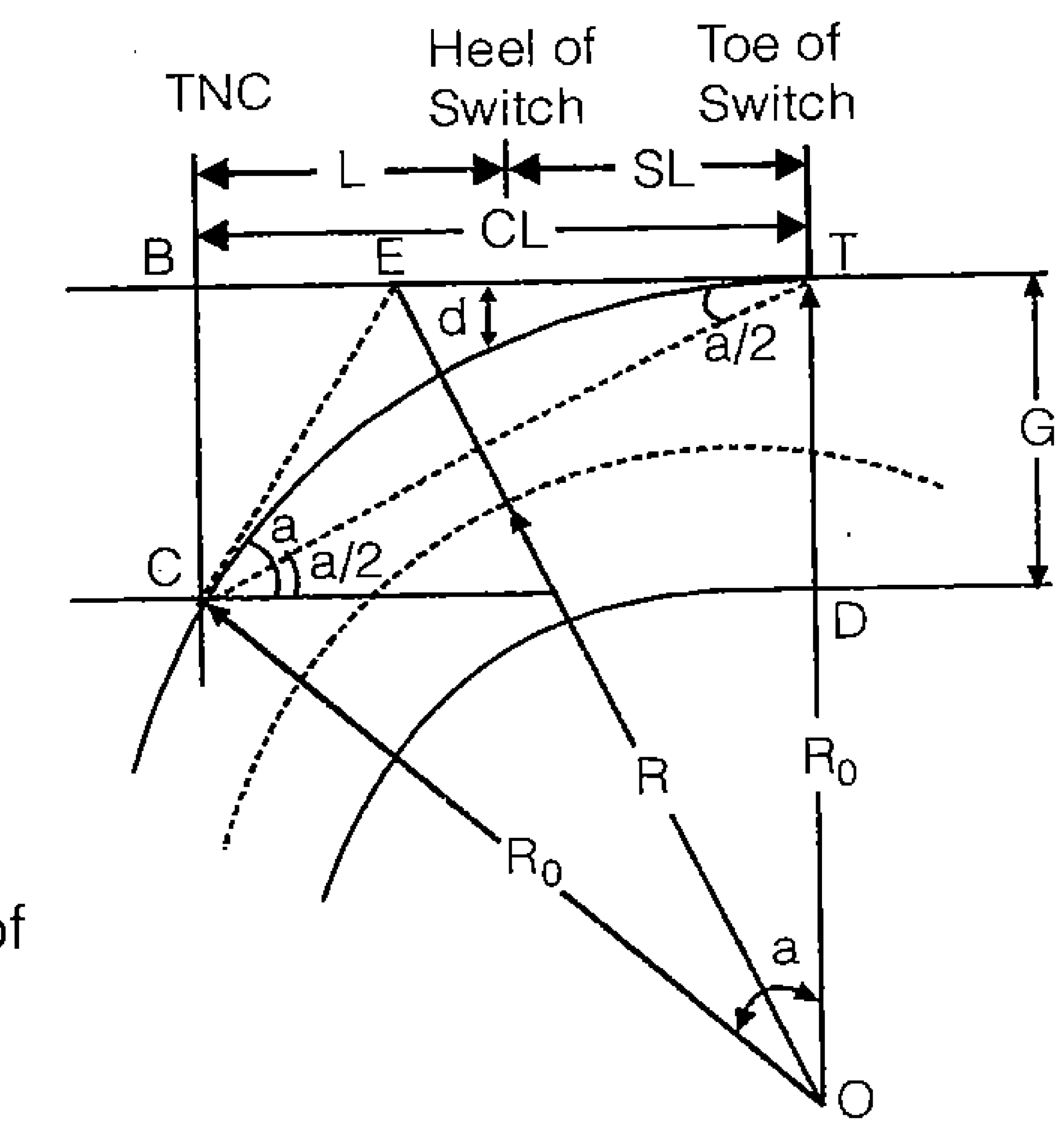


Design of Turnout

$$CL = L + SL$$

where, CL = Curve lead
L = Lead
SL = Switch lead

Method I :



(i)

$$CL = G\cot\frac{\alpha}{2}$$

$$CL = \sqrt{2R_0G}$$

$$CL \simeq 2GN$$

where,
α = Angle of crossing.
d = Heel divergence.
R_0 = Radius of outer curve of turnout.
G = Gauge of track.
N = Number of crossing.
R = Radius of centre line of turnoff.
D = Distance between T.N.C & Tangent point of curve.
β = Angle of switch.

(ii) Central radius (R)

$$R = R_0 - \frac{G}{2} \qquad R_0 = G + 2GN^2$$

For Indian Railway

$$R_0 = 1.5G + 2GN^2 \qquad R_0 = CL\,\text{cosec}\,\alpha$$

(iii) Switch Lead (SL)

$$SL = \sqrt{2R_0d}$$

(iv) Lead or Crossing Lead

$$L = CL - SL$$

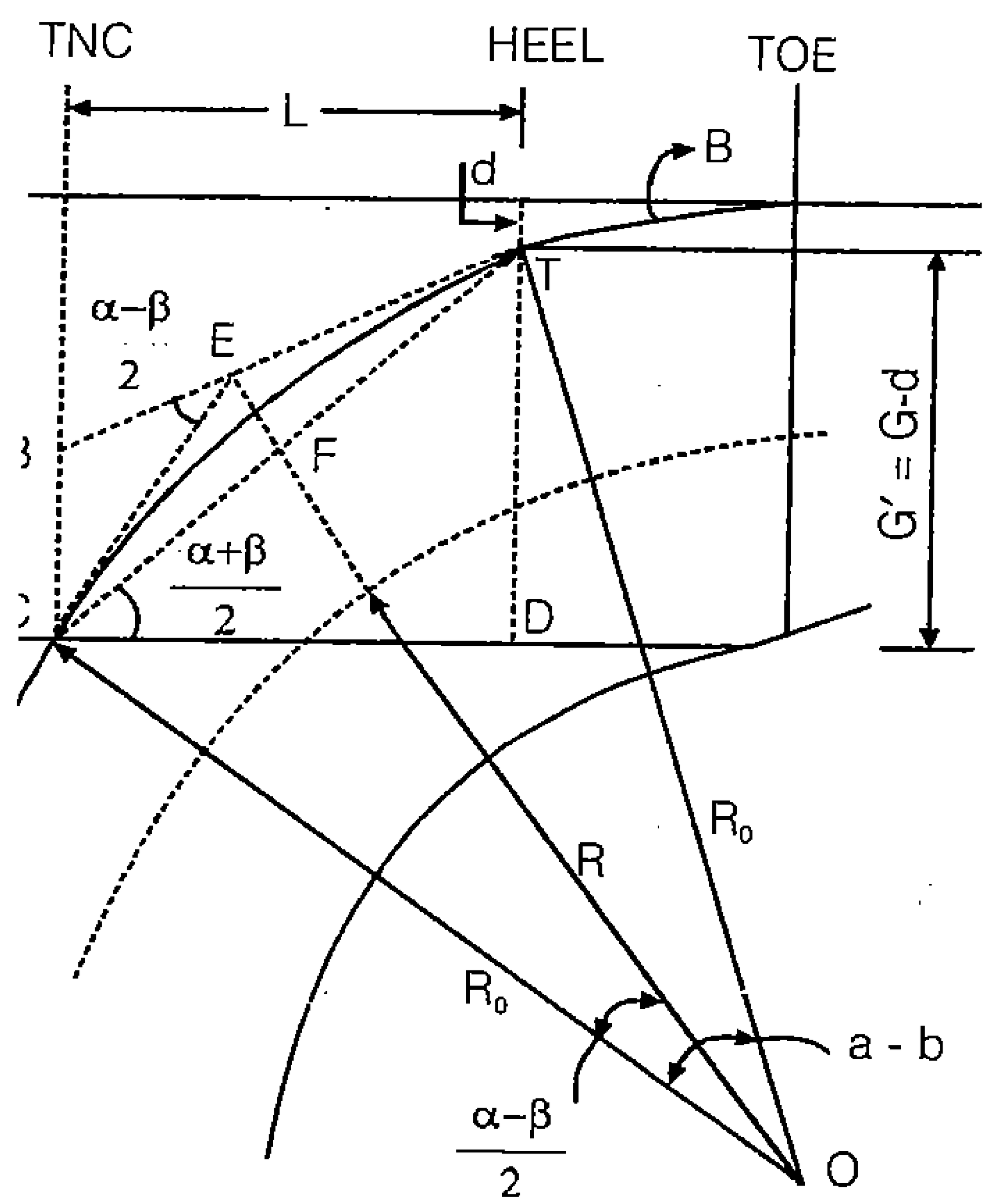
(v) Heel Divergence (d)

$$d = \frac{SL^2}{2R_0}$$

Method II :

(i) Lead or Crossing Lead (L)

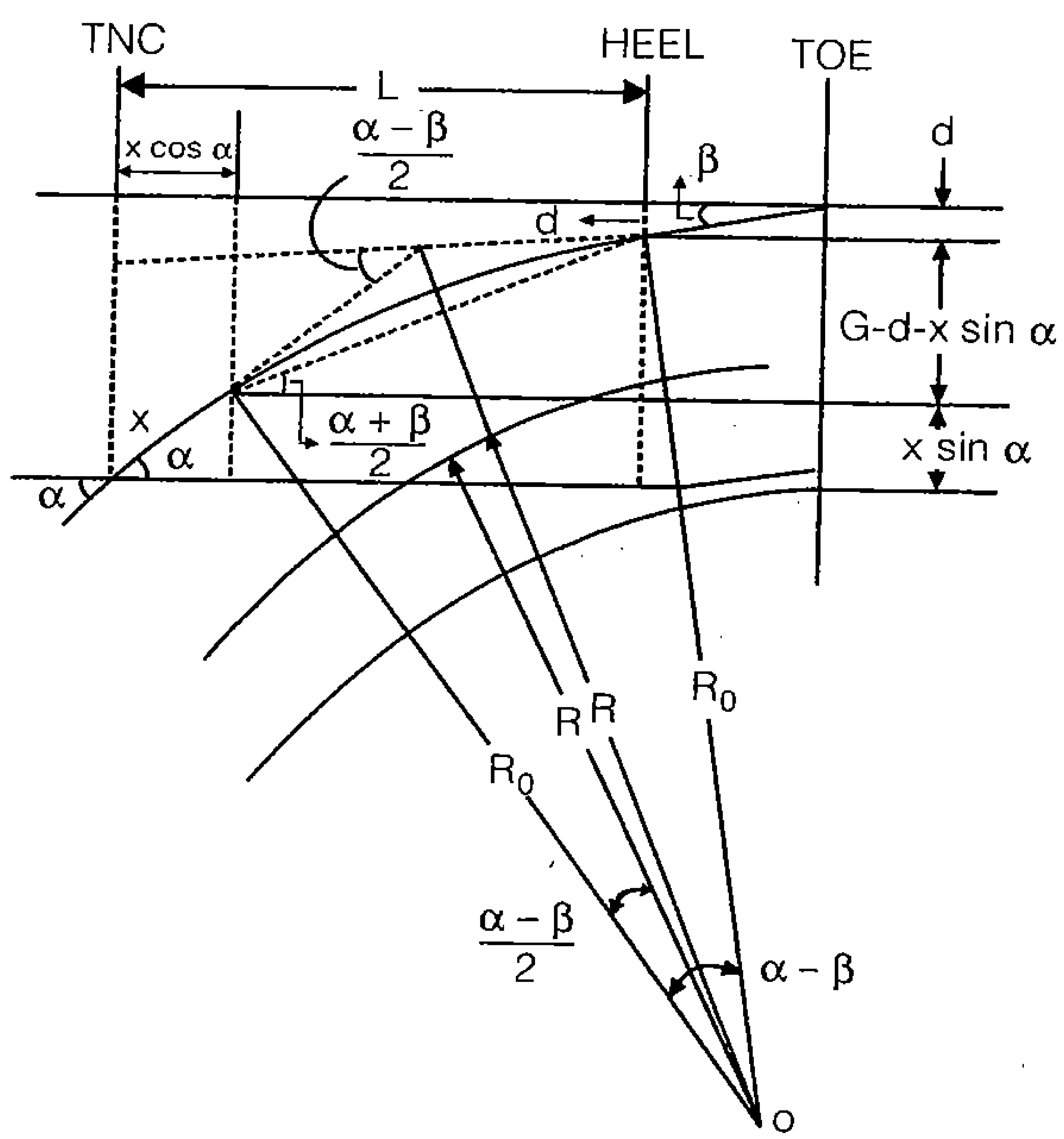
$$L = (G - d)\cot\left(\frac{\alpha + \beta}{2}\right)$$



(ii) Radius (R)

$$\boxed{R = R_0 - \frac{G}{2}} \quad \boxed{R_0 = \frac{G-d}{\cos\beta - \cos\alpha}}$$

Method III :



(i) Crossing Lead (L)

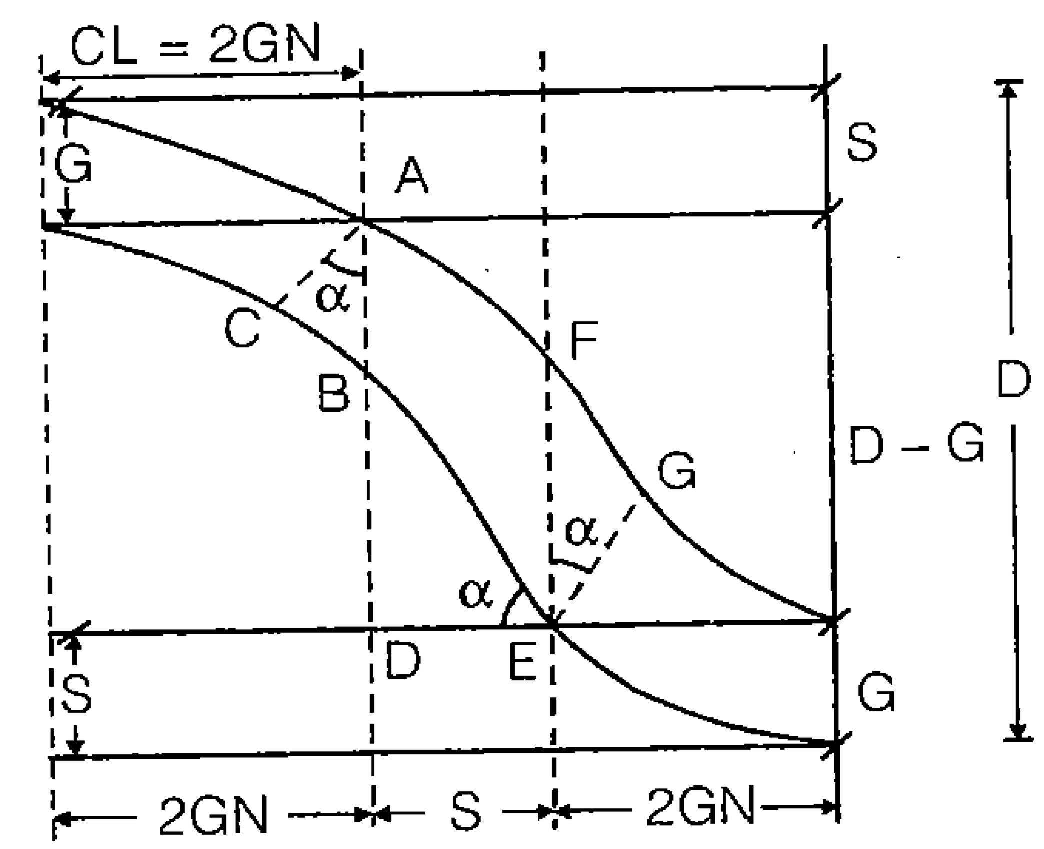
$$\boxed{L = x\cos\alpha + (G - d - x\sin\alpha)\cot\left(\frac{\alpha+\beta}{2}\right)}$$

(ii) Radius (R & R_0)

$$\boxed{R_0 = \frac{G-d-x\sin\alpha}{\cos\beta - \cos\alpha}} \quad \boxed{R = R_0 - \frac{G}{2}}$$

Cross Over

Type (I) :Two turn out provided on two tracks joint with a straight portion between two turnoffs.



$$\boxed{N = \cot\alpha}$$

where,

N = Number of crossing

α = Crossing angle

D = Centre to centre distance between two tracks

G = Gauge.

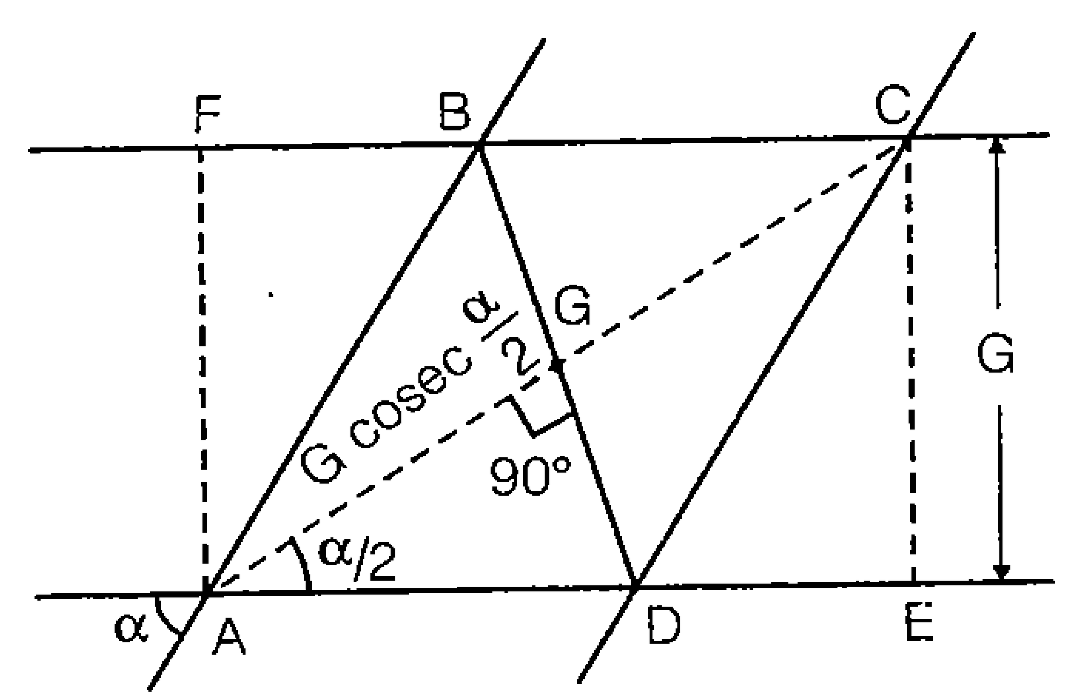
$$\boxed{\text{Length of one turn out} = CL = 2GN}$$

Length of straight portion of cross over along the track

$$\boxed{S = (D-G)N - G\sqrt{1+N^2}}$$

$$\boxed{\text{Overall length of turnout} = 4NG + (D-G)N - G\sqrt{1+N^2}}$$

Design of Diamond Crossing



(i) AB = BC = CD = AD = G cosec α

(ii) DE = BF = G cot α

(iii) AC = G cosec $\frac{\alpha}{2}$

(iv) BD = G sec $\frac{\alpha}{2}$.

Track Stresses

5

Dynamic Augment (speed effect)

- For N.G

$$\text{Speed factor} = \frac{V}{18.2\sqrt{\mu}}$$ where, V = Speed in km/hr.
μ = Track modulus in kg/cm/cm.

- For B.G & M.G

$$\text{Speed factor} = \frac{V^2}{30,000} \quad (V \leq 100 \text{ km/hr})$$

$$\text{Speed factor} = \frac{4.5V^2}{10^5} - \frac{1.5V^2}{10^7} \quad (V > 100 \text{ km/hr})$$

Hammer Blow Effect

$$\text{Hammer blow} = 0.14 \cdot \frac{M}{g} \cdot (2\pi n)^2 \cdot \sin\theta$$

where, M = Net over weight in kg.
r = Crank pin diameter in m.
n = Number of revolutions of wheel per sec.
θ = Crank angle.

Steam Effect

The vertical component of pressure of steam acting on piston is given in F.P.S units.

$$= \frac{\pi}{4} \cdot d^2 \cdot P \cdot \frac{r\sin\theta \pm h}{L}$$

where, L = Length of connecting rod in inches.
d = Diameter of piston in inches.
h = Height of cross head above the centre line of driving wheels in inches.
θ = Crank angle.

Inertia of Reciprocating Forces

$$F_v = \frac{M}{g} \cdot r(2\pi n)^2 \left(\cos\theta + \frac{r}{L}\cos 2\theta\right)\frac{r\sin\theta \pm h}{L}$$

where, F_v = The vertical component of the accelerating force in the connecting rod (Fv) at crank
M = Mass of reciprocating Parts
L = Length of connecting rod.
N = Number of revolutions per sec.
h = Height of cross head above the centre line of driving wheel in inches.
θ = Crank angle.

Method of calculating longitudinal bending stress in rail

- $$x_i = 42.33\sqrt{\frac{I}{\mu}}$$ where x_i = Distance from the load to the point of contraflexure of the rail in cm.
I = Vertical moment of inertia of rail section in cm^4.
μ = Track modulus in kg/cm/cm.

- $$f_{comp} = \frac{M_0}{Z_{comp}} \text{ tonnes / cm}^2$$

- $$f_{tension} = \frac{M_0}{Z_{tension}} \text{ tonnes / cm}^2$$

- $$d = \frac{9.25P}{\sqrt[4]{I\mu^3}}$$ where, M_o = The bending moment in tonne-cm immediately under an isolated load P tonne on one rail.

f_{comp} = The consequent compressive stress in the rail head under the load P in tonne per square cm.
$f_{tension}$ = The consequent tensile stress in the rail foot, under the load P, in tonne per square cm.
d = Deflection of track in cm.
P = Load on one rail in tonnes.
z_{comp} = Section modulus of rail in compression (cm^3).
$z_{tension}$ = Section modulus of rail in tension (cm^3).

Rail wheel contact stresses

The maximum contact shear stress which occur in the transverse direction at right angle to the rail is,

$$T_{max} = 4.13\sqrt{\frac{Q}{R}}$$

where, T_{max} = Maximum shear stress in kg/mm².
Q = Static wheel load in kg (P), increased for on-loading on curves. This on-loading is taken as 1 ton (1000 kg)
Q = (P + 100) kg.
R = Wheel radius in mm (fully worn condition).

Formation Pressure

$$P_{max} = \frac{2PS}{\pi DL}\sqrt[4]{\frac{\mu}{64EI}}$$

where, P_{max} = Maximum formation pressure
S = Sleeper spacing
D = Depth of ballast under sleeper
L = Effective length of sleeper under one rail seat.
= 76 cm for BG & 63 cm for MG
μ = Track modulus (only track modulus in elastic range is considered)
I = Moment of inertia of worn rail along horizontal axis.
E = Modulus of elasticity of rail steel.

Track deterioration and Maintainability of Track

$$TD = P^3(1 + 3S^2)$$

where, TD = Track deterioration factor
P = Nominal wheel load

$$S = \frac{\text{Standard deviation of wheel load}}{\text{Nominal wheel load}}$$

$$\therefore \quad TD \propto P^3$$

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6 Traction and Tractive Resistance

Tractive Effort (T_e)

$$T_e = \frac{nPd^2L}{2 \cdot D}$$

where, P = Difference of pressure on two sides of the piston.
A = Area of piston
d = Diameter of piston
L = Length of stoke
D = Diameter of driving wheel
T_e = Tractive effort on the wheel.

Hauling Capacity (H.C)

$$H \cdot C = \mu \cdot n \cdot w$$

where, h = Number of driving wheels.
w = Weight of one pair of driving wheels (or on each axle)
μ = Coefficient of friction

$$\mu \,\alpha\, \frac{1}{V}$$

μ = 0.1 → for high speeds
= 0.2 → for low speeds

Generally $\mu = \frac{1}{6}$.

Train Resistance (R_T)

$$R_T = R_{T_1} + R_{T_2} + R_{T_3}$$

↓

$$R_T = 0.0016\,w + 0.00008\,w \cdot v + 0.0000006\,wv^2$$

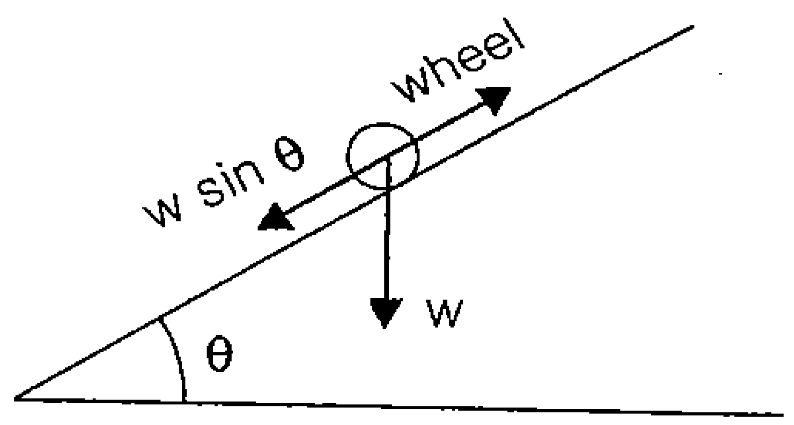
where, R_{T_1} = Resistance independent of speed.
w = Weight of train in tonnes.
= Weight of locomotive + wagons.
R_{T_2} = Resistance dependent on speed.
v = Speed of train in km/hr.
R_{T_3} = Atmospheric resistance.

Resistance due to Track Profile

(a) Resistance due to Gradient (R_g)

$$R_g \simeq w \tan\theta$$

where, w = Weight of train



(b) Resistance due to curve (R_C)

R_C = 0.0004 W.D → B.G track

= 0.0003 W.D → M.G track

= 0.0002 W.D → N.G track

where, W = Weight of train in Tonnes.

D = Degree of curve.

Resistance due to Starting & Acceleration

(a) Resistance due to Starting (R_S)

For locomotive, $\boxed{R_{LS} = 0.15\,W_1}$ W_1 = wt. of locomotive in tonnes

For wagons, $\boxed{R_{VS} = 0.005 W_2}$ W_2 = wt. of wagon in tonnes

(b) Due to Acceleration (R_a)

$$\boxed{R_a = 0.028W\left(\frac{v_2 - v_1}{t}\right)}$$

where, W = weight of train in tonne v_2 & v_1 are speed in km/hr & t is time in 'sec'.

Wind Resistance (R_w)

$\boxed{R_w = 0.000017av^2}$ where, a = Exposed area of train in (m^2)

v = Speed in km/hr.

Total Resistance (R_T)

$$\boxed{\begin{aligned} R_T &= (0.0016\,w + 0.00008\,wv + 0.0000006\,wv^2) + \\ &\begin{bmatrix} 0.0004\,wD \to \text{for B}\cdot\text{G} \\ 0.0003\,wD \to \text{for M}\cdot\text{G} \\ 0.0002\,wD \to \text{for N}\cdot\text{G} \end{bmatrix} + \begin{bmatrix} 0.15\,w \to \text{locomotive} \\ 0.005\,w \to \text{vehicle} \end{bmatrix} + \\ &\left[0.028w\left(\frac{v_2 - v_1}{t}\right)\right] + (0.000017av^2) \end{aligned}}$$

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A Handbook on

Civil Engineering

15

Airport, Dock, Harbour & Tunnelling Engineering

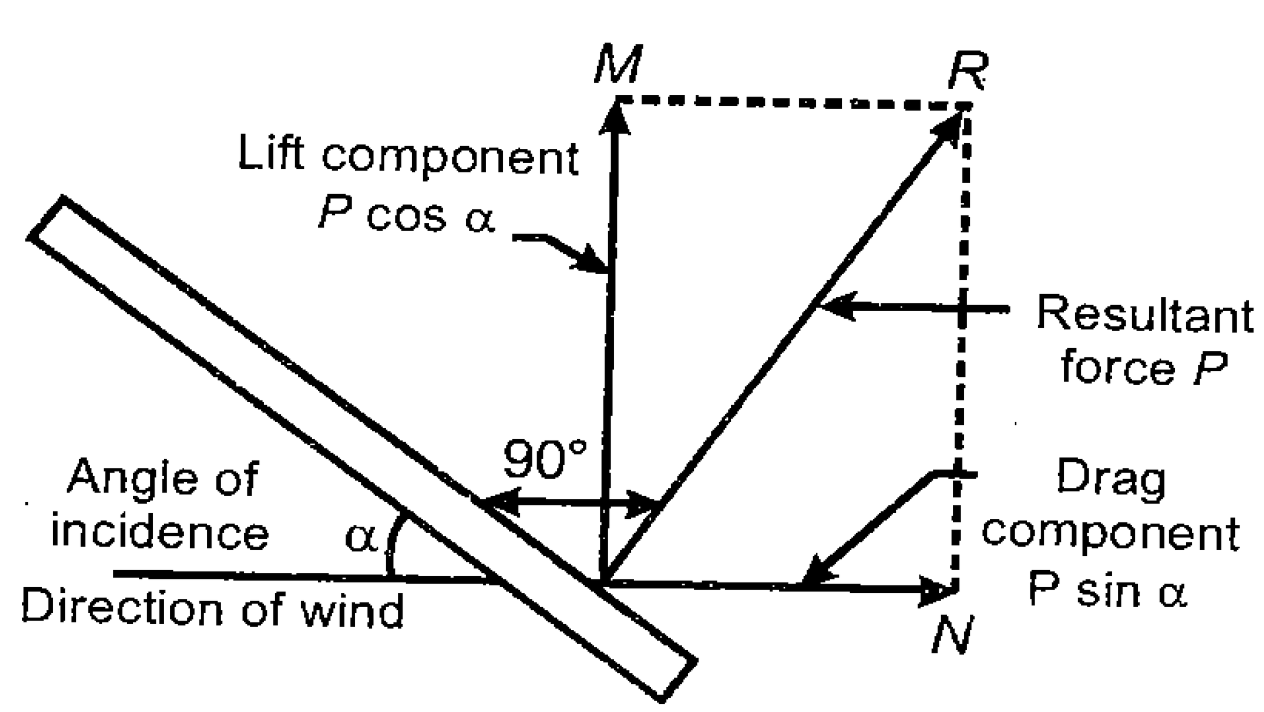
CONTENTS

Aircraft Characteristics and Planning

1

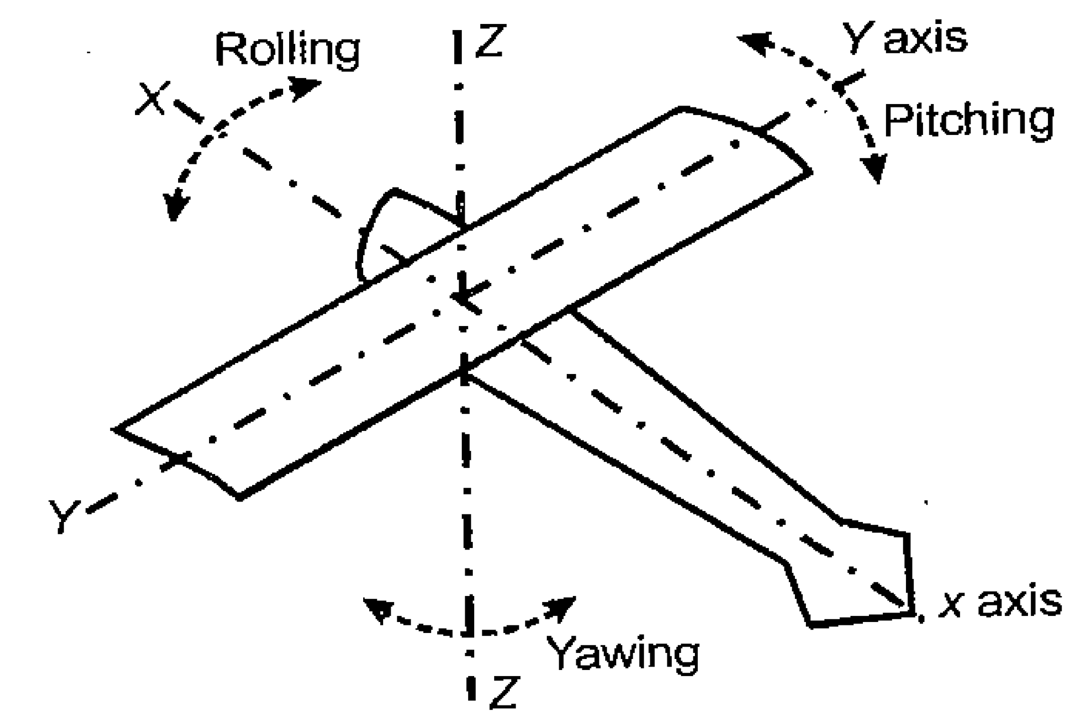
Aeroplane Component Parts

- Engine
  - (i) **Piston engine:** is driven by propeller. Suitable to operate at low altitudes and moderate speeds.
  - (ii) **Turbojet:** efficiency is higher at high altitudes owing to the drop in the atmospheric density and greater temperature difference through the turbine.
  - (iii) **Turbo Prop:** performs well at low altitudes as well as high altitude.
  - (iv) **Ram jet:** no moving parts, must be operated at high speed.
  - (v) **Rocket engine:** no limit on altitude since oxygen in the atmosphere is not relied upon for combustion. Engine carries its own supply of oxygen.
- Fuselage
  - main body of the aircraft
  - provides for power plant, fuel, cockpit, passengers, cargo etc.
- **Wings:** The purpose of an aircraft wing is to support the machine in the air when the engine has given it the necessary forward speed.



NOTE: As the angle of incidence increase, the drag component also increases and the lift component reduces.

- Three control



- The movement of aircraft about the X-axis is called rolling movement.
- The movement about y and z axes are called pitching and yawing movements respectively.
- **Elevator:** controls the pitching or up and down movements of the aircraft.
- **Rudder:** It is used for turning or yawing movement of the aircraft.
- **Aileron:** It is used to control of rolling movement about longitudinal axis.

AirPort Planning

- The regional plan usually provides the following information:
  1. Approximate locations of the airports in national map.
  2. Classification of airports
  3. Location of air strips
  4. Routes of air Travel.
- Following data is collected for regional planning:
  1. Population
  2. Topographical and geographical features
  3. Existing airports in the vicinity
  4. Air traffic characteristics
- Minimum spacings from existing airports:
  - (i) for Airports serving small general aviation aircrafts under VFR conditions = 3.2 km (2 Miles)
  - (ii) for airports serving bigger aircrafts under VFR conditions = 6.4 km (4 Miles)
  - (iii) for airports operating piston engine aircrafts under IFR conditions = 25.6 km (16 Miles)
  - (iv) for jet aircrafts under IFR conditions = 160 km (100 Miles)

  The best location is a site adjacent to the main highway.

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Runway Design

2

Orientation

- Runway is usually oriented in the direction of prevailing winds.
- The head wind i.e. the direction of wind opposite to the direction of landing and take off, provides greater lift on the wings of the aircraft when it is taking off.
- Cross wind component = V sin θ where θ = Angle of wind direction to runway centreline
- Normal component of the wind is called Cross wind component.
- The maximum permissible cross wind component depends upon the size of aircraft and the wing configuration.

Airports Serving	Max. limit of C.W.C.
1. For small aircrafts.	15 kmph
– for mixed traffic	25 kmph
2. For big Aircrafts	35 kmph

Basic Runway Length

It is the length of runway under the following assumed conditions at the airport:

1. Airport altitude is at sea level
2. Temperature at the airport is standard (15°C)
3. Runway is levelled in the longitudinal direction.
4. No wind is blowing on runway.
5. Aircraft is loaded to its full loading capacity.
6. There is no wind blowing enroute to the destination.
7. Enroute temperature is standard.

Corrections for Elevation, Temperature and Gradient

(a) **Correction of Elevation:** Basic runway length is increased at the rate of 7% per 300 m rise in elevation above the mean sea level.

(b) Correction for Temperature

$$\text{Airport reference temperature} = T_a + \frac{T_m - T_a}{3}$$

where T_a = monthly mean of average daily temperature
T_m = monthly mean of the max daily temperature for the same month of the year.

Total correction for elevation plus temp. ⇒ 35% of basic runway length.

(c) Correction for Gradient

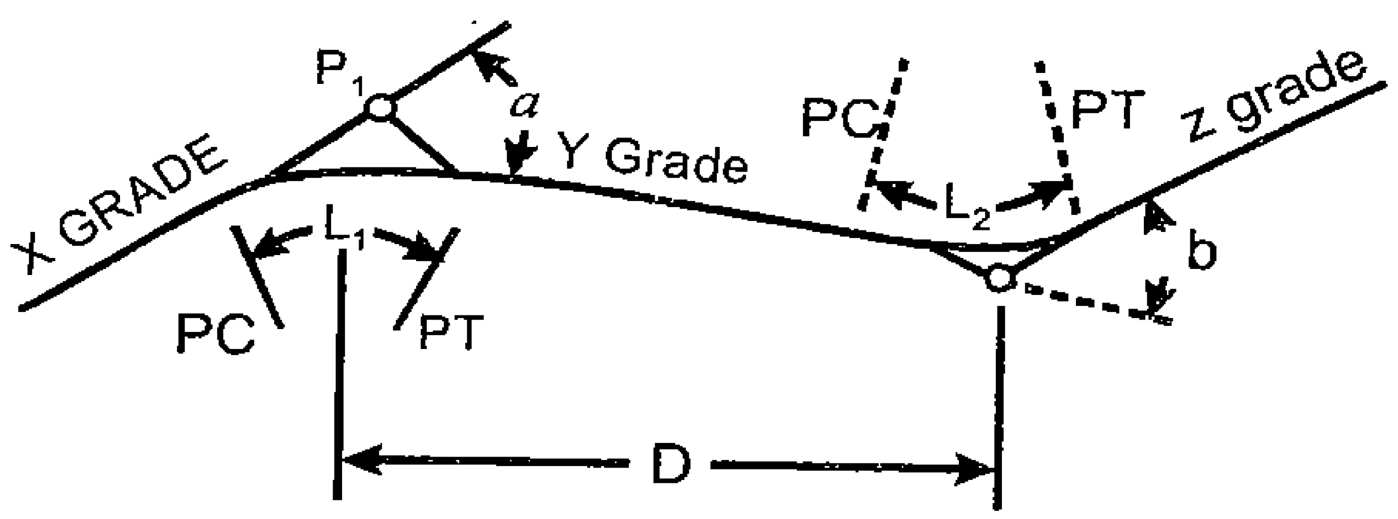
- Steeper gradient results in greater consumption of energy and as such longer length of runway is required to attain the desired ground speed.
- After having been corrected for elevation and temperature should be further increased at the rate of 20% for every 1% of effective gradient.
- Effective gradient is defined as the maximum difference in elevation between the highest and lowest points of runway divided by the total length of runway.

Runway Geometric Design

- **Runway Width:** ICAD recommends the percent with varying from 45 m to 18 m for different types of airport.
- **Safety Area:** Consists of the runway, which is paved area plus the shoulder on either side of runway plus the area that is cleared, graded and drained.
  For non-instrumental runway, the width of safety area should be at least 150 m for A, B, C and 78 m for D and E types and for instrumental runway, it should be minimum 300 m
  The length of safety area is equal to the length of runway plus 120 m
- **Transverse Gradient:** Essential for quick drainage of surface water.
  For A, B, C types of Airports ⥣ 1.5%
  For D and E types of Airports ⥣ 2%
  Transverse gradient ⇔ 0.5%
- **Longitudinal Gradient**
  Max. limit
  A, B, C types of Airports = 1.5%
  D and E types of Airports = 2.0%

For effective gradient: Max limit
A, B and C type of airports = 1.0%
D and E types of airports = 2.0%

- **Rate of Change of Gradient:** Should be limited to a maximum of 0.1% per 30 m length of vertical curve for A and B types, 0.2% for C type and 0.4% for D and E types of airports.
  Vertical curves are generally not necessary if the change in slope is not more than 0.4%.
- **Sight Distance:** For A, B, C types of airports, any two points 3 m above the surface of runway should be mutually visible from a distance equal to half the runway length.
  For D and E types of runway there should be unobstructed line of sight from any point 3 m above runway and to all other point 2.1 m above runway within a distance of at least one half the length of runway.



Description	Small airport	Large airport
1. Maximum grade change such as (a) or (b) should not exceed	2 percent	1.5 percent
2. Length of vertical curve (L_1 or L_2) for each one percent grade change	90 m	300 m
3. Distance between points of intersection of grade lines (D)	75(a + b)m	300(a + b)m

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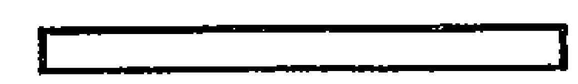
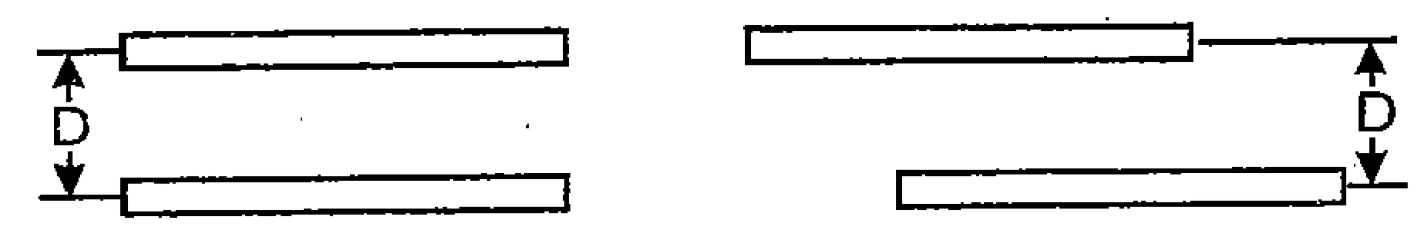
3 Airport Capacity and Tunneling Area

Airport Capacity

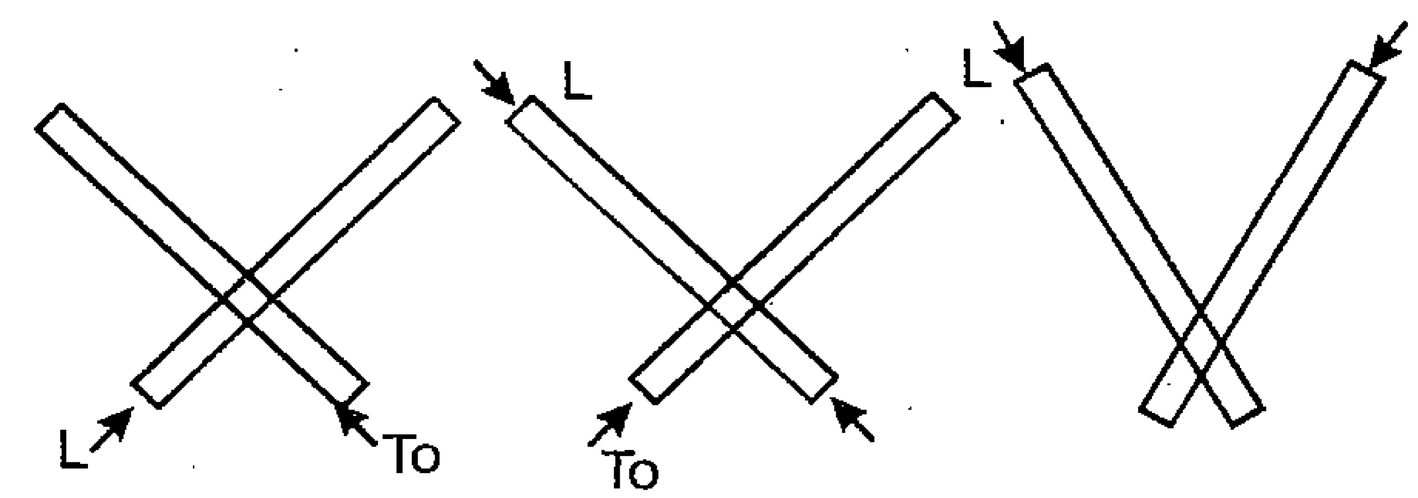
- The number of aircraft movements which an airport can process within a specified period of time with an average delay to the departing aircraft within the acceptable time limit is defined as airport capacity.
- The following factors affect the airport operating capacity:
  Runway configurations and the connected taxiways



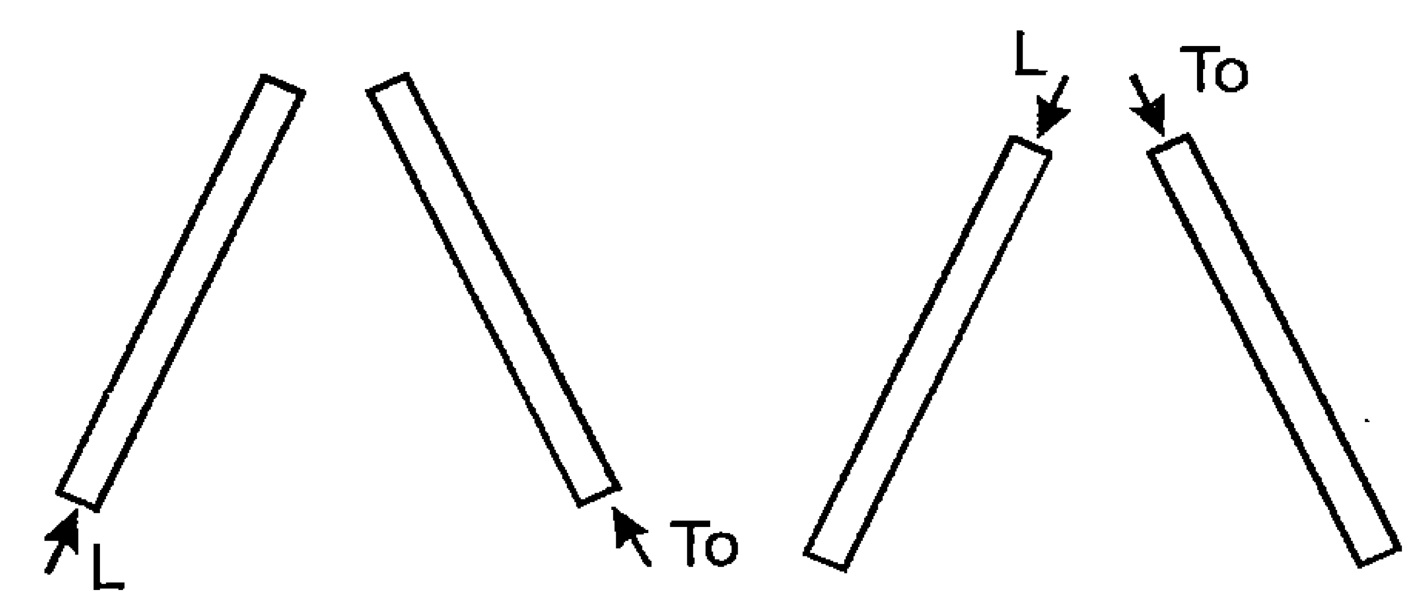
(i) Single runway



(ii) Parallel runway



(iii) Intersecting runways



L = Landing T.o. = Taking off.

- **Single Runway:** is usually adopted when the wind blows in one direction for most of the time in a year and air traffic requirement does not exceed the capacity of such pattern.

- **Parallel Runway:** The capacity of this pattern depends upon the lateral spacing between the two runways, the weather conditions and the navigational aids available at the airport.
- **Intersecting Runways:** is usually adopted when wind in a particular direction does not provide the required coverage. Whether both the runways can be used for simultaneous landing and take off, depends upon the cross wind component on each runway.
- **Non-intersecting Runways:** Capacity depends upon the wind conditions and visibility.

Runway Capacity

- It is defined as the ability of a runway system to accommodate aircraft landings and take offs.
- Expressed as operations/hour or operations per year.

Saturation Capacity

- The ultimate or saturation capacity of a runway is the maximum number of aircraft that can be handled during a given period under conditions of continuous demand.
- **Factors affecting runway capacity**
  Runway capacity depends upon
  1. Air traffic control
  2. Characterization of demand
  3. Environmental factors
  4. Layout and design of the runway system
- **Average landing delay (Steady State)**

$$W = \frac{\rho}{2\mu(1-\rho)}$$ (Bowen & Pearcy equation).

where, ρ = the load factor = 1 μ

λ = arrival rate (aircraft/unit time)

μ = Service rate (aircraft/unit time)

$= \frac{1}{b}$

b = mean service time.

$$W = \frac{\rho(1+C_b^2)}{2\mu(1-\rho)}$$ (Pollaczek-Khinchin formula)

where, C_b = coefficient of variation of service time = $\frac{\sigma_b}{b}$

σ_b = Standard deviation of service time

- **The weighted hourly capacity (WHC)**

$$\frac{\Sigma \text{capacity} \times \% \text{ use} \times \text{weighting factor}}{\Sigma \% \text{ use} \times \text{weighting factor}}$$

- **Practical Annual Capacity**

$$PAC = WHC \times \text{annual utilization} \times \text{percentage use of airport}$$

GATE Capacity

- **Gate:** is defined by an aircraft parking space, adjacent to a terminal building and used by a single aircraft for the loading and unloading of passengers, baggage and mail.
- **Gate capacity** is defined as the ability of a specified number of gates to accommodate aircraft loading and unloading operations under conditions of continuous demand.
- It is the inverse of the weighted average gate occupancy time for all the aircraft served.

  The gate capacity for a single gate

$$C_{sg} = \frac{1}{\text{weight service time}}$$

= ....aircraft/minute/gate.

If G = the total number of gates

The capacity for all gates is C = G C_{sg} = ... aircraft/hr

The capacity of the gate system

$$C = \min_{\text{all}} \left[\frac{G_i}{T_i M_i}\right]$$

where, G_i = the number of gates that can accommodate aircraft of class i.

T_i = mean gate occupancy time of aircraft of class i

M_i = fraction of aircraft class *i* demanding service.

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Taxiway Design

4

Turning Radius

- $R = \dfrac{V^2}{125\,f}$ where R is the radius of curve in m, V is the speed in kmph and f is the coefficient of friction between the tyre and pavement surface. The value of f may be assumed as 0.13.
- For airports serving large subsonic jet transports, minimum value of radius of curvature is 120 m whatever be the speed.
- For supersonic transports, a minimum radius of 180 m is suggested.
- Horonieff equation

$$\text{Radius of taxiway in metre} = \frac{0.388\,W^2}{T/2 - S}$$

where, W = Wheel base of aircraft in m.
T = Width of taxiway pavement in m.
S = distance between midway point of the main gear and the edge of the taxiway pavement in m.

Design of Exit Taxiway Connecting Runway and Parallel Taxiway

The following principles govern the design of taxiway.

(i) The most significant factor effecting the turning radius is the exit speed of aircraft.

(ii) Slightly widened entrance of 30 m gradually tapering to the normal width of taxiway is preferred. The widened entrance gives to the pilot more latitude in using the exit taxiway.

(iii) Total angle of turn of 30° to 45° can be negotiated satisfactorily. The smaller angle seems to be preferable because the length of curved path is reduced.

(iv) For smooth and comfortable turn, the turning radius should be determined from the Equation

$$R = \frac{V^2}{125\,f}$$

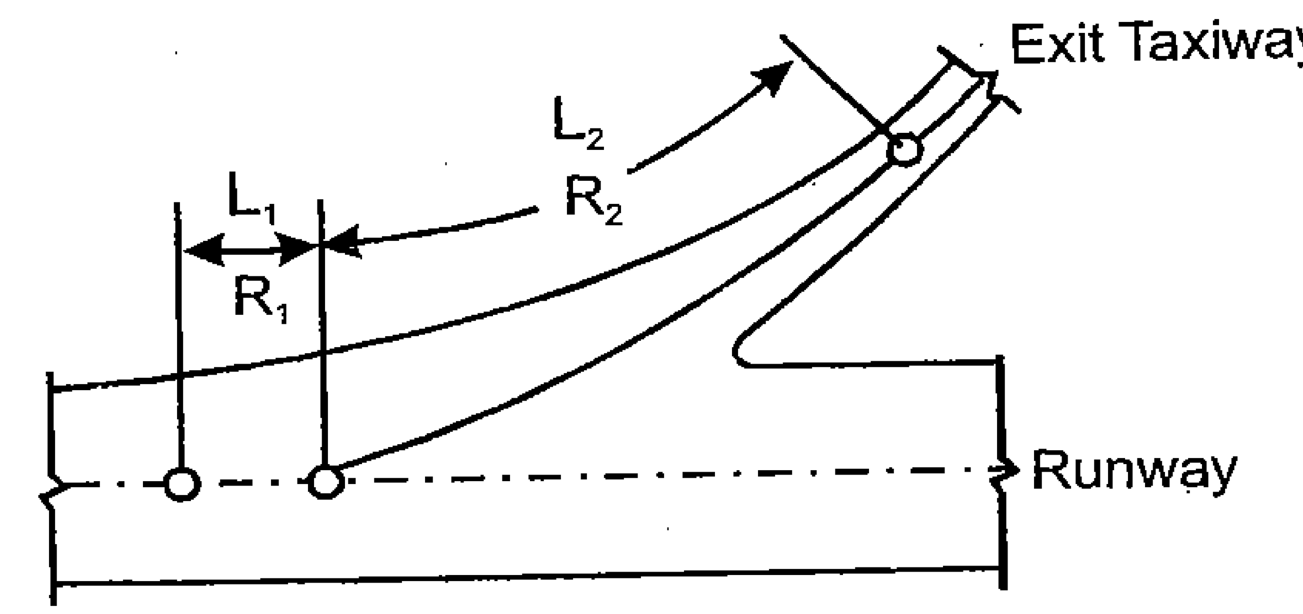
(v) At high turn-off speeds of 65 to 95 kmph (40 to 60 mph), a

compound curve is necessary to minimize the tire wear on the nose gear. Therefore the main curve radius R_2 should be preceded by a larger radius curve R_1 as shown in Figure. Aircraft path approximates a spiral. But still a compound curve is preferred as it is relatively easier to establish it in the field and its shape is similar to that of a spiral.

The following radius were found experimentally suitable.

Speed		Radius	
kmph	mph	m	ft.
65	40	517	1724
80	50	731	2436
95	60	941	3138

R_1 = radius of entrance curve
L_1 = length of entrance curve
R_2 = radius of central curve
L_2 = length of central curve



Radius of curvature for exit taxiway

(vi) The length of larger radius curve can be roughly obtained from the following relation:

$$L_1 = \frac{(0.28\,V)^3}{CR_2} \qquad L_1 = \frac{V^3}{45.5 \times C \times R_2}$$

The value of C is 0.39

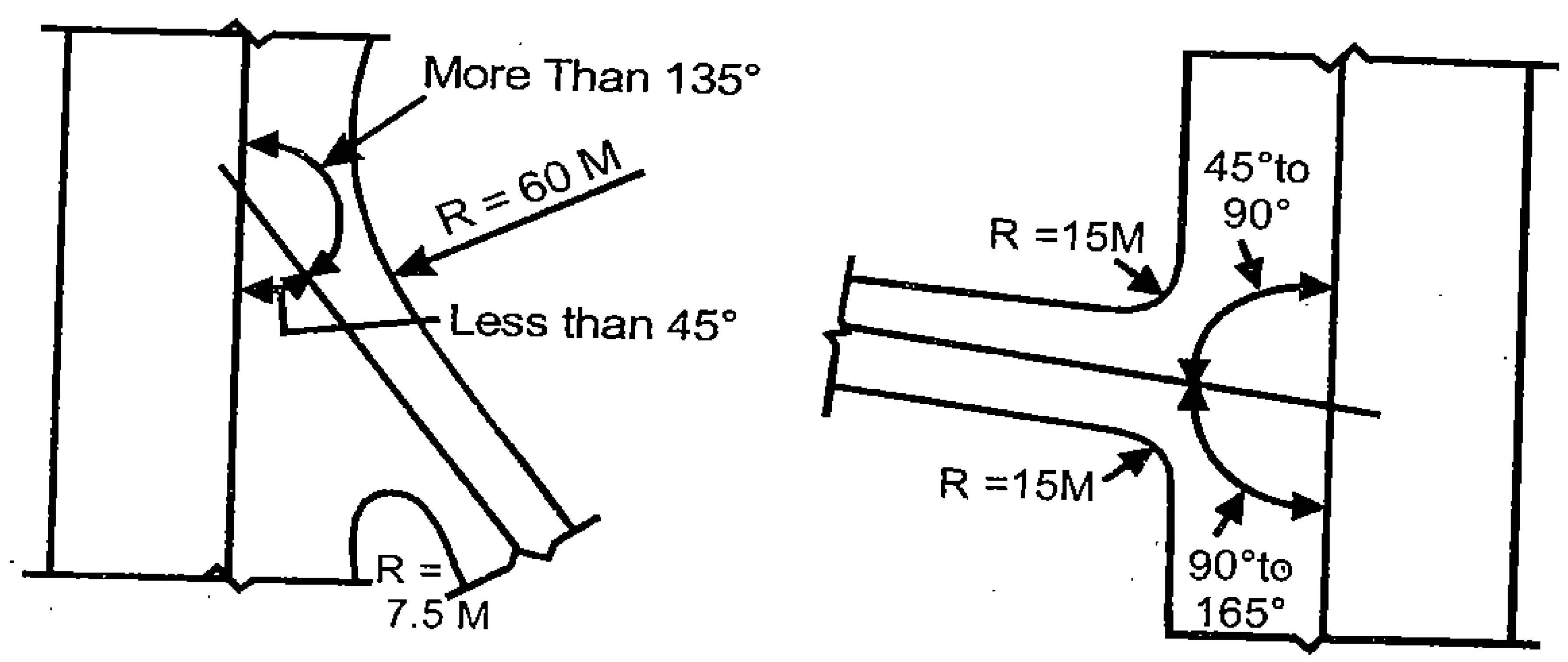
(vii) Sufficient distance must be provided to comfortably decelerate an aircraft after it leaves the runway. This distance may be based on an average deceleration rate of 1 m/sec² (3.3 ft/sec²). The stopping distance may be obtained from the following equation:

$$SD = \frac{(0.28V)^2}{2d} \qquad SD = \frac{V^2}{25.5d}$$

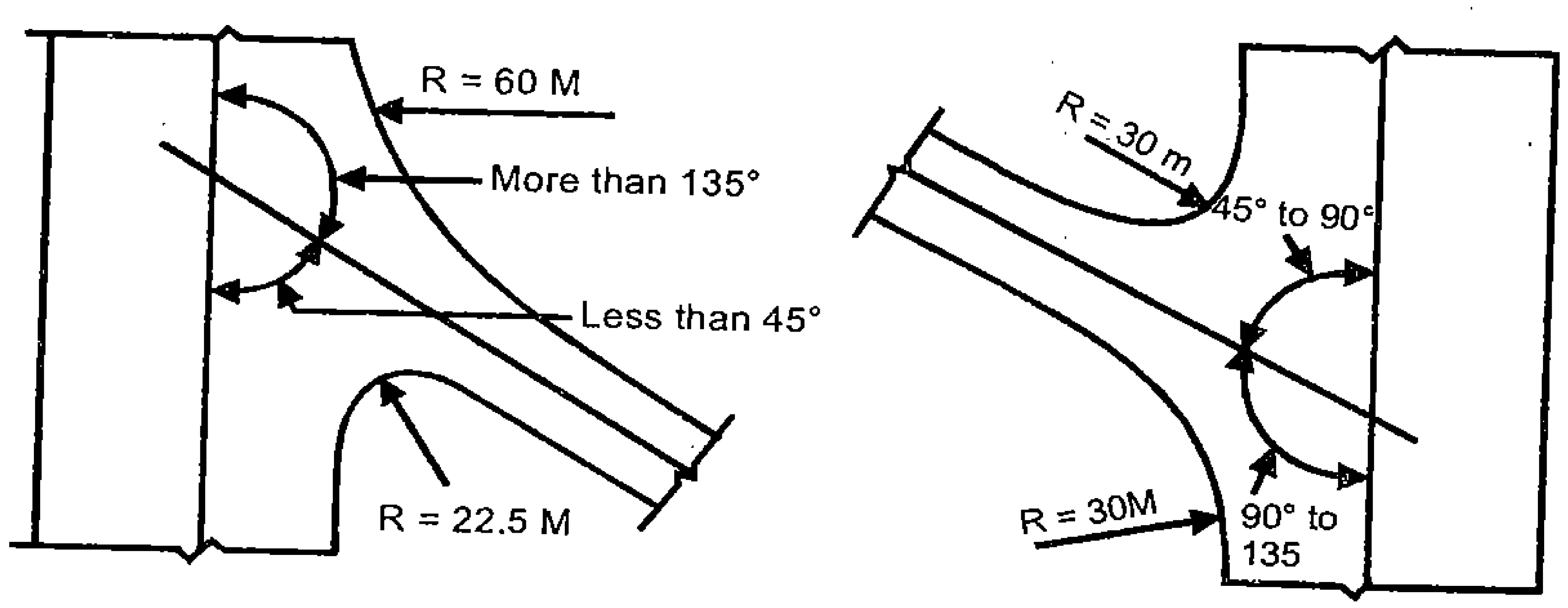
where d is the deceleration in m/sec². The stopping distance would be measured from the edge of the runway pavement along the exit taxiway.

Fillets

- Fillets are provided at the junction or intersection of two or more number of traffic ways e.g. runways, taxiways or aprons.
- ICAO recommends that the radius of the fillet should not be less than the width of taxiway.
- Typical Runway and Taxiway Fillets



For small airports



For large airports

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5 Terminal Area and Airport Layout

Terminal Area

- It is the portion of an airport other than the landing area.
- It includes terminal and operational building, vehicle parking area aircraft service hangars etc.

Planning

- There are two concepts for planning of the terminal building viz. Centralization and decentralization
- In the centralized plan, all passengers, baggage and cargo are funneled through a central building and are then dispersed to the respective aircraft positions.
- In the decentralized plan, the passengers and baggage arrive at a point near the departing plane.

Apron

It is a paved area for parking of aircrafts and loading and unloading of passengers & cargo.

Number of Gate Positions

Depends upon the peak hourly aircraft movernents and the time during which each aircraft remains in a gate position. The time is known as *ramp time*.

$$\text{Number of gate positions} = \frac{\text{Capacity of runway}}{60 \times 2} \times \text{average gate occupancy time}$$

For design, Gate occupancy time for big air crafts = 60 Minutes. for small aircrafts = 10 Minutes.

Blast Considerations

Wake velocity along the axis of the engine at a distance x from the exhaust end

$$V_x = \frac{4DV_o}{x}$$

where, D = Dia of jet exhaust pipe

V_0 = Velocity of wake at $x = 0$

x = Distance from the engine along the axis at which the wake velocity is to be determined.

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Airport Grading and Drainage

6

Airport Grading

- The grading is done with minimum cost of earthwork.
- For preparation of grading plan, the topographic map of the area is required.

Airport Drainage

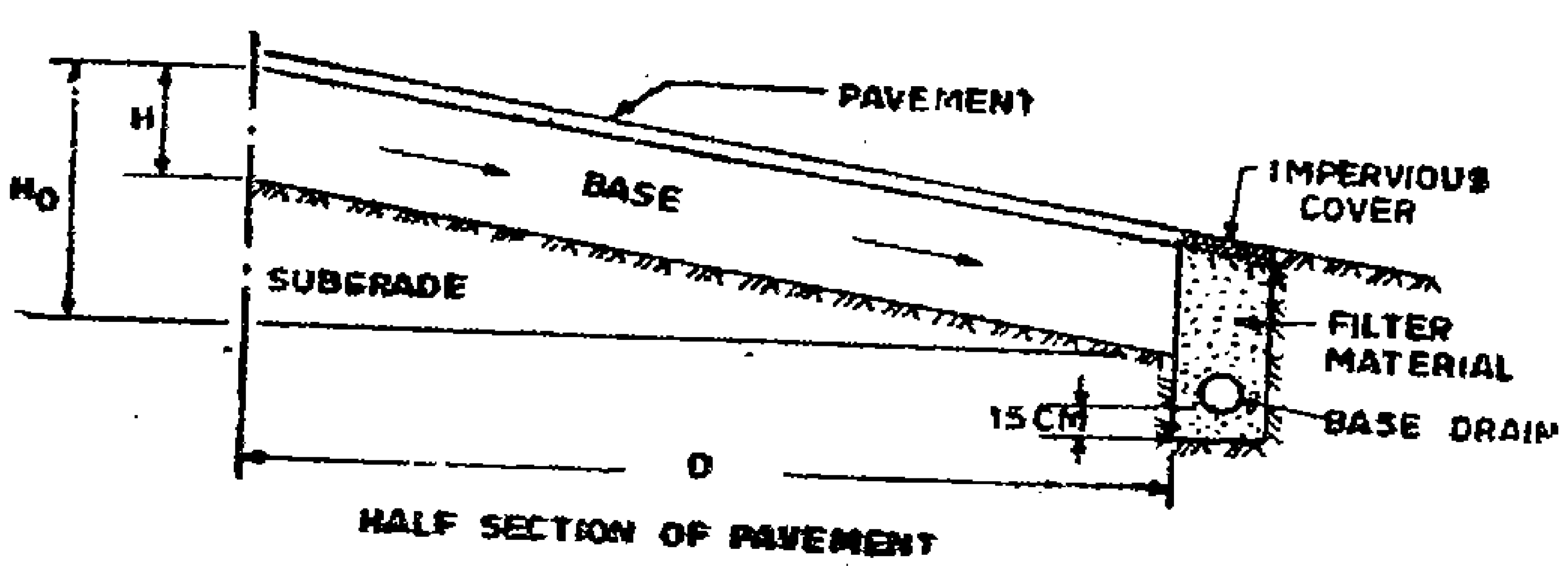
A drainage system for an airfield normally serves three functions:

(i) Removal of surface runoff the airfield.
(ii) Interception and diversion of surface and ground water flow originating from lands adjacent to the airfield area.
(iii) Lowering of subsurface water level in the airfield area.

- The volume of possible storage

where, V = Vol. of pond in cubic meter
b = height of pond in meter
A = area of the upper surface of the pond in sq. m
B = area of the lower surface of the pond in sq. m.

- Base Course Drainage



- Max. rate of see page from base course into the base drain

where, q = max. discharge in cubic meter/minute/meter of drain
K = coefficient of horizontal permeability of base course in meter per minute

- Time for 50% drainage of base course

where, t_{50} = time in days for 50% drainage
n_e = effective porosity of soil

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Harbours and Ports

7

Classification of harbour depending upon the protection needed

Depending upon the protection needed, harbours are broadly classified as:

(i) Natural Harbours or Natural roadsteads
(ii) Semi-natural harbours
(iii) Artificial harbours or artificial roadsteads

Classification of harbour depending upon the utility

From their utility, harbours are further classified into five major types

(i) Harbours of refuge
(ii) Commercial harbours
(iii) Fishery harbours
(iv) Military harbours
(v) Marine harbours.

Classification of harbour based upon the location

The layout of a harbour is greatly influenced by its location and based on the location, harbours are further classified into the following four major types:

(i) Canal harbour
(ii) Lake harbour
(iii) River or estuary harbour
(iv) Sea or ocean harbour

Harbour depth

The channel depth is generally determined by the following formula:

$$D = D_1 + \frac{H}{3} + D_2$$

where, D_1 = draft of the largest ship to be accommodated
D_2 = allowance for squat of the moving ship
H = height of storm waves.

The max. harbour depth below lowest low water is achieved as follows:
Max. harbour depth = loaded draft + 1.2 m when bottom is soft
Max. harbour depth = loaded draft + 1.8 m when bottom is rock.
The depths of sea bottom are obtained by use of fathometer or echo sounder.

Wave parameters

- **Height and length of waves:** Waves being generated by wind, their development depends upon the surface area of sea exposed to wind action. Such a surface giving rise to a wave is called a Fetch and is usually measured in km, denoting the length across which the wave action is generated and is active.
- **The height of the wave in metres** = $0.34\sqrt{F}$, **where F is the fetch in km.**
- This is an empirical formula given by Thomas.
- The length could be defined as the distance between adjacent crests of a wave. The length influences the force of the wave.
- **Bertin's formula,**

$$L = \frac{t^2}{2\pi}.g \quad \text{or} \quad L = 1.56t^2$$

where, L = length in metres and t is the period in seconds for two successive waves to pass the same section.

Dynamic Effect of Wave Action

1. **In deep water:** When the depth of water is great compared to the length of wave, **Velocity** $v = \sqrt{2g\frac{l}{4\pi}} = 1.25\sqrt{l}$

   - Considering the wave as a cycloidal curve, the height h of the wave = $1/\pi$, where l is the length of the wave.

   $v = \sqrt{5h}$ and pressure on unit surface

   $$p = 5\frac{wv}{g} = \frac{wh}{2}\text{nearly.}$$

2. **In shallow water:** In shallow water, if depth of water is d, it has been found that the velocity $v = 3.16\sqrt{d}$ approximately.

$$p = \frac{w}{g} \times 10 \times d \text{ or } wd \text{ approximately.}$$

- **Height of wave:** $hp = h\left\{\sqrt{b/B} - 0.027\sqrt[4]{D}\left(1+\sqrt{b/B}\right)\right\}$

where, h_p = height of wave at the point under consideration
h = height of wave at the entrance of the harbour.
b = width of entrance
B = width of basin at point under consideration
D = distance of point of consideration from entrance.

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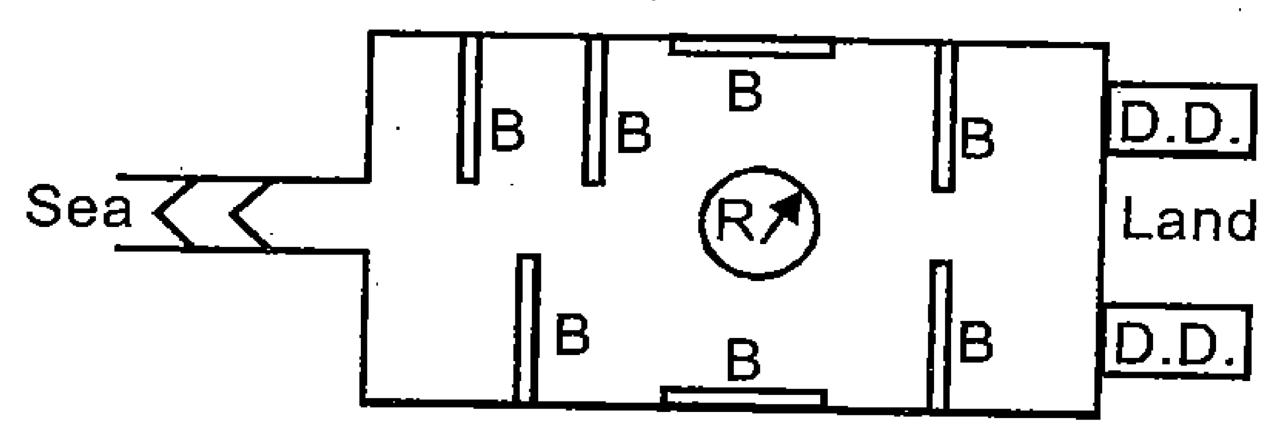
Ports and Docks

8

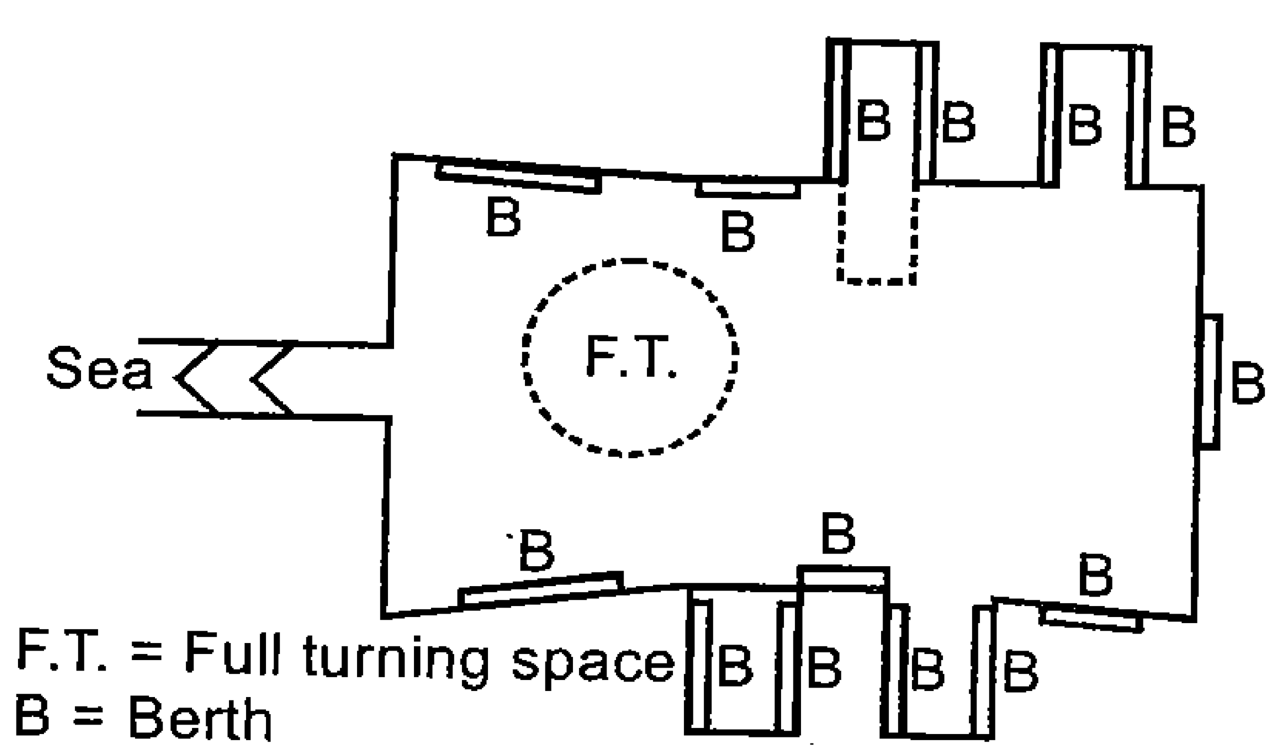
Layout of Ports

Quayage for different geometrical shapes adopted for suitable layouts are:

1. Square layout
2. Rectangular layout
   (i) *Simple rectangular layout:* Quayage for rectangular layout is greater than quayage for square layout but this nearly reduces chances of free turning.
   (ii) Rectangular layout with central pier:
3. **Machicolated layout:** To make a rectangular layout with large quayage for the rectangular layout dimensions of length and breadth should be increased and piers should be made projecting from longer side in staggering style with a central open space for free turning.



4. **Tridentine layout:** The machicolated layout can further be modified to improve quayage by making lengths of layout flaring outside instead of at right angles to width side, partly and making use of excavations.



docks

- Docks which are used for berthing of vessels to facilitate loading and unloading of passengers and cargo are known as wet docks and those used for repairs of the vessels are known as dry docks.
- The wet docks are sometimes known as the harbour docks.

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Tunnel and Docks

9

Stages in Tunnel Construction

- **Investigations:** Investigations are made to find necessary information for proposed tunnel site. Informations to be collected are:
  - (i) Origin of soil mass.
  - (ii) Hydrology in surrounding proposed tunnel site.
  - (iii) Presence of foul gases.
  - (iv) Temperature of soil nearby.
  - (v) Physical and mechanical strength of soil mass existing at proposed site.
  - (vi) Location of weak geological features like faults, folds, etc.
- Investigations to collect information are made in the following stages:
  1. Investigations made prior to planning the project.
  2. Investigations made at the time of planning the project.
  3. Investigations made at the time of construction.

1. **Investigations before planning:** Geological investigations are made to determine relation between bed rock and top soil when exploration at the surface in form of knowing morphology, petrology, stratigraphy, etc. Geophysical methods like electrical resistivity methods are used to locate positions of weak zones like faults, folds and shear zones.
2. **Investigations at the time of planning:** Investigations at the time of planning are made through drilling holes either by:
   - (i) Percussion
   - (ii) Rotary percussion
   - (iii) Rotary

   Rotary or Rotary percussion methods are used for investigating loose soils while rotary drilling method is used for rocky soils.
3. **Investigations at the time of construction:** Information at the time of construction is achieved by driving either of the following:
   - **(i) Heading:** Heading is part of tunnel cross-section excavated for small lengths.
   - **(ii) Driving drift:** Heading and drift give information regarding stratification, fault, fold, presence of foul gases, etc. exactly in tunnel alignment. Heading gives information at the time of construction, while drift gives complete information prior to construction of tunnel.
     - When such holes are drilled as the work proceeds, they are known as headings.
     - Drift or heading provide space for setting service lines.
     - When these holes are drilled for the entire length of tunnel in the beginning, the holes are known as drifts.

Blasting

- Nowadays non-explosive techniques of fragmentation like laser and electron beams are available, the primary methods of drilling and blasting are yet used for rock release.

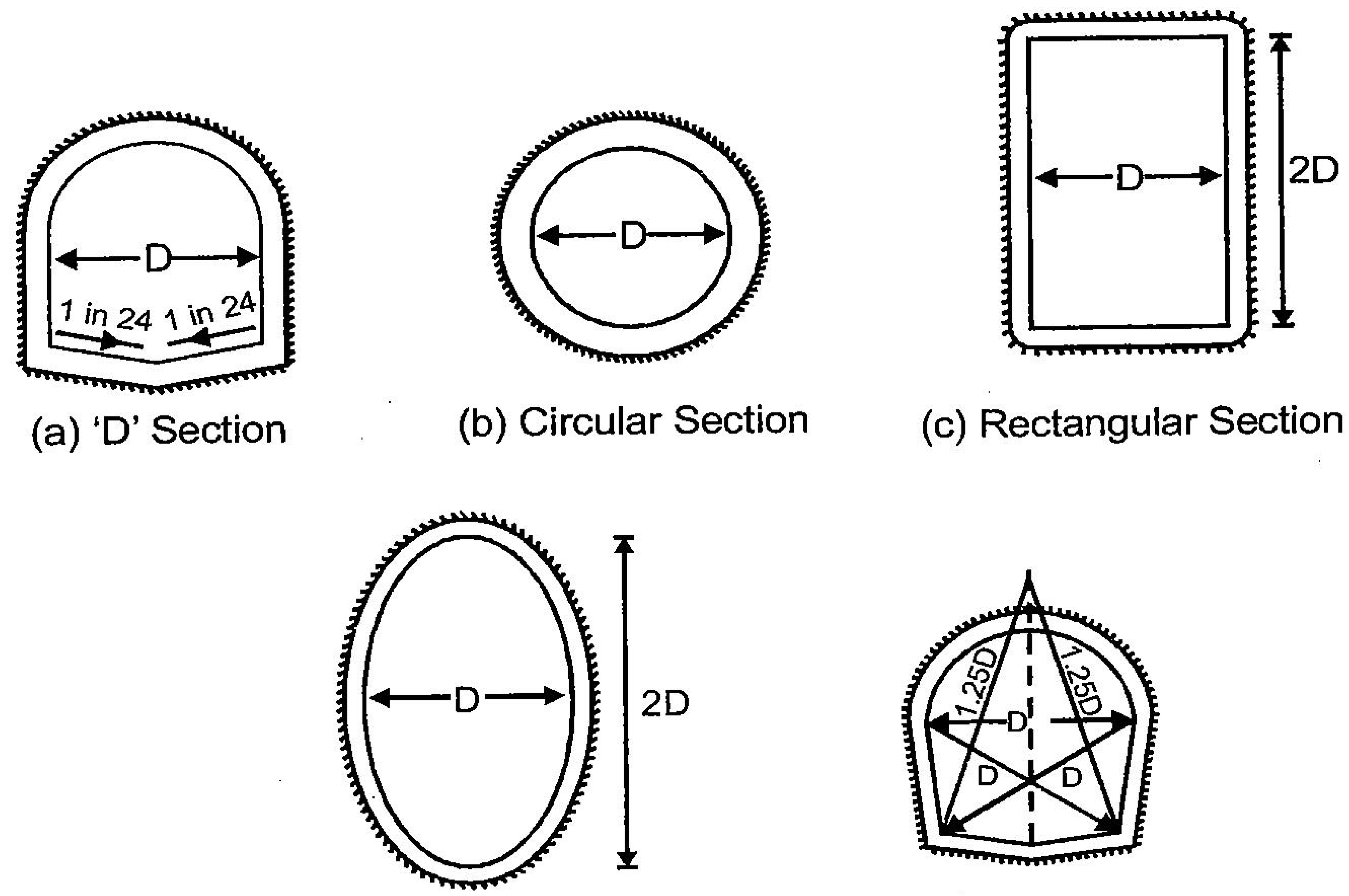
1. **Types of explosives**
   - (i) Straight dynamites
   - (ii) Ammonia dynamites
   - (iii) Ammonia gelatine
   - (iv) Semi-gelatine
   - (v) Blasting agents
   - (vi) Slurries or water gels
2. **Cuts:** The blast is made to get initial cut or void of rock release with creation of fresh free face so that other following cut or cuts shall release more and more rock i.e. to say initial blast must release satisfactory amount of rock and free face for blast to follow.
   - (i) Angle cut
   - (ii) Burn cut
3. **Theory of blasting**: Process by which rock can be blasted is divided into following groups:
   - (i) Impact
   - (ii) Abrasion
   - (iii) Thermally induced spalling
   - (iv) Fusion and vaporization
   - (v) Chemical reaction

   When excavation is to be done in rock usually blasting is resorted.

Shape and Size

Following are the usual cross-sections adopted for tunnels.

1. **'D' section :** In rock tunnels, the risk of failure or collapse caused by external pressure from water or loose or unstable soil conditions on tunnel lining is practically non-existent and it is then convenient to have a section with an arched roof and straight sides, which is called the 'D' section or segmental-roof section.



(a) 'D' Section (b) Circular Section (c) Rectangular Section

(d) Egg. Shaped section (e) Horse -shoe section

Shapes for tunnel cross-sections

This section is suitable for sub-ways or navigation tunnels.

2. **Circular section:** For tunnels which may have to withstand heavy internal or external radial pressures, this form is the most desirable.
3. **Rectangular section:** This section is suitable only in case of hard rocks.
4. **Egg-shaped section:** This section is commonly used for carrying sewage because it gives self-cleansing velocity even in dry weather flow.
5. **Horse-shoe form:** It is the best shape suited for traffic purposes and as the floor of the tunnel is nearly flat, it gives working space to the contractor to store materials during construction.

Portals

- The actual doorways or main entrances of the tunnels are known as portals and thus, a portal indicates the intersection point between the underground opening and the ground surface.

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